Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 October 2019 Worked Solutions

October 2019 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 October 2019

October 2019

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Applications of Differentiation

1. A curve C has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 October 2019

1 Expand the curve

$y = 2x^2(x - 5) = 2x^3 - 10x^2$.

2 Differentiate

$\frac{dy}{dx} = 6x^2 - 20x = 2x(3x - 10)$.

3 Find the stationary values of x

At a stationary point, $\frac{dy}{dx} = 0$. Hence $2x(3x - 10) = 0$, so $x = 0$ or $x = \frac{10}{3}$.

4 Use the sign of the derivative

Since $2x(3x - 10)$ is positive outside its two roots and negative between them, y is increasing for $x < 0$ and for $x > \frac{10}{3}$.

FINAL ANSWER

$(a)\ x = 0,\ \frac{10}{3}$. $(b)\ x < 0$ or $x > \frac{10}{3}$.

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Question 2

6 marks | Modelling with Sequences & Series

2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

WORKED SOLUTION

Question 2 · WMA12/01 October 2019

1 Identify the common ratio

An increase of 2% each year gives the geometric ratio $r = 1.02$.

2 Count the years to 2032

From the start of 2019 to the start of 2032 there are 13 yearly increases, so the population is $25000(1.02)^{13}$.

3 Calculate the 2032 population

$25000(1.02)^{13} \approx 32340.7$, so the model gives about 32341 adults.

4 Set up the total number of adult payments

From 2019 to 2032 inclusive there are 14 yearly populations, so the total number of adult payments is $25000\dfrac{1.02^{14}-1}{1.02-1}$.

5 Multiply by the charity amount

The total money is $5 \times 25000\dfrac{1.02^{14}-1}{0.02} \approx 1\,996\,766$, so this is about £1,996,766.

6 **Round as requested**

To the nearest £1000, this is £1,997,000.

✓ **FINAL ANSWER**

(a) 32341 adults, approximately.

(b) £1,997,000.

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Question 3

6 marks | Binomial Expansion

3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1+\frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{x}{4}\right)^{12}$$

(3)

WORKED SOLUTION

Question 3 · WMA12/01 October 2019

1 Write the binomial terms

$(1+\frac{x}{4})^{12} = 1 + \binom{12}{1}\frac{x}{4} + \binom{12}{2}\left(\frac{x}{4}\right)^2 + \binom{12}{3}\left(\frac{x}{4}\right)^3 + \cdots$.

2 Simplify the first four terms

This gives $1 + 3x + \frac{33}{8}x^2 + \frac{55}{16}x^3$.

3 Rewrite the prefactor

$\frac{x^2+8}{x^5} = x^{-3} + 8x^{-5}$.

4 Find the constant contributions

The x^{-3} part needs the x^3 coefficient, which is $\frac{55}{16}$. The $8x^{-5}$ part needs the x^5 coefficient, which is $8\binom{12}{5}\left(\frac{1}{4}\right)^5 = 8 \cdot \frac{99}{128} = \frac{99}{16}$.

5 Add the constant terms

The term independent of x is $\frac{55}{16} + \frac{99}{16} = \frac{154}{16} = \frac{77}{8}$.

FINAL ANSWER

$(a)\ 1+3x+\frac{33}{8}x^2+\frac{55}{16}x^3.\quad (b)\ \frac{77}{8}.$

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Question 4

8 marks | Polynomials

4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $f(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $f(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $f(x)$.

(5)

WORKED SOLUTION

Question 4 · WMA12/01 October 2019

1 Use the remainder theorem

When $f(x)$ is divided by $(x - 3)$, the remainder is $f(3)$. Since $f(3) = (3 - 3)(3 \cdot 3^2 + 3 + a) - 35$, the remainder is -35.

2 Use the given factor

Since $(3x - 2)$ is a factor, $x = \frac{2}{3}$ is a root and $f\left(\frac{2}{3}\right) = 0$.

3 Solve for a

$0 = \left(\frac{2}{3} - 3\right)\left(3\left(\frac{2}{3}\right)^2 + \frac{2}{3} + a\right) - 35 = -\frac{7}{3}(a + 2) - 35$. Hence $-7(a + 2) = 105$, so $a = -17$.

4 Substitute a into f(x)

$f(x) = (x - 3)(3x^2 + x - 17) - 35 = 3x^3 - 8x^2 - 20x + 16$.

5 Divide by the known factor

Dividing $3x^3 - 8x^2 - 20x + 16$ by $(3x - 2)$ gives $x^2 - 2x - 8$.

6 Factorise fully

$x^2 - 2x - 8 = (x + 2)(x - 4)$, so $f(x) = (3x - 2)(x + 2)(x - 4)$.

FINAL ANSWER

$(a)\ -35.\quad (b)\ a=-17.\quad (c)\ f(x)=(3x-2)(x+2)(x-4).$

Question 5

7 marks | Integration

5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(3)

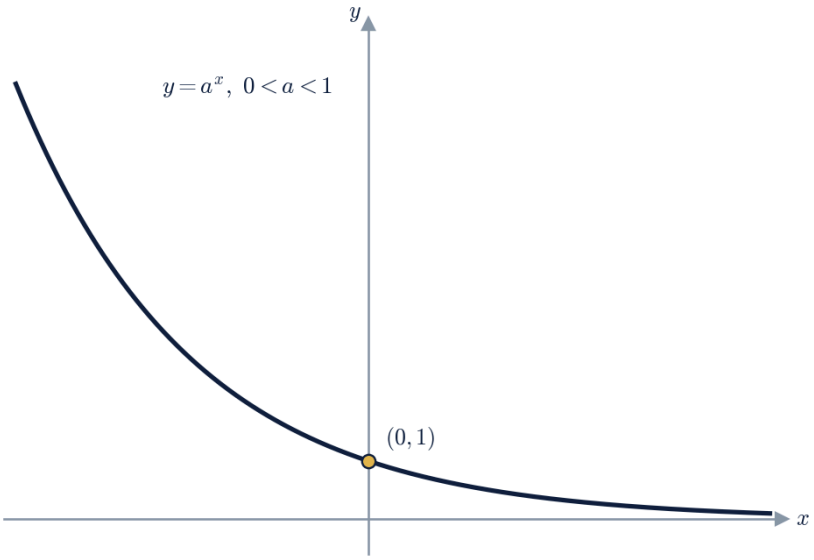
Using your answer to part (b) and making your method clear, estimate

(c) $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(2)

1 **Describe the sketch**

For $0 < a < 1$, $y = a^x$ is a decreasing exponential curve. At $x = 0$, $y = a^0 = 1$, so it crosses the y-axis at $(0, 1)$.



2 **Find the strip width**

The x values increase by 0.5, so $h = 0.5$.

3 **Apply the trapezium rule**

$A \approx \frac{0.5}{2}\{4.25 + 16.06 + 2(6.427 + 9.125 + 12.34)\}$.

4 **Calculate the estimate**

$A \approx 0.25(76.094) = 19.0235$, so the estimate is 19.0 to 3 significant figures.

5 **Relate the two integrands**

$x(x - 3) + \left(\frac{1}{2}\right)^x = x^2 + \left(\frac{1}{2}\right)^x - 3x$.

6 **Subtract the exact integral of 3x**

$\int_2^4 3x\,dx = \left[\frac{3}{2}x^2\right]_2^4 = 24 - 6 = 18$.

7 **Estimate the new integral**

Using the estimate from part (b), the required value is $19.0235 - 18 = 1.0235$, so the estimate is 1.0.

FINAL ANSWER

(a) A decreasing exponential curve crossing the y-axis at $(0, 1)$.

(b) 19.0. (c) 1.0.

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Question 6

7 marks | Circles

6.

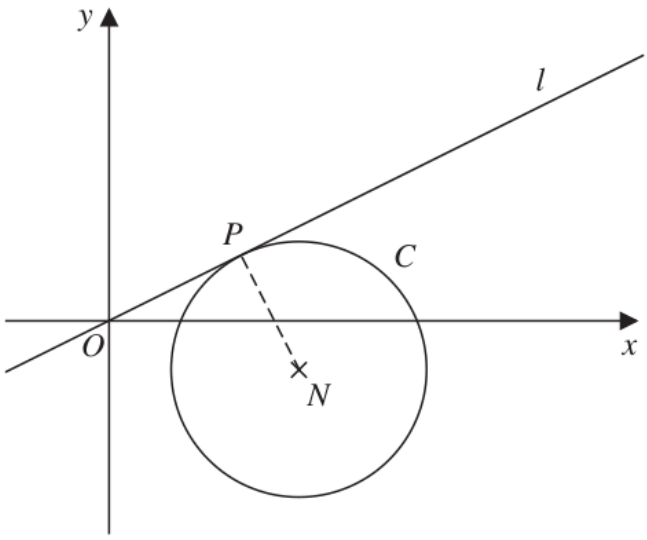


Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

WORKED SOLUTION

Question 6 · WMA12/01 October 2019

1 Use perpendicular gradients

The tangent has gradient $\frac{1}{2}$, so the radius PN has gradient -2.

2 Find the equation of PN

Line PN passes through $N(4, -1)$, so $y + 1 = -2(x - 4)$. Therefore $y = -2x + 7$.

3 **Find point P**

At P, the tangent $y = \frac{1}{2}x$ meets the radius $y = -2x + 7$. Thus $\frac{1}{2}x = -2x + 7$, so $x = \frac{14}{5}$ and $y = \frac{7}{5}$.

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4 **Find the radius squared**

$$r^2 = \left(\frac{14}{5} - 4\right)^2 + \left(\frac{7}{5} + 1\right)^2 = \left(-\frac{6}{5}\right)^2 + \left(\frac{12}{5}\right)^2 = \frac{36}{5}.$$

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5 **Write the circle equation**

With centre $(4, -1)$ and radius squared $\frac{36}{5}$, the circle is $(x-4)^2 + (y+1)^2 = \frac{36}{5}$.

✓ **FINAL ANSWER**

$(a)\ y = -2x + 7. \quad (b)\ (x-4)^2 + (y+1)^2 = \frac{36}{5}.$

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Question 7

7 marks | Laws of Logarithms

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

WORKED SOLUTION

Question 7 · WMA12/01 October 2019

1 Use the given logarithm

Since $\log_a b = k$, we can replace every $\log_a b$ by k.

2 Simplify part (i)

$\log_a\left(\frac{\sqrt{a}}{b}\right) = \log_a(a^{1/2}) - \log_a b = \frac{1}{2} - k.$

3 Simplify the numerator and denominator

$\log_a(a^2 b) = \log_a a^2 + \log_a b = 2 + k$, and $\log_a(b^3) = 3\log_a b = 3k.$

4 Write part (ii)

Therefore $\frac{\log_a(a^2 b)}{\log_a(b^3)} = \frac{k+2}{3k}$.

5 Simplify the summand

$\log_a(b^n) = n\log_a b = nk$, so $k + \log_a(b^n) = k + nk = (n+1)k.$

6 Sum the series

$\sum_{n=1}^{50}(n+1)k = k\left(\sum_{n=1}^{50} n + 50\right) = k(1275 + 50) = 1325k.$

FINAL ANSWER

$(i)\ \frac{1}{2} - k.\quad (ii)\ \frac{k+2}{3k}.\quad (iii)\ 1325k.$

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Question 8

9 marks | Integration

8. Solutions relying on calculator technology are not acceptable in this question.

(i)

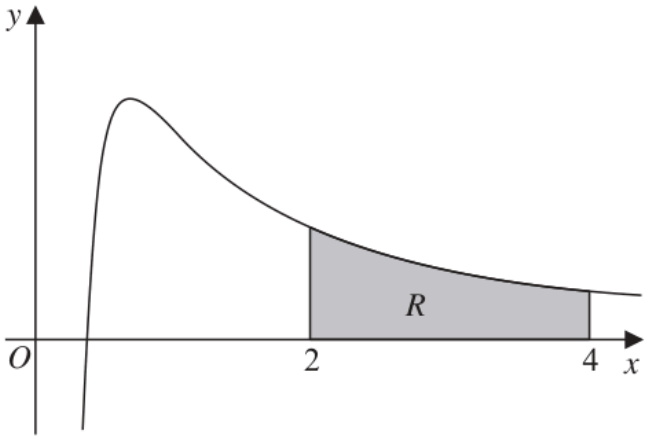


Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) \mathrm{d}x = 55$$

(4)

WORKED SOLUTION

Question 8 · WMA12/01 October 2019

1 **Rewrite the curve**

$$y = \frac{8\sqrt{x} - 5}{2x^2} = 4x^{-3/2} - \frac{5}{2}x^{-2}.$$

2 **Integrate the curve**

$$\int \left(4x^{-3/2} - \frac{5}{2}x^{-2}\right)dx = -8x^{-1/2} + \frac{5}{2x}.$$

3 **Apply the limits for the area**

$$\left[-8x^{-1/2} + \frac{5}{2x}\right]_2^4 = \left(-4 + \frac{5}{8}\right) - \left(-4\sqrt{2} + \frac{5}{4}\right).$$

4 **Simplify the exact area**

This gives $4\sqrt{2} - \frac{37}{8}$.

5 **Integrate the second expression**

$$\int \left(\frac{1}{2}x^2 + k\right)dx = \frac{x^3}{6} + kx.$$

6 **Use the limits**

$$\left[\frac{x^3}{6} + kx\right]_{-3}^{6} = (36 + 6k) - \left(-\frac{9}{2} - 3k\right) = \frac{81}{2} + 9k.$$

7 **Solve for k**

$\frac{81}{2} + 9k = 55$, so $9k = \frac{29}{2}$ and $k = \frac{29}{18}$.

✓ **FINAL ANSWER**

$(i)\ 4\sqrt{2} - \frac{37}{8}$. $\quad (ii)\ k = \frac{29}{18}$.

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Question 9

12 marks | Trigonometric Equations

9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

WORKED SOLUTION

Question 9 · WMA12/01 October 2019

1 Solve the sine equation

Let $u = 2\theta - 10°$. Then $\sin u = \frac{1}{3}$.

2 Find the valid angle values

Since $0 \leq \theta < 180°$, $-10° \leq u < 350°$. In this interval, $u = 19.471\ldots°$ or $u = 160.529\ldots°$.

3 Return to theta

$\theta = \frac{u + 10°}{2}$, giving $\theta = 14.7°$ and $85.3°$ to one decimal place.

4 **Use the arithmetic sequence condition**

For three consecutive terms of an arithmetic sequence, second minus first equals third minus second. Hence $\frac{1}{\tan\alpha} - \sin\alpha = 2\sin\alpha - \frac{1}{\tan\alpha}$.

5 **Prove the required identity**

This gives $\frac{2}{\tan\alpha} = 3\sin\alpha$. Since $\frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha}$, we get $\frac{2\cos\alpha}{\sin\alpha} = 3\sin\alpha$, so $2\cos\alpha = 3\sin^2\alpha$.

6 **Form a quadratic in cos alpha**

Using $\sin^2\alpha = 1 - \cos^2\alpha$, $2\cos\alpha = 3(1-\cos^2\alpha)$, so $3\cos^2\alpha + 2\cos\alpha - 3 = 0$.

7 **Solve in the given interval**

$\cos\alpha = \frac{-1+\sqrt{10}}{3}$; the other root is less than -1. Since $\pi < \alpha < 2\pi$ and cosine is positive, α is in the fourth quadrant.

8 **State alpha**

$\alpha = 2\pi - \cos^{-1}\left(\frac{-1+\sqrt{10}}{3}\right) = 5.517$ radians to 3 decimal places.

✓ **FINAL ANSWER**

$(i)\ \theta = 14.7°,\ 85.3°.$ $(ii)(a)$ Proven.

$(ii)(b)\ \alpha = 5.517$ radians.

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Question 10

7 marks | Applications of Differentiation

10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

WORKED SOLUTION

Question 10 · WMA12/01 October 2019

1 Use the point on the curve

Substitute $(2, 5)$ into $y = ax^3 - 3x^2 + 3x + b$: $5 = 8a - 12 + 6 + b$, so $8a + b = 11$.

2 Differentiate

$\frac{dy}{dx} = 3ax^2 - 6x + 3$.

3 Use the given gradient

At $x = 2$, the gradient is 7, so $12a - 12 + 3 = 7$. Hence $12a = 16$ and $a = \frac{4}{3}$.

4 Find b

Using $8a + b = 11$, $b = 11 - 8\left(\frac{4}{3}\right) = \frac{1}{3}$.

5 Check for turning points

With $a = \frac{4}{3}$, $\frac{dy}{dx} = 4x^2 - 6x + 3$.

6 **Use the discriminant**

For $4x^2 - 6x + 3 = 0$, the discriminant is $(-6)^2 - 4(4)(3) = 36 - 48 = -12 < 0$.

7 **Conclude**

The derivative is never zero, so the curve has no turning points.

✓ **FINAL ANSWER**

(a) $a = \frac{4}{3}$, $b = \frac{1}{3}$. (b) Proven: C has no turning points.