Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 May/June R 2024 Worked Solutions

May/June R 2024 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 May/June R 2024

May/June R 2024

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

5 marks | Sequences & Series

1. The sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^{4} (u_r + 2r)$

(3)

WORKED SOLUTION

Question 1 · WMA12/01 May/June R 2024

1 Use the recurrence at n equals one

$u_3 = 3u_2 - 2u_1$.

2 Substitute given values

$4 = 3u_2 - 2(7)$, so $4 = 3u_2 - 14$.

3 Find u2

$3u_2 = 18$, hence $u_2 = 6$.

4 Find u4

$u_4 = 3u_3 - 2u_2 = 3(4) - 2(6) = 0$.

5 Evaluate the sum

$\sum_{r=1}^{4}(u_r + 2r) = (7+2) + (6+4) + (4+6) + (0+8)$.

6 Add the terms

The sum is $9 + 10 + 10 + 8 = 37$.

FINAL ANSWER

(a) $u_2 = 6$.

(b) 37.

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Question 2

5 marks | Integration

2. The table shows corresponding values of x and y for a continuous curve with equation $y = \mathrm{f}(x)$ between $x = -4$ and $x = 5$, where a is a constant.

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} \mathrm{f}(x)\,\mathrm{d}x$$

Given that the value of this approximation is 19.3

(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} \left(2\mathrm{f}(x) - 3\right)\mathrm{d}x$$

(2)

WORKED SOLUTION

Question 2 · WMA12/01 May/June R 2024

1 Use the trapezium width

The x values increase by 1.5, so $h = 1.5$.

2 Set up the trapezium estimate

$$19.3 = \frac{1.5}{2}[4.16 + 2.28 + 2(2.91 + a + 1.73 + 1.37 + 1.43)].$$

3 Solve for a

This simplifies to $19.3 = 15.99 + 1.5a$, so $a = \dfrac{3.31}{1.5} = 2.206\ldots$.

4 Round a

Therefore $a = 2.21$ to 3 significant figures.

5 **Use the transformed integral**

$$\int_{-4}^{5}(2f(x)-3)dx = 2\int_{-4}^{5}f(x)dx - \int_{-4}^{5}3\,dx.$$

6 **Substitute the given estimate**

This is $2(19.3)-3(9) = 38.6-27 = 11.6$.

✓ **FINAL ANSWER**

(a) $a = 2.21$ to 3 s.f.

(b) 11.6.

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Question 3

5 marks | Circles

3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is nearest the origin O,

(b) find the exact length of OP

(2)

WORKED SOLUTION

Question 3 · WMA12/01 May/June R 2024

1 Complete the square

$x^2 + y^2 + 8x - 14y - 79 = 0$ becomes $(x+4)^2 - 16 + (y-7)^2 - 49 - 79 = 0$.

2 Write standard form

$(x+4)^2 + (y-7)^2 = 144$.

3 Read centre and radius

The centre is $(-4, 7)$ and the radius is 12.

4 Find distance from centre to origin

The distance from $(-4, 7)$ to $O(0, 0)$ is $\sqrt{(-4)^2 + 7^2} = \sqrt{65}$.

5 Use origin inside the circle

Since $\sqrt{65} < 12$, the origin is inside the circle.

6 Find nearest distance to the circle

The nearest point on the circle lies along the line from the centre to the origin, so $OP = 12 - \sqrt{65}$.

FINAL ANSWER

$(a)(i)\ (-4, 7)$.

$(a)(ii)\ 12$.

$(b)\ 12 - \sqrt{65}$.

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Question 4

7 marks | Binomial Expansion

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3+2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2-\frac{1}{6x}\right)(3+2x)^6$$

(3)

WORKED SOLUTION

Question 4 · WMA12/01 May/June R 2024

1 Write the expansion terms

$(3+2x)^6 = 3^6 + \binom{6}{1}3^5(2x) + \binom{6}{2}3^4(2x)^2 + \binom{6}{3}3^3(2x)^3 + \cdots$.

2 Simplify the first four terms

This gives $729 + 2916x + 4860x^2 + 4320x^3$.

3 Identify x squared contributions

In $\left(2x^2-\frac{1}{6x}\right)(3+2x)^6$, the x^2 term comes from $2x^2$ times the constant term and $-\frac{1}{6x}$ times the x^3 term.

4 Calculate the coefficient

The coefficient is $2(729) - \frac{1}{6}(4320) = 1458 - 720 = 738$.

FINAL ANSWER

(a) $729 + 2916x + 4860x^2 + 4320x^3$.

(b) 738.

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Question 5

6 marks | Trigonometric Equations

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

WORKED SOLUTION

Question 5 · WMA12/01 May/June R 2024

1 **Substitute t equals two**

$D = 8 + 5\sin\left(\frac{\pi(2)}{6} + 3\right) = 8 + 5\sin\left(\frac{\pi}{3} + 3\right)$.

2 **Evaluate**

$D = 4.064\ldots$, so the depth at 2 am is just over 4 metres.

3 **Set depth equal to six**

$6 = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right)$, so $\sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$.

4 **Use the time interval**

After midday means $t > 12$. Then $\frac{\pi t}{6} + 3$ is greater than $2\pi + 3$.

5 **Choose the first suitable angle**

The first angle after $2\pi + 3$ with sine $-\frac{2}{5}$ is $3\pi + \sin^{-1}\left(\frac{2}{5}\right)$.

6 **Solve for t**

$\frac{\pi t}{6} + 3 = 3\pi + \sin^{-1}\left(\frac{2}{5}\right)$, so $t = 13.056\ldots$ hours after midnight.

7 **Convert to clock time**

$0.056\ldots$ hours is about 3.38 minutes, so the time to the nearest minute is $13{:}03$, or $1{:}03$ pm.

✓ **FINAL ANSWER**

(a) $D(2) = 4.06\ldots$ m, just over 4 m.

(b) $13{:}03$, or $1{:}03$ pm.

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Question 6

9 marks | Applications of Differentiation

6. $f(x) = 4x^3 + px^2 + 8x + q$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

WORKED SOLUTION

Question 6 · WMA12/01 May/June R 2024

1 Use the factor theorem

Since $(2x+3)$ is a factor, $x = -\frac{3}{2}$ is a root.

2 Substitute x equals negative three halves

$4\left(-\frac{3}{2}\right)^3 + p\left(-\frac{3}{2}\right)^2 + 8\left(-\frac{3}{2}\right) + q = 0.$

3 Simplify the first condition

$-\frac{27}{2} + \frac{9}{4}p - 12 + q = 0$, so $\frac{9}{4}p + q = \frac{51}{2}$.

4 Use the remainder theorem

The remainder on division by $(x+2)$ is $f(-2)$.

5 Substitute x equals negative two

$f(-2) = 4(-8) + 4p - 16 + q = 4p + q - 48.$

6 **Use the given remainder**

$4p + q - 48 = -5$, so $4p + q = 43$.

7 **Solve for p and q**

Subtract $\frac{9}{4}p + q = \frac{51}{2}$ from $4p + q = 43$: $\frac{7}{4}p = \frac{35}{2}$, so $p = 10$. Then $4(10) + q = 43$, so $q = 3$.

8 **Differentiate**

With $p = 10$, $f(x) = 4x^3 + 10x^2 + 8x + 3$, so $f'(x) = 12x^2 + 20x + 8$.

9 **Factor the derivative**

$f'(x) = 4(3x + 2)(x + 1)$.

10 **Find where decreasing**

The quadratic derivative is negative between its roots, so $f(x)$ is decreasing for $-1 < x < -\frac{2}{3}$.

✓ **FINAL ANSWER**

$(a)(i)\ p = 10$.

$(a)(ii)\ q = 3$.

$(b)\ -1 < x < -\frac{2}{3}$.

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Question 7

7 marks | Trigonometric Equations

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

WORKED SOLUTION

Question 7 · WMA12/01 May/June R 2024

1 Rewrite tan x

$3\sin x \tan x = 11 + \cos x$ becomes $\dfrac{3\sin^2 x}{\cos x} = 11 + \cos x$.

2 Multiply by cos x

$3\sin^2 x = 11\cos x + \cos^2 x$.

3 Use sine squared

$3(1 - \cos^2 x) = 11\cos x + \cos^2 x$, so $4\cos^2 x + 11\cos x - 3 = 0$.

4 Solve for cosine

$(4\cos x - 1)(\cos x + 3) = 0$, so $\cos x = \dfrac{1}{4}$ or $\cos x = -3$.

5 Reject impossible value

$\cos x = -3$ is impossible, so $\cos x = \dfrac{1}{4}$.

6 Find x values

For $0 \leq x < 2\pi$, this gives $x = 1.318\ldots$ or $x = 4.965\ldots$, so $x = 1.318, 4.965$ radians.

7 **Use the right triangle identity**

Given $0 < \theta < 90°$ and $\cos\theta = \frac{1}{3}$, $\sin\theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$.

8 **Find tan theta**

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$.

✓ **FINAL ANSWER**

$(i)\ x = 1.318,\ 4.965$.

$(ii)\ 2\sqrt{2}$.

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Question 8

13 marks | Arithmetic Sequences

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k+4 \qquad k-2 \qquad 11-k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

WORKED SOLUTION

Question 8 · WMA12/01 May/June R 2024

1 Write the first and last terms

The first term is a and the nth term is $a + (n-1)d$.

2 Use the arithmetic sum formula

$S_n = \frac{n}{2}(\text{first term} + \text{last term})$.

3 Show the formula

Therefore $S_n = \frac{n}{2}\{a + [a + (n-1)d]\} = \frac{n}{2}\{2a + (n-1)d\}$.

4 Identify the arithmetic series

For $900 + 892 + 884 + \cdots + 500$, the first term is 900, the common difference is -8, and the last term is 500.

5 **Find the number of terms**

$500 = 900 + (n-1)(-8)$, so $n - 1 = 50$ and $n = 51$.

6 **Find the sum**

$S = \frac{51}{2}(900 + 500) = 35700$.

7 **Use the geometric condition**

For $k + 4$, $k - 2$, $11 - k$ to be consecutive geometric terms, $(k-2)^2 = (k+4)(11-k)$.

8 **Show the quadratic**

$k^2 - 4k + 4 = -k^2 + 7k + 44$, so $2k^2 - 11k - 40 = 0$.

9 **Solve for k**

$2k^2 - 11k - 40 = 0$ gives $k = 8$ or $k = -\frac{5}{2}$.

10 **Use convergence**

If $k = 8$, the terms are $12, 6, 3$ and the common ratio is $\frac{1}{2}$, so the series is convergent. If $k = -\frac{5}{2}$, the common ratio is -3, so it is not convergent.

11 **Find S infinity**

Use $k = 8$: $S_\infty = \frac{12}{1 - 1/2} = 24$.

FINAL ANSWER

$(i)(a)$ Proven.

$(i)(b)$ 35700.

$(ii)(a)$ Proven.

$(ii)(b)$ $S_\infty = 24$.

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Question 9

9 marks | Laws of Logarithms

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

WORKED SOLUTION

Question 9 · WMA12/01 May/June R 2024

1 Verify t equals four

Substitute $t = 4$: $3\log_2(8) - 2\log_2(2) = 3(3) - 2(1) = 7$.

2 Combine the logarithms

$3\log_2(t+4) - 2\log_2(t-2) = \log_2\left(\dfrac{(t+4)^3}{(t-2)^2}\right)$.

3 Use seven as a logarithm

$7 = \log_2(128)$, so $\dfrac{(t+4)^3}{(t-2)^2} = 128$.

4 Clear the denominator

$(t+4)^3 = 128(t-2)^2$.

5 Expand

$t^3 + 12t^2 + 48t + 64 = 128t^2 - 512t + 512$.

6 **Show the cubic**

Rearranging gives $t^3 - 116t^2 + 560t - 448 = 0$.

7 **Use the known root**

Since $t = 4$ is a root, $(t - 4)$ is a factor.

8 **Factor the cubic**

$t^3 - 116t^2 + 560t - 448 = (t - 4)(t^2 - 112t + 112)$.

9 **Solve the quadratic**

$t^2 - 112t + 112 = 0$ gives $t = \dfrac{112 \pm \sqrt{112^2 - 4(112)}}{2} = 56 \pm 12\sqrt{21}$.

10 **Use the log domain**

The original equation requires $t > 2$. The value $56 - 12\sqrt{21}$ is less than 2, so it is rejected. Hence $t = 4$ or $t = 56 + 12\sqrt{21}$.

✓ **FINAL ANSWER**

(a) Verified.

(b) Proven.

(c) $t = 4$ or $t = 56 + 12\sqrt{21}$.

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Question 10

9 marks | Applications of Differentiation

10.

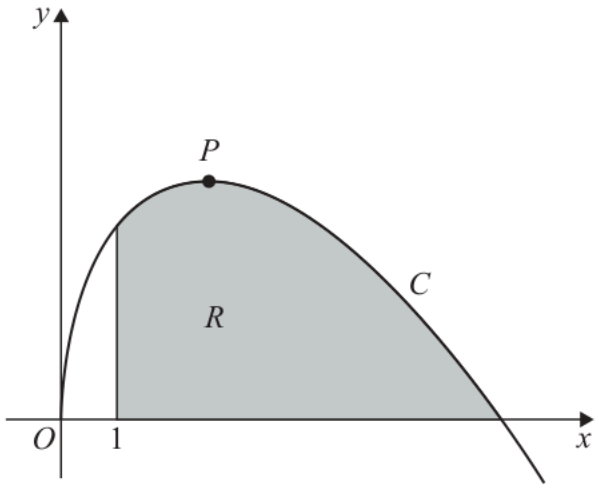


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

WORKED SOLUTION

Question 10 · WMA12/01 May/June R 2024

1 **Rewrite the curve**

$$y = \frac{9x - x^2}{2\sqrt{x}} = \frac{1}{2}(9x^{1/2} - x^{3/2}).$$

2 **Differentiate**

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{9}{2}x^{-1/2} - \frac{3}{2}x^{1/2}\right) = \frac{3(3-x)}{4\sqrt{x}}.$$

3 **Use the stationary condition**

For $x > 0$, the denominator is not zero. So $\frac{dy}{dx} = 0$ when $3 - x = 0$.

4 **Find P**

Therefore the x coordinate of P is 3.

5 **Find where the curve meets the x-axis**

$y = 0$ when $9x - x^2 = 0$, so $x = 9$ for the positive intercept.

6 **Set up the area**

The region is under the curve from $x = 1$ to $x = 9$, so $R = \int_1^9 \frac{9x - x^2}{2\sqrt{x}}dx$.

7 **Integrate**

$$R = \int_1^9 \frac{1}{2}(9x^{1/2} - x^{3/2})dx = \left[3x^{3/2} - \frac{1}{5}x^{5/2}\right]_1^9.$$

8 **Evaluate**

$$R = \left(3(27) - \frac{1}{5}(243)\right) - \left(3 - \frac{1}{5}\right) = \frac{148}{5}.$$

✓ **FINAL ANSWER**

(a) $x = 3$.

(b) $\frac{148}{5}$.