Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 January 2022

January 2022

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2022

Question 1

4 marks | Integration

1. Find

$$\int\left(\frac{8x^3}{5}-\frac{2}{3x^4}-1\right)\mathrm{d}x$$

giving each term in simplest form.

(4)

(Total 4 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2022

1 **Rewrite using powers**

$\frac{8x^3}{5}-\frac{2}{3x^4}-1=\frac{8}{5}x^3-\frac{2}{3}x^{-4}-1.$

2 **Integrate term by term**

$\int\frac{8}{5}x^3dx=\frac{2}{5}x^4$, $\int-\frac{2}{3}x^{-4}dx=\frac{2}{9}x^{-3}$, and $\int-1dx=-x.$

3 **Write in simplest form**

Therefore the integral is $\frac{2x^4}{5}+\frac{2}{9x^3}-x+c.$

✓ **FINAL ANSWER**

$$\frac{2x^4}{5}+\frac{2}{9x^3}-x+c.$$

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Question 2

6 marks | Quadratics

2. $f(x) = 11 - 4x - 2x^2$

(a) Express $f(x)$ in the form

$$a + b(x + c)^2$$

where a, b and c are integers to be found.

(3)

(b) Sketch the graph of the curve C with equation $y = f(x)$, showing clearly the coordinates of the point where the curve crosses the y-axis.

(2)

(c) Write down the equation of the line of symmetry of C.

(1)

(Total 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2022

1 Complete the square

$f(x) = 11 - 4x - 2x^2 = -2(x^2 + 2x) + 11$.

2 Write in vertex form

$-2(x^2 + 2x) + 11 = -2\big((x+1)^2 - 1\big) + 11 = 13 - 2(x+1)^2$.

3 Identify the graph features

The graph is a downward-opening parabola with maximum point $(-1, 13)$. The y-intercept is $f(0) = 11$, so it crosses the y-axis at $(0, 11)$.

4 State the line of symmetry

From $13 - 2(x+1)^2$, the line of symmetry is $x = -1$.

✓ **FINAL ANSWER**

$(a)\ 13 - 2(x+1)^2$. $(b)\ y$-intercept $(0, 11)$. $(c)\ x = -1$.

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Question 3

7 marks | Quadratics

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express $f(x)$ in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

(Total 7 marks)

WORKED SOLUTION

Question 3 · WMA11/01 January 2022

1 **Expand the first square**

$(x + \sqrt{2})^2 = x^2 + 2\sqrt{2}x + 2$.

2 **Expand the second square**

Since $\sqrt{8} = 2\sqrt{2}$, $(3x - 5\sqrt{8})^2 = (3x - 10\sqrt{2})^2 = 9x^2 - 60\sqrt{2}x + 200$.

3 **Collect terms**

$f(x) = x^2 + 2\sqrt{2}x + 2 + 9x^2 - 60\sqrt{2}x + 200 = 10x^2 - 58\sqrt{2}x + 202$.

4 **Expand the surd equation**

$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$ gives $4\sqrt{3}y - 9 = 5y + \sqrt{3}$.

5 **Solve for y**

$y(4\sqrt{3}-5) = 9+\sqrt{3}$, so $y = \dfrac{9+\sqrt{3}}{4\sqrt{3}-5}$.

6 **Rationalise the denominator**

$$y = \frac{(9+\sqrt{3})(4\sqrt{3}+5)}{(4\sqrt{3})^2-5^2} = \frac{57+41\sqrt{3}}{23} = \frac{57}{23}+\frac{41}{23}\sqrt{3}.$$

✓ **FINAL ANSWER**

$(i)\ f(x) = 10x^2 - 58\sqrt{2}x + 202.$ $\quad (ii)\ y = \dfrac{57}{23}+\dfrac{41}{23}\sqrt{3}.$

Question 4

7 marks | Graphs of Functions

4.

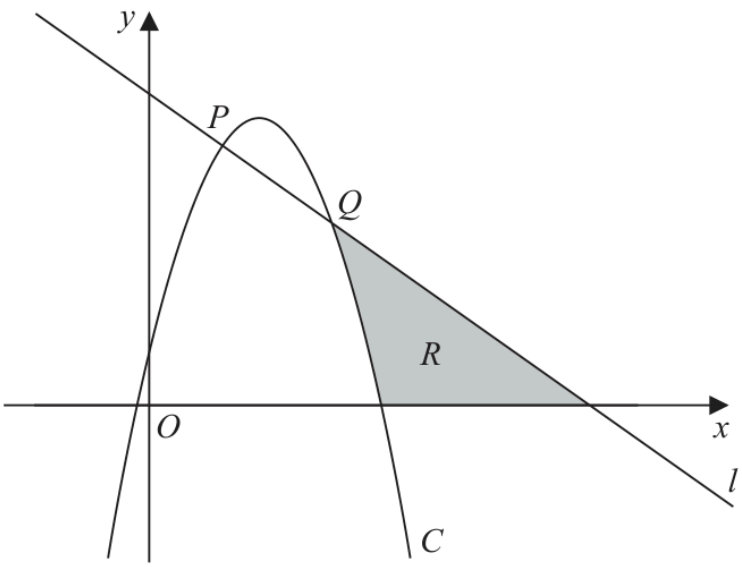


Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

(Total 7 marks)

1 **Set line equal to curve**

The line $x + y = 6$ has equation $y = 6 - x$. Intersections satisfy $6 - x = 6x - 2x^2 + 1$.

2 **Solve the quadratic**

Rearranging gives $2x^2 - 7x + 5 = 0$, so $(2x - 5)(x - 1) = 0$. Therefore $x = 1$ or $x = \frac{5}{2}$.

3 **Find the coordinates**

Using $y = 6 - x$, the points are $P = (1, 5)$ and $Q = \left(\frac{5}{2}, \frac{7}{2}\right)$.

4 **Read the bounded region**

Region R is above the x-axis, below the line, and above the curve.

5 **Write the inequalities**

Thus $y \geq 0$, $y \leq 6 - x$, and $y \geq 6x - 2x^2 + 1$.

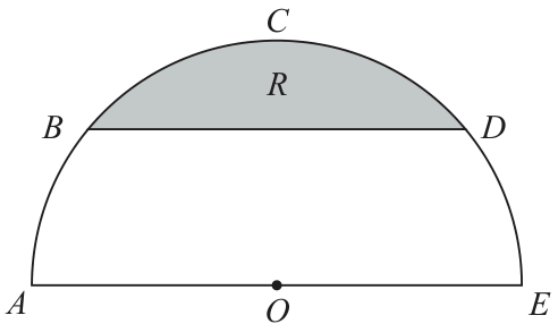
✓ **FINAL ANSWER**

$(a)\ P = (1, 5),\ Q = \left(\frac{5}{2}, \frac{7}{2}\right).$ $(b)\ y \geq 0,\ y \leq 6 - x,\ y \geq 6x - 2x^2 + 1.$

Question 5

7 marks | Radians

5.



Not to scale

Figure 2

Figure 2 shows a plan view of a semicircular garden $ABCDEOA$

The semicircle has

- centre O
- diameter AOE
- radius 3 m

The straight line BD is parallel to AE and angle BOA is 0.7 radians.

(a) Show that, to 4 significant figures, angle BOD is 1.742 radians.

(1)

The flowerbed R, shown shaded in Figure 2, is bounded by BD and the arc BCD.

(b) Find the area of the flowerbed, giving your answer in square metres to one decimal place.

(3)

(c) Find the perimeter of the flowerbed, giving your answer in metres to one decimal place.

(3)

(Total 7 marks)

1 Find angle BOD

Since BD is parallel to the diameter AE, the two side angles at the centre are both 0.7 radians. Therefore $\angle BOD = \pi - 0.7 - 0.7 = \pi - 1.4 = 1.74159\ldots$, so 1.742 radians to 4 significant figures.

2 Find the sector area

With $r = 3$ and $\theta = 1.74159\ldots$, sector area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(3)^2(1.74159\ldots) = 7.837\ldots$.

3 Subtract triangle BOD

Triangle area $= \dfrac{1}{2}(3)(3)\sin(1.74159\ldots) = 4.434\ldots$. The segment area is $7.837\ldots - 4.434\ldots = 3.403\ldots$, so $3.4\ \text{m}^2$.

4 Find the arc length

Arc $BCD = r\theta = 3(1.74159\ldots) = 5.225\ldots$.

5 Find the chord and perimeter

$BD = 2r\sin\dfrac{\theta}{2} = 6\sin(0.87079\ldots) = 4.590\ldots$. Perimeter $= 5.225\ldots + 4.590\ldots = 9.8\ \text{m}$.

FINAL ANSWER

$(a)\ \angle BOD = 1.742\ \text{rad}.\quad (b)\ 3.4\ \text{m}^2.\quad (c)\ 9.8\ \text{m}.$

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Question 6

11 marks | Integration

6. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$, simplifying your answer.

(7)

(Total 11 marks)

WORKED SOLUTION

Question 6 · WMA11/01 January 2022

1 Simplify f'(x)

$$f'(x) = \frac{(x+3)^2}{x\sqrt{x}} = \frac{x^2+6x+9}{x^{3/2}} = x^{1/2} + 6x^{-1/2} + 9x^{-3/2}.$$

2 Find the gradient at P

At $x = 4$, $f'(4) = 2 + 3 + \dfrac{9}{8} = \dfrac{49}{8}$.

3 Find the tangent equation

Using $P(4, 20)$, $y - 20 = \dfrac{49}{8}(x - 4)$. Multiplying out gives $49x - 8y - 36 = 0$.

4 Integrate f'(x)

$$f(x) = \int \left(x^{1/2} + 6x^{-1/2} + 9x^{-3/2}\right) dx = \frac{2}{3}x^{3/2} + 12x^{1/2} - 18x^{-1/2} + c.$$

5 **Use P(4,20)**

$20 = \frac{2}{3}(8) + 12(2) - \frac{18}{2} + c = \frac{61}{3} + c$, so $c = -\frac{1}{3}$.

6 **Write f(x)**

$f(x) = \frac{2}{3}x\sqrt{x} + 12\sqrt{x} - \frac{18}{\sqrt{x}} - \frac{1}{3}$.

✓ **FINAL ANSWER**

$(a)(i)\ \frac{49}{8}$. $(a)(ii)\ 49x - 8y - 36 = 0$. $(b)\ f(x) = \frac{2}{3}x\sqrt{x} + 12\sqrt{x} - \frac{18}{\sqrt{x}} - \frac{1}{3}$.

Question 7

11 marks | Differentiation

7.

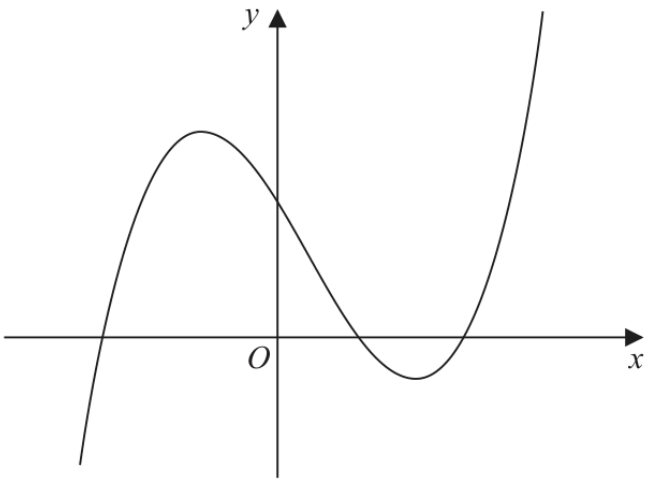


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = f(x) - p$ passes through the point with coordinates $(0, 50)$

(a) find the value of the constant p.

(2)

Given that the curve with equation $y = f(x + q)$ passes through the origin,

(b) write down the possible values of the constant q.

(2)

(c) Find $f'(x)$.

(4)

(d) Hence find the range of values of x for which the gradient of the curve with equation $y = f(x)$ is less than -18

(3)

(Total 11 marks)

1 Find p from the translated curve

$f(0) = (4)(-2)(-9) = 72$. Since $y = f(x) - p$ passes through $(0, 50)$, $72 - p = 50$, so $p = 22$.

2 Use roots for f(x + q)

If $y = f(x + q)$ passes through the origin, then $f(q) = 0$. From $f(x) = (x + 4)(x - 2)(2x - 9)$, the possible values are $q = -4, 2, \frac{9}{2}$.

3 Expand f(x)

$(x + 4)(x - 2)(2x - 9) = 2x^3 - 5x^2 - 34x + 72$.

4 Differentiate

$f'(x) = 6x^2 - 10x - 34$.

5 Set the gradient less than -18

$6x^2 - 10x - 34 < -18$, so $6x^2 - 10x - 16 < 0$, or $3x^2 - 5x - 8 < 0$.

6 Solve the inequality

$3x^2 - 5x - 8 = (3x - 8)(x + 1)$. Since the quadratic opens upward, it is negative between the roots: $-1 < x < \frac{8}{3}$.

FINAL ANSWER

$(a)\ p = 22.$ $(b)\ q = -4, 2, \frac{9}{2}.$ $(c)\ f'(x) = 6x^2 - 10x - 34.$ $(d)\ -1 < x < \frac{8}{3}.$

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Question 8

9 marks | Straight Line

8. The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates $(6, -2)$
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

(Total 9 marks)

WORKED SOLUTION

Question 8 · WMA11/01 January 2022

1 Find the gradient of l1

$2x - 5y + 7 = 0$ gives $y = \frac{2}{5}x + \frac{7}{5}$, so the gradient is $\frac{2}{5}$.

2 Find l2

The gradient perpendicular to $\frac{2}{5}$ is $-\frac{5}{2}$. Through $A(6, -2)$, $y + 2 = -\frac{5}{2}(x - 6)$, so $y = -\frac{5}{2}x + 13$.

3 **Find M**

At the intersection, $\frac{2}{5}x + \frac{7}{5} = -\frac{5}{2}x + 13$. Multiplying by 10 gives $4x + 14 = -25x + 130$, so $x = 4$.

4 **Find the y-coordinate of M**

$y = \frac{2}{5}(4) + \frac{7}{5} = 3$, so $M = (4, 3)$.

5 **Use midpoint of the diagonal**

In a square, the diagonals bisect each other, so M is the midpoint of AC. Thus $C = 2M - A = (8, 6) - (6, -2) = (2, 8)$.

✓ **FINAL ANSWER**

$(a)\ \frac{2}{5}.\quad (b)\ y = -\frac{5}{2}x + 13.\quad (c)\ M = (4, 3).\quad (d)\ C = (2, 8).$

Question 9

4 marks | Trigonometric Functions

9.

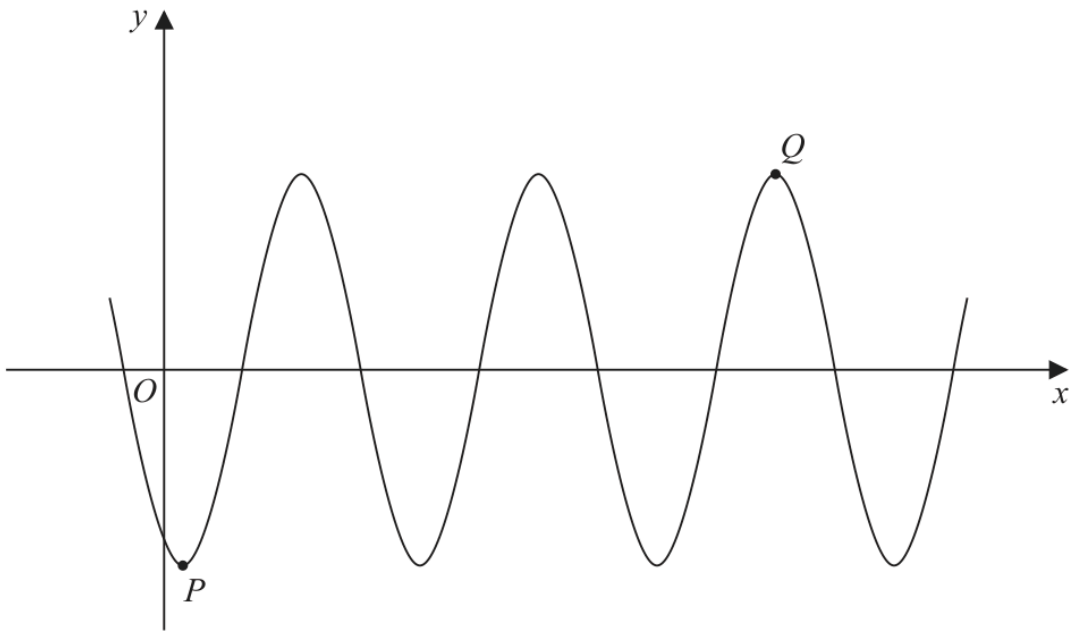


Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates $(30, -3)$ as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

(Total 4 marks)

1 **Use the minimum point**

At $P(30, -3)$, $x - 30 = 0$, so $\cos 0° = 1$. Therefore $A = -3$.

2 **Find maximum points**

Since $A = -3$, maximum points occur when $\cos(x - 30)° = -1$.

3 **Use the shown maximum Q**

$x - 30 = 180 + 360n$, so $x = 210 + 360n$. The maximum shown at Q is two full periods after the first maximum, so $x = 210 + 720 = 930$.

4 **Write Q**

The maximum y-value is 3, hence $Q = (930, 3)$.

✓ **FINAL ANSWER**

$(a)\ A = -3.$ $(b)\ Q = (930, 3)$.

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Question 10

9 marks | Graphs of Functions

10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

(a) Sketch the graph of C.

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

(b) find the range of possible values for k.

(5)

(Total 9 marks)

WORKED SOLUTION

Question 10 · WMA11/01 January 2022

1 Find intercepts

For x-intercepts, set $\frac{1}{x^2} - 9 = 0$. Then $x^2 = \frac{1}{9}$, so $x = \pm\frac{1}{3}$. There is no y-intercept because $x = 0$ is not in the domain.

2 State asymptotes

The vertical asymptote is $x = 0$. As $|x|$ becomes large, $\frac{1}{x^2} \to 0$, so the horizontal asymptote is $y = -9$.

3 Set up intersections with D

For intersections with $y = kx^2$, solve $\frac{1}{x^2} - 9 = kx^2$.

4 **Use u = x squared**

Let $u = x^2$, where $u > 0$. Multiplying by x^2 gives $1 - 9u = ku^2$, so $ku^2 + 9u - 1 = 0$.

5 **Require two positive u-values**

Four distinct x-values require two distinct positive values of u, since each positive u gives $x = \pm\sqrt{u}$.

6 **Find the range of k**

For two positive roots, the product $-\frac{1}{k}$ and sum $-\frac{9}{k}$ must both be positive, so $k < 0$. Also the discriminant must be positive: $81 + 4k > 0$, so $k > -\frac{81}{4}$. Therefore $-\frac{81}{4} < k < 0$.

✓ **FINAL ANSWER**

(a) x-intercepts $\left(-\frac{1}{3}, 0\right), \left(\frac{1}{3}, 0\right)$; asymptotes $x = 0$, $y = -9$. (b) $-\frac{81}{4} < k < 0$.