Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 May/June 2024 Worked Solutions

May/June 2024 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 May/June 2024

May/June 2024

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 MAY/JUNE 2024

Question 1

5 marks | Binomial Expansion

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{1}{6}x\right)^9$$

giving each term in simplest form.

(3)

(b) Hence find the coefficient of x^3 in the expansion of

$$(10x+3)\left(1-\frac{1}{6}x\right)^9$$

giving the answer in simplest form.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 May/June 2024

1 **Expand the first terms**

$$\left(1-\frac{1}{6}x\right)^9 = 1+9\left(-\frac{1}{6}x\right)+\binom{9}{2}\left(-\frac{1}{6}x\right)^2+\binom{9}{3}\left(-\frac{1}{6}x\right)^3+\cdots.$$

2 **Simplify**

The first four terms are $1-\frac{3}{2}x+x^2-\frac{7}{18}x^3$.

3 **Identify contributions to x cubed**

In $(10x+3)\left(1-\frac{1}{6}x\right)^9$, the x^3 coefficient comes from 3 times the x^3 coefficient and $10x$ times the x^2 coefficient.

4 **Calculate the coefficient**

$$3\left(-\frac{7}{18}\right)+10(1)=-\frac{7}{6}+10=\frac{53}{6}.$$

FINAL ANSWER

$(a)\ 1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3.$

$(b)\ \frac{53}{6}.$

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Question 2

7 marks | Arithmetic Sequences

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.

(4)

(b) Hence find the smallest value of n for which

$$S_n > 8000$$

(3)

WORKED SOLUTION

Question 2 · WMA12/01 May/June 2024

1 **Use the sixth term**

For an arithmetic series, $u_6 = a + 5d = 2$.

2 **Use the sum of ten terms**

$S_{10} = \frac{10}{2}(2a + 9d) = -80$, so $2a + 9d = -16$.

3 **Solve simultaneously**

From $a + 5d = 2$, $a = 2 - 5d$. Substitute into $2a + 9d = -16$.

4 **Find a and d**

$2(2 - 5d) + 9d = -16$, so $4 - d = -16$ and $d = 20$. Then $a = 2 - 5(20) = -98$.

5 **Write S n**

$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(-196 + 20n - 20) = 10n^2 - 108n$.

6 **Set up the inequality**

We need $10n^2 - 108n > 8000$.

7 **Solve the boundary**

$10n^2 - 108n - 8000 = 0$ gives $5n^2 - 54n - 4000 = 0$.

8 **Find the positive threshold**

$$n = \frac{54 + \sqrt{82916}}{10} = 34.195\ldots.$$

9 **Choose the smallest integer**

Therefore the smallest integer value with $S_n > 8000$ is $n = 35$.

✓ **FINAL ANSWER**

(a) First term $= -98$, common difference $= 20$.

(b) $n = 35$.

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Question 3

6 marks | Laws of Logarithms

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10)$$

(5)

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where a is a positive constant greater than 1

(1)

WORKED SOLUTION

Question 3 · WMA12/01 May/June 2024

1 **State the domain**

The logarithms require $2 - x > 0$ and $x + 10 > 0$, so $-10 < x < 2$.

2 **Combine powers**

$2\log_2(2-x) = \log_2((2-x)^2)$.

3 **Rewrite four**

$4 = \log_2 16$, so $4 + \log_2(x+10) = \log_2(16(x+10))$.

4 **Equate arguments**

$(2-x)^2 = 16(x+10)$.

5 **Solve the quadratic**

$x^2 - 4x + 4 = 16x + 160$, so $x^2 - 20x - 156 = 0$.

6 **Use the domain**

$x = 26$ or $x = -6$. The value 26 is outside the domain, so $x = -6$.

7 **Evaluate the logarithm**

$\log_{\sqrt{a}}(a^6) = \log_{a^{1/2}}(a^6) = \dfrac{6}{1/2} = 12$.

FINAL ANSWER

$(i)\ x = -6.$

$(ii)\ 12.$

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Question 4

8 marks | Polynomials

4. $f(x) = (x - 2)(2x^2 + 5x + k) + 21$

where k is a constant.

(a) State the remainder when $f(x)$ is divided by $(x - 2)$

(1)

Given that $(2x - 1)$ is a factor of $f(x)$

(b) show that $k = 11$

(2)

(c) Hence

(i) fully factorise $f(x)$,

(ii) find the number of real solutions of the equation

$$f(x) = 0$$

giving a reason for your answer.

(5)

WORKED SOLUTION

Question 4 · WMA12/01 May/June 2024

1 Use the remainder theorem

The remainder on division by $(x - 2)$ is $f(2)$.

2 Find f of two

$f(2) = (2 - 2)(2(2)^2 + 5(2) + k) + 21 = 21$.

3 Use the factor theorem

Since $(2x - 1)$ is a factor, $x = \frac{1}{2}$ is a root, so $f\left(\frac{1}{2}\right) = 0$.

4 Substitute x equals half

$\left(\frac{1}{2} - 2\right)\left(2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + k\right) + 21 = 0$.

5 **Solve for k**

$-\frac{3}{2}(3+k)+21=0$. Multiplying by 2 gives $-3(k+3)+42=0$, so $k=11$.

6 **Write f with k equals eleven**

$f(x)=(x-2)(2x^2+5x+11)+21=2x^3+x^2+x-1$.

7 **Factorise**

$2x^3+x^2+x-1=(2x-1)(x^2+x+1)$.

8 **Count real solutions**

The quadratic x^2+x+1 has discriminant $1-4=-3$, so it has no real roots. Therefore $f(x)=0$ has one real solution.

✓ **FINAL ANSWER**

(a) 21.

(b) $k=11$.

$(c)(i)$ $(2x-1)(x^2+x+1)$.

$(c)(ii)$ One real solution.

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Question 5

6 marks | Proof

5. **In this question you must show detailed reasoning.**

(a) Given that x and y are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

(4)

(b) Using a counter example, show that the result in part (a) is not true for all real numbers.

(2)

WORKED SOLUTION

Question 5 · WMA12/01 May/June 2024

1 Start from the inequality

$(x-y)^3 > x^3 - y^3$.

2 Expand the left side

$x^3 - 3x^2y + 3xy^2 - y^3 > x^3 - y^3$.

3 Cancel common terms

Subtract $x^3 - y^3$ from both sides to get $-3x^2y + 3xy^2 > 0$.

4 Factor

$3xy(y-x) > 0$.

5 Use positivity

Since x and y are positive, $3xy > 0$. Therefore $y - x > 0$, so $y > x$.

6 Give a real-number counterexample

Take $x = 1$ and $y = -1$. Then $(x-y)^3 = 2^3 = 8$ and $x^3 - y^3 = 1 - (-1) = 2$, so the given inequality is true.

7 Show the conclusion fails

But $y > x$ would mean $-1 > 1$, which is false. Therefore the result is not true for all real numbers.

FINAL ANSWER

(a) Proven.

(b) For example, $x = 1, y = -1$ satisfies the inequality but not $y > x$.

Question 6

9 marks | Integration

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

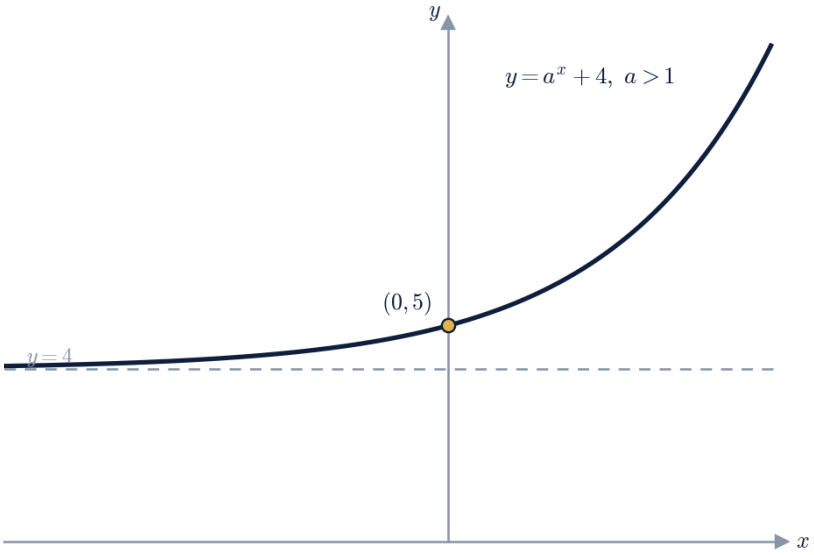
(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

1 **Sketch key features**

For $y = a^x + 4$ with $a > 1$, the curve is increasing.



2 **Find the y-intercept**

At $x = 0$, $y = a^0 + 4 = 5$, so the y-intercept is $(0, 5)$.

3 **Find the asymptote**

As $x \to -\infty$, $a^x \to 0$, so the horizontal asymptote is $y = 4$.

4 **Use the trapezium width**

The x values increase by 0.3, so $h = 0.3$.

5 **Apply the trapezium rule**

$$\int_2^{3.5} (2^x - 2x)dx \approx \frac{0.3}{2}[0 + 4.3137 + 2(0.3246 + 0.8629 + 1.6643 + 2.7896)].$$

6 **Round part b**

This gives 2.339475, so the estimate is 2.34 to 2 decimal places.

7 **Estimate part c i**

$2^x + 2x = (2^x - 2x) + 4x$, so $\int_2^{3.5} (2^x + 2x)dx \approx 2.34 + \int_2^{3.5} 4x\,dx$.

8 **Add the extra integral**

$\int_2^{3.5} 4x\,dx = [2x^2]_2^{3.5} = 16.5$, so the estimate is $2.34 + 16.5 = 18.84$.

9 **Estimate part c ii**

$2^{x+1} - 4x = 2(2^x - 2x)$, so the estimate is $2(2.34) = 4.68$.

✓ **FINAL ANSWER**

(a) y-intercept $(0, 5)$ and asymptote $y = 4$.

(b) 2.34.

$(c)(i)$ 18.84.

$(c)(ii)$ 4.68.

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Question 7

6 marks | Circles

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x-5)^2 + (y+8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

WORKED SOLUTION

Question 7 · WMA12/01 May/June 2024

1 Complete the square

$x^2 + y^2 + 8x - 10y = 29$ becomes $(x+4)^2 - 16 + (y-5)^2 - 25 = 29$.

2 Write circle C1 in standard form

$(x+4)^2 + (y-5)^2 = 70$.

3 Read centre and radius

The centre of C_1 is $(-4, 5)$ and its radius is $\sqrt{70}$.

4 Read C2

C_2 has equation $(x-5)^2 + (y+8)^2 = 52$, so its centre is $(5, -8)$ and its radius is $\sqrt{52} = 2\sqrt{13}$.

5 Find distance between centres

The distance between $(-4, 5)$ and $(5, -8)$ is $\sqrt{9^2 + (-13)^2} = \sqrt{250} = 5\sqrt{10}$.

6 Compare with radii

$\sqrt{70} + 2\sqrt{13} = 15.58\ldots$, while $5\sqrt{10} = 15.81\ldots$.

7 **Conclude**

The distance between centres is greater than the sum of the radii, so the circles are separate.

Therefore they neither touch nor intersect.

✓ **FINAL ANSWER**

$(a)(i)$ $(-4, 5)$.

$(a)(ii)$ $\sqrt{70}$.

(b) The circles neither touch nor intersect.

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Question 8

12 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H°C$, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

WORKED SOLUTION

Question 8 · WMA12/01 May/June 2024

1 **Rewrite the trig equation**

$5\sin x \tan x + 13 = \cos x$ becomes $\dfrac{5\sin^2 x}{\cos x} + 13 = \cos x$.

2 **Multiply by cos x**

$5\sin^2 x + 13\cos x = \cos^2 x$.

3 **Use sine squared**

$5(1 - \cos^2 x) + 13\cos x = \cos^2 x$, so $6\cos^2 x - 13\cos x - 5 = 0$.

4 **Solve for cos x**

$(2\cos x - 5)(3\cos x + 1) = 0$, so $\cos x = \frac{5}{2}$ or $\cos x = -\frac{1}{3}$.

5 **Choose the valid root**

$\cos x = \frac{5}{2}$ is impossible, so $\cos x = -\frac{1}{3}$. For $0 < x \leq \pi$, $x = 1.9106\ldots$, so $x = 1.91$ radians to 3 significant figures.

6 **Use the temperature at 6 am**

At $t = 6$, $20 = 10 + 12\sin(6k + 18)°$, so $\sin(6k + 18)° = \frac{5}{6}$.

7 **Solve for k**

$\sin^{-1}\left(\frac{5}{6}\right) = 56.442\ldots°$. Since $0 < k < 20$, $18 < 6k + 18 < 138$, so $6k + 18 = 56.442\ldots$ or $123.557\ldots$.

8 **Find possible k values**

This gives $k = 6.407\ldots$ or $k = 17.592\ldots$, so $k = 6.41$ or 17.59 to 2 decimal places.

9 **Use the tighter condition**

Given further that $0 < k < 10$, use $k = 6.407\ldots$.

10 **Find the maximum temperature**

The maximum value of $\sin$ is 1, so the maximum temperature is $10 + 12 = 22°\text{C}$.

11 **Find the time of maximum**

The maximum occurs when $kt + 18 = 90$, so $t = \frac{72}{6.407\ldots} = 11.2375\ldots$ hours after midnight.

12 **Convert to time of day**

$0.2375\ldots$ hours is $14.25\ldots$ minutes, so the maximum occurs at about $11{:}14$ am.

FINAL ANSWER

(i) $x = 1.91$ radians.

$(ii)(a)$ $k = 6.41$ or 17.59.

$(ii)(b)$ $22°\text{C}$.

$(ii)(c)$ $11{:}14$ am.

Question 9

8 marks | Applications of Differentiation

9.

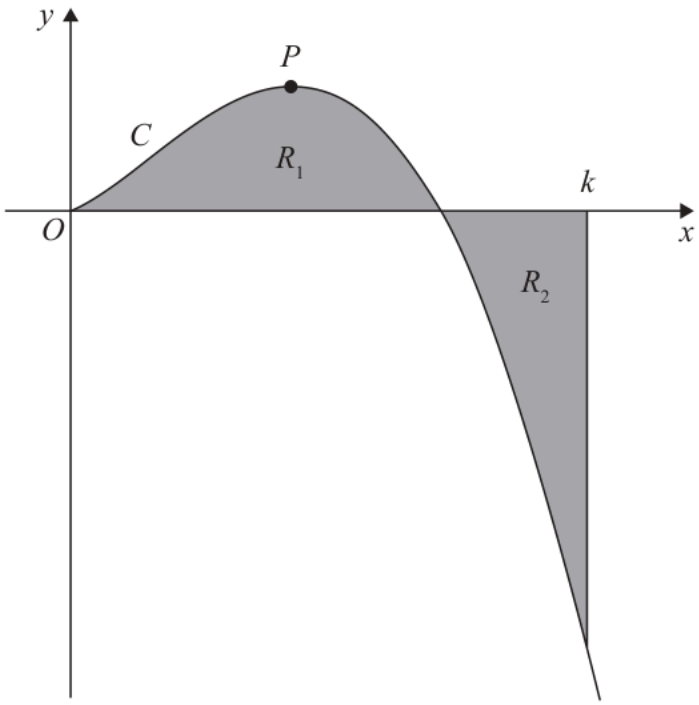


Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

1 Rewrite the curve

$y = 2x^{3/2}(4 - x) = 8x^{3/2} - 2x^{5/2}$.

2 Differentiate

$\dfrac{dy}{dx} = 12x^{1/2} - 5x^{3/2} = \sqrt{x}(12 - 5x)$.

3 Find the stationary point

For the interior stationary point P, $12 - 5x = 0$, so $x = \dfrac{12}{5}$.

4 Set up the area condition

R_1 is the area from $x = 0$ to $x = 4$ above the x-axis. R_2 is the area from $x = 4$ to $x = k$ below the x-axis.

5 Use equal areas

Equal areas mean $\displaystyle\int_0^4 y\,dx = -\int_4^k y\,dx$, so $\displaystyle\int_0^k y\,dx = 0$.

6 Integrate y

$\displaystyle\int 2x^{3/2}(4 - x)dx = \int (8x^{3/2} - 2x^{5/2})dx = \frac{16}{5}x^{5/2} - \frac{4}{7}x^{7/2}$.

7 Solve for k

$\dfrac{16}{5}k^{5/2} - \dfrac{4}{7}k^{7/2} = 0$.

8 Factor

$4k^{5/2}\left(\dfrac{4}{5} - \dfrac{k}{7}\right) = 0$.

9 Choose the value beyond four

Since $k > 4$, $\dfrac{4}{5} - \dfrac{k}{7} = 0$, so $k = \dfrac{28}{5}$.

FINAL ANSWER

$(a)\ x = \dfrac{12}{5}$.

$(b)\ k = \dfrac{28}{5}$.

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Question 10

8 marks | Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that
- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

WORKED SOLUTION

Question 10 · WMA12/01 May/June 2024

1 **Model the dormice**

The dormice model is $D = 2000(1.03)^t$.

2 **Find the dormice after six years**

$D = 2000(1.03)^6 = 2388.10\ldots$, so there are 2390 dormice to 3 significant figures.

3 **Set up the vole model**

Let the vole model be $N = ab^t$.

4 **Use the two data points**

At $t = 4$, $ab^4 = 3690$. At $t = 7$, $ab^7 = 3470$.

5 **Find b**

Dividing gives $b^3 = \dfrac{3470}{3690}$, so $b = 0.9797\ldots$, which is 0.98 to 2 significant figures.

6 **Find a**

$a = \dfrac{3690}{b^4} = 4005.18\ldots$, which is 4000 to 2 significant figures.

7 **Write the vole model**

So the model is approximately $N = 4000(0.98)^t$.

8 **Set the two models equal**

Use $2000(1.03)^T = ab^T$ with the unrounded constants from the vole model.

9 **Solve using logarithms**

$\left(\dfrac{1.03}{b}\right)^T = \dfrac{a}{2000}$, so $T = \dfrac{\log(a/2000)}{\log(1.03/b)}$.

10 **Calculate T**

Substituting $a = 4005.18\ldots$ and $b = 0.9797\ldots$ gives $T = 13.875\ldots$, so $T = 13.9$ to one decimal place.

✓ **FINAL ANSWER**

(a) 2390 dormice.

(b) $N = 4000(0.98)^t$ to 2 s.f. constants.

(c) $T = 13.9$.