Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 May/June 2021 Worked Solutions

May/June 2021 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 May/June 2021

May/June 2021

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 MAY/JUNE 2021

Question 1

7 marks | Modelling with Sequences & Series

1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

WORKED SOLUTION

Question 1 · WMA12/01 May/June 2021

1 Choose the model

The weekly amounts are 5, 5.25, 5.50, and so on. The increase is a constant 0.25, so an arithmetic series should be used.

2 Write the nth amount

The first amount is 5 and the common difference is 0.25, so the amount saved in week n is $5 + 0.25(n-1)$.

3 Set up the total

After n weeks, $S_n = \frac{n}{2}\{2(5) + (n-1)(0.25)\}$.

4 Form the inequality

Adina needs $S_n \geq 350$, so $\frac{n}{2}(9.75 + 0.25n) \geq 350$.

5 **Solve for n**

This is $n^2 + 39n - 2800 \geq 0$. The positive root is $36.89\ldots$, so the first whole number that works is $n = 37$.

6 **Check the week**

After 36 weeks the total is below 350, and after 37 weeks it is 351.50, so it takes 37 weeks.

✓ **FINAL ANSWER**

$(a)(i)$ Arithmetic series.

$(a)(ii)$ $5 + 0.25(n - 1)$.

(b) 37 weeks.

Question 2

8 marks | Laws of Logarithms

2.

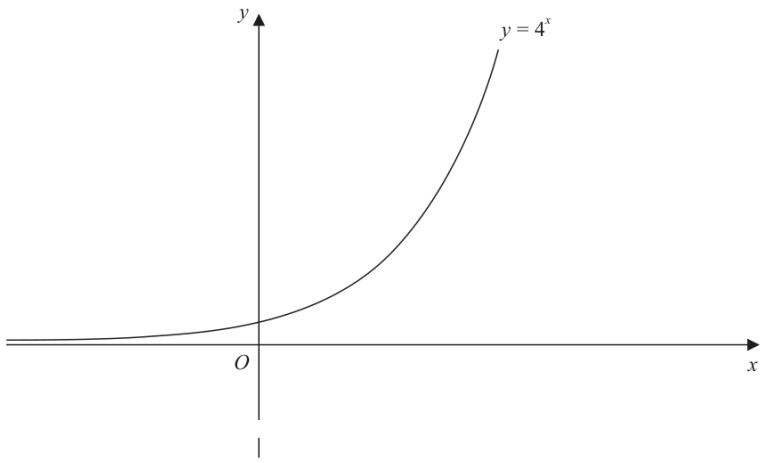


Figure 1

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P.

(b) Using algebra, find the exact coordinates of P.

(4)

Question 2 continued

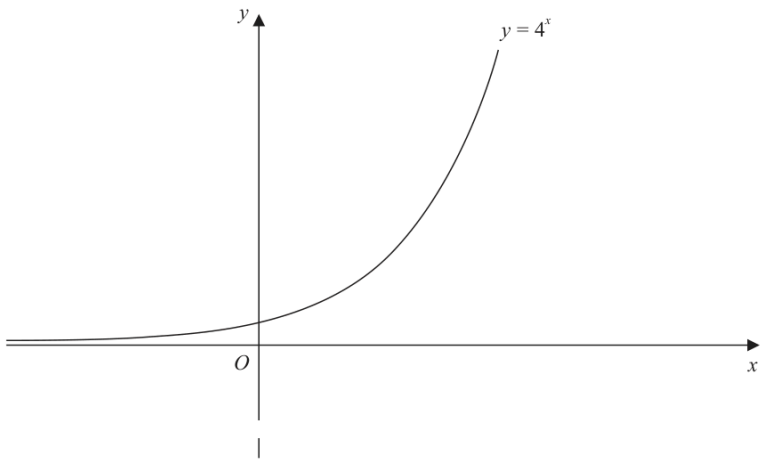


Diagram 1

1 **Describe y equals two to the x**

$y = 2^x$ is an increasing exponential curve. It crosses the y-axis at $(0, 1)$ and has no x-axis intercept.

2 **Describe y equals four to the x minus six**

$y = 4^x - 6$ is the graph of $y = 4^x$ translated 6 units down. It crosses the y-axis at $(0, -5)$.

3 **Find the x-axis intercept**

For $y = 4^x - 6$, set $4^x - 6 = 0$. Hence $4^x = 6$ and $x = \log_4 6$.

4 **Set up the intersection equation**

At P, $2^x = 4^x - 6$.

5 **Use a substitution**

Let $u = 2^x$. Then $4^x = (2^x)^2 = u^2$, so $u = u^2 - 6$.

6 **Solve for u**

$u^2 - u - 6 = 0$, so $(u - 3)(u + 2) = 0$. Since $u = 2^x > 0$, $u = 3$.

7 **State P**

$2^x = 3$, so $x = \log_2 3$, and $y = 3$. Therefore $P = (\log_2 3, 3)$.

✓ **FINAL ANSWER**

$(a)(i)$ Sketch $y = 2^x$ through $(0, 1)$ with no x-axis intercept.

$(a)(ii)$ Sketch $y = 4^x - 6$ through $(0, -5)$ and $(\log_4 6, 0)$.

(b) $P = (\log_2 3, 3)$.

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Question 3

5 marks | Proof

3. (i) Prove that for all single digit prime numbers, p,

$$p^3 + p \text{ is a multiple of } 10$$

(2)

(ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n + 1)^3 - n^3 \text{ is not a multiple of } 3$$

(3)

WORKED SOLUTION

Question 3 · WMA12/01 May/June 2021

1 List the cases

The single digit prime numbers are 2, 3, 5, and 7.

2 Check each prime

$2^3 + 2 = 10$, $3^3 + 3 = 30$, $5^3 + 5 = 130$, and $7^3 + 7 = 350$.

3 Conclude part one

Each result is a multiple of 10, so $p^3 + p$ is a multiple of 10 for all single digit primes.

4 Expand the difference of cubes

$(n+1)^3 - n^3 = (n^3 + 3n^2 + 3n + 1) - n^3 = 3n^2 + 3n + 1$.

5 Factor the multiple of three

$3n^2 + 3n + 1 = 3n(n+1) + 1$.

6 Use the remainder

$3n(n+1)$ is a multiple of 3, so $3n(n+1) + 1$ is one more than a multiple of 3. Therefore it is not a multiple of 3.

FINAL ANSWER

(i) Proven by checking $p = 2, 3, 5, 7$.

(ii) Proven: $(n+1)^3 - n^3$ leaves remainder 1 on division by 3.

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Question 4

8 marks | Binomial Expansion

4. (a) Find, in ascending powers of x, up to and including the term in x^3, the binomial expansion of

$$\left(2+\frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

(b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

(c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

WORKED SOLUTION

Question 4 · WMA12/01 May/June 2021

1 Write the binomial terms

$(2+\frac{x}{8})^{13} = 2^{13} + \binom{13}{1}2^{12}\frac{x}{8} + \binom{13}{2}2^{11}\left(\frac{x}{8}\right)^2 + \binom{13}{3}2^{10}\left(\frac{x}{8}\right)^3 + \cdots$.

2 Simplify the expansion

This gives $8192 + 6656x + 2496x^2 + 572x^3 + \cdots$.

3 Match the value

Since $2.0125 = 2 + 0.0125 = 2 + \dfrac{0.1}{8}$, use $x = 0.1$ in the expansion.

4 Calculate the approximation

$8192 + 6656(0.1) + 2496(0.1)^2 + 572(0.1)^3 = 8883.132$.

5 Decide the direction

The next terms in the expansion are positive because $x = 0.1 > 0$, so stopping after the x^3 term gives an underestimate.

FINAL ANSWER

$(a)\ 8192 + 6656x + 2496x^2 + 572x^3.$ $(b)\ 8883.132.$ (c) Underestimate.

Question 5

10 marks | Integration

5.

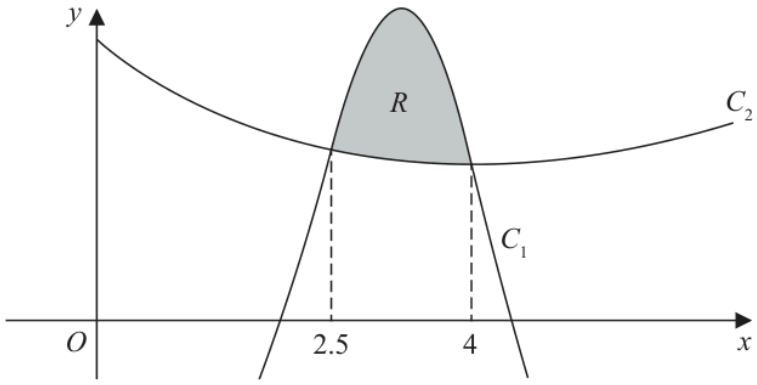


Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\int\left(x^{\frac{3}{2}} - 3x + 9\right)\mathrm{d}x$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

1 Identify the strip width

The x values increase by 0.25, so $h = 0.25$.

2 Apply the trapezium rule

The area under C_1 is approximately
$\frac{0.25}{2}\{5.453 + 5 + 2(7.764 + 9.375 + 9.964 + 9.367 + 7.626)\}$.

3 Calculate the estimate

This gives 12.330625, so the estimate is 12.33 to 2 decimal places.

4 Integrate C two

$\int (x^{3/2} - 3x + 9)\,dx = \frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x + c$.

5 Find the area under C two

Between $x = 2.5$ and $x = 4$, the exact integral of C_2 is $\left[\frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x\right]_{2.5}^{4} = 7.722\ldots$.

6 Find the shaded area

The shaded region is between C_1 and C_2, so its area is approximately
$12.330625 - 7.722\ldots = 4.608\ldots$.

7 Round the result

To one decimal place, the area of region R is 4.6.

FINAL ANSWER

(a) 12.33. (b) $\frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x + c$. (c) 4.6.

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Question 6

7 marks | Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

WORKED SOLUTION

Question 6 · WMA12/01 May/June 2021

1 Complete the square

$x^2 - 6x = (x-3)^2 - 9$ and $y^2 + 8y = (y+4)^2 - 16$.

2 Write centre-radius form

The circle becomes $(x-3)^2 + (y+4)^2 = 25 - k$.

3 Use the tangent condition

The centre is $(3, -4)$. Since the x-axis is tangent, the radius is the distance from the centre to the x-axis, which is 4.

4 Find k

Thus $25 - k = 4^2 = 16$, so $k = 9$.

5 Find the intercepts

With $k = 9$, the circle is $(x-3)^2 + (y+4)^2 = 16$. On the x-axis, $y = 0$, giving $x = 3$.

6 Find the y-axis points

On the y-axis, $x = 0$, so $9 + (y+4)^2 = 16$. Hence $y = -4 \pm \sqrt{7}$.

7 **Calculate the triangle area**

The vertical base on the y-axis has length $2\sqrt{7}$, and its perpendicular distance to $(3,0)$ is 3. The area is $\frac{1}{2}(2\sqrt{7})(3) = 3\sqrt{7}$.

✓ **FINAL ANSWER**

$(a)\ k = 9.\quad (b)\ 3\sqrt{7}.$

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Question 7

10 marks | Laws of Logarithms

7. (a) Given that

$$3\log_3(2x-1) = 2 + \log_3(14x-25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x-1) = 2 + \log_3(14x-25)$$

(4)

WORKED SOLUTION

Question 7 · WMA12/01 May/June 2021

1 Use the power law

$3\log_3(2x-1) = \log_3(2x-1)^3$.

2 Rewrite two as a logarithm

Since $2 = \log_3 9$, the right side is $\log_3 9 + \log_3(14x-25) = \log_3\{9(14x-25)\}$.

3 Equate the arguments

So $(2x-1)^3 = 9(14x-25)$.

4 Expand and simplify

$8x^3 - 12x^2 + 6x - 1 = 126x - 225$, which simplifies to $2x^3 - 3x^2 - 30x + 56 = 0$.

5 Check the given root

Substituting $x = -4$ gives $2(-64) - 3(16) - 30(-4) + 56 = 0$, so -4 is a root.

6 Factorise the cubic

Dividing by $(x+4)$ gives $2x^2 - 11x + 14 = (2x-7)(x-2)$.

7 **Apply the logarithm domain**

The possible roots are -4, 2, and $\frac{7}{2}$. The original logarithms require $2x-1>0$ and $14x-25>0$, so $x>\frac{25}{14}$.

8 **State the valid solutions**

Therefore the solutions are $x=2$ and $x=\frac{7}{2}$.

✓ **FINAL ANSWER**

(a) Proven.

(b) -4 is a root of the cubic.

(c) $x=2,\ \frac{7}{2}$.

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Question 8

10 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

WORKED SOLUTION

Question 8 · WMA12/01 May/June 2021

1 **Solve for theta plus thirty**

Let $u = \theta + 30°$. Then $3\sin u = 7\cos u$, so $\tan u = \frac{7}{3}$.

2 **Find u in the interval**

Since $0 < \theta < 360°$, $30° < u < 390°$. Thus $u = 66.801\ldots°$ or $246.801\ldots°$.

3 **Return to theta**

$\theta = u - 30°$, giving $\theta = 36.8°$ and $216.8°$ to one decimal place.

4 **Rearrange the second equation**

$3\sin^3 x = 5\sin x - 7\sin x\cos x$ gives $3\sin^3 x - 5\sin x + 7\sin x\cos x = 0$.

5 **Factor out sine**

$\sin x(3\sin^2 x - 5 + 7\cos x) = 0$.

6 **Use sin squared**

Since $\sin^2 x = 1 - \cos^2 x$, this becomes $\sin x(-3\cos^2 x + 7\cos x - 2) = 0$, or $\sin x(3\cos^2 x - 7\cos x + 2) = 0$.

7 **Solve in the interval**

For $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $\sin x = 0$ gives $x = 0$. Also $3\cos^2 x - 7\cos x + 2 = 0$ gives $(3\cos x - 1)(\cos x - 2) = 0$.

8 **State the solutions**

$\cos x = 2$ is impossible, so $\cos x = \frac{1}{3}$. The solutions are $x = -\cos^{-1}\left(\frac{1}{3}\right)$, 0, and $\cos^{-1}\left(\frac{1}{3}\right)$.

✓ **FINAL ANSWER**

$(i)\ \theta = 36.8°,\ 216.8°.\quad (ii)(a)\ a = 3,\ b = -7,\ c = 2.\quad (ii)(b)\ x = -\cos^{-1}\left(\frac{1}{3}\right),\ 0,\ \cos^{-1}\left(\frac{1}{3}\right)$

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Question 9

10 marks | Applications of Differentiation

9.

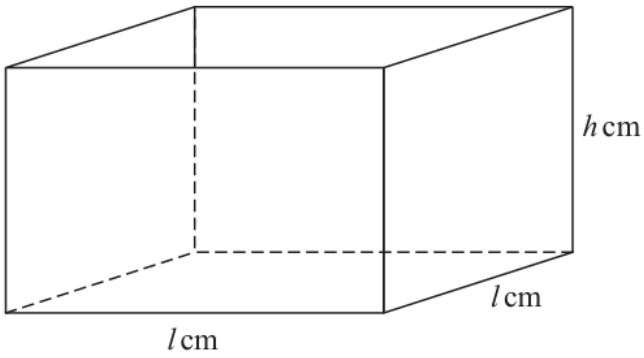


Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, S cm^2, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

WORKED SOLUTION

Question 9 · WMA12/01 May/June 2021

1 Use the volume

The volume is $l^2h = 250000$, so $l^2 = \dfrac{250000}{h}$.

2 Write the surface area

The open box has one square base and four side faces, so $S = l^2 + 4lh$.

3 **Substitute l**

Since $l = \sqrt{\dfrac{250000}{h}} = \dfrac{500}{\sqrt{h}}$, $4lh = 4 \cdot \dfrac{500}{\sqrt{h}} \cdot h = 2000\sqrt{h}$.

4 **Show S**

Therefore $S = \dfrac{250000}{h} + 2000\sqrt{h}$.

5 **Differentiate S**

$S = 250000h^{-1} + 2000h^{1/2}$, so $\dfrac{dS}{dh} = -250000h^{-2} + 1000h^{-1/2}$.

6 **Set the derivative to zero**

$-250000h^{-2} + 1000h^{-1/2} = 0$, so $1000h^{-1/2} = 250000h^{-2}$.

7 **Solve for h**

Multiplying by h^2 gives $1000h^{3/2} = 250000$, so $h^{3/2} = 250$ and $h = 250^{2/3}$.

8 **Find k**

Since $h = 250^k$, we have $k = \dfrac{2}{3}$.

9 **Use the second derivative**

$\dfrac{d^2S}{dh^2} = 500000h^{-3} - 500h^{-3/2}$.

10 **Check the sign**

At $h = 250^{2/3}$, $h^{3/2} = 250$, so the second derivative is $\dfrac{500000}{250^2} - \dfrac{500}{250} = 8 - 2 = 6 > 0$.

11 **Conclude**

The stationary point is therefore a minimum.

✓ **FINAL ANSWER**

(a) Proven: $S = \dfrac{250000}{h} + 2000\sqrt{h}$.

(b) $h = 250^{2/3}$, so $k = \dfrac{2}{3}$.

(c) Minimum.