Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2025 Worked Solutions

January 2025 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

WMA11/01 January 2025

January 2025

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2025

Question 1

4 marks | Integration

1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) \mathrm{d}x$$

giving your answer in simplest form.

(4)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2025

1 **Rewrite with powers**

$8x^3 - 6\sqrt{x} - \dfrac{2}{5x^3} = 8x^3 - 6x^{1/2} - \dfrac{2}{5}x^{-3}$.

2 **Integrate term by term**

$\int 8x^3\,dx = 2x^4$, $\int -6x^{1/2}\,dx = -4x^{3/2}$, and $\int -\dfrac{2}{5}x^{-3}\,dx = \dfrac{1}{5}x^{-2}$.

3 **Write in simplest form**

$2x^4 - 4x^{3/2} + \dfrac{1}{5x^2} + c$.

✓ **FINAL ANSWER**

$2x^4 - 4x^{3/2} + \dfrac{1}{5x^2} + c$.

WMA11/01 JANUARY 2025

Question 2

5 marks | Straight Line

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $\left(-2\sqrt{3}, 5\right)$
- the point B has coordinates $\left(7\sqrt{3}, 8\right)$
- the straight line l_1 passes through A and B

(a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A.

(b) Find the equation of l_2, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2025

1 **Find the gradient of l1**

$$m = \frac{8-5}{7\sqrt{3}-(-2\sqrt{3})} = \frac{3}{9\sqrt{3}} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}.$$

2 **Find the perpendicular gradient**

The gradient of l_2 is the negative reciprocal of $\frac{\sqrt{3}}{9}$, so $m_2 = -\frac{9}{\sqrt{3}} = -3\sqrt{3}$.

3 **Use point A**

l_2 passes through $A(-2\sqrt{3}, 5)$, so $y - 5 = -3\sqrt{3}(x + 2\sqrt{3})$.

4 **Simplify**

$y - 5 = -3\sqrt{3}x - 18$, hence $y = -3\sqrt{3}x - 13$.

✓ **FINAL ANSWER**

(a) gradient $= \frac{1}{9}\sqrt{3}$. (b) $y = -3\sqrt{3}x - 13$.

WMA11/01 JANUARY 2025

Question 3

5 marks | Simultaneous Equations

3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · WMA11/01 January 2025

1 Use t equals 5

The population is 58 thousand, so $58^2 = a + b(5^3)$, giving $3364 = a + 125b$.

2 Use t equals 10

The population is 65 thousand, so $65^2 = a + b(10^3)$, giving $4225 = a + 1000b$.

3 Solve for b

Subtracting gives $861 = 875b$, so $b = \frac{861}{875} = \frac{123}{125}$.

4 Solve for a

$a = 3364 - 125\left(\frac{123}{125}\right) = 3241$.

5 Use population 85 thousand

$85^2 = 3241 + \frac{123}{125}T^3$.

6 **Find T**

$T^3 = \dfrac{125(7225 - 3241)}{123}$, so $T \approx 15.938$ and $T = 15.9$ to one decimal place.

FINAL ANSWER

$(a)\ a = 3241,\ b = \dfrac{123}{125}.$ $(b)\ T = 15.9$ years.

WMA11/01 JANUARY 2025

Question 4

9 marks | Indices & Surds

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \qquad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1}

(1)

(b) $\dfrac{5}{\left(3a^{1-x}\right)^{-2}}$

(3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$

(2)

(b) Hence solve

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

(3)

(Total for Question 4 is 9 marks)

WORKED SOLUTION

Question 4 · WMA11/01 January 2025

1 **Use y equals a to the x**

$a^{3x+1} = a(a^x)^3 = ay^3$.

2 **Rewrite the denominator**

$\dfrac{5}{(3a^{1-x})^{-2}} = 5(3a^{1-x})^2 = 45a^{2-2x}$.

3 **Write in terms of y**

$45a^{2-2x} = \dfrac{45a^2}{(a^x)^2} = \dfrac{45a^2}{y^2}$.

4 **Use p equals 9 to the t**

$p = 9^t = 3^{2t}$, so $3^{4t+2} = 9(3^{2t})^2 = 9p^2$.

5 **Rewrite the equation**

$3(3^{4t+2} + 1) = 82 \times 9^t$ becomes $3(9p^2 + 1) = 82p$.

6 **Obtain the quadratic**

$27p^2 + 3 = 82p$, so $27p^2 - 82p + 3 = 0$.

7 **Solve for p**

$p = \dfrac{82 \pm 80}{54}$, so $p = 3$ or $p = \dfrac{1}{27}$.

8 **Return to t**

$9^t = 3$ gives $3^{2t} = 3^1$, so $t = \dfrac{1}{2}$. Also $9^t = \dfrac{1}{27} = 3^{-3}$ gives $2t = -3$, so $t = -\dfrac{3}{2}$.

✓ **FINAL ANSWER**

$(i)(a)\ ay^3.$ $(i)(b)\ \dfrac{45a^2}{y^2}.$ $(ii)(b)\ t = \dfrac{1}{2}$ or $t = -\dfrac{3}{2}$.

Question 5

8 marks | Differentiation

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

(Total for Question 5 is 8 marks)

WORKED SOLUTION

Question 5 · WMA11/01 January 2025

1 Differentiate

$y = 4x^3 + 2x^{-1} + 9$, so $\frac{dy}{dx} = 12x^2 - 2x^{-2} = 12x^2 - \frac{2}{x^2}$.

2 Use the tangent gradient

The tangent line $y = k - 5x$ has gradient -5, so at P, $12x^2 - \frac{2}{x^2} = -5$.

3 Clear the denominator

Multiplying by x^2 gives $12x^4 - 2 = -5x^2$, hence $12x^4 + 5x^2 - 2 = 0$.

4 Solve for x squared

Let $u = x^2$. Then $12u^2 + 5u - 2 = (3u + 2)(4u - 1) = 0$.

5 Use x greater than zero

$x^2 = \frac{1}{4}$, so $x = \frac{1}{2}$.

6 **Find the y-coordinate of P**

$y = 4\left(\frac{1}{2}\right)^3 + \frac{2}{1/2} + 9 = \frac{1}{2} + 4 + 9 = \frac{27}{2}$.

7 **Find k**

Since P lies on $y = k - 5x$, $\frac{27}{2} = k - \frac{5}{2}$, so $k = 16$.

FINAL ANSWER

$(a)\ \frac{dy}{dx} = 12x^2 - \frac{2}{x^2}$. $(c)\ k = 16$.

Question 6

10 marks | Integration

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x),\ x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $\mathrm{f}'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $\mathrm{f}(x)$

(7)

Curve C is transformed to the curve with equation $y = \mathrm{f}(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

(Total for Question 6 is 10 marks)

WORKED SOLUTION

Question 6 · WMA11/01 January 2025

1 **Use the gradient at P**

At $x = 4$, $f'(4) = 7$. Since $f'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, we get $\dfrac{32 + 4a + b}{8} = 7$.

2 **Show the required equation**

$32 + 4a + b = 56$, so $4a + b = 24$.

3 **Solve for a and b**

Together with $a + b = -9$, subtracting gives $3a = 33$, so $a = 11$ and $b = -20$.

4 **Rewrite f prime**

$$f'(x) = \frac{2x^2 + 11x - 20}{4\sqrt{x}} = \frac{1}{2}x^{3/2} + \frac{11}{4}x^{1/2} - 5x^{-1/2}.$$

5 **Integrate**

$$f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} + c.$$

6 **Use P to find c**

$$-5 = \frac{1}{5}(4^{5/2}) + \frac{11}{6}(4^{3/2}) - 10(4^{1/2}) + c = \frac{16}{15} + c.$$

7 **State f(x)**

$c = -\frac{91}{15}$, so $f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} - \frac{91}{15}$.

8 **Transform P**

$y = f(x - 3)$ translates the curve 3 units to the right, so $P(4, -5)$ becomes $Q(7, -5)$.

✓ **FINAL ANSWER**

$(b)\ f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} - \frac{91}{15}.$ $(c)\ Q = (7, -5).$

WMA11/01 JANUARY 2025

Question 7

8 marks | Graphs of Functions

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C.

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k.

(3)

The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k.

(5)

(Total for Question 7 is 8 marks)

WORKED SOLUTION

Question 7 · WMA11/01 January 2025

1 **Identify the horizontal asymptote**

$y = \frac{2}{x} - k$ has horizontal asymptote $y = -k$.

2 **Find axis intersections**

Setting $y = 0$ gives $\frac{2}{x} = k$, so $x = \frac{2}{k}$. There is no y-intercept because $x = 0$ is not in the domain.

3 **Set up line-curve intersections**

$\frac{2}{x} - k = -kx - 6$.

4 **Form a quadratic**

Multiplying by x gives $2 - kx = -kx^2 - 6x$, so $kx^2 + (6 - k)x + 2 = 0$.

5 **Use two distinct intersections**

For two distinct points, the discriminant must be positive.

6 **Solve the inequality**

$(6-k)^2-8k>0$, so $k^2-20k+36>0$. Since $k^2-20k+36=(k-2)(k-18)$, $k<2$ or $k>18$.

7 **Use k positive**

As $k>0$, the range is $0<k<2$ or $k>18$.

✓ **FINAL ANSWER**

(a) x-intercept $(\frac{2}{k},0)$, no y-intercept, horizontal asymptote $y=-k$.

(b) $0<k<2$ or $k>18$.

WMA11/01 JANUARY 2025

Question 8

13 marks | Radians

8.

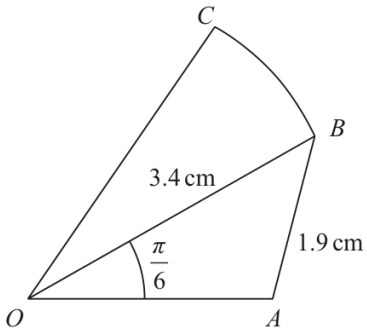


Figure 1

Figure 1 shows a sketch of a design for a badge.
The design consists of a triangle OAB joined to a sector OBC of a circle with centre O
In the design

- $OB = 3.4\,\text{cm}$
- $AB = 1.9\,\text{cm}$
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB, giving your answer in radians to 4 significant figures,

(3)

(b) find the area of triangle OAB, in cm^2, giving your answer to 3 significant figures.

(2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is $3:2$

(c) show that angle BOC is 0.462 radians to 3 significant figures.

(3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer.

(5)

(Total for Question 8 is 13 marks)

WORKED SOLUTION

Question 8 · WMA11/01 January 2025

1 Use the sine rule

In triangle OAB, $AB = 1.9$, $OB = 3.4$ and $\angle AOB = \frac{\pi}{6}$.

2 Find sine of angle OAB

$\frac{\sin \angle OAB}{3.4} = \frac{\sin(\pi/6)}{1.9}$, so $\sin \angle OAB = \frac{17}{19}$.

3 Choose the obtuse angle

Since $\angle OAB > \frac{\pi}{2}$, $\angle OAB = \pi - \sin^{-1}\left(\frac{17}{19}\right) \approx 2.034$ radians.

4 **Find OA**

Using the sine rule again, $OA \approx 2.096$ cm.

5 **Find the triangle area**

$\text{Area } OAB = \frac{1}{2}(OA)(OB)\sin(\pi/6) \approx 1.78 \text{ cm}^2$.

6 **Use the area ratio**

$\text{Area sector } OBC : \text{Area triangle } OAB = 3 : 2$, so sector area $= \frac{3}{2}(1.78156\ldots)$.

7 **Find angle BOC**

$\frac{1}{2}(3.4)^2\theta = \frac{3}{2}(1.78156\ldots)$, so $\theta \approx 0.462$ radians.

8 **Find the perimeter**

The perimeter is $OA + AB + OC + \text{arc } BC = 2.096 + 1.9 + 3.4 + 3.4(0.46234\ldots) \approx 8.968$ cm.

9 **Round**

The perimeter is 9 cm to the nearest integer.

FINAL ANSWER

$(a)\ \angle OAB = 2.034 \text{ radians.}$ $(b)\ 1.78 \text{ cm}^2.$ $(c)\ \angle BOC = 0.462 \text{ radians.}$ $(d)\ 9 \text{ cm}.$

Question 9

13 marks | Trigonometric Functions

9.

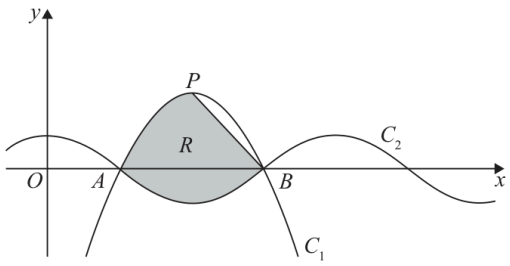


Figure 2

(a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a, b and c are constants to be found.

(3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

(b) state the coordinates of P

(2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x-axis at point A and at point B, as shown in Figure 2,

(c) (i) state the x coordinate of B

(ii) state the value of k

(iii) state the period of C_2

(3)

The line segment L joins P and B.
The region R, shown shaded in Figure 2, is bounded by L, C_1 and C_2

(d) Use inequalities to define R.

(5)

(Total for Question 9 is 13 marks)

WORKED SOLUTION

Question 9 · WMA11/01 January 2025

1 **Complete the square**

$$6x - \frac{27}{4} - x^2 = -(x^2 - 6x) - \frac{27}{4} = -(x-3)^2 + 9 - \frac{27}{4}.$$

2 **State the completed-square form**

$$6x - \frac{27}{4} - x^2 = \frac{9}{4} - (x-3)^2.$$

3 **Find the maximum point**

The maximum point is $P = (3, \frac{9}{4})$.

4 **Find the x-intercepts of C1**

$\frac{9}{4} - (x-3)^2 = 0$ gives $x = 3 \pm \frac{3}{2}$. Hence A has $x = \frac{3}{2}$ and B has $x = \frac{9}{2}$.

5 **Use consecutive zeros of cosine**

The distance between consecutive zeros of $\cos(kx)$ is $\frac{\pi}{k}$.

6 **Find k and the period**

Since $x_B - x_A = \frac{9}{2} - \frac{3}{2} = 3$, $\frac{\pi}{k} = 3$, so $k = \frac{\pi}{3}$. The period is $\frac{2\pi}{k} = 6$.

7 **Find the line through P and B**

The line through $P(3, \frac{9}{4})$ and $B(\frac{9}{2}, 0)$ has gradient $-\frac{3}{2}$, so L is $y = -\frac{3}{2}x + \frac{27}{4}$.

8 **Define the x-range**

The region lies between A and B, so $\frac{3}{2} \le x \le \frac{9}{2}$.

9 **Define the lower boundary**

The lower boundary is C_2, so $y \ge \cos\left(\frac{\pi}{3}x\right)$.

10 **Define the upper boundaries**

The region is below both C_1 and the line segment L, so $y \le \frac{9}{4} - (x-3)^2$ and $y \le -\frac{3}{2}x + \frac{27}{4}$.

FINAL ANSWER

$(a)\ \frac{9}{4} - (x-3)^2$. $(b)\ P = (3, \frac{9}{4})$. $(c)(i)\ x_B = \frac{9}{2}$. $(c)(ii)\ k = \frac{\pi}{3}$. $(c)(iii)\ 6$. $(d)\ \frac{3}{2} \le x \le \frac{9}{2},\ y \ge \cos(\frac{\pi}{3}x),\ y \le \frac{9}{4} - (x-3)^2,\ y \le -\frac{3}{2}x + \frac{27}{4}$

.