Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 October 2022 Worked Solutions

October 2022 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 October 2022

October 2022

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Differentiation

1. The curve C has equation

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} \qquad x > 0$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in simplest form.

(3)

The point $R\left(2, \dfrac{13}{2}\right)$ lies on C.

(b) Find the equation of the tangent to C at the point R. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · WMA11/01 October 2022

1 Rewrite the curve

$y = \dfrac{x^3}{4} - x^2 + \dfrac{17}{x} = \dfrac{1}{4}x^3 - x^2 + 17x^{-1}$.

2 Differentiate

$\dfrac{dy}{dx} = \dfrac{3}{4}x^2 - 2x - 17x^{-2} = \dfrac{3}{4}x^2 - 2x - \dfrac{17}{x^2}$.

3 Find the gradient at R

At $x = 2$, the tangent gradient is $\dfrac{3}{4}(4) - 2(2) - \dfrac{17}{4} = 3 - 4 - \dfrac{17}{4} = -\dfrac{21}{4}$.

4 Use the point R

The point is $R\left(2, \dfrac{13}{2}\right)$, so $y - \dfrac{13}{2} = -\dfrac{21}{4}(x - 2)$.

5 Write in integer form

Multiplying by 4 gives $4y - 26 = -21x + 42$, hence $21x + 4y - 68 = 0$.

FINAL ANSWER

$(a)\ \dfrac{dy}{dx} = \dfrac{3}{4}x^2 - 2x - \dfrac{17}{x^2}.$ $(b)\ 21x + 4y - 68 = 0.$

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Question 2

7 marks | Integration

2. Given that

$$(x-5)(2x+1)(x+3) \equiv ax^3 + bx^2 - 32x - 15$$

where a and b are constants,

(a) find the value of a and the value of b.

(2)

(b) Hence find

$$\int \frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}}\,\mathrm{d}x$$

writing each term in simplest form.

(5)

(Total for Question 2 is 7 marks)

WORKED SOLUTION

Question 2 · WMA11/01 October 2022

1 Expand the identity

$(2x+1)(x+3) = 2x^2 + 7x + 3$, so $(x-5)(2x^2+7x+3) = 2x^3 - 3x^2 - 32x - 15$.

2 Read off a and b

Comparing with $ax^3 + bx^2 - 32x - 15$ gives $a = 2$ and $b = -3$.

3 Rewrite the integrand

$$\frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}} = \frac{1}{5}(2x^{5/2} - 3x^{3/2} - 32x^{1/2} - 15x^{-1/2}).$$

4 Integrate term by term

$\int \frac{1}{5}(2x^{5/2})dx = \frac{4}{35}x^{7/2}$ and $\int \frac{1}{5}(-3x^{3/2})dx = -\frac{6}{25}x^{5/2}$.

5 Complete the integral

$\int \frac{1}{5}(-32x^{1/2})dx = -\frac{64}{15}x^{3/2}$ and $\int \frac{1}{5}(-15x^{-1/2})dx = -6x^{1/2}$, then add c.

FINAL ANSWER

$(a)\ a = 2,\ b = -3.$ $(b)\ \frac{4}{35}x^{7/2} - \frac{6}{25}x^{5/2} - \frac{64}{15}x^{3/2} - 6\sqrt{x} + c.$

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Question 3

5 marks | Straight Line

3. The share price of a company is monitored.

Exactly 3 years after monitoring began, the share price was £1.05

Exactly 5 years after monitoring began, the share price was £1.65

The share price, £V, of the company is modelled by the equation

$$V = pt + q$$

where t is the number of years after monitoring began and p and q are constants.

(a) Find the value of p and the value of q.

(3)

Exactly T years after monitoring began, the share price was £2.50

(b) Find the value of T, according to the model, giving your answer to one decimal place.

(2)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · WMA11/01 October 2022

1 Use the two data points

The model $V = pt + q$ passes through $(3, 1.05)$ and $(5, 1.65)$.

2 Find the gradient

$$p = \frac{1.65 - 1.05}{5 - 3} = \frac{0.60}{2} = 0.30.$$

3 Find the intercept

Using $1.05 = 0.30(3) + q$, we get $q = 0.15$.

4 Find T

For $V = 2.50$, $2.50 = 0.30T + 0.15$, so $T = \frac{2.35}{0.30} = 7.833\ldots$.

5 Round correctly

To one decimal place, $T = 7.8$.

FINAL ANSWER

$(a)\ p = 0.3,\ q = 0.15.$ $(b)\ T = 7.8$ years.

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Question 4

6 marks | Indices & Surds

4. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^2(2x + 1) - 15x$$

(a) Solve

$$f(x) = 0$$

(4)

(b) Hence solve

$$y^{\frac{4}{3}}\left(2y^{\frac{2}{3}} + 1\right) - 15y^{\frac{2}{3}} = 0 \qquad y > 0$$

giving your answer in simplified surd form.

(2)

(Total for Question 4 is 6 marks)

WORKED SOLUTION

Question 4 · WMA11/01 October 2022

1 **Factor f(x)**

$f(x) = x^2(2x+1) - 15x = x(2x^2 + x - 15)$.

2 **Factor the quadratic**

$2x^2 + x - 15 = (2x - 5)(x + 3)$.

3 **Solve f(x) = 0**

$x(2x-5)(x+3) = 0$, so $x = 0$, $x = \dfrac{5}{2}$, or $x = -3$.

4 **Use the substitution for part (b)**

The equation is the same as $f(u) = 0$ with $u = y^{2/3}$.

5 **Choose the valid root**

Since $y > 0$, $u = y^{2/3} > 0$. The valid root is $u = \dfrac{5}{2}$, so $y^{2/3} = \dfrac{5}{2}$.

6 **Write y in surd form**

$$y = \left(\frac{5}{2}\right)^{3/2} = \sqrt{\frac{125}{8}} = \frac{5\sqrt{10}}{4}.$$

✓ **FINAL ANSWER**

$(a)\ x = -3,\ 0,\ \frac{5}{2}.$ $(b)\ y = \frac{5\sqrt{10}}{4}.$

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Question 5

9 marks | Integration

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

(Total for Question 5 is 9 marks)

WORKED SOLUTION

Question 5 · WMA11/01 October 2022

1 Integrate f'(x)

$f'(x) = 12x^{-1/2} + \frac{x}{3} - 4$, so $f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x + c$.

2 Use P(9, 8)

$8 = 24\sqrt{9} + \frac{9^2}{6} - 4(9) + c = 72 + \frac{81}{6} - 36 + c$.

3 Find c

$72 + \frac{81}{6} - 36 = \frac{99}{2}$, so $c = 8 - \frac{99}{2} = -\frac{83}{2}$.

4 Find the tangent gradient at P

$f'(9) = \frac{12}{3} + \frac{9}{3} - 4 = 3$.

5 Find the normal gradient

The normal gradient is the negative reciprocal of 3, so it is $-\frac{1}{3}$.

6 **Find the y-intercept**

The normal through $(9, 8)$ is $y - 8 = -\frac{1}{3}(x - 9)$, so $y = -\frac{1}{3}x + 11$. It crosses the y-axis at $(0, 11)$.

✓ **FINAL ANSWER**

$(a)\ f(x) = 24\sqrt{x} + \frac{x^2}{6} - 4x - \frac{83}{2}.$ $(b)\ (0, 11).$

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Question 6

6 marks | Graphs of Functions

6. (a) Given that k is a positive constant such that $0 < k < 4$ sketch, on **separate axes**, the graphs of

(i) $y = (2x - k)(x + 4)^2$

(ii) $y = \dfrac{k}{x^2}$

showing the coordinates of any points where the graphs cross or meet the coordinate axes, leaving coordinates in terms of k, where appropriate.

(5)

(b) State, with a reason, the number of roots of the equation

$$(2x - k)(x + 4)^2 = \frac{k}{x^2}$$

(1)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · WMA11/01 October 2022

1 Key points for y = (2x-k)(x+4)^2

The graph has x-intercepts $(-4, 0)$ and $\left(\frac{k}{2}, 0\right)$. The root at $x = -4$ is a repeated root, so the graph touches the x-axis there.

2 Use 0 < k < 4

Since $0 < k < 4$, the positive intercept $x = \frac{k}{2}$ lies between 0 and 2. The y-intercept is $(0, -16k)$.

3 Key features for y = k/x^2

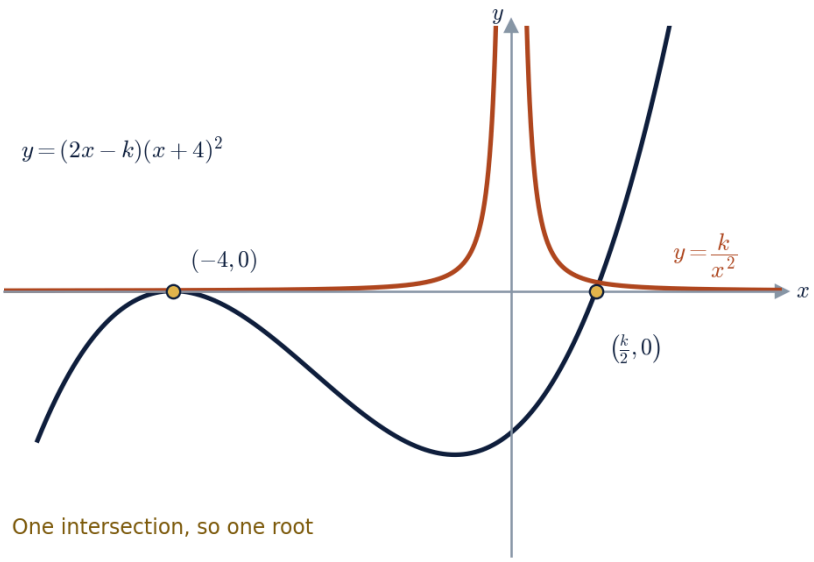
The graph is positive on both sides of the y-axis, with asymptotes $x = 0$ and $y = 0$. It has no intercepts with the coordinate axes.

4 Compare the two graphs

The equation $(2x - k)(x + 4)^2 = \frac{k}{x^2}$ represents intersections between the two graphs.

5 State the number of roots

From the sketches, the positive branch of $\dfrac{k}{x^2}$ meets the increasing positive branch of $(2x-k)(x+4)^2$ once only. Therefore there is one root.



One intersection, so one root

FINAL ANSWER

The graphs intersect once, so the equation has one root.

Question 7

8 marks | Transformations

7.

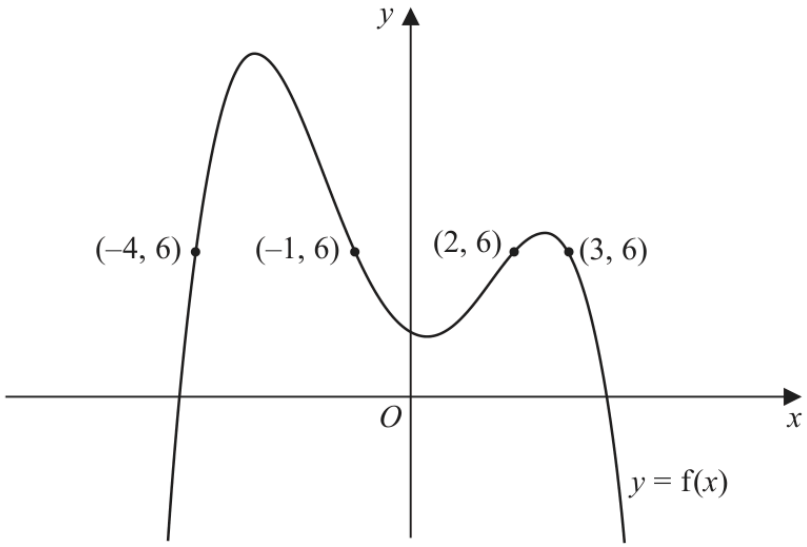


Figure 1

Figure 1 shows the curve with equation $y = f(x)$.

The points $P(-4, 6)$, $Q(-1, 6)$, $R(2, 6)$ and $S(3, 6)$ lie on the curve.

(a) Using Figure 1, find the range of values of x for which

$$f(x) < 6$$

(3)

(b) State the largest solution of the equation

$$f(2x) = 6$$

(1)

(c) (i) Sketch the curve with equation $y = f(-x)$.

On your sketch, state the coordinates of the points to which P, Q, R and S are transformed.

(ii) Hence find the set of values of x for which

$$f(-x) \geqslant 6 \text{ and } x < 0$$

(4)

(Total for Question 7 is 8 marks)

1 Read f(x) < 6 from the graph

The curve is below the line $y = 6$ for $x < -4$, for $-1 < x < 2$, and for $x > 3$.

2 Solve f(2x) = 6

Since $f(u) = 6$ at $u = -4, -1, 2, 3$, we set $2x = -4, -1, 2, 3$. The solutions are $x = -2, -\frac{1}{2}, 1, \frac{3}{2}$.

3 State the largest solution

The largest solution is $x = \frac{3}{2}$.

4 Reflect the points in the y-axis

For $y = f(-x)$, each point (x, y) becomes $(-x, y)$.

5 List the transformed points

$P(-4, 6) \to (4, 6)$, $Q(-1, 6) \to (1, 6)$, $R(2, 6) \to (-2, 6)$, and $S(3, 6) \to (-3, 6)$.

6 Use the reflected graph

For $x < 0$, the part of the reflected graph on or above $y = 6$ is from $x = -3$ to $x = -2$.

FINAL ANSWER

$(a)\ x < -4,\ -1 < x < 2,\ x > 3.\quad (b)\ \frac{3}{2}.\quad (c)(ii)\ -3 \le x \le -2.$

Question 8

14 marks | Radians

8.

Diagram **NOT** to scale

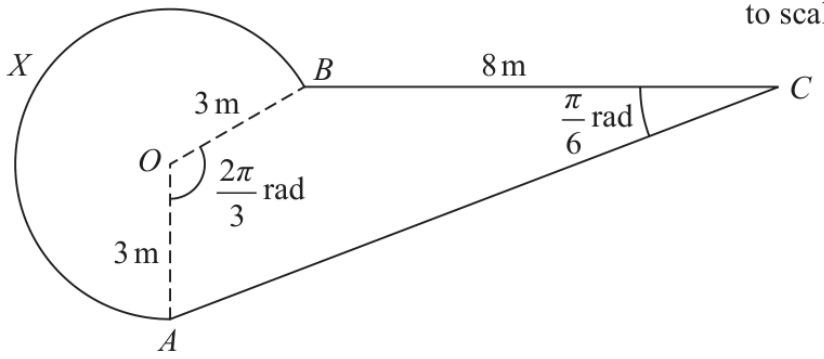


Figure 2

Figure 2 shows the plan view of a design for a pond.

The design consists of a sector $AOBX$ of a circle centre O joined to a quadrilateral $AOBC$.

- $BC = 8\,\text{m}$
- $OA = OB = 3\,\text{m}$
- angle AOB is $\frac{2\pi}{3}$ radians
- angle BCA is $\frac{\pi}{6}$ radians

(a) Calculate (i) the exact area of the sector $AOBX$,

(ii) the exact perimeter of the sector $AOBX$.

(5)

(b) Calculate the exact area of the triangle AOB.

(2)

(c) Show that the length AB is $3\sqrt{3}$ m.

(2)

(d) Find the total surface area of the pond. Give your answer in m^2 correct to 2 significant figures.

(5)

(Total for Question 8 is 14 marks)

1 **Find the angle of the sector AOBX**

The sector AOBX is the major sector, so its angle is $2\pi - \dfrac{2\pi}{3} = \dfrac{4\pi}{3}$.

2 **Find the sector area**

$\text{Area} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(3^2)\left(\dfrac{4\pi}{3}\right) = 6\pi\ \text{m}^2$.

3 **Find the sector perimeter**

The arc length is $r\theta = 3\left(\dfrac{4\pi}{3}\right) = 4\pi$. Adding the two radii gives $4\pi + 6\ \text{m}$.

4 **Find the area of triangle AOB**

$\text{Area} = \dfrac{1}{2}(3)(3)\sin\left(\dfrac{2\pi}{3}\right) = \dfrac{9}{2}\cdot\dfrac{\sqrt{3}}{2} = \dfrac{9\sqrt{3}}{4}\ \text{m}^2$.

5 **Show AB**

By the cosine rule, $AB^2 = 3^2 + 3^2 - 2(3)(3)\cos\left(\dfrac{2\pi}{3}\right) = 18 + 9 = 27$, so $AB = 3\sqrt{3}$.

6 **Find AC**

In triangle ABC, $AB^2 = AC^2 + 8^2 - 2(AC)(8)\cos\left(\dfrac{\pi}{6}\right)$, so $27 = AC^2 - 8\sqrt{3}\,AC + 64$.

7 **Choose the correct length**

Solving gives $AC = 4\sqrt{3} \pm \sqrt{11}$. From the diagram, AC is the longer value, so $AC = 4\sqrt{3} + \sqrt{11}$.

8 **Find the total area**

Area of triangle ABC is $\dfrac{1}{2}(8)(AC)\sin\left(\dfrac{\pi}{6}\right) = 2AC = 8\sqrt{3} + 2\sqrt{11}$. Total area is $6\pi + \dfrac{9\sqrt{3}}{4} + 8\sqrt{3} + 2\sqrt{11} = 43.236\ldots$, so $43\ \text{m}^2$ to 2 significant figures.

FINAL ANSWER

$(a)(i)\ 6\pi\ \text{m}^2,\ (ii)\ 4\pi + 6\ \text{m}.\quad (b)\ \dfrac{9\sqrt{3}}{4}\ \text{m}^2.\quad (c)\ AB = 3\sqrt{3}\ \text{m}.\quad (d)\ 43\ \text{m}^2.$

Question 9

14 marks | Differentiation

9.

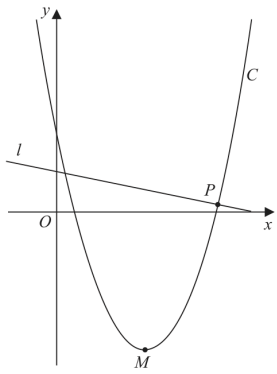


Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{1}{2}x^2 - 10x + 22$$

(a) Write $\frac{1}{2}x^2 - 10x + 22$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point M is the minimum turning point of C, as shown in Figure 3.

(b) Deduce the coordinates of M

(2)

The line l is the normal to C at the point P, as shown in Figure 3.

Given that l has equation $y = k - \frac{1}{8}x$, where k is a constant,

(c) (i) find the coordinates of P

(ii) find the value of k

(6)

Question 9 continues on the next page

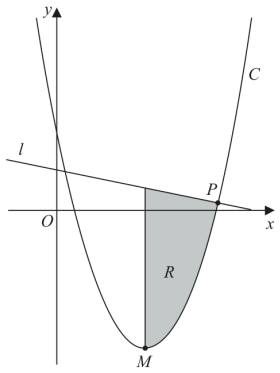


Figure 4

Figure 4 is a copy of Figure 3. The finite region R, shown shaded in Figure 4, is bounded by l, C and the line through M parallel to the y-axis.

(d) Identify the inequalities that define R.

(3)

(Total for Question 9 is 14 marks)

1 **Complete the square**

$\frac{1}{2}x^2 - 10x + 22 = \frac{1}{2}(x^2 - 20x) + 22 = \frac{1}{2}\left((x-10)^2 - 100\right) + 22.$

2 **Simplify the completed square form**

$\frac{1}{2}\left((x-10)^2 - 100\right) + 22 = \frac{1}{2}(x-10)^2 - 28.$

3 **Read the minimum point**

The minimum point is $M(10, -28)$.

4 **Use the normal gradient**

The line l has gradient $-\frac{1}{8}$, so the tangent gradient at P is 8.

5 **Differentiate the curve**

$\frac{dy}{dx} = x - 10$. At P, $x - 10 = 8$, so $x = 18$.

6 **Find P**

When $x = 18$, $y = \frac{1}{2}(18)^2 - 10(18) + 22 = 4$, so $P = (18, 4)$.

7 **Find k**

Since P lies on $y = k - \frac{1}{8}x$, $4 = k - \frac{1}{8}(18) = k - \frac{9}{4}$, so $k = \frac{25}{4}$.

8 **Write the upper boundary**

The line is $y = \frac{25}{4} - \frac{x}{8}$.

9 **Write the lower boundary**

The curve is $y = \frac{1}{2}x^2 - 10x + 22$.

10 **Identify the region**

The region lies to the right of the vertical line through M and between the curve and the normal line: $10 \le x \le 18$ and $\frac{1}{2}x^2 - 10x + 22 \le y \le \frac{25}{4} - \frac{x}{8}$.

✓ **FINAL ANSWER**

$(a)\ \frac{1}{2}(x-10)^2 - 28.\quad (b)\ M(10, -28).\quad (c)\ P(18, 4),\ k = \frac{25}{4}.\quad (d)\ 10 \le x \le 18,\ \frac{1}{2}x^2 - 10x + 22 \le y \le \frac{25}{4} - \frac{x}{8}$

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