Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 October 2024 Worked Solutions

October 2024 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 October 2024

October 2024

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Straight Line

1. The line l_1 passes through the point $A(-5, 20)$ and the point $B(3, -4)$.

(a) Find an equation for l_1 giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB

(b) Find an equation for l_2 giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

(3)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · WMA11/01 October 2024

1 Find the gradient of l1

$$m = \frac{-4 - 20}{3 - (-5)} = \frac{-24}{8} = -3.$$

2 Find the equation of l1

Using $B(3, -4)$, $y = -3x + c$. Then $-4 = -9 + c$, so $c = 5$.

3 Find the midpoint of AB

$$M = \left(\frac{-5 + 3}{2}, \frac{20 + (-4)}{2}\right) = (-1, 8).$$

4 Use perpendicular gradient

Since l_1 has gradient -3, l_2 has gradient $\frac{1}{3}$.

5 Find the equation of l2

$y - 8 = \frac{1}{3}(x + 1)$, so $3y - 24 = x + 1$ and $x - 3y + 25 = 0$.

FINAL ANSWER

$(a)\ y = -3x + 5. \quad (b)\ x - 3y + 25 = 0.$

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Question 2

7 marks | Indices & Surds

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Simplify fully

$$\frac{3y^3(2x^4)^3}{4x^2y^4}$$

(3)

(ii) Find the exact value of a such that

$$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$$

Write your answer in the form $p\sqrt{3} + q$ where p and q are fully simplified rational constants.

(4)

(Total for Question 2 is 7 marks)

WORKED SOLUTION

Question 2 · WMA11/01 October 2024

1 **Simplify the power bracket**

$(2x^4)^3 = 8x^{12}$.

2 **Simplify the algebraic fraction**

$$\frac{3y^3(2x^4)^3}{4x^2y^4} = \frac{3y^3(8x^{12})}{4x^2y^4} = \frac{6x^{10}}{y}.$$

3 **Rationalise the left side**

$$\frac{16}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 8\sqrt{3} - 8.$$

4 **Equate the surd parts**

$8\sqrt{3} - 8 = a\sqrt{27} + 4 = 3a\sqrt{3} + 4$.

5 **Solve for a**

$3a\sqrt{3} = 8\sqrt{3} - 12$, so $a = \dfrac{8\sqrt{3} - 12}{3\sqrt{3}} = \dfrac{8}{3} - \dfrac{4}{3}\sqrt{3}$.

✓ **FINAL ANSWER**

$(i)\ \dfrac{6x^{10}}{y}.\quad (ii)\ a = -\dfrac{4}{3}\sqrt{3} + \dfrac{8}{3}.$

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Question 3

9 marks | Integration

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(a) Find $\int f(x)\,dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

(Total for Question 3 is 9 marks)

WORKED SOLUTION

Question 3 · WMA11/01 October 2024

1 **Rewrite f(x)**

$f(x) = \dfrac{(x+5)^2}{\sqrt{x}} = \dfrac{x^2+10x+25}{x^{1/2}} = x^{3/2} + 10x^{1/2} + 25x^{-1/2}.$

2 **Integrate**

$\int f(x)\,dx = \dfrac{2}{5}x^{5/2} + \dfrac{20}{3}x^{3/2} + 50x^{1/2} + c.$

3 **Differentiate f(x)**

$f'(x) = \dfrac{3}{2}x^{1/2} + 5x^{-1/2} - \dfrac{25}{2}x^{-3/2}.$

4 **Set f prime equal to zero**

$\dfrac{3}{2}x^{1/2} + 5x^{-1/2} - \dfrac{25}{2}x^{-3/2} = 0.$

5 **Clear powers of x**

Multiplying by $2x^{3/2}$ gives $3x^2 + 10x - 25 = 0$.

6 **Use x greater than zero**

$3x^2+10x-25=(3x-5)(x+5)$, so $x=\dfrac{5}{3}$ or $x=-5$. Since $x>0$, $x=\dfrac{5}{3}$.

FINAL ANSWER

$(a)\ \dfrac{2}{5}x^{5/2}+\dfrac{20}{3}x^{3/2}+50x^{1/2}+c.\quad (b)\ x=\dfrac{5}{3}.$

Question 4

10 marks | Polynomials

4.

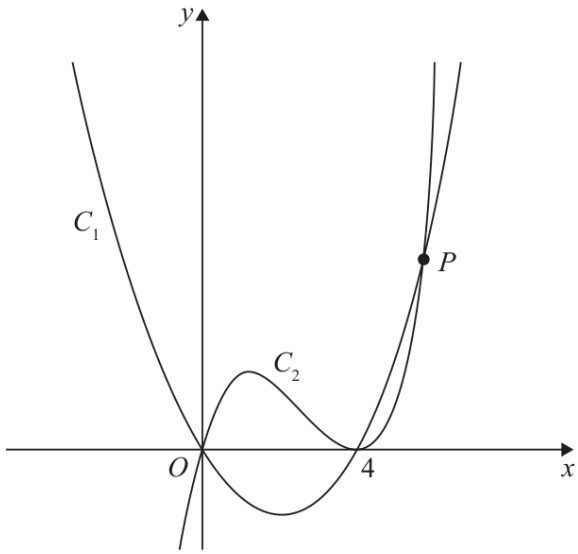


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a quadratic function
- cuts the x-axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $f(x)$

(3)

Given that C_2

- has equation $y = g(x)$ where $g(x)$ is a cubic function
- cuts the x-axis at the origin and meets the x-axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $g(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P, shown in Figure 1.

(c) Use algebra to find the coordinates of P.

(4)

(Total for Question 4 is 10 marks)

1 **Use the roots of C1**

Since C_1 cuts the x-axis at 0 and 4, let $f(x) = ax(x-4)$.

2 **Use the minimum point of C1**

At $x = 2$, $f(2) = -4a = -4.8 = -\frac{24}{5}$, so $a = \frac{6}{5}$.

3 **Write f(x)**

$f(x) = \frac{6}{5}x(x-4)$.

4 **Use the roots of C2**

The cubic cuts the x-axis at the origin and meets the x-axis at $x = 4$, so $x = 4$ is a repeated root. Let $g(x) = bx(x-4)^2$.

5 **Use the point (6, 7.2)**

$7.2 = b(6)(2^2) = 24b$, so $b = 0.3 = \frac{3}{10}$.

6 **Equate the curves**

$\frac{6}{5}x(x-4) = \frac{3}{10}x(x-4)^2$.

7 **Find the first-quadrant intersection**

Ignoring the shared roots on the x-axis, divide by $x(x-4)$ to get $\frac{6}{5} = \frac{3}{10}(x-4)$. Hence $x = 8$.

8 **Find y**

$y = f(8) = \frac{6}{5}(8)(4) = \frac{192}{5}$.

FINAL ANSWER

$(a)\ f(x) = \frac{6}{5}x(x-4).\quad (b)\ g(x) = \frac{3}{10}x(x-4)^2.\quad (c)\ P = (8, \frac{192}{5})$.

Question 5

9 marks | Radians

5. A plot of land OAB is in the shape of a sector of a circle with centre O.

Given

- $OA = OB = 5\,\text{km}$
- angle $AOB = 1.2$ radians

(a) find the perimeter of the plot of land.

(2)

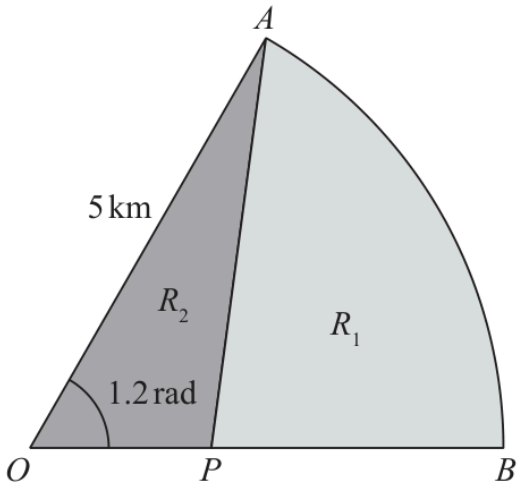


Figure 2

Diagram **NO**
accurately dr

A point P lies on OB such that the line AP divides the plot of land into two regions R_1 and R_2 as shown in Figure 2.

Given that

$$\text{area of } R_1 = 3 \times \text{area of } R_2$$

(b) show that the area of $R_2 = 3.75\,\text{km}^2$

(3)

(c) Find the length of AP, giving your answer to the nearest 100 m.

(4)

(Total for Question 5 is 9 marks)

1 **Find the arc length**

$\text{arc } AB = r\theta = 5(1.2) = 6 \text{ km}$.

2 **Find the perimeter**

$OA + OB + \text{arc } AB = 5 + 5 + 6 = 16 \text{ km}$.

3 **Find the total sector area**

$\text{Area of } OAB = \frac{1}{2}(5^2)(1.2) = 15 \text{ km}^2$.

4 **Use the area ratio**

Since $\text{area } R_1 = 3 \times \text{area } R_2$, the total area is $4 \times \text{area } R_2$.

5 **Find area R2**

$\text{area } R_2 = \frac{15}{4} = 3.75 \text{ km}^2$.

6 **Find OP from triangle OAP**

$\frac{1}{2}(5)(OP)\sin 1.2 = 3.75$, so $OP \approx 1.609$ km.

7 **Use the cosine rule**

$AP^2 = 5^2 + OP^2 - 2(5)(OP)\cos 1.2$, so $AP \approx 4.665$ km.

8 **Round to the nearest 100 m**

$AP = 4.7 \text{ km}$ to the nearest 0.1 km.

✓ **FINAL ANSWER**

(a) 16 km. (b) 3.75 km^2. (c) $AP = 4.7 \text{ km}$.

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Question 6

9 marks | Graphs of Functions

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{2 - x} \qquad x \neq 2$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the intersection of C with the y-axis

(3)

The straight line l has equation $y = kx - 4$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 - 5k + 4 \geqslant 0$$

(ii) find the range of possible values for k.

(6)

(Total for Question 6 is 9 marks)

WORKED SOLUTION

Question 6 · WMA11/01 October 2024

1 **Identify the graph features**

$y = \dfrac{1}{2 - x}$ has vertical asymptote $x = 2$ and horizontal asymptote $y = 0$.

2 **Find the y-intercept**

At $x = 0$, $y = \dfrac{1}{2}$, so the y-intercept is $(0, \dfrac{1}{2})$.

3 **Set up the intersection with the line**

If $y = kx - 4$ cuts C, then $kx - 4 = \dfrac{1}{2 - x}$.

4 **Form the quadratic**

$(kx-4)(2-x)=1$, so $kx^2-(2k+4)x+9=0$.

5 **Use the discriminant**

For at least one intersection, the discriminant is non-negative.

6 **Derive the given inequality**

$(2k+4)^2-4(k)(9)\geq 0$, so $4(k^2-5k+4)\geq 0$, hence $k^2-5k+4\geq 0$.

7 **Solve the inequality**

$k^2-5k+4=(k-1)(k-4)$, so $k\leq 1$ or $k\geq 4$.

✓ **FINAL ANSWER**

(a) vertical asymptote $x=2$, y-intercept $(0,\frac{1}{2})$.

(b) $k\leq 1$ or $k\geq 4$.

Question 7

8 marks | Trigonometric Functions

7.

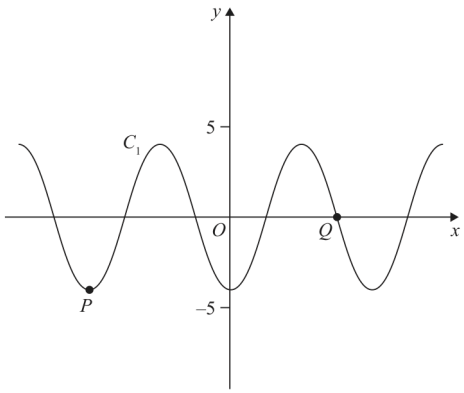


Figure 3

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4\cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

(i) the coordinates of P

(ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4\cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

(b) (i) state the value of k

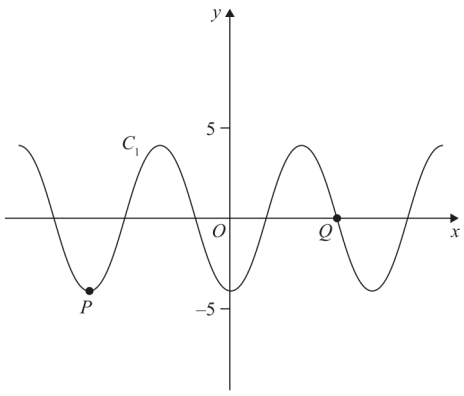
(ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4\cos x = 5 - \frac{10}{\pi}x$$



(Total for Question 7 is 8 marks)

1 **Use y equals negative 4 cos x**

For $y = -4\cos x$, minimum points occur at $x = 2\pi n$ with value -4.

2 **Read P**

The displayed minimum to the left of the y-axis is $P = (-2\pi, -4)$.

3 **Read Q**

The point Q is the positive x-intercept after the maximum at $x = \pi$, so $Q = (\frac{3\pi}{2}, 0)$.

4 **Find k for C2**

For $y = -4\cos x + k$, the maximum value is $4 + k$. Given the maximum is 11, $k = 7$.

5 **Find the smallest positive minimum on C2**

The minimum value is $-4 + 7 = 3$. The first positive minimum is at $x = 2\pi$, so the point is $(2\pi, 3)$.

6 **Interpret the equation graphically**

$-4\cos x = 5 - \frac{10}{\pi}x$ is the intersection of the cosine curve with the straight line $y = 5 - \frac{10}{\pi}x$.

7 **Count intersections**

The line passes through $(0, 5)$ and meets the displayed curve once, at the x-axis point, so there is 1 solution.

✓ **FINAL ANSWER**

$(a)(i)\ P = (-2\pi, -4).$ $(a)(ii)\ Q = (\frac{3\pi}{2}, 0).$ $(b)(i)\ k = 7.$ $(b)(ii)\ (2\pi, 3).$ $(c)\ 1$ solution .

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Question 8

7 marks | Integration

8. A curve C has equation $y = \mathrm{f}(x)$.

The point P with x coordinate 3 lies on C

Given

- $\mathrm{f}'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $\mathrm{f}(x)$.

(4)

(Total for Question 8 is 7 marks)

WORKED SOLUTION

Question 8 · WMA11/01 October 2024

1 Find the y-coordinate of P

The normal is $y = -\frac{1}{24}x + 5$. At $x = 3$, $y = -\frac{3}{24} + 5 = \frac{39}{8}$.

2 Use the normal gradient

The normal gradient is $-\frac{1}{24}$, so the tangent gradient is 24.

3 Use f prime at x equals 3

$f'(3) = 4(3)^2 + 3k + 3 = 39 + 3k$.

4 Find k

$39 + 3k = 24$, so $k = -5$.

5 Integrate f prime

With $k = -5$, $f'(x) = 4x^2 - 5x + 3$, so $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x + c$.

6 Use point P

$\frac{39}{8} = \frac{4}{3}(27) - \frac{5}{2}(9) + 3(3) + c = \frac{45}{2} + c$.

7 **Find c**

$$c = \frac{39}{8} - \frac{45}{2} = -\frac{141}{8}.$$

FINAL ANSWER

$k = -5$ and $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x - \frac{141}{8}$.

Question 9

10 marks | Differentiation

9.

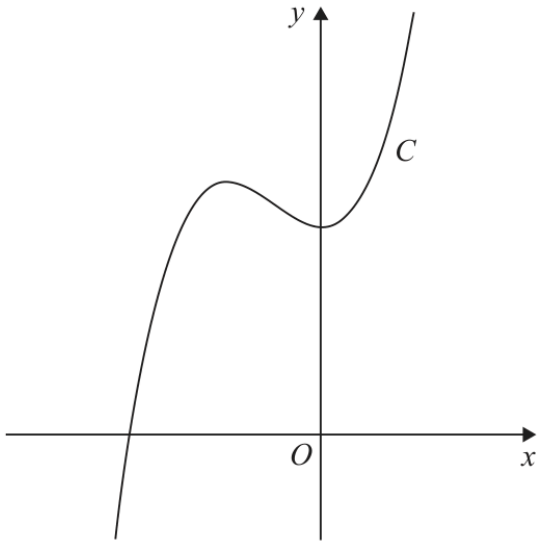


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

(Total for Question 9 is 10 marks)

1 Use the positive quadratic factor

$3x^2 - 4x + 20$ has discriminant $(-4)^2 - 4(3)(20) < 0$ and positive leading coefficient, so it is always positive.

2 Deduce the sign of f(x)

The sign of $f(x) = (x + 5)(3x^2 - 4x + 20)$ is therefore the sign of $x + 5$. Hence $f(x) \geq 0$ for $x \geq -5$.

3 Expand f(x)

$f(x) = 3x^3 + 11x^2 + 100$.

4 Differentiate

$f'(x) = 9x^2 + 22x$.

5 Find the gradient at R

At $R(-4, 84)$, $f'(-4) = 9(16) + 22(-4) = 56$.

6 Use parallel tangents

For the tangent at P to be parallel, $f'(x) = 56$.

7 Solve for x

$9x^2 + 22x - 56 = 0$, so $x = \dfrac{-22 \pm 50}{18}$. The roots are $x = -4$ and $x = \dfrac{14}{9}$.

8 Choose the point P

$x = -4$ is the point R, so $x_P = \dfrac{14}{9}$.

9 Translate point R

$y = f(x - 3)$ translates the curve 3 units to the right, so $(-4, 84)$ maps to $(-1, 84)$.

10 Transform R under y equals 4f(x)

$y = 4f(x)$ stretches vertically by scale factor 4, so $(-4, 84)$ maps to $(-4, 336)$.

FINAL ANSWER

$(a)\ x \geq -5.$ $(b)\ f'(x) = 9x^2 + 22x.$ $(c)\ x_P = \dfrac{14}{9}.$ $(d)(i)\ (-1, 84).$ $(d)(ii)\ (-4, 336).$