Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 May/June 2026 Worked Solutions

May/June 2026 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 May/June 2026

May/June 2026

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 MAY/JUNE 2026

Question 1

5 marks | Integration

1. Find

$$\int \left(3x + \frac{2}{x}\right)^2 \mathrm{d}x$$

giving the answer in simplest form.

(5)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · WMA11/01 May/June 2026

1 Expand the square

$\left(3x + \frac{2}{x}\right)^2 = 9x^2 + 12 + \frac{4}{x^2} = 9x^2 + 12 + 4x^{-2}$.

2 Integrate term by term

$\int \left(9x^2 + 12 + 4x^{-2}\right) \mathrm{d}x = 3x^3 + 12x - 4x^{-1} + C$.

3 Write the result in simplest form

Since $x^{-1} = \frac{1}{x}$, the integral is $3x^3 + 12x - \frac{4}{x} + C$.

FINAL ANSWER

$$3x^3 + 12x - \frac{4}{x} + C$$

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Question 2

6 marks | Straight Line

2. The line l_1 has equation $y = 10 - 3x$

The line l_2

- is perpendicular to l_1
- intersects l_1 at the point P with x coordinate 4

(a) Find an equation for l_2 writing the answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

Given that

- line l_1 cuts the y-axis at the point M
- line l_2 cuts the y-axis at the point N

(b) find the area of triangle MPN, making your method clear.

(3)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 May/June 2026

1 **(a) Locate the intersection point and perpendicular gradient**

At $x = 4$ on l_1, $y = 10 - 3(4) = -2$, so $P = (4, -2)$. The gradient of l_1 is -3, so the gradient of the perpendicular line l_2 is $\frac{1}{3}$.

2 **(a) Form the equation of the second line**

$y + 2 = \frac{1}{3}(x - 4)$. Multiplying by 3 and rearranging gives $x - 3y - 10 = 0$.

3 **(b) Find the two intercepts on the y-axis**

For l_1, setting $x = 0$ gives $M = (0, 10)$. From l_2, $y = \frac{x}{3} - \frac{10}{3}$, so $N = \left(0, -\frac{10}{3}\right)$.

4 **(b) Use a vertical base and horizontal height**

$MN = 10 - \left(-\frac{10}{3}\right) = \frac{40}{3}$. The perpendicular distance from P to the y-axis is 4, so $\text{Area} = \frac{1}{2} \times \frac{40}{3} \times 4 = \frac{80}{3}$.

FINAL ANSWER

$(a)\ x - 3y - 10 = 0.$ $(b)\ \dfrac{80}{3}$ square units.

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Question 3

6 marks | Basic Trigonometry

3. A triangular plot of land, ABC, has area $60\ \text{m}^2$

Given

- $AB = 15\ \text{m}$
- $AC = 10\ \text{m}$
- angle $BAC = \theta°$

(a) show that $\sin\theta° = \dfrac{4}{5}$

(1)

Given also that $\theta°$ is the **largest** angle in the triangle,

(b) find the length of the side BC, in metres, to 3 significant figures,

(3)

(c) find the size of the **smallest** angle in the triangle, in degrees, to one decimal place.

(2)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · WMA11/01 May/June 2026

1 **(a) Use the area formula**

$60 = \dfrac{1}{2}(15)(10)\sin\theta° = 75\sin\theta°$, so $\sin\theta° = \dfrac{60}{75} = \dfrac{4}{5}$.

2 **(b) Select the angle consistent with the information**

$\sin^{-1}\left(\dfrac{4}{5}\right) = 53.130\ldots°$, but θ is the largest angle, so it must be the obtuse alternative:

$\theta = 180° - 53.130\ldots° = 126.869\ldots°$. Hence $\cos\theta = -\dfrac{3}{5}$.

3 **(b) Apply the cosine rule**

$BC^2 = 15^2 + 10^2 - 2(15)(10)\cos\theta = 225 + 100 + 180 = 505$. Therefore

$BC = \sqrt{505} = 22.472\ldots\ \text{m} = 22.5\ \text{m}$ to 3 significant figures.

4 **(c) Identify the smallest angle**

The shortest side is $AC = 10$, so the smallest angle is $\angle ABC$.

5 **(c) Use the sine rule**

$\dfrac{\sin \angle ABC}{10} = \dfrac{\sin \theta}{\sqrt{505}}$, so $\sin \angle ABC = \dfrac{8}{\sqrt{505}}$. Thus $\angle ABC = 20.854\ldots° = 20.9°$ to one decimal place.

✓ **FINAL ANSWER**

$(a)\ \sin \theta° = \dfrac{4}{5}.$ $(b)\ BC = 22.5\ \text{m}.$ $(c)\ \angle ABC = 20.9°.$

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Question 4

10 marks | Differentiation

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^3 - 2x^2\sqrt{x} + 14 \qquad x > 0$$

(a) Solve the equation

$$f'(x) = 0$$

(5)

(b) (i) Show that the equation

$$f''(x) = 66$$

can be written as

$$4x - 5x^{\frac{1}{2}} - 44 = 0$$

(ii) Hence, using algebra, show that $x = 16$ is the only solution of the equation

$$f''(x) = 66$$

(5)

(Total for Question 4 is 10 marks)

WORKED SOLUTION

Question 4 · WMA11/01 May/June 2026

1 **(a) Rewrite the radical as a power**

$f(x) = x^3 - 2x^{5/2} + 14$.

2 **(a) Differentiate and solve**

$f'(x) = 3x^2 - 5x^{3/2} = x^{3/2}(3x^{1/2} - 5)$. Since $x > 0$, the factor $x^{3/2}$ is non-zero. Therefore $3\sqrt{x} - 5 = 0$, so $\sqrt{x} = \frac{5}{3}$ and $x = \frac{25}{9}$.

3 **(b)(i) Differentiate again**

$f''(x) = 6x - \frac{15}{2}x^{1/2}$.

4 **(b)(i) Rearrange the given equation**

$6x - \frac{15}{2}x^{1/2} = 66$. Multiplying by 2 and dividing by 3 gives $4x - 5x^{1/2} - 44 = 0$, as required.

5 **(b)(ii) Factorise as a quadratic in the square root**

Let $u = \sqrt{x}$, where $u \geq 0$. Then $4u^2 - 5u - 44 = 0$, and $(u - 4)(4u + 11) = 0$. Hence $u = 4$ or $u = -\dfrac{11}{4}$.

6 **(b)(ii) Apply the square-root restriction**

Because $u = \sqrt{x} \geq 0$, the value $-\dfrac{11}{4}$ is impossible. Thus $\sqrt{x} = 4$ and $x = 16$, which is the only solution.

FINAL ANSWER

$(a)\ x = \dfrac{25}{9}.$ $\quad (b)(i)\ 4x - 5\sqrt{x} - 44 = 0.$ $\quad (b)(ii)\ x = 16$ **is the only solution.**

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Question 5

7 marks | Quadratics

5. The curve C has equation

$$y = -2x^2 + 12x + 7$$

(a) Write $-2x^2 + 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

(3)

The point P is the maximum point on C.

(b) Find the coordinates of P.

(2)

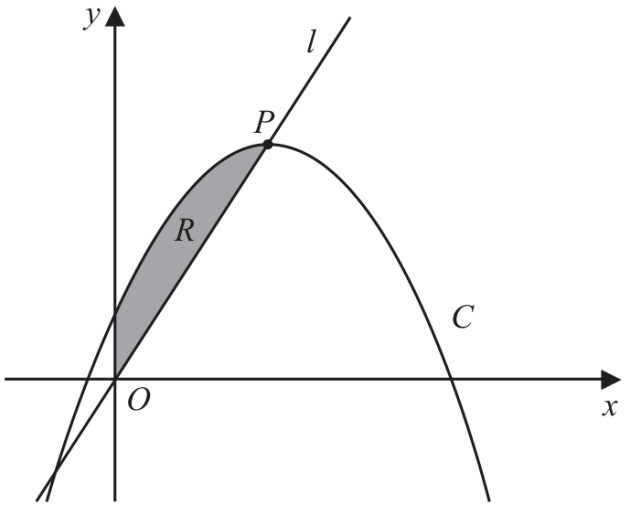


Figure 1

Figure 1 shows a sketch of the curve C and a line l.

The line l passes through the point P and the origin O.

The region R, shown shaded in Figure 1, is bounded by C, l and the y-axis.

(c) Use inequalities to define R.

(2)

(Total for Question 5 is 7 marks)

WORKED SOLUTION

Question 5 · WMA11/01 May/June 2026

1 **(a) Factor out the leading coefficient**

$-2x^2 + 12x + 7 = -2(x^2 - 6x) + 7$.

2 **(a) Complete the square**

$x^2 - 6x = (x-3)^2 - 9$, so $-2x^2 + 12x + 7 = -2\big((x-3)^2 - 9\big) + 7 = -2(x-3)^2 + 25$.

---

3 **(b) Read the maximum from completed-square form**

Because the coefficient of $(x-3)^2$ is negative, the curve has its maximum when $(x-3)^2 = 0$. Hence $x = 3$, and the maximum value is $y = 25$, so $P = (3, 25)$.

---

4 **(c) Find the equation of the boundary line**

The line through $O = (0, 0)$ and $P = (3, 25)$ has gradient $\dfrac{25}{3}$, so l has equation $y = \dfrac{25}{3}x$.

---

5 **(c) Describe the shaded region**

Points in R are below the curve, above the line, and on or to the right of the y-axis. Therefore $y \leq -2x^2 + 12x + 7$, $y \geq \dfrac{25}{3}x$, and $x \geq 0$.

✓ **FINAL ANSWER**

$(a)\ -2(x-3)^2 + 25.$ $(b)\ P = (3, 25).$ $(c)\ y \leq -2x^2 + 12x + 7,\ y \geq \dfrac{25}{3}x,\ x \geq 0.$

Question 6

7 marks | Polynomials

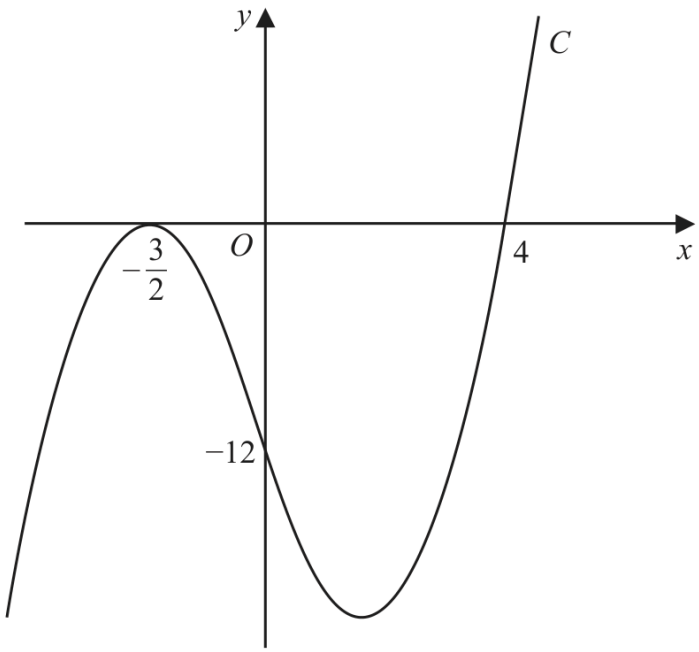


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve C

- touches the x-axis at $-\frac{3}{2}$
- cuts the x-axis at 4
- cuts the y-axis at -12

Given that $\mathrm{f}(x)$ is a cubic function,

(a) find $\mathrm{f}(x)$. You may leave the answer in a factorised form.

(3)

The line with equation $y = 2x - 12$ cuts C at $(0, -12)$ and at two further points, P and Q.

(b) Use algebra to find the exact x coordinates of P and Q.
Give each answer in simplest form.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(Total for Question 6 is 7 marks)

1 **(a) Build the cubic from its roots**

The curve touches the x-axis at $x = -\frac{3}{2}$, so this is a repeated root. It crosses the x-axis at $x = 4$, so this is a simple root. Hence

$$f(x) = k(2x+3)^2(x-4).$$

2 **(a) Use the y-intercept**

The curve passes through $(0, -12)$. Substituting $x = 0$ gives

$$-12 = k(3)^2(-4) = -36k,$$

so $k = \frac{1}{3}$. Therefore

$$f(x) = \frac{1}{3}(2x+3)^2(x-4).$$

3 **(b) Form the intersection equation**

At an intersection with the line $y = 2x - 12$,

$$\frac{1}{3}(2x+3)^2(x-4) = 2x - 12.$$

Multiplying by 3 and expanding gives

$$(4x^2 + 12x + 9)(x-4) = 6x - 36,$$

so

$$4x^3 - 4x^2 - 45x = 0.$$

4 **(b) Remove the known intersection and solve**

Factor out the known root $x = 0$:

$$x(4x^2 - 4x - 45) = 0.$$

The two further intersections satisfy

$$4x^2 - 4x - 45 = 0.$$

Using the quadratic formula,

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-45)}}{8} = \frac{4 \pm 4\sqrt{46}}{8} = \frac{1 \pm \sqrt{46}}{2}.$$

FINAL ANSWER

$(a)\ f(x) = \dfrac{1}{3}(2x+3)^2(x-4).$ $(b)\ x = \dfrac{1-\sqrt{46}}{2}$ or $x = \dfrac{1+\sqrt{46}}{2}.$

Question 7

7 marks | Radians

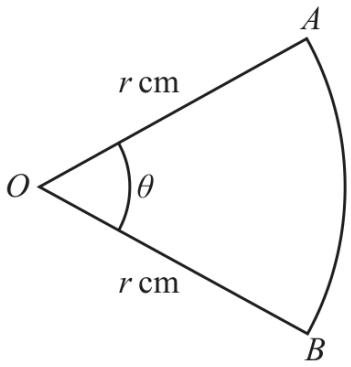


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sector AOB of a circle, centre O.

The radius OA of the sector is r cm and angle AOB is θ radians.

Given that

- the **area** of sector AOB is $1\,\text{cm}^2$
- the **perimeter** of sector AOB is $8\,\text{cm}$

(a) write down two different equations involving r and θ.

(2)

Given that $r > 2$

(b) solve the two simultaneous equations in part (a) to find the exact value of r and the exact value of θ.
Write each answer in the form $a + b\sqrt{c}$ where a, b and c are integers.

(*Solutions relying on calculator technology are not acceptable.*)

(5)

(Total for Question 7 is 7 marks)

1

(a) Translate the area and perimeter

For a sector of radius r and angle θ radians,

$$\frac{1}{2}r^2\theta = 1.$$

Its perimeter is two radii plus the arc length, so

$$2r + r\theta = 8.$$

2

(b) Eliminate the angle

From the area equation,

$$\theta = \frac{2}{r^2}.$$

Substitute this into the perimeter equation:

$$2r + r\left(\frac{2}{r^2}\right) = 8.$$

Therefore

$$2r + \frac{2}{r} = 8,$$

and, since $r > 0$,

$$r^2 - 4r + 1 = 0.$$

3

(b) Solve for the radius and apply the condition

Complete the square:

$$(r-2)^2 = 3,$$

so

$$r = 2 \pm \sqrt{3}.$$

The condition $r > 2$ selects

$$r = 2 + \sqrt{3}.$$

4 **(b) Find the exact angle**

Using $\theta = \frac{2}{r^2}$,

$$\theta = \frac{2}{(2+\sqrt{3})^2}.$$

Since $\frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$,

$$\theta = 2(2-\sqrt{3})^2 = 2(7-4\sqrt{3}) = 14-8\sqrt{3}.$$

FINAL ANSWER

$(a)\ \frac{1}{2}r^2\theta = 1, \quad 2r + r\theta = 8. \qquad (b)\ r = 2+\sqrt{3}\ \text{cm}, \quad \theta = 14-8\sqrt{3}\ \text{radians}.$

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Question 8

11 marks | Integration

8. A curve C has equation $y = \mathrm{f}(x),\ x > 0$

The point P with x coordinate $\frac{1}{4}$ lies on C.

Given that

- $\mathrm{f}'(x) = 10 + \dfrac{5\sqrt{x} - 4}{2x^2}$
- the tangent to C at P has equation $y = kx + 2$, where k is a constant

(a) (i) find the value of k

(ii) find the equation of the normal to C at P.
Give the answer in the form $y = mx + c$, where m and c are constants.

(6)

(b) Find $\mathrm{f}(x)$.

(5)

(Total for Question 8 is 11 marks)

WORKED SOLUTION

Question 8 · WMA11/01 May/June 2026

1 **(a)(i) Evaluate the tangent gradient**

The tangent gradient at P is $f'\left(\frac{1}{4}\right)$. Using

$$f'(x) = 10 + \frac{5\sqrt{x} - 4}{2x^2},$$

we get

$$f'\left(\frac{1}{4}\right) = 10 + \frac{5\left(\frac{1}{2}\right) - 4}{2\left(\frac{1}{4}\right)^2} = 10 + \frac{-\frac{3}{2}}{\frac{1}{8}} = -2.$$

Therefore $k = -2$.

2 **(a)(ii) Find the point of contact**

The tangent is $y = -2x + 2$. At $x = \frac{1}{4}$,

$$y = -2\left(\frac{1}{4}\right) + 2 = \frac{3}{2},$$

so $P = \left(\frac{1}{4}, \frac{3}{2}\right)$.

---

3 **(a)(ii) Form the normal**

The tangent gradient is -2, so the normal gradient is $\frac{1}{2}$. Through P,

$$y - \frac{3}{2} = \frac{1}{2}\left(x - \frac{1}{4}\right).$$

Hence

$$y = \frac{1}{2}x + \frac{11}{8}.$$

---

4 **(b) Rewrite and integrate**

First split the fractional term:

$$f'(x) = 10 + \frac{5}{2}x^{-3/2} - 2x^{-2}.$$

Integrating term by term gives

$$f(x) = 10x - 5x^{-1/2} + 2x^{-1} + c = 10x - \frac{5}{\sqrt{x}} + \frac{2}{x} + c.$$

---

5 **(b) Use the point P to find the constant**

Since $P = \left(\frac{1}{4}, \frac{3}{2}\right)$ lies on the curve,

$$\frac{3}{2} = 10\left(\frac{1}{4}\right) - 5\left(\frac{1}{4}\right)^{-1/2} + 2\left(\frac{1}{4}\right)^{-1} + c.$$

Thus

$$\frac{3}{2} = \frac{5}{2} - 10 + 8 + c = \frac{1}{2} + c,$$

so $c = 1$.

✓ **FINAL ANSWER**

$(a)(i)\ k = -2.$ $(a)(ii)\ y = \frac{1}{2}x + \frac{11}{8}.$ $(b)\ f(x) = 10x - \frac{5}{\sqrt{x}} + \frac{2}{x} + 1.$

Question 9

5 marks | Trigonometric Functions

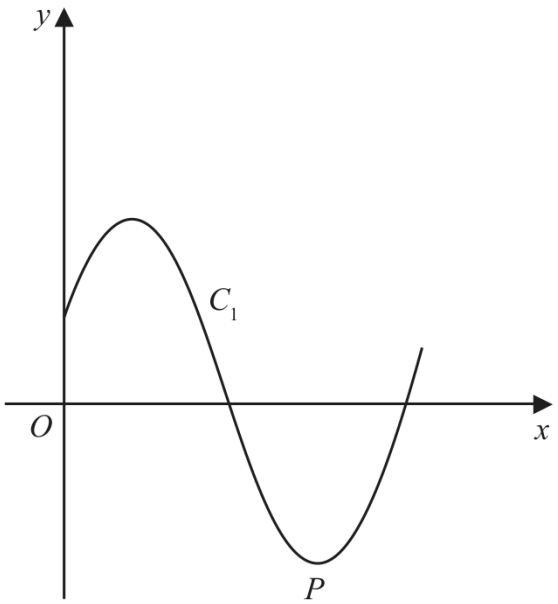


Figure 4

The curve C_1 has equation $y = \sin\left(x + \frac{\pi}{8}\right)$

Figure 4 shows a sketch of C_1 in the interval $0 \leqslant x < 2\pi$

The point P is a minimum turning point on C_1 and is shown in Figure 4.

(a) State the x coordinate of P.

(1)

(b) State the number of roots of the equation

$$\sin\left(x + \frac{\pi}{8}\right) = \sin\left(\frac{\pi}{8}\right)$$

in the interval $0 \leqslant x < 101\pi$, giving a reason for your answer.

(2)

The curve C_2 has equation $y = -5\sin\left(x + \frac{\pi}{8}\right)$

The point Q is the minimum turning point on C_2 in the interval $0 \leqslant x < 2\pi$

(c) State the coordinates of Q.

(2)

(Total for Question 9 is 5 marks)

1 (a) Locate the minimum of C1

A sine curve reaches its minimum when its angle is $\frac{3\pi}{2}$. Therefore

$$x + \frac{\pi}{8} = \frac{3\pi}{2},$$

so

$$x = \frac{12\pi}{8} - \frac{\pi}{8} = \frac{11\pi}{8}.$$

2 (b) Write both families of roots

For $\sin u = \sin\left(\frac{\pi}{8}\right)$,

$$u = \frac{\pi}{8} + 2n\pi \quad \text{or} \quad u = \frac{7\pi}{8} + 2n\pi.$$

With $u = x + \frac{\pi}{8}$, this gives

$$x = 2n\pi \quad \text{or} \quad x = \frac{3\pi}{4} + 2n\pi.$$

3 (b) Count the roots in the interval

For $0 \le x < 101\pi$, each family has $n = 0, 1, \ldots, 50$, giving 51 roots from each family. Hence the total number of roots is

$$51 + 51 = 102.$$

4 (c) Locate the minimum of C2

Because $y = -5\sin\left(x + \frac{\pi}{8}\right)$, the minimum value -5 occurs when the sine value is 1. Thus

$$x + \frac{\pi}{8} = \frac{\pi}{2},$$

so $x = \frac{3\pi}{8}$. Therefore

$$Q = \left(\frac{3\pi}{8}, -5\right).$$

FINAL ANSWER

$(a)\ x = \frac{11\pi}{8}.$ $(b)\ 102$ roots. $(c)\ Q = \left(\frac{3\pi}{8}, -5\right).$

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Question 10

11 marks | Graphs of Functions

10. The curve C has equation

$$y = \frac{3k}{x - 2k} \qquad x \neq 2k$$

where k is a positive constant.

(a) Sketch a graph of C.
Show on your sketch the coordinates of the point where C crosses a coordinate axis.

(3)

The line l has equation $x + y = 12$

Given that l is a tangent to C,

(b) show that

$$k^2 - 15k + 36 = 0$$

(4)

The line l meets C at a point P.

Given that P lies below the x-axis,

(c) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(Total for Question 10 is 11 marks)

WORKED SOLUTION

Question 10 · WMA11/01 May/June 2026

1 (a) Identify the graph features

The curve

$$y = \frac{3k}{x - 2k}$$

is a rectangular hyperbola with vertical asymptote $x = 2k$ and horizontal asymptote $y = 0$. Since $k > 0$, the branch for $x > 2k$ lies above the x-axis. The other branch lies below the x-axis and passes through the y-axis.

2 **(a) Find the axis intercept**

At $x = 0$,

$$y = \frac{3k}{-2k} = -\frac{3}{2}.$$

So the curve crosses the y-axis at

$$\left(0, -\frac{3}{2}\right).$$

There is no x-intercept because the numerator $3k$ is non-zero.

---

3 **(b) Form the intersection quadratic**

The line is $y = 12 - x$. At an intersection,

$$\frac{3k}{x - 2k} = 12 - x.$$

Cross-multiplying gives

$$3k = (12 - x)(x - 2k) = 12x - x^2 - 24k + 2kx.$$

Hence

$$x^2 - (12 + 2k)x + 27k = 0.$$

---

4 **(b) Use the tangency condition**

A tangent gives a repeated solution for x, so the discriminant is zero:

$$(12 + 2k)^2 - 4(1)(27k) = 0.$$

Expanding and simplifying,

$$144 + 48k + 4k^2 - 108k = 0,$$

$$4k^2 - 60k + 144 = 0,$$

and therefore

$$k^2 - 15k + 36 = 0,$$

as required.

---

5 **(c) Find the possible values of k**

Factor the equation from part (b):

$$k^2 - 15k + 36 = (k - 3)(k - 12) = 0.$$

Thus $k = 3$ or $k = 12$.

---

6 **(c) Find and select the tangent point**

The repeated root of

$$x^2 - (12 + 2k)x + 27k = 0$$

is

$$x = \frac{12 + 2k}{2} = 6 + k.$$

Then $y = 12 - x = 6 - k$. For $k = 3$, the point is $(9, 3)$, which is above the x-axis. For $k = 12$, the point is $(18, -6)$, which is below the x-axis. Therefore

$$P = (18, -6).$$

✓ **FINAL ANSWER**

(a) Rectangular hyperbola with asymptotes $x = 2k$ and $y = 0$, crossing the y-axis at $\left(0, -\frac{3}{2}\right)$.

(b) $k^2 - 15k + 36 = 0$.

(c) $P = (18, -6)$.