Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 May/June 2021 Worked Solutions

May/June 2021 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 May/June 2021

May/June 2021

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 MAY/JUNE 2021

Question 1

8 marks | Differentiation

1. The curve C has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(4)

The point $P(4, 3)$ lies on C.

(b) Find the equation of the normal to C at the point P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(Total 8 marks)

WORKED SOLUTION

Question 1 · WMA11/01 May/June 2021

1 Rewrite using powers

$y = \frac{x^2}{3} + 4x^{-1/2} + \frac{8}{3}x^{-1} - 5$.

2 Differentiate term by term

$\frac{dy}{dx} = \frac{2x}{3} - 2x^{-3/2} - \frac{8}{3}x^{-2} = \frac{2x}{3} - \frac{2}{x\sqrt{x}} - \frac{8}{3x^2}$.

3 Find the tangent gradient at P

At $x = 4$, $\frac{dy}{dx} = \frac{8}{3} - \frac{1}{4} - \frac{1}{6} = \frac{9}{4}$.

4 Find the normal gradient

The normal gradient is the negative reciprocal of $\frac{9}{4}$, so $m_n = -\frac{4}{9}$.

5 Write the normal equation

Using $P(4, 3)$, $y - 3 = -\frac{4}{9}(x - 4)$. Multiplying by 9 gives $9y - 27 = -4x + 16$, hence $4x + 9y - 43 = 0$.

FINAL ANSWER

$(a)\ \dfrac{dy}{dx} = \dfrac{2x}{3} - \dfrac{2}{x\sqrt{x}} - \dfrac{8}{3x^2}.$ $(b)\ 4x + 9y - 43 = 0.$

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Question 2

10 marks | Polynomials

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3(9^{3z}) + 26(9^{2z}) - 9(9^{z}) = 0$

(5)

(Total 10 marks)

WORKED SOLUTION

Question 2 · WMA11/01 May/June 2021

1 **Use f(-1) = 32**

$f(-1) = -a + (6a + 8) + a^2 = a^2 + 5a + 8$. Since $f(-1) = 32$, $a^2 + 5a - 24 = 0$.

2 **Choose the positive value of a**

$a^2 + 5a - 24 = (a - 3)(a + 8) = 0$. Since a is positive, the only possible value is $a = 3$.

3 **Solve f(x) = 0**

Using $a = 3$, $f(x) = 3x^3 + 26x^2 - 9x = x(3x^2 + 26x - 9) = x(3x - 1)(x + 9)$.

4 **List the roots**

$x = 0$, $x = \frac{1}{3}$, or $x = -9$.

5 **Solve the equation in y**

Let $u = y^{1/3}$. Then $y = u^3$ and $y^{2/3} = u^2$, so $3y + 26y^{2/3} - 9y^{1/3} = 0$ becomes $3u^3 + 26u^2 - 9u = 0$.

6 **Convert back to y**

From part (a), $u = -9, 0, \frac{1}{3}$. Therefore $y = u^3$, giving $y = -729, 0, \frac{1}{27}$.

7 **Solve the exponential equation**

Let $u = 9^z$. Then $3(9^{3z}) + 26(9^{2z}) - 9(9^z) = 0$ becomes $3u^3 + 26u^2 - 9u = 0$.

8 **Use positivity of 9^z**

Since $9^z > 0$, the only valid root is $u = \frac{1}{3}$. Hence $9^z = \frac{1}{3}$, so $(3^2)^z = 3^{-1}$ and $2z = -1$. Therefore $z = -\frac{1}{2}$.

✓ **FINAL ANSWER**

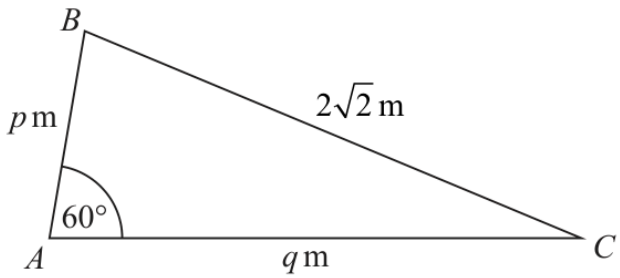
$(a)(i)\ a = 3.$ $(a)(ii)\ x = -9, 0, \frac{1}{3}.$ $(b)(i)\ y = -729, 0, \frac{1}{27}.$ $(b)(ii)\ z = -\frac{1}{2}.$

Question 3

9 marks | Basic Trigonometry

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.



Not drawn to scale

Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle ABC with

- $AB = p$ metres
- $AC = q$ metres
- $BC = 2\sqrt{2}$ metres
- angle $BAC = 60°$

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side AC is 2 metres longer than side AB, use algebra to find

(b) (i) the exact value of p,

(ii) the exact value of q.

(5)

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

(Total 9 marks)

1 **Apply the cosine rule**

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 60°$. Since $BC = 2\sqrt{2}$, $AB = p$ and $AC = q$, this gives $8 = p^2 + q^2 - 2pq\left(\frac{1}{2}\right)$.

2 **Show the required result**

Therefore $p^2 + q^2 - pq = 8$.

3 **Use q = p + 2**

Side AC is 2 metres longer than AB, so $q = p + 2$. Substitute into $p^2 + q^2 - pq = 8$.

4 **Solve for p**

$p^2 + (p+2)^2 - p(p+2) = 8$, so $p^2 + 2p - 4 = 0$. Hence $p = -1 \pm \sqrt{5}$. Since $p > 0$, $p = \sqrt{5} - 1$.

5 **Find q**

$q = p + 2 = \sqrt{5} + 1$.

6 **Find the exact area**

Area $= \frac{1}{2}pq\sin 60°$. Now $pq = (\sqrt{5}-1)(\sqrt{5}+1) = 4$, so area $= \frac{1}{2}(4)\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,\text{m}^2$.

FINAL ANSWER

$(a)\ p^2 + q^2 - pq = 8.\quad (b)\ p = \sqrt{5} - 1,\ q = \sqrt{5} + 1.\quad (c)\ \sqrt{3}\,\text{m}^2.$

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Question 4

6 marks | Integration

4. Find

$$\int \frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}}\,\mathrm{d}x$$

writing each term in simplest form.

(6)

(Total 6 marks)

WORKED SOLUTION

Question 4 · WMA11/01 May/June 2021

1 Expand the numerator

$(3\sqrt{x}+2)(x-5) = 3x\sqrt{x} - 15\sqrt{x} + 2x - 10.$

2 Divide by 4sqrt(x)

$$\frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}} = \frac{3x}{4} - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}.$$

3 Integrate term by term

$$\int \left(\frac{3x}{4} - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}\right)dx = \frac{3x^2}{8} - \frac{15x}{4} + \frac{1}{3}x^{3/2} - 5x^{1/2} + c.$$

4 Write each term simply

So the integral is $\frac{3x^2}{8} - \frac{15x}{4} + \frac{x\sqrt{x}}{3} - 5\sqrt{x} + c.$

✓ **FINAL ANSWER**

$$\frac{3x^2}{8} - \frac{15x}{4} + \frac{x\sqrt{x}}{3} - 5\sqrt{x} + c.$$

Question 5

8 marks | Graphs of Functions

5.

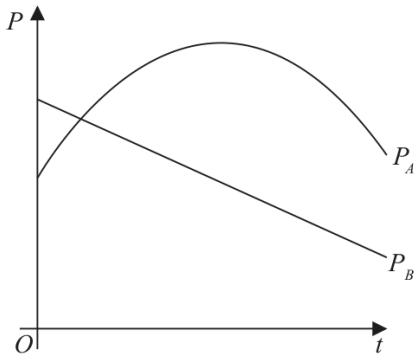


Figure 2

The share value of two companies, company A and company B, has been monitored over a 15-year period.

The share value P_A of **company *A***, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

The share value P_B of **company *B***, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

(a) Find the difference between the share value of **company *A*** and the share value of **company *B*** at the point monitoring began.

(2)

(b) State the maximum share value of **company *A*** during the 15-year period.

(1)

(c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company *A*** was greater than the share value of **company *B***.

(4)

(d) Explain why the model for **company *A*** should not be used to predict its share value when $t = 20$

(1)

(Total 8 marks)

1 Evaluate both models at t = 0

$P_A(0) = 53 - 0.4(0-8)^2 = 53 - 25.6 = 27.4$. Also $P_B(0) = -1.6(0) + 44.2 = 44.2$.

2 Find the initial difference

The difference is $44.2 - 27.4 = 16.8$, so the share values differ by 16.8 million pounds.

3 State the maximum of company A

$P_A = 53 - 0.4(t-8)^2$ has maximum value 53, reached when $t = 8$, which is within the 15-year period.

4 Compare company costs

$53 - 0.4(t-8)^2 > -1.6t + 44.2$.

5 Rearrange to a quadratic inequality

Expanding gives $-0.4t^2 + 8t - 16.8 > 0$. Multiplying by -2.5 reverses the inequality: $t^2 - 20t + 42 < 0$.

6 Find the crossing times

$t = \dfrac{20 \pm \sqrt{400 - 168}}{2} = 10 \pm \sqrt{58}$. Therefore company A is greater between the two roots.

7 Restrict to the 15-year period

Since the model is monitored over $0 \le t \le 15$, the required times are $10 - \sqrt{58} < t \le 15$.

8 Explain the limitation

The model should not be used at $t = 20$ because it was only built from a 15-year monitoring period; 20 years is outside that range.

✓ FINAL ANSWER

(a) 16.8 million pounds. (b) 53 million pounds. (c) $10 - \sqrt{58} < t \le 15$. (d) $t = 20$ is outside the 15-year model .

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Question 6

8 marks | Integration

6. The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point $P(8, 2)$
- $f'(x) = \frac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $f(x)$.

(5)

(Total 8 marks)

WORKED SOLUTION

Question 6 · WMA11/01 May/June 2021

1 Find the gradient at P

$f'(x) = \frac{32}{3x^2} + 3 - 2\sqrt[3]{x}$. At $x = 8$, $f'(8) = \frac{32}{3(64)} + 3 - 2(2) = \frac{1}{6} - 1 = -\frac{5}{6}$.

2 Write the tangent equation

Using $P(8, 2)$, $y - 2 = -\frac{5}{6}(x - 8)$, hence $y = -\frac{5}{6}x + \frac{26}{3}$.

3 Integrate f'(x)

$f'(x) = \frac{32}{3}x^{-2} + 3 - 2x^{1/3}$, so $f(x) = -\frac{32}{3x} + 3x - \frac{3}{2}x^{4/3} + c$.

4 Use P(8,2) to find c

$2 = -\frac{32}{24} + 24 - \frac{3}{2}(16) + c = -\frac{4}{3} + c$, so $c = \frac{10}{3}$.

5 Write f(x)

$f(x) = -\frac{32}{3x} + 3x - \frac{3}{2}x^{4/3} + \frac{10}{3}$.

FINAL ANSWER

$(a)\ y = -\dfrac{5}{6}x + \dfrac{26}{3}.$ $(b)\ f(x) = -\dfrac{32}{3x} + 3x - \dfrac{3}{2}x^{4/3} + \dfrac{10}{3}.$

Question 7

10 marks | Radians

7.

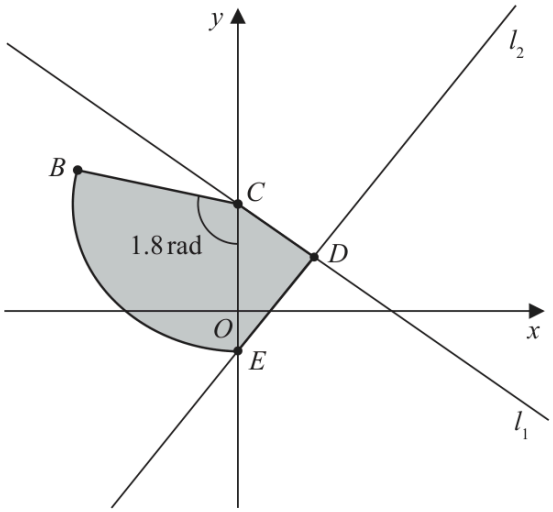


Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y-axis at the point C, as shown in Figure 3.

(a) State the y coordinate of C.

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y-axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\frac{14}{3}$

(3)

A sector BCE of a circle with centre C is also shown in Figure 3.

Given that angle BCE is 1.8 radians,

(c) find the length of arc BE.

(3)

The region $CBED$, shown shaded in Figure 3, consists of the sector BCE joined to the triangle CDE.

(d) Calculate the exact area of the region $CBED$.

(3)

(Total 10 marks)

1 Find C

On the y-axis, $x = 0$. From $4y + 3x = 48$, $4y = 48$, so the y-coordinate of C is 12.

2 Find the gradient of l1

$4y + 3x = 48$ gives $y = -\frac{3}{4}x + 12$, so the gradient of l_1 is $-\frac{3}{4}$.

3 Use perpendicular gradients for l2

Since l_2 is perpendicular to l_1, its gradient is $\frac{4}{3}$. Through $D(8, 6)$, $y - 6 = \frac{4}{3}(x - 8)$.

4 Show the y-coordinate of E

$y = \frac{4}{3}x - \frac{32}{3} + 6 = \frac{4}{3}x - \frac{14}{3}$. Therefore when $x = 0$, $E_y = -\frac{14}{3}$.

5 Find the radius of the sector

$C = (0, 12)$ and $E = \left(0, -\frac{14}{3}\right)$, so $CE = 12 + \frac{14}{3} = \frac{50}{3}$.

6 Find arc BE

For sector BCE, $r = \frac{50}{3}$ and $\theta = 1.8 = \frac{9}{5}$. Hence arc $BE = r\theta = \frac{50}{3} \cdot \frac{9}{5} = 30$.

7 Find the sector area

Sector area $= \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{50}{3}\right)^2\left(\frac{9}{5}\right) = 250$.

8 Find the triangle area and total area

Triangle CDE has vertical base $CE = \frac{50}{3}$ and horizontal height 8, so its area is $\frac{1}{2} \cdot \frac{50}{3} \cdot 8 = \frac{200}{3}$.

Total area $= 250 + \frac{200}{3} = \frac{950}{3}$.

✓ FINAL ANSWER

$(a)\ 12.$ $(b)\ E_y = -\frac{14}{3}.$ $(c)\ 30.$ $(d)\ \frac{950}{3}.$

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Question 8

9 marks | Polynomials

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

(Total 9 marks)

WORKED SOLUTION

Question 8 · WMA11/01 May/June 2021

1 Complete the square

$3x^2 + 6x + 9 = 3(x^2 + 2x) + 9 = 3\big((x+1)^2 - 1\big) + 9 = 3(x+1)^2 + 6$.

2 Deduce the minimum point

The minimum occurs when $(x+1)^2 = 0$, so $x = -1$ and $y = 6$. Therefore $P = (-1, 6)$.

3 **Use the x-intercepts of C2**

Since C_2 intersects the x-axis at $-4, -2$ and 3, it has form $y = k(x+4)(x+2)(x-3)$.

4 **Use point P to find k**

Substitute $P = (-1, 6)$: $6 = k(3)(1)(-4) = -12k$, so $k = -\dfrac{1}{2}$.

5 **Expand the cubic**

$(x+4)(x+2)(x-3) = (x^2+6x+8)(x-3) = x^3 + 3x^2 - 10x - 24$.

6 **Read off A, B, C and D**

Thus $y = -\dfrac{1}{2}x^3 - \dfrac{3}{2}x^2 + 5x + 12$, so $A = -\dfrac{1}{2}$, $B = -\dfrac{3}{2}$, $C = 5$ and $D = 12$.

✓ **FINAL ANSWER**

$(a)\ 3(x+1)^2 + 6. \quad (b)\ P = (-1, 6). \quad (c)\ A = -\dfrac{1}{2},\ B = -\dfrac{3}{2},\ C = 5,\ D = 12.$

Question 9

7 marks | Trigonometric Functions

9.

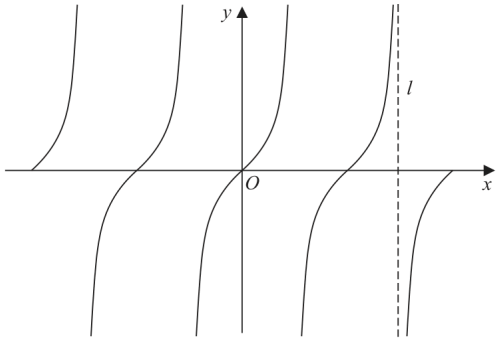


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

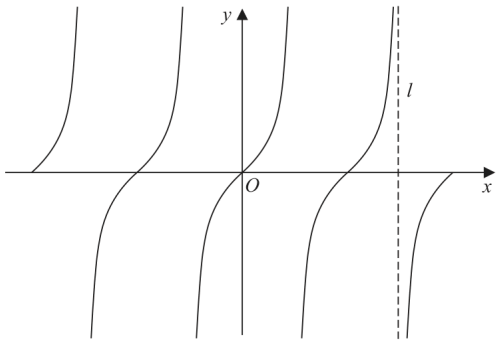


Diagram 1

(Total 7 marks)

1 State the shown tangent asymptote

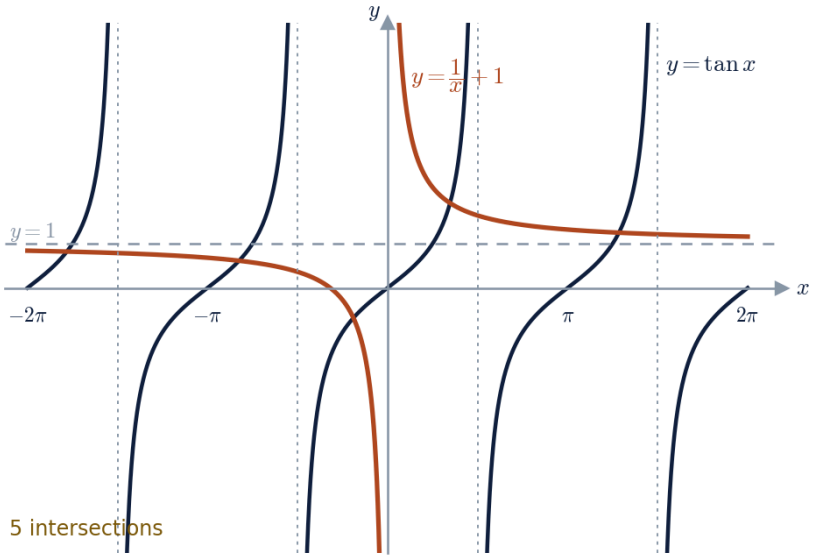
The shown dashed asymptote is the right-hand positive asymptote before 2π, so l has equation $x = \frac{3\pi}{2}$.

2 Describe the reciprocal graph

For $y = \frac{1}{x} + 1$, the vertical asymptote is $x = 0$ and the horizontal asymptote is $y = 1$. The right branch lies above $y = 1$ and the left branch lies below $y = 1$.

3 Count intersections on -2pi to 2pi

The number of solutions of $\tan x = \frac{1}{x} + 1$ is the number of intersections of the two graphs. From the sketch on $-2\pi \le x \le 2\pi$, there are 5 intersections.



4 Count first interval solutions

From the repeated tangent branches and the reciprocal curve, there is one intersection in each positive interval between consecutive tangent asymptotes. From 0 to 40π, this gives 40 solutions.

5 Count second interval solutions

Extending the same intersection pattern across the stated interval gives 14 intersections, so there are 14 solutions.

FINAL ANSWER

$(a)\ x = \frac{3\pi}{2}.$ $(b)(i)\ y = 1$ is the horizontal asymptote. $(b)(ii)\ 5.$ $(c)(i)\ 40.$ $(c)(ii)\ 14.$