Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 January 2021 Worked Solutions

January 2021 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 January 2021

January 2021

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 JANUARY 2021

Question 1

6 marks | Polynomials

1. $$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

WORKED SOLUTION

Question 1 · WMA12/01 January 2021

1 Use the first remainder

Dividing by $(x+1)$ means use $x = -1$. Hence $f(-1) = 1 - a - 3 - b + 5 = 3 - a - b$.

2 Show the required relation

The remainder is 4, so $3 - a - b = 4$. Therefore $a + b = -1$.

3 Use the second remainder

Dividing by $(x-2)$ means use $x = 2$. Hence $f(2) = 16 + 8a - 12 + 2b + 5 = 9 + 8a + 2b$.

4 Form the second equation

The remainder is -23, so $9 + 8a + 2b = -23$, which gives $4a + b = -16$.

5 Solve simultaneously

Subtract $a + b = -1$ from $4a + b = -16$ to get $3a = -15$, so $a = -5$.

6 Find b

Using $a + b = -1$, $-5 + b = -1$, hence $b = 4$.

FINAL ANSWER

(a) Proven: $a + b = -1$.

(b) $a = -5,\ b = 4$.

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Question 2

7 marks | Applications of Differentiation

2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

WORKED SOLUTION

Question 2 · WMA12/01 January 2021

1 Differentiate the curve

For $y = x^3 - x^2 - 16x + 2$, $\dfrac{dy}{dx} = 3x^2 - 2x - 16$.

2 Set the gradient to zero

Stationary points occur when $3x^2 - 2x - 16 = 0$.

3 Factorise

$3x^2 - 2x - 16 = (3x - 8)(x + 2)$, so $x = \dfrac{8}{3}$ or $x = -2$.

4 Differentiate again

$\dfrac{d^2y}{dx^2} = 6x - 2$.

5 Classify x equals negative two

At $x = -2$, $6x - 2 = -14 < 0$, so the stationary point is a local maximum.

6 Classify x equals eight over three

At $x = \dfrac{8}{3}$, $6x - 2 = 14 > 0$, so the stationary point is a local minimum.

FINAL ANSWER

(a) $x = -2,\ \dfrac{8}{3}$. (b) local maximum at $x = -2$ and local minimum at $x = \dfrac{8}{3}$.

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Question 3

8 marks | Laws of Logarithms

3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

WORKED SOLUTION

Question 3 · WMA12/01 January 2021

1 Rewrite the exponential equation

From $7^{x+2} = 3$, divide by 7^2 to get $7^x = \frac{3}{49}$.

2 Write x as a logarithm

Therefore $x = \log_7\left(\frac{3}{49}\right)$.

3 Convert one to a logarithm

Since $1 = \log_2 2$, the equation becomes $\log_2 2 + \log_2 y + \log_2(y+4) = \log_2(5-y)$.

4 Combine the logarithms

$\log_2\{2y(y+4)\} = \log_2(5-y)$.

5 Solve the quadratic

Thus $2y(y+4) = 5 - y$, so $2y^2 + 9y - 5 = 0 = (2y-1)(y+5)$.

6 Check the domain

The original logarithms require $y > 0$ and $y < 5$, so $y = -5$ is invalid and $y = \frac{1}{2}$.

FINAL ANSWER

$(i)\ x = \log_7\left(\frac{3}{49}\right).\quad (ii)\ y = \frac{1}{2}.$

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Question 4

8 marks | Binomial Expansion

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

(b) find the possible values of p.

(4)

WORKED SOLUTION

Question 4 · WMA12/01 January 2021

1 Write the first binomial terms

$(2+px)^6 = 2^6 + \binom{6}{1}2^5(px) + \binom{6}{2}2^4(px)^2 + \cdots$.

2 Simplify the expansion

The first three terms are $64 + 192px + 240p^2x^2$.

3 Find the coefficient of x squared

In $(3 - \frac{1}{2}x)(2+px)^6$, the x^2 coefficient is $3(240p^2) - \frac{1}{2}(192p)$.

4 Form the equation

So $720p^2 - 96p = -\frac{3}{4}$.

5 Solve the quadratic

Multiplying by 4 gives $2880p^2 - 384p + 3 = 0$, or $960p^2 - 128p + 1 = 0$.

6 State the values of p

$p = \frac{128 \pm 112}{1920}$, so $p = \frac{1}{8}$ or $p = \frac{1}{120}$.

FINAL ANSWER

$(a)\ 64 + 192px + 240p^2x^2.$ $\quad (b)\ p = \dfrac{1}{8}$ or $p = \dfrac{1}{120}.$

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Question 5

4 marks | Proof

5. (i) Use algebra to prove that for all $x \geqslant 0$

$$3x + 1 \geqslant 2\sqrt{3x}$$

(3)

(ii) Show that the following statement is not true.

"The sum of three consecutive prime numbers is always a multiple of 5"

(1)

WORKED SOLUTION

Question 5 · WMA12/01 January 2021

1 Start from a square

For $x \geq 0$, $\sqrt{3x}$ is real and $(\sqrt{3x} - 1)^2 \geq 0$.

2 Expand the square

$(\sqrt{3x} - 1)^2 = 3x - 2\sqrt{3x} + 1 \geq 0$.

3 Rearrange the inequality

Therefore $3x + 1 \geq 2\sqrt{3x}$, as required.

4 Give a counterexample

The consecutive prime numbers 5, 7, and 11 have sum $5 + 7 + 11 = 23$.

5 Complete the disproof

23 is not a multiple of 5, so the statement is not true.

FINAL ANSWER

(i) Proven.

(ii) For example, $5 + 7 + 11 = 23$, which is not a multiple of 5.

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Question 6

8 marks | Trigonometric Equations

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

WORKED SOLUTION

Question 6 · WMA12/01 January 2021

1 Clear the fraction

Starting with $\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta$, multiply by $(2\sin\theta - 1)\cos\theta$.

2 Use tan theta

This gives $3\sin\theta\cos^2\theta = 5\sin\theta(2\sin\theta - 1)$.

3 Use cos squared

Substitute $\cos^2\theta = 1 - \sin^2\theta$: $3\sin\theta(1 - \sin^2\theta) = 10\sin^2\theta - 5\sin\theta$.

4 Collect terms

Rearranging gives $3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$.

5 Factor the sine equation

$3s^3 + 10s^2 - 8s = 0$ gives $s(3s - 2)(s + 4) = 0$, where $s = \sin 2x$.

6 **Use the valid sine values**

Since $-\frac{\pi}{2} < 2x < \frac{\pi}{2}$, the valid values are $\sin 2x = 0$ and $\sin 2x = \frac{2}{3}$.

7 **Find x**

$2x = 0$ gives $x = 0$. Also $2x = \sin^{-1}\left(\frac{2}{3}\right)$, so $x = 0.364863\ldots = 0.365$ to 3 decimal places.

✓ **FINAL ANSWER**

(a) Proven.

(b) $x = 0,\ 0.365$.

Question 7

6 marks | Integration

7.

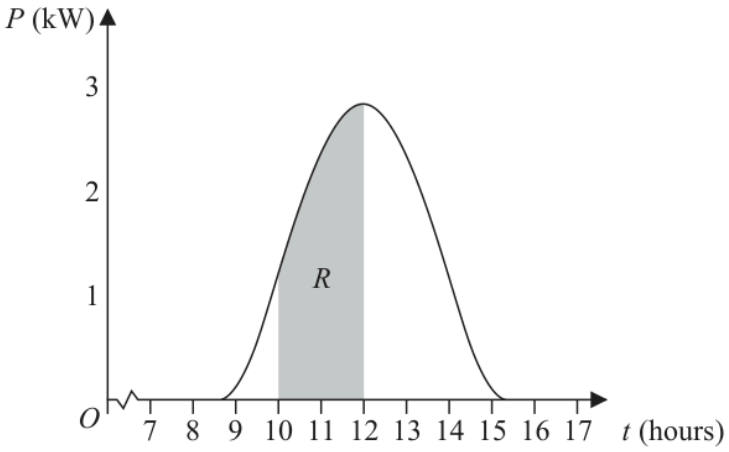


Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

1 **Use the model at t equals ten**

$P(10) = 0.95 + 2^{-2} + 2^{2} - (10 - 12)^{2} = 0.95 + 0.25 + 4 - 4 = 1.20$.

2 **Use the model at t equals eleven point five**

$P(11.5) = 0.95 + 2^{-0.5} + 2^{0.5} - (11.5 - 12)^{2} = 2.821\ldots$.

3 **Identify the trapezium width**

The time values increase by 0.5, so $h = 0.5$.

4 **Apply the trapezium rule**

The estimate is $\frac{0.5}{2}\{1.20 + 2.95 + 2(1.882 + 2.45 + 2.821\ldots)\}$.

5 **Calculate the energy**

This gives $4.614\ldots$, so the estimated energy is $4.61\ \text{kWh}$ to 2 decimal places.

✓ **FINAL ANSWER**

$(a)\ P(10) = 1.20,\ P(11.5) = 2.821.\quad (b)\ 4.61\ \text{kWh}$.

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Question 8

7 marks | Sequences & Series

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

WORKED SOLUTION

Question 8 · WMA12/01 January 2021

1 Find a two

Since $a_1 = p - 3$, we have $a_1 + 3 = p$. Therefore $a_2 = 2(a_1 + 3)^2 - 7 = 2p^2 - 7$.

2 Find a three

$a_2 + 3 = 2p^2 - 4$, so $a_3 = 2(2p^2 - 4)^2 - 7 = 8(p^2 - 2)^2 - 7$.

3 Use the given sum

$a_1 + a_2 + a_3 = p + 15$.

4 Substitute the expressions

$(p - 3) + (2p^2 - 7) + 8(p^2 - 2)^2 - 7 = p + 15$.

5 Simplify

Cancelling p and expanding gives $8p^4 - 30p^2 = 0$, so $2p^2(4p^2 - 15) = 0$.

6 Find the possible a two values

If $p = 0$, then $a_2 = -7$. If $p^2 = \frac{15}{4}$, then $a_2 = 2 \cdot \frac{15}{4} - 7 = \frac{1}{2}$.

FINAL ANSWER

$(a)\ a_2 = 2p^2 - 7. \quad (b)\ a_2 = -7 \text{ or } \dfrac{1}{2}.$

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Question 9

10 marks | Circles

9. A circle C has equation

$$(x-k)^2+(y-2k)^2=k+7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2,3)$ lies on C

(b) (i) show that $5k^2-17k+6=0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k<2$

(c) calculate the exact area of triangle OPT.

(5)

WORKED SOLUTION

Question 9 · WMA12/01 January 2021

1 Read the centre and radius

From $(x-k)^2+(y-2k)^2=k+7$, the centre is $(k,2k)$ and the radius is $\sqrt{k+7}$.

2 Substitute point P

Since $P(2,3)$ lies on the circle, $(2-k)^2+(3-2k)^2=k+7$.

3 Show the quadratic

Expanding gives $k^2-4k+4+4k^2-12k+9=k+7$, hence $5k^2-17k+6=0$.

4 **Solve for k**

$5k^2 - 17k + 6 = (5k - 2)(k - 3)$, so $k = \dfrac{2}{5}$ or $k = 3$.

5 **Use the condition k less than two**

For the tangent calculation, $k < 2$, so the centre is $\left(\dfrac{2}{5}, \dfrac{4}{5}\right)$.

6 **Find the tangent gradient**

The radius from the centre to P has gradient $\dfrac{3 - 4/5}{2 - 2/5} = \dfrac{11}{8}$, so the tangent gradient is $-\dfrac{8}{11}$.

7 **Find the x intercept**

The tangent is $y - 3 = -\dfrac{8}{11}(x - 2)$. Setting $y = 0$ gives $x = \dfrac{49}{8}$, so $T = \left(\dfrac{49}{8}, 0\right)$.

8 **Calculate the area**

Triangle OPT has base $\dfrac{49}{8}$ on the x-axis and height 3, so its area is $\dfrac{1}{2} \cdot \dfrac{49}{8} \cdot 3 = \dfrac{147}{16}$.

✓ **FINAL ANSWER**

$(a)(i)\ (k, 2k).$ $(a)(ii)\ \sqrt{k + 7}.$ $(b)(ii)\ k = 3 \text{ or } \dfrac{2}{5}.$ $(c)\ \dfrac{147}{16}.$

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Question 10

11 marks | Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

WORKED SOLUTION

Question 10 · WMA12/01 January 2021

1 **Find the arithmetic difference**

Plan A has 12 Sunday runs from 15 km to 37 km, so there are 11 equal increases. The common difference is $\frac{37-15}{11}=2$.

2 **Find week five for plan A**

Week 5 is the fifth term, so the distance is $15 + 4(2) = 23\ \text{km}$.

3 **Find the geometric ratio**

For plan B, $15r^{11} = 37$, so $r = \left(\dfrac{37}{15}\right)^{1/11}$.

4 **Find week five for plan B**

The fifth Sunday distance is $15r^4 = 15\left(\dfrac{37}{15}\right)^{4/11} = 20.829\ldots$, so it is $20.8\ \text{km}$.

5 **Sum the Sunday runs for plan A**

The arithmetic total is $\dfrac{12}{2}(15 + 37) = 312$ km.

6 **Find the Wednesday limit for plan A**

The total distance condition gives $312 + 12x \leq 360$, so $x \leq 4$. The maximum integer is 4.

7 **Sum the Sunday runs for plan B**

The geometric total is $15\dfrac{r^{12} - 1}{r - 1} = 294.185\ldots$ km.

8 **Find the Wednesday limit for plan B**

Now $294.185\ldots + 12x \leq 360$, so $x \leq 5.484\ldots$. The maximum integer is 5.

✓ **FINAL ANSWER**

$(a)\ 23\ \text{km}.$ $(b)\ 20.8\ \text{km}.$ $(c)(i)\ x = 4.$ $(c)(ii)\ x = 5.$