Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 May/June 2024 Worked Solutions

May/June 2024 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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PAST PAPER

WMA11/01 May/June 2024

May/June 2024

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

3 marks | Integration

1. Find

$$\int \left(10x^4 - \frac{3}{2x^3} - 7\right) \mathrm{d}x$$

giving each term in simplest form.

(3)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · WMA11/01 May/June 2024

1 **Rewrite the fraction**

$10x^4 - \dfrac{3}{2x^3} - 7 = 10x^4 - \dfrac{3}{2}x^{-3} - 7.$

2 **Integrate term by term**

$\int 10x^4\,dx = 2x^5$, $\int -\dfrac{3}{2}x^{-3}\,dx = \dfrac{3}{4}x^{-2}$, and $\int -7\,dx = -7x$.

3 **Write in simplest form**

$2x^5 + \dfrac{3}{4x^2} - 7x + c.$

✓ **FINAL ANSWER**

$2x^5 + \dfrac{3}{4x^2} - 7x + c.$

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Question 2

6 marks | Indices & Surds

2. (i) Given that $m = 2^n$, express each of the following in simplest form in terms of m.

(a) 2^{n+3}

(1)

(b) 16^{3n}

(2)

(ii) **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$x\sqrt{3} - 3 = x + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(3)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 May/June 2024

1 **Use m equals 2 to the n**

$2^{n+3} = 2^n \times 2^3 = 8m.$

2 **Rewrite 16 as a power of 2**

$16^{3n} = (2^4)^{3n} = 2^{12n} = (2^n)^{12} = m^{12}.$

3 **Collect the x terms**

$x\sqrt{3} - 3 = x + \sqrt{3}$ gives $x(\sqrt{3} - 1) = 3 + \sqrt{3}.$

4 **Rationalise**

$$x = \frac{3 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{(3 + \sqrt{3})(\sqrt{3} + 1)}{2}.$$

5 **Simplify**

$$x = \frac{4\sqrt{3} + 6}{2} = 3 + 2\sqrt{3}.$$

FINAL ANSWER

$(i)(a)\ 8m. \quad (i)(b)\ m^{12}. \quad (ii)\ x = 3 + 2\sqrt{3}.$

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Question 3

6 marks | Transformations

3.

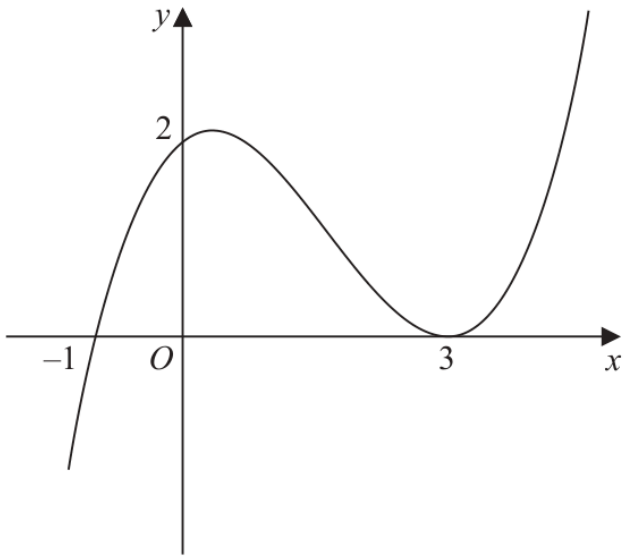


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-1, 0)$ and $(0, 2)$ and touches the x-axis at the point $(3, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · WMA11/01 May/June 2024

1 **Read the original key points**

The original curve passes through $(-1, 0)$ and $(0, 2)$, and touches the x-axis at $(3, 0)$.

2 **Translate the graph left**

$y = f(x + 3)$ is a translation 3 units left. So $(-1, 0)$ moves to $(-4, 0)$, $(0, 2)$ moves to $(-3, 2)$, and $(3, 0)$ moves to $(0, 0)$.

---

3 **State the axis intercepts for part a**

The curve cuts the x-axis at $(-4, 0)$ and touches it at $(0, 0)$. The y-intercept is also $(0, 0)$.

---

4 **Transform y equals f of negative 3x**

For $y = f(-3x)$, a point with original x-coordinate X moves to $x = -\frac{X}{3}$.

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5 **State the axis intercepts for part b**

$(-1, 0)$ moves to $(\frac{1}{3}, 0)$, $(3, 0)$ moves to $(-1, 0)$, and $(0, 2)$ stays at $(0, 2)$.

✓ **FINAL ANSWER**

(a) intercepts $(-4, 0)$ and $(0, 0)$, with $(0, 0)$ a touch point.

(b) intercepts $(-1, 0)$, $(\frac{1}{3}, 0)$ and $(0, 2)$, with $(-1, 0)$ a touch point.

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Question 4

6 marks | Quadratics

4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

(Total for Question 4 is 6 marks)

WORKED SOLUTION

Question 4 · WMA11/01 May/June 2024

1 Equate the curves

$x^2 + kx - 9 = -3x^2 - 5x + k$ gives $4x^2 + (k+5)x - (k+9) = 0$.

2 Use one intersection only

For a single point of intersection, the discriminant is zero.

3 Form the condition on k

$(k+5)^2 - 4(4)(-(k+9)) = 0$, so $k^2 + 26k + 169 = 0$.

4 Find k

$k^2 + 26k + 169 = (k+13)^2$, so $k = -13$.

5 Find x

Substituting $k = -13$ gives $4x^2 - 8x + 4 = 0$, so $4(x-1)^2 = 0$ and $x = 1$.

6 **Find y**

$y = x^2 + kx - 9 = 1 - 13 - 9 = -21$, so $P = (1, -21)$.

✓ **FINAL ANSWER**

$P = (1, -21)$.

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Question 5

7 marks | Radians

5.

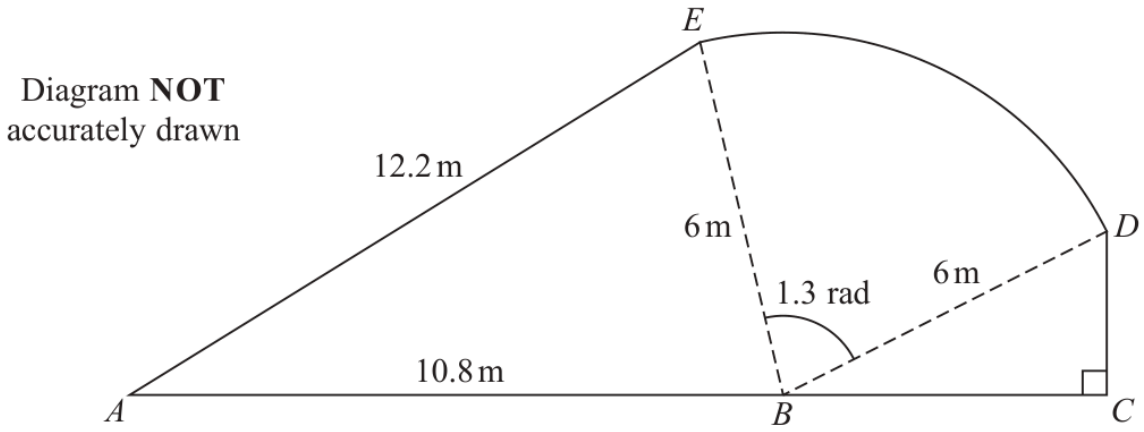


Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden $ABCDEA$ consists of a triangle ABE and a right-angled triangle BCD joined to a sector BDE of a circle with radius 6 m and centre B.

The points A, B and C lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

(a) Find the area of the sector BDE, giving your answer in m^2

(2)

(b) Find the size of angle ABE, giving your answer in radians to 2 decimal places.

(2)

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures.

(3)

(Total for Question 5 is 7 marks)

WORKED SOLUTION

Question 5 · WMA11/01 May/June 2024

1 Find the sector area

Area of sector $BDE = \frac{1}{2}(6^2)(1.3) = 23.4\ \text{m}^2$.

2 Use the cosine rule in triangle ABE

$12.2^2 = 10.8^2 + 6^2 - 2(10.8)(6)\cos\angle ABE$.

3 **Find angle ABE**

$\cos\angle ABE = \dfrac{10.8^2 + 6^2 - 12.2^2}{2(10.8)(6)}$, so $\angle ABE \approx 1.54$ radians.

4 **Find the area of triangle ABE**

$\text{Area } ABE = \dfrac{1}{2}(10.8)(6)\sin(1.54147\ldots) \approx 32.386$.

5 **Find angle DBC**

$\angle DBC = \pi - 1.3 - \angle ABE \approx 0.30012$ radians.

6 **Find the area of triangle BCD**

$\text{Area } BCD = \dfrac{1}{2}(6\cos\angle DBC)(6\sin\angle DBC) \approx 5.084$.

7 **Add the three areas**

$23.4 + 32.386 + 5.084 \approx 60.9\ \text{m}^2$ to 3 significant figures.

✓ **FINAL ANSWER**

$(a)\ 23.4\ \text{m}^2.$ $(b)\ 1.54\ \text{radians}.$ $(c)\ 60.9\ \text{m}^2.$

Question 6

7 marks | Inequalities

6.

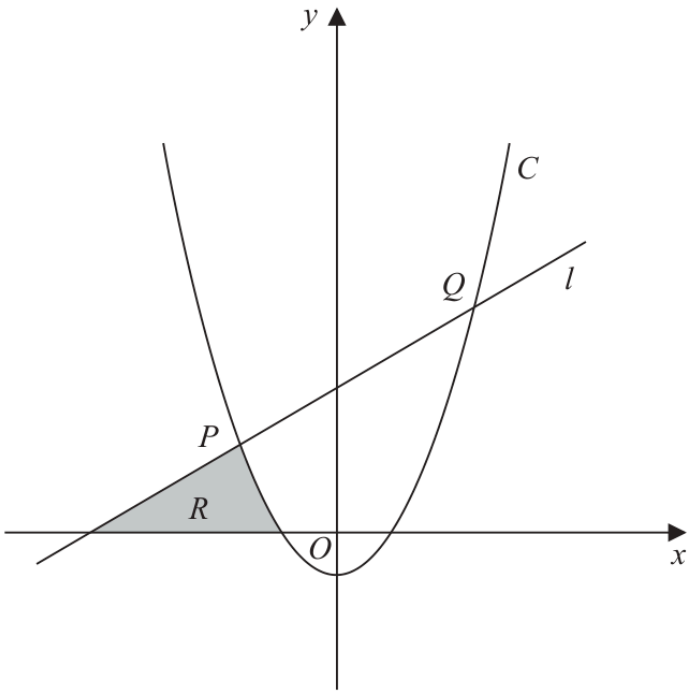


Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(Total for Question 6 is 7 marks)

1 **Write the line as y equals**

$y - 5x = 75$, so $y = 5x + 75$.

2 **Equate line and curve**

$5x + 75 = 2x^2 + x - 21$, so $2x^2 - 4x - 96 = 0$.

3 **Solve the quadratic**

$x^2 - 2x - 48 = 0 = (x - 8)(x + 6)$, so $x = -6$ or $x = 8$.

4 **Find the y-coordinates**

Using $y = 5x + 75$, $x = -6$ gives $y = 45$ and $x = 8$ gives $y = 115$.

5 **State P and Q**

From the diagram, $P = (-6, 45)$ and $Q = (8, 115)$.

6 **Use the x-axis and left branch**

The shaded region is above the x-axis and on the left part of the diagram, so $y \geq 0$ and $x \leq 0$.

7 **Apply both boundaries**

The region is below both the line and the curve, so $y \leq 5x + 75$ and $y \leq 2x^2 + x - 21$.

✓ **FINAL ANSWER**

$(a)\ P = (-6, 45),\ Q = (8, 115).$ $(b)\ y \geq 0,\ y \leq 5x + 75,\ y \leq 2x^2 + x - 21,\ x \leq 0.$

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Question 7

8 marks | Differentiation

7. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $\mathrm{f}'(x)$

(ii) $\mathrm{f}''(x)$

(3)

The curve with equation $y = \mathrm{f}'(x)$ intersects the curve with equation $y = \mathrm{f}''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

(Total for Question 7 is 8 marks)

WORKED SOLUTION

Question 7 · WMA11/01 May/June 2024

1 Differentiate once

$f'(x) = 6x^2 - 2kx + 14$.

2 Differentiate again

$f''(x) = 12x - 2k$.

3 Use the intersection at A

At an intersection of $y = f'(x)$ and $y = f''(x)$, $f'(x) = f''(x)$. Given $x_A = 5$, substitute $x = 5$.

4 Find k

$6(5)^2 - 2k(5) + 14 = 12(5) - 2k$, so $164 - 10k = 60 - 2k$ and $k = 13$.

5 **Find the other x-coordinate**

With $k = 13$, $6x^2 - 26x + 14 = 12x - 26$, so $6x^2 - 38x + 40 = 0$. Hence $(3x - 4)(x - 5) = 0$.

6 **Use the point B**

The other point has $x = \frac{4}{3}$. Then $y = f''(\frac{4}{3}) = 12(\frac{4}{3}) - 26 = -10$.

FINAL ANSWER

$(a)(i)\ f'(x) = 6x^2 - 2kx + 14.$ $(a)(ii)\ f''(x) = 12x - 2k.$ $(b)\ k = 13.$ $(c)\ B = (\frac{4}{3}, -10)$

.

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Question 8

7 marks | Graphs of Functions

8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \frac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

(1)

(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

(Total for Question 8 is 7 marks)

WORKED SOLUTION

Question 8 · WMA11/01 May/June 2024

1 Find the intercepts of C1

$y = x(4 - x^2)$, so $y = 0$ when $x = 0$ or $x = \pm 2$. The curve crosses the axes at $(-2, 0)$, $(0, 0)$ and $(2, 0)$.

2 Use the cubic shape

$y = 4x - x^3$ has negative leading cubic term, so the left tail rises and the right tail falls.

3 Equate C1 and C2

$x(4 - x^2) = \frac{A}{x}$.

4 Multiply by x

$x^2(4 - x^2) = A$, so $4x^2 - x^4 = A$ and therefore $x^4 - 4x^2 + A = 0$.

5 Let u equal x squared

Let $u = x^2$. Then $u^2 - 4u + A = 0$.

6 **Require two positive u-values**

For 4 distinct x-values, the quadratic in u must have two distinct positive roots.

7 **Find the range of A**

The discriminant gives $16 - 4A > 0$, so $A < 4$. The product of the two roots is A, so $A > 0$. Therefore $0 < A < 4$.

✓ **FINAL ANSWER**

(a) x-intercepts $(-2, 0)$, $(0, 0)$, $(2, 0)$.

(b) $x^4 - 4x^2 + A = 0$. (c) $0 < A < 4$.

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Question 9

9 marks | Straight Line

9. Given that

- the point A has coordinates (4, 2)
- the point B has coordinates (15, 7)
- the line l_1 passes through A and B

(a) find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(3)

The line l_2 passes through A and is parallel to the x-axis.

The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

(b) Find both possible pairs of coordinates of the point C.

(4)

(c) Hence find the minimum possible area of triangle ABC.

(2)

(Total for Question 9 is 9 marks)

WORKED SOLUTION

Question 9 · WMA11/01 May/June 2024

1 Find the gradient of l1

$m = \dfrac{7-2}{15-4} = \dfrac{5}{11}$.

2 Use point A

$y - 2 = \dfrac{5}{11}(x - 4)$.

3 Rearrange

$11y - 22 = 5x - 20$, so $5x - 11y + 2 = 0$.

4 Use the horizontal line l2

Since l_2 passes through $A(4, 2)$ and is parallel to the x-axis, point C has coordinates $(c, 2)$.

5 Use BC equals 5 root 5

$(c - 15)^2 + (2 - 7)^2 = (5\sqrt{5})^2$.

6 **Solve for c**

$(c-15)^2+25=125$, so $(c-15)^2=100$ and $c=5$ or 25.

7 **Find the smaller area**

The height from B to $y=2$ is 5. For $C=(5,2)$, $AC=1$, so the minimum area is $\frac{1}{2}(1)(5)=\frac{5}{2}$.

FINAL ANSWER

$(a)\ 5x-11y+2=0.$ $(b)\ C=(5,2)$ or $(25,2).$ $(c)\ \frac{5}{2}.$

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Question 10

10 marks | Integration

10. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(6)

(Total for Question 10 is 10 marks)

WORKED SOLUTION

Question 10 · WMA11/01 May/June 2024

1 Expand the numerator

$(2x-1)(3x+2) = 6x^2 + x - 2$.

2 Simplify f prime

$f'(x) = 6x - \dfrac{6x^2 + x - 2}{2\sqrt{x}} = 6x - 3x^{3/2} - \dfrac{1}{2}x^{1/2} + x^{-1/2}$.

3 Find the tangent gradient at P

$f'(4) = 24 - 3(8) - \dfrac{1}{2}(2) + \dfrac{1}{2} = -\dfrac{1}{2}$.

4 Use the normal gradient

The normal gradient is the negative reciprocal of $-\dfrac{1}{2}$, so it is 2.

5 Find the normal equation

Through $P(4, 12)$, $y - 12 = 2(x - 4)$, hence $y = 2x + 4$.

6 Integrate f prime

$f(x) = 3x^2 - \dfrac{6}{5}x^{5/2} - \dfrac{1}{3}x^{3/2} + 2x^{1/2} + c$.

7 **Use P to find c**

$$12 = 3(4)^2 - \frac{6}{5}(4^{5/2}) - \frac{1}{3}(4^{3/2}) + 2(4^{1/2}) + c.$$

8 **Simplify c**

$$12 = 48 - \frac{192}{5} - \frac{8}{3} + 4 + c = \frac{164}{15} + c, \text{ so } c = \frac{16}{15}.$$

✓ **FINAL ANSWER**

$$(a)\ y = 2x + 4. \quad (b)\ f(x) = 3x^2 - \frac{6}{5}x^{5/2} - \frac{1}{3}x^{3/2} + 2x^{1/2} + \frac{16}{15}.$$

Question 11

6 marks | Trigonometric Functions

11.

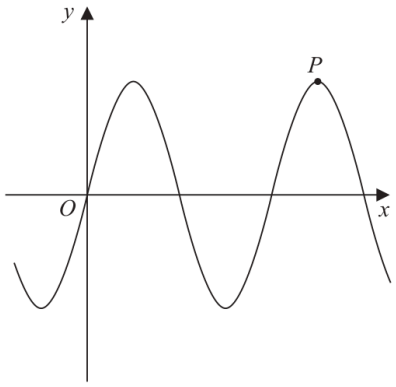


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

1 Use maxima of sine

For $y = 12\sin x$, maxima occur when $x = \frac{\pi}{2} + 2\pi n$, with value 12.

2 Read point P

The maximum point P shown is the second positive maximum, so $P = (\frac{5\pi}{2}, 12)$.

3 Find k for C2

For $y = 12\sin x + k$, the maximum value is $12 + k$. Given the maximum is 3, $k = -9$.

4 Find the smallest positive minimum on C2

The minimum value is $-12 - 9 = -21$, and it first occurs at $x = \frac{3\pi}{2}$. So the point is $(\frac{3\pi}{2}, -21)$.

5 Find A for C3

For $y = 12\sin(x + B)$, any minimum has value -12, so $A = -12$.

6 Find the smallest positive B

At the minimum $x = \frac{\pi}{4}$, so $\frac{\pi}{4} + B = \frac{3\pi}{2}$. Hence $B = \frac{5\pi}{4}$.

✓ **FINAL ANSWER**

$(a)\ P = (\frac{5\pi}{2}, 12).$ $(b)\ (\frac{3\pi}{2}, -21).$ $(c)(i)\ A = -12.$ $(c)(ii)\ B = \frac{5\pi}{4}.$