Elite Mathematics
Edexcel International A Level

IAL PURE 1 · WMA11

WMA11/01 October 2019 Worked Solutions

October 2019 paper rebuilt with the worked solution after each question.

11	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

WMA11/01 October 2019

October 2019

11	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA11/01 OCTOBER 2019

Question 1

5 marks | Radians

1.

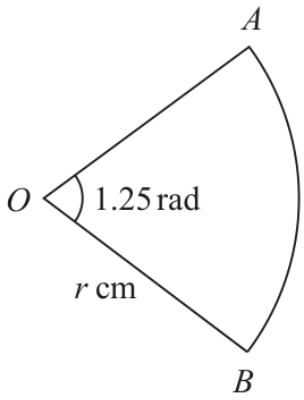


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is 1.25 radians.

Given that the area of the sector AOB is 15 cm^2

(a) find the exact value of r,

(2)

(b) find the exact length of the perimeter of the sector. Write your answer in simplest form.

(3)

(Total 5 marks)

WORKED SOLUTION

Question 1 · WMA11/01 October 2019

1 Use the sector area formula

For a sector, $A = \frac{1}{2}r^2\theta$. Here $A = 15$ and $\theta = 1.25 = \frac{5}{4}$, so $15 = \frac{1}{2}r^2 \cdot \frac{5}{4}$.

2 Find the exact radius

$r^2 = \dfrac{30}{5/4} = 24$, so $r = \sqrt{24} = 2\sqrt{6}$ because the radius is positive.

3 Find the perimeter

Arc length is $s = r\theta = (2\sqrt{6})\left(\frac{5}{4}\right) = \frac{5\sqrt{6}}{2}$. The perimeter is $2r + s = 4\sqrt{6} + \frac{5\sqrt{6}}{2} = \frac{13\sqrt{6}}{2}$.

FINAL ANSWER

$(a)\ r = 2\sqrt{6}\ \text{cm}. \quad (b)\ \dfrac{13\sqrt{6}}{2}\ \text{cm}.$

WMA11/01 OCTOBER 2019

Question 2

5 marks | Straight Line

2. A tree was planted in the ground.

Exactly 2 years after it was planted, the height of the tree was 1.85 m.

Exactly 7 years after it was planted, the height of the tree was 3.45 m.

Given that the height, H metres, of the tree, t years after it was planted in the ground, can be modelled by the equation

$$H = at + b$$

where a and b are constants,

(a) find the value of a and the value of b.

(4)

(b) State, according to the model, the height of the tree when it was planted.

(1)

(Total 5 marks)

WORKED SOLUTION

Question 2 · WMA11/01 October 2019

1 Form two equations

Using $H = at + b$: when $t = 2$, $2a + b = 1.85$; when $t = 7$, $7a + b = 3.45$.

2 Solve for the gradient of the model

Subtract the first equation from the second: $5a = 1.60$, so $a = 0.32$.

3 Find the starting height

Substitute into $2a + b = 1.85$: $0.64 + b = 1.85$, hence $b = 1.21$. At planting, $t = 0$, so $H = b = 1.21$ m.

FINAL ANSWER

$a = 0.32,\ b = 1.21.$ Height when planted: 1.21 m.

Question 3

10 marks | Graphs of Functions

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

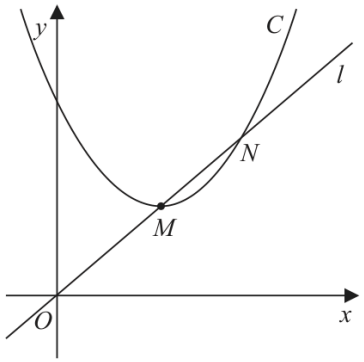


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = x^2 - 5x + 13$

The point M is the minimum point of C.

The straight line l passes through the origin O and intersects C at the points M and N as shown.

Find, showing your working,

(a) the coordinates of M,

(3)

(b) the coordinates of N.

(5)

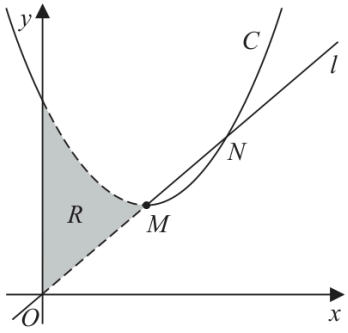


Figure 3

Figure 3 shows the curve C and the line l. The finite region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(Total 10 marks)

1 **Complete the square to locate the minimum**

$y = x^2 - 5x + 13 = \left(x - \frac{5}{2}\right)^2 + \frac{27}{4}$. Therefore $M = \left(\frac{5}{2}, \frac{27}{4}\right)$.

2 **Find line OM**

The gradient of OM is $\dfrac{27/4}{5/2} = \dfrac{27}{10}$, so l has equation $y = \frac{27}{10}x$.

3 **Find the second intersection**

Solve $x^2 - 5x + 13 = \frac{27}{10}x$. This gives $x^2 - \frac{77}{10}x + 13 = 0$. One root is $x = \frac{5}{2}$, so the other is $x = \frac{26}{5}$. Then $y = \frac{27}{10} \cdot \frac{26}{5} = \frac{351}{25}$.

4 **Describe the shaded region**

The region is between the y-axis, the line, and the curve up to M: $0 \le x \le \frac{5}{2}$ and $\frac{27}{10}x \le y \le x^2 - 5x + 13$.

✓ **FINAL ANSWER**

$M = \left(\frac{5}{2}, \frac{27}{4}\right)$, $N = \left(\frac{26}{5}, \frac{351}{25}\right)$, and $0 \le x \le \frac{5}{2}$, $\frac{27}{10}x \le y \le x^2 - 5x + 13$.

WMA11/01 OCTOBER 2019

Question 4

5 marks | Basic Trigonometry

4. A parallelogram $ABCD$ has area $40\,\text{cm}^2$

Given that AB has length $10\,\text{cm}$, BC has length $6\,\text{cm}$ and angle DAB is obtuse, find

(a) the size of angle DAB, in degrees, to 2 decimal places,

(3)

(b) the length of diagonal BD, in cm, to one decimal place.

(2)

(Total 5 marks)

WORKED SOLUTION

Question 4 · WMA11/01 October 2019

1 Use the area of a parallelogram

Area $= ab\sin\theta$, so $40 = 10 \cdot 6\sin\theta$. Hence $\sin\theta = \frac{2}{3}$.

2 Choose the obtuse angle

$\theta = 180° - \sin^{-1}\left(\frac{2}{3}\right) = 138.19°$ to 2 decimal places.

3 Use the cosine rule for BD

$BD^2 = 10^2 + 6^2 - 2(10)(6)\cos(138.19°)$. Therefore $BD \approx 15.014\ldots$, so $BD = 15.0\ \text{cm}$ to 1 decimal place.

FINAL ANSWER

$(a)\ \angle DAB = 138.19°.$ $(b)\ BD = 15.0\ \text{cm}.$

WMA11/01 OCTOBER 2019

Question 5

7 marks | Differentiation

5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

(Total 7 marks)

WORKED SOLUTION

Question 5 · WMA11/01 October 2019

1 Rewrite before differentiating

$y = \frac{x^3}{6} + 4x^{1/2} - 15$.

2 Differentiate term by term

$\frac{dy}{dx} = \frac{3x^2}{6} + 4 \cdot \frac{1}{2}x^{-1/2} = \frac{x^2}{2} + \frac{2}{\sqrt{x}}$.

3 Find tangent and normal gradients

At $x = 4$, $\frac{dy}{dx} = \frac{16}{2} + \frac{2}{2} = 9$. The normal gradient is therefore $-\frac{1}{9}$.

4 Build the normal equation

Using $P\left(4, \frac{11}{3}\right)$: $y - \frac{11}{3} = -\frac{1}{9}(x - 4)$. Multiplying out gives $x + 9y - 37 = 0$.

✓ FINAL ANSWER

$\frac{dy}{dx} = \frac{x^2}{2} + \frac{2}{\sqrt{x}}$, and the normal is $x + 9y - 37 = 0$.

WMA11/01 OCTOBER 2019

Question 6

8 marks | Graphs of Functions

6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.

(a) Sketch a graph of C, stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x-axis.

(3)

The line with equation $y = 10 - 2x$ is a tangent to C.

(b) Find the possible values for k.

(5)

(Total 8 marks)

WORKED SOLUTION

Question 6 · WMA11/01 October 2019

1 Read the graph features

For $y = \frac{4}{x} + k$, the horizontal asymptote is $y = k$. Setting $y = 0$ gives $0 = \frac{4}{x} + k$, so the x-intercept is $x = -\frac{4}{k}$.

2 Equate line and curve

A tangent touches once, so solve $10 - 2x = \frac{4}{x} + k$. Multiplying by x gives $2x^2 + (k - 10)x + 4 = 0$.

3 Use the discriminant condition

For one repeated intersection, discriminant $= 0$: $(k - 10)^2 - 4(2)(4) = 0$. Hence $(k - 10)^2 = 32$.

4 Find k

$k - 10 = \pm 4\sqrt{2}$, so $k = 10 \pm 4\sqrt{2}$. Both are positive, so both satisfy the condition in the question.

✓ FINAL ANSWER

Asymptote $y = k$, x-intercept $\left(-\frac{4}{k}, 0\right)$, and $k = 10 \pm 4\sqrt{2}$.

WMA11/01 OCTOBER 2019

Question 7

6 marks | Differentiation

7.

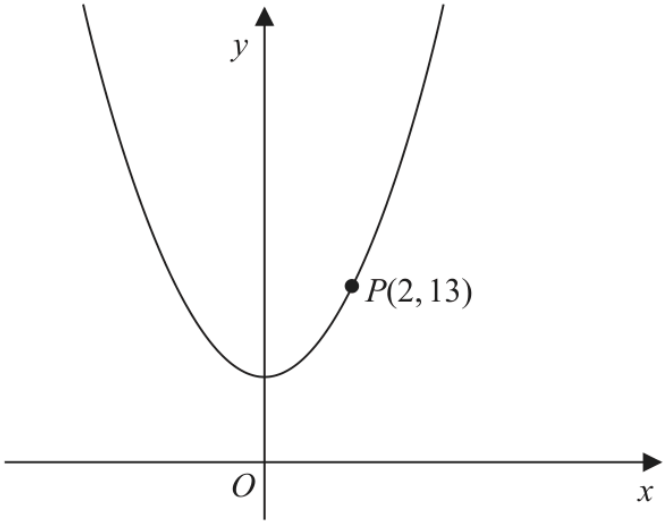


Figure 4

Figure 4 shows part of the curve with equation $y = 2x^2 + 5$

The point $P(2,13)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Give your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

(Total 6 marks)

WORKED SOLUTION

Question 7 · WMA11/01 October 2019

1 Differentiate for the tangent gradient

For $y = 2x^2 + 5$, $\frac{dy}{dx} = 4x$. At P, $x = 2$, so the tangent gradient is 8.

2 **Find the coordinates of Q**

If $x_Q = 2 + h$, then $y_Q = 2(2+h)^2 + 5 = 13 + 8h + 2h^2$.

3 **Find the gradient of PQ**

$$\text{gradient } PQ = \frac{(13 + 8h + 2h^2) - 13}{(2+h) - 2} = \frac{8h + 2h^2}{h} = 8 + 2h.$$

4 **Explain the link**

As h gets closer to 0, point Q gets closer to P, and the chord gradient $8 + 2h$ tends to the tangent gradient 8.

✓ **FINAL ANSWER**

$(a)\ 8. \quad (b)\ 8 + 2h. \quad (c)\ As\ h \to 0,\ 8 + 2h \to 8.$

WMA11/01 OCTOBER 2019

Question 8

5 marks | Quadratics

8. Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

(Total 5 marks)

WORKED SOLUTION

Question 8 · WMA11/01 October 2019

1 Use a substitution

Let $u = \sqrt{x}$. Then $x = u^2$, so the equation becomes $u^2 - 6u + 4 = 0$.

2 Solve the quadratic

$$u = \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}.$$

3 Return to x

Since $u = \sqrt{x}$, $x = u^2 = (3 \pm \sqrt{5})^2 = 9 + 5 \pm 6\sqrt{5} = 14 \pm 6\sqrt{5}$.

FINAL ANSWER

$x = 14 + 6\sqrt{5}$ or $x = 14 - 6\sqrt{5}$.

Question 9

4 marks | Trigonometric Functions

9.

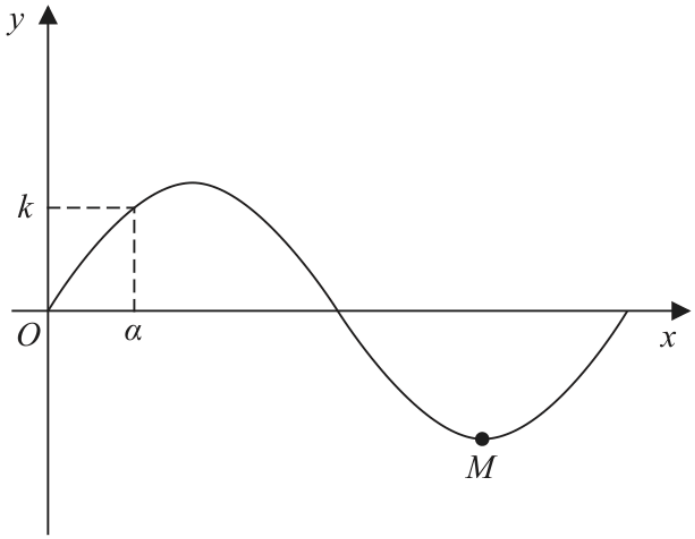


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \sin\left(\frac{x}{12}\right)$, where x is measured in radians. The point M shown in Figure 5 is a minimum point on C.

(a) State the period of C.

(1)

(b) State the coordinates of M.

(1)

The smallest positive solution of the equation $\sin\left(\frac{x}{12}\right) = k$, where k is a constant, is α.

Find, in terms of α,

(c) (i) the negative solution of the equation $\sin\left(\frac{x}{12}\right) = k$ that is closest to zero,

(ii) the smallest positive solution of the equation $\cos\left(\frac{x}{12}\right) = k$.

(2)

(Total 4 marks)

1 **Use the scale factor inside the sine**

For $y = \sin\left(\frac{x}{12}\right)$, one full sine cycle needs $\frac{x}{12}$ to increase by 2π, so the period is 24π.

2 **Locate the minimum point**

The sine function is -1 when its angle is $\frac{3\pi}{2}$. Therefore $\frac{x}{12} = \frac{3\pi}{2}$, so $x = 18\pi$ and $M = (18\pi, -1)$.

3 **Use sine symmetry**

If the smallest positive solution of $\sin\left(\frac{x}{12}\right) = k$ is α, then the closest negative solution is one half-period before the opposite sine position: $-12\pi - \alpha$.

4 **Convert to cosine**

Since $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$, the corresponding smallest positive cosine solution is $6\pi - \alpha$.

✓ **FINAL ANSWER**

$(a)\ 24\pi.\quad (b)\ (18\pi, -1).\quad (c)(i)\ -12\pi - \alpha.\quad (ii)\ 6\pi - \alpha.$

Question 10

10 marks | Graphs of Functions

10.

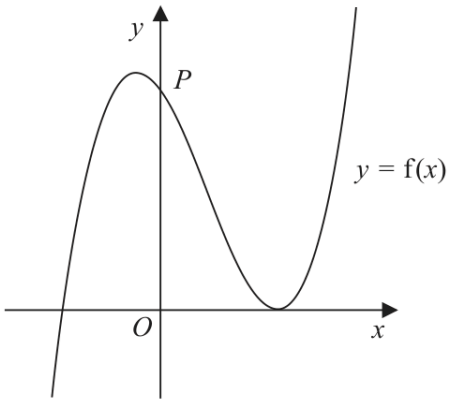


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

(Total 10 marks)

1 **Use the factorised form for the inequality**

$f(x) = (2x + 5)(x - 3)^2$. Since $(x - 3)^2 \geq 0$, the sign is controlled by $2x + 5$, with an extra zero at $x = 3$. Hence $f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$.

2 **Expand the cubic**

$(x - 3)^2 = x^2 - 6x + 9$, so $f(x) = (2x + 5)(x^2 - 6x + 9) = 2x^3 - 7x^2 - 12x + 45$.

3 **Find P and the gradient at P**

At the y-axis, $x = 0$, so $P = (0, 45)$. Also $f'(x) = 6x^2 - 14x - 12$, so the gradient at P is $f'(0) = -12$.

4 **Translate the graph**

A translation 2 units in the positive x direction means $g(x) = f(x - 2)$. Therefore $g(x) = (2(x - 2) + 5)((x - 2) - 3)^2 = (2x + 1)(x - 5)^2$.

5 **Find the new y-intercept**

Set $x = 0$: $g(0) = (1)(25) = 25$, so the y-intercept is 25.

✓ **FINAL ANSWER**

$f(x) \leq 0$ for $x \leq -\frac{5}{2}$ or $x = 3$; $f(x) = 2x^3 - 7x^2 - 12x + 45$; $P = (0, 45)$; gradient $= -12$; $g(x) = (2x + 1)(x - 5)^2$; y-intercept 25.

WMA11/01 OCTOBER 2019

Question 11

10 marks | Integration

11. A curve has equation $y = f(x)$.

The point $P\left(4, \frac{32}{3}\right)$ lies on the curve.

Given that

- $f''(x) = \frac{4}{\sqrt{x}} - 3$
- $f'(x) = 5$ at P

find

(a) the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(2)

(b) $f(x)$.

(8)

(Total 10 marks)

WORKED SOLUTION

Question 11 · WMA11/01 October 2019

1 Use the given gradient for the tangent

At $P\left(4, \frac{32}{3}\right)$, $f'(4) = 5$, so the tangent has gradient 5.

2 Find the tangent equation

$y - \frac{32}{3} = 5(x - 4)$. Hence $y = 5x - \frac{28}{3}$.

3 Integrate f'' to find f'

$f''(x) = 4x^{-1/2} - 3$, so $f'(x) = 8x^{1/2} - 3x + C$.

4 Use f'(4)=5

$5 = 8(2) - 12 + C = 4 + C$, so $C = 1$. Therefore $f'(x) = 8\sqrt{x} - 3x + 1$.

5 Integrate again

$f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x + D$.

6 **Use the point on the curve**

Substitute $x = 4$, $f(4) = \frac{32}{3}$: $\frac{32}{3} = \frac{16}{3}(8) - \frac{3}{2}(16) + 4 + D = \frac{68}{3} + D$, so $D = -12$.

FINAL ANSWER

$(a)\ y = 5x - \frac{28}{3}$. $\quad (b)\ f(x) = \frac{16}{3}x^{3/2} - \frac{3}{2}x^2 + x - 12$.