ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Pure 1

*Complete strategy booklet collected from the individual topic notes, with hard-variant coverage preserved.*

Pure 1 Strategy Booklet · WMA11 · Complete

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Dr Eslam Ahmed · Cairo University Faculty of Engineering

# Booklet Map

This complete booklet collects all Pure 1 individual strategy notes into one printable file. The individual topic PDFs remain available separately.

COURSE EVIDENCE  Topics: 13. Topic-note pages: 145. The combined PDF also includes this cover and map.

ELITE IGCSE MATHEMATICS
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# Laws of Indices & Surds

*Read the trigger, name the repeated structure, then use one clean line of algebra before calculating.*

Laws of Indices & Surds · P1 · 01

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Dr Eslam Ahmed · Cairo University Faculty of Engineering

# The Map

These notes are built from the Pure 1 topic booklet, its worked answers, and the WMA11 website solution bank. The whole topic reduces to five moves. Each move has a trigger, a first line, a short model, and a practice page.

**01** POWER: Clean Powers

TRIGGER *simplify, divide powers, negative/fractional indices, form* $ax^n$

**02** INDEX: Name The Target

TRIGGER *given* $m = 2^n$, $y = a^x$, *express in terms of the named letter*

**03** SURD: Surd Clean-Up

TRIGGER *form* $a\sqrt{b}$, *collect like surds, expand surd brackets*

**04** PAIR: Conjugate And Compare

TRIGGER *rationalise,* $a + b\sqrt{k}$, *linear equations with surds, exact geometry*

**05** HIDE: Hidden Quadratic

TRIGGER *same exponential repeated,* $y^{2/3}$, $x^4$ *with* $x^2$

BANK EVIDENCE Local source: 36 questions across 26 problem pages and 55 answer pages. Website source: 13 checked WMA11 primary Indices & Surds questions, with step-by-step solution data.

FORMULA BANK

$$a^m a^n = a^{m+n}, \qquad \frac{a^m}{a^n} = a^{m-n}, \qquad (a^m)^n = a^{mn}, \qquad a^{-n} = \frac{1}{a^n}, \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m.$$

$$\sqrt{ab} = \sqrt{a}\,\sqrt{b}, \qquad (u+v)(u-v) = u^2 - v^2, \qquad \sqrt{a} + \sqrt{b} \neq \sqrt{a+b} \text{ in general.}$$

# 01 POWER: Clean Powers

TRIGGER *You see powers being multiplied, divided, raised again, or written in the form $ax^n$.*

BECOMES A bookkeeping question: coefficient with coefficient, exponent with exponent.

### FIRST LINE TO WRITE

$$\sqrt{x} = x^{1/2}, \qquad \frac{1}{x^n} = x^{-n}, \qquad \frac{x^a x^b}{x^c} = x^{a+b-c}.$$

### SIMPLEST STRATEGY

1. **P**ut roots and denominators into power form.
2. **O**perate on exponents: add, subtract, or multiply.
3. **W**ork coefficients separately from powers.
4. **E**xpress the result in the requested form.
5. **R**emove negative powers only if the question wants no negative indices.

### WORKED MODEL

$$\frac{y^{5/7} \times y^{1/7}}{y^{3/7}} = y^{5/7+1/7-3/7} = y^{3/7}.$$

$$\frac{3y^3(2x^4)^3}{4x^2y^4} = \frac{3y^3(8x^{12})}{4x^2y^4} = \frac{6x^{10}}{y}.$$

### HARD VARIANTS

1. If powers sit inside brackets, expand the power across every factor before collecting exponents.
2. If coefficients and powers are mixed, simplify coefficients separately and only then simplify the variable powers.
3. If the requested form bans negative or fractional powers, convert the final answer after the exponent work is complete.

### BOTTOM LINE

Do the exponent arithmetic in one line. Most mistakes here come from moving the coefficient with the exponent.

# Practice 01

SAME IDEA Clean every power before simplifying.

QUESTION

> Simplify each expression.
>
> $$(a)\quad \frac{x^4x^{-7}}{x^{-2}}, \qquad (b)\quad \left(\frac{1}{8}x^{3/2}\right)^{-4/3}, \qquad (c)\quad \frac{5z^2(3z^{-1})^2}{15z^4}.$$

YOUR SOLUTION

# 02 INDEX: Name The Target

TRIGGER *The question says "given $m = 2^n$" or "given $y = a^x$, express in terms of $m$ or $y$".*

BECOMES A replacement problem. Build the exact target power, then exchange it for the named letter.

### FIRST LINE TO WRITE

$$\text{Target: } 2^n = m. \qquad 2^{n+3} = 2^n \cdot 2^3.$$

### SIMPLEST STRATEGY

1. **I**solate the target power: $2^n$, $3^x$, or $a^x$.
2. **N**ew base: rewrite $16, 27, 81$ as powers of the target base.
3. **D**istribute the exponent if it contains a sum or difference.
4. **E**xchange powers: $2^{12n} = (2^n)^{12}$.
5. **X**change the target for the named letter and simplify.

### WORKED MODEL

Given $m = 2^n$, express in terms of $m$.

$$2^{n+3} = 2^n \cdot 2^3 = 8m.$$

$$16^{3n} = (2^4)^{3n} = 2^{12n} = (2^n)^{12} = m^{12}.$$

### HARD VARIANTS

1. For bases like $16, \frac{1}{8}, \sqrt{2}$, rewrite everything as powers of the named target base first.
2. If the target appears in a denominator or negative power, substitute first, then simplify the fraction.
3. If substitution creates an equation, solve it after the expression is in the named letter.

### BOTTOM LINE

Name the target first. If the target appears, the substitution is almost finished.

# Practice 02

SAME IDEA Use INDEX: isolate, new base, distribute, exchange, substitute.

QUESTION

> Given that $k = 3^x$, express each expression in terms of $k$.
>
> $$(a)\quad 3^{x+2}, \qquad (b)\quad 9^{x-1}, \qquad (c)\quad 27^{2x/3}.$$

YOUR SOLUTION

# 03 SURD: Surd Clean-Up

TRIGGER *The answer must be in the form $a\sqrt{b}$, or the question asks you to simplify surds.*

BECOMES A common-surd question. Pull out square factors, then collect only matching surds.

### FIRST LINE TO WRITE

$$\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}.$$

### SIMPLEST STRATEGY

1. **S**plit each root using the largest square factor.
2. **U**nify terms by rewriting them with the same root where possible.
3. **R**educe by collecting only like surds.
4. **D**eliver the final form, usually $a\sqrt{b}$.

### WORKED MODEL

$$4\sqrt{3} - \sqrt{12} + 4\sqrt{48} = 4\sqrt{3} - 2\sqrt{3} + 16\sqrt{3} = 18\sqrt{3}.$$
$$(2\sqrt{2})^3 + 3\sqrt{2} = 8(\sqrt{2})^3 + 3\sqrt{2} = 16\sqrt{2} + 3\sqrt{2} = 19\sqrt{2}.$$

### HARD VARIANTS

1. Expand surd brackets before collecting, especially $(a + b\sqrt{k})^2$.
2. Only like surds combine; $\sqrt{2}$ and $\sqrt{3}$ must stay separate.
3. For exact geometry or area answers, keep the surd exact until the final requested form.

### BOTTOM LINE

A surd expression becomes easy only after every root is simplified.

# Practice 03

SAME IDEA Simplify first, then collect like surds.

QUESTION

> Write each expression in the form $a\sqrt{b}$, where possible.
>
> $$(a)\quad \sqrt{50}-2\sqrt{8}+3\sqrt{18}, \qquad (b)\quad (3\sqrt{5})^2-4\sqrt{45}+7\sqrt{5}.$$

YOUR SOLUTION

# 04 PAIR: Conjugate And Compare

TRIGGER *There is a surd in a denominator, or the answer must be $a + b\sqrt{k}$, or a surd equation is linear in $x$.*

BECOMES A denominator-clearing question. Use the conjugate, then compare rational and surd parts if needed.

### FIRST LINE TO WRITE

$$\frac{1}{a+\sqrt{b}} \times \frac{a-\sqrt{b}}{a-\sqrt{b}}.$$

### SIMPLEST STRATEGY

1. **P**ick the matching partner: the surd itself or the conjugate.
2. **A**pply it to the numerator and denominator.
3. **I**dentify the difference of squares in the denominator.
4. **R**educe, then compare rational and surd parts if needed.

### WORKED MODEL

$$\frac{14}{4+\sqrt{2}} \times \frac{4-\sqrt{2}}{4-\sqrt{2}} = \frac{56-14\sqrt{2}}{16-2} = 4-\sqrt{2}.$$

$$x\sqrt{3} - 3 = x + \sqrt{3} \quad \Rightarrow \quad x(\sqrt{3}-1) = 3+\sqrt{3}.$$

$$x = \frac{3+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 3+2\sqrt{3}.$$

### HARD VARIANTS

1. For $a + b\sqrt{k}$ comparison, expand fully and match the rational part and the surd part separately.
2. For denominators with two terms, use the conjugate every time; do not try to cancel a single surd term.
3. For surd equations, rationalise only after isolating the factor multiplying $x$.

### BOTTOM LINE

The conjugate is not a trick; it is the fastest way to make a denominator rational.

# Practice 04

SAME IDEA Use the conjugate, then simplify.

QUESTION

$$(a)\quad \text{Rationalise } \frac{5}{2+\sqrt{3}}.$$

$$(b)\quad \text{Solve } x\sqrt{5} - 2 = x + \sqrt{5}, \text{ giving an exact answer.}$$

YOUR SOLUTION

# 05 HIDE: Hidden Quadratic

TRIGGER *You see $9^x, 3^x, 3^{2x}$, or $x^4, x^2$, or $y^{2/3}$ repeated.*

BECOMES A quadratic in one repeated expression. Name the repeated expression, solve, reject invalid values, then return to the original variable.

### FIRST LINE TO WRITE

$$p = 3^x \quad \Rightarrow \quad 9^x = p^2, \qquad 3^{x+2} = 9p, \qquad 3^{x-1} = \frac{p}{3}.$$

### SIMPLEST STRATEGY

1. **H**ighlight the repeated expression.
2. **I**ntroduce one new letter for it.
3. **D**o the quadratic in that new letter.
4. **E**xit back to the original variable, rejecting impossible values.

### WORKED MODEL

Let $p = 3^x$. Then $9^x = p^2$, $3^{x+2} = 9p$, and $3^{x-1} = p/3$.

$$3p^2 + 9p = 1 + \frac{p}{3} \quad \Rightarrow \quad 9p^2 + 26p - 3 = 0$$

$$(9p-1)(p+3) = 0.$$

Since $p = 3^x > 0$, $p = \frac{1}{9}$. Therefore $3^x = 3^{-2}$, so $x = -2$.

### HARD VARIANTS

1. For exponential quadratics, reject negative substitution values because $a^x > 0$.
2. For $x^4, x^2$ substitutions, return from $u = x^2$ by taking both square-root signs when $u > 0$.
3. For fractional powers such as $y^{2/3}$, name the repeated power and check that the returned values satisfy the original equation.

### BOTTOM LINE

The substitution is the question. After that, it is just a quadratic.

# Practice 05

SAME IDEA Name the repeated expression and return at the end.

QUESTION

Solve exactly.

$$(a) \quad 2(4^x) - 8(2^x) = 17(2^{x-1}) - 4.$$

Find the real values of $x$.

$$(b) \quad x^4 - 3x^2 - 40 = 0.$$

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| $x^a x^b$, $\frac{x^a}{x^b}$, $(x^a)^b$ | Write the operation above the expression: add for multiply, subtract for divide, multiply for a power. |
| $m = 2^n$, $y = a^x$ | Write the target power first, then rebuild the expression around it. |
| $\sqrt{48}$, $\sqrt{75}$, form $a\sqrt{b}$ | Pull out the largest square factor before collecting terms. |
| $\frac{1}{a+\sqrt{b}}$, $a + b\sqrt{k}$ | Multiply by the conjugate and compare rational/surd parts. |
| $9^x$ with $3^x$, or $x^4$ with $x^2$ | Let the repeated expression be one letter and solve a quadratic. |

COMMON RESCUE LINES

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}, \qquad \sqrt{a} - \sqrt{b} \neq \sqrt{a-b}.$$

$$(a + b\sqrt{5})^2 = a^2 + 5b^2 + 2ab\sqrt{5}.$$

$$Y = a^2 \quad \Rightarrow \quad a^4 - 49a^2 + 180 = 0 \text{ becomes } Y^2 - 49Y + 180 = 0.$$

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# Quadratics

*Turn every quadratic into a form you can read: roots, vertex, sign, or graph.*

Quadratics · P1 · 02

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** ROOT: Factor To Solve

TRIGGER *the quadratic is equal to zero*

**02** SQUARE: Complete The Square

TRIGGER *minimum, maximum, turning point, vertex*

**03** FORM: Formula And Discriminant

TRIGGER *not factorisable, exact roots, number of roots*

**04** GRAPH: Read The Parabola

TRIGGER *sketch, intersections, above or below the axis*

BANK EVIDENCE Local pages: 35 problem, 72 answer, 20 notes. Website primary entries: 16.

# 01 ROOT: Factor To Solve

TRIGGER *the quadratic is equal to zero*

BECOMES A product where one factor must be zero.

### FIRST LINE TO WRITE

$$ax^2 + bx + c = 0$$

### SIMPLEST STRATEGY

1 **R**earrange so one side is zero.

2 **O**pen or factor the expression.

3 **O**ne bracket equal to zero.

4 **T**ake the roots and check them.

### WORKED MODEL

$$x^2 - 5x + 6 = 0 \quad \Rightarrow \quad (x-2)(x-3) = 0, \quad x = 2, 3.$$

### HARD VARIANTS

1 Hidden quadratic: if you see $x^4, x^2$ or $a^{2x}, a^x$, let one repeated expression be a new letter first.

2 Parameter question: factor normally, then use the required number of roots or repeated-root condition.

3 After solving, return to the original variable and reject values that do not fit the substitution.

### BOTTOM LINE

A factorised quadratic is solved by setting each factor equal to zero.

# Practice 01

SAME IDEA Use ROOT: Factor To Solve.

QUESTION

> Solve $x^2 - 7x + 10 = 0$.

YOUR SOLUTION

# 02 SQUARE: Complete The Square

TRIGGER *minimum, maximum, turning point, vertex*

BECOMES A quadratic written as one square plus a constant.

FIRST LINE TO WRITE

$$x^2 + bx + c = (x + \frac{b}{2})^2 + c - \left(\frac{b}{2}\right)^2$$

SIMPLEST STRATEGY

1. **S**tart with the $x^2 + bx$ part.
2. **Q**uarter the coefficient effect by halving $b$.
3. **U**se the bracket square.
4. **A**djust the constant.
5. **R**ead the vertex.
6. **E**xtract roots if required.

WORKED MODEL

$$x^2 + 6x + 5 = (x+3)^2 - 9 + 5 = (x+3)^2 - 4.$$

HARD VARIANTS

1. If $a \neq 1$, factor $a$ from the $x^2, x$ terms before completing the square.
2. For range questions, read the vertex and decide whether the parabola opens up or down.
3. For tangent/minimum wording, complete the square first, then use the vertex condition.

BOTTOM LINE

The number inside the bracket gives the vertex after changing sign.

# Practice 02

SAME IDEA Use SQUARE: Complete The Square.

QUESTION

> Write $x^2 - 8x + 11$ in completed square form.

YOUR SOLUTION

# 03 FORM: Formula And Discriminant

TRIGGER *not factorisable, exact roots, number of roots*

BECOMES A discriminant check or formula substitution.

### FIRST LINE TO WRITE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### SIMPLEST STRATEGY

1. **F**ind $a, b, c$.
2. **O**btain $b^2 - 4ac$.
3. **R**oots use the formula.
4. **M**eaning: positive, zero, or negative discriminant.

### WORKED MODEL

$$2x^2 - 3x - 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}.$$

### HARD VARIANTS

1. When coefficients contain letters, keep $a, b, c$ symbolic and use the discriminant condition.
2. For equal roots use $b^2 - 4ac = 0$; for no real roots use $b^2 - 4ac < 0$.
3. Simplify exact roots only after the formula is correct.

### BOTTOM LINE

The discriminant tells you the type of roots before solving.

# Practice 03

SAME IDEA Use FORM: Formula And Discriminant.

QUESTION

> Use the formula to solve $3x^2 + 2x - 5 = 0$.

YOUR SOLUTION

# 04 GRAPH: Read The Parabola

TRIGGER *sketch, intersections, above or below the axis*

BECOMES A curve whose roots and vertex control the answer.

## FIRST LINE TO WRITE

$$y = a(x - r_1)(x - r_2)$$

## SIMPLEST STRATEGY

1. **G**et the roots or intercepts.
2. **R**ead the direction from $a$.
3. **A**xis of symmetry is halfway between roots.
4. **P**lot the vertex/intercepts.
5. **H**ighlight the required region.

## WORKED MODEL

$$y = (x - 1)(x - 5)$$

has roots $1, 5$, axis $x = 3$, and opens upwards.

## HARD VARIANTS

1. For inequalities, roots are boundaries; the graph decides inside or outside.
2. If the curve is transformed, find the new vertex/intercepts before shading.
3. Check open or closed endpoints when the sign is strict or inclusive.

## BOTTOM LINE

For inequalities, the graph tells you which interval is wanted.

# Practice 04

SAME IDEA Use GRAPH: Read The Parabola.

QUESTION

> Sketch $y = (x+2)(x-4)$ and state where $y < 0$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| the quadratic is equal to zero | $ax^2 + bx + c = 0$ |
| minimum, maximum, turning point, vertex | $x^2 + bx + c = (x + \frac{b}{2})^2 + c - \left(\frac{b}{2}\right)^2$ |
| not factorisable, exact roots, number of roots | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| sketch, intersections, above or below the axis | $y = a(x - r_1)(x - r_2)$ |

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# Simultaneous Equations

*Choose the method that removes one unknown fastest.*

Simultaneous Equations · P1 · 03

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** ELIM: Eliminate A Letter

TRIGGER *two linear equations*

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**02** SUBS: Substitute Directly

TRIGGER *one equation already gives $x =$ or $y =$*

---

**03** CURVE: Line Meets Curve

TRIGGER *linear and quadratic equation together*

---

**04** CHECK: Check The Pair

TRIGGER *answer is an ordered pair*

---

BANK EVIDENCE Local pages: 27 problem, 48 answer, 9 notes. Website primary entries: 3.

# 01 ELIM: Eliminate A Letter

TRIGGER *two linear equations*

BECOMES Two equations become one equation in one unknown.

### FIRST LINE TO WRITE

$$2x + 3y = 13$$
$$4x - y = 5$$

### SIMPLEST STRATEGY

1. **E**qualise one coefficient.
2. **L**ine up the equations.
3. **I**dentify add or subtract.
4. **M**ake one letter disappear.

### WORKED MODEL

$$2(2x + 3y = 13) \Rightarrow 4x + 6y = 26. \quad (4x + 6y) - (4x - y) = 21, \ y = 3.$$

### HARD VARIANTS

1. If coefficients are awkward, multiply both equations until one letter has matching coefficients.
2. If equations contain fractions, clear denominators before eliminating.
3. After solving, substitute into both original equations, not the simplified copies only.

### BOTTOM LINE

Elimination is safest when coefficients can be matched quickly.

# Practice 01

SAME IDEA Use ELIM: Eliminate A Letter.

QUESTION

> Solve $3x + 2y = 16,\ 2x - y = 3$.

YOUR SOLUTION

# 02 SUBS: Substitute Directly

TRIGGER *one equation already gives $x =$ or $y =$*

BECOMES One unknown is replaced by an expression.

### FIRST LINE TO WRITE

$$y = 2x + 1$$

### SIMPLEST STRATEGY

1. **S**elect the isolated equation.
2. **U**se it inside the other equation.
3. **B**uild one equation.
4. **S**ubstitute back.

### WORKED MODEL

$$y = 2x + 1, \quad x + y = 10 \Rightarrow x + 2x + 1 = 10, \; x = 3, \; y = 7.$$

### HARD VARIANTS

1. If nothing is isolated, isolate the variable with the smallest coefficient first.
2. When substitution creates a quadratic, keep both roots and find both coordinate pairs.
3. Reject any pair that fails one original equation.

### BOTTOM LINE

Substitute when a variable is already isolated.

# Practice 02

SAME IDEA Use SUBS: Substitute Directly.

QUESTION

Solve $y = 3x - 2,\ x + y = 14$.

YOUR SOLUTION

# 03 CURVE: Line Meets Curve

TRIGGER *linear and quadratic equation together*

BECOMES Substitute the line into the curve to get a quadratic.

### FIRST LINE TO WRITE

$$y = mx + c \quad \text{goes into} \quad y = x^2 + ax + b$$

### SIMPLEST STRATEGY

1 **C**hoose the line equation.

2 **U**se it in the curve.

3 **R**earrange to a quadratic.

4 **V**erify both coordinates.

5 **E**xpress both intersection points.

### WORKED MODEL

$$y = x + 2,\ y = x^2 \Rightarrow x^2 = x + 2 \Rightarrow (x-2)(x+1) = 0.$$

### HARD VARIANTS

1 Line-curve hard versions usually become a quadratic; keep both intersections.

2 For tangency, use the repeated-root condition or discriminant $= 0$.

3 Always turn each $x$-value back into a full coordinate.

### BOTTOM LINE

Intersections are found by making the two expressions for $y$ equal.

# Practice 03

SAME IDEA Use CURVE: Line Meets Curve.

QUESTION

> Find where $y = x + 6$ meets $y = x^2$.

YOUR SOLUTION

# 04 CHECK: Check The Pair

TRIGGER *answer is an ordered pair*

BECOMES The pair must satisfy both original equations.

### FIRST LINE TO WRITE

$$(x, y) = (\cdot, \cdot)$$

### SIMPLEST STRATEGY

1. **C**hoose the original equations.
2. **H**it both equations with the pair.
3. **E**valuate left and right sides.
4. **C**orrect if one equation fails.
5. **K**eep the ordered pair notation.

### WORKED MODEL

$$(3, 7): \quad 3 + 7 = 10, \quad 7 = 2(3) + 1.$$

### HARD VARIANTS

1. For parameter questions, check the pair before using it to find the parameter.
2. If a point is claimed to lie on two curves, test both equations separately.
3. State the ordered pair in the requested order.

### BOTTOM LINE

A simultaneous answer is not finished until both equations agree.

# Practice 04

SAME IDEA Use CHECK: Check The Pair.

QUESTION

> Check whether $(2, 5)$ solves $x + y = 7,\ 2x - y = -1$.

YOUR SOLUTION

# Quick Reference

## TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| two linear equations | $2x + 3y = 13$<br>$4x - y = 5$ |
| one equation already gives $x =$ or $y =$ | $y = 2x + 1$ |
| linear and quadratic equation together | $y = mx + c$ goes into $y = x^2 + ax + b$ |
| answer is an ordered pair | $(x, y) = (\cdot, \cdot)$ |

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# Inequalities

*Solve the boundary first, then decide which side survives.*

INEQUALITIES · P1 · 04

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DR ESLAM AHMED · CAIRO UNIVERSITY FACULTY OF ENGINEERING

# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** FLIP: Linear Inequality
TRIGGER *solve an inequality with brackets or fractions*

**02** SIGN: Quadratic Sign Chart
TRIGGER *quadratic inequality*

**03** SHADE: Graph Region
TRIGGER *region satisfies inequalities*

**04** RANGE: Read The Range
TRIGGER *find possible values*

BANK EVIDENCE Local pages: 34 problem, 59 answer, 12 notes. Website primary entries: 5.

# 01 FLIP: Linear Inequality

TRIGGER *solve an inequality with brackets or fractions*

BECOMES A normal equation with one sign rule.

### FIRST LINE TO WRITE

$$ax + b < c$$

### SIMPLEST STRATEGY

1. **F**ree brackets and fractions.
2. **L**ine up $x$-terms on one side.
3. **I**nvert the sign only when multiplying or dividing by a negative.
4. **P**resent the final interval.

### WORKED MODEL

$$-2x + 5 < 11 \Rightarrow -2x < 6 \Rightarrow x > -3.$$

### HARD VARIANTS

1. Clear fractions carefully; multiply by a negative only when its sign is known.
2. If the multiplier contains $x$, do not multiply across until you know its sign.
3. Write the final answer as an interval and check one test value.

### BOTTOM LINE

Only a negative multiply/divide flips the inequality sign.

# Practice 01

SAME IDEA Use FLIP: Linear Inequality.

QUESTION

Solve $7 - 3x \leq 16$.

YOUR SOLUTION

# 02 SIGN: Quadratic Sign Chart

TRIGGER *quadratic inequality*

BECOMES A number line split by the roots.

### FIRST LINE TO WRITE

$$ax^2 + bx + c = 0$$

### SIMPLEST STRATEGY

1. **S**olve the boundary equation.
2. **I**nsert roots on a number line.
3. **G**et the sign of each interval.
4. **N**ame the interval that matches the inequality.

### WORKED MODEL

$$(x-1)(x-4) < 0 \Rightarrow 1 < x < 4.$$

### HARD VARIANTS

1. Repeated roots only touch the axis; the sign may not change there.
2. If the leading coefficient is negative, the outside/inside pattern reverses.
3. For rational inequalities, include denominator zeros as forbidden boundaries.

### BOTTOM LINE

Quadratic inequalities are interval questions.

# Practice 02

SAME IDEA Use SIGN: Quadratic Sign Chart.

QUESTION

> Solve $(x+2)(x-5) \geq 0$.

YOUR SOLUTION

# 03 SHADE: Graph Region

TRIGGER *region satisfies inequalities*

BECOMES Draw boundaries, then shade the wanted side.

### FIRST LINE TO WRITE

$$y = mx + c$$

### SIMPLEST STRATEGY

1. **S**ketch the boundary line.
2. **H**andle solid or dashed boundary.
3. **A**pply a test point.
4. **D**arken the correct side.
5. **E**xpress the region clearly.

### WORKED MODEL

> For $y > 2x + 1$, test $(0, 0)$: $0 > 1$ false, so shade away from the origin.

### HARD VARIANTS

1. Use a dashed line for strict inequalities and a solid line for inclusive ones.
2. For several inequalities, shade one at a time and keep only the overlap.
3. If the boundary is curved, test a point after drawing the curve.

### BOTTOM LINE

A test point prevents shading the wrong side.

# Practice 03

SAME IDEA Use SHADE: Graph Region.

QUESTION

> Show the region $y \leq x + 3$ on a sketch.

YOUR SOLUTION

# 04 RANGE: Read The Range

TRIGGER *find possible values*

BECOMES Turn the expression into a boundary inequality.

### FIRST LINE TO WRITE

> boundary first, then direction

### SIMPLEST STRATEGY

1. **R**ewrite the condition.
2. **A**nalyse the boundary.
3. **N**ote excluded values.
4. **G**ive interval notation if needed.
5. **E**ndpoints checked.

### WORKED MODEL

$$x^2 \geq 0 \Rightarrow x^2 + 3 \geq 3.$$

### HARD VARIANTS

1. Complete the square or use a graph before writing the range.
2. Check whether endpoints are included from the domain and the inequality sign.
3. For transformed functions, move the range after moving the graph.

### BOTTOM LINE

Always ask whether the endpoint is included.

# Practice 04

SAME IDEA Use RANGE: Read The Range.

QUESTION

> Find the range of $x^2 - 4$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| solve an inequality with brackets or fractions | $ax + b < c$ |
| quadratic inequality | $ax^2 + bx + c = 0$ |
| region satisfies inequalities | $y = mx + c$ |
| find possible values | boundary first, then direction |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Polynomials

*Use factors, remainders, and roots as the language of polynomial questions.*

Polynomials · P1 · 05

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** REMA: Remainder Theorem

TRIGGER *remainder when divided by $x - a$*

**02** FACTOR: Factor Theorem

TRIGGER *show $x - a$ is a factor*

**03** DIVIDE: Polynomial Division

TRIGGER *divide by a linear or quadratic factor*

**04** ROOTS: Use Known Roots

TRIGGER *roots, factorise completely*

BANK EVIDENCE Local pages: 11 problem, 21 answer, 6 notes. Website primary entries: 4.

# 01 REMA: Remainder Theorem

TRIGGER *remainder when divided by $x - a$*

BECOMES Substitute $x = a$.

FIRST LINE TO WRITE

$$\text{remainder} = f(a)$$

SIMPLEST STRATEGY

1. **R**ead the divisor $x - a$.
2. **E**valuate $f(a)$.
3. **M**atch this value to the remainder.
4. **A**nswer without full division.

WORKED MODEL

If divisor is $x - 2$, remainder is $f(2)$.

HARD VARIANTS

1. For divisor $ax - b$, substitute $x = b/a$, not just $b$.
2. If the remainder is given as a condition, set $f(b/a)$ equal to that value.
3. Use substitution when long division would waste time.

BOTTOM LINE

For $x - a$, the remainder is $f(a)$.

# Practice 01

SAME IDEA Use REMA: Remainder Theorem.

QUESTION

> Find the remainder when $f(x) = x^3 - 4x + 1$ is divided by $x - 2$.

YOUR SOLUTION

# 02 FACTOR: Factor Theorem

TRIGGER *show $x - a$ is a factor*

BECOMES A zero remainder proves a factor.

### FIRST LINE TO WRITE

$$f(a) = 0$$

### SIMPLEST STRATEGY

1. **F**ind the value from the divisor.
2. **A**pply substitution.
3. **C**heck for zero.
4. **T**ell the conclusion.
5. **O**btain the remaining factor.
6. **R**oots follow from factors.

### WORKED MODEL

$$f(3) = 0 \Rightarrow (x-3) \text{ is a factor.}$$

### HARD VARIANTS

1. For $ax - b$, prove $f(b/a) = 0$ before claiming the factor.
2. If a parameter appears, use the zero-remainder condition to find it.
3. After proving one factor, divide to reduce the polynomial.

### BOTTOM LINE

A factor is just a divisor with zero remainder.

# Practice 02

SAME IDEA  Use FACTOR: Factor Theorem.

QUESTION

Show that $x - 1$ is a factor of $x^3 - 4x^2 + x + 2$.

YOUR SOLUTION

# 03 DIVIDE: Polynomial Division

TRIGGER *divide by a linear or quadratic factor*

BECOMES Long division or comparison of coefficients.

FIRST LINE TO WRITE

$$f(x) = (\text{divisor})(\text{quotient}) + \text{remainder}$$

SIMPLEST STRATEGY

1. **D**ecide quotient degree.
2. **I**nsert unknown coefficients.
3. **V**alue-match by expanding.
4. **I**dentify the quotient.
5. **D**eclare remainder.
6. **E**xtract roots if possible.

WORKED MODEL

$$x^3 + 2x^2 - x - 2 = (x+2)(x^2-1).$$

HARD VARIANTS

1. If the quotient has unknown coefficients, compare powers of $x$ after expanding.
2. Keep the remainder term until the final comparison is complete.
3. Use the reduced polynomial to find remaining roots.

BOTTOM LINE

Comparison of coefficients often beats long division.

# Practice 03

SAME IDEA Use DIVIDE: Polynomial Division.

QUESTION

> Divide $x^3 + 3x^2 - 4x - 12$ by $x + 3$.

YOUR SOLUTION

# 04 ROOTS: Use Known Roots

TRIGGER *roots, factorise completely*

BECOMES Each root gives a factor.

FIRST LINE TO WRITE

$$x = a \Rightarrow (x - a)$$

SIMPLEST STRATEGY

1. **R**ecord each root.
2. **O**btain its factor.
3. **O**rganise the product.
4. **T**est the leading coefficient.
5. **S**tate all roots/factors.

WORKED MODEL

> Roots 2, $-1$ give factors $(x-2)(x+1)$.

HARD VARIANTS

1. Repeated roots create repeated factors; do not list them once if multiplicity matters.
2. If roots are surds or complex-looking pairs, use conjugate/symmetric factors.
3. Match the leading coefficient before expanding the final polynomial.

BOTTOM LINE

Roots and factors are the same information in different forms.

# Practice 04

SAME IDEA Use ROOTS: Use Known Roots.

QUESTION

> Form a quadratic with roots 3 and $-5$.

YOUR SOLUTION

# Quick Reference

## TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| remainder when divided by $x - a$ | remainder $= f(a)$ |
| show $x - a$ is a factor | $f(a) = 0$ |
| divide by a linear or quadratic factor | $f(x) = (\text{divisor})(\text{quotient}) + \text{remainder}$ |
| roots, factorise completely | $x = a \Rightarrow (x - a)$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Graphs of Functions

*Read a graph by its shape, intercepts, intersections, and domain.*

Graphs of Functions · P1 · 06

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Graphs of Functions Complete · 06

# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** SHAPE: Know The Parent Shape
TRIGGER *sketch a standard function*

**02** ZERO: Find Roots And Intercepts
TRIGGER *where the graph crosses axes*

**03** MEET: Intersections
TRIGGER *two graphs meet*

**04** ASYM: Asymptotes And Domain
TRIGGER *reciprocal, square root, denominator zero*

BANK EVIDENCE Local pages: 25 problem, 47 answer, 25 notes. Website primary entries: 18.


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# 01 SHAPE: Know The Parent Shape

TRIGGER *sketch a standard function*

BECOMES Identify the parent curve before plotting details.

### FIRST LINE TO WRITE

$$y = x^2, \quad y = x^3, \quad y = \frac{1}{x}, \quad y = \sqrt{x}$$

### SIMPLEST STRATEGY

1. **S**elect the parent graph.
2. **H**ighlight symmetry/asymptotes.
3. **A**dd intercepts.
4. **P**lot key points.
5. **E**nd with the correct curve shape.

### WORKED MODEL

$$y = (x - 2)^2$$

is the $x^2$ shape shifted right by 2.

### HARD VARIANTS

1. Transform the parent graph before plotting extra points.
2. Mark asymptotes/endpoints before drawing reciprocal or root curves.
3. Use symmetry to reduce work, but check the domain.

### BOTTOM LINE

The parent shape is the memory anchor.

# Practice 01

SAME IDEA Use SHAPE: Know The Parent Shape.

QUESTION

> Sketch $y = (x+1)^2 - 4$.

YOUR SOLUTION

# 02 ZERO: Find Roots And Intercepts

TRIGGER *where the graph crosses axes*

BECOMES Set $y = 0$ for roots and $x = 0$ for y-intercept.

### FIRST LINE TO WRITE

$$y = 0 \quad \text{or} \quad x = 0$$

### SIMPLEST STRATEGY

1. **Z**ero the correct variable.
2. **E**quation solve.
3. **R**ecord coordinates.
4. **O**rder points clearly.

### WORKED MODEL

For $y = x^2 - 9$, roots: $x = \pm 3$, y-intercept: $-9$.

### HARD VARIANTS

1. If factorisation is hard, use a substitution or formula before writing intercepts.
2. Intercepts are coordinates; include $y = 0$ or $x = 0$ correctly.
3. Check whether any intercept is outside the stated domain.

### BOTTOM LINE

Intercepts are coordinates, not just numbers.

# Practice 02

SAME IDEA Use ZERO: Find Roots And Intercepts.

QUESTION

> Find the intercepts of $y = x^2 - 4x$.

YOUR SOLUTION

# 03 MEET: Intersections

TRIGGER *two graphs meet*

BECOMES Equate the two formulas.

FIRST LINE TO WRITE

$$f(x) = g(x)$$

SIMPLEST STRATEGY

1. **M**ake the two expressions equal.
2. **E**xpand/rearrange.
3. **E**valuate $x$.
4. **T**urn each $x$ into a coordinate.

WORKED MODEL

$$x^2 = x + 2 \Rightarrow x = 2, -1.$$

HARD VARIANTS

1. Equating two functions may create a quadratic or cubic; solve all valid roots.
2. For tangency, look for one repeated intersection.
3. Use either graph to find $y$, then verify with the other.

BOTTOM LINE

Meeting points need both $x$ and $y$.

# Practice 03

SAME IDEA Use MEET: Intersections.

QUESTION

> Find where $y = x^2$ meets $y = 2x + 3$.

YOUR SOLUTION

# 04 ASYM: Asymptotes And Domain

TRIGGER *reciprocal, square root, denominator zero*

BECOMES Find forbidden or endpoint values first.

### FIRST LINE TO WRITE

$$\text{denominator} \neq 0, \qquad \text{inside root} \geq 0$$

### SIMPLEST STRATEGY

1. **A**sk what values are forbidden.
2. **S**et denominator/root condition.
3. **Y**ield domain/asymptote.
4. **M**ark it before sketching.

### WORKED MODEL

$$y = \frac{1}{x-3}$$

has vertical asymptote $x = 3$.

### HARD VARIANTS

1. Denominator zeros give vertical asymptotes unless the factor cancels first.
2. Square-root domains come from inside root $\geq 0$.
3. Horizontal shifts move asymptotes and endpoints too.

### BOTTOM LINE

Domain restrictions shape the whole graph.

# Practice 04

SAME IDEA Use ASYM: Asymptotes And Domain.

QUESTION

> State the domain of $y = \sqrt{x-2}$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| sketch a standard function | $y = x^2, \quad y = x^3, \quad y = \dfrac{1}{x}, \quad y = \sqrt{x}$ |
| where the graph crosses axes | $y = 0 \quad \text{or} \quad x = 0$ |
| two graphs meet | $f(x) = g(x)$ |
| reciprocal, square root, denominator zero | denominator $\neq 0$, inside root $\geq 0$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Transformations of Functions

*Name the parent graph, then apply the transformation in the right direction.*

Transformations of Functions · P1 · 07

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** SHIFT: Translations

TRIGGER $f(x)+a$, $f(x-a)$

**02** SCALE: Stretches

TRIGGER $af(x)$, $f(ax)$

**03** FLIP: Reflections

TRIGGER $-f(x)$, $f(-x)$

**04** ORDER: Combined Transformations

TRIGGER *more than one transformation*

BANK EVIDENCE Local pages: 33 problem, 50 answer, 18 notes. Website primary entries: 7.

# 01 SHIFT: Translations

TRIGGER $f(x) + a$, $f(x - a)$

BECOMES Move the whole graph without changing its shape.

FIRST LINE TO WRITE

$$y = f(x - a) + b$$

SIMPLEST STRATEGY

1. **S**tart from $y = f(x)$.
2. **H**orizontal move comes from inside the bracket.
3. **I**nside sign works opposite.
4. **F**inal vertical move is outside.
5. **T**ranslate every key point.

WORKED MODEL

$$y = f(x - 3) + 2$$

moves right 3 and up 2.

HARD VARIANTS

1. Move key points, endpoints, and asymptotes, not just the visible curve.
2. Inside bracket moves are opposite: $f(x - a)$ moves right $a$.
3. Update domain and range after the shift.

BOTTOM LINE

Inside changes are horizontal and opposite.

# Practice 01

SAME IDEA Use SHIFT: Translations.

QUESTION

> Describe $y = f(x+4) - 1$.

YOUR SOLUTION

# 02 SCALE: Stretches

TRIGGER $af(x)$, $f(ax)$

BECOMES Multiply distances from an axis.

FIRST LINE TO WRITE

$$y = af(x), \qquad y = f(ax)$$

SIMPLEST STRATEGY

1. **S**elect vertical or horizontal.
2. **C**heck whether the multiplier is outside or inside.
3. **A**pply reciprocal effect for inside.
4. **L**ocate transformed key points.
5. **E**nd with the same shape.

WORKED MODEL

$$y = 2f(x)$$

doubles y-values; $y = f(2x)$ halves x-values.

HARD VARIANTS

1. Inside scale factors act reciprocally on $x$-coordinates.
2. Vertical stretch changes range; horizontal stretch changes domain/endpoints.
3. Asymptotes and intercepts must be stretched too.

BOTTOM LINE

Inside stretch factors act reciprocally.

# Practice 02

SAME IDEA Use SCALE: Stretches.

QUESTION

> Describe $y = 3f(x)$ and $y = f(\frac{1}{2}x)$.

YOUR SOLUTION

# 03 FLIP: Reflections

TRIGGER $-f(x)$, $f(-x)$

BECOMES Reflect in the correct axis.

## FIRST LINE TO WRITE

$$y = -f(x), \qquad y = f(-x)$$

## SIMPLEST STRATEGY

1 **F**ind whether the minus is outside or inside.

2 **L**ink outside to x-axis.

3 **I**nside to y-axis.

4 **P**lot reflected key points.

## WORKED MODEL

$$y = -f(x)$$

reflects in the x-axis; $y = f(-x)$ reflects in the y-axis.

## HARD VARIANTS

1 Outside minus reflects in the $x$-axis; inside minus reflects in the $y$-axis.

2 Reflect endpoints and asymptotes together with the graph.

3 Combined reflection and shift: transform key points in order.

## BOTTOM LINE

Outside controls $y$; inside controls $x$.

# Practice 03

SAME IDEA Use FLIP: Reflections.

QUESTION

> Describe $y = -f(x+2)$.

YOUR SOLUTION

# 04 ORDER: Combined Transformations

TRIGGER *more than one transformation*

BECOMES Apply inside changes to x-values and outside changes to y-values.

FIRST LINE TO WRITE

$$(x, y) \mapsto (\text{new } x, \text{new } y)$$

SIMPLEST STRATEGY

1. **O**rganise inside and outside changes.
2. **R**ewrite key points.
3. **D**o horizontal changes carefully.
4. **E**xecute vertical changes.
5. **R**edraw with labelled points.

WORKED MODEL

> For $y = 2f(x-1) - 3$, point $(a, b)$ becomes $(a+1, 2b-3)$.

HARD VARIANTS

1. For a point $(a, b)$, transform $x$ using inside changes and $y$ using outside changes.
2. Do not rely on drawing first; calculate key-point images first.
3. Check the final domain and range.

BOTTOM LINE

Transform key points; the curve follows.

# Practice 04

SAME IDEA Use ORDER: Combined Transformations.

QUESTION

> If $(2, 5)$ is on $y = f(x)$, find its image on $y = 3f(x - 4)$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| $f(x)+a$, $f(x-a)$ | $y = f(x-a)+b$ |
| $af(x)$, $f(ax)$ | $y = af(x), \qquad y = f(ax)$ |
| $-f(x)$, $f(-x)$ | $y = -f(x), \qquad y = f(-x)$ |
| more than one transformation | $(x,y) \mapsto (\text{new } x, \text{new } y)$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Equation of a Straight Line

*A line is controlled by gradient and one point.*

Equation of a Straight Line · P1 · 08

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Equation of a Straight Line Complete · 08

# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** GRAD: Find The Gradient

TRIGGER *two points, parallel line, perpendicular line*

**02** POINT: Build The Line

TRIGGER *gradient and point are known*

**03** PARA: Parallel And Perpendicular

TRIGGER *parallel or perpendicular*

**04** MEET: Intersection Of Lines

TRIGGER *where two lines meet*

BANK EVIDENCE Local pages: 26 problem, 57 answer, 18 notes. Website primary entries: 17.

# 01 GRAD: Find The Gradient

TRIGGER *two points, parallel line, perpendicular line*

BECOMES Gradient is change in $y$ over change in $x$.

### FIRST LINE TO WRITE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### SIMPLEST STRATEGY

1. **G**et two points.
2. **R**ise is change in y.
3. **A**cross is change in x.
4. **D**ivide rise by across.

### WORKED MODEL

$$m = \frac{7 - 1}{4 - 2} = 3.$$

### HARD VARIANTS

1. Parallel gradients are equal; perpendicular gradients are negative reciprocals.
2. If coordinates contain letters, calculate the gradient symbolically first.
3. Vertical lines have undefined gradient and need $x =$ constant.

### BOTTOM LINE

Gradient is a ratio, not just a difference.

# Practice 01

SAME IDEA Use GRAD: Find The Gradient.

QUESTION

> Find the gradient through $(1, 4)$ and $(5, 12)$.

YOUR SOLUTION

# 02 POINT: Build The Line

TRIGGER *gradient and point are known*

BECOMES Use point-gradient form.

FIRST LINE TO WRITE

$$y - y_1 = m(x - x_1)$$

SIMPLEST STRATEGY

1. **P**ick the known point.
2. **O**btain the gradient.
3. **I**nsert into point-gradient form.
4. **N**eaten into $y = mx + c$ if needed.
5. **T**est with the point.

WORKED MODEL

$$m = 2, (3, 5):\ y - 5 = 2(x - 3) \Rightarrow y = 2x - 1.$$

HARD VARIANTS

1. If the point is an intercept, convert it to coordinates before using the formula.
2. For parameter lines, substitute the given point to find the parameter.
3. Return to the requested form: $y = mx + c$, $ax + by + c = 0$, or coordinate form.

BOTTOM LINE

One point plus one gradient fixes the line.

# Practice 02

SAME IDEA Use POINT: Build The Line.

QUESTION

> Find the line with gradient $-3$ through $(2, 7)$.

YOUR SOLUTION

# 03 PARA: Parallel And Perpendicular

TRIGGER *parallel or perpendicular*

BECOMES Parallel gradients match; perpendicular gradients multiply to $-1$.

### FIRST LINE TO WRITE

$$m_{\parallel} = m, \qquad m_{\perp} = -\frac{1}{m}$$

### SIMPLEST STRATEGY

1. **P**ick the original gradient.
2. **A**dapt it for parallel/perpendicular.
3. **R**eplace into line equation.
4. **A**nswer in required form.

### WORKED MODEL

If $m = 4$, a perpendicular line has gradient $-\frac{1}{4}$.

### HARD VARIANTS

1. For perpendicular lines, invert and change sign before building the new line.
2. If the original line is not in gradient form, rearrange it first.
3. Use the given point after finding the new gradient.

### BOTTOM LINE

Perpendicular means negative reciprocal.

# Practice 03

SAME IDEA Use PARA: Parallel And Perpendicular.

QUESTION

> Find the gradient perpendicular to $y = 2x + 5$.

YOUR SOLUTION

# 04 MEET: Intersection Of Lines

TRIGGER *where two lines meet*

BECOMES Set the two equations equal.

FIRST LINE TO WRITE

$$m_1x + c_1 = m_2x + c_2$$

SIMPLEST STRATEGY

1 **M**ake y-values equal.

2 **E**valuate $x$.

3 **E**valuate $y$.

4 **T**urn into coordinate form.

WORKED MODEL

$$2x + 1 = 5 - x \Rightarrow 3x = 4,\ x = \frac{4}{3}.$$

HARD VARIANTS

1 If the lines are parallel, there may be no meeting point.

2 With parameters, solve the intersection condition to find the unknown first.

3 Give the answer as a coordinate and verify both equations.

BOTTOM LINE

The meeting point is an ordered pair.

# Practice 04

SAME IDEA Use MEET: Intersection Of Lines.

QUESTION

> Find where $y = 3x - 2$ and $y = x + 6$ meet.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| two points, parallel line, perpendicular line | $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ |
| gradient and point are known | $y - y_1 = m(x - x_1)$ |
| parallel or perpendicular | $m_{\parallel} = m, \qquad m_{\perp} = -\dfrac{1}{m}$ |
| where two lines meet | $m_1 x + c_1 = m_2 x + c_2$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Basic Trigonometry

*Choose the triangle rule from the sides and angle you can see.*

Basic Trigonometry · P1 · 09

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** RATIO: Right-Triangle Ratios
TRIGGER *right-angled triangle*

**02** SINE: Sine Rule
TRIGGER *non-right triangle with matching side-angle pair*

**03** COS: Cosine Rule
TRIGGER *two sides and included angle, or three sides*

**04** AREA: Triangle Area
TRIGGER *two sides and included angle*

BANK EVIDENCE Local pages: 29 problem, 55 answer, 14 notes. Website primary entries: 6.

# 01 RATIO: Right-Triangle Ratios

TRIGGER *right-angled triangle*

BECOMES Label opposite, adjacent, hypotenuse.

### FIRST LINE TO WRITE

$$\sin\theta = \frac{O}{H}, \quad \cos\theta = \frac{A}{H}, \quad \tan\theta = \frac{O}{A}$$

### SIMPLEST STRATEGY

1. **R**ead the angle.
2. **A**ssign O/A/H.
3. **T**ake the ratio with the knowns.
4. **I**nsert values.
5. **O**btain side or angle.

### WORKED MODEL

$$\sin 30^\circ = \frac{x}{10} \Rightarrow x = 5.$$

### HARD VARIANTS

1. In compound diagrams, split the shape into right triangles before using SOH CAH TOA.
2. If the angle is not the reference angle, relabel opposite and adjacent from the correct angle.
3. Use exact surd values for special angles when the question asks for exact form.

### BOTTOM LINE

SOH CAH TOA starts with labels, not the calculator.

# Practice 01

SAME IDEA Use RATIO: Right-Triangle Ratios.

QUESTION

> In a right triangle, $H = 12$ and $\theta = 35^\circ$. Find the opposite side.

YOUR SOLUTION

# 02 SINE: Sine Rule

TRIGGER *non-right triangle with matching side-angle pair*

BECOMES Use side over sine of opposite angle.

### FIRST LINE TO WRITE

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

### SIMPLEST STRATEGY

1. **S**pot an opposite pair.
2. **I**nsert known side-angle pair.
3. **N**ame the unknown pair.
4. **E**valuate carefully.

### WORKED MODEL

$$\frac{x}{\sin 40^\circ} = \frac{8}{\sin 70^\circ}.$$

### HARD VARIANTS

1. Ambiguous SSA cases can give two possible angles; check the diagram and angle sum.
2. Pair each side only with its opposite angle.
3. After finding an angle, confirm the triangle angle sum is possible.

### BOTTOM LINE

Sine rule needs opposite pairs.

# Practice 02

SAME IDEA Use SINE: Sine Rule.

QUESTION

> Use sine rule to find $x$ when $x$ is opposite $42^\circ$, and side 9 is opposite $68^\circ$.

YOUR SOLUTION

# 03 COS: Cosine Rule

TRIGGER *two sides and included angle, or three sides*

BECOMES Use the formula with the opposite side first.

### FIRST LINE TO WRITE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

### SIMPLEST STRATEGY

1. **C**hoose the side opposite the angle.
2. **O**rganise $b, c, A$.
3. **S**ubstitute into cosine rule.

### WORKED MODEL

$$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60^\circ.$$

### HARD VARIANTS

1. Use cosine rule for SAS or SSS cases, especially when Pythagoras almost fits but no right angle is given.
2. When finding an angle, rearrange for $\cos A$ before applying inverse cosine.
3. Check whether the answer should be acute or obtuse from the side lengths.

### BOTTOM LINE

Cosine rule replaces Pythagoras when the angle is not $90^\circ$.

# Practice 03

SAME IDEA Use COS: Cosine Rule.

QUESTION

> Find the side opposite 50° between sides 6 and 8.

YOUR SOLUTION

# 04 AREA: Triangle Area

TRIGGER *two sides and included angle*

BECOMES Use $\frac{1}{2}ab\sin C$.

### FIRST LINE TO WRITE

$$\text{Area} = \frac{1}{2}ab\sin C$$

### SIMPLEST STRATEGY

1. **A**ssign the two sides around the angle.
2. **R**ead the included angle.
3. **E**valuate $\frac{1}{2}ab\sin C$.
4. **A**dd units squared.

### WORKED MODEL

$$\text{Area} = \frac{1}{2}(6)(9)\sin 40^\circ.$$

### HARD VARIANTS

1. The angle must be included between the two sides used in $\frac{1}{2}ab\sin C$.
2. For compound areas, subtract or add triangle areas after finding missing sides.
3. Give square units and keep exact form if requested.

### BOTTOM LINE

The angle must sit between the two sides.

# Practice 04

SAME IDEA Use AREA: Triangle Area.

QUESTION

> Find the area with sides 7 and 10 including angle 35°.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| right-angled triangle | $\sin\theta = \frac{O}{H}, \quad \cos\theta = \frac{A}{H}, \quad \tan\theta = \frac{O}{A}$ |
| non-right triangle with matching side-angle pair | $\frac{a}{\sin A} = \frac{b}{\sin B}$ |
| two sides and included angle, or three sides | $a^2 = b^2 + c^2 - 2bc\cos A$ |
| two sides and included angle | $\text{Area} = \frac{1}{2}ab\sin C$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Radian Measure

*Radian questions are sector questions: angle, radius, arc, area.*

Radian Measure · P1 · 10

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** RAD: Degrees To Radians

TRIGGER *convert degrees/radians*

**02** ARC: Arc Length

TRIGGER *arc length or perimeter of sector*

**03** SECT: Sector Area

TRIGGER *sector area*

**04** SEG: Segment Problems

TRIGGER *minor segment, shaded area*

BANK EVIDENCE Local pages: 26 problem, 38 answer, 14 notes. Website primary entries: 18.

# 01 RAD: Degrees To Radians

TRIGGER *convert degrees/radians*

BECOMES Use $\pi$ radians equals $180^\circ$.

FIRST LINE TO WRITE

$$180^\circ = \pi \text{ radians}$$

SIMPLEST STRATEGY

1. **R**ead the starting unit.
2. **A**pply the conversion factor.
3. **D**eliver exact multiples of $\pi$ where possible.

WORKED MODEL

$$60^\circ = 60 \cdot \frac{\pi}{180} = \frac{\pi}{3}.$$

HARD VARIANTS

1. Convert to radians before using sector formulas.
2. Keep exact multiples of $\pi$ until the final answer.
3. If an angle is reflex or major, decide whether to use $\theta$ or $2\pi - \theta$.

BOTTOM LINE

Keep exact $\pi$ unless decimals are requested.

# Practice 01

SAME IDEA Use RAD: Degrees To Radians.

QUESTION

Convert 135° to radians.

YOUR SOLUTION

# 02 ARC: Arc Length

TRIGGER *arc length or perimeter of sector*

BECOMES Use $s = r\theta$ with $\theta$ in radians.

FIRST LINE TO WRITE

$$s = r\theta$$

SIMPLEST STRATEGY

1. **A**ngle must be in radians.
2. **R**adius identified.
3. **C**alculate arc, then add straight sides if perimeter.

WORKED MODEL

$$r = 5, \theta = 1.2 \Rightarrow s = 6.$$

HARD VARIANTS

1. For perimeter, add the two radii after finding the arc length.
2. If given chord/sector information, find $\theta$ first in radians.
3. Use major/minor wording to choose the correct angle.

BOTTOM LINE

Arc length uses radians only.

# Practice 02

SAME IDEA Use ARC: Arc Length.

QUESTION

> Find arc length for $r = 8, \theta = \frac{3\pi}{4}$.

YOUR SOLUTION

# 03 SECT: Sector Area

TRIGGER *sector area*

BECOMES Use $\frac{1}{2}r^2\theta$.

FIRST LINE TO WRITE

$$A = \frac{1}{2}r^2\theta$$

SIMPLEST STRATEGY

1. **S**ecure radians.
2. **E**nter radius squared.
3. **C**alculate sector area.
4. **T**ake away triangle if segment is needed.

WORKED MODEL

$$A = \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right) = \frac{8\pi}{3}.$$

HARD VARIANTS

1. Sector area uses $\frac{1}{2}r^2\theta$, never degrees.
2. For shaded regions, compute sector and triangle separately before subtracting.
3. If radius or angle is unknown, set up the sector formula and solve symbolically.

BOTTOM LINE

Sector area is half $r^2\theta$, not $\pi r^2\theta$.

# Practice 03

SAME IDEA Use SECT: Sector Area.

QUESTION

Find sector area for $r = 6, \theta = 2$.

YOUR SOLUTION

# 04 SEG: Segment Problems

TRIGGER *minor segment, shaded area*

BECOMES Sector minus triangle.

FIRST LINE TO WRITE

$$\text{segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

SIMPLEST STRATEGY

1. **S**ector area first.
2. **E**valuate triangle area.
3. **G**et difference.

WORKED MODEL

$$\text{segment} = \frac{1}{2}r^2(\theta - \sin\theta).$$

HARD VARIANTS

1. Minor segment is sector minus triangle; major segment is circle minus minor segment.
2. Use $\frac{1}{2}r^2\sin\theta$ for the triangle only when $\theta$ is included.
3. Check whether the shaded part is the small segment or its complement.

BOTTOM LINE

Most shaded segment questions are subtraction questions.

# Practice 04

SAME IDEA Use SEG: Segment Problems.

QUESTION

> Write the segment area for radius 5, angle $\theta$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| convert degrees/radians | $180^\circ = \pi$ radians |
| arc length or perimeter of sector | $s = r\theta$ |
| sector area | $A = \dfrac{1}{2}r^2\theta$ |
| minor segment, shaded area | segment $= \dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2\sin\theta$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Trigonometric Functions

*Use period, amplitude, CAST, and identities to control trig graphs and equations.*

Trigonometric Functions · P1 · 11

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** WAVE: Sketch The Wave

TRIGGER *sketch sine, cosine, tangent*

**02** CAST: Solve With Quadrants

TRIGGER *solve trig equation in interval*

**03** IDENT: Use Identities

TRIGGER *simplify or prove trig identity*

**04** TRANS: Transform Trig Graphs

TRIGGER $a \sin bx + c$

BANK EVIDENCE Local pages: 27 problem, 50 answer, 12 notes. Website primary entries: 16.

# 01 WAVE: Sketch The Wave

TRIGGER *sketch sine, cosine, tangent*

BECOMES Start with parent period and key points.

### FIRST LINE TO WRITE

$$y = \sin x, \quad y = \cos x, \quad y = \tan x$$

### SIMPLEST STRATEGY

1. **W**rite the parent graph.
2. **A**mplitude or asymptotes.
3. **V**ertical/horizontal shifts.
4. **E**ndpoints and key values.

### WORKED MODEL

$$y = 2\sin x$$

has amplitude 2 and period 360°.

### HARD VARIANTS

1. Tangent has asymptotes, not maximum and minimum points.
2. Transform amplitude, period, and midline before plotting key points.
3. Respect the stated interval and include endpoints only when allowed.

### BOTTOM LINE

A trig sketch is key points plus smooth wave.

# Practice 01

SAME IDEA Use WAVE: Sketch The Wave.

QUESTION

> Sketch $y = \cos x + 1$ for $0 \leq x \leq 360°$.

YOUR SOLUTION

# 02 CAST: Solve With Quadrants

TRIGGER *solve trig equation in interval*

BECOMES Find reference angle, then use quadrant signs.

### FIRST LINE TO WRITE

$$\sin x = a, \quad \cos x = a, \quad \tan x = a$$

### SIMPLEST STRATEGY

1. **C**alculate the reference angle.
2. **A**sk which trig sign is positive/negative.
3. **S**elect quadrants.
4. **T**urn into all interval solutions.

### WORKED MODEL

$$\sin x = \frac{1}{2} \Rightarrow x = 30^\circ, 150^\circ.$$

### HARD VARIANTS

1. For $0^\circ \leq x \leq 360^\circ$, find all quadrant solutions, not just calculator output.
2. For $kx$, solve for $kx$ over the expanded interval first.
3. Reject solutions outside the required interval after dividing by $k$.

### BOTTOM LINE

CAST gives all solutions in the interval.

# Practice 02

SAME IDEA Use CAST: Solve With Quadrants.

QUESTION

> Solve $\cos x = -\frac{1}{2}$ for $0 \leq x \leq 360°$.

YOUR SOLUTION

# 03 IDENT: Use Identities

TRIGGER *simplify or prove trig identity*

BECOMES Replace using the identity that changes the problem shape.

FIRST LINE TO WRITE

$$\sin^2 x + \cos^2 x = 1$$

SIMPLEST STRATEGY

1. **I**dentify the target form.
2. **D**ecide which identity changes the expression.
3. **E**xchange terms.
4. **N**eatens fractions.
5. **T**arget reached.

WORKED MODEL

$$1 - \cos^2 x = \sin^2 x.$$

HARD VARIANTS

1. Choose the identity that moves toward the target expression.
2. For equations, identity changes may create factorisation opportunities.
3. Check restrictions when dividing by a trig expression.

BOTTOM LINE

Use identities to move toward the target, not away from it.

# Practice 03

SAME IDEA Use IDENT: Use Identities.

QUESTION

> Simplify $1 - \sin^2 x$.

YOUR SOLUTION

# 04 TRANS: Transform Trig Graphs

TRIGGER $a \sin bx + c$

BECOMES Read amplitude, period, and shift.

FIRST LINE TO WRITE

$$\text{period} = \frac{360^\circ}{b}$$

SIMPLEST STRATEGY

1. **T**ake amplitude from outside multiplier.
2. **R**ead period from inside multiplier.
3. **A**dd vertical shift.
4. **N**ote range.
5. **S**ketch key points.

WORKED MODEL

$$y = 3 \sin 2x - 1$$

has amplitude 3, period $180^\circ$, midline $-1$.

HARD VARIANTS

1. Period is $360^\circ/b$ or $2\pi/b$, depending on units.
2. Vertical shift changes the midline and range.
3. Use transformed key points to avoid drawing a phase or period incorrectly.

BOTTOM LINE

The multiplier inside controls period.

# Practice 04

SAME IDEA Use TRANS: Transform Trig Graphs.

QUESTION

> State amplitude and period of $y = 2\cos 3x$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| sketch sine, cosine, tangent | $y = \sin x, \quad y = \cos x, \quad y = \tan x$ |
| solve trig equation in interval | $\sin x = a, \quad \cos x = a, \quad \tan x = a$ |
| simplify or prove trig identity | $\sin^2 x + \cos^2 x = 1$ |
| $a \sin bx + c$ | $\text{period} = \dfrac{360^\circ}{b}$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Differentiation

*Differentiate to read gradient, tangent, normal, and stationary behaviour.*

DIFFERENTIATION · P1 · 12

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# The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA11 website answer bank. The topic reduces to these exam moves.

**01** POWER: Differentiate Powers

TRIGGER *find* $\frac{dy}{dx}$

**02** TANG: Tangent Line

TRIGGER *tangent at a point*

**03** NORM: Normal Line

TRIGGER *normal at a point*

**04** STAT: Stationary Points

TRIGGER *stationary point, increasing/decreasing*

BANK EVIDENCE Local pages: 27 problem, 54 answer, 17 notes. Website primary entries: 24.

# 01 POWER: Differentiate Powers

TRIGGER *find* $\frac{dy}{dx}$

BECOMES Bring the power down and reduce it by 1.

### FIRST LINE TO WRITE

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

### SIMPLEST STRATEGY

1. **P**ut terms into power form.
2. **O**perate: multiply by the power.
3. **W**rite the new coefficient.
4. **E**xponent decreases by 1.
5. **R**ewrite neatly.

### WORKED MODEL

$$\frac{d}{dx}(3x^4 - 2x^{1/2}) = 12x^3 - x^{-1/2}.$$

### HARD VARIANTS

1. Rewrite roots and denominators as powers before differentiating.
2. If the expression is expanded brackets over roots, simplify first.
3. Constants differentiate to zero, but parameters multiply like constants.

### BOTTOM LINE

Rewrite roots and fractions before differentiating.

# Practice 01

SAME IDEA Use POWER: Differentiate Powers.

QUESTION

> Differentiate $5x^3 - 4\sqrt{x}$.

YOUR SOLUTION

# 02 TANG: Tangent Line

TRIGGER *tangent at a point*

BECOMES Differentiate, substitute x, then use line equation.

### FIRST LINE TO WRITE

$$m = f'(a)$$

### SIMPLEST STRATEGY

1. **T**ake derivative.
2. **A**t the point, find gradient.
3. **N**ame the point.
4. **G**enerate $y - y_1 = m(x - x_1)$.

### WORKED MODEL

$$f'(2) = 5,\ P(2,7) \Rightarrow y - 7 = 5(x - 2).$$

### HARD VARIANTS

1. If only $x$ is given, find $y$ from the original curve first.
2. For parallel tangents, set $f'(x)$ equal to the required gradient.
3. For parameter tangents, use both point-on-curve and gradient conditions.

### BOTTOM LINE

A tangent is just a line with gradient $f'(a)$.

# Practice 02

SAME IDEA Use TANG: Tangent Line.

QUESTION

> Find the tangent to $y = x^2 + 1$ at $x = 3$.

YOUR SOLUTION

# 03 NORM: Normal Line

TRIGGER *normal at a point*

BECOMES Use the negative reciprocal of tangent gradient.

### FIRST LINE TO WRITE

$$m_n = -\frac{1}{m_t}$$

### SIMPLEST STRATEGY

1. **N**eed tangent gradient first.
2. **O**btain negative reciprocal.
3. **R**ecord point.
4. **M**ake the line equation.

### WORKED MODEL

If $m_t = 4$, then $m_n = -\frac{1}{4}$.

### HARD VARIANTS

1. Find the tangent gradient first, then use the negative reciprocal.
2. If tangent gradient is zero, the normal is vertical: $x = a$.
3. If tangent is vertical, the normal is horizontal.

### BOTTOM LINE

Normal gradient is negative reciprocal.

# Practice 03

SAME IDEA Use NORM: Normal Line.

QUESTION

> Find the normal gradient when $f'(a) = -2$.

YOUR SOLUTION

# 04 STAT: Stationary Points

TRIGGER *stationary point, increasing/decreasing*

BECOMES Set derivative equal to zero.

### FIRST LINE TO WRITE

$$f'(x) = 0$$

### SIMPLEST STRATEGY

1. **S**et derivative to zero.
2. **T**ake roots.
3. **A**ssess sign or second derivative.
4. **T**ell max/min/increasing/decreasing.

### WORKED MODEL

$$f'(x) = 2x - 6 = 0 \Rightarrow x = 3.$$

### HARD VARIANTS

1. Stationary points come from $f'(x) = 0$, then $y$ from the original curve.
2. Use sign change or second derivative to classify max/min.
3. For increasing/decreasing intervals, test the derivative sign between critical values.

### BOTTOM LINE

Stationary means gradient zero.

# Practice 04

SAME IDEA Use STAT: Stationary Points.

QUESTION

> Find stationary points of $y = x^2 - 8x + 1$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| find $\frac{dy}{dx}$ | $\dfrac{d}{dx}(ax^n) = anx^{n-1}$ |
| tangent at a point | $m = f'(a)$ |
| normal at a point | $m_n = -\dfrac{1}{m_t}$ |
| stationary point, increasing/decreasing | $f'(x) = 0$ |

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

# Integration

*Integrate by reversing differentiation, then use constants or limits.*

Integration · P1 · 13

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# The Map

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**01** POWER: Integrate Powers

TRIGGER *integrate polynomial, roots, fractions*

**02** EXPAND: Expand Before Integrating

TRIGGER *brackets or fractions before integration*

**03** CONST: Find The Constant

TRIGGER *curve passes through a point*

**04** AREA: Area Under Curve

TRIGGER *area bounded by curve and axis*

BANK EVIDENCE Local pages: 13 problem, 26 answer, 7 notes. Website primary entries: 32.

# 01 POWER: Integrate Powers

TRIGGER *integrate polynomial, roots, fractions*

BECOMES Increase the power by 1 and divide by the new power.

### FIRST LINE TO WRITE

$$\int ax^n\,dx = \frac{a}{n+1}x^{n+1} + c$$

### SIMPLEST STRATEGY

1. **P**ut terms into power form.
2. **O**ne is added to each power.
3. **W**rite coefficient divided by new power.
4. **E**nd with $+c$.
5. **R**ewrite neatly.

### WORKED MODEL

$$\int 6x^2\,dx = 2x^3 + c.$$

### HARD VARIANTS

1. Rewrite roots and denominators as powers before integrating.
2. Do not use the power rule for $x^{-1}$; that special case is outside this Pure 1 move.
3. Include $+c$ unless limits are present.

### BOTTOM LINE

The $+c$ belongs to indefinite integration.

# Practice 01

SAME IDEA Use POWER: Integrate Powers.

QUESTION

> Integrate $4x^3 - 3x^{-2}$.

YOUR SOLUTION

# 02 EXPAND: Expand Before Integrating

TRIGGER *brackets or fractions before integration*

BECOMES Make each term a simple power.

### FIRST LINE TO WRITE

$$\frac{x^2 + 3x}{\sqrt{x}} = x^{3/2} + 3x^{1/2}$$

### SIMPLEST STRATEGY

1. **E**xpand brackets.
2. **X**press roots/fractions as powers.
3. **P**ut each term separately.
4. **A**pply power rule.
5. **N**eaten coefficients.
6. **D**on't forget $+c$.

### WORKED MODEL

$$\frac{(x+2)^2}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 4x^{-1/2}.$$

### HARD VARIANTS

1. Expand brackets and split fractions before integrating term by term.
2. If the numerator is a product, multiply out first unless a substitution is clearly intended.
3. Keep fractional powers exact until the final simplification.

### BOTTOM LINE

Integration starts after simplification.

# Practice 02

SAME IDEA Use EXPAND: Expand Before Integrating.

QUESTION

> Rewrite and integrate $\frac{(x+1)^2}{\sqrt{x}}$.

YOUR SOLUTION

# 03 CONST: Find The Constant

TRIGGER *curve passes through a point*

BECOMES Integrate first, then substitute the point.

FIRST LINE TO WRITE

$$y = F(x) + c, \quad (x, y) \text{ known}$$

SIMPLEST STRATEGY

1 **C**alculate the integral.

2 **O**btain $+c$.

3 **N**ame the point.

4 **S**ubstitute $x, y$.

5 **T**ell final function.

WORKED MODEL

$$y = x^2 + c,\ (3, 10) \Rightarrow 10 = 9 + c,\ c = 1.$$

HARD VARIANTS

1 Integrate first, then use the point to find $c$.

2 If given gradient and point, do not substitute into the derivative.

3 For two conditions, use them after the general function is written.

BOTTOM LINE

The constant is found from a point on the curve.

# Practice 03

SAME IDEA Use CONST: Find The Constant.

QUESTION

> Given $f'(x) = 2x$ and $f(4) = 9$, find $f(x)$.

YOUR SOLUTION

# 04 AREA: Area Under Curve

TRIGGER *area bounded by curve and axis*

BECOMES Integrate between the limits.

### FIRST LINE TO WRITE

$$\text{Area} = \int_a^b y \, dx$$

### SIMPLEST STRATEGY

1. **A**ssign lower and upper limits.
2. **R**ead whether area needs splitting.
3. **E**valuate the integral.
4. **A**nswer positive area.

### WORKED MODEL

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3}.$$

### HARD VARIANTS

1. If the curve crosses the axis, split the integral and take positive area.
2. For area between two curves, integrate top minus bottom.
3. Use exact limits and state square units if the context needs them.

### BOTTOM LINE

Area is positive; split if curve crosses the axis.

# Practice 04

SAME IDEA Use AREA: Area Under Curve.

QUESTION

> Find the area under $y = x + 1$ from $x = 0$ to $x = 3$.

YOUR SOLUTION

# Quick Reference

TRIGGER → FIRST ACTION

| TRIGGER | FIRST ACTION |
|---|---|
| integrate polynomial, roots, fractions | $\int ax^n \, dx = \frac{a}{n+1} x^{n+1} + c$ |
| brackets or fractions before integration | $\frac{x^2 + 3x}{\sqrt{x}} = x^{3/2} + 3x^{1/2}$ |
| curve passes through a point | $y = F(x) + c, \quad (x, y)$ known |
| area bounded by curve and axis | $\text{Area} = \int_a^b y \, dx$ |