Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

WMA12/01 January 2022

January 2022

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 JANUARY 2022

Question 1

7 marks | Integration

1. The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

(i) $\int_5^7 2^{6-\sqrt{x}}\,\mathrm{d}x$

(ii) $\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)\mathrm{d}x$

(4)

WORKED SOLUTION

Question 1 · WMA12/01 January 2022

1 Identify the strip width

The x values increase by 0.5, so $h = 0.5$.

2 Apply the trapezium rule

$\int_5^7 2^{5-\sqrt{x}}\,dx \approx \frac{0.5}{2}\{6.792 + 5.113 + 2(6.298 + 5.858 + 5.466)\}$.

3 Calculate the estimate

This gives 11.78725, so the estimate is 11.79 to 2 decimal places.

4 **Use the factor of two**

$2^{6-\sqrt{x}} = 2 \cdot 2^{5-\sqrt{x}}$, so $\int_5^7 2^{6-\sqrt{x}}\,dx \approx 2(11.78725) = 23.5745$.

5 **Estimate part i**

This is 23.57 to 2 decimal places.

6 **Add the constant integral**

$\int_5^7 (3 + 2^{5-\sqrt{x}})\,dx = \int_5^7 3\,dx + \int_5^7 2^{5-\sqrt{x}}\,dx$.

7 **Estimate part ii**

The estimate is $3(7-5) + 11.78725 = 17.78725$, so 17.79 to 2 decimal places.

✓ **FINAL ANSWER**

$(a)\ 11.79.$ $(b)(i)\ 23.57.$ $(b)(ii)\ 17.79.$

WMA12/01 JANUARY 2022

Question 2

8 marks | Applications of Differentiation

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in simplest form.

(2)

(b) Hence find the coordinates of the stationary point of C.

(4)

(c) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of the stationary point of C.

(2)

WORKED SOLUTION

Question 2 · WMA12/01 January 2022

1 **Differentiate**

For $y = 27x^{1/2} - x^{3/2} - 20$, $\frac{dy}{dx} = \frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$.

2 **Set the gradient to zero**

$\frac{27}{2}x^{-1/2} - \frac{3}{2}x^{1/2} = 0$.

3 **Clear the powers**

Multiplying by $2x^{1/2}$ gives $27 - 3x = 0$, so $x = 9$.

4 **Find y**

At $x = 9$, $y = 27\sqrt{9} - 9^{3/2} - 20 = 81 - 27 - 20 = 34$.

5 **State the stationary point**

The stationary point is $(9, 34)$.

6 **Differentiate again**

$\frac{d^2y}{dx^2} = -\frac{27}{4}x^{-3/2} - \frac{3}{4}x^{-1/2}$.

7 **Determine the nature**

At $x = 9$, both terms in $\dfrac{d^2y}{dx^2}$ are negative, so the stationary point is a local maximum.

FINAL ANSWER

$(a)\ \dfrac{dy}{dx} = \dfrac{27}{2}x^{-1/2} - \dfrac{3}{2}x^{1/2}.$ $(b)\ (9, 34).$ (c) local maximum.

WMA12/01 JANUARY 2022

Question 3

7 marks | Binomial Expansion

3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2-\frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form.

(4)

$$f(x)=(5-3x)\left(2-\frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

WORKED SOLUTION

Question 3 · WMA12/01 January 2022

1 Write the first terms

$$(2-\frac{kx}{4})^8=2^8+\binom{8}{1}2^7\left(-\frac{kx}{4}\right)+\binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2+\binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3+\cdots.$$

2 Simplify the expansion

The first four terms are $256-256kx+112k^2x^2-28k^3x^3$.

3 Find the constant term in f

For $f(x)=(5-3x)(2-\frac{kx}{4})^8$, the constant term is $5(256)=1280$.

4 Find the coefficient of x

The coefficient of x is $5(-256k)-3(256)=-1280k-768$.

5 Use the condition

The constant term is 3 times the coefficient of x, so $1280=3(-1280k-768)$.

6 Solve for k

$1280=-3840k-2304$, hence $k=-\dfrac{3584}{3840}=-\dfrac{14}{15}$.

FINAL ANSWER

$(a)\ 256 - 256kx + 112k^2x^2 - 28k^3x^3.$ $\quad (b)\ k = -\dfrac{14}{15}.$

WMA12/01 JANUARY 2022

Question 4

5 marks | Laws of Logarithms

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

WORKED SOLUTION

Question 4 · WMA12/01 January 2022

1 Rewrite the right side

$2\log_3(1-x) = \log_3(1-x)^2$ and $3 = \log_3 27$.

2 Combine the logarithms

The equation becomes $\log_3(32 - 12x) = \log_3\{27(1-x)^2\}$.

3 Equate the arguments

$32 - 12x = 27(1-x)^2$.

4 Form the quadratic

$32 - 12x = 27 - 54x + 27x^2$, so $27x^2 - 42x - 5 = 0$.

5 Solve

$x = \dfrac{42 \pm 48}{54}$, giving $x = \dfrac{5}{3}$ or $x = -\dfrac{1}{9}$.

6 Check the domain

The original logarithms require $1 - x > 0$, so $x < 1$. Therefore $x = \dfrac{5}{3}$ is invalid and $x = -\dfrac{1}{9}$.

FINAL ANSWER

$x = -\dfrac{1}{9}$.

WMA12/01 JANUARY 2022

Question 5

8 marks | Polynomials

5. $f(x) = 3x^3 + Ax^2 + Bx - 10$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)g(x)$$

(2)

WORKED SOLUTION

Question 5 · WMA12/01 January 2022

1 Use x equals one

$f(1) = 3 + A + B - 10 = A + B - 7$, so $k = A + B - 7$.

2 Use x equals negative one

$f(-1) = -3 + A - B - 10 = A - B - 13$.

3 Use the second remainder relation

Since $f(-1) = -10k$, $A - B - 13 = -10(A + B - 7)$.

4 Show the relation

Rearranging gives $11A + 9B = 83$.

5 **Use the factor**

Since $(3x-2)$ is a factor, $f\left(\frac{2}{3}\right)=0$.

6 **Form the second equation**

$3\left(\frac{2}{3}\right)^3+A\left(\frac{2}{3}\right)^2+B\left(\frac{2}{3}\right)-10=0$. Multiplying by 9 gives $2A+3B=41$.

7 **Solve for A and B**

Solving $11A+9B=83$ with $2A+3B=41$ gives $A=-8$ and $B=19$.

8 **Divide by the factor**

With these values, $f(x)=3x^3-8x^2+19x-10=(3x-2)(x^2-2x+5)$.

✓ **FINAL ANSWER**

(a) Proven: $11A+9B=83$.

(b) $A=-8,\ B=19$. (c) $g(x)=x^2-2x+5$.

WMA12/01 JANUARY 2022

Question 6

8 marks | Circles

6.

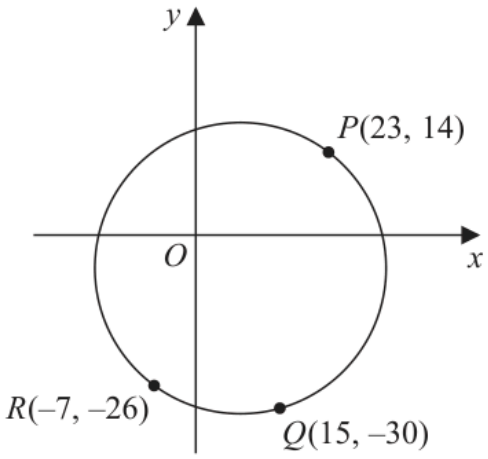


Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

WORKED SOLUTION

Question 6 · WMA12/01 January 2022

1 Find the gradient QP

The gradient of QP is $\dfrac{14-(-30)}{23-15} = \dfrac{44}{8} = \dfrac{11}{2}$.

2 Find the gradient QR

The gradient of QR is $\dfrac{-26-(-30)}{-7-15} = \dfrac{4}{-22} = -\dfrac{2}{11}$.

3 **Show the right angle**

The product of the two gradients is -1, so QP is perpendicular to QR and $\angle PQR = 90°$.

4 **Use the diameter**

Since the angle at Q is a right angle, PR is a diameter of the circle.

5 **Find the centre**

The centre is the midpoint of $P(23, 14)$ and $R(-7, -26)$, so it is $(8, -6)$.

6 **Find the radius**

$PR = \sqrt{30^2 + 40^2} = 50$, so the radius is 25.

7 **Find the farthest point from Q**

The greatest distance from Q occurs at the point opposite Q on the diameter through the centre. Thus $S = 2(8, -6) - (15, -30) = (1, 18)$.

8 **Find the tangent gradient**

The radius from the centre to S has gradient $\dfrac{18 - (-6)}{1 - 8} = -\dfrac{24}{7}$, so the tangent gradient is $\dfrac{7}{24}$.

9 **Write the tangent equation**

$y - 18 = \dfrac{7}{24}(x - 1)$, which rearranges to $7x - 24y + 425 = 0$.

✓ **FINAL ANSWER**

(a) Proven: $\angle PQR = 90°$.

$(b)(i)$ $(8, -6)$. $(b)(ii)$ 25. (c) $7x - 24y + 425 = 0$.

WMA12/01 JANUARY 2022

Question 7

8 marks | Trigonometric Equations

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

WORKED SOLUTION

Question 7 · WMA12/01 January 2022

1 **Solve the first tangent equation**

Let $u = 2x - 15°$. Then $3\sin u = \cos u$, so $\tan u = \frac{1}{3}$.

2 **Use the interval for u**

Since $-90° < x < 90°$, $-195° < u < 165°$.

3 **Find x**

$u = 18.434\ldots°$ or $u = -161.565\ldots°$, so $x = 16.7°$ or $x = -73.3°$.

4 **Rewrite the second equation**

$4\sin^2\theta + 8\cos\theta = 3$.

5 **Use sin squared**

Substitute $\sin^2\theta = 1 - \cos^2\theta$ to get $4(1 - \cos^2\theta) + 8\cos\theta = 3$.

6 **Solve for cos theta**

This gives $4\cos^2\theta - 8\cos\theta - 1 = 0$, so $\cos\theta = 1 \pm \frac{\sqrt{5}}{2}$.

7 **Choose the valid cosine**

Only $\cos\theta = 1 - \frac{\sqrt{5}}{2}$ lies between -1 and 1.

8 **State theta**

For $0 < \theta < 2\pi$, $\theta = 1.69$ or 4.59 to 3 significant figures.

FINAL ANSWER

$(i)\ x = -73.3°,\ 16.7°.\quad (ii)\ \theta = 1.69,\ 4.59.$

WMA12/01 JANUARY 2022

Question 8

9 marks | Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

WORKED SOLUTION

Question 8 · WMA12/01 January 2022

1 **Use the arithmetic rule**

For the first 20 hits, the additional distances form an arithmetic sequence with first term 100 and common difference -2.

2 **Show the twentieth hit**

The twentieth additional distance is $100 + 19(-2) = 62$ mm.

3 **Sum the first twenty hits**

$S_{20} = \frac{20}{2}(100 + 62) = 1620$ mm.

4 **Use the geometric information**

After the twentieth hit, the distances follow the same ratio, so the twenty second distance is $62r^2$.

5 **Find r**

$62r^2 = 60$, hence $r = \sqrt{\dfrac{30}{31}} = 0.984$ to 3 decimal places.

6 **Set up the remaining distance**

More than 3 m means more than 3000 mm. After 20 hits, the remaining amount to exceed is $3000 - 1620 = 1380$ mm.

7 **Sum the geometric tail**

If there are m further hits, their total is $62r + 62r^2 + \cdots + 62r^m = 62r\dfrac{1 - r^m}{1 - r}$.

8 **Find the first m that works**

Using $r = \sqrt{30/31}$, $62r\dfrac{1 - r^{27}}{1 - r}$ is not enough, but $62r\dfrac{1 - r^{28}}{1 - r}$ makes the total exceed 3000 mm.

9 **State N**

Therefore $m = 28$ further hits are needed after the first 20, so $N = 20 + 28 = 48$.

✓ **FINAL ANSWER**

(a) Proven: 62 mm.

(b) 1620 mm.

(c) $r = 0.984$. (d) $N = 48$.

Question 9

10 marks | Integration

9.

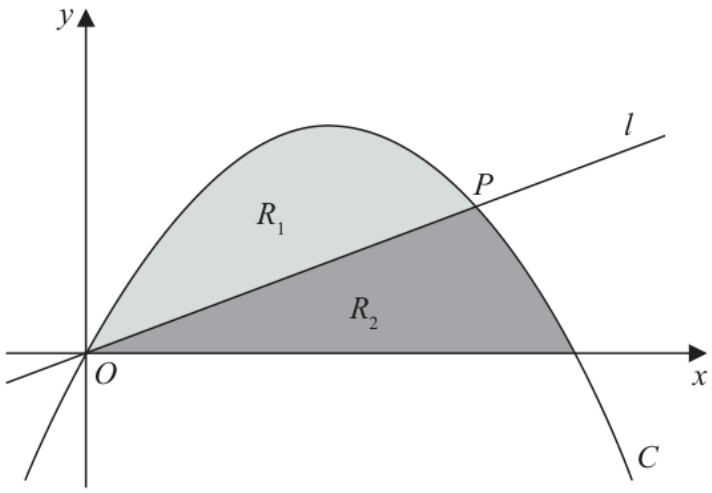


Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

1 Find the second intersection

At an intersection, $x - x^2 = mx$, so $x(1 - m - x) = 0$.

2 State P

The non-zero intersection has $x = 1 - m$, and then $y = m(1 - m)$, so $P = (1 - m, m(1 - m))$.

3 Set up area R one

Region R_1 is between the curve and the line from $x = 0$ to $x = 1 - m$.

4 Integrate the difference

$A_{R_1} = \int_0^{1-m} \{(x - x^2) - mx\}\,dx = \int_0^{1-m} \{(1 - m)x - x^2\}\,dx$.

5 Evaluate

$$A_{R_1} = \left[\frac{1 - m}{2}x^2 - \frac{1}{3}x^3\right]_0^{1-m} = \frac{(1 - m)^3}{2} - \frac{(1 - m)^3}{3} = \frac{(1 - m)^3}{6}.$$

6 Use the total curve area

The total area under $y = x - x^2$ from $x = 0$ to $x = 1$ is $\int_0^1 (x - x^2)\,dx = \frac{1}{6}$.

7 Use equal regions

Since R_1 and R_2 are equal and together make this total area, each has area $\frac{1}{12}$.

8 Solve for m

$\frac{(1 - m)^3}{6} = \frac{1}{12}$, so $(1 - m)^3 = \frac{1}{2}$ and $m = 1 - \frac{1}{\sqrt[3]{2}}$.

FINAL ANSWER

(a) $P = (1 - m,\ m(1 - m))$. (b) Proven: $\frac{(1 - m)^3}{6}$.

(c) $m = 1 - \frac{1}{\sqrt[3]{2}}$.

WMA12/01 JANUARY 2022

Question 10

5 marks | Proof

10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

(Total 5 marks)

WORKED SOLUTION

Question 10 · WMA12/01 January 2022

1 Give a counterexample

Take $p = 7$, which is prime.

2 Show the statement fails

$2p + 1 = 2(7) + 1 = 15$, and 15 is not prime. So the statement is false.

3 Case one: n even

If n is even, let $n = 2k$. Then

$5n^2 + n + 12 = 5(2k)^2 + 2k + 12 = 20k^2 + 2k + 12 = 2(10k^2 + k + 6)$, which is even.

4 Case two: n odd

If n is odd, let $n = 2k + 1$. Then $5n^2 + n + 12 = 5(2k + 1)^2 + (2k + 1) + 12$.

5 Simplify the odd case

$5(2k + 1)^2 + (2k + 1) + 12 = 20k^2 + 22k + 18 = 2(10k^2 + 11k + 9)$, which is even.

6 Conclude by exhaustion

Every integer is either even or odd, and both cases give an even result. Therefore $5n^2 + n + 12$ is always even.

FINAL ANSWER

(i) For $p = 7$, $2p + 1 = 15$, not prime.

(ii) Proven.