Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 October 2023 Worked Solutions

October 2023 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 October 2023

October 2023

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

5 marks | Differentiation

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(2)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · WMA11/01 October 2023

1 Rewrite the powers

$y = 5x^3 + 3x^{-2} - 7x$.

2 Differentiate once

$$\frac{dy}{dx} = 15x^2 - 6x^{-3} - 7 = 15x^2 - \frac{6}{x^3} - 7.$$

3 Differentiate again

$$\frac{d^2y}{dx^2} = 30x + 18x^{-4} = 30x + \frac{18}{x^4}.$$

FINAL ANSWER

$\dfrac{dy}{dx} = 15x^2 - \dfrac{6}{x^3} - 7$, and $\dfrac{d^2y}{dx^2} = 30x + \dfrac{18}{x^4}$.

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Question 2

4 marks | Indices & Surds

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\frac{16}{b^3}$

(1)

(c) $\left(\frac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · WMA11/01 October 2023

1 Square root a

$a^{1/2} = \left(\frac{1}{64}x^2\right)^{1/2} = \frac{1}{8}x.$

2 Cube b

$b = 16x^{-1/2}$, so $b^3 = 4096x^{-3/2}$.

3 Find 16 divided by b cubed

$\frac{16}{b^3} = \frac{16}{4096x^{-3/2}} = \frac{1}{256}x^{3/2}.$

4 Simplify ab over 2

$\frac{ab}{2} = \frac{1}{2}\left(\frac{1}{64}x^2\right)\left(16x^{-1/2}\right) = \frac{1}{8}x^{3/2}.$

5 Raise to the power -4 over 3

$\left(\frac{1}{8}x^{3/2}\right)^{-4/3} = 8^{4/3}x^{-2} = 16x^{-2}.$

FINAL ANSWER

$(a)\ \frac{1}{8}x.\quad (b)\ \frac{1}{256}x^{3/2}.\quad (c)\ 16x^{-2}.$

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Question 3

6 marks | Indices & Surds

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where a and b are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}$$

giving your answer in simplest form.

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · WMA11/01 October 2023

1 **Rationalise the denominator**

$$\frac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}\times\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}}=\frac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{12-5}.$$

2 **Simplify the numerator**

$$(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})=16\sqrt{3}-8\sqrt{5}-3\sqrt{5}+5\sqrt{3}=21\sqrt{3}-14\sqrt{5}.$$

3 **Divide by 7**

$$\frac{21\sqrt{3}-14\sqrt{5}}{7}=3\sqrt{3}-2\sqrt{5}.$$

4 **Rearrange the equation**

$(x+5\sqrt{3})\sqrt{5}=40-2x\sqrt{3}$ gives $x(\sqrt{5}+2\sqrt{3})=40-5\sqrt{15}$.

5 **Use the result from part (a)**

$$x=\frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}=5\left(3\sqrt{3}-2\sqrt{5}\right).$$

FINAL ANSWER

$(a)\ 3\sqrt{3}-2\sqrt{5}.\quad (b)\ x=15\sqrt{3}-10\sqrt{5}.$

Question 4

7 marks | Graphs of Functions

4.

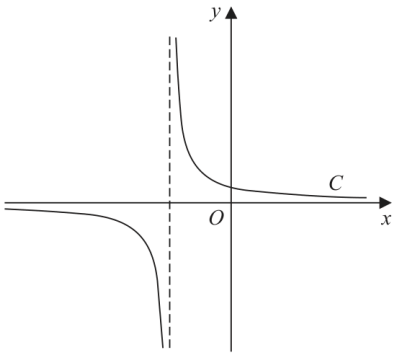


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y-axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x + 2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

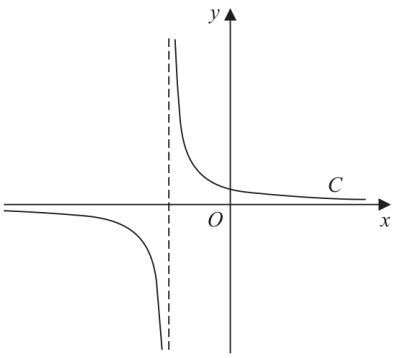
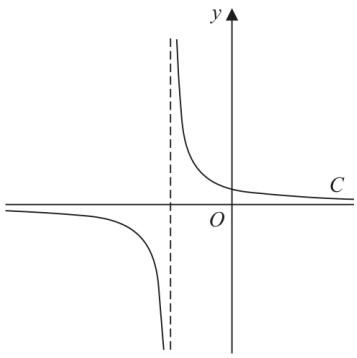


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(Total for Question 4 is 7 marks)

1 Find the vertical asymptote

For $y = \dfrac{1}{x+2}$, the denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$.

2 Factorise the cubic

$x^3 + 4x^2 + 4x = x(x^2 + 4x + 4) = x(x+2)^2$.

3 Use the factorised form for the sketch

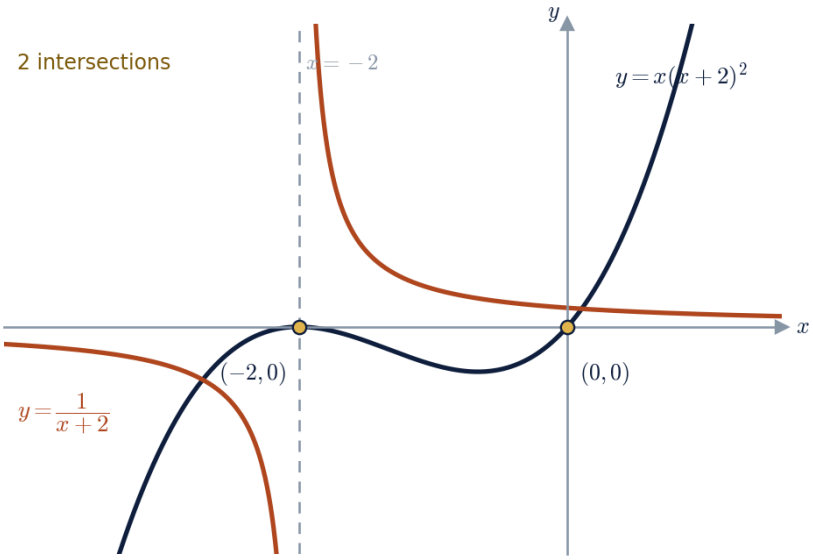
$x = 0$ is a single root, so the curve crosses the x-axis at $(0, 0)$. $x = -2$ is a repeated root, so the curve touches the x-axis at $(-2, 0)$.

4 Connect the equation to intersections

$(x+2)(x^3 + 4x^2 + 4x) = 1$ is equivalent to $x^3 + 4x^2 + 4x = \dfrac{1}{x+2}$.

5 Count the intersections

The number of real solutions is the number of intersections of the cubic $y = x(x+2)^2$ and the reciprocal curve $y = \dfrac{1}{x+2}$. The sketches show 2 intersections.



FINAL ANSWER

(a) $x = -2$. (b) $x(x+2)^2$. (c) roots at $(-2, 0)$ and $(0, 0)$. $x = -2$ is a repeated root.
(d) 2 real roots.

Question 5

7 marks | Basic Trigonometry

5.

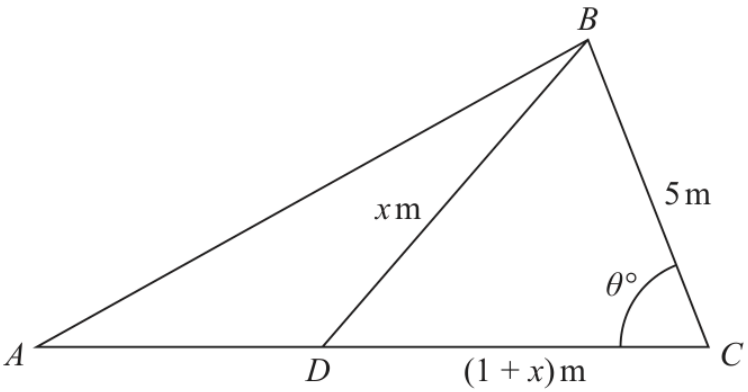


Diagram **NO** accurately dr

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle ABD joined to triangle BCD.

Given that

- $BD = x\,\text{m}$
- $CD = (1 + x)\,\text{m}$
- $BC = 5\,\text{m}$
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- ADC is a straight line

(b) find the area of triangle ABC, giving your answer, in m^2, to one decimal place.

(5)

(Total for Question 5 is 7 marks)

1 **Apply the cosine rule in triangle BCD**

$x^2 = 5^2 + (1+x)^2 - 2(5)(1+x)\cos\theta$.

2 **Rearrange for cos theta**

$10(1+x)\cos\theta = 25 + (1+x)^2 - x^2 = 26 + 2x$, so $\cos\theta = \dfrac{26+2x}{10(1+x)} = \dfrac{13+x}{5+5x}$.

3 **Substitute x equals 2 root 3**

$\cos C = \dfrac{13+2\sqrt{3}}{5+10\sqrt{3}}$, so $C \approx 42.47°$.

4 **Use angle sum**

$A = 30°$, so $B = 180° - 30° - C \approx 107.53°$.

5 **Use the sine rule for the area**

Since $BC = 5$ and $A = 30°$, $\dfrac{BC}{\sin A} = 10$. Hence $AB = 10\sin C$ and $AC = 10\sin B$.

6 **Calculate the area**

$\text{Area} = \dfrac{1}{2}(AB)(AC)\sin A = 50\sin C\sin B \approx 32.2\ \text{m}^2$.

✓ **FINAL ANSWER**

$(a)\ \cos\theta = \dfrac{13+x}{5+5x}$. $(b)\ \text{Area} = 32.2\ \text{m}^2$.

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Question 6

6 marks | Quadratics

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p-2x)=\frac{12+15p}{x+p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2-10p-8>0$$

(3)

(b) Hence, using algebra, find the range of possible values of p

(3)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · WMA11/01 October 2023

1 **Clear the denominator**

$4(p-2x)=\dfrac{12+15p}{x+p}$ gives $4(p-2x)(x+p)=12+15p$.

2 **Form a quadratic in x**

Expanding gives $4p^2-4px-8x^2=12+15p$, so $8x^2+4px+12+15p-4p^2=0$.

3 **Use the discriminant condition**

For two distinct real roots, $b^2-4ac>0$.

4 **Simplify the inequality**

$(4p)^2-4(8)(12+15p-4p^2)>0$, so $144p^2-480p-384>0$.

5 **Divide and factorise**

Dividing by 48 gives $3p^2-10p-8>0$, and $3p^2-10p-8=(3p+2)(p-4)$.

6 State the range

The upward-opening quadratic is positive outside its roots, so $p < -\frac{2}{3}$ or $p > 4$.

FINAL ANSWER

$p < -\frac{2}{3}$ or $p > 4$.

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Question 7

10 marks | Integration

7. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$.

(6)

(Total for Question 7 is 10 marks)

WORKED SOLUTION

Question 7 · WMA11/01 October 2023

1 Simplify f prime

$$f'(x) = \frac{4x^2 + 10 - 7x^{1/2}}{4x^{1/2}} = x^{3/2} + \frac{5}{2}x^{-1/2} - \frac{7}{4}.$$

2 Find the gradient at P

At $x = 4$, $f'(4) = 4^{3/2} + \frac{5}{2}(4^{-1/2}) - \frac{7}{4} = 8 + \frac{5}{4} - \frac{7}{4} = \frac{15}{2}$.

3 Find the normal gradient

The normal gradient is the negative reciprocal, so $m_n = -\frac{2}{15}$.

4 Use point P

Using $P(4, -1)$, $y + 1 = -\frac{2}{15}(x - 4)$.

5 Write the normal in integer form

$15y + 15 = -2x + 8$, hence $2x + 15y + 7 = 0$.

6 **Integrate f prime**

$$f(x)=\int\left(x^{3/2}+\frac{5}{2}x^{-1/2}-\frac{7}{4}\right)dx=\frac{2}{5}x^{5/2}+5x^{1/2}-\frac{7}{4}x+c.$$

7 **Use P to find c**

$-1=\frac{2}{5}(4^{5/2})+5(4^{1/2})-\frac{7}{4}(4)+c=\frac{64}{5}+10-7+c$, so $c=-\frac{84}{5}$.

✓ **FINAL ANSWER**

$(a)(i)\ \frac{15}{2}.\quad (a)(ii)\ 2x+15y+7=0.\quad (b)\ f(x)=\frac{2}{5}x^{5/2}+5x^{1/2}-\frac{7}{4}x-\frac{84}{5}.$

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Question 8

7 marks | Simultaneous Equations

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0$$

(2)

Given that C_1 and C_2 meet at points P and Q

(b) find, using algebra, the exact distance PQ

(5)

(Total for Question 8 is 7 marks)

WORKED SOLUTION

Question 8 · WMA11/01 October 2023

1 **Write C1 in terms of y**

$xy = \frac{15}{2} - 5x$, so $y = \frac{15}{2x} - 5$.

2 **Equate the two curves**

$\frac{15}{2x} - 5 = x^3 - \frac{7}{2}x - 5$.

3 **Multiply by 2x**

$15 = 2x^4 - 7x^2$, hence $2x^4 - 7x^2 - 15 = 0$.

4 **Solve for x**

Let $u = x^2$. Then $2u^2 - 7u - 15 = 0 = (2u + 3)(u - 5)$, so $x^2 = 5$ and $x = \pm\sqrt{5}$.

5 **Find the y-coordinates**

Using $y = \dfrac{15}{2x} - 5$, the points are $\left(\sqrt{5}, \dfrac{3\sqrt{5}}{2} - 5\right)$ and $\left(-\sqrt{5}, -\dfrac{3\sqrt{5}}{2} - 5\right)$.

6 **Use the distance formula**

$\Delta x = 2\sqrt{5}$ and $\Delta y = 3\sqrt{5}$, so $PQ = \sqrt{(2\sqrt{5})^2 + (3\sqrt{5})^2} = \sqrt{65}$.

✓ **FINAL ANSWER**

$(a)\ 2x^4 - 7x^2 - 15 = 0.$ $(b)\ PQ = \sqrt{65}.$

Question 9

7 marks | Radians

9. Diagram **NO** accurately dr

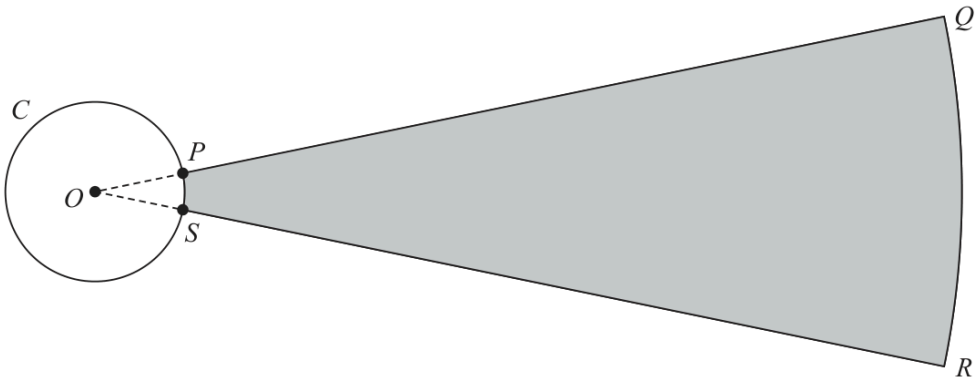


Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2\,\text{m}$

(1)

Given also that

- the target area, $PQRS$, is $90\,\text{m}^2$
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)

(Total for Question 9 is 7 marks)

1 Use arc length

$s = r\theta$, so $0.72 = OP \times 0.6$. Hence $OP = 1.2\ \text{m}$.

2 Set up the target area

The outer sector has radius $x + 1.2$ and the inner sector has radius 1.2, with angle 0.6 radians.

3 Use sector area

$\frac{1}{2}(x+1.2)^2(0.6) - \frac{1}{2}(1.2)^2(0.6) = 90$.

4 Simplify to the given quadratic

$0.3\left((x+1.2)^2 - 1.44\right) = 90$, so $0.3x^2 + 0.72x - 90 = 0$. Multiplying by $\frac{50}{3}$ gives $5x^2 + 12x - 1500 = 0$.

5 Find the positive value of x

$$x = \frac{-12 + \sqrt{12^2 + 4(5)(1500)}}{10} = \frac{-6 + 4\sqrt{471}}{5} \approx 16.16.$$

6 Find the perimeter

$\text{Perimeter} = PQ + SR + \text{arc } QR + \text{arc } PS = x + x + 0.6(x + 1.2) + 0.72 \approx 43.46\ \text{m}$.

7 Round to the nearest metre

The total perimeter is $43\ \text{m}$ to the nearest metre.

FINAL ANSWER

$(a)\ OP = 1.2\ \text{m}.$ $(b)\ 5x^2 + 12x - 1500 = 0.$ $(c)\ 43\ \text{m}.$

Question 10

6 marks | Trigonometric Functions

10.

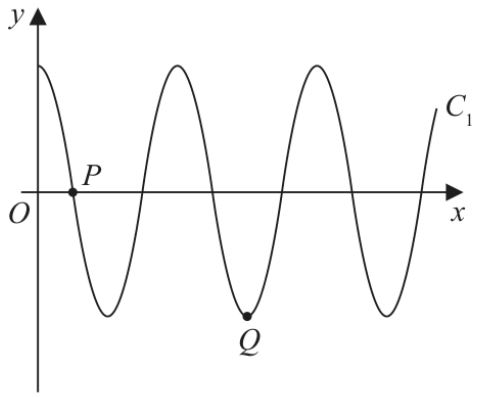


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3\cos\left(\frac{x}{n}\right)^\circ \qquad x \geqslant 0$$

where n is a constant.

The curve C_1 cuts the positive x-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n

(ii) State the period of C_1

(2)

The point Q, shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q.

(2)

The curve C_2 has equation $y = 2\sin x^\circ + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k.

(2)

(Total for Question 10 is 6 marks)

1 Use the first positive x-intercept

For $y = 3\cos\left(\frac{x}{n}\right)^\circ$, the first positive zero occurs when $\frac{x}{n} = 90°$.

2 Find n

Since $P = (270, 0)$, $\frac{270}{n} = 90$, so $n = 3$.

3 Find the period

The period is $360n = 360(3) = 1080°$.

4 Read the shown minimum point

A minimum occurs when $\frac{x}{3} = 180° + 360°k$. The point Q shown is the next displayed minimum after one full period, so $x = 1620$ and $y = -3$.

5 Use the two points on C2

For $y = 2\sin x° + k$, the points give $2\sin a + k = \frac{12}{5}$ and $-2\sin a + k = -\frac{3}{5}$.

6 Add the equations

Adding gives $2k = \frac{12}{5} - \frac{3}{5} = \frac{9}{5}$, so $k = \frac{9}{10}$.

FINAL ANSWER

$(a)(i)\ n = 3.$ $(a)(ii)\ 1080°.$ $(b)\ Q = (1620, -3).$ $(c)\ k = \frac{9}{10}.$

Question 11

10 marks | Quadratics

11.

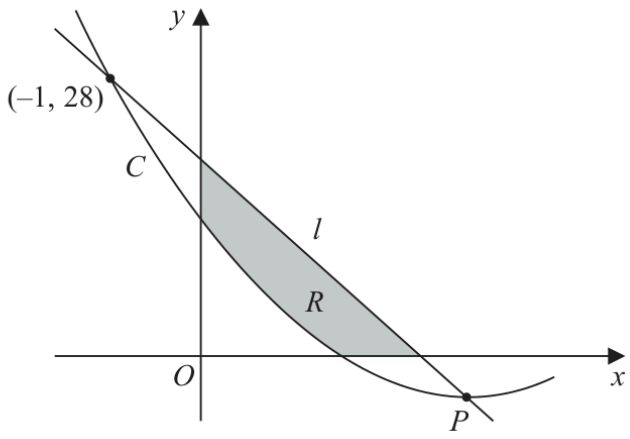


Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

(Total for Question 11 is 10 marks)

1 **Complete the square**

$2x^2 - 12x + 14 = 2(x^2 - 6x) + 14 = 2\left((x-3)^2 - 9\right) + 14 = 2(x-3)^2 - 4$.

2 **State the minimum point**

The minimum of $2(x-3)^2 - 4$ is at $P = (3, -4)$.

3 **Find the gradient of l**

The line passes through $(-1, 28)$ and $P(3, -4)$, so $m = \dfrac{-4 - 28}{3 - (-1)} = \dfrac{-32}{4} = -8$.

4 **Find the equation of l**

Using $P(3, -4)$, $y + 4 = -8(x - 3)$, so $y = -8x + 20$.

5 **Use the axes as boundaries**

The region is to the right of the y-axis and above the x-axis, so $x \geq 0$ and $y \geq 0$.

6 **Use the line and curve as boundaries**

The region is below the line and above the curve, so $y \leq -8x + 20$ and $y \geq 2x^2 - 12x + 14$.

✓ **FINAL ANSWER**

$(a)\ 2(x-3)^2 - 4$. $(b)\ P = (3, -4)$. $(c)\ y = -8x + 20$. $(d)\ x \geq 0,\ y \geq 0,\ y \leq -8x + 20,\ y \geq 2x^2 - 12x + 14$.