E

Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 October 2024 Worked Solutions

October 2024 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

WMA12/01 October 2024

October 2024

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 OCTOBER 2024

Question 1

5 marks | Integration

1. A continuous curve has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below.

x	0.5	1.75	3	4.25	5.5
y	3.479	6.101	7.448	6.823	5.182

Using the trapezium rule with all the values of y in the given table,

(a) find an estimate for

$$\int_{0.5}^{5.5} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to one decimal place.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

$$\int_{0.5}^{5.5} \left(\mathrm{f}(x) + 4x\right)\mathrm{d}x$$

(2)

WORKED SOLUTION

Question 1 · WMA12/01 October 2024

1 Find the strip width

The x values increase by 1.25, so the trapezium width is $h = 1.25$.

2 Apply the trapezium rule

$$\int_{0.5}^{5.5} f(x)\,dx \approx \frac{1.25}{2}\{3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)\}.$$

3 Evaluate the estimate

This gives 30.878125, which is 30.9 to one decimal place.

4 **Split the second integral**

$$\int_{0.5}^{5.5} (f(x) + 4x)\,dx = \int_{0.5}^{5.5} f(x)\,dx + \int_{0.5}^{5.5} 4x\,dx.$$

5 **Integrate 4x**

$$\int_{0.5}^{5.5} 4x\,dx = [2x^2]_{0.5}^{5.5} = 2(5.5)^2 - 2(0.5)^2 = 60.$$

6 **Use part a**

Using the estimate from part (a), the required value is $30.9 + 60 = 90.9$.

✓ **FINAL ANSWER**

(a) 30.9.

(b) 90.9.

WMA12/01 OCTOBER 2024

Question 2

7 marks | Sequences & Series

2. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$u_{n+1} = (-1)^n u_n + k$$

where k is a constant.

(a) Show that $u_5 = 7$ **(3)**

Given that $\sum_{r=1}^{4} u_r = 30$

(b) find the value of k. **(2)**

(c) Hence find the value of $\sum_{r=1}^{150} u_r$ **(2)**

WORKED SOLUTION

Question 2 · WMA12/01 October 2024

1 Find the first few terms

$u_1 = 7$, $u_2 = (-1)^1 u_1 + k = k - 7$, $u_3 = (-1)^2 u_2 + k = 2k - 7$, and $u_4 = (-1)^3 u_3 + k = 7 - k$.

2 Show u five

$u_5 = (-1)^4 u_4 + k = (7 - k) + k = 7$.

3 Use the given sum

$\sum_{r=1}^{4} u_r = 7 + (k - 7) + (2k - 7) + (7 - k) = 2k$.

4 Find k

Since $\sum_{r=1}^{4} u_r = 30$, we have $2k = 30$, so $k = 15$.

5 **Write one repeated block**

With $k = 15$, the first four terms are $7, 8, 23, -8$, and their sum is 30. Since $u_5 = u_1$, the pattern repeats every four terms.

---

6 **Sum 150 terms**

There are 37 complete blocks in the first 150 terms, with the first two terms left over. Hence $\sum_{r=1}^{150} u_r = 37(30) + 7 + 8 = 1125$.

✓ **FINAL ANSWER**

(a) Proven.

(b) $k = 15$.

(c) 1125.

WMA12/01 OCTOBER 2024

Question 3

7 marks | Polynomials

3. $f(x) = 2x^3 - x^2 + Ax + B$

where A and B are integers.

Given that when $f(x)$ is divided by $(x + 3)$ the remainder is 55

(a) show that

$$3A - B = -118$$

(2)

Given also that $(2x - 5)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quotient when $f(x)$ is divided by $(x - 7)$

(2)

WORKED SOLUTION

Question 3 · WMA12/01 October 2024

1 Use the remainder theorem

Dividing by $x + 3$ leaves remainder 55, so $f(-3) = 55$.

2 Show the given equation

$f(-3) = 2(-3)^3 - (-3)^2 - 3A + B = -63 - 3A + B$. Therefore $-63 - 3A + B = 55$, so $3A - B = -118$.

3 Use the factor theorem

Since $2x - 5$ is a factor, $x = \frac{5}{2}$ is a root and $f\left(\frac{5}{2}\right) = 0$.

4 Form a second equation

$2\left(\frac{5}{2}\right)^3 - \left(\frac{5}{2}\right)^2 + \frac{5}{2}A + B = 0$, so $25 + \frac{5}{2}A + B = 0$. Multiplying by 2 gives $5A + 2B = -50$.

5 **Solve simultaneously**

From $3A - B = -118$, $B = 3A + 118$. Substitute into $5A + 2B = -50$ to get $11A = -286$, so $A = -26$ and $B = 40$.

6 **Write the polynomial**

$f(x) = 2x^3 - x^2 - 26x + 40$.

7 **Divide by x minus seven**

$2x^3 - x^2 - 26x + 40 = (x - 7)(2x^2 + 13x + 65) + 495$, so the quotient is $2x^2 + 13x + 65$.

✓ **FINAL ANSWER**

(a) Proven.

(b) $A = -26$, $B = 40$.

(c) $2x^2 + 13x + 65$.

WMA12/01 OCTOBER 2024

Question 4

7 marks | Applications of Differentiation

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving each term in simplest form.

(2)

(b) Hence find the x coordinate of the stationary point of C.

(2)

(c) (i) Find $\frac{d^2y}{dx^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of C, giving a reason for your answer.

(2)

(d) State the range of values of x for which y is decreasing.

(1)

WORKED SOLUTION

Question 4 · WMA12/01 October 2024

1 **Rewrite using powers**

$y = 4x^{1/2} + 9x^{-1/2} + 3$.

2 **Differentiate**

$\frac{dy}{dx} = 2x^{-1/2} - \frac{9}{2}x^{-3/2} = \frac{2}{\sqrt{x}} - \frac{9}{2x^{3/2}}$.

3 **Set the derivative to zero**

$\frac{dy}{dx} = \frac{4x - 9}{2x^{3/2}}$. Since $x > 0$, the stationary point occurs when $4x - 9 = 0$.

4 **Find the x coordinate**

Therefore $x = \frac{9}{4}$.

5 **Differentiate again**

$\dfrac{d^2y}{dx^2} = -x^{-3/2} + \dfrac{27}{4}x^{-5/2} = -\dfrac{1}{x^{3/2}} + \dfrac{27}{4x^{5/2}}.$

6 **Test the stationary point**

$\dfrac{d^2y}{dx^2} = \dfrac{27 - 4x}{4x^{5/2}}$. At $x = \dfrac{9}{4}$, the numerator is $27 - 9 = 18 > 0$, so the stationary point is a minimum.

7 **Find where y is decreasing**

For $x > 0$, $2x^{3/2} > 0$, so $\dfrac{dy}{dx} < 0$ when $4x - 9 < 0$. Hence y is decreasing for $0 < x < \dfrac{9}{4}$.

FINAL ANSWER

(a) $\dfrac{2}{\sqrt{x}} - \dfrac{9}{2x^{3/2}}.$

(b) $x = \dfrac{9}{4}.$

$(c)(i)$ $-\dfrac{1}{x^{3/2}} + \dfrac{27}{4x^{5/2}}.$

$(c)(ii)$ Minimum.

(d) $0 < x < \dfrac{9}{4}.$

WMA12/01 OCTOBER 2024

Question 5

7 marks | Binomial Expansion

5. (a) Find, in terms of a, the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in simplest form.

(3)

$$f(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

Given that the constant term in the expansion of $f(x)$ is 576

(b) find the value of a.

(4)

WORKED SOLUTION

Question 5 · WMA12/01 October 2024

1 Write the first three binomial terms

$(2 + ax)^6 = 2^6 + 6(2^5)(ax) + \binom{6}{2}(2^4)(ax)^2 + \cdots$.

2 Simplify the terms

The first three terms are $64 + 192ax + 240a^2x^2$.

3 Expand the other factor

$\left(3 + \frac{1}{x}\right)^2 = 9 + 6x^{-1} + x^{-2}$.

4 Collect constant contributions

The constant term in $\left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$ is $9(64) + 6(192a) + 240a^2$.

5 Use the given constant term

$576 + 1152a + 240a^2 = 576$, so $240a^2 + 1152a = 0$.

6 Solve for a

$48a(5a + 24) = 0$. Since $a \neq 0$, $5a + 24 = 0$, so $a = -\frac{24}{5}$.

FINAL ANSWER

$(a)\ 64 + 192ax + 240a^2x^2.$

$(b)\ a = -\dfrac{24}{5}.$

WMA12/01 OCTOBER 2024

Question 6

5 marks | Laws of Logarithms

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

WORKED SOLUTION

Question 6 · WMA12/01 October 2024

1 State the domain

The logarithms require $12 - 2x > 0$ and $x + 1 > 0$, so $-1 < x < 6$.

2 Combine the right hand side

$2 = \log_4(16)$, so $2 + 2\log_4(x + 1) = \log_4\left(16(x + 1)^2\right)$.

3 Equate the arguments

$\log_4(12 - 2x) = \log_4\left(16(x + 1)^2\right)$ gives $12 - 2x = 16(x + 1)^2$.

4 Form the quadratic

$12 - 2x = 16x^2 + 32x + 16$, so $8x^2 + 17x + 2 = 0$.

5 Factor and solve

$8x^2 + 17x + 2 = (8x + 1)(x + 2) = 0$, so $x = -\frac{1}{8}$ or $x = -2$.

6 Check the domain

The value $x = -2$ is outside $-1 < x < 6$, so the solution is $x = -\frac{1}{8}$.

✓ **FINAL ANSWER**

$x = -\frac{1}{8}$.

WMA12/01 OCTOBER 2024

Question 7

6 marks | Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

WORKED SOLUTION

Question 7 · WMA12/01 October 2024

1 Identify Jem's sequence

Jem's payments are arithmetic with first term 20 and common difference 0.50.

2 Show month 100

The month 100 payment is $20 + 99(0.50) = 69.50$, so Jem pays £69.50.

3 Sum Jem's payments

$S_{300} = \frac{300}{2}\{2(20) + 299(0.50)\} = 150(189.5) = 28425$.

4 Model Kim's payments

Let the common ratio be r. Since the first payment is 20 and the month 300 payment is 250, $20r^{299} = 250$.

5 **Find the ratio**

Thus $r^{299} = 12.5$ and $r = 12.5^{1/299}$.

6 **Sum Kim's payments**

$S_B = 20\dfrac{r^{300} - 1}{r - 1}$ with $r = 12.5^{1/299}$, giving $S_B \approx 27362.95$.

7 **Find the difference**

The difference is $28425 - 27362.95 \approx 1062.05$, which is £1,060 to the nearest £10.

✓ **FINAL ANSWER**

(a) Proven.

(b) £28,425.

(c) £1,060.

Question 8

8 marks | Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

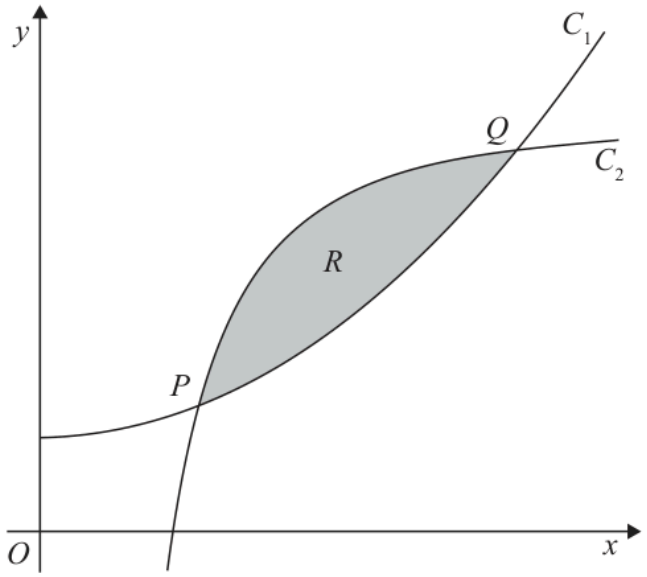


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

1 **Equate the curves**

$x^2 + 3 = 13 - \dfrac{9}{x^2}$.

2 **Clear the denominator**

Multiplying by x^2 gives $x^4 + 3x^2 = 13x^2 - 9$, so $x^4 - 10x^2 + 9 = 0$.

3 **Factor**

Let $u = x^2$. Then $u^2 - 10u + 9 = (u - 1)(u - 9) = 0$, so $x^2 = 1$ or $x^2 = 9$.

4 **Use x greater than zero**

Since $x > 0$, the intersections have $x = 1$ and $x = 3$. Thus $x_P = 1$ and $x_Q = 3$.

5 **Set up the area**

Between $x = 1$ and $x = 3$, curve C_2 is above curve C_1, so $R = \displaystyle\int_1^3 \left(13 - \frac{9}{x^2} - (x^2 + 3)\right) dx$.

6 **Simplify the integrand**

$R = \displaystyle\int_1^3 (10 - x^2 - 9x^{-2})\, dx$.

7 **Integrate**

$R = \left[10x - \dfrac{x^3}{3} + \dfrac{9}{x}\right]_1^3$.

8 **Evaluate**

$R = (30 - 9 + 3) - \left(10 - \dfrac{1}{3} + 9\right) = 24 - \dfrac{56}{3} = \dfrac{16}{3}$.

FINAL ANSWER

(a) $x_P = 1$, $x_Q = 3$.

(b) $\dfrac{16}{3}$.

WMA12/01 OCTOBER 2024

Question 9

7 marks | Trigonometric Equations

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

WORKED SOLUTION

Question 9 · WMA12/01 October 2024

1 Write tan in terms of sin and cos

$2\tan\theta = 3\cos\theta$ gives $2\dfrac{\sin\theta}{\cos\theta} = 3\cos\theta$.

2 Multiply by cos theta

Since $\tan\theta$ is defined, $\cos\theta \neq 0$. Hence $2\sin\theta = 3\cos^2\theta$.

3 Use the identity

$2\sin\theta = 3(1 - \sin^2\theta)$, so $3\sin^2\theta + 2\sin\theta - 3 = 0$.

4 Substitute the new angle

Let $\theta = 2x + \dfrac{\pi}{3}$. Then $3\sin^2\theta + 2\sin\theta - 3 = 0$.

5 Solve for sin theta

$\sin\theta = \dfrac{-2 \pm \sqrt{40}}{6} = \dfrac{-1 \pm \sqrt{10}}{3}$. The root $\dfrac{-1 - \sqrt{10}}{3}$ is less than -1, so reject it.

6 **Find the angle range**

Since $-\pi < x < \pi$, $-\dfrac{5\pi}{3} < \theta < \dfrac{7\pi}{3}$.

7 **Find the theta values**

For $\sin\theta = \dfrac{-1+\sqrt{10}}{3}$ in this range, $\theta \approx -3.94649$, 0.804897, 2.33670, and 7.08808.

8 **Convert back to x**

Using $x = \dfrac{\theta - \pi/3}{2}$ gives $x \approx -2.49684$, -0.121150, 0.644749, and 3.02044.

9 **Round the answers**

To 3 significant figures, $x = -2.50$, -0.121, 0.645, and 3.02.

✓ **FINAL ANSWER**

(a) Proven.

(b) $x = -2.50$, -0.121, 0.645, 3.02.

WMA12/01 OCTOBER 2024

Question 10

10 marks | Circles

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

WORKED SOLUTION

Question 10 · WMA12/01 October 2024

1 Complete the square

$x^2 + y^2 + 4x - 30y + 209 = 0$ becomes $(x+2)^2 + (y-15)^2 = 20$.

2 Read the centre and radius

The centre is $(-2, 15)$ and the radius is $\sqrt{20} = 2\sqrt{5}$.

3 Use the tangent distance

The line $y = mx + 1$ can be written as $mx - y + 1 = 0$. Its distance from $(-2, 15)$ must be $2\sqrt{5}$.

4 Set up the equation for m

$\dfrac{|m(-2) - 15 + 1|}{\sqrt{m^2+1}} = 2\sqrt{5}$, so $\dfrac{|-2m - 14|}{\sqrt{m^2+1}} = 2\sqrt{5}$.

5 Show the quadratic

Squaring gives $4(m+7)^2 = 20(m^2+1)$. Hence $(m+7)^2 = 5(m^2+1)$, so $2m^2 - 7m - 22 = 0$.

6 **Find the possible gradients**

$2m^2 - 7m - 22 = 0$ gives $m = -2$ or $m = \frac{11}{2}$.

7 **Use perpendicular radii**

A tangent is perpendicular to the radius at the point of contact.

8 **Find the first point**

For $m = -2$, the radius gradient is $\frac{1}{2}$. The radius through the centre is $y - 15 = \frac{1}{2}(x + 2)$. Solving with $y = -2x + 1$ gives $x = -6$ and $y = 13$.

9 **Find the second point**

For $m = \frac{11}{2}$, the radius gradient is $-\frac{2}{11}$. The radius through the centre is $y - 15 = -\frac{2}{11}(x + 2)$. Solving with $y = \frac{11}{2}x + 1$ gives $x = \frac{12}{5}$ and $y = \frac{71}{5}$.

✓ **FINAL ANSWER**

$(a)(i)$ $(-2, 15)$.

$(a)(ii)$ $2\sqrt{5}$.

(b) Proven.

(c) $(-6, 13)$ and $\left(\frac{12}{5}, \frac{71}{5}\right)$.

WMA12/01 OCTOBER 2024

Question 11

6 marks | Proof

11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

WORKED SOLUTION

Question 11 · WMA12/01 October 2024

1 Choose a prime value

Take $n = 5$, which is prime.

2 Evaluate the expression

$3^5 + 2 = 243 + 2 = 245 = 5 \times 49$, so it is not prime. This disproves the statement.

3 List the possible forms of m

If integer m is not divisible by 3, then m has remainder 1 or 2 on division by 3.

4 Case one

If $m = 3q + 1$, then $m^2 - 1 = (3q + 1)^2 - 1 = 9q^2 + 6q = 3(3q^2 + 2q)$, which is divisible by 3.

5 Case two

If $m = 3q + 2$, then $m^2 - 1 = (3q + 2)^2 - 1 = 9q^2 + 12q + 3 = 3(3q^2 + 4q + 1)$, which is divisible by 3.

6 Conclude by exhaustion

Both possible cases for an integer not divisible by 3 have been checked, so $m^2 - 1$ is divisible by 3.

FINAL ANSWER

(i) Counterexample: $n = 5$ gives $3^5 + 2 = 245$, not prime.

(ii) Proven.