Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 January 2020 Worked Solutions

January 2020 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA11/01 January 2020

January 2020

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

4 marks | Integration

1. Find, in simplest form,

$$\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 \right) \mathrm{d}x$$

(4)

(Total 4 marks)

WORKED SOLUTION

Question 1 · WMA11/01 January 2020

1 **Rewrite the integrand using powers**

$\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 = \frac{8}{3}x^3 - \frac{1}{2}x^{-1/2} - 5.$

2 **Integrate term by term**

$\int \frac{8}{3}x^3\,dx = \frac{2}{3}x^4$, $\int -\frac{1}{2}x^{-1/2}\,dx = -x^{1/2}$, and $\int -5\,dx = -5x$.

3 **Include the constant**

Therefore $\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 \right) dx = \frac{2}{3}x^4 - \sqrt{x} - 5x + c.$

✓ **FINAL ANSWER**

$\frac{2}{3}x^4 - \sqrt{x} - 5x + c.$

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Question 2

5 marks | Indices & Surds

2. Given $y = 3^x$, express each of the following in terms of y. Write each expression in its simplest form.

(a) 3^{3x}

(1)

(b) $\dfrac{1}{3^{x-2}}$

(2)

(c) $\dfrac{81}{9^{2-3x}}$

(2)

(Total 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 January 2020

1 Use y = 3^x

Since $y = 3^x$, powers of 3^x can be replaced by powers of y.

2 Simplify part (a)

$3^{3x} = (3^x)^3 = y^3$.

3 Simplify part (b)

$$\frac{1}{3^{x-2}} = 3^{2-x} = \frac{3^2}{3^x} = \frac{9}{y}.$$

4 Simplify part (c)

$$\frac{81}{9^{2-3x}} = \frac{3^4}{(3^2)^{2-3x}} = \frac{3^4}{3^{4-6x}} = 3^{6x} = (3^x)^6 = y^6.$$

✓ **FINAL ANSWER**

$(a)\ y^3.\quad (b)\ \dfrac{9}{y}.\quad (c)\ y^6.$

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Question 3

6 marks | Differentiation

3.

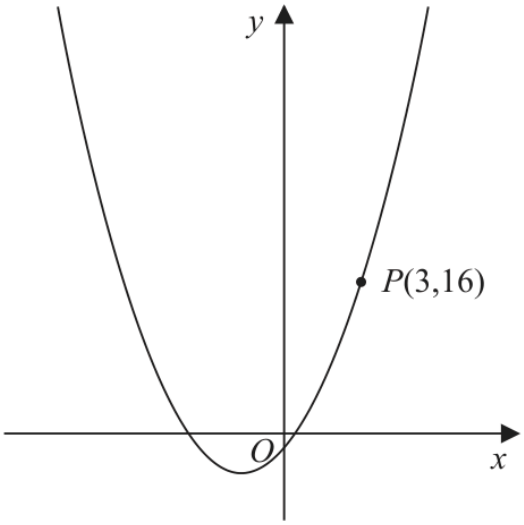


Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3,16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Write your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total 6 marks)

WORKED SOLUTION

Question 3 · WMA11/01 January 2020

1 Differentiate for the tangent gradient

For $y = x^2 + 3x - 2$, $\frac{dy}{dx} = 2x + 3$. At $x = 3$, the gradient is $2(3) + 3 = 9$.

2 **Find the y-coordinate of Q**

For $x_Q = 3 + h$, $y_Q = (3+h)^2 + 3(3+h) - 2 = 16 + 9h + h^2$.

3 **Calculate the gradient of PQ**

$$\text{gradient } PQ = \frac{(16 + 9h + h^2) - 16}{(3+h) - 3} = \frac{9h + h^2}{h} = 9 + h.$$

4 **Explain the relationship**

As $h \to 0$, point Q approaches P, and the chord gradient $9 + h$ approaches the tangent gradient 9.

✓ **FINAL ANSWER**

$(a)\ 9. \quad (b)\ 9 + h. \quad (c)\ As\ h \to 0,\ 9 + h \to 9.$

Question 4

9 marks | Radians

4.

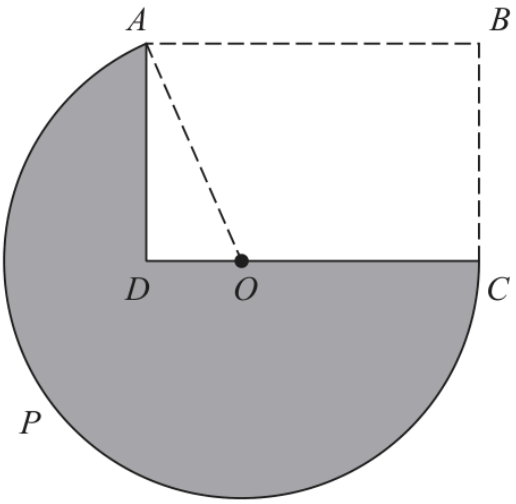


Figure 2

Figure 2 shows the plan view of a house $ABCD$ and a lawn $APCDA$.

$ABCD$ is a rectangle with $AB = 16\,\text{m}$.

$APCOA$ is a sector of a circle centre O with radius $12\,\text{m}$.

The point O lies on the line DC, as shown in Figure 2.

(a) Show that the size of angle AOD is 1.231 radians to 3 decimal places.

(2)

The lawn $APCDA$ is shown shaded in Figure 2.

(b) Find the area of the lawn, in m^2, to one decimal place.

(4)

(c) Find the perimeter of the lawn, in metres, to one decimal place.

(3)

(Total 9 marks)

1 Find angle AOD

Since $OC = 12$ and $DC = AB = 16$, $OD = 4$. In right triangle AOD, $\cos \angle AOD = \dfrac{OD}{OA} = \dfrac{4}{12} = \dfrac{1}{3}$. Hence $\angle AOD = \cos^{-1}\left(\dfrac{1}{3}\right) = 1.231$ radians to 3 d.p.

2 Identify the major sector

The shaded lawn uses the major sector from A to C, so its angle is $\pi + 1.231$ radians.

3 Subtract the unshaded triangle

$AD = \sqrt{12^2 - 4^2} = 8\sqrt{2}$. Area of major sector $= \frac{1}{2}(12)^2(\pi + 1.231)$. Area of triangle $AOD = \frac{1}{2}(4)(8\sqrt{2})$.

4 Find the lawn area

Area $= \frac{1}{2}(12)^2(\pi + 1.231) - \frac{1}{2}(4)(8\sqrt{2}) = 292.2\ \text{m}^2$ to 1 d.p.

5 Find the lawn perimeter

Perimeter $=$ major arc $+ CD + AD = 12(\pi + 1.231) + 16 + 8\sqrt{2} = 79.8\ \text{m}$ to 1 d.p.

FINAL ANSWER

$(a)\ \angle AOD = 1.231\ \text{rad}.$ $(b)\ 292.2\ \text{m}^2.$ $(c)\ 79.8\ \text{m}.$

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Question 5

7 marks | Quadratics

5. (a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0$$

(3)

(b) Hence find all real solutions of

$$20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{\frac{1}{2}} = 0$$

(4)

(Total 7 marks)

WORKED SOLUTION

Question 5 · WMA11/01 January 2020

1 Factorise the cubic

$20x^3 - 50x^2 - 30x = 10x(2x^2 - 5x - 3) = 10x(2x+1)(x-3)$.

2 Solve part (a)

$10x(2x+1)(x-3) = 0$, so $x = 0$, $x = -\frac{1}{2}$, or $x = 3$.

3 Use the same structure for part (b)

Let $u = (y+3)^{1/2}$. Then the equation becomes $20u^3 - 50u^2 - 30u = 0$, so $u = 0$, $u = -\frac{1}{2}$, or $u = 3$.

4 Reject the invalid square-root value

Since $u = \sqrt{y+3} \geq 0$, reject $u = -\frac{1}{2}$. If $u = 0$, then $y = -3$. If $u = 3$, then $y + 3 = 9$, so $y = 6$.

FINAL ANSWER

$(a)\ x = 0, -\frac{1}{2}, 3.$ $(b)\ y = -3, 6.$

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Question 6

8 marks | Straight Line

6. The line l_1 has equation $3x - 4y + 20 = 0$

The line l_2 cuts the x-axis at $R(8,0)$ and is parallel to l_1

(a) Find the equation of l_2, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

The line l_1 cuts the x-axis at P and the y-axis at Q.

Given that $PQRS$ is a parallelogram, find

(b) the area of $PQRS$,

(3)

(c) the coordinates of S.

(2)

(Total 8 marks)

WORKED SOLUTION

Question 6 · WMA11/01 January 2020

1 Find the gradient of l1

$3x - 4y + 20 = 0$ gives $y = \frac{3}{4}x + 5$, so the gradient is $\frac{3}{4}$.

2 Find l2 through R

Line l_2 is parallel to l_1 and passes through $R(8, 0)$, so $y = \frac{3}{4}(x - 8) = \frac{3}{4}x - 6$. Therefore $3x - 4y - 24 = 0$.

3 Find P and Q

For l_1, the x-intercept is $P\left(-\frac{20}{3}, 0\right)$ and the y-intercept is $Q = (0, 5)$.

4 Find the area of PQRS

Base $PR = 8 - \left(-\frac{20}{3}\right) = \frac{44}{3}$ and height $OQ = 5$. Area $= \frac{44}{3} \cdot 5 = \frac{220}{3}$.

5 **Find S using a vector**

In parallelogram $PQRS$, $\vec{PS} = \vec{QR} = (8, -5)$. Hence $S = P + (8, -5) = \left(-\frac{20}{3} + 8, -5\right) = \left(\frac{4}{3}, -5\right)$.

✓ **FINAL ANSWER**

$(a)\ 3x - 4y - 24 = 0.\quad (b)\ \dfrac{220}{3}.\quad (c)\ S = \left(\dfrac{4}{3}, -5\right).$

Question 7

5 marks | Trigonometric Functions

7.

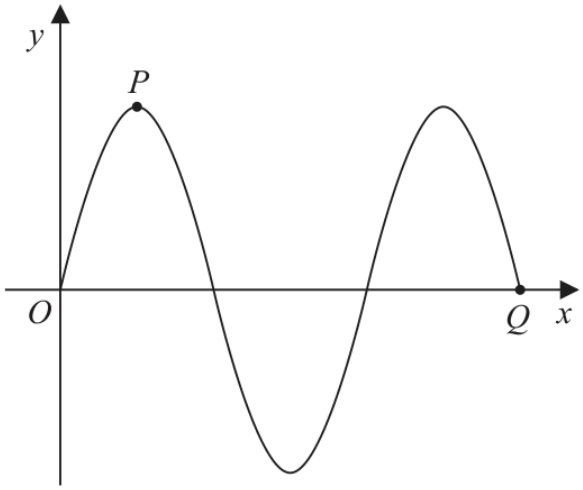


Figure 3

Figure 3 shows part of the curve C_1 with equation $y = 3\sin x$, where x is measured in degrees.

The point P and the point Q lie on C_1 and are shown in Figure 3.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

A different curve C_2 has equation $y = 3\sin x + k$, where k is a constant.

The curve C_2 has a maximum y value of 10

The point R is the minimum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

(Total 5 marks)

1 **Read point P**

The first maximum of $y = 3\sin x$ occurs at $x = 90°$, with $y = 3$. So $P = (90°, 3)$.

2 **Read Q from the next shown zero**

After the second maximum, the curve returns to the x-axis at $x = 540°$. So $Q = (540°, 0)$.

3 **Find the vertical shift**

For C_2, $y = 3\sin x + k$. Its maximum is $3 + k = 10$, so $k = 7$.

4 **Find the minimum point R**

The minimum value is $-3 + 7 = 4$. The smallest positive x-coordinate for a sine minimum is $270°$, so $R = (270°, 4)$.

✓ **FINAL ANSWER**

$(a)(i)\ P = (90°, 3).$ $(a)(ii)\ Q = (540°, 0).$ $(b)\ R = (270°, 4).$

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Question 8

6 marks | Inequalities

8. The straight line l has equation $y = k(2x - 1)$, where k is a constant.

The curve C has equation $y = x^2 + 2x + 11$

Find the set of values of k for which l does not cross or touch C.

(6)

(Total 6 marks)

WORKED SOLUTION

Question 8 · WMA11/01 January 2020

1 Equate the line and curve

$k(2x - 1) = x^2 + 2x + 11$, so $x^2 + (2 - 2k)x + (11 + k) = 0$.

2 Use the no-intersection condition

For the line not to cross or touch the curve, the quadratic in x must have no real roots, so its discriminant is less than zero.

3 Form the inequality in k

$(2 - 2k)^2 - 4(1)(11 + k) < 0$. Dividing by 4 gives $(1 - k)^2 - (11 + k) < 0$, so $k^2 - 3k - 10 < 0$.

4 Solve the quadratic inequality

$(k - 5)(k + 2) < 0$, so k lies between the two roots: $-2 < k < 5$.

✓ **FINAL ANSWER**

$-2 < k < 5$.

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Question 9

6 marks | Differentiation

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the x coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

(6)

(Total 6 marks)

WORKED SOLUTION

Question 9 · WMA11/01 January 2020

1 Rewrite the function

$y = \frac{4x^2 + 9}{2\sqrt{x}} = 2x^{3/2} + \frac{9}{2}x^{-1/2}$.

2 Differentiate

$\frac{dy}{dx} = 3x^{1/2} - \frac{9}{4}x^{-3/2}$.

3 Set the derivative equal to zero

$3x^{1/2} - \frac{9}{4}x^{-3/2} = 0$, so $3\sqrt{x} = \frac{9}{4x^{3/2}}$.

4 Solve for x

Multiply by $4x^{3/2}$: $12x^2 = 9$. Hence $x^2 = \frac{3}{4}$. Since $x > 0$, $x = \frac{\sqrt{3}}{2}$.

FINAL ANSWER

$x = \frac{\sqrt{3}}{2}$.

Question 10

8 marks | Graphs of Functions

10. The curve C_1 has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (4x - 3)(x - 5)^2$$

(a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes.

(3)

(b) Hence or otherwise

(i) find the values of x for which $\mathrm{f}\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant p such that the curve with equation $y = \mathrm{f}(x) + p$ passes through the origin.

(2)

A second curve C_2 has equation $y = \mathrm{g}(x)$, where $\mathrm{g}(x) = \mathrm{f}(x + 1)$

(c) (i) Find, in simplest form, $\mathrm{g}(x)$. You may leave your answer in a factorised form.

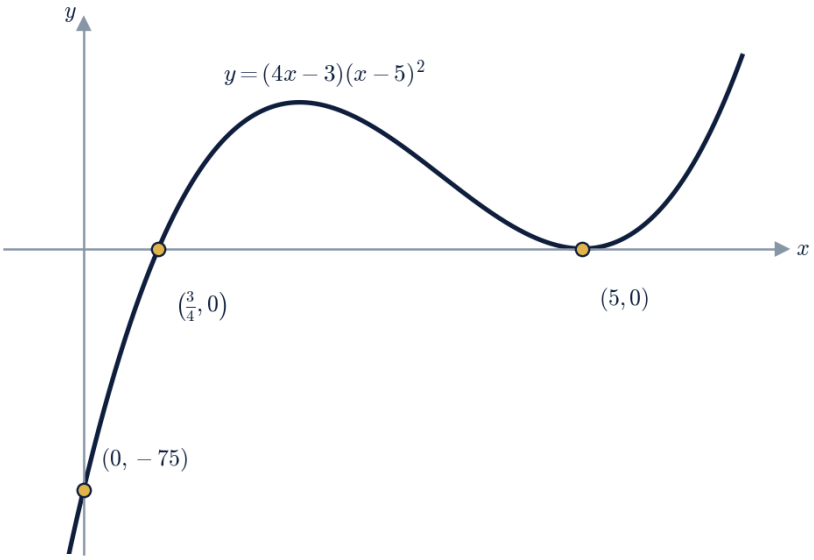
(ii) Hence, or otherwise, find the y intercept of curve C_2

(3)

(Total 8 marks)

1 Use the factorised form for the sketch

$f(x) = (4x - 3)(x - 5)^2$. The curve crosses the x-axis at $x = \frac{3}{4}$ and touches the x-axis at $x = 5$. The y-intercept is $f(0) = (-3)(25) = -75$.



2 Solve f(x/4)=0

For $f\left(\frac{1}{4}x\right) = 0$, either $4\left(\frac{x}{4}\right) - 3 = 0$ or $\frac{x}{4} - 5 = 0$. Therefore $x = 3$ or $x = 20$.

3 Find p

The curve $y = f(x) + p$ passes through the origin, so $0 = f(0) + p = -75 + p$. Hence $p = 75$.

4 Find g(x)

$g(x) = f(x + 1) = (4(x + 1) - 3)((x + 1) - 5)^2 = (4x + 1)(x - 4)^2$.

5 Find the y-intercept of C2

At $x = 0$, $g(0) = (1)(16) = 16$. So the y-intercept is 16.

FINAL ANSWER

Sketch: crosses at $\left(\frac{3}{4}, 0\right)$, touches at $(5, 0)$, crosses y-axis at $(0, -75)$. Also $x = 3, 20$, $p = 75$, $g(x) = (4x + 1)(x - 4)^2$, y-intercept 16.

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Question 11

11 marks | Integration

11. A curve has equation $y = f(x)$, where

$$f''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.

Given that $f'(x) = -4$ at P,

(a) find the equation of the normal at P, writing your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find $f(x)$.

(8)

(Total 11 marks)

WORKED SOLUTION

Question 11 · WMA11/01 January 2020

1 Find the normal gradient

At P, $f'(4) = -4$, so the tangent gradient is -4. The normal gradient is therefore $\frac{1}{4}$.

2 Find the normal equation

Using $P(4, -50)$: $y + 50 = \frac{1}{4}(x - 4)$, so $y = \frac{1}{4}x - 51$.

3 Integrate f'' to find f'

$f''(x) = 6x^{-3/2} + x$. Therefore $f'(x) = -12x^{-1/2} + \frac{1}{2}x^2 + k$.

4 Use f'(4) = -4

$-4 = -\frac{12}{2} + \frac{1}{2}(16) + k = -6 + 8 + k = 2 + k$, so $k = -6$.

5 Integrate again

$f(x) = -24x^{1/2} + \frac{x^3}{6} - 6x + d$.

6 **Use the point on the curve**

Substitute $P(4,-50)$: $-50 = -24(2) + \frac{64}{6} - 24 + d = -72 + \frac{32}{3} + d = -\frac{184}{3} + d$. Hence $d = \frac{34}{3}$.

✓ **FINAL ANSWER**

$(a)\ y = \dfrac{1}{4}x - 51. \quad (b)\ f(x) = -24\sqrt{x} + \dfrac{x^3}{6} - 6x + \dfrac{34}{3}.$