Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 May/June 2023 Worked Solutions

May/June 2023 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 May/June 2023

May/June 2023

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 MAY/JUNE 2023

Question 1

4 marks | Inequalities

1. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the inequality

$$4x^2 - 3x + 7 \geqslant 4x + 9$$

(4)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · WMA11/01 May/June 2023

1 **Move all terms to one side**

$4x^2 - 3x + 7 \geq 4x + 9$ gives $4x^2 - 7x - 2 \geq 0$.

2 **Factorise**

$4x^2 - 7x - 2 = (4x + 1)(x - 2)$.

3 **Use the sign of the quadratic**

The roots are $x = -\frac{1}{4}$ and $x = 2$. Since the quadratic opens upwards, it is non-negative outside the roots.

FINAL ANSWER

$x \leq -\frac{1}{4}$ or $x \geq 2$.

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Question 2

6 marks | Quadratics

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A rectangular sports pitch has length x metres and width y metres, where $x > y$

Given that the perimeter of the pitch is 350 m,

(a) write down an equation linking x and y

(1)

Given also that the area of the pitch is $7350\,\text{m}^2$

(b) write down a second equation linking x and y

(1)

(c) hence find the value of x and the value of y

(4)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 May/June 2023

1 **Use the perimeter**

$2x + 2y = 350$, so $x + y = 175$.

2 **Use the area**

$xy = 7350$.

3 **Form a quadratic**

Using $y = 175 - x$, $x(175 - x) = 7350$, so $x^2 - 175x + 7350 = 0$.

4 **Solve**

$x = \dfrac{175 \pm \sqrt{175^2 - 4(7350)}}{2} = \dfrac{175 \pm 35}{2}$, giving $x = 105$ or 70.

5 **Use x > y**

Since $x > y$, $x = 105$ and $y = 70$.

FINAL ANSWER

$x = 105,\ y = 70.$

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Question 3

6 marks | Quadratics

3. (a) Express $3x^2 + 12x + 13$ in the form

$$a(x + b)^2 + c$$

where a, b and c are integers to be found.

(3)

(b) Hence sketch the curve with equation $y = 3x^2 + 12x + 13$

On your sketch show clearly

- the coordinates of the y intercept
- the coordinates of the turning point of the curve

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · WMA11/01 May/June 2023

1 Complete the square

$3x^2 + 12x + 13 = 3(x^2 + 4x) + 13$.

2 Use (x + 2)^2

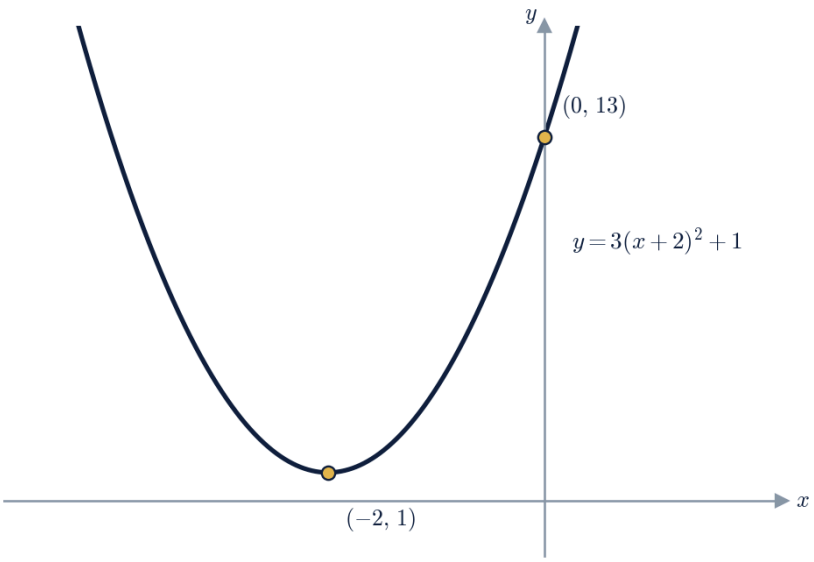
$x^2 + 4x = (x + 2)^2 - 4$, so $3(x^2 + 4x) + 13 = 3(x + 2)^2 - 12 + 13$.

3 Read the features

$y = 3(x + 2)^2 + 1$ has turning point $(-2, 1)$. The y-intercept is found from $x = 0$, giving $y = 13$.

4 Sketch description

The curve is an upward-opening parabola with minimum $(-2, 1)$ and y-intercept $(0, 13)$.



FINAL ANSWER

(a) $3(x+2)^2+1$. (b) y-intercept $(0, 13)$ and turning point $(-2, 1)$.

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Question 4

7 marks | Differentiation

4. **In this question you must show all stages of your working.**

(a) Write

$$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}}$$

in the form

$$y = Ax^p + Bx^q$$

where A, B, p and q are constants to be found.

(4)

(b) Hence find $\frac{dy}{dx}$ giving each coefficient in simplest form.

(3)

(Total for Question 4 is 7 marks)

WORKED SOLUTION

Question 4 · WMA11/01 May/June 2023

1 **Rewrite the denominator**

$\sqrt[3]{8x} = 2x^{1/3}$.

2 **Rewrite the numerator**

$\sqrt{x^3} = x^{3/2}$.

3 **Divide each term**

$$y = \frac{5x^2}{2x^{1/3}} + \frac{x^{3/2}}{2x^{1/3}} = \frac{5}{2}x^{5/3} + \frac{1}{2}x^{7/6}.$$

4 **Differentiate**

$$\frac{dy}{dx} = \frac{5}{2}\cdot\frac{5}{3}x^{2/3} + \frac{1}{2}\cdot\frac{7}{6}x^{1/6}.$$

5 **Simplify coefficients**

$$\frac{dy}{dx} = \frac{25}{6}x^{2/3} + \frac{7}{12}x^{1/6}.$$

FINAL ANSWER

$(a)\ y = \frac{5}{2}x^{5/3} + \frac{1}{2}x^{7/6}.$ $(b)\ \frac{dy}{dx} = \frac{25}{6}x^{2/3} + \frac{7}{12}x^{1/6}.$

Question 5

10 marks | Radians

5.

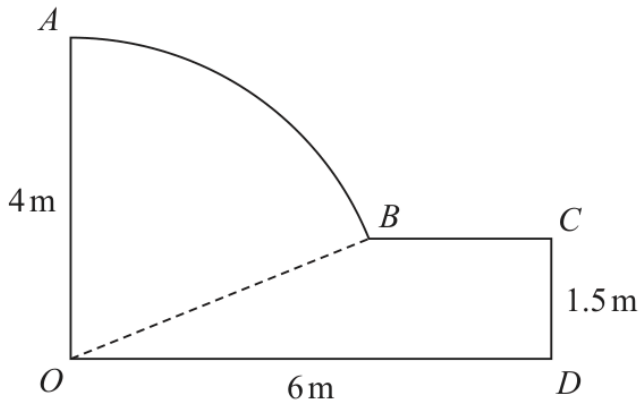


Figure 1

Figure 1 shows the plan for a garden.

In the plan

- OA and CD are perpendicular to OD
- AB is an arc of the circle with centre O and radius 4 metres
- BC is parallel to OD
- OD is 6 metres, OA is 4 metres and CD is 1.5 metres

(a) Show that angle AOB is 1.186 radians to 4 significant figures.

(2)

(b) Find the perimeter of the garden, giving your answer in metres to 3 significant figures.

(4)

(c) Find the area of the garden, giving your answer in square metres to 3 significant figures.

(4)

(Total for Question 5 is 10 marks)

1 Find the angle AOB

Since $OB = 4$ and the height of B is 1.5, $\cos \angle AOB = \dfrac{1.5}{4} = 0.375$.

2 Calculate the angle

$\angle AOB = \cos^{-1}(0.375) = 1.186\ldots$ rad, which is 1.186 rad to 4 significant figures.

3 Find BC

The horizontal coordinate of B is $4\sin(1.186\ldots) = 3.708\ldots$, so $BC = 6 - 3.708\ldots = 2.2919\ldots$ m.

4 Find the arc length

Arc $AB = r\theta = 4(1.186\ldots) = 4.7456\ldots$ m.

5 Find the perimeter

Perimeter $= OA + \text{arc } AB + BC + CD + OD = 4 + 4.7456\ldots + 2.2919\ldots + 1.5 + 6 = 18.537\ldots$ m.

6 Find the area

Area of sector $AOB = \dfrac{1}{2}(4^2)(1.186\ldots) = 9.491\ldots$.

7 Add the quadrilateral area

Area of $OBCD$ is the trapezium area $\dfrac{1}{2}(6 + 2.2919\ldots)(1.5) = 6.2189\ldots$. Total area $= 15.710\ldots = 15.7\ \text{m}^2$.

✓ **FINAL ANSWER**

$(a)\ \angle AOB = 1.186\ \text{rad}.$ $(b)\ 18.5\ \text{m}.$ $(c)\ 15.7\ \text{m}^2.$

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Question 6

7 marks | Indices & Surds

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Expand and simplify

$$\left(r - \frac{1}{r}\right)^2$$

(2)

(b) Express $\dfrac{1}{3 + 2\sqrt{2}}$ in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

(c) Use the results of parts (a) and (b), or otherwise, to show that

$$\sqrt{3 + 2\sqrt{2}} - \frac{1}{\sqrt{3 + 2\sqrt{2}}} = 2$$

(3)

(Total for Question 6 is 7 marks)

WORKED SOLUTION

Question 6 · WMA11/01 May/June 2023

1 **Expand the square**

$\left(r - \dfrac{1}{r}\right)^2 = r^2 - 2 + \dfrac{1}{r^2}$.

2 **Rationalise the denominator**

$\dfrac{1}{3+2\sqrt{2}} \times \dfrac{3-2\sqrt{2}}{3-2\sqrt{2}} = \dfrac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$.

3 **Let r be the square root**

Let $r = \sqrt{3+2\sqrt{2}}$. Then $r^2 = 3 + 2\sqrt{2}$ and $\dfrac{1}{r^2} = 3 - 2\sqrt{2}$.

4 **Use part (a)**

$\left(r - \dfrac{1}{r}\right)^2 = r^2 - 2 + \dfrac{1}{r^2} = (3 + 2\sqrt{2}) - 2 + (3 - 2\sqrt{2}) = 4$.

5 **Take the positive square root**

Since $r > 1$, $r - \dfrac{1}{r} > 0$, so $r - \dfrac{1}{r} = 2$.

✓ **FINAL ANSWER**

$(a)\ r^2 - 2 + \dfrac{1}{r^2}.$ $(b)\ 3 - 2\sqrt{2}.$ (c) The expression equals 2.

Question 7

6 marks | Inequalities

7.

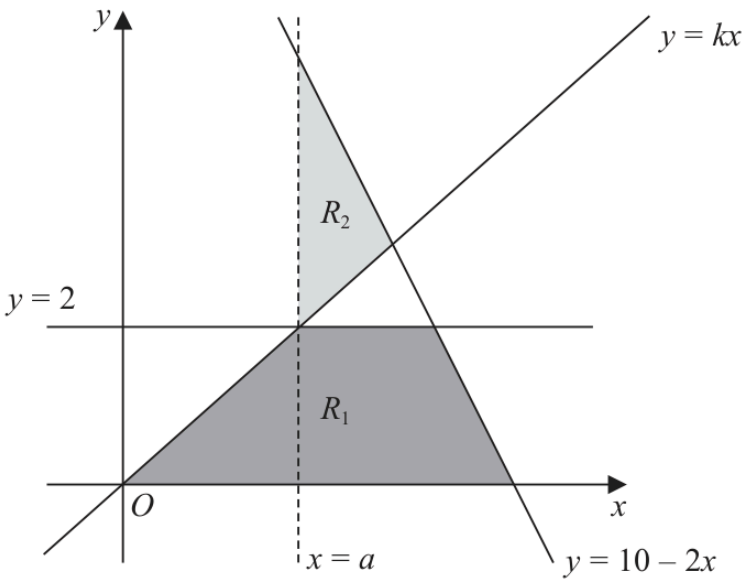


Figure 2

The region R_1, shown shaded in Figure 2, is defined by the inequalities

$$0 \leqslant y \leqslant 2 \qquad y \leqslant 10 - 2x \qquad y \leqslant kx$$

where k is a constant.

The line $x = a$, where a is a constant, passes through the intersection of the lines $y = 2$ and $y = kx$

Given that the area of R_1 is $\frac{27}{4}$ square units,

(a) find

(i) the value of a

(ii) the value of k

(4)

(b) Define the region R_2, also shown shaded in Figure 2, using inequalities.

(2)

(Total for Question 7 is 6 marks)

1 **Find the area of R1**

The bottom horizontal length is 5, from $x = 0$ to the intercept of $y = 10 - 2x$ at $x = 5$.

2 **Use the top length**

The top horizontal length is $4 - a$, because $y = 2$ meets $y = 10 - 2x$ at $x = 4$.

3 **Solve for a**

Area $R_1 = \frac{1}{2}(5 + 4 - a)(2) = 9 - a = \frac{27}{4}$, so $a = \frac{9}{4}$.

4 **Find k**

The line $x = a$ passes through the intersection of $y = 2$ and $y = kx$, so $2 = ka$. Hence $k = \frac{2}{9/4} = \frac{8}{9}$.

5 **Define R2**

Region R_2 is to the right of $x = a$, above $y = kx$, and below $y = 10 - 2x$.

✓ **FINAL ANSWER**

$(a)\ a = \frac{9}{4},\ k = \frac{8}{9}.\quad (b)\ x \geq \frac{9}{4},\ y \geq \frac{8}{9}x,\ y \leq 10 - 2x.$

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Question 8

10 marks | Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

The curve with equation $y = f(x)$ also passes through the point $P(4, 12)$

Given that

$$f'(x) = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

(b) find $f(x)$ giving the coefficients in simplest form.

(5)

(Total for Question 8 is 10 marks)

WORKED SOLUTION

Question 8 · WMA11/01 May/June 2023

1 **Differentiate the curve**

For $y = \frac{1}{4}x^3 - 8x^{-1/2}$, $\frac{dy}{dx} = \frac{3}{4}x^2 + 4x^{-3/2}$.

2 **Find the gradient at P**

At $x = 4$, $\frac{dy}{dx} = \frac{3}{4}(16) + 4(4^{-3/2}) = 12 + \frac{1}{2} = \frac{25}{2}$.

3 **Find the tangent**

Using $P(4, 12)$, $y - 12 = \frac{25}{2}(x - 4)$.

4 **Write in integer form**

Multiplying by 2 gives $2y - 24 = 25x - 100$, hence $25x - 2y - 76 = 0$.

5 **Integrate f'(x)**

$f'(x) = \frac{1}{4}x^3 - 8x^{-1/2}$ gives $f(x) = \frac{x^4}{16} - 16\sqrt{x} + c$.

6 **Use P(4, 12)**

$12 = \frac{4^4}{16} - 16\sqrt{4} + c = 16 - 32 + c$, so $c = 28$.

✓ **FINAL ANSWER**

$(a)\ 25x - 2y - 76 = 0.$ $(b)\ f(x) = \frac{x^4}{16} - 16\sqrt{x} + 28.$

Question 9

9 marks | Trigonometric Functions

9. (i)

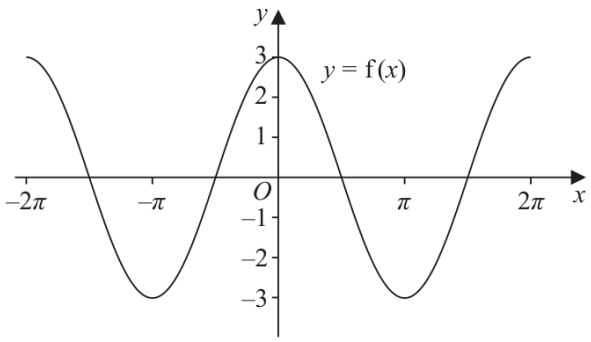


Figure 3

Figure 3 shows part of the graph of the trigonometric function with equation $y = f(x)$

(a) Write down an expression for $f(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = f\left(x + \frac{\pi}{4}\right)$

Show clearly the coordinates of all the points where the curve intersects the coordinate axes.

(3)

(ii)

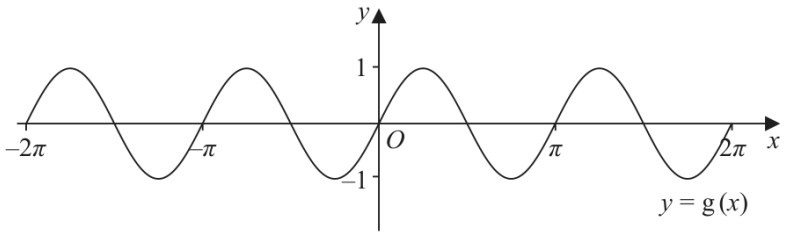


Figure 4

Figure 4 shows part of the graph of the trigonometric function with equation $y = g(x)$

(a) Write down an expression for $g(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = g(x) - 2$

Show clearly the coordinates of the y intercept.

(2)

(Total for Question 9 is 9 marks)

1 Identify f(x)

The first graph has amplitude 3, period 2π, and maximum at $x = 0$, so $f(x) = 3\cos x$.

2 Transform f

$y = f\left(x + \frac{\pi}{4}\right) = 3\cos\left(x + \frac{\pi}{4}\right)$, a shift left by $\frac{\pi}{4}$.

3 Find the x-intercepts

$\cos\left(x + \frac{\pi}{4}\right) = 0$ gives $x = \frac{\pi}{4} + n\pi$. In $-2\pi < x < 2\pi$, these are $-\frac{7\pi}{4}$, $-\frac{3\pi}{4}$, $\frac{\pi}{4}$, and $\frac{5\pi}{4}$.

4 Find the y-intercept

At $x = 0$, $y = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$.

5 Identify g(x)

The second graph has amplitude 1, period π, and passes through the origin with positive gradient, so $g(x) = \sin 2x$.

6 Transform g

$y = g(x) - 2 = \sin 2x - 2$, so the graph is shifted down by 2. Its y-intercept is $(0, -2)$.

FINAL ANSWER

$(i)(a)$ $f(x) = 3\cos x$. $(i)(b)$ x-intercepts $-\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$ and y-intercept $\frac{3\sqrt{2}}{2}$.

$(ii)(a)$ $g(x) = \sin 2x$. $(ii)(b)$ y-intercept $(0, -2)$.

Question 10

10 marks | Straight Line

10.

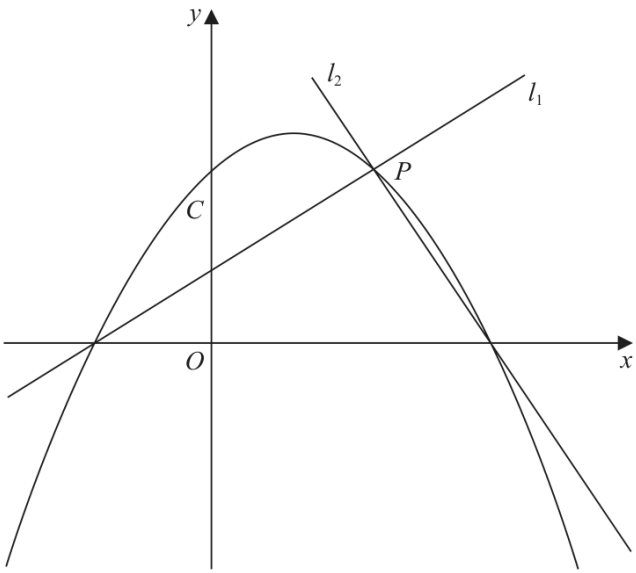


Figure 5

Figure 5 shows a sketch of the quadratic curve C with equation

$$y = -\frac{1}{4}(x+2)(x-b) \qquad \text{where } b \text{ is a positive constant}$$

The line l_1 also shown in Figure 5,

- has gradient $\frac{1}{2}$
- intersects C on the negative x-axis and at the point P

(a) (i) Write down an equation for l_1

(1)

(ii) Find, in terms of b, the coordinates of P

(3)

Given that the line l_2 is perpendicular to l_1 and intersects C on the positive x-axis,

(b) find, in terms of b, an equation for l_2

(2)

Given also that l_2 intersects C at the point P

(c) show that another equation for l_2 is

$$y = -2x + \frac{5b}{2} - 4$$

(2)

(d) Hence, or otherwise, find the value of b

(Total for Question 10 is 10 marks)

1 Find l1

Line l_1 has gradient $\frac{1}{2}$ and passes through the negative x-axis root $(-2, 0)$, so $y = \frac{1}{2}(x + 2) = \frac{1}{2}x + 1$.

2 Find P

Set $-\frac{1}{4}(x + 2)(x - b) = \frac{1}{2}(x + 2)$.

3 Solve the intersection

Multiplying by -4 gives $(x + 2)(x - b) = -2(x + 2)$, so $(x + 2)(x - b + 2) = 0$.

4 Choose the second point

The root $x = -2$ is the x-axis point, so P has $x = b - 2$. Then $y = \frac{1}{2}(b - 2) + 1 = \frac{b}{2}$.

5 Find l2 from the positive root

Since l_2 is perpendicular to l_1, its gradient is -2. It passes through the positive x-axis root $(b, 0)$, so $y = -2(x - b) = -2x + 2b$.

6 Use P to write another l2

Through $P\left(b - 2, \frac{b}{2}\right)$ with gradient -2: $y - \frac{b}{2} = -2(x - (b - 2))$, so $y = -2x + \frac{5b}{2} - 4$.

7 Find b

Both equations represent l_2, so $2b = \frac{5b}{2} - 4$. Hence $b = 8$.

FINAL ANSWER

$(a)(i)\ y = \frac{1}{2}x + 1.$ $(a)(ii)\ P = \left(b - 2, \frac{b}{2}\right).$ $(b)\ y = -2x + 2b.$ $(d)\ b = 8.$