Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 May/June 2022 Worked Solutions

May/June 2022 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 May/June 2022

May/June 2022

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 MAY/JUNE 2022

Question 1

4 marks | Integration

1. Find

$$\int \left(10x^5 + 6x^3 - \frac{3}{x^2}\right) \mathrm{d}x$$

giving your answer in simplest form.

(4)

(Total 4 marks)

WORKED SOLUTION

Question 1 · WMA11/01 May/June 2022

1 Rewrite the negative power

$\int \left(10x^5 + 6x^3 - \frac{3}{x^2}\right) dx = \int (10x^5 + 6x^3 - 3x^{-2}) dx.$

2 Integrate term by term

$\int 10x^5 dx = \frac{10x^6}{6} = \frac{5}{3}x^6$, $\int 6x^3 dx = \frac{6x^4}{4} = \frac{3}{2}x^4$, and $\int -3x^{-2} dx = 3x^{-1}$.

3 Add the constant

Therefore the integral is $\frac{5}{3}x^6 + \frac{3}{2}x^4 + \frac{3}{x} + c$.

✓ **FINAL ANSWER**

$$\frac{5}{3}x^6 + \frac{3}{2}x^4 + \frac{3}{x} + c.$$

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Question 2

5 marks | Basic Trigonometry

2. In the triangle ABC,

- $AB = 21\,\text{cm}$
- $BC = 13\,\text{cm}$
- angle $BAC = 25°$
- angle $ACB = x°$

(a) Use the sine rule to find the value of $\sin x°$, giving your answer to 4 decimal places.

(2)

Given also that AB is the longest side of the triangle,

(b) find the value of x, giving your answer to 2 decimal places.

(3)

(Total 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 May/June 2022

1 Use the sine rule

The side opposite x is $AB = 21$, and the side opposite $25°$ is $BC = 13$, so $\dfrac{\sin x}{21} = \dfrac{\sin 25°}{13}$.

2 Find the sine value

$$\sin x = \frac{21 \sin 25°}{13} = 0.68269\ldots.$$

3 Choose the correct angle

The angle from the calculator is $43.05°$, but AB is the longest side, so x is obtuse. Hence $x = 180° - 43.05° = 136.95°$.

FINAL ANSWER

$x = 137°$ to 3 significant figures.

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Question 3

5 marks | Indices & Surds

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Show that $\dfrac{\sqrt{180}-\sqrt{80}}{\sqrt{5}}$ is an integer and find its value.

(2)

(ii) Simplify

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}}$$

giving your answer in the form $a+b\sqrt{5}$ where a and b are rational numbers.

(3)

(Total 5 marks)

WORKED SOLUTION

Question 3 · WMA11/01 May/June 2022

1 **Simplify the square roots**

$\sqrt{180}=6\sqrt{5}$ and $\sqrt{80}=4\sqrt{5}$.

2 **Complete part (a)**

$$\frac{\sqrt{180}-\sqrt{80}}{\sqrt{5}}=\frac{6\sqrt{5}-4\sqrt{5}}{\sqrt{5}}=2.$$

3 **Rationalise part (b)**

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}}\times\frac{7+3\sqrt{5}}{7+3\sqrt{5}}=\frac{(4\sqrt{5}-5)(7+3\sqrt{5})}{49-45}.$$

4 **Expand and simplify**

The numerator is $28\sqrt{5}+60-35-15\sqrt{5}=25+13\sqrt{5}$, so the expression is $\dfrac{25+13\sqrt{5}}{4}$.

FINAL ANSWER

$(a)\ 2. \quad (b)\ \frac{25}{4} + \frac{13}{4}\sqrt{5}.$

Question 4

6 marks | Transformations

4.

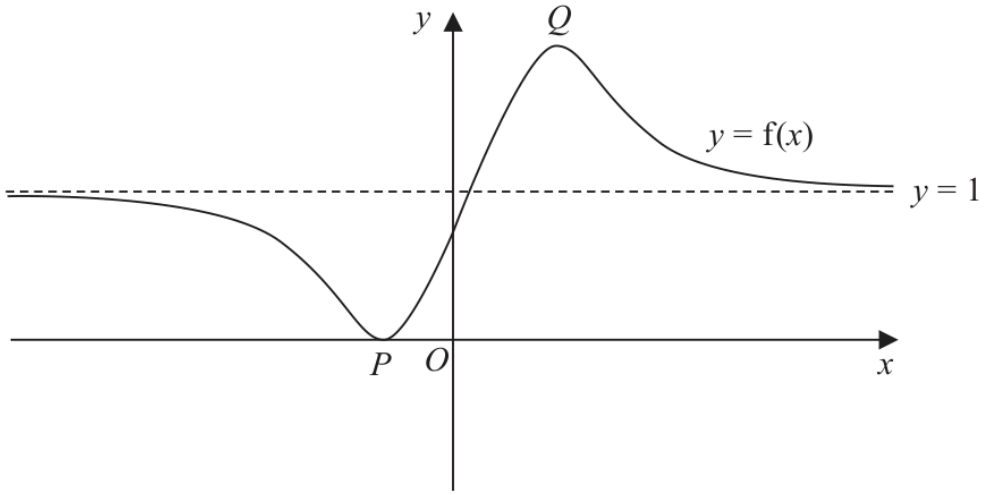


Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

(i) $y = f(x) - 2$

(3)

(ii) $y = f(-x)$

(3)

On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

(Total 6 marks)

1 **Translate for y = f(x) - 2**

Subtracting 2 moves every point of the graph down by 2 units.

2 **State the new key features**

The minimum $P(-1, 0)$ becomes $(-1, -2)$, the maximum $Q(\frac{3}{2}, 2)$ becomes $(\frac{3}{2}, 0)$, and the asymptote $y = 1$ becomes $y = -1$.

3 **Reflect for y = f(-x)**

Replacing x by $-x$ reflects the graph in the y-axis.

4 **State the reflected key features**

The minimum $P(-1, 0)$ becomes $(1, 0)$, the maximum $Q(\frac{3}{2}, 2)$ becomes $(-\frac{3}{2}, 2)$, and the horizontal asymptote remains $y = 1$.

✓ **FINAL ANSWER**

(i) minimum $(-1, -2)$, maximum $(\frac{3}{2}, 0)$, asymptote $y = -1$. (ii) minimum $(1, 0)$, maximum $(-\frac{3}{2}, 2)$, asymptote $y = 1$.

Question 5

9 marks | Straight Line

5. The curve C has equation $y = f(x)$

Given that

- $f(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $f(x)$

(4)

The line l_1 has equation $y = \frac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

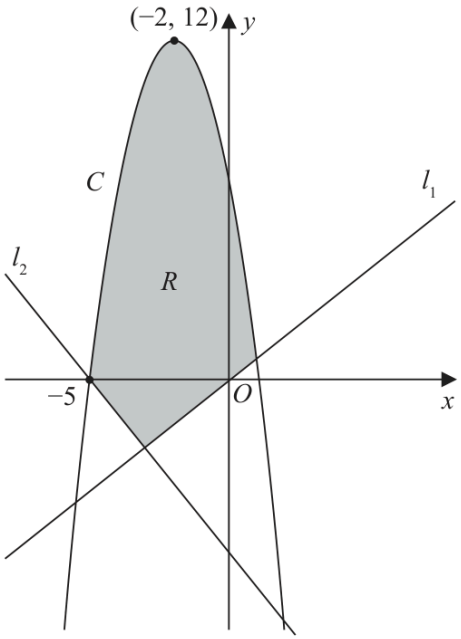


Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(c) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

(Total 9 marks)

1 **Use symmetry of the quadratic**

The maximum is at $x = -2$, and one root is $x = -5$. The other root is the same distance to the right of -2, so it is $x = 1$.

2 **Find the quadratic**

Let $f(x) = a(x + 5)(x - 1)$. Since the maximum point is $(-2, 12)$, $12 = a(3)(-3) = -9a$, so $a = -\frac{4}{3}$.

3 **Find the perpendicular gradient**

The gradient of l_1 is $\frac{4}{5}$, so the gradient of a perpendicular line is $-\frac{5}{4}$.

4 **Find l2**

Using the point $(-5, 0)$, $y = -\frac{5}{4}x + c$. Then $0 = \frac{25}{4} + c$, so $c = -\frac{25}{4}$ and l_2 is $y = -\frac{5}{4}x - \frac{25}{4}$.

5 **Write the inequalities for R**

The region is below the curve and above both straight lines, so $y \leq f(x)$, $y \geq \frac{4}{5}x$, and $y \geq -\frac{5}{4}x - \frac{25}{4}$.

✓ **FINAL ANSWER**

$(a)\ f(x) = -\frac{4}{3}(x + 5)(x - 1).$ $(b)\ y = -\frac{5}{4}x - \frac{25}{4}.$ $(c)\ y \leq f(x),\ y \geq \frac{4}{5}x,\ y \geq -\frac{5}{4}x - \frac{25}{4}$

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Question 6

7 marks | Quadratics

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

(Total 7 marks)

WORKED SOLUTION

Question 6 · WMA11/01 May/June 2022

1 **Rearrange the second equation**

From $y - x^3 + 6x = 0$, we get $y = x^3 - 6x$.

2 **Substitute into the first equation**

$2x(x^3 - 6x) - 3x^2 = 50$, so $2x^4 - 15x^2 - 50 = 0$.

3 **Solve the quadratic in x squared**

Let $u = x^2$. Then $2u^2 - 15u - 50 = 0$, giving $u = 10$ or $u = -\frac{5}{2}$.

4 **Reject the negative value**

Since x^2 cannot be negative, $x^2 = 10$, so $x = \pm\sqrt{10}$.

5 **Find y**

$y = x^3 - 6x = x(x^2 - 6) = x(10 - 6) = 4x$. Therefore $y = 4\sqrt{10}$ when $x = \sqrt{10}$ and $y = -4\sqrt{10}$ when $x = -\sqrt{10}$.

✓ **FINAL ANSWER**

$(x, y) = (\sqrt{10}, 4\sqrt{10})$ or $(-\sqrt{10}, -4\sqrt{10})$.

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Question 7

9 marks | Integration

7. The curve C has equation $y = f(x),\ x > 0$

Given that

- $f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $f''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $f(x) = 8\sqrt{3}$, when $x = 12$

(b) find $f(x)$, giving each term in simplest form.

(5)

(Total 9 marks)

WORKED SOLUTION

Question 7 · WMA11/01 May/June 2022

1 Differentiate f'(x)

$f'(x) = 2x^{-1/2} + Ax^{-2} + 3$, so $f''(x) = -x^{-3/2} - 2Ax^{-3}$.

2 Use f''(4) = 0

$-4^{-3/2} - 2A(4^{-3}) = 0$, so $-\frac{1}{8} - \frac{A}{32} = 0$ and $A = -4$.

3 Substitute A into f'(x)

$f'(x) = \frac{2}{\sqrt{x}} - \frac{4}{x^2} + 3$.

4 Integrate to find f(x)

$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x + c$.

5 **Use f(12) = 8sqrt3**

$f(12) = 8\sqrt{3} + \frac{1}{3} + 36 + c = 8\sqrt{3}$, so $c = -\frac{109}{3}$.

✓ **FINAL ANSWER**

$$f(x) = 4\sqrt{x} + \frac{4}{x} + 3x - \frac{109}{3}.$$

Question 8

10 marks | Radians

8.

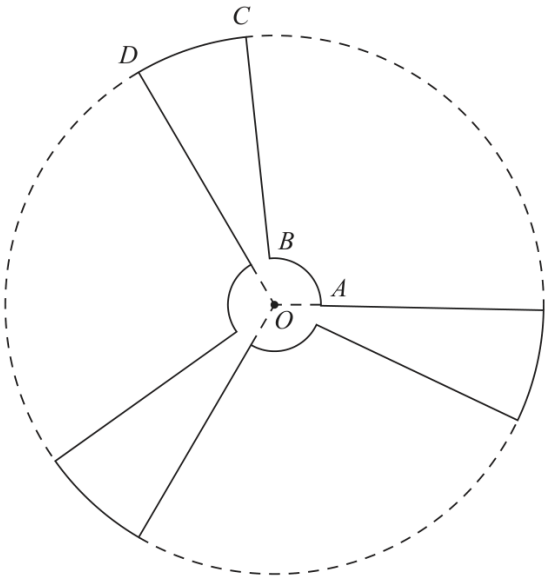


Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to $OABCDO$, shown in Figure 3, where

- $OABO$ is a sector of a circle with centre O and radius 9 cm
- $OBCDO$ is a sector of a circle with centre O and radius 84 cm
- angle $AOD = \frac{2\pi}{3}$ radians

Given that the length of the arc AB is 15 cm,

(a) show that the length of the arc CD is 35.9 cm to one decimal place.

(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

(b) the perimeter of the face of the fan, giving your answer to the nearest cm,

(2)

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.

(5)

(Total 10 marks)

1 Find the small sector angle

For arc AB, $s = r\theta$, so $15 = 9\theta$ and $\theta = \dfrac{5}{3}$ radians.

2 Find the outer arc angle

Each identical section has total angle $\dfrac{2\pi}{3}$, so the angle for arc CD is $\dfrac{2\pi}{3} - \dfrac{5}{3} = \dfrac{2\pi - 5}{3}$.

3 Find CD

$CD = 84\left(\dfrac{2\pi - 5}{3}\right) = 28(2\pi - 5) = 35.9\ \text{cm}$ to 3 significant figures.

4 Find the perimeter

There are 3 inner arcs, 3 outer arcs, and 6 straight radial edges of length $84 - 9 = 75$. Thus $P = 3(15) + 3(35.929\ldots) + 6(75) = 602.8\ldots\ \text{cm}$.

5 Find the area of one section

One section has area $\dfrac{1}{2}(9^2)\left(\dfrac{5}{3}\right) + \dfrac{1}{2}(84^2)\left(\dfrac{2\pi - 5}{3}\right) = 1576.525\ldots\ \text{cm}^2$.

6 Find the total area

The total area is $3(1576.525\ldots) = 4729.57\ldots\ \text{cm}^2 = 4.73 \times 10^3\ \text{cm}^2$ to 3 significant figures.

FINAL ANSWER

$(a)\ CD = 35.9\ \text{cm}.$ $(b)\ 603\ \text{cm}.$ $(c)\ 4.73 \times 10^3\ \text{cm}^2.$

Question 9

8 marks | Trigonometric Functions

9.

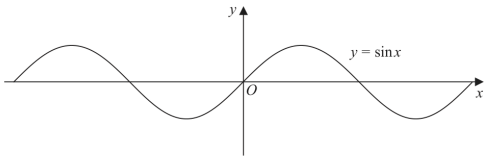


Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin\alpha = p$, where $0 < \alpha < 90°$

(a) state, in terms of p, the value of

(i) $2\sin(180° - \alpha)$

(ii) $\sin(\alpha - 180°)$

(iii) $3 + \sin(180° + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α, the x coordinates of any points in the interval $0 < x < 180°$ where

$$\sin 2x = p$$

(3)

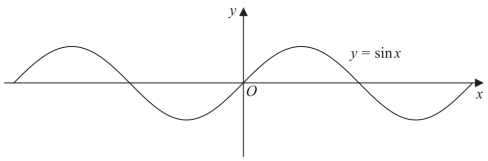


Diagram 1

(Total 8 marks)

1 **Use sine symmetry**

$\sin(180° - \alpha) = \sin\alpha = p$, so $2\sin(180° - \alpha) = 2p$.

2 **Use odd symmetry**

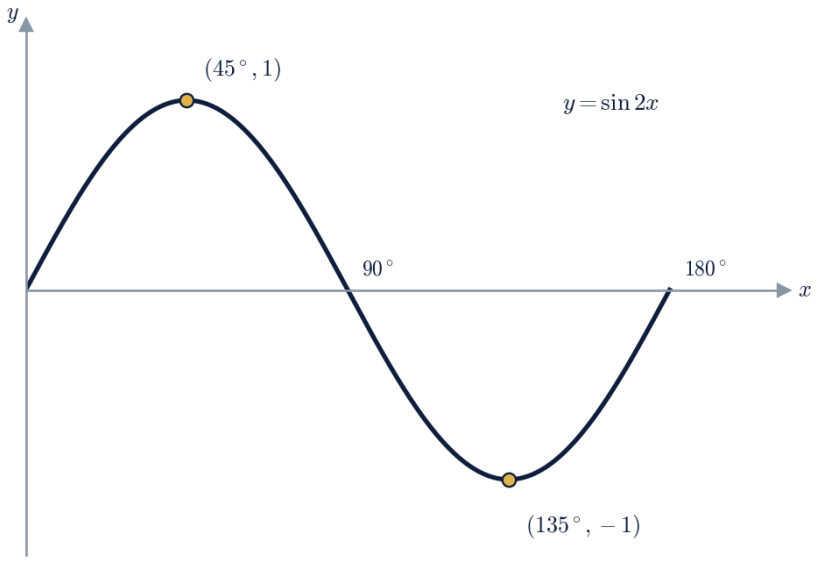
$\sin(\alpha - 180°) = -\sin(180° - \alpha) = -p$.

3 **Use the third quadrant identity**

$\sin(180° + \alpha) = -\sin\alpha = -p$, so $3 + \sin(180° + \alpha) = 3 - p$.

4 **Sketch y = sin 2x**

The graph has amplitude 1 and period $180°$. It crosses the x-axis at $0°, 90°, 180°$, has a maximum at $45°$, and a minimum at $135°$.



5 **Solve sin 2x = p**

Since $\sin\alpha = p$ and $0 < 2x < 360°$, the possible values are $2x = \alpha$ or $2x = 180° - \alpha$.

6 **Find x**

Therefore $x = \dfrac{\alpha}{2}$ or $x = 90° - \dfrac{\alpha}{2}$.

✓ **FINAL ANSWER**

$(a)(i)\ 2p,\ (ii)\ -p,\ (iii)\ 3 - p.\quad (c)\ x = \dfrac{\alpha}{2},\ 90° - \dfrac{\alpha}{2}.$

Question 10

12 marks | Differentiation

10.

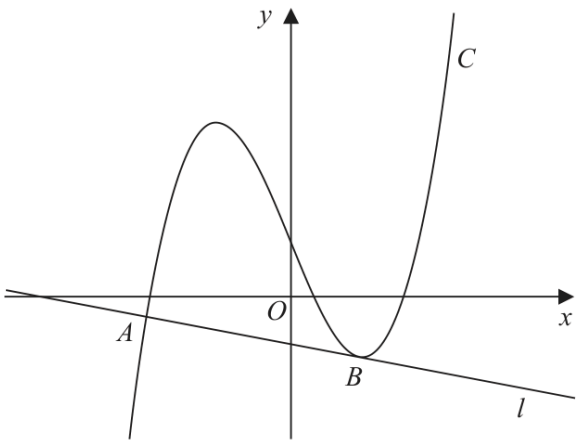


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

(Total 12 marks)

1 **Differentiate the curve**

For $y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$, $\frac{dy}{dx} = \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2}$.

2 **Find the tangent gradient at A**

At $x = -\frac{7}{2}$, $\frac{dy}{dx} = \frac{6}{7}\left(\frac{49}{4}\right) + \frac{2}{7}\left(-\frac{7}{2}\right) - \frac{5}{2} = \frac{21}{2} - 1 - \frac{5}{2} = 7$.

3 **Find the gradient of l**

Since l is the normal at A, its gradient is the negative reciprocal of 7, so $m_l = -\frac{1}{7}$.

4 **Use that l is tangent at B**

At B, the tangent gradient is also $-\frac{1}{7}$, so $\frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} = -\frac{1}{7}$.

5 **Reach the given quadratic**

Multiplying by 14 gives $12x^2 + 4x - 35 = -2$, hence $12x^2 + 4x - 33 = 0$.

6 **Solve for the x-coordinate of B**

$12x^2 + 4x - 33 = 0$ gives $x = \frac{-4 \pm 40}{24}$, so $x = \frac{3}{2}$ or $x = -\frac{11}{6}$. From the diagram, B is the point to the right of the y-axis, so $x_B = \frac{3}{2}$.

7 **Use the y-intercept of l**

Since the y-intercept is -1 and the gradient is $-\frac{1}{7}$, the line is $y = -\frac{1}{7}x - 1$.

8 **Find the y-coordinate of A**

At $x = -\frac{7}{2}$ on the line, $y = -\frac{1}{7}\left(-\frac{7}{2}\right) - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$.

9 **Substitute A into the curve**

$$-\frac{1}{2} = \frac{2}{7}\left(-\frac{7}{2}\right)^3 + \frac{1}{7}\left(-\frac{7}{2}\right)^2 - \frac{5}{2}\left(-\frac{7}{2}\right) + k = -\frac{7}{4} + k.$$

10 **Find k**

Therefore $k = \frac{5}{4}$.

✓ **FINAL ANSWER**

$(a)\ \frac{dy}{dx} = \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2}.$ $(c)\ x_B = \frac{3}{2}.$ $(d)\ k = \frac{5}{4}.$