Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 May/June 2022 Worked Solutions

May/June 2022 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 May/June 2022

May/June 2022

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 MAY/JUNE 2022

Question 1

4 marks | Binomial Expansion

1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)

WORKED SOLUTION

Question 1 · WMA12/01 May/June 2022

1 Write the binomial terms

$(2+\frac{3}{8}x)^{10} = 2^{10} + \binom{10}{1}2^9\left(\frac{3}{8}x\right) + \binom{10}{2}2^8\left(\frac{3}{8}x\right)^2 + \binom{10}{3}2^7\left(\frac{3}{8}x\right)^3 + \cdots$.

2 Simplify the constant and linear terms

$2^{10} = 1024$ and $\binom{10}{1}2^9\left(\frac{3}{8}\right)x = 1920x$.

3 Simplify the quadratic term

$\binom{10}{2}2^8\left(\frac{3}{8}\right)^2 x^2 = 1620x^2$.

4 Simplify the cubic term

$\binom{10}{3}2^7\left(\frac{3}{8}\right)^3 x^3 = 810x^3$.

5 State the expansion

The first four terms are $1024 + 1920x + 1620x^2 + 810x^3$.

FINAL ANSWER

$1024 + 1920x + 1620x^2 + 810x^3$.

Question 2

8 marks | Integration

2.

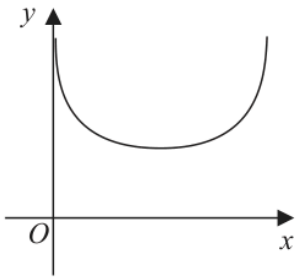


Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319		1.001		1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) \mathrm{d}x$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) \mathrm{d}x$$

(3)

1 **Complete the missing table values**

Using $y = 1 - \log_{10}(\sin x)$ gives $y(1) = 1.075$ and $y(2) = 1.041$ to 3 decimal places.

2 **Identify the strip width**

The x values increase by 0.5, so $h = 0.5$.

3 **Apply the trapezium rule**

$A \approx \frac{0.5}{2}\{1.319 + 1.850 + 2(1.075 + 1.001 + 1.041 + 1.223)\}$.

4 **Calculate part b**

This gives $2.9625\ldots$, so the estimate is 2.96 to 2 decimal places.

5 **Rewrite the new integrand**

Since $y = 1 - \log_{10}(\sin x)$, we have $\log_{10}(\sin x) = 1 - y$.

6 **Use the interval length**

$3 + \log_{10}(\sin x) = 4 - y$, and the interval from 0.5 to 3 has length 2.5.

7 **Estimate part c**

$\int_{0.5}^{3}(3 + \log_{10}(\sin x))\,dx \approx 4(2.5) - 2.9625\ldots = 7.0374\ldots$, so 7.04.

✓ **FINAL ANSWER**

$(a)\ y(1) = 1.075,\ y(2) = 1.041.$ $(b)\ 2.96.$ $(c)\ 7.04.$

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Question 3

7 marks | Proof

3. (i) Show that the following statement is **false**:

"$(n+1)^3 - n^3$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

WORKED SOLUTION

Question 3 · WMA12/01 May/June 2022

1 Choose a counterexample

Take $n = 5$.

2 Evaluate the expression

$(5+1)^3 - 5^3 = 216 - 125 = 91$.

3 Show it is not prime

$91 = 7 \times 13$, so the statement is false.

4 Find the gradient CA

For $A(1, 0)$ and $C(7, -6)$, the gradient of CA is $\dfrac{0-(-6)}{1-7} = -1$.

5 Find the gradient CB

For $B(3, -10)$ and $C(7, -6)$, the gradient of CB is $\dfrac{-10-(-6)}{3-7} = 1$.

6 Show the right angle

The product of the gradients is -1, so CA is perpendicular to CB and $\angle ACB = 90°$.

7 Conclude the diameter

An angle of $90°$ at the circumference subtends a diameter, so AB is a diameter of the circle.

FINAL ANSWER

(i) Counterexample: $n = 5$ gives 91, not prime.

(ii) Proven: AB is a diameter.

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Question 4

6 marks | Laws of Logarithms

4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

WORKED SOLUTION

Question 4 · WMA12/01 May/June 2022

1 Combine the logarithms

$\log_4 a + \log_4 b = \log_4(ab) = 3$.

2 Find ab

Therefore $ab = 4^3 = 64$.

3 Use a minus b

From $a - b = 8$, $b = a - 8$.

4 Form a quadratic

$a(a - 8) = 64$, so $a^2 - 8a - 64 = 0$.

5 Solve for a

$$a = \frac{8 \pm \sqrt{64 + 256}}{2} = 4 \pm 4\sqrt{5}.$$

6 Choose positive constants

Since $b = a - 8$ must also be positive, take $a = 4 + 4\sqrt{5}$.

7 Find b

$b = a - 8 = 4\sqrt{5} - 4$.

FINAL ANSWER

$a = 4 + 4\sqrt{5}, \quad b = 4\sqrt{5} - 4.$

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Question 5

6 marks | Trigonometric Equations

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

WORKED SOLUTION

Question 5 · WMA12/01 May/June 2022

1 Substitute u

Let $u = \theta + 43°$. The equation becomes $3\tan u = 2\cos u$.

2 Clear the cosine denominator

$3\dfrac{\sin u}{\cos u} = 2\cos u$, so $3\sin u = 2\cos^2 u$.

3 Use cos squared

$3\sin u = 2(1 - \sin^2 u)$, so $2\sin^2 u + 3\sin u - 2 = 0$.

4 Factor the quadratic

$(2\sin u - 1)(\sin u + 2) = 0$.

5 Use the valid sine value

$\sin u = -2$ is impossible, so $\sin u = \dfrac{1}{2}$.

6 Find u in the interval

Since $-180° < \theta \leq 180°$, $-137° < u \leq 223°$. Thus $u = 30°$ or $150°$.

7 Return to theta

$\theta = u - 43°$, so $\theta = -13°$ or $107°$.

FINAL ANSWER

$\theta = -13^\circ,\ 107^\circ.$

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Question 6

8 marks | Geometric Sequences

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

WORKED SOLUTION

Question 6 · WMA12/01 May/June 2022

1 Write the terms

$u_2 = u_1 r$, $u_3 = u_1 r^2$, and $u_4 = u_1 r^3$.

2 Use u four

Since $u_4 = 8$, $u_1 r^3 = 8$, so $u_1 r = \dfrac{8}{r^2}$.

3 Use u two plus u three

$u_2 + u_3 = u_1 r(1 + r) = 6$.

4 Show the quadratic

Substitute $u_1 r = \dfrac{8}{r^2}$ to get $\dfrac{8}{r^2}(1 + r) = 6$. Hence $8 + 8r = 6r^2$, so $3r^2 - 4r - 4 = 0$.

5 Solve for r

$3r^2 - 4r - 4 = 0$ gives $r = 2$ or $r = -\dfrac{2}{3}$.

6 **Choose the sum-to-infinity ratio**

A sum to infinity requires $|r| < 1$, so $r = -\frac{2}{3}$.

7 **Find u one**

$u_1 r^3 = 8$, so $u_1\left(-\frac{2}{3}\right)^3 = 8$ and $u_1 = -27$.

8 **Find the infinite sum**

$$S_\infty = \frac{u_1}{1-r} = \frac{-27}{1+2/3} = -\frac{81}{5}.$$

FINAL ANSWER

(a) Proven: $3r^2 - 4r - 4 = 0$.

(b) $u_1 = -27$. (c) $S_\infty = -\frac{81}{5}$.

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Question 7

7 marks | Integration

7. $\mathrm{f}(x) = Ax^3 + 6x^2 - 4x + B$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $\mathrm{f}(x)$
- $\int_3^5 \mathrm{f}(x)\,\mathrm{d}x = 176$

find the value of A and the value of B.

(7)

WORKED SOLUTION

Question 7 · WMA12/01 May/June 2022

1 Use the factor condition

Since $(x + 2)$ is a factor, $f(-2) = 0$.

2 Form the first equation

$A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$, so $-8A + 32 + B = 0$ and $B = 8A - 32$.

3 Set up the integral

$\int_3^5 (Ax^3 + 6x^2 - 4x + B)\,dx = 176$.

4 Integrate

$$\left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx\right]_3^5 = 176.$$

5 Simplify the second equation

This gives $136A + 164 + 2B = 176$, so $68A + B = 6$.

6 Solve simultaneously

Using $B = 8A - 32$ in $68A + B = 6$ gives $76A = 38$, so $A = \frac{1}{2}$.

7 **Find B**

$$B = 8\left(\frac{1}{2}\right) - 32 = -28.$$

✓ **FINAL ANSWER**

$$A = \frac{1}{2}, \quad B = -28.$$

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Question 8

8 marks | Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

WORKED SOLUTION

Question 8 · WMA12/01 May/June 2022

1 Rewrite with powers

$y = 256x^4 - 304x - 35 + 27x^{-2}$.

2 Differentiate

$\dfrac{dy}{dx} = 1024x^3 - 304 - 54x^{-3} = 1024x^3 - 304 - \dfrac{54}{x^3}$.

3 Set the derivative to zero

$1024x^3 - 304 - \dfrac{54}{x^3} = 0$.

4 Use a substitution

Multiplying by x^3 gives $1024x^6 - 304x^3 - 54 = 0$. Let $u = x^3$.

5 Solve for u

$1024u^2 - 304u - 54 = 0$, or $512u^2 - 152u - 27 = 0 = (8u + 1)(64u - 27)$.

6 Find x

Thus $x^3 = -\dfrac{1}{8}$ or $x^3 = \dfrac{27}{64}$, giving $x = -\dfrac{1}{2}$ or $x = \dfrac{3}{4}$.

7 **Find the y values**

Substitution into $y = 256x^4 - 304x - 35 + \dfrac{27}{x^2}$ gives $y = 241$ at $x = -\dfrac{1}{2}$ and $y = -134$ at $x = \dfrac{3}{4}$.

8 **State the stationary points**

The stationary points are $\left(-\dfrac{1}{2}, 241\right)$ and $\left(\dfrac{3}{4}, -134\right)$.

✓ **FINAL ANSWER**

$(a)\ \dfrac{dy}{dx} = 1024x^3 - 304 - \dfrac{54}{x^3}.$ $\quad (b)\ \left(-\dfrac{1}{2}, 241\right),\ \left(\dfrac{3}{4}, -134\right).$

Question 9

9 marks | Modelling with Sequences & Series

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

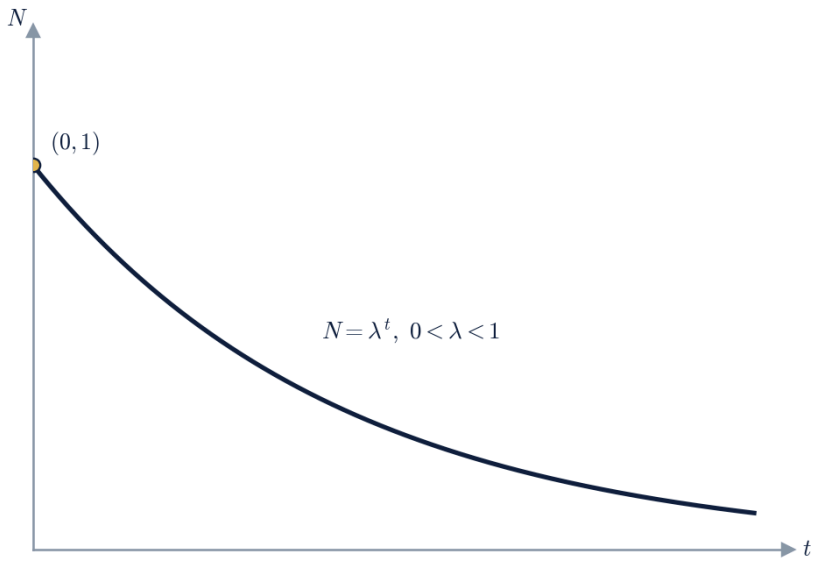
Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

1 **Sketch the model for k equals one**

For $k = 1$, $N = \lambda^t$ with $0 < \lambda < 1$. The graph starts at $(0, 1)$, decreases as t increases, and tends towards 0.



2 **Use the half-life information**

After 5700 years, $N = \dfrac{k}{2}$, so $k\lambda^{5700} = \dfrac{k}{2}$.

3 **Find lambda**

$\lambda^{5700} = \dfrac{1}{2}$, hence $\lambda = \left(\dfrac{1}{2}\right)^{1/5700} = 0.999878\ldots$, so $\lambda = 0.999878$ to 6 decimal places.

4 **Model item A**

For item A, $k = 15$ and $t = 3250$, so $N = 15\lambda^{3250}$.

5 **Calculate item A**

$N = 15(0.9998784026\ldots)^{3250} = 10.103\ldots$, so the current amount is 10.1 g to 3 significant figures.

6 **Set up item B**

For item B, $18 = 25\lambda^t$, so $\lambda^t = \dfrac{18}{25}$.

7 **Take logarithms**

$t \log \lambda = \log\left(\dfrac{18}{25}\right)$, so $t = \dfrac{\log(18/25)}{\log \lambda}$.

8 **Round the age**

Using $\lambda = (1/2)^{1/5700}$ gives $t = 2701\ldots$, so the age is 2700 years to the nearest 100 years.

FINAL ANSWER

(a) Decreasing exponential through $(0, 1)$.

(b) $\lambda = 0.999878$.

(c) $10.1\ \text{g}$. (d) 2700 years.

Question 10

12 marks | Circles

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

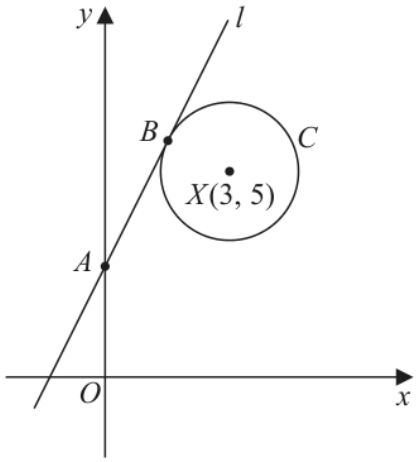


Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total 12 marks)

1 **Write the circle equation**

The circle has centre $(3, 5)$ and radius r, so $(x-3)^2 + (y-5)^2 = r^2$.

2 **Substitute the line**

On the line, $y = 2x + k$. Substitution gives $(x-3)^2 + (2x+k-5)^2 = r^2$.

3 **Expand**

$x^2 - 6x + 9 + 4x^2 + 4kx - 20x + k^2 - 10k + 25 - r^2 = 0$.

4 **Show the intersection equation**

Collecting terms gives $5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$.

5 **Use tangency**

For a tangent, the quadratic has one repeated root, so its discriminant is zero.

6 **Set the discriminant to zero**

$(4k-26)^2 - 20(k^2 - 10k + 34 - r^2) = 0$.

7 **Simplify**

This reduces to $20r^2 - 4k^2 - 8k - 4 = 0$, hence $5r^2 = (k+1)^2$. Therefore $p = 1$.

8 **Find the tangent point x coordinate**

For the repeated root, $x_B = -\dfrac{4k-26}{2 \cdot 5} = \dfrac{13-2k}{5}$.

9 **Use distance AB**

The point A is $(0, k)$ and B lies on a line of gradient 2, so $AB = \sqrt{x_B^2 + (2x_B)^2} = \sqrt{5}\,x_B$.

10 **Use the radius relation**

From $5r^2 = (k+1)^2$, $r = \dfrac{k+1}{\sqrt{5}}$ in the shown configuration.

11 **Apply AB equals two r**

$\sqrt{5}\left(\dfrac{13-2k}{5}\right) = 2\left(\dfrac{k+1}{\sqrt{5}}\right)$.

12 **Solve for k**

Multiplying by $\sqrt{5}$ gives $13 - 2k = 2k + 2$, so $k = \dfrac{11}{4}$.

FINAL ANSWER

(a) Proven.

(b) $5r^2 = (k+1)^2$, so $p = 1$.

(c) $k = \dfrac{11}{4}$.