Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2026 Worked Solutions

January 2026 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 January 2026

January 2026

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2026

Question 1

6 marks | Integration

1. $y = 4x^3 + \dfrac{3}{2\sqrt{x}} - 8 \qquad x > 0$

Find, in simplest form,

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) $\displaystyle\int y \, \mathrm{d}x$

(3)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2026

1 Rewrite with powers

$y = 4x^3 + \frac{3}{2}x^{-1/2} - 8.$

2 Differentiate

$\frac{dy}{dx} = 12x^2 + \frac{3}{2}\left(-\frac{1}{2}\right)x^{-3/2}.$

3 Simplify derivative

$\frac{dy}{dx} = 12x^2 - \frac{3}{4}x^{-3/2} = 12x^2 - \frac{3}{4x^{3/2}}.$

4 Integrate

$\int y\,dx = \int \left(4x^3 + \frac{3}{2}x^{-1/2} - 8\right)dx.$

5 Simplify integral

$\int y\,dx = x^4 + 3x^{1/2} - 8x + c = x^4 + 3\sqrt{x} - 8x + c.$

FINAL ANSWER

$(a)\ \frac{dy}{dx} = 12x^2 - \frac{3}{4x^{3/2}}. \quad (b)\ \int y\,dx = x^4 + 3\sqrt{x} - 8x + c.$

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Question 2

6 marks | Simultaneous Equations

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Given

$$y - x = 4$$

and

$$2x^2 - xy = 8$$

show that

$$x^2 - 4x - 8 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$y - x = 4$$

$$2x^2 - xy = 8$$

writing the answers in the form $p + q\sqrt{3}$, where p and q are integers to be found.

(4)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2026

1 **Use the linear equation**

$y - x = 4$, so $y = x + 4$.

2 **Substitute into the second equation**

$2x^2 - xy = 8$ becomes $2x^2 - x(x + 4) = 8$.

3 **Show the quadratic**

$2x^2 - x^2 - 4x = 8$, so $x^2 - 4x - 8 = 0$.

4 **Complete the square**

$x^2 - 4x - 8 = 0$ gives $(x - 2)^2 = 12$.

5 **Find x**

$x = 2 \pm \sqrt{12} = 2 \pm 2\sqrt{3}$.

6 **Find y**

Since $y = x + 4$, $y = 6 \pm 2\sqrt{3}$ with matching signs.

7 **State both solutions**

$(x, y) = (2 + 2\sqrt{3}, 6 + 2\sqrt{3})$ or $(2 - 2\sqrt{3}, 6 - 2\sqrt{3})$.

✓ **FINAL ANSWER**

$(x, y) = (2 + 2\sqrt{3},\ 6 + 2\sqrt{3})$ or $(2 - 2\sqrt{3},\ 6 - 2\sqrt{3})$.

Question 3

5 marks | Inequalities

3. (i) Find the range of values of z for which

$$\frac{4}{z} < 8$$

giving the answer in simplest form.

(2)

(ii)

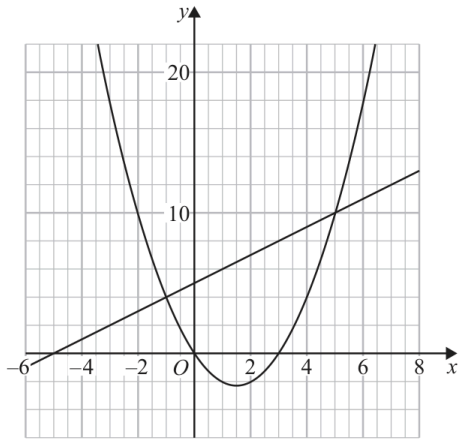


Figure 1

Figure 1 shows a plot of

- the line with equation $y = x + 5$
- the curve with equation $y = x(x - 3)$

On the opposite page there is a copy of Figure 1 labelled Diagram 1.
On Diagram 1 represent the following inequality graphically.

$$x + 5 \leqslant y \leqslant x(x - 3) \quad \text{for} \quad x \geqslant -2$$

(3)

Question 3 continued

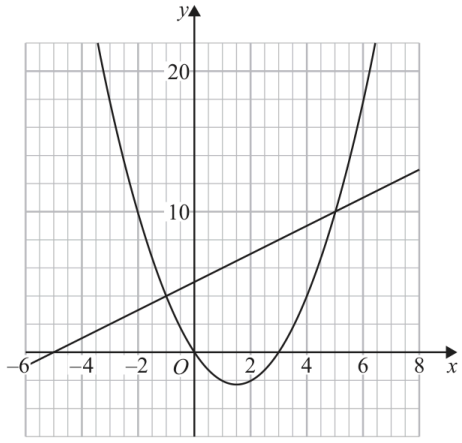


Diagram 1

(Total for Question 3 is 5 marks)

1 **Solve the rational inequality**

$\frac{4}{z} < 8$ gives $\frac{4}{z} - 8 < 0$, so $\frac{4 - 8z}{z} < 0$.

2 **Use critical values**

The critical values are $z = 0$ and $z = \frac{1}{2}$.

3 **State the range**

A sign check gives $z < 0$ or $z > \frac{1}{2}$.

4 **Find where line and curve meet**

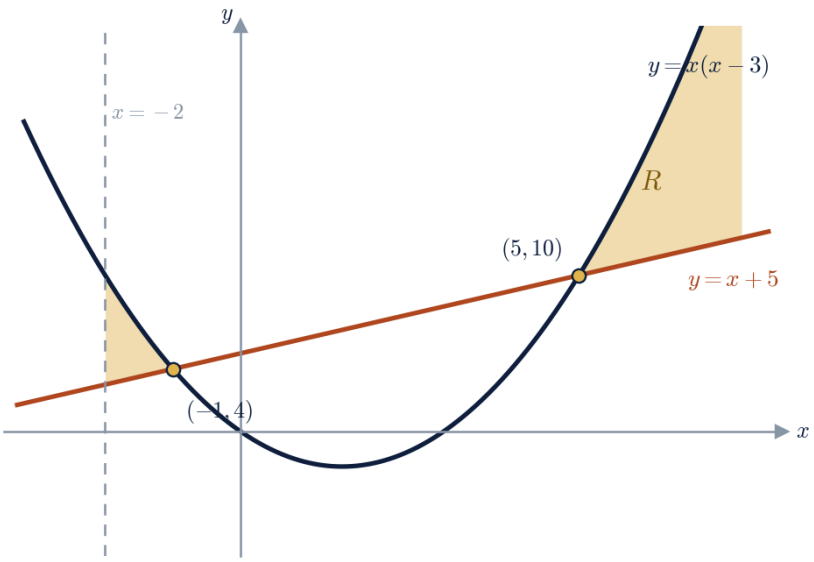
$x + 5 = x(x - 3)$ gives $x^2 - 4x - 5 = 0$, so $(x - 5)(x + 1) = 0$.

5 **Combine with the domain**

The inequality $x + 5 \leq y \leq x(x - 3)$ is possible when $x \leq -1$ or $x \geq 5$. With $x \geq -2$, the shaded x-ranges are $-2 \leq x \leq -1$ and $x \geq 5$.

6 **Graphical representation**

Draw the vertical boundary $x = -2$ and shade the two regions between the line and the parabola: from $x = -2$ to $x = -1$, and to the right of $x = 5$.



FINAL ANSWER

(i) $z < 0$ or $z > \frac{1}{2}$. (ii) shade the region above $y = x + 5$, below $y = x(x - 3)$, with $x \geq -2$.

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Question 4

9 marks | Straight Line

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The point $A(-1, 8)$ and the point $B(7, 2)$ lie on the line L_1

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line L_2

- passes through A and is perpendicular to L_1
- cuts the x-axis at the point C

(b) Find the coordinates of C.

(3)

(c) Hence find the area of triangle ABC.

(3)

(Total for Question 4 is 9 marks)

WORKED SOLUTION

Question 4 · WMA11/01 January 2026

1 Gradient of L1

Using $A(-1, 8)$ and $B(7, 2)$, $m = \dfrac{2-8}{7-(-1)} = -\dfrac{6}{8} = -\dfrac{3}{4}$.

2 Equation of L1

$y - 2 = -\dfrac{3}{4}(x - 7)$.

3 Rearrange

$4y - 8 = -3x + 21$, hence $3x + 4y - 29 = 0$.

4 Gradient of L2

Since L_2 is perpendicular to L_1, its gradient is $\dfrac{4}{3}$.

5 Equation of L2

L_2 passes through $A(-1, 8)$, so $y - 8 = \dfrac{4}{3}(x + 1)$.

6 **Find C**

At the x-axis, $y = 0$, so $-8 = \frac{4}{3}(x+1)$. Hence $x + 1 = -6$ and $x = -7$, giving $C = (-7, 0)$.

7 **Find perpendicular side lengths**

$AB = \sqrt{8^2 + (-6)^2} = 10$ and $AC = \sqrt{(-6)^2 + (-8)^2} = 10$.

8 **Area of triangle ABC**

Since $AB \perp AC$, $\text{Area} = \frac{1}{2}(10)(10) = 50$.

✓ **FINAL ANSWER**

$(a)\ 3x + 4y - 29 = 0.$ $(b)\ C = (-7, 0).$ $(c)\ 50.$

Question 5

9 marks | Radians

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

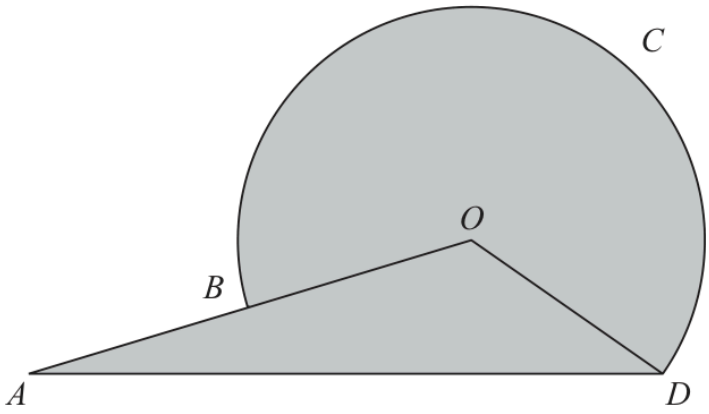


Diagram NOT to scale

Figure 2

Figure 2 shows the logo for the tourist board of an island.

The logo consists of a sector $BODCB$ of a circle centre O joined to a triangle AOD.

Given that

- ABO is a straight line
- $OD = 3\,\text{cm}$
- $AD = 8\,\text{cm}$
- angle $AOD = 2.5$ radians

(a) show that angle ODA is 0.415 radians to 3 significant figures.

(3)

(b) Find the total area of the logo, in cm^2, to one decimal place.

(3)

(c) Find the perimeter of the logo, $ABCDA$, in cm, to one decimal place.

(3)

(Total for Question 5 is 9 marks)

1 Use triangle AOD

In triangle AOD, $OD = 3$, $AD = 8$ and $\angle AOD = 2.5$ radians.

2 Use the sine rule

$\dfrac{\sin \angle OAD}{3} = \dfrac{\sin 2.5}{8}$, so $\angle OAD = \sin^{-1}\left(\dfrac{3 \sin 2.5}{8}\right) = 0.2263...$.

3 Show angle ODA

$\angle ODA = \pi - 2.5 - 0.2263... = 0.4152...$, so $\angle ODA = 0.415$ radians to 3 significant figures.

4 Area of the major sector

The sector has radius 3 and angle $2\pi - 2.5$, so its area is $\dfrac{1}{2}(3)^2(2\pi - 2.5) = 17.02...$.

5 Area of triangle AOD

$\text{Area of triangle } AOD = \dfrac{1}{2}(3)(8)\sin 0.4152... = 4.84...$.

6 Total area

$17.02... + 4.84... = 21.86...$, so the area is $21.9\ \text{cm}^2$ to one decimal place.

7 Find AO

By the cosine rule, $AO^2 = 3^2 + 8^2 - 2(3)(8)\cos 0.4152...$, so $AO \approx 5.39$ cm.

8 Find AB

Since A, B, O are collinear and $OB = 3$, $AB = AO - OB \approx 5.39 - 3 = 2.39$ cm.

9 Perimeter

$AB + \text{major arc } BD + AD = 2.39... + 3(2\pi - 2.5) + 8 = 21.7\ \text{cm}$ to one decimal place.

FINAL ANSWER

$(a)\ \angle ODA = 0.415$ rad. $(b)\ 21.9\ \text{cm}^2$. $(c)\ 21.7$ cm.

Question 6

9 marks | Polynomials

6.

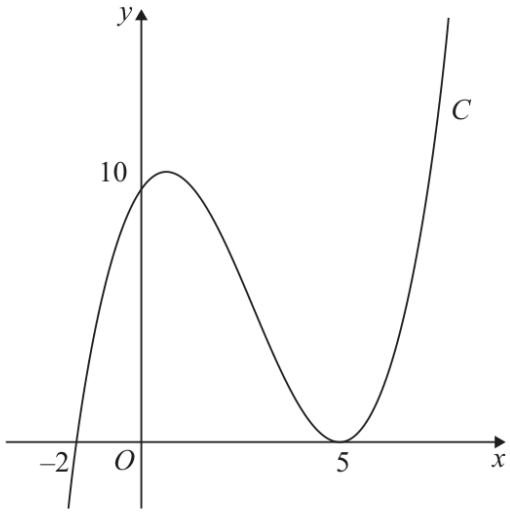


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic function in x.

The curve C

- cuts the x-axis at $(-2, 0)$ and cuts the y-axis at $(0, 10)$
- touches the x-axis at $(5, 0)$

as shown in Figure 3.

(a) Deduce the roots of the equation

(i) $\mathrm{f}\left(\frac{1}{3}x\right) = 0$

(ii) $\mathrm{f}(x - 3) = 0$

(2)

(b) Find an expression for $\mathrm{f}(x)$. You should leave your answer in factorised form.

(3)

The curve C intersects the straight line $y = 10(x + 2)$ at exactly three points.

(c) Use algebra to find the exact x coordinates of the three points of intersection.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

(Total for Question 6 is 9 marks)

1 **Roots of f one third x**

The roots of $f(x) = 0$ are $x = -2$ and $x = 5$, with $x = 5$ repeated. For $f\left(\frac{1}{3}x\right) = 0$, set $\frac{1}{3}x = -2$ or 5, giving $x = -6$ or 15.

2 **Roots of f of x minus 3**

For $f(x - 3) = 0$, set $x - 3 = -2$ or 5, giving $x = 1$ or 8.

3 **Use the roots and repeated root**

Since the curve cuts at $(-2, 0)$ and touches at $(5, 0)$, write $f(x) = k(x + 2)(x - 5)^2$.

4 **Use the y-intercept**

The curve passes through $(0, 10)$, so $10 = k(2)(25) = 50k$.

5 **State f(x)**

$k = \frac{1}{5}$, hence $f(x) = \frac{1}{5}(x + 2)(x - 5)^2$.

6 **Set equal to the line**

$\frac{1}{5}(x + 2)(x - 5)^2 = 10(x + 2)$.

7 **First solution**

The factor $x + 2$ gives one solution $x = -2$.

8 **Find the other solutions**

For $x \neq -2$, divide by $x + 2$: $\frac{1}{5}(x - 5)^2 = 10$, so $(x - 5)^2 = 50$.

9 **State all intersections**

$x = 5 \pm \sqrt{50} = 5 \pm 5\sqrt{2}$. Therefore the three x-coordinates are -2, $5 - 5\sqrt{2}$ and $5 + 5\sqrt{2}$.

✓ **FINAL ANSWER**

$(a)(i)\ -6, 15.$ $(a)(ii)\ 1, 8.$ $(b)\ f(x) = \frac{1}{5}(x + 2)(x - 5)^2.$ $(c)\ x = -2,\ 5 \pm 5\sqrt{2}.$

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Question 7

10 marks | Differentiation

7. The curve C has equation

$$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$$

(a) Show that $\frac{dy}{dx}$ can be written in the form

$$p(x+q)^2 + r$$

where p, q and r are constants to be found.

(5)

(b) Hence state

(i) the minimum value of $\frac{dy}{dx}$

(ii) the value of x at which this minimum value occurs.

(2)

Given that S is the point on C at which the gradient is a minimum,

(c) find the equation of the tangent to C at S, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 7 is 10 marks)

WORKED SOLUTION

Question 7 · WMA11/01 January 2026

1 Differentiate

$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$ gives $\frac{dy}{dx} = 2x^2 - 16x + 43$.

2 Complete the square

$2x^2 - 16x + 43 = 2(x^2 - 8x) + 43 = 2[(x-4)^2 - 16] + 43$.

3 Simplify

$\frac{dy}{dx} = 2(x-4)^2 + 11$.

4 **Minimum gradient**

Since $2(x-4)^2 \geq 0$, the minimum value of $\frac{dy}{dx}$ is 11.

5 **Where it occurs**

The minimum occurs when $x-4=0$, so $x=4$.

6 **Find the point S**

At $x=4$, $y=\frac{2}{3}(4)^3-8(4)^2+43(4)-\frac{20}{3}=80$.

7 **Tangent equation**

The gradient at S is 11, so $y-80=11(x-4)$.

8 **Simplify**

$y=11x+36$.

✓ **FINAL ANSWER**

$(a)\ \frac{dy}{dx}=2(x-4)^2+11. \quad (b)\ \min \frac{dy}{dx}=11 \text{ at } x=4. \quad (c)\ y=11x+36.$

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Question 8

12 marks | Integration

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x), \quad x > 0$

A point P lies on C.

Given that

- $f'(x) = 4x^2 + \frac{6}{x^2} - 4$
- the equation of the tangent to C at P is $y = 10x - 6\sqrt{3}$

(a) (i) verify that $\sqrt{3}$ is a possible x coordinate of P,

(ii) find, using algebra, the other possible x coordinate of P.

(6)

Given that the x coordinate of P is $\sqrt{3}$

(b) find an equation of the normal to C at P.

(2)

(c) Find $f(x)$, writing your answer in simplest form.
You must show each stage of your working.

(4)

(Total for Question 8 is 12 marks)

WORKED SOLUTION

Question 8 · WMA11/01 January 2026

1 **Verify x equals root 3**

$f'(\sqrt{3}) = 4(\sqrt{3})^2 + \frac{6}{(\sqrt{3})^2} - 4 = 12 + 2 - 4 = 10$, which matches the tangent gradient.

2 **Find all possible x-coordinates**

Set $4x^2 + \frac{6}{x^2} - 4 = 10$.

3 **Form a quadratic in x squared**

Multiplying by x^2 gives $4x^4 - 14x^2 + 6 = 0$, so $2x^4 - 7x^2 + 3 = 0$.

4 **Factor**

$2x^4 - 7x^2 + 3 = (2x^2 - 1)(x^2 - 3) = 0$.

5 **Use x greater than zero**

$x = \sqrt{3}$ or $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$. The other possible x-coordinate is $\dfrac{\sqrt{2}}{2}$.

6 **Find the point P**

Given $x_P = \sqrt{3}$, the tangent gives $y_P = 10\sqrt{3} - 6\sqrt{3} = 4\sqrt{3}$.

7 **Normal equation**

The normal gradient is $-\dfrac{1}{10}$, so $y - 4\sqrt{3} = -\dfrac{1}{10}(x - \sqrt{3})$.

8 **Integrate f prime**

$f(x) = \displaystyle\int \left(4x^2 + \frac{6}{x^2} - 4\right) dx = \frac{4}{3}x^3 - \frac{6}{x} - 4x + c$.

9 **Use P to find c**

$4\sqrt{3} = \dfrac{4}{3}(\sqrt{3})^3 - \dfrac{6}{\sqrt{3}} - 4\sqrt{3} + c = 4\sqrt{3} - 2\sqrt{3} - 4\sqrt{3} + c$.

10 **State f(x)**

$c = 6\sqrt{3}$, so $f(x) = \dfrac{4}{3}x^3 - \dfrac{6}{x} - 4x + 6\sqrt{3}$.

✓ **FINAL ANSWER**

$(a)(i)$ verified.

$(a)(ii)\ x = \dfrac{\sqrt{2}}{2}$. $\quad (b)\ y - 4\sqrt{3} = -\dfrac{1}{10}(x - \sqrt{3})$. $\quad (c)\ f(x) = \dfrac{4}{3}x^3 - \dfrac{6}{x} - 4x + 6\sqrt{3}$.

Question 9

9 marks | Trigonometric Functions

9.

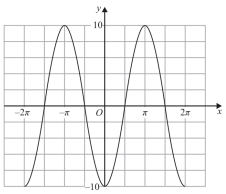


Figure 4

Figure 4 shows a sketch of part of the graph of the trigonometric function with equation $y = f(x)$.

(a) Write down an expression for $f(x)$.

(2)

Copies of Figure 4 (labelled Diagram 1 and Diagram 2) can be found on the following pages.

(b) (i) On Diagram 1 sketch a graph of the curve with equation

$$y = 10 - x^2$$

(ii) Hence find the **number** of solutions of the equation

$$f(x) = 10 - x^2 \quad \text{in the interval} \quad -100\pi \leqslant x \leqslant 100\pi$$

(3)

(c) (i) On Diagram 2 sketch a graph of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

(ii) Hence find the **number** of solutions of the equation

$$f(x) = \tan x \quad \text{in the interval} \quad -100\pi \leqslant x \leqslant 100\pi$$

giving a reason for your answer.

(4)

(a)

(b)(i)

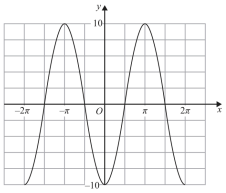
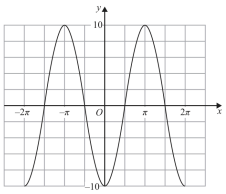


Diagram 1



Copy of Diagram 1

Only use this copy if you need to redraw your graph.

(b)(ii)

(c)(i)

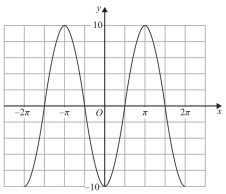
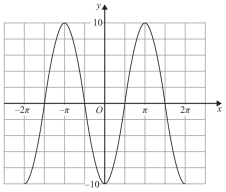


Diagram 2



Copy of Diagram 2

Only use this copy if you need to redraw your graph.

(c)(ii)

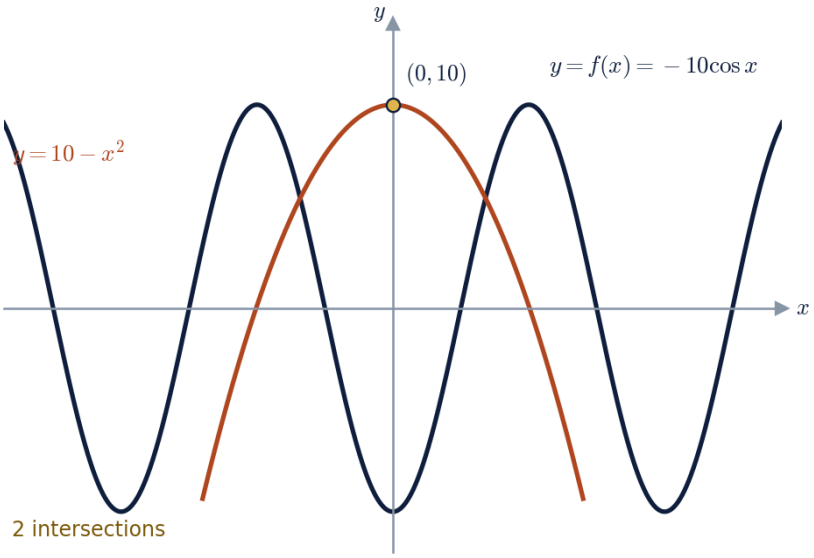
(Total for Question 9 is 9 marks)

1 Identify f(x)

The graph has amplitude 10, period 2π, and value -10 at $x = 0$, so $f(x) = -10\cos x$.

2 Sketch y equals 10 minus x squared

The graph $y = 10 - x^2$ is a downward-opening parabola with vertex $(0, 10)$ and x-intercepts at $x = \pm\sqrt{10}$.



3 Count intersections with the parabola

On the given sketch, the parabola meets $f(x)$ twice. Outside the central part the parabola is below the range of $f(x)$, so there are 2 solutions in $-100\pi \le x \le 100\pi$.

4 Sketch y equals tan x

The graph $y = \tan x$ has zeros at integer multiples of π and vertical asymptotes at $x = \dfrac{\pi}{2} + n\pi$.

5 Count in the reference interval

In $-2\pi \le x \le 2\pi$, the graph $y = \tan x$ intersects $f(x)$ four times.

6 Scale to the required interval

The interval $-100\pi \le x \le 100\pi$ is 50 times as long as $-2\pi \le x \le 2\pi$, so the number of solutions is $50 \times 4 = 200$.

FINAL ANSWER

$(a)\ f(x) = -10\cos x.$ $(b)(ii)\ 2.$ $(c)(ii)\ 200.$