**Elite Mathematics**
Edexcel International A Level

IAL PURE 1 · WMA11

# WMA11/01 January 2024 Worked Solutions

January 2024 paper rebuilt with the worked solution after each question.

| 10 | 2019–2026 | Single-paper solutions |
|---|---|---|
| paper questions | 19 exam sessions | checked by Dr Eslam |



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PAST PAPER

# WMA11/01 January 2024

January 2024

| 10 | 75 | 1 h 30 |
| --- | --- | --- |
| questions | marks | exam time |

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2024

# Question 1

5 marks | Integration

**1.** Find

$$\int (2x - 5)(3x + 2)(2x + 5)\mathrm{d}x$$

writing your answer in simplest form.

**(5)**

**(Total for Question 1 is 5 marks)**

## WORKED SOLUTION

Question 1 · WMA11/01 January 2024

**1** **Expand two brackets first**

$(2x - 5)(3x + 2) = 6x^2 - 11x - 10$.

**2** **Multiply by the third bracket**

$(6x^2 - 11x - 10)(2x + 5) = 12x^3 + 8x^2 - 75x - 50$.

**3** **Integrate term by term**

$\int (12x^3 + 8x^2 - 75x - 50)\, dx = 3x^4 + \frac{8}{3}x^3 - \frac{75}{2}x^2 - 50x + c$.

✓ **FINAL ANSWER**

$3x^4 + \frac{8}{3}x^3 - \frac{75}{2}x^2 - 50x + c$.

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# Question 2

5 marks | Basic Trigonometry

**2.** The triangle $ABC$ is such that

- $AB = 15\,\text{cm}$
- $AC = 25\,\text{cm}$
- angle $BAC = \theta°$
- area triangle $ABC = 100\,\text{cm}^2$

(a) Find the value of $\sin\theta°$

**(2)**

Given that $\theta > 90$

(b) find the length of $BC$, in cm, to 3 significant figures.

**(3)**

**(Total for Question 2 is 5 marks)**

## WORKED SOLUTION

Question 2 · WMA11/01 January 2024

**1 Use the triangle area formula**

$100 = \frac{1}{2}(15)(25)\sin\theta$.

**2 Find sine theta**

$\sin\theta = \frac{200}{375} = \frac{8}{15}$.

**3 Use theta greater than 90 degrees**

Since $\theta > 90°$, $\cos\theta$ is negative. So $\cos\theta = -\sqrt{1-\left(\frac{8}{15}\right)^2} = -\frac{\sqrt{161}}{15}$.

**4 Apply the cosine rule**

$BC^2 = 15^2 + 25^2 - 2(15)(25)\cos\theta = 850 + 50\sqrt{161}$.

**5 Round the length**

$BC = \sqrt{850 + 50\sqrt{161}} \approx 38.5\ \text{cm}$ to 3 significant figures.

**FINAL ANSWER**

$(a)\ \sin\theta = \dfrac{8}{15}.\quad (b)\ BC = 38.5\ \text{cm}.$

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# Question 3

7 marks | Differentiation

**3.** The curve $C$ has equation

$$y = \frac{5x^3 - 8}{2x^2} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

**(4)**

The point $P(2, 4)$ lies on $C$.

(b) Find an equation for the tangent to $C$ at $P$ writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(3)**

**(Total for Question 3 is 7 marks)**

## WORKED SOLUTION

Question 3 · WMA11/01 January 2024

**1 Rewrite y**

$y = \frac{5x^3 - 8}{2x^2} = \frac{5}{2}x - 4x^{-2}$.

**2 Differentiate**

$\frac{dy}{dx} = \frac{5}{2} + 8x^{-3} = \frac{5}{2} + \frac{8}{x^3}$.

**3 Find the gradient at P**

At $x = 2$, $\frac{dy}{dx} = \frac{5}{2} + \frac{8}{8} = \frac{7}{2}$.

**4 Use point-gradient form**

The tangent through $P(2, 4)$ is $y - 4 = \frac{7}{2}(x - 2)$.

**5 Rearrange**

$2y - 8 = 7x - 14$, so $7x - 2y - 6 = 0$.

**FINAL ANSWER**

$(a)\ \frac{dy}{dx} = \frac{5}{2} + \frac{8}{x^3}$. $(b)\ 7x - 2y - 6 = 0$.

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# Question 4

6 marks | Indices & Surds

**4.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) By substituting $p = 2^x$, show that the equation

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

can be written in the form

$$4p^2 - 33p + 8 = 0$$

**(3)**

(b) Hence solve

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

**(3)**

**(Total for Question 4 is 6 marks)**

## WORKED SOLUTION

Question 4 · WMA11/01 January 2024

**1 Substitute p equals 2 to the x**

If $p = 2^x$, then $4^x = p^2$, $2^{x+3} = 8p$, and $2^{x-1} = \frac{p}{2}$.

**2 Rewrite the equation**

$2p^2 - 8p = 17\left(\frac{p}{2}\right) - 4$.

**3 Clear the fraction**

Multiplying by 2 gives $4p^2 - 16p = 17p - 8$, so $4p^2 - 33p + 8 = 0$.

**4 Solve the quadratic**

$4p^2 - 33p + 8 = (4p - 1)(p - 8)$, so $p = \frac{1}{4}$ or $p = 8$.

**5 Return to x**

$2^x = \frac{1}{4} = 2^{-2}$ gives $x = -2$, and $2^x = 8 = 2^3$ gives $x = 3$.

**FINAL ANSWER**

$x = 3$ or $x = -2$.

# Question 5

10 marks | Straight Line

**5.**

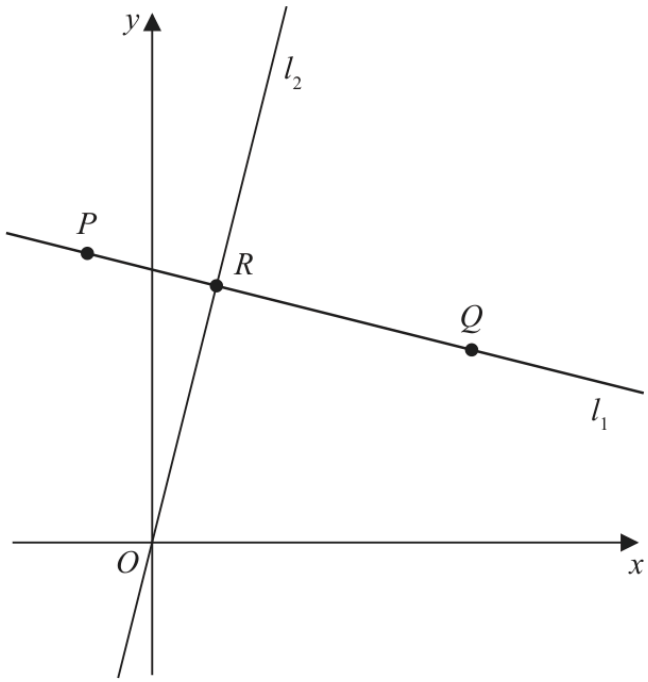



**Figure 1**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

The straight line $l_1$, shown in Figure 1, passes through the points $P(-2, 9)$ and $Q(10, 6)$.

(a) Find the equation of $l_1$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(3)**

The straight line $l_2$ passes through the origin $O$ and is perpendicular to $l_1$

The lines $l_1$ and $l_2$ meet at the point $R$ as shown in Figure 1.

(b) Find the coordinates of $R$

**(4)**

(c) Find the exact area of triangle $OPQ$.

**(3)**

**(Total for Question 5 is 10 marks)**

**1** **Find the gradient of l1**

$m = \dfrac{6-9}{10-(-2)} = -\dfrac{3}{12} = -\dfrac{1}{4}$.

**2** **Find the equation of l1**

Using $P(-2,9)$, $y = -\dfrac{1}{4}x + c$. Then $9 = \dfrac{1}{2} + c$, so $c = \dfrac{17}{2}$.

**3** **Find the equation of l2**

A line perpendicular to gradient $-\dfrac{1}{4}$ has gradient $4$. Since $l_2$ passes through the origin, $l_2$ is $y = 4x$.

**4** **Find R**

At the intersection, $4x = -\dfrac{1}{4}x + \dfrac{17}{2}$. Hence $\dfrac{17}{4}x = \dfrac{17}{2}$, so $x = 2$ and $y = 8$.

**5** **Use coordinate area**

For $O(0,0)$, $P(-2,9)$ and $Q(10,6)$, the area is $\dfrac{1}{2}|(-2)(6) - 9(10)| = 51$.

✓ **FINAL ANSWER**

$(a)\ y = -\dfrac{1}{4}x + \dfrac{17}{2}$. $\quad (b)\ R = (2,8)$. $\quad (c)\ \text{Area} = 51$.

# Question 6

6 marks | Trigonometric Functions

**6.**

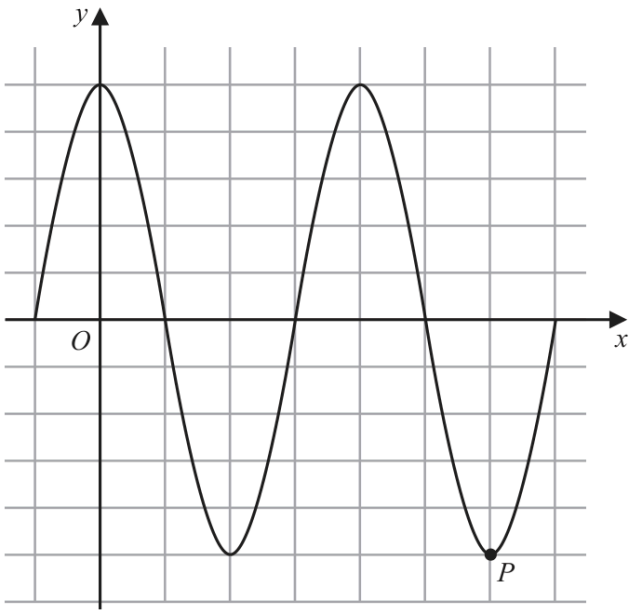



**Figure 2**

Figure 2 shows a plot of part of the curve $C_1$ with equation

$$y = 5\cos x$$

with $x$ being measured in degrees.

The point $P$, shown in Figure 2, is a minimum point on $C_1$

(a) State the coordinates of $P$

**(2)**

The point $Q$ lies on a different curve $C_2$

Given that point $Q$

- is a maximum point on the curve
- is the maximum point with the **smallest** $x$ coordinate, $x > 0$

(b) find the coordinates of $Q$ when

(i) $C_2$ has equation $y = 5\cos x - 2$

(ii) $C_2$ has equation $y = -5\cos x$

**(4)**

**(Total for Question 6 is 6 marks)**

**1** **Use the cosine graph**

For $y = 5\cos x$, the minimum value is $-5$ and it occurs at $x = 180° + 360°n$.

**2** **Read P from the displayed minimum**

The plotted point $P$ is the displayed minimum at $x = 540°$, so $P = (540°, -5)$.

**3** **Translate the graph down**

For $y = 5\cos x - 2$, the maximum value is $5 - 2 = 3$. The smallest positive x-coordinate for a maximum shown is $x = 360°$, so $Q = (360°, 3)$.

**4** **Reflect in the x-axis**

For $y = -5\cos x$, a maximum occurs where $\cos x = -1$, first at $x = 180°$. The maximum value is $5$, so $Q = (180°, 5)$.

✓ **FINAL ANSWER**

$(a)\ P = (540°, -5).$ $(b)(i)\ Q = (360°, 3).$ $(b)(ii)\ Q = (180°, 5).$

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# Question 7

9 marks | Graphs of Functions

**7.** (a) Sketch the graph of the curve $C$ with equation

$$y = \frac{4}{x - k}$$

where $k$ is a positive constant.

Show on your sketch

- the coordinates of any points where $C$ cuts the coordinate axes
- the equation of the vertical asymptote to $C$

**(4)**

Given that the straight line with equation $y = 9 - x$ does not cross or touch $C$

(b) find the range of values of $k$.

**(5)**

**(Total for Question 7 is 9 marks)**

## WORKED SOLUTION

Question 7 · WMA11/01 January 2024

**1 Identify the asymptote**

For $y = \frac{4}{x - k}$, the vertical asymptote is where the denominator is zero, so $x = k$.

**2 Find axis intercepts**

At $x = 0$, $y = -\frac{4}{k}$, so the y-intercept is $(0, -\frac{4}{k})$. The graph has no x-intercept because the numerator is never zero.

**3 Set up intersection with the line**

If the line $y = 9 - x$ meets the curve, then $9 - x = \frac{4}{x - k}$.

**4 Form a quadratic**

$(9 - x)(x - k) = 4$, which simplifies to $x^2 - (9 + k)x + 9k + 4 = 0$.

**5 Use no real intersection**

For the line not to cross or touch the curve, the discriminant must be negative.

**6** **Solve the inequality**

$(9+k)^2 - 4(9k+4) < 0$, so $k^2 - 18k + 65 < 0$. Since $k^2 - 18k + 65 = (k-5)(k-13)$, the range is $5 < k < 13$.

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✓ **FINAL ANSWER**

$(a)$ y-intercept $(0, -\frac{4}{k})$, no x-intercept, vertical asymptote $x = k$.

$(b)$ $5 < k < 13$.

# Question 8

9 marks | Radians

**8.**

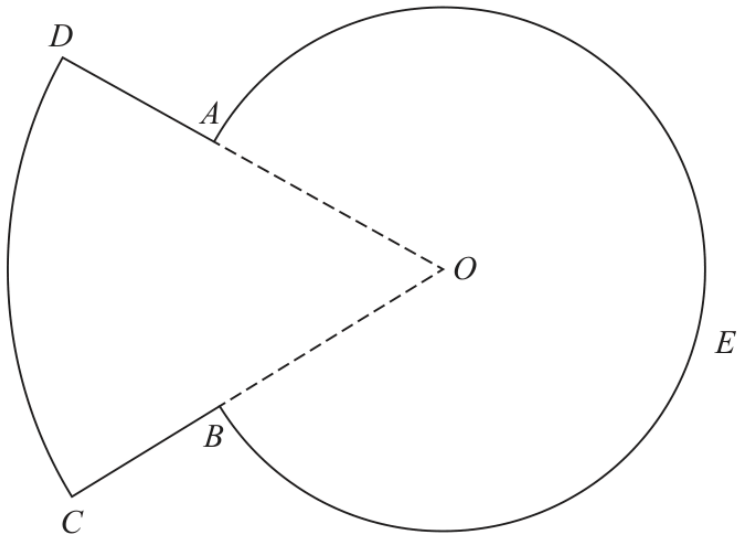



**Figure 3**

Figure 3 shows a sketch of the plan view of a platform.

The plan view of the platform consists of a sector $DOC$ of a circle centre $O$ joined to a sector $AOBEA$ of a different circle, also with centre $O$.

Given that

- angle $AOB = 0.8$ radians
- arc length $CD = 9\,\text{m}$
- $DA : AO = 3 : 5$

(a) show that $AO = 7.03\,\text{m}$ to 3 significant figures.

**(3)**

(b) Find the perimeter of the platform, in m, to 3 significant figures.

**(3)**

(c) Find the total area of the platform, giving your answer in $\text{m}^2$ to the nearest whole number.

**(3)**

**(Total for Question 8 is 9 marks)**

**1 Find the outer radius**

For sector $DOC$, arc length $CD = 9$ and angle $0.8$ radians, so $OD = \dfrac{9}{0.8} = 11.25\ \text{m}$.

**2 Use the ratio DA to AO**

$DA : AO = 3 : 5$, so $OD = DA + AO$ represents $8$ parts. Hence

$AO = \dfrac{5}{8}(11.25) = 7.03125\ \text{m} \approx 7.03\ \text{m}$.

**3 Find the straight side lengths**

$DA = CB = OD - AO = 11.25 - 7.03125 = 4.21875\ \text{m}$.

**4 Find the major arc**

The major sector $AOBEA$ has angle $2\pi - 0.8$, so arc $AEB = 7.03125(2\pi - 0.8)$.

**5 Calculate the perimeter**

$\text{Perimeter} = 9 + 2(4.21875) + 7.03125(2\pi - 0.8) \approx 56.0\ \text{m}$.

**6 Calculate the total area**

$\text{Area} = \dfrac{1}{2}(11.25)^2(0.8) + \dfrac{1}{2}(7.03125)^2(2\pi - 0.8) \approx 186\ \text{m}^2$ to the nearest whole number.

**FINAL ANSWER**

$(a)\ AO = 7.03\ \text{m}.$ $(b)\ 56.0\ \text{m}.$ $(c)\ 186\ \text{m}^2$.

# Question 9

10 marks | Quadratics

**9.** The curve $C_1$ has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- $C_1$ has a maximum turning point at (2, 20)
- $C_1$ passes through the origin

(a) sketch a graph of $C_1$ showing the coordinates of any points where $C_1$ cuts the coordinate axes,

**(2)**

(b) find an expression for $f(x)$.

**(3)**

The curve $C_2$ has equation $y = x(x^2 - 4)$

Curve $C_1$ and $C_2$ meet at the origin, and at the points $P$ and $Q$

Given that the $x$ coordinate of the point $P$ is negative,

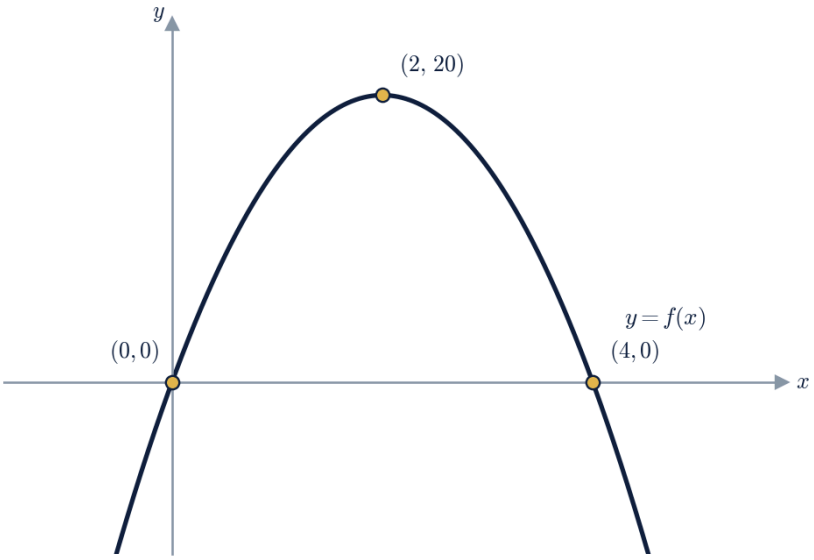
(c) using algebra and showing all stages of your working, find the coordinates of $P$

**(5)**

**(Total for Question 9 is 10 marks)**

**1** **Use symmetry for the sketch**

The maximum is at $(2, 20)$ and the curve passes through $(0, 0)$. Since the axis of symmetry is $x = 2$, the other x-intercept is $(4, 0)$.




**2** **Use completed-square form**

Let $f(x) = a(x - 2)^2 + 20$.

**3** **Use the origin**

$0 = a(0 - 2)^2 + 20$, so $4a = -20$ and $a = -5$.

**4** **Write f(x)**

$f(x) = -5(x - 2)^2 + 20 = -5x^2 + 20x = -5x(x - 4)$.

**5** **Equate the two curves**

$-5x^2 + 20x = x(x^2 - 4) = x^3 - 4x$.

**6** **Factorise**

$x^3 + 5x^2 - 24x = 0$, so $x(x^2 + 5x - 24) = 0$ and $x(x + 8)(x - 3) = 0$.

**7** **Use the negative x-coordinate**

The negative x-coordinate is $x = -8$.

**8** **Find y**

$y = x(x^2 - 4) = (-8)(64 - 4) = -480$, so $P = (-8, -480)$.

**FINAL ANSWER**

$(a)$ downward parabola through $(0, 0)$ and $(4, 0)$ with maximum $(2, 20)$.

$(b)$ $f(x) = -5x(x - 4)$.

$(c)$ $P = (-8, -480)$.

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# Question 10

8 marks | Integration

**10.** **In this question you must show all stages of your working.**

The curve $C$ has equation $y = f(x), x > 0$

Given that

- the point $P\left(2, 8\sqrt{2}\right)$ lies on $C$
- $f'(x) = 4\sqrt{x^3} + \frac{k}{x^2}$ where $k$ is a constant
- $f''(x) = 0$ at $P$

(a) find the exact value of $k$,

**(4)**

(b) find $f(x)$, giving your answer in simplest form.

**(4)**

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**(Total for Question 10 is 8 marks)**

## WORKED SOLUTION

Question 10 · WMA11/01 January 2024

**1** **Differentiate f prime**

$f'(x) = 4x^{3/2} + kx^{-2}$, so $f''(x) = 6x^{1/2} - 2kx^{-3}$.

**2** **Use f''(P) = 0**

At $P$, $x = 2$ and $f''(2) = 0$, so $6\sqrt{2} - 2k(2^{-3}) = 0$.

**3** **Find k**

$6\sqrt{2} - \frac{k}{4} = 0$, hence $k = 24\sqrt{2}$.

**4** **Integrate f prime**

$f(x) = \int \left(4x^{3/2} + 24\sqrt{2}x^{-2}\right)dx = \frac{8}{5}x^{5/2} - 24\sqrt{2}x^{-1} + c$.

**5** **Use the point P**

$P(2, 8\sqrt{2})$ lies on $C$, so $8\sqrt{2} = \frac{8}{5}(2^{5/2}) - \frac{24\sqrt{2}}{2} + c$.

**6** **Find c**

$8\sqrt{2} = \frac{32\sqrt{2}}{5} - 12\sqrt{2} + c$, so $c = \frac{68\sqrt{2}}{5}$.

✓ **FINAL ANSWER**

$(a)\ k = 24\sqrt{2}.$ $(b)\ f(x) = \frac{8}{5}x^{5/2} - \frac{24\sqrt{2}}{x} + \frac{68\sqrt{2}}{5}.$