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WMA11 Pure 1 Classified With Answers

Each placement has the question followed by a worked-solution page with bordered steps.

202
topic placements

WMA11
Pure 1

Question bank with worked answers
standalone IAL route

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P1

TOPIC

Indices & Surds

WMA11/01 JANUARY 2020

Question 2

5 marks

Indices & Surds

2. Given $y = 3^x$, express each of the following in terms of y. Write each expression in its simplest form.

(a) 3^{3x}

(1)

(b) $\dfrac{1}{3^{x-2}}$

(2)

(c) $\dfrac{81}{9^{2-3x}}$

(2)

(Total 5 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 2

Indices & Surds

Topic group

1. Use y = 3^x

Since $y = 3^x$, powers of 3^x can be replaced by powers of y.

2. Part (a)

$3^{3x} = (3^x)^3 = y^3$.

3. Part (b)

$$\frac{1}{3^{x-2}} = 3^{2-x} = \frac{3^2}{3^x} = \frac{9}{y}.$$

4. Part (c)

$$\frac{81}{9^{2-3x}} = \frac{3^4}{(3^2)^{2-3x}} = \frac{3^4}{3^{4-6x}} = 3^{6x} = (3^x)^6 = y^6.$$

Final answer

$(a)\ y^3.$

$(b)\ \frac{9}{y}.$

$(c)\ y^6.$

WMA11/01 MAY/JUNE 2022

Question 3

5 marks

Indices & Surds

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Show that $\dfrac{\sqrt{180}-\sqrt{80}}{\sqrt{5}}$ is an integer and find its value.

(2)

(ii) Simplify

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}}$$

giving your answer in the form $a+b\sqrt{5}$ where a and b are rational numbers.

(3)

(Total 5 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 3

Indices & Surds

Topic group

1. Simplify the square roots

$\sqrt{180} = 6\sqrt{5}$ and $\sqrt{80} = 4\sqrt{5}$.

2. Complete part (a)

$$\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} = \frac{6\sqrt{5} - 4\sqrt{5}}{\sqrt{5}} = 2.$$

3. Rationalise part (b)

$$\frac{4\sqrt{5} - 5}{7 - 3\sqrt{5}} \times \frac{7 + 3\sqrt{5}}{7 + 3\sqrt{5}} = \frac{(4\sqrt{5} - 5)(7 + 3\sqrt{5})}{49 - 45}.$$

4. Expand and simplify

The numerator is $28\sqrt{5} + 60 - 35 - 15\sqrt{5} = 25 + 13\sqrt{5}$, so the expression is $\frac{25 + 13\sqrt{5}}{4}$.

Final answer

$(a)\ 2.$

$(b)\ \frac{25}{4} + \frac{13}{4}\sqrt{5}.$

WMA11/01 OCTOBER 2022

Question 4

6 marks

Indices & Surds

4. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^2(2x + 1) - 15x$$

(a) Solve

$$f(x) = 0$$

(4)

(b) Hence solve

$$y^{\frac{4}{3}}\left(2y^{\frac{2}{3}} + 1\right) - 15y^{\frac{2}{3}} = 0 \qquad y > 0$$

giving your answer in simplified surd form.

(2)

(Total for Question 4 is 6 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 4

Indices & Surds

Topic group

1. Factor f(x)

$f(x) = x^2(2x+1) - 15x = x(2x^2 + x - 15)$.

2. Factor the quadratic

$2x^2 + x - 15 = (2x-5)(x+3)$.

3. Solve f(x) = 0

$x(2x-5)(x+3) = 0$, so $x = 0$, $x = \dfrac{5}{2}$, or $x = -3$.

4. Use the substitution for part (b)

The equation is the same as $f(u) = 0$ with $u = y^{2/3}$.

5. Choose the valid root

Since $y > 0$, $u = y^{2/3} > 0$. The valid root is $u = \dfrac{5}{2}$, so $y^{2/3} = \dfrac{5}{2}$.

6. Write y in surd form

$$y = \left(\frac{5}{2}\right)^{3/2} = \sqrt{\frac{125}{8}} = \frac{5\sqrt{10}}{4}.$$

Final answer

$(a)\ x = -3,\ 0,\ \dfrac{5}{2}.$

$(b)\ y = \dfrac{5\sqrt{10}}{4}.$

WMA11/01 JANUARY 2023

Question 5

6 marks

Indices & Surds

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 3^x$, show that the equation

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

can be rewritten in the form

$$9p^2 + 26p - 3 = 0$$

(3)

(b) Hence solve

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

(3)

(Total for Question 5 is 6 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 5

Indices & Surds

Topic group

1. Let p = 3^x

Then $9^x = 3^{2x} = p^2$, $3^{x+2} = 9p$, and $3^{x-1} = \frac{p}{3}$.

2. Substitute into the equation

$3p^2 + 9p = 1 + \frac{p}{3}$.

3. Clear the fraction

Multiplying by 3 gives $9p^2 + 27p = 3 + p$, hence $9p^2 + 26p - 3 = 0$.

4. Solve for p

$9p^2 + 26p - 3 = (9p - 1)(p + 3) = 0$, so $p = \frac{1}{9}$ or $p = -3$.

5. Reject the negative value

Since $p = 3^x > 0$, we reject $p = -3$. Therefore $3^x = \frac{1}{9} = 3^{-2}$.

6. Find x

$x = -2$.

Final answer

$x = -2$.

WMA11/01 MAY/JUNE 2023

Question 6

7 marks

Indices & Surds

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Expand and simplify

$$\left(r - \frac{1}{r}\right)^2$$

(2)

(b) Express $\frac{1}{3 + 2\sqrt{2}}$ in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

(c) Use the results of parts (a) and (b), or otherwise, to show that

$$\sqrt{3 + 2\sqrt{2}} - \frac{1}{\sqrt{3 + 2\sqrt{2}}} = 2$$

(3)

(Total for Question 6 is 7 marks)

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Worked Solution - Question 6

Indices & Surds

Topic group

1. Expand the square

$$\left(r-\frac{1}{r}\right)^2 = r^2 - 2 + \frac{1}{r^2}.$$

2. Rationalise the denominator

$$\frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}.$$

3. Let r be the square root

Let $r = \sqrt{3+2\sqrt{2}}$. Then $r^2 = 3+2\sqrt{2}$ and $\frac{1}{r^2} = 3-2\sqrt{2}$.

4. Use part (a)

$$\left(r-\frac{1}{r}\right)^2 = r^2 - 2 + \frac{1}{r^2} = (3+2\sqrt{2}) - 2 + (3-2\sqrt{2}) = 4.$$

5. Take the positive square root

Since $r > 1$, $r - \frac{1}{r} > 0$, so $r - \frac{1}{r} = 2$.

Final answer

(a) $r^2 - 2 + \frac{1}{r^2}$.

(b) $3-2\sqrt{2}$.

(c) The expression equals 2.

WMA11/01 OCTOBER 2023

Question 2

4 marks

Indices & Surds

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\frac{16}{b^3}$

(1)

(c) $\left(\frac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

(Total for Question 2 is 4 marks)

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Worked Solution - Question 2

Indices & Surds

Topic group

1. Square root a

$$a^{1/2} = \left(\frac{1}{64}x^2\right)^{1/2} = \frac{1}{8}x.$$

2. Cube b

$b = 16x^{-1/2}$, so $b^3 = 4096x^{-3/2}$.

3. Find 16 divided by b cubed

$$\frac{16}{b^3} = \frac{16}{4096x^{-3/2}} = \frac{1}{256}x^{3/2}.$$

4. Simplify ab over 2

$$\frac{ab}{2} = \frac{1}{2}\left(\frac{1}{64}x^2\right)\left(16x^{-1/2}\right) = \frac{1}{8}x^{3/2}.$$

5. Raise to the power -4 over 3

$$\left(\frac{1}{8}x^{3/2}\right)^{-4/3} = 8^{4/3}x^{-2} = 16x^{-2}.$$

Final answer

(a) $\frac{1}{8}x.$

(b) $\frac{1}{256}x^{3/2}.$

(c) $16x^{-2}.$

WMA11/01 OCTOBER 2023

Question 3

6 marks

Indices & Surds

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where a and b are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}$$

giving your answer in simplest form.

(3)

(Total for Question 3 is 6 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 3

Indices & Surds

Topic group

1. Rationalise the denominator

$$\frac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} \times \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}} = \frac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{12-5}.$$

2. Simplify the numerator

$$(8-\sqrt{15})(2\sqrt{3}-\sqrt{5}) = 16\sqrt{3}-8\sqrt{5}-3\sqrt{5}+5\sqrt{3} = 21\sqrt{3}-14\sqrt{5}.$$

3. Divide by 7

$$\frac{21\sqrt{3}-14\sqrt{5}}{7} = 3\sqrt{3}-2\sqrt{5}.$$

4. Rearrange the equation

$(x+5\sqrt{3})\sqrt{5} = 40-2x\sqrt{3}$ gives $x(\sqrt{5}+2\sqrt{3}) = 40-5\sqrt{15}$.

5. Use the result from part (a)

$$x = \frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}} = 5\left(3\sqrt{3}-2\sqrt{5}\right).$$

Final answer

$(a)\ 3\sqrt{3}-2\sqrt{5}.$

$(b)\ x = 15\sqrt{3}-10\sqrt{5}.$

WMA11/01 JANUARY 2024

Question 4

6 marks

Indices & Surds

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 2^x$, show that the equation

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

can be written in the form

$$4p^2 - 33p + 8 = 0$$

(3)

(b) Hence solve

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

(3)

(Total for Question 4 is 6 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 4

Indices & Surds

Topic group

1. Substitute p equals 2 to the x

If $p = 2^x$, then $4^x = p^2$, $2^{x+3} = 8p$, and $2^{x-1} = \frac{p}{2}$.

2. Rewrite the equation

$2p^2 - 8p = 17\left(\frac{p}{2}\right) - 4$.

3. Clear the fraction

Multiplying by 2 gives $4p^2 - 16p = 17p - 8$, so $4p^2 - 33p + 8 = 0$.

4. Solve the quadratic

$4p^2 - 33p + 8 = (4p - 1)(p - 8)$, so $p = \frac{1}{4}$ or $p = 8$.

5. Return to x

$2^x = \frac{1}{4} = 2^{-2}$ gives $x = -2$, and $2^x = 8 = 2^3$ gives $x = 3$.

Final answer

$x = 3$ or $x = -2$.

WMA11/01 MAY/JUNE 2024

Question 2

6 marks

Indices & Surds

2. (i) Given that $m = 2^n$, express each of the following in simplest form in terms of m.

(a) 2^{n+3}

(1)

(b) 16^{3n}

(2)

(ii) **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$x\sqrt{3} - 3 = x + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(3)

(Total for Question 2 is 6 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 2

Indices & Surds

Topic group

1. Use m equals 2 to the n

$2^{n+3} = 2^n \times 2^3 = 8m.$

2. Rewrite 16 as a power of 2

$16^{3n} = (2^4)^{3n} = 2^{12n} = (2^n)^{12} = m^{12}.$

3. Collect the x terms

$x\sqrt{3} - 3 = x + \sqrt{3}$ gives $x(\sqrt{3} - 1) = 3 + \sqrt{3}.$

4. Rationalise

$$x = \frac{3 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{(3 + \sqrt{3})(\sqrt{3} + 1)}{2}.$$

5. Simplify

$$x = \frac{4\sqrt{3} + 6}{2} = 3 + 2\sqrt{3}.$$

Final answer

$(i)(a)\ 8m. \quad (i)(b)\ m^{12}. \quad (ii)\ x = 3 + 2\sqrt{3}.$

WMA11/01 OCTOBER 2024

Question 2

7 marks

Indices & Surds

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Simplify fully

$$\frac{3y^3(2x^4)^3}{4x^2y^4}$$

(3)

(ii) Find the exact value of a such that

$$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$$

Write your answer in the form $p\sqrt{3} + q$ where p and q are fully simplified rational constants.

(4)

(Total for Question 2 is 7 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 2

Indices & Surds

Topic group

1. Simplify the power bracket

$(2x^4)^3 = 8x^{12}$.

2. Simplify the algebraic fraction

$$\frac{3y^3(2x^4)^3}{4x^2y^4} = \frac{3y^3(8x^{12})}{4x^2y^4} = \frac{6x^{10}}{y}.$$

3. Rationalise the left side

$$\frac{16}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 8\sqrt{3} - 8.$$

4. Equate the surd parts

$8\sqrt{3} - 8 = a\sqrt{27} + 4 = 3a\sqrt{3} + 4$.

5. Solve for a

$3a\sqrt{3} = 8\sqrt{3} - 12$, so $a = \dfrac{8\sqrt{3}-12}{3\sqrt{3}} = \dfrac{8}{3} - \dfrac{4}{3}\sqrt{3}$.

Final answer

$(i)\ \dfrac{6x^{10}}{y}.\quad (ii)\ a = -\dfrac{4}{3}\sqrt{3} + \dfrac{8}{3}.$

WMA11/01 JANUARY 2025

Question 4

9 marks

Indices & Surds

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \qquad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1}

(1)

(b) $\dfrac{5}{\left(3a^{1-x}\right)^{-2}}$

(3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$

(2)

(b) Hence solve

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

(3)

(Total for Question 4 is 9 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 4

Indices & Surds

Topic group

1. Use y equals a to the x

$a^{3x+1} = a(a^x)^3 = ay^3$.

2. Rewrite the denominator

$\dfrac{5}{(3a^{1-x})^{-2}} = 5(3a^{1-x})^2 = 45a^{2-2x}$.

3. Write in terms of y

$45a^{2-2x} = \dfrac{45a^2}{(a^x)^2} = \dfrac{45a^2}{y^2}$.

4. Use p equals 9 to the t

$p = 9^t = 3^{2t}$, so $3^{4t+2} = 9(3^{2t})^2 = 9p^2$.

5. Rewrite the equation

$3(3^{4t+2} + 1) = 82 \times 9^t$ becomes $3(9p^2 + 1) = 82p$.

6. Obtain the quadratic

$27p^2 + 3 = 82p$, so $27p^2 - 82p + 3 = 0$.

7. Solve for p

$p = \dfrac{82 \pm 80}{54}$, so $p = 3$ or $p = \dfrac{1}{27}$.

8. Return to t

$9^t = 3$ gives $3^{2t} = 3^1$, so $t = \dfrac{1}{2}$. Also $9^t = \dfrac{1}{27} = 3^{-3}$ gives $2t = -3$, so $t = -\dfrac{3}{2}$.

Final answer

$(i)(a)\ ay^3.\quad (i)(b)\ \dfrac{45a^2}{y^2}.\quad (ii)(b)\ t=\dfrac{1}{2}$ or $t=-\dfrac{3}{2}$.

WMA11/01 MAY/JUNE 2025

Question 1

5 marks

Indices & Surds

1. Given that

$$p = \frac{1}{16}x^4 \qquad q = \frac{40}{x^3}$$

express each of the following in the form kx^n where k and n are fully simplified constants.

(a) $p^{\frac{1}{2}}$

(1)

(b) $(pq)^{-1}$

(2)

(c) pq^2

(2)

(Total for Question 1 is 5 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 1

Indices & Surds

Topic group

1. Simplify p

$p = \frac{1}{16}x^4.$

2. Part (a)

$p^{1/2} = \left(\frac{1}{16}x^4\right)^{1/2} = \frac{1}{4}x^2.$

3. Simplify pq

$pq = \frac{1}{16}x^4 \cdot \frac{40}{x^3} = \frac{5}{2}x.$

4. Part (b)

$(pq)^{-1} = \left(\frac{5}{2}x\right)^{-1} = \frac{2}{5}x^{-1}.$

5. Part (c)

$pq^2 = \frac{1}{16}x^4\left(\frac{40}{x^3}\right)^2 = \frac{1600}{16}x^{-2} = 100x^{-2}.$

Final answer

$(a)\ \frac{1}{4}x^2.$

$(b)\ \frac{2}{5}x^{-1}.$

$(c)\ 100x^{-2}.$

WMA11/01 OCTOBER 2025

Question 4

7 marks

Indices & Surds

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Using the laws of indices, solve

$$3^{2p-5} = \frac{1}{9}\sqrt{27}$$

(3)

(ii) Find the real roots of the equation

$$x^2 + 1 = \frac{36}{x^2 - 4}$$

(4)

(Total for Question 4 is 7 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 4

Indices & Surds

Topic group

1. Rewrite the right-hand side

$\frac{1}{9}\sqrt{27} = 3^{-2} \cdot 3^{3/2} = 3^{-1/2}$.

2. Equate powers of 3

$3^{2p-5} = 3^{-1/2}$, so $2p - 5 = -\frac{1}{2}$.

3. Solve for p

$2p = \frac{9}{2}$, hence $p = \frac{9}{4}$.

4. Clear the denominator

$x^2 + 1 = \frac{36}{x^2 - 4}$ gives $(x^2 + 1)(x^2 - 4) = 36$.

5. Form a quadratic in x squared

$x^4 - 3x^2 - 4 = 36$, so $x^4 - 3x^2 - 40 = 0$.

6. Factor

$(x^2 - 8)(x^2 + 5) = 0$.

7. Use real roots

$x^2 = 8$, so $x = \pm 2\sqrt{2}$. The equation $x^2 = -5$ gives no real roots.

Final answer

$(i)\ p = \frac{9}{4}$. $\quad (ii)\ x = \pm 2\sqrt{2}$.

TOPIC

Quadratics

WMA11/01 OCTOBER 2019

Question 8

5 marks

Quadratics

8. Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

(Total 5 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 8

Quadratics

Topic group

1. Use a substitution

Let $u = \sqrt{x}$. Then $x = u^2$, so the equation becomes $u^2 - 6u + 4 = 0$.

2. Solve the quadratic

$$u = \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}.$$

3. Return to x

Since $u = \sqrt{x}$, $x = u^2 = (3 \pm \sqrt{5})^2 = 9 + 5 \pm 6\sqrt{5} = 14 \pm 6\sqrt{5}$.

Final answer

$x = 14 + 6\sqrt{5}$ or $x = 14 - 6\sqrt{5}$.

WMA11/01 JANUARY 2020

Question 5

7 marks

Quadratics

5. (a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0$$

(3)

(b) Hence find all real solutions of

$$20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{\frac{1}{2}} = 0$$

(4)

(Total 7 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 5

Quadratics

Topic group

1. Factorise the cubic

$20x^3 - 50x^2 - 30x = 10x(2x^2 - 5x - 3) = 10x(2x + 1)(x - 3)$.

2. Solve part (a)

$10x(2x + 1)(x - 3) = 0$, so $x = 0$, $x = -\frac{1}{2}$, or $x = 3$.

3. Use the same structure for part (b)

Let $u = (y + 3)^{1/2}$. Then the equation becomes $20u^3 - 50u^2 - 30u = 0$, so $u = 0$, $u = -\frac{1}{2}$, or $u = 3$.

4. Reject the invalid square-root value

Since $u = \sqrt{y + 3} \geq 0$, reject $u = -\frac{1}{2}$. If $u = 0$, then $y = -3$. If $u = 3$, then $y + 3 = 9$, so $y = 6$.

Final answer

$(a)\ x = 0, -\dfrac{1}{2}, 3.$

$(b)\ y = -3, 6.$

WMA11/01 JANUARY 2021

Question 2

6 marks

Quadratics

2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q.

(4)

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place.

(2)

(Total 6 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 2

Quadratics

Topic group

1. Substitute the two given heights

When $t = 3$ and $H = 2$, $2^3 = 9p + q$, so $8 = 9p + q$. When $t = 5$ and $H = 2.4$, $2.4^3 = 25p + q$, so $13.824 = 25p + q$.

2. Solve for p and q

Subtracting the two equations gives $16p = 5.824$, so $p = 0.364$. Then $8 = 9(0.364) + q$, giving $q = 4.724$, so $q = 4.72$ to 3 significant figures.

3. Use H = 5

For $H = 5$, $125 = 0.364T^2 + 4.724$.

4. Find T

$T^2 = \dfrac{125 - 4.724}{0.364} = 330.428\ldots$, so $T = 18.177\ldots$. Therefore $T = 18.2$ years to one decimal place.

Final answer

(a) $p = 0.364,\ q = 4.72.$

(b) $T = 18.2$ years.

WMA11/01 JANUARY 2022

Question 2

6 marks

Quadratics

2. $f(x) = 11 - 4x - 2x^2$

(a) Express $f(x)$ in the form

$$a + b(x + c)^2$$

where a, b and c are integers to be found.

(3)

(b) Sketch the graph of the curve C with equation $y = f(x)$, showing clearly the coordinates of the point where the curve crosses the y-axis.

(2)

(c) Write down the equation of the line of symmetry of C.

(1)

(Total 6 marks)

WMA11/01 JANUARY 2022

Worked Solution - Question 2

Quadratics

Topic group

1. Complete the square

$f(x) = 11 - 4x - 2x^2 = -2(x^2 + 2x) + 11$.

2. Write in vertex form

$-2(x^2 + 2x) + 11 = -2\big((x+1)^2 - 1\big) + 11 = 13 - 2(x+1)^2$.

3. Identify the graph features

The graph is a downward-opening parabola with maximum point $(-1, 13)$. The y-intercept is $f(0) = 11$, so it crosses the y-axis at $(0, 11)$.

4. State the line of symmetry

From $13 - 2(x+1)^2$, the line of symmetry is $x = -1$.

Final answer

$(a)\ 13 - 2(x+1)^2$.

$(b)\ y$-intercept $(0, 11)$.

$(c)\ x = -1$.

WMA11/01 JANUARY 2022

Question 3

7 marks

Quadratics

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express f(x) in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

(Total 7 marks)

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Worked Solution - Question 3

Quadratics

Topic group

1. Expand the first square

$(x+\sqrt{2})^2 = x^2 + 2\sqrt{2}x + 2$.

2. Expand the second square

Since $\sqrt{8} = 2\sqrt{2}$, $(3x - 5\sqrt{8})^2 = (3x - 10\sqrt{2})^2 = 9x^2 - 60\sqrt{2}x + 200$.

3. Collect terms

$f(x) = x^2 + 2\sqrt{2}x + 2 + 9x^2 - 60\sqrt{2}x + 200 = 10x^2 - 58\sqrt{2}x + 202$.

4. Expand the surd equation

$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$ gives $4\sqrt{3}y - 9 = 5y + \sqrt{3}$.

5. Solve for y

$y(4\sqrt{3} - 5) = 9 + \sqrt{3}$, so $y = \dfrac{9+\sqrt{3}}{4\sqrt{3}-5}$.

6. Rationalise the denominator

$$y = \frac{(9+\sqrt{3})(4\sqrt{3}+5)}{(4\sqrt{3})^2 - 5^2} = \frac{57 + 41\sqrt{3}}{23} = \frac{57}{23} + \frac{41}{23}\sqrt{3}.$$

Final answer

$(i)\ f(x) = 10x^2 - 58\sqrt{2}x + 202.$ $\quad (ii)\ y = \dfrac{57}{23} + \dfrac{41}{23}\sqrt{3}.$

WMA11/01 MAY/JUNE 2022

Question 6

7 marks

Quadratics

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

(Total 7 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 6

Quadratics

Topic group

1. Rearrange the second equation

From $y - x^3 + 6x = 0$, we get $y = x^3 - 6x$.

2. Substitute into the first equation

$2x(x^3 - 6x) - 3x^2 = 50$, so $2x^4 - 15x^2 - 50 = 0$.

3. Solve the quadratic in x squared

Let $u = x^2$. Then $2u^2 - 15u - 50 = 0$, giving $u = 10$ or $u = -\frac{5}{2}$.

4. Reject the negative value

Since x^2 cannot be negative, $x^2 = 10$, so $x = \pm\sqrt{10}$.

5. Find y

$y = x^3 - 6x = x(x^2 - 6) = x(10 - 6) = 4x$. Therefore $y = 4\sqrt{10}$ when $x = \sqrt{10}$ and $y = -4\sqrt{10}$ when $x = -\sqrt{10}$.

Final answer

$(x, y) = (\sqrt{10}, 4\sqrt{10})$ or $(-\sqrt{10}, -4\sqrt{10})$.

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Question 4

4 marks

Quadratics

4. Given that the equation

$$kx^2 + 6kx + 5 = 0 \qquad \text{where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(4)

(Total for Question 4 is 4 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 4

Quadratics

Topic group

1. Use the discriminant condition

For no real roots, the discriminant must be negative: $b^2 - 4ac < 0$.

2. Substitute a, b, and c

For $kx^2 + 6kx + 5 = 0$, $a = k$, $b = 6k$, and $c = 5$.

3. Form the inequality

$(6k)^2 - 4(k)(5) < 0$, so $36k^2 - 20k < 0$.

4. Solve the inequality

$4k(9k - 5) < 0$, so the product is negative when $0

Final answer

$0

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Question 8

8 marks

Quadratics

8.

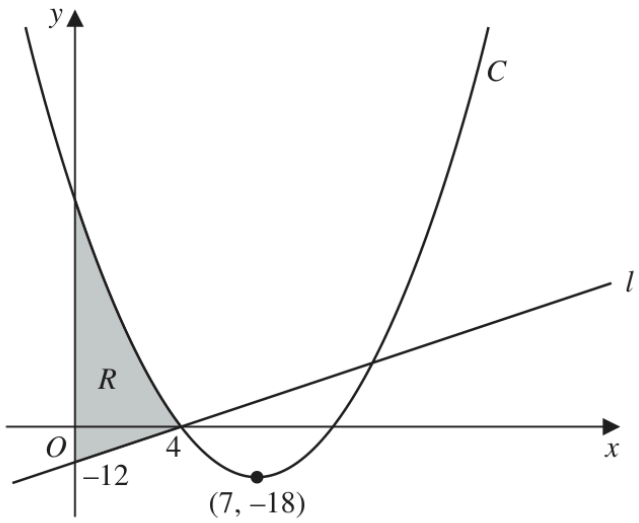


Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at -12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression
- has a minimum point at $(7, -18)$
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $\mathrm{f}(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

(Total for Question 8 is 8 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 8

Quadratics

Topic group

1. Find the straight line

The line cuts the y-axis at -12 and the x-axis at 4, so its gradient is $\frac{0-(-12)}{4-0}=3$. Hence l is $y=3x-12$.

2. Use symmetry of the quadratic

The minimum point has x-coordinate 7. The roots are 4 and k, so their midpoint is 7.

3. Find k

$\frac{4+k}{2}=7$, so $k=10$.

4. Build the quadratic

Let $f(x)=a(x-4)(x-10)$. Since the minimum point is $(7,-18)$, $-18=a(3)(-3)=-9a$, so $a=2$.

5. Write f(x)

$f(x)=2(x-4)(x-10)=2x^2-28x+80$.

6. Identify the shaded region

The region lies between the y-axis and $x=4$, above the line and below the curve: $0\le x\le 4$ and $3x-12\le y\le 2x^2-28x+80$.

Final answer

$(a)\ y = 3x - 12.$

$(b)\ k = 10.$

$(c)\ f(x) = 2(x-4)(x-10).$

$(d)\ 0 \le x \le 4,\ 3x - 12 \le y \le 2x^2 - 28x + 80.$

WMA11/01 MAY/JUNE 2023

Question 2

6 marks

Quadratics

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A rectangular sports pitch has length x metres and width y metres, where $x > y$

Given that the perimeter of the pitch is 350 m,

(a) write down an equation linking x and y

(1)

Given also that the area of the pitch is $7350\,m^2$

(b) write down a second equation linking x and y

(1)

(c) hence find the value of x and the value of y

(4)

(Total for Question 2 is 6 marks)

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Worked Solution - Question 2

Quadratics

Topic group

1. Use the perimeter

$2x + 2y = 350$, so $x + y = 175$.

2. Use the area

$xy = 7350$.

3. Form a quadratic

Using $y = 175 - x$, $x(175 - x) = 7350$, so $x^2 - 175x + 7350 = 0$.

4. Solve

$$x = \frac{175 \pm \sqrt{175^2 - 4(7350)}}{2} = \frac{175 \pm 35}{2},$$ giving $x = 105$ or 70.

5. Use x > y

Since $x > y$, $x = 105$ and $y = 70$.

Final answer

$x = 105,\ y = 70$.

WMA11/01 MAY/JUNE 2023

Question 3

6 marks

Quadratics

3. (a) Express $3x^2 + 12x + 13$ in the form

$$a(x + b)^2 + c$$

where a, b and c are integers to be found.

(3)

(b) Hence sketch the curve with equation $y = 3x^2 + 12x + 13$

On your sketch show clearly

- the coordinates of the y intercept
- the coordinates of the turning point of the curve

(3)

(Total for Question 3 is 6 marks)

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Worked Solution - Question 3

Quadratics

Topic group

1. Complete the square

$3x^2 + 12x + 13 = 3(x^2 + 4x) + 13$.

2. Use (x + 2)^2

$x^2 + 4x = (x + 2)^2 - 4$, so $3(x^2 + 4x) + 13 = 3(x + 2)^2 - 12 + 13$.

3. Read the features

$y = 3(x + 2)^2 + 1$ has turning point $(-2, 1)$. The y-intercept is found from $x = 0$, giving $y = 13$.

4. Sketch description

The curve is an upward-opening parabola with minimum $(-2, 1)$ and y-intercept $(0, 13)$.

Final answer

(a) $3(x + 2)^2 + 1$.

(b) y-intercept $(0, 13)$ and turning point $(-2, 1)$.

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Question 6

6 marks

Quadratics

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

(3)

(b) Hence, using algebra, find the range of possible values of p

(3)

(Total for Question 6 is 6 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 6

Quadratics

Topic group

1. Clear the denominator

$4(p-2x)=\dfrac{12+15p}{x+p}$ gives $4(p-2x)(x+p)=12+15p$.

2. Form a quadratic in x

Expanding gives $4p^2-4px-8x^2=12+15p$, so
$8x^2+4px+12+15p-4p^2=0$.

3. Use the discriminant condition

For two distinct real roots, $b^2-4ac>0$.

4. Simplify the inequality

$(4p)^2-4(8)(12+15p-4p^2)>0$, so $144p^2-480p-384>0$.

5. Divide and factorise

Dividing by 48 gives $3p^2-10p-8>0$, and $3p^2-10p-8=(3p+2)(p-4)$.

6. State the range

The upward-opening quadratic is positive outside its roots, so $p<-\dfrac{2}{3}$ or $p>4$.

Final answer

$p<-\dfrac{2}{3}$ or $p>4$.

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Question 11

10 marks

Quadratics

11.

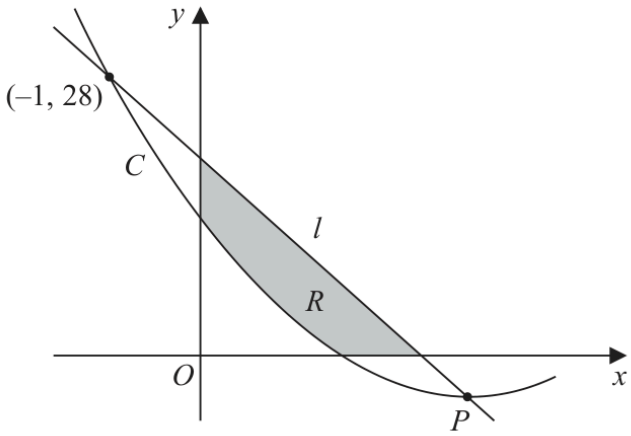


Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

(Total for Question 11 is 10 marks)

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Worked Solution - Question 11

Quadratics

Topic group

1. Complete the square

$2x^2 - 12x + 14 = 2(x^2 - 6x) + 14 = 2\left((x-3)^2 - 9\right) + 14 = 2(x-3)^2 - 4$

.

2. State the minimum point

The minimum of $2(x-3)^2 - 4$ is at $P = (3, -4)$.

3. Find the gradient of l

The line passes through $(-1, 28)$ and $P(3, -4)$, so
$m = \dfrac{-4 - 28}{3 - (-1)} = \dfrac{-32}{4} = -8$.

4. Find the equation of l

Using $P(3, -4)$, $y + 4 = -8(x - 3)$, so $y = -8x + 20$.

5. Use the axes as boundaries

The region is to the right of the y-axis and above the x-axis, so $x \geq 0$ and $y \geq 0$.

6. Use the line and curve as boundaries

The region is below the line and above the curve, so $y \leq -8x + 20$ and
$y \geq 2x^2 - 12x + 14$.

Final answer

(a) $2(x-3)^2-4$.

(b) $P=(3,-4)$.

(c) $y=-8x+20$.

(d) $x\geq 0,\ y\geq 0,\ y\leq -8x+20,\ y\geq 2x^2-12x+14$.

WMA11/01 JANUARY 2024

Question 9

10 marks

Quadratics

9. The curve C_1 has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- C_1 has a maximum turning point at (2, 20)
- C_1 passes through the origin

(a) sketch a graph of C_1 showing the coordinates of any points where C_1 cuts the coordinate axes,

(2)

(b) find an expression for $f(x)$.

(3)

The curve C_2 has equation $y = x(x^2 - 4)$

Curve C_1 and C_2 meet at the origin, and at the points P and Q

Given that the x coordinate of the point P is negative,

(c) using algebra and showing all stages of your working, find the coordinates of P

(5)

(Total for Question 9 is 10 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 9

Quadratics

Topic group

1. Use symmetry for the sketch

The maximum is at $(2, 20)$ and the curve passes through $(0, 0)$. Since the axis of symmetry is $x = 2$, the other x-intercept is $(4, 0)$.

2. Use completed-square form

Let $f(x) = a(x - 2)^2 + 20$.

3. Use the origin

$0 = a(0 - 2)^2 + 20$, so $4a = -20$ and $a = -5$.

4. Write f(x)

$f(x) = -5(x - 2)^2 + 20 = -5x^2 + 20x = -5x(x - 4)$.

5. Equate the two curves

$-5x^2 + 20x = x(x^2 - 4) = x^3 - 4x$.

6. Factorise

$x^3 + 5x^2 - 24x = 0$, so $x(x^2 + 5x - 24) = 0$ and $x(x + 8)(x - 3) = 0$.

7. Use the negative x-coordinate

The negative x-coordinate is $x = -8$.

8. Find y

$y = x(x^2 - 4) = (-8)(64 - 4) = -480$, so $P = (-8, -480)$.

Final answer

(a) downward parabola through $(0,0)$ and $(4,0)$ with maximum $(2,20)$.

(b) $f(x) = -5x(x-4)$.

(c) $P = (-8,-480)$.

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Question 4

6 marks

Quadratics

4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

(Total for Question 4 is 6 marks)

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Worked Solution - Question 4

Quadratics

Topic group

1. Equate the curves

$x^2 + kx - 9 = -3x^2 - 5x + k$ gives $4x^2 + (k+5)x - (k+9) = 0$.

2. Use one intersection only

For a single point of intersection, the discriminant is zero.

3. Form the condition on k

$(k+5)^2 - 4(4)(-(k+9)) = 0$, so $k^2 + 26k + 169 = 0$.

4. Find k

$k^2 + 26k + 169 = (k+13)^2$, so $k = -13$.

5. Find x

Substituting $k = -13$ gives $4x^2 - 8x + 4 = 0$, so $4(x-1)^2 = 0$ and $x = 1$.

6. Find y

$y = x^2 + kx - 9 = 1 - 13 - 9 = -21$, so $P = (1, -21)$.

Final answer

$P = (1, -21)$.

WMA11/01 MAY/JUNE 2025

Question 10

7 marks

Quadratics

10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$(k-1)x^6 + 4x^3 + (k-4) = 0$ where k is a constant

(a) Find the exact solutions to the given equation for $k = 4.5$

(3)

(b) Find the set of possible values of k for which the given equation has no real roots.

(4)

(Total for Question 10 is 7 marks)

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Worked Solution - Question 10

Quadratics

Topic group

1. Substitute k equals 4.5

The equation becomes $3.5x^6 + 4x^3 + 0.5 = 0$.

2. Clear decimals

Multiplying by 2 gives $7x^6 + 8x^3 + 1 = 0$.

3. Use a quadratic in x cubed

$7x^6 + 8x^3 + 1 = (x^3 + 1)(7x^3 + 1) = 0$.

4. Solve for x

$x^3 = -1$ or $x^3 = -\frac{1}{7}$, so $x = -1$ or $x = -\sqrt[3]{\frac{1}{7}}$.

5. View the equation as a quadratic

Let $u = x^3$. Then the equation is $(k-1)u^2 + 4u + (k-4) = 0$.

6. Use no real roots

Since every real value of u gives a real x, the original equation has no real roots exactly when this quadratic in u has no real roots.

7. Use the discriminant

Require $b^2 - 4ac < 0$: $16 - 4(k-1)(k-4) < 0$.

8. Simplify

$16 - 4(k^2 - 5k + 4) < 0$ gives $20k - 4k^2 < 0$, so $4k(5-k) < 0$.

9. Solve the inequality

$4k(5-k) < 0$ for $k < 0$ or $k > 5$.

Final answer

$(a)\ x = -1,\ -\sqrt[3]{\frac{1}{7}}.$

$(b)\ k < 0$ or $k > 5$.

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Question 5

10 marks

Quadratics

5.

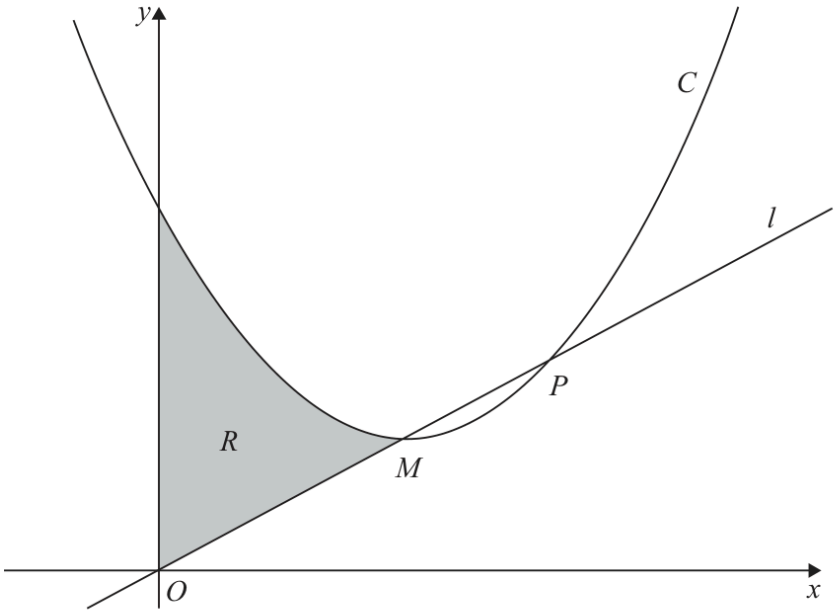


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(a) Express $2x^2 - 16x + 50$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Figure 1 shows a sketch of the curve C with equation $y = 2x^2 - 16x + 50$

Given that in Figure 1

- M is the minimum point on C
- line l passes through the origin and intersects C at the points M and P

(b) find, using algebra and showing your working, the x coordinate of P.

(5)

The region R is shown shaded in Figure 1.

(c) Use inequalities to fully define R.

(2)

(Total for Question 5 is 10 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 5

Quadratics

Topic group

1. Complete the square

$2x^2 - 16x + 50 = 2(x^2 - 8x) + 50 = 2[(x-4)^2 - 16] + 50$.

2. Simplify the completed square

$2x^2 - 16x + 50 = 2(x-4)^2 + 18$.

3. Find M

The minimum point is $M = (4, 18)$.

4. Find the line through O and M

Since l passes through $(0, 0)$ and $(4, 18)$, its gradient is $\frac{18}{4} = \frac{9}{2}$, so $l : y = \frac{9}{2}x$.

5. Find the second intersection

Set $2x^2 - 16x + 50 = \frac{9}{2}x$.

6. Form the quadratic

Multiplying by 2 gives $4x^2 - 32x + 100 = 9x$, so $4x^2 - 41x + 100 = 0$.

7. Factor and select P

$4x^2 - 41x + 100 = (x-4)(4x-25) = 0$. The point M has $x = 4$, so P has $x = \frac{25}{4}$.

8. Define the region

The shaded region is above the line, below the curve, and between the y-axis and M.

9. Write inequalities

$\frac{9}{2}x \le y \le 2x^2 - 16x + 50$ with $0 \le x \le 4$.

Final answer

$(a)\ 2(x-4)^2 + 18.$

$(b)\ x_P = \frac{25}{4}.$

$(c)\ \frac{9}{2}x \le y \le 2x^2 - 16x + 50,\ 0 \le x \le 4.$

TOPIC

Simultaneous Equations

WMA11/01 OCTOBER 2023

Question 8

7 marks

Simultaneous Equations

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0$$

(2)

Given that C_1 and C_2 meet at points P and Q

(b) find, using algebra, the exact distance PQ

(5)

(Total for Question 8 is 7 marks)

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Worked Solution - Question 8

Simultaneous Equations

Topic group

1. Write C1 in terms of y

$xy = \frac{15}{2} - 5x$, so $y = \frac{15}{2x} - 5$.

2. Equate the two curves

$\frac{15}{2x} - 5 = x^3 - \frac{7}{2}x - 5$.

3. Multiply by 2x

$15 = 2x^4 - 7x^2$, hence $2x^4 - 7x^2 - 15 = 0$.

4. Solve for x

Let $u = x^2$. Then $2u^2 - 7u - 15 = 0 = (2u + 3)(u - 5)$, so $x^2 = 5$ and $x = \pm\sqrt{5}$.

5. Find the y-coordinates

Using $y = \frac{15}{2x} - 5$, the points are $\left(\sqrt{5}, \frac{3\sqrt{5}}{2} - 5\right)$ and $\left(-\sqrt{5}, -\frac{3\sqrt{5}}{2} - 5\right)$.

6. Use the distance formula

$\Delta x = 2\sqrt{5}$ and $\Delta y = 3\sqrt{5}$, so $PQ = \sqrt{(2\sqrt{5})^2 + (3\sqrt{5})^2} = \sqrt{65}$.

Final answer

$(a)\ 2x^4 - 7x^2 - 15 = 0.$

$(b)\ PQ = \sqrt{65}.$

Question 3

5 marks

Simultaneous Equations

3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

(Total for Question 3 is 5 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 3

Simultaneous Equations

Topic group

1. Use t equals 5

The population is 58 thousand, so $58^2 = a + b(5^3)$, giving $3364 = a + 125b$.

2. Use t equals 10

The population is 65 thousand, so $65^2 = a + b(10^3)$, giving $4225 = a + 1000b$.

3. Solve for b

Subtracting gives $861 = 875b$, so $b = \dfrac{861}{875} = \dfrac{123}{125}$.

4. Solve for a

$a = 3364 - 125\left(\dfrac{123}{125}\right) = 3241$.

5. Use population 85 thousand

$85^2 = 3241 + \dfrac{123}{125}T^3$.

6. Find T

$T^3 = \dfrac{125(7225 - 3241)}{123}$, so $T \approx 15.938$ and $T = 15.9$ to one decimal place.

Final answer

$(a)\ a = 3241,\ b = \dfrac{123}{125}$.

$(b)\ T = 15.9$ years.

WMA11/01 JANUARY 2026

Question 2

6 marks

Simultaneous Equations

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Given

$$y - x = 4$$

and

$$2x^2 - xy = 8$$

show that

$$x^2 - 4x - 8 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$y - x = 4$$

$$2x^2 - xy = 8$$

writing the answers in the form $p + q\sqrt{3}$, where p and q are integers to be found.

(4)

(Total for Question 2 is 6 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 2

Simultaneous Equations

Topic group

1. Use the linear equation

$y - x = 4$, so $y = x + 4$.

2. Substitute into the second equation

$2x^2 - xy = 8$ becomes $2x^2 - x(x + 4) = 8$.

3. Show the quadratic

$2x^2 - x^2 - 4x = 8$, so $x^2 - 4x - 8 = 0$.

4. Complete the square

$x^2 - 4x - 8 = 0$ gives $(x - 2)^2 = 12$.

5. Find x

$x = 2 \pm \sqrt{12} = 2 \pm 2\sqrt{3}$.

6. Find y

Since $y = x + 4$, $y = 6 \pm 2\sqrt{3}$ with matching signs.

7. State both solutions

$(x, y) = (2 + 2\sqrt{3}, 6 + 2\sqrt{3})$ or $(2 - 2\sqrt{3}, 6 - 2\sqrt{3})$.

Final answer

$(x, y) = (2 + 2\sqrt{3},\ 6 + 2\sqrt{3})$ or $(2 - 2\sqrt{3},\ 6 - 2\sqrt{3})$.

TOPIC

Inequalities

WMA11/01 JANUARY 2020

Question 8

6 marks

Inequalities

8. The straight line l has equation $y = k(2x - 1)$, where k is a constant.

The curve C has equation $y = x^2 + 2x + 11$

Find the set of values of k for which l does not cross or touch C.

(6)

(Total 6 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 8

Inequalities

Topic group

1. Equate the line and curve

$k(2x-1) = x^2 + 2x + 11$, so $x^2 + (2-2k)x + (11+k) = 0$.

2. Use the no-intersection condition

For the line not to cross or touch the curve, the quadratic in x must have no real roots, so its discriminant is less than zero.

3. Form the inequality in k

$(2-2k)^2 - 4(1)(11+k) < 0$. Dividing by 4 gives $(1-k)^2 - (11+k) < 0$, so $k^2 - 3k - 10 < 0$.

4. Solve the quadratic inequality

$(k-5)(k+2) < 0$, so k lies between the two roots: $-2

Final answer

$-2

WMA11/01 MAY/JUNE 2023

Question 1

4 marks

Inequalities

1. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the inequality

$$4x^2 - 3x + 7 \geqslant 4x + 9$$

(4)

(Total for Question 1 is 4 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 1

Inequalities

Topic group

1. Move all terms to one side

$4x^2 - 3x + 7 \geq 4x + 9$ gives $4x^2 - 7x - 2 \geq 0$.

2. Factorise

$4x^2 - 7x - 2 = (4x + 1)(x - 2)$.

3. Use the sign of the quadratic

The roots are $x = -\frac{1}{4}$ and $x = 2$. Since the quadratic opens upwards, it is non-negative outside the roots.

Final answer

$x \leq -\frac{1}{4}$ or $x \geq 2$.

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Question 7

6 marks

Inequalities

7.

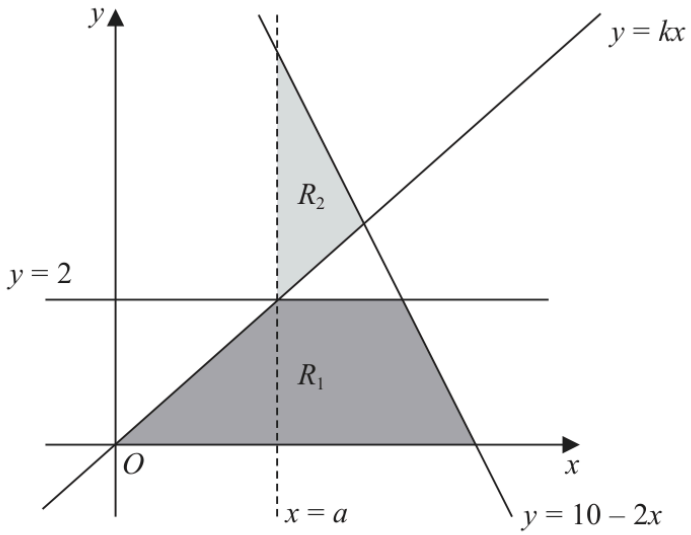


Figure 2

The region R_1, shown shaded in Figure 2, is defined by the inequalities

$$0 \leqslant y \leqslant 2 \qquad y \leqslant 10 - 2x \qquad y \leqslant kx$$

where k is a constant.

The line $x = a$, where a is a constant, passes through the intersection of the lines $y = 2$ and $y = kx$

Given that the area of R_1 is $\frac{27}{4}$ square units,

(a) find

(i) the value of a

(ii) the value of k

(4)

(b) Define the region R_2, also shown shaded in Figure 2, using inequalities.

(2)

(Total for Question 7 is 6 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 7

Inequalities

Topic group

1. Find the area of R1

The bottom horizontal length is 5, from $x = 0$ to the intercept of $y = 10 - 2x$ at $x = 5$.

2. Use the top length

The top horizontal length is $4 - a$, because $y = 2$ meets $y = 10 - 2x$ at $x = 4$.

3. Solve for a

Area $R_1 = \frac{1}{2}(5 + 4 - a)(2) = 9 - a = \frac{27}{4}$, so $a = \frac{9}{4}$.

4. Find k

The line $x = a$ passes through the intersection of $y = 2$ and $y = kx$, so $2 = ka$. Hence $k = \frac{2}{9/4} = \frac{8}{9}$.

5. Define R2

Region R_2 is to the right of $x = a$, above $y = kx$, and below $y = 10 - 2x$.

Final answer

$(a)\ a = \frac{9}{4},\ k = \frac{8}{9}.$

$(b)\ x \geq \frac{9}{4},\ y \geq \frac{8}{9}x,\ y \leq 10 - 2x.$

WMA11/01 MAY/JUNE 2024

Question 6

7 marks

Inequalities

6.

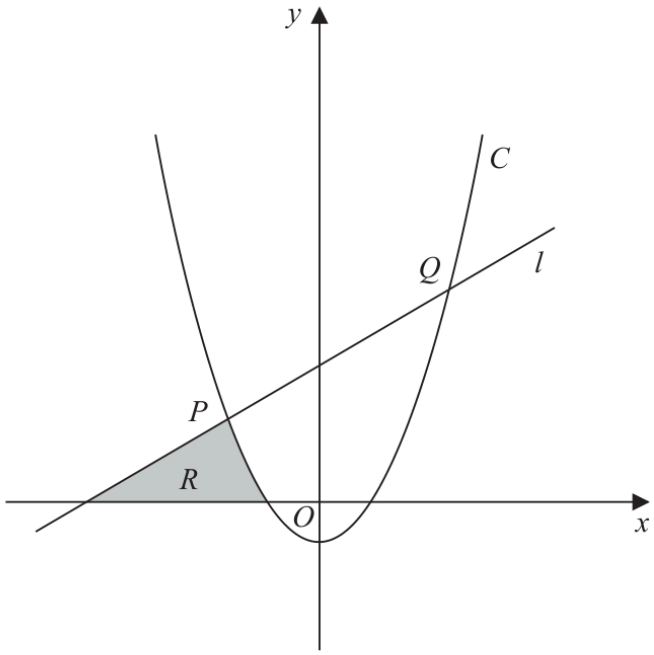


Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(Total for Question 6 is 7 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 6

Inequalities

Topic group

1. Write the line as y equals

$y - 5x = 75$, so $y = 5x + 75$.

2. Equate line and curve

$5x + 75 = 2x^2 + x - 21$, so $2x^2 - 4x - 96 = 0$.

3. Solve the quadratic

$x^2 - 2x - 48 = 0 = (x - 8)(x + 6)$, so $x = -6$ or $x = 8$.

4. Find the y-coordinates

Using $y = 5x + 75$, $x = -6$ gives $y = 45$ and $x = 8$ gives $y = 115$.

5. State P and Q

From the diagram, $P = (-6, 45)$ and $Q = (8, 115)$.

6. Use the x-axis and left branch

The shaded region is above the x-axis and on the left part of the diagram, so $y \geq 0$ and $x \leq 0$.

7. Use the line and curve as upper boundaries

The region is below both the line and the curve, so $y \leq 5x + 75$ and $y \leq 2x^2 + x - 21$.

Final answer

$(a)\ P = (-6, 45),\ Q = (8, 115).$

$(b)\ y \geq 0,\ y \leq 5x + 75,\ y \leq 2x^2 + x - 21,\ x \leq 0.$

WMA11/01 JANUARY 2026

Question 3

5 marks

Inequalities

3. (i) Find the range of values of z for which

$$\frac{4}{z} < 8$$

giving the answer in simplest form.

(2)

(ii)

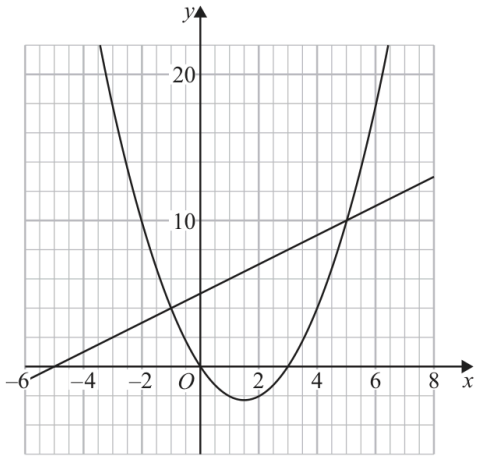


Figure 1

Figure 1 shows a plot of

- the line with equation $y = x + 5$
- the curve with equation $y = x(x - 3)$

On the opposite page there is a copy of Figure 1 labelled Diagram 1.
On Diagram 1 represent the following inequality graphically.

$$x + 5 \leqslant y \leqslant x(x - 3) \quad \text{for} \quad x \geqslant -2$$

(3)

Question 3 continued

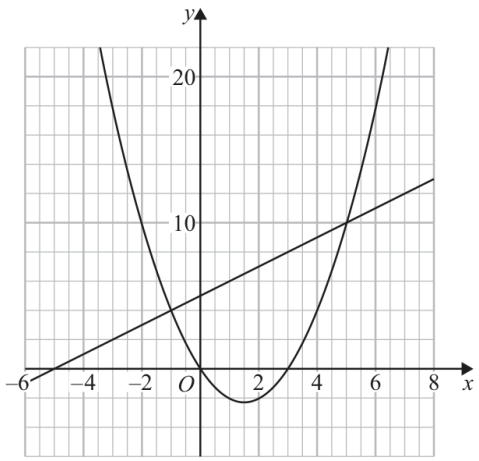


Diagram 1

(Total for Question 3 is 5 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 3

Inequalities

Topic group

1. Solve the rational inequality

$\frac{4}{z} < 8$ gives $\frac{4}{z} - 8 < 0$, so $\frac{4 - 8z}{z} < 0$.

2. Use critical values

The critical values are $z = 0$ and $z = \frac{1}{2}$.

3. State the range

A sign check gives $z < 0$ or $z > \frac{1}{2}$.

4. Find where line and curve meet

$x + 5 = x(x - 3)$ gives $x^2 - 4x - 5 = 0$, so $(x - 5)(x + 1) = 0$.

5. Combine with x greater than or equal to -2

The inequality $x + 5 \leq y \leq x(x - 3)$ is possible when $x \leq -1$ or $x \geq 5$. With $x \geq -2$, the shaded x-ranges are $-2 \leq x \leq -1$ and $x \geq 5$.

6. Graphical representation

Draw the vertical boundary $x = -2$ and shade the two regions between the line and the parabola: from $x = -2$ to $x = -1$, and to the right of $x = 5$.

Final answer

(i) $z < 0$ or $z > \frac{1}{2}$. (ii) shade the region above $y = x + 5$, below $y = x(x - 3)$, with $x \geq -2$.

TOPIC

Polynomials

WMA11/01 MAY/JUNE 2021

Question 2

10 marks

Polynomials

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3\left(9^{3z}\right) + 26\left(9^{2z}\right) - 9\left(9^{z}\right) = 0$

(5)

(Total 10 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 2

Polynomials

Topic group

1. Use f(-1) = 32

$f(-1) = -a + (6a + 8) + a^2 = a^2 + 5a + 8$. Since $f(-1) = 32$, $a^2 + 5a - 24 = 0$.

2. Choose the positive value of a

$a^2 + 5a - 24 = (a - 3)(a + 8) = 0$. Since a is positive, the only possible value is $a = 3$.

3. Solve f(x) = 0

Using $a = 3$,
$f(x) = 3x^3 + 26x^2 - 9x = x(3x^2 + 26x - 9) = x(3x - 1)(x + 9)$.

4. List the roots

$x = 0$, $x = \frac{1}{3}$, or $x = -9$.

5. Solve the equation in y

Let $u = y^{1/3}$. Then $y = u^3$ and $y^{2/3} = u^2$, so $3y + 26y^{2/3} - 9y^{1/3} = 0$ becomes $3u^3 + 26u^2 - 9u = 0$.

6. Convert back to y

From part (a), $u = -9, 0, \frac{1}{3}$. Therefore $y = u^3$, giving $y = -729, 0, \frac{1}{27}$.

7. Solve the exponential equation

Let $u = 9^z$. Then $3(9^{3z}) + 26(9^{2z}) - 9(9^z) = 0$ becomes $3u^3 + 26u^2 - 9u = 0$.

8. Use positivity of 9^z

Since $9^z > 0$, the only valid root is $u = \frac{1}{3}$. Hence $9^z = \frac{1}{3}$, so $(3^2)^z = 3^{-1}$ and $2z = -1$. Therefore $z = -\frac{1}{2}$.

Final answer

$(a)(i)\ a = 3.$ $(a)(ii)\ x = -9, 0, \frac{1}{3}.$

$(b)(i)\ y = -729, 0, \frac{1}{27}.$

$(b)(ii)\ z = -\frac{1}{2}.$

WMA11/01 MAY/JUNE 2021

Question 8

9 marks

Polynomials

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

(Total 9 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 8

Polynomials

Topic group

1. Complete the square

$3x^2+6x+9=3(x^2+2x)+9=3\big((x+1)^2-1\big)+9=3(x+1)^2+6$.

2. Deduce the minimum point

The minimum occurs when $(x+1)^2=0$, so $x=-1$ and $y=6$. Therefore $P=(-1,6)$.

3. Use the x-intercepts of C2

Since C_2 intersects the x-axis at $-4, -2$ and 3, it has form $y=k(x+4)(x+2)(x-3)$.

4. Use point P to find k

Substitute $P=(-1,6)$: $6=k(3)(1)(-4)=-12k$, so $k=-\frac{1}{2}$.

5. Expand the cubic

$(x+4)(x+2)(x-3)=(x^2+6x+8)(x-3)=x^3+3x^2-10x-24$.

6. Read off A, B, C and D

Thus $y=-\frac{1}{2}x^3-\frac{3}{2}x^2+5x+12$, so $A=-\frac{1}{2}$, $B=-\frac{3}{2}$, $C=5$ and $D=12$.

Final answer

$(a)\ 3(x+1)^2+6.$

$(b)\ P=(-1,6).$

$(c)\ A=-\frac{1}{2},\ B=-\frac{3}{2},\ C=5,\ D=12.$

WMA11/01 OCTOBER 2024

Question 4

10 marks

Polynomials

4.

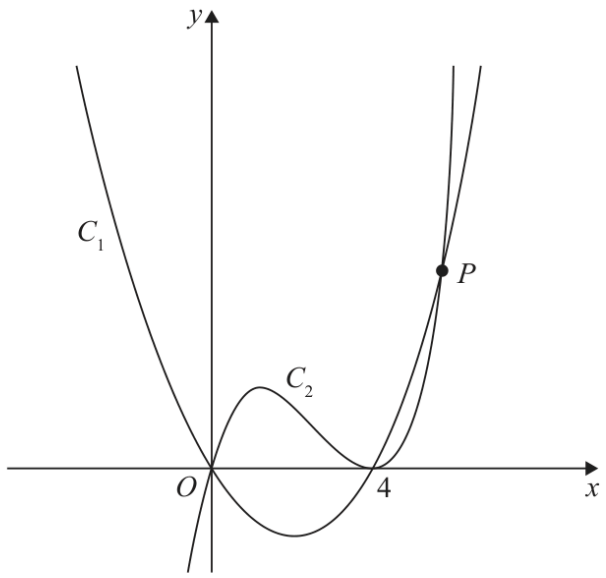


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a quadratic function
- cuts the x-axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $f(x)$

(3)

Given that C_2

- has equation $y = g(x)$ where $g(x)$ is a cubic function
- cuts the x-axis at the origin and meets the x-axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $g(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P, shown in Figure 1.

(c) Use algebra to find the coordinates of P.

(4)

(Total for Question 4 is 10 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 4

Polynomials

Topic group

1. Use the roots of C1

Since C_1 cuts the x-axis at 0 and 4, let $f(x) = ax(x-4)$.

2. Use the minimum point of C1

At $x = 2$, $f(2) = -4a = -4.8 = -\frac{24}{5}$, so $a = \frac{6}{5}$.

3. Write f(x)

$f(x) = \frac{6}{5}x(x-4)$.

4. Use the roots of C2

The cubic cuts the x-axis at the origin and meets the x-axis at $x = 4$, so $x = 4$ is a repeated root. Let $g(x) = bx(x-4)^2$.

5. Use the point (6, 7.2)

$7.2 = b(6)(2^2) = 24b$, so $b = 0.3 = \frac{3}{10}$.

6. Equate the curves

$\frac{6}{5}x(x-4) = \frac{3}{10}x(x-4)^2$.

7. Find the first-quadrant intersection

Ignoring the shared roots on the x-axis, divide by $x(x-4)$ to get $\frac{6}{5} = \frac{3}{10}(x-4)$. Hence $x = 8$.

8. Find y

$y = f(8) = \frac{6}{5}(8)(4) = \frac{192}{5}$.

Final answer

$(a)\ f(x) = \frac{6}{5}x(x-4).$

$(b)\ g(x) = \frac{3}{10}x(x-4)^2.$

$(c)\ P = (8, \frac{192}{5}).$

WMA11/01 JANUARY 2026

Question 6

9 marks

Polynomials

6.

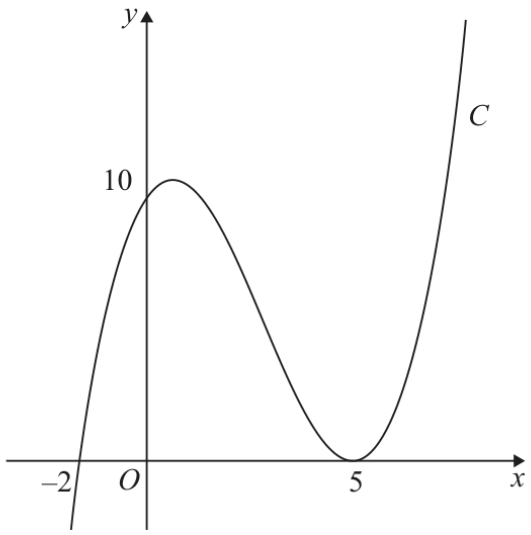


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$ where $f(x)$ is a cubic function in x.

The curve C

- cuts the x-axis at $(–2, 0)$ and cuts the y-axis at $(0, 10)$
- touches the x-axis at $(5, 0)$

as shown in Figure 3.

(a) Deduce the roots of the equation

(i) $f\left(\frac{1}{3}x\right) = 0$

(ii) $f(x – 3) = 0$

(2)

(b) Find an expression for $f(x)$. You should leave your answer in factorised form.

(3)

The curve C intersects the straight line $y = 10(x + 2)$ at exactly three points.

(c) Use algebra to find the exact x coordinates of the three points of intersection.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

(Total for Question 6 is 9 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 6

Polynomials

Topic group

1. Roots of f one third x

The roots of $f(x) = 0$ are $x = -2$ and $x = 5$, with $x = 5$ repeated. For $f\left(\frac{1}{3}x\right) = 0$, set $\frac{1}{3}x = -2$ or 5, giving $x = -6$ or 15.

2. Roots of f of x minus 3

For $f(x - 3) = 0$, set $x - 3 = -2$ or 5, giving $x = 1$ or 8.

3. Use the roots and repeated root

Since the curve cuts at $(-2, 0)$ and touches at $(5, 0)$, write $f(x) = k(x + 2)(x - 5)^2$.

4. Use the y-intercept

The curve passes through $(0, 10)$, so $10 = k(2)(25) = 50k$.

5. State f(x)

$k = \frac{1}{5}$, hence $f(x) = \frac{1}{5}(x + 2)(x - 5)^2$.

6. Set equal to the line

$\frac{1}{5}(x + 2)(x - 5)^2 = 10(x + 2)$.

7. First solution

The factor $x + 2$ gives one solution $x = -2$.

8. Find the other solutions

For $x \neq -2$, divide by $x + 2$: $\frac{1}{5}(x - 5)^2 = 10$, so $(x - 5)^2 = 50$.

9. State all intersections

$x = 5 \pm \sqrt{50} = 5 \pm 5\sqrt{2}$. Therefore the three x-coordinates are -2, $5 - 5\sqrt{2}$ and $5 + 5\sqrt{2}$.

Final answer

$(a)(i)\ -6, 15.\quad (a)(ii)\ 1, 8.$

$(b)\ f(x) = \frac{1}{5}(x+2)(x-5)^2.$

$(c)\ x = -2,\ 5 \pm 5\sqrt{2}.$

TOPIC

Graphs of Functions

WMA11/01 OCTOBER 2019

Question 3

10 marks

Graphs of Functions

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

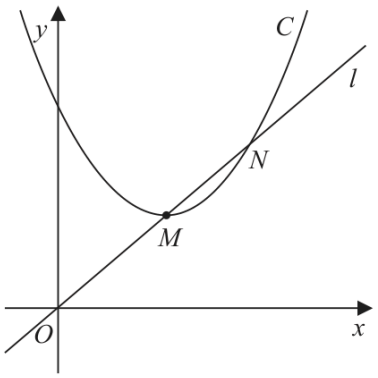


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = x^2 - 5x + 13$

The point M is the minimum point of C.

The straight line l passes through the origin O and intersects C at the points M and N as shown.

Find, showing your working,

(a) the coordinates of M,

(3)

(b) the coordinates of N.

(5)

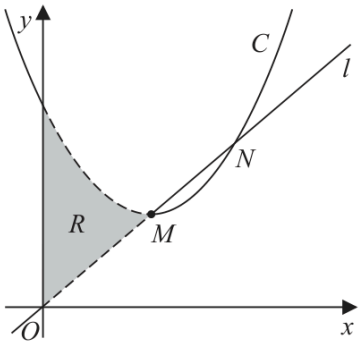


Figure 3

Figure 3 shows the curve C and the line l. The finite region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(Total 10 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 3

Graphs of Functions

Topic group

1. Complete the square to locate the minimum

$y = x^2 - 5x + 13 = \left(x - \frac{5}{2}\right)^2 + \frac{27}{4}$. Therefore $M = \left(\frac{5}{2}, \frac{27}{4}\right)$.

2. Find the equation of the line through O and M

The gradient of OM is $\dfrac{27/4}{5/2} = \dfrac{27}{10}$, so l has equation $y = \frac{27}{10}x$.

3. Find the second intersection

Solve $x^2 - 5x + 13 = \frac{27}{10}x$. This gives $x^2 - \frac{77}{10}x + 13 = 0$. One root is $x = \frac{5}{2}$, so the other is $x = \frac{26}{5}$. Then $y = \frac{27}{10} \cdot \frac{26}{5} = \frac{351}{25}$.

4. Describe the shaded region

The region is between the y-axis, the line, and the curve up to M: $0 \le x \le \frac{5}{2}$ and $\frac{27}{10}x \le y \le x^2 - 5x + 13$.

Final answer

$M = \left(\frac{5}{2}, \frac{27}{4}\right)$, $N = \left(\frac{26}{5}, \frac{351}{25}\right)$, and $0 \le x \le \frac{5}{2}$, $\frac{27}{10}x \le y \le x^2 - 5x + 13$.

WMA11/01 OCTOBER 2019

Question 6

8 marks

Graphs of Functions

6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.

(a) Sketch a graph of C, stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x-axis.

(3)

The line with equation $y = 10 - 2x$ is a tangent to C.

(b) Find the possible values for k.

(5)

(Total 8 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Read the graph features

For $y = \frac{4}{x} + k$, the horizontal asymptote is $y = k$. Setting $y = 0$ gives $0 = \frac{4}{x} + k$, so the x-intercept is $x = -\frac{4}{k}$.

2. Set the tangent line equal to the curve

A tangent touches once, so solve $10 - 2x = \frac{4}{x} + k$. Multiplying by x gives $2x^2 + (k - 10)x + 4 = 0$.

3. Use the discriminant condition

For one repeated intersection, discriminant $= 0$: $(k - 10)^2 - 4(2)(4) = 0$. Hence $(k - 10)^2 = 32$.

4. Find k

$k - 10 = \pm 4\sqrt{2}$, so $k = 10 \pm 4\sqrt{2}$. Both are positive, so both satisfy the condition in the question.

Final answer

Asymptote $y = k$, x-intercept $\left(-\frac{4}{k}, 0\right)$, and $k = 10 \pm 4\sqrt{2}$.

WMA11/01 OCTOBER 2019

Question 10

10 marks

Graphs of Functions

10.

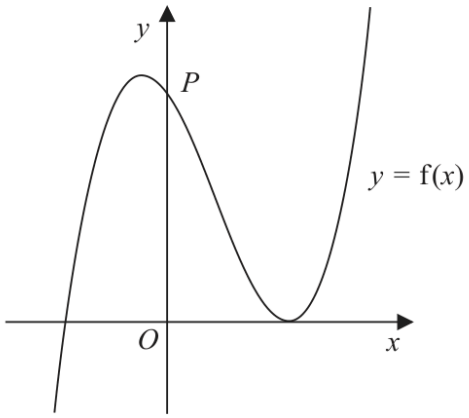


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

(Total 10 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 10

Graphs of Functions

Topic group

1. Use the factorised form for the inequality

$f(x) = (2x+5)(x-3)^2$. Since $(x-3)^2 \geq 0$, the sign is controlled by $2x+5$, with an extra zero at $x = 3$. Hence $f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$.

2. Expand the cubic

$(x-3)^2 = x^2 - 6x + 9$, so
$f(x) = (2x+5)(x^2-6x+9) = 2x^3 - 7x^2 - 12x + 45$.

3. Find P and the gradient at P

At the y-axis, $x = 0$, so $P = (0, 45)$. Also $f'(x) = 6x^2 - 14x - 12$, so the gradient at P is $f'(0) = -12$.

4. Translate the graph

A translation 2 units in the positive x direction means $g(x) = f(x-2)$. Therefore $g(x) = (2(x-2)+5)((x-2)-3)^2 = (2x+1)(x-5)^2$.

5. Find the new y-intercept

Set $x = 0$: $g(0) = (1)(25) = 25$, so the y-intercept is 25.

Final answer

$f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$; $f(x) = 2x^3 - 7x^2 - 12x + 45$; $P = (0, 45)$; gradient $= -12$; $g(x) = (2x+1)(x-5)^2$; y-intercept 25.

WMA11/01 JANUARY 2020

Question 10

8 marks

Graphs of Functions

10. The curve C_1 has equation $y = f(x)$, where

$$f(x) = (4x - 3)(x - 5)^2$$

(a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes.

(3)

(b) Hence or otherwise

(i) find the values of x for which $f\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant p such that the curve with equation $y = f(x) + p$ passes through the origin.

(2)

A second curve C_2 has equation $y = g(x)$, where $g(x) = f(x + 1)$

(c) (i) Find, in simplest form, $g(x)$. You may leave your answer in a factorised form.

(ii) Hence, or otherwise, find the y intercept of curve C_2

(3)

(Total 8 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 10

Graphs of Functions

Topic group

1. Use the factorised form for the sketch

$f(x) = (4x - 3)(x - 5)^2$. The curve crosses the x-axis at $x = \frac{3}{4}$ and touches the x-axis at $x = 5$. The y-intercept is $f(0) = (-3)(25) = -75$.

2. Solve f(x/4)=0

For $f\left(\frac{1}{4}x\right) = 0$, either $4\left(\frac{x}{4}\right) - 3 = 0$ or $\frac{x}{4} - 5 = 0$. Therefore $x = 3$ or $x = 20$.

3. Find p

The curve $y = f(x) + p$ passes through the origin, so $0 = f(0) + p = -75 + p$. Hence $p = 75$.

4. Find g(x)

$g(x) = f(x + 1) = (4(x + 1) - 3)((x + 1) - 5)^2 = (4x + 1)(x - 4)^2$.

5. Find the y-intercept of C2

At $x = 0$, $g(0) = (1)(16) = 16$. So the y-intercept is 16.

Final answer

Sketch: crosses at $\left(\frac{3}{4}, 0\right)$, touches at $(5, 0)$, crosses y-axis at $(0, -75)$.
Also $x = 3, 20$, $p = 75$, $g(x) = (4x + 1)(x - 4)^2$, y-intercept 16.

WMA11/01 JANUARY 2021

Question 6

10 marks

Graphs of Functions

6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

(Total 10 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Sketch y = -k/x

Because $k > 0$, $y = -\frac{k}{x}$ has one branch in quadrant II and one branch in quadrant IV, with asymptotes $x = 0$ and $y = 0$.

2. Translate the graph upward

$y = -\frac{k}{x} + k$ is $y = -\frac{k}{x}$ translated up by k units. The vertical asymptote is still $x = 0$, and the horizontal asymptote is $y = k$.

3. Find the x-intercept

Set $y = 0$: $0 = -\frac{k}{x} + k$. Since $k > 0$, $\frac{k}{x} = k$, so $x = 1$. The x-intercept is $(1, 0)$.

4. Set up the intersection equation

For intersections with $y = 3x + 4$, solve $-\frac{k}{x} + k = 3x + 4$. Multiplying by x gives $-k + kx = 3x^2 + 4x$.

5. Use the discriminant

This rearranges to $3x^2 + (4 - k)x + k = 0$. For no touch or intersection, the discriminant must be negative: $(4 - k)^2 - 12k < 0$.

6. Solve the quadratic inequality in k

$k^2 - 20k + 16 < 0$. The roots are $k = 10 \pm 2\sqrt{21}$, so the required range is $10-2\sqrt{21}

Final answer

$(b)\ (1,0),\ y=k.$

$(c)\ 10-2\sqrt{21}$

Question 8

9 marks

Graphs of Functions

8.

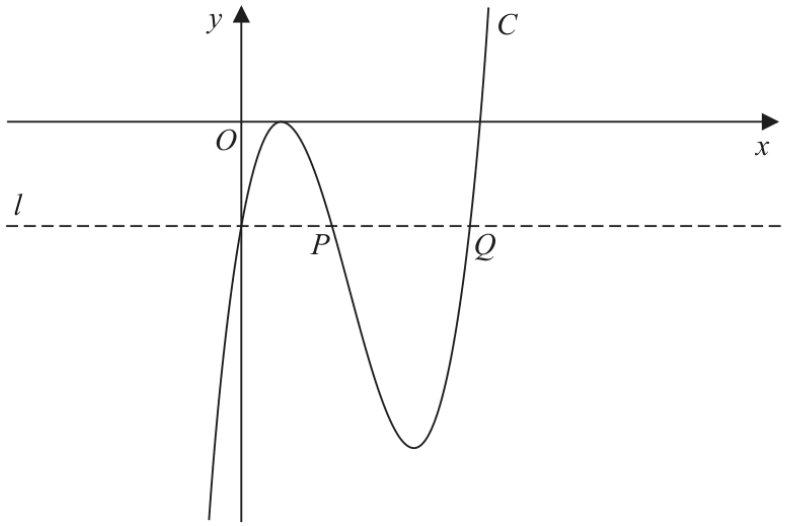


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x - 2)^2 (x - 4)$$

(a) Deduce the values of x for which $f(x) > 0$

(1)

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

The line l, also shown in Figure 4, passes through the y intercept of C and is parallel to the x-axis.

The line l cuts C again at points P and Q, also shown in Figure 4.

(c) Using algebra and showing your working, find the length of line PQ. Write your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total 9 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 8

Graphs of Functions

Topic group

1. Use the factorised form

$f(x) = (3x-2)^2(x-4)$. The squared factor is never negative, and it is zero at $x = \frac{2}{3}$. Therefore the sign of $f(x)$ away from the repeated root is controlled by $x-4$.

2. Find where f(x) is positive

For $f(x) > 0$, we need $x - 4 > 0$, so $x > 4$.

3. Expand f(x)

$(3x-2)^2 = 9x^2 - 12x + 4$. Therefore
$f(x) = (9x^2 - 12x + 4)(x-4) = 9x^3 - 48x^2 + 52x - 16$.

4. Find the line l

The y-intercept of C is $f(0) = -16$, so the horizontal line l is $y = -16$.

5. Find the other intersections with l

Solve $f(x) = -16$: $9x^3 - 48x^2 + 52x - 16 = -16$, so $x(9x^2 - 48x + 52) = 0$. The root $x = 0$ is the y-intercept, and the two other roots give P and Q.

6. Solve the quadratic

$9x^2 - 48x + 52 = 0$. Hence

$$x = \frac{48 \pm \sqrt{48^2 - 4(9)(52)}}{18} = \frac{48 \pm 12\sqrt{3}}{18} = \frac{8 \pm 2\sqrt{3}}{3}.$$

7. Find PQ

Because P and Q lie on the same horizontal line, PQ is the difference in their x-coordinates: $PQ = \frac{8+2\sqrt{3}}{3} - \frac{8-2\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$.

Final answer

(a) $x > 4$.

(b) $f(x) = 9x^3 - 48x^2 + 52x - 16$.

(c) $PQ = \dfrac{4\sqrt{3}}{3}$.

WMA11/01 MAY/JUNE 2021

Question 5

8 marks

Graphs of Functions

5.

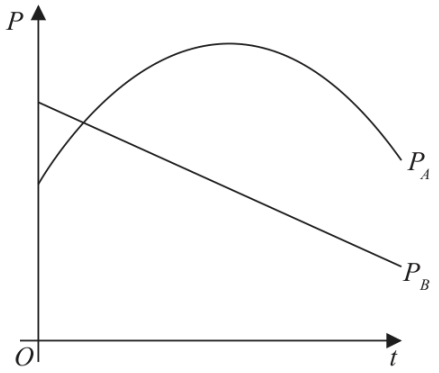


Figure 2

The share value of two companies, company A and company B, has been monitored over a 15-year period.

The share value P_A of **company *A***, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

The share value P_B of **company *B***, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

(a) Find the difference between the share value of **company *A*** and the share value of **company *B*** at the point monitoring began.

(2)

(b) State the maximum share value of **company *A*** during the 15-year period.

(1)

(c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company *A*** was greater than the share value of **company *B***.

(4)

(d) Explain why the model for **company *A*** should not be used to predict its share value when $t = 20$

(1)

(Total 8 marks)

WMA11/01 MAY/JUNE 2021

Graphs of Functions

Topic group

Worked Solution - Question 5

1. Evaluate both models at t = 0

$P_A(0) = 53 - 0.4(0-8)^2 = 53 - 25.6 = 27.4$. Also $P_B(0) = -1.6(0) + 44.2 = 44.2$.

2. Find the initial difference

The difference is $44.2 - 27.4 = 16.8$, so the share values differ by 16.8 million pounds.

3. State the maximum of company A

$P_A = 53 - 0.4(t-8)^2$ has maximum value 53, reached when $t = 8$, which is within the 15-year period.

4. Set up the inequality for company A greater than company B

$53 - 0.4(t-8)^2 > -1.6t + 44.2$.

5. Rearrange to a quadratic inequality

Expanding gives $-0.4t^2 + 8t - 16.8 > 0$. Multiplying by -2.5 reverses the inequality: $t^2 - 20t + 42 < 0$.

6. Find the crossing times

$t = \dfrac{20 \pm \sqrt{400 - 168}}{2} = 10 \pm \sqrt{58}$. Therefore company A is greater between the two roots.

7. Restrict to the 15-year period

Since the model is monitored over $0 \le t \le 15$, the required times are $10-\sqrt{58}$

8. Explain the limitation

The model should not be used at $t = 20$ because it was only built from a 15-year monitoring period; 20 years is outside that range.

Final answer

(a) 16.8 million pounds.

(b) 53 million pounds.

$(c)\ 10-\sqrt{58}$

(d) $t = 20$ is outside the 15-year model.

WMA11/01 OCTOBER 2021

Question 6

10 marks

Graphs of Functions

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2(x + 1)(x - 3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $\mathrm{f}(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

(Total 10 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Use the factorised form for the sketch

$f(x) = 2(x+1)(x-3)^2$. The curve cuts the x-axis at $(-1, 0)$ and meets the x-axis at $(3, 0)$ because $x = 3$ is a repeated root.

2. Find the y-intercept

$f(0) = 2(1)(9) = 18$, so the y-intercept is $(0, 18)$.

3. Describe the cubic shape

The leading term is positive, so the cubic goes down to the left and up to the right. It crosses at $x = -1$ and touches at $x = 3$.

4. Expand f(x)

$(x+1)(x-3)^2 = (x+1)(x^2 - 6x + 9) = x^3 - 5x^2 + 3x + 9$, so $f(x) = 2x^3 - 10x^2 + 6x + 18$.

5. Differentiate

$f'(x) = 6x^2 - 20x + 6$.

6. Find the gradient at x = 1/3

$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{9}\right) - 20\left(\frac{1}{3}\right) + 6 = 0$, so the tangent is horizontal.

7. Find the y-coordinate

$$f\left(\frac{1}{3}\right) = 2\left(\frac{4}{3}\right)\left(-\frac{8}{3}\right)^2 = \frac{512}{27}.$$

8. Write the tangent

A horizontal tangent through this point has equation $y = \frac{512}{27}$.

Final answer

(a) x-intercepts $(-1, 0), (3, 0)$, y-intercept $(0, 18)$.

(b) $f(x) = 2x^3 - 10x^2 + 6x + 18$.

(c) $y = \dfrac{512}{27}$.

WMA11/01 JANUARY 2022

Question 4

7 marks

Graphs of Functions

4.

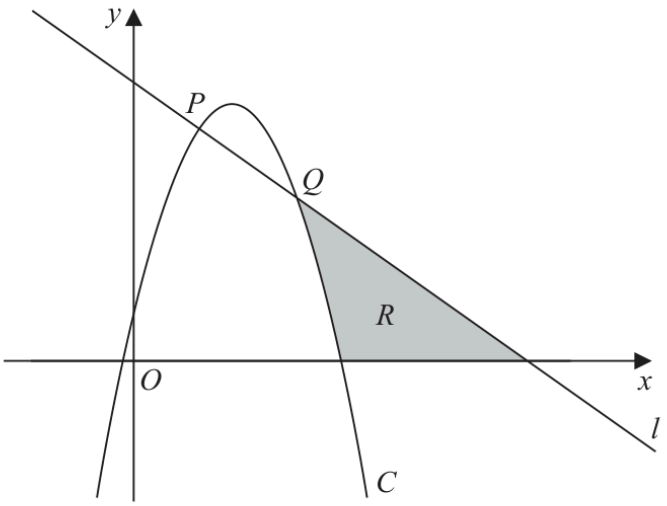


Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(Total 7 marks)

WMA11/01 JANUARY 2022

Worked Solution - Question 4

Graphs of Functions

Topic group

1. Set line equal to curve

The line $x + y = 6$ has equation $y = 6 - x$. Intersections satisfy $6 - x = 6x - 2x^2 + 1$.

2. Solve the quadratic

Rearranging gives $2x^2 - 7x + 5 = 0$, so $(2x - 5)(x - 1) = 0$. Therefore $x = 1$ or $x = \frac{5}{2}$.

3. Find the coordinates

Using $y = 6 - x$, the points are $P = (1, 5)$ and $Q = \left(\frac{5}{2}, \frac{7}{2}\right)$.

4. Read the bounded region

Region R is above the x-axis, below the line, and above the curve.

5. Write the inequalities

Thus $y \geq 0$, $y \leq 6 - x$, and $y \geq 6x - 2x^2 + 1$.

Final answer

$(a)\ P = (1, 5),\ Q = \left(\frac{5}{2}, \frac{7}{2}\right).$

$(b)\ y \geq 0,\ y \leq 6 - x,\ y \geq 6x - 2x^2 + 1.$

WMA11/01 JANUARY 2022

Question 10

9 marks

Graphs of Functions

10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

(a) Sketch the graph of C.

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

(b) find the range of possible values for k.

(5)

(Total 9 marks)

WMA11/01 JANUARY 2022

Worked Solution - Question 10

Graphs of Functions

Topic group

1. Find intercepts

For x-intercepts, set $\frac{1}{x^2} - 9 = 0$. Then $x^2 = \frac{1}{9}$, so $x = \pm\frac{1}{3}$. There is no y-intercept because $x = 0$ is not in the domain.

2. State asymptotes

The vertical asymptote is $x = 0$. As $|x|$ becomes large, $\frac{1}{x^2} \to 0$, so the horizontal asymptote is $y = -9$.

3. Set up intersections with D

For intersections with $y = kx^2$, solve $\frac{1}{x^2} - 9 = kx^2$.

4. Use u = x squared

Let $u = x^2$, where $u > 0$. Multiplying by x^2 gives $1 - 9u = ku^2$, so $ku^2 + 9u - 1 = 0$.

5. Require two positive u-values

Four distinct x-values require two distinct positive values of u, since each positive u gives $x = \pm\sqrt{u}$.

6. Find the range of k

For two positive roots, the product $-\frac{1}{k}$ and sum $-\frac{9}{k}$ must both be positive, so $k < 0$. Also the discriminant must be positive: $81 + 4k > 0$, so $k > -\frac{81}{4}$.

Therefore $-\dfrac{81}{4}

Final answer

(a) x-intercepts $\left(-\frac{1}{3}, 0\right), \left(\frac{1}{3}, 0\right)$; asymptotes $x = 0,\ y = -9.$

$(b)\ -\dfrac{81}{4}

WMA11/01 OCTOBER 2022

Question 6

6 marks

Graphs of Functions

6. (a) Given that k is a positive constant such that $0 < k < 4$ sketch, on **separate axes**, the graphs of

(i) $y = (2x - k)(x + 4)^2$

(ii) $y = \dfrac{k}{x^2}$

showing the coordinates of any points where the graphs cross or meet the coordinate axes, leaving coordinates in terms of k, where appropriate.

(5)

(b) State, with a reason, the number of roots of the equation

$$(2x - k)(x + 4)^2 = \frac{k}{x^2}$$

(1)

(Total for Question 6 is 6 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Key points for y = (2x-k)(x+4)^2

The graph has x-intercepts $(-4, 0)$ and $\left(\frac{k}{2}, 0\right)$. The root at $x = -4$ is a repeated root, so the graph touches the x-axis there.

2. Use 0 < k < 4

Since $0

3. Key features for y = k/x^2

The graph is positive on both sides of the y-axis, with asymptotes $x = 0$ and $y = 0$. It has no intercepts with the coordinate axes.

4. Compare the two graphs

The equation $(2x - k)(x + 4)^2 = \frac{k}{x^2}$ represents intersections between the two graphs.

5. State the number of roots

From the sketches, the positive branch of $\frac{k}{x^2}$ meets the increasing positive branch of $(2x - k)(x + 4)^2$ once only. Therefore there is one root.

Final answer

The graphs intersect once, so the equation has one root.

WMA11/01 OCTOBER 2023

Question 4

7 marks

Graphs of Functions

4.

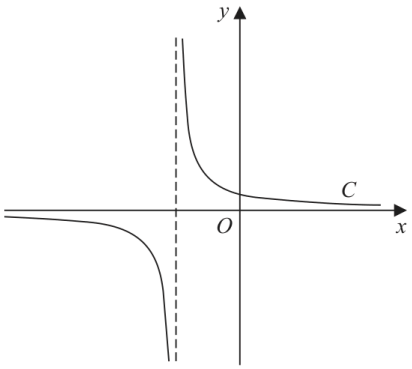


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y-axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x + 2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

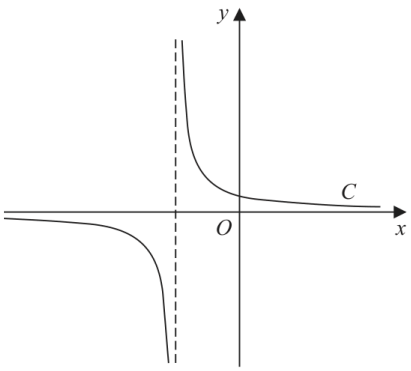
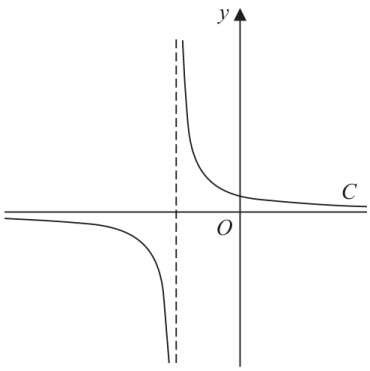


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(Total for Question 4 is 7 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 4

Graphs of Functions

Topic group

1. Find the vertical asymptote

For $y = \dfrac{1}{x+2}$, the denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$.

2. Factorise the cubic

$x^3 + 4x^2 + 4x = x(x^2 + 4x + 4) = x(x+2)^2$.

3. Use the factorised form for the sketch

$x = 0$ is a single root, so the curve crosses the x-axis at $(0, 0)$. $x = -2$ is a repeated root, so the curve touches the x-axis at $(-2, 0)$.

4. Connect the equation to intersections

$(x+2)(x^3 + 4x^2 + 4x) = 1$ is equivalent to $x^3 + 4x^2 + 4x = \dfrac{1}{x+2}$.

5. Count the intersections

The number of real solutions is the number of intersections of the cubic $y = x(x+2)^2$ and the reciprocal curve $y = \dfrac{1}{x+2}$. The sketches show 2 intersections.

Final answer

(a) $x = -2$.

(b) $x(x+2)^2$.

(c) roots at $(-2, 0)$ and $(0, 0)$. $x = -2$ is a repeated root.

(d) 2 real roots.

WMA11/01 JANUARY 2024

Question 7

9 marks

Graphs of Functions

7. (a) Sketch the graph of the curve C with equation

$$y = \frac{4}{x - k}$$

where k is a positive constant.

Show on your sketch

- the coordinates of any points where C cuts the coordinate axes
- the equation of the vertical asymptote to C

(4)

Given that the straight line with equation $y = 9 - x$ does not cross or touch C

(b) find the range of values of k.

(5)

(Total for Question 7 is 9 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 7

Graphs of Functions

Topic group

1. Identify the asymptote

For $y = \frac{4}{x-k}$, the vertical asymptote is where the denominator is zero, so $x = k$.

2. Find axis intercepts

At $x = 0$, $y = -\frac{4}{k}$, so the y-intercept is $(0, -\frac{4}{k})$. The graph has no x-intercept because the numerator is never zero.

3. Set up intersection with the line

If the line $y = 9 - x$ meets the curve, then $9 - x = \frac{4}{x-k}$.

4. Form a quadratic

$(9-x)(x-k) = 4$, which simplifies to $x^2 - (9+k)x + 9k + 4 = 0$.

5. Use no real intersection

For the line not to cross or touch the curve, the discriminant must be negative.

6. Solve the inequality

$(9+k)^2 - 4(9k+4) < 0$, so $k^2 - 18k + 65 < 0$. Since $k^2 - 18k + 65 = (k-5)(k-13)$, the range is $5

Final answer

(a) y-intercept $(0, -\frac{4}{k})$, no x-intercept, vertical asymptote $x = k$.

$(b)\ 5

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Question 8

7 marks

Graphs of Functions

8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \frac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

(1)

(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

(Total for Question 8 is 7 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 8

Graphs of Functions

Topic group

1. Find the intercepts of C1

$y = x(4 - x^2)$, so $y = 0$ when $x = 0$ or $x = \pm 2$. The curve crosses the axes at $(-2, 0)$, $(0, 0)$ and $(2, 0)$.

2. Use the cubic shape

$y = 4x - x^3$ has negative leading cubic term, so the left tail rises and the right tail falls.

3. Equate C1 and C2

$x(4 - x^2) = \dfrac{A}{x}$.

4. Multiply by x

$x^2(4 - x^2) = A$, so $4x^2 - x^4 = A$ and therefore $x^4 - 4x^2 + A = 0$.

5. Let u equal x squared

Let $u = x^2$. Then $u^2 - 4u + A = 0$.

6. Require two positive u-values

For 4 distinct x-values, the quadratic in u must have two distinct positive roots.

7. Find the range of A

The discriminant gives $16 - 4A > 0$, so $A < 4$. The product of the two roots is A, so $A > 0$. Therefore $0

Final answer

(a) x-intercepts $(-2, 0)$, $(0, 0)$, $(2, 0)$.

(b) $x^4 - 4x^2 + A = 0.$

$(c)\ 0

WMA11/01 OCTOBER 2024

Question 6

9 marks

Graphs of Functions

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{2 - x} \qquad x \neq 2$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the intersection of C with the y-axis

(3)

The straight line l has equation $y = kx - 4$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 - 5k + 4 \geqslant 0$$

(ii) find the range of possible values for k.

(6)

(Total for Question 6 is 9 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Identify the graph features

$y = \dfrac{1}{2-x}$ has vertical asymptote $x = 2$ and horizontal asymptote $y = 0$.

2. Find the y-intercept

At $x = 0$, $y = \dfrac{1}{2}$, so the y-intercept is $(0, \dfrac{1}{2})$.

3. Set up the intersection with the line

If $y = kx - 4$ cuts C, then $kx - 4 = \dfrac{1}{2-x}$.

4. Form the quadratic

$(kx - 4)(2 - x) = 1$, so $kx^2 - (2k + 4)x + 9 = 0$.

5. Use the discriminant

For at least one intersection, the discriminant is non-negative.

6. Derive the given inequality

$(2k + 4)^2 - 4(k)(9) \geq 0$, so $4(k^2 - 5k + 4) \geq 0$, hence $k^2 - 5k + 4 \geq 0$.

7. Solve the inequality

$k^2 - 5k + 4 = (k - 1)(k - 4)$, so $k \leq 1$ or $k \geq 4$.

Final answer

(a) vertical asymptote $x = 2$, y-intercept $(0, \dfrac{1}{2})$.

(b) $k \leq 1$ or $k \geq 4$.

WMA11/01 JANUARY 2025

Question 7

8 marks

Graphs of Functions

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C.

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k.

(3)

The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k.

(5)

(Total for Question 7 is 8 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 7

Graphs of Functions

Topic group

1. Identify the horizontal asymptote

$y = \frac{2}{x} - k$ has horizontal asymptote $y = -k$.

2. Find axis intersections

Setting $y = 0$ gives $\frac{2}{x} = k$, so $x = \frac{2}{k}$. There is no y-intercept because $x = 0$ is not in the domain.

3. Set up line-curve intersections

$\frac{2}{x} - k = -kx - 6$.

4. Form a quadratic

Multiplying by x gives $2 - kx = -kx^2 - 6x$, so $kx^2 + (6 - k)x + 2 = 0$.

5. Use two distinct intersections

For two distinct points, the discriminant must be positive.

6. Solve the inequality

$(6 - k)^2 - 8k > 0$, so $k^2 - 20k + 36 > 0$. Since $k^2 - 20k + 36 = (k - 2)(k - 18)$, $k < 2$ or $k > 18$.

7. Use k positive

As $k > 0$, the range is \$018\$.

Final answer

(a) x-intercept $(\frac{2}{k}, 0)$, no y-intercept, horizontal asymptote $y = -k$.

$(b)\ 018$.

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Question 7

10 marks

Graphs of Functions

7.

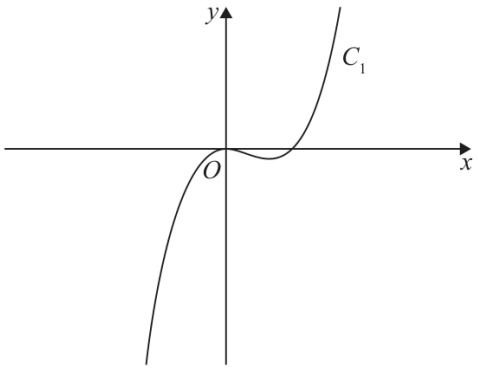


Figure 3

Figure 3 shows a sketch of part of the curve C_1

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a cubic function
- touches the x-axis at the origin and cuts the x-axis at $x = 4$
- passes through the point $(10, 120)$

(a) find $f(x)$

(3)

The curve C_2 has equation $y = 1.2x(8 - x)$

On the following page there is a copy of Figure 3 called Diagram 1.

(b) On Diagram 1 sketch a graph of the curve C_2

(2)

(c) Use algebra to find the coordinates of the points where C_1 and C_2 intersect. Show each stage of your working.

(5)

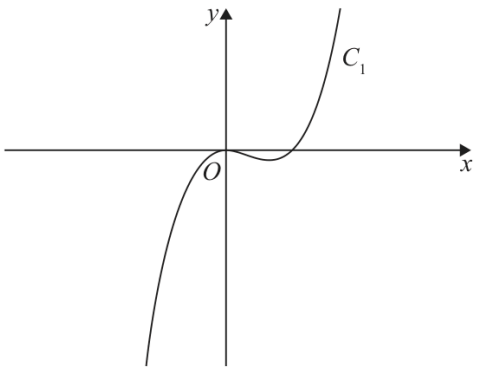


Diagram 1

(Total for Question 7 is 10 marks)

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Worked Solution - Question 7

Graphs of Functions

Topic group

1. Use the repeated root at the origin

Since the cubic touches the x-axis at the origin and cuts it at $x = 4$, write $f(x) = \lambda x^2(x - 4)$.

2. Use the point (10,120)

$120 = \lambda(10)^2(10 - 4) = 600\lambda$, so $\lambda = 0.2$.

3. State f(x)

$f(x) = 0.2x^2(x - 4)$.

4. Sketch C2

$C_2 : y = 1.2x(8 - x)$ is a downward-opening parabola through $x = 0$ and $x = 8$, with its maximum between them.

5. Set intersections equal

$1.2x(8 - x) = 0.2x^2(x - 4)$.

6. Clear decimals

Divide by 0.2 to get $6x(8 - x) = x^2(x - 4)$.

7. Factor

$48x - 6x^2 = x^3 - 4x^2$, so $x^3 + 2x^2 - 48x = 0$ and $x(x^2 + 2x - 48) = 0$.

8. Solve for x

$x = 0$, or $x^2 + 2x - 48 = 0 = (x + 8)(x - 6)$, so $x = -8$ or $x = 6$.

9. Find y-values

Using $y = 1.2x(8 - x)$ gives $(-8, -153.6)$, $(0, 0)$ and $(6, 14.4)$.

Final answer

$(a)\ f(x) = 0.2x^2(x-4).$

$(c)\ (-8, -153.6),\ (0, 0),\ (6, 14.4).$

WMA11/01 OCTOBER 2025

Question 6

9 marks

Graphs of Functions

6. (a) Sketch the graph of the curve C with equation

$$y = \frac{4k}{x - 2k}$$

where k is a positive constant.

On your sketch show

- the coordinates of any points where C cuts the coordinate axes
- the equation of the vertical asymptote to C

(4)

The straight line l has equation

$$y = 6 - 2x$$

Given that there is at least one point of intersection between l and C,

(b) find the range of possible values of k.

(5)

(Total for Question 6 is 9 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 6

Graphs of Functions

Topic group

1. Find coordinate-axis intersections

$y = \dfrac{4k}{x - 2k}$ has no x-intercept because the numerator is non-zero. At $x = 0$, $y = \dfrac{4k}{-2k} = -2$, so the y-intercept is $(0, -2)$.

2. Find the vertical asymptote

The denominator is zero when $x = 2k$, so the vertical asymptote is $x = 2k$.

3. Sketch shape

The curve has one branch in quadrant I to the right of $x = 2k$, and one branch passing through $(0, -2)$ and approaching the x-axis from below as $x \to -\infty$.

4. Set line and curve equal

$\dfrac{4k}{x - 2k} = 6 - 2x$.

5. Form a quadratic in x

$4k = (6 - 2x)(x - 2k) = 6x - 12k - 2x^2 + 4kx$, so $2x^2 - (4k + 6)x + 16k = 0$.

6. Use at least one intersection

For at least one real intersection, the discriminant must satisfy $b^2 - 4ac \geq 0$.

7. Find the discriminant

$(4k + 6)^2 - 4(2)(16k) \geq 0$, so $16k^2 - 80k + 36 \geq 0$.

8. Solve in k

$4k^2 - 20k + 9 \geq 0$, so $(2k - 1)(2k - 9) \geq 0$.

9. Use k positive

Therefore $k \leq \frac{1}{2}$ or $k \geq \frac{9}{2}$. Since k is positive, $0

Final answer

(a) y-intercept $(0, -2)$, vertical asymptote $x = 2k$.

$(b)\ 0

TOPIC

Transformations

WMA11/01 OCTOBER 2021

Question 9

9 marks

Transformations

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

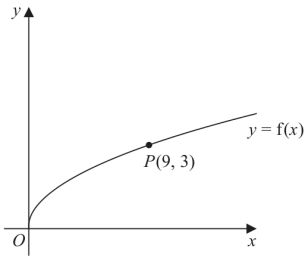


Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \sqrt{x} \qquad x > 0$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 5.

On the next page there is a copy of Figure 5 called Diagram 1.

(a) On Diagram 1, sketch and clearly label the graphs of

$$y = f(2x) \quad \text{and} \quad y = f(x) + 3$$

Show on each graph the coordinates of the point to which P is transformed.

(3)

The graph of $y = f(2x)$ meets the graph of $y = f(x) + 3$ at the point Q.

(b) Show that the x coordinate of Q is the solution of

$$\sqrt{x} = 3\left(\sqrt{2} + 1\right)$$

(3)

(c) Hence find, in simplest form, the coordinates of Q.

(3)

Question 9 continued

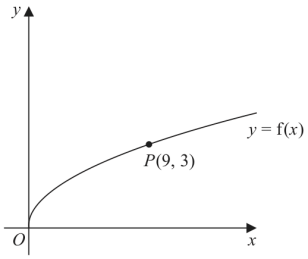
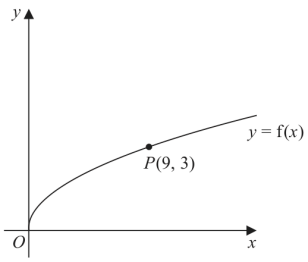


Diagram 1

Turn over for a copy of Diagram 1 if you need to redraw your graphs.

Only use this copy if you need to redraw your graphs.



Copy of Diagram 1

(Total 9 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 9

Transformations

Topic group

1. Transform P on y = f(2x)

$y = f(2x)$ is a horizontal scale factor of $\frac{1}{2}$, so $P(9, 3)$ maps to $\left(\frac{9}{2}, 3\right)$.

2. Transform P on y = f(x) + 3

$y = f(x) + 3$ is a translation 3 units upward, so $P(9, 3)$ maps to $(9, 6)$.

3. Set up the intersection equation

At Q, $f(2x) = f(x) + 3$. Since $f(x) = \sqrt{x}$, this gives $\sqrt{2x} = \sqrt{x} + 3$.

4. Show the required equation

Let $s = \sqrt{x}$. Then $\sqrt{2x} = \sqrt{2}\,s$, so $\sqrt{2}s = s + 3$. Hence $(\sqrt{2} - 1)s = 3$ and $s = \dfrac{3}{\sqrt{2} - 1} = 3(\sqrt{2} + 1)$.

5. Find the x-coordinate

$\sqrt{x} = 3(\sqrt{2} + 1)$, so $x = 9(\sqrt{2} + 1)^2 = 9(3 + 2\sqrt{2}) = 27 + 18\sqrt{2}$.

6. Find the y-coordinate

$y = \sqrt{x} + 3 = 3(\sqrt{2} + 1) + 3 = 6 + 3\sqrt{2}$. Therefore $Q = (27 + 18\sqrt{2},\ 6 + 3\sqrt{2})$.

Final answer

$(a)\ P \mapsto \left(\frac{9}{2}, 3\right)$ on $y = f(2x)$, $P \mapsto (9, 6)$ on $y = f(x) + 3$.

$(c)\ Q = (27 + 18\sqrt{2},\ 6 + 3\sqrt{2})$.

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Question 4

6 marks

Transformations

4.

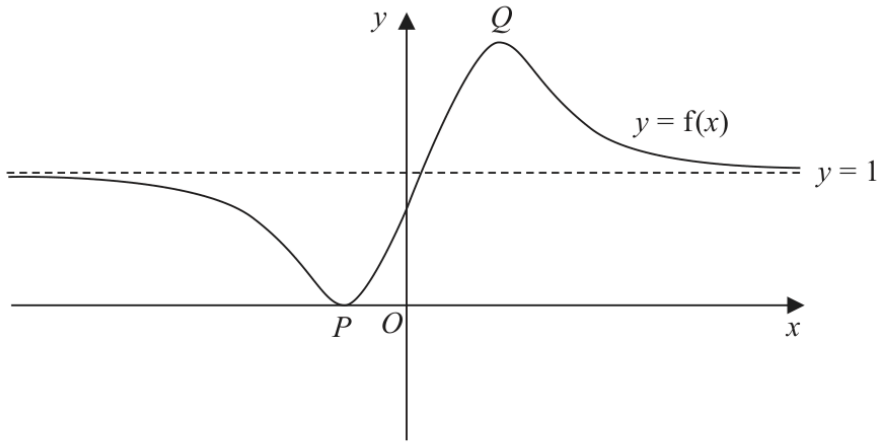


Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

(i) $y = f(x) - 2$

(3)

(ii) $y = f(-x)$

(3)

On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

(Total 6 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 4

Transformations

Topic group

1. Translate for y = f(x) - 2

Subtracting 2 moves every point of the graph down by 2 units.

2. State the new key features

The minimum $P(-1, 0)$ becomes $(-1, -2)$, the maximum $Q(\frac{3}{2}, 2)$ becomes $(\frac{3}{2}, 0)$, and the asymptote $y = 1$ becomes $y = -1$.

3. Reflect for y = f(-x)

Replacing x by $-x$ reflects the graph in the y-axis.

4. State the reflected key features

The minimum $P(-1, 0)$ becomes $(1, 0)$, the maximum $Q(\frac{3}{2}, 2)$ becomes $(-\frac{3}{2}, 2)$, and the horizontal asymptote remains $y = 1$.

Final answer

(i) minimum $(-1, -2)$, maximum $(\frac{3}{2}, 0)$, asymptote $y = -1$. (ii) minimum $(1, 0)$, maximum $(-\frac{3}{2}, 2)$, asymptote $y = 1$.

WMA11/01 OCTOBER 2022

Question 7

8 marks

Transformations

7.

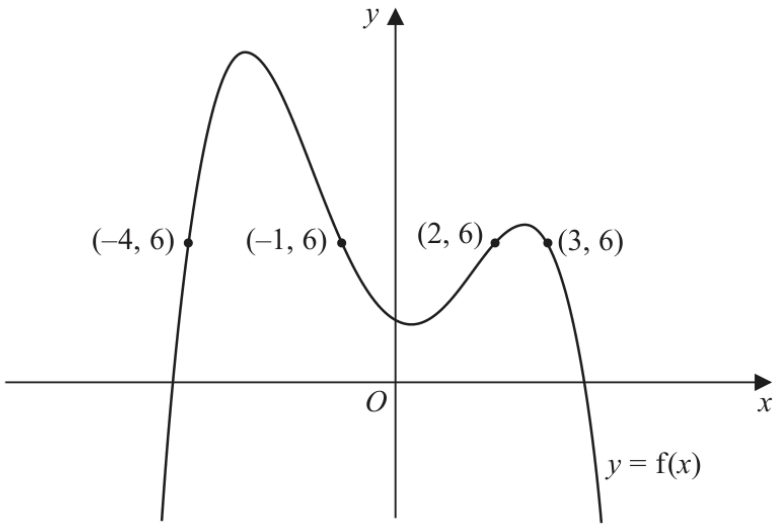


Figure 1

Figure 1 shows the curve with equation $y = f(x)$.

The points $P(-4, 6)$, $Q(-1, 6)$, $R(2, 6)$ and $S(3, 6)$ lie on the curve.

(a) Using Figure 1, find the range of values of x for which

$$f(x) < 6$$

(3)

(b) State the largest solution of the equation

$$f(2x) = 6$$

(1)

(c) (i) Sketch the curve with equation $y = f(-x)$.

On your sketch, state the coordinates of the points to which P, Q, R and S are transformed.

(ii) Hence find the set of values of x for which

$$f(-x) \geqslant 6 \text{ and } x < 0$$

(4)

(Total for Question 7 is 8 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 7

Transformations

Topic group

1. Read f(x) < 6 from the graph

The curve is below the line $y = 6$ for $x < -4$, for -13.

2. Solve f(2x) = 6

Since $f(u) = 6$ at $u = -4, -1, 2, 3$, we set $2x = -4, -1, 2, 3$. The solutions are $x = -2, -\frac{1}{2}, 1, \frac{3}{2}$.

3. State the largest solution

The largest solution is $x = \frac{3}{2}$.

4. Reflect the points in the y-axis

For $y = f(-x)$, each point (x, y) becomes $(-x, y)$.

5. List the transformed points

$P(-4, 6) \to (4, 6)$, $Q(-1, 6) \to (1, 6)$, $R(2, 6) \to (-2, 6)$, and $S(3, 6) \to (-3, 6)$.

6. Use the reflected graph

For $x < 0$, the part of the reflected graph on or above $y = 6$ is from $x = -3$ to $x = -2$.

Final answer

$(a)\ x<-4,\ -13.$

$(b)\ \frac{3}{2}.$

$(c)(ii)\ -3 \leq x \leq -2.$

WMA11/01 JANUARY 2023

Question 7

10 marks

Transformations

7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \qquad x \neq 0$$

(2)

The curve C is transformed onto the curve with equation $y = \dfrac{6}{x-2} \qquad x \neq 2$

(b) Fully describe this transformation.

(2)

The curve with equation

$$y = \frac{6}{x-2} \qquad x \neq 2$$

and the line with equation

$$y = kx + 7 \qquad \text{where } k \text{ is a constant}$$

intersect at exactly two points, P and Q.

Given that the x coordinate of point P is -4

(c) find the value of k,

(2)

(d) find, using algebra, the coordinates of point Q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

Question 7 continued

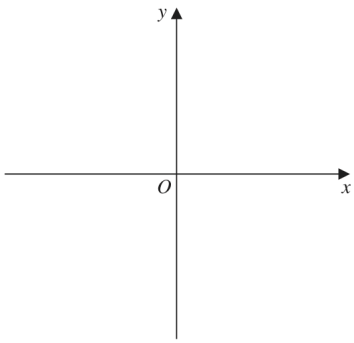
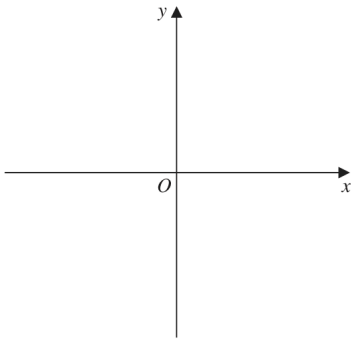


Diagram 1

Only use this copy of Diagram 1 if you need to redraw your graph.



Copy of Diagram 1

(Total for Question 7 is 10 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 7

Transformations

Topic group

1. Sketch y = 6/x

The graph is a rectangular hyperbola with asymptotes $x = 0$ and $y = 0$, with branches in quadrants I and III.

2. Describe the transformation

$y = \dfrac{6}{x-2}$ is obtained from $y = \dfrac{6}{x}$ by a translation 2 units in the positive x-direction.

3. Use P to find k

At $x = -4$ on $y = \dfrac{6}{x-2}$, $y = \dfrac{6}{-6} = -1$.

4. Substitute into the line

Since P lies on $y = kx + 7$, $-1 = -4k + 7$, so $k = 2$.

5. Find the second intersection

Solve $\dfrac{6}{x-2} = 2x + 7$. Multiplying by $x - 2$ gives $6 = (2x + 7)(x - 2)$.

6. Factor and choose Q

$6 = 2x^2 + 3x - 14$, so $2x^2 + 3x - 20 = 0 = (2x - 5)(x + 4)$. The root $x = -4$ is P, so Q has $x = \dfrac{5}{2}$.

7. Find y for Q

$y = 2\left(\dfrac{5}{2}\right) + 7 = 12$, so $Q = \left(\dfrac{5}{2}, 12\right)$.

Final answer

(b) Translation 2 units right.

(c) $k = 2$.

(d) $Q = \left(\frac{5}{2}, 12\right)$.

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Question 3

6 marks

Transformations

3.

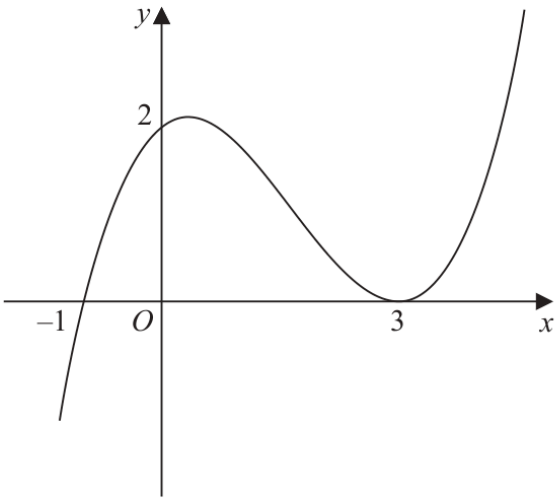


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-1, 0)$ and $(0, 2)$ and touches the x-axis at the point $(3, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(Total for Question 3 is 6 marks)

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Worked Solution - Question 3

Transformations

Topic group

1. Read the original key points

The original curve passes through $(-1, 0)$ and $(0, 2)$, and touches the x-axis at $(3, 0)$.

2. Transform y equals f of x plus 3

$y = f(x + 3)$ is a translation 3 units left. So $(-1, 0)$ moves to $(-4, 0)$, $(0, 2)$ moves to $(-3, 2)$, and $(3, 0)$ moves to $(0, 0)$.

3. State the axis intercepts for part a

The curve cuts the x-axis at $(-4, 0)$ and touches it at $(0, 0)$. The y-intercept is also $(0, 0)$.

4. Transform y equals f of negative 3x

For $y = f(-3x)$, a point with original x-coordinate X moves to $x = -\frac{X}{3}$.

5. State the axis intercepts for part b

$(-1, 0)$ moves to $(\frac{1}{3}, 0)$, $(3, 0)$ moves to $(-1, 0)$, and $(0, 2)$ stays at $(0, 2)$.

Final answer

(a) intercepts $(-4, 0)$ and $(0, 0)$, with $(0, 0)$ a touch point.

(b) intercepts $(-1, 0)$, $(\frac{1}{3}, 0)$ and $(0, 2)$, with $(-1, 0)$ a touch point.

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Question 9

10 marks

Transformations

9.

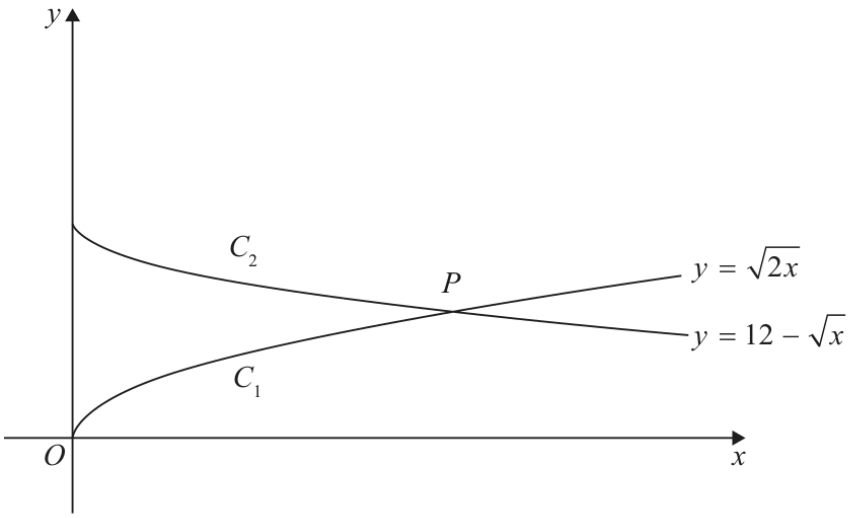


Figure 4

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 4 shows a sketch of

- the graph C_1 with equation $y = \sqrt{2x}$
- the graph C_2 with equation $y = 12 - \sqrt{x}$

(a) Describe fully the single transformation that would transform

(i) the graph with equation $y = \sqrt{x}$ onto C_1

(ii) the graph with equation $y = -\sqrt{x}$ onto C_2

(4)

The graphs C_1 and C_2 meet at the point P, as shown in Figure 4.

(b) (i) Show that the x coordinate of P is a solution of

$$\sqrt{x} = 12\left(\sqrt{2} - 1\right)$$

(ii) Hence find, in simplest form, the exact coordinates of P.

(6)

(Total for Question 9 is 10 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 9

Transformations

Topic group

1. Transform y equals root x to C1

C_1 has equation $y = \sqrt{2x}$. This is obtained from $y = \sqrt{x}$ by a stretch parallel to the x-axis with scale factor $\frac{1}{2}$.

2. Equivalent description

Equivalently, $y = \sqrt{2x} = \sqrt{2}\sqrt{x}$ is a stretch parallel to the y-axis with scale factor $\sqrt{2}$. The mark scheme also accepts the horizontal stretch description.

3. Transform y equals minus root x to C2

C_2 has equation $y = 12 - \sqrt{x} = -\sqrt{x} + 12$, so it is a translation of $y = -\sqrt{x}$ by $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$.

4. Set the curves equal

At P, $\sqrt{2x} = 12 - \sqrt{x}$.

5. Factor out root x

$\sqrt{2}\sqrt{x} + \sqrt{x} = 12$, so $(\sqrt{2}+1)\sqrt{x} = 12$.

6. Rationalise

$$\sqrt{x} = \frac{12}{\sqrt{2}+1} = \frac{12(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = 12(\sqrt{2}-1).$$

7. Find x

$x = [12(\sqrt{2}-1)]^2 = 144(3-2\sqrt{2})$.

8. Find y

$y = \sqrt{2x} = \sqrt{2}\sqrt{x} = \sqrt{2}\cdot 12(\sqrt{2}-1) = 24 - 12\sqrt{2}$.

Final answer

$(a)(i)$ stretch parallel to the x-axis scale factor $\frac{1}{2}$ (equivalently vertical stretch scale factor $\sqrt{2}$).

$(a)(ii)$ translation by $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$.

(b) $P = (144(3 - 2\sqrt{2}),\ 24 - 12\sqrt{2})$.

WMA11/01 OCTOBER 2025

Question 9

6 marks

Transformations

9.

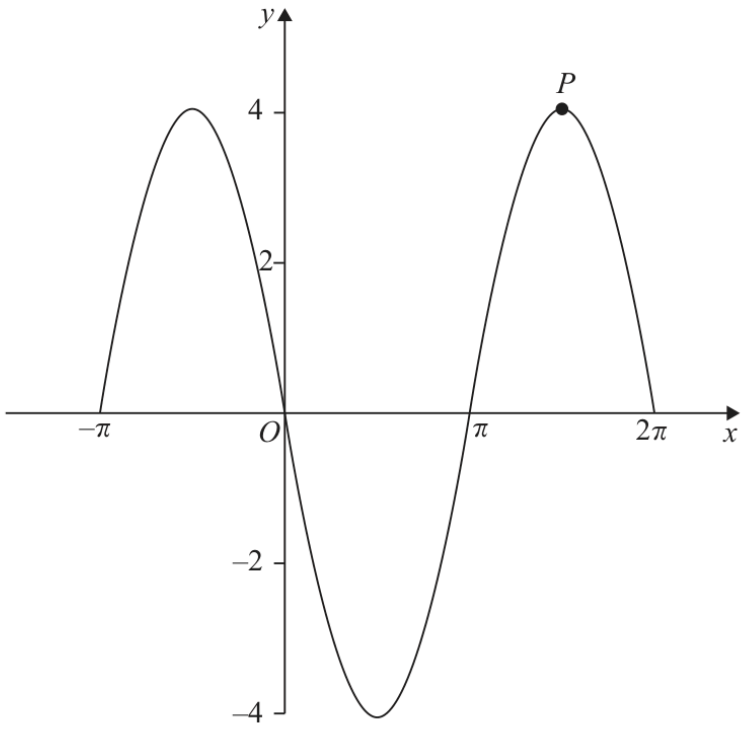


Figure 3

Figure 3 shows a sketch of part of the graph of the trigonometric function with equation $y = \mathrm{f}(x)$

(a) Write down an expression for $\mathrm{f}(x)$

(2)

The point P lies on $y = \mathrm{f}(x)$ and is shown in Figure 3.

(b) State the coordinates of the point to which P is transformed when the graph of $y = \mathrm{f}(x)$ is transformed to the graph with equation

(i) $y = \mathrm{f}\left(x - \frac{\pi}{6}\right)$

(2)

(ii) $y = -\frac{1}{2}\mathrm{f}(x)$

(2)

(Total for Question 9 is 6 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 9

Transformations

Topic group

1. Identify amplitude and direction

The graph has amplitude 4, crosses the origin, and initially moves downwards, so $f(x) = -4\sin x$.

2. Identify P

The labelled maximum is at $P = \left(\frac{3\pi}{2}, 4\right)$.

3. Transform y equals f of x minus pi over 6

$y = f\left(x - \frac{\pi}{6}\right)$ translates the graph right by $\frac{\pi}{6}$.

4. New coordinates for part b(i)

P becomes $\left(\frac{3\pi}{2} + \frac{\pi}{6}, 4\right) = \left(\frac{5\pi}{3}, 4\right)$.

5. Transform y equals negative half f(x)

$y = -\frac{1}{2}f(x)$ leaves x-coordinates unchanged and multiplies y-coordinates by $-\frac{1}{2}$.

6. New coordinates for part b(ii)

P becomes $\left(\frac{3\pi}{2}, -2\right)$.

Final answer

$(a)\ f(x) = -4\sin x.$

$(b)(i)\ \left(\frac{5\pi}{3}, 4\right).$

$(b)(ii)\ \left(\frac{3\pi}{2}, -2\right).$

TOPIC

Straight Line

WMA11/01 OCTOBER 2019

Question 2

5 marks

Straight Line

2. A tree was planted in the ground.

Exactly 2 years after it was planted, the height of the tree was 1.85 m.

Exactly 7 years after it was planted, the height of the tree was 3.45 m.

Given that the height, H metres, of the tree, t years after it was planted in the ground, can be modelled by the equation

$$H = at + b$$

where a and b are constants,

(a) find the value of a and the value of b.

(4)

(b) State, according to the model, the height of the tree when it was planted.

(1)

(Total 5 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 2

Straight Line

Topic group

1. Form two equations

Using $H = at + b$: when $t = 2$, $2a + b = 1.85$; when $t = 7$, $7a + b = 3.45$.

2. Solve for the gradient of the model

Subtract the first equation from the second: $5a = 1.60$, so $a = 0.32$.

3. Find the starting height

Substitute into $2a + b = 1.85$: $0.64 + b = 1.85$, hence $b = 1.21$. At planting, $t = 0$, so $H = b = 1.21\ \text{m}$.

Final answer

$a = 0.32,\ b = 1.21.$ Height when planted: $1.21\ \text{m}$.

WMA11/01 JANUARY 2020

Question 6

8 marks

Straight Line

6. The line l_1 has equation $3x - 4y + 20 = 0$

The line l_2 cuts the x-axis at $R(8,0)$ and is parallel to l_1

(a) Find the equation of l_2, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

The line l_1 cuts the x-axis at P and the y-axis at Q.

Given that $PQRS$ is a parallelogram, find

(b) the area of $PQRS$,

(3)

(c) the coordinates of S.

(2)

(Total 8 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 6

Straight Line

Topic group

1. Find the gradient of l1

$3x - 4y + 20 = 0$ gives $y = \frac{3}{4}x + 5$, so the gradient is $\frac{3}{4}$.

2. Find l2 through R

Line l_2 is parallel to l_1 and passes through $R(8, 0)$, so $y = \frac{3}{4}(x - 8) = \frac{3}{4}x - 6$. Therefore $3x - 4y - 24 = 0$.

3. Find P and Q

For l_1, the x-intercept is $P\left(-\frac{20}{3}, 0\right)$ and the y-intercept is $Q = (0, 5)$.

4. Find the area of PQRS

Base $PR = 8 - \left(-\frac{20}{3}\right) = \frac{44}{3}$ and height $OQ = 5$. Area $= \frac{44}{3} \cdot 5 = \frac{220}{3}$.

5. Find S using a vector

In parallelogram $PQRS$, $\vec{PS} = \vec{QR} = (8, -5)$. Hence $S = P + (8, -5) = \left(-\frac{20}{3} + 8, -5\right) = \left(\frac{4}{3}, -5\right)$.

Final answer

(a) $3x - 4y - 24 = 0$.

(b) $\dfrac{220}{3}$.

(c) $S = \left(\dfrac{4}{3}, -5\right)$.

WMA11/01 JANUARY 2021

Question 4

8 marks

Straight Line

4.

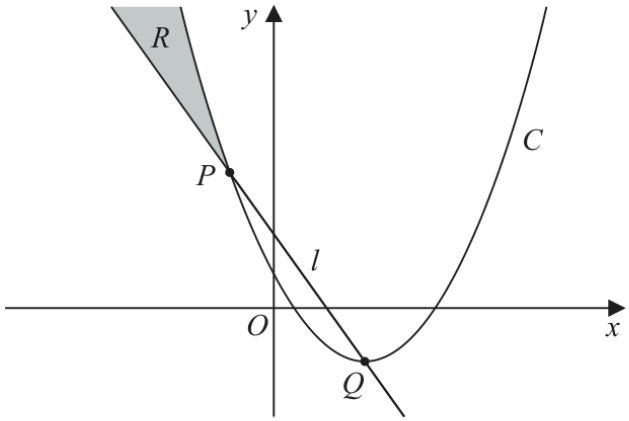


Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(3)

The quadratic curve C passes through P and has a minimum point at Q.

(b) Find an equation for C.

(3)

The region R, shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

(c) Use inequalities to define region R.

(2)

(Total 8 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 4

Straight Line

Topic group

1. Find the line through P and Q

The gradient is $\frac{-5-13}{4-(-2)} = \frac{-18}{6} = -3$. Using $P(-2, 13)$, $y - 13 = -3(x + 2)$, hence $y = -3x + 7$.

2. Use vertex form for the quadratic

The minimum point is $Q(4, -5)$, so the curve has form $y = a(x - 4)^2 - 5$.

3. Substitute P to find a

Since $P(-2, 13)$ lies on the curve, $13 = a(-6)^2 - 5$. Therefore $18 = 36a$ and $a = \frac{1}{2}$.

4. Write the curve

C has equation $y = \frac{1}{2}(x - 4)^2 - 5$.

5. Define the shaded region

In the shaded second-quadrant region, points are above the line, below the curve, and to the left of P. Therefore $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$, and $x < -2$.

Final answer

$(a)\ y = -3x + 7.$

$(b)\ y = \frac{1}{2}(x - 4)^2 - 5.$

$(c)\ y > -3x + 7,\ y < \frac{1}{2}(x - 4)^2 - 5,\ x < -2.$

WMA11/01 OCTOBER 2021

Question 3

6 marks

Straight Line

3. (i) Solve

$$\frac{3}{x} > 4$$

(3)

(ii)

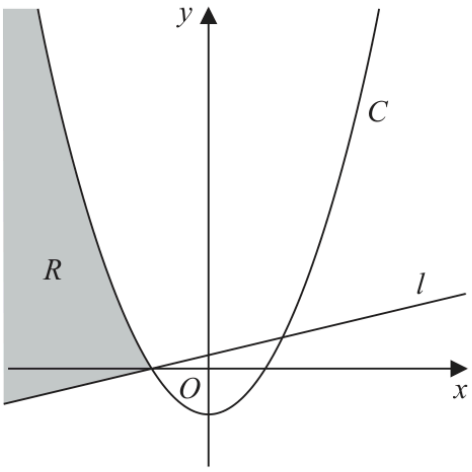


Figure 1

Figure 1 shows a sketch of the curve C and the straight line l.

The infinite region R, shown shaded in Figure 1, lies in quadrants 2 and 3 and is bounded by C and l only.

Given that

- l has a gradient of 3
- C has equation $y = 2x^2 - 50$
- C and l intersect on the negative x-axis

use inequalities to define the region R.

(3)

(Total 6 marks)

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Worked Solution - Question 3

Straight Line

Topic group

1. Solve the rational inequality

$\frac{3}{x} > 4$ is equivalent to $\frac{3-4x}{x} > 0$.

2. Use critical values

The critical values are $x = 0$ and $x = \frac{3}{4}$. A sign check gives the positive interval $0

3. Find the equation of l

The curve C meets the negative x-axis when $2x^2 - 50 = 0$, so $x = -5$. Since l has gradient 3 and passes through $(-5, 0)$, $y = 3(x + 5) = 3x + 15$.

4. Define the shaded region

The shaded region lies above the line, below the curve, and to the left of their negative x-axis intersection. Hence $y > 3x + 15$, $y < 2x^2 - 50$, and $x < -5$.

Final answer

$(i)\ 03x+15,\ y<2x^2-50,\ x<-5$.

WMA11/01 OCTOBER 2021

Question 5

8 marks

Straight Line

5. The line l_1 has equation $3y - 2x = 30$

The line l_2 passes through the point $A(24, 0)$ and is perpendicular to l_1

Lines l_1 and l_2 meet at the point P

(a) Find, using algebra and showing your working, the coordinates of P.

(5)

Given that l_1 meets the x-axis at the point B,

(b) find the area of triangle BPA.

(3)

(Total 8 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 5

Straight Line

Topic group

1. Rewrite l1

$3y - 2x = 30$ gives $y = \frac{2}{3}x + 10$, so the gradient of l_1 is $\frac{2}{3}$.

2. Find l2

Since l_2 is perpendicular to l_1, its gradient is $-\frac{3}{2}$. Through $A(24, 0)$, $y = -\frac{3}{2}(x - 24) = -\frac{3}{2}x + 36$.

3. Solve for P

At the intersection, $\frac{2}{3}x + 10 = -\frac{3}{2}x + 36$. Multiplying by 6 gives $4x + 60 = -9x + 216$, so $13x = 156$ and $x = 12$.

4. Find the y-coordinate

$y = \frac{2}{3}(12) + 10 = 18$, so $P = (12, 18)$.

5. Find B

For the x-intercept of l_1, set $y = 0$: $-2x = 30$, so $x = -15$. Hence $B = (-15, 0)$.

6. Find the triangle area

The base BA has length $24 - (-15) = 39$, and the height from P to the x-axis is 18. Area $= \frac{1}{2}(39)(18) = 351$.

Final answer

(a) $P = (12, 18).$

(b) $\mathbf{351}$ square units.

Question 8

10 marks

Straight Line

8.

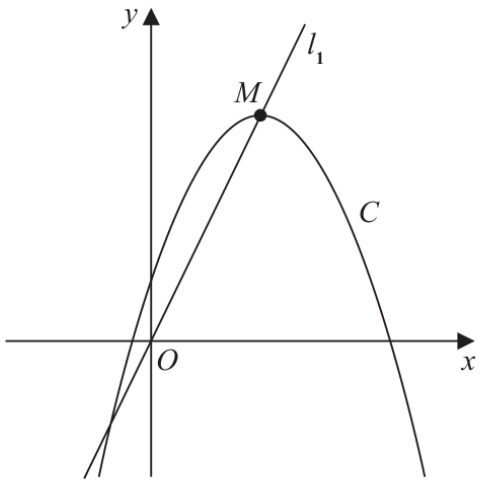


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 4 + 12x - 3x^2$$

The point M is the maximum turning point on C.

(a) (i) Write $4 + 12x - 3x^2$ in the form

$$a + b(x + c)^2$$

where a, b and c are constants to be found.

(ii) Hence, or otherwise, state the coordinates of M.

(5)

The line l_1 passes through O and M, as shown in Figure 4.

A line l_2 touches C and is parallel to l_1

(b) Find an equation for l_2

(5)

(Total 10 marks)

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Worked Solution - Question 8

Straight Line

Topic group

1. Complete the square

$4+12x-3x^2=-3(x^2-4x)+4=-3\big((x-2)^2-4\big)+4=16-3(x-2)^2$.

2. State the maximum point

The maximum occurs when $(x-2)^2=0$, so $M=(2,16)$.

3. Find the gradient of l1

Line l_1 passes through $O(0,0)$ and $M(2,16)$, so its gradient is $\frac{16}{2}=8$.

4. Use the parallel tangent gradient

Line l_2 is parallel to l_1, so its gradient is also 8. For $y=4+12x-3x^2$, $\frac{dy}{dx}=12-6x$.

5. Find the point of contact

Set $12-6x=8$, giving $x=\frac{2}{3}$. Then $y=4+12\left(\frac{2}{3}\right)-3\left(\frac{4}{9}\right)=\frac{32}{3}$.

6. Write l2

Using gradient 8 through $\left(\frac{2}{3},\frac{32}{3}\right)$, $y-\frac{32}{3}=8\left(x-\frac{2}{3}\right)$. Hence $y=8x+\frac{16}{3}$.

Final answer

$(a)(i)\ 16 - 3(x-2)^2$. $(a)(ii)\ M = (2, 16)$.

$(b)\ y = 8x + \dfrac{16}{3}$.

WMA11/01 JANUARY 2022

Question 8

9 marks

Straight Line

8. The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates $(6, -2)$
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

(Total 9 marks)

WMA11/01 JANUARY 2022

Worked Solution - Question 8

Straight Line

Topic group

1. Find the gradient of l1

$2x - 5y + 7 = 0$ gives $y = \frac{2}{5}x + \frac{7}{5}$, so the gradient is $\frac{2}{5}$.

2. Find l2

The gradient perpendicular to $\frac{2}{5}$ is $-\frac{5}{2}$. Through $A(6, -2)$, $y + 2 = -\frac{5}{2}(x - 6)$, so $y = -\frac{5}{2}x + 13$.

3. Find M

At the intersection, $\frac{2}{5}x + \frac{7}{5} = -\frac{5}{2}x + 13$. Multiplying by 10 gives $4x + 14 = -25x + 130$, so $x = 4$.

4. Find the y-coordinate of M

$y = \frac{2}{5}(4) + \frac{7}{5} = 3$, so $M = (4, 3)$.

5. Use midpoint of the diagonal

In a square, the diagonals bisect each other, so M is the midpoint of AC. Thus $C = 2M - A = (8, 6) - (6, -2) = (2, 8)$.

Final answer

(a) $\frac{2}{5}$.

(b) $y = -\frac{5}{2}x + 13$.

(c) $M = (4, 3)$.

(d) $C = (2, 8)$.

WMA11/01 MAY/JUNE 2022

Question 5

9 marks

Straight Line

5. The curve C has equation $y = f(x)$

Given that

- $f(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $f(x)$

(4)

The line l_1 has equation $y = \frac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

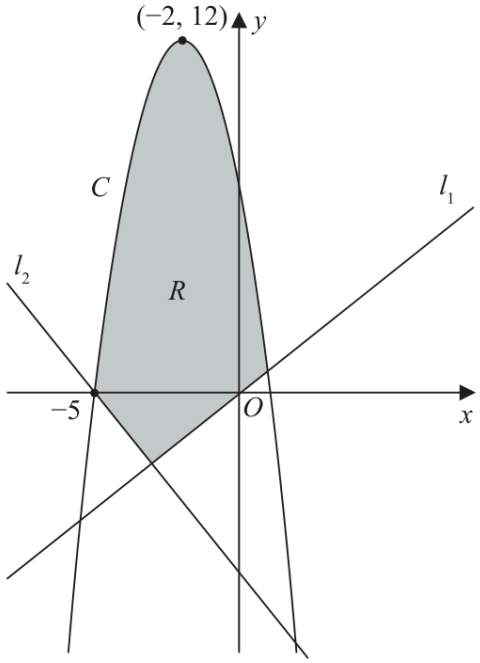


Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(c) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

(Total 9 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 5

Straight Line

Topic group

1. Use symmetry of the quadratic

The maximum is at $x = -2$, and one root is $x = -5$. The other root is the same distance to the right of -2, so it is $x = 1$.

2. Find the quadratic

Let $f(x) = a(x+5)(x-1)$. Since the maximum point is $(-2, 12)$, $12 = a(3)(-3) = -9a$, so $a = -\frac{4}{3}$.

3. Find the perpendicular gradient

The gradient of l_1 is $\frac{4}{5}$, so the gradient of a perpendicular line is $-\frac{5}{4}$.

4. Find l2

Using the point $(-5, 0)$, $y = -\frac{5}{4}x + c$. Then $0 = \frac{25}{4} + c$, so $c = -\frac{25}{4}$ and l_2 is $y = -\frac{5}{4}x - \frac{25}{4}$.

5. Write the inequalities for R

The region is below the curve and above both straight lines, so $y \leq f(x)$, $y \geq \frac{4}{5}x$, and $y \geq -\frac{5}{4}x - \frac{25}{4}$.

Final answer

$(a)\ f(x) = -\frac{4}{3}(x+5)(x-1).$

$(b)\ y = -\frac{5}{4}x - \frac{25}{4}.$

$(c)\ y \leq f(x),\ y \geq \frac{4}{5}x,\ y \geq -\frac{5}{4}x - \frac{25}{4}.$

WMA11/01 OCTOBER 2022

Question 3

5 marks

Straight Line

3. The share price of a company is monitored.

Exactly 3 years after monitoring began, the share price was £1.05

Exactly 5 years after monitoring began, the share price was £1.65

The share price, £V, of the company is modelled by the equation

$$V = pt + q$$

where t is the number of years after monitoring began and p and q are constants.

(a) Find the value of p and the value of q.

(3)

Exactly T years after monitoring began, the share price was £2.50

(b) Find the value of T, according to the model, giving your answer to one decimal place.

(2)

(Total for Question 3 is 5 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 3

Straight Line

Topic group

1. Use the two data points

The model $V = pt + q$ passes through $(3, 1.05)$ and $(5, 1.65)$.

2. Find the gradient

$$p = \frac{1.65 - 1.05}{5 - 3} = \frac{0.60}{2} = 0.30.$$

3. Find the intercept

Using $1.05 = 0.30(3) + q$, we get $q = 0.15$.

4. Find T

For $V = 2.50$, $2.50 = 0.30T + 0.15$, so $T = \frac{2.35}{0.30} = 7.833\ldots$.

5. Round correctly

To one decimal place, $T = 7.8$.

Final answer

(a) $p = 0.3,\ q = 0.15.$

(b) $T = 7.8$ years.

WMA11/01 JANUARY 2023

Question 2

5 marks

Straight Line

2. The points P, Q and R have coordinates $(-3, 7)$, $(9, 11)$ and $(12, 2)$ respectively.

(a) Prove that angle $PQR = 90°$

(3)

Given that the point S is such that $PQRS$ forms a rectangle,

(b) find the coordinates of S.

(2)

(Total for Question 2 is 5 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 2

Straight Line

Topic group

1. Find the gradient of PQ

$$m_{PQ} = \frac{11 - 7}{9 - (-3)} = \frac{4}{12} = \frac{1}{3}.$$

2. Find the gradient of QR

$$m_{QR} = \frac{2 - 11}{12 - 9} = \frac{-9}{3} = -3.$$

3. Prove the right angle

$m_{PQ}m_{QR} = \frac{1}{3}(-3) = -1$, so PQ and QR are perpendicular. Hence angle $PQR = 90°$.

4. Use the rectangle vector

For rectangle $PQRS$, $\overrightarrow{PS} = \overrightarrow{QR} = (12 - 9, 2 - 11) = (3, -9)$.

5. Find S

$$S = P + \overrightarrow{QR} = (-3, 7) + (3, -9) = (0, -2).$$

Final answer

(a) Angle $PQR = 90°$.

(b) $S = (0, -2)$.

WMA11/01 MAY/JUNE 2023

Question 10

10 marks

Straight Line

10.

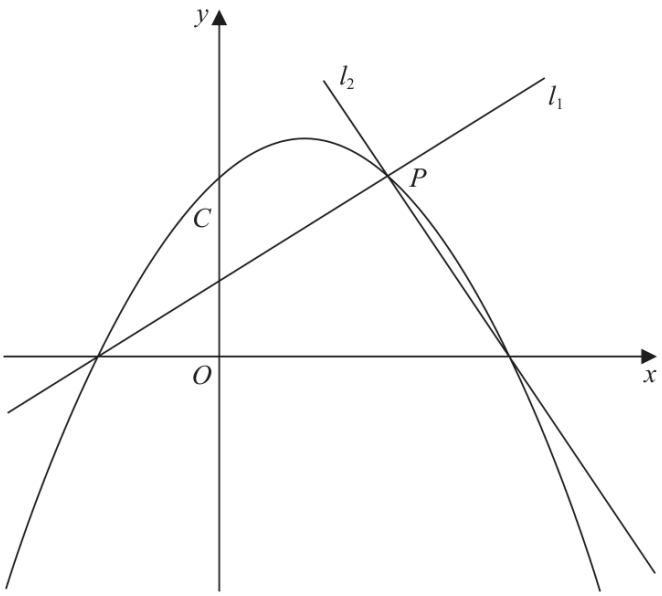


Figure 5

Figure 5 shows a sketch of the quadratic curve C with equation

$$y = -\frac{1}{4}(x+2)(x-b) \qquad \text{where } b \text{ is a positive constant}$$

The line l_1 also shown in Figure 5,

- has gradient $\frac{1}{2}$
- intersects C on the negative x-axis and at the point P

(a) (i) Write down an equation for l_1

(1)

(ii) Find, in terms of b, the coordinates of P

(3)

Given that the line l_2 is perpendicular to l_1 and intersects C on the positive x-axis,

(b) find, in terms of b, an equation for l_2

(2)

Given also that l_2 intersects C at the point P

(c) show that another equation for l_2 is

$$y = -2x + \frac{5b}{2} - 4$$

(2)

(d) Hence, or otherwise, find the value of b

(Total for Question 10 is 10 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 10

Straight Line

Topic group

1. Find l1

Line l_1 has gradient $\frac{1}{2}$ and passes through the negative x-axis root $(-2, 0)$, so $y = \frac{1}{2}(x+2) = \frac{1}{2}x + 1$.

2. Find P

Set $-\frac{1}{4}(x+2)(x-b) = \frac{1}{2}(x+2)$.

3. Solve the intersection

Multiplying by -4 gives $(x+2)(x-b) = -2(x+2)$, so $(x+2)(x-b+2) = 0$.

4. Choose the second point

The root $x = -2$ is the x-axis point, so P has $x = b - 2$. Then $y = \frac{1}{2}(b-2) + 1 = \frac{b}{2}$.

5. Find l2 from the positive root

Since l_2 is perpendicular to l_1, its gradient is -2. It passes through the positive x-axis root $(b, 0)$, so $y = -2(x-b) = -2x + 2b$.

6. Use P to write another l2

Through $P\left(b-2, \frac{b}{2}\right)$ with gradient -2: $y - \frac{b}{2} = -2(x - (b-2))$, so $y = -2x + \frac{5b}{2} - 4$.

7. Find b

Both equations represent l_2, so $2b = \frac{5b}{2} - 4$. Hence $b = 8$.

Final answer

$(a)(i)\ y = \frac{1}{2}x + 1.$ $(a)(ii)\ P = \left(b - 2, \frac{b}{2}\right).$

$(b)\ y = -2x + 2b.$

$(d)\ b = 8.$

WMA11/01 JANUARY 2024

Question 5

10 marks

Straight Line

5.

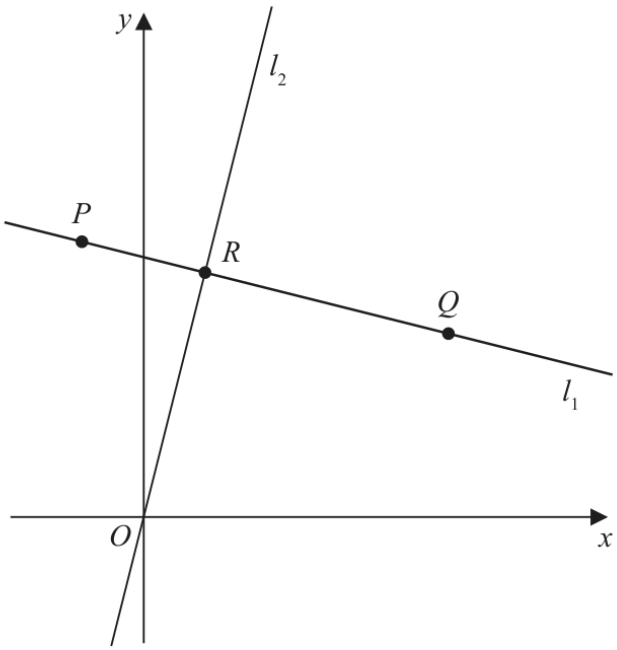


Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The straight line l_1, shown in Figure 1, passes through the points $P(-2, 9)$ and $Q(10, 6)$.

(a) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The straight line l_2 passes through the origin O and is perpendicular to l_1

The lines l_1 and l_2 meet at the point R as shown in Figure 1.

(b) Find the coordinates of R

(4)

(c) Find the exact area of triangle OPQ.

(3)

(Total for Question 5 is 10 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 5

Straight Line

Topic group

1. Find the gradient of l1

$m = \dfrac{6-9}{10-(-2)} = -\dfrac{3}{12} = -\dfrac{1}{4}$.

2. Find the equation of l1

Using $P(-2,9)$, $y = -\dfrac{1}{4}x + c$. Then $9 = \dfrac{1}{2} + c$, so $c = \dfrac{17}{2}$.

3. Find the equation of l2

A line perpendicular to gradient $-\dfrac{1}{4}$ has gradient 4. Since l_2 passes through the origin, l_2 is $y = 4x$.

4. Find R

At the intersection, $4x = -\dfrac{1}{4}x + \dfrac{17}{2}$. Hence $\dfrac{17}{4}x = \dfrac{17}{2}$, so $x = 2$ and $y = 8$.

5. Use coordinate area

For $O(0,0)$, $P(-2,9)$ and $Q(10,6)$, the area is $\dfrac{1}{2}|(-2)(6) - 9(10)| = 51$.

Final answer

$(a)\ y = -\dfrac{1}{4}x + \dfrac{17}{2}$.

$(b)\ R = (2,8)$.

$(c)\ \text{Area} = 51$.

WMA11/01 MAY/JUNE 2024

Question 9

9 marks

Straight Line

9. Given that

- the point A has coordinates (4, 2)
- the point B has coordinates (15, 7)
- the line l_1 passes through A and B

(a) find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(3)

The line l_2 passes through A and is parallel to the x-axis.

The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

(b) Find both possible pairs of coordinates of the point C.

(4)

(c) Hence find the minimum possible area of triangle ABC.

(2)

(Total for Question 9 is 9 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 9

Straight Line

Topic group

1. Find the gradient of l1

$m = \dfrac{7-2}{15-4} = \dfrac{5}{11}$.

2. Use point A

$y - 2 = \dfrac{5}{11}(x - 4)$.

3. Rearrange

$11y - 22 = 5x - 20$, so $5x - 11y + 2 = 0$.

4. Use the horizontal line l2

Since l_2 passes through $A(4, 2)$ and is parallel to the x-axis, point C has coordinates $(c, 2)$.

5. Use BC equals 5 root 5

$(c - 15)^2 + (2 - 7)^2 = (5\sqrt{5})^2$.

6. Solve for c

$(c - 15)^2 + 25 = 125$, so $(c - 15)^2 = 100$ and $c = 5$ or 25.

7. Find the smaller area

The height from B to $y = 2$ is 5. For $C = (5, 2)$, $AC = 1$, so the minimum area is $\dfrac{1}{2}(1)(5) = \dfrac{5}{2}$.

Final answer

(a) $5x - 11y + 2 = 0$.

(b) $C = (5, 2)$ or $(25, 2)$.

(c) $\dfrac{5}{2}$.

WMA11/01 OCTOBER 2024

Question 1

6 marks

Straight Line

1. The line l_1 passes through the point $A(-5, 20)$ and the point $B(3, -4)$.

(a) Find an equation for l_1 giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB

(b) Find an equation for l_2 giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

(3)

(Total for Question 1 is 6 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 1

Straight Line

Topic group

1. Find the gradient of l1

$m = \dfrac{-4-20}{3-(-5)} = \dfrac{-24}{8} = -3.$

2. Find the equation of l1

Using $B(3,-4)$, $y = -3x + c$. Then $-4 = -9 + c$, so $c = 5$.

3. Find the midpoint of AB

$M = \left(\dfrac{-5+3}{2}, \dfrac{20+(-4)}{2}\right) = (-1, 8).$

4. Use perpendicular gradient

Since l_1 has gradient -3, l_2 has gradient $\dfrac{1}{3}$.

5. Find the equation of l2

$y - 8 = \dfrac{1}{3}(x+1)$, so $3y - 24 = x + 1$ and $x - 3y + 25 = 0$.

Final answer

$(a)\ y = -3x + 5.$

$(b)\ x - 3y + 25 = 0.$

WMA11/01 JANUARY 2025

Question 2

5 marks

Straight Line

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $\left(-2\sqrt{3}, 5\right)$
- the point B has coordinates $\left(7\sqrt{3}, 8\right)$
- the straight line l_1 passes through A and B

(a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A.

(b) Find the equation of l_2, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 2 is 5 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 2

Straight Line

Topic group

1. Find the gradient of l1

$$m = \frac{8-5}{7\sqrt{3}-(-2\sqrt{3})} = \frac{3}{9\sqrt{3}} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}.$$

2. Find the perpendicular gradient

The gradient of l_2 is the negative reciprocal of $\frac{\sqrt{3}}{9}$, so $m_2 = -\frac{9}{\sqrt{3}} = -3\sqrt{3}$.

3. Use point A

l_2 passes through $A(-2\sqrt{3}, 5)$, so $y - 5 = -3\sqrt{3}(x + 2\sqrt{3})$.

4. Simplify

$y - 5 = -3\sqrt{3}x - 18$, hence $y = -3\sqrt{3}x - 13$.

Final answer

(a) gradient $= \frac{1}{9}\sqrt{3}$.

(b) $y = -3\sqrt{3}x - 13$.

WMA11/01 MAY/JUNE 2025

Question 5

7 marks

Straight Line

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The line l_1 has equation

$$x - 2y + 25 = 0$$

The line l_2 passes through the origin and is perpendicular to l_1

(a) Find an equation for l_2

(2)

The lines l_1 and l_2 intersect at the point P.

(b) Use algebra to find the coordinates of P.

(3)

(c) Hence find the shortest distance from l_1 to the origin.

Write your answer as a fully simplified surd.

(2)

(Total for Question 5 is 7 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 5

Straight Line

Topic group

1. Find the gradient of l1

$l_1 : x - 2y + 25 = 0$ gives $y = \frac{1}{2}x + \frac{25}{2}$, so its gradient is $\frac{1}{2}$.

2. Use perpendicular gradient

A line perpendicular to l_1 has gradient -2.

3. Use the origin

Since l_2 passes through the origin, its equation is $y = -2x$.

4. Find the intersection

Substitute $y = -2x$ into $x - 2y + 25 = 0$ to get $x + 4x + 25 = 0$.

5. Coordinates of I

$5x + 25 = 0$, so $x = -5$ and $y = 10$. Hence $I = (-5, 10)$.

6. Shortest distance from I to origin

The distance is $\sqrt{(-5)^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$.

Final answer

$(a)\ y = -2x.$

$(b)\ I = (-5, 10).$

$(c)\ 5\sqrt{5}.$

WMA11/01 JANUARY 2026

Question 4

9 marks

Straight Line

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The point $A(-1, 8)$ and the point $B(7, 2)$ lie on the line L_1

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line L_2

- passes through A and is perpendicular to L_1
- cuts the x-axis at the point C

(b) Find the coordinates of C.

(3)

(c) Hence find the area of triangle ABC.

(3)

(Total for Question 4 is 9 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 4

Straight Line

Topic group

1. Gradient of L1

Using $A(-1,8)$ and $B(7,2)$, $m = \dfrac{2-8}{7-(-1)} = -\dfrac{6}{8} = -\dfrac{3}{4}$.

2. Equation of L1

$y - 2 = -\dfrac{3}{4}(x-7)$.

3. Rearrange

$4y - 8 = -3x + 21$, hence $3x + 4y - 29 = 0$.

4. Gradient of L2

Since L_2 is perpendicular to L_1, its gradient is $\dfrac{4}{3}$.

5. Equation of L2

L_2 passes through $A(-1,8)$, so $y - 8 = \dfrac{4}{3}(x+1)$.

6. Find C

At the x-axis, $y = 0$, so $-8 = \dfrac{4}{3}(x+1)$. Hence $x + 1 = -6$ and $x = -7$, giving $C = (-7, 0)$.

7. Find perpendicular side lengths

$AB = \sqrt{8^2 + (-6)^2} = 10$ and $AC = \sqrt{(-6)^2 + (-8)^2} = 10$.

8. Area of triangle ABC

Since $AB \perp AC$, $\text{Area} = \dfrac{1}{2}(10)(10) = 50$.

Final answer

$(a)\ 3x + 4y - 29 = 0.$

$(b)\ C = (-7, 0).$

$(c)\ 50.$

TOPIC

Basic Trigonometry

WMA11/01 OCTOBER 2019

Question 4

5 marks

Basic Trigonometry

4. A parallelogram $ABCD$ has area $40\,\text{cm}^2$

Given that AB has length $10\,\text{cm}$, BC has length $6\,\text{cm}$ and angle DAB is obtuse, find

(a) the size of angle DAB, in degrees, to 2 decimal places,

(3)

(b) the length of diagonal BD, in cm, to one decimal place.

(2)

(Total 5 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 4

Basic Trigonometry

Topic group

1. Use the area of a parallelogram

Area $= ab\sin\theta$, so $40 = 10 \cdot 6\sin\theta$. Hence $\sin\theta = \frac{2}{3}$.

2. Choose the obtuse angle

$\theta = 180° - \sin^{-1}\left(\frac{2}{3}\right) = 138.19°$ to 2 decimal places.

3. Use the cosine rule for BD

$BD^2 = 10^2 + 6^2 - 2(10)(6)\cos(138.19°)$. Therefore $BD \approx 15.014\ldots$, so $BD = 15.0\ \text{cm}$ to 1 decimal place.

Final answer

$(a)\ \angle DAB = 138.19°.$

$(b)\ BD = 15.0\ \text{cm}.$

WMA11/01 MAY/JUNE 2021

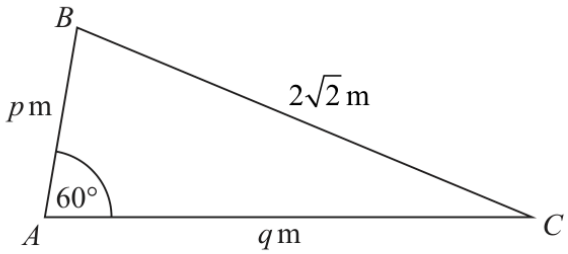
Question 3

9 marks

Basic Trigonometry

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.



Not drawn to scale

Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle ABC with

- $AB = p$ metres
- $AC = q$ metres
- $BC = 2\sqrt{2}$ metres
- angle $BAC = 60°$

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side AC is 2 metres longer than side AB, use algebra to find

(b) (i) the exact value of p,

(ii) the exact value of q.

(5)

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

(Total 9 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 3

Basic Trigonometry

Topic group

1. Apply the cosine rule

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 60°$. Since $BC = 2\sqrt{2}$, $AB = p$ and $AC = q$, this gives $8 = p^2 + q^2 - 2pq\left(\frac{1}{2}\right)$.

2. Show the required result

Therefore $p^2 + q^2 - pq = 8$.

3. Use q = p + 2

Side AC is 2 metres longer than AB, so $q = p + 2$. Substitute into $p^2 + q^2 - pq = 8$.

4. Solve for p

$p^2 + (p+2)^2 - p(p+2) = 8$, so $p^2 + 2p - 4 = 0$. Hence $p = -1 \pm \sqrt{5}$. Since $p > 0$, $p = \sqrt{5} - 1$.

5. Find q

$q = p + 2 = \sqrt{5} + 1$.

6. Find the exact area

Area $= \frac{1}{2}pq\sin 60°$. Now $pq = (\sqrt{5} - 1)(\sqrt{5} + 1) = 4$, so area $= \frac{1}{2}(4)\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,\text{m}^2$.

Final answer

$(a)\ p^2 + q^2 - pq = 8.$

$(b)\ p = \sqrt{5} - 1,\ q = \sqrt{5} + 1.$

$(c)\ \sqrt{3}\,\mathrm{m}^2.$

WMA11/01 MAY/JUNE 2022

Question 2

5 marks

Basic Trigonometry

2. In the triangle ABC,

- $AB = 21\,\text{cm}$
- $BC = 13\,\text{cm}$
- angle $BAC = 25°$
- angle $ACB = x°$

(a) Use the sine rule to find the value of $\sin x°$, giving your answer to 4 decimal places.

(2)

Given also that AB is the longest side of the triangle,

(b) find the value of x, giving your answer to 2 decimal places.

(3)

(Total 5 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 2

Basic Trigonometry

Topic group

1. Use the sine rule

The side opposite x is $AB = 21$, and the side opposite $25°$ is $BC = 13$, so $\dfrac{\sin x}{21} = \dfrac{\sin 25°}{13}$.

2. Find the sine value

$$\sin x = \frac{21 \sin 25°}{13} = 0.68269\ldots.$$

3. Choose the correct angle

The angle from the calculator is $43.05°$, but AB is the longest side, so x is obtuse.

Hence $x = 180° - 43.05° = 136.95°$.

Final answer

$x = 137°$ to 3 significant figures.

WMA11/01 OCTOBER 2023

Question 5

7 marks

Basic Trigonometry

5.

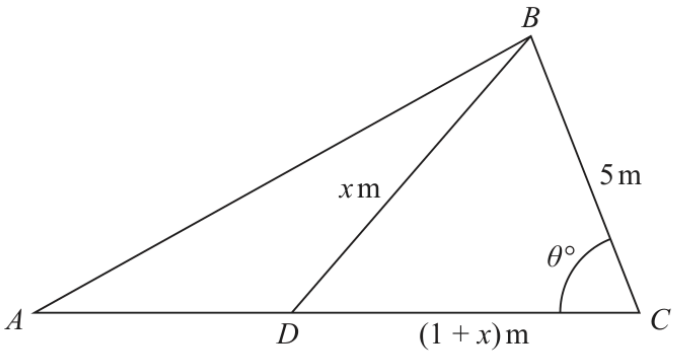


Diagram **NO** accurately dr

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle ABD joined to triangle BCD.

Given that

- $BD = x\,\text{m}$
- $CD = (1 + x)\,\text{m}$
- $BC = 5\,\text{m}$
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- ADC is a straight line

(b) find the area of triangle ABC, giving your answer, in m^2, to one decimal place.

(5)

(Total for Question 5 is 7 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 5

Basic Trigonometry

Topic group

1. Apply the cosine rule in triangle BCD

$x^2 = 5^2 + (1+x)^2 - 2(5)(1+x)\cos\theta$.

2. Rearrange for cos theta

$10(1+x)\cos\theta = 25 + (1+x)^2 - x^2 = 26 + 2x$, so $\cos\theta = \dfrac{26+2x}{10(1+x)} = \dfrac{13+x}{5+5x}$.

3. Substitute x equals 2 root 3

$\cos C = \dfrac{13+2\sqrt{3}}{5+10\sqrt{3}}$, so $C \approx 42.47°$.

4. Use angle sum

$A = 30°$, so $B = 180° - 30° - C \approx 107.53°$.

5. Use the sine rule for the area

Since $BC = 5$ and $A = 30°$, $\dfrac{BC}{\sin A} = 10$. Hence $AB = 10\sin C$ and $AC = 10\sin B$.

6. Calculate the area

$\text{Area} = \dfrac{1}{2}(AB)(AC)\sin A = 50\sin C\sin B \approx 32.2\ \text{m}^2$.

Final answer

$(a)\ \cos\theta = \dfrac{13+x}{5+5x}$.

$(b)\ \text{Area} = 32.2\ \text{m}^2$.

WMA11/01 JANUARY 2024

Question 2

5 marks

Basic Trigonometry

2. The triangle ABC is such that

- $AB = 15\,\text{cm}$
- $AC = 25\,\text{cm}$
- angle $BAC = \theta°$
- area triangle $ABC = 100\,\text{cm}^2$

(a) Find the value of $\sin \theta°$

(2)

Given that $\theta > 90$

(b) find the length of BC, in cm, to 3 significant figures.

(3)

(Total for Question 2 is 5 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 2

Basic Trigonometry

Topic group

1. Use the triangle area formula

$100 = \frac{1}{2}(15)(25)\sin\theta$.

2. Find sine theta

$\sin\theta = \frac{200}{375} = \frac{8}{15}$.

3. Use theta greater than 90 degrees

Since $\theta > 90°$, $\cos\theta$ is negative. So $\cos\theta = -\sqrt{1-\left(\frac{8}{15}\right)^2} = -\frac{\sqrt{161}}{15}$.

4. Apply the cosine rule

$BC^2 = 15^2 + 25^2 - 2(15)(25)\cos\theta = 850 + 50\sqrt{161}$.

5. Round the length

$BC = \sqrt{850 + 50\sqrt{161}} \approx 38.5\ \text{cm}$ to 3 significant figures.

Final answer

$(a)\ \sin\theta = \frac{8}{15}$.

$(b)\ BC = 38.5\ \text{cm}$.

WMA11/01 OCTOBER 2025

Question 3

5 marks

Basic Trigonometry

3. The sides of a triangle are in the ratio $2:4:5$

(a) Find the size of the **largest** angle of the triangle, in degrees, to one decimal place.

(2)

Given that the area of the triangle is $140\,\text{cm}^2$

(b) find the length of the **shortest** side, in cm, to one decimal place.

(3)

(Total for Question 3 is 5 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 3

Basic Trigonometry

Topic group

1. Use the side ratio

Take the side lengths as $2x$, $4x$ and $5x$. The largest angle is opposite the side $5x$.

2. Use the cosine rule

$(5x)^2 = (2x)^2 + (4x)^2 - 2(2x)(4x)\cos\theta$.

3. Find the angle

$25 = 4 + 16 - 16\cos\theta$, so $\cos\theta = -\dfrac{5}{16}$ and $\theta = 108.2°$ to one decimal place.

4. Use the area formula

$140 = \dfrac{1}{2}(2x)(4x)\sin 108.2° = 4x^2 \sin 108.2°$.

5. Find the scale factor

$x^2 = \dfrac{140}{4\sin 108.2°}$, so $x \approx 6.070$.

6. Shortest side

The shortest side is $2x \approx 12.1\ \text{cm}$.

Final answer

$(a)\ 108.2°$.

$(b)\ 12.1\ \text{cm}$.

TOPIC

Radians

WMA11/01 OCTOBER 2019

Question 1

5 marks

Radians

1.

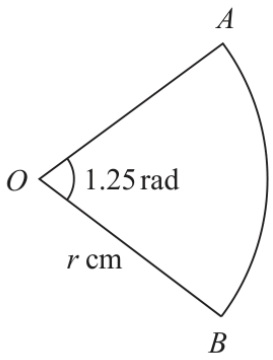


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is 1.25 radians.

Given that the area of the sector AOB is $15\,\text{cm}^2$

(a) find the exact value of r,

(2)

(b) find the exact length of the perimeter of the sector. Write your answer in simplest form.

(3)

(Total 5 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 1

Radians

Topic group

1. Use the sector area formula

For a sector, $A = \frac{1}{2}r^2\theta$. Here $A = 15$ and $\theta = 1.25 = \frac{5}{4}$, so $15 = \frac{1}{2}r^2 \cdot \frac{5}{4}$.

2. Find the exact radius

$r^2 = \dfrac{30}{5/4} = 24$, so $r = \sqrt{24} = 2\sqrt{6}$ because the radius is positive.

3. Find the perimeter

Arc length is $s = r\theta = (2\sqrt{6})\left(\frac{5}{4}\right) = \frac{5\sqrt{6}}{2}$. The perimeter is $2r + s = 4\sqrt{6} + \frac{5\sqrt{6}}{2} = \frac{13\sqrt{6}}{2}$.

Final answer

$(a)\ r = 2\sqrt{6}\ \text{cm}.$

$(b)\ \dfrac{13\sqrt{6}}{2}\ \text{cm}.$

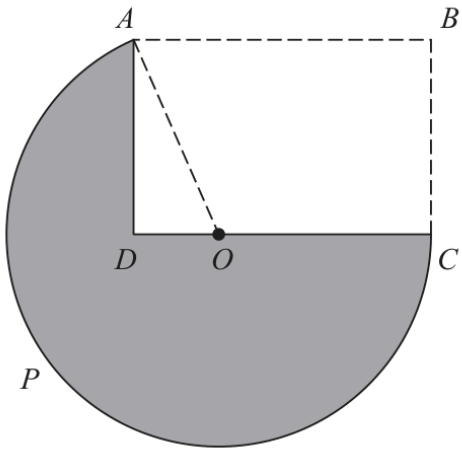
WMA11/01 JANUARY 2020

Question 4

9 marks

Radians

4.



Not to scale

Figure 2

Figure 2 shows the plan view of a house *ABCD* and a lawn *APCDA*.

ABCD is a rectangle with $AB = 16\,\text{m}$.

APCOA is a sector of a circle centre *O* with radius 12 m.

The point *O* lies on the line *DC*, as shown in Figure 2.

(a) Show that the size of angle *AOD* is 1.231 radians to 3 decimal places.

(2)

The lawn *APCDA* is shown shaded in Figure 2.

(b) Find the area of the lawn, in m^2, to one decimal place.

(4)

(c) Find the perimeter of the lawn, in metres, to one decimal place.

(3)

(Total 9 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 4

Radians

Topic group

1. Find angle AOD

Since $OC = 12$ and $DC = AB = 16$, $OD = 4$. In right triangle AOD, $\cos \angle AOD = \dfrac{OD}{OA} = \dfrac{4}{12} = \dfrac{1}{3}$. Hence $\angle AOD = \cos^{-1}\left(\dfrac{1}{3}\right) = 1.231$ radians to 3 d.p.

2. Identify the major sector used for the lawn

The shaded lawn uses the major sector from A to C, so its angle is $\pi + 1.231$ radians.

3. Subtract the unshaded triangle

$AD = \sqrt{12^2 - 4^2} = 8\sqrt{2}$. Area of major sector $= \frac{1}{2}(12)^2(\pi + 1.231)$. Area of triangle $AOD = \frac{1}{2}(4)(8\sqrt{2})$.

4. Find the lawn area

Area $= \frac{1}{2}(12)^2(\pi + 1.231) - \frac{1}{2}(4)(8\sqrt{2}) = 292.2\ \text{m}^2$ to 1 d.p.

5. Find the lawn perimeter

Perimeter $=$ major arc $+ CD + AD = 12(\pi + 1.231) + 16 + 8\sqrt{2} = 79.8\ \text{m}$ to 1 d.p.

Final answer

$(a)\ \angle AOD = 1.231\ \text{rad}.$

$(b)\ 292.2\ \text{m}^2.$

$(c)\ 79.8\ \text{m}.$

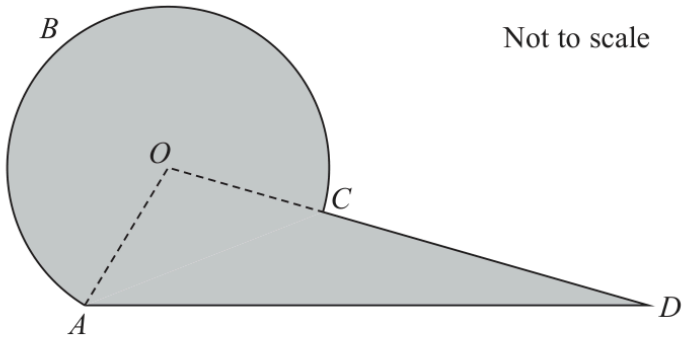
WMA11/01 JANUARY 2021

Question 5

9 marks

Radians

5.



Not to scale

Figure 3

Figure 3 shows the plan view of a viewing platform at a tourist site.

The shape of the viewing platform consists of a sector $ABCOA$ of a circle, centre O, joined to a triangle AOD.

Given that

- $OA = OC = 6\,\text{m}$
- $AD = 14\,\text{m}$
- angle $ADC = 0.43$ radians
- angle AOD is an obtuse angle
- OCD is a straight line

find

(a) the size of angle AOD, in radians, to 3 decimal places,

(3)

(b) the length of arc ABC, in metres, to one decimal place,

(2)

(c) the total area of the viewing platform, in m^2, to one decimal place.

(4)

(Total 9 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 5

Radians

Topic group

1. Use the sine rule in triangle AOD

Since O, C, D are in a straight line, $\angle ADO = 0.43$. By the sine rule, $\dfrac{14}{\sin \angle AOD} = \dfrac{6}{\sin 0.43}$, so $\sin \angle AOD = \dfrac{14 \sin 0.43}{6}$.

2. Choose the obtuse angle

The calculator gives the acute value $1.337\ldots$, but $\angle AOD$ is obtuse. Hence $\angle AOD = \pi - 1.337\ldots = 1.805\ldots$, so $\angle AOD = 1.805$ radians.

3. Find the major-sector angle

Arc ABC is the major arc, so its angle is $2\pi - 1.805\ldots = 4.478\ldots$ radians.

4. Find the arc length

$s = r\theta = 6(4.478\ldots) = 26.868\ldots$, so the length of arc ABC is 26.9 m.

5. Find the sector area

The sector area is $\dfrac{1}{2}r^2\theta = \dfrac{1}{2}(6)^2(4.478\ldots) = 80.607\ldots$.

6. Find the triangle area and total area

$\angle OAD = \pi - 0.43 - 1.805\ldots = 0.9066\ldots$. The triangle area is $\dfrac{1}{2}(6)(14)\sin(0.9066\ldots) = 33.093\ldots$. Total area $= 80.607\ldots + 33.093\ldots = 113.7\ \text{m}^2$.

Final answer

(a) $1.805\ \text{rad}$.

(b) $26.9\ \text{m}$.

(c) $113.7\ \text{m}^2$.

WMA11/01 MAY/JUNE 2021

Question 7

10 marks

Radians

7.

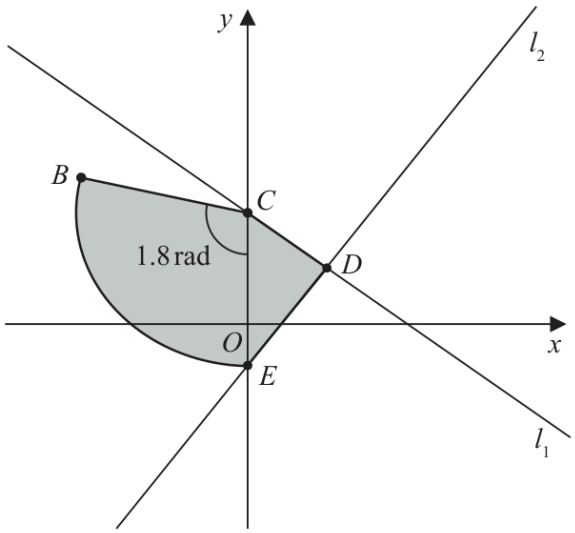


Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y-axis at the point C, as shown in Figure 3.

(a) State the y coordinate of C.

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y-axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\frac{14}{3}$

(3)

A sector BCE of a circle with centre C is also shown in Figure 3.

Given that angle BCE is 1.8 radians,

(c) find the length of arc BE.

(3)

The region $CBED$, shown shaded in Figure 3, consists of the sector BCE joined to the triangle CDE.

(d) Calculate the exact area of the region $CBED$.

(3)

(Total 10 marks)

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Worked Solution - Question 7

Radians

Topic group

1. Find C

On the y-axis, $x = 0$. From $4y + 3x = 48$, $4y = 48$, so the y-coordinate of C is 12.

2. Find the gradient of l1

$4y + 3x = 48$ gives $y = -\frac{3}{4}x + 12$, so the gradient of l_1 is $-\frac{3}{4}$.

3. Use perpendicular gradients for l2

Since l_2 is perpendicular to l_1, its gradient is $\frac{4}{3}$. Through $D(8, 6)$, $y - 6 = \frac{4}{3}(x - 8)$.

4. Show the y-coordinate of E

$y = \frac{4}{3}x - \frac{32}{3} + 6 = \frac{4}{3}x - \frac{14}{3}$. Therefore when $x = 0$, $E_y = -\frac{14}{3}$.

5. Find the radius of the sector

$C = (0, 12)$ and $E = \left(0, -\frac{14}{3}\right)$, so $CE = 12 + \frac{14}{3} = \frac{50}{3}$.

6. Find arc BE

For sector BCE, $r = \frac{50}{3}$ and $\theta = 1.8 = \frac{9}{5}$. Hence arc $BE = r\theta = \frac{50}{3} \cdot \frac{9}{5} = 30$.

7. Find the sector area

Sector area $= \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{50}{3}\right)^2\left(\frac{9}{5}\right) = 250$.

8. Find the triangle area and total area

Triangle CDE has vertical base $CE = \dfrac{50}{3}$ and horizontal height 8, so its area is $\dfrac{1}{2} \cdot \dfrac{50}{3} \cdot 8 = \dfrac{200}{3}$. Total area $= 250 + \dfrac{200}{3} = \dfrac{950}{3}$.

Final answer

(a) 12.

(b) $E_y = -\dfrac{14}{3}$.

(c) 30.

(d) $\dfrac{950}{3}$.

WMA11/01 OCTOBER 2021

Question 7

10 marks

Radians

7.

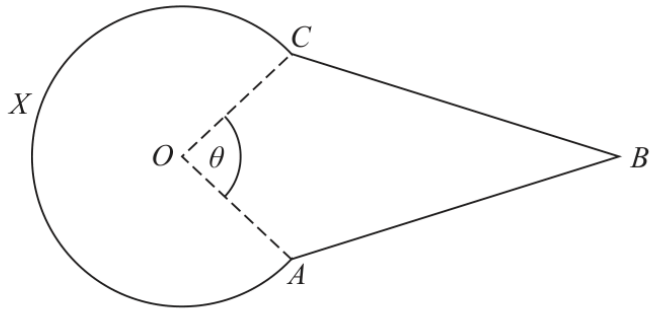


Figure 3

Figure 3 shows the design for a sign at a bird sanctuary.

The design consists of a kite $OABC$ joined to a sector $OCXA$ of a circle centre O.

In the design

- $OA = OC = 0.6\,\text{m}$
- $AB = CB = 1.4\,\text{m}$
- Angle OAB = Angle OCB = 2 radians
- Angle $AOC = \theta$ radians, as shown in Figure 3

Making your method clear,

(a) show that $\theta = 1.64$ radians to 3 significant figures,

(4)

(b) find the perimeter of the sign, in metres to 2 significant figures,

(2)

(c) find the area of the sign, in m^2 to 2 significant figures.

(4)

(Total 10 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 7

Radians

Topic group

1. Work in triangle OAB

In triangle OAB, $OA = 0.6$, $AB = 1.4$, and $\angle OAB = 2$ radians.

2. Find OB

By the cosine rule, $OB^2 = 0.6^2 + 1.4^2 - 2(0.6)(1.4)\cos 2$, so $OB = 1.73756\ldots$.

3. Find half of theta

Using the cosine rule again, $\cos \angle AOB = \dfrac{0.6^2 + OB^2 - 1.4^2}{2(0.6)(OB)}$, giving $\angle AOB = 0.822199\ldots$ radians.

4. Use symmetry of the kite

The kite is symmetric, so $\theta = 2\angle AOB = 1.644398\ldots$, hence $\theta = 1.64$ radians to 3 significant figures.

5. Find the outside arc length

The sector $OCXA$ uses the major angle $2\pi - \theta$. Its arc length is $0.6(2\pi - 1.644398\ldots) = 2.78327\ldots$.

6. Find the perimeter

Perimeter $= AB + CB + \text{major arc} = 1.4 + 1.4 + 2.78327\ldots = 5.58327\ldots$, so the perimeter is 5.6 m.

7. Find the kite area

The kite consists of two congruent triangles like OAB, so area of kite $= 2\left(\dfrac{1}{2}(0.6)(1.4)\sin 2\right) = 0.76381\ldots$.

8. Find the sector area and total area

Major sector area $= \frac{1}{2}(0.6)^2(2\pi - 1.644398\ldots) = 0.83498\ldots$. Total area $= 0.76381\ldots + 0.83498\ldots = 1.59879\ldots$, so the area is $1.6\ \mathrm{m}^2$.

Final answer

$(a)\ \theta = 1.64\ \mathrm{rad}.$

$(b)\ 5.6\ \mathrm{m}.$

$(c)\ 1.6\ \mathrm{m}^2.$

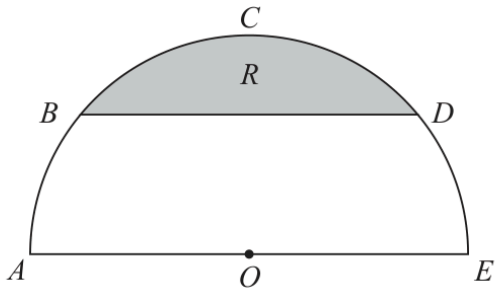
WMA11/01 JANUARY 2022

Question 5

7 marks

Radians

5.



Not to scale

Figure 2

Figure 2 shows a plan view of a semicircular garden *ABCDEOA*

The semicircle has

- centre *O*
- diameter *AOE*
- radius 3 m

The straight line *BD* is parallel to *AE* and angle *BOA* is 0.7 radians.

(a) Show that, to 4 significant figures, angle *BOD* is 1.742 radians.

(1)

The flowerbed *R*, shown shaded in Figure 2, is bounded by *BD* and the arc *BCD*.

(b) Find the area of the flowerbed, giving your answer in square metres to one decimal place.

(3)

(c) Find the perimeter of the flowerbed, giving your answer in metres to one decimal place.

(3)

(Total 7 marks)

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Worked Solution - Question 5

Radians

Topic group

1. Find angle BOD

Since BD is parallel to the diameter AE, the two side angles at the centre are both 0.7 radians. Therefore $\angle BOD = \pi - 0.7 - 0.7 = \pi - 1.4 = 1.74159\ldots$, so 1.742 radians to 4 significant figures.

2. Find the sector area

With $r = 3$ and $\theta = 1.74159\ldots$, sector area $= \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2(1.74159\ldots) = 7.837\ldots$.

3. Subtract triangle BOD

Triangle area $= \frac{1}{2}(3)(3)\sin(1.74159\ldots) = 4.434\ldots$. The segment area is $7.837\ldots - 4.434\ldots = 3.403\ldots$, so $3.4\ \text{m}^2$.

4. Find the arc length

Arc $BCD = r\theta = 3(1.74159\ldots) = 5.225\ldots$.

5. Find the chord and perimeter

$BD = 2r\sin\frac{\theta}{2} = 6\sin(0.87079\ldots) = 4.590\ldots$. Perimeter $= 5.225\ldots + 4.590\ldots = 9.8\ \text{m}$.

Final answer

$(a)\ \angle BOD = 1.742\ \text{rad}.$

$(b)\ 3.4\ \text{m}^2.$

$(c)\ 9.8\ \text{m}.$

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Question 8

10 marks

Radians

8.

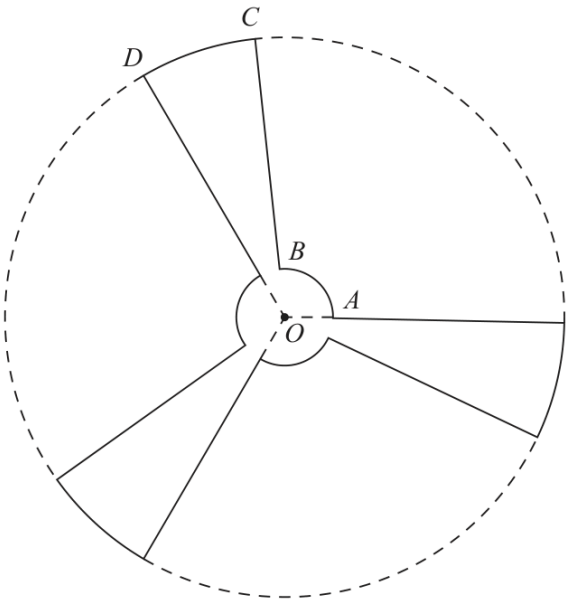


Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to $OABCDO$, shown in Figure 3, where

- $OABO$ is a sector of a circle with centre O and radius 9 cm
- $OBCDO$ is a sector of a circle with centre O and radius 84 cm
- angle $AOD = \frac{2\pi}{3}$ radians

Given that the length of the arc AB is 15 cm,

(a) show that the length of the arc CD is 35.9 cm to one decimal place.

(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

(b) the perimeter of the face of the fan, giving your answer to the nearest cm,

(2)

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.

(5)

(Total 10 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 8

Radians

Topic group

1. Find the small sector angle

For arc AB, $s = r\theta$, so $15 = 9\theta$ and $\theta = \frac{5}{3}$ radians.

2. Find the outer arc angle

Each identical section has total angle $\frac{2\pi}{3}$, so the angle for arc CD is $\frac{2\pi}{3} - \frac{5}{3} = \frac{2\pi - 5}{3}$.

3. Find CD

$CD = 84\left(\frac{2\pi - 5}{3}\right) = 28(2\pi - 5) = 35.9\ \text{cm}$ to 3 significant figures.

4. Find the perimeter

There are 3 inner arcs, 3 outer arcs, and 6 straight radial edges of length $84 - 9 = 75$. Thus $P = 3(15) + 3(35.929\ldots) + 6(75) = 602.8\ldots\ \text{cm}$.

5. Find the area of one section

One section has area $\frac{1}{2}(9^2)\left(\frac{5}{3}\right) + \frac{1}{2}(84^2)\left(\frac{2\pi - 5}{3}\right) = 1576.525\ldots\ \text{cm}^2$.

6. Find the total area

The total area is $3(1576.525\ldots) = 4729.57\ldots\ \text{cm}^2 = 4.73 \times 10^3\ \text{cm}^2$ to 3 significant figures.

Final answer

(a) $CD = 35.9\ \text{cm}$.

(b) $603\ \text{cm}$.

(c) $4.73 \times 10^3\ \text{cm}^2$.

WMA11/01 OCTOBER 2022

Question 8

14 marks

Radians

8.

Diagram **NOT** to scale

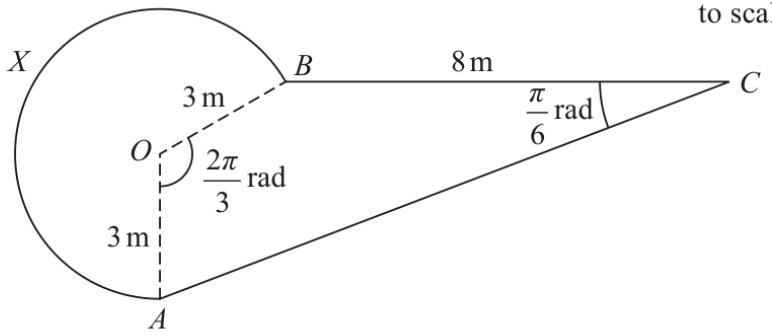


Figure 2

Figure 2 shows the plan view of a design for a pond.

The design consists of a sector $AOBX$ of a circle centre O joined to a quadrilateral $AOBC$.

- $BC = 8\,\text{m}$
- $OA = OB = 3\,\text{m}$
- angle AOB is $\frac{2\pi}{3}$ radians
- angle BCA is $\frac{\pi}{6}$ radians

(a) Calculate (i) the exact area of the sector $AOBX$,

(ii) the exact perimeter of the sector $AOBX$.

(5)

(b) Calculate the exact area of the triangle AOB.

(2)

(c) Show that the length AB is $3\sqrt{3}$ m.

(2)

(d) Find the total surface area of the pond. Give your answer in m^2 correct to 2 significant figures.

(5)

(Total for Question 8 is 14 marks)

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Worked Solution - Question 8

Radians

Topic group

1. Find the angle of the sector AOBX

The sector AOBX is the major sector, so its angle is $2\pi - \dfrac{2\pi}{3} = \dfrac{4\pi}{3}$.

2. Find the sector area

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(3^2)\left(\frac{4\pi}{3}\right) = 6\pi \text{ m}^2.$$

3. Find the sector perimeter

The arc length is $r\theta = 3\left(\dfrac{4\pi}{3}\right) = 4\pi$. Adding the two radii gives $4\pi + 6$ m.

4. Find the area of triangle AOB

$$\text{Area} = \frac{1}{2}(3)(3)\sin\left(\frac{2\pi}{3}\right) = \frac{9}{2}\cdot\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4} \text{ m}^2.$$

5. Show AB

By the cosine rule, $AB^2 = 3^2 + 3^2 - 2(3)(3)\cos\left(\dfrac{2\pi}{3}\right) = 18 + 9 = 27$, so $AB = 3\sqrt{3}$.

6. Find AC

In triangle ABC, $AB^2 = AC^2 + 8^2 - 2(AC)(8)\cos\left(\dfrac{\pi}{6}\right)$, so $27 = AC^2 - 8\sqrt{3}\,AC + 64$.

7. Choose the correct length

Solving gives $AC = 4\sqrt{3} \pm \sqrt{11}$. From the diagram, AC is the longer value, so $AC = 4\sqrt{3} + \sqrt{11}$.

8. Find the total area

Area of triangle ABC is $\frac{1}{2}(8)(AC)\sin\left(\frac{\pi}{6}\right) = 2AC = 8\sqrt{3} + 2\sqrt{11}$. Total area is $6\pi + \frac{9\sqrt{3}}{4} + 8\sqrt{3} + 2\sqrt{11} = 43.236\ldots$, so $43\ \mathrm{m}^2$ to 2 significant figures.

Final answer

$(a)(i)\ 6\pi\ \mathrm{m}^2,\ (ii)\ 4\pi + 6\ \mathrm{m}.$

$(b)\ \frac{9\sqrt{3}}{4}\ \mathrm{m}^2.$

$(c)\ AB = 3\sqrt{3}\ \mathrm{m}.$

$(d)\ 43\ \mathrm{m}^2.$

Question 6

10 marks

Radians

6.

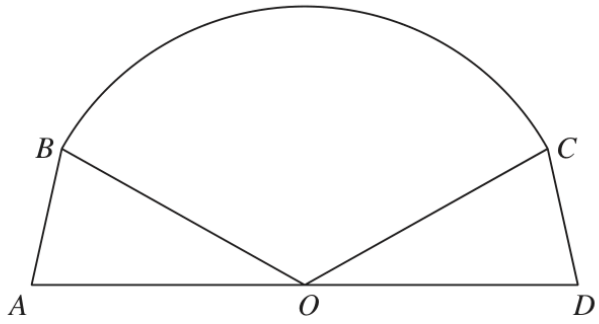


Diagram **NO** accurately dr

Figure 1

Figure 1 shows the plan view for the design of a stage.

The design consists of a sector OBC of a circle, with centre O, joined to two congruent triangles OAB and ODC.

Given that

- angle $BOC = 2.4$ radians
- area of sector $BOC = 40\,\text{m}^2$
- AOD is a straight line of length 12.5 m

(a) find the radius of the sector, giving your answer, in m, to 2 decimal places,

(2)

(b) find the size of angle AOB, in radians, to 2 decimal places.

(1)

Hence find

(c) the total area of the stage, giving your answer, in m^2, to one decimal place,

(3)

(d) the total perimeter of the stage, giving your answer, in m, to one decimal place.

(4)

(Total for Question 6 is 10 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 6

Radians

Topic group

1. Find the radius

Area of sector BOC is $\frac{1}{2}r^2\theta$. So $40 = \frac{1}{2}r^2(2.4) = 1.2r^2$, giving $r = \sqrt{\frac{40}{1.2}} = 5.77$ m.

2. Find angle AOB

The triangles OAB and ODC are congruent, so $\angle AOB = \angle COD$. Since AOD is a straight line, $2\angle AOB + 2.4 = \pi$.

3. Calculate the angle

$\angle AOB = \frac{\pi - 2.4}{2} = 0.3708\ldots$, so 0.37 rad.

4. Find the total area

The two congruent triangle areas together are $2\left(\frac{1}{2}(6.25)(5.7735\ldots)\sin 0.3708\ldots\right) = 13.075\ldots$.

5. Add the sector area

Total area $= 40 + 13.075\ldots = 53.1\ \text{m}^2$ to one decimal place.

6. Find AB

Using the cosine rule, $AB^2 = 6.25^2 + r^2 - 2(6.25)(r)\cos 0.3708\ldots$, giving $AB = 2.265\ldots$ m.

7. Find the perimeter

Perimeter $= AD + AB + CD + \text{arc } BC = 12.5 + 2(2.265\ldots) + 5.7735\ldots(2.4) = 30.9$ m.

Final answer

(a) $r = 5.77\ \text{m}$.

(b) $0.37\ \text{rad}$.

(c) $53.1\ \text{m}^2$.

(d) $30.9\ \text{m}$.

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Question 5

10 marks

Radians

5.

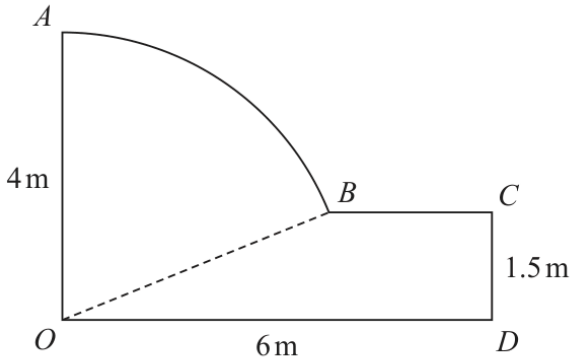


Figure 1

Figure 1 shows the plan for a garden.

In the plan

- OA and CD are perpendicular to OD
- AB is an arc of the circle with centre O and radius 4 metres
- BC is parallel to OD
- OD is 6 metres, OA is 4 metres and CD is 1.5 metres

(a) Show that angle AOB is 1.186 radians to 4 significant figures.

(2)

(b) Find the perimeter of the garden, giving your answer in metres to 3 significant figures.

(4)

(c) Find the area of the garden, giving your answer in square metres to 3 significant figures.

(4)

(Total for Question 5 is 10 marks)

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Worked Solution - Question 5

Radians

Topic group

1. Find the angle AOB

Since $OB = 4$ and the height of B is 1.5, $\cos \angle AOB = \dfrac{1.5}{4} = 0.375$.

2. Calculate the angle

$\angle AOB = \cos^{-1}(0.375) = 1.186\ldots$ rad, which is 1.186 rad to 4 significant figures.

3. Find BC

The horizontal coordinate of B is $4\sin(1.186\ldots) = 3.708\ldots$, so $BC = 6 - 3.708\ldots = 2.2919\ldots$ m.

4. Find the arc length

Arc $AB = r\theta = 4(1.186\ldots) = 4.7456\ldots$ m.

5. Find the perimeter

Perimeter $= OA + \text{arc } AB + BC + CD + OD = 4 + 4.7456\ldots + 2.2919\ldots + 1.5 + 6 =$ m.

6. Find the area

Area of sector $AOB = \dfrac{1}{2}(4^2)(1.186\ldots) = 9.491\ldots$.

7. Add the quadrilateral area

Area of $OBCD$ is the trapezium area $\dfrac{1}{2}(6 + 2.2919\ldots)(1.5) = 6.2189\ldots$.

Total area $= 15.710\ldots = 15.7\ \text{m}^2$.

Final answer

$(a)\ \angle AOB = 1.186\ \text{rad}.$

$(b)\ 18.5\ \text{m}.$

$(c)\ 15.7\ \text{m}^2.$

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Question 9

7 marks

Radians

9.

Diagram **NO**
accurately dr

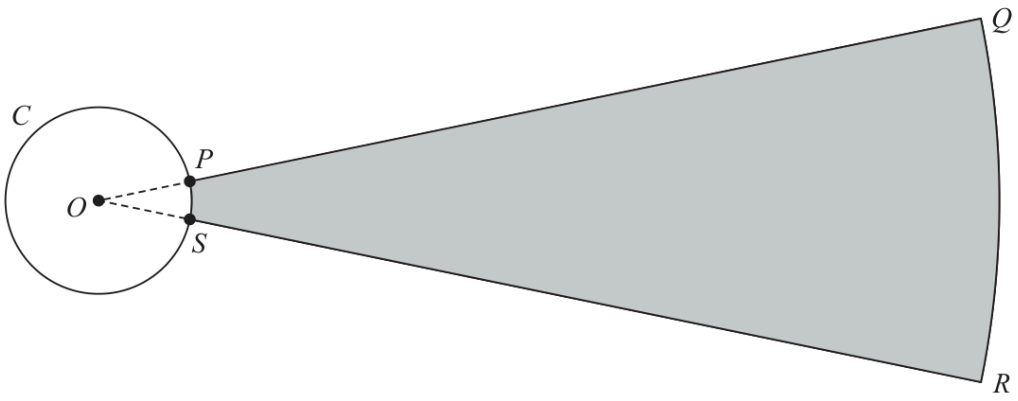


Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2\,\text{m}$

(1)

Given also that

- the target area, $PQRS$, is $90\,\text{m}^2$
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)

(Total for Question 9 is 7 marks)

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Worked Solution - Question 9

Radians

Topic group

1. Use arc length

$s = r\theta$, so $0.72 = OP \times 0.6$. Hence $OP = 1.2\ \text{m}$.

2. Set up the target area

The outer sector has radius $x + 1.2$ and the inner sector has radius 1.2, with angle 0.6 radians.

3. Use sector area

$\frac{1}{2}(x+1.2)^2(0.6) - \frac{1}{2}(1.2)^2(0.6) = 90$.

4. Simplify to the given quadratic

$0.3\left((x+1.2)^2 - 1.44\right) = 90$, so $0.3x^2 + 0.72x - 90 = 0$. Multiplying by $\frac{50}{3}$ gives $5x^2 + 12x - 1500 = 0$.

5. Find the positive value of x

$$x = \frac{-12 + \sqrt{12^2 + 4(5)(1500)}}{10} = \frac{-6 + 4\sqrt{471}}{5} \approx 16.16.$$

6. Find the perimeter

$\text{Perimeter} = PQ + SR + \text{arc } QR + \text{arc } PS = x + x + 0.6(x + 1.2) + 0.72 \approx 43$

.

7. Round to the nearest metre

The total perimeter is $43\ \text{m}$ to the nearest metre.

Final answer

(a) $OP = 1.2\ \text{m}$.

(b) $5x^2 + 12x - 1500 = 0$.

(c) $43\ \text{m}$.

WMA11/01 JANUARY 2024

Question 8

9 marks

Radians

8.

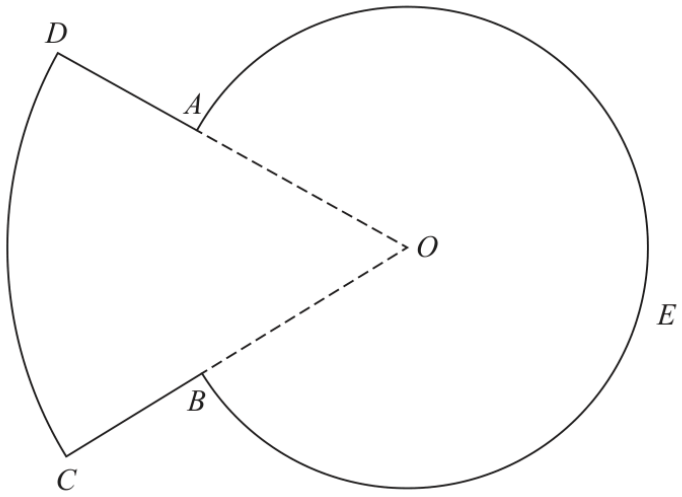


Figure 3

Figure 3 shows a sketch of the plan view of a platform.

The plan view of the platform consists of a sector DOC of a circle centre O joined to a sector $AOBEA$ of a different circle, also with centre O.

Given that

- angle $AOB = 0.8$ radians
- arc length $CD = 9\,\text{m}$
- $DA : AO = 3 : 5$

(a) show that $AO = 7.03\,\text{m}$ to 3 significant figures.

(3)

(b) Find the perimeter of the platform, in m, to 3 significant figures.

(3)

(c) Find the total area of the platform, giving your answer in m^2 to the nearest whole number.

(3)

(Total for Question 8 is 9 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 8

Radians

Topic group

1. Find the outer radius

For sector DOC, arc length $CD = 9$ and angle 0.8 radians, so $OD = \dfrac{9}{0.8} = 11.25\ \text{m}$.

2. Use the ratio DA to AO

$DA : AO = 3 : 5$, so $OD = DA + AO$ represents 8 parts. Hence $AO = \dfrac{5}{8}(11.25) = 7.03125\ \text{m} \approx 7.03\ \text{m}$.

3. Find the straight side lengths

$DA = CB = OD - AO = 11.25 - 7.03125 = 4.21875\ \text{m}$.

4. Find the major arc

The major sector $AOBEA$ has angle $2\pi - 0.8$, so arc $AEB = 7.03125(2\pi - 0.8)$.

5. Calculate the perimeter

$\text{Perimeter} = 9 + 2(4.21875) + 7.03125(2\pi - 0.8) \approx 56.0\ \text{m}$.

6. Calculate the total area

$\text{Area} = \dfrac{1}{2}(11.25)^2(0.8) + \dfrac{1}{2}(7.03125)^2(2\pi - 0.8) \approx 186\ \text{m}^2$ to the nearest whole number.

Final answer

(a) $AO = 7.03\ \text{m}$.

(b) $56.0\ \text{m}$.

(c) $186\ \text{m}^2$.

WMA11/01 MAY/JUNE 2024

Question 5

7 marks

Radians

5.

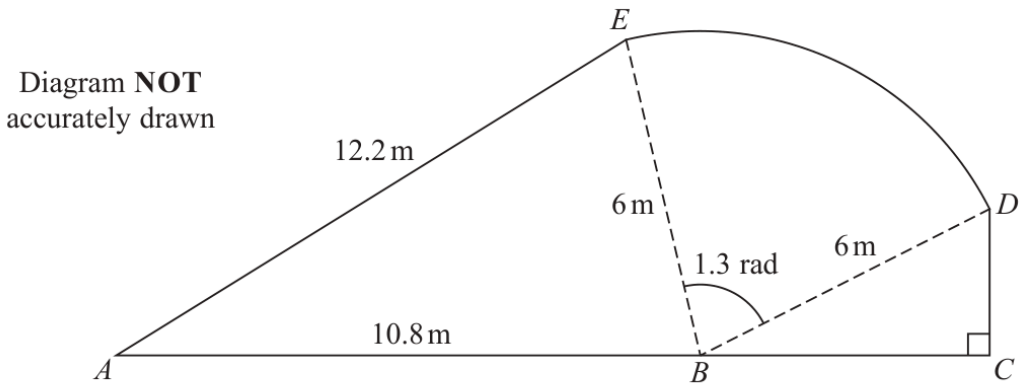


Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden $ABCDEA$ consists of a triangle ABE and a right-angled triangle BCD joined to a sector BDE of a circle with radius 6 m and centre B.

The points A, B and C lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

(a) Find the area of the sector BDE, giving your answer in m^2

(2)

(b) Find the size of angle ABE, giving your answer in radians to 2 decimal places.

(2)

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures.

(3)

(Total for Question 5 is 7 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 5

Radians

Topic group

1. Find the sector area

$\text{Area of sector } BDE = \frac{1}{2}(6^2)(1.3) = 23.4\ \text{m}^2.$

2. Use the cosine rule in triangle ABE

$12.2^2 = 10.8^2 + 6^2 - 2(10.8)(6)\cos\angle ABE.$

3. Find angle ABE

$\cos\angle ABE = \frac{10.8^2 + 6^2 - 12.2^2}{2(10.8)(6)}$, so $\angle ABE \approx 1.54$ radians.

4. Find the area of triangle ABE

$\text{Area } ABE = \frac{1}{2}(10.8)(6)\sin(1.54147\ldots) \approx 32.386.$

5. Find angle DBC

$\angle DBC = \pi - 1.3 - \angle ABE \approx 0.30012$ radians.

6. Find the area of triangle BCD

$\text{Area } BCD = \frac{1}{2}(6\cos\angle DBC)(6\sin\angle DBC) \approx 5.084.$

7. Add the three areas

$23.4 + 32.386 + 5.084 \approx 60.9\ \text{m}^2$ to 3 significant figures.

Final answer

(a) $23.4\ \mathrm{m}^2$.

(b) 1.54 radians.

(c) $60.9\ \mathrm{m}^2$.

WMA11/01 OCTOBER 2024

Question 5

9 marks

Radians

5. A plot of land OAB is in the shape of a sector of a circle with centre O.

Given

- $OA = OB = 5\,\text{km}$
- angle $AOB = 1.2$ radians

(a) find the perimeter of the plot of land.

(2)

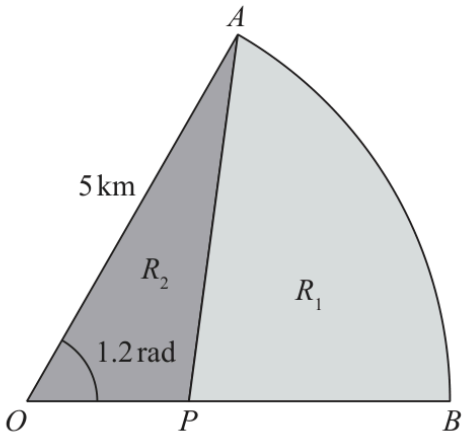


Figure 2

Diagram **NO** accurately dr

A point P lies on OB such that the line AP divides the plot of land into two regions R_1 and R_2 as shown in Figure 2.

Given that

$$\text{area of } R_1 = 3 \times \text{area of } R_2$$

(b) show that the area of $R_2 = 3.75\,\text{km}^2$

(3)

(c) Find the length of AP, giving your answer to the nearest 100 m.

(4)

(Total for Question 5 is 9 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 5

Radians

Topic group

1. Find the arc length

$\text{arc } AB = r\theta = 5(1.2) = 6 \text{ km}$.

2. Find the perimeter

$OA + OB + \text{arc } AB = 5 + 5 + 6 = 16 \text{ km}$.

3. Find the total sector area

$\text{Area of } OAB = \frac{1}{2}(5^2)(1.2) = 15 \text{ km}^2$.

4. Use the area ratio

Since $\text{area } R_1 = 3 \times \text{area } R_2$, the total area is $4 \times \text{area } R_2$.

5. Find area R2

$\text{area } R_2 = \frac{15}{4} = 3.75 \text{ km}^2$.

6. Find OP from triangle OAP

$\frac{1}{2}(5)(OP)\sin 1.2 = 3.75$, so $OP \approx 1.609$ km.

7. Use the cosine rule

$AP^2 = 5^2 + OP^2 - 2(5)(OP)\cos 1.2$, so $AP \approx 4.665$ km.

8. Round to the nearest 100 m

$AP = 4.7 \text{ km}$ to the nearest 0.1 km.

Final answer

(a) $16\ \text{km}$.

(b) $3.75\ \text{km}^2$.

(c) $AP = 4.7\ \text{km}$.

WMA11/01 JANUARY 2025

Question 8

13 marks

Radians

8.

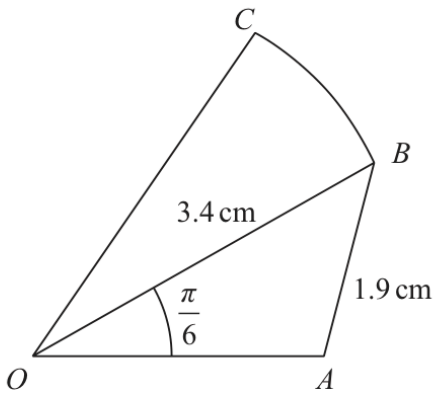


Figure 1

Figure 1 shows a sketch of a design for a badge.
The design consists of a triangle OAB joined to a sector OBC of a circle with centre O
In the design

- $OB = 3.4\,\text{cm}$
- $AB = 1.9\,\text{cm}$
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB, giving your answer in radians to 4 significant figures,

(3)

(b) find the area of triangle OAB, in cm^2, giving your answer to 3 significant figures.

(2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is $3:2$

(c) show that angle BOC is 0.462 radians to 3 significant figures.

(3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer.

(5)

(Total for Question 8 is 13 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 8

Radians

Topic group

1. Use the sine rule

In triangle OAB, $AB = 1.9$, $OB = 3.4$ and $\angle AOB = \dfrac{\pi}{6}$.

2. Find sine of angle OAB

$\dfrac{\sin \angle OAB}{3.4} = \dfrac{\sin(\pi/6)}{1.9}$, so $\sin \angle OAB = \dfrac{17}{19}$.

3. Choose the obtuse angle

Since $\angle OAB > \dfrac{\pi}{2}$, $\angle OAB = \pi - \sin^{-1}\left(\dfrac{17}{19}\right) \approx 2.034$ radians.

4. Find OA

Using the sine rule again, $OA \approx 2.096$ cm.

5. Find the triangle area

$\text{Area } OAB = \dfrac{1}{2}(OA)(OB)\sin(\pi/6) \approx 1.78 \text{ cm}^2$.

6. Use the area ratio

$\text{Area sector } OBC : \text{Area triangle } OAB = 3 : 2$, so sector area $= \dfrac{3}{2}(1.78156\ldots)$.

7. Find angle BOC

$\dfrac{1}{2}(3.4)^2\theta = \dfrac{3}{2}(1.78156\ldots)$, so $\theta \approx 0.462$ radians.

8. Find the perimeter

The perimeter is $OA + AB + OC + \text{arc } BC = 2.096 + 1.9 + 3.4 + 3.4(0.46234\ldots) \approx 8.968$ cm.

9. Round

The perimeter is 9 cm to the nearest integer.

Final answer

$(a)\ \angle OAB = 2.034$ radians.

$(b)\ 1.78\ \text{cm}^2$.

$(c)\ \angle BOC = 0.462$ radians.

$(d)\ 9$ cm.

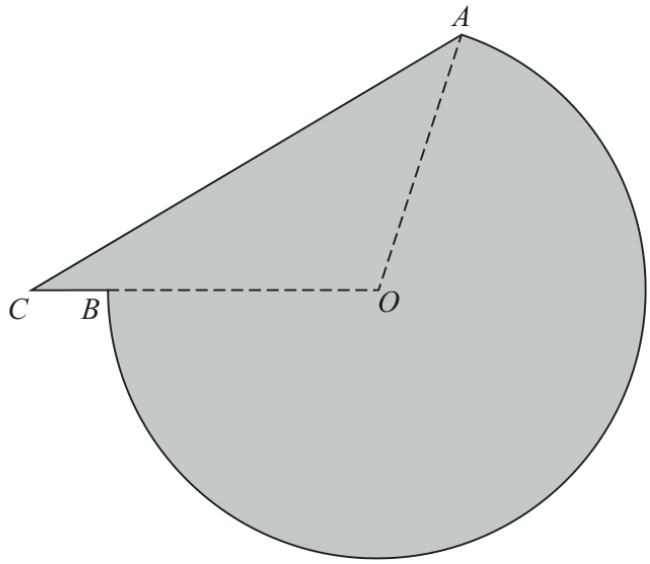
WMA11/01 MAY/JUNE 2025

Question 6

9 marks

Radians

6.



Not to scale

Figure 2

The shaded area in Figure 2 shows the plan view of a helicopter landing pad.

The area consists of the major sector AOB of a circle centre O joined to a triangle AOC.

Given that

- $AO = OB = 15\,\text{m}$
- $BC = 2\,\text{m}$
- CBO is a straight line
- angle $ACO = 0.6$ radians

(a) show that angle COA is 1.847 radians to 3 decimal places.

(3)

(b) Find the total area of the helicopter landing pad.
Give your answer in m^2 to 3 significant figures.

(3)

(c) Find the perimeter of the helicopter landing pad.
Give your answer in metres to 3 significant figures.

(3)

(Total for Question 6 is 9 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 6

Radians

Topic group

1. Set up triangle AOC

$AO = 15$, $OC = OB + BC = 17$, and $\angle ACO = 0.6$.

2. Use the sine rule

$\dfrac{\sin \angle CAO}{17} = \dfrac{\sin 0.6}{15}$, so $\angle CAO \approx 0.6944$ radians.

3. Show angle COA

$\angle COA = \pi - 0.6 - 0.6944\ldots = 1.847\ldots$, so $\angle COA = 1.847$ radians to 3 decimal places.

4. Major sector angle

The shaded sector is the major sector AOB, so its angle is $2\pi - 1.847\ldots$.

5. Sector area

$$\text{Sector area} = \frac{1}{2}(15)^2(2\pi - 1.847\ldots) = 499.15\ldots.$$

6. Triangle area

$$\text{Area } AOC = \frac{1}{2}(15)(17)\sin(1.847\ldots) = 122.73\ldots.$$

7. Total area

$499.15\ldots + 122.73\ldots = 621.88\ldots$, so the total area is $622\ \text{m}^2$ to 3 significant figures.

8. Find AC

By the cosine rule, $AC^2 = 15^2 + 17^2 - 2(15)(17)\cos(1.847\ldots)$, so $AC \approx 27.57$ m.

9. Major arc length

$\text{arc } AB = 15(2\pi - 1.847\ldots) = 66.54\ldots$ m.

10. Perimeter

$66.54\ldots + 27.57\ldots = 94.11\ldots$, so the perimeter is $94.1\ \text{m}$.

Final answer

$(a)\ \angle COA = 1.847\ \text{rad}.$

$(b)\ 622\ \text{m}^2.$

$(c)\ 94.1\ \text{m}.$

WMA11/01 OCTOBER 2025

Question 7

9 marks

Radians

7.

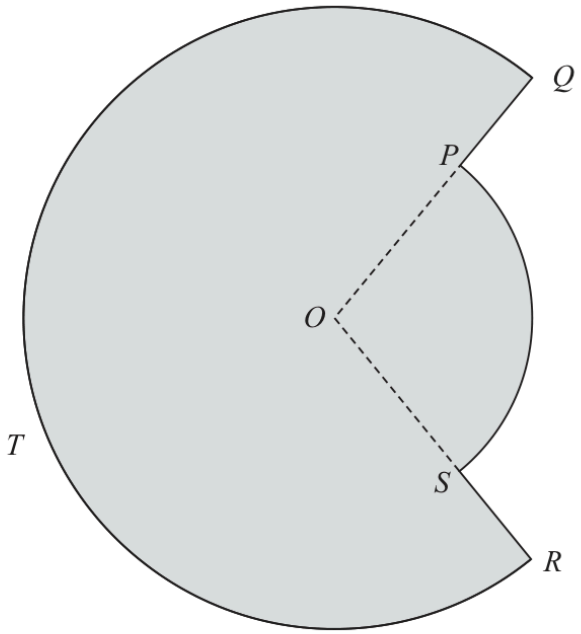


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows the plan view of a design for a swimming pool.

The design consists of a sector POS of a circle centre O joined to a major sector $QORTQ$ of a different circle, also with centre O.

Given that

- angle POS is 1.65 radians
- the area of sector POS is $30\,m^2$
- $PQ = 2.8\,m$

(a) show that, to 3 significant figures, $OQ = 8.83\,m$

(3)

(b) Find the total surface area of the swimming pool in m^2 to the nearest integer.

(3)

(c) Find the total perimeter of the swimming pool in metres to 2 significant figures.

(3)

(Total for Question 7 is 9 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 7

Radians

Topic group

1. Find OP from the small sector

Let $OP = r$. The area of sector POS is 30, so $\frac{1}{2}r^2(1.65) = 30$.

2. Calculate OP

$r^2 = \frac{60}{1.65}$, so $r = 6.030...$ m.

3. Show OQ

$OQ = OP + PQ = 6.030... + 2.8 = 8.830...$, so $OQ = 8.83$ m to 3 significant figures.

4. Major sector angle

The major sector $QORTQ$ has angle $2\pi - 1.65$ radians.

5. Total area

$\text{Area} = 30 + \frac{1}{2}(8.830...)^2(2\pi - 1.65) = 210.6...$.

6. Round area

The total surface area is $211\ \text{m}^2$ to the nearest integer.

7. Arc lengths

The inner arc PS has length $6.030...(1.65) = 9.95...$, and the outer major arc has length $8.830...(2\pi - 1.65) = 40.91...$.

8. Add straight sides

The straight sides PQ and SR each have length 2.8 m.

9. Perimeter

$9.95... + 40.91... + 2.8 + 2.8 = 56.46...$, so the perimeter is $56\ \text{m}$ to 2 significant figures.

Final answer

$(a)\ OQ = 8.83\ \text{m}.$

$(b)\ 211\ \text{m}^2.$

$(c)\ 56\ \text{m}.$

WMA11/01 JANUARY 2026

Question 5

9 marks

Radians

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

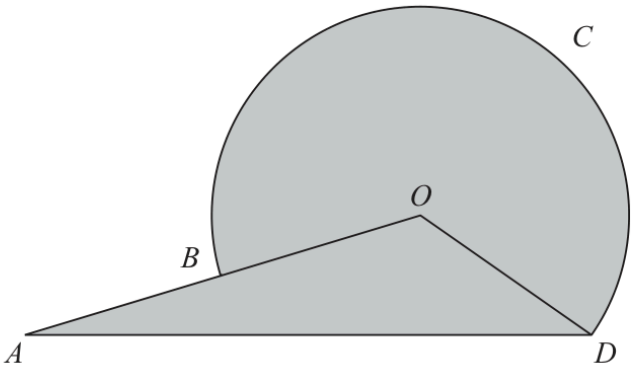


Diagram NOT to scale

Figure 2

Figure 2 shows the logo for the tourist board of an island.

The logo consists of a sector $BODCB$ of a circle centre O joined to a triangle AOD.

Given that

- ABO is a straight line
- $OD = 3\,\text{cm}$
- $AD = 8\,\text{cm}$
- angle $AOD = 2.5$ radians

(a) show that angle ODA is 0.415 radians to 3 significant figures.

(3)

(b) Find the total area of the logo, in cm^2, to one decimal place.

(3)

(c) Find the perimeter of the logo, $ABCDA$, in cm, to one decimal place.

(3)

(Total for Question 5 is 9 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 5

Radians

Topic group

1. Use triangle AOD

In triangle AOD, $OD = 3$, $AD = 8$ and $\angle AOD = 2.5$ radians.

2. Use the sine rule

$\dfrac{\sin \angle OAD}{3} = \dfrac{\sin 2.5}{8}$, so $\angle OAD = \sin^{-1}\left(\dfrac{3 \sin 2.5}{8}\right) = 0.2263\ldots$.

3. Show angle ODA

$\angle ODA = \pi - 2.5 - 0.2263\ldots = 0.4152\ldots$, so $\angle ODA = 0.415$ radians to 3 significant figures.

4. Area of the major sector

The sector has radius 3 and angle $2\pi - 2.5$, so its area is $\dfrac{1}{2}(3)^2(2\pi - 2.5) = 17.02\ldots$.

5. Area of triangle AOD

$\text{Area of triangle } AOD = \dfrac{1}{2}(3)(8)\sin 0.4152\ldots = 4.84\ldots$.

6. Total area

$17.02\ldots + 4.84\ldots = 21.86\ldots$, so the area is $21.9\ \text{cm}^2$ to one decimal place.

7. Find AO

By the cosine rule, $AO^2 = 3^2 + 8^2 - 2(3)(8)\cos 0.4152\ldots$, so $AO \approx 5.39$ cm.

8. Find AB

Since A, B, O are collinear and $OB = 3$, $AB = AO - OB \approx 5.39 - 3 = 2.39$ cm.

9. Perimeter

$AB + \text{major arc } BD + AD = 2.39... + 3(2\pi - 2.5) + 8 = 21.7\ \text{cm}$ to one decimal place.

Final answer

$(a)\ \angle ODA = 0.415\ \text{rad}.$

$(b)\ 21.9\ \text{cm}^2.$

$(c)\ 21.7\ \text{cm}.$

TOPIC

Trigonometric Functions

Question 9

4 marks

Trigonometric Functions

9.

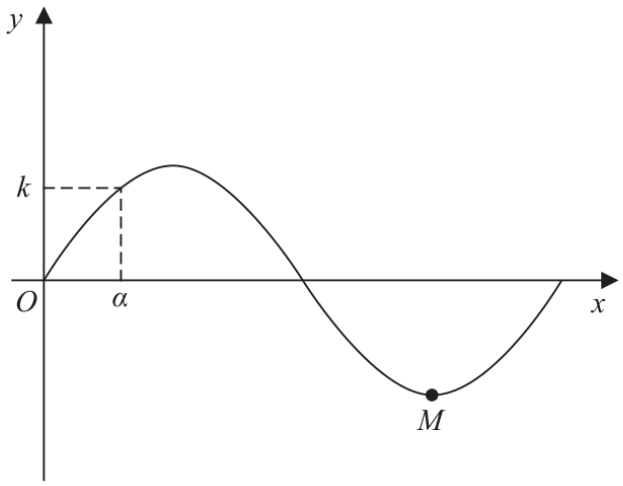


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \sin\left(\frac{x}{12}\right)$, where x is measured in radians. The point M shown in Figure 5 is a minimum point on C.

(a) State the period of C.

(1)

(b) State the coordinates of M.

(1)

The smallest positive solution of the equation $\sin\left(\frac{x}{12}\right) = k$, where k is a constant, is α.

Find, in terms of α,

(c) (i) the negative solution of the equation $\sin\left(\frac{x}{12}\right) = k$ that is closest to zero,

(ii) the smallest positive solution of the equation $\cos\left(\frac{x}{12}\right) = k$.

(2)

(Total 4 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Use the scale factor inside the sine

For $y = \sin\left(\frac{x}{12}\right)$, one full sine cycle needs $\frac{x}{12}$ to increase by 2π, so the period is 24π.

2. Locate the minimum point

The sine function is -1 when its angle is $\frac{3\pi}{2}$. Therefore $\frac{x}{12} = \frac{3\pi}{2}$, so $x = 18\pi$ and $M = (18\pi, -1)$.

3. Use sine symmetry

If the smallest positive solution of $\sin\left(\frac{x}{12}\right) = k$ is α, then the closest negative solution is one half-period before the opposite sine position: $-12\pi - \alpha$.

4. Convert to cosine

Since $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$, the corresponding smallest positive cosine solution is $6\pi - \alpha$.

Final answer

(a) 24π.

(b) $(18\pi, -1)$.

$(c)(i)$ $-12\pi - \alpha$. (ii) $6\pi - \alpha$.

WMA11/01 JANUARY 2020

Question 7

5 marks

Trigonometric Functions

7.

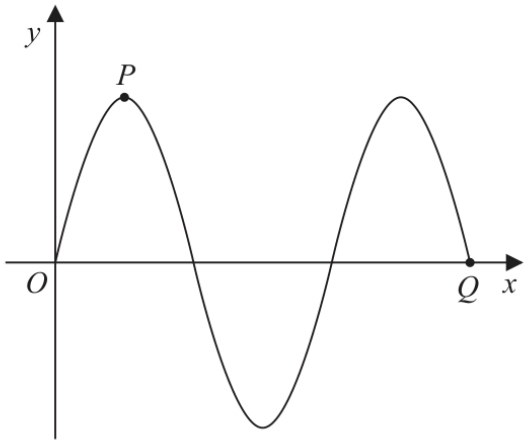


Figure 3

Figure 3 shows part of the curve C_1 with equation $y = 3\sin x$, where x is measured in degrees.

The point P and the point Q lie on C_1 and are shown in Figure 3.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

A different curve C_2 has equation $y = 3\sin x + k$, where k is a constant.

The curve C_2 has a maximum y value of 10

The point R is the minimum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(Total 5 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 7

Trigonometric Functions

Topic group

1. Read P from y = 3 sin x

The first maximum of $y = 3\sin x$ occurs at $x = 90°$, with $y = 3$. So $P = (90°, 3)$.

2. Read Q from the next shown zero

After the second maximum, the curve returns to the x-axis at $x = 540°$. So $Q = (540°, 0)$.

3. Find the vertical shift

For C_2, $y = 3\sin x + k$. Its maximum is $3 + k = 10$, so $k = 7$.

4. Find the minimum point R

The minimum value is $-3 + 7 = 4$. The smallest positive x-coordinate for a sine minimum is $270°$, so $R = (270°, 4)$.

Final answer

$(a)(i)\ P = (90°, 3).$ $(a)(ii)\ Q = (540°, 0).$

$(b)\ R = (270°, 4).$

WMA11/01 JANUARY 2021

Question 3

5 marks

Trigonometric Functions

3.

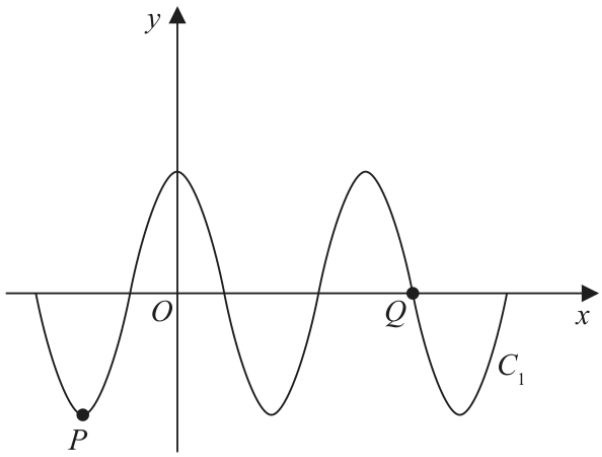


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4\cos x°$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4\cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(Total 5 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 3

Trigonometric Functions

Topic group

1. Read the key points of y = 4 cos x

For $y = 4\cos x°$, the maximum value is 4 and the minimum value is -4. The minimum immediately to the left of the y-axis is at $x = -180°$, so $P = (-180, -4)$.

2. Find Q from the cosine zero

The x-intercepts of $4\cos x°$ occur at $x = 90°, 270°, 450°, \ldots$. The point shown to the right of the next maximum is $Q = (450, 0)$.

3. Use the shifted curve C2

For C_2, $y = 4\cos x° + k$. Its minimum is $-4 + k$. Since the minimum value is -1, $-4 + k = -1$, so $k = 3$.

4. Find R

The maximum value of C_2 is $4 + 3 = 7$. The maximum point with the smallest positive x-coordinate occurs at $x = 360°$, so $R = (360, 7)$.

Final answer

$(a)(i)\ P = (-180, -4).$ $(a)(ii)\ Q = (450, 0).$

$(b)\ R = (360, 7).$

WMA11/01 MAY/JUNE 2021

Question 9

7 marks

Trigonometric Functions

9.

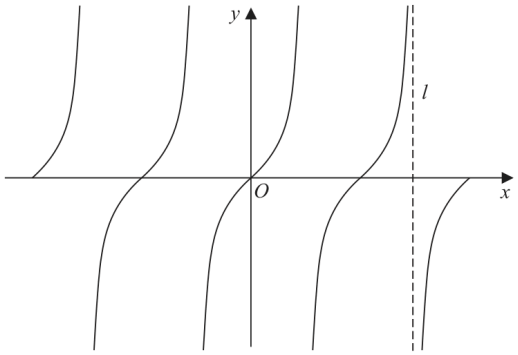


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

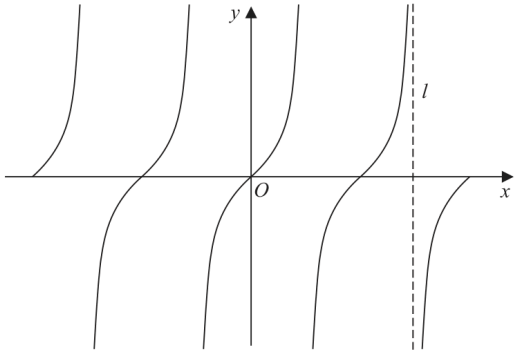


Diagram 1

(Total 7 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. State the shown tangent asymptote

The shown dashed asymptote is the right-hand positive asymptote before 2π, so l has equation $x = \frac{3\pi}{2}$.

2. Describe the reciprocal graph

For $y = \frac{1}{x} + 1$, the vertical asymptote is $x = 0$ and the horizontal asymptote is $y = 1$. The right branch lies above $y = 1$ and the left branch lies below $y = 1$.

3. Count intersections on -2pi to 2pi

The number of solutions of $\tan x = \frac{1}{x} + 1$ is the number of intersections of the two graphs. From the sketch on $-2\pi \leq x \leq 2\pi$, there are 5 intersections.

4. Count solutions for 0 <= x < 40pi

From the repeated tangent branches and the reciprocal curve, there is one intersection in each positive interval between consecutive tangent asymptotes. From 0 to 40π, this gives 40 solutions.

5. Count solutions for -10pi <= x <= 5pi/2

Extending the same intersection pattern across the stated interval gives 14 intersections, so there are 14 solutions.

Final answer

$(a)\ x = \dfrac{3\pi}{2}.$

$(b)(i)\ y = 1$ is the horizontal asymptote.

$(b)(ii)\ 5.$

$(c)(i)\ 40.$

$(c)(ii)\ 14.$

WMA11/01 OCTOBER 2021

Question 4

5 marks

Trigonometric Functions

4.

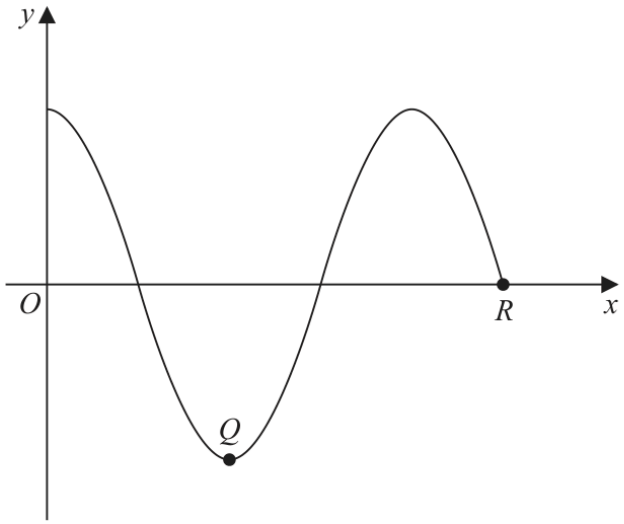


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \cos 2x^{\circ} \qquad 0 \leqslant x \leqslant k$$

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

(i) the coordinates of Q,

(ii) the value of k.

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^{\circ} = p \qquad \text{in the region } 0 \leqslant x \leqslant k$$

find the range of possible values for p.

(2)

(Total 5 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 4

Trigonometric Functions

Topic group

1. Find Q

For $f(x) = \cos 2x°$, the first minimum occurs when $2x = 180°$, so $x = 90°$ and $y = -1$. Therefore $Q = (90, -1)$.

2. Find k

The point $R(k, 0)$ is the x-intercept after the next maximum. This occurs when $2x = 450°$, so $x = 225°$. Hence $k = 225$.

3. Count horizontal intersections

A horizontal line $y = p$ gives exactly two intersections on $0 \leq x \leq 225$ when it cuts the two negative branches only, so $-1

4. Include the maximum case

There are also exactly two solutions when $p = 1$, at $x = 0$ and $x = 180$. Therefore $-1

Final answer

$(a)(i)\ Q = (90, -1). \quad (a)(ii)\ k = 225.$

$(b)\ -1

WMA11/01 JANUARY 2022

Question 9

4 marks

Trigonometric Functions

9.

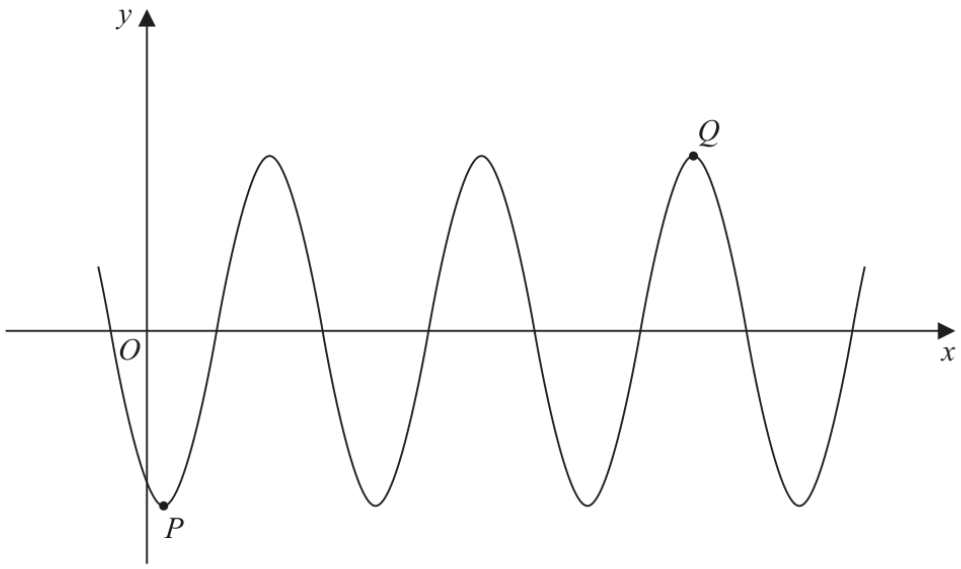


Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates $(30, -3)$ as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

(Total 4 marks)

WMA11/01 JANUARY 2022

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Use the minimum point

At $P(30, -3)$, $x - 30 = 0$, so $\cos 0° = 1$. Therefore $A = -3$.

2. Find maximum points

Since $A = -3$, maximum points occur when $\cos(x - 30)° = -1$.

3. Use the shown maximum Q

$x - 30 = 180 + 360n$, so $x = 210 + 360n$. The maximum shown at Q is two full periods after the first maximum, so $x = 210 + 720 = 930$.

4. Write Q

The maximum y-value is 3, hence $Q = (930, 3)$.

Final answer

$(a)\ A = -3.$

$(b)\ Q = (930, 3).$

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Question 9

8 marks

Trigonometric Functions

9.

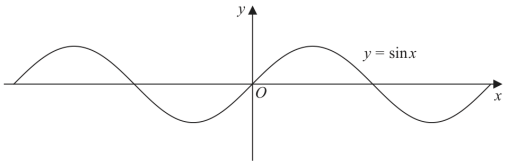


Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin\alpha = p$, where $0 < \alpha < 90°$

(a) state, in terms of p, the value of

(i) $2\sin(180° - \alpha)$

(ii) $\sin(\alpha - 180°)$

(iii) $3 + \sin(180° + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α, the x coordinates of any points in the interval $0 < x < 180°$ where

$$\sin 2x = p$$

(3)

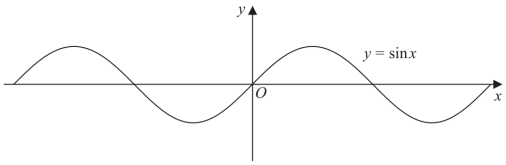


Diagram 1

(Total 8 marks)

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Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Use sine symmetry

$\sin(180° - \alpha) = \sin\alpha = p$, so $2\sin(180° - \alpha) = 2p$.

2. Use odd symmetry

$\sin(\alpha - 180°) = -\sin(180° - \alpha) = -p$.

3. Use the third quadrant identity

$\sin(180° + \alpha) = -\sin\alpha = -p$, so $3 + \sin(180° + \alpha) = 3 - p$.

4. Sketch y = sin 2x

The graph has amplitude 1 and period $180°$. It crosses the x-axis at $0°, 90°, 180°$, has a maximum at $45°$, and a minimum at $135°$.

5. Solve sin 2x = p

Since $\sin\alpha = p$ and $0 < 2x < 360°$, the possible values are $2x = \alpha$ or $2x = 180° - \alpha$.

6. Find x

Therefore $x = \frac{\alpha}{2}$ or $x = 90° - \frac{\alpha}{2}$.

Final answer

$(a)(i)\ 2p,\ (ii)\ -p,\ (iii)\ 3 - p.$

$(c)\ x = \frac{\alpha}{2},\ 90° - \frac{\alpha}{2}.$

WMA11/01 JANUARY 2023

Question 9

4 marks

Trigonometric Functions

9.

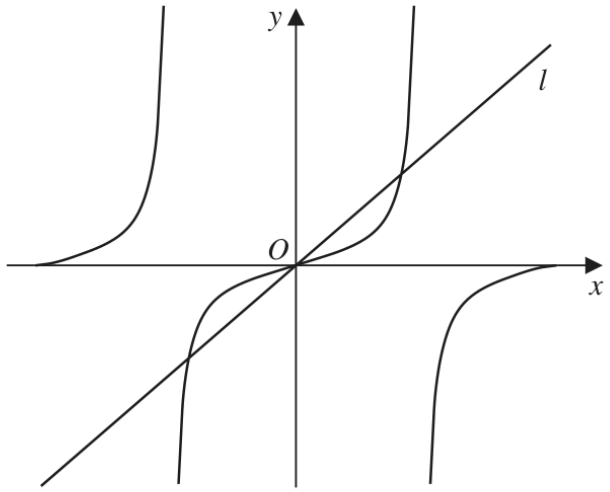


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$

(a) State the period of $\tan x$

(1)

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

(Total for Question 9 is 4 marks)

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Worked Solution - Question 9

Trigonometric Functions

Topic group

1. State the period

The period of $\tan x$ is π.

2. Count roots for the steeper line

In $-\pi

3. Extend to -2pi < x < 2pi

For $y = \pi x$, the interval $-2\pi

4. Extend to -100pi < x < 100pi

There are 99 positive outer roots, 99 negative outer roots, and 3 central roots, so the total is $99 + 99 + 3 = 201$.

Final answer

$(a)\ \pi.$

$(b)(i)\ 3,\ (ii)\ 5,\ (iii)\ 201.$

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Question 9

9 marks

Trigonometric Functions

9. (i)

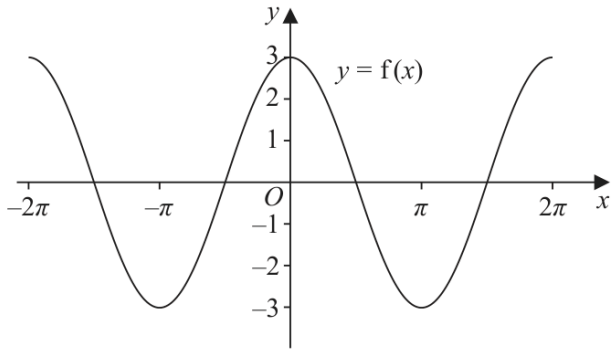


Figure 3

Figure 3 shows part of the graph of the trigonometric function with equation $y = f(x)$

(a) Write down an expression for $f(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = f\left(x + \frac{\pi}{4}\right)$

Show clearly the coordinates of all the points where the curve intersects the coordinate axes.

(3)

(ii)

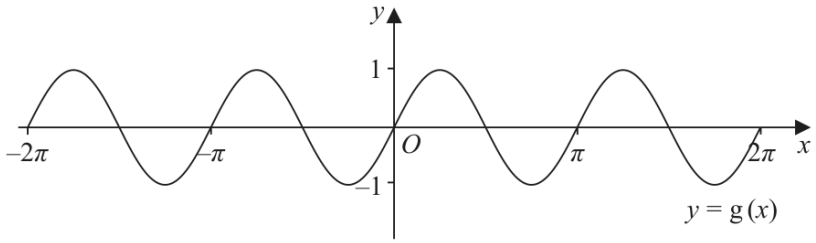


Figure 4

Figure 4 shows part of the graph of the trigonometric function with equation $y = g(x)$

(a) Write down an expression for $g(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = g(x) - 2$

Show clearly the coordinates of the y intercept.

(2)

(Total for Question 9 is 9 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Identify f(x)

The first graph has amplitude 3, period 2π, and maximum at $x = 0$, so $f(x) = 3\cos x$.

2. Transform f

$y = f\left(x + \frac{\pi}{4}\right) = 3\cos\left(x + \frac{\pi}{4}\right)$, a shift left by $\frac{\pi}{4}$.

3. Find the x-intercepts

$\cos\left(x + \frac{\pi}{4}\right) = 0$ gives $x = \frac{\pi}{4} + n\pi$. In $-2\pi

4. Find the y-intercept

At $x = 0$, $y = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$.

5. Identify g(x)

The second graph has amplitude 1, period π, and passes through the origin with positive gradient, so $g(x) = \sin 2x$.

6. Transform g

$y = g(x) - 2 = \sin 2x - 2$, so the graph is shifted down by 2. Its y-intercept is $(0, -2)$.

Final answer

$(i)(a)\ f(x) = 3\cos x.$ $(i)(b)$ x-intercepts $-\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$ and y-intercept $\frac{3\sqrt{2}}{2}$. $(ii)(a)\ g(x) = \sin 2x.$ $(ii)(b)$ y-intercept $(0, -2)$.

Question 10

6 marks

Trigonometric Functions

10.

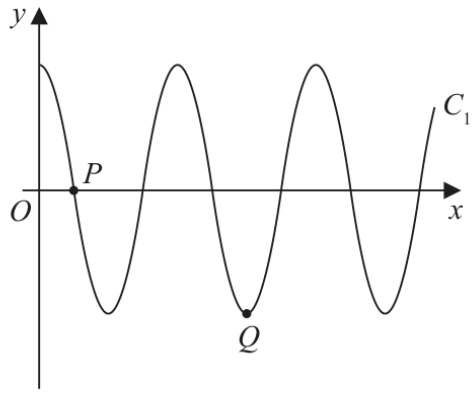


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3\cos\left(\frac{x}{n}\right)^\circ \qquad x \geqslant 0$$

where n is a constant.

The curve C_1 cuts the positive x-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n

(ii) State the period of C_1

(2)

The point Q, shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q.

(2)

The curve C_2 has equation $y = 2\sin x^\circ + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k.

(2)

(Total for Question 10 is 6 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 10

Trigonometric Functions

Topic group

1. Use the first positive x-intercept

For $y = 3\cos\left(\frac{x}{n}\right)^\circ$, the first positive zero occurs when $\frac{x}{n} = 90°$.

2. Find n

Since $P = (270, 0)$, $\frac{270}{n} = 90$, so $n = 3$.

3. Find the period

The period is $360n = 360(3) = 1080°$.

4. Read the shown minimum point

A minimum occurs when $\frac{x}{3} = 180° + 360°k$. The point Q shown is the next displayed minimum after one full period, so $x = 1620$ and $y = -3$.

5. Use the two points on C2

For $y = 2\sin x° + k$, the points give $2\sin a + k = \frac{12}{5}$ and $-2\sin a + k = -\frac{3}{5}$.

6. Add the equations

Adding gives $2k = \frac{12}{5} - \frac{3}{5} = \frac{9}{5}$, so $k = \frac{9}{10}$.

Final answer

$(a)(i)\ n = 3.\quad (a)(ii)\ 1080°.$

$(b)\ Q = (1620, -3).$

$(c)\ k = \frac{9}{10}.$

WMA11/01 JANUARY 2024

Question 6

6 marks

Trigonometric Functions

6.

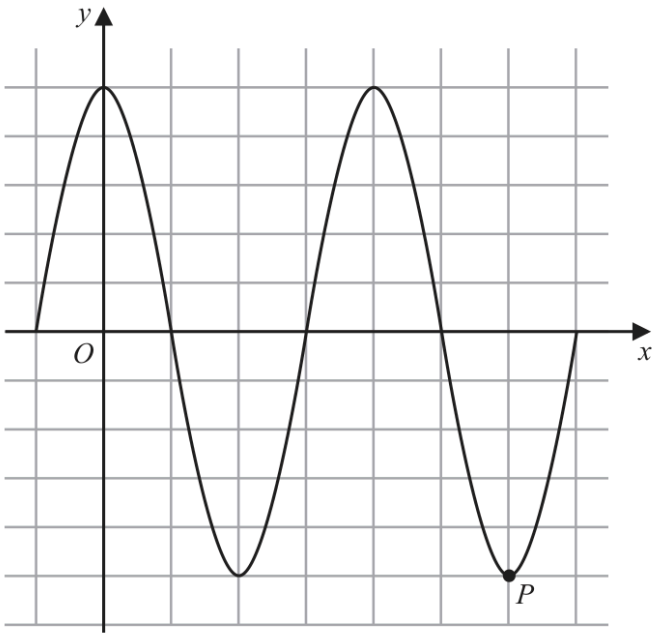


Figure 2

Figure 2 shows a plot of part of the curve C_1 with equation

$$y = 5\cos x$$

with x being measured in degrees.

The point P, shown in Figure 2, is a minimum point on C_1

(a) State the coordinates of P

(2)

The point Q lies on a different curve C_2

Given that point Q

- is a maximum point on the curve
- is the maximum point with the **smallest** x coordinate, $x > 0$

(b) find the coordinates of Q when

(i) C_2 has equation $y = 5\cos x - 2$

(ii) C_2 has equation $y = -5\cos x$

(4)

(Total for Question 6 is 6 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 6

Trigonometric Functions

Topic group

1. Use the cosine graph

For $y = 5\cos x$, the minimum value is -5 and it occurs at $x = 180° + 360°n$.

2. Read P from the displayed minimum

The plotted point P is the displayed minimum at $x = 540°$, so $P = (540°, -5)$.

3. Translate the graph down

For $y = 5\cos x - 2$, the maximum value is $5 - 2 = 3$. The smallest positive x-coordinate for a maximum shown is $x = 360°$, so $Q = (360°, 3)$.

4. Reflect in the x-axis

For $y = -5\cos x$, a maximum occurs where $\cos x = -1$, first at $x = 180°$. The maximum value is 5, so $Q = (180°, 5)$.

Final answer

$(a)\ P = (540°, -5).$

$(b)(i)\ Q = (360°, 3).$

$(b)(ii)\ Q = (180°, 5).$

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Question 11

6 marks

Trigonometric Functions

11.

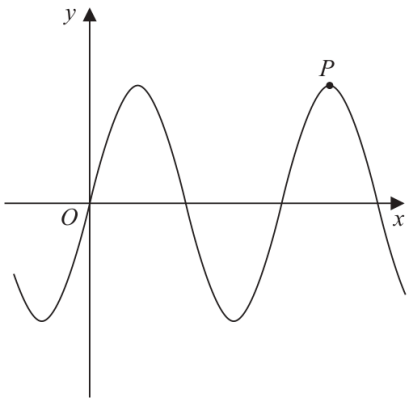


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 11

Trigonometric Functions

Topic group

1. Use maxima of sine

For $y = 12\sin x$, maxima occur when $x = \frac{\pi}{2} + 2\pi n$, with value 12.

2. Read point P

The maximum point P shown is the second positive maximum, so $P = (\frac{5\pi}{2}, 12)$.

3. Find k for C2

For $y = 12\sin x + k$, the maximum value is $12 + k$. Given the maximum is 3, $k = -9$.

4. Find the smallest positive minimum on C2

The minimum value is $-12 - 9 = -21$, and it first occurs at $x = \frac{3\pi}{2}$. So the point is $(\frac{3\pi}{2}, -21)$.

5. Find A for C3

For $y = 12\sin(x + B)$, any minimum has value -12, so $A = -12$.

6. Find the smallest positive B

At the minimum $x = \frac{\pi}{4}$, so $\frac{\pi}{4} + B = \frac{3\pi}{2}$. Hence $B = \frac{5\pi}{4}$.

Final answer

$(a)\ P = (\frac{5\pi}{2}, 12).$

$(b)\ (\frac{3\pi}{2}, -21).$

$(c)(i)\ A = -12.$

$(c)(ii)\ B = \frac{5\pi}{4}.$

WMA11/01 OCTOBER 2024

Question 7

8 marks

Trigonometric Functions

7.

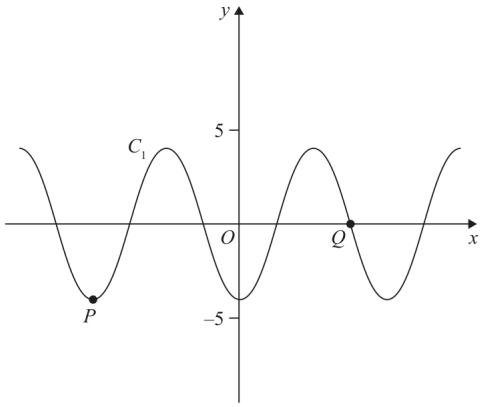


Figure 3

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4\cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

(i) the coordinates of P

(ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4\cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

(b) (i) state the value of k

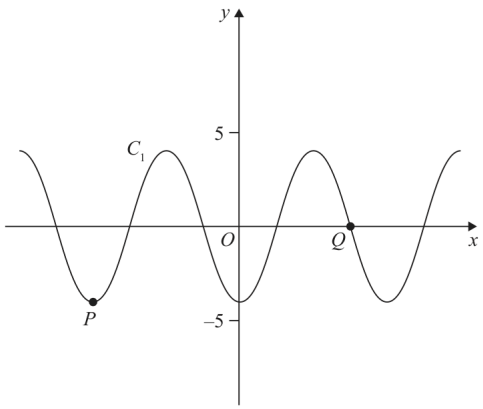
(ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4\cos x = 5 - \frac{10}{\pi}x$$



(Total for Question 7 is 8 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 7

Trigonometric Functions

Topic group

1. Use y equals negative 4 cos x

For $y = -4\cos x$, minimum points occur at $x = 2\pi n$ with value -4.

2. Read P

The displayed minimum to the left of the y-axis is $P = (-2\pi, -4)$.

3. Read Q

The point Q is the positive x-intercept after the maximum at $x = \pi$, so $Q = (\frac{3\pi}{2}, 0)$.

4. Find k for C2

For $y = -4\cos x + k$, the maximum value is $4 + k$. Given the maximum is 11, $k = 7$.

5. Find the smallest positive minimum on C2

The minimum value is $-4 + 7 = 3$. The first positive minimum is at $x = 2\pi$, so the point is $(2\pi, 3)$.

6. Interpret the equation graphically

$-4\cos x = 5 - \frac{10}{\pi}x$ is the intersection of the cosine curve with the straight line $y = 5 - \frac{10}{\pi}x$.

7. Count intersections

The line passes through $(0, 5)$ and meets the displayed curve once, at the x-axis point, so there is 1 solution.

Final answer

$(a)(i)\ P = (-2\pi, -4).$ $(a)(ii)\ Q = (\frac{3\pi}{2}, 0).$

$(b)(i)\ k = 7.$

$(b)(ii)\ (2\pi, 3).$

(c) 1 solution.

WMA11/01 JANUARY 2025

Question 9

13 marks

Trigonometric Functions

9.

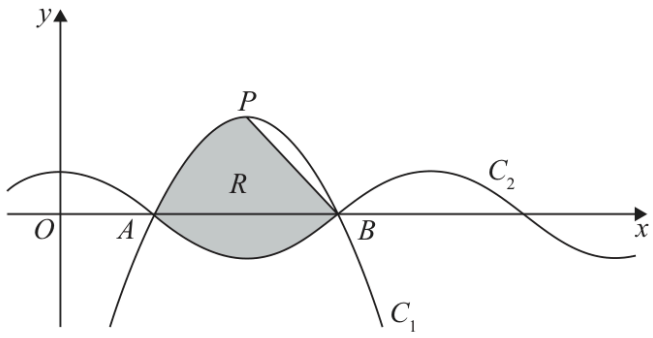


Figure 2

(a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a, b and c are constants to be found.

(3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

(b) state the coordinates of P

(2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x-axis at point A and at point B, as shown in Figure 2,

(c) (i) state the x coordinate of B

(ii) state the value of k

(iii) state the period of C_2

(3)

The line segment L joins P and B.
The region R, shown shaded in Figure 2, is bounded by L, C_1 and C_2

(d) Use inequalities to define R.

(5)

(Total for Question 9 is 13 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Complete the square

$6x - \frac{27}{4} - x^2 = -(x^2 - 6x) - \frac{27}{4} = -(x-3)^2 + 9 - \frac{27}{4}$.

2. State the completed-square form

$6x - \frac{27}{4} - x^2 = \frac{9}{4} - (x-3)^2$.

3. Find the maximum point

The maximum point is $P = (3, \frac{9}{4})$.

4. Find the x-intercepts of C1

$\frac{9}{4} - (x-3)^2 = 0$ gives $x = 3 \pm \frac{3}{2}$. Hence A has $x = \frac{3}{2}$ and B has $x = \frac{9}{2}$.

5. Use consecutive zeros of cosine

The distance between consecutive zeros of $\cos(kx)$ is $\frac{\pi}{k}$.

6. Find k and the period

Since $x_B - x_A = \frac{9}{2} - \frac{3}{2} = 3$, $\frac{\pi}{k} = 3$, so $k = \frac{\pi}{3}$. The period is $\frac{2\pi}{k} = 6$.

7. Find the line through P and B

The line through $P(3, \frac{9}{4})$ and $B(\frac{9}{2}, 0)$ has gradient $-\frac{3}{2}$, so L is $y = -\frac{3}{2}x + \frac{27}{4}$.

8. Define the x-range

The region lies between A and B, so $\frac{3}{2} \leq x \leq \frac{9}{2}$.

9. Define the lower boundary

The lower boundary is C_2, so $y \geq \cos\left(\frac{\pi}{3}x\right)$.

10. Define the upper boundaries

The region is below both C_1 and the line segment L, so $y \leq \frac{9}{4} - (x-3)^2$ and $y \leq -\frac{3}{2}x + \frac{27}{4}$.

Final answer

(a) $\frac{9}{4} - (x-3)^2$.

(b) $P = (3, \frac{9}{4})$.

$(c)(i)$ $x_B = \frac{9}{2}$.

$(c)(ii)$ $k = \frac{\pi}{3}$.

$(c)(iii)$ 6.

(d) $\frac{3}{2} \leq x \leq \frac{9}{2}$, $y \geq \cos(\frac{\pi}{3}x)$, $y \leq \frac{9}{4} - (x-3)^2$, $y \leq -\frac{3}{2}x + \frac{27}{4}$.

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Question 4

6 marks

Trigonometric Functions

4.

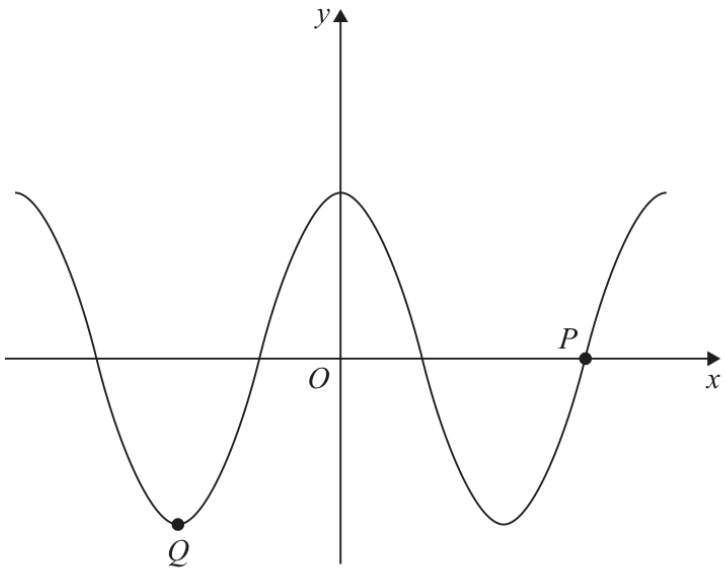


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4\cos x$$

where x is measured in degrees.

The points P and Q lie on the curve and are shown in Figure 1.

(a) State the coordinates of P.

(1)

(b) State the coordinates of Q.

(2)

(c) State the **number** of solutions of the equation

(i) $4\cos x = 3$ in the interval $0 < x \leqslant 18\,000°$

(ii) $5 + 4\cos x = 1$ in the interval $-720° < x \leqslant 720°$

(iii) $4\cos x - 3 = 1$ in the interval $-1080° \leqslant x \leqslant 1080°$

(3)

(Total for Question 4 is 6 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 4

Trigonometric Functions

Topic group

1. Point P

For $y = 4\cos x$, the x-intercepts occur at $x = 90° + 180°n$. The labelled point P is the right-hand intercept after $180°$, so $P = (270°, 0)$.

2. Point Q

The minimum value is -4, reached at $x = 180° + 360°n$. The labelled point on the left is $Q = (-180°, -4)$.

3. Part c(i)

$4\cos x = 3$ has two solutions every $360°$. The interval \$0

4. Part c(ii)

$5 + 4\cos x = 1$ gives $\cos x = -1$. This occurs once every $360°$. In \$-720^\circ

5. Part c(iii)

$4\cos x - 3 = 1$ gives $\cos x = 1$. In $-1080° \le x \le 1080°$, the multiples of $360°$ are $-1080, -720, -360, 0, 360, 720, 1080$, so there are 7.

Final answer

$(a)\ P = (270°, 0).$

$(b)\ Q = (-180°, -4).$

$(c)(i)\ 100,\ (ii)\ 4,\ (iii)\ 7.$

WMA11/01 JANUARY 2026

Question 9

9 marks

Trigonometric Functions

9.

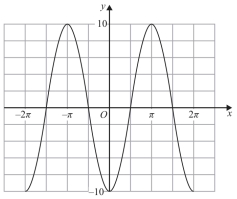


Figure 4

Figure 4 shows a sketch of part of the graph of the trigonometric function with equation $y = f(x)$.

(a) Write down an expression for $f(x)$.

(2)

Copies of Figure 4 (labelled Diagram 1 and Diagram 2) can be found on the following pages.

(b) (i) On Diagram 1 sketch a graph of the curve with equation

$$y = 10 - x^2$$

(ii) Hence find the **number** of solutions of the equation

$$f(x) = 10 - x^2 \quad \text{in the interval} \quad -100\pi \leqslant x \leqslant 100\pi$$

(3)

(c) (i) On Diagram 2 sketch a graph of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

(ii) Hence find the **number** of solutions of the equation

$$f(x) = \tan x \quad \text{in the interval} \quad -100\pi \leqslant x \leqslant 100\pi$$

giving a reason for your answer.

(4)

(a) _____

_____

(b)(i)

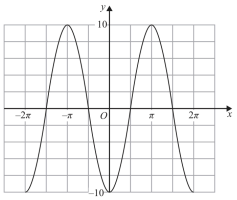
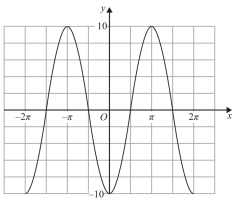


Diagram 1



Copy of Diagram 1

Only use this copy if you need to redraw your graph.

(b)(ii) _____

(c)(i)

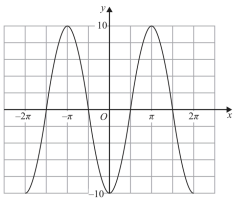
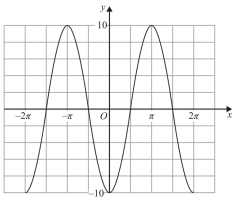


Diagram 2



Copy of Diagram 2

Only use this copy if you need to redraw your graph.

(c)(ii) _____

_____

(Total for Question 9 is 9 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 9

Trigonometric Functions

Topic group

1. Identify f(x)

The graph has amplitude 10, period 2π, and value -10 at $x = 0$, so $f(x) = -10\cos x$.

2. Sketch y equals 10 minus x squared

The graph $y = 10 - x^2$ is a downward-opening parabola with vertex $(0, 10)$ and x-intercepts at $x = \pm\sqrt{10}$.

3. Count intersections with the parabola

On the given sketch, the parabola meets $f(x)$ twice. Outside the central part the parabola is below the range of $f(x)$, so there are 2 solutions in $-100\pi \leq x \leq 100\pi$.

4. Sketch y equals tan x

The graph $y = \tan x$ has zeros at integer multiples of π and vertical asymptotes at $x = \dfrac{\pi}{2} + n\pi$.

5. Count in the reference interval

In $-2\pi \leq x \leq 2\pi$, the graph $y = \tan x$ intersects $f(x)$ four times.

6. Scale to the required interval

The interval $-100\pi \leq x \leq 100\pi$ is 50 times as long as $-2\pi \leq x \leq 2\pi$, so the number of solutions is $50 \times 4 = 200$.

Final answer

$(a)\ f(x) = -10\cos x.$

$(b)(ii)\ 2.$

$(c)(ii)\ 200.$

TOPIC

Differentiation

WMA11/01 OCTOBER 2019

Question 5

7 marks

Differentiation

5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(Total 7 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 5

Differentiation

Topic group

1. Rewrite before differentiating

$y = \frac{x^3}{6} + 4x^{1/2} - 15$.

2. Differentiate term by term

$\frac{dy}{dx} = \frac{3x^2}{6} + 4 \cdot \frac{1}{2}x^{-1/2} = \frac{x^2}{2} + \frac{2}{\sqrt{x}}$.

3. Find the tangent and normal gradients at P

At $x = 4$, $\frac{dy}{dx} = \frac{16}{2} + \frac{2}{2} = 9$. The normal gradient is therefore $-\frac{1}{9}$.

4. Build the normal equation

Using $P\left(4, \frac{11}{3}\right)$: $y - \frac{11}{3} = -\frac{1}{9}(x - 4)$. Multiplying out gives $x + 9y - 37 = 0$.

Final answer

$\frac{dy}{dx} = \frac{x^2}{2} + \frac{2}{\sqrt{x}}$, and the normal is $x + 9y - 37 = 0$.

WMA11/01 OCTOBER 2019

Question 7

6 marks

Differentiation

7.

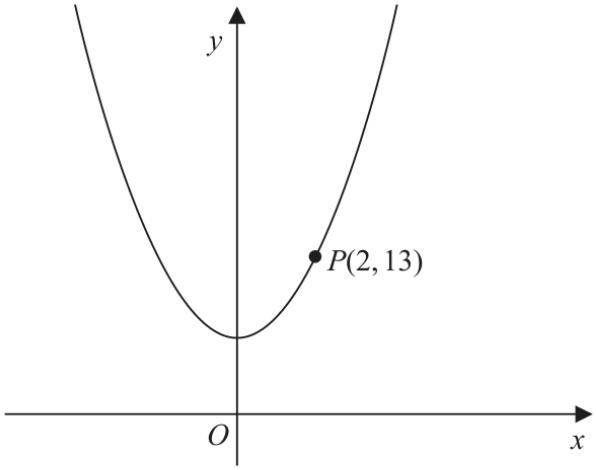


Figure 4

Figure 4 shows part of the curve with equation $y = 2x^2 + 5$

The point $P(2,13)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Give your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total 6 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 7

Differentiation

Topic group

1. Differentiate for the tangent gradient

For $y = 2x^2 + 5$, $\dfrac{dy}{dx} = 4x$. At P, $x = 2$, so the tangent gradient is 8.

2. Find the coordinates of Q

If $x_Q = 2 + h$, then $y_Q = 2(2 + h)^2 + 5 = 13 + 8h + 2h^2$.

3. Find the gradient of PQ

$$\text{gradient } PQ = \frac{(13 + 8h + 2h^2) - 13}{(2 + h) - 2} = \frac{8h + 2h^2}{h} = 8 + 2h.$$

4. Explain the link

As h gets closer to 0, point Q gets closer to P, and the chord gradient $8 + 2h$ tends to the tangent gradient 8.

Final answer

(a) 8.

(b) $8 + 2h$.

(c) $As\ h \to 0,\ 8 + 2h \to 8$.

WMA11/01 JANUARY 2020

Question 3

6 marks

Differentiation

3.

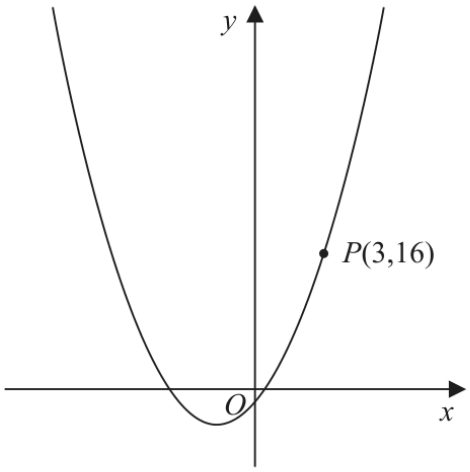


Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3,16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Write your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total 6 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 3

Differentiation

Topic group

1. Differentiate for the tangent gradient

For $y = x^2 + 3x - 2$, $\dfrac{dy}{dx} = 2x + 3$. At $x = 3$, the gradient is $2(3) + 3 = 9$.

2. Find the y-coordinate of Q

For $x_Q = 3 + h$, $y_Q = (3+h)^2 + 3(3+h) - 2 = 16 + 9h + h^2$.

3. Calculate the gradient of PQ

$$\text{gradient } PQ = \frac{(16 + 9h + h^2) - 16}{(3+h) - 3} = \frac{9h + h^2}{h} = 9 + h.$$

4. Explain the relationship

As $h \to 0$, point Q approaches P, and the chord gradient $9 + h$ approaches the tangent gradient 9.

Final answer

$(a)\ 9.$

$(b)\ 9 + h.$

$(c)\ As\ h \to 0,\ 9 + h \to 9.$

WMA11/01 JANUARY 2020

Question 9

6 marks

Differentiation

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the x coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

(6)

(Total 6 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 9

Differentiation

Topic group

1. Rewrite the function

$$y = \frac{4x^2 + 9}{2\sqrt{x}} = 2x^{3/2} + \frac{9}{2}x^{-1/2}.$$

2. Differentiate

$$\frac{dy}{dx} = 3x^{1/2} - \frac{9}{4}x^{-3/2}.$$

3. Set the derivative equal to zero

$3x^{1/2} - \frac{9}{4}x^{-3/2} = 0$, so $3\sqrt{x} = \frac{9}{4x^{3/2}}$.

4. Solve for x

Multiply by $4x^{3/2}$: $12x^2 = 9$. Hence $x^2 = \frac{3}{4}$. Since $x > 0$, $x = \frac{\sqrt{3}}{2}$.

Final answer

$$x = \frac{\sqrt{3}}{2}.$$

WMA11/01 JANUARY 2021

Question 1

8 marks

Differentiation

1. A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

The point P lies on the curve and has x coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(Total 8 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 1

Differentiation

Topic group

1. Rewrite the negative power term

$y = 2x^3 - 5x^2 - \dfrac{3}{2x} + 7 = 2x^3 - 5x^2 - \dfrac{3}{2}x^{-1} + 7.$

2. Differentiate term by term

$\dfrac{dy}{dx} = 6x^2 - 10x + \dfrac{3}{2}x^{-2} = 6x^2 - 10x + \dfrac{3}{2x^2}.$

3. Find the point P

At $x = \dfrac{1}{2}$, $y = 2\left(\dfrac{1}{8}\right) - 5\left(\dfrac{1}{4}\right) - \dfrac{3}{1} + 7 = 3$, so $P = \left(\dfrac{1}{2}, 3\right)$.

4. Find the normal gradient

At $x = \dfrac{1}{2}$, $\dfrac{dy}{dx} = 6\left(\dfrac{1}{4}\right) - 10\left(\dfrac{1}{2}\right) + \dfrac{3}{2(1/4)} = \dfrac{5}{2}$. The normal gradient is therefore $-\dfrac{2}{5}$.

5. Write the normal in integer form

$y - 3 = -\dfrac{2}{5}\left(x - \dfrac{1}{2}\right)$. Multiplying by 5 gives $5y - 15 = -2x + 1$, hence $2x + 5y - 16 = 0$.

Final answer

(a) $\dfrac{dy}{dx} = 6x^2 - 10x + \dfrac{3}{2x^2}.$

(b) $2x + 5y - 16 = 0.$

WMA11/01 JANUARY 2021

Question 7

9 marks

Differentiation

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

(Total 9 marks)

WMA11/01 JANUARY 2021

Worked Solution - Question 7

Differentiation

Topic group

1. Solve f(x) = 9 without calculator methods

$2x - 3\sqrt{x} - 5 = 9$, so $2x - 3\sqrt{x} - 14 = 0$.

2. Use u = sqrt(x)

Let $u = \sqrt{x}$, where $u > 0$. Then $x = u^2$, so $2u^2 - 3u - 14 = 0$.

3. Factorise and reject the invalid root

$2u^2 - 3u - 14 = (2u - 7)(u + 2)$. Since $u > 0$, $u = \frac{7}{2}$, so $x = u^2 = \frac{49}{4}$.

4. Differentiate twice

$f(x) = 2x - 3x^{1/2} - 5$, so $f'(x) = 2 - \frac{3}{2}x^{-1/2}$ and $f''(x) = \frac{3}{4}x^{-3/2}$.

5. Solve f''(x) = 6

$\frac{3}{4}x^{-3/2} = 6$, so $x^{-3/2} = 8$. Therefore $x^{3/2} = \frac{1}{8}$.

6. Find x

Since $x^{3/2} = (\sqrt{x})^3$, we get $\sqrt{x} = \frac{1}{2}$, hence $x = \frac{1}{4}$.

Final answer

$(a)\ x = \frac{49}{4}$.

$(b)\ x = \frac{1}{4}$.

WMA11/01 MAY/JUNE 2021

Question 1

8 marks

Differentiation

1. The curve C has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(4)

The point $P(4, 3)$ lies on C.

(b) Find the equation of the normal to C at the point P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(Total 8 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 1

Differentiation

Topic group

1. Rewrite using powers

$y = \frac{x^2}{3} + 4x^{-1/2} + \frac{8}{3}x^{-1} - 5.$

2. Differentiate term by term

$\frac{dy}{dx} = \frac{2x}{3} - 2x^{-3/2} - \frac{8}{3}x^{-2} = \frac{2x}{3} - \frac{2}{x\sqrt{x}} - \frac{8}{3x^2}.$

3. Find the tangent gradient at P

At $x = 4$, $\frac{dy}{dx} = \frac{8}{3} - \frac{1}{4} - \frac{1}{6} = \frac{9}{4}.$

4. Find the normal gradient

The normal gradient is the negative reciprocal of $\frac{9}{4}$, so $m_n = -\frac{4}{9}.$

5. Write the normal equation

Using $P(4, 3)$, $y - 3 = -\frac{4}{9}(x - 4)$. Multiplying by 9 gives $9y - 27 = -4x + 16$, hence $4x + 9y - 43 = 0$.

Final answer

(a) $\frac{dy}{dx} = \frac{2x}{3} - \frac{2}{x\sqrt{x}} - \frac{8}{3x^2}.$

(b) $4x + 9y - 43 = 0.$

WMA11/01 OCTOBER 2021

Question 2

5 marks

Differentiation

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = 3x^5 + 4x^3 - x + 5$$

The points P and Q lie on the curve.

The gradient of the curve at both point P and point Q is 2

Find the x coordinates of P and Q.

(5)

(Total 5 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 2

Differentiation

Topic group

1. Differentiate the curve

For $y = 3x^5 + 4x^3 - x + 5$, $\dfrac{dy}{dx} = 15x^4 + 12x^2 - 1$.

2. Set the gradient equal to 2

$15x^4 + 12x^2 - 1 = 2$, so $15x^4 + 12x^2 - 3 = 0$.

3. Reduce to a quadratic in x squared

Dividing by 3 gives $5x^4 + 4x^2 - 1 = 0$. Let $u = x^2$, giving $5u^2 + 4u - 1 = 0$.

4. Solve and reject the impossible value

$5u^2 + 4u - 1 = (5u - 1)(u + 1) = 0$. Since $u = x^2 \geq 0$, $u = \dfrac{1}{5}$.

5. Find x

$x^2 = \dfrac{1}{5}$, so $x = \pm\dfrac{1}{\sqrt{5}} = \pm\dfrac{\sqrt{5}}{5}$.

Final answer

$x = \pm\dfrac{\sqrt{5}}{5}$.

WMA11/01 JANUARY 2022

Question 7

11 marks

Differentiation

7.

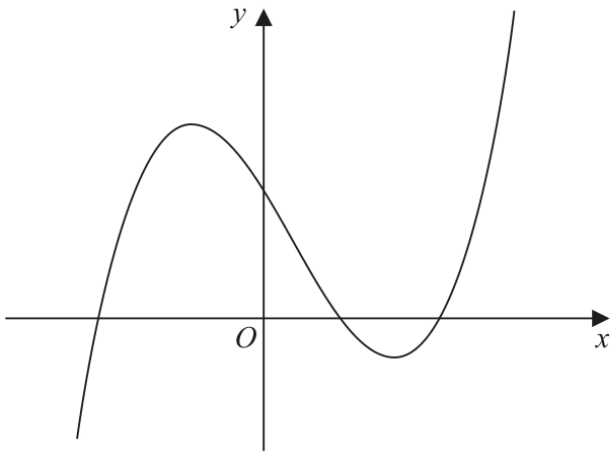


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = f(x) - p$ passes through the point with coordinates (0, 50)

(a) find the value of the constant p.

(2)

Given that the curve with equation $y = f(x + q)$ passes through the origin,

(b) write down the possible values of the constant q.

(2)

(c) Find $f'(x)$.

(4)

(d) Hence find the range of values of x for which the gradient of the curve with equation $y = f(x)$ is less than -18

(3)

(Total 11 marks)

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Worked Solution - Question 7

Differentiation

Topic group

1. Find p from the translated curve

$f(0) = (4)(-2)(-9) = 72$. Since $y = f(x) - p$ passes through $(0, 50)$, $72 - p = 50$, so $p = 22$.

2. Use roots for f(x + q)

If $y = f(x + q)$ passes through the origin, then $f(q) = 0$. From $f(x) = (x + 4)(x - 2)(2x - 9)$, the possible values are $q = -4, 2, \frac{9}{2}$.

3. Expand f(x)

$(x + 4)(x - 2)(2x - 9) = 2x^3 - 5x^2 - 34x + 72$.

4. Differentiate

$f'(x) = 6x^2 - 10x - 34$.

5. Set the gradient less than -18

$6x^2 - 10x - 34 < -18$, so $6x^2 - 10x - 16 < 0$, or $3x^2 - 5x - 8 < 0$.

6. Solve the inequality

$3x^2 - 5x - 8 = (3x - 8)(x + 1)$. Since the quadratic opens upward, it is negative between the roots: $-1

Final answer

$(a)\ p = 22.$

$(b)\ q = -4, 2, \frac{9}{2}.$

$(c)\ f'(x) = 6x^2 - 10x - 34.$

$(d)\ -1

WMA11/01 MAY/JUNE 2022

Question 10

12 marks

Differentiation

10.

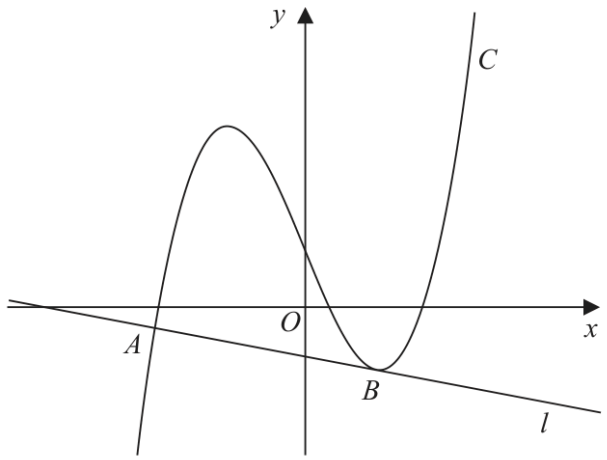


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

(Total 12 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 10

Differentiation

Topic group

1. Differentiate the curve

For $y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$, $\frac{dy}{dx} = \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2}$.

2. Find the tangent gradient at A

At $x = -\frac{7}{2}$, $\frac{dy}{dx} = \frac{6}{7}\left(\frac{49}{4}\right) + \frac{2}{7}\left(-\frac{7}{2}\right) - \frac{5}{2} = \frac{21}{2} - 1 - \frac{5}{2} = 7$.

3. Find the gradient of l

Since l is the normal at A, its gradient is the negative reciprocal of 7, so $m_l = -\frac{1}{7}$.

4. Use that l is tangent at B

At B, the tangent gradient is also $-\frac{1}{7}$, so $\frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} = -\frac{1}{7}$.

5. Reach the given quadratic

Multiplying by 14 gives $12x^2 + 4x - 35 = -2$, hence $12x^2 + 4x - 33 = 0$.

6. Solve for the x-coordinate of B

$12x^2 + 4x - 33 = 0$ gives $x = \frac{-4 \pm 40}{24}$, so $x = \frac{3}{2}$ or $x = -\frac{11}{6}$. From the diagram, B is the point to the right of the y-axis, so $x_B = \frac{3}{2}$.

7. Use the y-intercept of l

Since the y-intercept is -1 and the gradient is $-\frac{1}{7}$, the line is $y = -\frac{1}{7}x - 1$.

8. Find the y-coordinate of A

At $x = -\frac{7}{2}$ on the line, $y = -\frac{1}{7}\left(-\frac{7}{2}\right) - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$.

9. Substitute A into the curve

$$-\frac{1}{2}=\frac{2}{7}\left(-\frac{7}{2}\right)^3+\frac{1}{7}\left(-\frac{7}{2}\right)^2-\frac{5}{2}\left(-\frac{7}{2}\right)+k=-\frac{7}{4}+k.$$

10. Find k

Therefore $k=\frac{5}{4}$.

Final answer

(a) $\frac{dy}{dx}=\frac{6}{7}x^2+\frac{2}{7}x-\frac{5}{2}$.

(c) $x_B=\frac{3}{2}$.

(d) $k=\frac{5}{4}$.

WMA11/01 OCTOBER 2022

Question 1

6 marks

Differentiation

1. The curve C has equation

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(3)

The point $R\left(2, \frac{13}{2}\right)$ lies on C.

(b) Find the equation of the tangent to C at the point R. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(Total for Question 1 is 6 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 1

Differentiation

Topic group

1. Rewrite the curve

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} = \frac{1}{4}x^3 - x^2 + 17x^{-1}.$$

2. Differentiate

$$\frac{dy}{dx} = \frac{3}{4}x^2 - 2x - 17x^{-2} = \frac{3}{4}x^2 - 2x - \frac{17}{x^2}.$$

3. Find the gradient at R

At $x = 2$, the tangent gradient is $\frac{3}{4}(4) - 2(2) - \frac{17}{4} = 3 - 4 - \frac{17}{4} = -\frac{21}{4}$.

4. Use the point R

The point is $R\left(2, \frac{13}{2}\right)$, so $y - \frac{13}{2} = -\frac{21}{4}(x - 2)$.

5. Write in integer form

Multiplying by 4 gives $4y - 26 = -21x + 42$, hence $21x + 4y - 68 = 0$.

Final answer

(a) $\frac{dy}{dx} = \frac{3}{4}x^2 - 2x - \frac{17}{x^2}$.

(b) $21x + 4y - 68 = 0$.

WMA11/01 OCTOBER 2022

Question 9

14 marks

Differentiation

9.

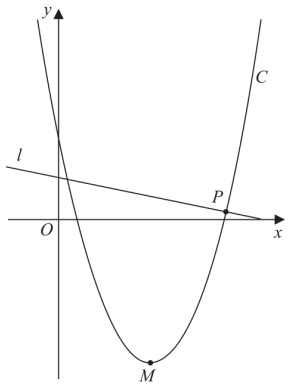


Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{1}{2}x^2 - 10x + 22$$

(a) Write $\frac{1}{2}x^2 - 10x + 22$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point M is the minimum turning point of C, as shown in Figure 3.

(b) Deduce the coordinates of M

(2)

The line l is the normal to C at the point P, as shown in Figure 3.

Given that l has equation $y = k - \frac{1}{8}x$, where k is a constant,

(c) (i) find the coordinates of P

(ii) find the value of k

(6)

Question 9 continues on the next page

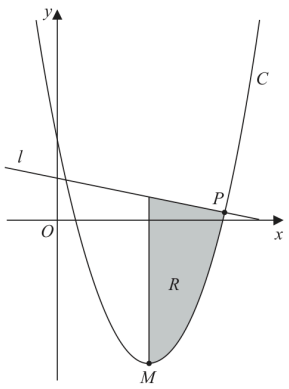


Figure 4

Figure 4 is a copy of Figure 3. The finite region R, shown shaded in Figure 4, is bounded by l, C and the line through M parallel to the y-axis.

(d) Identify the inequalities that define R.

(3)

(Total for Question 9 is 14 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 9

Differentiation

Topic group

1. Complete the square

$\frac{1}{2}x^2 - 10x + 22 = \frac{1}{2}(x^2 - 20x) + 22 = \frac{1}{2}\left((x-10)^2 - 100\right) + 22.$

2. Simplify the completed square form

$\frac{1}{2}\left((x-10)^2 - 100\right) + 22 = \frac{1}{2}(x-10)^2 - 28.$

3. Read the minimum point

The minimum point is $M(10, -28)$.

4. Use the normal gradient

The line l has gradient $-\frac{1}{8}$, so the tangent gradient at P is 8.

5. Differentiate the curve

$\frac{dy}{dx} = x - 10$. At P, $x - 10 = 8$, so $x = 18$.

6. Find P

When $x = 18$, $y = \frac{1}{2}(18)^2 - 10(18) + 22 = 4$, so $P = (18, 4)$.

7. Find k

Since P lies on $y = k - \frac{1}{8}x$, $4 = k - \frac{1}{8}(18) = k - \frac{9}{4}$, so $k = \frac{25}{4}$.

8. Write the upper boundary

The line is $y = \frac{25}{4} - \frac{x}{8}$.

9. Write the lower boundary

The curve is $y = \frac{1}{2}x^2 - 10x + 22$.

10. Identify the region

The region lies to the right of the vertical line through M and between the curve and the normal line: $10 \le x \le 18$ and $\frac{1}{2}x^2 - 10x + 22 \le y \le \frac{25}{4} - \frac{x}{8}$.

Final answer

(a) $\frac{1}{2}(x-10)^2 - 28$.

(b) $M(10, -28)$.

(c) $P(18, 4)$, $k = \frac{25}{4}$.

(d) $10 \le x \le 18$, $\frac{1}{2}x^2 - 10x + 22 \le y \le \frac{25}{4} - \frac{x}{8}$.

WMA11/01 JANUARY 2023

Question 1

5 marks

Differentiation

1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(2)

(Total for Question 1 is 5 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 1

Differentiation

Topic group

1. Differentiate term by term

$y = 2 + 10x^{1/2} - 2x^{3/2}$, so $\dfrac{dy}{dx} = 5x^{-1/2} - 3x^{1/2}$.

2. Write in simplest form

$\dfrac{dy}{dx} = \dfrac{5}{\sqrt{x}} - 3\sqrt{x}$.

3. Substitute x = 2

At $x = 2$, the gradient is $\dfrac{5}{\sqrt{2}} - 3\sqrt{2} = \dfrac{5}{\sqrt{2}} - \dfrac{6}{\sqrt{2}} = -\dfrac{1}{\sqrt{2}}$.

4. Rationalise

$-\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$.

Final answer

(a) $\dfrac{dy}{dx} = \dfrac{5}{\sqrt{x}} - 3\sqrt{x}$.

(b) $-\dfrac{\sqrt{2}}{2}$.

Question 10

10 marks

Differentiation

10.

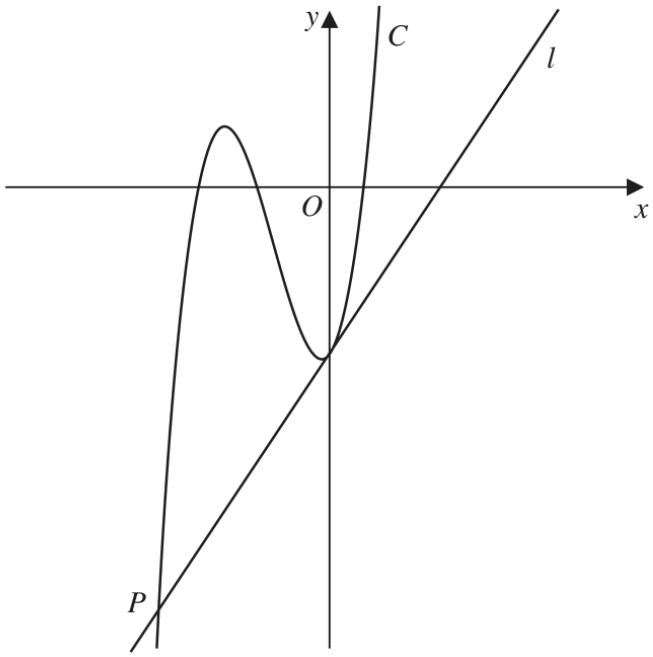


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (3x + 20)(x + 6)(2x - 3)$$

(a) Use the given information to state the values of x for which

$$\mathrm{f}(x) > 0$$

(2)

(b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

The straight line l is the tangent to C at the point where C cuts the y-axis.

Given that l cuts C at the point P, as shown in Figure 4,

(c) find, using algebra, the x coordinate of P

(*Solutions based on calculator technology are not acceptable.*)

(5)

(Total for Question 10 is 10 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 10

Differentiation

Topic group

1. Use the roots

$f(x) = (3x + 20)(x + 6)(2x - 3)$ has roots $x = -\dfrac{20}{3}$, $x = -6$, and $x = \dfrac{3}{2}$.

2. State where f(x) is positive

Since the cubic has positive leading coefficient, $f(x) > 0$ for $-\dfrac{20}{3}\dfrac32$.

3. Expand the first two brackets

$(3x + 20)(x + 6) = 3x^2 + 38x + 120$.

4. Complete the expansion

$(3x^2 + 38x + 120)(2x - 3) = 6x^3 + 67x^2 + 126x - 360$.

5. Find the tangent at the y-axis

$f'(x) = 18x^2 + 134x + 126$, so at $x = 0$ the tangent gradient is 126. Also $f(0) = -360$.

6. Write the tangent equation

The tangent line is $y = 126x - 360$.

7. Find the other intersection

Set $6x^3 + 67x^2 + 126x - 360 = 126x - 360$. This gives $6x^3 + 67x^2 = 0$, so $x^2(6x + 67) = 0$.

8. Choose point P

$x = 0$ is the tangent point on the y-axis, so the other intersection P has $x = -\dfrac{67}{6}$.

Final answer

$(a)\ -\dfrac{20}{3}\dfrac32.$

$(b)\ 6x^3+67x^2+126x-360.$

$(c)\ x_P=-\dfrac{67}{6}.$

WMA11/01 MAY/JUNE 2023

Question 4

7 marks

Differentiation

4. **In this question you must show all stages of your working.**

(a) Write

$$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}}$$

in the form

$$y = Ax^p + Bx^q$$

where A, B, p and q are constants to be found.

(4)

(b) Hence find $\frac{dy}{dx}$ giving each coefficient in simplest form.

(3)

(Total for Question 4 is 7 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 4

Differentiation

Topic group

1. Rewrite the denominator

$\sqrt[3]{8x} = 2x^{1/3}$.

2. Rewrite the numerator

$\sqrt{x^3} = x^{3/2}$.

3. Divide each term

$$y = \frac{5x^2}{2x^{1/3}} + \frac{x^{3/2}}{2x^{1/3}} = \frac{5}{2}x^{5/3} + \frac{1}{2}x^{7/6}.$$

4. Differentiate

$$\frac{dy}{dx} = \frac{5}{2}\cdot\frac{5}{3}x^{2/3} + \frac{1}{2}\cdot\frac{7}{6}x^{1/6}.$$

5. Simplify coefficients

$$\frac{dy}{dx} = \frac{25}{6}x^{2/3} + \frac{7}{12}x^{1/6}.$$

Final answer

$(a)\ y = \frac{5}{2}x^{5/3} + \frac{1}{2}x^{7/6}.$

$(b)\ \frac{dy}{dx} = \frac{25}{6}x^{2/3} + \frac{7}{12}x^{1/6}.$

WMA11/01 OCTOBER 2023

Question 1

5 marks

Differentiation

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$

(3)

(b) $\frac{d^2y}{dx^2}$

(2)

(Total for Question 1 is 5 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 1

Differentiation

Topic group

1. Rewrite the powers

$y = 5x^3 + 3x^{-2} - 7x.$

2. Differentiate once

$$\frac{dy}{dx} = 15x^2 - 6x^{-3} - 7 = 15x^2 - \frac{6}{x^3} - 7.$$

3. Differentiate again

$$\frac{d^2y}{dx^2} = 30x + 18x^{-4} = 30x + \frac{18}{x^4}.$$

Final answer

$\frac{dy}{dx} = 15x^2 - \frac{6}{x^3} - 7$, and $\frac{d^2y}{dx^2} = 30x + \frac{18}{x^4}$.

WMA11/01 JANUARY 2024

Question 3

7 marks

Differentiation

3. The curve C has equation

$$y = \frac{5x^3 - 8}{2x^2} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(4)

The point $P(2, 4)$ lies on C.

(b) Find an equation for the tangent to C at P writing your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(Total for Question 3 is 7 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 3

Differentiation

Topic group

1. Rewrite y

$$y = \frac{5x^3 - 8}{2x^2} = \frac{5}{2}x - 4x^{-2}.$$

2. Differentiate

$$\frac{dy}{dx} = \frac{5}{2} + 8x^{-3} = \frac{5}{2} + \frac{8}{x^3}.$$

3. Find the gradient at P

At $x = 2$, $\frac{dy}{dx} = \frac{5}{2} + \frac{8}{8} = \frac{7}{2}$.

4. Use point-gradient form

The tangent through $P(2, 4)$ is $y - 4 = \frac{7}{2}(x - 2)$.

5. Rearrange

$2y - 8 = 7x - 14$, so $7x - 2y - 6 = 0$.

Final answer

(a) $\frac{dy}{dx} = \frac{5}{2} + \frac{8}{x^3}$.

(b) $7x - 2y - 6 = 0$.

WMA11/01 MAY/JUNE 2024

Question 7

8 marks

Differentiation

7. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $\mathrm{f}'(x)$

(ii) $\mathrm{f}''(x)$

(3)

The curve with equation $y = \mathrm{f}'(x)$ intersects the curve with equation $y = \mathrm{f}''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

(Total for Question 7 is 8 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 7

Differentiation

Topic group

1. Differentiate once

$f'(x) = 6x^2 - 2kx + 14$.

2. Differentiate again

$f''(x) = 12x - 2k$.

3. Use the intersection at A

At an intersection of $y = f'(x)$ and $y = f''(x)$, $f'(x) = f''(x)$. Given $x_A = 5$, substitute $x = 5$.

4. Find k

$6(5)^2 - 2k(5) + 14 = 12(5) - 2k$, so $164 - 10k = 60 - 2k$ and $k = 13$.

5. Find the other x-coordinate

With $k = 13$, $6x^2 - 26x + 14 = 12x - 26$, so $6x^2 - 38x + 40 = 0$. Hence $(3x - 4)(x - 5) = 0$.

6. Use the point B

The other point has $x = \frac{4}{3}$. Then $y = f''(\frac{4}{3}) = 12(\frac{4}{3}) - 26 = -10$.

Final answer

$(a)(i)\ f'(x) = 6x^2 - 2kx + 14.$ $(a)(ii)\ f''(x) = 12x - 2k.$

$(b)\ k = 13.$

$(c)\ B = (\frac{4}{3}, -10).$

WMA11/01 OCTOBER 2024

Question 9

10 marks

Differentiation

9.

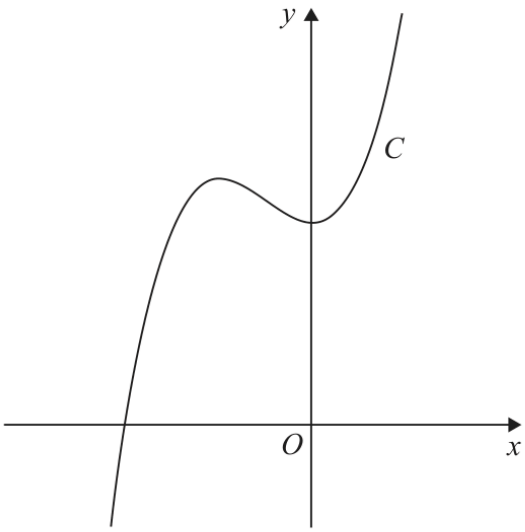


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

(Total for Question 9 is 10 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 9

Differentiation

Topic group

1. Use the positive quadratic factor

$3x^2 - 4x + 20$ has discriminant $(-4)^2 - 4(3)(20) < 0$ and positive leading coefficient, so it is always positive.

2. Deduce the sign of f(x)

The sign of $f(x) = (x + 5)(3x^2 - 4x + 20)$ is therefore the sign of $x + 5$. Hence $f(x) \geq 0$ for $x \geq -5$.

3. Expand f(x)

$f(x) = 3x^3 + 11x^2 + 100$.

4. Differentiate

$f'(x) = 9x^2 + 22x$.

5. Find the gradient at R

At $R(-4, 84)$, $f'(-4) = 9(16) + 22(-4) = 56$.

6. Use parallel tangents

For the tangent at P to be parallel, $f'(x) = 56$.

7. Solve for x

$9x^2 + 22x - 56 = 0$, so $x = \dfrac{-22 \pm 50}{18}$. The roots are $x = -4$ and $x = \dfrac{14}{9}$.

8. Choose the point P

$x = -4$ is the point R, so $x_P = \dfrac{14}{9}$.

9. Transform R under y equals f of x minus 3

$y = f(x - 3)$ translates the curve 3 units to the right, so $(-4, 84)$ maps to $(-1, 84)$.

10. Transform R under y equals 4f(x)

$y = 4f(x)$ stretches vertically by scale factor 4, so $(-4, 84)$ maps to $(-4, 336)$.

Final answer

$(a)\ x \geq -5.$

$(b)\ f'(x) = 9x^2 + 22x.$

$(c)\ x_P = \frac{14}{9}.$

$(d)(i)\ (-1, 84).$

$(d)(ii)\ (-4, 336).$

WMA11/01 JANUARY 2025

Question 5

8 marks

Differentiation

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

(Total for Question 5 is 8 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 5

Differentiation

Topic group

1. Differentiate

$y = 4x^3 + 2x^{-1} + 9$, so $\dfrac{dy}{dx} = 12x^2 - 2x^{-2} = 12x^2 - \dfrac{2}{x^2}$.

2. Use the tangent gradient

The tangent line $y = k - 5x$ has gradient -5, so at P, $12x^2 - \dfrac{2}{x^2} = -5$.

3. Clear the denominator

Multiplying by x^2 gives $12x^4 - 2 = -5x^2$, hence $12x^4 + 5x^2 - 2 = 0$.

4. Solve for x squared

Let $u = x^2$. Then $12u^2 + 5u - 2 = (3u + 2)(4u - 1) = 0$.

5. Use x greater than zero

$x^2 = \dfrac{1}{4}$, so $x = \dfrac{1}{2}$.

6. Find the y-coordinate of P

$y = 4\left(\dfrac{1}{2}\right)^3 + \dfrac{2}{1/2} + 9 = \dfrac{1}{2} + 4 + 9 = \dfrac{27}{2}$.

7. Find k

Since P lies on $y = k - 5x$, $\dfrac{27}{2} = k - \dfrac{5}{2}$, so $k = 16$.

Final answer

$(a)\ \dfrac{dy}{dx} = 12x^2 - \dfrac{2}{x^2}.$

$(c)\ k = 16.$

WMA11/01 MAY/JUNE 2025

Question 2

5 marks

Differentiation

2. The curve C has equation

$$y = 2x^{\frac{5}{2}} - 4x + 3$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

The point P lies on C.

Given that

- the x coordinate of P is 2^k where k is a constant
- the gradient of C at the point P is 16

(b) find the value of k.

(3)

(Total for Question 2 is 5 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 2

Differentiation

Topic group

1. Differentiate

$y = 2x^{5/2} - 4x + 3$, so $\dfrac{dy}{dx} = 2 \cdot \dfrac{5}{2}x^{3/2} - 4 = 5x^{3/2} - 4$.

2. Use the gradient at P

At P, the gradient is 16, so $5x^{3/2} - 4 = 16$.

3. Find x

$5x^{3/2} = 20$, hence $x^{3/2} = 4$.

4. Use x equals 2 to the k

Since the x-coordinate of P is 2^k, we have $(2^k)^{3/2} = 4 = 2^2$.

5. Solve for k

$2^{3k/2} = 2^2$, so $\dfrac{3k}{2} = 2$ and $k = \dfrac{4}{3}$.

Final answer

$(a)\ \dfrac{dy}{dx} = 5x^{3/2} - 4.$

$(b)\ k = \dfrac{4}{3}.$

WMA11/01 OCTOBER 2025

Question 1

6 marks

Differentiation

1. The curve C has equation

$$y = 6x^2 + 3\sqrt{x} + \frac{5}{8} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ writing the answer in simplest form.

(3)

The point $P\left(\frac{1}{4}, \frac{5}{2}\right)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$ where m and c are integers.

(3)

(Total for Question 1 is 6 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 1

Differentiation

Topic group

1. Rewrite the curve

$y = 6x^2 + 3x^{1/2} + \frac{5}{8}$.

2. Differentiate

$\frac{dy}{dx} = 12x + \frac{3}{2}x^{-1/2} = 12x + \frac{3}{2\sqrt{x}}$.

3. Find the tangent gradient

At $x = \frac{1}{4}$, $\frac{dy}{dx} = 12\left(\frac{1}{4}\right) + \frac{3}{2}\left(\frac{1}{4}\right)^{-1/2} = 3 + 3 = 6$.

4. Use point P

The tangent passes through $P\left(\frac{1}{4}, \frac{5}{2}\right)$, so $y - \frac{5}{2} = 6\left(x - \frac{1}{4}\right)$.

5. Simplify

$y - \frac{5}{2} = 6x - \frac{3}{2}$, hence $y = 6x + 1$.

Final answer

(a) $\frac{dy}{dx} = 12x + \frac{3}{2\sqrt{x}}$.

(b) $y = 6x + 1$.

WMA11/01 OCTOBER 2025

Question 10

10 marks

Differentiation

10.

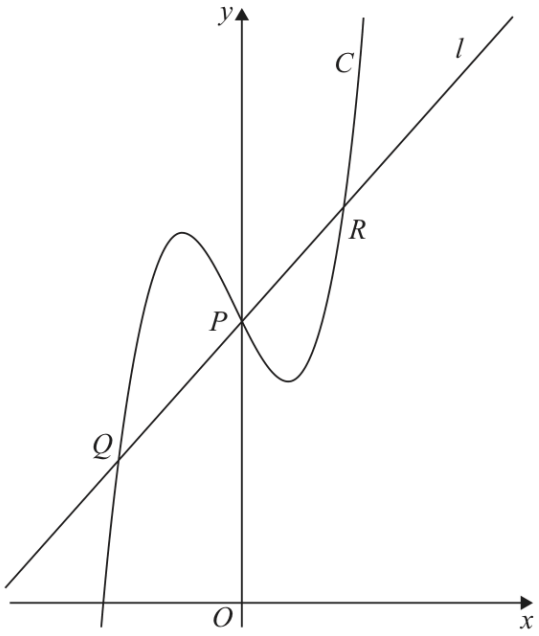


Figure 4

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 4 shows a sketch of the curve C with equation

$$y = 2x^3 + \frac{1}{2}x^2 - 2x + 5$$

The line l is the normal to C at the point P where $x = 0$

The line l also intersects C at points Q and R as shown in Figure 4.

(a) Find, using algebra, the x coordinate of point Q.

(6)

The point T lies on C.

Given that

- the tangent to C at T is parallel to l
- the x coordinate of T is positive

(b) find, using algebra, the exact x coordinate of T.

(4)

(Total for Question 10 is 10 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 10

Differentiation

Topic group

1. Differentiate C

$y = 2x^3 + \frac{1}{2}x^2 - 2x + 5$, so $\frac{dy}{dx} = 6x^2 + x - 2$.

2. Find the tangent gradient at P

At P, $x = 0$, so the tangent gradient is -2.

3. Equation of the normal

The normal gradient is $\frac{1}{2}$ and $P = (0, 5)$, so $l : y = \frac{1}{2}x + 5$.

4. Find intersections with the normal

Set $2x^3 + \frac{1}{2}x^2 - 2x + 5 = \frac{1}{2}x + 5$.

5. Form and factor the cubic

$2x^3 + \frac{1}{2}x^2 - \frac{5}{2}x = 0$. Multiplying by 2 gives
$4x^3 + x^2 - 5x = 0 = x(4x^2 + x - 5)$.

6. Solve for x

$x(4x + 5)(x - 1) = 0$, so the intersections have x-coordinates 0, $-\frac{5}{4}$ and 1.

7. Select point Q

The point Q is the left-hand intersection, so $x_Q = -\frac{5}{4}$.

8. Parallel tangent condition

The tangent at T is parallel to l, so its gradient is $\frac{1}{2}$.

9. Set derivative equal to one half

$6x^2 + x - 2 = \frac{1}{2}$, so $12x^2 + 2x - 5 = 0$.

10. Use the positive solution

$x = \frac{-2 \pm \sqrt{4 + 240}}{24} = \frac{-1 \pm \sqrt{61}}{12}$. Since x_T is positive, $x_T = \frac{-1 + \sqrt{61}}{12}$.

Final answer

$(a)\ x_Q = -\frac{5}{4}$.

$(b)\ x_T = \frac{-1 + \sqrt{61}}{12}$.

WMA11/01 JANUARY 2026

Question 7

10 marks

Differentiation

7. The curve C has equation

$$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$$

(a) Show that $\frac{dy}{dx}$ can be written in the form

$$p(x+q)^2 + r$$

where p, q and r are constants to be found.

(5)

(b) Hence state

(i) the minimum value of $\frac{dy}{dx}$

(ii) the value of x at which this minimum value occurs.

(2)

Given that S is the point on C at which the gradient is a minimum,

(c) find the equation of the tangent to C at S, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 7 is 10 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 7

Differentiation

Topic group

1. Differentiate

$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$ gives $\frac{dy}{dx} = 2x^2 - 16x + 43$.

2. Complete the square

$2x^2 - 16x + 43 = 2(x^2 - 8x) + 43 = 2[(x-4)^2 - 16] + 43$.

3. Simplify

$\frac{dy}{dx} = 2(x-4)^2 + 11$.

4. Minimum gradient

Since $2(x-4)^2 \geq 0$, the minimum value of $\frac{dy}{dx}$ is 11.

5. Where it occurs

The minimum occurs when $x - 4 = 0$, so $x = 4$.

6. Find the point S

At $x = 4$, $y = \frac{2}{3}(4)^3 - 8(4)^2 + 43(4) - \frac{20}{3} = 80$.

7. Tangent equation

The gradient at S is 11, so $y - 80 = 11(x - 4)$.

8. Simplify

$y = 11x + 36$.

Final answer

$(a)\ \dfrac{dy}{dx} = 2(x-4)^2 + 11.$

$(b)\ \min \dfrac{dy}{dx} = 11 \text{ at } x = 4.$

$(c)\ y = 11x + 36.$

WMA11/01 OCTOBER 2019

Question 10

10 marks

Differentiation

Also in Differentiation

Primary: Graphs of Functions

10.

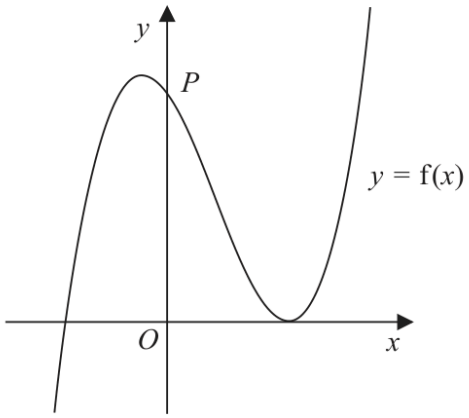


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

(Total 10 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 10

Differentiation

Primary group: Graphs of Functions

1. Use the factorised form for the inequality

$f(x) = (2x+5)(x-3)^2$. Since $(x-3)^2 \geq 0$, the sign is controlled by $2x+5$, with an extra zero at $x = 3$. Hence $f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$.

2. Expand the cubic

$(x-3)^2 = x^2 - 6x + 9$, so
$f(x) = (2x+5)(x^2-6x+9) = 2x^3 - 7x^2 - 12x + 45$.

3. Find P and the gradient at P

At the y-axis, $x = 0$, so $P = (0, 45)$. Also $f'(x) = 6x^2 - 14x - 12$, so the gradient at P is $f'(0) = -12$.

4. Translate the graph

A translation 2 units in the positive x direction means $g(x) = f(x-2)$. Therefore $g(x) = (2(x-2)+5)((x-2)-3)^2 = (2x+1)(x-5)^2$.

5. Find the new y-intercept

Set $x = 0$: $g(0) = (1)(25) = 25$, so the y-intercept is 25.

Final answer

$f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$; $f(x) = 2x^3 - 7x^2 - 12x + 45$; $P = (0, 45)$; gradient $= -12$; $g(x) = (2x+1)(x-5)^2$; y-intercept 25.

10 marks

Differentiation

Also in Differentiation

Primary: Integration

WMA11/01 OCTOBER 2019

Question 11

11. A curve has equation $y = f(x)$.

The point $P\left(4, \frac{32}{3}\right)$ lies on the curve.

Given that

- $f''(x) = \frac{4}{\sqrt{x}} - 3$
- $f'(x) = 5$ at P

find

(a) the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(2)

(b) $f(x)$.

(8)

(Total 10 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 11

Differentiation

Primary group: Integration

1. Use the given gradient for the tangent

At $P\left(4, \frac{32}{3}\right)$, $f'(4) = 5$, so the tangent has gradient 5.

2. Find the tangent equation

$y - \frac{32}{3} = 5(x - 4)$. Hence $y = 5x - \frac{28}{3}$.

3. Integrate f'' to find f'

$f''(x) = 4x^{-1/2} - 3$, so $f'(x) = 8x^{1/2} - 3x + C$.

4. Use f'(4)=5

$5 = 8(2) - 12 + C = 4 + C$, so $C = 1$. Therefore $f'(x) = 8\sqrt{x} - 3x + 1$.

5. Integrate again

$f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x + D$.

6. Use the point on the curve

Substitute $x = 4$, $f(4) = \frac{32}{3}$: $\frac{32}{3} = \frac{16}{3}(8) - \frac{3}{2}(16) + 4 + D = \frac{68}{3} + D$, so $D = -12$.

Final answer

$(a)\ y = 5x - \frac{28}{3}$.

$(b)\ f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x - 12$.

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Question 11

11 marks

Differentiation

Also in Differentiation

Primary: Integration

11. A curve has equation $y = f(x)$, where

$$f''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.

Given that $f'(x) = -4$ at P,

(a) find the equation of the normal at P, writing your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find $f(x)$.

(8)

(Total 11 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 11

Differentiation

Primary group: Integration

1. Find the normal gradient

At P, $f'(4) = -4$, so the tangent gradient is -4. The normal gradient is therefore $\frac{1}{4}$.

2. Find the normal equation

Using $P(4, -50)$: $y + 50 = \frac{1}{4}(x - 4)$, so $y = \frac{1}{4}x - 51$.

3. Integrate f'' to find f'

$f''(x) = 6x^{-3/2} + x$. Therefore $f'(x) = -12x^{-1/2} + \frac{1}{2}x^2 + k$.

4. Use f'(4) = -4

$-4 = -\frac{12}{2} + \frac{1}{2}(16) + k = -6 + 8 + k = 2 + k$, so $k = -6$.

5. Integrate again

$f(x) = -24x^{1/2} + \frac{x^3}{6} - 6x + d$.

6. Use the point on the curve

Substitute $P(4, -50)$:
$-50 = -24(2) + \frac{64}{6} - 24 + d = -72 + \frac{32}{3} + d = -\frac{184}{3} + d$. Hence $d = \frac{34}{3}$.

Final answer

$(a)\ y = \dfrac{1}{4}x - 51.$

$(b)\ f(x) = -24\sqrt{x} + \dfrac{x^3}{6} - 6x + \dfrac{34}{3}.$

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Question 9

11 marks

Differentiation

Also in Differentiation

Primary: Integration

9. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(5)

(ii) A curve C has equation $y = \mathrm{f}(x)$.

Given

- $\mathrm{f}'(x) = x^2 + ax + b$ where a and b are constants
- the y intercept of C is -8
- the point $P(3,-2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $\mathrm{f}(x)$.

(6)

(Total 11 marks)

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Worked Solution - Question 9

Differentiation

Primary group: Integration

1. Expand and simplify the integrand

$$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2+12x+4}{4x^{1/2}} = \frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}.$$

2. Integrate term by term

$$\int \left(\frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}\right)dx = \frac{9}{10}x^{5/2} + 2x^{3/2} + 2x^{1/2} + c.$$

3. Write in simplest form

Factoring out $\sqrt{x}$ gives $\dfrac{\sqrt{x}(9x^2+20x+20)}{10} + c$.

4. Use the gradient condition at P

Since $f'(x) = x^2 + ax + b$ and the gradient at $P(3,-2)$ is 2, $f'(3) = 2$. Hence $9 + 3a + b = 2$, so $3a + b = -7$.

5. Integrate f'(x)

$f(x) = \dfrac{x^3}{3} + \dfrac{a}{2}x^2 + bx + c$. The y-intercept is -8, so $c = -8$.

6. Use P(3,-2)

$-2 = \dfrac{27}{3} + \dfrac{9a}{2} + 3b - 8$, so $\dfrac{9a}{2} + 3b = -3$.

7. Solve for a and b

From $3a + b = -7$, $b = -7 - 3a$. Substitute into $\dfrac{9a}{2} + 3b = -3$ to get $a = -4$, and then $b = 5$.

8. Write f(x)

Therefore $f(x) = \dfrac{x^3}{3} - 2x^2 + 5x - 8$.

Final answer

$(i)\ \dfrac{\sqrt{x}(9x^2+20x+20)}{10}+c. \quad (ii)\ f(x)=\dfrac{x^3}{3}-2x^2+5x-8.$

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Question 6

8 marks

Differentiation

Also in Differentiation

Primary: Integration

6. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- C passes through the point $P(8, 2)$
- $\mathrm{f}'(x) = \dfrac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $\mathrm{f}(x)$.

(5)

(Total 8 marks)

WMA11/01 MAY/JUNE 2021

Worked Solution - Question 6

Differentiation

Primary group: Integration

1. Find the gradient at P

$f'(x)=\frac{32}{3x^2}+3-2\sqrt[3]{x}$. At $x=8$,

$f'(8)=\frac{32}{3(64)}+3-2(2)=\frac{1}{6}-1=-\frac{5}{6}$.

2. Write the tangent equation

Using $P(8,2)$, $y-2=-\frac{5}{6}(x-8)$, hence $y=-\frac{5}{6}x+\frac{26}{3}$.

3. Integrate f'(x)

$f'(x)=\frac{32}{3}x^{-2}+3-2x^{1/3}$, so $f(x)=-\frac{32}{3x}+3x-\frac{3}{2}x^{4/3}+c$.

4. Use P(8,2) to find c

$2=-\frac{32}{24}+24-\frac{3}{2}(16)+c=-\frac{4}{3}+c$, so $c=\frac{10}{3}$.

5. Write f(x)

$f(x)=-\frac{32}{3x}+3x-\frac{3}{2}x^{4/3}+\frac{10}{3}$.

Final answer

$(a)\ y=-\frac{5}{6}x+\frac{26}{3}$.

$(b)\ f(x)=-\frac{32}{3x}+3x-\frac{3}{2}x^{4/3}+\frac{10}{3}$.

WMA11/01 OCTOBER 2021

Question 6

10 marks

Differentiation

Also in Differentiation

Primary: Graphs of Functions

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

(Total 10 marks)

WMA11/01 OCTOBER 2021

Worked Solution - Question 6

Differentiation

Primary group: Graphs of Functions

1. Use the factorised form for the sketch

$f(x) = 2(x+1)(x-3)^2$. The curve cuts the x-axis at $(-1, 0)$ and meets the x-axis at $(3, 0)$ because $x = 3$ is a repeated root.

2. Find the y-intercept

$f(0) = 2(1)(9) = 18$, so the y-intercept is $(0, 18)$.

3. Describe the cubic shape

The leading term is positive, so the cubic goes down to the left and up to the right. It crosses at $x = -1$ and touches at $x = 3$.

4. Expand f(x)

$(x+1)(x-3)^2 = (x+1)(x^2 - 6x + 9) = x^3 - 5x^2 + 3x + 9$, so $f(x) = 2x^3 - 10x^2 + 6x + 18$.

5. Differentiate

$f'(x) = 6x^2 - 20x + 6$.

6. Find the gradient at x = 1/3

$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{9}\right) - 20\left(\frac{1}{3}\right) + 6 = 0$, so the tangent is horizontal.

7. Find the y-coordinate

$$f\left(\frac{1}{3}\right) = 2\left(\frac{4}{3}\right)\left(-\frac{8}{3}\right)^2 = \frac{512}{27}.$$

8. Write the tangent

A horizontal tangent through this point has equation $y = \frac{512}{27}$.

Final answer

(a) x-intercepts $(-1, 0), (3, 0)$, y-intercept $(0, 18)$.

(b) $f(x) = 2x^3 - 10x^2 + 6x + 18$.

(c) $y = \dfrac{512}{27}$.

10 marks

Differentiation

Also in Differentiation

Primary: Straight Line

WMA11/01 OCTOBER 2021

Question 8

8.

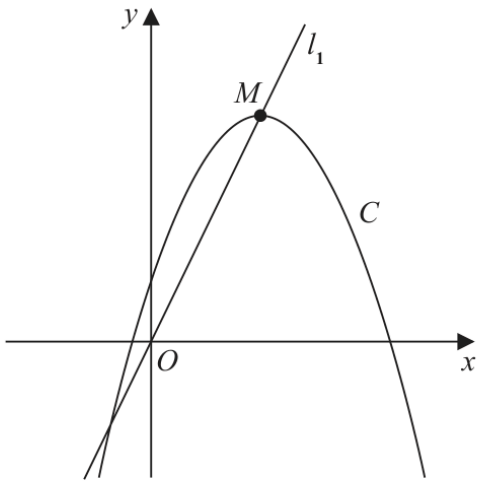


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 4 + 12x - 3x^2$$

The point M is the maximum turning point on C.

(a) (i) Write $4 + 12x - 3x^2$ in the form

$$a + b(x + c)^2$$

where a, b and c are constants to be found.

(ii) Hence, or otherwise, state the coordinates of M.

(5)

The line l_1 passes through O and M, as shown in Figure 4.

A line l_2 touches C and is parallel to l_1

(b) Find an equation for l_2

(5)

(Total 10 marks)

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Worked Solution - Question 8

Differentiation

Primary group: Straight Line

1. Complete the square

$4+12x-3x^2=-3(x^2-4x)+4=-3\big((x-2)^2-4\big)+4=16-3(x-2)^2$.

2. State the maximum point

The maximum occurs when $(x-2)^2=0$, so $M=(2,16)$.

3. Find the gradient of l1

Line l_1 passes through $O(0,0)$ and $M(2,16)$, so its gradient is $\frac{16}{2}=8$.

4. Use the parallel tangent gradient

Line l_2 is parallel to l_1, so its gradient is also 8. For $y=4+12x-3x^2$, $\frac{dy}{dx}=12-6x$.

5. Find the point of contact

Set $12-6x=8$, giving $x=\frac{2}{3}$. Then $y=4+12\left(\frac{2}{3}\right)-3\left(\frac{4}{9}\right)=\frac{32}{3}$.

6. Write l2

Using gradient 8 through $\left(\frac{2}{3},\frac{32}{3}\right)$, $y-\frac{32}{3}=8\left(x-\frac{2}{3}\right)$. Hence $y=8x+\frac{16}{3}$.

Final answer

$(a)(i)\ 16 - 3(x-2)^2.$ $(a)(ii)\ M = (2, 16).$

$(b)\ y = 8x + \dfrac{16}{3}.$

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Question 10

7 marks

Differentiation

Also in Differentiation

Primary: Integration

10. A curve has equation $y = f(x), x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a.

(3)

(b) Hence find $f(x)$.

(4)

(Total 7 marks)

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Worked Solution - Question 10

Differentiation

Primary group: Integration

1. Differentiate f'(x)

$f'(x) = ax - 12x^{1/3}$, so $f''(x) = a - 4x^{-2/3}$.

2. Use f''(27) = 0

Since $27^{2/3} = 9$, $0 = a - \frac{4}{9}$, so $a = \frac{4}{9}$.

3. Integrate f'(x)

With $a = \frac{4}{9}$, $f'(x) = \frac{4}{9}x - 12x^{1/3}$. Therefore $f(x) = \frac{2}{9}x^2 - 9x^{4/3} + c$.

4. Use the point (1, -8)

$-8 = \frac{2}{9} - 9 + c$, so $c = -8 + 9 - \frac{2}{9} = \frac{7}{9}$.

5. Write f(x)

$f(x) = \frac{2}{9}x^2 - 9x^{4/3} + \frac{7}{9}$.

Final answer

$(a)\ a = \frac{4}{9}$.

$(b)\ f(x) = \frac{2}{9}x^2 - 9x^{4/3} + \frac{7}{9}$.

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Question 6

11 marks

Differentiation

Also in Differentiation

Primary: Integration

6. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$, simplifying your answer.

(7)

(Total 11 marks)

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Worked Solution - Question 6

Differentiation

Primary group: Integration

1. Simplify f'(x)

$$f'(x)=\frac{(x+3)^2}{x\sqrt{x}}=\frac{x^2+6x+9}{x^{3/2}}=x^{1/2}+6x^{-1/2}+9x^{-3/2}.$$

2. Find the gradient at P

At $x=4$, $f'(4)=2+3+\frac{9}{8}=\frac{49}{8}$.

3. Find the tangent equation

Using $P(4,20)$, $y-20=\frac{49}{8}(x-4)$. Multiplying out gives $49x-8y-36=0$.

4. Integrate f'(x)

$$f(x)=\int\left(x^{1/2}+6x^{-1/2}+9x^{-3/2}\right)dx=\frac{2}{3}x^{3/2}+12x^{1/2}-18x^{-1/2}+c.$$

5. Use P(4,20)

$20=\frac{2}{3}(8)+12(2)-\frac{18}{2}+c=\frac{61}{3}+c$, so $c=-\frac{1}{3}$.

6. Write f(x)

$$f(x)=\frac{2}{3}x\sqrt{x}+12\sqrt{x}-\frac{18}{\sqrt{x}}-\frac{1}{3}.$$

Final answer

$(a)(i)\ \frac{49}{8}.\quad (a)(ii)\ 49x-8y-36=0.$

$(b)\ f(x)=\frac{2}{3}x\sqrt{x}+12\sqrt{x}-\frac{18}{\sqrt{x}}-\frac{1}{3}.$

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Question 7

9 marks

Differentiation

Also in Differentiation

Primary: Integration

7. The curve C has equation $y = f(x),\ x > 0$

Given that

- $f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $f''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $f(x) = 8\sqrt{3}$, when $x = 12$

(b) find $f(x)$, giving each term in simplest form.

(5)

(Total 9 marks)

WMA11/01 MAY/JUNE 2022

Worked Solution - Question 7

Differentiation

Primary group: Integration

1. Differentiate f'(x)

$f'(x) = 2x^{-1/2} + Ax^{-2} + 3$, so $f''(x) = -x^{-3/2} - 2Ax^{-3}$.

2. Use f''(4) = 0

$-4^{-3/2} - 2A(4^{-3}) = 0$, so $-\frac{1}{8} - \frac{A}{32} = 0$ and $A = -4$.

3. Substitute A into f'(x)

$f'(x) = \frac{2}{\sqrt{x}} - \frac{4}{x^2} + 3$.

4. Integrate to find f(x)

$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x + c$.

5. Use f(12) = 8sqrt3

$f(12) = 8\sqrt{3} + \frac{1}{3} + 36 + c = 8\sqrt{3}$, so $c = -\frac{109}{3}$.

Final answer

$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x - \frac{109}{3}$.

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Question 5

9 marks

Differentiation

Also in Differentiation

Primary: Integration

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x), x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

(Total for Question 5 is 9 marks)

WMA11/01 OCTOBER 2022

Worked Solution - Question 5

Differentiation

Primary group: Integration

1. Integrate f'(x)

$f'(x) = 12x^{-1/2} + \frac{x}{3} - 4$, so $f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x + c$.

2. Use P(9, 8)

$8 = 24\sqrt{9} + \frac{9^2}{6} - 4(9) + c = 72 + \frac{81}{6} - 36 + c$.

3. Find c

$72 + \frac{81}{6} - 36 = \frac{99}{2}$, so $c = 8 - \frac{99}{2} = -\frac{83}{2}$.

4. Find the tangent gradient at P

$f'(9) = \frac{12}{3} + \frac{9}{3} - 4 = 3$.

5. Find the normal gradient

The normal gradient is the negative reciprocal of 3, so it is $-\frac{1}{3}$.

6. Find the y-intercept

The normal through $(9, 8)$ is $y - 8 = -\frac{1}{3}(x - 9)$, so $y = -\frac{1}{3}x + 11$. It crosses the y-axis at $(0, 11)$.

Final answer

$(a)\ f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x - \frac{83}{2}$.

$(b)\ (0, 11)$.

WMA11/01 JANUARY 2023

Question 11

8 marks

Differentiation

Also in Differentiation

Primary: Integration

11. A curve C has equation $y = f(x), \quad x > 0$

Given that

- $f''(x) = 4x + \frac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

(Total for Question 11 is 8 marks)

WMA11/01 JANUARY 2023

Worked Solution - Question 11

Differentiation

Primary group: Integration

1. Find point P

Since P has x-coordinate 4 and lies on the tangent $y = 3x + 4$, $P = (4, 16)$.

2. Find the normal

The tangent gradient is 3, so the normal gradient is $-\frac{1}{3}$. Hence the normal is $y - 16 = -\frac{1}{3}(x - 4)$.

3. Integrate f''(x)

$f''(x) = 4x + x^{-1/2}$, so $f'(x) = 2x^2 + 2\sqrt{x} + A$.

4. Use the tangent gradient

At $x = 4$, $f'(4) = 3$. Thus $2(16) + 2(2) + A = 3$, so $A = -33$.

5. Integrate again

$f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + B$.

6. Use P(4, 16)

$16 = \frac{2}{3}(64) + \frac{4}{3}(8) - 33(4) + B = \frac{160}{3} - 132 + B$.

7. Find B

$B = \frac{284}{3}$, so $f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}$.

Final answer

$(a)\ y - 16 = -\frac{1}{3}(x - 4).$

$(b)\ f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}.$

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Question 8

10 marks

Differentiation

Also in Differentiation

Primary: Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

The curve with equation $y = f(x)$ also passes through the point $P(4, 12)$

Given that

$$f'(x) = \frac{1}{4}x^3 - 8x^{\frac{1}{2}}$$

(b) find $f(x)$ giving the coefficients in simplest form.

(5)

(Total for Question 8 is 10 marks)

WMA11/01 MAY/JUNE 2023

Worked Solution - Question 8

Differentiation

Primary group: Integration

1. Differentiate the curve

For $y = \frac{1}{4}x^3 - 8x^{-1/2}$, $\frac{dy}{dx} = \frac{3}{4}x^2 + 4x^{-3/2}$.

2. Find the gradient at P

At $x = 4$, $\frac{dy}{dx} = \frac{3}{4}(16) + 4(4^{-3/2}) = 12 + \frac{1}{2} = \frac{25}{2}$.

3. Find the tangent

Using $P(4, 12)$, $y - 12 = \frac{25}{2}(x - 4)$.

4. Write in integer form

Multiplying by 2 gives $2y - 24 = 25x - 100$, hence $25x - 2y - 76 = 0$.

5. Integrate f'(x)

$f'(x) = \frac{1}{4}x^3 - 8x^{-1/2}$ gives $f(x) = \frac{x^4}{16} - 16\sqrt{x} + c$.

6. Use P(4, 12)

$12 = \frac{4^4}{16} - 16\sqrt{4} + c = 16 - 32 + c$, so $c = 28$.

Final answer

$(a)\ 25x - 2y - 76 = 0.$

$(b)\ f(x) = \frac{x^4}{16} - 16\sqrt{x} + 28.$

WMA11/01 OCTOBER 2023

Question 7

10 marks

Differentiation

Also in Differentiation

Primary: Integration

7. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $f(x)$.

(6)

(Total for Question 7 is 10 marks)

WMA11/01 OCTOBER 2023

Worked Solution - Question 7

Differentiation

Primary group: Integration

1. Simplify f prime

$$f'(x)=\frac{4x^2+10-7x^{1/2}}{4x^{1/2}}=x^{3/2}+\frac{5}{2}x^{-1/2}-\frac{7}{4}.$$

2. Find the gradient at P

At $x=4$, $f'(4)=4^{3/2}+\frac{5}{2}(4^{-1/2})-\frac{7}{4}=8+\frac{5}{4}-\frac{7}{4}=\frac{15}{2}$.

3. Find the normal gradient

The normal gradient is the negative reciprocal, so $m_n=-\frac{2}{15}$.

4. Use point P

Using $P(4,-1)$, $y+1=-\frac{2}{15}(x-4)$.

5. Write the normal in integer form

$15y+15=-2x+8$, hence $2x+15y+7=0$.

6. Integrate f prime

$$f(x)=\int\left(x^{3/2}+\frac{5}{2}x^{-1/2}-\frac{7}{4}\right)dx=\frac{2}{5}x^{5/2}+5x^{1/2}-\frac{7}{4}x+c.$$

7. Use P to find c

$-1=\frac{2}{5}(4^{5/2})+5(4^{1/2})-\frac{7}{4}(4)+c=\frac{64}{5}+10-7+c$, so $c=-\frac{84}{5}$.

Final answer

$(a)(i)\ \dfrac{15}{2}.\quad (a)(ii)\ 2x+15y+7=0.$

$(b)\ f(x)=\dfrac{2}{5}x^{5/2}+5x^{1/2}-\dfrac{7}{4}x-\dfrac{84}{5}.$

WMA11/01 JANUARY 2024

Question 10

8 marks

Differentiation

Also in Differentiation

Primary: Integration

10. **In this question you must show all stages of your working.**

The curve C has equation $y = f(x), x > 0$

Given that

- the point $P\left(2, 8\sqrt{2}\right)$ lies on C
- $f'(x) = 4\sqrt{x^3} + \frac{k}{x^2}$ where k is a constant
- $f''(x) = 0$ at P

(a) find the exact value of k,

(4)

(b) find $f(x)$, giving your answer in simplest form.

(4)

First released on AP - Edexcel Discord
https://sites.google.com/view/ap-edexcel

(Total for Question 10 is 8 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 10

Differentiation

Primary group: Integration

1. Differentiate f prime

$f'(x) = 4x^{3/2} + kx^{-2}$, so $f''(x) = 6x^{1/2} - 2kx^{-3}$.

2. Use f double prime equals zero at P

At P, $x = 2$ and $f''(2) = 0$, so $6\sqrt{2} - 2k(2^{-3}) = 0$.

3. Find k

$6\sqrt{2} - \frac{k}{4} = 0$, hence $k = 24\sqrt{2}$.

4. Integrate f prime

$$f(x) = \int \left(4x^{3/2} + 24\sqrt{2}x^{-2}\right)dx = \frac{8}{5}x^{5/2} - 24\sqrt{2}x^{-1} + c.$$

5. Use the point P

$P(2, 8\sqrt{2})$ lies on C, so $8\sqrt{2} = \frac{8}{5}(2^{5/2}) - \frac{24\sqrt{2}}{2} + c$.

6. Find c

$8\sqrt{2} = \frac{32\sqrt{2}}{5} - 12\sqrt{2} + c$, so $c = \frac{68\sqrt{2}}{5}$.

Final answer

(a) $k = 24\sqrt{2}$.

(b) $f(x) = \frac{8}{5}x^{5/2} - \frac{24\sqrt{2}}{x} + \frac{68\sqrt{2}}{5}$.

WMA11/01 MAY/JUNE 2024

Question 10

10 marks

Differentiation

Also in Differentiation

Primary: Integration

10. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(6)

(Total for Question 10 is 10 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 10

Differentiation

Primary group: Integration

1. Expand the numerator

$(2x-1)(3x+2) = 6x^2 + x - 2$.

2. Simplify f prime

$$f'(x) = 6x - \frac{6x^2 + x - 2}{2\sqrt{x}} = 6x - 3x^{3/2} - \frac{1}{2}x^{1/2} + x^{-1/2}.$$

3. Find the tangent gradient at P

$$f'(4) = 24 - 3(8) - \frac{1}{2}(2) + \frac{1}{2} = -\frac{1}{2}.$$

4. Use the normal gradient

The normal gradient is the negative reciprocal of $-\frac{1}{2}$, so it is 2.

5. Find the normal equation

Through $P(4, 12)$, $y - 12 = 2(x - 4)$, hence $y = 2x + 4$.

6. Integrate f prime

$$f(x) = 3x^2 - \frac{6}{5}x^{5/2} - \frac{1}{3}x^{3/2} + 2x^{1/2} + c.$$

7. Use P to find c

$$12 = 3(4)^2 - \frac{6}{5}(4^{5/2}) - \frac{1}{3}(4^{3/2}) + 2(4^{1/2}) + c.$$

8. Simplify c

$12 = 48 - \frac{192}{5} - \frac{8}{3} + 4 + c = \frac{164}{15} + c$, so $c = \frac{16}{15}$.

Final answer

$(a)\ y = 2x + 4.$

$(b)\ f(x) = 3x^2 - \frac{6}{5}x^{5/2} - \frac{1}{3}x^{3/2} + 2x^{1/2} + \frac{16}{15}.$

WMA11/01 OCTOBER 2024

Question 3

9 marks

Differentiation

Also in Differentiation

Primary: Integration

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(a) Find $\int f(x)\,dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

(Total for Question 3 is 9 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 3

Differentiation

Primary group: Integration

1. Rewrite f(x)

$$f(x)=\frac{(x+5)^2}{\sqrt{x}}=\frac{x^2+10x+25}{x^{1/2}}=x^{3/2}+10x^{1/2}+25x^{-1/2}.$$

2. Integrate

$$\int f(x)\,dx=\frac{2}{5}x^{5/2}+\frac{20}{3}x^{3/2}+50x^{1/2}+c.$$

3. Differentiate f(x)

$$f'(x)=\frac{3}{2}x^{1/2}+5x^{-1/2}-\frac{25}{2}x^{-3/2}.$$

4. Set f prime equal to zero

$$\frac{3}{2}x^{1/2}+5x^{-1/2}-\frac{25}{2}x^{-3/2}=0.$$

5. Clear powers of x

Multiplying by $2x^{3/2}$ gives $3x^2+10x-25=0$.

6. Use x greater than zero

$3x^2+10x-25=(3x-5)(x+5)$, so $x=\frac{5}{3}$ or $x=-5$. Since $x>0$, $x=\frac{5}{3}$.

Final answer

$(a)\ \frac{2}{5}x^{5/2}+\frac{20}{3}x^{3/2}+50x^{1/2}+c.$

$(b)\ x=\frac{5}{3}.$

WMA11/01 OCTOBER 2024

Question 8

7 marks

Differentiation

Also in Differentiation

Primary: Integration

8. A curve C has equation $y = f(x)$.

The point P with x coordinate 3 lies on C

Given

- $f'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $f(x)$.

(4)

(Total for Question 8 is 7 marks)

WMA11/01 OCTOBER 2024

Worked Solution - Question 8

Differentiation

Primary group: Integration

1. Find the y-coordinate of P

The normal is $y = -\frac{1}{24}x + 5$. At $x = 3$, $y = -\frac{3}{24} + 5 = \frac{39}{8}$.

2. Use the normal gradient

The normal gradient is $-\frac{1}{24}$, so the tangent gradient is 24.

3. Use f prime at x equals 3

$f'(3) = 4(3)^2 + 3k + 3 = 39 + 3k$.

4. Find k

$39 + 3k = 24$, so $k = -5$.

5. Integrate f prime

With $k = -5$, $f'(x) = 4x^2 - 5x + 3$, so $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x + c$.

6. Use point P

$\frac{39}{8} = \frac{4}{3}(27) - \frac{5}{2}(9) + 3(3) + c = \frac{45}{2} + c$.

7. Find c

$c = \frac{39}{8} - \frac{45}{2} = -\frac{141}{8}$.

Final answer

$k = -5$ and $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x - \frac{141}{8}$.

WMA11/01 JANUARY 2025

Question 6

10 marks

Differentiation

Also in Differentiation

Primary: Integration

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $\mathrm{f}'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $\mathrm{f}(x)$

(7)

Curve C is transformed to the curve with equation $y = \mathrm{f}(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

(Total for Question 6 is 10 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 6

Differentiation

Primary group: Integration

1. Use the gradient at P

At $x = 4$, $f'(4) = 7$. Since $f'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, we get $\dfrac{32 + 4a + b}{8} = 7$.

2. Show the required equation

$32 + 4a + b = 56$, so $4a + b = 24$.

3. Solve for a and b

Together with $a + b = -9$, subtracting gives $3a = 33$, so $a = 11$ and $b = -20$.

4. Rewrite f prime

$$f'(x) = \frac{2x^2 + 11x - 20}{4\sqrt{x}} = \frac{1}{2}x^{3/2} + \frac{11}{4}x^{1/2} - 5x^{-1/2}.$$

5. Integrate

$$f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} + c.$$

6. Use P to find c

$$-5 = \frac{1}{5}(4^{5/2}) + \frac{11}{6}(4^{3/2}) - 10(4^{1/2}) + c = \frac{16}{15} + c.$$

7. State f(x)

$c = -\dfrac{91}{15}$, so $f(x) = \dfrac{1}{5}x^{5/2} + \dfrac{11}{6}x^{3/2} - 10x^{1/2} - \dfrac{91}{15}$.

8. Transform P

$y = f(x - 3)$ translates the curve 3 units to the right, so $P(4, -5)$ becomes $Q(7, -5)$.

Final answer

$(b)\ f(x) = \dfrac{1}{5}x^{5/2} + \dfrac{11}{6}x^{3/2} - 10x^{1/2} - \dfrac{91}{15}.$

$(c)\ Q = (7, -5).$

WMA11/01 MAY/JUNE 2025

Question 8

11 marks

Differentiation

Also in Differentiation

Primary: Integration

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve has equation $y = f(x),\ x > 0$

The point $P(4, 12)$ lies on the curve.

Given that

- $f'(x) = 3\sqrt{x} + kx^2$ where k is a constant
- the equation of the tangent to the curve at P has equation $y = 10x + c$ where c is a constant

(a) (i) show that $k = \frac{1}{4}$

(ii) find the value of c

(4)

(b) Hence find the value of $f''(x)$ at P.

(3)

(c) Find $f(x)$.

(4)

(Total for Question 8 is 11 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 8

Differentiation

Primary group: Integration

1. Use the tangent gradient

The tangent at P has equation $y = 10x + c$, so the gradient at P is 10.

2. Show k

$f'(4) = 3\sqrt{4} + k(4)^2 = 6 + 16k = 10$, so $16k = 4$ and $k = \frac{1}{4}$.

3. Find c

Substitute $P(4, 12)$ into $y = 10x + c$: $12 = 40 + c$, so $c = -28$.

4. Differentiate f prime

With $k = \frac{1}{4}$, $f'(x) = 3x^{1/2} + \frac{1}{4}x^2$.

5. Find f double prime

$f''(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x$.

6. Evaluate at P

$f''(4) = \frac{3}{2} \cdot \frac{1}{2} + 2 = \frac{3}{4} + 2 = \frac{11}{4}$.

7. Integrate f prime

$f(x) = \int \left(3x^{1/2} + \frac{1}{4}x^2\right) dx = 2x^{3/2} + \frac{1}{12}x^3 + d$.

8. Use P to find d

$12 = 2(4^{3/2}) + \frac{1}{12}(4^3) + d = 16 + \frac{16}{3} + d$.

9. State f(x)

$d = -\dfrac{28}{3}$, so $f(x) = 2x^{3/2} + \dfrac{1}{12}x^3 - \dfrac{28}{3}$.

Final answer

$(a)(i)\ k = \dfrac{1}{4}.$ $(a)(ii)\ c = -28.$

$(b)\ f''(P) = \dfrac{11}{4}.$

$(c)\ f(x) = 2x^{3/2} + \dfrac{1}{12}x^3 - \dfrac{28}{3}.$

WMA11/01 OCTOBER 2025

Question 8

8 marks

Differentiation

Also in Differentiation

Primary: Integration

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve has equation $y = f(x), x > 0$

Given that

- $f'(x) = 2x + \frac{8}{x^2} + k$, where k is a constant
- the equation of the tangent to the curve at $x = \sqrt{2}$ is $y = 5x - 3\sqrt{2}$

(a) find the exact value of k.

(2)

(b) Find an equation of the normal to the curve at $x = \sqrt{2}$

(2)

(c) Find $f(x)$, writing your answer in simplest form.

(4)

(Total for Question 8 is 8 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 8

Differentiation

Primary group: Integration

1. Use tangent gradient

The tangent has equation $y = 5x - 3\sqrt{2}$, so the gradient at $x = \sqrt{2}$ is 5.

2. Find k

$f'(\sqrt{2}) = 2\sqrt{2} + \dfrac{8}{(\sqrt{2})^2} + k = 2\sqrt{2} + 4 + k = 5$, so $k = 1 - 2\sqrt{2}$.

3. Find the point on the curve

At $x = \sqrt{2}$, the tangent gives $y = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$.

4. Normal gradient

The tangent gradient is 5, so the normal gradient is $-\dfrac{1}{5}$.

5. Equation of the normal

$y - 2\sqrt{2} = -\dfrac{1}{5}(x - \sqrt{2})$, hence $y = -\dfrac{1}{5}x + \dfrac{11\sqrt{2}}{5}$.

6. Integrate f prime

$f(x) = \displaystyle\int \left(2x + \frac{8}{x^2} + k\right) dx = x^2 - \frac{8}{x} + kx + c.$

7. Substitute k

$f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + c.$

8. Use the point

$2\sqrt{2} = 2 - \dfrac{8}{\sqrt{2}} + (1 - 2\sqrt{2})\sqrt{2} + c = -2 - 3\sqrt{2} + c.$

9. Find c

$c = 2 + 5\sqrt{2}$, so $f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + 2 + 5\sqrt{2}$.

Final answer

$(a)\ k = 1 - 2\sqrt{2}.$

$(b)\ y = -\dfrac{1}{5}x + \dfrac{11\sqrt{2}}{5}.$

$(c)\ f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + 2 + 5\sqrt{2}.$

WMA11/01 JANUARY 2026

Question 1

6 marks

Differentiation

Also in Differentiation

Primary: Integration

1. $y = 4x^3 + \dfrac{3}{2\sqrt{x}} - 8 \qquad x > 0$

Find, in simplest form,

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) $\displaystyle\int y \, \mathrm{d}x$

(3)

(Total for Question 1 is 6 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 1

Differentiation

Primary group: Integration

1. Rewrite with powers

$$y = 4x^3 + \frac{3}{2}x^{-1/2} - 8.$$

2. Differentiate

$$\frac{dy}{dx} = 12x^2 + \frac{3}{2}\left(-\frac{1}{2}\right)x^{-3/2}.$$

3. Simplify derivative

$$\frac{dy}{dx} = 12x^2 - \frac{3}{4}x^{-3/2} = 12x^2 - \frac{3}{4x^{3/2}}.$$

4. Integrate

$$\int y\,dx = \int \left(4x^3 + \frac{3}{2}x^{-1/2} - 8\right)dx.$$

5. Simplify integral

$$\int y\,dx = x^4 + 3x^{1/2} - 8x + c = x^4 + 3\sqrt{x} - 8x + c.$$

Final answer

(a) $\dfrac{dy}{dx} = 12x^2 - \dfrac{3}{4x^{3/2}}.$

(b) $\int y\,dx = x^4 + 3\sqrt{x} - 8x + c.$

WMA11/01 JANUARY 2026

Question 8

12 marks

Differentiation

Also in Differentiation

Primary: Integration

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x), \quad x > 0$

A point P lies on C.

Given that

- $\mathrm{f}'(x) = 4x^2 + \dfrac{6}{x^2} - 4$
- the equation of the tangent to C at P is $y = 10x - 6\sqrt{3}$

(a) (i) verify that $\sqrt{3}$ is a possible x coordinate of P,

(ii) find, using algebra, the other possible x coordinate of P.

(6)

Given that the x coordinate of P is $\sqrt{3}$

(b) find an equation of the normal to C at P.

(2)

(c) Find $\mathrm{f}(x)$, writing your answer in simplest form.
You must show each stage of your working.

(4)

(Total for Question 8 is 12 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 8

Differentiation

Primary group: Integration

1. Verify x equals root 3

$f'(\sqrt{3}) = 4(\sqrt{3})^2 + \dfrac{6}{(\sqrt{3})^2} - 4 = 12 + 2 - 4 = 10$, which matches the tangent gradient.

2. Find all possible x-coordinates

Set $4x^2 + \dfrac{6}{x^2} - 4 = 10$.

3. Form a quadratic in x squared

Multiplying by x^2 gives $4x^4 - 14x^2 + 6 = 0$, so $2x^4 - 7x^2 + 3 = 0$.

4. Factor

$2x^4 - 7x^2 + 3 = (2x^2 - 1)(x^2 - 3) = 0$.

5. Use x greater than zero

$x = \sqrt{3}$ or $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$. The other possible x-coordinate is $\dfrac{\sqrt{2}}{2}$.

6. Find the point P

Given $x_P = \sqrt{3}$, the tangent gives $y_P = 10\sqrt{3} - 6\sqrt{3} = 4\sqrt{3}$.

7. Normal equation

The normal gradient is $-\dfrac{1}{10}$, so $y - 4\sqrt{3} = -\dfrac{1}{10}(x - \sqrt{3})$.

8. Integrate f prime

$f(x) = \displaystyle\int \left(4x^2 + \frac{6}{x^2} - 4\right) dx = \frac{4}{3}x^3 - \frac{6}{x} - 4x + c$.

9. Use P to find c

$$4\sqrt{3} = \frac{4}{3}(\sqrt{3})^3 - \frac{6}{\sqrt{3}} - 4\sqrt{3} + c = 4\sqrt{3} - 2\sqrt{3} - 4\sqrt{3} + c.$$

10. State f(x)

$c = 6\sqrt{3}$, so $f(x) = \frac{4}{3}x^3 - \frac{6}{x} - 4x + 6\sqrt{3}$.

Final answer

$(a)(i)$ verified.

$(a)(ii)\ x = \frac{\sqrt{2}}{2}.$

$(b)\ y - 4\sqrt{3} = -\frac{1}{10}(x - \sqrt{3}).$

$(c)\ f(x) = \frac{4}{3}x^3 - \frac{6}{x} - 4x + 6\sqrt{3}.$

TOPIC

Integration

WMA11/01 OCTOBER 2019

Question 11

10 marks

Integration

11. A curve has equation $y = f(x)$.

The point $P\left(4, \frac{32}{3}\right)$ lies on the curve.

Given that

- $f''(x) = \frac{4}{\sqrt{x}} - 3$
- $f'(x) = 5$ at P

find

(a) the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(2)

(b) $f(x)$.

(8)

(Total 10 marks)

WMA11/01 OCTOBER 2019

Worked Solution - Question 11

Integration

Topic group

1. Use the given gradient for the tangent

At $P\left(4, \frac{32}{3}\right)$, $f'(4) = 5$, so the tangent has gradient 5.

2. Find the tangent equation

$y - \frac{32}{3} = 5(x - 4)$. Hence $y = 5x - \frac{28}{3}$.

3. Integrate f'' to find f'

$f''(x) = 4x^{-1/2} - 3$, so $f'(x) = 8x^{1/2} - 3x + C$.

4. Use f'(4)=5

$5 = 8(2) - 12 + C = 4 + C$, so $C = 1$. Therefore $f'(x) = 8\sqrt{x} - 3x + 1$.

5. Integrate again

$f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x + D$.

6. Use the point on the curve

Substitute $x = 4$, $f(4) = \frac{32}{3}$: $\frac{32}{3} = \frac{16}{3}(8) - \frac{3}{2}(16) + 4 + D = \frac{68}{3} + D$, so $D = -12$.

Final answer

$(a)\ y = 5x - \frac{28}{3}$.

$(b)\ f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x - 12$.

WMA11/01 JANUARY 2020

Question 1

4 marks

Integration

1. Find, in simplest form,

$$\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 \right) \mathrm{d}x$$

(4)

(Total 4 marks)

WMA11/01 JANUARY 2020

Worked Solution - Question 1

Integration

Topic group

1. Rewrite the integrand using powers

$$\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 = \frac{8}{3}x^3 - \frac{1}{2}x^{-1/2} - 5.$$

2. Integrate term by term

$\int \frac{8}{3}x^3\,dx = \frac{2}{3}x^4$, $\int -\frac{1}{2}x^{-1/2}\,dx = -x^{1/2}$, and $\int -5\,dx = -5x$.

3. Include the constant

Therefore $\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5\right)dx = \frac{2}{3}x^4 - \sqrt{x} - 5x + c.$

Final answer

$$\frac{2}{3}x^4 - \sqrt{x} - 5x + c.$$

WMA11/01 JANUARY 2020

Question 11

11 marks

Integration

11. A curve has equation $y = f(x)$, where

$$f''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.

Given that $f'(x) = -4$ at P,

(a) find the equation of the normal at P, writing your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find $f(x)$.

(8)

(Total 11 marks)

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Worked Solution - Question 11

Integration

Topic group

1. Find the normal gradient

At P, $f'(4) = -4$, so the tangent gradient is -4. The normal gradient is therefore $\frac{1}{4}$.

2. Find the normal equation

Using $P(4, -50)$: $y + 50 = \frac{1}{4}(x - 4)$, so $y = \frac{1}{4}x - 51$.

3. Integrate f'' to find f'

$f''(x) = 6x^{-3/2} + x$. Therefore $f'(x) = -12x^{-1/2} + \frac{1}{2}x^2 + k$.

4. Use f'(4) = -4

$-4 = -\frac{12}{2} + \frac{1}{2}(16) + k = -6 + 8 + k = 2 + k$, so $k = -6$.

5. Integrate again

$f(x) = -24x^{1/2} + \frac{x^3}{6} - 6x + d$.

6. Use the point on the curve

Substitute $P(4, -50)$:
$-50 = -24(2) + \frac{64}{6} - 24 + d = -72 + \frac{32}{3} + d = -\frac{184}{3} + d$. Hence $d = \frac{34}{3}$.

Final answer

$(a)\ y = \dfrac{1}{4}x - 51.$

$(b)\ f(x) = -24\sqrt{x} + \dfrac{x^3}{6} - 6x + \dfrac{34}{3}.$

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Question 9

11 marks

Integration

9. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(5)

(ii) A curve C has equation $y = \mathrm{f}(x)$.

Given

- $\mathrm{f}'(x) = x^2 + ax + b$ where a and b are constants
- the y intercept of C is -8
- the point $P(3,-2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $\mathrm{f}(x)$.

(6)

(Total 11 marks)

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Worked Solution - Question 9

Integration

Topic group

1. Expand and simplify the integrand

$$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2+12x+4}{4x^{1/2}} = \frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}.$$

2. Integrate term by term

$$\int \left(\frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}\right)dx = \frac{9}{10}x^{5/2} + 2x^{3/2} + 2x^{1/2} + c.$$

3. Write in simplest form

Factoring out $\sqrt{x}$ gives $\dfrac{\sqrt{x}(9x^2+20x+20)}{10} + c$.

4. Use the gradient condition at P

Since $f'(x) = x^2 + ax + b$ and the gradient at $P(3,-2)$ is 2, $f'(3) = 2$. Hence $9 + 3a + b = 2$, so $3a + b = -7$.

5. Integrate f'(x)

$f(x) = \dfrac{x^3}{3} + \dfrac{a}{2}x^2 + bx + c$. The y-intercept is -8, so $c = -8$.

6. Use P(3,-2)

$-2 = \dfrac{27}{3} + \dfrac{9a}{2} + 3b - 8$, so $\dfrac{9a}{2} + 3b = -3$.

7. Solve for a and b

From $3a + b = -7$, $b = -7 - 3a$. Substitute into $\dfrac{9a}{2} + 3b = -3$ to get $a = -4$, and then $b = 5$.

8. Write f(x)

Therefore $f(x) = \dfrac{x^3}{3} - 2x^2 + 5x - 8$.

Final answer

$(i)\ \dfrac{\sqrt{x}(9x^2+20x+20)}{10}+c. \quad (ii)\ f(x)=\dfrac{x^3}{3}-2x^2+5x-8.$

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Question 4

6 marks

Integration

4. Find

$$\int \frac{(3\sqrt{x} + 2)(x - 5)}{4\sqrt{x}} \, dx$$

writing each term in simplest form.

(6)

(Total 6 marks)

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Worked Solution - Question 4

Integration

Topic group

1. Expand the numerator

$(3\sqrt{x}+2)(x-5) = 3x\sqrt{x} - 15\sqrt{x} + 2x - 10.$

2. Divide by 4sqrt(x)

$$\frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}} = \frac{3x}{4} - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}.$$

3. Integrate term by term

$$\int \left(\frac{3x}{4} - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}\right)dx = \frac{3x^2}{8} - \frac{15x}{4} + \frac{1}{3}x^{3/2} - 5x^{1/2} + c.$$

4. Write each term simply

So the integral is $\frac{3x^2}{8} - \frac{15x}{4} + \frac{x\sqrt{x}}{3} - 5\sqrt{x} + c.$

Final answer

$$\frac{3x^2}{8} - \frac{15x}{4} + \frac{x\sqrt{x}}{3} - 5\sqrt{x} + c.$$

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Question 6

8 marks

Integration

6. The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point $P(8, 2)$
- $f'(x) = \frac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $f(x)$.

(5)

(Total 8 marks)

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Worked Solution - Question 6

Integration

Topic group

1. Find the gradient at P

$f'(x) = \dfrac{32}{3x^2} + 3 - 2\sqrt[3]{x}$. At $x = 8$,

$f'(8) = \dfrac{32}{3(64)} + 3 - 2(2) = \dfrac{1}{6} - 1 = -\dfrac{5}{6}$.

2. Write the tangent equation

Using $P(8, 2)$, $y - 2 = -\dfrac{5}{6}(x - 8)$, hence $y = -\dfrac{5}{6}x + \dfrac{26}{3}$.

3. Integrate f'(x)

$f'(x) = \dfrac{32}{3}x^{-2} + 3 - 2x^{1/3}$, so $f(x) = -\dfrac{32}{3x} + 3x - \dfrac{3}{2}x^{4/3} + c$.

4. Use P(8,2) to find c

$2 = -\dfrac{32}{24} + 24 - \dfrac{3}{2}(16) + c = -\dfrac{4}{3} + c$, so $c = \dfrac{10}{3}$.

5. Write f(x)

$f(x) = -\dfrac{32}{3x} + 3x - \dfrac{3}{2}x^{4/3} + \dfrac{10}{3}$.

Final answer

$(a)\ y = -\dfrac{5}{6}x + \dfrac{26}{3}$.

$(b)\ f(x) = -\dfrac{32}{3x} + 3x - \dfrac{3}{2}x^{4/3} + \dfrac{10}{3}$.

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Question 1

5 marks

Integration

1. Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx$$

giving each term in simplest form.

(5)

(Total 5 marks)

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Worked Solution - Question 1

Integration

Topic group

1. Rewrite with powers

$12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4} = 12x^3 + \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-4}$.

2. Integrate term by term

$\int 12x^3 dx = 3x^4$, $\int \frac{1}{6}x^{-1/2}dx = \frac{1}{3}x^{1/2}$, and $\int -\frac{3}{2}x^{-4}dx = \frac{1}{2}x^{-3}$.

3. Write in simplest form

Therefore the integral is $3x^4 + \frac{\sqrt{x}}{3} + \frac{1}{2x^3} + c$.

Final answer

$3x^4 + \frac{\sqrt{x}}{3} + \frac{1}{2x^3} + c$.

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Question 10

7 marks

Integration

10. A curve has equation $y = f(x), x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a.

(3)

(b) Hence find $f(x)$.

(4)

(Total 7 marks)

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Worked Solution - Question 10

Integration

Topic group

1. Differentiate f'(x)

$f'(x) = ax - 12x^{1/3}$, so $f''(x) = a - 4x^{-2/3}$.

2. Use f''(27) = 0

Since $27^{2/3} = 9$, $0 = a - \frac{4}{9}$, so $a = \frac{4}{9}$.

3. Integrate f'(x)

With $a = \frac{4}{9}$, $f'(x) = \frac{4}{9}x - 12x^{1/3}$. Therefore $f(x) = \frac{2}{9}x^2 - 9x^{4/3} + c$.

4. Use the point (1, -8)

$-8 = \frac{2}{9} - 9 + c$, so $c = -8 + 9 - \frac{2}{9} = \frac{7}{9}$.

5. Write f(x)

$f(x) = \frac{2}{9}x^2 - 9x^{4/3} + \frac{7}{9}$.

Final answer

$(a)\ a = \frac{4}{9}$.

$(b)\ f(x) = \frac{2}{9}x^2 - 9x^{4/3} + \frac{7}{9}$.

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Question 1

4 marks

Integration

1. Find

$$\int \left(\frac{8x^3}{5} - \frac{2}{3x^4} - 1 \right) \mathrm{d}x$$

giving each term in simplest form.

(4)

(Total 4 marks)

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Worked Solution - Question 1

Integration

Topic group

1. Rewrite using powers

$\frac{8x^3}{5} - \frac{2}{3x^4} - 1 = \frac{8}{5}x^3 - \frac{2}{3}x^{-4} - 1.$

2. Integrate term by term

$\int \frac{8}{5}x^3 dx = \frac{2}{5}x^4$, $\int -\frac{2}{3}x^{-4} dx = \frac{2}{9}x^{-3}$, and $\int -1 dx = -x.$

3. Write in simplest form

Therefore the integral is $\frac{2x^4}{5} + \frac{2}{9x^3} - x + c.$

Final answer

$\frac{2x^4}{5} + \frac{2}{9x^3} - x + c.$

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Question 6

11 marks

Integration

6. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $f(x)$, simplifying your answer.

(7)

(Total 11 marks)

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Worked Solution - Question 6

Integration

Topic group

1. Simplify f'(x)

$$f'(x)=\frac{(x+3)^2}{x\sqrt{x}}=\frac{x^2+6x+9}{x^{3/2}}=x^{1/2}+6x^{-1/2}+9x^{-3/2}.$$

2. Find the gradient at P

At $x=4$, $f'(4)=2+3+\frac{9}{8}=\frac{49}{8}$.

3. Find the tangent equation

Using $P(4,20)$, $y-20=\frac{49}{8}(x-4)$. Multiplying out gives $49x-8y-36=0$.

4. Integrate f'(x)

$$f(x)=\int\left(x^{1/2}+6x^{-1/2}+9x^{-3/2}\right)dx=\frac{2}{3}x^{3/2}+12x^{1/2}-18x^{-1/2}+c.$$

5. Use P(4,20)

$20=\frac{2}{3}(8)+12(2)-\frac{18}{2}+c=\frac{61}{3}+c$, so $c=-\frac{1}{3}$.

6. Write f(x)

$$f(x)=\frac{2}{3}x\sqrt{x}+12\sqrt{x}-\frac{18}{\sqrt{x}}-\frac{1}{3}.$$

Final answer

$(a)(i)\ \frac{49}{8}.\quad (a)(ii)\ 49x-8y-36=0.$

$(b)\ f(x)=\frac{2}{3}x\sqrt{x}+12\sqrt{x}-\frac{18}{\sqrt{x}}-\frac{1}{3}.$

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Question 1

4 marks

Integration

1. Find

$$\int \left(10x^5 + 6x^3 - \frac{3}{x^2}\right) \mathrm{d}x$$

giving your answer in simplest form.

(4)

(Total 4 marks)

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Worked Solution - Question 1

Integration

Topic group

1. Rewrite the negative power

$$\int \left(10x^5 + 6x^3 - \frac{3}{x^2}\right)dx = \int (10x^5 + 6x^3 - 3x^{-2})dx.$$

2. Integrate term by term

$\int 10x^5 dx = \frac{10x^6}{6} = \frac{5}{3}x^6$, $\int 6x^3 dx = \frac{6x^4}{4} = \frac{3}{2}x^4$, and $\int -3x^{-2} dx = 3x^{-1}$.

3. Add the constant

Therefore the integral is $\frac{5}{3}x^6 + \frac{3}{2}x^4 + \frac{3}{x} + c$.

Final answer

$$\frac{5}{3}x^6 + \frac{3}{2}x^4 + \frac{3}{x} + c.$$

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Question 7

9 marks

Integration

7. The curve C has equation $y = \mathrm{f}(x),\ x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{2}{\sqrt{x}} + \dfrac{A}{x^2} + 3$, where A is a constant
- $\mathrm{f}''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $\mathrm{f}(x) = 8\sqrt{3}$, when $x = 12$

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(5)

(Total 9 marks)

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Worked Solution - Question 7

Integration

Topic group

1. Differentiate f'(x)

$f'(x) = 2x^{-1/2} + Ax^{-2} + 3$, so $f''(x) = -x^{-3/2} - 2Ax^{-3}$.

2. Use f''(4) = 0

$-4^{-3/2} - 2A(4^{-3}) = 0$, so $-\frac{1}{8} - \frac{A}{32} = 0$ and $A = -4$.

3. Substitute A into f'(x)

$f'(x) = \frac{2}{\sqrt{x}} - \frac{4}{x^2} + 3$.

4. Integrate to find f(x)

$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x + c$.

5. Use f(12) = 8sqrt3

$f(12) = 8\sqrt{3} + \frac{1}{3} + 36 + c = 8\sqrt{3}$, so $c = -\frac{109}{3}$.

Final answer

$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x - \frac{109}{3}$.

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Question 2

7 marks

Integration

2. Given that

$$(x - 5)(2x + 1)(x + 3) \equiv ax^3 + bx^2 - 32x - 15$$

where a and b are constants,

(a) find the value of a and the value of b.

(2)

(b) Hence find

$$\int \frac{(x - 5)(2x + 1)(x + 3)}{5\sqrt{x}} \, dx$$

writing each term in simplest form.

(5)

(Total for Question 2 is 7 marks)

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Worked Solution - Question 2

Integration

Topic group

1. Expand the identity

$(2x+1)(x+3) = 2x^2+7x+3$, so
$(x-5)(2x^2+7x+3) = 2x^3-3x^2-32x-15$.

2. Read off a and b

Comparing with $ax^3+bx^2-32x-15$ gives $a=2$ and $b=-3$.

3. Rewrite the integrand

$$\frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}} = \frac{1}{5}(2x^{5/2}-3x^{3/2}-32x^{1/2}-15x^{-1/2}).$$

4. Integrate term by term

$\int \frac{1}{5}(2x^{5/2})dx = \frac{4}{35}x^{7/2}$ and $\int \frac{1}{5}(-3x^{3/2})dx = -\frac{6}{25}x^{5/2}$.

5. Complete the integral

$\int \frac{1}{5}(-32x^{1/2})dx = -\frac{64}{15}x^{3/2}$ and $\int \frac{1}{5}(-15x^{-1/2})dx = -6x^{1/2}$, then add c.

Final answer

$(a)\ a=2,\ b=-3.$

$(b)\ \frac{4}{35}x^{7/2}-\frac{6}{25}x^{5/2}-\frac{64}{15}x^{3/2}-6\sqrt{x}+c.$

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Question 5

9 marks

Integration

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x), x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

(Total for Question 5 is 9 marks)

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Worked Solution - Question 5

Integration

Topic group

1. Integrate f'(x)

$f'(x) = 12x^{-1/2} + \frac{x}{3} - 4$, so $f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x + c$.

2. Use P(9, 8)

$8 = 24\sqrt{9} + \frac{9^2}{6} - 4(9) + c = 72 + \frac{81}{6} - 36 + c$.

3. Find c

$72 + \frac{81}{6} - 36 = \frac{99}{2}$, so $c = 8 - \frac{99}{2} = -\frac{83}{2}$.

4. Find the tangent gradient at P

$f'(9) = \frac{12}{3} + \frac{9}{3} - 4 = 3$.

5. Find the normal gradient

The normal gradient is the negative reciprocal of 3, so it is $-\frac{1}{3}$.

6. Find the y-intercept

The normal through $(9, 8)$ is $y - 8 = -\frac{1}{3}(x - 9)$, so $y = -\frac{1}{3}x + 11$. It crosses the y-axis at $(0, 11)$.

Final answer

(a) $f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x - \frac{83}{2}$.

(b) $(0, 11)$.

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Question 3

5 marks

Integration

3. Find

$$\int \frac{4x^5 + 3}{2x^2} \, dx$$

giving your answer in simplest form.

(5)

(Total for Question 3 is 5 marks)

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Worked Solution - Question 3

Integration

Topic group

1. Split the fraction

$$\frac{4x^5+3}{2x^2} = 2x^3 + \frac{3}{2}x^{-2}.$$

2. Integrate the first term

$$\int 2x^3\,dx = \frac{x^4}{2}.$$

3. Integrate the second term

$$\int \frac{3}{2}x^{-2}\,dx = -\frac{3}{2}x^{-1}.$$

4. Write in simplest form

The integral is $\frac{x^4}{2} - \frac{3}{2x} + c$.

Final answer

$$\frac{x^4}{2} - \frac{3}{2x} + c.$$

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Question 11

8 marks

Integration

11. A curve C has equation $y = f(x), \quad x > 0$

Given that

- $f''(x) = 4x + \frac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

(Total for Question 11 is 8 marks)

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Worked Solution - Question 11

Integration

Topic group

1. Find point P

Since P has x-coordinate 4 and lies on the tangent $y = 3x + 4$, $P = (4, 16)$.

2. Find the normal

The tangent gradient is 3, so the normal gradient is $-\frac{1}{3}$. Hence the normal is $y - 16 = -\frac{1}{3}(x - 4)$.

3. Integrate f''(x)

$f''(x) = 4x + x^{-1/2}$, so $f'(x) = 2x^2 + 2\sqrt{x} + A$.

4. Use the tangent gradient

At $x = 4$, $f'(4) = 3$. Thus $2(16) + 2(2) + A = 3$, so $A = -33$.

5. Integrate again

$f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + B$.

6. Use P(4, 16)

$16 = \frac{2}{3}(64) + \frac{4}{3}(8) - 33(4) + B = \frac{160}{3} - 132 + B$.

7. Find B

$B = \frac{284}{3}$, so $f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}$.

Final answer

$(a)\ y - 16 = -\frac{1}{3}(x - 4).$

$(b)\ f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + \frac{284}{3}.$

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Question 8

10 marks

Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

The curve with equation $y = \mathrm{f}(x)$ also passes through the point $P(4, 12)$

Given that

$$\mathrm{f}'(x) = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

(b) find $\mathrm{f}(x)$ giving the coefficients in simplest form.

(5)

(Total for Question 8 is 10 marks)

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Worked Solution - Question 8

Integration

Topic group

1. Differentiate the curve

For $y = \frac{1}{4}x^3 - 8x^{-1/2}$, $\frac{dy}{dx} = \frac{3}{4}x^2 + 4x^{-3/2}$.

2. Find the gradient at P

At $x = 4$, $\frac{dy}{dx} = \frac{3}{4}(16) + 4(4^{-3/2}) = 12 + \frac{1}{2} = \frac{25}{2}$.

3. Find the tangent

Using $P(4, 12)$, $y - 12 = \frac{25}{2}(x - 4)$.

4. Write in integer form

Multiplying by 2 gives $2y - 24 = 25x - 100$, hence $25x - 2y - 76 = 0$.

5. Integrate f'(x)

$f'(x) = \frac{1}{4}x^3 - 8x^{-1/2}$ gives $f(x) = \frac{x^4}{16} - 16\sqrt{x} + c$.

6. Use P(4, 12)

$12 = \frac{4^4}{16} - 16\sqrt{4} + c = 16 - 32 + c$, so $c = 28$.

Final answer

$(a)\ 25x - 2y - 76 = 0.$

$(b)\ f(x) = \frac{x^4}{16} - 16\sqrt{x} + 28.$

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Question 7

10 marks

Integration

7. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $f(x)$.

(6)

(Total for Question 7 is 10 marks)

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Worked Solution - Question 7

Integration

Topic group

1. Simplify f prime

$$f'(x)=\frac{4x^2+10-7x^{1/2}}{4x^{1/2}}=x^{3/2}+\frac{5}{2}x^{-1/2}-\frac{7}{4}.$$

2. Find the gradient at P

At $x=4$, $f'(4)=4^{3/2}+\frac{5}{2}(4^{-1/2})-\frac{7}{4}=8+\frac{5}{4}-\frac{7}{4}=\frac{15}{2}$.

3. Find the normal gradient

The normal gradient is the negative reciprocal, so $m_n=-\frac{2}{15}$.

4. Use point P

Using $P(4,-1)$, $y+1=-\frac{2}{15}(x-4)$.

5. Write the normal in integer form

$15y+15=-2x+8$, hence $2x+15y+7=0$.

6. Integrate f prime

$$f(x)=\int\left(x^{3/2}+\frac{5}{2}x^{-1/2}-\frac{7}{4}\right)dx=\frac{2}{5}x^{5/2}+5x^{1/2}-\frac{7}{4}x+c.$$

7. Use P to find c

$-1=\frac{2}{5}(4^{5/2})+5(4^{1/2})-\frac{7}{4}(4)+c=\frac{64}{5}+10-7+c$, so $c=-\frac{84}{5}$.

Final answer

$(a)(i)\ \dfrac{15}{2}.\quad (a)(ii)\ 2x+15y+7=0.$

$(b)\ f(x)=\dfrac{2}{5}x^{5/2}+5x^{1/2}-\dfrac{7}{4}x-\dfrac{84}{5}.$

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Question 1

5 marks

Integration

1. Find

$$\int (2x - 5)(3x + 2)(2x + 5)\mathrm{d}x$$

writing your answer in simplest form.

(5)

(Total for Question 1 is 5 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 1

Integration

Topic group

1. Expand two brackets first

$(2x-5)(3x+2) = 6x^2 - 11x - 10.$

2. Multiply by the third bracket

$(6x^2 - 11x - 10)(2x+5) = 12x^3 + 8x^2 - 75x - 50.$

3. Integrate term by term

$$\int (12x^3 + 8x^2 - 75x - 50)\,dx = 3x^4 + \frac{8}{3}x^3 - \frac{75}{2}x^2 - 50x + c.$$

Final answer

$$3x^4 + \frac{8}{3}x^3 - \frac{75}{2}x^2 - 50x + c.$$

WMA11/01 JANUARY 2024

Question 10

8 marks

Integration

10. **In this question you must show all stages of your working.**

The curve C has equation $y = f(x), x > 0$

Given that

- the point $P\left(2, 8\sqrt{2}\right)$ lies on C
- $f'(x) = 4\sqrt{x^3} + \frac{k}{x^2}$ where k is a constant
- $f''(x) = 0$ at P

(a) find the exact value of k,

(4)

(b) find $f(x)$, giving your answer in simplest form.

(4)

First released on AP - Edexcel Discord
https://sites.google.com/view/ap-edexcel

(Total for Question 10 is 8 marks)

WMA11/01 JANUARY 2024

Worked Solution - Question 10

Integration

Topic group

1. Differentiate f prime

$f'(x) = 4x^{3/2} + kx^{-2}$, so $f''(x) = 6x^{1/2} - 2kx^{-3}$.

2. Use f double prime equals zero at P

At P, $x = 2$ and $f''(2) = 0$, so $6\sqrt{2} - 2k(2^{-3}) = 0$.

3. Find k

$6\sqrt{2} - \dfrac{k}{4} = 0$, hence $k = 24\sqrt{2}$.

4. Integrate f prime

$$f(x) = \int \left(4x^{3/2} + 24\sqrt{2}x^{-2}\right)dx = \frac{8}{5}x^{5/2} - 24\sqrt{2}x^{-1} + c.$$

5. Use the point P

$P(2, 8\sqrt{2})$ lies on C, so $8\sqrt{2} = \dfrac{8}{5}(2^{5/2}) - \dfrac{24\sqrt{2}}{2} + c$.

6. Find c

$8\sqrt{2} = \dfrac{32\sqrt{2}}{5} - 12\sqrt{2} + c$, so $c = \dfrac{68\sqrt{2}}{5}$.

Final answer

$(a)\ k = 24\sqrt{2}.$

$(b)\ f(x) = \dfrac{8}{5}x^{5/2} - \dfrac{24\sqrt{2}}{x} + \dfrac{68\sqrt{2}}{5}.$

WMA11/01 MAY/JUNE 2024

Question 1

3 marks

Integration

1. Find

$$\int \left(10x^4 - \frac{3}{2x^3} - 7\right) \mathrm{d}x$$

giving each term in simplest form.

(3)

(Total for Question 1 is 3 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 1

Integration

Topic group

1. Rewrite the fraction

$10x^4 - \dfrac{3}{2x^3} - 7 = 10x^4 - \dfrac{3}{2}x^{-3} - 7.$

2. Integrate term by term

$\int 10x^4\,dx = 2x^5$, $\int -\dfrac{3}{2}x^{-3}\,dx = \dfrac{3}{4}x^{-2}$, and $\int -7\,dx = -7x$.

3. Write in simplest form

$2x^5 + \dfrac{3}{4x^2} - 7x + c.$

Final answer

$2x^5 + \dfrac{3}{4x^2} - 7x + c.$

WMA11/01 MAY/JUNE 2024

Question 10

10 marks

Integration

10. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(6)

(Total for Question 10 is 10 marks)

WMA11/01 MAY/JUNE 2024

Worked Solution - Question 10

Integration

Topic group

1. Expand the numerator

$(2x-1)(3x+2) = 6x^2 + x - 2$.

2. Simplify f prime

$f'(x) = 6x - \dfrac{6x^2 + x - 2}{2\sqrt{x}} = 6x - 3x^{3/2} - \dfrac{1}{2}x^{1/2} + x^{-1/2}$.

3. Find the tangent gradient at P

$f'(4) = 24 - 3(8) - \dfrac{1}{2}(2) + \dfrac{1}{2} = -\dfrac{1}{2}$.

4. Use the normal gradient

The normal gradient is the negative reciprocal of $-\dfrac{1}{2}$, so it is 2.

5. Find the normal equation

Through $P(4, 12)$, $y - 12 = 2(x - 4)$, hence $y = 2x + 4$.

6. Integrate f prime

$f(x) = 3x^2 - \dfrac{6}{5}x^{5/2} - \dfrac{1}{3}x^{3/2} + 2x^{1/2} + c$.

7. Use P to find c

$12 = 3(4)^2 - \dfrac{6}{5}(4^{5/2}) - \dfrac{1}{3}(4^{3/2}) + 2(4^{1/2}) + c$.

8. Simplify c

$12 = 48 - \dfrac{192}{5} - \dfrac{8}{3} + 4 + c = \dfrac{164}{15} + c$, so $c = \dfrac{16}{15}$.

Final answer

$(a)\ y = 2x + 4.$

$(b)\ f(x) = 3x^2 - \frac{6}{5}x^{5/2} - \frac{1}{3}x^{3/2} + 2x^{1/2} + \frac{16}{15}.$

WMA11/01 OCTOBER 2024

Question 3

9 marks

Integration

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(a) Find $\int f(x)\,dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

(Total for Question 3 is 9 marks)

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Worked Solution - Question 3

Integration

Topic group

1. Rewrite f(x)

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} = \frac{x^2 + 10x + 25}{x^{1/2}} = x^{3/2} + 10x^{1/2} + 25x^{-1/2}.$$

2. Integrate

$$\int f(x)\,dx = \frac{2}{5}x^{5/2} + \frac{20}{3}x^{3/2} + 50x^{1/2} + c.$$

3. Differentiate f(x)

$$f'(x) = \frac{3}{2}x^{1/2} + 5x^{-1/2} - \frac{25}{2}x^{-3/2}.$$

4. Set f prime equal to zero

$$\frac{3}{2}x^{1/2} + 5x^{-1/2} - \frac{25}{2}x^{-3/2} = 0.$$

5. Clear powers of x

Multiplying by $2x^{3/2}$ gives $3x^2 + 10x - 25 = 0$.

6. Use x greater than zero

$3x^2 + 10x - 25 = (3x - 5)(x + 5)$, so $x = \frac{5}{3}$ or $x = -5$. Since $x > 0$, $x = \frac{5}{3}$.

Final answer

(a) $\frac{2}{5}x^{5/2} + \frac{20}{3}x^{3/2} + 50x^{1/2} + c.$

(b) $x = \frac{5}{3}.$

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Question 8

7 marks

Integration

8. A curve C has equation $y = \mathrm{f}(x)$.

The point P with x coordinate 3 lies on C

Given

- $\mathrm{f}'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $\mathrm{f}(x)$.

(4)

(Total for Question 8 is 7 marks)

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Worked Solution - Question 8

Integration

Topic group

1. Find the y-coordinate of P

The normal is $y = -\frac{1}{24}x + 5$. At $x = 3$, $y = -\frac{3}{24} + 5 = \frac{39}{8}$.

2. Use the normal gradient

The normal gradient is $-\frac{1}{24}$, so the tangent gradient is 24.

3. Use f prime at x equals 3

$f'(3) = 4(3)^2 + 3k + 3 = 39 + 3k$.

4. Find k

$39 + 3k = 24$, so $k = -5$.

5. Integrate f prime

With $k = -5$, $f'(x) = 4x^2 - 5x + 3$, so $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x + c$.

6. Use point P

$\frac{39}{8} = \frac{4}{3}(27) - \frac{5}{2}(9) + 3(3) + c = \frac{45}{2} + c$.

7. Find c

$c = \frac{39}{8} - \frac{45}{2} = -\frac{141}{8}$.

Final answer

$k = -5$ and $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x - \frac{141}{8}$.

WMA11/01 JANUARY 2025

Question 1

4 marks

Integration

1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) \mathrm{d}x$$

giving your answer in simplest form.

(4)

(Total for Question 1 is 4 marks)

WMA11/01 JANUARY 2025

Worked Solution - Question 1

Integration

Topic group

1. Rewrite with powers

$$8x^3 - 6\sqrt{x} - \frac{2}{5x^3} = 8x^3 - 6x^{1/2} - \frac{2}{5}x^{-3}.$$

2. Integrate term by term

$\int 8x^3\,dx = 2x^4$, $\int -6x^{1/2}\,dx = -4x^{3/2}$, and $\int -\frac{2}{5}x^{-3}\,dx = \frac{1}{5}x^{-2}$.

3. Write in simplest form

$$2x^4 - 4x^{3/2} + \frac{1}{5x^2} + c.$$

Final answer

$$2x^4 - 4x^{3/2} + \frac{1}{5x^2} + c.$$

Question 6

10 marks

Integration

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $f'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $f(x)$

(7)

Curve C is transformed to the curve with equation $y = f(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

(Total for Question 6 is 10 marks)

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Worked Solution - Question 6

Integration

Topic group

1. Use the gradient at P

At $x = 4$, $f'(4) = 7$. Since $f'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, we get $\dfrac{32 + 4a + b}{8} = 7$.

2. Show the required equation

$32 + 4a + b = 56$, so $4a + b = 24$.

3. Solve for a and b

Together with $a + b = -9$, subtracting gives $3a = 33$, so $a = 11$ and $b = -20$.

4. Rewrite f prime

$$f'(x) = \frac{2x^2 + 11x - 20}{4\sqrt{x}} = \frac{1}{2}x^{3/2} + \frac{11}{4}x^{1/2} - 5x^{-1/2}.$$

5. Integrate

$$f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} + c.$$

6. Use P to find c

$$-5 = \frac{1}{5}(4^{5/2}) + \frac{11}{6}(4^{3/2}) - 10(4^{1/2}) + c = \frac{16}{15} + c.$$

7. State f(x)

$c = -\dfrac{91}{15}$, so $f(x) = \dfrac{1}{5}x^{5/2} + \dfrac{11}{6}x^{3/2} - 10x^{1/2} - \dfrac{91}{15}$.

8. Transform P

$y = f(x - 3)$ translates the curve 3 units to the right, so $P(4, -5)$ becomes $Q(7, -5)$.

Final answer

$(b)\ f(x) = \dfrac{1}{5}x^{5/2} + \dfrac{11}{6}x^{3/2} - 10x^{1/2} - \dfrac{91}{15}.$

$(c)\ Q = (7, -5).$

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Question 3

5 marks

Integration

3. Find

$$\int \frac{(x+3)^2}{3\sqrt{x}}\,\mathrm{d}x$$

writing your answer in simplest form.

(5)

(Total for Question 3 is 5 marks)

WMA11/01 MAY/JUNE 2025

Worked Solution - Question 3

Integration

Topic group

1. Expand the numerator

$(x+3)^2 = x^2 + 6x + 9.$

2. Divide by the denominator

$$\frac{(x+3)^2}{3\sqrt{x}} = \frac{x^2+6x+9}{3x^{1/2}} = \frac{1}{3}x^{3/2} + 2x^{1/2} + 3x^{-1/2}.$$

3. Integrate term by term

$$\int \left(\frac{1}{3}x^{3/2} + 2x^{1/2} + 3x^{-1/2}\right) dx = \frac{1}{3}\cdot\frac{2}{5}x^{5/2} + 2\cdot\frac{2}{3}x^{3/2} + 3\cdot 2x^{1/2} + c.$$

4. Simplify

$$\frac{2}{15}x^{5/2} + \frac{4}{3}x^{3/2} + 6x^{1/2} + c.$$

Final answer

$$\frac{2}{15}x^{5/2} + \frac{4}{3}x^{3/2} + 6x^{1/2} + c.$$

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Question 8

11 marks

Integration

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve has equation $y = \mathrm{f}(x),\ x > 0$

The point $P(4, 12)$ lies on the curve.

Given that

- $\mathrm{f}'(x) = 3\sqrt{x} + kx^2$ where k is a constant
- the equation of the tangent to the curve at P has equation $y = 10x + c$ where c is a constant

(a) (i) show that $k = \frac{1}{4}$

(ii) find the value of c

(4)

(b) Hence find the value of $\mathrm{f}''(x)$ at P.

(3)

(c) Find $\mathrm{f}(x)$.

(4)

(Total for Question 8 is 11 marks)

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Worked Solution - Question 8

Integration

Topic group

1. Use the tangent gradient

The tangent at P has equation $y = 10x + c$, so the gradient at P is 10.

2. Show k

$f'(4) = 3\sqrt{4} + k(4)^2 = 6 + 16k = 10$, so $16k = 4$ and $k = \frac{1}{4}$.

3. Find c

Substitute $P(4, 12)$ into $y = 10x + c$: $12 = 40 + c$, so $c = -28$.

4. Differentiate f prime

With $k = \frac{1}{4}$, $f'(x) = 3x^{1/2} + \frac{1}{4}x^2$.

5. Find f double prime

$f''(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x$.

6. Evaluate at P

$f''(4) = \frac{3}{2} \cdot \frac{1}{2} + 2 = \frac{3}{4} + 2 = \frac{11}{4}$.

7. Integrate f prime

$f(x) = \int \left(3x^{1/2} + \frac{1}{4}x^2\right) dx = 2x^{3/2} + \frac{1}{12}x^3 + d$.

8. Use P to find d

$12 = 2(4^{3/2}) + \frac{1}{12}(4^3) + d = 16 + \frac{16}{3} + d$.

9. State f(x)

$d = -\dfrac{28}{3}$, so $f(x) = 2x^{3/2} + \dfrac{1}{12}x^3 - \dfrac{28}{3}$.

Final answer

$(a)(i)\ k = \dfrac{1}{4}.$ $(a)(ii)\ c = -28.$

$(b)\ f''(P) = \dfrac{11}{4}.$

$(c)\ f(x) = 2x^{3/2} + \dfrac{1}{12}x^3 - \dfrac{28}{3}.$

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Question 2

5 marks

Integration

2. Find

$$\int (x-3)^2(2x+5)\,\mathrm{d}x$$

writing the answer in simplest form.

(5)

(Total for Question 2 is 5 marks)

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Worked Solution - Question 2

Integration

Topic group

1. Expand the square

$(x-3)^2 = x^2 - 6x + 9.$

2. Multiply out

$(x^2 - 6x + 9)(2x + 5) = 2x^3 + 5x^2 - 12x^2 - 30x + 18x + 45.$

3. Collect terms

$(x-3)^2(2x+5) = 2x^3 - 7x^2 - 12x + 45.$

4. Integrate term by term

$$\int (2x^3 - 7x^2 - 12x + 45)\,dx = \frac{1}{2}x^4 - \frac{7}{3}x^3 - 6x^2 + 45x + c.$$

Final answer

$$\frac{1}{2}x^4 - \frac{7}{3}x^3 - 6x^2 + 45x + c.$$

WMA11/01 OCTOBER 2025

Question 8

8 marks

Integration

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve has equation $y = f(x), x > 0$

Given that

- $f'(x) = 2x + \frac{8}{x^2} + k$, where k is a constant
- the equation of the tangent to the curve at $x = \sqrt{2}$ is $y = 5x - 3\sqrt{2}$

(a) find the exact value of k.

(2)

(b) Find an equation of the normal to the curve at $x = \sqrt{2}$

(2)

(c) Find $f(x)$, writing your answer in simplest form.

(4)

(Total for Question 8 is 8 marks)

WMA11/01 OCTOBER 2025

Worked Solution - Question 8

Integration

Topic group

1. Use tangent gradient

The tangent has equation $y = 5x - 3\sqrt{2}$, so the gradient at $x = \sqrt{2}$ is 5.

2. Find k

$f'(\sqrt{2}) = 2\sqrt{2} + \dfrac{8}{(\sqrt{2})^2} + k = 2\sqrt{2} + 4 + k = 5$, so $k = 1 - 2\sqrt{2}$.

3. Find the point on the curve

At $x = \sqrt{2}$, the tangent gives $y = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$.

4. Normal gradient

The tangent gradient is 5, so the normal gradient is $-\dfrac{1}{5}$.

5. Equation of the normal

$y - 2\sqrt{2} = -\dfrac{1}{5}(x - \sqrt{2})$, hence $y = -\dfrac{1}{5}x + \dfrac{11\sqrt{2}}{5}$.

6. Integrate f prime

$f(x) = \displaystyle\int \left(2x + \frac{8}{x^2} + k\right) dx = x^2 - \frac{8}{x} + kx + c$.

7. Substitute k

$f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + c$.

8. Use the point

$2\sqrt{2} = 2 - \dfrac{8}{\sqrt{2}} + (1 - 2\sqrt{2})\sqrt{2} + c = -2 - 3\sqrt{2} + c$.

9. Find c

$c = 2 + 5\sqrt{2}$, so $f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + 2 + 5\sqrt{2}$.

Final answer

$(a)\ k = 1 - 2\sqrt{2}.$

$(b)\ y = -\dfrac{1}{5}x + \dfrac{11\sqrt{2}}{5}.$

$(c)\ f(x) = x^2 - \dfrac{8}{x} + (1 - 2\sqrt{2})x + 2 + 5\sqrt{2}.$

WMA11/01 JANUARY 2026

Question 1

6 marks

Integration

1. $y = 4x^3 + \dfrac{3}{2\sqrt{x}} - 8 \qquad x > 0$

Find, in simplest form,

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) $\displaystyle\int y \, \mathrm{d}x$

(3)

(Total for Question 1 is 6 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 1

Integration

Topic group

1. Rewrite with powers

$$y = 4x^3 + \frac{3}{2}x^{-1/2} - 8.$$

2. Differentiate

$$\frac{dy}{dx} = 12x^2 + \frac{3}{2}\left(-\frac{1}{2}\right)x^{-3/2}.$$

3. Simplify derivative

$$\frac{dy}{dx} = 12x^2 - \frac{3}{4}x^{-3/2} = 12x^2 - \frac{3}{4x^{3/2}}.$$

4. Integrate

$$\int y\,dx = \int \left(4x^3 + \frac{3}{2}x^{-1/2} - 8\right)dx.$$

5. Simplify integral

$$\int y\,dx = x^4 + 3x^{1/2} - 8x + c = x^4 + 3\sqrt{x} - 8x + c.$$

Final answer

(a) $\dfrac{dy}{dx} = 12x^2 - \dfrac{3}{4x^{3/2}}.$

(b) $\int y\,dx = x^4 + 3\sqrt{x} - 8x + c.$

WMA11/01 JANUARY 2026

Question 8

12 marks

Integration

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x), \quad x > 0$

A point P lies on C.

Given that

- $\mathrm{f}'(x) = 4x^2 + \dfrac{6}{x^2} - 4$
- the equation of the tangent to C at P is $y = 10x - 6\sqrt{3}$

(a) (i) verify that $\sqrt{3}$ is a possible x coordinate of P,

(ii) find, using algebra, the other possible x coordinate of P.

(6)

Given that the x coordinate of P is $\sqrt{3}$

(b) find an equation of the normal to C at P.

(2)

(c) Find $\mathrm{f}(x)$, writing your answer in simplest form.
You must show each stage of your working.

(4)

(Total for Question 8 is 12 marks)

WMA11/01 JANUARY 2026

Worked Solution - Question 8

Integration

Topic group

1. Verify x equals root 3

$f'(\sqrt{3}) = 4(\sqrt{3})^2 + \dfrac{6}{(\sqrt{3})^2} - 4 = 12 + 2 - 4 = 10$, which matches the tangent gradient.

2. Find all possible x-coordinates

Set $4x^2 + \dfrac{6}{x^2} - 4 = 10$.

3. Form a quadratic in x squared

Multiplying by x^2 gives $4x^4 - 14x^2 + 6 = 0$, so $2x^4 - 7x^2 + 3 = 0$.

4. Factor

$2x^4 - 7x^2 + 3 = (2x^2 - 1)(x^2 - 3) = 0$.

5. Use x greater than zero

$x = \sqrt{3}$ or $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$. The other possible x-coordinate is $\dfrac{\sqrt{2}}{2}$.

6. Find the point P

Given $x_P = \sqrt{3}$, the tangent gives $y_P = 10\sqrt{3} - 6\sqrt{3} = 4\sqrt{3}$.

7. Normal equation

The normal gradient is $-\dfrac{1}{10}$, so $y - 4\sqrt{3} = -\dfrac{1}{10}(x - \sqrt{3})$.

8. Integrate f prime

$f(x) = \displaystyle\int \left(4x^2 + \frac{6}{x^2} - 4\right) dx = \frac{4}{3}x^3 - \frac{6}{x} - 4x + c$.

9. Use P to find c

$$4\sqrt{3} = \frac{4}{3}(\sqrt{3})^3 - \frac{6}{\sqrt{3}} - 4\sqrt{3} + c = 4\sqrt{3} - 2\sqrt{3} - 4\sqrt{3} + c.$$

10. State f(x)

$c = 6\sqrt{3}$, so $f(x) = \frac{4}{3}x^3 - \frac{6}{x} - 4x + 6\sqrt{3}$.

Final answer

$(a)(i)$ verified.

$(a)(ii)\ x = \frac{\sqrt{2}}{2}.$

$(b)\ y - 4\sqrt{3} = -\frac{1}{10}(x - \sqrt{3}).$

$(c)\ f(x) = \frac{4}{3}x^3 - \frac{6}{x} - 4x + 6\sqrt{3}.$