Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2023 Worked Solutions

January 2023 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 January 2023

January 2023

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2023

Question 1

5 marks | Differentiation

1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(Solutions relying on calculator technology are not acceptable.)

(2)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2023

1 Differentiate term by term

$y = 2 + 10x^{1/2} - 2x^{3/2}$, so $\dfrac{dy}{dx} = 5x^{-1/2} - 3x^{1/2}$.

2 Write in simplest form

$\dfrac{dy}{dx} = \dfrac{5}{\sqrt{x}} - 3\sqrt{x}$.

3 Substitute x = 2

At $x = 2$, the gradient is $\dfrac{5}{\sqrt{2}} - 3\sqrt{2} = \dfrac{5}{\sqrt{2}} - \dfrac{6}{\sqrt{2}} = -\dfrac{1}{\sqrt{2}}$.

4 Rationalise

$-\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$.

FINAL ANSWER

$(a)\ \dfrac{dy}{dx} = \dfrac{5}{\sqrt{x}} - 3\sqrt{x}. \quad (b)\ -\dfrac{\sqrt{2}}{2}.$

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Question 2

5 marks | Straight Line

2. The points P, Q and R have coordinates (–3, 7), (9, 11) and (12, 2) respectively.

(a) Prove that angle $PQR = 90°$

(3)

Given that the point S is such that $PQRS$ forms a rectangle,

(b) find the coordinates of S.

(2)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2023

1 Find the gradient of PQ

$m_{PQ} = \dfrac{11-7}{9-(-3)} = \dfrac{4}{12} = \dfrac{1}{3}$.

2 Find the gradient of QR

$m_{QR} = \dfrac{2-11}{12-9} = \dfrac{-9}{3} = -3$.

3 Prove the right angle

$m_{PQ}m_{QR} = \dfrac{1}{3}(-3) = -1$, so PQ and QR are perpendicular. Hence angle $PQR = 90°$.

4 Use the rectangle vector

For rectangle $PQRS$, $\overrightarrow{PS} = \overrightarrow{QR} = (12-9, 2-11) = (3, -9)$.

5 Find S

$S = P + \overrightarrow{QR} = (-3, 7) + (3, -9) = (0, -2)$.

✓ **FINAL ANSWER**

(a) Angle $PQR = 90°$.

(b) $S = (0, -2)$.

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Question 3

5 marks | Integration

3. Find

$$\int \frac{4x^5+3}{2x^2}\,\mathrm{d}x$$

giving your answer in simplest form.

(5)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · WMA11/01 January 2023

1 Split the fraction

$\frac{4x^5+3}{2x^2} = 2x^3 + \frac{3}{2}x^{-2}$.

2 Integrate the first term

$\int 2x^3 dx = \frac{x^4}{2}$.

3 Integrate the second term

$\int \frac{3}{2}x^{-2}dx = -\frac{3}{2}x^{-1}$.

4 Write in simplest form

The integral is $\frac{x^4}{2} - \frac{3}{2x} + c$.

✓ **FINAL ANSWER**

$\frac{x^4}{2} - \frac{3}{2x} + c$.

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Question 4

4 marks | Quadratics

4. Given that the equation

$$kx^2 + 6kx + 5 = 0 \qquad \text{where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(4)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · WMA11/01 January 2023

1 Use the discriminant condition

For no real roots, the discriminant must be negative: $b^2 - 4ac < 0$.

2 Substitute a, b, and c

For $kx^2 + 6kx + 5 = 0$, $a = k$, $b = 6k$, and $c = 5$.

3 Form the inequality

$(6k)^2 - 4(k)(5) < 0$, so $36k^2 - 20k < 0$.

4 Solve the inequality

$4k(9k - 5) < 0$, so the product is negative when $0 < k < \frac{5}{9}$.

FINAL ANSWER

$0 < k < \frac{5}{9}$.

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Question 5

6 marks | Indices & Surds

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 3^x$, show that the equation

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

can be rewritten in the form

$$9p^2 + 26p - 3 = 0$$

(3)

(b) Hence solve

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · WMA11/01 January 2023

1 **Let p = 3^x**

Then $9^x = 3^{2x} = p^2$, $3^{x+2} = 9p$, and $3^{x-1} = \frac{p}{3}$.

2 **Substitute into the equation**

$3p^2 + 9p = 1 + \frac{p}{3}$.

3 **Clear the fraction**

Multiplying by 3 gives $9p^2 + 27p = 3 + p$, hence $9p^2 + 26p - 3 = 0$.

4 **Solve for p**

$9p^2 + 26p - 3 = (9p - 1)(p + 3) = 0$, so $p = \frac{1}{9}$ or $p = -3$.

5 **Reject the negative value**

Since $p = 3^x > 0$, we reject $p = -3$. Therefore $3^x = \frac{1}{9} = 3^{-2}$.

6 **Find x**

$x = -2.$

✓ **FINAL ANSWER**

$x = -2.$

Question 6

10 marks | Radians

6.

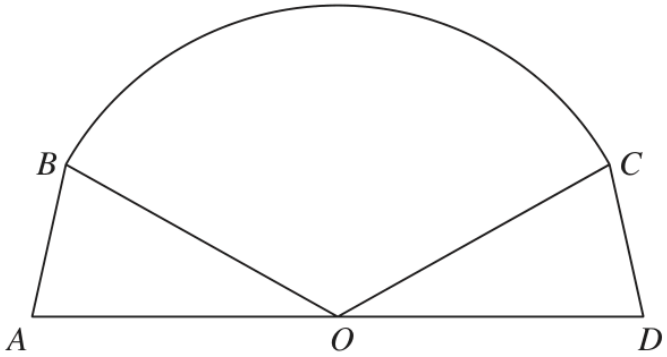


Diagram **NO** accurately dr

Figure 1

Figure 1 shows the plan view for the design of a stage.

The design consists of a sector OBC of a circle, with centre O, joined to two congruent triangles OAB and ODC.

Given that

- angle $BOC = 2.4$ radians
- area of sector $BOC = 40\,m^2$
- AOD is a straight line of length 12.5 m

(a) find the radius of the sector, giving your answer, in m, to 2 decimal places,

(2)

(b) find the size of angle AOB, in radians, to 2 decimal places.

(1)

Hence find

(c) the total area of the stage, giving your answer, in m^2, to one decimal place,

(3)

(d) the total perimeter of the stage, giving your answer, in m, to one decimal place.

(4)

(Total for Question 6 is 10 marks)

1 Find the radius

Area of sector BOC is $\frac{1}{2}r^2\theta$. So $40 = \frac{1}{2}r^2(2.4) = 1.2r^2$, giving $r = \sqrt{\frac{40}{1.2}} = 5.77$ m.

2 Find angle AOB

The triangles OAB and ODC are congruent, so $\angle AOB = \angle COD$. Since AOD is a straight line, $2\angle AOB + 2.4 = \pi$.

3 Calculate the angle

$\angle AOB = \frac{\pi - 2.4}{2} = 0.3708\ldots$, so 0.37 rad.

4 Find the total area

The two congruent triangle areas together are

$2\left(\frac{1}{2}(6.25)(5.7735\ldots)\sin 0.3708\ldots\right) = 13.075\ldots$.

5 Add the sector area

Total area $= 40 + 13.075\ldots = 53.1\ \text{m}^2$ to one decimal place.

6 Find AB

Using the cosine rule, $AB^2 = 6.25^2 + r^2 - 2(6.25)(r)\cos 0.3708\ldots$, giving $AB = 2.265\ldots$ m.

7 Find the perimeter

Perimeter $= AD + AB + CD + \text{arc } BC = 12.5 + 2(2.265\ldots) + 5.7735\ldots(2.4) = 30.9$ m.

FINAL ANSWER

$(a)\ r = 5.77\ \text{m}.$ $(b)\ 0.37\ \text{rad}.$ $(c)\ 53.1\ \text{m}^2.$ $(d)\ 30.9\ \text{m}.$

Question 7

10 marks | Transformations

7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \qquad x \neq 0$$

(2)

The curve C is transformed onto the curve with equation $y = \frac{6}{x-2} \qquad x \neq 2$

(b) Fully describe this transformation.

(2)

The curve with equation

$$y = \frac{6}{x-2} \qquad x \neq 2$$

and the line with equation

$$y = kx + 7 \qquad \text{where } k \text{ is a constant}$$

intersect at exactly two points, P and Q.

Given that the x coordinate of point P is -4

(c) find the value of k,

(2)

(d) find, using algebra, the coordinates of point Q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

Question 7 continued

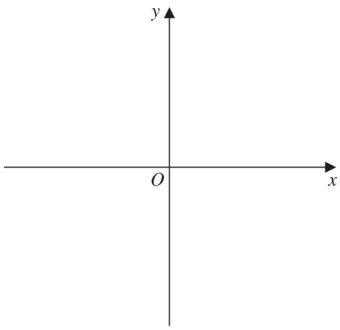
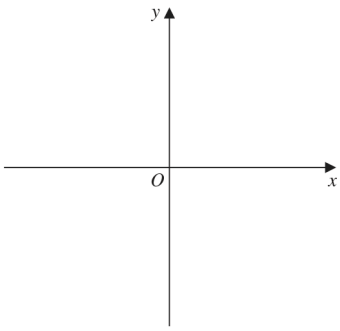


Diagram 1

Only use this copy of Diagram 1 if you need to redraw your graph.

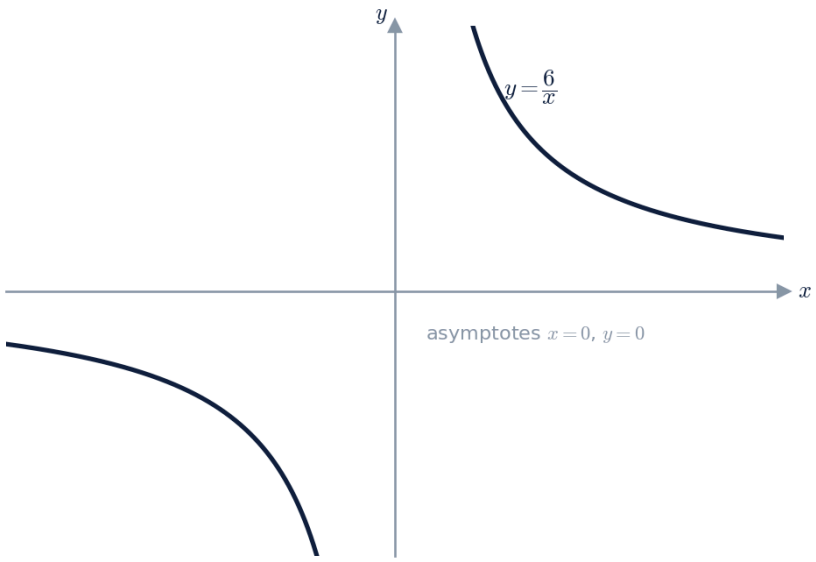


Copy of Diagram 1

(Total for Question 7 is 10 marks)

1 **Sketch y = 6/x**

The graph is a rectangular hyperbola with asymptotes $x = 0$ and $y = 0$, with branches in quadrants I and III.



2 **Describe the transformation**

$y = \dfrac{6}{x-2}$ is obtained from $y = \dfrac{6}{x}$ by a translation 2 units in the positive x-direction.

3 **Use P to find k**

At $x = -4$ on $y = \dfrac{6}{x-2}$, $y = \dfrac{6}{-6} = -1$.

4 **Substitute into the line**

Since P lies on $y = kx + 7$, $-1 = -4k + 7$, so $k = 2$.

5 **Find the second intersection**

Solve $\dfrac{6}{x-2} = 2x + 7$. Multiplying by $x - 2$ gives $6 = (2x + 7)(x - 2)$.

6 **Factor and choose Q**

$6 = 2x^2 + 3x - 14$, so $2x^2 + 3x - 20 = 0 = (2x - 5)(x + 4)$. The root $x = -4$ is P, so Q has $x = \dfrac{5}{2}$.

7 **Find y for Q**

$y = 2\left(\dfrac{5}{2}\right) + 7 = 12$, so $Q = \left(\dfrac{5}{2}, 12\right)$.

FINAL ANSWER

(b) Translation 2 units right.

$(c)\ k = 2.\quad (d)\ Q = \left(\frac{5}{2}, 12\right).$

Question 8

8 marks | Quadratics

8.

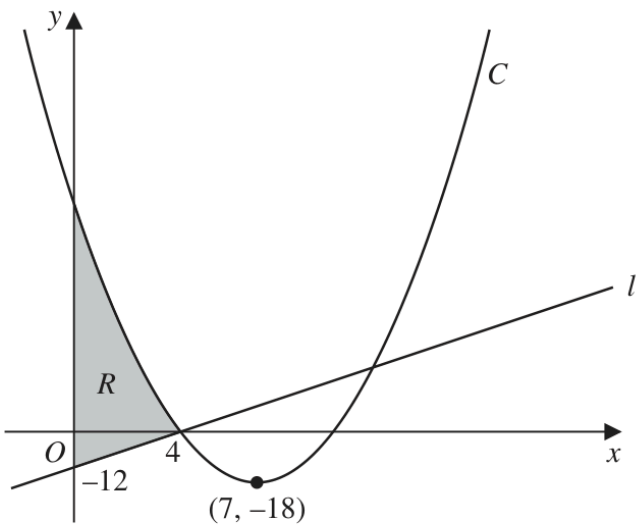


Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at -12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression
- has a minimum point at $(7, -18)$
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $\mathrm{f}(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

(Total for Question 8 is 8 marks)

1 **Find the straight line**

The line cuts the y-axis at -12 and the x-axis at 4, so its gradient is $\frac{0-(-12)}{4-0}=3$. Hence l is $y=3x-12$.

2 **Use symmetry of the quadratic**

The minimum point has x-coordinate 7. The roots are 4 and k, so their midpoint is 7.

3 **Find k**

$\frac{4+k}{2}=7$, so $k=10$.

4 **Build the quadratic**

Let $f(x)=a(x-4)(x-10)$. Since the minimum point is $(7,-18)$, $-18=a(3)(-3)=-9a$, so $a=2$.

5 **Write f(x)**

$f(x)=2(x-4)(x-10)=2x^2-28x+80$.

6 **Identify the shaded region**

The region lies between the y-axis and $x=4$, above the line and below the curve: $0\le x\le 4$ and $3x-12\le y\le 2x^2-28x+80$.

✓ **FINAL ANSWER**

$(a)\ y=3x-12.\quad (b)\ k=10.\quad (c)\ f(x)=2(x-4)(x-10).\quad (d)\ 0\le x\le 4,\ 3x-12\le y\le 2x^2-28x+80$.

Question 9

4 marks | Trigonometric Functions

9.

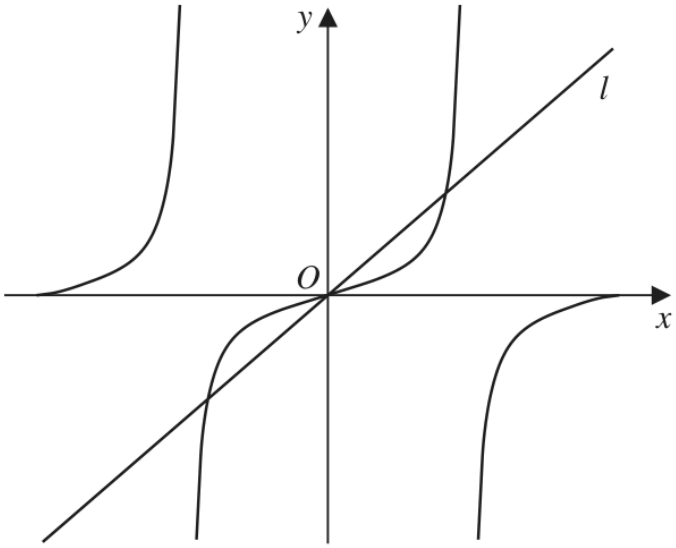


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan x$
- the straight line l with equation $y = \pi x$

in the interval $-\pi < x < \pi$

(a) State the period of $\tan x$

(1)

(b) Write down the number of roots of the equation

(i) $\tan x = (\pi + 2)x$ in the interval $-\pi < x < \pi$

(1)

(ii) $\tan x = \pi x$ in the interval $-2\pi < x < 2\pi$

(1)

(iii) $\tan x = \pi x$ in the interval $-100\pi < x < 100\pi$

(1)

(Total for Question 9 is 4 marks)

1 **State the period**

The period of $\tan x$ is π.

2 **Count roots for the steeper line**

In $-\pi < x < \pi$, the line $y = (\pi + 2)x$ meets $y = \tan x$ once at the origin and once on each side of the origin, giving 3 roots.

3 **Extend to -2pi < x < 2pi**

For $y = \pi x$, the interval $-2\pi < x < 2\pi$ has the 3 central roots plus one further root on each neighbouring branch, giving 5 roots.

4 **Extend to -100pi < x < 100pi**

There are 99 positive outer roots, 99 negative outer roots, and 3 central roots, so the total is $99 + 99 + 3 = 201$.

✓ **FINAL ANSWER**

$(a)\ \pi.\quad (b)(i)\ 3,\ (ii)\ 5,\ (iii)\ 201.$

Question 10

10 marks | Differentiation

10.

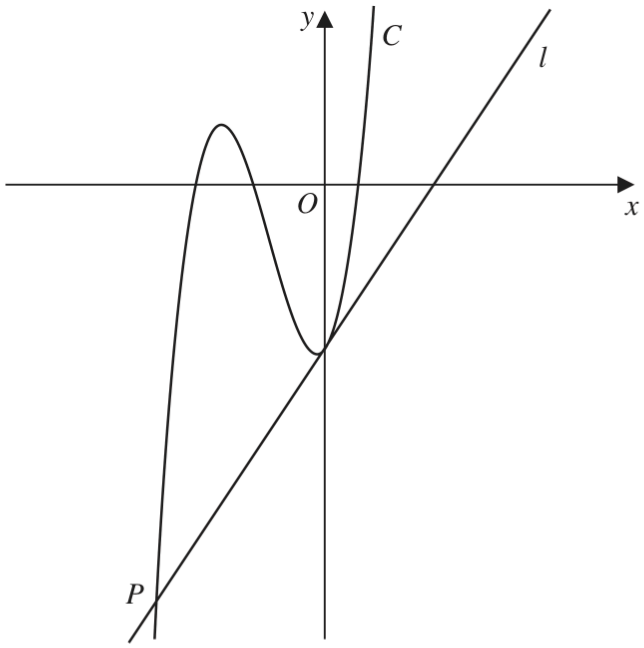


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (3x + 20)(x + 6)(2x - 3)$$

(a) Use the given information to state the values of x for which

$$\mathrm{f}(x) > 0$$

(2)

(b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

The straight line l is the tangent to C at the point where C cuts the y-axis.

Given that l cuts C at the point P, as shown in Figure 4,

(c) find, using algebra, the x coordinate of P

(*Solutions based on calculator technology are not acceptable.*)

(5)

(Total for Question 10 is 10 marks)

1 Use the roots

$f(x) = (3x + 20)(x + 6)(2x - 3)$ has roots $x = -\frac{20}{3}$, $x = -6$, and $x = \frac{3}{2}$.

2 State where f(x) is positive

Since the cubic has positive leading coefficient, $f(x) > 0$ for $-\frac{20}{3} < x < -6$ and for $x > \frac{3}{2}$.

3 Expand the first two brackets

$(3x + 20)(x + 6) = 3x^2 + 38x + 120$.

4 Complete the expansion

$(3x^2 + 38x + 120)(2x - 3) = 6x^3 + 67x^2 + 126x - 360$.

5 Find the tangent at the y-axis

$f'(x) = 18x^2 + 134x + 126$, so at $x = 0$ the tangent gradient is 126. Also $f(0) = -360$.

6 Write the tangent equation

The tangent line is $y = 126x - 360$.

7 Find the other intersection

Set $6x^3 + 67x^2 + 126x - 360 = 126x - 360$. This gives $6x^3 + 67x^2 = 0$, so $x^2(6x + 67) = 0$.

8 Choose point P

$x = 0$ is the tangent point on the y-axis, so the other intersection P has $x = -\frac{67}{6}$.

FINAL ANSWER

$(a)\ -\frac{20}{3} < x < -6$ or $x > \frac{3}{2}$. $(b)\ 6x^3 + 67x^2 + 126x - 360$. $(c)\ x_P = -\frac{67}{6}$.

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Question 11

8 marks | Integration

11. A curve C has equation $y = f(x), \quad x > 0$

Given that

- $f''(x) = 4x + \frac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

(Total for Question 11 is 8 marks)

WORKED SOLUTION

Question 11 · WMA11/01 January 2023

1 Find point P

Since P has x-coordinate 4 and lies on the tangent $y = 3x + 4$, $P = (4, 16)$.

2 Find the normal

The tangent gradient is 3, so the normal gradient is $-\frac{1}{3}$. Hence the normal is $y - 16 = -\frac{1}{3}(x - 4)$.

3 Integrate f''(x)

$f''(x) = 4x + x^{-1/2}$, so $f'(x) = 2x^2 + 2\sqrt{x} + A$.

4 Use the tangent gradient

At $x = 4$, $f'(4) = 3$. Thus $2(16) + 2(2) + A = 3$, so $A = -33$.

5 Integrate again

$f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - 33x + B$.

6 Use P(4, 16)

$16 = \frac{2}{3}(64) + \frac{4}{3}(8) - 33(4) + B = \frac{160}{3} - 132 + B$.

7 **Find B**

$B = \dfrac{284}{3}$, so $f(x) = \dfrac{2}{3}x^3 + \dfrac{4}{3}x^{3/2} - 33x + \dfrac{284}{3}$.

FINAL ANSWER

$(a)\ y - 16 = -\dfrac{1}{3}(x - 4).$ $(b)\ f(x) = \dfrac{2}{3}x^3 + \dfrac{4}{3}x^{3/2} - 33x + \dfrac{284}{3}.$