Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2021 Worked Solutions

January 2021 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 January 2021

January 2021

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 JANUARY 2021

Question 1

8 marks | Differentiation

1. A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

The point P lies on the curve and has x coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(Total 8 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2021

1 Rewrite the negative power term

$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 = 2x^3 - 5x^2 - \frac{3}{2}x^{-1} + 7$.

2 Differentiate term by term

$\frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2}x^{-2} = 6x^2 - 10x + \frac{3}{2x^2}$.

3 Find the point P

At $x = \frac{1}{2}$, $y = 2\left(\frac{1}{8}\right) - 5\left(\frac{1}{4}\right) - \frac{3}{1} + 7 = 3$, so $P = \left(\frac{1}{2}, 3\right)$.

4 Find the normal gradient

At $x = \frac{1}{2}$, $\frac{dy}{dx} = 6\left(\frac{1}{4}\right) - 10\left(\frac{1}{2}\right) + \frac{3}{2(1/4)} = \frac{5}{2}$. The normal gradient is therefore $-\frac{2}{5}$.

5 Write the normal in integer form

$y - 3 = -\frac{2}{5}\left(x - \frac{1}{2}\right)$. Multiplying by 5 gives $5y - 15 = -2x + 1$, hence $2x + 5y - 16 = 0$.

FINAL ANSWER

$(a)\ \dfrac{dy}{dx} = 6x^2 - 10x + \dfrac{3}{2x^2}.$ $\quad (b)\ 2x + 5y - 16 = 0.$

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Question 2

6 marks | Quadratics

2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q.

(4)

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place.

(2)

(Total 6 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2021

1 Substitute the two given heights

When $t = 3$ and $H = 2$, $2^3 = 9p + q$, so $8 = 9p + q$. When $t = 5$ and $H = 2.4$, $2.4^3 = 25p + q$, so $13.824 = 25p + q$.

2 Solve for p and q

Subtracting the two equations gives $16p = 5.824$, so $p = 0.364$. Then $8 = 9(0.364) + q$, giving $q = 4.724$, so $q = 4.72$ to 3 significant figures.

3 Use H = 5

For $H = 5$, $125 = 0.364T^2 + 4.724$.

4 Find T

$T^2 = \dfrac{125 - 4.724}{0.364} = 330.428\ldots$, so $T = 18.177\ldots$. Therefore $T = 18.2$ years to one decimal place.

FINAL ANSWER

$(a)\ p = 0.364,\ q = 4.72.$ $(b)\ T = 18.2$ years.

Question 3

5 marks | Trigonometric Functions

3.

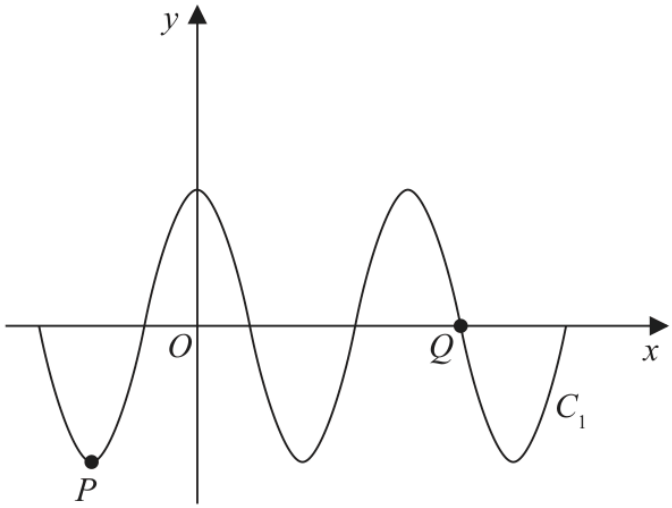


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4\cos x°$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4\cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(Total 5 marks)

1 Read cosine key points

For $y = 4\cos x°$, the maximum value is 4 and the minimum value is -4. The minimum immediately to the left of the y-axis is at $x = -180°$, so $P = (-180, -4)$.

2 Find Q from the cosine zero

The x-intercepts of $4\cos x°$ occur at $x = 90°, 270°, 450°, \ldots$. The point shown to the right of the next maximum is $Q = (450, 0)$.

3 Use the shifted curve C2

For C_2, $y = 4\cos x° + k$. Its minimum is $-4 + k$. Since the minimum value is -1, $-4 + k = -1$, so $k = 3$.

4 Find R

The maximum value of C_2 is $4 + 3 = 7$. The maximum point with the smallest positive x-coordinate occurs at $x = 360°$, so $R = (360, 7)$.

FINAL ANSWER

$(a)(i)\ P = (-180, -4).$ $(a)(ii)\ Q = (450, 0).$ $(b)\ R = (360, 7).$

Question 4

8 marks | Straight Line

4.

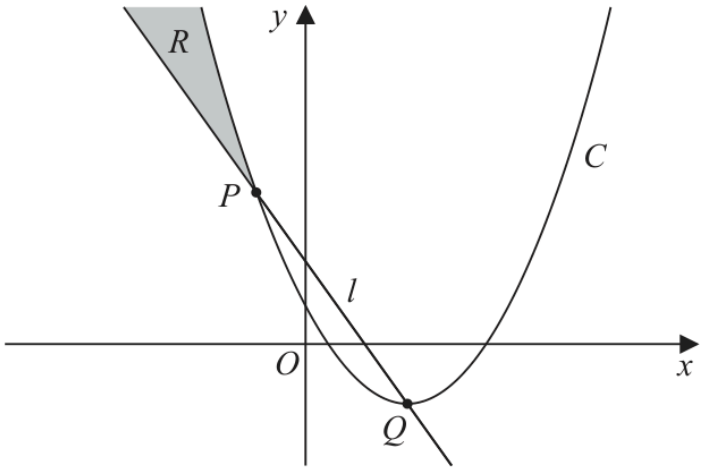


Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(3)

The quadratic curve C passes through P and has a minimum point at Q.

(b) Find an equation for C.

(3)

The region R, shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

(c) Use inequalities to define region R.

(2)

(Total 8 marks)

1 Find the line through P and Q

The gradient is $\frac{-5-13}{4-(-2)} = \frac{-18}{6} = -3$. Using $P(-2, 13)$, $y - 13 = -3(x + 2)$, hence $y = -3x + 7$.

2 Use vertex form for the quadratic

The minimum point is $Q(4, -5)$, so the curve has form $y = a(x - 4)^2 - 5$.

3 Substitute P to find a

Since $P(-2, 13)$ lies on the curve, $13 = a(-6)^2 - 5$. Therefore $18 = 36a$ and $a = \frac{1}{2}$.

4 Write the curve

C has equation $y = \frac{1}{2}(x - 4)^2 - 5$.

5 Define the shaded region

In the shaded second-quadrant region, points are above the line, below the curve, and to the left of P. Therefore $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$, and $x < -2$.

FINAL ANSWER

$(a)\ y = -3x + 7. \quad (b)\ y = \frac{1}{2}(x - 4)^2 - 5. \quad (c)\ y > -3x + 7,\ y < \frac{1}{2}(x - 4)^2 - 5,\ x < -2.$

Question 5

9 marks | Radians

5.

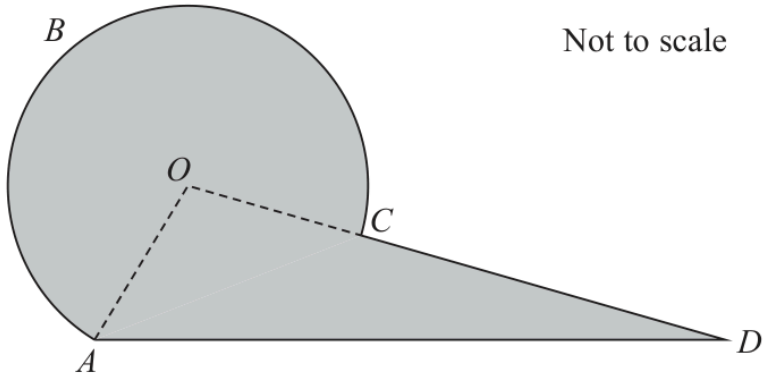


Figure 3

Figure 3 shows the plan view of a viewing platform at a tourist site.

The shape of the viewing platform consists of a sector $ABCOA$ of a circle, centre O, joined to a triangle AOD.

Given that

- $OA = OC = 6\,\text{m}$
- $AD = 14\,\text{m}$
- angle $ADC = 0.43$ radians
- angle AOD is an obtuse angle
- OCD is a straight line

find

(a) the size of angle AOD, in radians, to 3 decimal places,

(3)

(b) the length of arc ABC, in metres, to one decimal place,

(2)

(c) the total area of the viewing platform, in m^2, to one decimal place.

(4)

(Total 9 marks)

1 Use the sine rule in triangle AOD

Since O, C, D are in a straight line, $\angle ADO = 0.43$. By the sine rule, $\dfrac{14}{\sin \angle AOD} = \dfrac{6}{\sin 0.43}$, so $\sin \angle AOD = \dfrac{14 \sin 0.43}{6}$.

2 Choose the obtuse angle

The calculator gives the acute value $1.337\ldots$, but $\angle AOD$ is obtuse. Hence $\angle AOD = \pi - 1.337\ldots = 1.805\ldots$, so $\angle AOD = 1.805$ radians.

3 Find the major-sector angle

Arc ABC is the major arc, so its angle is $2\pi - 1.805\ldots = 4.478\ldots$ radians.

4 Find the arc length

$s = r\theta = 6(4.478\ldots) = 26.868\ldots$, so the length of arc ABC is 26.9 m.

5 Find the sector area

The sector area is $\dfrac{1}{2}r^2\theta = \dfrac{1}{2}(6)^2(4.478\ldots) = 80.607\ldots$.

6 Find the triangle area and total area

$\angle OAD = \pi - 0.43 - 1.805\ldots = 0.9066\ldots$. The triangle area is $\dfrac{1}{2}(6)(14)\sin(0.9066\ldots) = 33.093\ldots$. Total area $= 80.607\ldots + 33.093\ldots = 113.7\ \text{m}^2$.

FINAL ANSWER

(a) 1.805 rad. (b) 26.9 m. (c) $113.7\ \text{m}^2$.

Question 6

10 marks | Graphs of Functions

6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

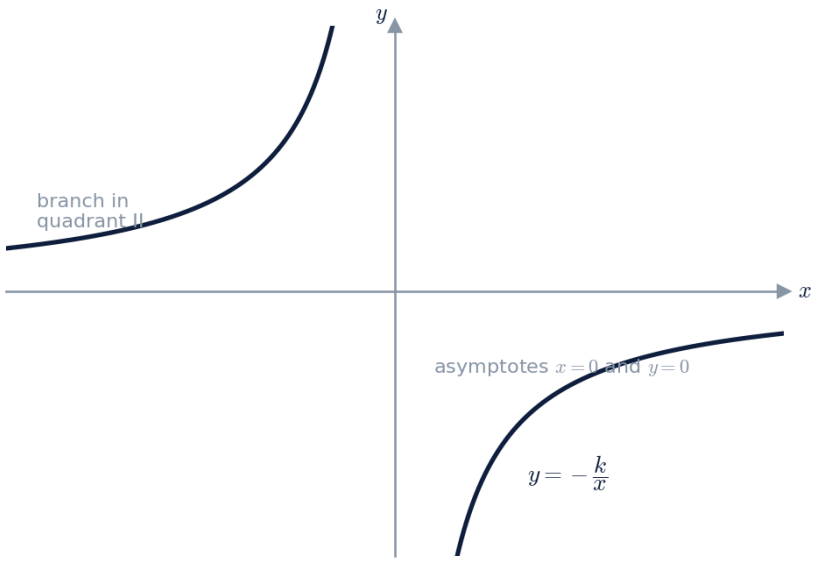
does not touch or intersect the line with equation $y = 3x + 4$

(5)

(Total 10 marks)

1 Sketch y = -k/x

Because $k > 0$, $y = -\dfrac{k}{x}$ has one branch in quadrant II and one branch in quadrant IV, with asymptotes $x = 0$ and $y = 0$.



2 Translate the graph upward

$y = -\dfrac{k}{x} + k$ is $y = -\dfrac{k}{x}$ translated up by k units. The vertical asymptote is still $x = 0$, and the horizontal asymptote is $y = k$.

3 Find the x-intercept

Set $y = 0$: $0 = -\dfrac{k}{x} + k$. Since $k > 0$, $\dfrac{k}{x} = k$, so $x = 1$. The x-intercept is $(1, 0)$.

4 Set up the intersection equation

For intersections with $y = 3x + 4$, solve $-\dfrac{k}{x} + k = 3x + 4$. Multiplying by x gives $-k + kx = 3x^2 + 4x$.

5 Use the discriminant

This rearranges to $3x^2 + (4 - k)x + k = 0$. For no touch or intersection, the discriminant must be negative: $(4 - k)^2 - 12k < 0$.

6 Solve the quadratic inequality in k

$k^2 - 20k + 16 < 0$. The roots are $k = 10 \pm 2\sqrt{21}$, so the required range is $10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}$.

FINAL ANSWER

$(b)\ (1, 0),\ y = k.$ $(c)\ 10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}.$

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Question 7

9 marks | Differentiation

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

(Total 9 marks)

WORKED SOLUTION

Question 7 · WMA11/01 January 2021

1 **Solve f(x) = 9 without calculator methods**

$2x - 3\sqrt{x} - 5 = 9$, so $2x - 3\sqrt{x} - 14 = 0$.

2 **Use u = sqrt(x)**

Let $u = \sqrt{x}$, where $u > 0$. Then $x = u^2$, so $2u^2 - 3u - 14 = 0$.

3 **Factorise and reject the invalid root**

$2u^2 - 3u - 14 = (2u - 7)(u + 2)$. Since $u > 0$, $u = \frac{7}{2}$, so $x = u^2 = \frac{49}{4}$.

4 **Differentiate twice**

$f(x) = 2x - 3x^{1/2} - 5$, so $f'(x) = 2 - \frac{3}{2}x^{-1/2}$ and $f''(x) = \frac{3}{4}x^{-3/2}$.

5 **Solve f''(x) = 6**

$\frac{3}{4}x^{-3/2} = 6$, so $x^{-3/2} = 8$. Therefore $x^{3/2} = \frac{1}{8}$.

6 **Find x**

Since $x^{3/2} = (\sqrt{x})^3$, we get $\sqrt{x} = \dfrac{1}{2}$, hence $x = \dfrac{1}{4}$.

✓ **FINAL ANSWER**

$(a)\ x = \dfrac{49}{4}.\quad (b)\ x = \dfrac{1}{4}.$

Question 8

9 marks | Graphs of Functions

8.

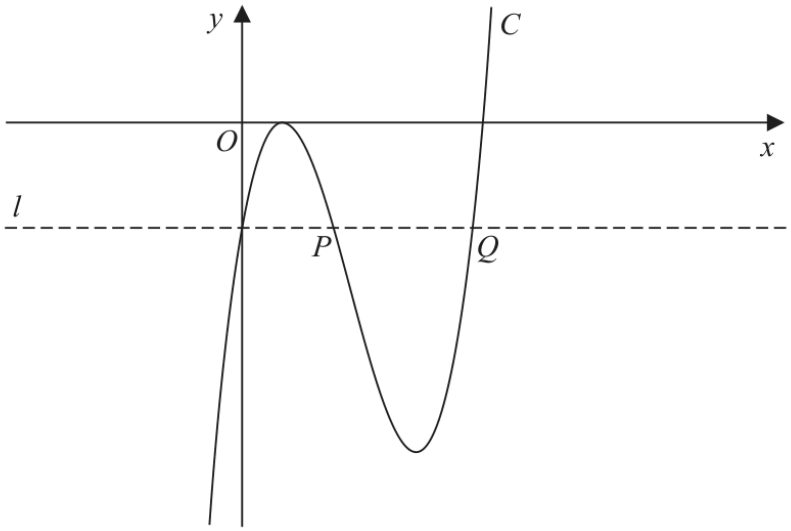


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (3x - 2)^2 (x - 4)$$

(a) Deduce the values of x for which $\mathrm{f}(x) > 0$

(1)

(b) Expand $\mathrm{f}(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

The line l, also shown in Figure 4, passes through the y intercept of C and is parallel to the x-axis.

The line l cuts C again at points P and Q, also shown in Figure 4.

(c) Using algebra and showing your working, find the length of line PQ. Write your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total 9 marks)

1 Use the factorised form

$f(x) = (3x-2)^2(x-4)$. The squared factor is never negative, and it is zero at $x = \frac{2}{3}$. Therefore the sign of $f(x)$ away from the repeated root is controlled by $x - 4$.

2 Find where f(x) is positive

For $f(x) > 0$, we need $x - 4 > 0$, so $x > 4$.

3 Expand f(x)

$(3x-2)^2 = 9x^2 - 12x + 4$. Therefore
$f(x) = (9x^2 - 12x + 4)(x-4) = 9x^3 - 48x^2 + 52x - 16$.

4 Find the line l

The y-intercept of C is $f(0) = -16$, so the horizontal line l is $y = -16$.

5 Find the other intersections with l

Solve $f(x) = -16$: $9x^3 - 48x^2 + 52x - 16 = -16$, so $x(9x^2 - 48x + 52) = 0$. The root $x = 0$ is the y-intercept, and the two other roots give P and Q.

6 Solve the quadratic

$9x^2 - 48x + 52 = 0$. Hence $x = \dfrac{48 \pm \sqrt{48^2 - 4(9)(52)}}{18} = \dfrac{48 \pm 12\sqrt{3}}{18} = \dfrac{8 \pm 2\sqrt{3}}{3}$.

7 Find PQ

Because P and Q lie on the same horizontal line, PQ is the difference in their x-coordinates:
$PQ = \dfrac{8+2\sqrt{3}}{3} - \dfrac{8-2\sqrt{3}}{3} = \dfrac{4\sqrt{3}}{3}$.

✓ **FINAL ANSWER**

$(a)\ x > 4.$ $(b)\ f(x) = 9x^3 - 48x^2 + 52x - 16.$ $(c)\ PQ = \dfrac{4\sqrt{3}}{3}.$

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Question 9

11 marks | Integration

9. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(5)

(ii) A curve C has equation $y = \mathrm{f}(x)$.

Given

- $\mathrm{f}'(x) = x^2 + ax + b$ where a and b are constants
- the y intercept of C is -8
- the point $P(3,-2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $\mathrm{f}(x)$.

(6)

(Total 11 marks)

WORKED SOLUTION

Question 9 · WMA11/01 January 2021

1 **Expand and simplify the integrand**

$$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2+12x+4}{4x^{1/2}} = \frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}.$$

2 **Integrate term by term**

$$\int \left(\frac{9}{4}x^{3/2} + 3x^{1/2} + x^{-1/2}\right)dx = \frac{9}{10}x^{5/2} + 2x^{3/2} + 2x^{1/2} + c.$$

3 **Write in simplest form**

Factoring out $\sqrt{x}$ gives $\dfrac{\sqrt{x}(9x^2+20x+20)}{10} + c.$

4 **Use the gradient condition at P**

Since $f'(x) = x^2 + ax + b$ and the gradient at $P(3,-2)$ is 2, $f'(3) = 2$. Hence $9 + 3a + b = 2$, so $3a + b = -7$.

5 **Integrate f'(x)**

$f(x) = \frac{x^3}{3} + \frac{a}{2}x^2 + bx + c$. The y-intercept is -8, so $c = -8$.

6 **Use P(3,-2)**

$-2 = \frac{27}{3} + \frac{9a}{2} + 3b - 8$, so $\frac{9a}{2} + 3b = -3$.

7 **Solve for a and b**

From $3a + b = -7$, $b = -7 - 3a$. Substitute into $\frac{9a}{2} + 3b = -3$ to get $a = -4$, and then $b = 5$.

8 **Write f(x)**

Therefore $f(x) = \frac{x^3}{3} - 2x^2 + 5x - 8$.

✓ **FINAL ANSWER**

$(i)\ \frac{\sqrt{x}(9x^2 + 20x + 20)}{10} + c. \quad (ii)\ f(x) = \frac{x^3}{3} - 2x^2 + 5x - 8.$