Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 October 2021 Worked Solutions

October 2021 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 October 2021

October 2021

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

5 marks | Integration

1. Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,\mathrm{d}x$$

giving each term in simplest form.

(5)

(Total 5 marks)

WORKED SOLUTION

Question 1 · WMA11/01 October 2021

1 **Rewrite with powers**

$12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4} = 12x^3 + \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-4}$.

2 **Integrate term by term**

$\int 12x^3 dx = 3x^4$, $\int \frac{1}{6}x^{-1/2}dx = \frac{1}{3}x^{1/2}$, and $\int -\frac{3}{2}x^{-4}dx = \frac{1}{2}x^{-3}$.

3 **Write in simplest form**

Therefore the integral is $3x^4 + \frac{\sqrt{x}}{3} + \frac{1}{2x^3} + c$.

✓ **FINAL ANSWER**

$3x^4 + \frac{\sqrt{x}}{3} + \frac{1}{2x^3} + c$.

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Question 2

5 marks | Differentiation

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = 3x^5 + 4x^3 - x + 5$$

The points P and Q lie on the curve.

The gradient of the curve at both point P and point Q is 2

Find the x coordinates of P and Q.

(5)

(Total 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 October 2021

1 **Differentiate the curve**

For $y = 3x^5 + 4x^3 - x + 5$, $\frac{dy}{dx} = 15x^4 + 12x^2 - 1$.

2 **Set the gradient equal to 2**

$15x^4 + 12x^2 - 1 = 2$, so $15x^4 + 12x^2 - 3 = 0$.

3 **Reduce to a quadratic in x squared**

Dividing by 3 gives $5x^4 + 4x^2 - 1 = 0$. Let $u = x^2$, giving $5u^2 + 4u - 1 = 0$.

4 **Solve and reject the impossible value**

$5u^2 + 4u - 1 = (5u - 1)(u + 1) = 0$. Since $u = x^2 \geq 0$, $u = \frac{1}{5}$.

5 **Find x**

$x^2 = \frac{1}{5}$, so $x = \pm\frac{1}{\sqrt{5}} = \pm\frac{\sqrt{5}}{5}$.

FINAL ANSWER

$$x = \pm \frac{\sqrt{5}}{5}.$$

Question 3

6 marks | Straight Line

3. (i) Solve

$$\frac{3}{x} > 4$$

(3)

(ii)

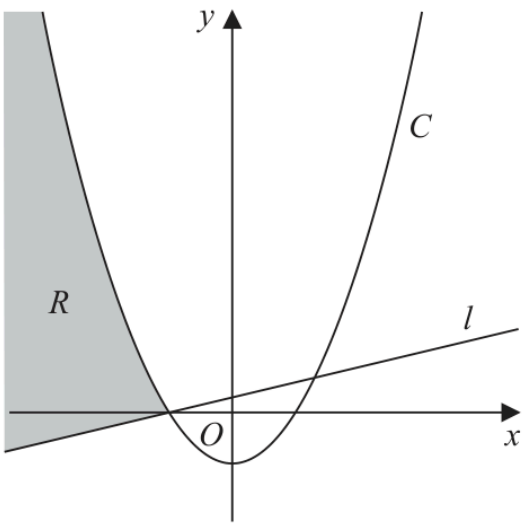


Figure 1

Figure 1 shows a sketch of the curve C and the straight line l.

The infinite region R, shown shaded in Figure 1, lies in quadrants 2 and 3 and is bounded by C and l only.

Given that

- l has a gradient of 3
- C has equation $y = 2x^2 - 50$
- C and l intersect on the negative x-axis

use inequalities to define the region R.

(3)

(Total 6 marks)

1 **Solve the rational inequality**

$\frac{3}{x} > 4$ is equivalent to $\frac{3-4x}{x} > 0$.

2 **Use critical values**

The critical values are $x = 0$ and $x = \frac{3}{4}$. A sign check gives the positive interval $0 < x < \frac{3}{4}$.

3 **Find the equation of l**

The curve C meets the negative x-axis when $2x^2 - 50 = 0$, so $x = -5$. Since l has gradient 3 and passes through $(-5, 0)$, $y = 3(x + 5) = 3x + 15$.

4 **Define the shaded region**

The shaded region lies above the line, below the curve, and to the left of their negative x-axis intersection. Hence $y > 3x + 15$, $y < 2x^2 - 50$, and $x < -5$.

FINAL ANSWER

$(i)\ 0 < x < \frac{3}{4}.\quad (ii)\ y > 3x + 15,\ y < 2x^2 - 50,\ x < -5.$

Question 4

5 marks | Trigonometric Functions

4.

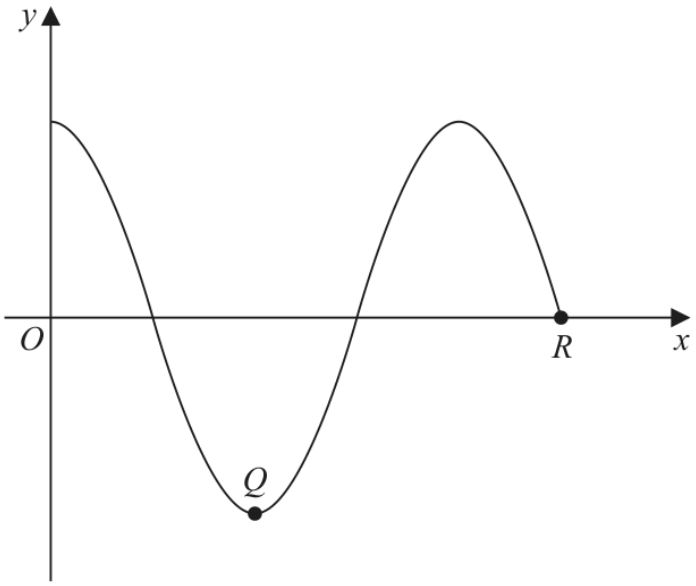


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \cos 2x^{\circ} \qquad 0 \leqslant x \leqslant k$$

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

(i) the coordinates of Q,

(ii) the value of k.

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^{\circ} = p \qquad \text{in the region } 0 \leqslant x \leqslant k$$

find the range of possible values for p.

(2)

(Total 5 marks)

1 **Find Q**

For $f(x) = \cos 2x°$, the first minimum occurs when $2x = 180°$, so $x = 90°$ and $y = -1$. Therefore $Q = (90, -1)$.

2 **Find k**

The point $R(k, 0)$ is the x-intercept after the next maximum. This occurs when $2x = 450°$, so $x = 225°$. Hence $k = 225$.

3 **Count horizontal intersections**

A horizontal line $y = p$ gives exactly two intersections on $0 \leq x \leq 225$ when it cuts the two negative branches only, so $-1 < p < 0$.

4 **Include the maximum case**

There are also exactly two solutions when $p = 1$, at $x = 0$ and $x = 180$. Therefore $-1 < p < 0$ or $p = 1$.

✓ **FINAL ANSWER**

$(a)(i)\ Q = (90, -1).\quad (a)(ii)\ k = 225.\quad (b)\ -1 < p < 0 \text{ or } p = 1.$

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Question 5

8 marks | Straight Line

5. The line l_1 has equation $3y - 2x = 30$

The line l_2 passes through the point $A(24, 0)$ and is perpendicular to l_1

Lines l_1 and l_2 meet at the point P

(a) Find, using algebra and showing your working, the coordinates of P.

(5)

Given that l_1 meets the x-axis at the point B,

(b) find the area of triangle BPA.

(3)

(Total 8 marks)

WORKED SOLUTION

Question 5 · WMA11/01 October 2021

1 Rewrite l1

$3y - 2x = 30$ gives $y = \frac{2}{3}x + 10$, so the gradient of l_1 is $\frac{2}{3}$.

2 Find l2

Since l_2 is perpendicular to l_1, its gradient is $-\frac{3}{2}$. Through $A(24, 0)$,
$y = -\frac{3}{2}(x - 24) = -\frac{3}{2}x + 36$.

3 Solve for P

At the intersection, $\frac{2}{3}x + 10 = -\frac{3}{2}x + 36$. Multiplying by 6 gives $4x + 60 = -9x + 216$, so $13x = 156$ and $x = 12$.

4 Find the y-coordinate

$y = \frac{2}{3}(12) + 10 = 18$, so $P = (12, 18)$.

5 Find B

For the x-intercept of l_1, set $y = 0$: $-2x = 30$, so $x = -15$. Hence $B = (-15, 0)$.

6 **Find the triangle area**

The base BA has length $24-(-15)=39$, and the height from P to the x-axis is 18. Area $=\frac{1}{2}(39)(18)=351$.

✓ **FINAL ANSWER**

$(a)\ P=(12,18).$ $(b)\ 351$ square units.

Question 6

10 marks | Graphs of Functions

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

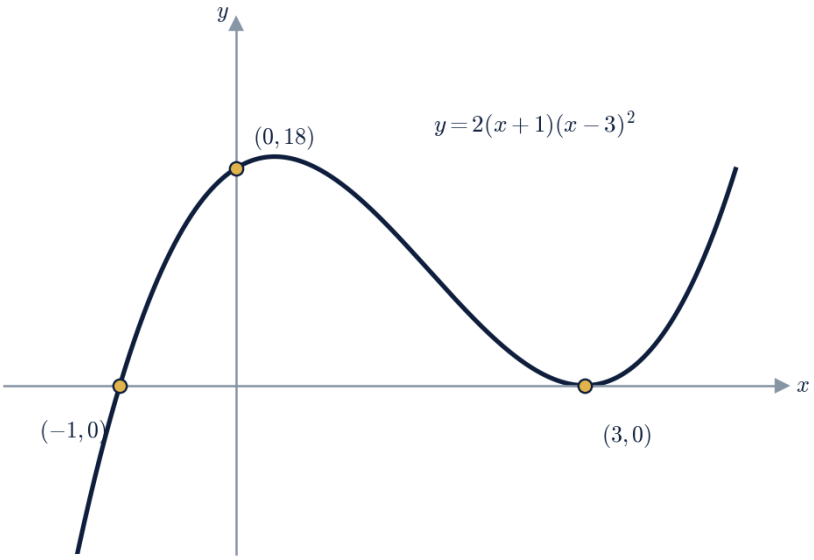
(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

(Total 10 marks)

1 **Use the factorised form for the sketch**

$f(x) = 2(x+1)(x-3)^2$. The curve cuts the x-axis at $(-1, 0)$ and meets the x-axis at $(3, 0)$ because $x = 3$ is a repeated root.



2 **Find the y-intercept**

$f(0) = 2(1)(9) = 18$, so the y-intercept is $(0, 18)$.

3 **Describe the cubic shape**

The leading term is positive, so the cubic goes down to the left and up to the right. It crosses at $x = -1$ and touches at $x = 3$.

4 **Expand f(x)**

$(x+1)(x-3)^2 = (x+1)(x^2 - 6x + 9) = x^3 - 5x^2 + 3x + 9$, so $f(x) = 2x^3 - 10x^2 + 6x + 18$.

5 **Differentiate**

$f'(x) = 6x^2 - 20x + 6$.

6 **Find the gradient at x = 1/3**

$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{9}\right) - 20\left(\frac{1}{3}\right) + 6 = 0$, so the tangent is horizontal.

7 **Find the y-coordinate**

$f\left(\frac{1}{3}\right) = 2\left(\frac{4}{3}\right)\left(-\frac{8}{3}\right)^2 = \frac{512}{27}$.

8 **Write the tangent**

A horizontal tangent through this point has equation $y = \dfrac{512}{27}$.

FINAL ANSWER

(a) x-intercepts $(-1, 0), (3, 0)$, y-intercept $(0, 18)$. (b) $f(x) = 2x^3 - 10x^2 + 6x + 18$. (c) $y = \dfrac{512}{27}$

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Question 7

10 marks | Radians

7.

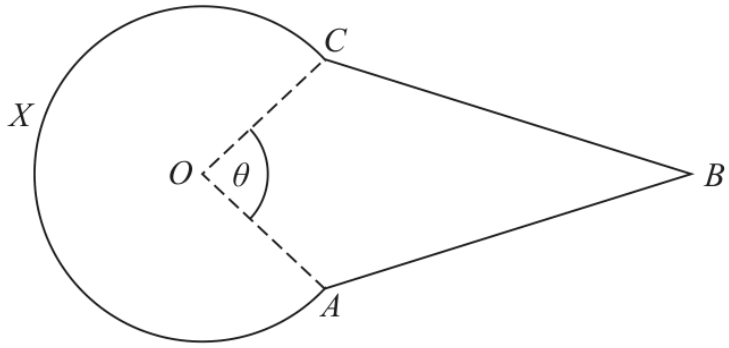


Figure 3

Figure 3 shows the design for a sign at a bird sanctuary.

The design consists of a kite $OABC$ joined to a sector $OCXA$ of a circle centre O.

In the design

- $OA = OC = 0.6\,\text{m}$
- $AB = CB = 1.4\,\text{m}$
- Angle OAB = Angle OCB = 2 radians
- Angle $AOC = \theta$ radians, as shown in Figure 3

Making your method clear,

(a) show that $\theta = 1.64$ radians to 3 significant figures,

(4)

(b) find the perimeter of the sign, in metres to 2 significant figures,

(2)

(c) find the area of the sign, in m^2 to 2 significant figures.

(4)

(Total 10 marks)

WORKED SOLUTION

Question 7 · WMA11/01 October 2021

1 **Work in triangle OAB**

In triangle OAB, $OA = 0.6$, $AB = 1.4$, and $\angle OAB = 2$ radians.

2 **Find OB**

By the cosine rule, $OB^2 = 0.6^2 + 1.4^2 - 2(0.6)(1.4)\cos 2$, so $OB = 1.73756\ldots$.

3 **Find half of theta**

Using the cosine rule again, $\cos\angle AOB = \dfrac{0.6^2 + OB^2 - 1.4^2}{2(0.6)(OB)}$, giving $\angle AOB = 0.822199\ldots$ radians.

4 **Use symmetry of the kite**

The kite is symmetric, so $\theta = 2\angle AOB = 1.644398\ldots$, hence $\theta = 1.64$ radians to 3 significant figures.

5 **Find the outside arc length**

The sector $OCXA$ uses the major angle $2\pi - \theta$. Its arc length is $0.6(2\pi - 1.644398\ldots) = 2.78327\ldots$.

6 **Find the perimeter**

Perimeter $= AB + CB + \text{major arc} = 1.4 + 1.4 + 2.78327\ldots = 5.58327\ldots$, so the perimeter is 5.6 m.

7 **Find the kite area**

The kite consists of two congruent triangles like OAB, so area of kite $= 2\left(\dfrac{1}{2}(0.6)(1.4)\sin 2\right) = 0.76381\ldots$.

8 **Find the sector area and total area**

Major sector area $= \dfrac{1}{2}(0.6)^2(2\pi - 1.644398\ldots) = 0.83498\ldots$. Total area $= 0.76381\ldots + 0.83498\ldots = 1.59879\ldots$, so the area is $1.6\ \text{m}^2$.

✓ **FINAL ANSWER**

$(a)\ \theta = 1.64$ rad. $\quad (b)\ 5.6$ m. $\quad (c)\ 1.6\ \text{m}^2$.

Question 8

10 marks | Straight Line

8.

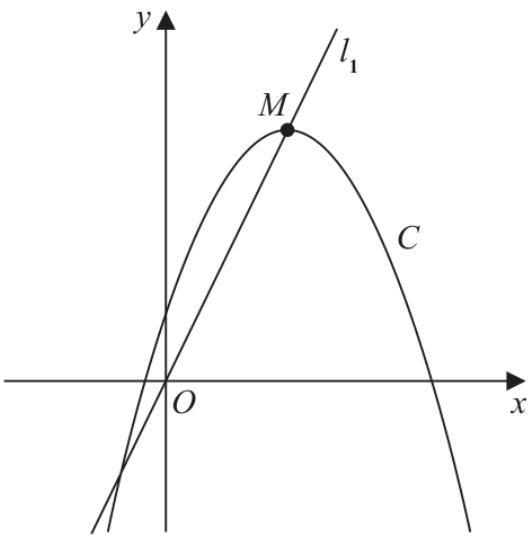


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 4 + 12x - 3x^2$$

The point M is the maximum turning point on C.

(a) (i) Write $4 + 12x - 3x^2$ in the form

$$a + b(x + c)^2$$

where a, b and c are constants to be found.

(ii) Hence, or otherwise, state the coordinates of M.

(5)

The line l_1 passes through O and M, as shown in Figure 4.

A line l_2 touches C and is parallel to l_1

(b) Find an equation for l_2

(5)

(Total 10 marks)

1 **Complete the square**

$4+12x-3x^2=-3(x^2-4x)+4=-3\big((x-2)^2-4\big)+4=16-3(x-2)^2$.

2 **State the maximum point**

The maximum occurs when $(x-2)^2=0$, so $M=(2,16)$.

3 **Find the gradient of l1**

Line l_1 passes through $O(0,0)$ and $M(2,16)$, so its gradient is $\dfrac{16}{2}=8$.

4 **Use the parallel tangent gradient**

Line l_2 is parallel to l_1, so its gradient is also 8. For $y=4+12x-3x^2$, $\dfrac{dy}{dx}=12-6x$.

5 **Find the point of contact**

Set $12-6x=8$, giving $x=\dfrac{2}{3}$. Then $y=4+12\left(\dfrac{2}{3}\right)-3\left(\dfrac{4}{9}\right)=\dfrac{32}{3}$.

6 **Write l2**

Using gradient 8 through $\left(\dfrac{2}{3},\dfrac{32}{3}\right)$, $y-\dfrac{32}{3}=8\left(x-\dfrac{2}{3}\right)$. Hence $y=8x+\dfrac{16}{3}$.

FINAL ANSWER

$(a)(i)\ 16-3(x-2)^2.\quad (a)(ii)\ M=(2,16).\quad (b)\ y=8x+\dfrac{16}{3}$.

Question 9

9 marks | Transformations

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

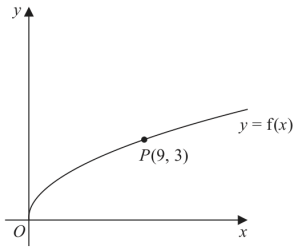


Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \sqrt{x} \qquad x > 0$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 5.

On the next page there is a copy of Figure 5 called Diagram 1.

(a) On Diagram 1, sketch and clearly label the graphs of

$$y = f(2x) \quad \text{and} \quad y = f(x) + 3$$

Show on each graph the coordinates of the point to which P is transformed.

(3)

The graph of $y = f(2x)$ meets the graph of $y = f(x) + 3$ at the point Q.

(b) Show that the x coordinate of Q is the solution of

$$\sqrt{x} = 3\left(\sqrt{2} + 1\right)$$

(3)

(c) Hence find, in simplest form, the coordinates of Q.

(3)

Question 9 continued

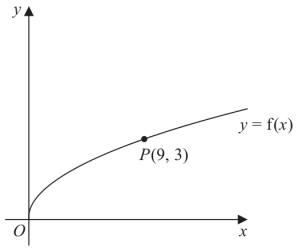
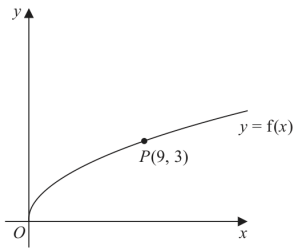


Diagram 1

Turn over for a copy of Diagram 1 if you need to redraw your graphs.

Only use this copy if you need to redraw your graphs.



Copy of Diagram 1

(Total 9 marks)

1 **Transform P on y = f(2x)**

$y = f(2x)$ is a horizontal scale factor of $\frac{1}{2}$, so $P(9, 3)$ maps to $\left(\frac{9}{2}, 3\right)$.

2 **Translate point P**

$y = f(x) + 3$ is a translation 3 units upward, so $P(9, 3)$ maps to $(9, 6)$.

3 **Set up the intersection equation**

At Q, $f(2x) = f(x) + 3$. Since $f(x) = \sqrt{x}$, this gives $\sqrt{2x} = \sqrt{x} + 3$.

4 **Show the required equation**

Let $s = \sqrt{x}$. Then $\sqrt{2x} = \sqrt{2}\,s$, so $\sqrt{2}s = s + 3$. Hence $(\sqrt{2} - 1)s = 3$ and $s = \dfrac{3}{\sqrt{2} - 1} = 3(\sqrt{2} + 1)$.

5 **Find the x-coordinate**

$\sqrt{x} = 3(\sqrt{2} + 1)$, so $x = 9(\sqrt{2} + 1)^2 = 9(3 + 2\sqrt{2}) = 27 + 18\sqrt{2}$.

6 **Find the y-coordinate**

$y = \sqrt{x} + 3 = 3(\sqrt{2} + 1) + 3 = 6 + 3\sqrt{2}$. Therefore $Q = (27 + 18\sqrt{2},\ 6 + 3\sqrt{2})$.

✓ **FINAL ANSWER**

$(a)\ P \mapsto \left(\frac{9}{2}, 3\right)$ on $y = f(2x)$, $P \mapsto (9, 6)$ on $y = f(x) + 3$. $(c)\ Q = (27 + 18\sqrt{2},\ 6 + 3\sqrt{2})$

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Question 10

7 marks | Integration

10. A curve has equation $y = f(x), x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a.

(3)

(b) Hence find $f(x)$.

(4)

(Total 7 marks)

WORKED SOLUTION

Question 10 · WMA11/01 October 2021

1 Differentiate f'(x)

$f'(x) = ax - 12x^{1/3}$, so $f''(x) = a - 4x^{-2/3}$.

2 Use f''(27) = 0

Since $27^{2/3} = 9$, $0 = a - \frac{4}{9}$, so $a = \frac{4}{9}$.

3 Integrate f'(x)

With $a = \frac{4}{9}$, $f'(x) = \frac{4}{9}x - 12x^{1/3}$. Therefore $f(x) = \frac{2}{9}x^2 - 9x^{4/3} + c$.

4 Use the point (1, -8)

$-8 = \frac{2}{9} - 9 + c$, so $c = -8 + 9 - \frac{2}{9} = \frac{7}{9}$.

5 Write f(x)

$f(x) = \frac{2}{9}x^2 - 9x^{4/3} + \frac{7}{9}$.

FINAL ANSWER

$(a)\ a = \dfrac{4}{9}.\quad (b)\ f(x) = \dfrac{2}{9}x^2 - 9x^{4/3} + \dfrac{7}{9}.$