Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

Linear Classified With Answers (Part 2 of 2)

Every question followed by Dr Eslam Ahmed's worked solution and final answer. Chapters: Chapter 4: Geometry & Trigonometry, Chapter 5: Vectors & Transformation Geometry, Chapter 6: Statistics & Probability.

539
topic placements

2020–2026
55 past papers

Questions with worked solutions
checked by Dr Eslam



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TOPIC

Angles in Polygons & Parallel Lines

Every past-paper question on this topic, 2020 to 2026.

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

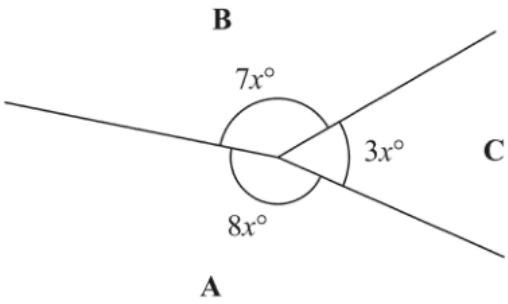


Diagram **NOT** accurately drawn

Polygon **B** has n sides.
Work out the value of n

$n =$ .....

(Total for Question 4 is 4 marks)

1 **Angles around a point add**

Angles around a point add to $360°$:

$$8x + 7x + 3x = 360$$

$$18x = 360$$

$$x = 20$$

2 **The interior angle of polygon**

The interior angle of polygon B is

$$7x = 7(20) = 140°$$

3 **Find the gradient**

For a regular n-sided polygon,

$$\frac{180(n-2)}{n} = 140$$

$$180n - 360 = 140n$$

$$40n = 360$$

$$n = 9$$

FINAL ANSWER

$n = 9$.

4MA1/2H JANUARY 2020

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows a triangle.

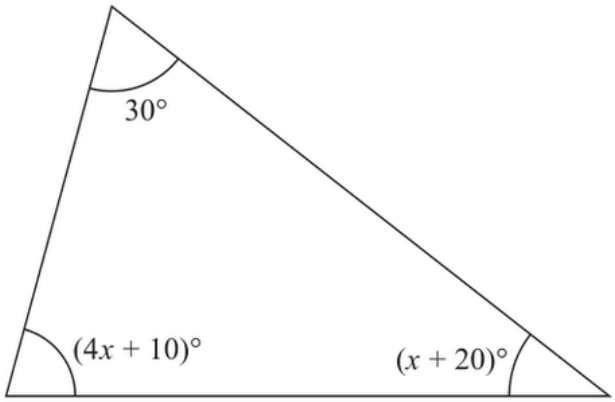


Diagram **NOT** accurately drawn

Work out the value of x.

$x =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2020

1 Angles in a triangle add

Angles in a triangle add to $180°$:

$$30 + (4x + 10) + (x + 20) = 180$$

$$5x + 60 = 180$$

$$5x = 120$$

$$x = 24$$

FINAL ANSWER

$x = 24$.

4MA1/1H MAY/NOVEMBER 2020

Question 8

4 marks | Angles in Polygons & Parallel Lines

8 Here is a 10-sided polygon.

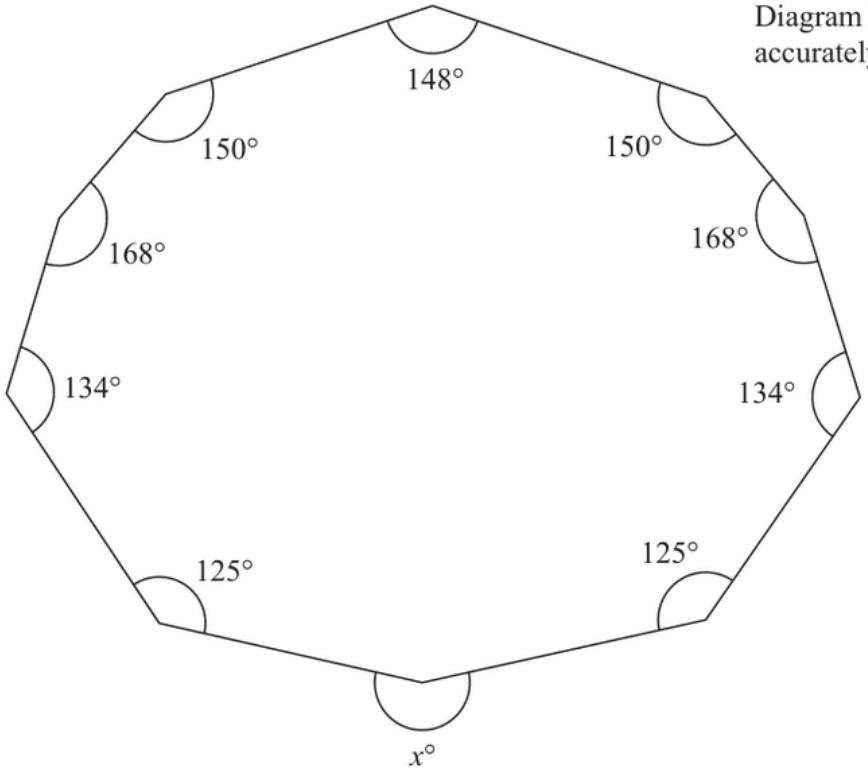


Diagram **NOT** accurately drawn

Work out the value of x.

$x =$ .....

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May/November 2020

1 **The sum of the interior**

The sum of the interior angles of a 10-sided polygon is

$$(10 - 2) \times 180 = 1440°$$

2 **The sum of the 9**

The sum of the 9 given interior angles is

$$150 + 168 + 134 + 125 + 125 + 134 + 168 + 150 + 148 = 1302°$$

3 Find the gradient

So the missing interior angle is

$$1440^\circ - 1302^\circ = 138^\circ$$

4 The angle is the reflex

The angle x is the reflex angle outside the polygon at this vertex:

$$x = 360^\circ - 138^\circ = 222^\circ$$

FINAL ANSWER

222°.

4MA1/1HR JANUARY 2021

Question 8

5 marks | Angles in Polygons & Parallel Lines

8 The diagram shows a regular pentagon, $ABCDE$, a regular hexagon, $CFGHID$, and a quadrilateral, $EDIJ$.

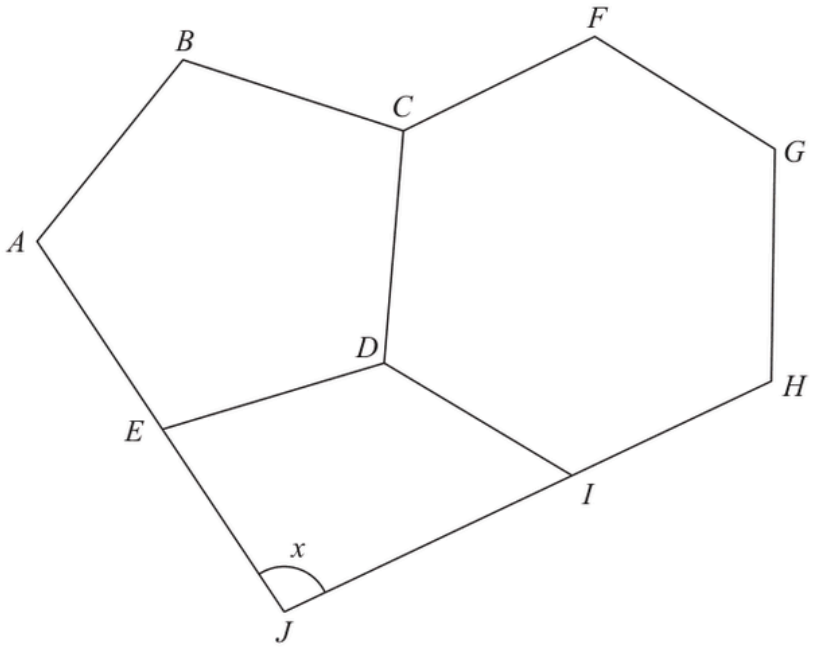


Diagram **NOT** accurately drawn

AEJ and HIJ are straight lines.

Work out the size of the angle marked x.
Show your working clearly.

.....°

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2021

1 **A regular pentagon has interior**

A regular pentagon has interior angle

$$\frac{(5-2)180}{5} = 108°$$

2 **A regular hexagon has interior**

A regular hexagon has interior angle

$$\frac{(6-2)180}{6} = 120°$$

3 **Find the gradient**

At E, since AEJ is a straight line,

$$\angle DEJ = 180 - 108 = 72^\circ$$

---

4 **Find the gradient**

At D, the angles around the point include the pentagon angle 108° and the hexagon angle 120°, so

$$\angle EDI = 360 - 108 - 120 = 132^\circ$$

---

5 **Find the gradient**

At I, since HIJ is a straight line,

$$\angle DIJ = 180 - 120 = 60^\circ$$

---

6 **Find the gradient**

In quadrilateral $EDIJ$, angles sum to 360°:

$$x = 360 - 72 - 132 - 60 = 96$$

✓ **FINAL ANSWER**

96°.

4MA1/2H MAY 2021

Question 5

5 marks | Angles in Polygons & Parallel Lines

5

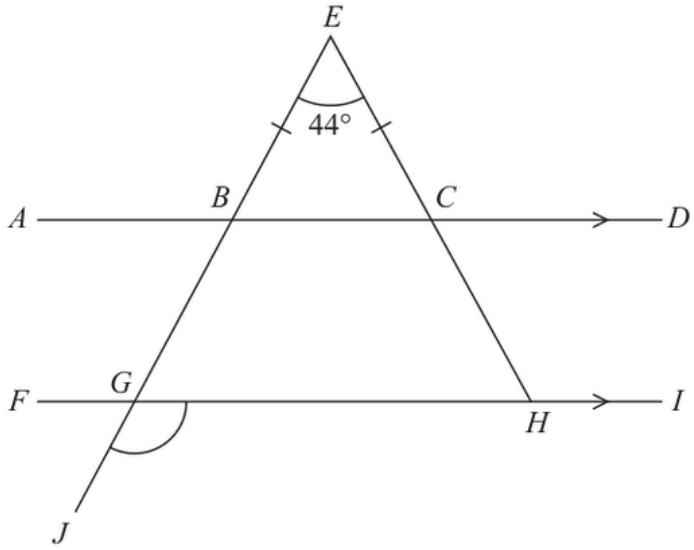


Diagram **NOT** accurately drawn

ABCD and *FGHI* are parallel straight lines.
EBGJ and *ECH* are straight lines.

$BE = CE$
Angle $BEC = 44°$

Work out the size of angle *JGH*.
Give a reason for each stage of your working.

.....°

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H May 2021

1 **Triangle is isosceles because**

Triangle EBC is isosceles because

$$BE = CE$$

2 **Find the gradient**

So the base angles are equal.

$$\angle EBC = \angle ECB = \frac{180° - 44°}{2} = 68°$$

3 **The two horizontal lines are**

The two horizontal lines are parallel, so the corresponding acute angle at G is $68°$.

4 **The required angle is supplementary**

The required angle is supplementary to this angle:

$$180° - 68° = 112°$$

✓ **FINAL ANSWER**

$112°$.

4MA1/2H NOVEMBER 2021

Question 5

5 marks | Angles in Polygons & Parallel Lines

5 The diagram shows a regular octagon $ABCDHIJK$ and a pentagon $DEFGH$.

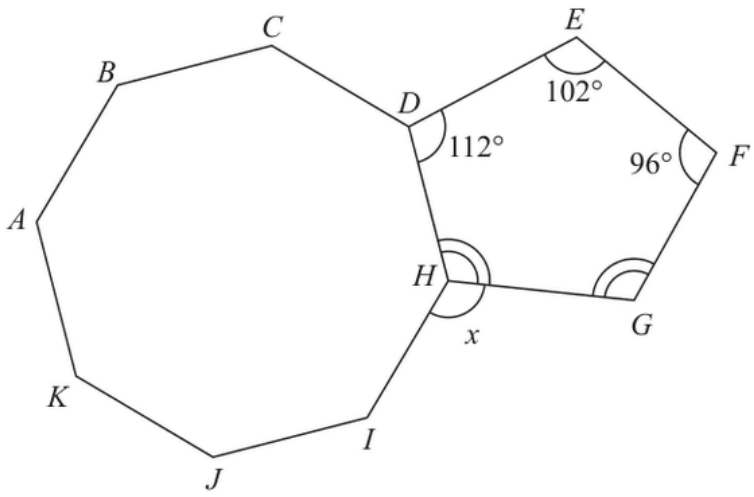


Diagram **NOT** accurately drawn

Angle GHD = angle FGH.

Work out the size of the angle marked x.
Show your working clearly.

.....°

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2021

1 Find the gradient

For pentagon $DEFGH$, the interior angle sum is

$$(5-2)\times 180 = 540°$$

2 Let

Let

$$\angle GHD = \angle FGH = a$$

3 **Find the gradient**

Then

$$112 + 102 + 96 + a + a = 540$$

$$2a = 230$$

$$a = 115°$$

---

4 **A regular octagon has interior**

A regular octagon has interior angle

$$\frac{(8-2) \times 180}{8} = 135°$$

---

5 **Find the gradient**

At H, angles around a point add to $360°$:

$$x = 360° - 135° - 115° = 110°$$

✓ **FINAL ANSWER**

$110°$.

4MA1/2HR JANUARY 2022

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows a regular octagon $ABCDEFGH$ and a regular pentagon $ABIJK$

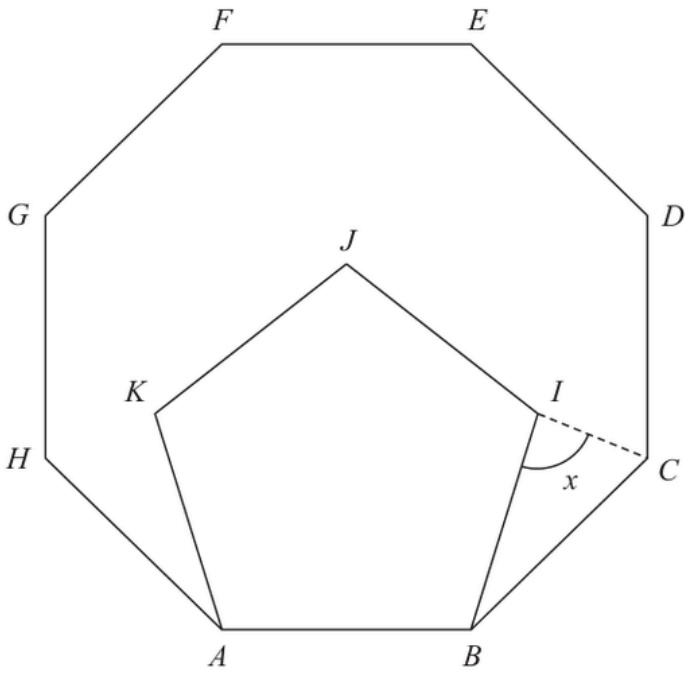


Diagram **NOT** accurately drawn

Work out the size of the angle x

.....°

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2022

1 **The interior angle of a**

The interior angle of a regular octagon is

$$\frac{(8-2)\times 180}{8} = 135°$$

2 **The interior angle of a**

The interior angle of a regular pentagon is

$$\frac{(5-2)\times 180}{5} = 108°$$

3 **Find the gradient**

At B,

$$\angle IBC = 135° - 108° = 27°$$

---

4 **Find the gradient**

Since both polygons share side AB,

$$BI = BC$$

---

5 **Find the gradient**

So triangle BIC is isosceles.

$$x = \frac{180° - 27°}{2} = 76.5°$$

✓ **FINAL ANSWER**

$76.5°$.

4MA1/2H JANUARY 2022

Question 11

4 marks | Angles in Polygons & Parallel Lines

11 The diagram shows a regular 10-sided polygon, *ABCDEFGHIJ*

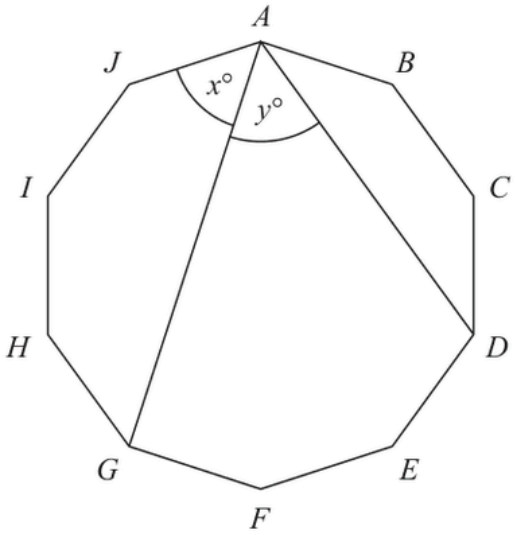


Diagram **NOT** accurately drawn

Show that $x = y$

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2022

1 **A regular 10 sided polygon**

A regular 10-sided polygon can be divided into 10 equal arcs around its circumcircle.

2 **The angle subtends the arc**

The angle x subtends the arc from J to G, which covers 3 equal sides of the decagon.

3 **The angle subtends the arc**

The angle y subtends the arc from G to D, which also covers 3 equal sides of the decagon.

4 **Equal arcs subtend equal angles**

Equal arcs subtend equal angles at the circumference, so

$$x = y$$

✓ **FINAL ANSWER**

$x = y$.

4MA1/1H JUNE 2022

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

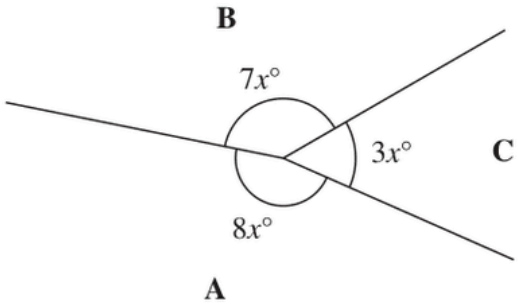


Diagram **NOT** accurately drawn

Polygon **B** has n sides.

Work out the value of n.

$n =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2022

1 **The angles around the point**

The angles around the point add to $360°$:

$$7x + 3x + 8x = 360$$

$$18x = 360$$

$$x = 20$$

2 **The interior angle of polygon**

The interior angle of polygon B is

$$7x = 7 \times 20 = 140°$$

3 **Find the gradient**

So the exterior angle is

$$180° - 140° = 40°$$

4 **Find the gradient**

For a regular polygon,

$$n = \frac{360}{40} = 9$$

✓ **FINAL ANSWER**

$n = 9$.

4MA1/1H JUNE 2022

Question 23

6 marks | Angles in Polygons & Parallel Lines

23 A polygon has n sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of n
Show clear algebraic working.

Question 23 continued

$n =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H June 2022

1 The interior angles form an

The interior angles form an arithmetic sequence:

$$177,\ 175,\ 173,\ 171,\ 169, \ldots$$

2 Calculate value

So

$$a = 177, \quad d = -2$$

3 Use trigonometry

For an n-sided polygon, the sum of the interior angles is

$$180(n-2)$$

4 **The sum of the arithmetic**

The sum of the arithmetic sequence is

$$\frac{n}{2}(2(177)+(n-1)(-2))$$

$$=\frac{n}{2}(354-2n+2)$$

$$=\frac{n}{2}(356-2n)$$

$$=n(178-n)$$

---

5 **Set this equal to the**

Set this equal to the polygon angle sum:

$$n(178-n)=180(n-2)$$

$$178n-n^2=180n-360$$

$$n^2+2n-360=0$$

---

6 **Factorise**

Factorise:

$$(n+20)(n-18)=0$$

---

7 **Solve inequality**

Since $n>5$,

$$n=18$$

✓ **FINAL ANSWER**

$n=18$

4MA1/2H JUNE 2023

Question 3

4 marks | Angles in Polygons & Parallel Lines

3 $ABCD$ is a trapezium.

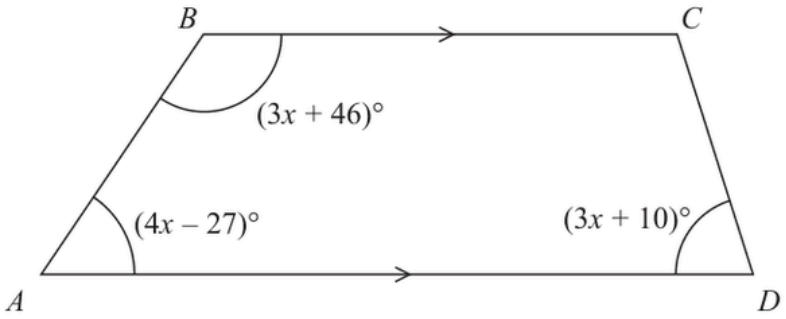


Diagram **NOT** accurately drawn

BC is parallel to AD

Find the size of the largest angle inside the trapezium.

.....°

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2023

1 **Find the gradient**

Since $BC \parallel AD$, the co-interior angles on side AB add to $180°$:

$$(4x - 27) + (3x + 46) = 180$$

$$7x + 19 = 180$$

$$7x = 161$$

$$x = 23$$

2 **Find the gradient**

Now find the angles:

$$\angle A = 4(23) - 27 = 65°$$

$$\angle B = 3(23) + 46 = 115°$$

$$\angle D = 3(23) + 10 = 79°$$

---

3 **Find the gradient**

Also, co-interior angles on side CD add to $180°$:

$$\angle C = 180° - 79° = 101°$$

---

4 **The largest angle is**

The largest angle is $115°$.

✓ **FINAL ANSWER**

$115°$.

4MA1/2H JUNE 2023

Question 11

3 marks | Angles in Polygons & Parallel Lines

11 Here is a 9-sided regular polygon $ABCDEFGHJ$, with centre O

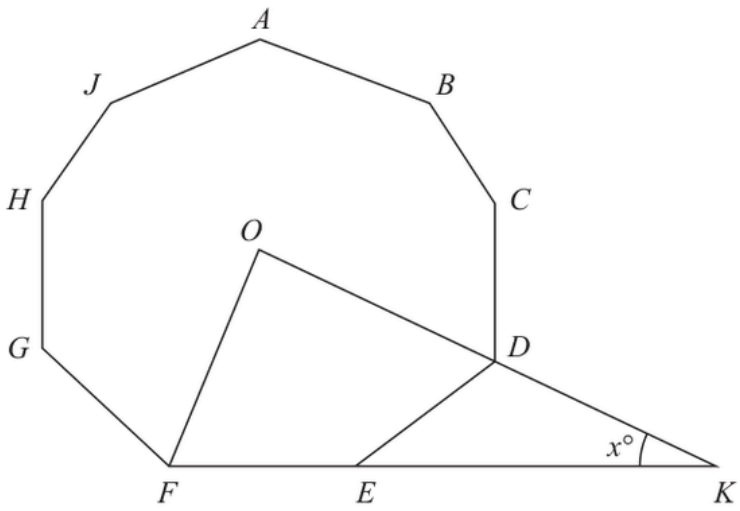


Diagram **NOT** accurately drawn

ODK and FEK are straight lines.

Work out the value of x

$x =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2023

1 **Find the gradient**

For a regular 9-sided polygon, the angle at the centre between adjacent vertices is

$$\frac{360}{9} = 40°$$

2 **Find the gradient**

The side FE is perpendicular to the radius drawn to the midpoint of side FE.

3 **From to this midpoint radius**

From OD to this midpoint radius is

$$40° + 20° = 60°$$

4 **Find the gradient**

So the angle between OD and the side FE is

$$90° - 60° = 30°$$

5 **Find the gradient**

Since ODK and FEK are straight lines, this is the angle x.

✓ **FINAL ANSWER**

$x = 30°$.

4MA1/1HR MAY 2023

Question 15

4 marks | Angles in Polygons & Parallel Lines

15

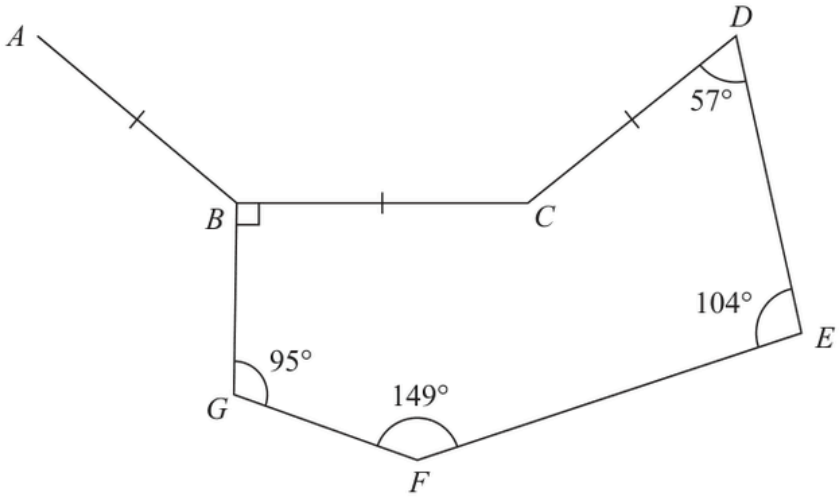


Diagram **NOT** accurately drawn

BCDEFG is a hexagon.
AB, *BC* and *CD* are three sides of a regular *n*-sided polygon.

Calculate the value of *n*
Show your working clearly.

$n =$ .....

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR May 2023

1 **The sum of the interior**

The sum of the interior angles of a hexagon is

$$(6-2)\times 180 = 720^\circ$$

2 **Find the gradient**

The reflex angle at C in the hexagon is

$$720^\circ - (90^\circ + 95^\circ + 149^\circ + 104^\circ + 57^\circ) = 225^\circ$$

3 **Find the gradient**

So the smaller angle BCD is

$$360^\circ - 225^\circ = 135^\circ$$

4 **This is the interior angle**

This is the interior angle of the regular n-sided polygon.

5 **Find the gradient**

So the exterior angle is

$$180^\circ - 135^\circ = 45^\circ$$

$$n = \frac{360}{45} = 8$$

✓ **FINAL ANSWER**

$n = 8$.

4MA1/1H MAY 2024

Question 5

4 marks | Angles in Polygons & Parallel Lines

5 In the diagram, $ABCDE$ is a regular pentagon.

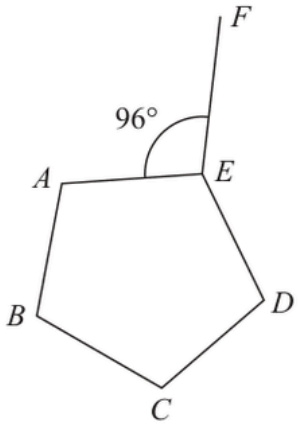


Diagram **NOT** accurately drawn

Angle $AEF = 96°$

Work out the size of the obtuse angle FED
Show your working clearly.

.....°

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2024

1 **The interior angle of a**

The interior angle of a regular pentagon is

$$\frac{(5-2)\times 180}{5} = 108°$$

2 **Find the gradient**

At E, angles around a point add to $360°$.

$$\angle FED = 360° - 96° - 108° = 156°$$

✓ **FINAL ANSWER**

$156°$.

4MA1/2H NOVEMBER 2024

Question 11

5 marks | Angles in Polygons & Parallel Lines

11

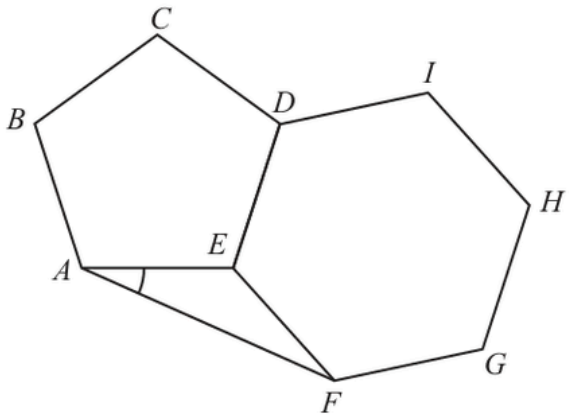


Diagram **NOT** accurately drawn

ABCDE is a regular pentagon.
DEFGHI is a regular hexagon.

AF is a straight line.

Work out the size of angle *EAF*

.....°

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2024

1 A regular pentagon has interior

A regular pentagon has interior angle

$$108°$$

2 A regular hexagon has interior

A regular hexagon has interior angle

$$120°$$

3 Use trigonometry

At E, the angle inside triangle AEF is

$$360° - 108° - 120° = 132°$$

4 **Use trigonometry**

Also $AE = ED$ and $ED = EF$, so

$$AE = EF$$

---

5 **Use trigonometry**

Triangle AEF is isosceles, so the base angles are equal:

$$\angle EAF = \frac{180^\circ - 132^\circ}{2} = 24^\circ$$

✓ **FINAL ANSWER**

24°.

4MA1/1H JUNE 2025

Question 3

4 marks | Angles in Polygons & Parallel Lines

3

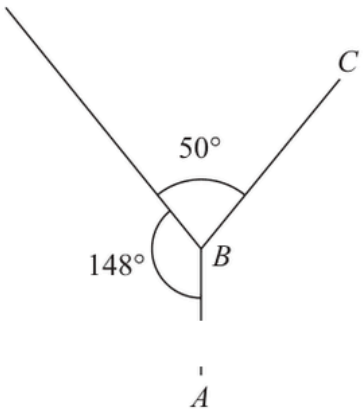


Diagram **NOT** accurately drawn

AB and BC are two sides of a regular polygon with n sides.

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2025

1 Find the gradient

Angles around point B add to $360°$.

2 The interior angle of the

The interior angle of the regular polygon is

$$360° - 140° - 50° = 170°$$

3 Find the gradient

So the exterior angle is

$$180° - 170° = 10°$$

4 Find the gradient

For a regular polygon,

$$n = \frac{360}{10} = 36$$

FINAL ANSWER

$n = 36$.

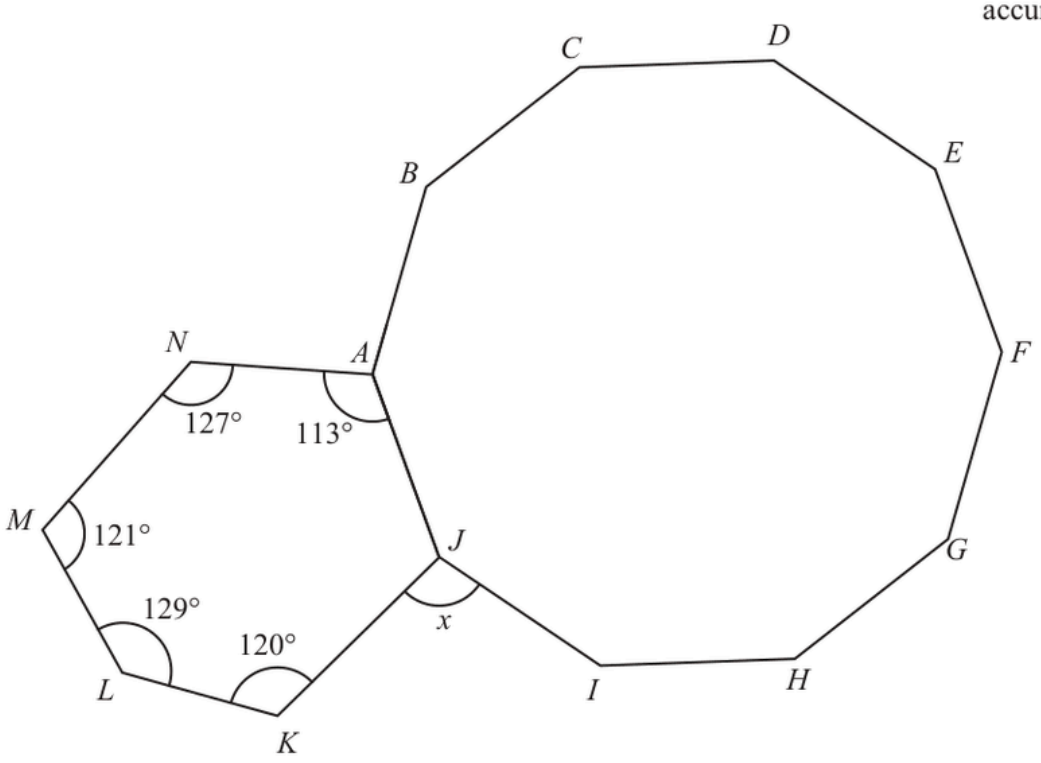
4WM2/01R MAY 2025

Question 10

5 marks | Angles in Polygons & Parallel Lines

10 The diagram shows a regular 10-sided polygon *ABCDEFGHIJ* and a hexagon *AJKLMN*

Diagram **NOT** accurately drawn



Work out the size of the angle marked x
Show your working clearly.

.....°

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM2/01R May 2025

1 **Required**

Find angle *x* in the figure combining a regular 10-gon (ABCDEFGHIJ) and hexagon AJKLMN.

2 **Interior angle of a regular**

Interior angle of a regular 10-sided polygon:

$$textInteriorangle = frac(10 - 2)times180^\circ 10 = frac1440^\circ 10 = 144^\circ$$

3 **Use trigonometry**

Find the interior angle of the hexagon at vertex *A*. At point *A*: decagon angle + 113° + hexagon angle at *A* = 360°:

∠*x* = 360° − 144° − 110° = 106°

✓ **FINAL ANSWER**

x = 106°

4WM2/01 NOVEMBER 2025

Question 7

4 marks | Angles in Polygons & Parallel Lines

7 *ABCDE* is a regular pentagon and *APQE* is a rectangle.

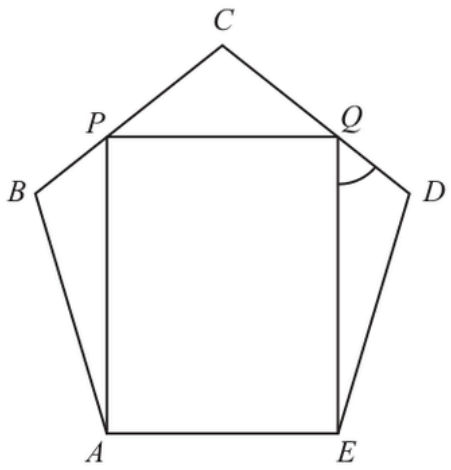


Diagram **NOT** accurately drawn

BPC and *DQC* are straight lines.

Work out the size of angle *EQD*
Show your working clearly.

.....°

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4WM2/01 November 2025

1 Use trigonometry

Missing open brace for superscript

So, ∠*A**E**D* = 108[∘].

Step 2: Find angles within the corner triangle △*DE**Q*.** Since *A**P**Q**E* is a rectangle, the angle ∠*A**E**Q* = 90[∘]. Therefore, the angle ∠*D*E**Q* is:

∠*D**E**Q* = ∠*A**E**D* − ∠*A**E**Q* = 108[∘] − 90[∘] = 18[∘]

Step 3: Analyze △*DE**Q*.** The problem states *D**Q*C* is a straight line, meaning *Q* lies on the side *D*C*. Therefore, △*D**E**Q* is formed by *D*, *E*, *Q*. In △*D**E**Q*, the angle at vertex *D* is the pentagon's interior angle ∠*C*D**E* = 108[∘].

Using angles in a triangle summing to 180[∘]:

∠*E**Q*D* = 180[∘] − (∠*D**E**Q*+∠*E**D*Q*) = 180[∘] − (18[∘]+108[∘]) = 180[∘] − 126[∘] = 54[∘]

FINAL ANSWER

54[∘]

4MA1/1H NOVEMBER 2025

Question 12

4 marks | Angles in Polygons & Parallel Lines

12

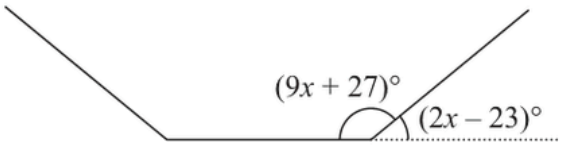


Diagram **NOT** accurately drawn

The diagram shows part of a regular n-sided polygon with

an interior angle of $(9x + 27)°$
an exterior angle of $(2x – 23)°$

Work out the value of n

$n =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H November 2025

1 The interior angle and exterior

The interior angle and exterior angle of a regular polygon add to $180°$:

$$(9x + 27) + (2x - 23) = 180$$

$$11x + 4 = 180$$

$$x = 16$$

2 The exterior angle is

The exterior angle is

$$2x - 23 = 2(16) - 23 = 9°$$

3 Find the gradient

For a regular polygon,

$$n = \frac{360}{9} = 40$$

FINAL ANSWER

$n = 40$.

4MA1/2H JUNE 2026

Question 6

3 marks | Angles in Polygons & Parallel Lines

6 $ABCDE$ is a pentagon.

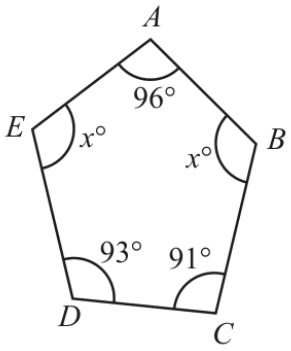


Diagram **NOT** accurately drawn

angle $EAB = 96°$ angle $BCD = 91°$ angle $CDE = 93°$ angle DEA = angle $ABC = x°$

Work out the value of x

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2026

1 **Solution | Step 1**

The sum of the interior angles of a pentagon is $(5-2)180° = 540°$.

Use the interior-angle sum formula.

2 **Solution | Step 2**

$$96 + 91 + 93 + x + x = 540 \quad \Rightarrow \quad 2x = 260 \quad \Rightarrow \quad x = 130.$$

Subtract the three known angles and divide the remainder equally.

✓ **FINAL ANSWER**

$x = 130$

4MA1/1HR JUNE 2026

Question 15

3 marks | Angles in Polygons & Parallel Lines

15

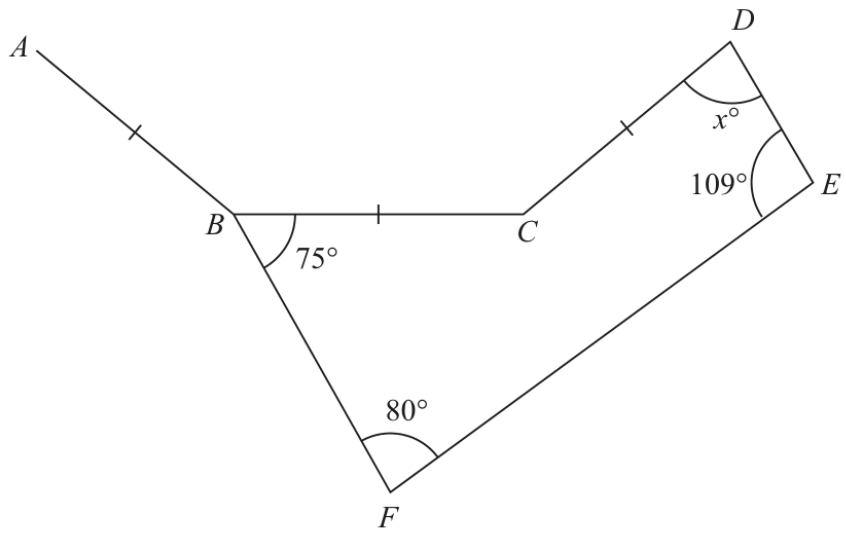


Diagram **NOT** accurately drawn

BCDEF is a pentagon.
AB, *BC* and *CD* are three sides of a regular 10-sided polygon.

Find the value of x

$x =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$\text{interior angle of a regular decagon} = \frac{(10-2)180^\circ}{10} = 144^\circ.$$

BC and CD are consecutive sides of the regular decagon.

2 **Solution | Step 2**

$$\text{reflex interior angle at C} = 360^\circ - 144^\circ = 216^\circ.$$

The pentagon BCDEF is concave at C, and its interior lies on the other side from the decagon interior angle.

3 **Solution | Step 3**

$$75^\circ + 216^\circ + x + 109^\circ + 80^\circ = (5-2)180^\circ = 540^\circ,$$

The interior-angle sum formula also applies to a simple concave pentagon.

4 **Solution | Step 4**

$$x = 540^\circ - 480^\circ = 60^\circ.$$

Subtract the four known interior angles.

✓ **FINAL ANSWER**

$x = 60^\circ$

4MA1/2HR JUNE 2026

Question 24

4 marks | Angles in Polygons & Parallel Lines

24

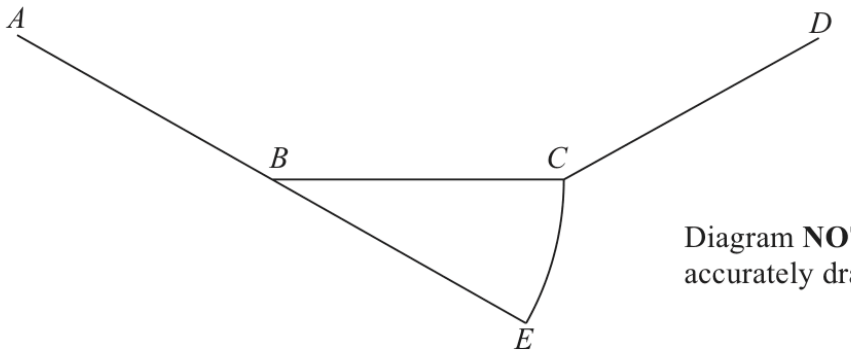


Diagram **NOT** accurately drawn

AB, BC and CD are three sides of a regular polygon.
ABE is a straight line.

BCE is a sector of a circle with centre B and radius 4.5 cm

The length of arc CE is $\frac{3}{10}\pi$ cm

Work out the sum of the interior angles of the regular polygon.

.....°

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2026

1 **Solution | Step 1**

Let $\theta = \angle CBE$. Then

$$\frac{\theta}{360}\,2\pi(4.5) = \frac{3}{10}\pi \quad \Rightarrow \quad \theta = 12°.$$

The arc is theta/360 of a full circumference.

2 **Solution | Step 2**

$\angle ABC = 180° - 12° = 168°$, so the regular polygon has exterior angle $12°$.
A, B and E are collinear, so the sector angle is supplementary to the interior angle.

3 **Solution | Step 3**

$$n = \frac{360}{12} = 30, \qquad \text{sum} = (30-2)180^\circ = 5040^\circ.$$

Exterior angles sum to 360 degrees, and an n-gon has interior-angle sum (n-2) times 180 degrees.

✓ **FINAL ANSWER**

5040°.

TOPIC

Constructions & Loci

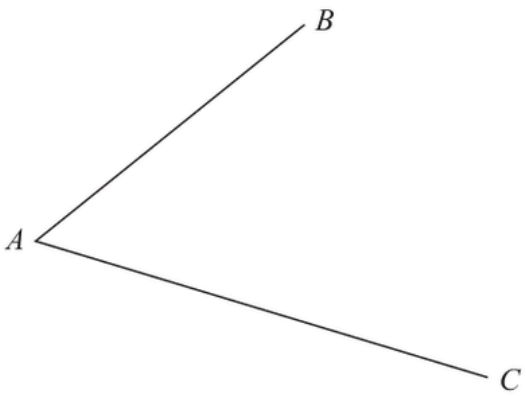
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 5

2 marks | Constructions & Loci

5 Use ruler and compasses to construct the bisector of angle BAC.
You must show all your construction lines.



(Total for Question 5 is 2 marks)

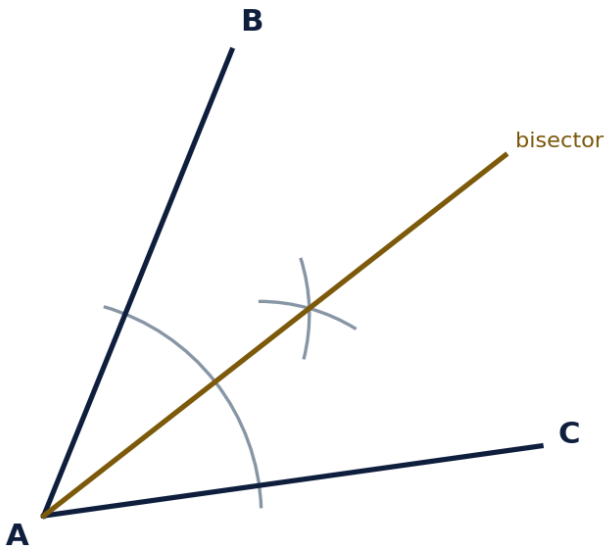
WORKED SOLUTION

Question 5 · 4MA1/2H January 2020

1 **Construction answer**

To construct the bisector of angle BAC:

1. Put the compass point at A and draw an arc that cuts AB and AC.
2. From the two points where the arc cuts the arms, draw equal arcs inside the angle.
3. Draw a straight line from A through the intersection of those two arcs.



FINAL ANSWER

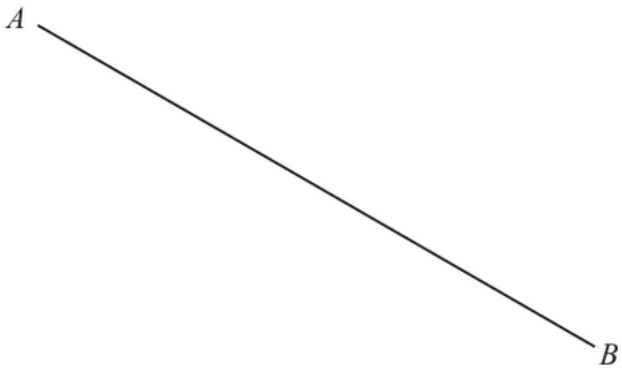
The ray from A through the arc intersection is the angle bisector of BAC.

4MA1/2HR JANUARY 2020

Question 6

2 marks | Constructions & Loci

6 Use ruler and compasses only to construct the perpendicular bisector of the line AB.
You must show all your construction lines.



(Total for Question 6 is 2 marks)

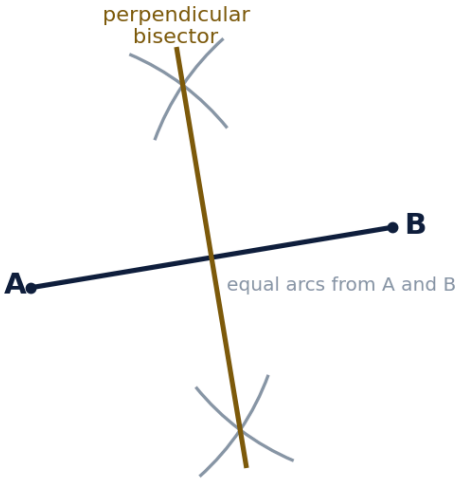
WORKED SOLUTION

Question 6 · 4MA1/2HR January 2020

1 Construction answer

To construct the perpendicular bisector of AB:

1. Set the compass radius to more than half of AB.
2. With centre A, draw arcs above and below the line.
3. With the same radius and centre B, draw arcs crossing the first pair.
4. Draw the straight line through the two arc intersections.



FINAL ANSWER

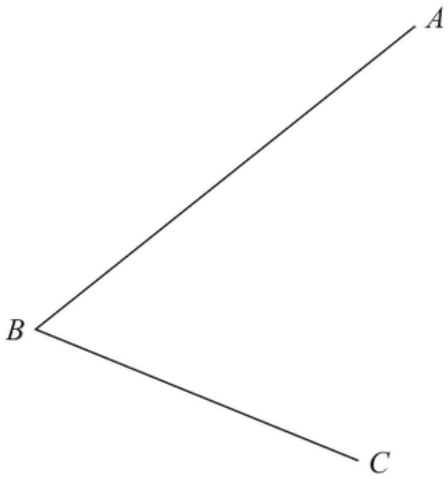
This line is the perpendicular bisector of AB.

4MA1/1HR MAY/NOVEMBER 2020

Question 2

2 marks | Constructions & Loci

2 Using ruler and compasses only, construct the bisector of angle ABC.
You must show all your construction lines.

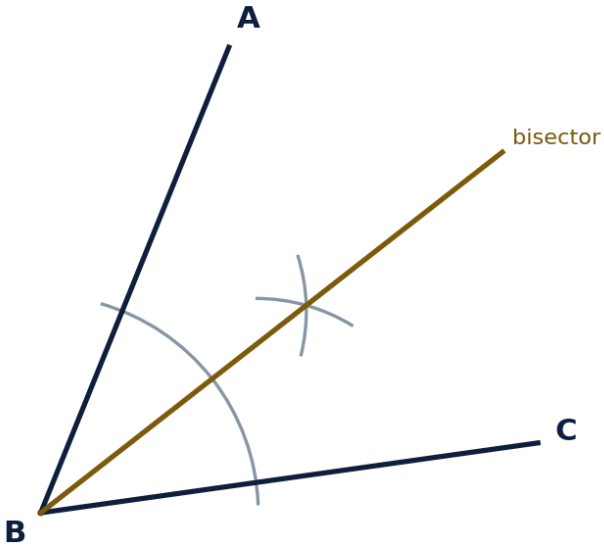


(Total for Question 2 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Put the compass point at B and draw an arc cutting BA and BC.
2. From those two cut points, draw equal arcs inside the angle.
3. Draw a straight line from B through the point where the two arcs cross.



FINAL ANSWER

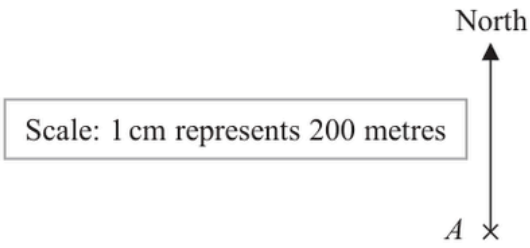
This line bisects angle ABC.

4MA1/2H JANUARY 2022

Question 5

4 marks | Constructions & Loci

5 The scale diagram shows the position on a map of a house, A



House C is on a bearing of 110° from A
The distance from A to C is 700 m

(a) Mark the position of C on the diagram with a cross (×)
Label your cross C

(3)

(b) Write the scale of the map in the form $1 : n$

1 : .....

(1)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H January 2022

1 The distance from to is

The distance from A to C is $700\ \text{m}$.

2 The scale is

The scale is

$$1\ \text{cm represents } 200\ \text{m}$$

3 Use bearings

So the map distance is

$$700 \div 200 = 3.5\ \text{cm}$$

4 **Use bearings**

Part (a): from A, draw a bearing of $110°$ and mark C at $3.5\ \text{cm}$ from A.

5 **Use bearings**

For part (b),

$$200\ \text{m} = 20000\ \text{cm}$$

6 **Use bearings**

So the scale is

$$1 : 20000$$

FINAL ANSWER

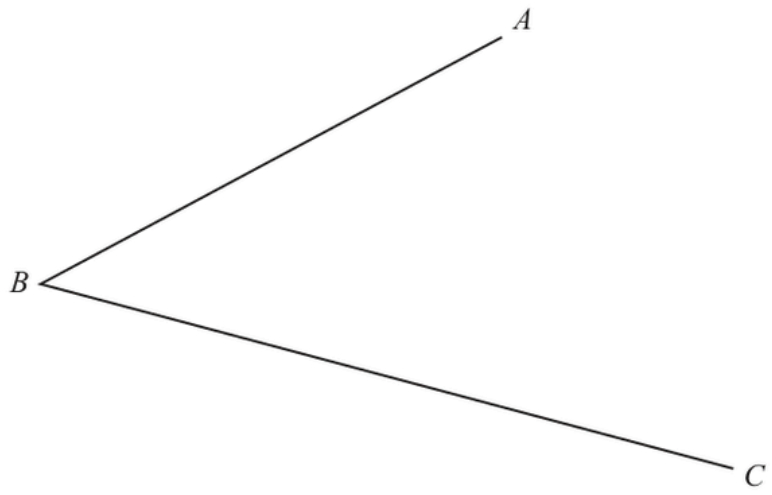
Draw C 3.5 cm from A on a bearing of $110°$, and the scale is $1 : 20000$.

4MA1/1H NOVEMBER 2024

Question 2

2 marks | Constructions & Loci

2 Use ruler and compasses only to construct the bisector of angle ABC
You must show all your construction lines.

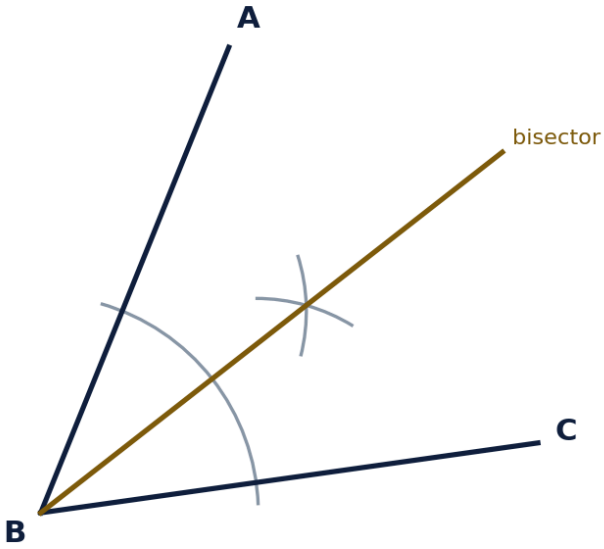


(Total for Question 2 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Draw an arc centred at B cutting both arms of the angle.
2. From the two cut points, draw equal arcs inside the angle.
3. Draw the line from B through the intersection of these arcs.



FINAL ANSWER

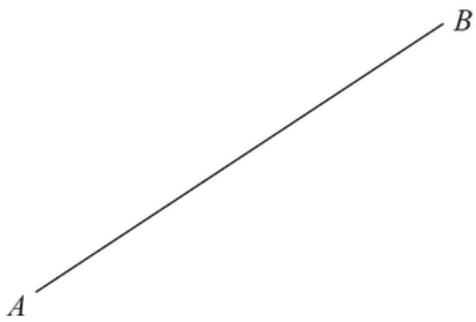
The constructed ray is the angle bisector of ABC.

4MA1/2H JUNE 2025

Question 4

2 marks | Constructions & Loci

4 Using ruler and compasses only, construct the perpendicular bisector of the line AB
Show all your construction lines.



(Total for Question 4 is 2 marks)

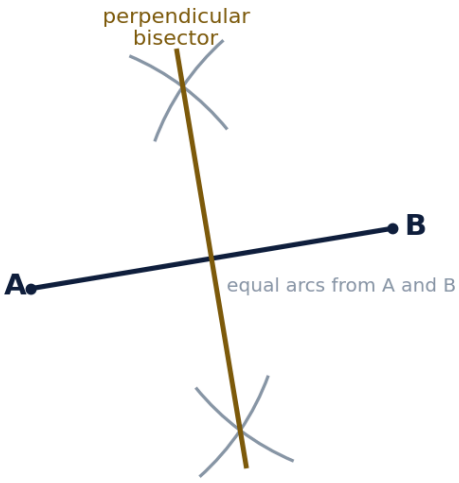
WORKED SOLUTION

Question 4 · 4MA1/2H June 2025

1 **Construction answer**

To construct the perpendicular bisector of AB:

1. Open the compass to more than half the length of AB.
2. Draw equal arcs from A above and below AB.
3. Without changing the compass width, draw equal arcs from B.
4. Join the two arc intersections with a ruler.



FINAL ANSWER

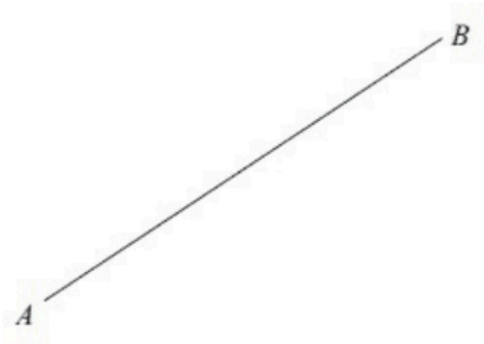
The joined line is the perpendicular bisector of AB.

4WM2/01 MAY 2025

Question 4

2 marks | Constructions & Loci

4 Using ruler and compasses only, construct the perpendicular bisector of the line AB
Show all your construction lines.



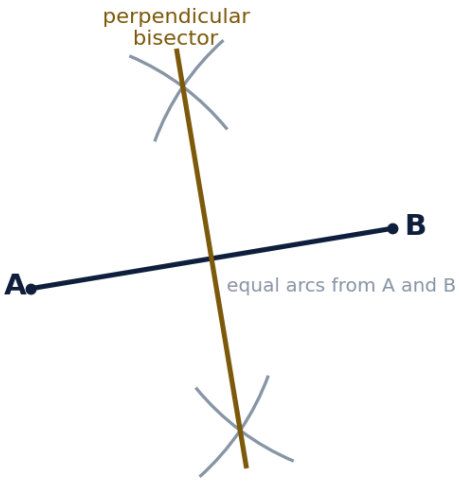
(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4WM2/01 May 2025

1 **Use a compass radius greater**

Use a compass radius greater than half of AB. Draw matching arcs from A above and below the line, then draw matching arcs from B with the same radius. Join the two arc-intersection points.



✓ **FINAL ANSWER**

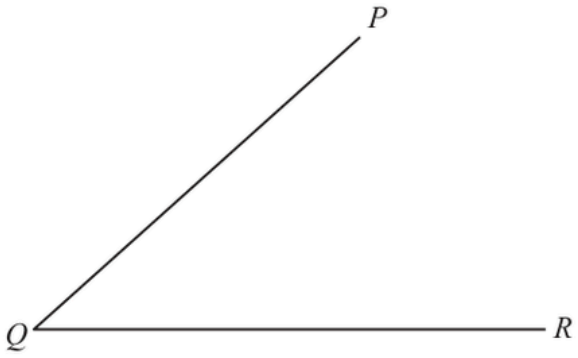
the joined line is the perpendicular bisector of AB.

4WM2/01 NOVEMBER 2025

Question 1

2 marks | Constructions & Loci

1 Using ruler and compasses only, construct the bisector of angle PQR
You must show all your construction lines.



(Total for Question 1 is 2 marks)

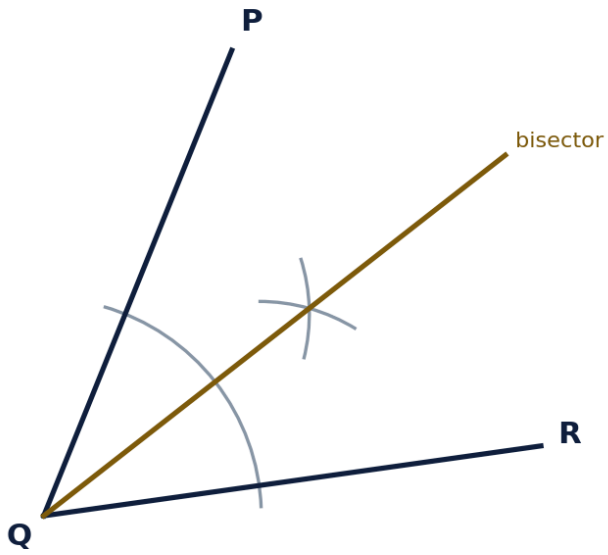
1 Construct angle bisector

Step 1: Place the point of the compass at the vertex *Q*. Draw an arc that intersects both arms *Q**P*** **and *Q**R***. Let's call these intersection points *A* and *B*.

Step 2: Without changing the compass width, place the compass point on *A* and draw an arc inside the angle.

Step 3: Place the compass point on *B* and draw another arc to intersect the previous arc. Let this intersection be point *C*.

Step 4: Draw a straight line from *Q* through *C*. This is the angle bisector.



FINAL ANSWER

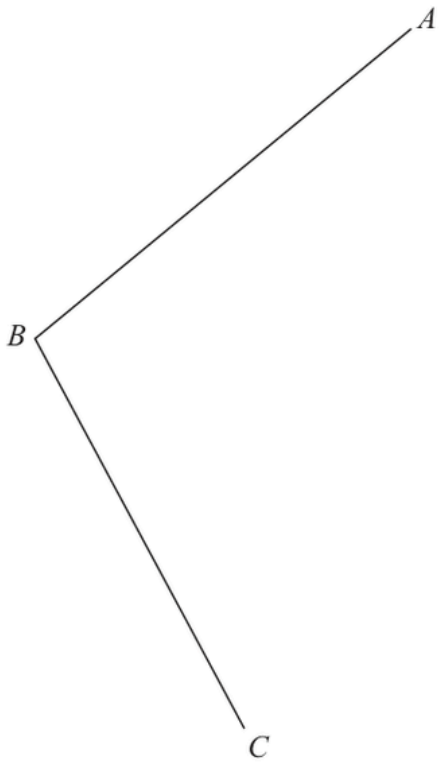
This is the angle bisector.

4MA1/1H NOVEMBER 2025

Question 3

2 marks | Constructions & Loci

3 Using ruler and compasses only, construct the bisector of angle ABC
Show all your construction lines.

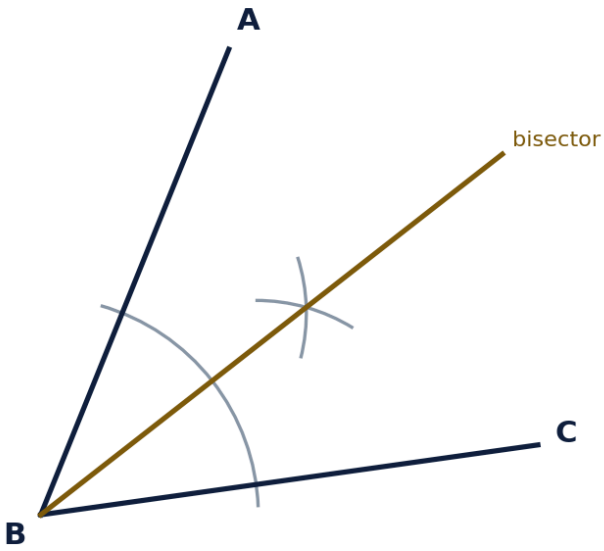


(Total for Question 3 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Use centre B to draw an arc crossing BA and BC.
2. Use the same compass width from the two crossing points to make two arcs inside the angle.
3. Join B to the point where these two arcs meet.



FINAL ANSWER

That line is the required angle bisector.

TOPIC

Perimeter & Area

Every past-paper question on this topic, 2020 to 2026.

Question 6

4 marks | Perimeter & Area

6 The diagram shows an 8-sided shape *ABCDEFGH*

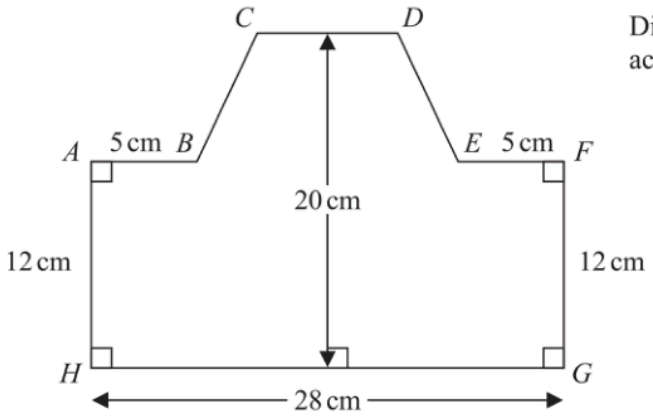


Diagram **NOT** accurately drawn

$HG = 28\,\text{cm}$ $AH = FG = 12\,\text{cm}$ $AB = EF = 5\,\text{cm}$
The height of the shape is 20 cm
CD is parallel to *HG*

The area of shape *ABCDEFGH* is $434\,\text{cm}^2$

Find the length of *CD*

1 Find the rectangular part

The rectangle at the bottom has width 28 cm and height 12 cm.

$$28 \times 12 = 336$$

2 Find the top area

$$434 - 336 = 98$$

3 Set up the trapezium

The top trapezium has height

$$20 - 12 = 8$$

Its lower parallel side is

$$28 - 5 - 5 = 18$$

4 Solve for the missing length

Let $CD = x$.

$$\frac{1}{2}(x + 18) \times 8 = 98$$

$$4(x + 18) = 98$$

$$x = 6.5$$

FINAL ANSWER

$CD = 6.5$ cm.

4MA1/1H JANUARY 2020

Question 3

4 marks | Perimeter & Area

3 The diagram shows a shape.

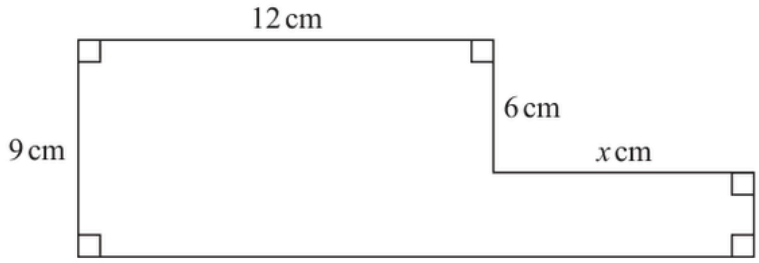


Diagram **NOT** accurately drawn

The shape has area 129 cm^2

Work out the value of x.

$x =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2020

1 Split the shape into two

Split the shape into two rectangles.

2 The left rectangle has area

The left rectangle has area

$$12 \times 9 = 108 \text{ cm}^2$$

3 The right rectangle has height

The right rectangle has height

$$9 - 6 = 3 \text{ cm}$$

4 Calculate area

So its area is

$$3x$$

5 **Total area**

Total area:

$$108 + 3x = 129$$

$$3x = 21$$

$$x = 7$$

✓ **FINAL ANSWER**

$7\ \text{cm}$.

4MA1/1HR JANUARY 2020

Question 5

3 marks | Perimeter & Area

5 The diagram shows Yuen's garden.

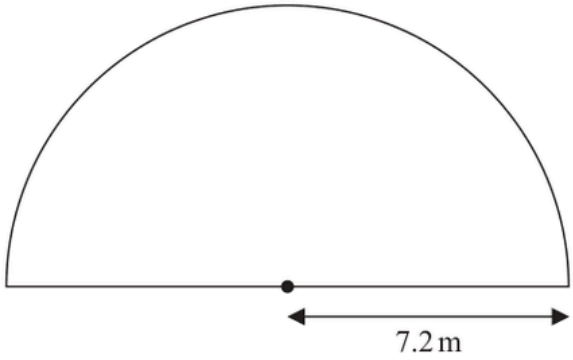


Diagram **NOT** accurately drawn

The garden is in the shape of a semicircle of radius 7.2 m.
Yuen is going to cover his garden with grass seed.

Yuen has 12 boxes of grass seed.
Each box of grass seed contains enough seed to cover 6 m² of the garden.

Has Yuen enough grass seed for his garden?
Show your working clearly.

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2020

1 The garden is a semicircle

The garden is a semicircle of radius $7.2\ \text{m}$.

$$\text{area} = \frac{1}{2}\pi r^2$$

$$\text{area} = \frac{1}{2}\pi(7.2)^2 = 81.43\ldots\ \text{m}^2$$

2 Calculate area

Yuen has 12 boxes, and each box covers $6\ \text{m}^2$:

$$12 \times 6 = 72\ \text{m}^2$$

3 Calculate area

Since

$$72 < 81.43\ldots$$

4

Yuen does not have enough

Yuen does not have enough grass seed.

✓ **FINAL ANSWER**

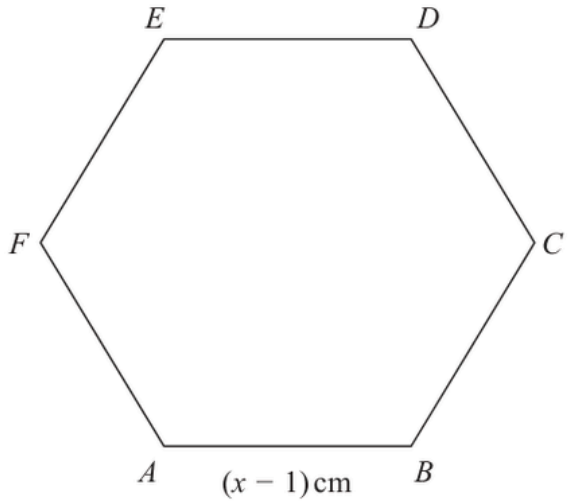
No, he does not have enough grass seed.

4MA1/1HR JANUARY 2020

Question 10

5 marks | Perimeter & Area

10 The diagram shows a regular hexagon, $ABCDEF$, and an isosceles triangle, GHJ.



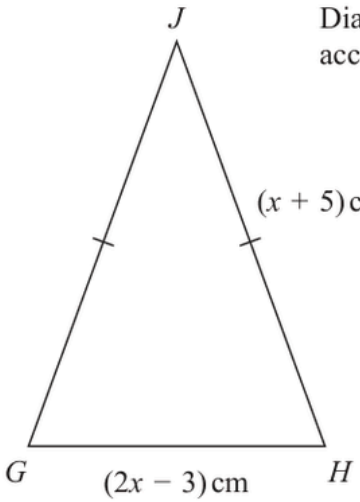


Diagram **NOT** accurately drawn

The perimeter of the hexagon is equal to the perimeter of the triangle.

Find the length of each side of the hexagon.
Show clear algebraic working.

..... cm

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2020

1 **The regular hexagon has side**

The regular hexagon has side length $(x - 1)$ cm, so its perimeter is

$$6(x - 1)$$

2 **The isosceles triangle has perimeter**

The isosceles triangle has perimeter

$$(x + 5) + (x + 5) + (2x - 3) = 4x + 7$$

3 The perimeters are equal

The perimeters are equal:

$$6(x-1) = 4x + 7$$

$$6x - 6 = 4x + 7$$

$$2x = 13$$

$$x = 6.5$$

---

4 Each side of the hexagon

Each side of the hexagon is

$$x - 1 = 6.5 - 1 = 5.5$$

FINAL ANSWER

$5.5\ \text{cm}$.

4MA1/2H JANUARY 2020

Question 11

5 marks | Perimeter & Area

11 The diagram shows a shaded shape $ABCD$ made from a semicircle ABC and a right-angled triangle ACD.

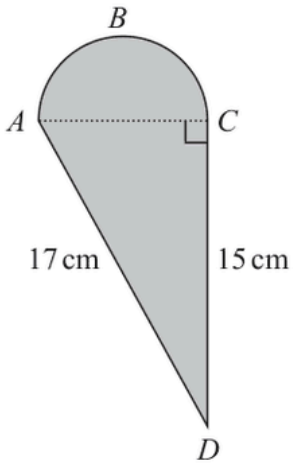


Diagram **NOT** accurately drawn

AC is the diameter of the semicircle ABC.

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2020

1 Use trigonometry

Triangle ACD is right-angled, with

$$AD = 17 \text{ cm}, \qquad CD = 15 \text{ cm}$$

2 Use Pythagoras to find

Use Pythagoras to find AC:

$$AC^2 = 17^2 - 15^2 = 289 - 225 = 64$$

$$AC = 8 \text{ cm}$$

3 **Calculate area**

So the semicircle has radius

$$r = 4 \text{ cm}$$

4 **The arc length of the**

The arc length of the semicircle is

$$\pi r = 4\pi$$

5 **The perimeter of the shaded**

The perimeter of the shaded shape is

$$AD + CD + \text{semicircle arc} = 17 + 15 + 4\pi = 44.566\ldots$$

✓ **FINAL ANSWER**

44.6 cm.

4MA1/1H MAY/NOVEMBER 2020

Question 11

6 marks | Perimeter & Area

11 The diagram shows trapezium $ABCD$ in which BC and AD are parallel.

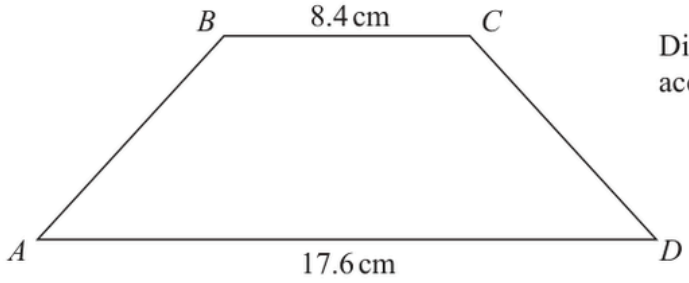


Diagram **NOT** accurately drawn

The trapezium has exactly one line of symmetry.

$BC = 8.4\,\text{cm}$
$AD = 17.6\,\text{cm}$

The trapezium has area $179.4\,\text{cm}^2$

Work out the size of angle ABC.
Give your answer correct to 1 decimal place.

.....°

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May/November 2020

1 **The average of the parallel**

The average of the parallel sides is

$$\frac{8.4 + 17.6}{2} = 13$$

2 **Find the gradient**

So the perpendicular height is

$$179.4 \div 13 = 13.8$$

3 **The horizontal offset on each**

The horizontal offset on each side is

$$\frac{17.6 - 8.4}{2} = 4.6$$

4 **Use trigonometry**

The acute angle between AB and the horizontal is

$$\tan^{-1}\left(\frac{13.8}{4.6}\right) = 71.565\ldots^\circ$$

5 **Use trigonometry**

So the interior angle ABC is

$$180^\circ - 71.565\ldots^\circ = 108.434\ldots^\circ$$

✓ **FINAL ANSWER**

108.4°.

4MA1/1HR MAY/NOVEMBER 2020

Question 22

6 marks | Perimeter & Area

22 The diagram shows a regular octagon *ABCDEFGH*.

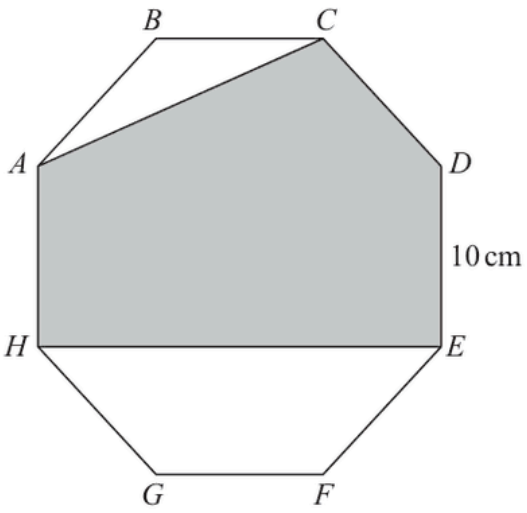


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May/November 2020

1 **Each side of the regular**

Each side of the regular octagon is 10 cm.

2 **Calculate area**

For a regular octagon with side length s,

$$\text{area} = 2(1+\sqrt{2})s^2$$

3 **Calculate area**

So the area of the whole octagon is

$$2(1+\sqrt{2})(10)^2 = 200 + 200\sqrt{2}$$

4 **The unshaded triangle has**

The unshaded triangle ABC has

$$AB = 10, \qquad BC = 10, \qquad \angle ABC = 135^\circ$$

5 **Use trigonometry**

So

$$\text{area of triangle } ABC = \frac{1}{2}(10)(10)\sin 135^\circ = 25\sqrt{2}$$

6 **Find the gradient**

At the bottom, the unshaded trapezium $HEFG$ has parallel sides

$$HE = 10 + 10\sqrt{2}, \qquad GF = 10$$

7 **Simplify surd**

and height

$$5\sqrt{2}$$

8 **Calculate area**

So

$$\text{area of trapezium } HEFG = \frac{1}{2}(10 + 10 + 10\sqrt{2})(5\sqrt{2})$$

$$= 50 + 50\sqrt{2}$$

9 **The shaded area is**

The shaded area is

$$(200 + 200\sqrt{2}) - 25\sqrt{2} - (50 + 50\sqrt{2})$$

$$= 150 + 125\sqrt{2}$$

$$= 326.7766\ldots$$

FINAL ANSWER

$\boxed{327\ \text{cm}^2}$

4MA1/2HR JANUARY 2021

Question 2

3 marks | Perimeter & Area

2 The diagram shows a rectangle $ABCD$ and a semicircle with diameter AB where $AB = 12\,\text{cm}$. The point E lies on DC and also on the semicircle.

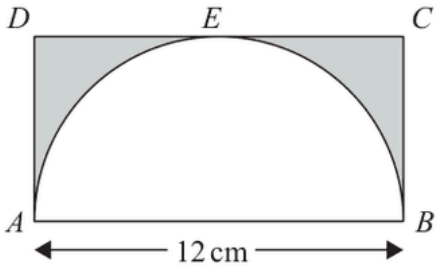


Diagram **NOT** accurately drawn

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR January 2021

1 Calculate area

The semicircle has diameter $12\ \text{cm}$, so its radius is $6\ \text{cm}$.

2 The rectangle has area

The rectangle has area

$$12 \times 6 = 72$$

3 Area of the semicircle

Area of the semicircle:

$$\frac{1}{2}\pi(6)^2 = 18\pi$$

4 Shaded area

Shaded area:

$$72 - 18\pi = 15.451\ldots$$

FINAL ANSWER

$15.5\ \text{cm}^2$.

4MA1/2H JANUARY 2021

Question 8

5 marks | Perimeter & Area

8 A, B and C are points on a circle with centre O.

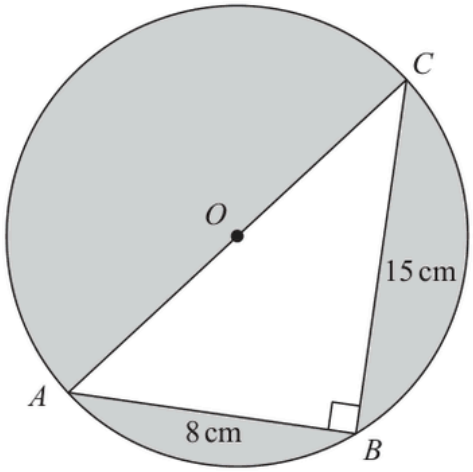


Diagram **NOT** accurately drawn

AOC is a diameter of the circle.

$AB = 8\,\text{cm}$ $BC = 15\,\text{cm}$

Angle $ABC = 90°$

Work out the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2021

1 **Triangle is right angled**

Triangle ABC is right-angled:

$$AC^2 = 8^2 + 15^2 = 64 + 225 = 289$$

$$AC = 17\,\text{cm}$$

2 **Calculate area**

Since AOC is a diameter, the radius of the circle is

$$17 \div 2 = 8.5 \text{ cm}$$

3 **Area of the circle**

Area of the circle:

$$\pi(8.5)^2 = 226.98\ldots \text{ cm}^2$$

4 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$$

5 **The shaded area is**

The shaded area is

$$226.98\ldots - 60 = 166.98\ldots$$

✓ **FINAL ANSWER**

167 cm^2.

4MA1/1HR JANUARY 2021

Question 10

5 marks | Perimeter & Area

10 The diagram shows one face of a wall.
This face is in the shape of a pentagon with exactly one line of symmetry.

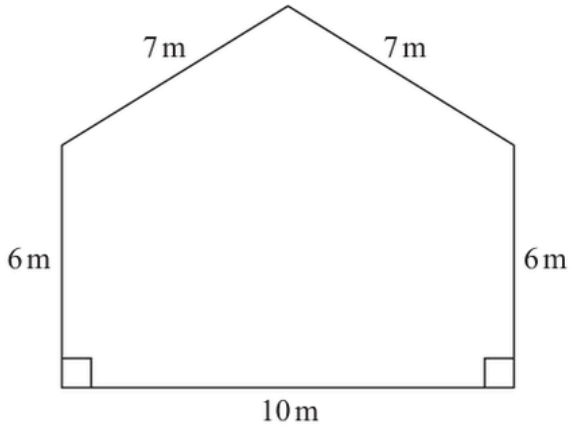


Diagram **NOT** accurately drawn

Omondi is going to paint this face of the wall once.
He has to buy all the paint that he needs to use.

The paint in each tin of paint Omondi is going to buy will cover $16\,m^2$ of the face of the wall.

Work out the least number of tins of paint Omondi will need to buy.
Show your working clearly.

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2021

1 **The wall is a rectangle**

The wall is a rectangle plus an isosceles triangle.

2 **Rectangle area**

Rectangle area:

$$10 \times 6 = 60$$

3 **Use trigonometry**

For the triangle, half the base is 5 m.

$$h^2 = 7^2 - 5^2 = 24$$

$$h = \sqrt{24}$$

4 **Triangle area**

Triangle area:

$$\frac{1}{2} \times 10 \times \sqrt{24} = 24.494\ldots$$

5 **Total area**

Total area:

$$60 + 24.494\ldots = 84.494\ldots$$

6 **Each tin covers**

Each tin covers $16\ \text{m}^2$:

$$\frac{84.494\ldots}{16} = 5.28\ldots$$

7 **Calculate area**

Omondi must buy whole tins, so he needs 6 tins.

✓ **FINAL ANSWER**

6 tins.

4MA1/2H JANUARY 2021

Question 20

4 marks | Perimeter & Area

20

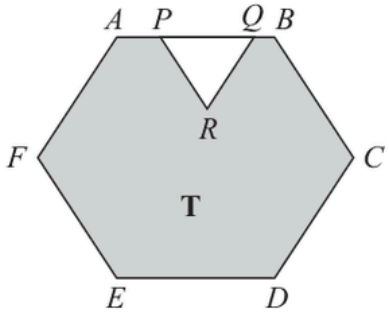


Diagram **NOT** accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle PQR from a regular hexagon $ABCDEF$.

The points P and Q lie on AB such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3}$ cm^2

work out the length of PQ.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2021

1 Let

Let

$$PQ = p$$

2 Given

Given

$$AB = 1.5PQ$$

3 Find the gradient

so the side length of the regular hexagon is

$$1.5p = \frac{3}{2}p$$

4 **Area of a regular hexagon**

Area of a regular hexagon with side length s is

$$\frac{3\sqrt{3}}{2}s^2$$

5 **Find the gradient**

So the area of the hexagon is

$$\frac{3\sqrt{3}}{2}\left(\frac{3}{2}p\right)^2$$

$$=\frac{27\sqrt{3}}{8}p^2$$

6 **Find the gradient**

The removed triangle PQR is equilateral, so its area is

$$\frac{\sqrt{3}}{4}p^2$$

7 **The shaded area is**

The shaded area is

$$\frac{27\sqrt{3}}{8}p^2-\frac{\sqrt{3}}{4}p^2$$

$$=\frac{25\sqrt{3}}{8}p^2$$

8 **Find the gradient**

This is $72\sqrt{3}$, so

$$\frac{25\sqrt{3}}{8}p^2=72\sqrt{3}$$

$$\frac{25}{8}p^2=72$$

$$p^2=\frac{576}{25}$$

$$p=4.8$$

FINAL ANSWER

$PQ = 4.8\ \text{cm}$

4MA1/1H MAY 2021

Question 3

4 marks | Perimeter & Area

3

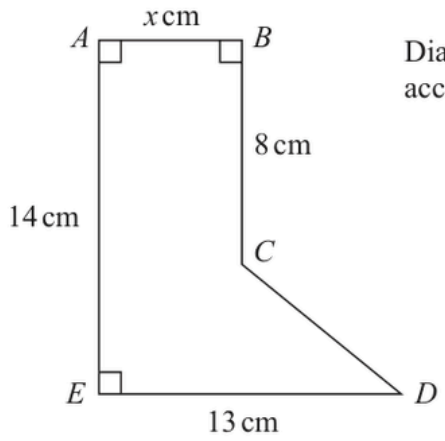


Diagram **NOT** accurately drawn

The diagram shows the shape $ABCDE$.

The area of the shape is $91.8\,\text{cm}^2$

Work out the value of x.

$x =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2021

1 Split the shape into a

Split the shape into a rectangle and a triangle.

2 Rectangle area

Rectangle area:

$$14x$$

3 Use trigonometry

The triangle has base $13 - x$ and height 6, so its area is

$$\frac{1}{2}(13 - x)(6) = 39 - 3x$$

4 **Total area**

Total area:

$$14x + 39 - 3x = 91.5$$

$$11x = 52.5$$

$$x = 4.772\ldots$$

✓ **FINAL ANSWER**

$4.77\ \text{cm}$.

4MA1/1H NOVEMBER 2021

Question 4

5 marks | Perimeter & Area

4 The diagram shows the front of a wooden door with a semicircular glass window.

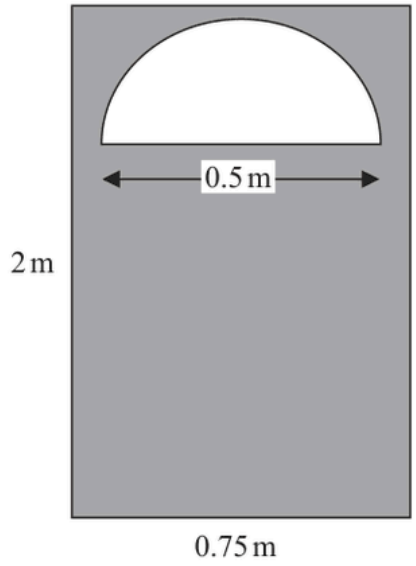


Diagram **NOT** accurately drawn

Julie wants to apply 2 coats of wood varnish to the front of the door, shown shaded in the diagram.

250 millilitres of wood varnish covers 4 m^2 of the wood.

Work out how many millilitres of wood varnish Julie will need.
Give your answer correct to the nearest millilitre.

..... millilitres

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2021

1 **Area of the rectangular door**

Area of the rectangular door:

$$0.75 \times 2 = 1.5 \text{ m}^2$$

2 **The semicircular window has diameter**

The semicircular window has diameter 0.5 m, so

$$r = 0.25 \text{ m}$$

3 Area of the semicircular window

Area of the semicircular window:

$$\frac{1}{2}\pi(0.25)^2 = 0.098174\ldots \text{ m}^2$$

4 Shaded area for one coat

Shaded area for one coat:

$$1.5 - 0.098174\ldots = 1.401825\ldots \text{ m}^2$$

5 Calculate area

For 2 coats:

$$2 \times 1.401825\ldots = 2.80365\ldots \text{ m}^2$$

6 Calculate area

Since 250 ml covers 4 m^2, varnish needed is

$$\frac{2.80365\ldots}{4} \times 250 = 175.22\ldots$$

FINAL ANSWER

175 millilitres.

4MA1/2H NOVEMBER 2021

Question 10

4 marks | Perimeter & Area

10 ABC is an isosceles triangle with $BA = BC$.

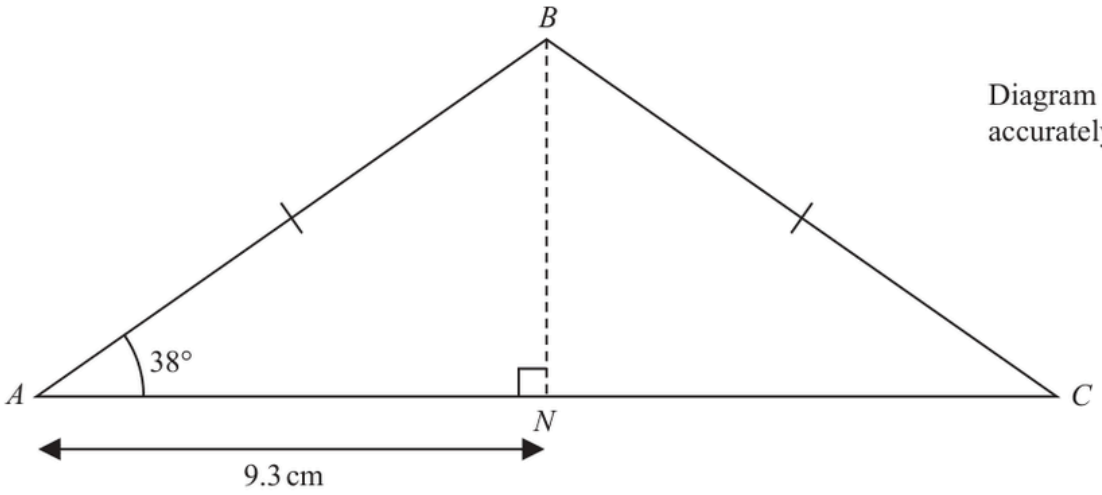


Diagram **NOT** accurately drawn

N is the point on AC such that $AN = 9.3$ cm and BN is perpendicular to AC.

Work out the perimeter of triangle ABC.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2021

1 Use trigonometry

In right-angled triangle ABN,

$$\cos 38° = \frac{9.3}{AB}$$

$$AB = \frac{9.3}{\cos 38°} = 11.802\ldots$$

2 Calculate area

Since ABC is isosceles,

$$BC = AB$$

3 **Calculate area**

Also,

$$AC = 2 \times 9.3 = 18.6$$

---

4 **Perimeter**

Perimeter:

$$11.802\ldots + 11.802\ldots + 18.6 = 42.204\ldots$$

✓ **FINAL ANSWER**

$42.2\ \text{cm}$.

4MA1/2H JUNE 2022

Question 5

4 marks | Perimeter & Area

5 The diagram shows an 8-sided shape *ABCDEFGH*.

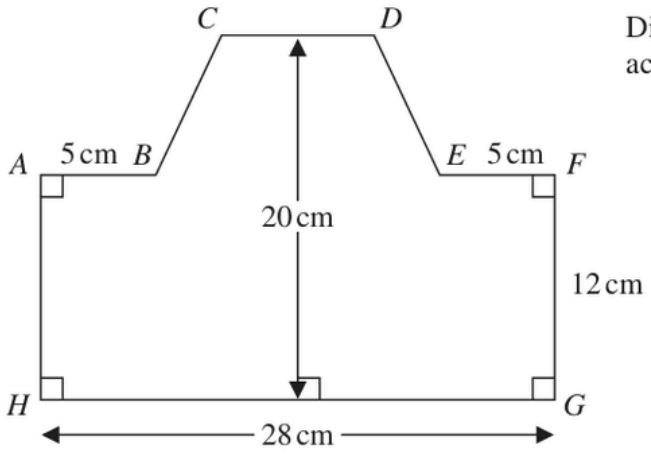


Diagram **NOT** accurately drawn

$HG = 28\,\text{cm}$ $FG = 12\,\text{cm}$ $AB = EF = 5\,\text{cm}$
The height of the shape is 20 cm
CD is parallel to *HG*

The area of shape *ABCDEFGH* is $434\,\text{cm}^2$

Find the length of *CD*.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2022

1 The lower rectangle has width

The lower rectangle has width $28\ \text{cm}$ and height $12\ \text{cm}$:

$$28 \times 12 = 336\ \text{cm}^2$$

2 The top part is a

The top part is a trapezium of height

$$20 - 12 = 8\ \text{cm}$$

3 Find the gradient

The lower parallel side of this trapezium is

$$28 - 5 - 5 = 18\ \text{cm}$$

4 **Area of the trapezium**

Area of the trapezium:

$$434 - 336 = 98 \text{ cm}^2$$

---

5 **Use**

Use

$$\text{area} = \frac{1}{2}(a+b)h$$

$$98 = \frac{1}{2}(18+CD)(8)$$

$$98 = 4(18+CD)$$

$$18+CD = 24.5$$

$$CD = 6.5$$

✓ **FINAL ANSWER**

6.5 cm.

4MA1/2HR JUNE 2022

Question 5

4 marks | Perimeter & Area

5 The diagram shows a shape made from a square $ABCD$ and 4 identical semicircles.

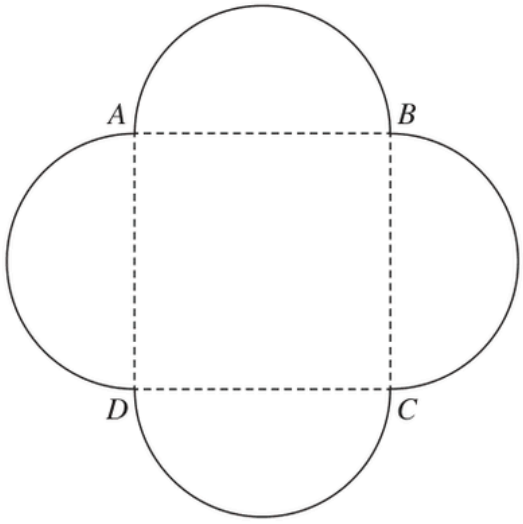


Diagram **NOT** accurately drawn

As shown in the diagram, the semicircles have AB, BC, CD and DA as diameters.

The area of the square is $36\,\text{cm}^2$

Calculate the total area of the shape.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2022

1 **Calculate area**

The square has area $36\ \text{cm}^2$, so its side length is

$$\sqrt{36} = 6\ \text{cm}$$

2 **Calculate area**

Each semicircle has diameter $6\ \text{cm}$, so radius

$$r = 3\ \text{cm}$$

3 Area of 4 semicircles

Area of 4 semicircles:

$$4 \times \frac{1}{2}\pi(3)^2 = 18\pi$$

4 Total area

Total area:

$$36 + 18\pi = 92.548\ldots$$

FINAL ANSWER

$92.5\ \text{cm}^2$.

Question 11

6 marks | Perimeter & Area

11 The diagram shows a quadrilateral $ABCD$

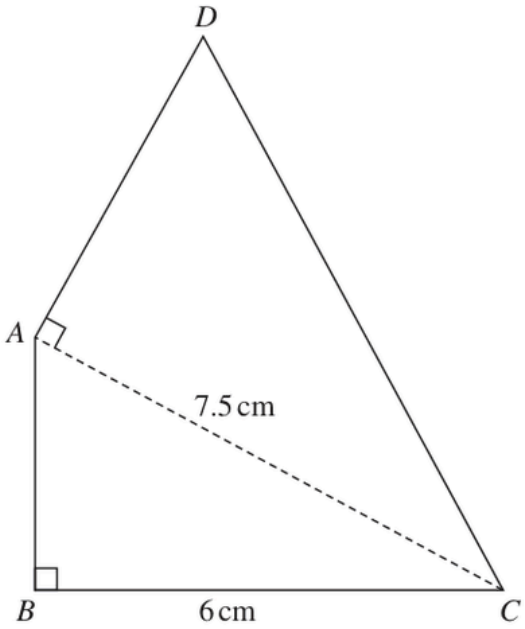


Diagram **NOT** accurately drawn

In the diagram, ABC and DAC are right-angled triangles.

$BC = 6\,\text{cm}$ $AC = 7.5\,\text{cm}$

The area of quadrilateral $ABCD$ is $31.5\,\text{cm}^2$

Work out the length of AD

Question 11 continued.

..... cm

(Total for Question 11 is 6 marks)

1 **Use trigonometry**

In right-angled triangle ABC,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 6^2 = 7.5^2$$

$$AB^2 = 56.25 - 36 = 20.25$$

$$AB = 4.5 \text{ cm}$$

2 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 4.5 = 13.5 \text{ cm}^2$$

3 **Area of triangle**

Area of triangle ADC:

$$31.5 - 13.5 = 18 \text{ cm}^2$$

4 **Use trigonometry**

Triangle ADC is right-angled, so

$$\frac{1}{2} \times AD \times 7.5 = 18$$

$$AD = \frac{36}{7.5} = 4.8$$

✓ **FINAL ANSWER**

4.8 cm.

4MA1/2HR JUNE 2023

Question 5

4 marks | Perimeter & Area

5 A field is in the shape of a trapezium.

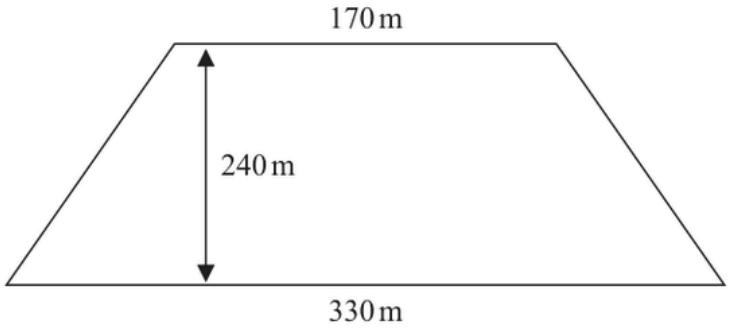


Diagram **NOT** accurately drawn

The field is sold for a price of \$49 650

Given that 1 hectare = 10 000 m²

work out the average price of the field per hectare.

\$.....

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2023

1 Calculate area

$$\text{area} = \frac{1}{2}(170 + 330) \times 240 = 60000 \text{ m}^2$$

$$60000 \text{ m}^2 = 6 \text{ hectares}$$

$$\text{price per hectare} = 49650 \div 6 = 8275$$

FINAL ANSWER

\$8275.

4MA1/2H NOVEMBER 2023

Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

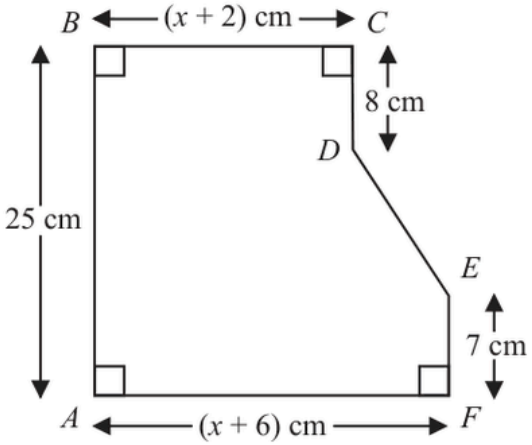


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad BC = (x + 2)$ cm $\quad CD = 8$ cm $\quad EF = 7$ cm $\quad AF = (x + 6)$ cm

The area of hexagon $ABCDEF$ is 258 cm²

Work out the value of x

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2023

1 The area of the hexagon

The area of the hexagon can be found from coordinates or by splitting the shape.

2 Calculate area

It simplifies to

$$\text{area} = 25x + 98$$

3 **Calculate area**

Given the area is $258\ \text{cm}^2$,

$$25x + 98 = 258$$

$$25x = 160$$

$$x = 6.4$$

FINAL ANSWER

$x = 6.4$.

4MA1/2H JUNE 2024

Question 7

4 marks | Perimeter & Area

7 The diagram shows a 6-sided shape $ABCDEF$

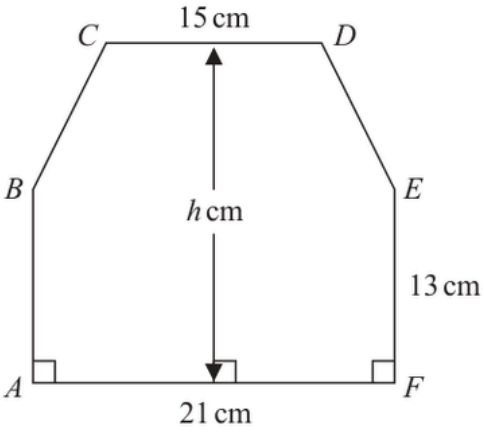


Diagram **NOT** accurately drawn

$AF = 21$ cm $\quad CD = 15$ cm $\quad AB = FE = 13$ cm

The perpendicular height of the shape is h cm
CD is parallel to AF

The area of the shape is $390\,\text{cm}^2$

Work out the value of h

$h =$ .....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2024

1 The bottom rectangle has area

The bottom rectangle has area

$$21 \times 13 = 273$$

2 **The top trapezium has height**

The top trapezium has height $h - 13$, with parallel sides 21 and 15.

$$273 + \frac{1}{2}(21 + 15)(h - 13) = 390$$

$$273 + 18(h - 13) = 390$$

$$18h + 39 = 390$$

$$h = 19.5$$

✓ **FINAL ANSWER**

$19.5\ \text{cm}$.

4MA1/1HR MAY 2024

Question 8

6 marks | Perimeter & Area

8 The diagram shows a trapezium, $ABCD$

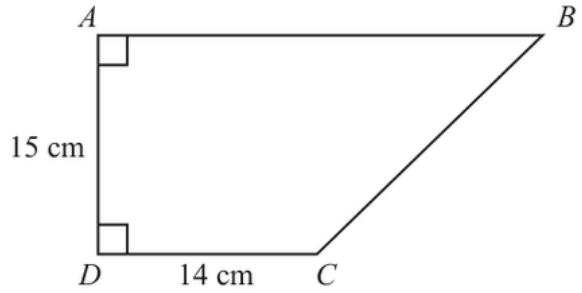


Diagram **NOT** accurately drawn

DAB and ADC are right angles.

$AD = 15$ cm $DC = 14$ cm

The area of the trapezium is 360 cm²

Work out the perimeter of the trapezium.

..... cm

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May 2024

1 Use the trapezium area formula

Use the trapezium area formula:

$$360 = \frac{1}{2}(AB + 14)(15)$$

$$AB + 14 = 48$$

$$AB = 34$$

2 Use trigonometry

Triangle ABC is right-angled with horizontal length

$$34 - 14 = 20$$

3 Calculate area

So

$$BC = \sqrt{20^2 + 15^2} = 25$$

4 Perimeter

Perimeter:

$$15 + 14 + 25 + 34 = 88$$

FINAL ANSWER

$88\ \text{cm}$.

Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

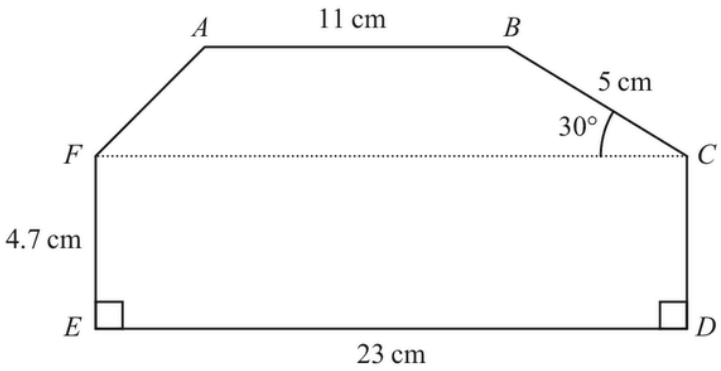


Diagram **NOT** accurately drawn

Angle $BCF = 30°$
AB, FC and ED are parallel.

Calculate the area of $ABCDEF$
Show your working clearly.

..... cm^2

(Total for Question 10 is 5 marks)

1 **The rectangle has area**

The rectangle $EDCF$ has area

$$23 \times 4.7 = 108.1$$

2 **The height of the top**

The height of the top trapezium is

$$5 \sin 30° = 2.5$$

3 **Area of the top trapezium**

Area of the top trapezium:

$$\frac{1}{2}(11 + 23) \times 2.5 = 42.5$$

4 **Total area**

Total area:

$$108.1 + 42.5 = 150.6$$

FINAL ANSWER

$150.6\ \text{cm}^2$.

4WM1/01 MAY 2025

Question 4

3 marks | Perimeter & Area

4 The diagram shows triangle ABC and a semicircle.

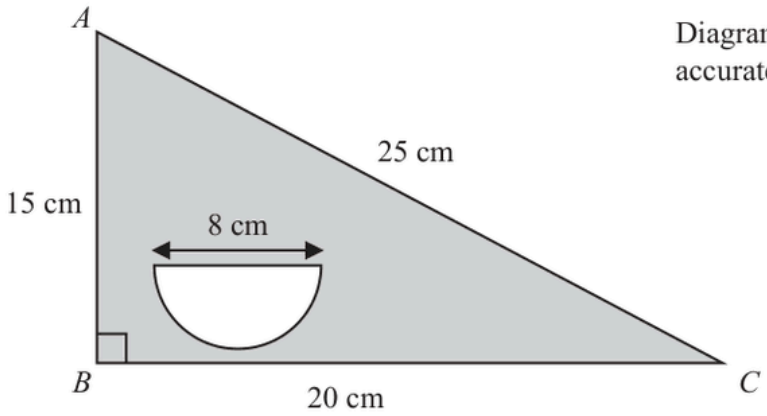


Diagram **NOT** accurately drawn

$AB = 15$ cm $\quad BC = 20$ cm $\quad AC = 25$ cm $\quad$ angle $ABC = 90°$

The diameter of the semicircle is 8 cm

Work out the area of the region shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4WM1/01 May 2025

1 **The shaded area is the**

The shaded area is the triangle area minus the semicircle area.

$$\text{triangle area} = \frac{1}{2} \times 15 \times 20 = 150$$

2 **Calculate area**

The semicircle has diameter 8, so radius 4.

$$\text{semicircle area} = \frac{1}{2}\pi(4)^2 = 8\pi$$

$$\text{shaded area} = 150 - 8\pi = 124.867\ldots$$

3 **Calculate area**

To 3 significant figures,

$$125$$

✓ **FINAL ANSWER**

$125\ \text{cm}^2$

4MA1/1H JUNE 2025

Question 9

5 marks | Perimeter & Area

9

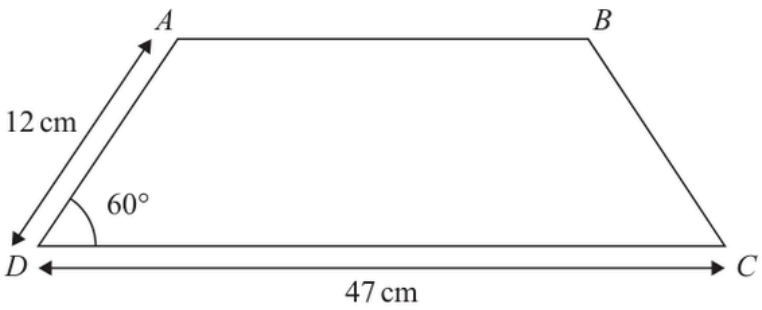


Diagram **NOT** accurately drawn

ABCD is a trapezium with one line of symmetry.

angle $ADC = 60°$ $AD = 12\,\text{cm}$ $DC = 47\,\text{cm}$

Work out the area of the trapezium.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2025

1 Use trigonometry

Because the trapezium has one line of symmetry, each sloping side makes the same $60°$ angle with the base.

2 Height

Height:

$$12\sin 60° = 10.392\ldots$$

3 The top parallel side is

The top parallel side is

$$47 - 2(12\cos 60°) = 35$$

4 **Area**

Area:

$$\frac{1}{2}(47+35)(10.392\ldots) = 426.084\ldots$$

✓ **FINAL ANSWER**

$426\ \text{cm}^2$.

4WM1/01 MAY 2025

Question 10

5 marks | Perimeter & Area

10

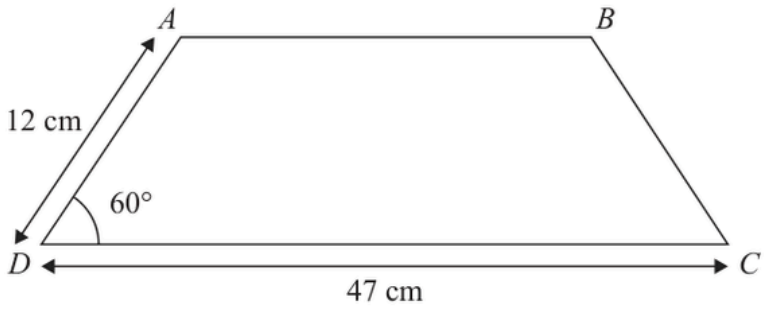


Diagram **NOT** accurately drawn

ABCD is a trapezium with one line of symmetry.

angle $ADC = 60°$ $AD = 12\,\text{cm}$ $DC = 47\,\text{cm}$

Work out the area of the trapezium.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM1/01 May 2025

1 The trapezium is symmetrical. The

The trapezium is symmetrical. The horizontal shift from each sloping side is

$$12\cos 60° = 6$$

2 Find the gradient

So the top parallel side is

$$47 - 6 - 6 = 35$$

3 The height is

The height is

$$12\sin 60° = 10.392\ldots$$

4 Area of the trapezium

Area of the trapezium:

$$\frac{1}{2}(47+35)(12\sin 60°) = 426.084\ldots$$

5 Calculate area

To 3 significant figures,

$$426$$

FINAL ANSWER

$426\ \text{cm}^2$

4WM1/01R MAY 2025

Question 19

5 marks | Perimeter & Area

19 The diagram shows a pentagon $ABCDE$

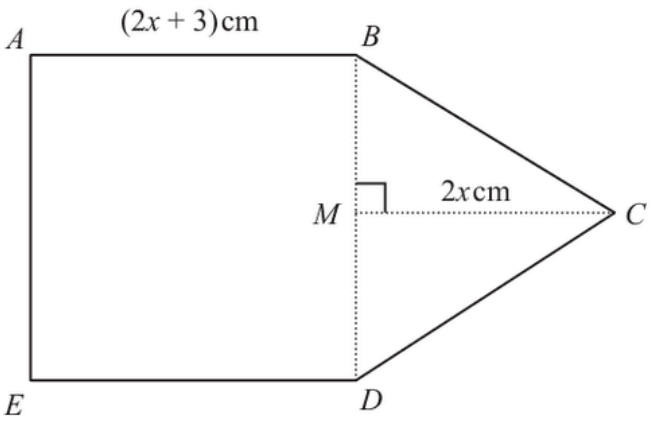


Diagram **NOT** accurately drawn

$ABCDE$ is made from a square $ABDE$ and a triangle BCD
M is the midpoint of BD

$AB = (2x + 3)$ cm $MC = 2x$ cm angle $BMC = 90°$

The area of pentagon $ABCDE$ is 84 cm^2

Work out the area of square $ABDE$
Show clear algebraic working.

..... cm^2

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4WM1/01R May 2025

1 **Square area**

Square area:

$$(2x + 3)^2$$

2 **Triangle area**

Triangle area:

$$\frac{1}{2}(2x + 3)(2x)$$

3 Evaluate fraction

So

$$(2x+3)^2+\frac{1}{2}(2x+3)(2x)=84$$

$$6x^2+15x-75=0$$

$$2x^2+5x-25=0$$

$$(2x-5)(x+5)=0$$

4 Calculate value

The positive solution is $x=2.5$, so

$$(2x+3)^2=8^2=64$$

FINAL ANSWER

$64\ \text{cm}^2$.

4WM1/01 NOVEMBER 2025

Question 1

2 marks | Perimeter & Area

1 The diagram shows a semicircle.

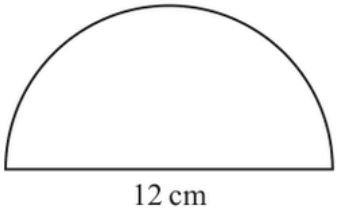


Diagram **NOT** accurately drawn

The semicircle has diameter 12 cm

Work out the perimeter of the semicircle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 November 2025

1 **Radius cm**

Radius $= 6$ cm.

$$\text{perimeter} = \pi r + 12 = 6\pi + 12 \approx 30.8496$$

FINAL ANSWER

30.8 cm to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 18

4 marks | Perimeter & Area

18 The diagram shows a rectangle.

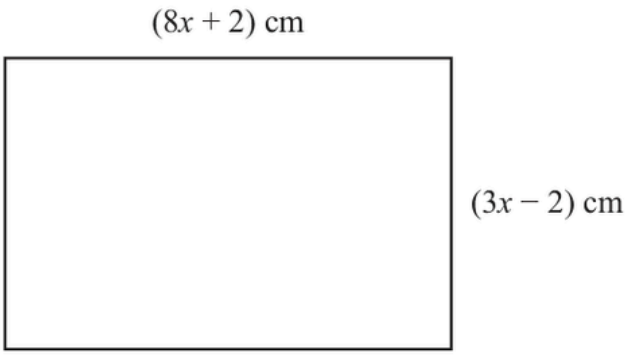


Diagram **NOT** accurately drawn

The rectangle has length $(8x + 2)$ cm and width $(3x - 2)$ cm
The area of the rectangle is $500\ \text{cm}^2$

(a) Show that $12x^2 - 5x - 252 = 0$

(1)

(b) Hence find the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(3)

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM1/01 November 2025

1 Calculate value

$$(8x + 2)(3x - 2) = 500$$

$$24x^2 - 10x - 4 = 500$$

$$12x^2 - 5x - 252 = 0$$

2 Solve quadratic equation

Using the quadratic formula,

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(12)(-252)}}{24}$$

3 **The positive solution is**

The positive solution is

$$x \approx 4.79564$$

✓ **FINAL ANSWER**

$x = 4.80$ to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 21

4 marks | Perimeter & Area

21 The diagram shows an equilateral triangle and a circle.

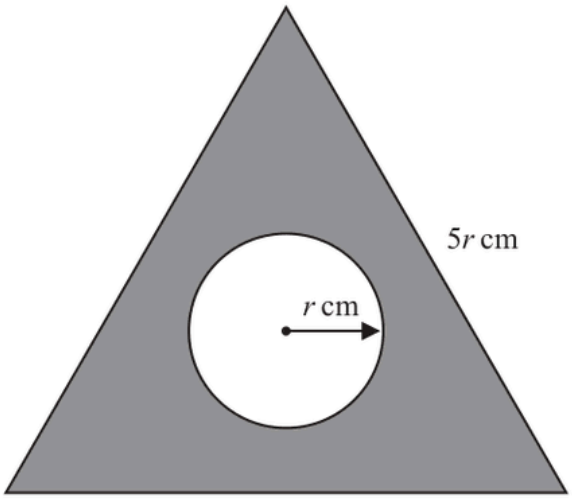


Diagram **NOT** accurately drawn

The equilateral triangle has sides of length $5r$ cm
The circle has radius r cm

The area of the shaded region is 610π cm^2
Work out the value of r

Give your answer correct to 3 significant figures.

$r =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 November 2025

1 **Triangle area**

Triangle area:

$$\frac{\sqrt{3}}{4}(5r)^2 = \frac{25\sqrt{3}}{4}r^2$$

2 **Circle area**

Circle area:

$$\pi r^2$$

3 **Simplify surd**

So

$$\left(\frac{25\sqrt{3}}{4} - \pi\right) r^2 = 610\pi$$

$$r \approx 15.7926$$

FINAL ANSWER

$r = 15.8$ to 3 significant figures.

Question 4

5 marks | Perimeter & Area

4 The diagram shows a shaded shape $ABCD$ made from a right-angled triangle ABD and a semicircle BCD

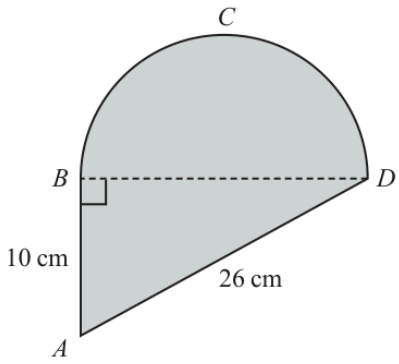


Diagram **NOT** accurately drawn

$AB = 10$ cm $AD = 26$ cm angle $ABD = 90°$

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 4 is 5 marks)

1 **Find BD with Pythagoras**

$$BD = \sqrt{26^2 - 10^2} = \sqrt{576} = 24 \text{ cm}$$

2 **Semicircle arc**

Diameter $BD = 24$, so the arc is

$$\tfrac{1}{2}\pi \times 24 = 12\pi = 37.699\ldots \text{ cm}$$

3 **Perimeter (BD is inside, not counted)**

$$10 + 26 + 12\pi = 73.699\ldots$$

✓ **FINAL ANSWER**

73.7 cm

4MA1/1H JUNE 2026

Question 11

5 marks | Perimeter & Area

11 The diagram shows a shaded shape $ABCDEO$ made from a semicircle BCD and two identical right-angled triangles ABO and EDO

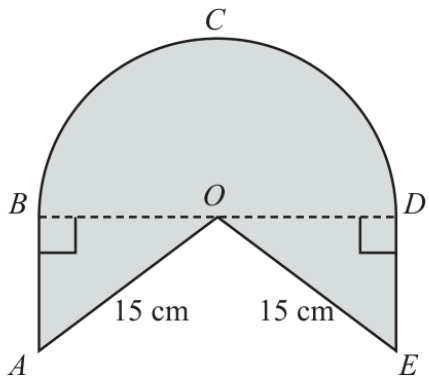


Diagram **NOT** accurately drawn

BD is the diameter of the semicircle.
O is the midpoint of BD

angle ABO = angle EDO = 90° $\quad AO = EO = 15$ cm

The area of semicircle BCD is 227 cm^2

Work out the perimeter of the shaded shape.
Show your working clearly.
Give your answer correct to one decimal place.

..... cm

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2026

1 **Solution | Step 1**

Let the radius be $r = OB = OD$. Then $\frac{1}{2}\pi r^2 = 227$, so $r = \sqrt{454/\pi} = 12.021343\ldots$ cm.
The given area belongs to the semicircle.

2 **Solution | Step 2**

$$AB = DE = \sqrt{15^2 - r^2} = 8.971472\ldots \text{ cm.}$$

The identical triangles have hypotenuse 15 cm and horizontal leg r.

3 Solution | Step 3

$$P = \pi r + AB + DE + AO + EO = \pi r + 2\sqrt{225 - r^2} + 30 = 85.709107\ldots \text{ cm}.$$

The boundary contains the semicircular arc and the four outer triangle sides, but not the internal diameter.

4 Solution | Step 4

$$P = 85.7 \text{ cm} \quad (1 \text{ d.p.}).$$

Retain unrounded intermediate values.

FINAL ANSWER

85.7 cm

TOPIC

Circles, Arcs & Sectors

Every past-paper question on this topic, 2020 to 2026.

Question 18

4 marks | Circles, Arcs & Sectors

18 The diagram shows two identical circles drawn inside a rectangle.

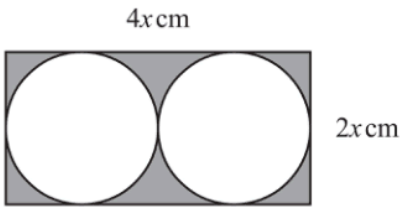


Diagram **NOT** accurately drawn

The length of the rectangle is $4x$ cm and the width of the rectangle is $2x$ cm
Each circle has a radius of x cm

The circles touch each other and each circle touches three sides of the rectangle.

The region inside the rectangle that is outside the circles, shown shaded in the diagram, has a total area of $20\,\text{cm}^2$

Work out the perimeter of the rectangle.
Give your answer correct to 3 significant figures.

..... cm

1 Write the shaded area

Rectangle area:

$$4x \times 2x = 8x^2$$

Area of the two circles:

$$2\pi x^2$$

So

$$8x^2 - 2\pi x^2 = 20$$

2 Solve for x

$$x^2(8 - 2\pi) = 20$$

$$x = \sqrt{\frac{20}{8 - 2\pi}} = 3.413\ldots$$

3 Find the perimeter

$$P = 2(4x + 2x) = 12x$$

$$12(3.413\ldots) = 40.96\ldots$$

4 Round to 3 significant figures

$$40.96\ldots \approx 41.0$$

FINAL ANSWER

41.0 cm.

4MA1/1H JANUARY 2021

Question 8

3 marks | Circles, Arcs & Sectors

8 The region, shown shaded in the diagram, is a path.

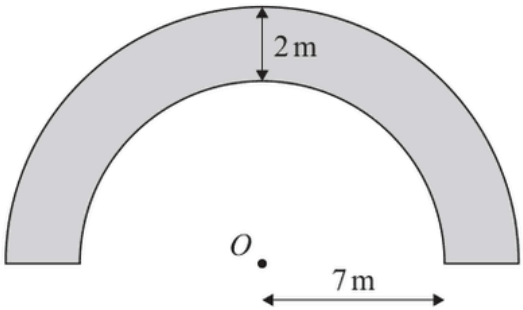


Diagram **NOT** accurately drawn

The boundary of the path is formed by two semicircles, with the same centre O, and two straight lines.

The inner semicircle has a radius of 7 metres.
The path has a width of 2 metres.

Work out the perimeter of the path.
Give your answer correct to one decimal place.

..... m

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2021

1 **The inner radius is m**

The inner radius is 7 m.

2 **Calculate value**

The path is 2 m wide, so the outer radius is

$$7 + 2 = 9 \text{ m}$$

3 Perimeter of the path

Perimeter of the path:

$$\text{inner semicircle} + \text{outer semicircle} + 2 \text{ straight ends}$$

$$= 7\pi + 9\pi + 2 + 2$$

$$= 16\pi + 4$$

$$= 54.265\ldots$$

FINAL ANSWER

$54.3\ \text{m}$.

4MA1/2HR JANUARY 2021

Question 11

4 marks | Circles, Arcs & Sectors

11 A circle centre O has radius 9 cm.

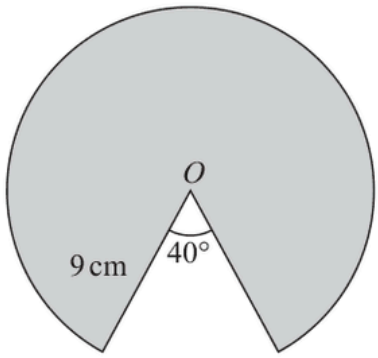


Diagram **NOT** accurately drawn

Calculate the perimeter of the shaded sector of the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR January 2021

1 **The shaded sector is the**

The shaded sector is the major sector:

$$360° - 40° = 320°$$

2 **Arc length**

Arc length:

$$\frac{320}{360} \times 2\pi(9) = 16\pi$$

3 **Perimeter**

Perimeter:

$$16\pi + 2(9) = 68.265\ldots$$

✓ **FINAL ANSWER**

68.3 cm.

4MA1/2HR JANUARY 2021

Question 20

5 marks | Circles, Arcs & Sectors

20 A, B and C are points on a circle with centre O.

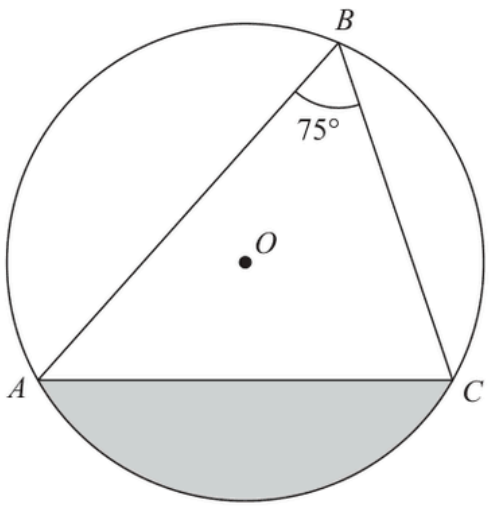


Diagram **NOT** accurately drawn

Angle $ABC = 75°$

The area of the shaded segment is $200\,\text{cm}^2$

Calculate the radius of the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2021

1 **Use trigonometry**

$$\angle ABC = 75°$$

2 **The angle at the centre**

The angle at the centre is twice the angle at the circumference, so

$$\angle AOC = 2(75°) = 150°$$

3 **The shaded segment is**

The shaded segment is:

$$\text{sector } AOC - \text{triangle } AOC$$

4 **Let the radius be**

Let the radius be r.

5 **Sector area**

Sector area:

$$\frac{150}{360}\pi r^2$$

6 **Triangle area**

Triangle area:

$$\frac{1}{2}r^2 \sin 150°$$

7 **Use trigonometry**

So

$$\frac{150}{360}\pi r^2 - \frac{1}{2}r^2 \sin 150° = 200$$

$$r^2\left(\frac{5\pi}{12} - \frac{1}{4}\right) = 200$$

$$r^2 = \frac{200}{\frac{5\pi}{12} - \frac{1}{4}}$$

$$r = 13.7426\ldots$$

✓ **FINAL ANSWER**

$\boxed{13.7 \text{ cm}}$

4MA1/2H JANUARY 2021

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows a sector OBC of a circle with centre O and radius $(6 + x)$ cm.

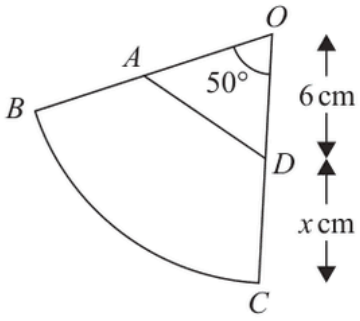


Diagram **NOT** accurately drawn

A is the point on OB and D is the point on OC such that $OA = OD = 6$ cm

Angle $BOC = 50°$

Given that

the perimeter of sector $OBC = 2 \times$ the perimeter of triangle OAD

find the value of x.
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2021

1 The radius of sector is

The radius of sector OBC is

$$6 + x$$

2 Use trigonometry

The angle is $50°$, so the arc length of sector OBC is

$$\frac{50}{360} \times 2\pi(6 + x)$$

3 **Evaluate fraction**

Therefore the perimeter of sector OBC is

$$2(6+x)+\frac{50}{360}\times 2\pi(6+x)$$

$$=(6+x)\left(2+\frac{5\pi}{18}\right)$$

4 **Use trigonometry**

In triangle OAD,

$$OA=OD=6, \qquad \angle AOD=50^\circ$$

5 **Use trigonometry**

So

$$AD=2(6)\sin 25^\circ=12\sin 25^\circ$$

6 **The perimeter of triangle is**

The perimeter of triangle OAD is

$$12+12\sin 25^\circ$$

7 **Given that**

Given that

$$\text{perimeter of sector } OBC=2\times\text{perimeter of triangle } OAD$$

$$(6+x)\left(2+\frac{5\pi}{18}\right)=2(12+12\sin 25^\circ)$$

$$x=\frac{2(12+12\sin 25^\circ)}{2+\frac{5\pi}{18}}-6$$

$$x=5.8854\ldots$$

✓ **FINAL ANSWER**

$\boxed{5.89}$

4MA1/1H MAY 2021

Question 12

2 marks | Circles, Arcs & Sectors

12 The diagram shows a sector of a circle with radius 7 cm.

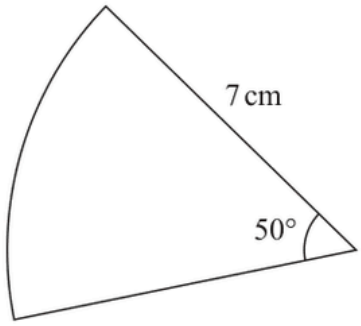


Diagram **NOT** accurately drawn

Work out the length of the arc of the sector.
Give your answer correct to one decimal place.

..... cm

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May 2021

1 **Evaluate fraction**

$$\text{arc length} = \frac{50}{360} \times 2\pi(7)$$

$$= 6.108\ldots$$

FINAL ANSWER

6.1 cm.

4MA1/1H MAY 2021

Question 25

6 marks | Circles, Arcs & Sectors

25 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

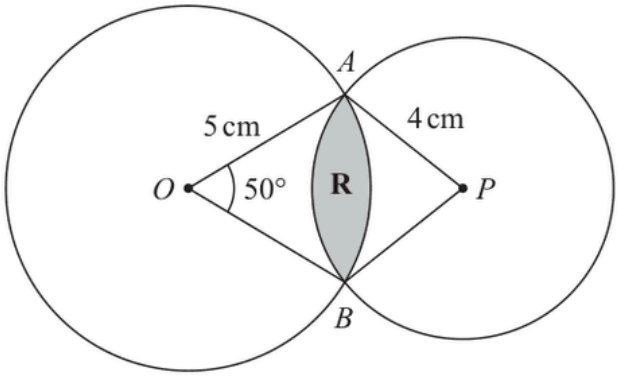


Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2021

1 **Use trigonometry**

For the circle with centre O:

$$\angle AOB = 50°, \qquad OA = OB = 5$$

2 **The half chord length is**

The half chord length is

$$\frac{AB}{2} = 5\sin 25°$$

3 **Use trigonometry**

For the circle with centre P, let

$$\angle APB = \theta$$

---

4 **Use trigonometry**

The radius is 4, so

$$4\sin\left(\frac{\theta}{2}\right) = 5\sin 25°$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{5\sin 25°}{4}$$

$$\theta = 63.7777\ldots°$$

---

5 **The common region is made**

The common region is made from two minor segments.

---

6 **Use trigonometry**

For the circle with centre O:

$$\text{segment area} = \frac{50}{360}\pi(5)^2 - \frac{1}{2}(5)^2\sin 50° = 1.3328\ldots$$

---

7 **Use trigonometry**

For the circle with centre P:

$$\text{segment area} = \frac{\theta}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin\theta$$

$$= 1.7284\ldots$$

---

8 **Total area of region**

Total area of region R:

$$1.3328\ldots + 1.7284\ldots = 3.0611\ldots$$

✓ **FINAL ANSWER**

$\boxed{3.06\ \text{cm}^2}$

4MA1/2HR JANUARY 2022

Question 9

5 marks | Circles, Arcs & Sectors

9 In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc.

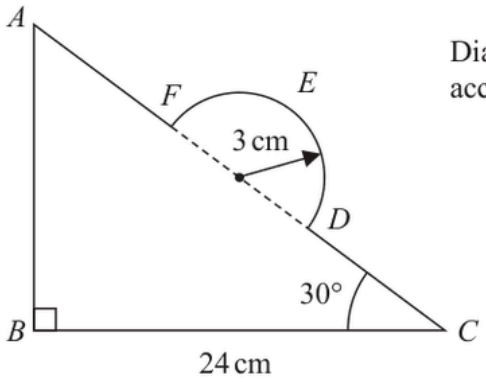


Diagram **NOT** accurately drawn

In triangle ABC

$BC = 24$ cm angle $ABC = 90°$ angle $BCA = 30°$

The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF
The radius of the semicircular arc is 3 cm.

Work out the length of $AFEDC$
Give your answer correct to 2 significant figures.

..... cm

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2022

1 **Use trigonometry**

In triangle ABC,

$$\cos 30° = \frac{24}{AC}$$

$$AC = \frac{24}{\cos 30°} = 27.712\ldots$$

2 **Calculate value**

The semicircle has radius $3\ \text{cm}$, so its diameter is $6\ \text{cm}$.

$$AF + DC = AC - DF = 27.712\ldots - 6$$

3 **Arc is a semicircle**

Arc FED is a semicircle:

$$\text{arc length} = 3\pi$$

4 **Calculate value**

Therefore

$$AFEDC = 27.712\ldots - 6 + 3\pi = 31.137\ldots$$

✓ **FINAL ANSWER**

$31\ \text{cm}$.

4MA1/1HR JANUARY 2022

Question 11

2 marks | Circles, Arcs & Sectors

11 The diagram shows sector OPQ of a circle, centre O

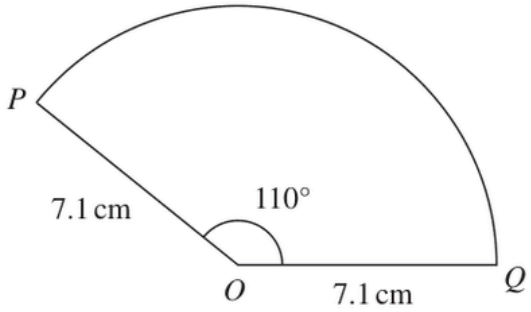


Diagram **NOT** accurately drawn

$OP = OQ = 7.1$ cm
Angle $POQ = 110°$

Calculate the area of sector OPQ
Give your answer correct to one decimal place.

..... cm²

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2022

1 Calculate area

$$\text{area} = \frac{110}{360} \times \pi(7.1)^2$$

$$= 48.390\ldots$$

FINAL ANSWER

$48.4\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 16

4 marks | Circles, Arcs & Sectors

16 The diagram shows a circle with centre O

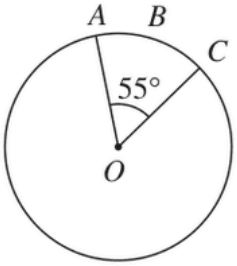


Diagram **NOT** accurately drawn

A, B and C are points on the circle so that the length of the arc ABC is 5 cm.

Given that angle $AOC = 55°$

work out the area of the circle.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2022

1 Use trigonometry

The arc length is $5\ \text{cm}$, and the angle is $55°$.

$$5 = \frac{55}{360} \times 2\pi r$$

$$r = 5.208\ldots$$

2 Area of the circle

Area of the circle:

$$\pi r^2 = \pi(5.208\ldots)^2 = 85.233\ldots$$

FINAL ANSWER

$85.2\ \text{cm}^2$.

4MA1/2HR JUNE 2022

Question 13

3 marks | Circles, Arcs & Sectors

13 The diagram shows a sector AOB of a circle with centre O

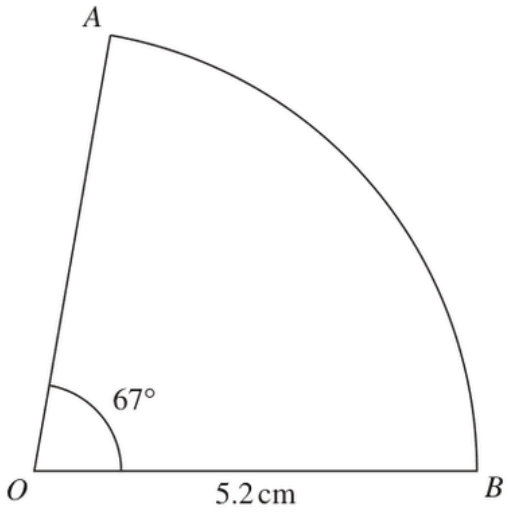


Diagram **NOT** accurately drawn

Angle $AOB = 67°$
$OA = OB = 5.2\,\text{cm}$

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2022

1 Arc length

Arc length:

$$\frac{67}{360} \times 2\pi(5.2) = 6.080\ldots$$

2 Perimeter

Perimeter:

$$5.2 + 5.2 + 6.080\ldots = 16.480\ldots$$

FINAL ANSWER

$16.5\ \text{cm}$.

4MA1/1H JUNE 2022

Question 19

5 marks | Circles, Arcs & Sectors

19

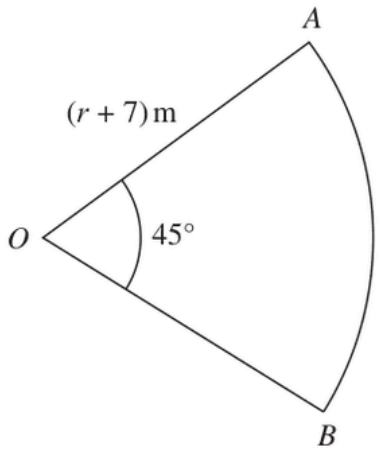


Diagram **NOT** accurately drawn

OAB is a sector **S** of a circle with centre *O* and radius $(r + 7)$ metres.
Angle $AOB = 45°$

A circle **C** has radius $(r - 2)$ metres.

The area of sector **S** is twice the area of circle **C**

Find the value of r
Show your working clearly.

Question 19 continued.

$r =$ .....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2022

1 **Area of sector**

Area of sector S:

$$\frac{45}{360}\pi(r+7)^2 = \frac{1}{8}\pi(r+7)^2$$

2 **Area of circle**

Area of circle C:

$$\pi(r-2)^2$$

---

3 **The sector area is twice**

The sector area is twice the circle area:

$$\frac{1}{8}\pi(r+7)^2 = 2\pi(r-2)^2$$

$$(r+7)^2 = 16(r-2)^2$$

---

4 **Solve inequality**

Since $r > 2$,

$$r+7 = 4(r-2)$$

$$r+7 = 4r-8$$

$$r = 5$$

✓ **FINAL ANSWER**

$r = 5$.

4MA1/2H JUNE 2022

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows four identical circles drawn inside a square.

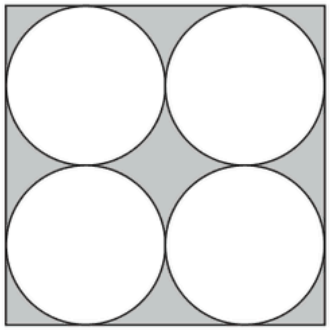


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of $40\,\text{cm}^2$

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2022

1 Let the radius of each

Let the radius of each circle be r.

2 There are two circles across

There are two circles across the width of the square, so the side length of the square is

$$4r$$

3 Area of the square

Area of the square:

$$(4r)^2 = 16r^2$$

4 Area of the four circles

Area of the four circles:

$$4\pi r^2$$

5 **Calculate area**

The shaded area is $40\ \text{cm}^2$, so

$$16r^2 - 4\pi r^2 = 40$$

$$r^2(16 - 4\pi) = 40$$

$$r = \sqrt{\frac{40}{16 - 4\pi}} = 3.4131\ldots$$

---

6 **The perimeter of the square**

The perimeter of the square is

$$4(4r) = 16r$$

$$16r = 54.6101\ldots$$

✓ **FINAL ANSWER**

$\boxed{54.6\ \text{cm}}$

4MA1/1HR MAY 2023

Question 4

5 marks | Circles, Arcs & Sectors

4 The diagram shows a shape made up of three semicircles, enclosing a right-angled triangle.

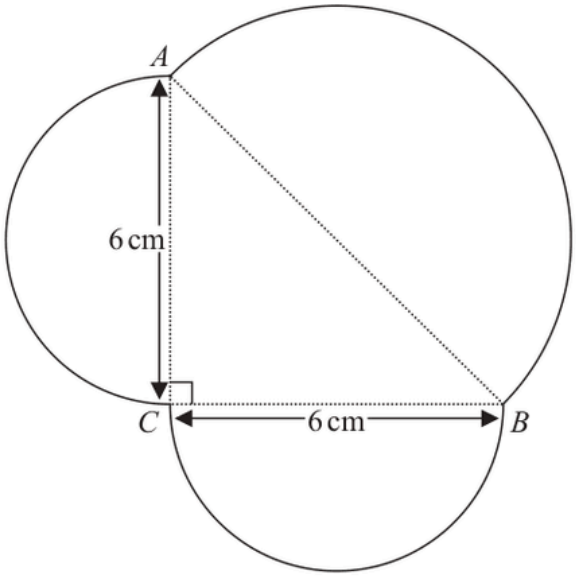


Diagram **NOT** accurately drawn

AB, BC and CA are each the diameter of a semicircle.

$BC = CA = 6$ cm.

Work out the perimeter of the shape.
Give your answer correct to one decimal place.

..... cm

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May 2023

1 **Simplify surd**

$$AB = \sqrt{6^2 + 6^2} = \sqrt{72} = 8.485\ldots$$

2 **The perimeter is the sum**

The perimeter is the sum of three semicircular arcs:

$$\frac{\pi}{2}(AB + BC + CA) = \frac{\pi}{2}(8.485\ldots + 6 + 6) = 32.178\ldots$$

FINAL ANSWER

$32.2\ \text{cm}$.

4MA1/1H MAY 2023

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

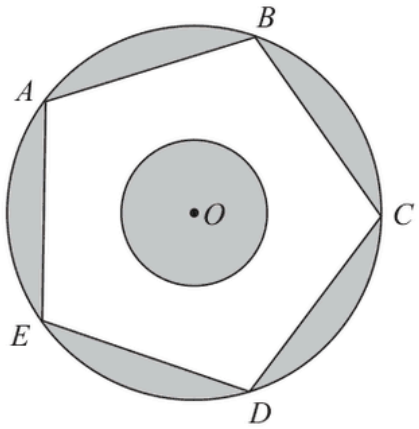


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2023

1 **The regular pentagon has side**

The regular pentagon has side length 8 cm.

2 **Use trigonometry**

For a regular pentagon,

$$R = \frac{s}{2\sin 36°}$$

3 **Use trigonometry**

where R is the radius of the larger circle.

$$R = \frac{8}{2\sin 36^\circ} = 6.8052\ldots$$

4 **Area of the larger circle**

Area of the larger circle:

$$\pi R^2 = 145.4898\ldots$$

5 **Area of the regular pentagon**

Area of the regular pentagon:

$$\frac{5s^2}{4\tan 36^\circ} = \frac{5(8)^2}{4\tan 36^\circ} = 110.1106\ldots$$

6 **Let the radius of the**

Let the radius of the smaller circle be r.

7 **The shaded area is**

The shaded area is:

$$\text{larger circle area} - \text{pentagon area} + \pi r^2$$

8 **The unshaded area is**

The unshaded area is:

$$\text{pentagon area} - \pi r^2$$

9 **Simplify surd**

These are equal, so

$$145.4898 - 110.1106 + \pi r^2 = 110.1106 - \pi r^2$$

$$2\pi r^2 = 2(110.1106) - 145.4898$$

$$\pi r^2 = 37.3657\ldots$$

$$r = \sqrt{\frac{37.3657\ldots}{\pi}} = 3.4487\ldots$$

✓ **FINAL ANSWER**

$$\boxed{3.45\text{ cm}}$$

4MA1/1H NOVEMBER 2023

Question 5

3 marks | Circles, Arcs & Sectors

5 The diagram shows a shaded shape made from three identical semicircles, X, Y and Z

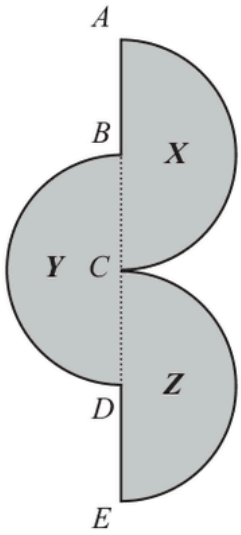


Diagram **NOT** accurately drawn

$ABCDE$ is a straight line.

AC is the diameter of semicircle X and B is the centre of semicircle X
BD is the diameter of semicircle Y and C is the centre of semicircle Y
CE is the diameter of semicircle Z and D is the centre of semicircle Z

$AC = BD = CE = 20$ cm

Work out the perimeter of the shaded shape.
Give your answer correct to the nearest whole number.

..... cm

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2023

1 Each semicircle has diameter

Each semicircle has diameter 20 cm, so each has radius 10 cm.

2 The total curved perimeter is

The total curved perimeter is three semicircles:

$$3 \times \pi r = 3 \times 10\pi = 30\pi$$

3 The two exposed straight parts

The two exposed straight parts are

$$AB = 10, \qquad DE = 10$$

4 Calculate value

So the perimeter is

$$30\pi + 20 = 114.2\ldots$$

FINAL ANSWER

$114\ \text{cm}$ to the nearest whole number.

4MA1/2H NOVEMBER 2023

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows equilateral triangle ABC with sides of length 10 cm.
A circle is drawn inside the triangle.

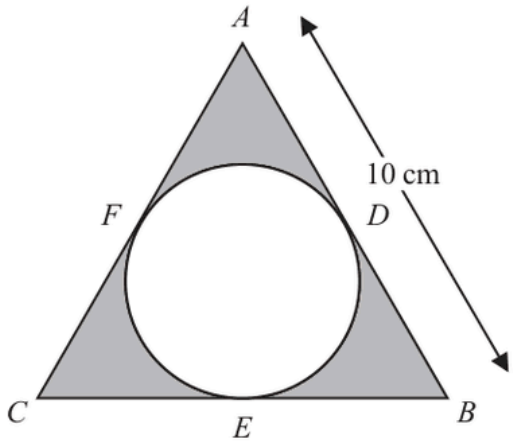


Diagram **NOT** accurately drawn

D, E and F are points on the circle.

ADB, BEC and CFA are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2023

1 **Use trigonometry**

The triangle is equilateral with side length 10 cm.

2 **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$$

3 **Use trigonometry**

For an equilateral triangle, the inradius is

$$r = \frac{s\sqrt{3}}{6}$$

4 **Simplify surd**

So

$$r = \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

5 **Area of the circle**

Area of the circle:

$$\pi r^2 = \pi\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{25\pi}{3}$$

6 **The shaded area is**

The shaded area is

$$25\sqrt{3} - \frac{25\pi}{3} = 17.1213\ldots$$

✓ **FINAL ANSWER**

$\boxed{17.1\ \text{cm}^2}$

4MA1/2HR JUNE 2024

Question 5

4 marks | Circles, Arcs & Sectors

5 The diagram shows the design for a badge, which will be made using wire.
The design is a circle inside a square $ABCD$

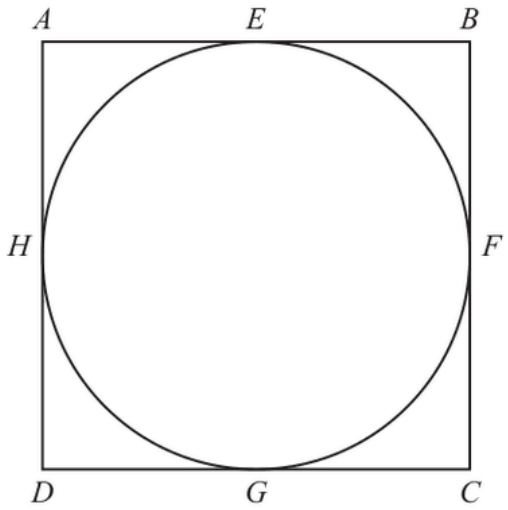


Diagram **NOT** accurately drawn

The circle touches the square at the points E, F, G and H

The area of the square is $81\ \text{cm}^2$

Calculate the total length of wire that will be needed to make the square and the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2024

1 **Calculate area**

The square has area $81\ \text{cm}^2$, so its side length is

$$\sqrt{81} = 9\ \text{cm}$$

2 **Perimeter of the square**

Perimeter of the square:

$$4 \times 9 = 36$$

3 **Calculate value**

The circle has diameter $9\ \text{cm}$, so its circumference is

$$\pi d = 9\pi$$

4 **Total wire length**

Total wire length:

$$36 + 9\pi = 64.274\ldots$$

✓ **FINAL ANSWER**

$64.3\ \text{cm}$ to 3 significant figures.

4MA1/2HR JUNE 2024

Question 13

3 marks | Circles, Arcs & Sectors

13 OAB is a sector of a circle with centre O and radius r cm.

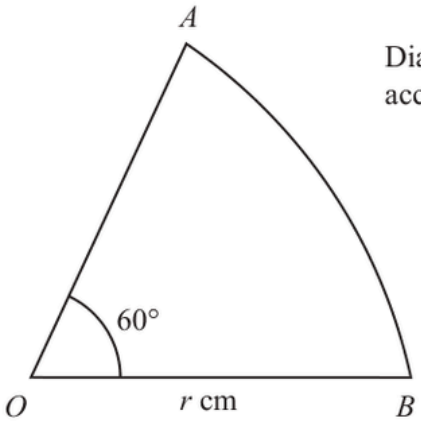


Diagram **NOT** accurately drawn

Angle $AOB = 60°$
The perimeter of the sector is P cm.

Find a formula for P in terms of r
Give your answer in the form $P = r(c\pi + k)$ where c and k are values to be found.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2024

1 The arc length is

The arc length is

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

2 The perimeter is two radii

The perimeter is two radii plus the arc:

$$P = 2r + \frac{\pi r}{3}$$

$$P = r\left(\frac{\pi}{3} + 2\right)$$

FINAL ANSWER

$P = r\left(\frac{1}{3}\pi + 2\right)$.

4MA1/1H MAY 2024

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows a sector $OABC$ of a circle centre O

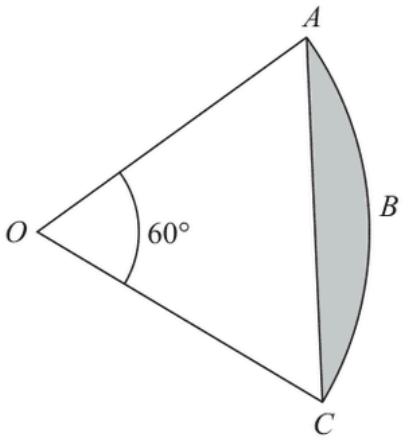


Diagram **NOT** accurately drawn

Angle $AOC = 60°$

The area of the shaded segment ABC is $38\,\text{cm}^2$

Work out the perimeter of the shaded segment ABC
Give your answer correct to one decimal place.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2024

1 **Let the radius of the**

Let the radius of the sector be r.

2 **The shaded segment is**

The shaded segment is

$$\text{sector area} - \text{triangle area}$$

3 **Use trigonometry**

The angle is $60°$, so

$$38 = \frac{60}{360}\pi r^2 - \frac{1}{2}r^2 \sin 60°$$

$$38 = r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$r = 20.4815\ldots$$

4 **The perimeter of the shaded**

The perimeter of the shaded segment is the arc AC plus the chord AC.

5 **Arc length**

Arc length:

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

6 **Chord length**

Chord length:

$$2r \sin 30° = r$$

7 **Evaluate fraction**

So the perimeter is

$$\frac{\pi r}{3} + r$$

$$= \frac{\pi(20.4815\ldots)}{3} + 20.4815\ldots$$

$$= 41.9296\ldots$$

✓ **FINAL ANSWER**

$\boxed{41.9\text{ cm}}$

4MA1/2H JUNE 2024

Question 21

5 marks | Circles, Arcs & Sectors

21 The diagram shows a square $ABCD$ and a circle.

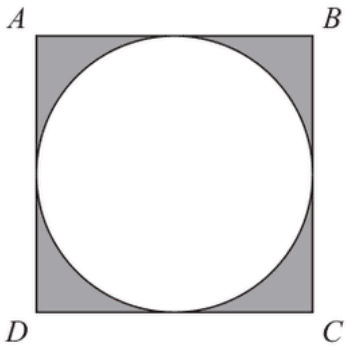


Diagram **NOT** accurately drawn

The sides of the square are tangents to the circle.

The total area of the shaded regions is $80\,\text{cm}^2$

Work out the length of AC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2024

1 **Let the side length**

Let the side length of the square be s.

2 **Square**

The circle is inscribed in the square, so its radius is

$$\frac{s}{2}$$

3 **The shaded area is the**

The shaded area is the area of the square minus the area of the circle:

$$s^2 - \pi\left(\frac{s}{2}\right)^2 = 80$$

$$s^2 - \frac{\pi s^2}{4} = 80$$

$$s^2\left(1 - \frac{\pi}{4}\right) = 80$$

$$s = \sqrt{\frac{80}{1 - \frac{\pi}{4}}} = 19.3076\ldots$$

---

4 **The diagonal of the square**

The diagonal of the square is

$$AC = s\sqrt{2}$$

$$AC = 19.3076\ldots\sqrt{2} = 27.3051\ldots$$

✓ **FINAL ANSWER**

$\boxed{27.3\text{ cm}}$

4MA1/1H NOVEMBER 2024

Question 16

4 marks | Circles, Arcs & Sectors

16 $OAPB$ is a sector of a circle, centre O

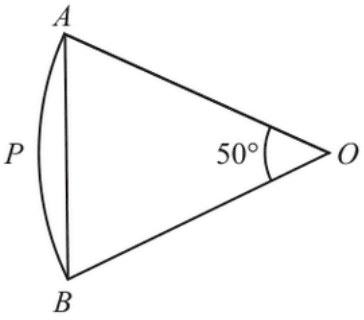


Diagram **NOT** accurately drawn

Angle $AOB = 50°$

Area of triangle $OAB = 120\ \text{cm}^2$

Work out the area of the sector $OAPB$
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2024

1 Use trigonometry

For triangle OAB,

$$120 = \frac{1}{2}r^2 \sin 50°$$

$$r^2 = \frac{240}{\sin 50°}$$

2 Area of sector

Area of sector $OAPB$:

$$\frac{50}{360}\pi r^2 = \frac{50}{360}\pi\left(\frac{240}{\sin 50°}\right) = 136.701\ldots$$

FINAL ANSWER

$137\ \text{cm}^2$.

4MA1/2H NOVEMBER 2024

Question 25

5 marks | Circles, Arcs & Sectors

25 The diagram shows an equilateral triangle ABC and a circle with centre O

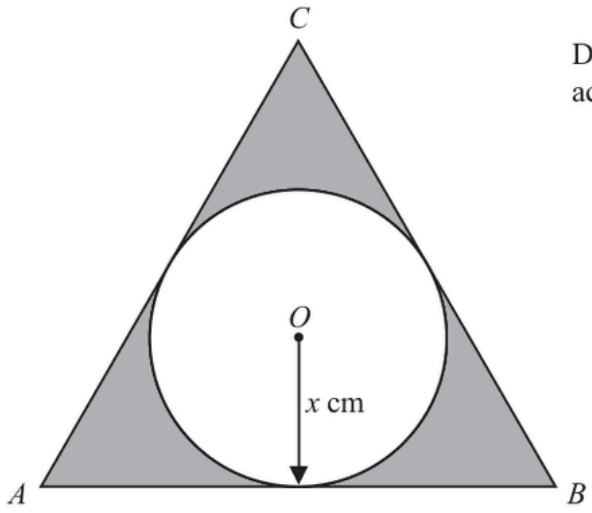


Diagram **NOT** accurately drawn

AB, BC and CA are tangents to the circle.

The radius of the circle is x cm

The total area, in cm^2, of the regions shown shaded in the diagram is nx^2

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2024

1 **Use trigonometry**

The circle has radius x, so the inradius of the equilateral triangle is x.

2 **Use trigonometry**

For an equilateral triangle with side length a,

$$\text{inradius} = \frac{a\sqrt{3}}{6}$$

3 **Simplify surd**

So

$$x = \frac{a\sqrt{3}}{6}$$

$$a = \frac{6x}{\sqrt{3}} = 2\sqrt{3}x$$

4 **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(2\sqrt{3}x)^2$$

$$= \frac{\sqrt{3}}{4}(12x^2) = 3\sqrt{3}x^2$$

5 **Area of the circle**

Area of the circle:

$$\pi x^2$$

6 **The shaded area is**

The shaded area is

$$3\sqrt{3}x^2 - \pi x^2$$

$$= (3\sqrt{3} - \pi)x^2$$

7 **Simplify surd**

Therefore

$$n = 3\sqrt{3} - \pi$$

✓ **FINAL ANSWER**

$$\boxed{3\sqrt{3} - \pi}$$

4MA1/1H JUNE 2025

Question 11

2 marks | Circles, Arcs & Sectors

11 $OABC$ is a sector of a circle, centre O

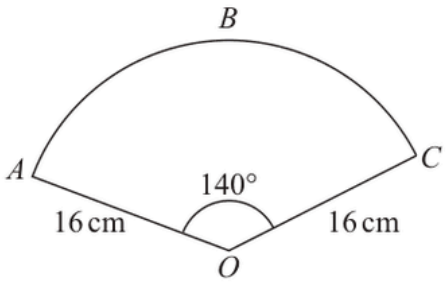


Diagram **NOT** accurately drawn

angle $AOC = 140°$
$OA = OC = 16\,\text{cm}$

Calculate the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2025

1 Calculate area

$$\text{area} = \frac{140}{360} \times \pi(16)^2$$

$$= 312.763\ldots$$

FINAL ANSWER

$313\ \text{cm}^2$.

Question 14

4 marks | Circles, Arcs & Sectors

14 The diagram shows sector OAB of a circle, centre O

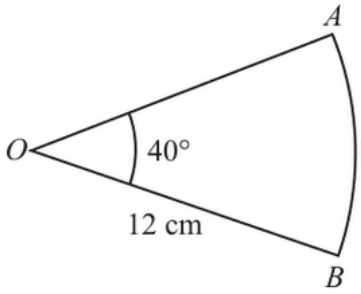


Diagram **NOT** accurately drawn

$OA = OB = 12$ cm angle $AOB = 40°$

(a) Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

The diagram shows triangle XYZ

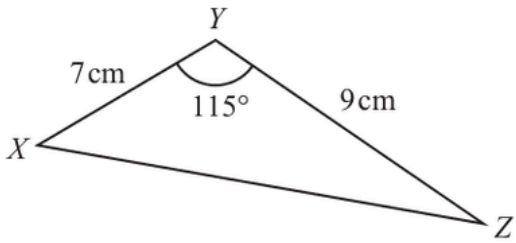


Diagram **NOT** accurately drawn

$XY = 7$ cm $YZ = 9$ cm angle $XYZ = 115°$

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 14 is 4 marks)

1 **Sector area**

Sector area:

$$\frac{40}{360}\pi(12)^2 = 16\pi = 50.265\ldots$$

2 **To 3 significant figures**

To 3 significant figures,

$$50.3\ \text{cm}^2$$

3 **Triangle area**

Triangle area:

$$\frac{1}{2}(7)(9)\sin 115° = 28.548\ldots$$

4 **To 3 significant figures**

To 3 significant figures,

$$28.5\ \text{cm}^2$$

✓ **FINAL ANSWER**

$50.3\ \text{cm}^2$ and $28.5\ \text{cm}^2$.

4WM1/01R MAY 2025

Question 22

6 marks | Circles, Arcs & Sectors

22

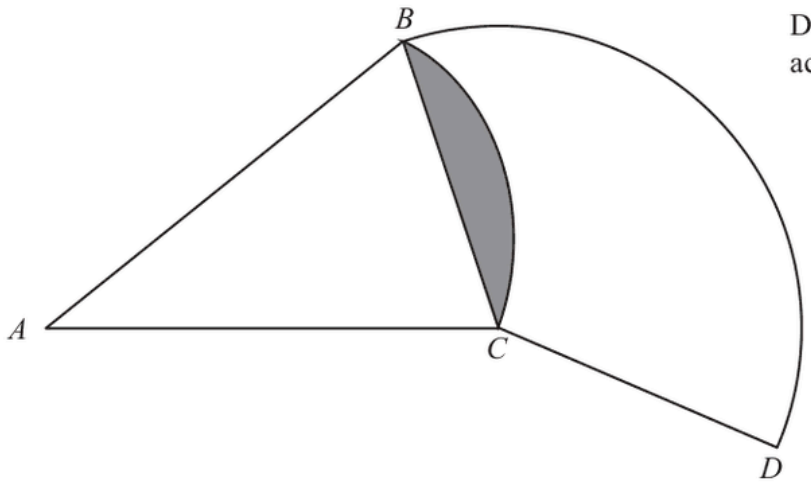


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre A

BCD is a sector of a circle, centre C

Angle $BAC = 40°$
Angle $BCD = 130°$
Area of shaded segment $= 28\,\text{cm}^2$

Find the length of the arc BD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM1/01R May 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/1HR JUNE 2025

Question 25

6 marks | Circles, Arcs & Sectors

25

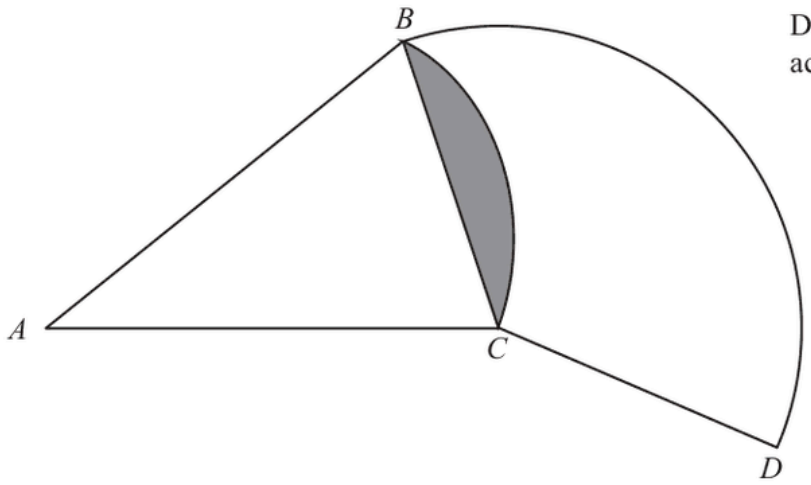


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre *A*

BCD is a sector of a circle, centre *C*

Angle *BAC* = 40°
Angle *BCD* = 130°
Area of shaded segment = 28 cm^2

Find the length of the arc *BD*
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/2H NOVEMBER 2025

Question 6

5 marks | Circles, Arcs & Sectors

6 A large circle with centre O contains 3 identical small circles.
Each circle touches two other circles.

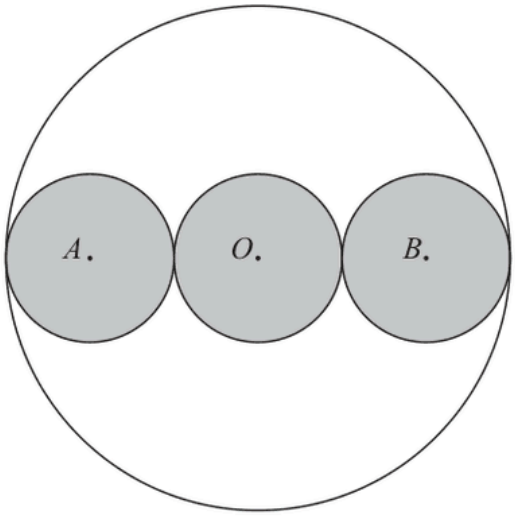


Diagram **NOT** accurately drawn

A, O and B are the centres of the small circles.
AOB is a straight line.

The circumference of the large circle is 160 cm

Work out the area of one small circle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2025

1 **The circumference of the large**

The circumference of the large circle is $160\ \text{cm}$, so

$$2\pi R = 160$$

$$R = \frac{80}{\pi}$$

2 The three equal small circles

The three equal small circles fit along the diameter of the large circle, so

$$2R = 6r$$

$$r = \frac{R}{3} = \frac{80}{3\pi}$$

3 Area of one small circle

Area of one small circle is

$$\pi r^2 = \pi\left(\frac{80}{3\pi}\right)^2$$

$$= 226.35\ldots$$

FINAL ANSWER

$226\ \text{cm}^2$ to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 14

2 marks | Circles, Arcs & Sectors

14 AOB is a sector of a circle with centre O and radius 12 cm

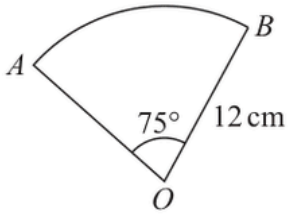


Diagram **NOT** accurately drawn

Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4WM1/01 November 2025

1 Calculate area

$$\text{area} = \frac{75}{360}\pi(12)^2$$

$$= 30\pi \approx 94.2478$$

FINAL ANSWER

$94.2\ \text{cm}^2$ to 3 significant figures.

4MA1/1H NOVEMBER 2025

Question 15

2 marks | Circles, Arcs & Sectors

15 AOB is a sector of a circle with centre O and radius 12 cm

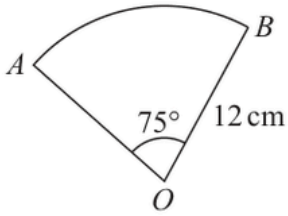


Diagram **NOT** accurately drawn

Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2025

1 Calculate area

$$\text{area} = \frac{75}{360} \times \pi(12)^2$$

$$= 94.247\ldots$$

FINAL ANSWER

$94.2\ \text{cm}^2$.

4MA1/2H JUNE 2026

Question 15

4 marks | Circles, Arcs & Sectors

15 The diagram shows a sector $OABC$ of a circle, centre O

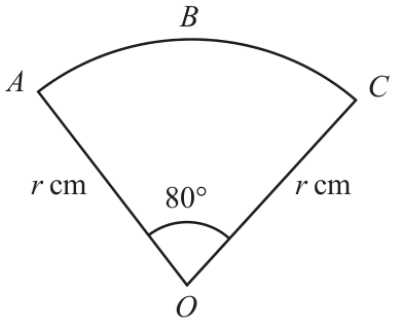


Diagram **NOT** accurately drawn

$OA = OC = r$ cm angle $AOC = 80°$

The area of the sector $OABC$ is 32π cm^2

Work out the perimeter of the sector $OABC$
Give your answer correct to one decimal place.

..... cm

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2026

1 **Solution | Step 1**

$$\frac{80}{360}\pi r^2 = 32\pi \quad \Rightarrow \quad r^2 = 32\frac{360}{80} = 144 \quad \Rightarrow \quad r = 12 \text{ cm}.$$

Use the fraction of the full circle and choose the positive radius.

2 **Solution | Step 2**

$$\text{Arc length} = \frac{80}{360}(2\pi \times 12) = \frac{16\pi}{3} \text{ cm}.$$

The same central-angle fraction applies to the circumference.

3 **Solution | Step 3**

$$P = 12 + 12 + \frac{16\pi}{3} = 40.7551608\ldots \text{ cm}.$$

The sector perimeter contains both radii as well as the arc.

FINAL ANSWER

$40.8\ \text{cm}$ (1 decimal place).

Question 23

6 marks | Circles, Arcs & Sectors

23 OPQ and OAB are sectors of a circle, centre O

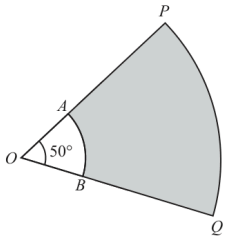


Diagram **NOT** accurately drawn

Angle $POQ = 50°$

A is the point on OP such that $OA = \frac{1}{4}OP$

The area of the shaded region is $\frac{243}{16}\pi\ \text{cm}^2$

Work out the perimeter of the shaded region.
Give your answer in the form $a\pi + b$ where a and b are constants.
Show your working clearly.

.....cm

(Total for Question 23 is 6 marks)

1 **Let OP = r**

Then $OA = \frac{r}{4}$. Shaded area:

$$\frac{50}{360}\pi\left(r^2 - \frac{r^2}{16}\right) = \frac{243}{16}\pi$$

2 **Solve for r**

$$\frac{75}{576}r^2 = \frac{243}{16} \Rightarrow r^2 = 116.64 \Rightarrow r = 10.8,\ OA = 2.7$$

3 **Arcs**

$$\text{arc } PQ = \frac{50}{360} \times 2\pi(10.8) = 3\pi, \qquad \text{arc } AB = \frac{50}{360} \times 2\pi(2.7) = \tfrac{3}{4}\pi$$

4 **Straight edges and total**

$AP = BQ = 10.8 - 2.7 = 8.1$:

$$3\pi + \tfrac{3}{4}\pi + 2(8.1) = \tfrac{15}{4}\pi + 16.2$$

FINAL ANSWER

$\frac{15}{4}\pi + 16.2$ cm

TOPIC

Volume & Surface Area

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

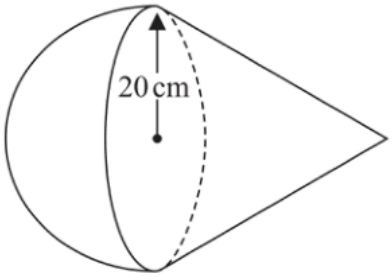


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 Specimen

1 **Calculate surface area**

The curved surface area of a cone is

$$\pi rl$$

2 **Calculate area**

Here $r = 20$ and $\pi rl = 580\pi$, so

$$20l = 580$$

$$l = 29$$

3 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 = 29^2 - 20^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

4 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (20)^2 (21) = 2800\pi$$

---

5 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi (20)^3 = \frac{16000}{3}\pi$$

---

6 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

7 **Calculate area**

Since the volume is $k\pi$,

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$k = \frac{24400}{3}$.

4MA1/2HR JANUARY 2020

Question 13

3 marks | Volume & Surface Area

13

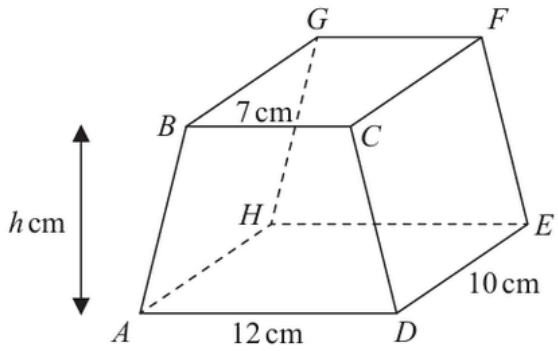


Diagram **NOT** accurately drawn

The diagram shows a prism $ABCDEFGH$ in which $ABCD$ is a trapezium with BC parallel to AD and $CDEF$ is a rectangle.

$BC = 7\,\text{cm}$ $AD = 12\,\text{cm}$ $DE = 10\,\text{cm}$

The height of trapezium $ABCD$ is $h\,\text{cm}$
The volume of the prism is $608\,\text{cm}^3$

Work out the value of h.

$h =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2020

1 Area of the trapezium cross

Area of the trapezium cross-section:

$$\frac{1}{2}(7+12)h = 9.5h$$

2 Volume of the prism

Volume of the prism:

$$9.5h \times 10 = 608$$

$$95h = 608$$

$$h = 6.4$$

FINAL ANSWER

$h = 6.4\ \text{cm}$.

Question 26

6 marks | Volume & Surface Area

26 Here is a sector, AOB, of a circle with centre O and angle $AOB = x°$

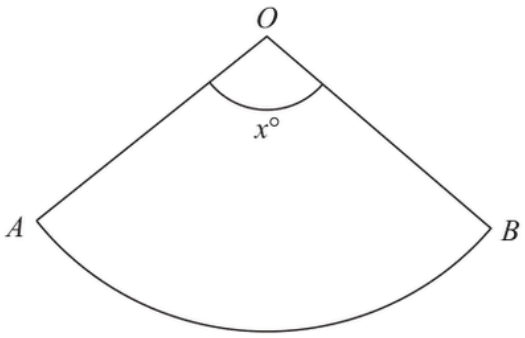


Diagram **NOT** accurately drawn

The sector can form the curved surface of a cone by joining OA to OB.

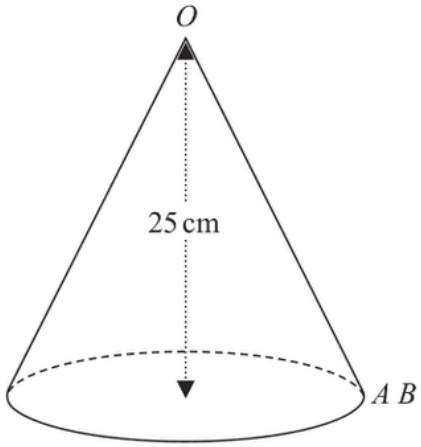


Diagram **NOT** accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 cm^3

Work out the value of x.
Give your answer correct to the nearest whole number.

$x =$ .....

(Total for Question 26 is 6 marks)

1 **Let the radius of the**

Let the radius of the base of the cone be r, and let the slant height be l.

2 **The volume of the cone**

The volume of the cone is

$$\frac{1}{3}\pi r^2 h = 1600$$

3 **Calculate area**

The height is 25 cm, so

$$\frac{1}{3}\pi r^2 (25) = 1600$$

$$r^2 = \frac{4800}{25\pi}$$

$$r = 7.8176\ldots$$

4 **The slant height is found**

The slant height is found using Pythagoras:

$$l^2 = r^2 + 25^2$$

$$l = 26.1938\ldots$$

5 **Calculate area**

When the sector is joined to form the cone, the arc length of the sector becomes the circumference of the cone base.

$$\frac{x}{360} \times 2\pi l = 2\pi r$$

6 **Cancel**

Cancel 2π:

$$\frac{x}{360}l = r$$

$$x = \frac{360r}{l}$$

$$x = \frac{360(7.8176\ldots)}{26.1938\ldots} = 107.4433\ldots$$

✓ **FINAL ANSWER**

$\boxed{107°}$

4MA1/1H JANUARY 2021

Question 3

4 marks | Volume & Surface Area

3 The diagram shows a container for water in the shape of a prism.

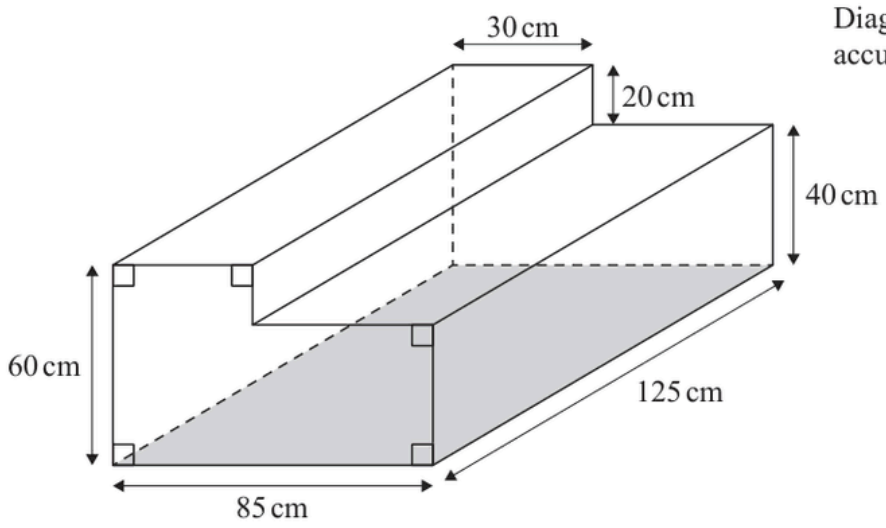


Diagram **NOT** accurately drawn

The rectangular base of the prism, shown shaded in the diagram, is horizontal.
The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 10 30 and at 12 00 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2021

1 The cross section is an

The cross-section is an L-shape.

2 Area of the lower rectangle

Area of the lower rectangle:

$$85 \times 40 = 3400$$

3 **Area of the upper rectangle**

Area of the upper rectangle:

$$30 \times 20 = 600$$

4 **Calculate area**

So the cross-sectional area is

$$3400 + 600 = 4000 \text{ cm}^2$$

5 **The full volume is**

The full volume is

$$4000 \times 125 = 500000 \text{ cm}^3$$

6 **Calculate area**

In the first 1.5 hours, the water level drops by 20 cm. This is the top rectangular part:

$$30 \times 20 \times 125 = 75000 \text{ cm}^3$$

7 **Calculate area**

So the pumping rate is

$$\frac{75000}{1.5} = 50000 \text{ cm}^3/\text{hour}$$

8 **Time to empty the whole**

Time to empty the whole container:

$$\frac{500000}{50000} = 10 \text{ hours}$$

9 **Calculate area**

Starting at $10{:}30$, ten hours later is

$$20{:}30$$

✓ **FINAL ANSWER**

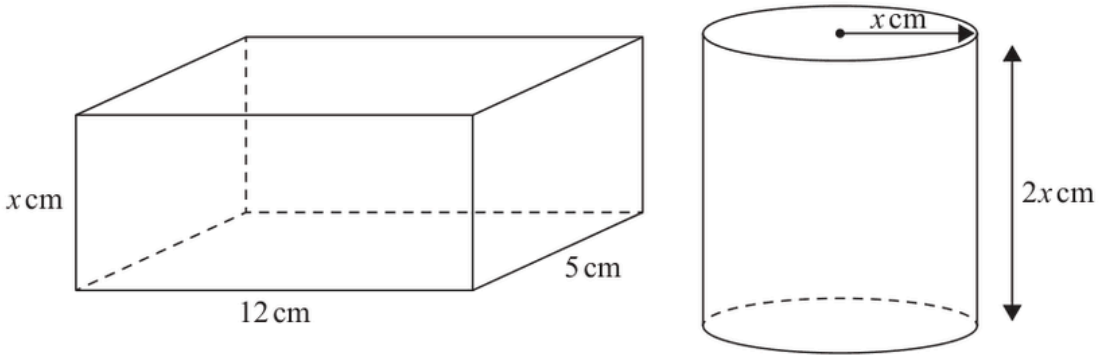
$20{:}30$

Question 5

4 marks | Volume & Surface Area

5 The diagram shows a cuboid and a cylinder.

Diagram **NOT** accurately drawn



The dimensions of the cuboid are x cm by 12 cm by 5 cm.
The volume of the cuboid is 270 cm^3

The radius of the cylinder is x cm.
The height of the cylinder is $2x$ cm.

(a) Work out the volume of the cylinder.
Give your answer correct to the nearest whole number.

..... cm^3

(3)

(b) Change 1 m^3 to cm^3

..... cm^3

(1)

(Total for Question 5 is 4 marks)

1 **Calculate area**

For the cuboid,

$$12 \times 5 \times x = 270$$

$$60x = 270$$

$$x = 4.5$$

2 **Calculate area**

The cylinder has radius $4.5\ \text{cm}$ and height $9\ \text{cm}$.

$$V = \pi r^2 h = \pi(4.5)^2(9) = 572.555\ldots$$

3 **Calculate area**

Also,

$$1\ \text{m}^3 = 100^3 = 1000000\ \text{cm}^3$$

FINAL ANSWER

$573\ \text{cm}^3$, and $1\ \text{m}^3 = 1000000\ \text{cm}^3$.

Question 17

7 marks | Volume & Surface Area

17 The diagram shows a solid prism *ABCDEFGH*.

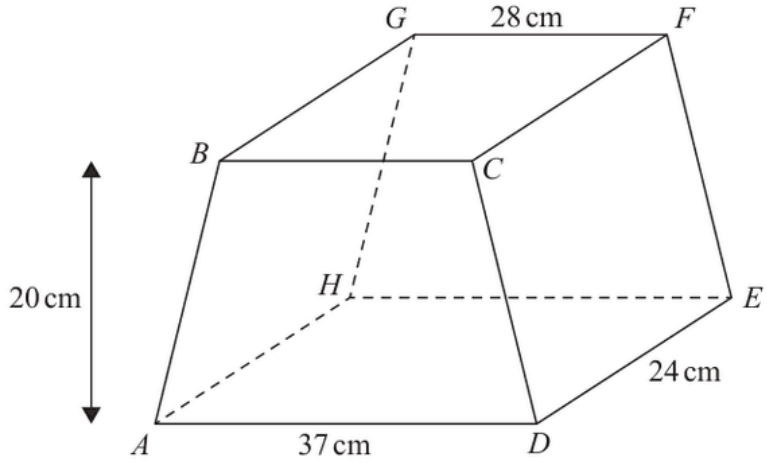


Diagram **NOT** accurately drawn

The trapezium *ABCD*, in which *AD* is parallel to *BC*, is a cross section of the prism.
The base *ADEH* of the prism is a horizontal plane.
ADEH and *BCFG* are rectangles.
The midpoint of *BC* is vertically above the midpoint of *AD* so that $BA = CD$.

$AD = 37$ cm $GF = 28$ cm $DE = 24$ cm

The perpendicular distance between edges *AD* and *BC* is 20 cm.

(a) Work out the total surface area of the prism.

..... cm^2

(4)

(b) Calculate the size of the angle between *AF* and the plane *ADEH*.
Give your answer correct to one decimal place.

..... °

(3)

(Total for Question 17 is 7 marks)

1 **The cross section is a**

The cross-section is a trapezium with parallel sides 37 cm and 28 cm, and perpendicular height 20 cm.

$$\text{Area of cross-section} = \frac{1}{2}(37+28)(20) = 650$$

2 **Calculate area**

Since the trapezium is isosceles, the horizontal difference on each side is

$$\frac{37-28}{2} = 4.5$$

3 **Calculate area**

So

$$AB = CD = \sqrt{20^2 + 4.5^2} = 20.5$$

4 **The perimeter of the cross**

The perimeter of the cross-section is

$$37 + 28 + 20.5 + 20.5 = 106$$

5 **The length of the prism**

The length of the prism is 24 cm, so the total surface area is

$$2(650) + 106(24) = 3844$$

6 **Use trigonometry**

For the angle between AF and the horizontal plane, the horizontal projection has length

$$\sqrt{24^2 + 4.5^2}$$

7 **Use trigonometry**

The vertical height is 20, so

$$\tan\theta = \frac{20}{\sqrt{24^2 + 4.5^2}}$$

$$\theta = 39.3°$$

FINAL ANSWER

$3844\ \text{cm}^2$, and $39.3°$.

4MA1/2H MAY 2021

Question 10

5 marks | Volume & Surface Area

10 Here is a triangular prism.

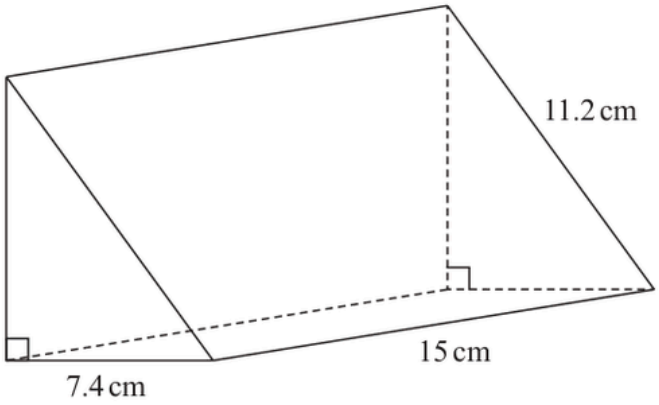


Diagram **NOT** accurately drawn

Work out the volume of the prism.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H May 2021

1 Find the height of the

Find the height of the triangular cross-section:

$$h^2 = 11.2^2 - 7.4^2$$

$$h = 8.407\ldots$$

2 Area of the triangular cross

Area of the triangular cross-section:

$$\frac{1}{2}(7.4)(8.407\ldots) = 31.106\ldots$$

3 Volume of the prism

Volume of the prism:

$$31.106\ldots \times 15 = 466.596\ldots$$

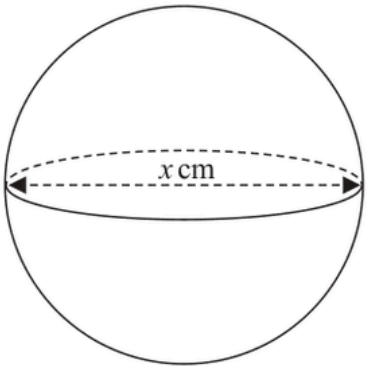
FINAL ANSWER

$467\ \text{cm}^3$.

Question 22

6 marks | Volume & Surface Area

22 The diagram shows a sphere of diameter x cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.



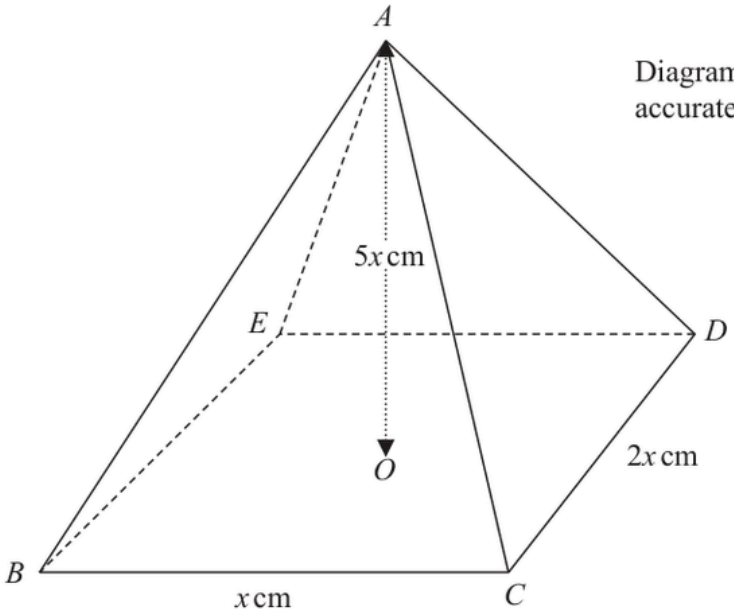


Diagram **NOT** accurately drawn

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is 288π cm^3

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

1 **Use trigonometry**

The sphere has diameter x, so its radius is

$$\frac{x}{2}$$

2 **The volume of the sphere**

The volume of the sphere is $288\pi\ \text{cm}^3$:

$$\frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 288\pi$$

$$\frac{\pi x^3}{6} = 288\pi$$

$$x^3 = 1728$$

$$x = 12$$

3 **The rectangular base of the**

The rectangular base of the pyramid has dimensions x and $2x$, so its area is

$$x(2x) = 2x^2$$

4 **Use trigonometry**

For the two faces with base x, the slant height is

$$\sqrt{(5x)^2 + x^2} = x\sqrt{26}$$

5 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}x \cdot x\sqrt{26}\right) = x^2\sqrt{26}$$

6 **Use trigonometry**

For the two faces with base $2x$, the slant height is

$$\sqrt{(5x)^2 + \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{101}}{2}$$

7 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}(2x)\cdot\frac{x\sqrt{101}}{2}\right)=x^2\sqrt{101}$$

8 **Total surface area**

Total surface area:

$$2x^2+x^2\sqrt{26}+x^2\sqrt{101}$$

9 **Substitute**

Substitute $x=12$:

$$144(2+\sqrt{26}+\sqrt{101})=2469.440\ldots$$

10 **Correct to the nearest**

Correct to the nearest cm^2:

$$2469$$

✓ **FINAL ANSWER**

$$\boxed{2469\ \mathrm{cm}^2}$$

4MA1/1HR JANUARY 2022

Question 10

6 marks | Volume & Surface Area

10 Jonty has a storage container in the shape of a cuboid, as shown in the diagram.

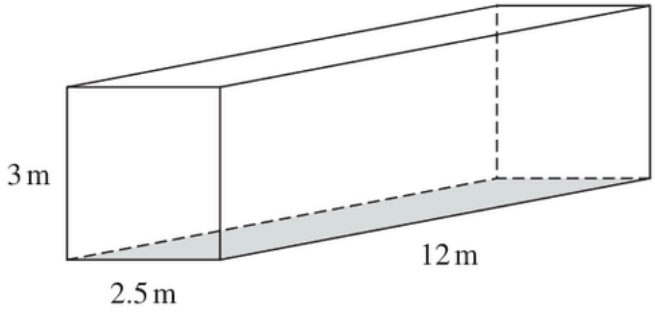


Diagram **NOT** accurately drawn

Jonty is going to paint the outside of his storage container, apart from the base which is shown shaded in the diagram.
He needs enough paint to cover the four sides and the top.

Each tin of paint covers an area of $15\,m^2$

The cost of each tin of paint recently increased by 10%
After the increase, the cost of each tin of paint is £26.95

Jonty says

"**Before** the increase, I could have bought enough paint for less than £200"

Show that Jonty is correct.
Show your working clearly.

(Total for Question 10 is 6 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2022

1 Jonty paints the four sides

Jonty paints the four sides and the top, but not the base.

2 Top

Top:

$$12 \times 2.5 = 30$$

3 **Two long sides**

Two long sides:

$$2(12 \times 3) = 72$$

4 **Two short sides**

Two short sides:

$$2(2.5 \times 3) = 15$$

5 **Total area**

Total area:

$$30 + 72 + 15 = 117\ \text{m}^2$$

6 **Calculate area**

Each tin covers $15\ \text{m}^2$, so the number of tins needed is

$$\frac{117}{15} = 7.8$$

7 **He must buy tins**

He must buy 8 tins.

8 **Calculate area**

After the 10% increase, each tin costs $\pounds 26.95$. Before the increase:

$$\frac{26.95}{1.10} = 24.50$$

9 **Calculate area**

Before the increase, 8 tins would cost

$$8 \times 24.50 = 196$$

10 **Calculate area**

Since $\pounds 196 < \pounds 200$, Jonty is correct.

✓ **FINAL ANSWER**

Jonty is correct; before the increase the paint would have cost $\pounds 196$.

4MA1/1H JUNE 2022

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

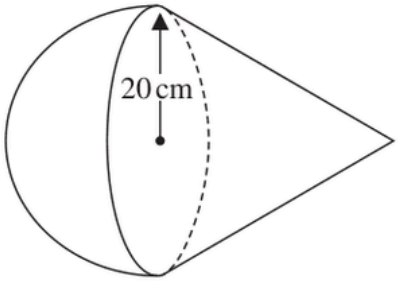


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi \text{ cm}^2$

The volume of the solid is $k\pi \text{ cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2022

1 The cone has radius cm

The cone has radius 20 cm.

2 Calculate surface area

The curved surface area of the cone is

$$\pi rl$$

3 Calculate value

So

$$\pi(20)l = 580\pi$$

$$l = 29$$

4 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 + 20^2 = 29^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

5 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(20)^2(21) = 2800\pi$$

---

6 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(20)^3 = \frac{16000}{3}\pi$$

---

7 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

8 **Calculate volume**

The volume is $k\pi$, so

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{24400}{3}}$$

4MA1/2HR JUNE 2023

Question 9

6 marks | Volume & Surface Area

9 The diagram shows a rectangular sheet of metal $ABCD$

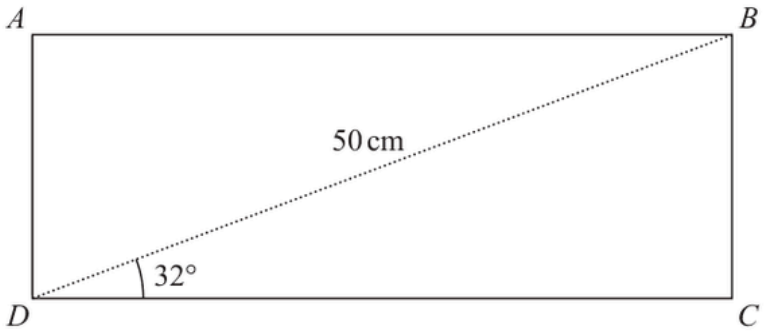


Diagram **NOT** accurately drawn

$BD = 50\,\text{cm}$ and angle $BDC = 32°$

Nasser joins side AD to side BC to form a cylinder.

BC is the height of the cylinder.
DC is the circumference of the cross section of the cylinder.

Work out the volume, in cm^3, of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 9 is 6 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2023

1 **Use trigonometry**

In triangle DBC,

$$DC = 50\cos 32° = 42.402\ldots, \qquad BC = 50\sin 32° = 26.496\ldots$$

2 **The circumference of the cylinder**

The circumference of the cylinder is DC, so

$$2\pi r = DC$$

$$r = \frac{42.402\ldots}{2\pi}$$

---

3 **Volume**

Volume:

$$V = \pi r^2 h = \pi\left(\frac{42.402\ldots}{2\pi}\right)^2 (26.496\ldots) = 3790.974\ldots$$

✓ **FINAL ANSWER**

$3.79 \times 10^3\ \mathrm{cm}^3$ to 3 significant figures.

4MA1/2HR JUNE 2023

Question 24

5 marks | Volume & Surface Area

24 The surface area of sphere **A** is nine times the surface area of sphere **B**
The difference between the volume of sphere **A** and the volume of sphere **B** is 117π cm^3

Find the radius of the smaller sphere.
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2023

1 **Let the radius of the**

Let the radius of the smaller sphere be r.

2 **Let the radius of the**

Let the radius of the larger sphere be R.

3 **Surface area of a sphere**

Surface area of a sphere is

$$4\pi r^2$$

4 **The surface area of sphere**

The surface area of sphere A is nine times the surface area of sphere B, so

$$4\pi R^2 = 9(4\pi r^2)$$

$$R^2 = 9r^2$$

$$R = 3r$$

5 The difference between the volumes

The difference between the volumes is 117π:

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

$$\frac{4}{3}\pi(27r^3 - r^3) = 117\pi$$

$$\frac{104}{3}\pi r^3 = 117\pi$$

6 Cancel

Cancel π:

$$\frac{104}{3}r^3 = 117$$

$$r^3 = \frac{117 \times 3}{104}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

FINAL ANSWER

$\boxed{1.5\text{ cm}}$

4MA1/1H NOVEMBER 2023

Question 21

5 marks | Volume & Surface Area

21 Here is a triangular prism $ABCDEF$

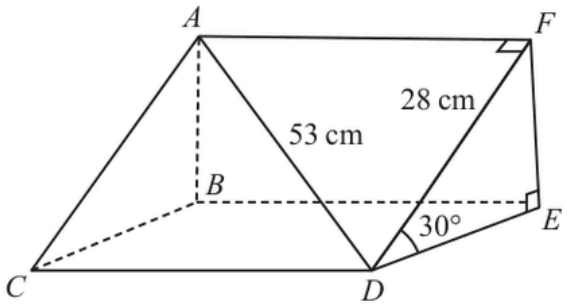


Diagram **NOT** accurately drawn

$AD = 53$ cm
$DF = 28$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2023

1 The triangular cross section is

The triangular cross-section is triangle DEF.

2 It is right angled at

It is right-angled at E, with

$$DF = 28$$

3 Use trigonometry

and

$$\angle FDE = 30°$$

4 Use trigonometry

So

$$FE = 28 \sin 30° = 14$$

5 **Use trigonometry**

and

$$DE = 28\cos 30^\circ = 14\sqrt{3}$$

6 **Area of triangle**

Area of triangle DEF:

$$\frac{1}{2}(14)(14\sqrt{3}) = 98\sqrt{3}$$

7 **The length of the prism**

The length of the prism is

$$AD = 53$$

8 **Volume of the prism**

Volume of the prism:

$$98\sqrt{3} \times 53$$

$$= 5194\sqrt{3}$$

$$= 8996.2719\ldots$$

✓ **FINAL ANSWER**

$\boxed{8996\ \text{cm}^3}$

4MA1/1H MAY 2024

Question 23

5 marks | Volume & Surface Area

23 A solid shape is made by removing a hemisphere, shown shaded, from a cone as shown in the diagram.

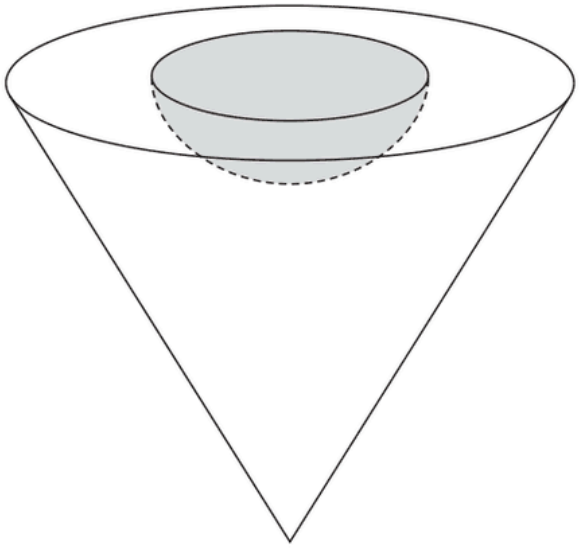


Diagram **NOT** accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the solid shape is 6948π cm^3

Work out the **total** surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2024

1 **Define cone dimensions**

The cone has radius $5x$ and height $6x$.

2 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi(5x)^2(6x)$$

$$= 50\pi x^3$$

3 **The removed hemisphere has radius**

The removed hemisphere has radius $2x$.

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi(2x)^3$$

$$= \frac{16}{3}\pi x^3$$

5 **The remaining solid has volume**

The remaining solid has volume 6948π, so

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\frac{134}{3}\pi x^3 = 6948\pi$$

6 **Cancel**

Cancel π:

$$\frac{134}{3}x^3 = 6948$$

$$x^3 = \frac{10422}{67}$$

$$x = 5.3781\ldots$$

7 **The solid hemisphere removed has**

The solid hemisphere removed has radius $2x$.

8 **Total surface area of a**

Total surface area of a solid hemisphere is

$$3\pi r^2$$

9 **Calculate value**

So

$$3\pi(2x)^2 = 12\pi x^2$$

$$12\pi(5.3781\ldots)^2 = 1090.3903\ldots$$

✓ **FINAL ANSWER**

$\boxed{1090\ \text{cm}^2}$

4MA1/1H NOVEMBER 2024

Question 5

4 marks | Volume & Surface Area

5 The diagram shows a cylinder.

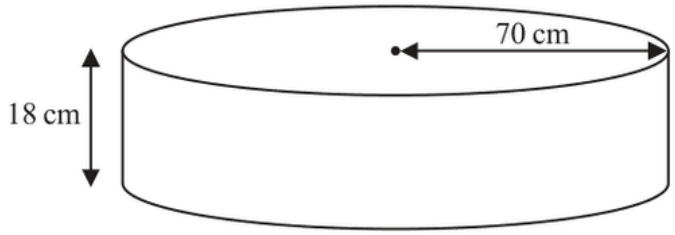


Diagram **NOT** accurately drawn

The radius of the cylinder is 70 cm
The height of the cylinder is 18 cm

Work out the volume of the cylinder.
Give your answer in litres correct to the nearest litre.

..... litres

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2024

1 Calculate area

$$V = \pi r^2 h = \pi(70)^2(18)$$

$$V = 277088.472\ldots \text{ cm}^3$$

2 Calculate area

Since $1000 \text{ cm}^3 = 1$ litre,

$$277088.472\ldots \text{ cm}^3 = 277.088\ldots \text{ litres}$$

FINAL ANSWER

277 litres.

4MA1/2H NOVEMBER 2024

Question 6

3 marks | Volume & Surface Area

6 The diagram shows a solid triangular prism.

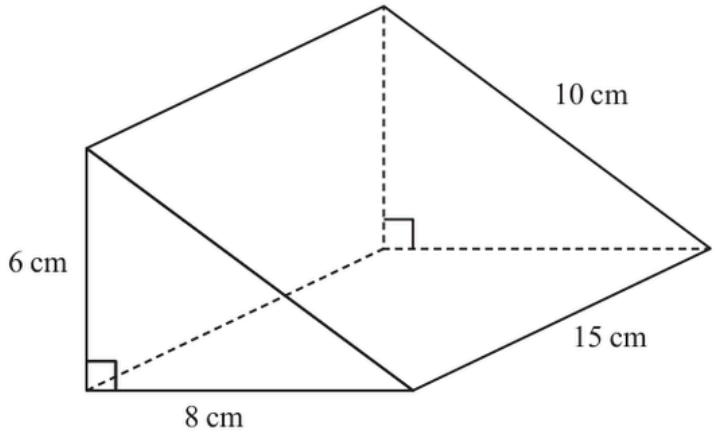


Diagram **NOT** accurately drawn

Work out the **total** surface area of the triangular prism.

..... cm^2

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2024

1 The triangular end is a

The triangular end is a right triangle with sides 6, 8, and 10.

2 Area of the two triangular

Area of the two triangular ends:

$$2\left(\frac{1}{2} \times 6 \times 8\right) = 48$$

3 Area of the three rectangles

Area of the three rectangles:

$$15(6 + 8 + 10) = 360$$

4 Total surface area

Total surface area:

$$48 + 360 = 408$$

FINAL ANSWER

$408\ \text{cm}^2$.

4MA1/1H NOVEMBER 2024

Question 22

3 marks | Volume & Surface Area

22 The diagram shows a bowl in the shape of a hemisphere.
The bowl is made from metal.

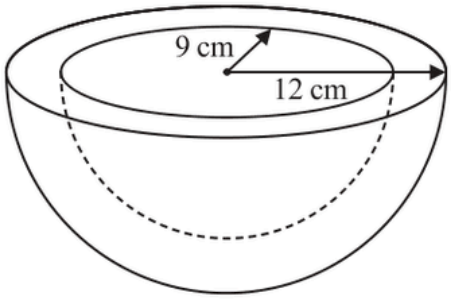


Diagram **NOT** accurately drawn

The outer radius of the bowl is 12 cm
The inner radius of the bowl is 9 cm

The thickness of the bowl is uniform.

Work out the volume of the metal.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2024

1 **The bowl is a hemispherical**

The bowl is a hemispherical shell.

2 **The volume of the metal**

The volume of the metal is:

$$\text{outer hemisphere volume} - \text{inner hemisphere volume}$$

3 **Volume of a hemisphere**

Volume of a hemisphere:

$$\frac{2}{3}\pi r^3$$

4 **Calculate volume**

So

$$\text{metal volume} = \frac{2}{3}\pi(12)^3 - \frac{2}{3}\pi(9)^3$$

$$= \frac{2}{3}\pi(12^3 - 9^3)$$

$$= \frac{2}{3}\pi(1728 - 729)$$

$$= 666\pi$$

$$= 2092.3007\ldots$$

✓ **FINAL ANSWER**

$\boxed{2092\ \text{cm}^3}$

4WM2/01 MAY 2025

Question 2

3 marks | Volume & Surface Area

2 The diagram shows a solid prism with a cross section in the shape of a trapezium.

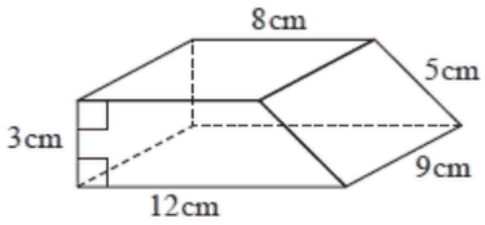


Diagram **NOT** accurately drawn

Work out the total surface area of the prism.

..... cm^2

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM2/01 May 2025

1 Area of one trapezium

Area of one trapezium:

$$\frac{1}{2}(8+12)\times 3 = 30$$

2 Perimeter of the trapezium

Perimeter of the trapezium:

$$8+12+3+5 = 28$$

3 Rectangular faces

Rectangular faces:

$$28 \times 9 = 252$$

4 Total surface area

Total surface area:

$$252 + 2(30) = 312$$

FINAL ANSWER

$312\ \text{cm}^2$.

4MA1/1H JUNE 2025

Question 7

3 marks | Volume & Surface Area

7 Here is a solid cylinder.

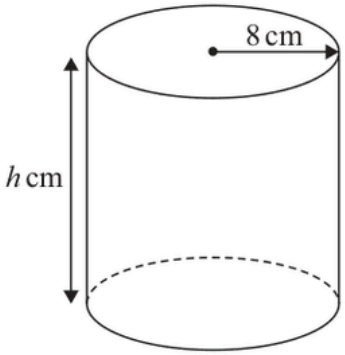


Diagram **NOT** accurately drawn

The radius of the cylinder is 8 cm
The height of the cylinder is h cm

The volume of the cylinder is $3892\,\text{cm}^3$

Work out the value of h
Give your answer correct to one decimal place.

$h =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2025

1 Calculate area

For a cylinder,

$$V = \pi r^2 h$$

$$3892 = \pi(8)^2 h$$

$$h = \frac{3892}{64\pi} = 19.357\ldots$$

FINAL ANSWER

$19.4\ \text{cm}$.

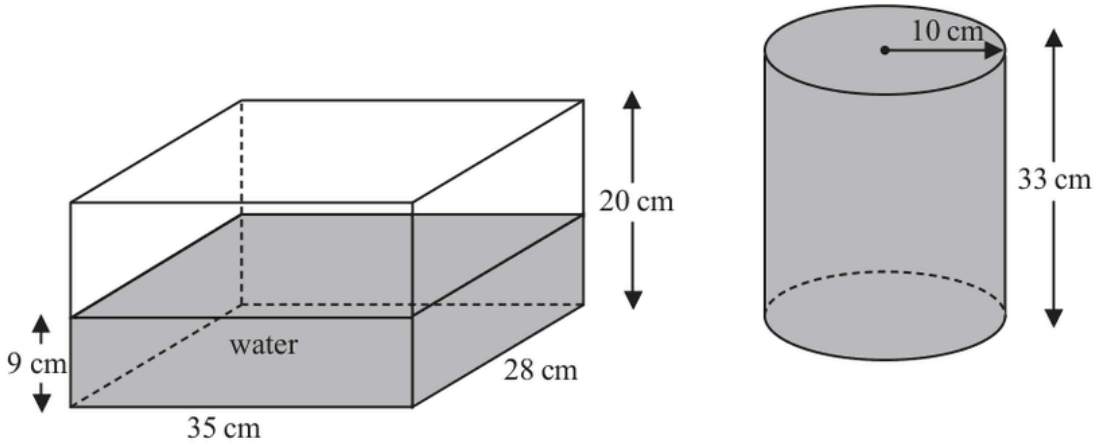
4MA1/1HR JUNE 2025

Question 10

3 marks | Volume & Surface Area

10 The diagram shows two water containers.
One is a cuboid and one is a cylinder.

Diagram **NOT** accurately drawn



The cuboid measures 35 cm by 28 cm by 20 cm
The surface of the water in the cuboid is 9 cm above the base of the cuboid.

The cylinder has a radius of 10 cm and a height of 33 cm
The cylinder is completely full of water.

Izzy is going to pour all the water from the cylinder into the cuboid.

Show that the cuboid will not be completely full of water.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2025

1 Capacity of the cuboid

Capacity of the cuboid:

$$35 \times 28 \times 20 = 19600 \text{ cm}^3$$

2 Water already in the cuboid

Water already in the cuboid:

$$35 \times 28 \times 9 = 8820 \text{ cm}^3$$

3 **Volume of the full cylinder**

Volume of the full cylinder:

$$\pi(10)^2(33) = 10367.255\ldots \text{ cm}^3$$

4 **Total water after pouring**

Total water after pouring:

$$8820 + 10367.255\ldots = 19187.255\ldots$$

5 **Calculate area**

Since

$$19187.255\ldots < 19600$$

6 **the cuboid will not be**

the cuboid will not be completely full.

✓ **FINAL ANSWER**

The cuboid will not be completely full.

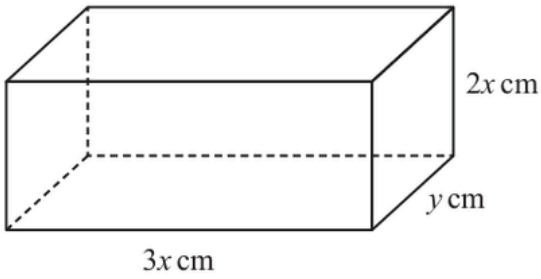
4MA1/2H JUNE 2025

Question 21

3 marks | Volume & Surface Area

21 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is $1014\,\text{cm}^3$
The total surface area of the cuboid is $A\,\text{cm}^2$

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2025

1 **The cuboid has dimensions**

The cuboid has dimensions

$$3x, \quad 2x, \quad y$$

2 **Calculate volume**

Its volume is $1014\ \text{cm}^3$, so

$$(3x)(2x)(y) = 1014$$

$$6x^2y = 1014$$

$$y = \frac{1014}{6x^2}$$

$$y = \frac{169}{x^2}$$

3 **The total surface area is**

The total surface area is

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$A = 2(6x^2 + 5xy)$$

$$A = 12x^2 + 10xy$$

4 **Substitute**

Substitute $y = \frac{169}{x^2}$:

$$A = 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$A = 12x^2 + \frac{1690}{x}$$

✓ **FINAL ANSWER**

$$\boxed{A = 12x^2 + \frac{1690}{x}}$$

4WM2/01R MAY 2025

Question 21

5 marks | Volume & Surface Area

21 The diagram shows a solid cuboid.

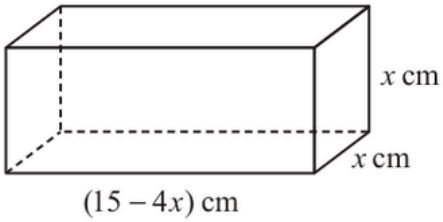


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM2/01R May 2025

1 Required

Cuboid has a square cross-section of side *x* cm and length (15−4*x*) cm. Find the maximum value of the volume *V*.

2 Write V as a function

Write *V* as a function of *x*:

V = *x* · *x* · (15−4*x*) = *x*²(15−4*x*) = 15*x*² − 4*x*³

3 Differentiate

Differentiate:

$$fracdVdx = 30x - 12x^2$$

4 Set (x 0 gives

Set $dfracdVdx = 0$:

$$30x - 12x^2 = 0implies6x(5 - 2x) = 0impliesx = 0; textor; x = tfrac52 = 2.5$$

(*x* = 0 gives zero volume, so the meaningful solution is *x* = 2.5.)

5 **Second derivative test to confirm**

Second derivative test to confirm maximum:

$$fracd^2Vdx^2 = 30 - 24x; Bigg|_x = 2.5 = 30 - 60 = -30\<0quadcheckmarktext(maximum)$$

6 **Calculate V max V max**

Calculate *V*_{max}:

*V*_{max} = (2.5)² × (15−4×2.5) = 6.25 × 5 = 31.25 cm³

7 **Note**

Calculus optimisation: always verify the nature of the stationary point using $dfracd^2Vdx^2$. Also check *x* \> 0 and 15 − 4*x* \> 0 (i.e. *x* \< 3.75) to ensure a valid cuboid.

✓ **FINAL ANSWER**

Maximum volume *V* = 31.25 cm³

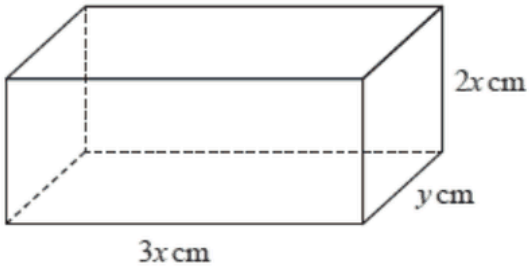
4WM2/01 MAY 2025

Question 23

3 marks | Volume & Surface Area

23 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is 1014 cm^3
The total surface area of the cuboid is A cm^2

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 May 2025

1 **Evaluate fraction**

$$3x \times 2x \times y = 1014$$

$$6x^2y = 1014$$

$$y = \frac{169}{x^2}$$

2 Surface area

Surface area:

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$= 12x^2 + 10xy$$

$$= 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$= 12x^2 + \frac{1690}{x}$$

FINAL ANSWER

$A = 12x^2 + \dfrac{1690}{x}$.

4WM2/01R MAY 2025

Question 25

5 marks | Volume & Surface Area

25 The diagram shows a solid hemisphere, H

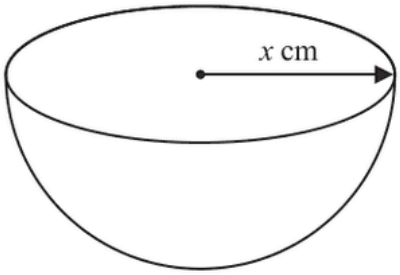


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

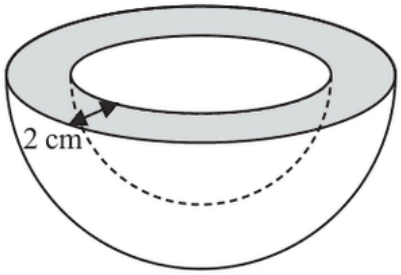


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM2/01R May 2025

1 **Required**

Hemisphere H has radius *x* cm and volume 6174*π* cm³. A bowl is made by removing an inner hemisphere, leaving 2 cm thickness. Find total surface area in terms of *π*.

2 **Calculate volume**

Find outer radius *x*. Volume of hemisphere $= \frac{2}{3}\pi r^3$:

$$\frac{2}{3}\pi x^3 = 6174\pi \implies x^3 = \frac{6174}{3} \times 2 = 9261 \implies x = \sqrt[3]{9261} = 21\,\text{cm}$$

3 **Find inner radius r inner**

Find inner radius:

*r*_{inner} = 21 − 2 = 19 cm

4 **Add 1**

Three surfaces to add:

1. Outer curved surface (half-sphere, radius 21):

*A*₁ = 2*π*(21)² = 2*π* × 441 = 882*π*

2. Inner curved surface (half-sphere, radius 19):

*A*₂ = 2*π*(19)² = 2*π* × 361 = 722*π*

3. Flat annular ring at the top rim:

*A*₃ = *π*(21)² − *π*(19)² = *π*(441−361) = 80*π*

5 **Total surface area A total**

Total surface area:

*A*_{total} = 882*π* + 722*π* + 80*π* = 1684*π*

6 **Note**

Three separate surfaces! Missing the flat annular ring is the most common error on this type of question.

✓ **FINAL ANSWER**

Total surface area = 1684*π* cm²

4MA1/2HR JUNE 2025

Question 26

5 marks | Volume & Surface Area

26 The diagram shows a solid hemisphere, H

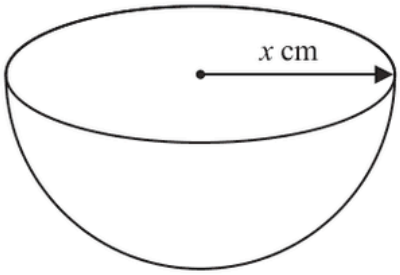


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

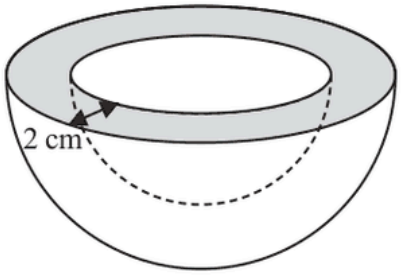


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2025

1 **The volume of a hemisphere**

The volume of a hemisphere is

$$\frac{2}{3}\pi r^3$$

2 **Evaluate fraction**

For the original hemisphere H,

$$\frac{2}{3}\pi x^3 = 6174\pi$$

3 **Cancel**

Cancel π:

$$\frac{2}{3}x^3 = 6174$$

$$x^3 = 9261$$

$$x = 21$$

4 **The outer radius of the**

The outer radius of the bowl is 21 cm.

5 **Calculate value**

The thickness is 2 cm, so the inner radius is

$$21 - 2 = 19$$

6 **Total surface area of the**

Total surface area of the bowl is:

7 **outer curved hemisphere inner curved**

- outer curved hemisphere
- inner curved hemisphere
- flat annular rim

8 **Outer curved surface area**

Outer curved surface area:

$$2\pi(21)^2 = 882\pi$$

9 **Inner curved surface area**

Inner curved surface area:

$$2\pi(19)^2 = 722\pi$$

10 **Flat rim area**

Flat rim area:

$$\pi(21^2 - 19^2) = 80\pi$$

---

11 **Total**

Total:

$$882\pi + 722\pi + 80\pi = 1684\pi$$

✓ **FINAL ANSWER**

$\boxed{1684\pi \text{ cm}^2}$

4WM2/01 NOVEMBER 2025

Question 19

2 marks | Volume & Surface Area

19 A sphere has a diameter of 24 cm

Find the surface area of the sphere.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4WM2/01 November 2025

1 **Evaluate fraction**

$$r = frac242 = 12textcm$$

2 **Calculate area**

Surface Area = 4*π**r*²

= 4 × *π* × 12²

= 4 × *π* × 144

= 576*π* ≈ 1809.557... cm²

Step 3: Round to 3 significant figures. 1809.557 → 1810 (the 1, 8, and 1 are the first three sig figs).

✓ **FINAL ANSWER**

1810 cm²

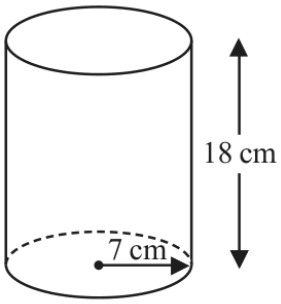
4MA1/2H JUNE 2026

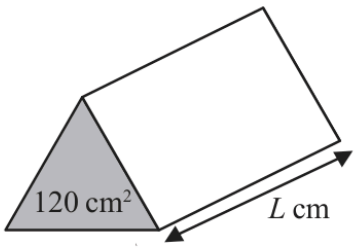
Question 7

3 marks | Volume & Surface Area

7 The diagram shows a cylinder and a prism.

Diagram **NOT** accurately drawn





The radius of the cylinder is 7 cm
The height of the cylinder is 18 cm

The area of the cross section of the prism, shown shaded in the diagram, is $120\ \text{cm}^2$
The length of the prism is L cm

The volume of the cylinder is equal to the volume of the prism.

Work out the value of L
Give your answer correct to 3 significant figures.

$L =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2026

1 Solution | Step 1

$$V_{\text{cylinder}} = \pi r^2 h = \pi(7)^2(18) = 882\pi\ \text{cm}^3.$$

Use the stated radius, not a diameter.

2 Solution | Step 2

$$V_{\text{prism}} = 120L, \qquad 120L = 882\pi \quad \Rightarrow \quad L = \frac{882\pi}{120} = 23.090706\ldots.$$

Prism volume equals cross-sectional area times length; equate the two volumes.

FINAL ANSWER

$L = 23.1\ \text{cm}$ (3 significant figures).

Question 11

4 marks | Volume & Surface Area

11 Here is a solid cylinder.

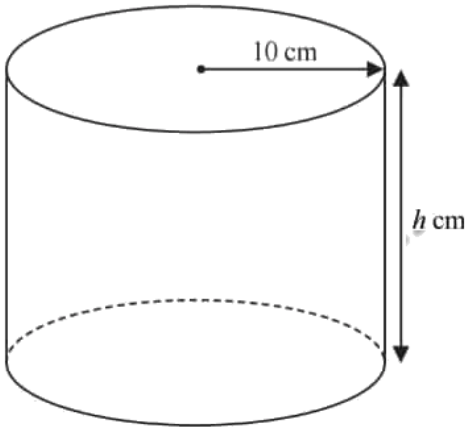


Diagram **NOT** accurately drawn

The radius of the cylinder is 10 cm
The height of the cylinder is h cm

The volume of the cylinder is 5341 cm^3

Work out the **total** surface area of the cylinder.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 11 is 4 marks)

1 Find h from the volume

$$\pi(10)^2 h = 5341 \Rightarrow h = \frac{5341}{100\pi} = 17.00\ldots \text{ cm}$$

2 Curved surface

$$2\pi r h = 2\pi(10)(17.00\ldots) = 1068.2$$

3 Two circular ends

$$2\pi r^2 = 2\pi(10)^2 = 200\pi = 628.31\ldots$$

4 Total

$$1068.2 + 628.31\ldots = 1696.5\ldots$$

Keep h unrounded; using $h = 17$ gives 1696.

FINAL ANSWER

1697 cm^2

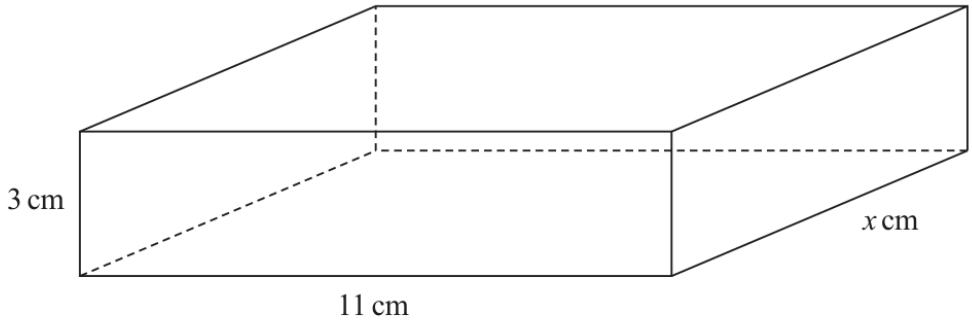
4MA1/1HR JUNE 2026

Question 11

4 marks | Volume & Surface Area

11 The diagram shows a solid cuboid.

Diagram **NOT** accurately drawn



The cuboid measures 3 cm by 11 cm by x cm

The volume of the cuboid is 264 cm^3

Work out the total surface area of the cuboid.

..... cm^2

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$3 \times 11 \times x = 264, \qquad x = \frac{264}{33} = 8 \text{ cm}.$$

The volume is the product of the three perpendicular edge lengths.

2 **Solution | Step 2**

$$S = 2(3 \times 11) + 2(3 \times 8) + 2(11 \times 8).$$

There are two faces of each of the three rectangular sizes.

3 **Solution | Step 3**

$$S = 66 + 48 + 176 = 290 \text{ cm}^2.$$

Add all six face areas.

FINAL ANSWER

$290\ \text{cm}^2$

Question 20

5 marks | Volume & Surface Area

20 The diagram shows a solid shape.

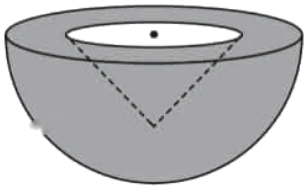


Diagram **NOT** accurately drawn

The solid shape is made by removing a cone from a hemisphere.

The radius of the cone is x cm
The height of the cone is x cm

The radius of the hemisphere is $2x$ cm

The volume of the solid shape is $\frac{3645}{8}\pi$ cm^3

Find the outer curved surface area of the solid shape.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 20 is 5 marks)

1 **Volume of the solid**

$$\tfrac{2}{3}\pi(2x)^3 - \tfrac{1}{3}\pi x^2(x) = \tfrac{16}{3}\pi x^3 - \tfrac{1}{3}\pi x^3 = 5\pi x^3$$

2 **Solve for x**

$$5\pi x^3 = \frac{3645}{8}\pi \Rightarrow x^3 = \frac{729}{8} \Rightarrow x = 4.5$$

3 **Outer curved surface = curved hemisphere**

Radius $2x = 9$:

$$2\pi(9)^2 = 162\pi = 508.9\ldots$$

✓ **FINAL ANSWER**

509 cm^2

TOPIC

Right-Angled Triangles - Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

Question 10

6 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 The diagram shows a quadrilateral $ABCD$

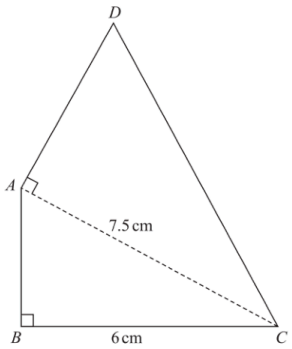


Diagram **NOT** accurately drawn

In the diagram, ABC and DAC are right-angled triangles.

$BC = 6$ cm $\quad AC = 7.5$ cm

The area of quadrilateral $ABCD$ is 31.5 cm^2

(a) Work out the length of AB

..... cm

(2)

(b) Work out the length of AD

..... cm

(4)

(Total for Question 10 is 6 marks)

1 **Use Pythagoras for AB**

In triangle ABC,

$$AB^2 + 6^2 = 7.5^2$$

$$AB^2 = 56.25 - 36 = 20.25$$

$$AB = 4.5$$

2 **Find the area of triangle ABC**

$$\frac{1}{2} \times 6 \times 4.5 = 13.5$$

3 **Find the area of triangle DAC**

$$31.5 - 13.5 = 18$$

4 **Use the triangle area formula**

In triangle DAC, AD and AC are perpendicular.

$$\frac{1}{2} \times AD \times 7.5 = 18$$

$$AD = \frac{36}{7.5} = 4.8$$

✓ **FINAL ANSWER**

$AB = 4.5$ cm, $AD = 4.8$ cm.

4MA1/2HR JANUARY 2020

Question 4

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

4 The diagram shows a rectangle and a diagonal of the rectangle.

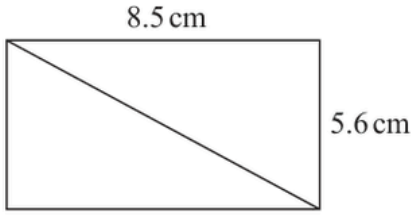


Diagram **NOT** accurately drawn

Work out the length of the diagonal of the rectangle.
Give your answer correct to 1 decimal place.

..... cm

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2020

1 Use trigonometry

$$d^2 = 8.5^2 + 5.6^2$$

$$d = \sqrt{103.61} = 10.178\ldots$$

FINAL ANSWER

10.2 cm.

Question 9

3 marks

Right-Angled Triangles - Pythagoras & Trigonometry

9 Here is a right-angled triangle.

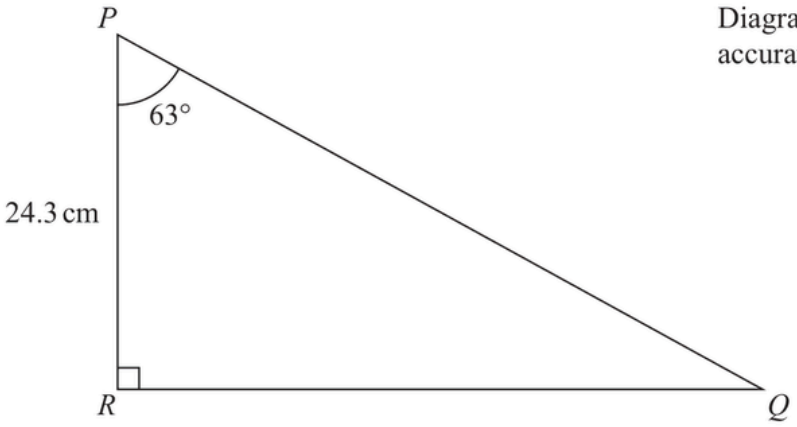


Diagram **NOT** accurately drawn

Calculate the length of PQ.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 9 is 3 marks)

1 Use trigonometry

$$\cos 63^\circ = \frac{24.3}{PQ}$$

$$PQ = \frac{24.3}{\cos 63^\circ} = 53.525\ldots$$

FINAL ANSWER

53.5 cm.

4MA1/1HR JANUARY 2020

Question 15

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

15 The diagram shows two right-angled triangles, DEF and EFG.

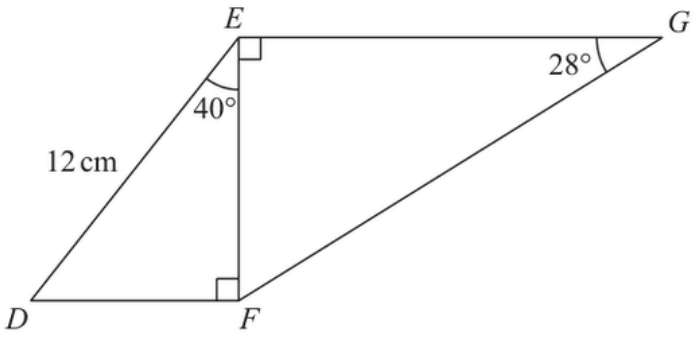


Diagram **NOT** accurately drawn

Work out the length of EG.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2020

1 **Use trigonometry**

In triangle DEF,

$$EF = 12\cos 40^\circ = 9.192\ldots$$

2 **Use trigonometry**

In triangle EFG,

$$\sin 28^\circ = \frac{EF}{EG}$$

$$EG = \frac{9.192\ldots}{\sin 28^\circ} = 19.580\ldots$$

✓ **FINAL ANSWER**

19.6 cm.

4MA1/1H JANUARY 2021

Question 10

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 Here is isosceles triangle ABC.

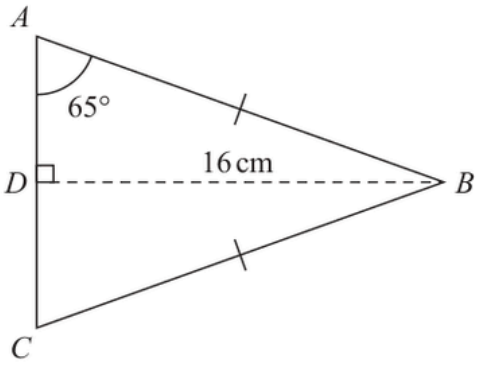


Diagram **NOT** accurately drawn

D is the midpoint of AC and $DB = 16$ cm.

Angle $DAB = 65°$

Work out the perimeter of triangle ABC.
Give your answer correct to one decimal place.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2021

1 Find the gradient

Since ABC is isosceles and D is the midpoint of AC, BD is perpendicular to AC.

2 Use trigonometry

In right triangle ABD,

$$BD = 16, \qquad \angle DAB = 65°$$

$$\sin 65° = \frac{16}{AB}$$

$$AB = \frac{16}{\sin 65°}$$

3 **Use trigonometry**

Also,

$$\tan 65^\circ = \frac{16}{AD}$$

$$AD = \frac{16}{\tan 65^\circ}$$

---

4 **The perimeter is**

The perimeter is

$$AB + BC + AC = 2AB + 2AD$$

$$= 2\left(\frac{16}{\sin 65^\circ}\right) + 2\left(\frac{16}{\tan 65^\circ}\right)$$

$$= 50.2299\ldots$$

✓ **FINAL ANSWER**

$50.2\ \text{cm}$.

4MA1/2HR JANUARY 2021

Question 13

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

13 The diagram shows two hot air balloons.
A is a point on the base of one of the balloons and B is a point on the base of the other balloon.

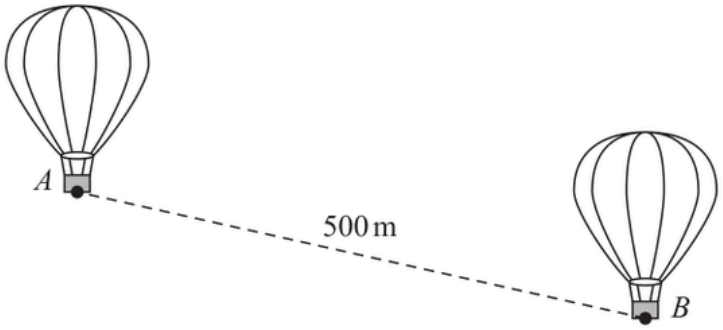


Diagram **NOT** accurately drawn

The distance between A and B is 500 metres.
The angle of depression of B from A is 23°

Calculate the vertical height of A above B.
Give your answer correct to one decimal place.

..... metres

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2021

1 **The vertical height is opposite**

The vertical height is opposite the $23°$ angle, and $AB = 500\ \text{m}$ is the hypotenuse.

$$\sin 23° = \frac{h}{500}$$

$$h = 500 \sin 23° = 195.365\ldots$$

FINAL ANSWER

$195.4\ \text{m}$.

4MA1/2H MAY 2021

Question 7

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

7 Here is a right-angled triangle.

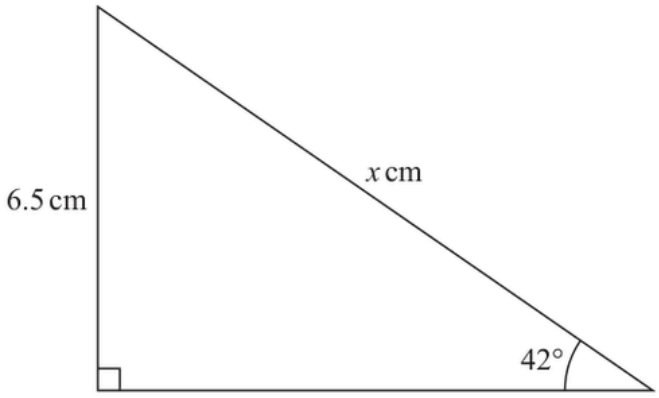


Diagram **NOT** accurately drawn

Work out the value of x.
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H May 2021

1 The side cm is opposite

The side 6.5 cm is opposite the $42°$ angle and x is the hypotenuse.

$$\sin 42° = \frac{6.5}{x}$$

$$x = \frac{6.5}{\sin 42°} = 9.714\ldots$$

✓ **FINAL ANSWER**

$x = 9.7$ cm.

4MA1/1H NOVEMBER 2021

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 Here is a right-angled triangle.

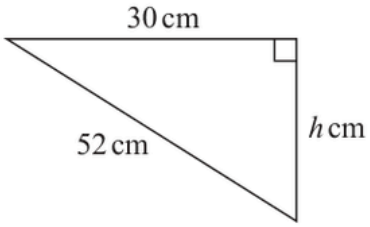


Diagram **NOT** accurately drawn

Calculate the value of h.
Give your answer correct to 3 significant figures.

h = ...

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2021

1 Use trigonometry

$$h^2 + 30^2 = 52^2$$

$$h^2 = 52^2 - 30^2 = 1804$$

$$h = \sqrt{1804} = 42.473\ldots$$

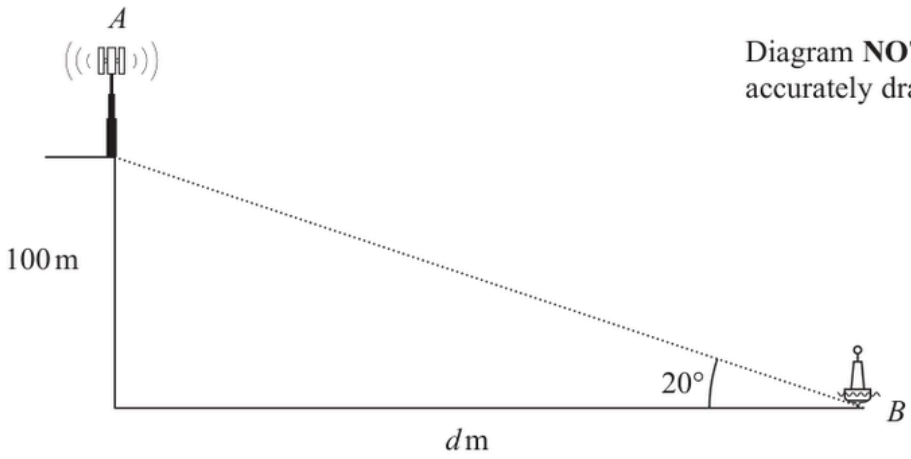
FINAL ANSWER

42.5 cm to 3 significant figures.

Question 12

6 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows a vertical cliff with a vertical radio mast on top of the cliff and a buoy in the sea.



The height of the cliff is 100 metres.
The buoy is at the point B that is d metres from the base of the cliff.
The angle of elevation from B to the top of the cliff is 20°

(a) Calculate the value of d.
Give your answer correct to 3 significant figures.

$d =$ .....

(3)

The point A at the top of the radio mast is vertically above the top of the cliff.
The angle of elevation from B to A is 25°

(b) Calculate the height of the radio mast.
Give your answer correct to 3 significant figures.

..... m

(3)

(Total for Question 12 is 6 marks)

1 (a)

(a)

$$\tan 20° = \frac{100}{d}$$

$$d = \frac{100}{\tan 20°} = 274.747\ldots$$

2 Use trigonometry

So

$$d = 275 \text{ m}$$

3 (b) The angle of elevation

(b) The angle of elevation to the top of the radio mast is $25°$.

$$\tan 25° = \frac{100 + h}{274.747\ldots}$$

$$100 + h = 274.747\ldots \tan 25° = 128.116\ldots$$

$$h = 28.116\ldots$$

FINAL ANSWER

(a) 275 m, (b) 28.1 m.

4MA1/1H JANUARY 2022

Question 3

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The diagram shows triangle ABC

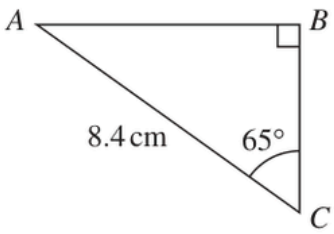


Diagram **NOT** accurately drawn

Work out the length of the side AB
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2022

1 **Use trigonometry**

Side AB is opposite the $65°$ angle, and $AC = 8.4$ cm is the hypotenuse.

$$\sin 65° = \frac{AB}{8.4}$$

$$AB = 8.4 \sin 65° = 7.613\ldots$$

✓ **FINAL ANSWER**

7.61 cm.

4MA1/1HR JANUARY 2022

Question 3

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The shaded shape is made using three identical right-angled triangles and a square.

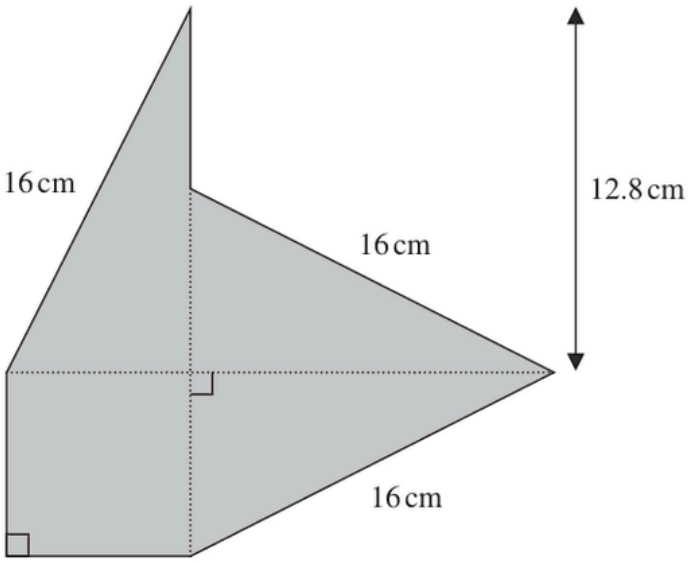


Diagram **NOT** accurately drawn

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2022

1 **Use trigonometry**

Each triangle has hypotenuse $16\ \text{cm}$ and one side $12.8\ \text{cm}$.

2 **Find the other side**

Find the other side:

$$\sqrt{16^2 - 12.8^2} = 9.6\ \text{cm}$$

3 **The exposed short vertical side**

The exposed short vertical side is

$$12.8 - 9.6 = 3.2\ \text{cm}$$

4 **Perimeter**

Perimeter:

$$3(16) + 9.6 + 9.6 + 3.2 = 70.4$$

✓ **FINAL ANSWER**

$70.4\ \text{cm}$.

4MA1/2HR JANUARY 2022

Question 12

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows two vertical phone masts, AB and CD, on horizontal ground.

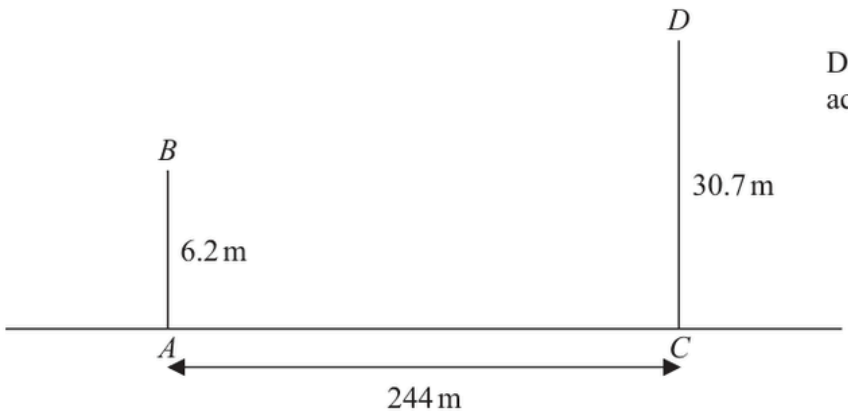


Diagram **NOT** accurately drawn

$AB = 6.2\,\text{m}$ $AC = 244\,\text{m}$ $CD = 30.7\,\text{m}$

Work out the size of the angle of depression of B from D
Give your answer correct to one decimal place.

.....°

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2022

1 **The vertical difference between the**

The vertical difference between the tops of the masts is

$$30.7 - 6.2 = 24.5\ \text{m}$$

2 **The horizontal distance is**

The horizontal distance is $244\ \text{m}$.

$$\tan\theta = \frac{24.5}{244}$$

$$\theta = 5.733\ldots^\circ$$

✓ **FINAL ANSWER**

5.7°.

4MA1/2H JANUARY 2022

Question 14

3 marks

Right-Angled Triangles - Pythagoras & Trigonometry

14 A zip wire is shown as the dashed line AC in the diagram.

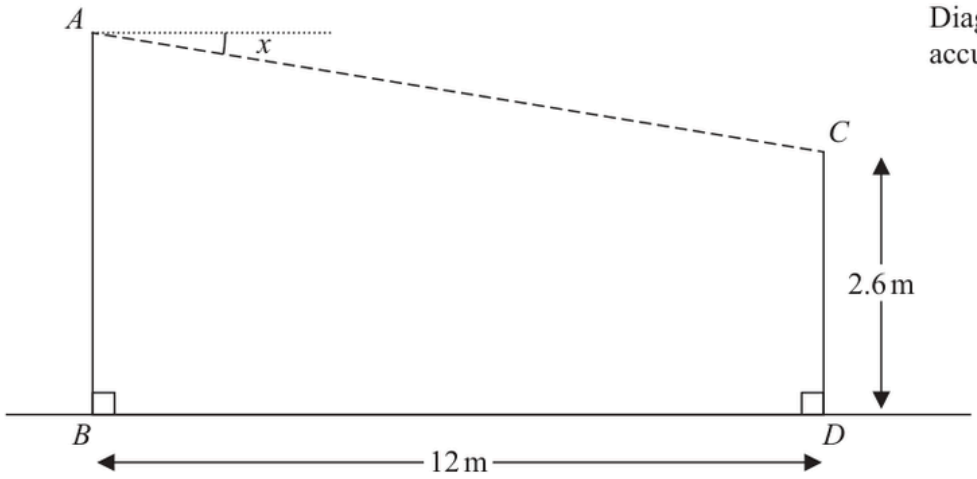


Diagram **NOT** accurately drawn

The zip wire is supported by two vertical posts AB and CD standing on horizontal ground.

$CD = 2.6\,\text{m}$ $BD = 12\,\text{m}$

The zip wire makes an angle x with the horizontal, as shown in the diagram.
The design of the zip wire requires the angle x to be at least 5°

Work out the least possible height of the post AB
Give your answer correct to 3 significant figures.

..... m

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2022

1 **Use trigonometry**

The horizontal distance is $12\ \text{m}$ and the zip wire is $13\ \text{m}$.

2 **Vertical difference**

Vertical difference:

$$\sqrt{13^2 - 12^2} = 5$$

3 **Use trigonometry**

Since the shorter post is $2.6\ \text{m}$,

$$AB = 2.6 + 5 = 7.6$$

FINAL ANSWER

$7.60\ \text{m}$.

4MA1/2H JUNE 2022

Question 6

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

6 The diagram shows triangle PQR.

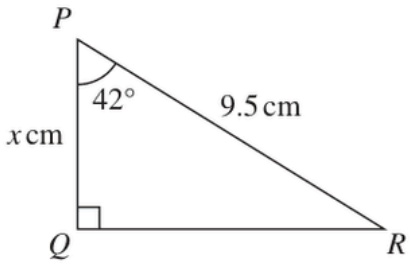


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2022

1 **Use trigonometry**

In the right-angled triangle, x is adjacent to the $42°$ angle and 9.5 cm is the hypotenuse.

$$\cos 42° = \frac{x}{9.5}$$

$$x = 9.5 \cos 42° = 7.059\ldots$$

✓ **FINAL ANSWER**

7.1 cm.

4MA1/2HR JUNE 2022

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 The diagram shows triangle ABP inside the regular hexagon $ABCDEF$

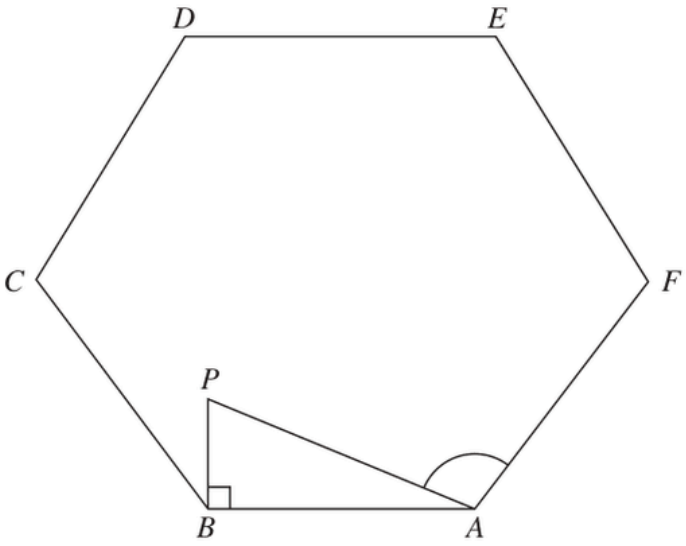


Diagram **NOT** accurately drawn

$AB = 5\,\text{cm}$ $BP = 2\,\text{cm}$ Angle $ABP = 90°$

Work out the size of angle PAF
Give your answer correct to 3 significant figures.

.....°

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2022

1 **A regular hexagon has interior**

A regular hexagon has interior angle

$$120°$$

2 **Use trigonometry**

In right-angled triangle ABP,

$$\tan(\angle PAB) = \frac{BP}{AB} = \frac{2}{5}$$

$$\angle PAB = \tan^{-1}\left(\frac{2}{5}\right) = 21.801\ldots°$$

3 Use trigonometry

Therefore

$$\angle PAF = 120° - 21.801\ldots° = 98.198\ldots°$$

FINAL ANSWER

$98.2°$.

4MA1/1H MAY 2023

Question 5

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

5 The diagram shows a shaded shape $AEBCD$ made by removing triangle AEB from rectangle $ABCD$

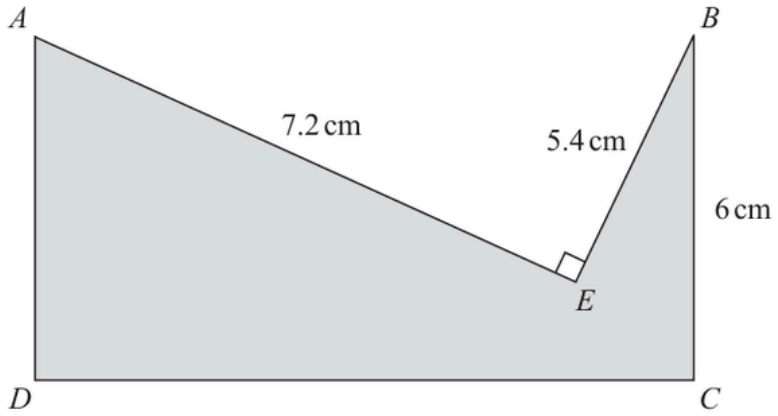


Diagram **NOT** accurately drawn

$AE = 7.2\,\text{cm}$ $BE = 5.4\,\text{cm}$ $BC = 6\,\text{cm}$ angle $AEB = 90°$

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2023

1 Triangle is right angled

Triangle AEB is right-angled:

$$AB^2 = 7.2^2 + 5.4^2 = 81$$

$$AB = 9$$

2 Use trigonometry

For rectangle $ABCD$,

$$DC = 9, \qquad AD = BC = 6$$

3 Perimeter of the shaded shape

Perimeter of the shaded shape:

$$7.2 + 5.4 + 6 + 9 + 6 = 33.6$$

FINAL ANSWER

$33.6\ \text{cm}$.

4MA1/2H JUNE 2023

Question 12

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows right-angled triangle ABD

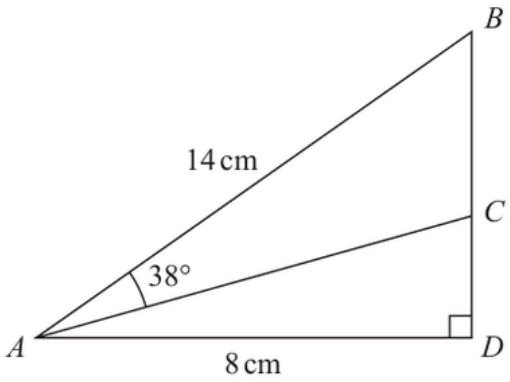


Diagram **NOT** accurately drawn

$AB = 14\,\text{cm}$ $AD = 8\,\text{cm}$

C is the point on BD such that angle $BAC = 38°$

Work out the length of CD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2023

1 Use trigonometry

First find angle DAB:

$$\cos(\angle DAB) = \frac{8}{14}$$

$$\angle DAB = 55.150\ldots°$$

2 Given

Given

$$\angle BAC = 38°$$

3 Use trigonometry

So

$$\angle CAD = 55.150\ldots° - 38° = 17.150\ldots°$$

4 **Use trigonometry**

In right-angled triangle ACD,

$$\tan(17.150\ldots^\circ) = \frac{CD}{8}$$

$$CD = 2.468\ldots$$

✓ **FINAL ANSWER**

$2.47\ \text{cm}$.

4MA1/2H NOVEMBER 2023

Question 2

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 The diagram shows triangle ABC and triangle ECD

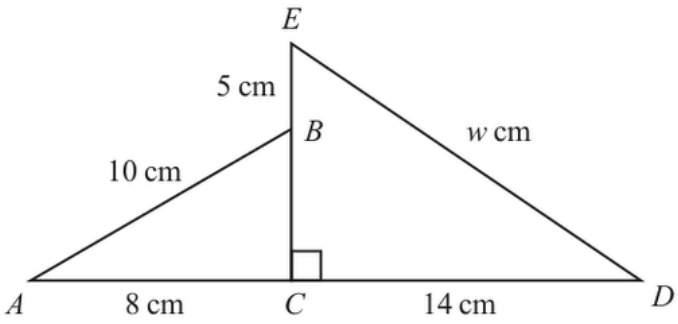


Diagram **NOT** accurately drawn

ACD and EBC are straight lines.

$AB = 10$ cm $\quad AC = 8$ cm $\quad EB = 5$ cm $\quad CD = 14$ cm $\quad ED = w$ cm

Work out the value of w
Give your answer correct to one decimal place.

$w =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2023

1 **Use trigonometry**

In triangle ABC,

$$BC^2 = 10^2 - 8^2$$

$$BC^2 = 36$$

$$BC = 6$$

2 **Use trigonometry**

So

$$EC = EB + BC = 5 + 6 = 11$$

3 **Use trigonometry**

In triangle ECD,

$$w^2 = 11^2 + 14^2$$

$$w^2 = 317$$

$$w = 17.804\ldots$$

✓ **FINAL ANSWER**

$w = 17.8\ \text{cm}$ to 1 decimal place.

4MA1/1H NOVEMBER 2023

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 Here is triangle ABC

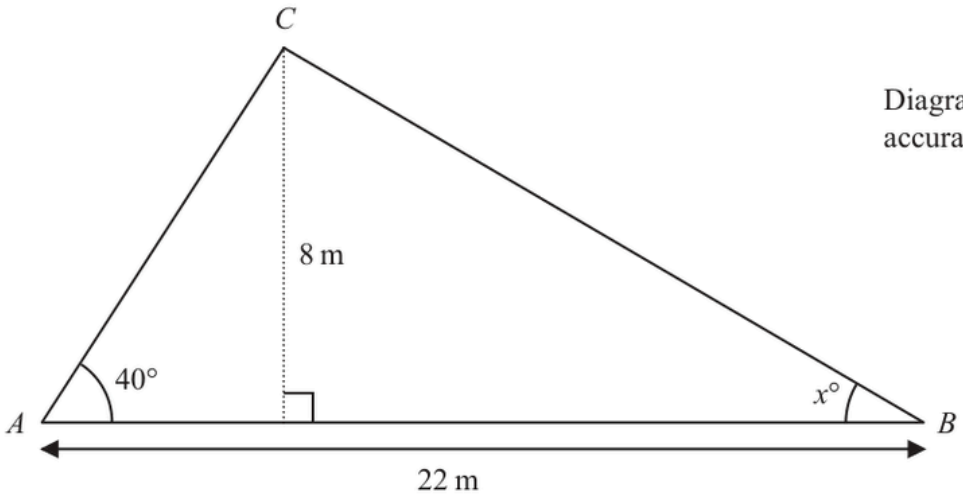


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.
Show your working clearly.

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2023

1 **Let the perpendicular from meet**

Let the perpendicular from C meet AB at F.

$$AF = \frac{8}{\tan 40^\circ} = 9.534\ldots$$

$$FB = 22 - 9.534\ldots = 12.465\ldots$$

2 **Use trigonometry**

In right-angled triangle CFB,

$$\tan x = \frac{8}{12.465\ldots}$$

$$x = 32.7^\circ$$

FINAL ANSWER

$32.7°$.

4MA1/2H JUNE 2024

Question 5

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

5 The diagram shows triangle ABC

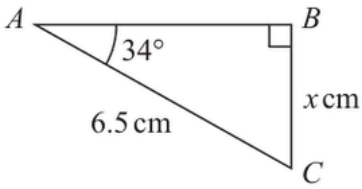


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2024

1 Use trigonometry

The hypotenuse is 6.5 cm, and x is opposite the $34°$ angle.

$$\sin 34° = \frac{x}{6.5}$$

$$x = 6.5 \sin 34° = 3.634\ldots$$

FINAL ANSWER

3.6 cm.

4MA1/1H MAY 2024

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a roof support.

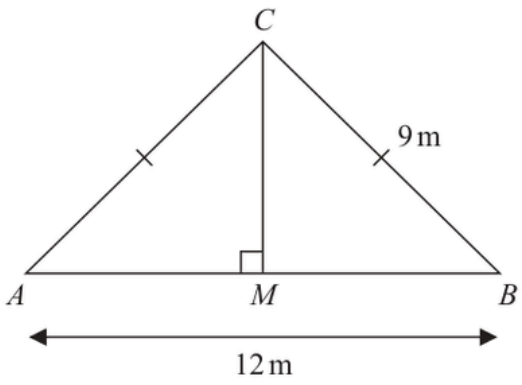


Diagram **NOT** accurately drawn

The roof support is made from four lengths of wood, AB, AC, BC and MC

$AC = BC = 9\,\text{m}$ $AB = 12\,\text{m}$

angle $AMC = 90°$

Lewis is going to buy lengths of wood to make the roof support.

The wood costs 21.50 euros per metre.
Each length of wood he buys has to be a whole number of metres.

Work out the total cost of the wood Lewis needs to buy.
Show your working clearly.

..... euros

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2024

1 **Find the gradient**

Since $AC = BC$, the perpendicular support from C meets AB at its midpoint.

$$AM = \frac{12}{2} = 6$$

2 **Use trigonometry**

In right-angled triangle AMC,

$$AC^2 = AM^2 + MC^2$$

$$9^2 = 6^2 + MC^2$$

$$MC^2 = 81 - 36 = 45$$

$$MC = \sqrt{45} = 6.708\ldots$$

---

3 **A whole number of metres**

A whole number of metres must be bought, so buy 7 m for MC.

---

4 **Total length needed**

Total length needed:

$$12 + 9 + 9 + 7 = 37$$

---

5 **Cost**

Cost:

$$37 \times 21.50 = 795.50$$

✓ **FINAL ANSWER**

795.50 euros.

4MA1/2H NOVEMBER 2024

Question 3

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The diagram shows triangle PQR

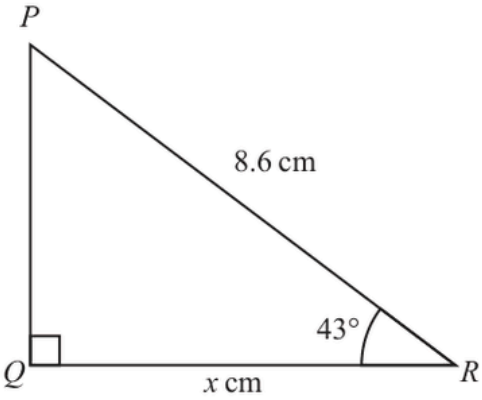


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2024

1 Use trigonometry

In the right-angled triangle, x is adjacent to the $43°$ angle.

$$\cos 43° = \frac{x}{8.6}$$

$$x = 8.6 \cos 43° = 6.289\ldots$$

FINAL ANSWER

6.3 cm.

4MA1/1H NOVEMBER 2024

Question 12

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 ABC is a right-angled triangle.
D is a point on BC

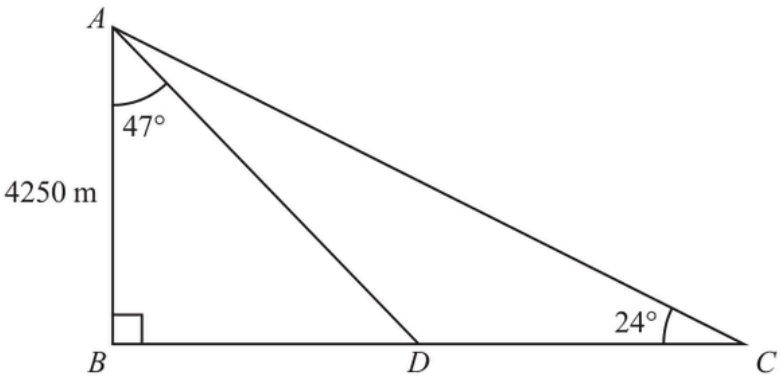


Diagram **NOT** accurately drawn

$AB = 4250$ m angle $BAD = 47°$ angle $BCA = 24°$

Work out the length of DC
Give your answer correct to the nearest integer.

..... m

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H November 2024

1 **Use trigonometry**

In triangle ABD,

$$\tan 47° = \frac{BD}{4250}$$

$$BD = 4250 \tan 47° = 4557.567\ldots$$

2 **Use trigonometry**

In triangle ABC,

$$\tan 24^\circ = \frac{4250}{BC}$$

$$BC = \frac{4250}{\tan 24^\circ} = 9545.656\ldots$$

$$DC = BC - BD = 4988.089\ldots$$

FINAL ANSWER

$4988\ \text{m}$.

4WM1/01R MAY 2025

Question 8

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

8

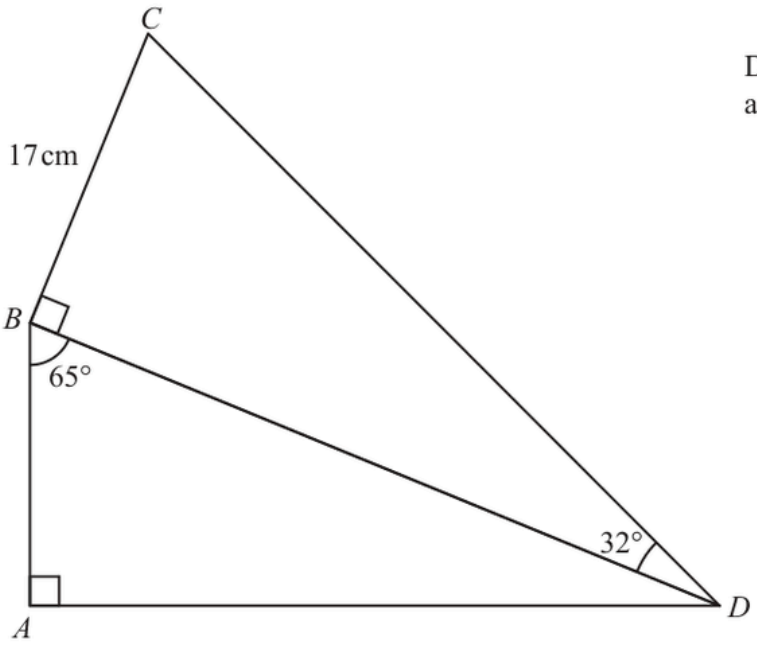


Diagram **NOT** accurately drawn

ABD and *BCD* are right-angled triangles.

angle $ABD = 65°$ angle $CDB = 32°$ $BC = 17$ cm

Work out the length of *AD*
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4WM1/01R May 2025

1 **Use trigonometry**

In right triangle BCD,

$$\tan 32° = \frac{17}{BD}$$

$$BD = \frac{17}{\tan 32°}$$

2 **Use trigonometry**

In right triangle ABD,

$$\sin 65^\circ = \frac{AD}{BD}$$

$$AD = \frac{17}{\tan 32^\circ} \sin 65^\circ \approx 24.6567$$

✓ **FINAL ANSWER**

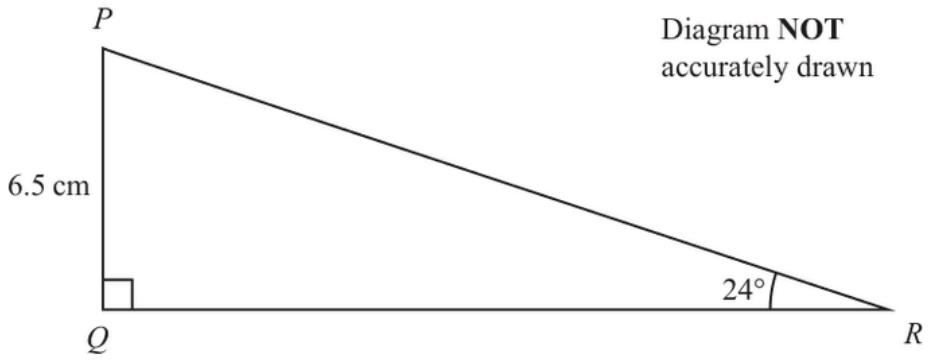
$AD = 24.7$ cm to 3 significant figures.

4MA1/1HR JUNE 2025

Question 9

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

9 The diagram shows triangle PQR



Work out the length of QR
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2025

1 Use trigonometry

$$\tan 24^\circ = \frac{6.5}{QR}$$

$$QR = \frac{6.5}{\tan 24^\circ} = 14.599\ldots$$

FINAL ANSWER

14.6 cm.

4MA1/2H JUNE 2025

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 ABD and ABC are right-angled triangles.

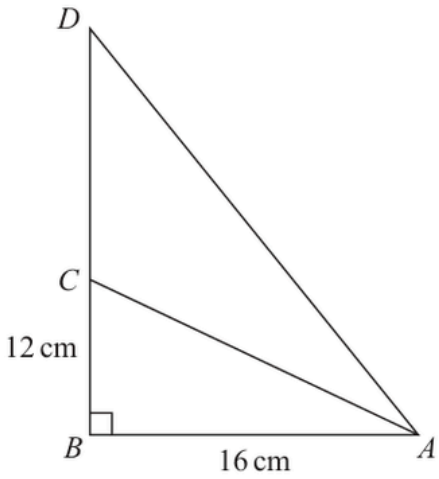


Diagram **NOT** accurately drawn

$AB = 16\,\text{cm}$ $BC = 12\,\text{cm}$ $AD = 1.5 \times AC$

Find the length of CD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2025

1 **Use trigonometry**

In triangle ABC,

$$AC^2 = 16^2 + 12^2 = 400$$

$$AC = 20$$

2 **Use trigonometry**

Given $AD = 1.5AC$,

$$AD = 30$$

3 **Use trigonometry**

In triangle ABD,

$$BD^2 = 30^2 - 16^2 = 644$$

$$BD = 25.377\ldots$$

---

4 **Use trigonometry**

So

$$CD = BD - BC = 25.377\ldots - 12 = 13.377\ldots$$

✓ **FINAL ANSWER**

$13.4\ \text{cm}$.

4MA1/2HR JUNE 2025

Question 11

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows two triangles, ADE and CDB

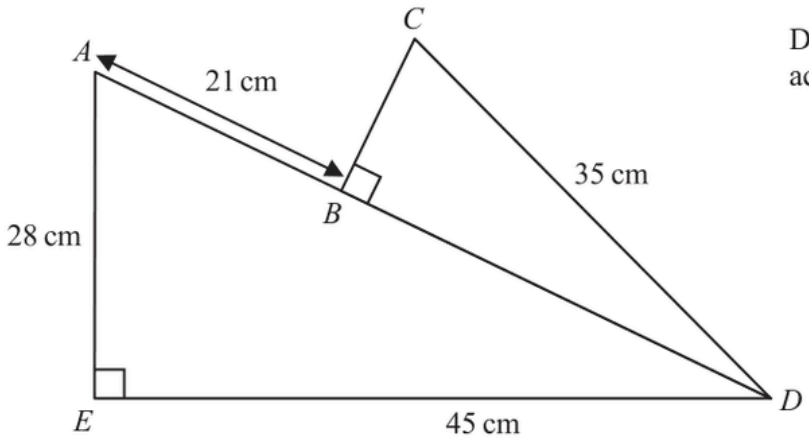


Diagram **NOT** accurately drawn

ABD is a straight line.

$AE = 28\,\text{cm}$ $ED = 45\,\text{cm}$ $AB = 21\,\text{cm}$ $CD = 35\,\text{cm}$

angle AED = angle $CBD = 90°$

Work out the area of triangle CDB
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2025

1 **Use trigonometry**

In right-angled triangle AFD,

$$AD^2 = 28^2 + 45^2$$

$$AD^2 = 2809$$

$$AD = 53$$

2 **Use trigonometry**

Since $AB = 21$,

$$BD = 53 - 21 = 32$$

3 **Use trigonometry**

In right-angled triangle CBD,

$$CB^2 = 35^2 - 32^2$$

$$CB^2 = 201$$

$$CB = \sqrt{201}$$

---

4 **Area of triangle is**

Area of triangle CDB is

$$\frac{1}{2} \times BD \times CB$$

$$\text{Area} = \frac{1}{2} \times 32 \times \sqrt{201} = 226.9\ldots$$

✓ **FINAL ANSWER**

$227\ \text{cm}^2$ to 3 significant figures.

Question 13

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

13 In the diagram, AB represents a vertical tower and CD represents a vertical building.

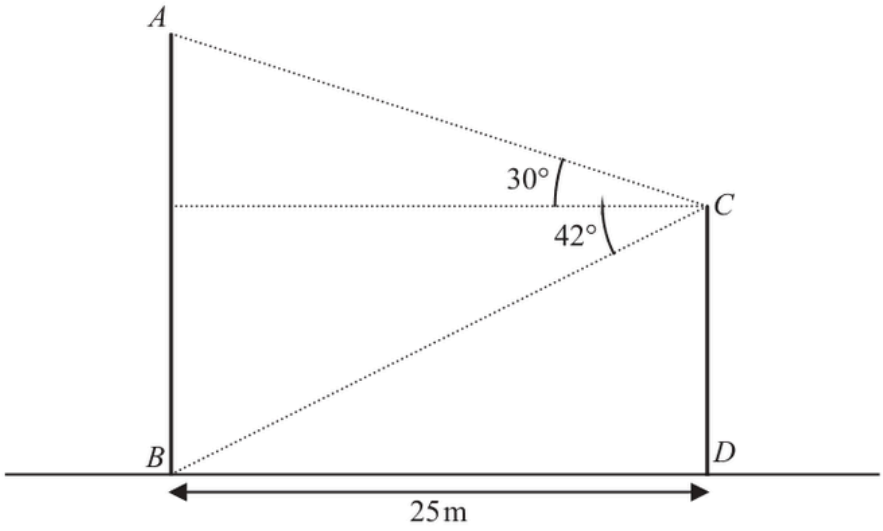


Diagram **NOT** accurately drawn

The bottom of the tower, B, and the bottom of the building, D, are on horizontal ground such that $BD = 25$ m

The angle of elevation of A from C is $30°$
The angle of depression of B from C is $42°$

(a) Work out the height of the tower AB
Give your answer correct to one decimal place.

..... m

(3)

(b) Work out the angle of elevation of A from D
Give your answer correct to one decimal place.

..... °

(2)

(Total for Question 13 is 5 marks)

1 **The height of the building**

The height of the building is

$$CD = 25\tan 42°$$

2 **Use trigonometry**

The extra height from C to A is

$$25\tan 30°$$

3 **Use trigonometry**

So

$$AB = 25\tan 42° + 25\tan 30° \approx 36.9439$$

4 **Use trigonometry**

For the angle of elevation θ from D,

$$\tan\theta = \frac{AB}{25}$$

$$\theta \approx 55.9137°$$

✓ **FINAL ANSWER**

$AB = 36.9$ m, angle of elevation $55.9°$.

4WM1/01 NOVEMBER 2025

Question 9

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

9 The diagram shows a sketch of a right-angled triangle, ABC

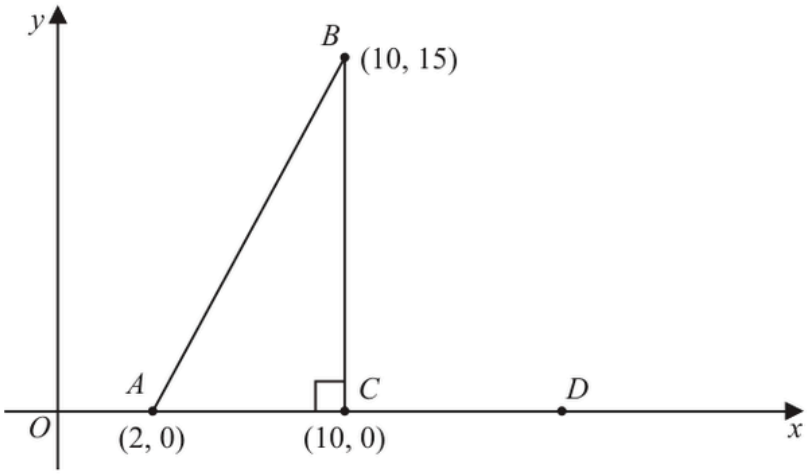


Diagram **NOT** accurately drawn

A is the point with coordinates $(2, 0)$
B is the point with coordinates $(10, 15)$
C is the point with coordinates $(10, 0)$

D is a point on the x-axis such that $AB = AD$

Find the coordinates of D

(..... , ...)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4WM1/01 November 2025

1 **Simplify surd**

$$AB = \sqrt{(10-2)^2 + (15-0)^2}$$

$$= \sqrt{64 + 225} = 17$$

2 **Calculate value**

Since D is on the positive x-axis and $AD = 17$,

$$D = (2 + 17, 0) = (19, 0)$$

FINAL ANSWER

$(19, 0)$.

4WM1/01 NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 ABC is a right-angled triangle.

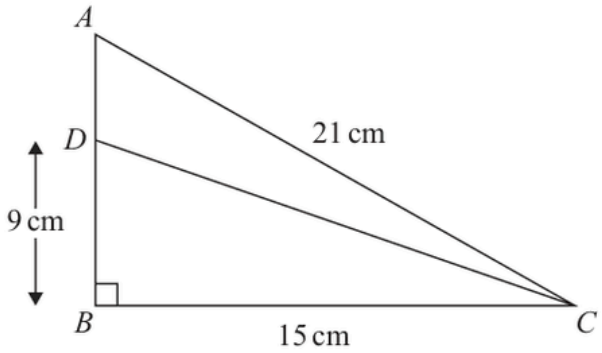


Diagram **NOT** accurately drawn

$AC = 21\,\text{cm}$ $BC = 15\,\text{cm}$ angle $ABC = 90°$

The point D lies on AB such that $DB = 9\,\text{cm}$

Work out the size of angle ACD
Give your answer correct to one decimal place.

.....°

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4WM1/01 November 2025

1 Use trigonometry

First find AB:

$$AB^2 = 21^2 - 15^2 = 216$$

$$AB = 14.696\ldots$$

$$AD = AB - DB = 14.696\ldots - 9 = 5.696\ldots$$

2 Angle

Angle ACB:

$$\cos ACB = \frac{15}{21}$$

$$ACB = 44.415\ldots°$$

3 **Angle**

Angle DCB:

$$\tan DCB = \frac{9}{15}$$

$$DCB = 30.963\ldots^\circ$$

$$ACD = 44.415\ldots - 30.963\ldots = 13.451\ldots^\circ$$

✓ **FINAL ANSWER**

13.5°.

4MA1/1H NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 ABC is a right-angled triangle.

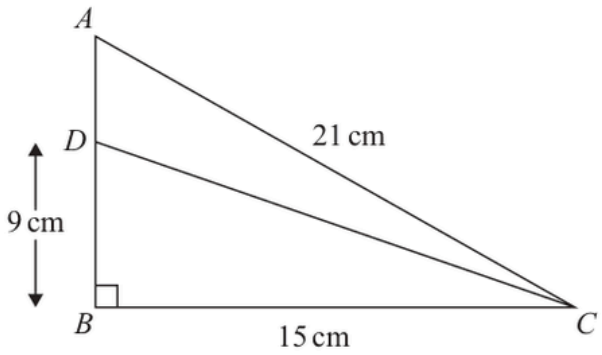


Diagram **NOT** accurately drawn

$AC = 21\,\text{cm}$ $BC = 15\,\text{cm}$ angle $ABC = 90°$

The point D lies on AB such that $DB = 9\,\text{cm}$

Work out the size of angle ACD
Give your answer correct to one decimal place.

.....°

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H November 2025

1 **Use trigonometry**

First find AB:

$$AB^2 = 21^2 - 15^2 = 216$$

$$AB = 14.696\ldots$$

$$AD = AB - DB = 14.696\ldots - 9 = 5.696\ldots$$

2 **Angle**

Angle ACB:

$$\cos ACB = \frac{15}{21}$$

$$ACB = 44.415\ldots^\circ$$

---

3 **Angle**

Angle DCB:

$$\tan DCB = \frac{9}{15}$$

$$DCB = 30.963\ldots^\circ$$

$$ACD = 44.415\ldots - 30.963\ldots = 13.451\ldots^\circ$$

✓ **FINAL ANSWER**

13.5°.

4MA1/2H NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a right-angled triangle ABC

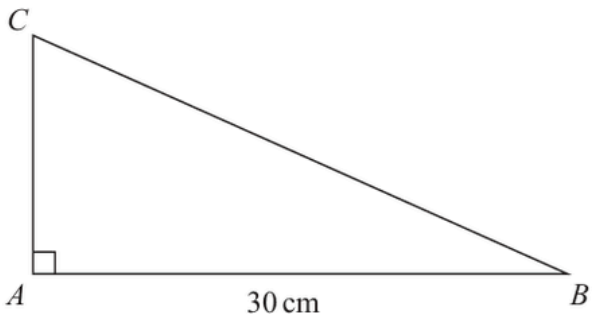


Diagram **NOT** accurately drawn

Area of triangle $ABC = 240\,\text{cm}^2$

Work out the perimeter of triangle ABC

..... cm

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2025

1 Use the area to find

Use the area to find AC:

$$\frac{1}{2} \times 30 \times AC = 240$$

$$15AC = 240$$

$$AC = 16$$

2 Use trigonometry

Then

$$BC^2 = 30^2 + 16^2 = 1156$$

$$BC = 34$$

3 **Perimeter**

Perimeter:

$$30 + 16 + 34 = 80$$

✓ **FINAL ANSWER**

$80\ \text{cm}$.

4MA1/2HR JUNE 2026

Question 1

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

1 The diagram shows a right-angled triangle.

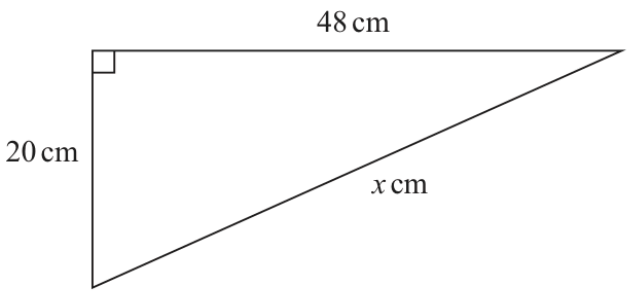


Diagram **NOT** accurately drawn

Work out the value of x

$x =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2026

1 Solution | Step 1

$$x^2 = 20^2 + 48^2 = 400 + 2304 = 2704.$$

Use Pythagoras because the 20 cm and 48 cm sides meet at the right angle.

2 Solution | Step 2

$$x = \sqrt{2704} = 52.$$

Take the positive square root because x is a length.

FINAL ANSWER

$x = 52$, so the hypotenuse is 52 cm.

4MA1/1H JUNE 2026

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2

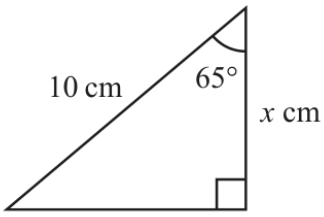


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2026

1 Solution | Step 1

$$\cos 65^\circ = \frac{x}{10}.$$

The side of length 10 cm is the hypotenuse and x is adjacent to the given angle.

2 Solution | Step 2

$$x = 10\cos 65^\circ = 4.22618\ldots = 4.23 \text{ cm} \quad (3 \text{ s.f.}).$$

Multiply by 10 and round only the final value.

✓ FINAL ANSWER

$x = 4.23$ cm

4WM1/01 MAY 2026

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2

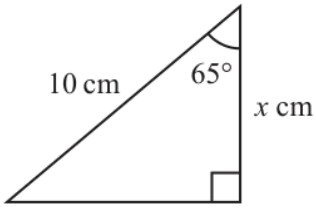


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 May 2026

1 Choose the ratio

x is next to the $65°$ angle and 10 cm is the hypotenuse, so use cosine.

2 Calculate

$$x = 10\cos 65° = 4.2261\ldots$$

FINAL ANSWER

$x = 4.23$

4MA1/2HR JUNE 2026

Question 11

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a square $ABCD$ and a right-angled triangle EFG

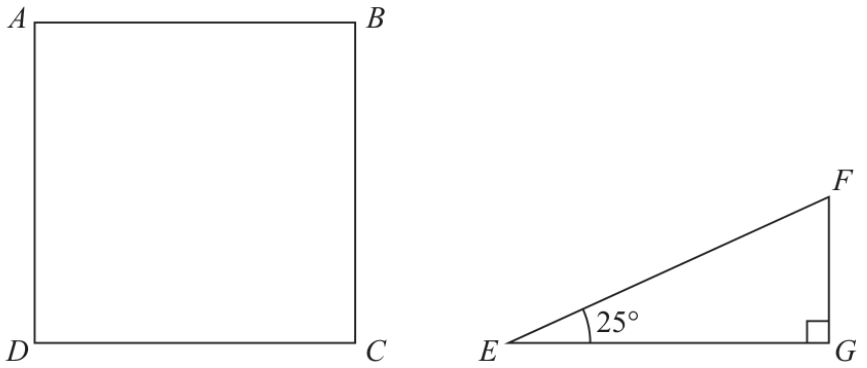


Diagram **NOT** accurately drawn

The area of square $ABCD$ is $36\,\text{cm}^2$
$DC = EG$

Work out the area of triangle EFG
Give your answer correct to one decimal place.
Show your working clearly.

..... cm^2

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$$DC = \sqrt{36} = 6\text{ cm}, \qquad EG = 6\text{ cm}.$$

All sides of the square have length equal to the square root of its area, and DC=EG.

2 **Solution | Step 2**

$$\tan 25^\circ = \frac{FG}{EG} \quad \Rightarrow \quad FG = 6\tan 25^\circ = 2.7978459\ldots\text{ cm}.$$

Relative to angle E, FG is opposite and EG is adjacent.

3 **Solution | Step 3**

$$\text{Area} = \frac{1}{2}(EG)(FG) = \frac{1}{2}(6)(6\tan 25^\circ) = 8.3935378\ldots\text{ cm}^2.$$

The base EG and height FG are perpendicular.

FINAL ANSWER

$8.4\ \text{cm}^2$ (1 decimal place).

TOPIC

3D Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

Question 19

4 marks | 3D Pythagoras & Trigonometry

19 The diagram shows a triangular prism, $ABCDEF$, with a rectangular base $ABCD$

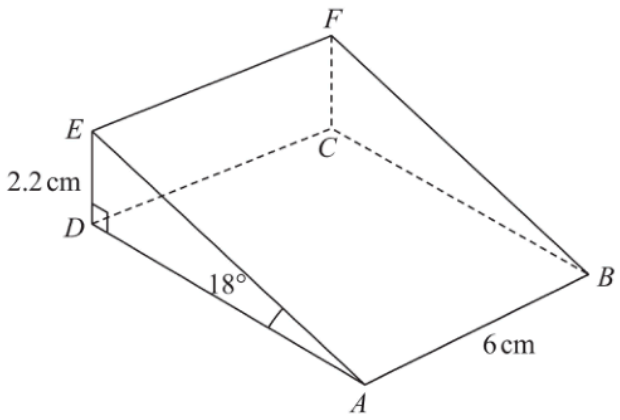


Diagram **NOT** accurately drawn

$AB = 6\,\text{cm}$ $DE = 2.2\,\text{cm}$ angle $DAE = 18°$ angle $ADE = 90°$

Work out the angle that BE makes with the plane $ABCD$
Give your answer correct to one decimal place.

1 Find AD

In triangle ADE,

$$\tan 18^\circ = \frac{DE}{AD}$$

$$AD = \frac{2.2}{\tan 18^\circ} = 6.77\ldots$$

2 Find DB in the base

The base $ABCD$ is a rectangle, so

$$DB = \sqrt{6^2 + 6.77\ldots^2} = 9.04\ldots$$

3 Use the projection triangle

The projection of BE on the plane is BD. Let the required angle be θ.

$$\tan\theta = \frac{DE}{DB} = \frac{2.2}{9.04\ldots}$$

4 Calculate the angle

$$\theta = 13.7^\circ$$

FINAL ANSWER

13.7°.

4MA1/1H JANUARY 2020

Question 19

4 marks | 3D Pythagoras & Trigonometry

19 The diagram shows a cuboid $ABCDEFGH$.

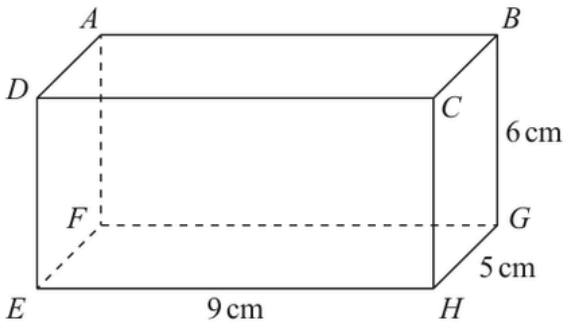


Diagram **NOT** accurately drawn

$EH = 9\,\text{cm}$, $HG = 5\,\text{cm}$ and $GB = 6\,\text{cm}$.

Work out the size of the angle between AH and the plane $EFGH$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2020

1 The projection of onto the

The projection of AH onto the base plane is the diagonal of the base rectangle:

$$\sqrt{9^2 + 5^2} = \sqrt{106}$$

2 Use trigonometry

The vertical height is $6\ \text{cm}$, so

$$\tan\theta = \frac{6}{\sqrt{106}}$$

$$\theta = 30.232\ldots^\circ$$

FINAL ANSWER

30.2°.

4MA1/1HR MAY/NOVEMBER 2020

Question 17

6 marks | 3D Pythagoras & Trigonometry

17 The diagram shows a prism $ABCDEFGH$ with a horizontal base.

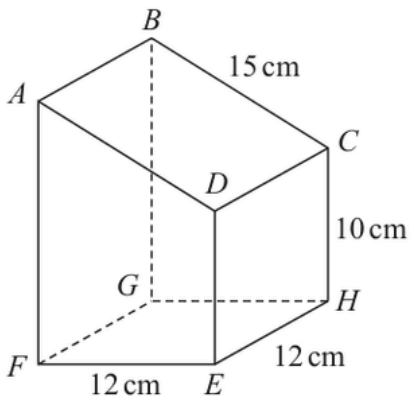


Diagram **NOT** accurately drawn

The base of the prism, $EFGH$, is a square of side 12 cm.
Trapezium $ADEF$ is a cross section of the prism where AF and DE are vertical edges.

$DE = CH = 10$ cm
$AD = BC = 15$ cm

(a) Work out the size of the angle between CF and the base $EFGH$.
Give your answer correct to one decimal place.

..... °

(3)

(b) Work out the length of BE.
Give your answer correct to one decimal place.

..... cm

(3)

(Total for Question 17 is 6 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May/November 2020

1 **Use trigonometry**

(a) C is vertically above H, so the vertical height from F to C is 10 cm.

2 **The projection of on the**

The projection of CF on the base is FH:

$$FH = 12\sqrt{2}$$

$$\tan\theta = \frac{10}{12\sqrt{2}}$$

$$\theta = 30.508\ldots^\circ$$

---

3 **Answer**

Answer: 30.5°.

---

4 **(b) In the trapezium cross**

(b) In the trapezium cross-section, $AD = 15$ and the horizontal distance is 12, so the difference in vertical heights is

$$\sqrt{15^2 - 12^2} = 9$$

---

5 **Use trigonometry**

Since $DE = 10$, the height $BG = 19\ \text{cm}$.

---

6 **The projection of on the**

The projection of BE on the base is $GE = 12\sqrt{2}$, so

$$BE = \sqrt{19^2 + (12\sqrt{2})^2} = 25.475\ldots$$

✓ **FINAL ANSWER**

$25.5\ \text{cm}$.

4MA1/2HR JANUARY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 ABC is an isosceles triangle in a horizontal plane.
The point T is vertically above B.

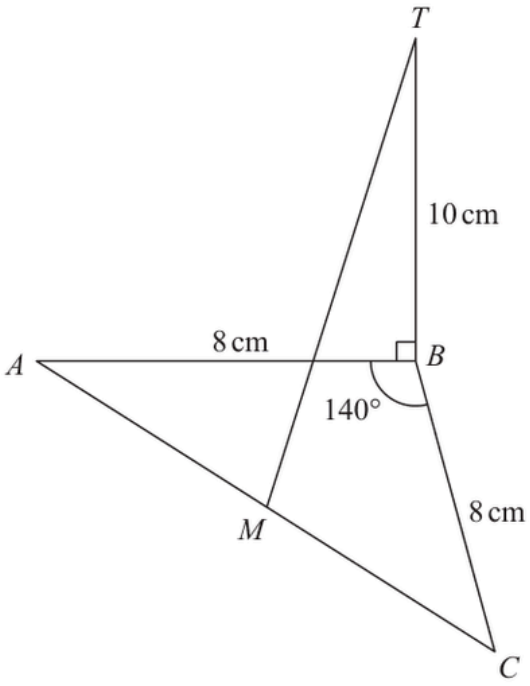


Diagram **NOT** accurately drawn

Angle $ABC = 140°$
$AB = BC = 8$ cm
$TB = 10$ cm
M is the midpoint of AC.

Calculate the size of the angle between MT and the horizontal plane ABC.
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2021

1 **Use trigonometry**

Since $AB = BC = 8$, triangle ABC is isosceles.

2 **The line from to the**

The line from B to the midpoint M bisects angle ABC.

$$\angle ABM = \frac{140^\circ}{2} = 70^\circ$$

3 **Use trigonometry**

In right triangle ABM,

$$\cos 70^\circ = \frac{BM}{8}$$

$$BM = 8\cos 70^\circ$$

4 **Use trigonometry**

Since T is vertically above B, the projection of MT onto the horizontal plane is MB.

5 **Let the required angle be**

Let the required angle be θ.

$$\tan\theta = \frac{TB}{BM}$$

$$\tan\theta = \frac{10}{8\cos 70^\circ}$$

$$\theta = 74.7^\circ$$

FINAL ANSWER

74.7°

4MA1/1H MAY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

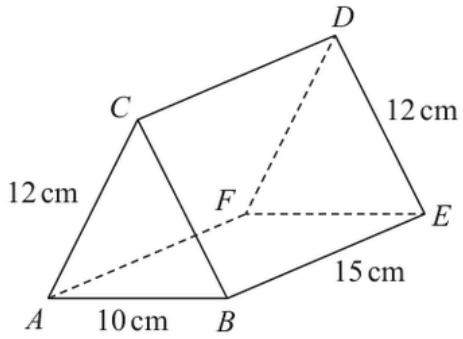


Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between AD and the base $ABEF$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2021

1 Calculate value

The triangular end is isosceles, with

$$AC = BC = 12, \qquad AB = 10$$

2 The perpendicular height of the

The perpendicular height of the triangle is found by halving AB:

$$AM = 5$$

$$CM^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$CM = \sqrt{119}$$

3 The projection of on the

The projection of AD on the horizontal base goes from A to the midpoint of FE. Its length is

$$\sqrt{15^2 + 5^2} = \sqrt{250}$$

4 Use trigonometry

So, for the angle θ between AD and the base,

$$\tan\theta = \frac{\sqrt{119}}{\sqrt{250}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.602\ldots$$

FINAL ANSWER

$34.6°$

4MA1/2HR JANUARY 2022

Question 19

3 marks | 3D Pythagoras & Trigonometry

19 $ABCD$ is a horizontal rectangular field.

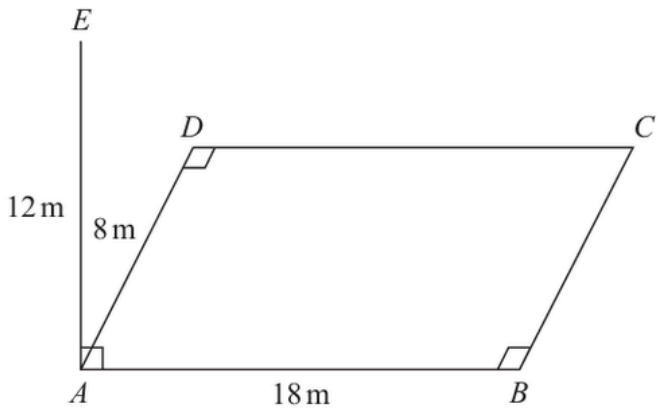


Diagram **NOT** accurately drawn

A vertical pole, AE, is placed at the corner A of the field.

$AE = 12\,\text{m}$ $AB = 18\,\text{m}$ $AD = 8\,\text{m}$

Calculate the size of the angle between EC and the plane $ABCD$
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2022

1 The projection of onto the

The projection of EC onto the field is AC.

$$AC = \sqrt{18^2 + 8^2} = \sqrt{388}$$

2 Use trigonometry

The vertical height is $AE = 12\ \text{m}$, so

$$\tan\theta = \frac{12}{\sqrt{388}}$$

$$\theta = 31.350\ldots^\circ$$

FINAL ANSWER

$31.4°$.

4MA1/1H JANUARY 2022

Question 21

5 marks | 3D Pythagoras & Trigonometry

21 The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

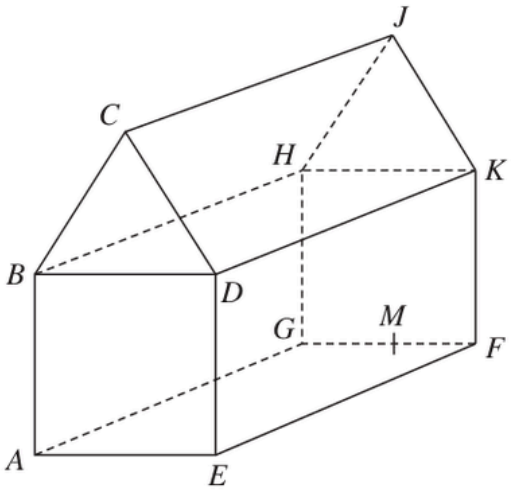


Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of T, giving your answer in the form $\frac{\sqrt{p} + \sqrt{q}}{17}$ where p and q are integers.

$T =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2022

1 Let

Let

$$AE = x$$

2 **Use trigonometry**

Since $ABDE$ is a square,

$$AB = BD = DE = AE = x$$

3 **Use trigonometry**

Since BCD is an equilateral triangle, the height from C to BD is

$$\frac{\sqrt{3}}{2}x$$

4 **Use trigonometry**

Therefore the vertical height from the base to J is

$$JM = x + \frac{\sqrt{3}}{2}x$$

$$JM = \frac{x(2+\sqrt{3})}{2}$$

5 **Use trigonometry**

Also,

$$EF = 2AE = 2x$$

6 **Find the midpoint**

Since M is the midpoint of GF, the horizontal distance from A to M is

$$AM = \sqrt{\left(\frac{x}{2}\right)^2 + (2x)^2}$$

$$AM = \sqrt{\frac{x^2}{4} + 4x^2}$$

$$AM = \frac{x\sqrt{17}}{2}$$

7 **Use trigonometry**

In right triangle AMJ,

$$\tan y^\circ = \frac{JM}{AM}$$

$$T = \frac{\frac{x(2+\sqrt{3})}{2}}{\frac{x\sqrt{17}}{2}}$$

$$T = \frac{2+\sqrt{3}}{\sqrt{17}}$$

---

8 **Rationalise the denominator**

Rationalise the denominator:

$$T = \frac{(2+\sqrt{3})\sqrt{17}}{17}$$

$$T = \frac{2\sqrt{17}+\sqrt{51}}{17}$$

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

✓ **FINAL ANSWER**

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

4MA1/1HR JUNE 2022

Question 18

4 marks | 3D Pythagoras & Trigonometry

18 The diagram shows a cube $ABCDEFGH$ with sides of length 6 cm.

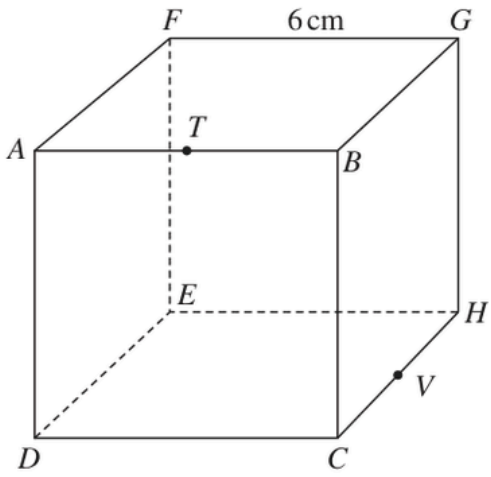


Diagram **NOT** accurately drawn

T is the midpoint of AB and V is the midpoint of CH

Work out the distance from T to V in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where a is an integer.

..... cm

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2022

1 The cube has side length

The cube has side length $\mathbf{6}$ cm.

2 Find the midpoint

Let T be the midpoint of AB and V be the midpoint of CH.

3 The changes in the three

The changes in the three perpendicular directions from T to V are

$$3, \quad 3, \quad 6$$

4 **Use trigonometry**

So by 3D Pythagoras,

$$TV = \sqrt{3^2 + 3^2 + 6^2}$$

$$TV = \sqrt{9 + 9 + 36} = \sqrt{54}$$

✓ **FINAL ANSWER**

$\sqrt{54}\ \text{cm}$.

Question 22

5 marks | 3D Pythagoras & Trigonometry

22 Here is a cuboid $ABCDEFGH$

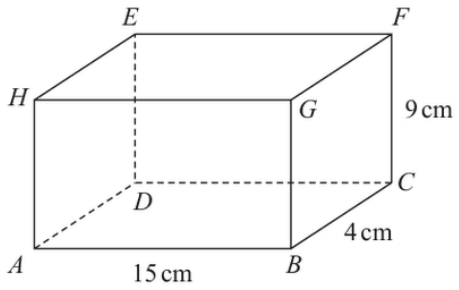


Diagram **NOT** accurately drawn

$AB = 15\,\text{cm}$ $BC = 4\,\text{cm}$ $CF = 9\,\text{cm}$

(a) Work out the length of BE
Give your answer correct to 3 significant figures.

..... cm

(2)

Here is a cuboid $PQRSTUVW$

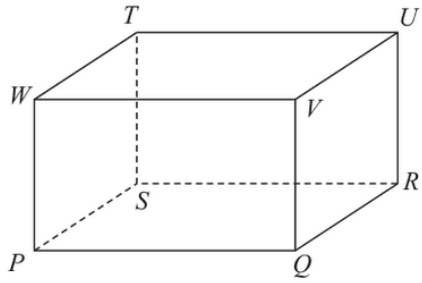


Diagram **NOT** accurately drawn

$PR = 42\,\text{cm}$

The size of the angle between PU and the plane $PQRS$ is 30°

M is the midpoint of PR

(b) Work out the size of angle UMR
Give your answer correct to 3 significant figures.

.....°

(3)

(Total for Question 22 is 5 marks)

1 **Find the gradient**

For part (a), BE is the space diagonal using the three perpendicular lengths:

$$AB = 15, \qquad BC = 4, \qquad CF = 9$$

$$BE^2 = 15^2 + 4^2 + 9^2$$

$$BE^2 = 225 + 16 + 81 = 322$$

$$BE = \sqrt{322} = 17.9 \text{ cm}$$

2 **Use trigonometry**

For part (b), the projection of PU onto the plane $PQRS$ is PR.

$$PR = 42$$

3 **The angle between and the**

The angle between PU and the plane is $30°$, so

$$\tan 30° = \frac{RU}{PR}$$

$$RU = 42 \tan 30°$$

4 **Find the midpoint**

Since M is the midpoint of PR,

$$MR = 21$$

5 **Use trigonometry**

In right triangle MRU,

$$\tan \angle UMR = \frac{RU}{MR}$$

$$\tan \angle UMR = \frac{42 \tan 30°}{21}$$

$$\tan \angle UMR = 2 \tan 30°$$

$$\angle UMR = 49.1°$$

FINAL ANSWER

$BE = 17.9\ \text{cm}$, $\angle UMR = 49.1°$

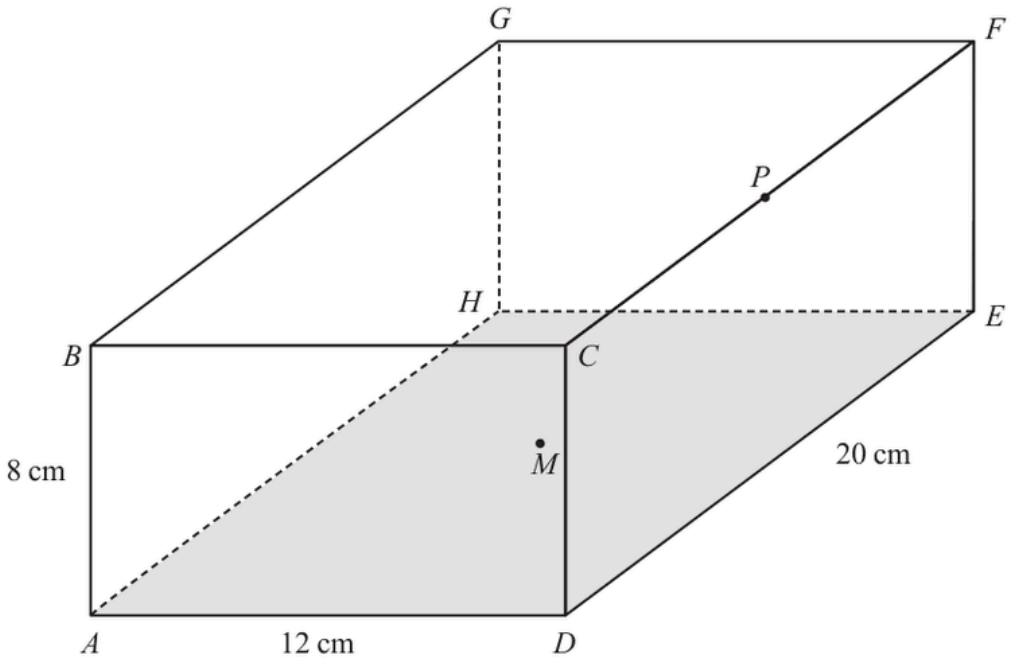
4MA1/1HR MAY 2024

Question 22

6 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a cuboid $ABCDEFGH$ with horizontal base $ADEH$

Diagram **NOT** accurately drawn



$AB = 8$ cm $\quad AD = 12$ cm $\quad DE = 20$ cm

M is the midpoint of the base $ADEH$ and P is the midpoint of the edge CF

Work out the size of angle BMP
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May 2024

1 **Use trigonometry**

Use coordinates with

$$A = (0, 0, 0)$$

2 **Use trigonometry**

Since

$$AD = 12, \qquad DE = 20, \qquad AB = 8$$

3 **take**

take

$$B = (0, 0, 8)$$

4 **The centre of the rectangular**

The centre of the rectangular base $ADEH$ is

$$M = (6, 10, 0)$$

5 **Find the midpoint**

Point P is the midpoint of CF.

$$C = (12, 0, 8), \qquad F = (12, 20, 8)$$

$$P = (12, 10, 8)$$

6 **Use trigonometry**

Now

$$\vec{MB} = B - M = (-6, -10, 8)$$

$$\vec{MP} = P - M = (6, 0, 8)$$

7 Use the scalar product formula

Use the scalar product formula:

$$\vec{MB} \cdot \vec{MP} = (-6)(6) + (-10)(0) + (8)(8)$$

$$\vec{MB} \cdot \vec{MP} = 28$$

$$|\vec{MB}| = \sqrt{(-6)^2 + (-10)^2 + 8^2} = \sqrt{200}$$

$$|\vec{MP}| = \sqrt{6^2 + 0^2 + 8^2} = 10$$

$$\cos \angle BMP = \frac{28}{10\sqrt{200}}$$

$$\angle BMP = 78.6°$$

FINAL ANSWER

$78.6°$

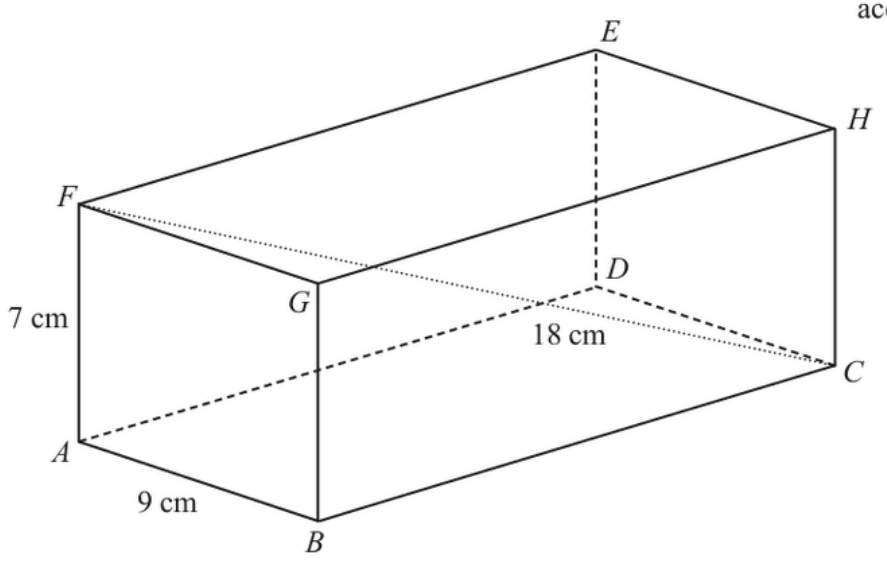
4MA1/1HR JUNE 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2025

1 Use trigonometry

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

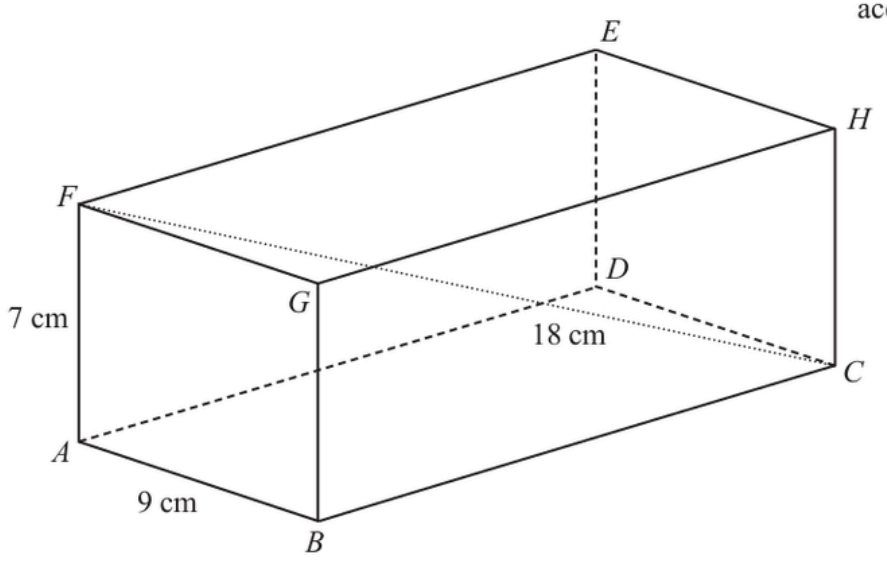
4WM1/01R MAY 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01R May 2025

1 **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

4MA1/2HR JUNE 2025

Question 23

3 marks | 3D Pythagoras & Trigonometry

23 The diagram shows a triangular prism, $ABCDEF$, with a horizontal rectangular base $ABCD$

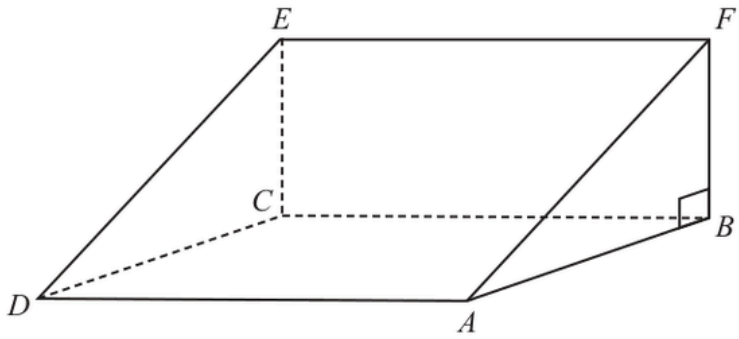


Diagram **NOT** accurately drawn

M is the midpoint of the line AC

$AC = 40\,\text{cm}$ angle $CAE = 35°$ angle $ABF = 90°$

Work out the size of angle CME
Give your answer correct to 3 significant figures.

.....°

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2025

1 Use trigonometry

$$AC = 40$$

2 The point is vertically above

The point E is vertically above C, so triangle ACE is right-angled at C.

$$\angle CAE = 35°$$

$$\tan 35° = \frac{CE}{AC}$$

$$CE = 40 \tan 35°$$

3 **Find the midpoint**

Since M is the midpoint of AC,

$$CM = 20$$

---

4 **Use trigonometry**

In right triangle CME,

$$\tan \angle CME = \frac{CE}{CM}$$

$$\tan \angle CME = \frac{40 \tan 35^\circ}{20}$$

$$\tan \angle CME = 2 \tan 35^\circ$$

$$\angle CME = 54.5^\circ$$

✓ **FINAL ANSWER**

54.5°

4WM1/01 MAY 2025

Question 23

6 marks | 3D Pythagoras & Trigonometry

23 The diagram shows cuboid $ABCDEFGH$

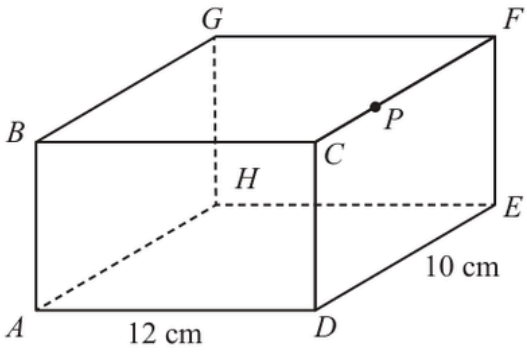


Diagram **NOT** accurately drawn

The point P lies on CF

$AD = 12$ cm $\quad DE = 10$ cm $\quad CP = 3$ cm

The angle of elevation of P from A is $24°$

Calculate the size of angle APG
Give your answer correct to the nearest degree.
Show your working clearly.

.....°

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 May 2025

1 **Point is cm from along**

Point P is 3 cm from C along CF. The horizontal distance from A to the point directly below P is

$$\sqrt{12^2 + 3^2} = \sqrt{153}$$

2 **Use trigonometry**

Using the angle of elevation,

$$h = \sqrt{153}\tan 24° = 5.507\ldots$$

3 **Use trigonometry**

So

$$AP = \frac{\sqrt{153}}{\cos 24^\circ} = 13.5399\ldots$$

4 **Use trigonometry**

On the top face,

$$PG = \sqrt{12^2 + 7^2} = \sqrt{193} = 13.8924\ldots$$

5 **Use trigonometry**

Also,

$$AG = \sqrt{12^2 + 10^2 + h^2} = 16.561\ldots$$

6 **Use cosine rule**

Use the cosine rule in triangle APG.

$$\cos \angle APG = \frac{AP^2 + PG^2 - AG^2}{2(AP)(PG)}$$

$$\angle APG = 49.163\ldots^\circ$$

7 **Use trigonometry**

To the nearest degree,

$$49^\circ$$

✓ **FINAL ANSWER**

49°

4MA1/2H JUNE 2025

Question 24

4 marks | 3D Pythagoras & Trigonometry

24 The diagram shows a square-based pyramid $ABCDE$

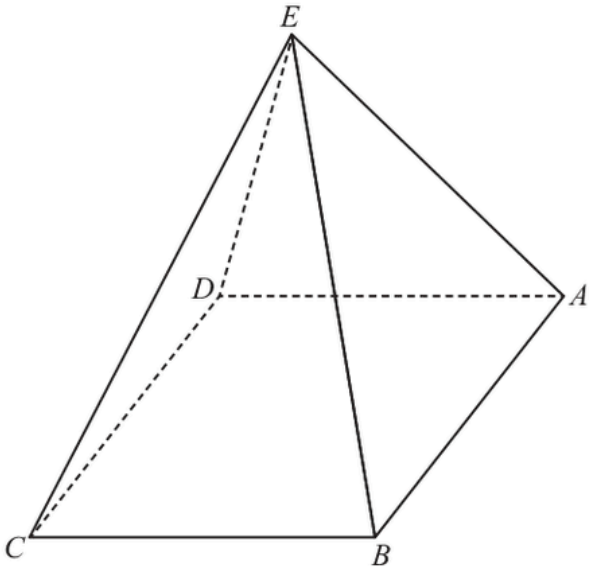


Diagram **NOT** accurately drawn

$EA = EB = EC = ED$

M is the centre of the horizontal square base $ABCD$
Q is the midpoint of AB
Angle $EQM = 80°$

$EA : AB = n : 1$

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2025

1 Let

Let

$$AB = s$$

2 **Find the midpoint**

Since M is the centre of the square base and Q is the midpoint of AB,

$$QM = \frac{s}{2}$$

3 **Use trigonometry**

Since $EA = EB = EC = ED$, the point E is vertically above M.

4 **Use trigonometry**

In right triangle EQM,

$$\tan 80° = \frac{EM}{QM}$$

$$EM = \frac{s}{2}\tan 80°$$

5 **The distance from the centre**

The distance from the centre of a square to a vertex is half the diagonal:

$$AM = \frac{s\sqrt{2}}{2}$$

6 **Use trigonometry**

In right triangle EMA,

$$EA^2 = EM^2 + AM^2$$

$$EA^2 = \left(\frac{s}{2}\tan 80°\right)^2 + \left(\frac{s\sqrt{2}}{2}\right)^2$$

$$EA = s\sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

7 **Use trigonometry**

So

$$\frac{EA}{AB} = \sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

$$n = 2.92$$

FINAL ANSWER

$n = 2.92$

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 The diagram shows prism $ABCDEFGH$

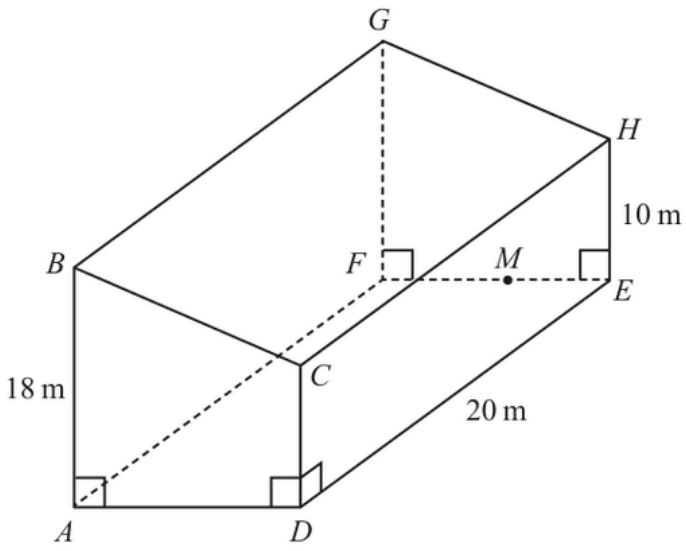


Diagram **NOT** accurately drawn

$$AB = FG = 18\,\text{m} \qquad DC = EH = 10\,\text{m} \qquad AD = FE$$
$$DE = CH = AF = BG = 20\,\text{m}$$
$$\text{angle } BAD = \text{angle } ADC = \text{angle } CDE = 90°$$

M is the point on FE such that $FM = \frac{3}{5}FE$

Given that angle $GMF = 60°$

calculate the angle of elevation of G from D
Give your answer correct to one decimal place.
Show your working clearly.

.....°

(Total for Question 26 is 5 marks)

1 **Use trigonometry**

From triangle GMF,

$$\tan 60^\circ = \frac{FG}{FM} = \frac{18}{FM}$$

$$FM = \frac{18}{\tan 60^\circ} = 6\sqrt{3}$$

2 **Simplify surd**

Since $FM = \frac{3}{5}FE$,

$$FE = 10\sqrt{3}$$

3 **Simplify surd**

The horizontal distance from D to the point below G is

$$\sqrt{20^2 + (10\sqrt{3})^2} = 10\sqrt{7}$$

4 **Use trigonometry**

If θ is the angle of elevation,

$$\tan\theta = \frac{18}{10\sqrt{7}}$$

$$\theta \approx 34.2289^\circ$$

✓ **FINAL ANSWER**

34.2°.

4MA1/1H NOVEMBER 2025

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 Here is a prism $ABCDEFGH$ with a horizontal, rectangular base $ABGF$

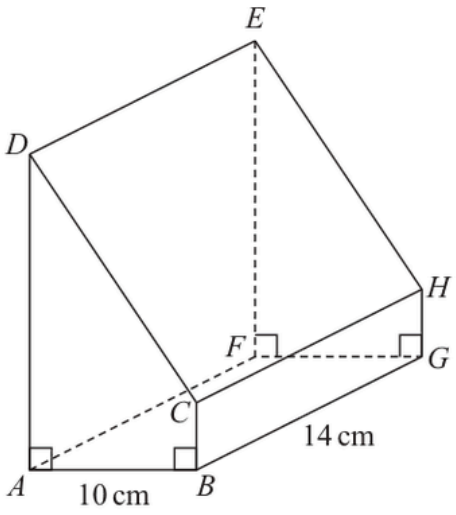


Diagram **NOT** accurately drawn

$AB = 10\,\text{cm}$ $BG = 14\,\text{cm}$

angle DAB = angle ABC = angle EFG = angle $FGH = 90°$

$BC = \frac{1}{5}AD$

The angle of elevation of E from B is $50°$

Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^3

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H November 2025

1 **The horizontal base is a**

The horizontal base $ABGF$ is a rectangle.

$$AB = 10, \qquad BG = 14$$

2 **Use trigonometry**

So the horizontal distance from B to F is

$$BF = \sqrt{10^2 + 14^2} = \sqrt{296}$$

3 **Use trigonometry**

The angle of elevation of E from B is $50°$.

4 **Use trigonometry**

Since E is vertically above F,

$$\tan 50° = \frac{FE}{BF}$$

$$FE = \sqrt{296} \tan 50°$$

5 **The prism has the same**

The prism has the same cross section throughout, so

$$AD = FE = \sqrt{296} \tan 50°$$

6 **Use trigonometry**

Also

$$BC = \frac{1}{5} AD$$

7 **Area of the trapezium cross**

Area of the trapezium cross section $ABCD$:

$$\frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}\left(AD + \frac{1}{5}AD\right) \times 10$$

$$= 6AD$$

8 **Volume of the prism**

Volume of the prism:

$$6AD \times 14 = 84AD$$

$$= 84\sqrt{296}\tan 50°$$

$$= 1722.311\ldots$$

9 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1720$$

✓ **FINAL ANSWER**

$$\boxed{1720\ \text{cm}^3}$$

Question 19

3 marks | 3D Pythagoras & Trigonometry

19 The diagram shows cuboid $PQRSTUVW$

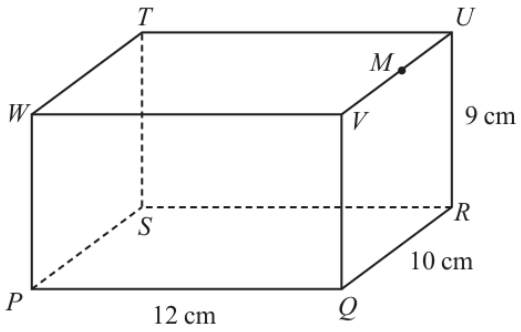


Diagram **NOT** accurately drawn

$PQ = 12$ cm $\quad QR = 10$ cm $\quad RU = 9$ cm

M is the midpoint of VU

Work out the size of the angle between line MP and plane $PQRS$
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

1 **Drop M to the base**

M is above the midpoint N of QR, so $QN = 5$ and $MN = 9$.

2 **Length PN**

$$PN = \sqrt{12^2 + 5^2} = 13$$

3 **Angle**

$$\tan\theta = \frac{9}{13} \Rightarrow \theta = 34.69\ldots^\circ$$

✓ **FINAL ANSWER**

34.7°

4MA1/1H JUNE 2026

Question 25

5 marks | 3D Pythagoras & Trigonometry

25 The diagram shows a triangular prism $ABCDEF$ with horizontal rectangular base $ABCD$
F is vertically above A

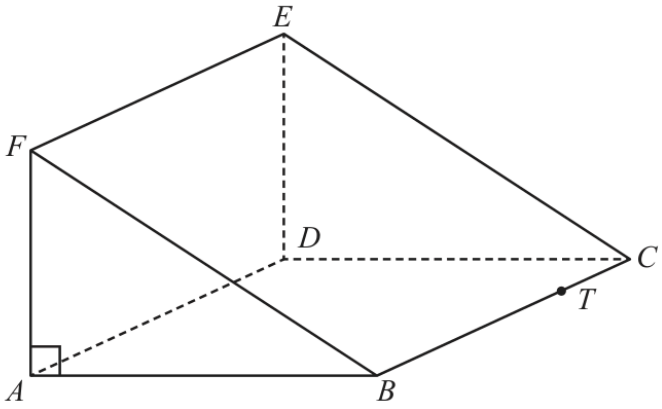


Diagram **NOT** accurately drawn

$AF = 12\,\text{cm}$ angle $ABF = 30°$ angle $BAF = 90°$

T is the point on the line BC such that $BF : BT = 3 : 2$

Work out the size of the angle between the line FT and the plane $ABCD$
Give your answer correct to one decimal place.
Show your working clearly.

.....°

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H June 2026

1 Solution | Step 1

$$BF = \frac{12}{\sin 30°} = 24\,\text{cm}, \qquad AB = \frac{12}{\tan 30°} = 12\sqrt{3}\,\text{cm}.$$

Use right triangle ABF.

2 Solution | Step 2

$$BT = \frac{2}{3}BF = 16\,\text{cm}.$$

Apply the given BF:BT ratio.

3 Solution | Step 3

$$AT = \sqrt{AB^2 + BT^2} = \sqrt{432 + 256} = \sqrt{688} = 4\sqrt{43}\ \text{cm}.$$

AB is perpendicular to BC in the rectangular base, so triangle ABT is right-angled at B.

4 Solution | Step 4

The projection of FT onto plane ABCD is AT because F lies vertically above A. Therefore the required angle is $\theta = \angle FTA$.

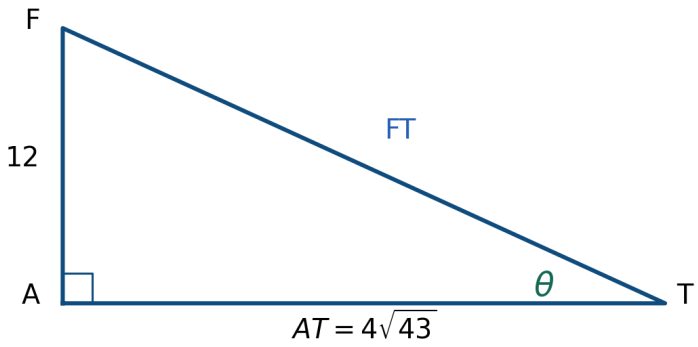
A line-plane angle is the angle between the line and its perpendicular projection onto the plane.

5 Solution | Step 5

$$\tan\theta = \frac{AF}{AT} = \frac{12}{4\sqrt{43}} = \frac{3}{\sqrt{43}}, \quad \theta = 24.583891\ldots^\circ = 24.6^\circ.$$

Use right triangle FAT and round to one decimal place.

FT projects onto AT in the base plane



FINAL ANSWER

24.6°

TOPIC

Sine & Cosine Rules

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 14

5 marks | Sine & Cosine Rules

14 The diagram shows parallelogram $EFGH$.

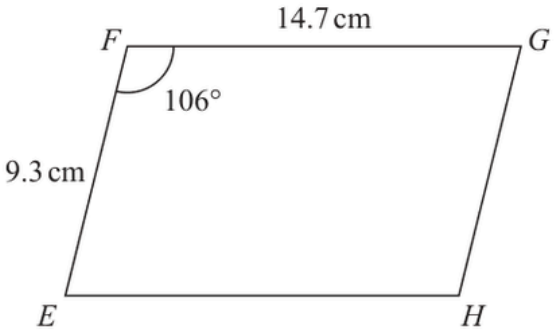


Diagram **NOT** accurately drawn

$EF = 9.3$ cm
$FG = 14.7$ cm
Angle $EFG = 106°$

(a) Work out the area of the parallelogram.
Give your answer correct to 3 significant figures.

..... cm^2
(2)

(b) Work out the length of the diagonal EG of the parallelogram.
Give your answer correct to 3 significant figures.

..... cm
(3)

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2020

1 **(a)**

(a)

$$\text{area} = 9.3 \times 14.7 \times \sin 106° = 131.414\ldots$$

2 **Answer**

Answer: $131\ cm^2$.

3 **Use trigonometry**

(b) In triangle EFG,

$$EG^2 = 9.3^2 + 14.7^2 - 2(9.3)(14.7)\cos 106°$$

$$EG = 19.440\ldots$$

✓ **FINAL ANSWER**

$19.4\ \text{cm}$.

4MA1/1HR JANUARY 2020

Question 21

6 marks | Sine & Cosine Rules

21 The diagram shows the prism $ABCDEF$ with cross section triangle ABC.

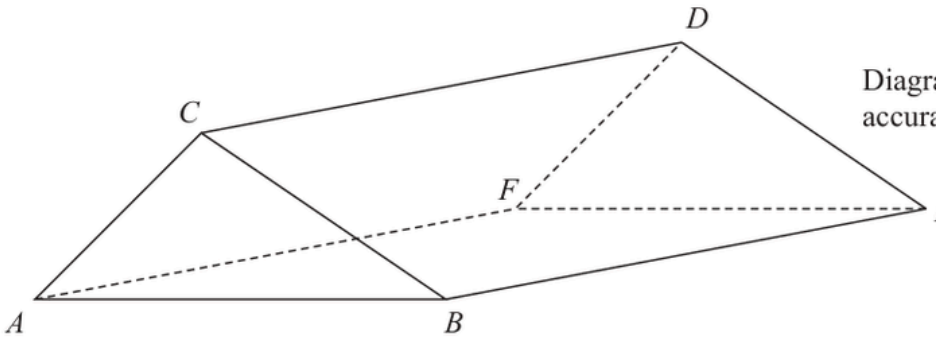


Diagram **NOT** accurately drawn

Angle $BEC = 40°$ and angle ACB is obtuse.
$AC = 6\,\text{cm}$ and $CE = 13\,\text{cm}$

The area of triangle ABC is $22\,\text{cm}^2$

Calculate the length of AB.
Give your answer correct to one decimal place.

..... cm

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2020

1 Use trigonometry

In triangle BEC, $CE = 13$ and $\angle BEC = 40°$.

2 Find the gradient

Since this is a prism, BE is perpendicular to the cross section, so triangle BEC is right-angled at B.

$$BC = 13 \sin 40°$$

$$BC = 8.356238\ldots$$

3 The area of triangle is

The area of triangle ABC is $22\ \text{cm}^2$.

4 **Use trigonometry**

Using

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$22 = \frac{1}{2}(AC)(BC)\sin(\angle ACB)$$

$$22 = \frac{1}{2}(6)(8.356238\ldots)\sin(\angle ACB)$$

$$\sin(\angle ACB) = 0.877587\ldots$$

---

5 **Use trigonometry**

The angle ACB is obtuse, so

$$\angle ACB = 180° - \sin^{-1}(0.877587\ldots)$$

$$\angle ACB = 118.647\ldots°$$

---

6 **Use the cosine rule**

Use the cosine rule:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos(\angle ACB)$$

$$AB^2 = 6^2 + (8.356238\ldots)^2 - 2(6)(8.356238\ldots)\cos(118.647\ldots°)$$

$$AB = 12.4056\ldots$$

---

7 **Correct to one decimal place**

Correct to one decimal place:

$$12.4$$

✓ **FINAL ANSWER**

$\boxed{12.4\text{ cm}}$

4MA1/2H NOVEMBER 2021

Question 18

5 marks | Sine & Cosine Rules

18 Here is a quadrilateral $ABCD$.

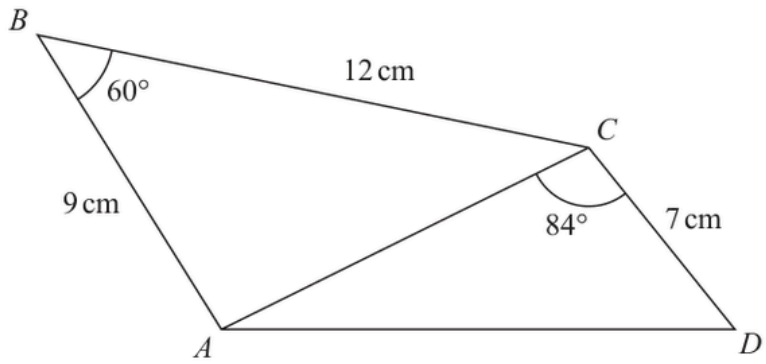


Diagram **NOT** accurately drawn

Calculate the area of quadrilateral $ABCD$.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2021

1 Area of triangle

Area of triangle ABC:

$$\frac{1}{2}(9)(12)\sin 60° = 46.765\ldots$$

2 Find using the cosine rule

Find AC using the cosine rule:

$$AC^2 = 9^2 + 12^2 - 2(9)(12)\cos 60°$$

$$AC = 10.816\ldots$$

3 Area of triangle

Area of triangle ACD:

$$\frac{1}{2}(10.816\ldots)(7)\sin 84° = 37.642\ldots$$

4 Total area

Total area:

$$46.765\ldots + 37.642\ldots = 84.407\ldots$$

FINAL ANSWER

$84.4\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 9

4 marks | Sine & Cosine Rules

9 The diagram shows isosceles triangle ABC

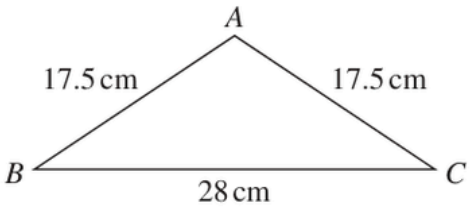


Diagram **NOT** accurately drawn

$AB = AC = 17.5$ cm $\quad\quad$ $BC = 28$ cm

Calculate the area of triangle ABC

..... cm^2

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2022

1 **Use cosine rule**

Split the isosceles triangle into two right-angled triangles.

$$\frac{BC}{2} = 14$$

$$h^2 = 17.5^2 - 14^2 = 110.25$$

$$h = 10.5$$

$$\text{area} = \frac{1}{2} \times 28 \times 10.5 = 147$$

FINAL ANSWER

$147\ \text{cm}^2$.

4MA1/2HR JANUARY 2022

Question 16

5 marks | Sine & Cosine Rules

16 The diagram shows quadrilateral $ABCD$

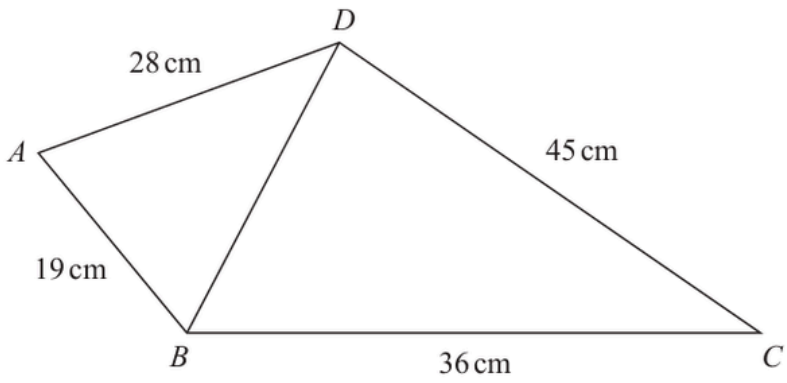


Diagram **NOT** accurately drawn

The angle BCD is acute.

Given that the area of triangle $BCD = 405\,\text{cm}^2$

work out the size of angle ABD
Give your answer correct to one decimal place.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR January 2022

1 **Use trigonometry**

For triangle BCD,

$$405 = \frac{1}{2}(36)(45)\sin BCD$$

$$\sin BCD = 0.5$$

2 **Use trigonometry**

The angle BCD is acute, so

$$BCD = 30°$$

3 **Find**

Find BD:

$$BD^2 = 36^2 + 45^2 - 2(36)(45)\cos 30°$$

$$BD = 22.695\ldots$$

---

4 **Use trigonometry**

Now use cosine rule in triangle ABD:

$$\cos ABD = \frac{19^2 + BD^2 - 28^2}{2(19)(BD)}$$

$$ABD = 83.871\ldots°$$

✓ **FINAL ANSWER**

$83.9°$.

4MA1/2H JANUARY 2022

Question 23

6 marks | Sine & Cosine Rules

23 The diagram shows triangle PQR

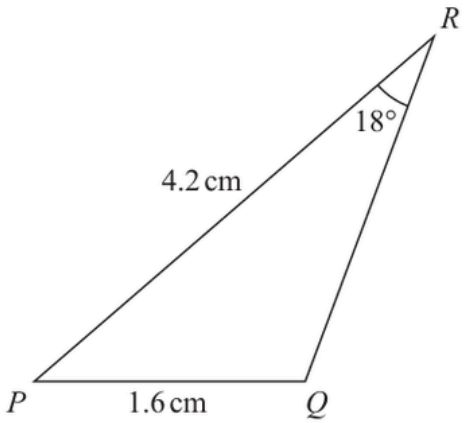


Diagram **NOT** accurately drawn

$PQ = 1.6\,\text{cm}$ $PR = 4.2\,\text{cm}$ Angle $PRQ = 18°$

Given that angle PQR is obtuse,

work out the area of triangle PQR
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2022

1 **Use trigonometry**

In triangle PQR,

$$PQ = 1.6, \quad PR = 4.2, \quad \angle PRQ = 18°$$

2 **Use the sine rule**

Use the sine rule:

$$\frac{\sin Q}{PR} = \frac{\sin R}{PQ}$$

$$\frac{\sin Q}{4.2} = \frac{\sin 18^\circ}{1.6}$$

$$\sin Q = \frac{4.2 \sin 18^\circ}{1.6}$$

$$\sin Q = 0.811169\ldots$$

---

3 **Use trigonometry**

Since angle PQR is obtuse,

$$Q = 180^\circ - \sin^{-1}(0.811169\ldots)$$

$$Q = 125.7896\ldots^\circ$$

---

4 **Use trigonometry**

Now find angle P:

$$P = 180^\circ - 18^\circ - 125.7896\ldots^\circ$$

$$P = 36.2103\ldots^\circ$$

---

5 **Area of triangle**

Area of triangle PQR:

$$\frac{1}{2}(PQ)(PR)\sin P$$

$$= \frac{1}{2}(1.6)(4.2)\sin(36.2103\ldots^\circ)$$

$$= 1.984925\ldots$$

---

6 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1.98$$

FINAL ANSWER

$\boxed{1.98\ \text{cm}^2}$

4MA1/2HR JUNE 2022

Question 15

3 marks | Sine & Cosine Rules

15 The diagram shows a kite $ABCD$

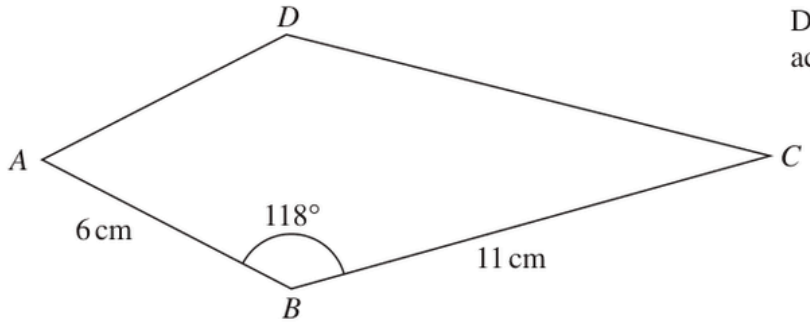


Diagram **NOT** accurately drawn

$AB = 6\,\text{cm}$ $BC = 11\,\text{cm}$ Angle $ABC = 118°$

Calculate the area of the kite.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2022

1 **A kite can be split**

A kite can be split into two congruent triangles.

2 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 11 \times \sin 118°$$

3 **Use trigonometry**

So the area of the kite is

$$2 \times \frac{1}{2} \times 6 \times 11 \times \sin 118°$$

$$= 6 \times 11 \times \sin 118° = 58.274\ldots$$

FINAL ANSWER

$58.3\ \text{cm}^2$.

4MA1/1HR JUNE 2022

Question 17

5 marks | Sine & Cosine Rules

17 Here is a parallelogram $PQRS$, in which angle SPQ is acute.

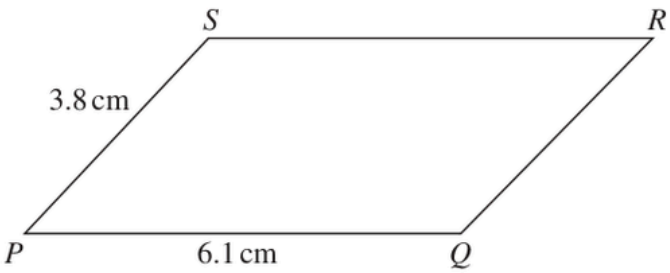


Diagram **NOT** accurately drawn

$PQ = 6.1\,\text{cm}$ $PS = 3.8\,\text{cm}$

The area of the parallelogram is $18\,\text{cm}^2$

Work out the length of QS
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2022

1 Height of the parallelogram

Height of the parallelogram:

$$h = \frac{18}{6.1} = 2.9508\ldots$$

2 Use cosine rule

The horizontal offset from P to S is

$$\sqrt{3.8^2 - h^2} = \sqrt{3.8^2 - 2.9508\ldots^2} = 2.394\ldots$$

3 Use trigonometry

So the horizontal distance from S to Q is

$$6.1 - 2.394\ldots = 3.705\ldots$$

$$QS = \sqrt{3.705\ldots^2 + 2.9508\ldots^2} = 4.737\ldots$$

FINAL ANSWER

4.74 cm.

4MA1/1H JUNE 2022

Question 18

5 marks | Sine & Cosine Rules

18 Here is triangle ABC

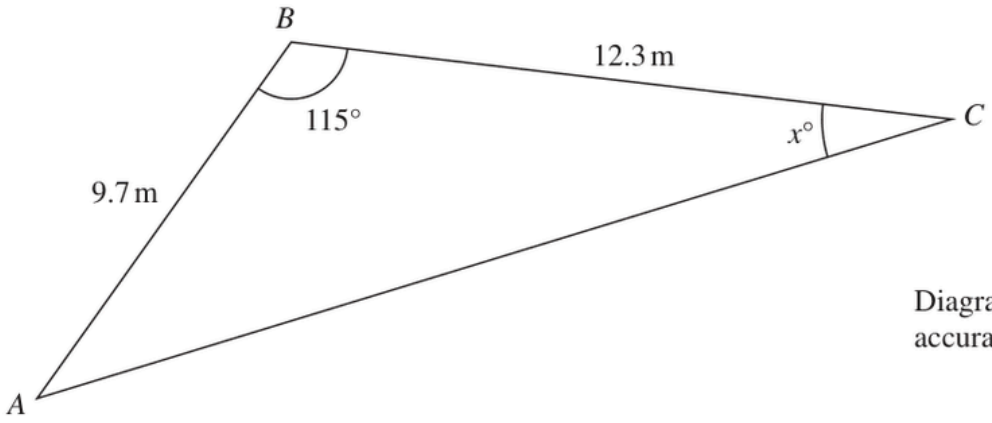


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2022

1 Use trigonometry

First use the cosine rule to find AC:

$$AC^2 = 9.7^2 + 12.3^2 - 2(9.7)(12.3)\cos 115°$$

$$AC = 18.607\ldots$$

2 Use trigonometry

Now use the sine rule:

$$\frac{\sin x}{9.7} = \frac{\sin 115°}{18.607\ldots}$$

$$\sin x = \frac{9.7 \sin 115°}{18.607\ldots}$$

$$x = 28.194\ldots°$$

FINAL ANSWER

$28.2°$.

4MA1/2H JUNE 2023

Question 22

5 marks | Sine & Cosine Rules

22 The diagram shows a triangle ABC and a flagpole BF

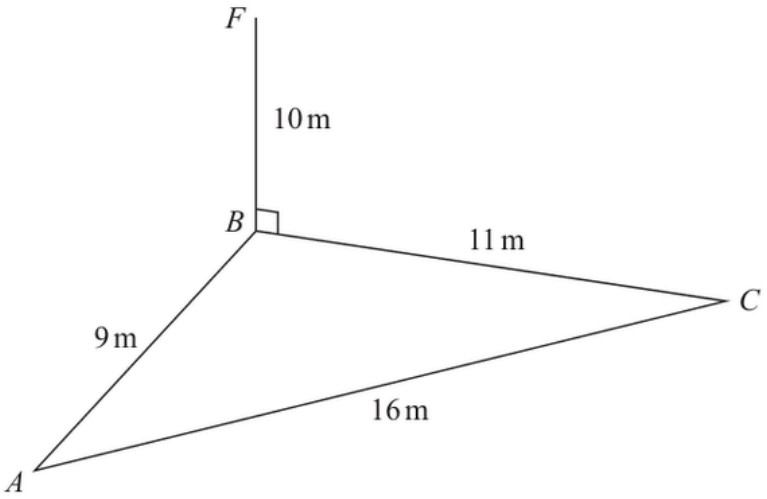


Diagram **NOT** accurately drawn

A, B and C are points on horizontal ground.

BF is vertical.

$AB = 9\,\text{m}$ $BC = 11\,\text{m}$ $AC = 16\,\text{m}$ $BF = 10\,\text{m}$

D is the point on AC such that angle $BDC = 90°$

Work out the size of the angle of elevation of the point F from the point D
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2023

1 **Use trigonometry**

In triangle ABC,

$$AB = 9, \qquad BC = 11, \qquad AC = 16$$

2 **Use Heron s formula**

Use Heron's formula.

$$s = \frac{9 + 11 + 16}{2} = 18$$

$$\text{area} = \sqrt{18(18 - 9)(18 - 11)(18 - 16)}$$

$$= \sqrt{18 \cdot 9 \cdot 7 \cdot 2} = 47.6235\ldots$$

3 **is the foot of the**

D is the foot of the perpendicular from B to AC, so BD is the height of triangle ABC.

$$\text{area} = \frac{1}{2} \times AC \times BD$$

$$47.6235\ldots = \frac{1}{2} \times 16 \times BD$$

$$BD = 5.9529\ldots$$

4 **Use trigonometry**

In right-angled triangle BDF,

$$\tan\theta = \frac{BF}{BD}$$

$$\tan\theta = \frac{10}{5.9529\ldots}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.9529\ldots}\right) = 59.2349\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{59.2^\circ}$

4MA1/1HR MAY 2023

Question 25

6 marks | Sine & Cosine Rules

25 Here is a triangle ABC

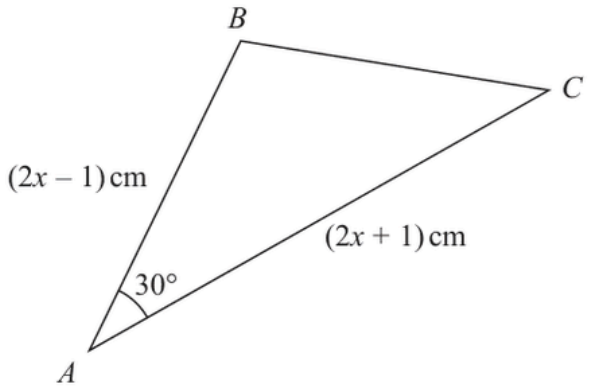


Diagram **NOT** accurately drawn

The area of the triangle is $(x^2 + x - 3.75)\,\text{cm}^2$

Find the size of the largest angle in triangle ABC
Give your answer correct to the nearest degree.

.....°

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR May 2023

1 The two sides around the

The two sides around the $30°$ angle are

$$2x - 1 \quad \text{and} \quad 2x + 1$$

2 Use trigonometry

Using the area formula,

$$\text{area} = \frac{1}{2}ab\sin C$$

$$x^2 + x - 3.75 = \frac{1}{2}(2x - 1)(2x + 1)\sin 30°$$

3 **Use trigonometry**

Since $\sin 30° = \frac{1}{2}$,

$$x^2 + x - 3.75 = \frac{1}{4}(2x - 1)(2x + 1)$$

$$x^2 + x - 3.75 = \frac{1}{4}(4x^2 - 1)$$

$$x^2 + x - 3.75 = x^2 - 0.25$$

$$x = 3.5$$

---

4 **Use trigonometry**

So

$$AB = 2x - 1 = 6, \qquad AC = 2x + 1 = 8$$

---

5 **Use cosine rule**

Use the cosine rule to find BC:

$$BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 30°$$

$$BC = 4.1063\ldots$$

---

6 **Use trigonometry**

The largest side is $AC = 8$, so the largest angle is $\angle B$.

---

7 **Use trigonometry**

Using the cosine rule again:

$$\cos B = \frac{6^2 + BC^2 - 8^2}{2(6)(BC)}$$

$$B = 103.0643\ldots°$$

✓ **FINAL ANSWER**

$\boxed{103°}$

4MA1/2HR JUNE 2024

Question 15

3 marks | Sine & Cosine Rules

15

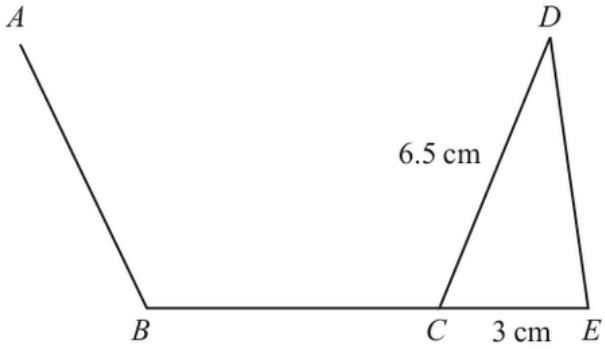


Diagram **NOT** accurately drawn

AB, *BC* and *CD* are three sides of a regular pentagon and *CDE* is a triangle.
BCE is a straight line.

$CD = 6.5\,\text{cm}$ $CE = 3\,\text{cm}$

Work out the area of triangle *CDE*
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2024

1 **A regular pentagon has interior**

A regular pentagon has interior angle

$$\frac{(5-2)\times 180}{5} = 108^\circ$$

2 **Use trigonometry**

Since BCE is a straight line,

$$\angle DCE = 180^\circ - 108^\circ = 72^\circ$$

3 **Area of triangle is**

Area of triangle CDE is

$$\frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}(6.5)(3)\sin 72°$$

$$\text{Area} = 9.27\ldots$$

✓ **FINAL ANSWER**

$9.27\ \text{cm}^2$ to 3 significant figures.

4MA1/1HR MAY 2024

Question 15

3 marks | Sine & Cosine Rules

15 The diagram shows isosceles triangle EFG

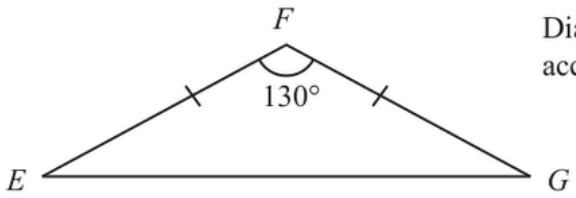


Diagram **NOT** accurately drawn

$EF = GF$
Angle $EFG = 130°$
The area of triangle EFG is 74 cm²

Work out the length of EF
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR May 2024

1 Let

Let

$$EF = FG = x$$

2 Use the area formula

Use the area formula:

$$74 = \frac{1}{2}x^2 \sin 130°$$

$$x^2 = \frac{148}{\sin 130°}$$

$$x = 13.899\ldots$$

FINAL ANSWER

13.9 cm.

4MA1/1H MAY 2024

Question 19

5 marks | Sine & Cosine Rules

19 Here is quadrilateral $ABCD$

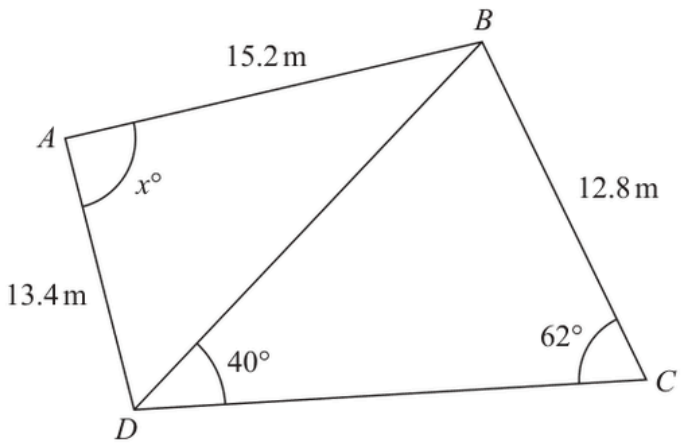


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2024

1 **Use trigonometry**

In triangle BCD,

$$\angle DBC = 180° - 40° - 62° = 78°$$

2 **Use trigonometry**

By the sine rule,

$$\frac{DB}{\sin 62°} = \frac{12.8}{\sin 40°}$$

$$DB = 17.582\ldots$$

3 Use trigonometry

Now use cosine rule in triangle ABD:

$$\cos x = \frac{15.2^2 + 13.4^2 - 17.582\ldots^2}{2(15.2)(13.4)}$$

$$x = 75.577\ldots^\circ$$

FINAL ANSWER

75.6°.

Question 20

5 marks | Sine & Cosine Rules

20

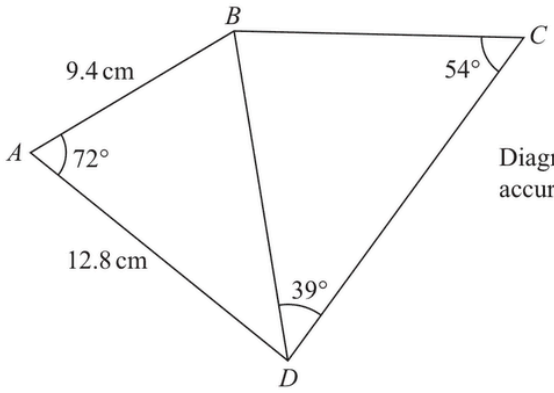


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

1 **Use trigonometry**

First use triangle ABD.

$$AB = 9.4, \quad AD = 12.8, \quad \angle BAD = 72^\circ$$

2 **Use cosine rule**

Use the cosine rule to find BD:

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72^\circ$$

$$BD = 13.3356\ldots$$

3 **Use trigonometry**

In triangle BCD,

$$\angle BDC = 39^\circ, \quad \angle BCD = 54^\circ$$

4 **Use the sine rule**

Use the sine rule:

$$\frac{BC}{\sin 39^\circ} = \frac{BD}{\sin 54^\circ}$$

$$BC = \frac{13.3356\ldots \sin 39^\circ}{\sin 54^\circ}$$

$$BC = 10.3735\ldots$$

5 **Use trigonometry**

Correct to 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

10.4 cm

4WM1/01 MAY 2025

Question 21

5 marks | Sine & Cosine Rules

21

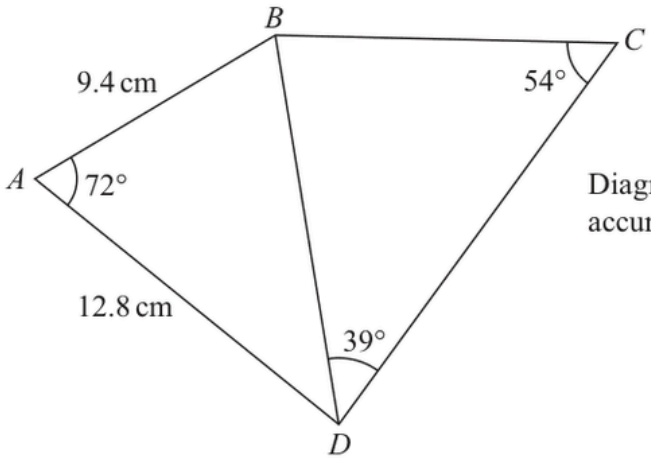


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 May 2025

1 **Use trigonometry**

First find BD in triangle ABD using the cosine rule.

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

2 **Use trigonometry**

In triangle BCD, use the sine rule.

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = 13.3356\ldots \times \frac{\sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

3 **Use trigonometry**

To 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

$10.4\ \text{cm}$

4WM1/01 NOVEMBER 2025

Question 19

3 marks | Sine & Cosine Rules

19 The diagram shows triangle ABC

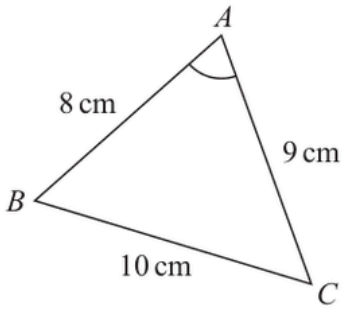


Diagram **NOT** accurately drawn

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$ $CA = 9\,\text{cm}$

Work out the size of angle BAC
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM1/01 November 2025

1 Use cosine rule

Use the cosine rule for angle BAC:

$$\cos A = \frac{8^2 + 9^2 - 10^2}{2(8)(9)}$$

$$\cos A = \frac{45}{144}$$

$$A = 71.790\ldots^\circ$$

FINAL ANSWER

71.8°.

4MA1/1H NOVEMBER 2025

Question 19

3 marks | Sine & Cosine Rules

19 The diagram shows triangle ABC

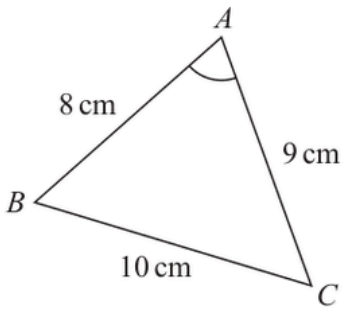


Diagram **NOT** accurately drawn

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$ $CA = 9\,\text{cm}$

Work out the size of angle BAC
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2025

1 Use cosine rule

Use the cosine rule for angle BAC:

$$\cos A = \frac{8^2 + 9^2 - 10^2}{2(8)(9)}$$

$$\cos A = \frac{45}{144}$$

$$A = 71.790\ldots^\circ$$

FINAL ANSWER

71.8°.

Question 15

5 marks | Sine & Cosine Rules

15 The diagram shows triangle ABC

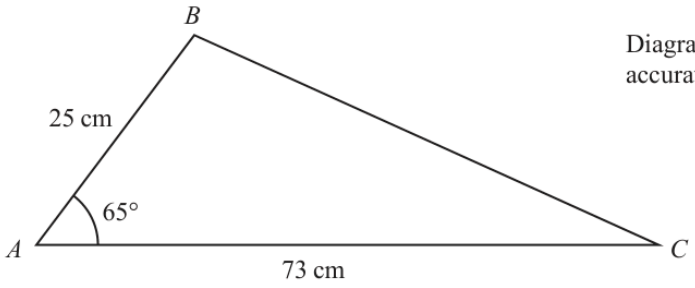


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad AC = 73$ cm $\quad$ angle $BAC = 65°$

(a) Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(3)

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 15 is 5 marks)

1 **(a) Cosine rule**

$$BC^2 = 25^2 + 73^2 - 2(25)(73)\cos 65° = 4411.4\ldots$$

2 **Square root**

$$BC = 66.418\ldots$$

3 **(b) Area formula**

$$\tfrac{1}{2}(25)(73)\sin 65° = 827.005\ldots$$

✓ **FINAL ANSWER**

(a) $\mathbf{66.4}$ cm (b) $\mathbf{827}$ cm^2

4MA1/1H JUNE 2026

Question 16

5 marks | Sine & Cosine Rules

16 The diagram shows triangle ABC

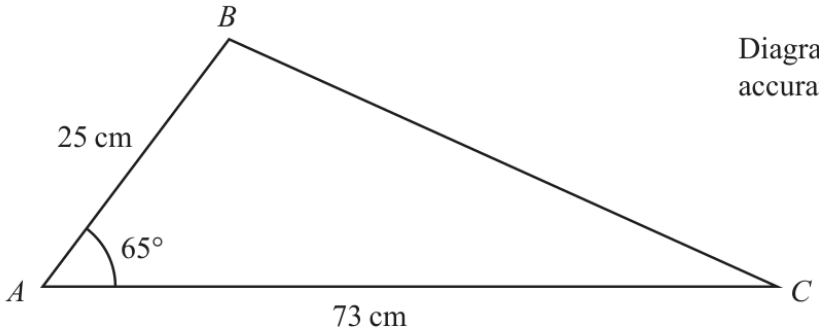


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad AC = 73$ cm $\quad$ angle $BAC = 65°$

(a) Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(3)

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$BC^2 = 25^2 + 73^2 - 2(25)(73)\cos 65°.$$

The included angle is between the two given sides.

2 **Part (a) | Step 2**

$$BC^2 = 4411.443344\ldots, \quad BC = \sqrt{4411.443344\ldots} = 66.418697\ldots = 66.4 \text{ cm} \quad (3 \text{ s.f.}).$$

Take the positive square root for a length.

3 **Part (b) | Step 1**

$$\text{Area} = \frac{1}{2}(25)(73)\sin 65^\circ = 827.005855\ldots = 827\ \text{cm}^2 \quad (3\ \text{s.f.}).$$

Use half the product of the two sides and the sine of their included angle.

✓ **FINAL ANSWER**

(a) $66.4\ \text{cm}$ (b) $827\ \text{cm}^2$

4MA1/1HR JUNE 2026

Question 22

4 marks | Sine & Cosine Rules

22 The diagram shows triangle ABC

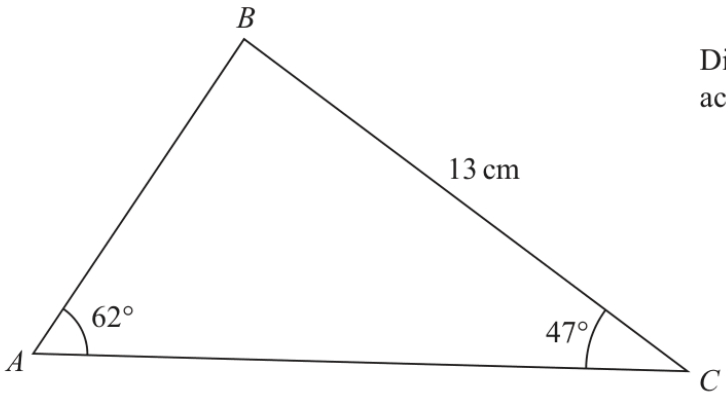


Diagram **NOT** accurately drawn

Calculate the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$\angle ABC = 180^\circ - 62^\circ - 47^\circ = 71^\circ.$$

The three triangle angles sum to 180 degrees.

2 **Solution | Step 2**

$$\frac{AB}{\sin 47^\circ} = \frac{13}{\sin 62^\circ} \quad \Longrightarrow \quad AB = \frac{13 \sin 47^\circ}{\sin 62^\circ} = 10.7680208859\ldots \text{ cm}.$$

Match each side to its opposite angle in the sine rule.

3 **Solution | Step 3**

$$\text{Area} = \frac{1}{2}(13)\left(\frac{13 \sin 47^\circ}{\sin 62^\circ}\right)\sin 71^\circ = 66.1788645053\ldots \text{ cm}^2.$$

Use half the product of two sides times the sine of their included angle; retain unrounded values.

4 **Solution | Step 4**

$$\text{Area} = 66.2\ \text{cm}^2 \quad (3\ \text{significant figures}).$$

Round only the final area.

✓ **FINAL ANSWER**

$66.2\ \text{cm}^2$

TOPIC

Congruent Shapes

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

Question 12

3 marks | Congruent Shapes

12 The diagram shows two congruent isosceles triangles and parts of two congruent regular polygons, **X** and **Y**.

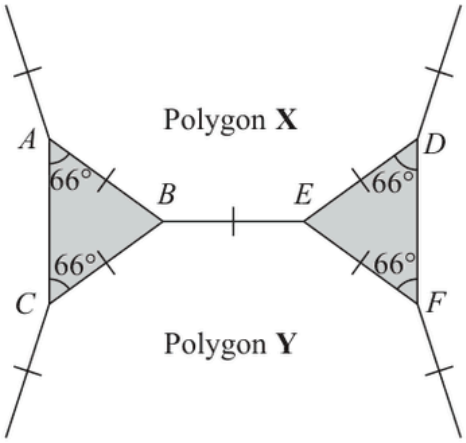


Diagram **NOT** accurately drawn

The two regular polygons each have n sides.

Work out the value of n.

$n =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2020

1 **Each congruent isosceles triangle has**

Each congruent isosceles triangle has two angles of $66°$.

2 **Use trigonometry**

So the angle at the point between the two polygons is

$$180° - 66° - 66° = 48°$$

3 **The two regular polygons have**

The two regular polygons have equal interior angles. Around the point:

$$48^\circ + I + I = 360^\circ$$

$$2I = 312^\circ$$

$$I = 156^\circ$$

4 **The exterior angle is**

The exterior angle is

$$180^\circ - 156^\circ = 24^\circ$$

$$n = \frac{360}{24} = 15$$

✓ **FINAL ANSWER**

$n = 15$.

TOPIC

Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2021

Question 4

4 marks | Similar Shapes

4 ABC and DEF are similar triangles.

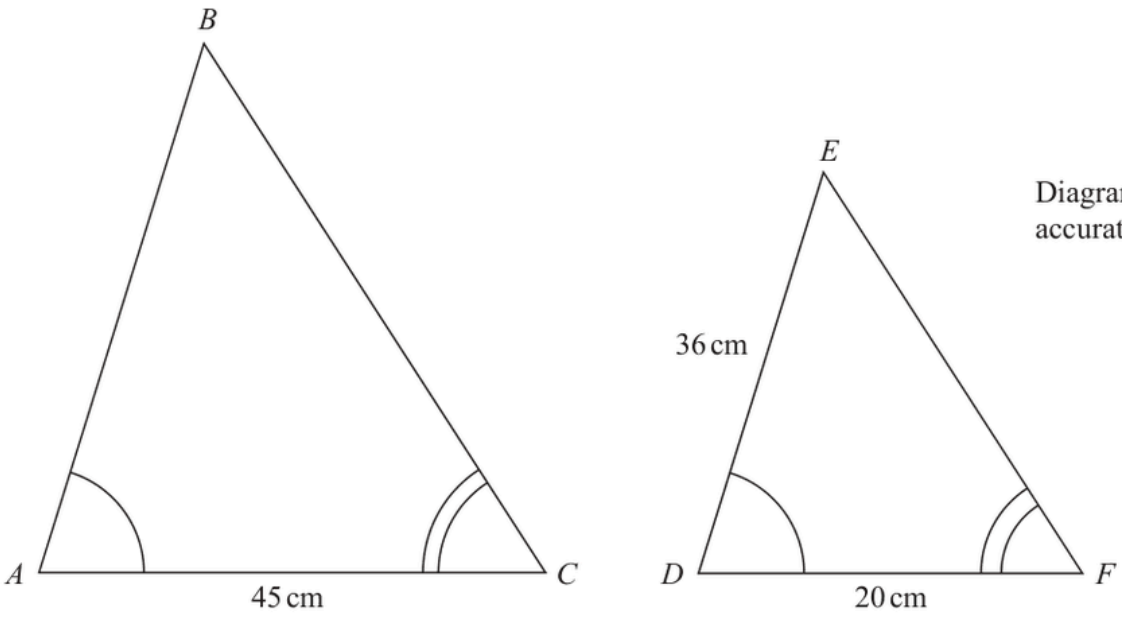


Diagram **NOT** accurately drawn

(a) Work out the length of AB.

..... cm

(2)

Given that $BC = 54$ cm,

(b) work out the length of EF.

..... cm

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2021

1 **The scale factor from triangle**

The scale factor from triangle DEF to triangle ABC is

$$\frac{AC}{DF} = \frac{45}{20} = 2.25$$

$$AB = DE \times 2.25 = 36 \times 2.25 = 81$$

2 **Evaluate fraction**

Given $BC = 54$,

$$EF = \frac{54}{2.25} = 24$$

✓ **FINAL ANSWER**

$AB = 81\ \text{cm}$, and $EF = 24\ \text{cm}$.

4MA1/2H JANUARY 2022

Question 3

3 marks | Similar Shapes

3

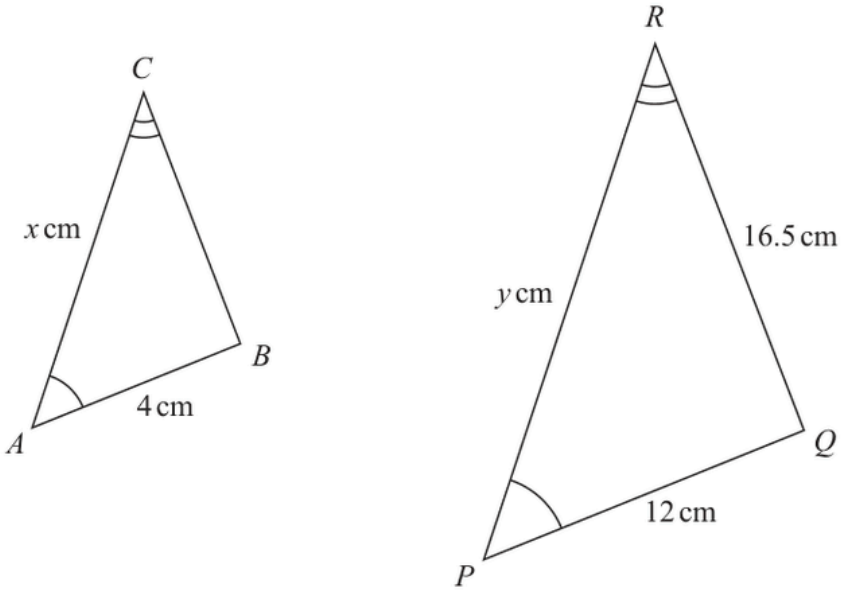


Diagram **NOT** accurately drawn

Triangle ABC is similar to triangle PQR

$AB = 4\,\text{cm}$ $PQ = 12\,\text{cm}$ $RQ = 16.5\,\text{cm}$ $AC = x\,\text{cm}$ $PR = y\,\text{cm}$

(a) Calculate the length of BC

..... cm

(2)

(b) Write down an expression for y in terms of x

$y =$ .....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2022

1 **The scale factor from triangle**

The scale factor from triangle ABC to triangle PQR is

$$\frac{PQ}{AB} = \frac{12}{4} = 3$$

2 **Evaluate fraction**

So

$$BC = \frac{RQ}{3} = \frac{16.5}{3} = 5.5$$

3 **Calculate value**

Also PR corresponds to AC, so

$$y = 3x$$

✓ **FINAL ANSWER**

$BC = 5.5\ \text{cm}$, and $y = 3x$.

4MA1/2HR JUNE 2022

Question 12

3 marks | Similar Shapes

12

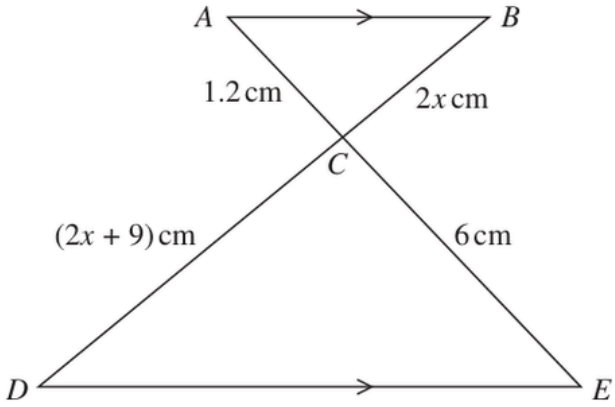


Diagram **NOT** accurately drawn

ACE and *BCD* are straight lines.
AB is parallel to *DE*

Work out the value of x

$x =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2022

1 Find the gradient

Since AB is parallel to DE, triangles ACB and ECD are similar.

2 Split the ratio

So corresponding sides are in the same ratio:

$$\frac{AC}{CE} = \frac{BC}{CD}$$

3 Substitute the lengths

Substitute the lengths:

$$\frac{1.2}{6} = \frac{2x}{2x+9}$$

$$\frac{1}{5} = \frac{2x}{2x+9}$$

$$2x + 9 = 10x$$

$$8x = 9$$

$$x = \frac{9}{8}$$

FINAL ANSWER

$x = \frac{9}{8}$.

4MA1/1HR MAY 2023

Question 3

4 marks | Similar Shapes

3 $ABCD$ and $EFGH$ are similar quadrilaterals.

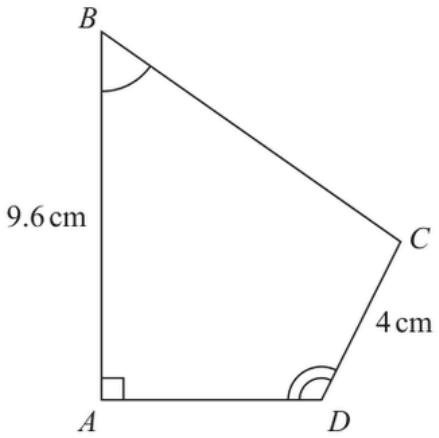
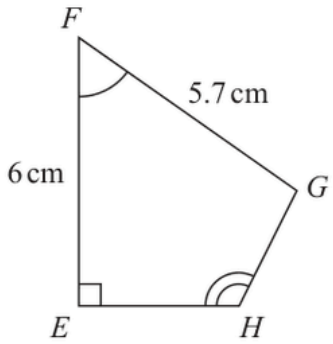


Diagram **NOT** accurately drawn



(a) Work out the length of GH

..... cm

(2)

(b) Work out the length of BC

..... cm

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May 2023

1 **Evaluate fraction**

The scale factor from $EFGH$ to $ABCD$ is

$$\frac{9.6}{6} = 1.6$$

2 **Evaluate fraction**

Since GH corresponds to CD,

$$GH = \frac{4}{1.6} = 2.5$$

3 **Calculate value**

Since FG corresponds to BC,

$$BC = 5.7 \times 1.6 = 9.12$$

✓ **FINAL ANSWER**

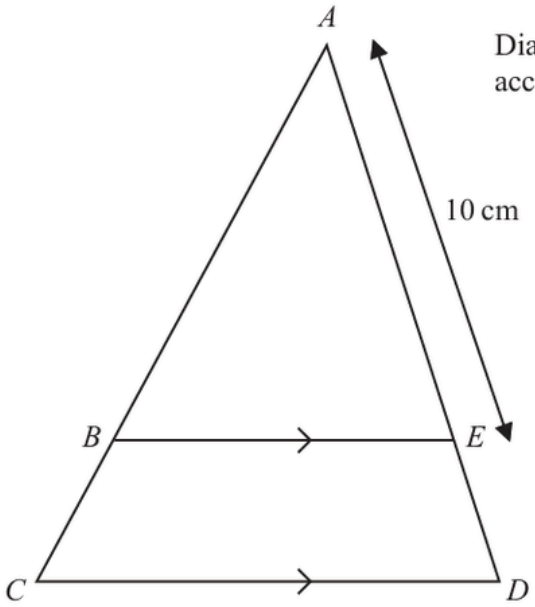
$GH = 2.5\ \text{cm}$, and $BC = 9.12\ \text{cm}$.

4MA1/2HR JUNE 2024

Question 12

4 marks | Similar Shapes

12



In the diagram, ABC and AED are straight lines.
BE is parallel to CD

$$AE = 10\,\text{cm} \qquad \text{and} \qquad CD = 1.5 \times BE$$

(a) Work out the length of ED

..... cm

(2)

$AB = (2x + 5)\,\text{cm}$ and $BC = (3x - 5)\,\text{cm}$

(b) Work out the value of x

$x =$ .....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2024

1 **Find the gradient**

Since $BE \parallel CD$, triangles ABE and ACD are similar.

2 **Find the gradient**

Also

$$CD = 1.5BE$$

so

$$\frac{BE}{CD} = \frac{1}{1.5} = \frac{2}{3}$$

---

3 **Find the gradient**

Therefore

$$\frac{AE}{AD} = \frac{2}{3}$$

---

4 **Find the gradient**

Given $AE = 10$,

$$\frac{10}{AD} = \frac{2}{3}$$

$$AD = 15$$

---

5 **Find the gradient**

So

$$ED = AD - AE = 15 - 10 = 5$$

---

6 **Find the gradient**

For part (b),

$$AB = 2x + 5, \qquad BC = 3x - 5$$

$$AC = AB + BC = 5x$$

---

7 **Find the gradient**

Using similarity,

$$\frac{AB}{AC} = \frac{2}{3}$$

$$\frac{2x+5}{5x} = \frac{2}{3}$$

$$3(2x+5) = 10x$$

$$6x + 15 = 10x$$

$$x = 3.75$$

✓ **FINAL ANSWER**

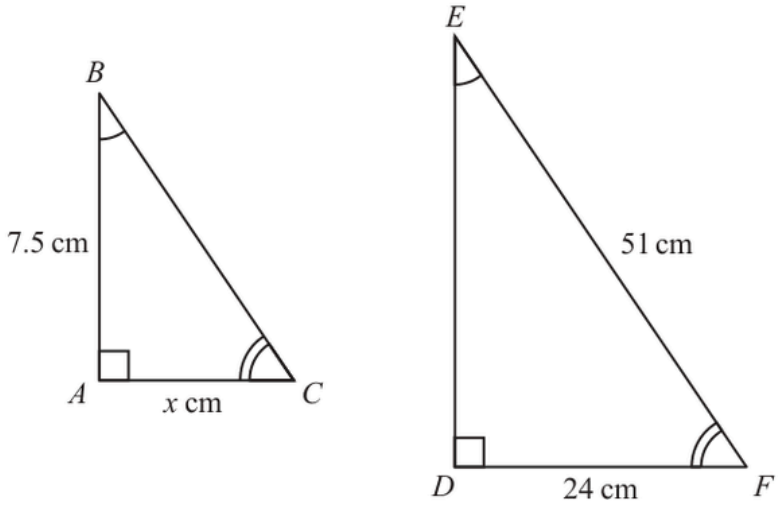
(a) $ED = 5\ \text{cm}$, (b) $x = 3.75$.

4MA1/1H NOVEMBER 2024

Question 10

5 marks | Similar Shapes

10 Here are two similar triangles.



Diagrams **NOT** accurately drawn

Work out the value of x
Show your working clearly.

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2024

1 Use trigonometry

In the larger triangle,

$$DE^2 = 51^2 - 24^2$$

$$DE = 45$$

2 The scale factor from the

The scale factor from the large triangle to the small triangle is

$$\frac{7.5}{45} = \frac{1}{6}$$

3 Evaluate fraction

So

$$x = 24 \times \frac{1}{6} = 4$$

FINAL ANSWER

$x = 4$.

4MA1/2H JUNE 2025

Question 5

4 marks | Similar Shapes

5 The diagram shows two similar quadrilaterals, $ABCD$ and $EFGH$

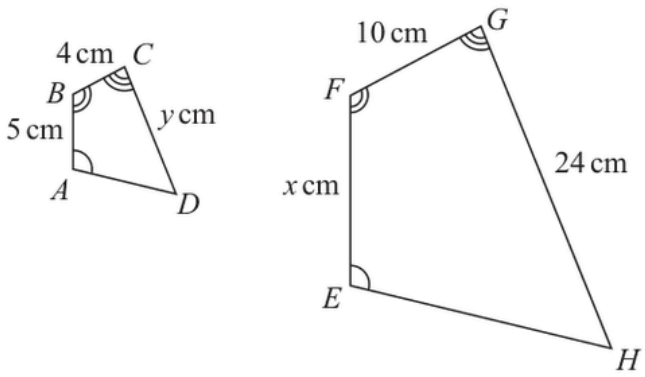


Diagram **NOT** accurately drawn

$AB = 5\,\text{cm}$ $BC = 4\,\text{cm}$ $CD = y\,\text{cm}$

$EF = x\,\text{cm}$ $FG = 10\,\text{cm}$ $GH = 24\,\text{cm}$

(a) Work out the value of x

$x =$ .....

(2)

(b) Work out the value of y

$y =$ .....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2025

1 **Evaluate fraction**

The scale factor from $ABCD$ to $EFGH$ is

$$\frac{FG}{BC} = \frac{10}{4} = 2.5$$

2 **Calculate value**

Since AB corresponds to EF,

$$x = EF = 5 \times 2.5 = 12.5$$

3 **Calculate value**

Since CD corresponds to GH,

$$2.5y = 24$$

$$y = 9.6$$

✓ **FINAL ANSWER**

$x = 12.5$, $y = 9.6$.

4WM2/01 MAY 2025

Question 6

4 marks | Similar Shapes

6 The diagram shows two similar quadrilaterals, $ABCD$ and $EFGH$

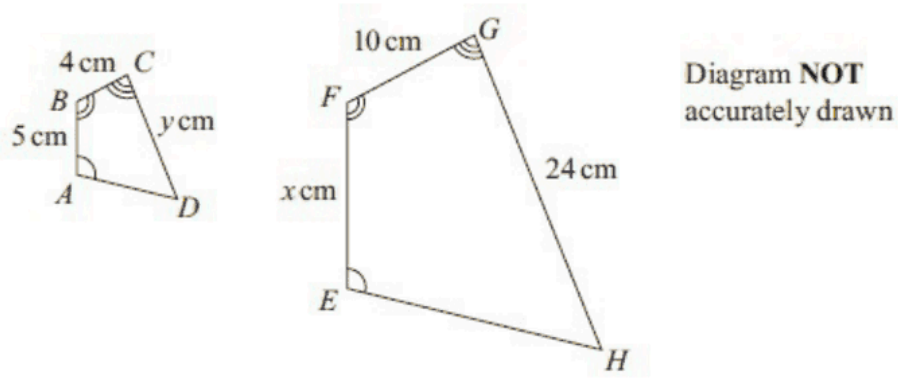


Diagram **NOT** accurately drawn

$AB = 5$ cm $\quad BC = 4$ cm $\quad CD = y$ cm

$EF = x$ cm $\quad FG = 10$ cm $\quad GH = 24$ cm

(a) Work out the value of x

$x =$ .....

(2)

(b) Work out the value of y

$y =$ .....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 May 2025

1 **Evaluate fraction**

The scale factor from $ABCD$ to $EFGH$ is

$$\frac{FG}{BC} = \frac{10}{4} = 2.5$$

2 **Calculate value**

So

$$x = EF = 5 \times 2.5 = 12.5$$

3 **Evaluate fraction**

and

$$y = CD = \frac{24}{2.5} = 9.6$$

✓ **FINAL ANSWER**

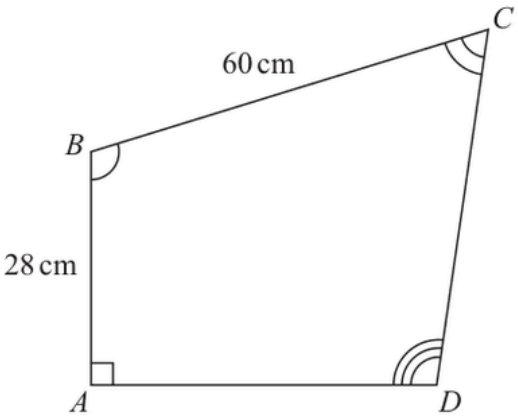
$x = 12.5$, $y = 9.6$.

4MA1/2H NOVEMBER 2025

Question 4

3 marks | Similar Shapes

4 Here are three similar quadrilaterals.



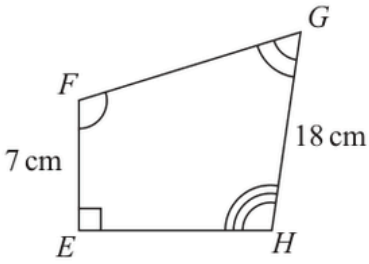
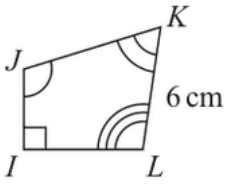


Diagram **NOT** accurately drawn



Work out the length of JK

..... cm

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2025

1 The scale factor from the

The scale factor from the largest quadrilateral to the middle quadrilateral is

$$\frac{7}{28} = \frac{1}{4}$$

2 Evaluate fraction

So the side corresponding to 60 cm in the middle shape is

$$60 \times \frac{1}{4} = 15$$

3 The scale factor from the

The scale factor from the middle quadrilateral to the smallest quadrilateral is

$$\frac{6}{18} = \frac{1}{3}$$

$$JK = 15 \times \frac{1}{3} = 5$$

FINAL ANSWER

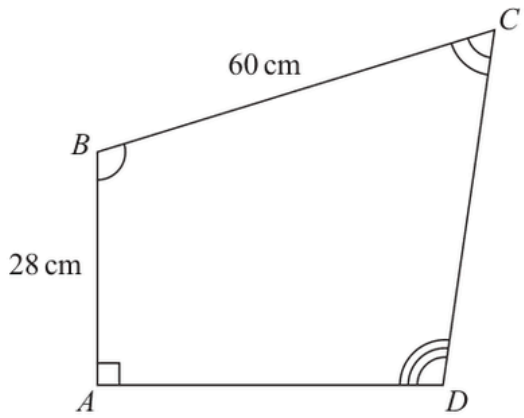
5 cm.

4WM2/01 NOVEMBER 2025

Question 5

3 marks | Similar Shapes

5 Here are three similar quadrilaterals.



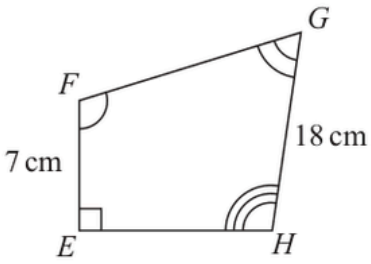
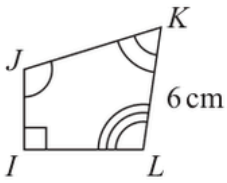


Diagram **NOT** accurately drawn



Work out the length of JK

..... cm

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM2/01 November 2025

1 Identify matching sides

Step 1: Identify corresponding sides from the similar shapes. Looking at the given angles/orientations:

2 Left side A B corresponds

Left side *A**B* corresponds to *E**F* and *I**J*.

3 Top side B C corresponds

Top side *B**C* corresponds to *F**G* and *J**K*.

4 Right side C D corresponds

Right side *C**D* corresponds to *G**H* and *K**L*.

5 **We are given C D**

We are given *C**D* = 60 (wait, diagram check: *B**C* is the top side labeled 60, *A**B* = 28. On middle shape *E**F* = 7, *G**H* = 18. On right shape *K**L* = 6, *J**K* is unknown). Let's use the explicit lengths:

Misplaced &

---

6 **Find scale factors**

Step 2: Find scale factors. Scale factor from *A**B**C**D* to *E**F**G**H* using the left sides:

$$\text{SF}_1 = \frac{EF}{AB} = \frac{7}{28} = \frac{1}{4}$$

So, $FG = BC \times \frac{1}{4} = 60 \times \frac{1}{4} = 15\text{cm}$.

---

7 **Scale factor from E F**

Scale factor from *E**F**G**H* to *I**J**K**L* using the right sides (*G**H* and *K**L*):

$$\text{SF}_2 = \frac{KL}{GH} = \frac{6}{18} = \frac{1}{3}$$

---

8 **Calculate J K**

Step 3: Calculate *JK*. Side *J**K* corresponds to *F**G*.

$$JK = FG \times \text{SF}_2 = 15 \times \frac{1}{3} = 5\text{cm}$$

✓ **FINAL ANSWER**

5 cm

TOPIC

Area & Volume of Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

Question 19

4 marks | Area & Volume of Similar Shapes

19 The diagram shows two similar metal statues.

Diagram **NOT** accurately drawn



A **B**

The volume of statue **B** is 20% less than the volume of statue **A**
The surface area of statue **B** is k% less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 19 is 4 marks)

1 **The volume of statue is**

The volume of statue B is 80% of the volume of statue A, so the volume scale factor is

$$0.8$$

2 **Calculate area**

For similar solids:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 **The surface area scale factor**

The surface area scale factor is therefore

$$\left(\sqrt[3]{0.8}\right)^2 = 0.8^{2/3}$$

$$0.8^{2/3} = 0.861773\ldots$$

4 **Convert standard form**

So the percentage decrease in surface area is

$$(1 - 0.861773\ldots) \times 100 = 13.8226\ldots$$

5 **Calculate area**

Correct to 3 significant figures,

$$k = 13.8$$

✓ **FINAL ANSWER**

$k = 13.8$.

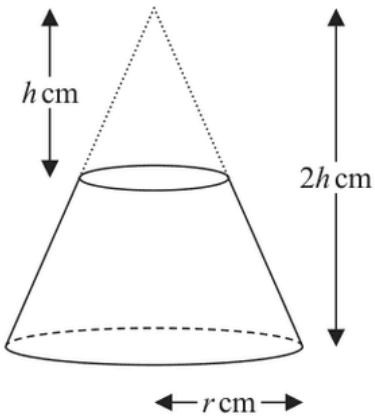
4MA1/1HR JANUARY 2020

Question 20

5 marks | Area & Volume of Similar Shapes

20 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.



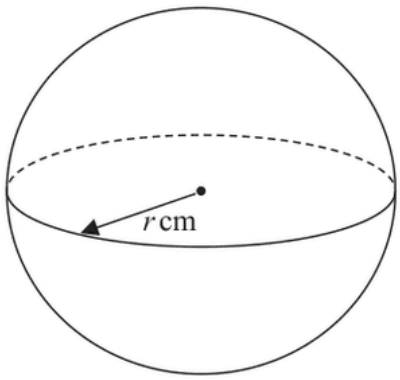


Diagram **NOT** accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h.
Give your expression in its simplest form.

$r =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2020

1 **The two cones are similar**

The two cones are similar.

2 **The large cone has height**

The large cone has height $2h$ and base radius r.

3 **The small cone has height**

The small cone has height h, so its linear scale factor compared with the large cone is

$$\frac{h}{2h} = \frac{1}{2}$$

4 **Define cone dimensions**

So the small cone has radius

$$\frac{r}{2}$$

5 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$

6 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h = \frac{1}{12}\pi r^2 h$$

7 **Calculate volume**

So the volume of the frustum is

$$\frac{2}{3}\pi r^2 h - \frac{1}{12}\pi r^2 h = \frac{7}{12}\pi r^2 h$$

8 **Calculate volume**

The sphere has radius r, so its volume is

$$\frac{4}{3}\pi r^3$$

9 **The two volumes are equal**

The two volumes are equal:

$$\frac{7}{12}\pi r^2 h = \frac{4}{3}\pi r^3$$

10 **Divide by**

Divide by πr^2:

$$\frac{7}{12}h = \frac{4}{3}r$$

$$7h = 16r$$

$$r = \frac{7h}{16}$$

✓ **FINAL ANSWER**

$$r = \frac{7h}{16}$$

4MA1/2HR JANUARY 2020

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar vases, **A** and **B**.

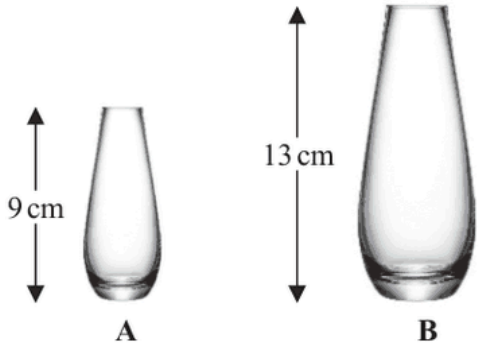


Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

surface area of vase **A** + surface area of vase **B** = 1800 cm^2

calculate the surface area of vase **A**.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2020

1 **The two vases are similar**

The two vases are similar.

2 **Their heights are in the**

Their heights are in the ratio

$$9 : 13$$

3 **Split the ratio**

So their surface areas are in the ratio

$$9^2 : 13^2 = 81 : 169$$

4 **The total surface area is**

The total surface area is $1800\ \text{cm}^2$, so there are

$$81 + 169 = 250$$

5 **parts altogether**

parts altogether.

6 **Surface area of vase A**

Surface area of vase A:

$$1800 \times \frac{81}{250} = 583.2$$

✓ **FINAL ANSWER**

$\boxed{583.2\ \text{cm}^2}$

4MA1/1H MAY/NOVEMBER 2020

Question 20

3 marks | Area & Volume of Similar Shapes

20 **R** and **S** are two similar solid shapes.

Shape **R** has surface area 108 cm^2 and volume 135 cm^3
Shape **S** has surface area 300 cm^2

Work out the volume of shape **S**.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May/November 2020

1 **Shape R has surface area**

Shape R has surface area $108\ \text{cm}^2$ and volume $135\ \text{cm}^3$.

2 **Shape S has surface area**

Shape S has surface area $300\ \text{cm}^2$.

3 **Surface area ratio S R**

Surface area ratio S:R:

$$300 : 108 = 25 : 9$$

4 **Split the ratio**

So the linear ratio S:R is

$$5 : 3$$

5 **Split the ratio**

Therefore the volume ratio S:R is

$$5^3 : 3^3 = 125 : 27$$

6 **Volume of S**

Volume of S:

$$135 \times \frac{125}{27} = 625$$

FINAL ANSWER

$\boxed{625\ \text{cm}^3}$

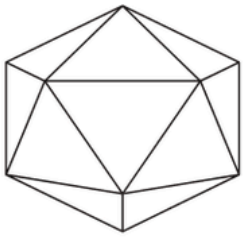
4MA1/1H JANUARY 2021

Question 20

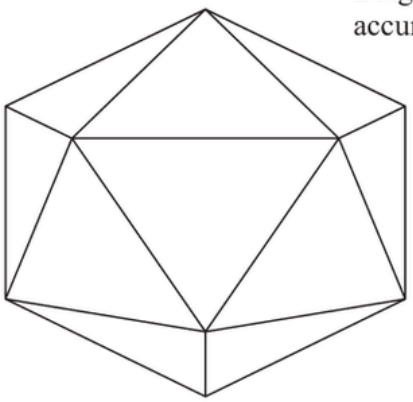
3 marks | Area & Volume of Similar Shapes

20 **A** and **B** are two similar solids.

Diagram **NOT** accurately drawn



A



B

A has a volume of $1836\,\text{cm}^3$
B has a volume of $4352\,\text{cm}^3$

B has a total surface area of $1120\,\text{cm}^2$

Work out the total surface area of **A**.

..... cm^2

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2021

1 The volume ratio of solid

The volume ratio of solid A to solid B is

$$1836 : 4352$$

2 Simplify

Simplify:

$$\frac{1836}{4352} = \frac{27}{64}$$

3 Split the ratio

So the linear ratio is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

4 **Surface area ratio**

Surface area ratio:

$$3^2 : 4^2 = 9 : 16$$

---

5 **Solid B has surface area**

Solid B has surface area $1120\ \text{cm}^2$, so solid A has surface area

$$1120 \times \frac{9}{16} = 630$$

✓ **FINAL ANSWER**

$\boxed{630\ \text{cm}^2}$

4MA1/2H MAY 2021

Question 20

3 marks | Area & Volume of Similar Shapes

20 Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is $300\,\text{cm}^3$

Given that

the surface area of each small block : the surface area of each large block = 25 : 36

work out the volume of each large block.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H May 2021

1 Split the ratio

The surface area ratio of the small block to the large block is

$$25 : 36$$

2 Split the ratio

So the linear ratio is

$$\sqrt{25} : \sqrt{36} = 5 : 6$$

3 Split the ratio

Therefore the volume ratio is

$$5^3 : 6^3 = 125 : 216$$

4 The small block has volume

The small block has volume $300\ \text{cm}^3$.

5 Large block volume

Large block volume:

$$300 \times \frac{216}{125} = 518.4$$

FINAL ANSWER

$\boxed{518.4\ \text{cm}^3}$

4MA1/1H JANUARY 2022

Question 17

4 marks | Area & Volume of Similar Shapes

17 **A** and **B** are two similar vases.

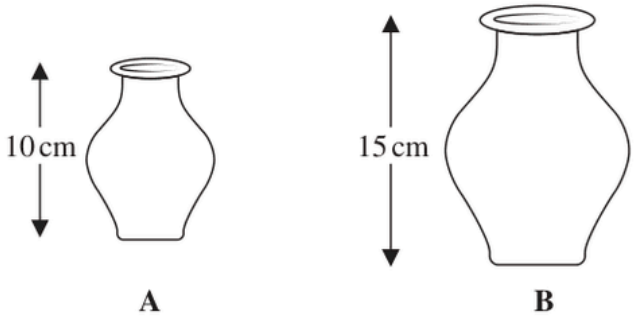


Diagram **NOT** accurately drawn

Vase **A** has height 10 cm.
Vase **B** has height 15 cm.

The difference between the volume of vase **A** and the volume of vase **B** is 1197 cm^3

Calculate the volume of vase **A**

..... cm^3

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H January 2022

1 **Calculate area**

The linear scale factor from vase A to vase B is

$$\frac{15}{10} = 1.5$$

2 **Calculate area**

So the volume scale factor is

$$1.5^3 = 3.375$$

3 **Let the volume of vase**

Let the volume of vase A be V.

$$3.375V - V = 1197$$

$$2.375V = 1197$$

$$V = 504$$

✓ **FINAL ANSWER**

$504\ \text{cm}^3$.

4MA1/1HR JANUARY 2022

Question 18

4 marks | Area & Volume of Similar Shapes

18 The three solids **A**, **B** and **C** are similar such that

the surface area of **A** : the surface area of **B** = 4 : 9

and

the volume of **B** : the volume of **C** = 125 : 343

Work out the ratio

the height of **A** : the height of **C**

Give your ratio in its simplest form.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2022

1 Split the ratio

Surface area ratio $A : B = 4 : 9$, so the length ratio is

$$\sqrt{4} : \sqrt{9} = 2 : 3$$

2 Split the ratio

Volume ratio $B : C = 125 : 343$, so the length ratio is

$$\sqrt[3]{125} : \sqrt[3]{343} = 5 : 7$$

3 Split the ratio

Therefore

$$\frac{A}{C} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

FINAL ANSWER

$10 : 21$

4MA1/1HR JUNE 2022

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar metal statues.



A



B

Diagram **NOT** accurately drawn

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2022

1 The volume of statue is

The volume of statue B is 20% less than statue A, so the volume scale factor is 0.8.

2 Calculate area

For similar solids, the linear scale factor is the cube root of the volume scale factor:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 Surface area scale factor is

Surface area scale factor is the square of the linear scale factor:

$$\text{surface area scale factor} = (\sqrt[3]{0.8})^2 = 0.8^{2/3} = 0.861773\ldots$$

4

Calculate area

So statue B has $86.1773...\%$ of the surface area of statue A.

$k = 100 - 86.1773... = 13.8226...$

✓ **FINAL ANSWER**

$k = 13.8$ to 3 significant figures.

Question 24

6 marks | Area & Volume of Similar Shapes

24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

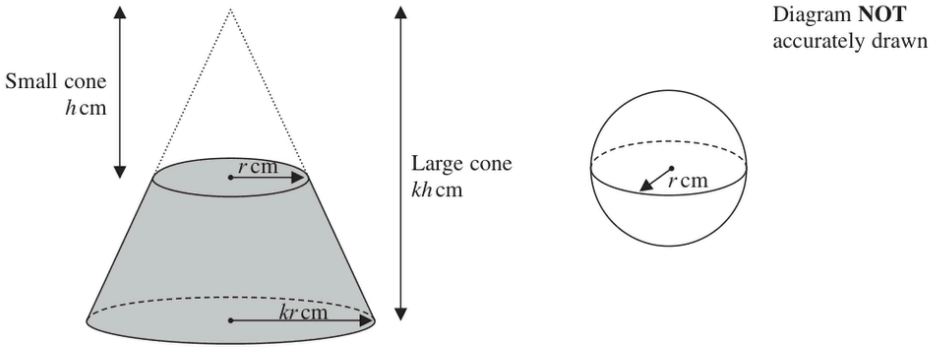


Diagram **NOT** accurately drawn

The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

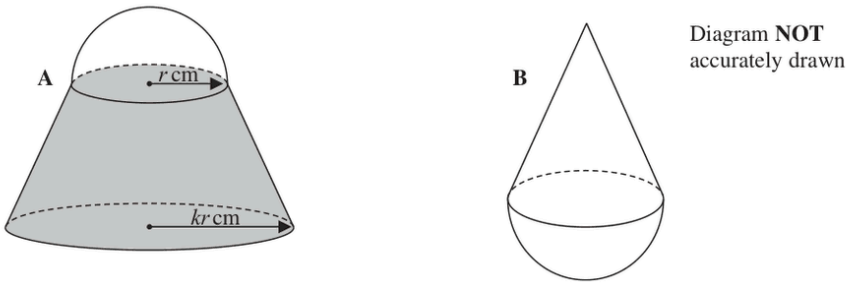


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

$h =$ .....

(Total for Question 24 is 6 marks)

1 Small cone volume

Small cone volume:

$$V_s = \frac{1}{3}\pi r^2 h$$

2 The larger similar cone has

The larger similar cone has radius kr and height kh, so its volume is

$$V_L = \frac{1}{3}\pi (kr)^2 (kh) = \frac{1}{3}\pi k^3 r^2 h$$

3 Calculate volume

So the frustum volume is

$$V_L - V_s = \frac{1}{3}\pi r^2 h(k^3 - 1)$$

4 The volume of one hemisphere

The volume of one hemisphere of radius r is

$$\frac{2}{3}\pi r^3$$

5 Solid A is the frustum

Solid A is the frustum plus one hemisphere:

$$V_A = \frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3$$

6 Solid B is the small

Solid B is the small cone plus one hemisphere:

$$V_B = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

7 Given

Given $V_A = 6V_B$:

$$\frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

8 **Divide by**

Divide by $\frac{1}{3}\pi r^2$:

$$h(k^3-1)+2r=6h+12r$$

$$h(k^3-7)=10r$$

$$h=\frac{10r}{k^3-7}$$

✓ **FINAL ANSWER**

$$h=\frac{10r}{k^3-7}$$

4MA1/1H MAY 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

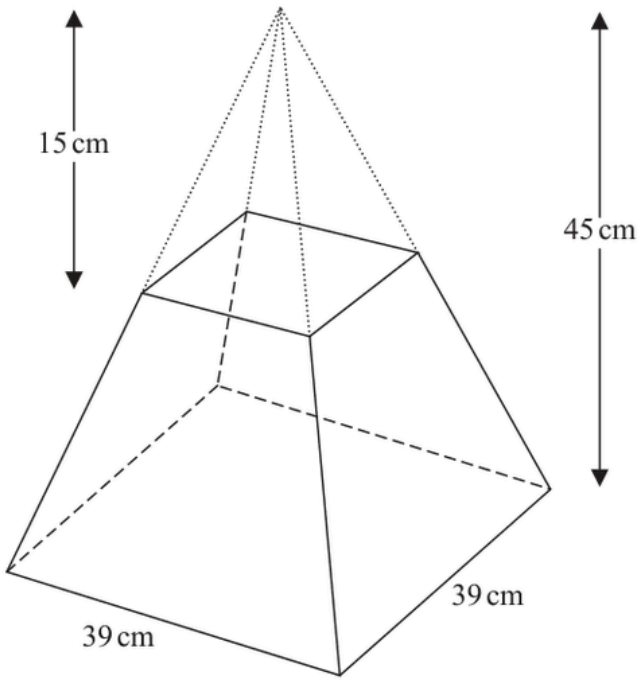


Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2023

1 **The small and large square**

The small and large square-based pyramids are similar.

2 **Their heights are**

Their heights are

$$15 : 45 = 1 : 3$$

---

3 **Square of the frustum**

So the side length of the top square of the frustum is

$$\frac{39}{3} = 13 \text{ cm}$$

---

4 **The vertical height of the**

The vertical height of the frustum is

$$45 - 15 = 30 \text{ cm}$$

---

5 **Evaluate fraction**

For one trapezium face, the difference between the half-side lengths is

$$\frac{39 - 13}{2} = 13 \text{ cm}$$

---

6 **Simplify surd**

So the slant height of the trapezium face is

$$\sqrt{30^2 + 13^2} = \sqrt{1069}$$

---

7 **Area of one trapezium face**

Area of one trapezium face:

$$\frac{1}{2}(39 + 13)\sqrt{1069} = 26\sqrt{1069}$$

---

8 **Area of four trapezium faces**

Area of four trapezium faces:

$$4(26\sqrt{1069}) = 104\sqrt{1069}$$

---

9 **Add the top and bottom**

Add the top and bottom square faces:

$$13^2 + 39^2 = 169 + 1521 = 1690$$

---

10 **Total surface area**

Total surface area:

$$1690 + 104\sqrt{1069} = 5090.338\ldots$$

---

11 Correct to the nearest whole

Correct to the nearest whole number:

$$5090$$

FINAL ANSWER

$$\boxed{5090\ \text{cm}^2}$$

4MA1/1H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Solid **A** is similar to solid **B**

Here is some information about solid **A** and solid **B**

	solid **A**	solid **B**
Height (cm)	3^x	
Area (cm^2)	7776	486
Volume (cm^3)	8^x	2^{x+4}

Work out the height of solid **B**
Give your answer as a decimal.

..... cm

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2023

1 **The area ratio of solid**

The area ratio of solid A to solid B is

$$7776 : 486 = 16 : 1$$

2 **Split the ratio**

So the linear ratio A:B is

$$\sqrt{16} : 1 = 4 : 1$$

3 **Split the ratio**

Therefore the volume ratio A:B is

$$4^3 : 1^3 = 64 : 1$$

4 **Calculate volume**

The volumes are 8^x and 2^{x+4}, so

$$\frac{8^x}{2^{x+4}} = 64$$

5 **Use index laws**

Write 8^x as a power of 2:

$$8^x = (2^3)^x = 2^{3x}$$

6 **Evaluate fraction**

So

$$\frac{2^{3x}}{2^{x+4}} = 2^6$$

$$2^{3x-(x+4)} = 2^6$$

$$2x - 4 = 6$$

$$x = 5$$

7 **The height of solid A**

The height of solid A is

$$3^x = 3^5 = 243$$

8 **Split the ratio**

Since the linear ratio A:B is $4 : 1$, the height of solid B is

$$\frac{243}{4} = 60.75$$

✓ **FINAL ANSWER**

$\boxed{60.75 \text{ cm}}$

4MA1/2H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Here is a frustum of a cone.
The frustum is made by removing a small cone from a similar large cone.

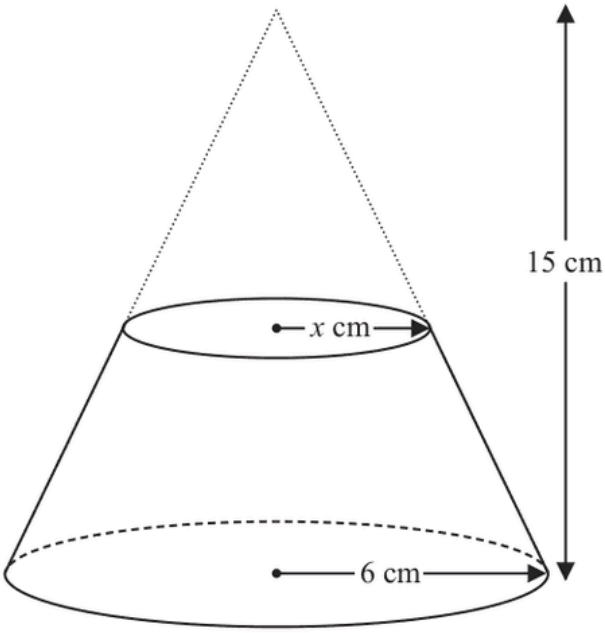


Diagram **NOT** accurately drawn

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is x cm.

Given that the volume of the frustum is $\frac{4212}{25}\pi$ cm^3

work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2023

1 **The large cone has height**

The large cone has height $\mathbf{15}$ cm and radius $\mathbf{6}$ cm.

2 **The small cone has radius**

The small cone has radius x. Since the cones are similar,

$$\frac{\text{radius}}{\text{height}} = \frac{6}{15} = \frac{2}{5}$$

3 **Evaluate fraction**

So for the small cone,

$$\frac{x}{h} = \frac{2}{5}$$

$$h = \frac{5}{2}x$$

4 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi(6)^2(15) = 180\pi$$

5 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$$

6 **The frustum volume is**

The frustum volume is

$$\frac{4212}{25}\pi$$

7 **Evaluate fraction**

So

$$180\pi - \frac{5}{6}\pi x^3 = \frac{4212}{25}\pi$$

8 **Cancel**

Cancel π:

$$180 - \frac{5}{6}x^3 = \frac{4212}{25}$$

$$\frac{5}{6}x^3 = 180 - \frac{4212}{25}$$

$$\frac{5}{6}x^3 = \frac{288}{25}$$

$$x^3 = \frac{1728}{125}$$

$$x = \frac{12}{5} = 2.4$$

✓ **FINAL ANSWER**

$\boxed{2.4 \text{ cm}}$

4MA1/1HR MAY 2024

Question 18

4 marks | Area & Volume of Similar Shapes

18 The diagram shows two similar vases, **A** and **B**

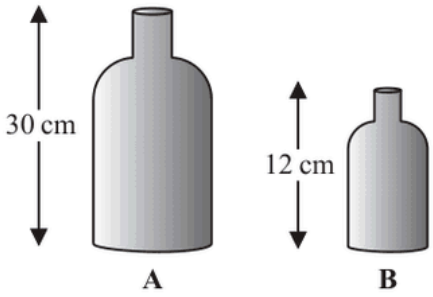


Diagram **NOT** accurately drawn

The height of vase **A** is 30 cm
The height of vase **B** is 12 cm

Given that

surface area of vase **A** – surface area of vase **B** = 178.5 cm²

find the surface area of vase **A**

..... cm²

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May 2024

1 **Calculate area**

The linear scale factor from vase B to vase A is

$$\frac{30}{12} = 2.5$$

2 **Calculate area**

So the surface area scale factor is

$$2.5^2 = 6.25$$

3 Calculate surface area

Let the surface area of vase B be S.

$$6.25S - S = 178.5$$

$$5.25S = 178.5$$

$$S = 34$$

4 Calculate area

Therefore the surface area of vase A is

$$6.25 \times 34 = 212.5$$

FINAL ANSWER

$212.5\ \text{cm}^2$.

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows a solid, **S**, made from a cone and a hemisphere.

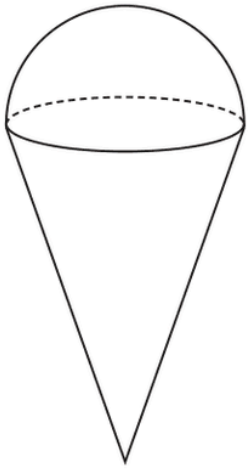


Diagram **NOT** accurately drawn

The centre of the circular face of the cone coincides with the centre of the flat surface of the hemisphere.

The radius of the circular face of the cone, x cm, is equal to the radius of the hemisphere.

The total height of **S** is 4 × the radius of the hemisphere.

A separate sphere has radius kx cm.
The volume of this sphere is 12.5 × the volume of **S**

(a) Work out the value of k

$k =$ .....

(4)

A solid, **T**, is similar to solid **S**

The volume of **T** is 512 × the volume of **S**

The total surface area of **T** is d × the total surface area of **S**

(b) Find the value of d

$d =$ .....

(1)

(Total for Question 24 is 5 marks)

1 **The hemisphere has radius**

The hemisphere has radius x, so its height is x.

2 **The total height of solid**

The total height of solid S is $4x$, so the cone height is

$$4x - x = 3x$$

3 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi x^2(3x) = \pi x^3$$

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

5 **Calculate volume**

So the volume of solid S is

$$\pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

6 **Calculate volume**

The sphere has radius kx, so its volume is

$$\frac{4}{3}\pi(kx)^3 = \frac{4}{3}\pi k^3 x^3$$

7 **This is times the volume**

This is 12.5 times the volume of S:

$$\frac{4}{3}\pi k^3 x^3 = 12.5\left(\frac{5}{3}\pi x^3\right)$$

8 **Cancel**

Cancel $\frac{1}{3}\pi x^3$:

$$4k^3 = 62.5$$

$$k^3 = 15.625 = \frac{125}{8}$$

$$k = 2.5$$

---

9 **Calculate volume**

For solid T, the volume scale factor is 512.

$$512 = 8^3$$

---

10 **Calculate area**

So the linear scale factor is 8, and the surface area scale factor is

$$8^2 = 64$$

✓ **FINAL ANSWER**

$$\boxed{k = 2.5,\ d = 64}$$

4WM2/01 MAY 2025

Question 25

4 marks | Area & Volume of Similar Shapes

25 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** − volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4WM2/01 May 2025

1 Linear scale factor from solid

Linear scale factor from solid A to solid B:

$$\frac{18.6}{31} = 0.6$$

2 Calculate volume

So the volume scale factor is

$$0.6^3 = 0.216$$

3 Let the volume of be

Let the volume of A be V. Then

$$V - 0.216V = 735$$

$$0.784V = 735$$

$$V = 937.5$$

FINAL ANSWER

$937.5\ \text{cm}^3$.

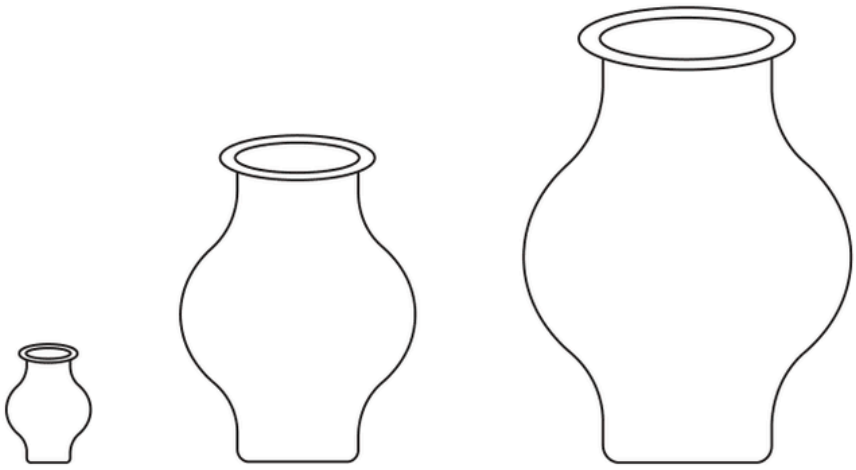
4MA1/1HR JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **R**, **S** and **T** are three similar vases.

Diagram **NOT** accurately drawn



The volume of vase **S** is 72.8% more than the volume of vase **R**

The height of vase **R** is h cm
The height of vase **T** is $6h$ cm

The surface area of vase **S** is A cm^2
The surface area of vase **T** is kA cm^2

Work out the value of k

$k =$ .....

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/1HR June 2025

1 **The volume of vase is**

The volume of vase S is 72.8% more than vase R, so

$$\frac{V_S}{V_R} = 1.728$$

2 **Calculate area**

For similar solids, volume scale factor is the cube of the linear scale factor:

$$\text{linear scale factor from } R \text{ to } S = \sqrt[3]{1.728} = 1.2$$

3 **The height of vase is**

The height of vase T is $6h$, so the linear scale factor from R to T is 6.

4 **Calculate area**

Therefore the linear scale factor from S to T is

$$\frac{6}{1.2} = 5$$

5 **Surface area scale factor is**

Surface area scale factor is the square of the linear scale factor:

$k = 5^2 = 25$

✓ **FINAL ANSWER**

$k = 25$

4MA1/2H JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** – volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H June 2025

1 **The heights of solids A**

The heights of solids A and B are 31 cm and 18.6 cm.

$$31 : 18.6 = 5 : 3$$

2 **Split the ratio**

So the volumes are in the ratio

$$5^3 : 3^3 = 125 : 27$$

3 **The difference in volume is**

The difference in volume is

$$125 - 27 = 98$$

4 **parts**

parts.

5 **Evaluate fraction**

This difference is $735\ \text{cm}^3$, so one part is

$$\frac{735}{98} = 7.5$$

6 Volume of solid A

Volume of solid A:

$$125 \times 7.5 = 937.5$$

FINAL ANSWER

$\boxed{937.5\ \text{cm}^3}$

4WM2/01 NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 November 2025

1 Calculate area

$$textASF = fractextAreaofQtextAreaofP = frac450200 = frac94 = 2.25$$

2 Simplify surd

$$textLSF = sqrttextASF = sqrtfrac94 = frac32 = 1.5$$

3 Calculate volume

$$textVSF = (textLSF)^3 = left(frac32right)^3 = frac278 = 3.375$$

4 Calculate volume

$$textVolumeofQ = textVolumeofPtimestextVSF = 672timesfrac278$$

= 84 × 27 = 2268

FINAL ANSWER

2268 cm³

4MA1/2H NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2025

1 Surface area ratio Q P

Surface area ratio Q:P:

$$450 : 200 = 9 : 4$$

2 Split the ratio

So the linear ratio Q:P is

$$3 : 2$$

3 Split the ratio

Therefore the volume ratio Q:P is

$$3^3 : 2^3 = 27 : 8$$

4 Shape P has volume

Shape P has volume $672\ cm^3$.

5 Volume of Q

Volume of Q:

$$672 \times \frac{27}{8} = 2268$$

FINAL ANSWER

$$\boxed{2268\ \text{cm}^3}$$

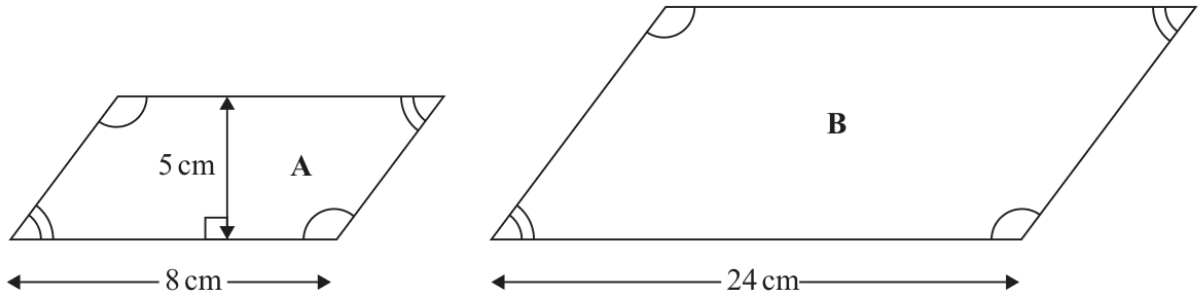
4MA1/1HR JUNE 2026

Question 6

3 marks | Area & Volume of Similar Shapes

6

Diagram **NOT** accurately drawn



A and **B** are similar parallelograms.

Work out the area of **B**

..... cm^2

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2026

1 Solution | Step 1

$$k = \frac{24}{8} = 3.$$

The marked corresponding bases give the linear scale factor from A to B.

2 Solution | Step 2

$$\text{area}(A) = 8 \times 5 = 40 \text{ cm}^2.$$

The 5 cm measurement is the perpendicular height.

3 Solution | Step 3

$$\text{area}(B) = k^2 \, \text{area}(A) = 3^2 \times 40 = 360 \text{ cm}^2.$$

Areas scale by the square of the linear scale factor.

FINAL ANSWER

$360\ \text{cm}^2$

4MA1/2HR JUNE 2026

Question 23

5 marks | Area & Volume of Similar Shapes

23 **K**, **L** and **M** are three similar solids.

volume of **K** : volume of **L** = 125 : 216

surface area of **K** : surface area of **M** = 4 : 9

The height of **M** is y% greater than the height of **L**

Work out the value of y
Show your working clearly.

$y =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2026

1 Solution | Step 1

$$h_K : h_L = \sqrt[3]{125} : \sqrt[3]{216} = 5 : 6.$$

Corresponding lengths have the cube-root ratio of volumes.

2 Solution | Step 2

$$h_K : h_M = \sqrt{4} : \sqrt{9} = 2 : 3.$$

Corresponding lengths have the square-root ratio of surface areas.

3 Solution | Step 3

Take $h_K = 10$ units. Then $h_L = 12$ and $h_M = 15$, so

$$y = \frac{15 - 12}{12} \times 100 = 25.$$

Align the shared K-length and compare the increase relative to L.

FINAL ANSWER

$y = 25$, so the height is 25% greater.

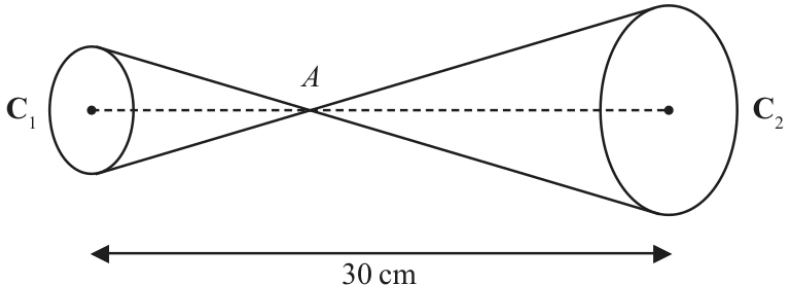
4MA1/2H JUNE 2026

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows two cones $\mathbf{C}_1$ and $\mathbf{C}_2$

Diagram **NOT** accurately drawn



The vertex of $\mathbf{C}_1$ coincides with the vertex of $\mathbf{C}_2$ at the point A
$\mathbf{C}_1$ and $\mathbf{C}_2$ have the same axis of symmetry.

$$\text{height of } \mathbf{C}_1 + \text{height of } \mathbf{C}_2 = 30 \text{ cm}$$

$$\text{total surface area of } \mathbf{C}_1 : \text{total surface area of } \mathbf{C}_2 = 4 : 9$$

$$\text{volume of } \mathbf{C}_1 = 23.04\pi \text{ cm}^3$$

Work out the radius of the base of $\mathbf{C}_2$
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2026

1 **Solution | Step 1**

The straight generators shown through A give equal vertex angles, so the cones are similar. The area ratio $4 : 9$ gives length ratio $\sqrt{4} : \sqrt{9} = 2 : 3$.
For similar solids, surface areas scale as the square of corresponding lengths.

2 **Solution | Step 2**

The heights total 30 cm, so

$$h_1 = \frac{2}{5}(30) = 12 \text{ cm}, \qquad h_2 = \frac{3}{5}(30) = 18 \text{ cm}.$$

Split the total in the common length ratio 2:3.

3 **Solution | Step 3**

$$\frac{1}{3}\pi r_1^2(12) = 23.04\pi \quad \Rightarrow \quad 4r_1^2 = 23.04 \quad \Rightarrow \quad r_1 = \sqrt{5.76} = 2.4 \text{ cm}.$$

Use the volume of the first cone and choose a positive radius.

---

4 **Solution | Step 4**

$$r_2 = \frac{3}{2}r_1 = \frac{3}{2}(2.4) = 3.6 \text{ cm}.$$

Radii scale by the same factor as the heights.

✓ **FINAL ANSWER**

3.6 cm

TOPIC

Circle Theorems

Every past-paper question on this topic, 2020 to 2026.

Question 14

5 marks | Circle Theorems

14 A, B, C and D are points on a circle, centre O

EBF is the tangent to the circle at B

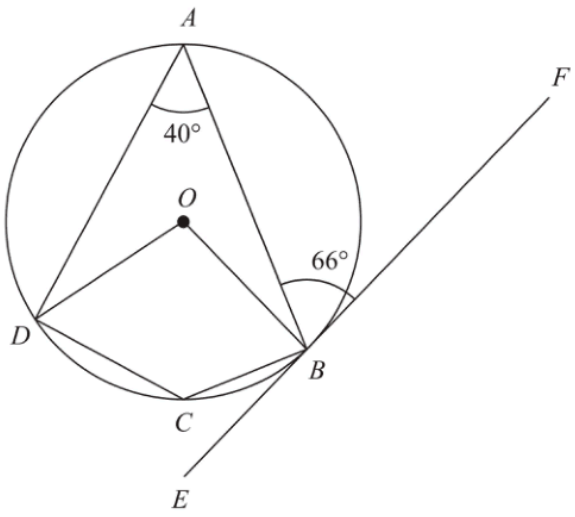


Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle DCB

.....

(1)

(ii) Give a reason for your answer to (a)(i)

.....

.....

(1)

(b) Work out the size of angle ADO

..... °

(3)

(Total for Question 14 is 5 marks)

1 **Use trigonometry**

In cyclic quadrilateral $ADCB$, opposite angles add to $180°$.

$$\angle DCB = 180° - 40° = 140°$$

2 **Reason opposite angles in a**

Reason: opposite angles in a cyclic quadrilateral add to $180°$.

3 **Use trigonometry**

By the alternate segment theorem,

$$\angle ADB = 66°$$

4 **Use trigonometry**

In triangle ADB,

$$\angle ABD = 180° - 40° - 66° = 74°$$

5 **The angle at the centre**

The angle at the centre is twice the angle at the circumference standing on the same chord AD:

$$\angle AOD = 2(74°) = 148°$$

6 **Use trigonometry**

Since $OA = OD$, triangle AOD is isosceles:

$$\angle ADO = \frac{180° - 148°}{2} = 16°$$

FINAL ANSWER

$\angle DCB = 140°$; reason: opposite angles in a cyclic quadrilateral; $\angle ADO = 16°$.

4MA1/2H JANUARY 2020

Question 13

4 marks | Circle Theorems

13

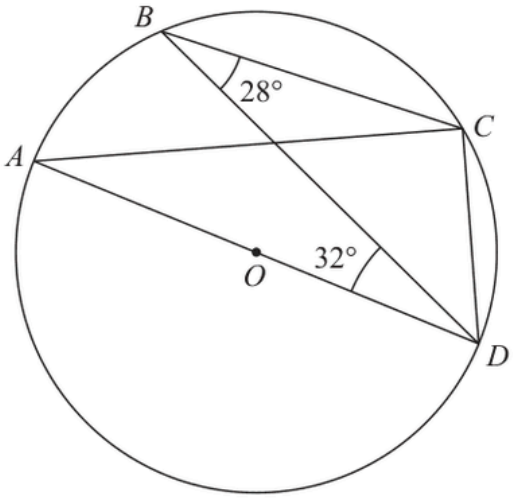


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O.
AOD is a diameter of the circle.

Angle $CBD = 28°$
Angle $BDA = 32°$

Find the size of angle BDC.
Give a reason for each stage of your working.

.....°

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2020

1 Use trigonometry

Since AD is a diameter,

$$\angle ABD = 90°$$

2 Given

Given

$$\angle CBD = 28°$$

3 **Use trigonometry**

So

$$\angle ABC = 90° - 28° = 62°$$

---

4 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$:

$$\angle ADC = 180° - 62° = 118°$$

---

5 **Use trigonometry**

At D,

$$\angle ADC = \angle BDA + \angle BDC$$

$$118° = 32° + \angle BDC$$

$$\angle BDC = 86°$$

✓ **FINAL ANSWER**

$86°$.

4MA1/1HR JANUARY 2020

Question 14

4 marks | Circle Theorems

14

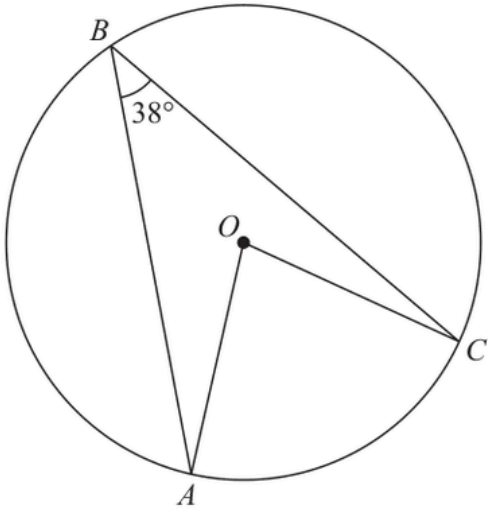


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O.
Angle $ABC = 38°$

Work out the size of angle OAC.
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2020

1 Angle and central angle stand

Angle ABC and central angle AOC stand on the same arc AC.

2 The angle at the centre

The angle at the centre is twice the angle at the circumference:

$$\angle AOC = 2 \times 38° = 76°$$

3 Use trigonometry

Since $OA = OC$, triangle AOC is isosceles.

$$\angle OAC = \angle OCA = \frac{180° - 76°}{2} = 52°$$

FINAL ANSWER

$52°$.

4MA1/2HR JANUARY 2020

Question 18

2 marks | Circle Theorems

18

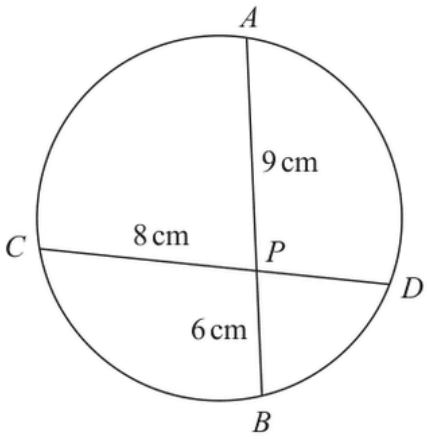


Diagram **NOT** accurately drawn

APB and *CPD* are chords of a circle.

$AP = 9\,\text{cm}$ $PB = 6\,\text{cm}$ $CP = 8\,\text{cm}$

Calculate the length of *PD*.

..... cm

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2020

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$AP \times PB = CP \times PD$$

2 Substitute the values

Substitute the values:

$$9 \times 6 = 8 \times PD$$

$$54 = 8PD$$

$$PD = 6.75$$

FINAL ANSWER

$6.75\ \text{cm}$.

Question 15

4 marks | Circle Theorems

15

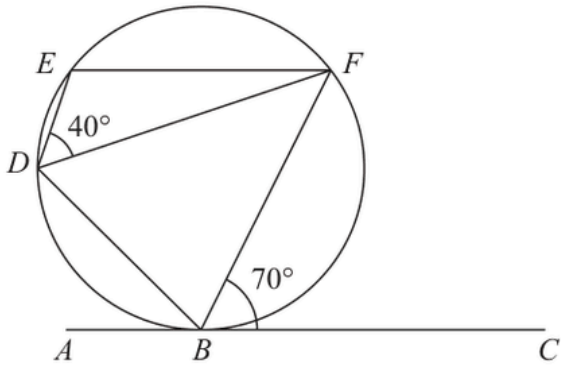


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle.
ABC is the tangent to the circle at B.

Angle $EDF = 40°$
Angle $FBC = 70°$

Prove that the tangent ABC is parallel to EF.
Give a reason for each stage of your working.

(Total for Question 15 is 4 marks)

1 **Proof**

By the alternate segment theorem, the angle between tangent BC and chord BF equals the angle in the opposite segment:

$$\angle BEF = 70°$$

Given

$$\angle EDF = 40°$$

Angles EDF and EBF stand on the same chord EF, so

$$\angle EBF = 40°$$

In triangle BEF,

$$\angle EFB = 180° - 70° - 40° = 70°$$

So

$$\angle EFB = \angle FBC = 70°$$

These are alternate interior angles, so

$$EF \parallel ABC$$

✓ **FINAL ANSWER**

The tangent ABC is parallel to EF.

4MA1/1H MAY/NOVEMBER 2020

Question 16

4 marks | Circle Theorems

16

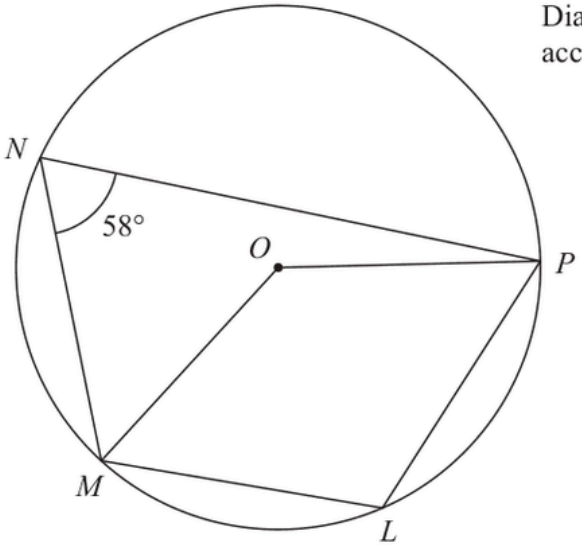


Diagram **NOT** accurately drawn

L, M, N and P are points on a circle, centre O

Angle $MNP = 58°$

(a) (i) Find the size of angle MLP

.....°

(ii) Give a reason for your answer.

.....

.....

(2)

(b) Find the size of the reflex angle MOP

.....°

.....

(2)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May/November 2020

1 (a) Opposite angles in cyclic

(a) Opposite angles in cyclic quadrilateral $MNPL$ add to $180°$:

$$\angle MLP = 180° - 58° = 122°$$

2 **(b) The angle at the**

(b) The angle at the centre is twice the angle at the circumference standing on the same arc MP:

$$\angle MOP = 2 \times 58^\circ = 116^\circ$$

---

3 **The reflex angle is**

The reflex angle is

$$360^\circ - 116^\circ = 244^\circ$$

✓ **FINAL ANSWER**

$\angle MLP = 122^\circ$, and reflex $\angle MOP = 244^\circ$.

4MA1/2H JANUARY 2021

Question 13

3 marks | Circle Theorems

13 P, Q, R, S and T are points on a circle with centre O.

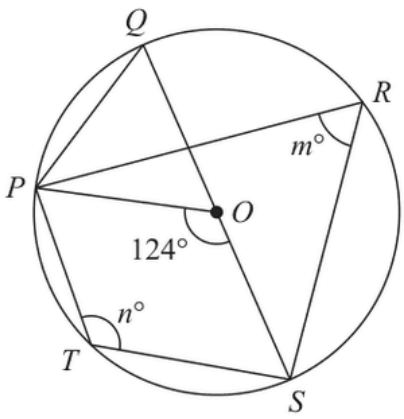


Diagram **NOT** accurately drawn

QOS is a diameter of the circle.

angle $POS = 124°$ angle $PRS = m°$ angle $PTS = n°$

(a) Find the value of

(i) m

.....

(ii) n

.....

(2)

(b) Find the size of angle QPO.

..... °

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2021

1 **Use trigonometry**

The central angle $\angle POS$ is $124°$. The angle at the circumference standing on the same chord PS is half this angle.

$$m = \frac{124}{2} = 62$$

2 **Use circle theorem**

Angles opposite each other in a cyclic quadrilateral add to $180°$, so

$$n = 180 - 62 = 118$$

3 **Use trigonometry**

Since QOS is a diameter,

$$\angle QOP = 180° - 124° = 56°$$

4 **Use trigonometry**

Triangle QOP is isosceles because $OQ = OP$. Therefore

$$\angle QPO = \frac{180 - 56}{2} = 62°$$

✓ **FINAL ANSWER**

$m = 62$, $n = 118$, and $\angle QPO = 62°$.

4MA1/1HR JANUARY 2021

Question 14

4 marks | Circle Theorems

14

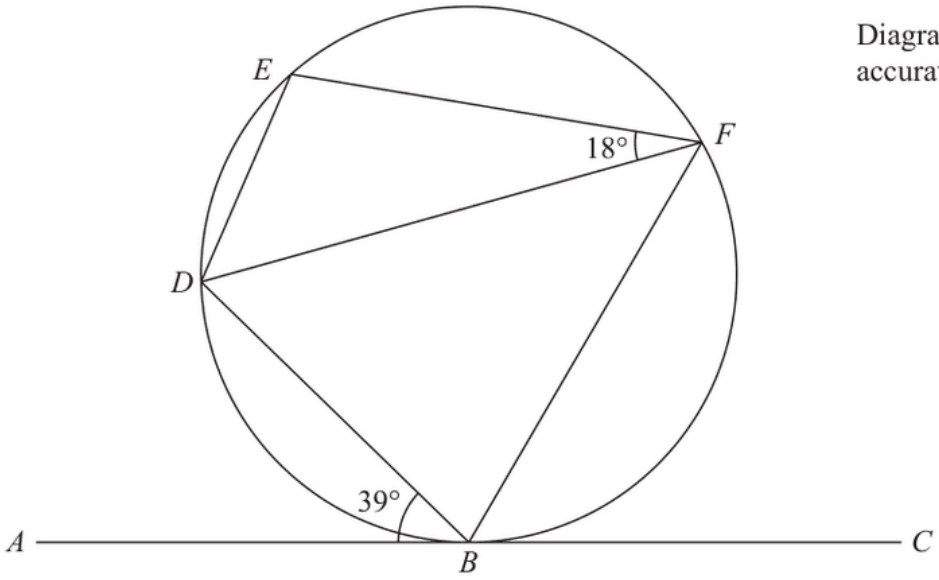


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle.

ABC is the tangent at B to the circle.
Angle $ABD = 39°$
Angle $EFD = 18°$

Work out the size of angle BDE.
Give reasons for your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2021

1 **Use trigonometry**

By the alternate segment theorem, the angle between the tangent and chord BD is equal to the angle in the opposite segment:

$$\angle BFD = 39°$$

2 **Given**

Given

$$\angle EFD = 18°$$

3 Use trigonometry

So

$$\angle BFE = 39^\circ + 18^\circ = 57^\circ$$

4 Opposite angles in cyclic quadrilateral

Opposite angles in cyclic quadrilateral $BDEF$ add to 180°:

$$\angle BDE = 180^\circ - 57^\circ = 123^\circ$$

FINAL ANSWER

123°.

4MA1/1H MAY 2021

Question 15

4 marks | Circle Theorems

15 P, Q and R are points on a circle, centre O.
TRV is the tangent to the circle at R.

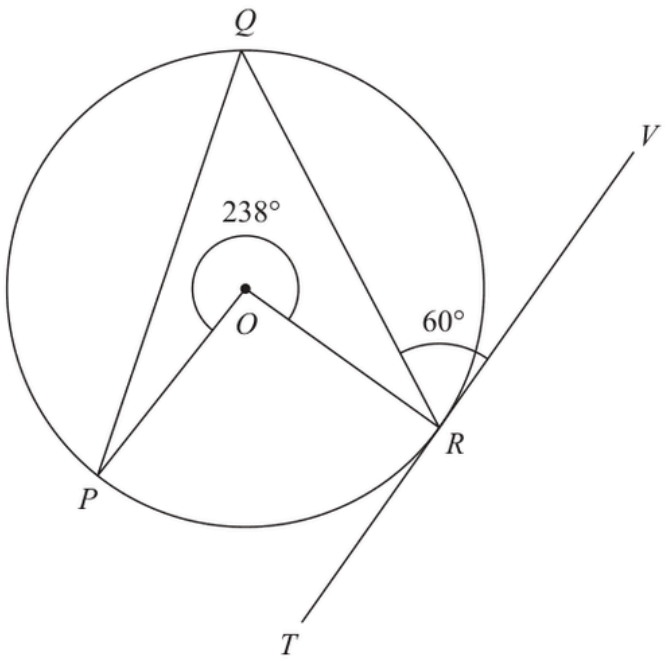


Diagram **NOT** accurately drawn

Reflex angle $POR = 238°$
Angle $QRV = 60°$

Calculate the size of angle OPQ.
Give a reason for each stage of your working.

.....°

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2021

1 **Use trigonometry**

The reflex angle POR is $238°$, so the smaller angle is

$$360° - 238° = 122°$$

2 **Use trigonometry**

Since $OP = OR$, triangle OPR is isosceles:

$$\angle OPR = \frac{180° - 122°}{2} = 29°$$

3 **Use trigonometry**

By the alternate segment theorem,

$$\angle QPR = 60°$$

---

4 **Use trigonometry**

Therefore

$$\angle QPO = 60° - 29° = 31°$$

✓ **FINAL ANSWER**

$31°$.

4MA1/2H MAY 2021

Question 18

6 marks | Circle Theorems

18

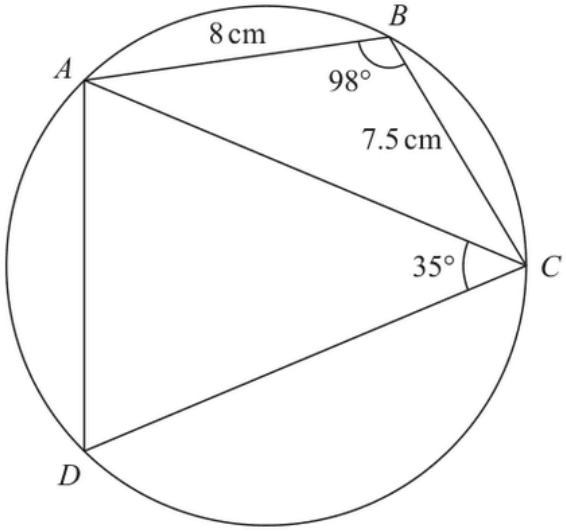


Diagram **NOT** accurately drawn

$ABCD$ is a quadrilateral where A, B, C and D are points on a circle.

$AB = 8$ cm
$BC = 7.5$ cm
Angle $ABC = 98°$
Angle $ACD = 35°$

Work out the perimeter of quadrilateral $ABCD$.
Give your answer correct to one decimal place.

..... cm

(Total for Question 18 is 6 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H May 2021

1 Use trigonometry

In triangle ABC, use the cosine rule:

$$AC^2 = 8^2 + 7.5^2 - 2(8)(7.5)\cos 98°$$

$$AC = 11.702\ldots$$

2 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$, so

$$\angle ADC = 180° - 98° = 82°$$

3 **Use trigonometry**

In triangle ACD,

$$\angle CAD = 180° - 82° - 35° = 63°$$

4 **Use the sine rule**

Use the sine rule:

$$AD = \frac{AC \sin 35°}{\sin 82°} = 6.778\ldots$$

$$CD = \frac{AC \sin 63°}{\sin 82°} = 10.529\ldots$$

5 **Perimeter**

Perimeter:

$$8 + 7.5 + 6.778\ldots + 10.529\ldots = 32.807\ldots$$

✓ **FINAL ANSWER**

$32.8\ \text{cm}$.

4MA1/2H NOVEMBER 2021

Question 11

3 marks | Circle Theorems

11

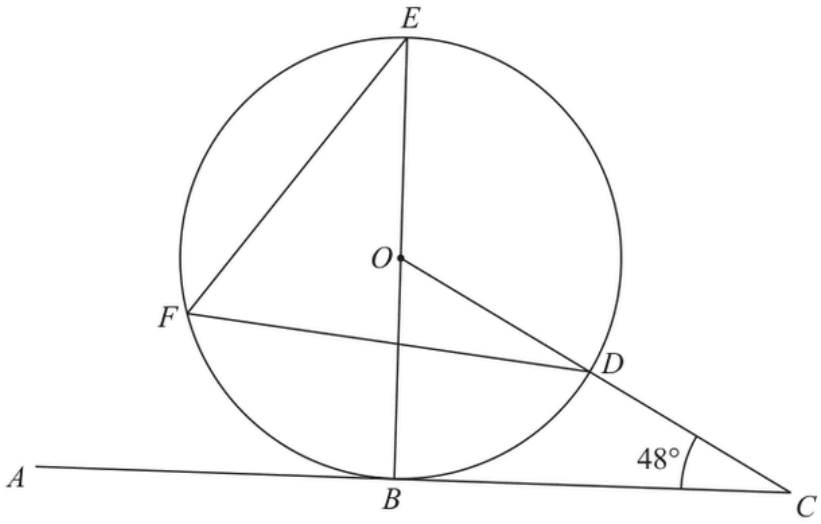


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle, centre O.
ABC is a tangent to the circle.
ODC is a straight line.

BOE is a diameter of the circle.

Angle $BCD = 48°$

Find the size of angle DFE.

.....°

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2021

1 The tangent at is perpendicular

The tangent at B is perpendicular to the radius OB, so

$$\angle OBC = 90°$$

2 Use trigonometry

In triangle OBC,

$$\angle BOC = 180° - 90° - 48° = 42°$$

3 **Use trigonometry**

Since O, D, C are on a straight line,

$$\angle BOD = 42°$$

---

4 **Use trigonometry**

BOK is a diameter, so BOE is a straight line and

$$\angle DOE = 180° - 42° = 138°$$

---

5 **The angle at the circumference**

The angle at the circumference is half the angle at the centre:

$$\angle DFE = \frac{138°}{2} = 69°$$

✓ **FINAL ANSWER**

$69°$.

4MA1/1HR JANUARY 2022

Question 14

4 marks | Circle Theorems

14

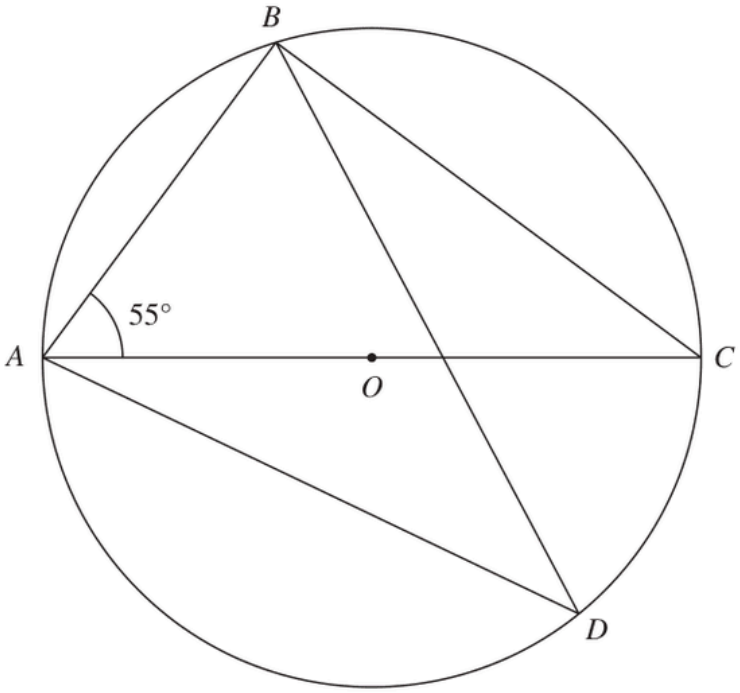


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O
AOC is a diameter of the circle.

Angle $BAC = 55°$

Work out the size of angle ADB
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2022

1 **Use trigonometry**

Since AC is a diameter,

$$\angle ABC = 90°$$

2 **Use trigonometry**

In triangle ABC,

$$\angle ACB = 180° - 90° - 55° = 35°$$

3

Angles in the same segment

Angles in the same segment are equal, so

$$\angle ADB = \angle ACB = 35^\circ$$

✓ **FINAL ANSWER**

35°.

4MA1/2H JANUARY 2022

Question 16

4 marks | Circle Theorems

16 D, E, F and G are points on a circle, centre O

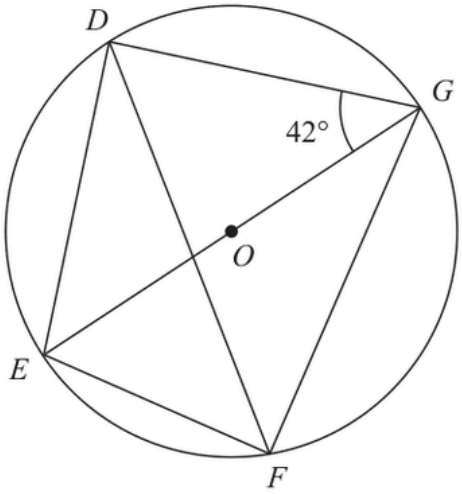


Diagram **NOT** accurately drawn

EOG is a diameter of the circle.

Angle $EGD = 42°$

Calculate the size of angle DFG
Give a reason for each stage of your working.

Angle DFG = ... °

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H January 2022

1 **Use trigonometry**

Since EOG is a diameter,

$$\angle EDG = 90°$$

2 **Use trigonometry**

In triangle EDG,

$$\angle DEG = 180° - 90° - 42° = 48°$$

3 **Angles and stand on the**

Angles DEG and DFG stand on the same chord DG, so they are equal.

$$\angle DFG = 48°$$

FINAL ANSWER

$48°$.

4MA1/2H JANUARY 2022

Question 19

3 marks | Circle Theorems

19

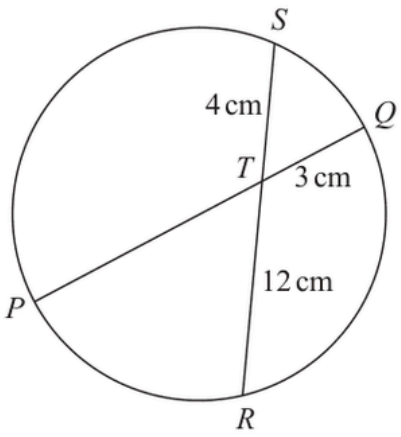


Diagram **NOT** accurately drawn

PTQ is a diameter of a circle.
RTS is a chord of the circle.

$TQ = 3\,\text{cm}$ $ST = 4\,\text{cm}$ $TR = 12\,\text{cm}$

Calculate the radius of the circle.

..... cm

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2022

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$PT \times TQ = ST \times TR$$

$$PT \times 3 = 4 \times 12$$

$$PT = 16$$

2 Use circle theorem

Since PTQ is a diameter,

$$PQ = PT + TQ = 16 + 3 = 19$$

3 **Use circle theorem**

So the radius is

$$19 \div 2 = 9.5$$

✓ **FINAL ANSWER**

$9.5\ \text{cm}$.

Question 15

3 marks | Circle Theorems

15

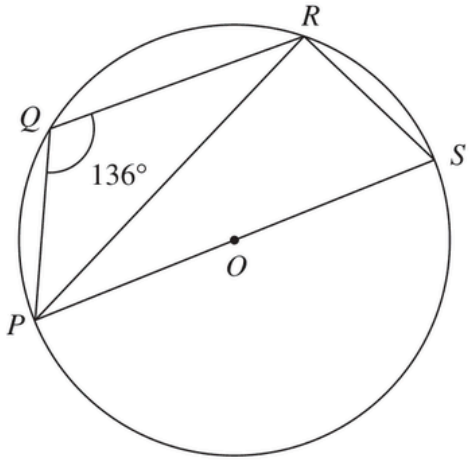


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136°$

Work out the size of angle RPS

.....°

(Total for Question 15 is 3 marks)

1 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$:

$$\angle PSR = 180° - 136° = 44°$$

2 **Use trigonometry**

Since PS is a diameter,

$$\angle PRS = 90°$$

3 **Use trigonometry**

In triangle PRS,

$$\angle RPS = 180° - 90° - 44° = 46°$$

✓ **FINAL ANSWER**

$46°$.

4MA1/2H JUNE 2023

Question 7

3 marks | Circle Theorems

7

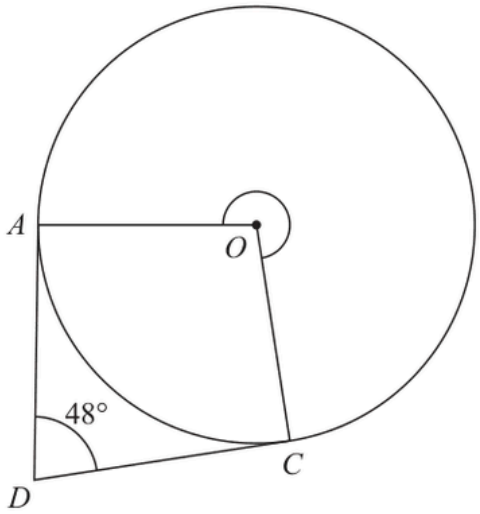


Diagram **NOT** accurately drawn

A and C are points on a circle, centre O
DA is the tangent to the circle at A and DC is the tangent to the circle at C

Angle $ADC = 48°$

Work out the size of reflex angle AOC

.....°

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2023

1 Use circle theorem

A radius is perpendicular to a tangent, so

$$\angle OAD = 90°$$

2 Use trigonometry

and

$$\angle OCD = 90°$$

3 Use trigonometry

In quadrilateral $AOCD$,

$$\angle AOC = 360° - 90° - 90° - 48° = 132°$$

4 **The question asks for the**

The question asks for the reflex angle:

$$360° - 132° = 228°$$

✓ **FINAL ANSWER**

$228°$.

Question 14

4 marks | Circle Theorems

14

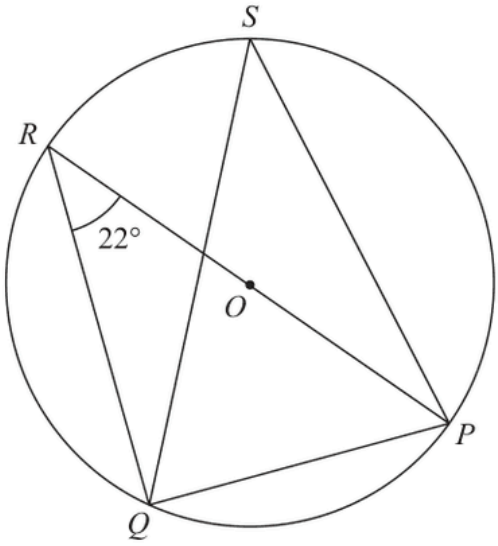


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle, centre O
ROP is a diameter of the circle.
Angle $PRQ = 22°$

(a) (i) Find the size of angle RQP

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Find the size of angle PSQ

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 14 is 4 marks)

1 **Use trigonometry**

(a) Since RP is a diameter,

$$\angle RQP = 90°$$

2 **Reason the angle in a**

Reason: the angle in a semicircle is $90°$.

3 **(b) Angles and stand on**

(b) Angles PRQ and PSQ stand on the same chord PQ, so they are equal.

$$\angle PSQ = 22°$$

FINAL ANSWER

$\angle RQP = 90°$, and $\angle PSQ = 22°$.

4MA1/2HR JUNE 2023

Question 18

3 marks | Circle Theorems

18

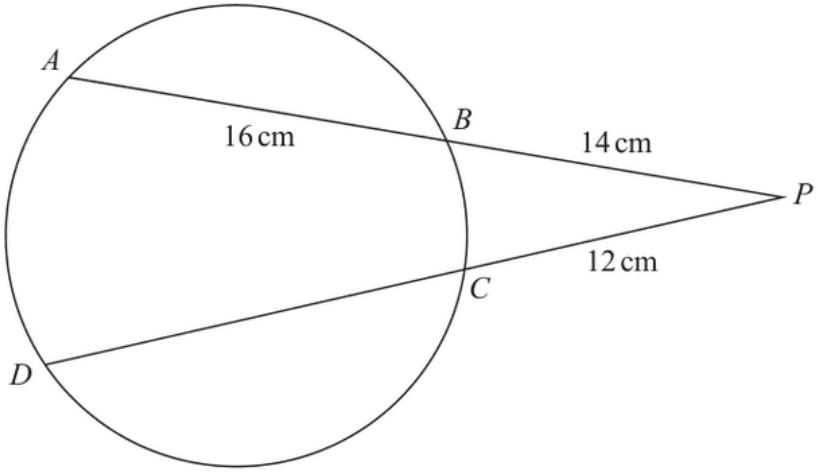


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle.

ABP and DCP are straight lines.

$AB = 16\,\text{cm}$ $BP = 14\,\text{cm}$ $CP = 12\,\text{cm}$

Work out the length of DC

..... cm

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2023

1 Use circle theorem

From the external point P, two secants meet the circle.

2 Use circle theorem

For the upper secant,

$$PB = 14, \qquad PA = 14 + 16 = 30$$

3 Use circle theorem

For the lower secant,

$$PC = 12, \qquad PD = 12 + DC$$

4 **Use circle theorem**

Using the secant theorem:

$$PB \times PA = PC \times PD$$

$$14 \times 30 = 12(12 + DC)$$

$$420 = 144 + 12DC$$

$$276 = 12DC$$

$$DC = 23$$

✓ **FINAL ANSWER**

$DC = 23$ cm.

4MA1/2HR JUNE 2023

Question 20

2 marks | Circle Theorems

20 A, B and C are points on a circle.

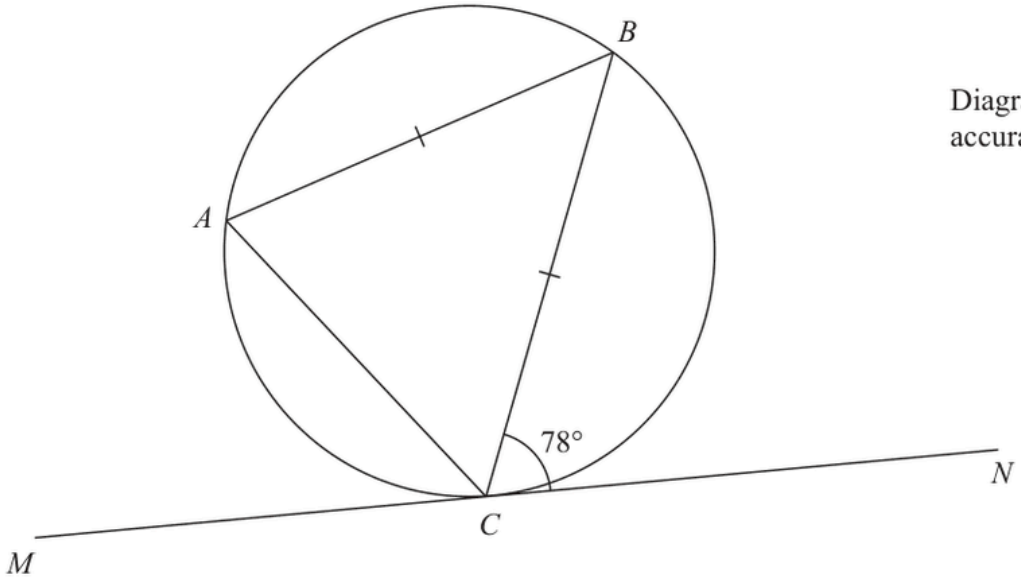


Diagram **NOT** accurately drawn

MN is the tangent to the circle at C

$AB = CB$

Angle $BCN = 78°$

Find the size of angle ABC

.....°

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2023

1 **The angle between the tangent**

The angle between the tangent CN and the chord CB is $78°$.

2 **Use trigonometry**

By the alternate segment theorem,

$$\angle BAC = 78°$$

3 **Calculate value**

Also,

$$AB = CB$$

---

4 **Use trigonometry**

so triangle ABC is isosceles and the base angles are equal:

$$\angle BAC = \angle ACB = 78^\circ$$

---

5 **Use trigonometry**

Using the angle sum of a triangle:

$$\angle ABC = 180^\circ - 78^\circ - 78^\circ$$

$$\angle ABC = 24^\circ$$

✓ **FINAL ANSWER**

$\boxed{24^\circ}$

4MA1/1H NOVEMBER 2023

Question 14

3 marks | Circle Theorems

14

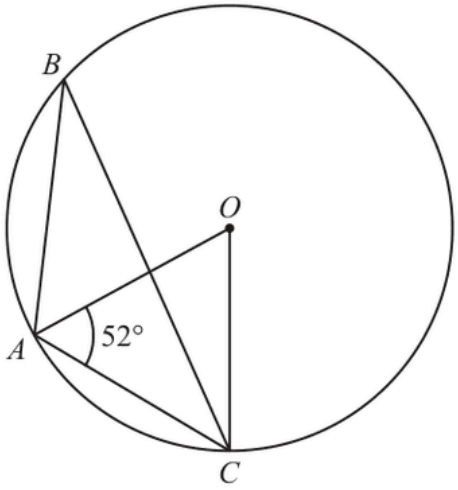


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O

Angle $OAC = 52°$

Find the size of angle ABC
Give reasons for your working.

.....°

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2023

1 Use trigonometry

Since $OA = OC$, triangle AOC is isosceles.

$$\angle ACO = \angle OAC = 52°$$

2 Use trigonometry

So

$$\angle AOC = 180° - 52° - 52° = 76°$$

3 The angle at the circumference

The angle at the circumference is half the angle at the centre standing on the same arc:

$$\angle ABC = \frac{76°}{2} = 38°$$

FINAL ANSWER

$38°$.

4MA1/1HR MAY 2024

Question 13

5 marks | Circle Theorems

13

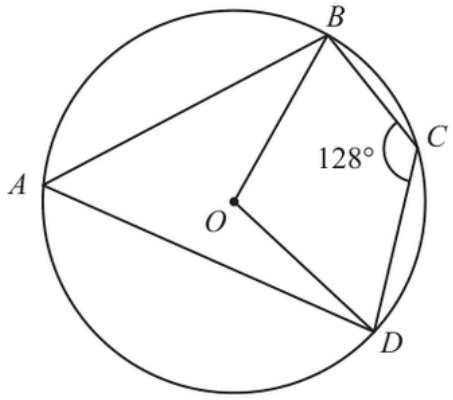


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

Angle $BCD = 128°$

Work out the size of angle OBD
Give a reason for each stage of your working.

angle OBD = ... °

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR May 2024

1 Opposite angles in cyclic quadrilateral

Opposite angles in cyclic quadrilateral $ABCD$ add to $180°$:

$$\angle BAD = 180° - 128° = 52°$$

2 The angle at the centre

The angle at the centre is twice the angle at the circumference standing on the same arc BD:

$$\angle BOD = 2 \times 52° = 104°$$

3 Use trigonometry

Since $OB = OD$, triangle OBD is isosceles:

$$\angle OBD = \frac{180° - 104°}{2} = 38°$$

FINAL ANSWER

$38°$.

Question 16

5 marks | Circle Theorems

16 A, B, C and D are points on a circle, centre O

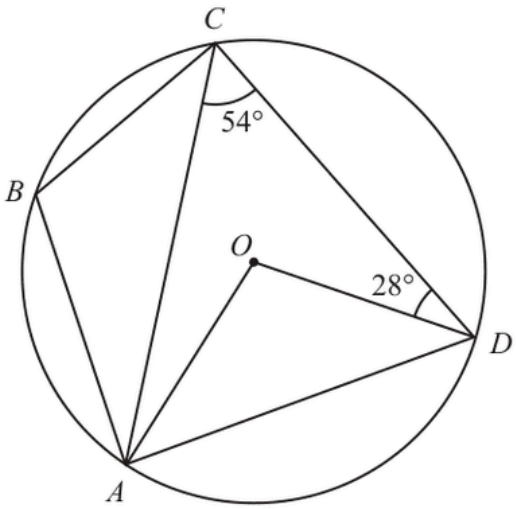


Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle AOD

.....°

(1)

(ii) Give a reason for your answer to part (a)(i)

.....

(1)

(b) Work out the size of angle CAO

.....°

(1)

(c) Work out the size of angle ABC

.....°

(2)

(Total for Question 16 is 5 marks)

1 **(a) The angle at the**

(a) The angle at the centre is twice the angle at the circumference standing on the same arc AD:

$$\angle AOD = 2 \times 54^\circ = 108^\circ$$

2 **Use trigonometry**

(b) Since $OC = OD$, triangle COD is isosceles:

$$\angle OCD = \angle CDO = 28^\circ$$

3 **Use trigonometry**

So

$$\angle COD = 180^\circ - 28^\circ - 28^\circ = 124^\circ$$

4 **Use circle theorem**

Angles around O add to 360°:

$$\angle AOC = 360^\circ - 108^\circ - 124^\circ = 128^\circ$$

5 **Use trigonometry**

Since $OA = OC$,

$$\angle CAO = \frac{180^\circ - 128^\circ}{2} = 26^\circ$$

6 **(c) Angle stands on the**

(c) Angle ABC stands on the major arc AC, so

$$\angle ABC = \frac{360^\circ - 128^\circ}{2} = 116^\circ$$

✓ **FINAL ANSWER**

$\angle AOD = 108^\circ$, $\angle CAO = 26^\circ$, and $\angle ABC = 116^\circ$.

Question 13

4 marks | Circle Theorems

13 The diagram shows a circle, centre O

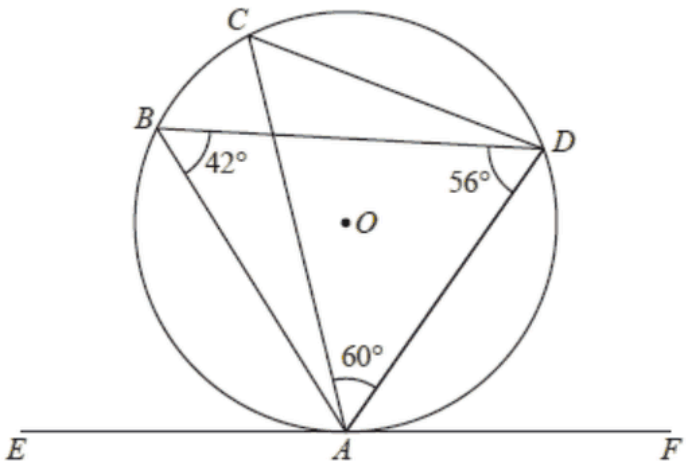


Diagram **NOT** accurately drawn

A, B, C and D are points on the circle.
EAF is the tangent to the circle at A

angle ABD = 42° **angle BDA = 56°** **angle CAD = 60°**

(a) (i) Find the size of angle EAB

..... °

(1)

(ii) Give a reason for your answer to (a) (i)

.....

.....

(1)

(b) Work out the size of angle CDB

..... °

(2)

(Total for Question 13 is 4 marks)

1 **Use trigonometry**

By the alternate segment theorem,

$$\angle EAB = \angle ADB = 56^\circ$$

2 **Use trigonometry**

In triangle ABD,

$$\angle BAD = 180^\circ - 42^\circ - 56^\circ = 82^\circ$$

3 **Use trigonometry**

So

$$\angle BAC = 82^\circ - 60^\circ = 22^\circ$$

4 **Angles subtended by the same**

Angles subtended by the same chord CB are equal, so

$$\angle CDB = 22^\circ$$

✓ **FINAL ANSWER**

56°, alternate segment theorem, 22°.

4MA1/2HR JUNE 2025

Question 15

3 marks | Circle Theorems

15 A, B and C are points on a circle, centre O

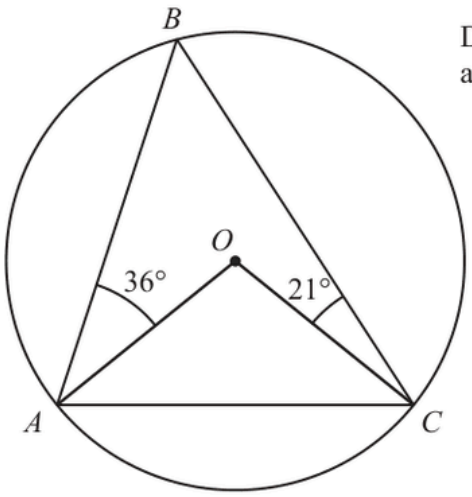


Diagram **NOT** accurately drawn

Angle $BAO = 36°$
Angle $BCO = 21°$

Work out the size of angle ACO

.....°

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2025

1 Use trigonometry

Since $OA = OB$, triangle AOB is isosceles:

$$\angle ABO = 36°$$

2 Use trigonometry

So

$$\angle AOB = 180° - 36° - 36° = 108°$$

3 Use trigonometry

Since $OB = OC$, triangle BOC is isosceles:

$$\angle OBC = 21°$$

4 **Use trigonometry**

So

$$\angle BOC = 180° - 21° - 21° = 138°$$

5 **Use circle theorem**

Angles around O add to $360°$:

$$\angle AOC = 360° - 108° - 138° = 114°$$

6 **Use trigonometry**

Since $OA = OC$,

$$\angle ACO = \frac{180° - 114°}{2} = 33°$$

✓ **FINAL ANSWER**

$33°$.

4WM2/01R MAY 2025

Question 15

3 marks | Circle Theorems

15 A, B and C are points on a circle, centre O

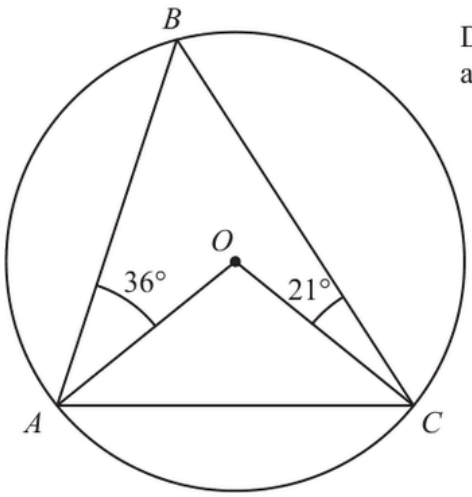


Diagram **NOT** accurately drawn

Angle $BAO = 36°$
Angle $BCO = 21°$

Work out the size of angle ACO

.....°

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM2/01R May 2025

1 Required

A, B, C on circle centre O. ∠*B**A**O* = 36°, ∠*B**C**O* = 21°. Find ∠*A**C**O*.

2 O A B is isosceles

△*O**A**B* is isosceles (*O**A*** = ***O**B*= radius):

∠*O**A**B* = ∠*O**B**A* = 36° ⟹ ∠*A**O**B* = 180° − 36° − 36° = 108°

3 O B C is isosceles

△*O**B**C* is isosceles (*O**B*** = ***O**C*):

∠*O**B**C* = ∠*O**C**B* = 21° ⟹ ∠*B**O**C* = 180° − 21° − 21° = 138°

4 Angles at centre O (on

Angles at centre O (on the same side):

∠*A**O**C* = 360° − 108° − 138° = 114°

5 O A C is isosceles

△*O**A**C* is isosceles (*O**A*** = ***O**C*):

$$angleOAC + angleOCA = 180^\circ - 114^\circ = 66^\circ impliesangleACO = frac66^\circ 2 = 33^\circ$$

FINAL ANSWER

∠*A**C**O* = 33°

4WM2/01R MAY 2025

Question 18

3 marks | Circle Theorems

18

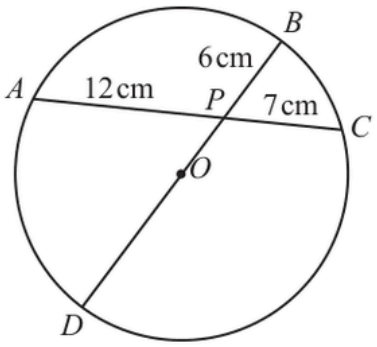


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

DB is a diameter of the circle.
AC and DB intersect at the point P

$AP = 12\,\text{cm}$ $PC = 7\,\text{cm}$ $PB = 6\,\text{cm}$

Work out the area of the circle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01R May 2025

1 **Required**

DB is a diameter; chords AC and DB intersect at P. AP = 12 cm, PC = 7 cm, PB = 6 cm. Find area of circle (3 s.f.).

2 **Apply the Intersecting Chords Theorem**

Apply the **Intersecting Chords Theorem**: *A**P** × *PC* = *B**P** × *PD*

$$12 times 7 = 6 times PD implies PD = frac{84}{6} = 14 textcm$$

3 **Diameter D B D B**

Diameter *D**B*:

*D**B** = *PD* + *P**B* = 14 + 6 = 20 cm ⟹ *r* = 10 cm

4 **Area of circle Area r**

Area of circle:

Area = *π**r*² = 100*π* ≈ 314.159… cm²

5 **Note**

Intersecting Chords Theorem: if two chords cross inside a circle at P, then *A**P* × **P**C* = *B**P* × **P**D*.

✓ **FINAL ANSWER**

Area of circle = 314 cm² (3 s.f.)

4MA1/1HR JUNE 2025

Question 23

5 marks | Circle Theorems

23

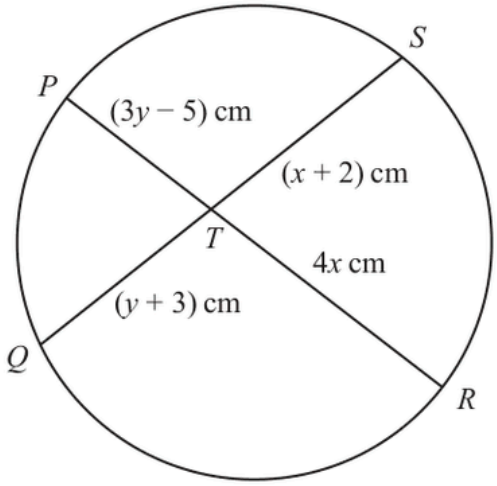


Diagram **NOT** accurately drawn

PTR and QTS are chords of a circle.

$PT = (3y - 5)\,\text{cm}$ $QT = (y + 3)\,\text{cm}$ $RT = 4x\,\text{cm}$ $ST = (x + 2)\,\text{cm}$

Find an expression for y in terms of x

$y =$ .....

Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2025

1 Use chord theorem

For two intersecting chords in a circle,

$$PT \times TR = QT \times TS$$

2 Substitute the given lengths

Substitute the given lengths:

$$(3y - 5)(4x) = (y + 3)(x + 2)$$

3 Expand

Expand:

$$12xy - 20x = xy + 2y + 3x + 6$$

4 **Collect the terms on one**

Collect the y terms on one side:

$$12xy - xy - 2y = 20x + 3x + 6$$

$$11xy - 2y = 23x + 6$$

---

5 **Factorise**

Factorise:

$$y(11x - 2) = 23x + 6$$

$$y = \frac{23x + 6}{11x - 2}$$

✓ **FINAL ANSWER**

$$\boxed{y = \frac{23x + 6}{11x - 2}}$$

Question 15

4 marks | Circle Theorems

15 A, B, C and D are points on a circle with centre O

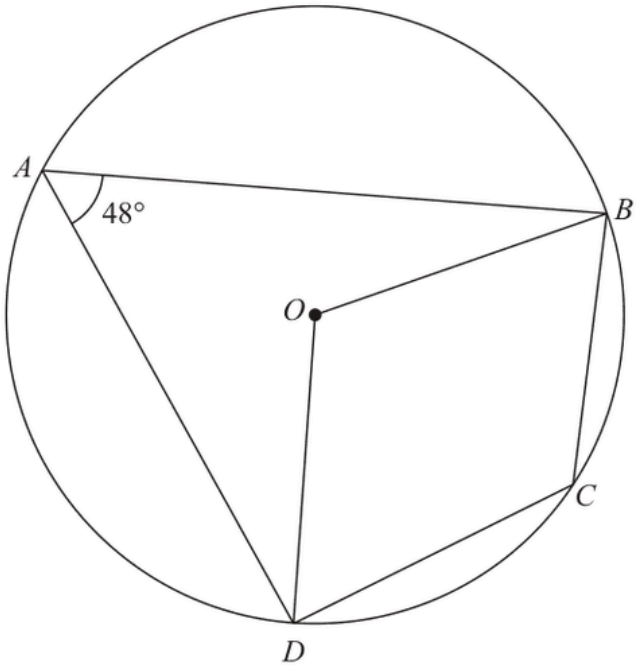


Diagram **NOT** accurately drawn

Angle $DAB = 48°$

(a) (i) Work out the size of the obtuse angle DOB

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Work out the size of angle BCD

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 15 is 4 marks)

1 **Part (a)(i) Angle D O**

Part (a)(i) Angle *DOB*:

∠*D**O**B* = 2 × 48[∘] = 96[∘]

2 **Answer**

(a)(i) 96[∘]

Part (a)(ii) Reason: The angle subtended by an arc at the centre is twice the angle subtended at the circumference.

Part (b)(i) Angle *BCD*:

∠*B**C**D* = 180[∘] − 48[∘] = 132[∘]

✓ **FINAL ANSWER**

(b)(i) 132[∘] **Part (b)(ii) Reason:** Opposite angles in a cyclic quadrilateral sum to 180[∘].

Question 15

4 marks | Circle Theorems

15 A, B, C and D are points on a circle with centre O

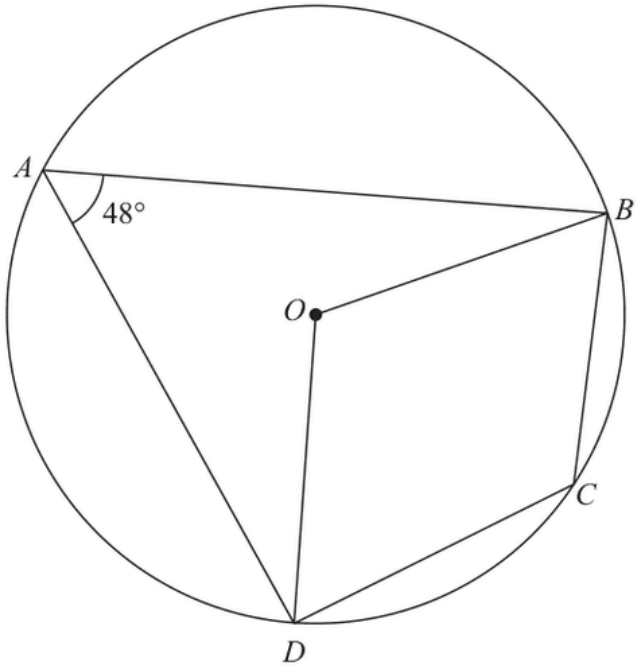


Diagram **NOT** accurately drawn

Angle $DAB = 48°$

(a) (i) Work out the size of the obtuse angle DOB

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Work out the size of angle BCD

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 15 is 4 marks)

1 **(a) The angle at the**

(a) The angle at the centre is twice the angle at the circumference standing on chord DB:

$$\angle DOB = 2 \times 48° = 96°$$

2 **(b) Opposite angles in cyclic**

(b) Opposite angles in cyclic quadrilateral $ABCD$ add to $180°$:

$$\angle BCD = 180° - 48° = 132°$$

✓ **FINAL ANSWER**

$\angle DOB = 96°$, and $\angle BCD = 132°$.

Question 14

4 marks | Circle Theorems

14

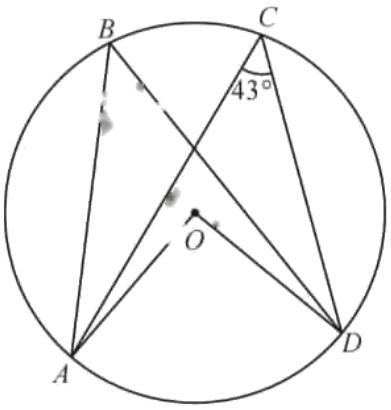


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

angle $ACD = 43°$

(a) (i) Find the size of angle ABD

..... °

(1)

(ii) Give a reason for your answer.

.....

.....

(1)

(b) Find the size of the reflex angle AOD

..... °

(2)

(Total for Question 14 is 4 marks)

1 **(a)(i) Same segment**

$$\angle ABD = \angle ACD = 43^\circ$$

2 **(a)(ii) Reason**

Angles in the same segment are equal (both stand on the arc AD).

3 **(b) Angle at the centre**

$$\angle AOD = 2 \times 43^\circ = 86^\circ \quad \text{(angle at centre is twice the angle at the circumference)}$$

4 **Reflex angle**

$$360^\circ - 86^\circ = 274^\circ$$

✓ **FINAL ANSWER**

(a) 43°, angles in the same segment are equal (b) 274°

4MA1/1HR JUNE 2026

Question 19

5 marks | Circle Theorems

19 P, Q, R and S are points on a circle, centre O

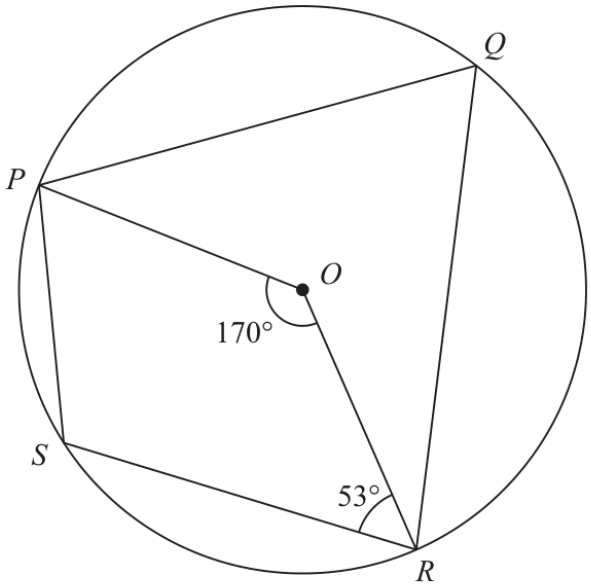


Diagram **NOT** accurately drawn

Angle $POR = 170°$ Angle $ORS = 53°$

Find the size of angle SPO
Give a reason for each stage of your working.

.....°

.....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$\angle PQR = \frac{170°}{2} = 85°.$$

The angle at the centre is twice the angle at the circumference subtended by the same arc PR containing S.

2 **Solution | Step 2**

$$\angle PSR = 180° - 85° = 95°.$$

Opposite angles of cyclic quadrilateral PQRS sum to 180 degrees.

3 Solution | Step 3

$$\angle SPO = 360^\circ - (170^\circ + 53^\circ + 95^\circ) = 42^\circ.$$

The interior angles of quadrilateral PSRO sum to 360 degrees; its other angles are POR, ORS and PSR.

FINAL ANSWER

$\angle SPO = 42^\circ$

4MA1/2H JUNE 2026

Question 19

2 marks | Circle Theorems

19 B, D and E are points on a circle.

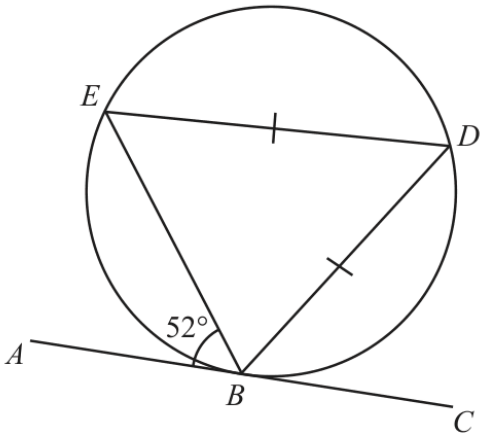


Diagram **NOT** accurately drawn

ABC is the tangent to the circle at B

angle $ABE = 52°$ $ED = BD$

Work out the size of angle BED

.....°

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2026

1 **Solution | Step 1**

$\angle BDE = \angle ABE = 52°$, by the alternate segment theorem for tangent AB and chord BE.
The angle between a tangent and chord equals the angle in the opposite segment.

2 **Solution | Step 2**

Since $ED = BD$, the base angles $\angle BED$ and $\angle EBD$ are equal. Thus

$$\angle BED = \frac{180° - 52°}{2} = 64°.$$

Equal sides in triangle EBD subtend equal opposite angles, and its angles sum to 180 degrees.

✓ **FINAL ANSWER**

$64°$

4MA1/1HR JUNE 2026

Question 24

4 marks | Circle Theorems

24 The centre of a circle is the point with coordinates (3, 1)

The point P with coordinates (7, 6) lies on the circle.

The line **L** is the tangent to the circle at P

Find an equation for **L**
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

.....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2026

1 Solution | Step 1

$$m_{CP} = \frac{6-1}{7-3} = \frac{5}{4}.$$

Find the gradient of the radius from the centre C=(3,1) to P=(7,6).

2 Solution | Step 2

$$m_L = -\frac{4}{5}.$$

The tangent is perpendicular to the radius at P, so its gradient is the negative reciprocal.

3 Solution | Step 3

$$y - 6 = -\frac{4}{5}(x - 7).$$

Use the tangent gradient and the given point of contact.

4 Solution | Step 4

$$5y - 30 = -4x + 28 \quad \Longrightarrow \quad 4x + 5y - 58 = 0.$$

Multiply by 5 and collect all terms on one side to obtain integer coefficients.

✓ **FINAL ANSWER**

$$4x + 5y - 58 = 0$$

TOPIC

Bearings

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2021

Question 16

5 marks | Bearings

16 The diagram shows the positions of three ships, A, B and C.

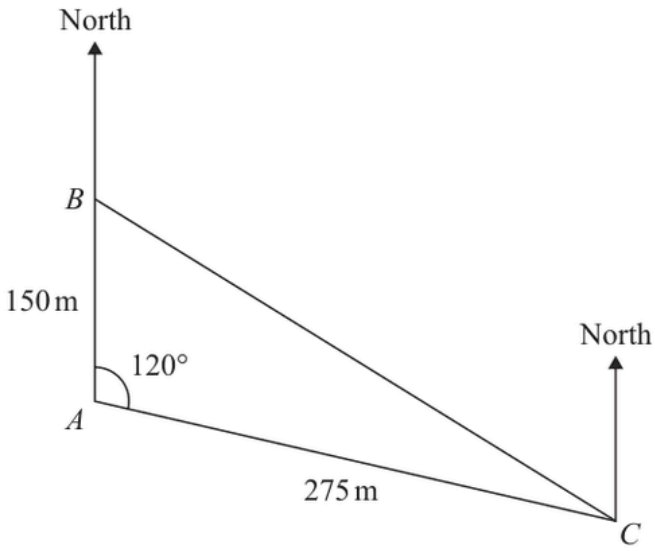


Diagram **NOT** accurately drawn

Ship B is due north of ship A.

The bearing of ship C from ship A is 120°

Calculate the bearing of ship C from ship B.
Give your answer correct to the nearest degree.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2021

1 **Use bearings**

From A, ship C is on a bearing of $120°$, and $AC = 275\ \text{m}$.

2 **Resolve**

Resolve AC:

$$\text{east component} = 275\sin 120^\circ = 238.16\ldots$$

$$\text{north component} = 275\cos 120^\circ = -137.5$$

---

3 **Use bearings**

Ship B is $150\ \text{m}$ north of A, so from B to C:

$$\text{east} = 238.16\ldots, \qquad \text{north} = -137.5 - 150 = -287.5$$

---

4 **Use bearings**

The bearing is therefore in the south-east quadrant:

$$180^\circ - \tan^{-1}\left(\frac{238.16\ldots}{287.5}\right) = 140.36\ldots^\circ$$

✓ **FINAL ANSWER**

140°.

4MA1/2H MAY 2021

Question 3

2 marks | Bearings

3 The bearing of Paris from London is 149°

Work out the bearing of London from Paris.

.....°

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H May 2021

1 Use bearings

The bearing of Paris from London is the reverse bearing of London from Paris.

2 Reverse bearings differ by

Reverse bearings differ by $180°$:

$$149° + 180° = 329°$$

FINAL ANSWER

$329°$.

4MA1/1H NOVEMBER 2023

Question 18

5 marks | Bearings

18 The diagram shows the positions of three villages, A, B and C

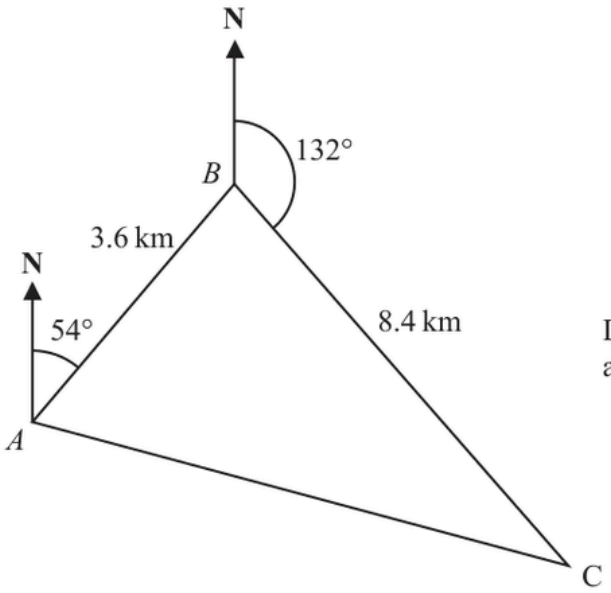


Diagram **NOT** accurately drawn

The bearing of B from A is 054°
The bearing of C from B is 132°

Melur walks from A to B
She then walks from B to C and from C to A

Melur walks at an average speed of 6 km/h

Work out the total time Melur takes.
Give your answer in hours and minutes.

..... hours ..... minutes

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2023

1 The bearing of from is

The bearing of B from A is $054°$, so the bearing of A from B is

$$054° + 180° = 234°$$

2 The bearing of from is

The bearing of C from B is $132°$.

3 **Use bearings**

So

$$\angle ABC = 234° - 132° = 102°$$

---

4 **Use cosine rule**

Use the cosine rule to find AC:

$$AC^2 = 3.6^2 + 8.4^2 - 2(3.6)(8.4)\cos 102°$$

$$AC = 9.8028\ldots \text{ km}$$

---

5 **Total distance walked**

Total distance walked:

$$3.6 + 8.4 + 9.8028\ldots = 21.8028\ldots \text{ km}$$

---

6 **Use bearings**

At 6 km/h, time is

$$21.8028\ldots \div 6 = 3.6338\ldots \text{ hours}$$

$$0.6338\ldots \times 60 = 38.0\ldots \text{ minutes}$$

✓ **FINAL ANSWER**

3 hours 38 minutes.

4MA1/2H NOVEMBER 2024

Question 24

6 marks | Bearings

24 A, B and C are three points on horizontal ground.

$AB = 8.4$ metres $BC = 9.2$ metres

B is on a bearing of 067° from A
C is on a bearing of 129° from B

Calculate the bearing of A from C
Give your answer correct to the nearest degree.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2024

1 Resolve each journey into east

Resolve each journey into east and north components.

2 Use trigonometry

From A to B:

$$\text{east} = 8.4 \sin 67° = 7.7322\ldots$$

$$\text{north} = 8.4 \cos 67° = 3.2821\ldots$$

3 Use trigonometry

From B to C:

$$\text{east} = 9.2 \sin 129° = 7.1497\ldots$$

$$\text{north} = 9.2 \cos 129° = -5.7897\ldots$$

4 **Calculate value**

So from A to C:

$$\text{east} = 14.8820\ldots$$

$$\text{north} = -2.5076\ldots$$

---

5 **Calculate value**

Therefore from C to A:

$$\text{east} = -14.8820\ldots, \qquad \text{north} = 2.5076\ldots$$

---

6 **Use trigonometry**

This direction is west of north.

$$\theta = \tan^{-1}\left(\frac{14.8820\ldots}{2.5076\ldots}\right) = 80.4355\ldots^\circ$$

---

7 **Use bearings**

Bearing of A from C:

$$360^\circ - 80.4355\ldots^\circ = 279.5644\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{280^\circ}$

TOPIC

Transformations

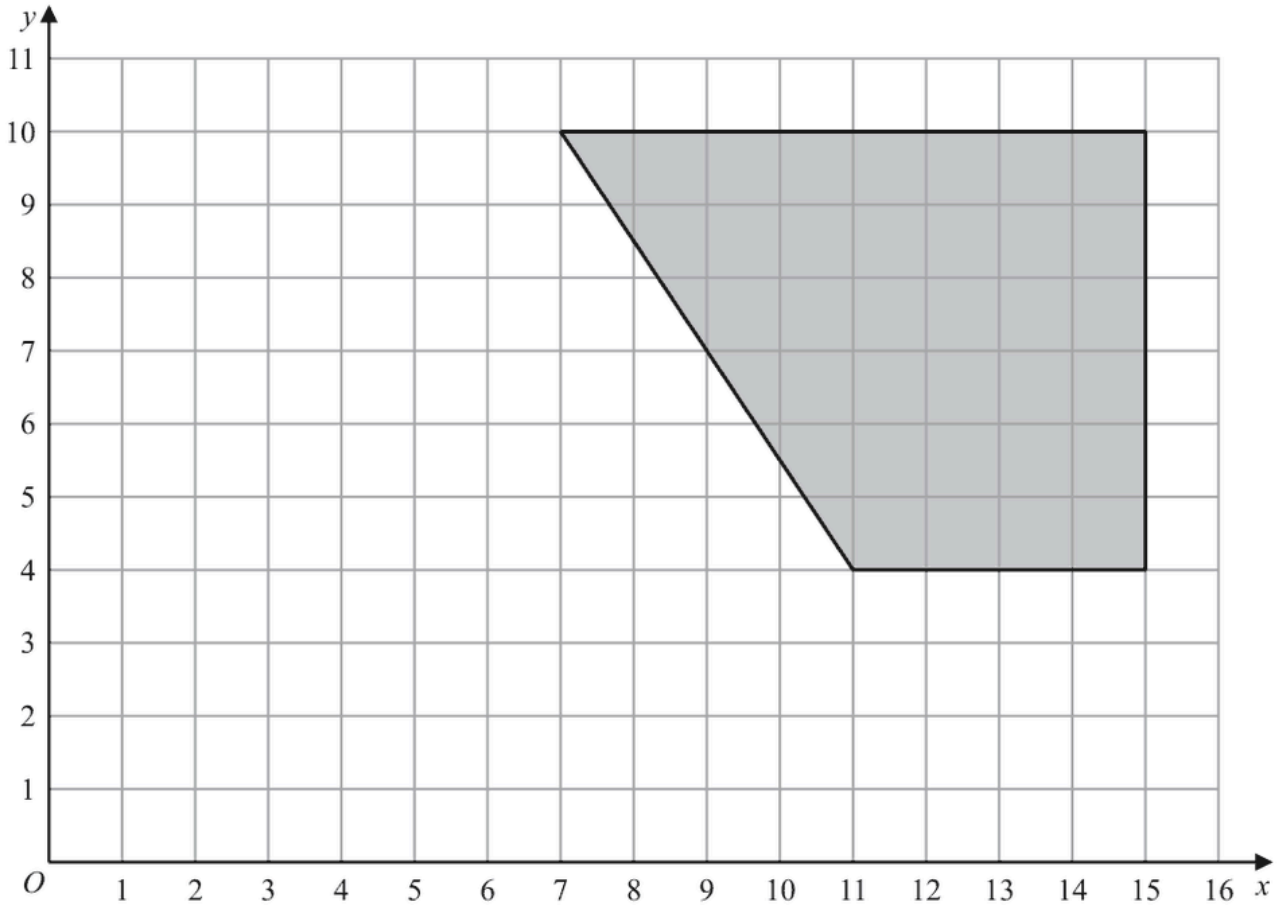
Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 8

2 marks | Transformations

8



On the grid, enlarge the shaded shape with scale factor $\frac{1}{2}$ and centre (1, 2)

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2020

1 **Use**

Use

$$\text{new point} = (1, 2) + \frac{1}{2}(\text{old point} - (1, 2))$$

2 **The vertices**

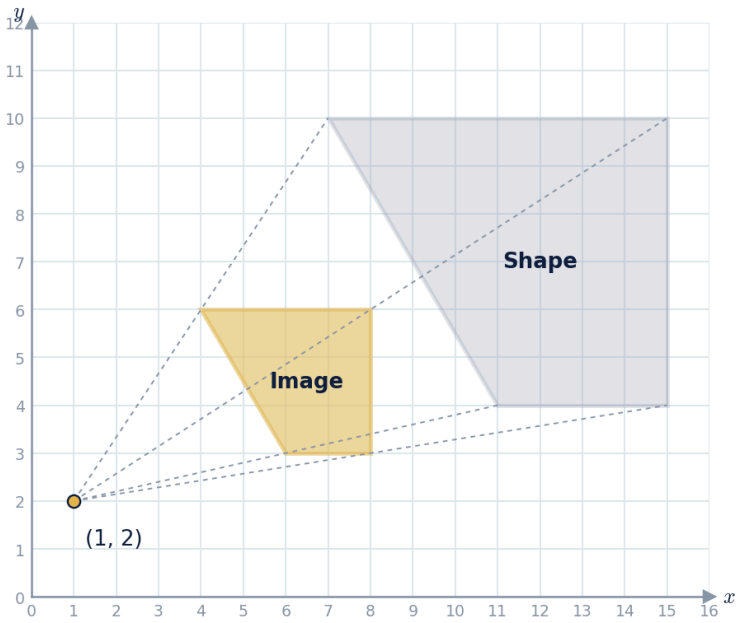
The vertices

$$(7, 10),\ (15, 10),\ (15, 4),\ (11, 4)$$

3 **become**

become

$$(4, 6),\ (8, 6),\ (8, 3),\ (6, 3)$$



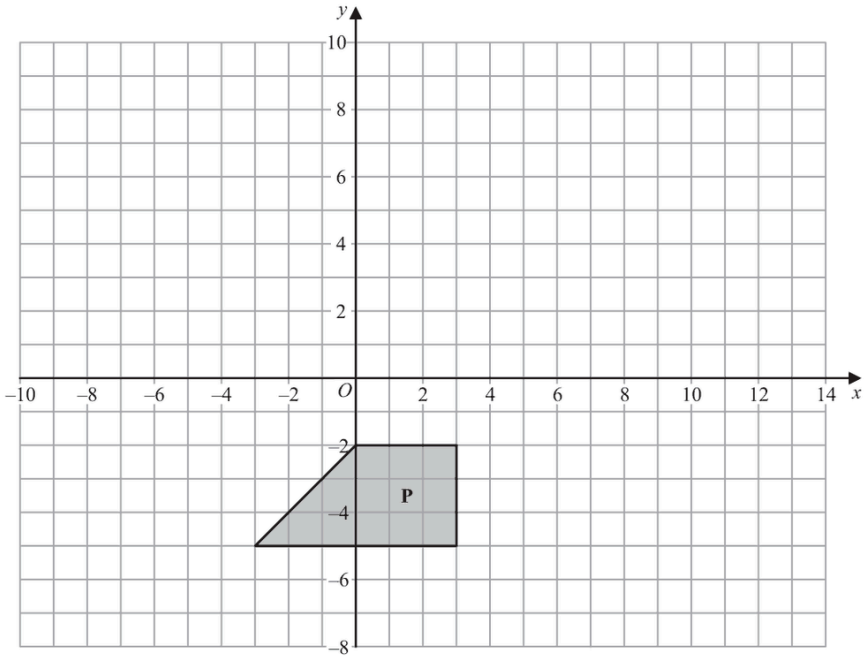
✓ **FINAL ANSWER**

draw the enlarged shape with vertices $(4, 6), (8, 6), (8, 3), (6, 3)$.

Question 5

6 marks | Transformations

5 Here is a shape **P** drawn on a grid of squares.



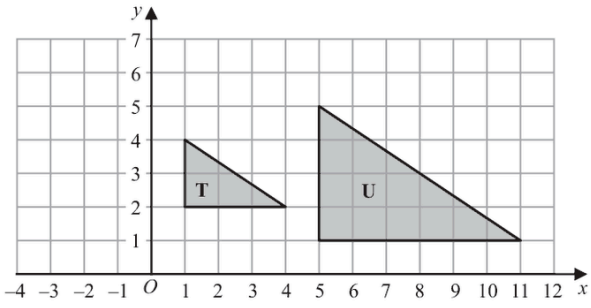
(a) On the grid, rotate shape **P** 180° about the point (–3, 2)
Label the new shape **Q**.

(2)

(b) On the grid, translate shape **P** by the vector $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$
Label the new shape **R**.

(1)

Here are triangle **T** and triangle **U** drawn on a grid of squares.



(c) Describe fully the single transformation that maps triangle **T** onto triangle **U**.

(3)

(Total for Question 5 is 6 marks)

1 **(a) For a rotation about**

(a) For a $180°$ rotation about $(-3, 2)$,

$$(x, y) \mapsto (-6 - x,\ 4 - y)$$

2 **The image has vertices**

The image has vertices

$$(-3, 9),\ (-9, 9),\ (-9, 6),\ (-6, 6)$$

3 **(b) Translate by**

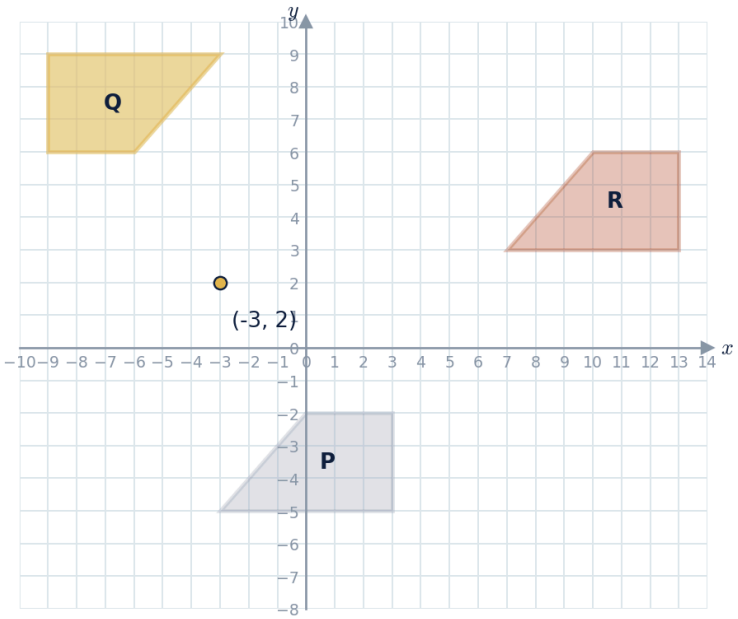
(b) Translate by

$$\begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

4 **The image has vertices**

The image has vertices

$$(7, 3),\ (13, 3),\ (13, 6),\ (10, 6)$$



5 **Use trigonometry**

(c) Triangle T is mapped to triangle U by an enlargement with scale factor 2 and centre $(-3, 3)$.

✓ **FINAL ANSWER**

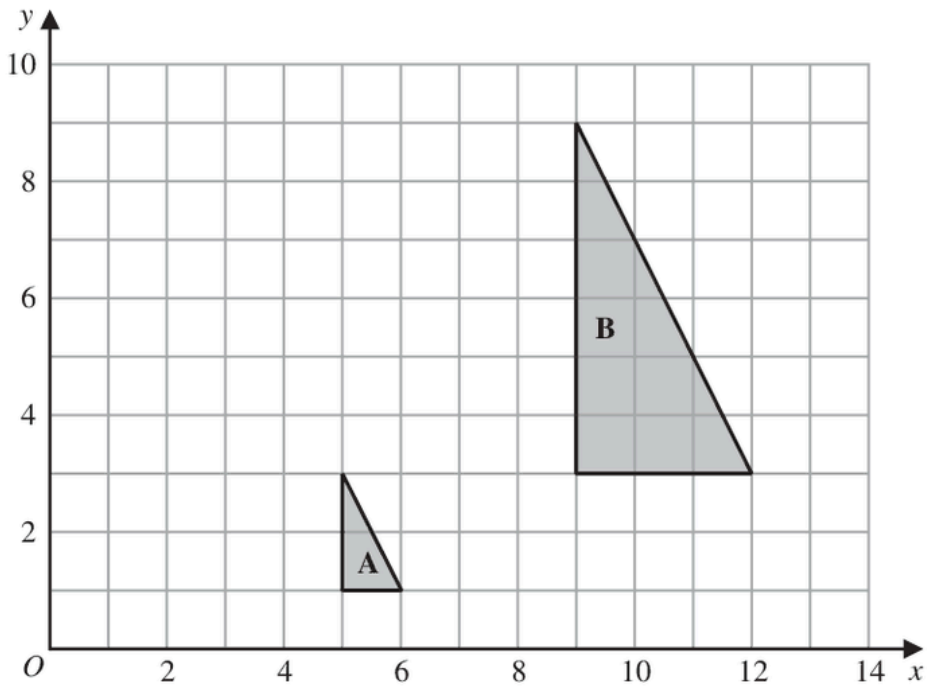
(a) vertices $(-3, 9), (-9, 9), (-9, 6), (-6, 6)$; (b) vertices $(7, 3), (13, 3), (13, 6), (10, 6)$; (c) enlargement, scale factor 2, centre $(-3, 3)$.

4MA1/2HR JUNE 2022

Question 1

4 marks | Transformations

1



(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(b) On the grid above, translate triangle **A** by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Label your triangle **C**

(1)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2022

1 **Use trigonometry**

(a) Triangle A is enlarged to triangle B.

2 **The scale factor is**

The scale factor is

3 **Use Transformations tag correct**

Using corresponding vertices gives centre $(3, 0)$.

4 **Use Transformations tag correct**

So the transformation is an enlargement, scale factor 3, centre $(3, 0)$.

5 **(b) Translating triangle by**

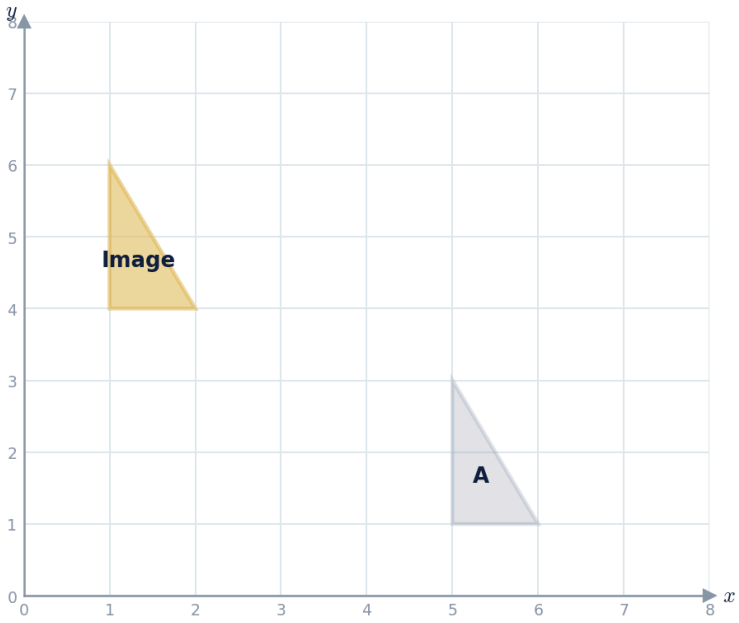
(b) Translating triangle A by

$$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

6 **gives vertices**

gives vertices

$$(1, 4),\ (2, 4),\ (1, 6)$$



FINAL ANSWER

(a) enlargement, scale factor 3, centre $(3, 0)$; (b) vertices $(1, 4), (2, 4), (1, 6)$.

Question 4

8 marks | Transformations

4 The diagram shows a sketch of triangle **A** and triangle **B** on a coordinate grid.

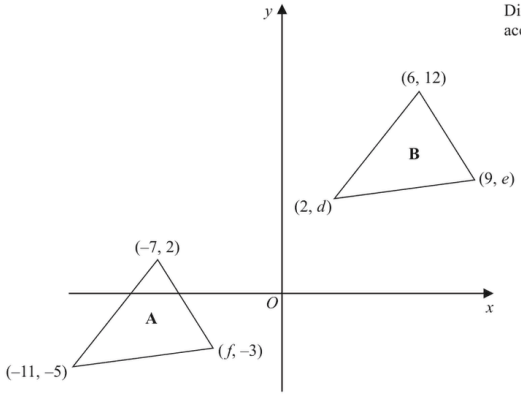


Diagram **NOT** accurately drawn

(a) Given that triangle **A** has been translated to give triangle **B**, work out the value of d, the value of e and the value of f

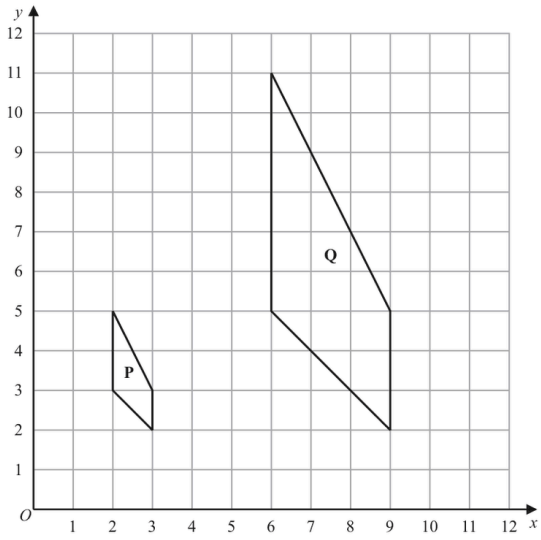
$d =$ .....

$e =$ .....

$f =$ .....

(3)

The diagram shows shape **P** and shape **Q** drawn on a grid.



(b) Describe fully the single transformation that maps shape **P** onto shape **Q**

.....

(3)

(c) On the grid above, rotate shape **P** 90° clockwise about (3, 5)
Label your shape **R**

(2)

(Total for Question 4 is 8 marks)

1 **Use matrices**

(a) The translation from $(-7, 2)$ to $(6, 12)$ is

$$\begin{pmatrix} 13 \\ 10 \end{pmatrix}$$

2 **Use Transformations tag correct**

So

$$(-11, -5) \mapsto (2, 5)$$

3 **Use Transformations tag correct**

and

$$(f, -3) \mapsto (9, 7)$$

4 **Apply transformation**

Therefore

$$d = 5, \qquad e = 7, \qquad f = -4$$

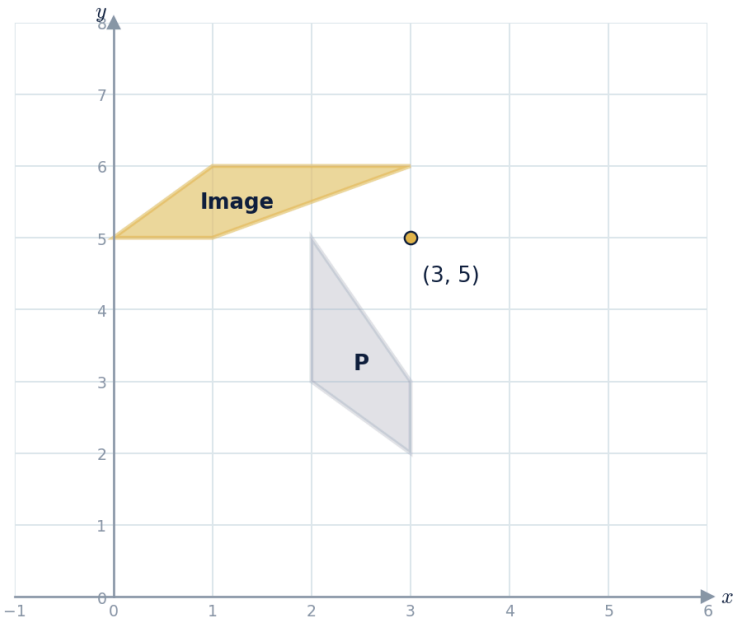
5 **(b) Shape maps to by**

(b) Shape P maps to Q by an enlargement with scale factor 3 and centre $(0, 2)$.

6 **(c) Rotate clockwise about**

(c) Rotate P $90°$ clockwise about $(3, 5)$. The image has vertices

$$(3, 6),\ (1, 6),\ (0, 5),\ (1, 5)$$



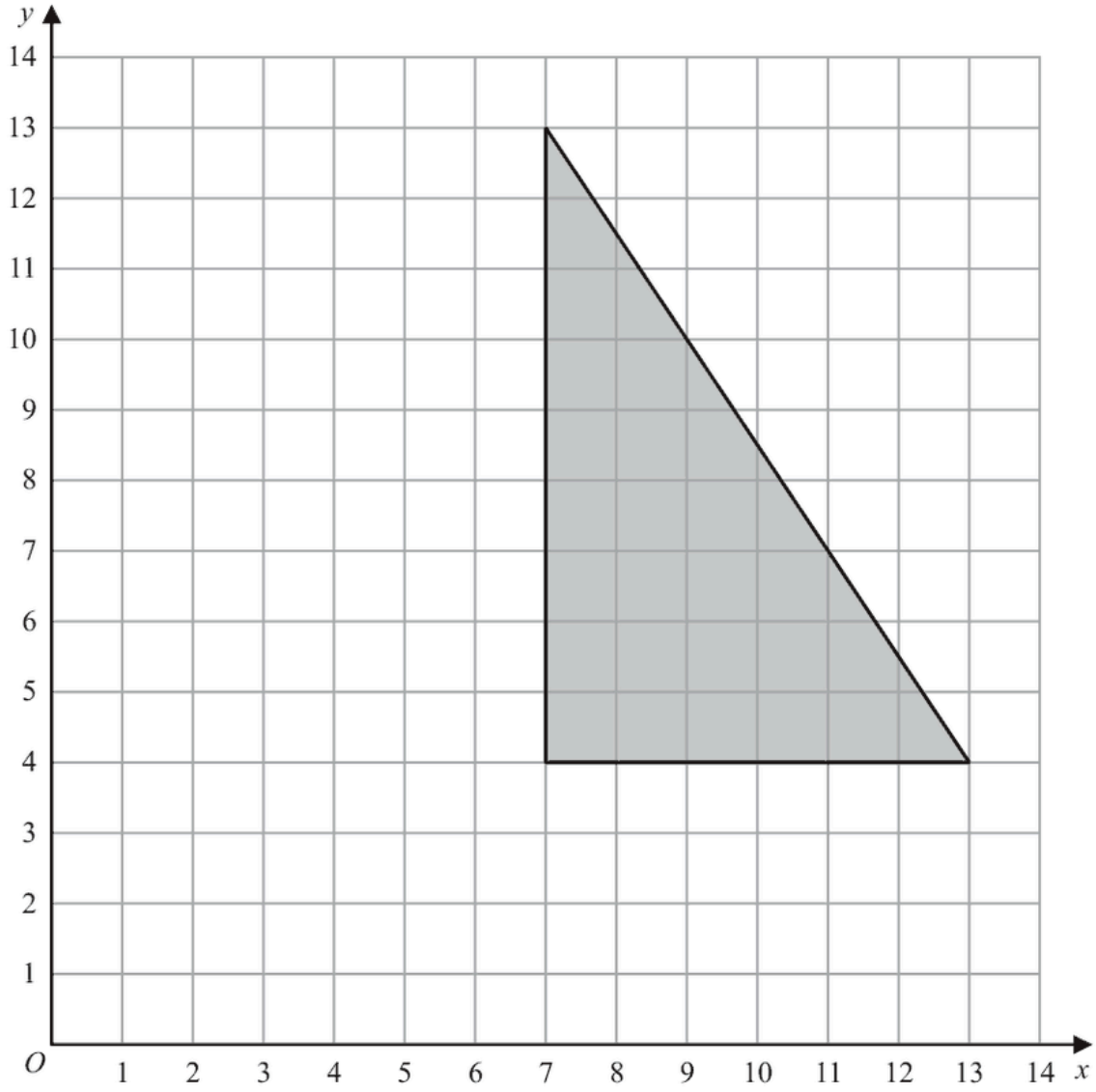
FINAL ANSWER

(a) $d = 5,\ e = 7,\ f = -4$; (b) enlargement, scale factor 3, centre $(0, 2)$; (c) vertices $(3, 6), (1, 6), (0, 5), (1, 5)$.

Question 10

2 marks | Transformations

10



On the grid, enlarge the shaded shape with scale factor $\frac{1}{3}$ and centre (1, 7)

(Total for Question 10 is 2 marks)

1 **Use**

Use

$$\text{new point} = (1,7) + \frac{1}{3}(\text{old point} - (1,7))$$

2 **The original vertices are**

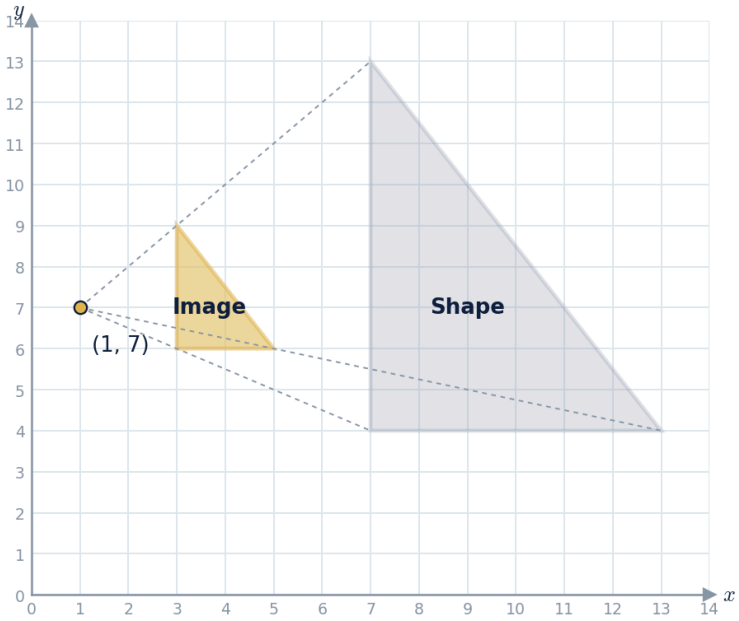
The original vertices are

$$(7,4),\ (13,4),\ (7,13)$$

3 **They become**

They become

$$(3,6),\ (5,6),\ (3,9)$$



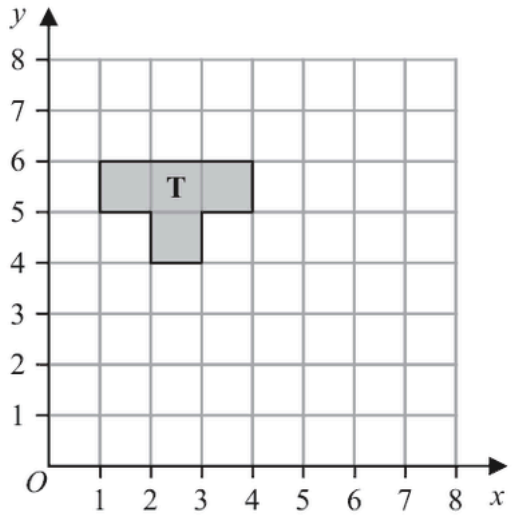
FINAL ANSWER

draw the triangle with vertices $(3,6), (5,6), (3,9)$.

Question 3

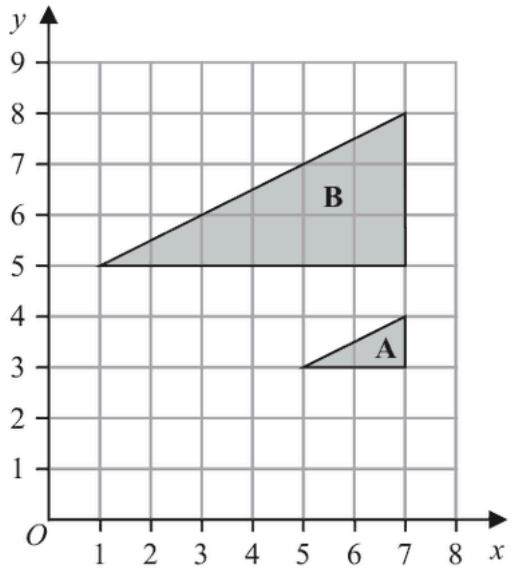
5 marks | Transformations

3



(a) Reflect shape **T** in the line $y = x$

(2)



(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(Total for Question 3 is 5 marks)

1 (a) Reflection in swaps the

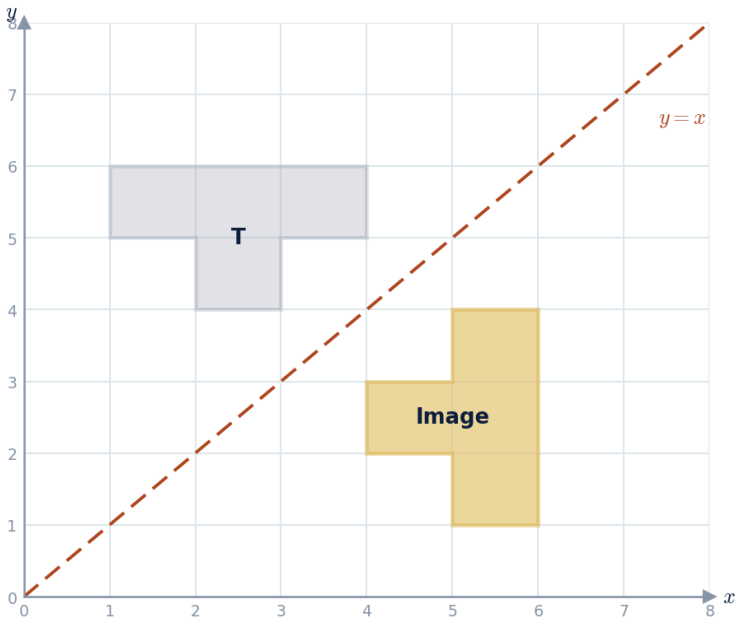
(a) Reflection in $y = x$ swaps the coordinates:

$$(x, y) \mapsto (y, x)$$

2 The reflected T shape has

The reflected T-shape has vertices

$$(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$$



3 Use trigonometry

(b) Triangle A is mapped to B by an enlargement with scale factor 3 and centre $(7, 2)$.

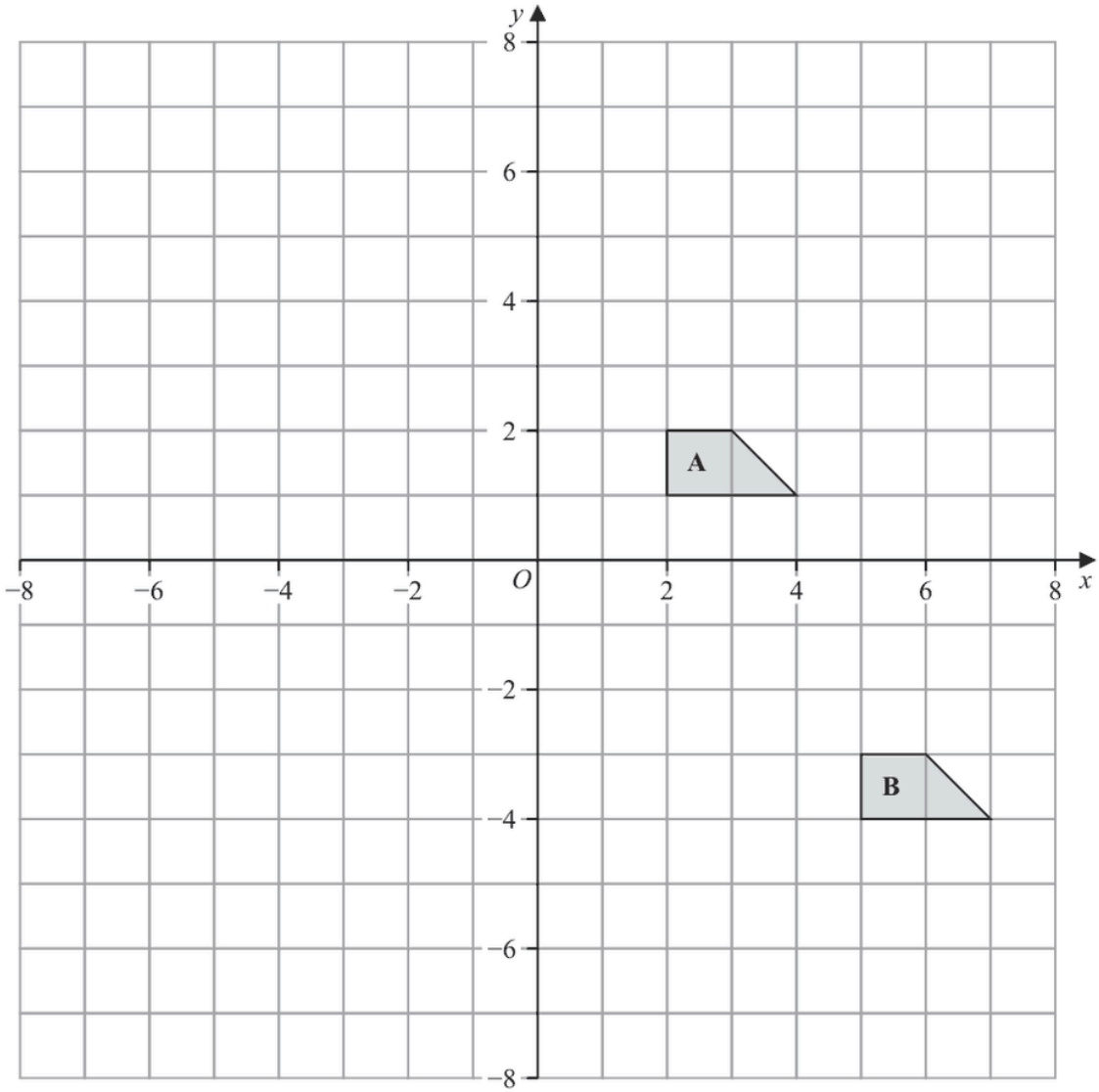
FINAL ANSWER

(a) reflected vertices $(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$; (b) enlargement, scale factor 3, centre $(7, 2)$.

Question 2

4 marks | Transformations

2



(a) Describe fully the single transformation that maps shape **A** onto shape **B**

.....

.....

(2)

(b) On the grid above, rotate shape **A** 180° about (−1, 0)
Label your shape **C**

(2)

(Total for Question 2 is 4 marks)

1 **(a) Shape moves to shape**

(a) Shape A moves to shape B by the vector

$$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

2 **Use matrices**

So the transformation is a translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

3 **(b) For a rotation about**

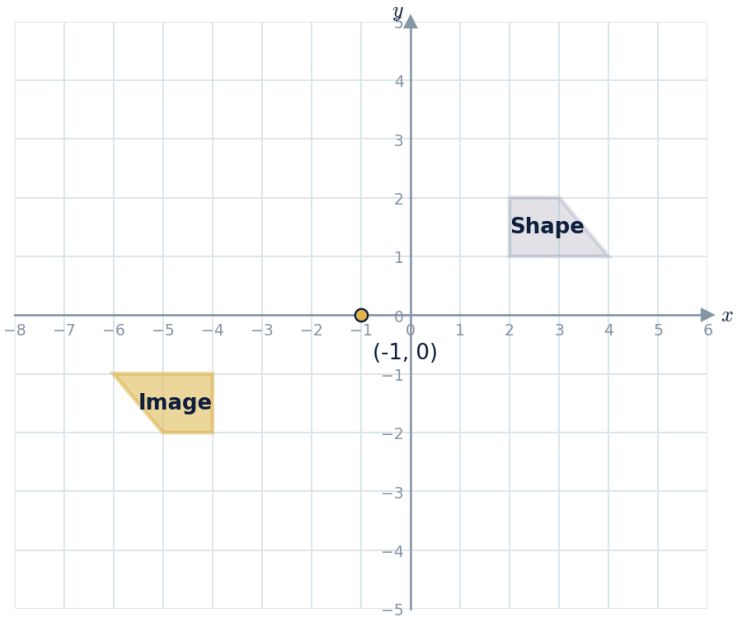
(b) For a $180°$ rotation about $(-1, 0)$,

$$(x, y) \mapsto (-2 - x, \ -y)$$

4 **The image has vertices**

The image has vertices

$$(-4, -1),\ (-6, -1),\ (-5, -2),\ (-4, -2)$$



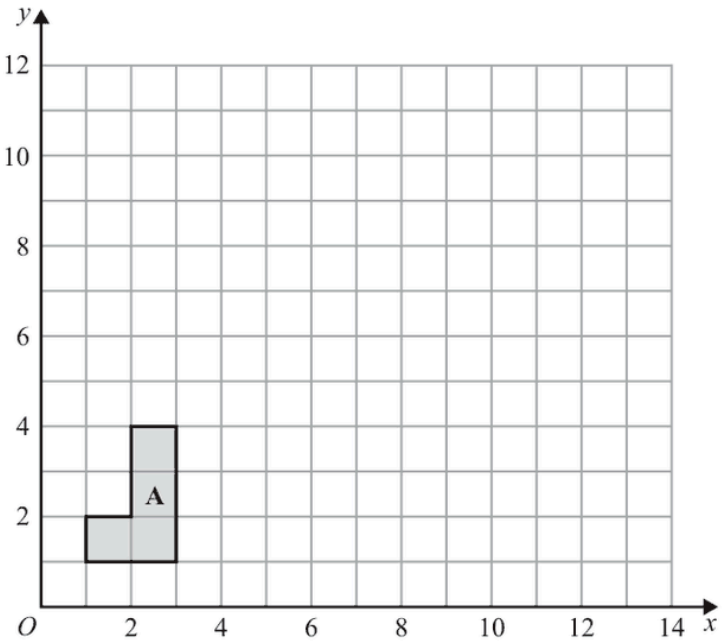
✓ **FINAL ANSWER**

(a) translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$; (b) vertices $(-4, -1), (-6, -1), (-5, -2), (-4, -2)$.

Question 2

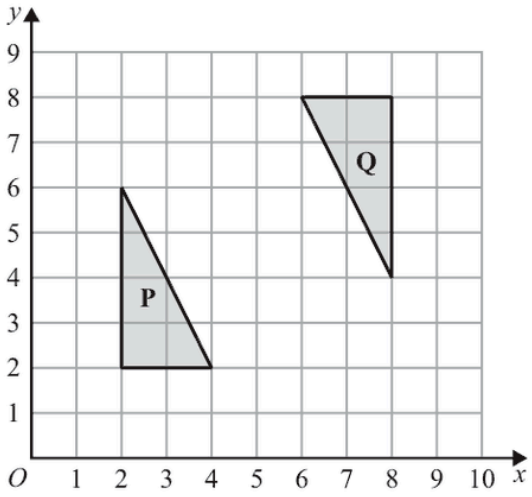
5 marks | Transformations

2



(a) On the grid, enlarge shape **A** with scale factor 3 and centre (0, 1)

(2)



(b) Describe fully the single transformation that maps triangle **P** onto triangle **Q**

.....

.....

(3)

(Total for Question 2 is 5 marks)

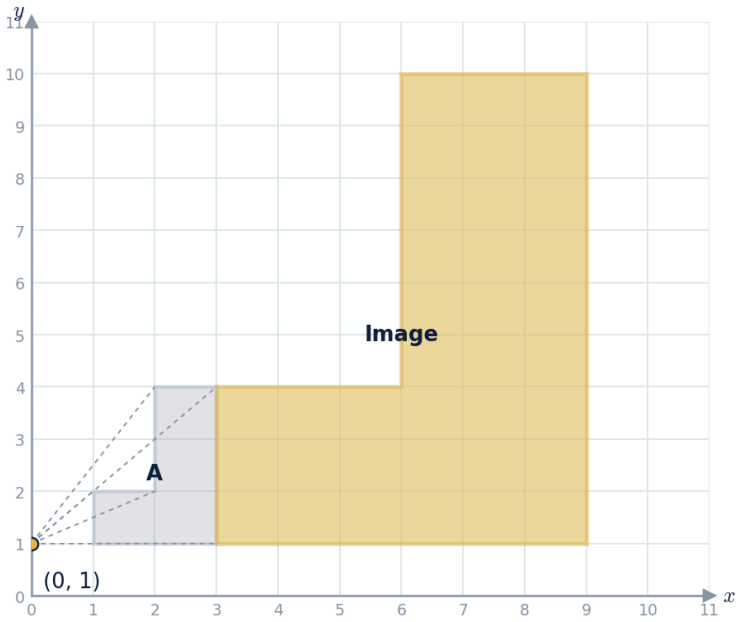
1 **Use Transformations correct**

(a) Enlarge from centre $(0, 1)$ with scale factor 3.

2 **The vertices become**

The vertices become

$$(3, 1),\ (9, 1),\ (9, 10),\ (6, 10),\ (6, 4),\ (3, 4)$$



3 **Use trigonometry**

(b) Triangle P is mapped to triangle Q by a $180°$ rotation about $(5, 5)$.

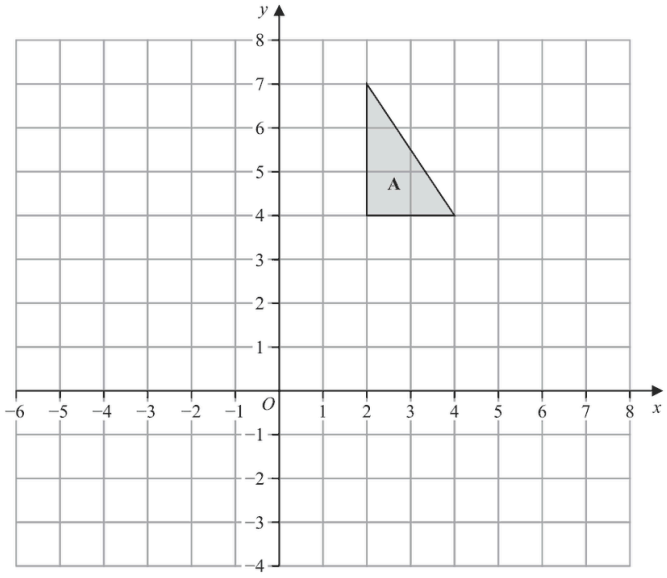
✓ **FINAL ANSWER**

(a) vertices $(3, 1), (9, 1), (9, 10), (6, 10), (6, 4), (3, 4)$; (b) rotation $180°$ about $(5, 5)$.

Question 4

4 marks | Transformations

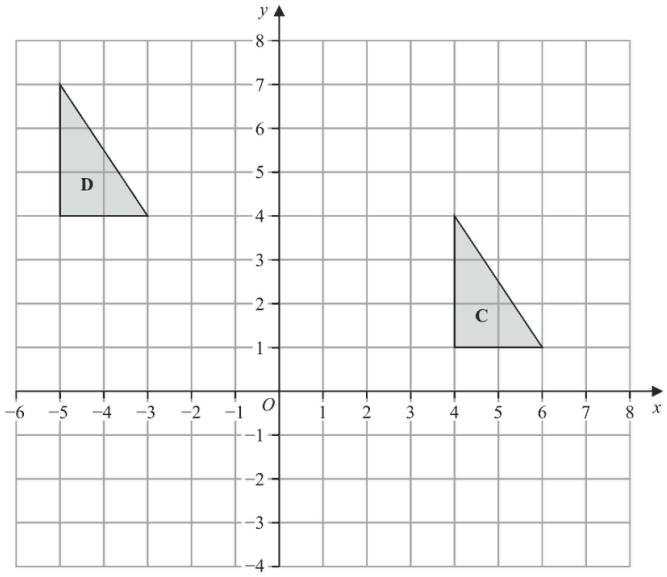
4



(a) On the grid above, rotate triangle **A** 90° anticlockwise about $(1, 2)$

Label your triangle **B**

(2)



(b) Describe fully the single transformation that maps triangle **C** onto triangle **D**

(2)

(Total for Question 4 is 4 marks)

1 Required

a Rotate triangle A 90° anticlockwise about (1,2); label triangle B. (b) Describe fully the transformation mapping C onto D.

2 The rule for 90 anticlockwise

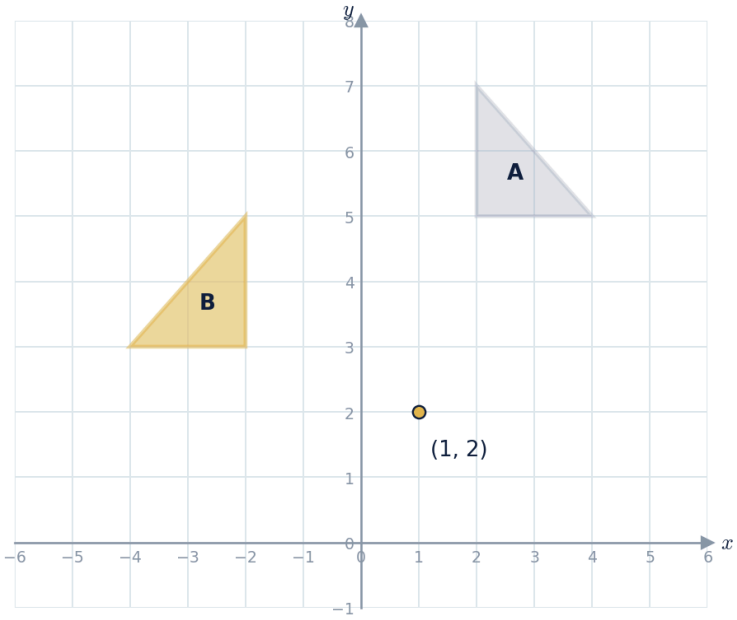
The rule for 90° anticlockwise rotation about (*a*,*b*):
(*x*, *y*) → (*a* − (*y*−*b*), *b* + (*x*−*a*))
About (1,2): (*x*,*y*) → (3−*y*, *x*+1)

3 Triangle A has vertices A

Triangle A has vertices *A*₁(2,7), *A*₂(4,5), *A*₃(2,5).
Apply the rule:
(2,7) → (−4,3), (4,5) → (−2,5), (2,5) → (−2,3)

4 Answer

Triangle B has vertices (−4,3), (−2,5), (−2,3)



5 Identify corresponding vertices of C

Identify corresponding vertices of C and D from the diagram and compute the vector:

$$(4, , 4)longrightarrow(-5, , 7) : quadvecv = beginpmatrix - 93endpmatrix$$

Check: (6,1) → (−3,4) gives the same vector $beginpmatrix - 93endpmatrix$

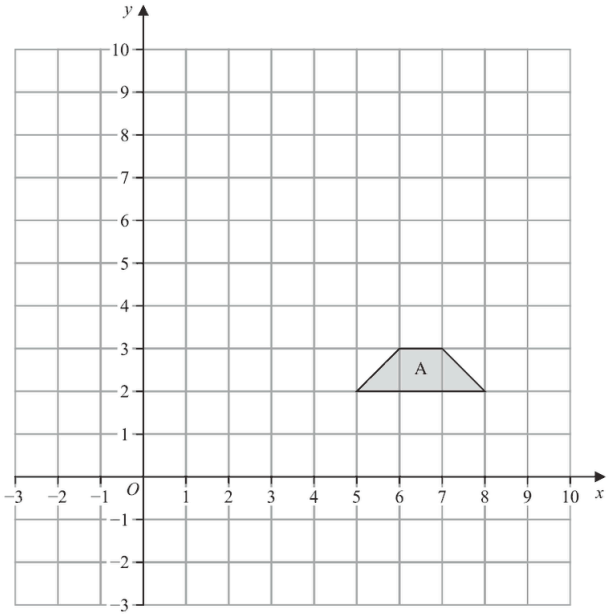
FINAL ANSWER

Translation by vector $beginpmatrix - 93endpmatrix$

Question 5

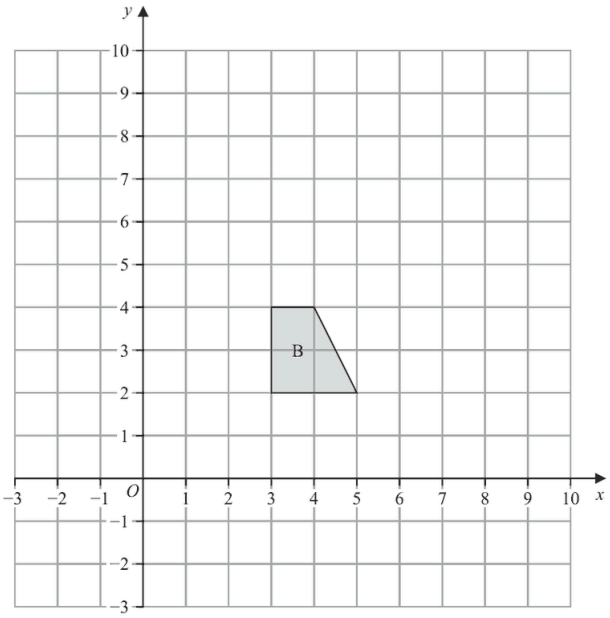
4 marks | Transformations

5



(a) On the grid above, reflect shape **A** in the line $y = x$

(2)



(b) On the grid above, enlarge shape **B** by scale factor 2 with centre (1, 1)

(2)

(Total for Question 5 is 4 marks)

1 (a) Reflection in swaps each

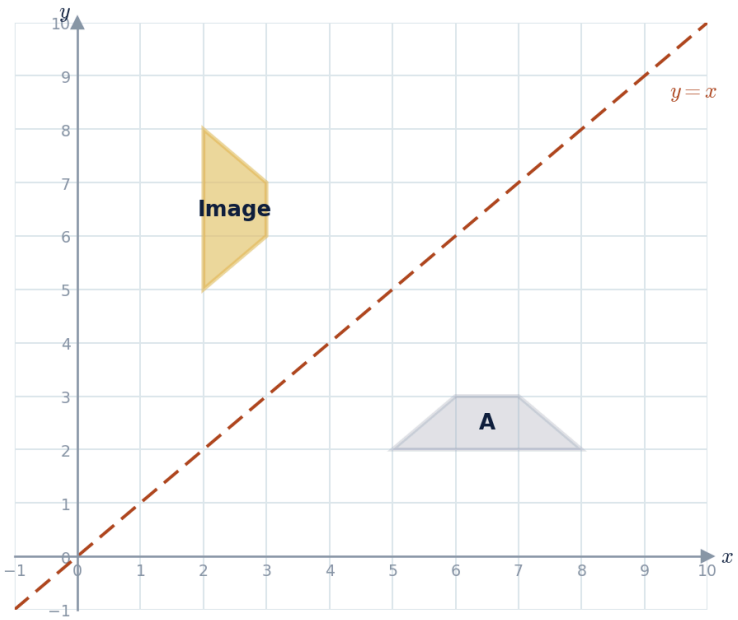
(a) Reflection in $y = x$ swaps each coordinate:

$$(x, y) \mapsto (y, x)$$

2 The image of has vertices

The image of A has vertices

$$(2, 5),\ (2, 8),\ (3, 7),\ (3, 6)$$



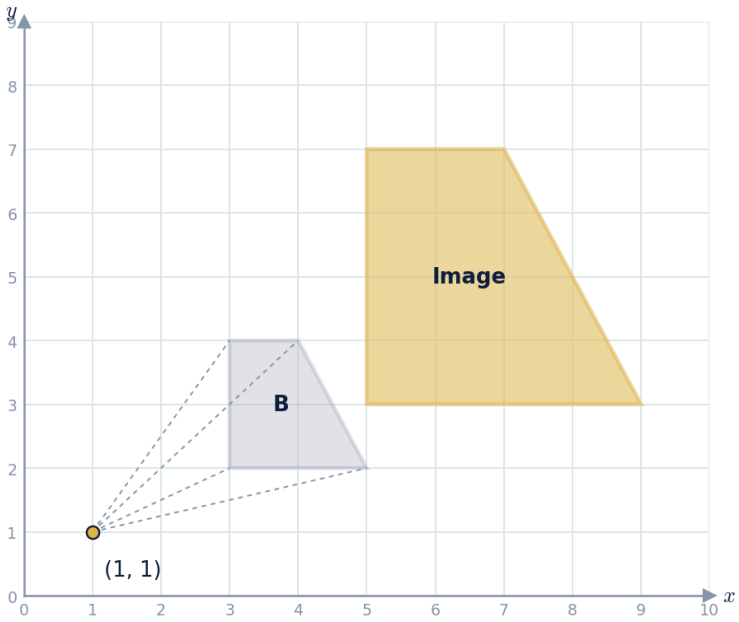
3 (b) Enlarge by scale factor

(b) Enlarge B by scale factor 2 from centre $(1, 1)$.

4 **The vertices become**

The vertices become

$$(5,3),\ (9,3),\ (7,7),\ (5,7)$$



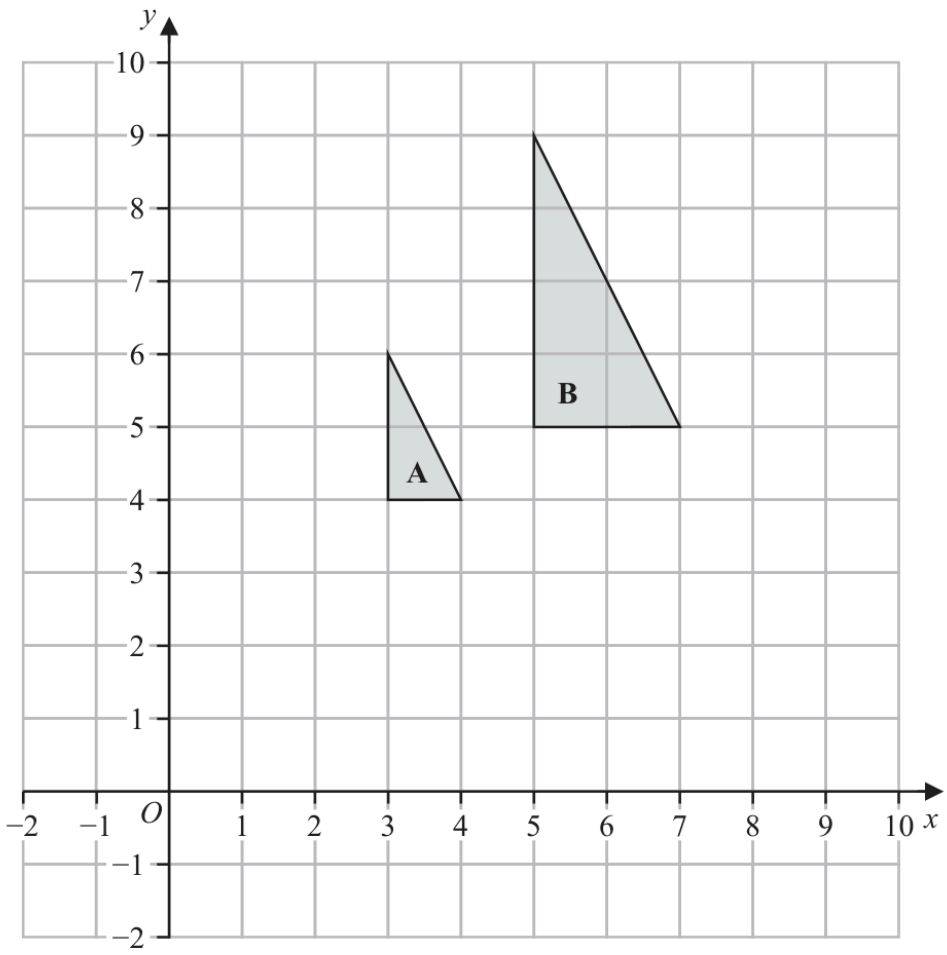
FINAL ANSWER

(a) vertices $(2,5),(2,8),(3,7),(3,6)$; (b) vertices $(5,3),(9,3),(7,7),(5,7)$.

Question 5

4 marks | Transformations

5



(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(b) On the grid, translate triangle **A** by the vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Label your triangle **C**

(1)

(Total for Question 5 is 4 marks)

1

Part (a) | Step 1

Triangle A has vertices $(3,4),(4,4),(3,6)$; the corresponding vertices of B are $(5,5),(7,5),(5,9)$.

The lengths have doubled.

Read corresponding vertices and compare side lengths.

2

Part (a) | Step 2

For enlargement factor 2, the centre O satisfies $B = O + 2(A - O)$, so $O = 2A - B$. Using $(3,4) \mapsto (5,5)$ gives $O = (6,8) - (5,5) = (1,3)$.

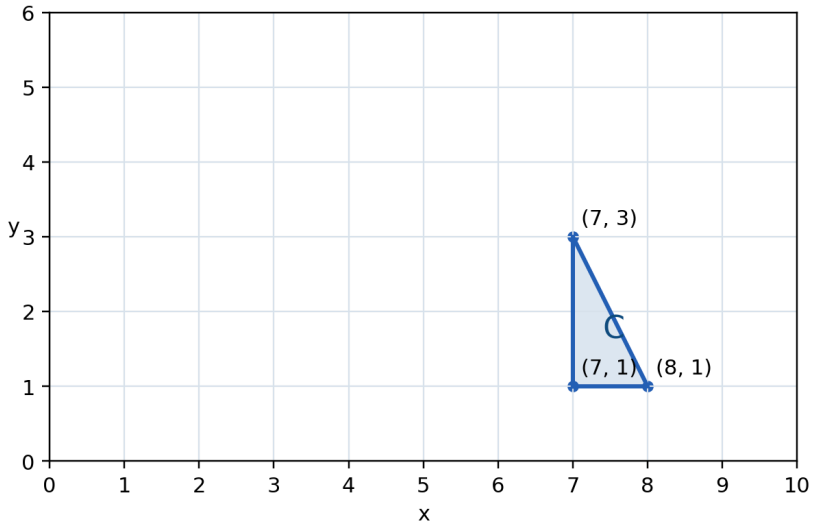
Find the fixed centre from a corresponding pair; the other pairs give the same centre.

3

Part (b) | Step 1

Add $(4,-3)$ to every vertex: $(3,4) \mapsto (7,1)$, $(4,4) \mapsto (8,1)$, and $(3,6) \mapsto (7,3)$. Plot these three points, join them with straight sides and label the triangle C.

A translation moves every point by the same vector.



FINAL ANSWER

(a) Enlargement with scale factor 2 and centre $(1,3)$. (b) $C : (7,1),(8,1),(7,3)$.

TOPIC

Vectors

Every past-paper question on this topic, 2020 to 2026.

Question 23

5 marks | Vectors

23

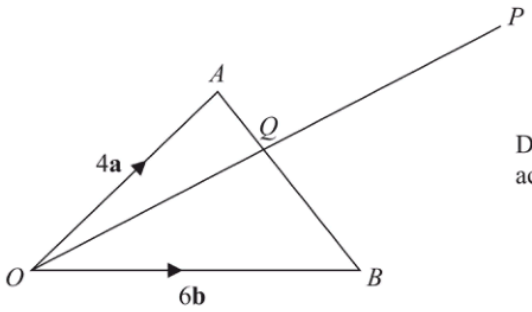


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = \mathbf{4a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AP} = \mathbf{2a} + \mathbf{8b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 23 is 5 marks)

1 **Use vectors**

$$\overrightarrow{OA} = 4a, \qquad \overrightarrow{OB} = 6b, \qquad \overrightarrow{AP} = 2a + 8b$$

2 **Use vectors**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4a + (2a + 8b) = 6a + 8b$$

3 **Use vectors**

Since O, Q, P are collinear, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Use vectors**

Also Q lies on AB. Let

$$\overrightarrow{AQ} = \mu\overrightarrow{AB}$$

5 **Use vectors**

Then

$$\overrightarrow{AB} = 6b - 4a$$

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a)$$

$$\overrightarrow{OQ} = (4 - 4\mu)a + 6\mu b$$

6 **Use vectors**

Compare coefficients with

$$\lambda(6a + 8b) = 6\lambda a + 8\lambda b$$

7 **Use vectors**

So

$$4 - 4\mu = 6\lambda, \qquad 6\mu = 8\lambda$$

8 **Use vectors**

From $6\mu = 8\lambda$,

$$\lambda = \frac{3}{4}\mu$$

---

9 **Substitute**

Substitute:

$$4 - 4\mu = 6\left(\frac{3}{4}\mu\right)$$

$$4 - 4\mu = \frac{9}{2}\mu$$

$$8 = 17\mu$$

$$\mu = \frac{8}{17}$$

---

10 **Use vectors**

Therefore

$$AQ : QB = \frac{8}{17} : \frac{9}{17} = 8 : 9$$

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$.

4MA1/2H JANUARY 2020

Question 23

3 marks | Vectors

23

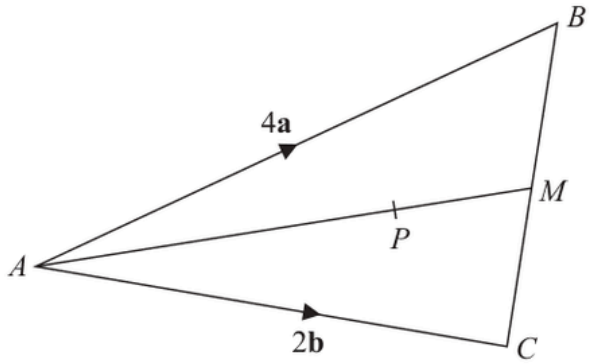


Diagram **NOT** accurately drawn

ABC is a triangle.
The midpoint of BC is M.
P is a point on AM.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2020

1 Split the ratio

Since M is the midpoint of BC,

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = \frac{4a + 2b}{2} = 2a + b$$

2 Given

Given

$$\overrightarrow{AP} = \frac{3}{2}a + \frac{3}{4}b$$

3 **Factorise**

Factorise:

$$\overrightarrow{AP} = \frac{3}{4}(2a + b) = \frac{3}{4}\overrightarrow{AM}$$

4 **Split the ratio**

So AP is $\frac{3}{4}$ of AM, leaving PM as $\frac{1}{4}$ of AM.

✓ **FINAL ANSWER**

$AP : PM = 3 : 1$

4MA1/2HR JANUARY 2020

Question 26

5 marks | Vectors

26 The diagram shows trapezium $OACB$.

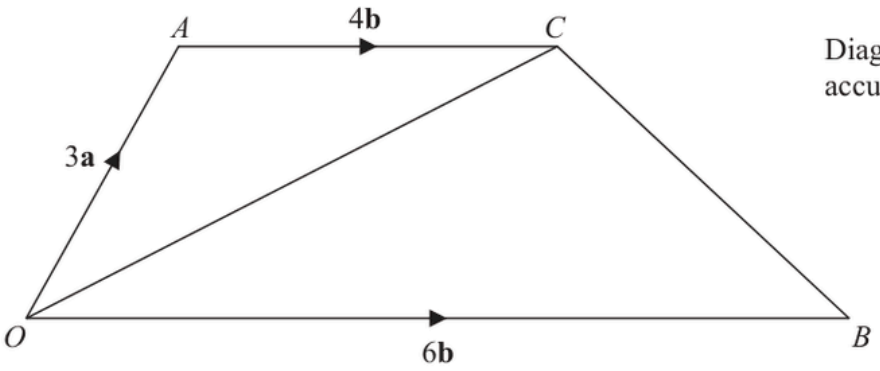


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AC} = \mathbf{4b}$

N is the point on OC such that ANB is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR January 2020

1 **Read the graph**

$$\vec{OA} = \mathbf{3a}, \quad \vec{OB} = \mathbf{6b}, \quad \vec{AC} = \mathbf{4b}$$

2 **Read the graph**

So

$$\vec{OC} = \mathbf{3a} + \mathbf{4b}$$

3 **Read the graph**

Point N lies on OC, so

$$\vec{ON} = t(\mathbf{3a} + \mathbf{4b})$$

4 **Read the graph**

Point N also lies on AB, so

$$\vec{ON} = 3\mathbf{a} + s(6\mathbf{b} - 3\mathbf{a})$$

$$\vec{ON} = (3 - 3s)\mathbf{a} + 6s\mathbf{b}$$

---

5 **Equate coefficients**

Equate coefficients:

$$3t = 3 - 3s$$

$$4t = 6s$$

---

6 **Read the graph**

So

$$t = 1 - s$$

---

7 **Read the graph**

and

$$t = \frac{3}{2}s$$

$$\frac{3}{2}s = 1 - s$$

$$5s = 2$$

$$s = \frac{2}{5}$$

---

8 **Read the graph**

Then

$$t = \frac{3}{5}$$

---

9 **Read the graph**

Therefore

$$\vec{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$$

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

4MA1/1HR JANUARY 2021

Question 13

2 marks | Vectors

13 Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Find, as a column vector, $\overrightarrow{AC}$

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR January 2021

1 **Use vectors**

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

2 **Use vectors**

and

$$\overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

3 **Use vectors**

So

$$\overrightarrow{BC} = -\overrightarrow{CB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

✓ **FINAL ANSWER**

$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$.

4MA1/1H JANUARY 2021

Question 23

6 marks | Vectors

23 OAB is a triangle.

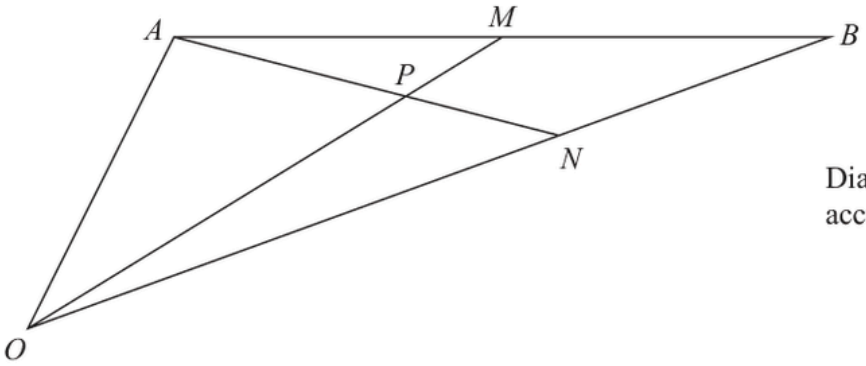


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

M is the midpoint of AB.
N is the point on OB such that $ON : NB = 2 : 1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP : PM$
Show your working clearly.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2021

1 **Use vectors**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{OB} = 2\mathbf{b}$$

2 **Find the midpoint**

Since M is the midpoint of AB,

$$\vec{OM} = \frac{2\mathbf{a} + 2\mathbf{b}}{2}$$

$$\vec{OM} = \mathbf{a} + \mathbf{b}$$

3 **Use vectors**

Since $ON : NB = 2 : 1$,

$$\vec{ON} = \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \frac{2}{3}(2\mathbf{b}) = \frac{4}{3}\mathbf{b}$$

4 **Use vectors**

Point P lies on OM, so

$$\vec{OP} = t(\mathbf{a} + \mathbf{b})$$

5 **Use vectors**

Point P also lies on AN, so

$$\vec{OP} = 2\mathbf{a} + s\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$$

$$\vec{OP} = (2 - 2s)\mathbf{a} + \frac{4s}{3}\mathbf{b}$$

6 **Use vectors**

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$:

$$t = 2 - 2s$$

$$t = \frac{4s}{3}$$

7 **Use vectors**

So

$$\frac{4s}{3} = 2 - 2s$$

$$4s = 6 - 6s$$

$$10s = 6$$

$$s = \frac{3}{5}$$

8 **Use vectors**

Then

$$t = \frac{4}{5}$$

9 **Use vectors**

So P is $\frac{4}{5}$ of the way from O to M.

✓ **FINAL ANSWER**

$OP : PM = 4 : 1$

4MA1/1H MAY 2021

Question 26

5 marks | Vectors

26 $OACB$ is a trapezium.

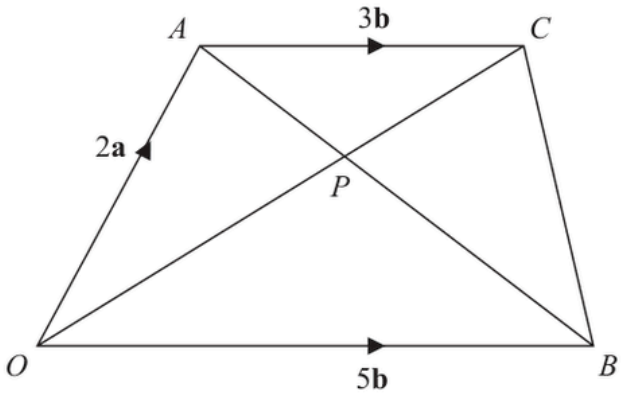


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{2a}$ $\overrightarrow{OB} = \mathbf{5b}$ $\overrightarrow{AC} = \mathbf{3b}$

The diagonals, OC and AB, of the trapezium intersect at the point P.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H May 2021

1 **Use vectors**

$$\overrightarrow{OA} = \mathbf{2a}, \quad \overrightarrow{OB} = \mathbf{5b}, \quad \overrightarrow{AC} = \mathbf{3b}$$

2 **Use vectors**

So

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{2a} + \mathbf{3b}$$

3 **Use vectors**

Since P lies on OC, let

$$\overrightarrow{OP} = t(2\mathbf{a} + 3\mathbf{b})$$

4 **Use vectors**

Since P also lies on AB, let

$$\overrightarrow{OP} = \overrightarrow{OA} + s\overrightarrow{AB}$$

5 **Use vectors**

Now

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{b} - 2\mathbf{a}$$

6 **Use vectors**

So

$$\overrightarrow{OP} = 2\mathbf{a} + s(5\mathbf{b} - 2\mathbf{a})$$

$$\overrightarrow{OP} = (2 - 2s)\mathbf{a} + 5s\mathbf{b}$$

7 **Use vectors**

Compare this with

$$t(2\mathbf{a} + 3\mathbf{b}) = 2t\mathbf{a} + 3t\mathbf{b}$$

8 **Equating coefficients**

Equating coefficients:

$$2t = 2 - 2s$$

$$3t = 5s$$

9 **Use vectors**

From $t = 1 - s$,

$$3(1 - s) = 5s$$

$$3 = 8s$$

$$s = \frac{3}{8}$$

---

10 **Use vectors**

So

$$t = 1 - \frac{3}{8} = \frac{5}{8}$$

---

11 **Use vectors**

Therefore

$$\overrightarrow{OP} = \frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$$

$$\overrightarrow{OP} = \frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}}$$

Question 14

8 marks | Vectors

14 *ABCDEF* and *GHIJKL* are regular hexagons each with centre *O*.

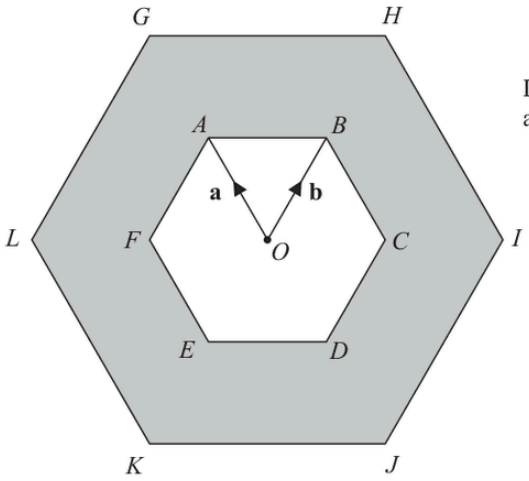


Diagram **NOT** accurately drawn

GHIJKL is an enlargement of *ABCDEF*, with centre *O* and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

(a) Write the following vectors, in terms of **a** and **b**.
Simplify your answers.

(i) $\overrightarrow{AB}$

.....

(1)

(ii) $\overrightarrow{KI}$

.....

(2)

(iii) $\overrightarrow{LD}$

.....

(2)

The triangle *OAB* has an area of 5 cm^2

(b) Calculate the area of the shaded region.

..... cm^2

(3)

(Total for Question 14 is 8 marks)

1 **Use vectors**

$$\overrightarrow{OA} = \mathbf{a}, \qquad \overrightarrow{OB} = \mathbf{b}$$

2 **(a)(i)**

(a)(i)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$$

3 **Use vectors**

For a regular hexagon,

$$\overrightarrow{OC} = \mathbf{b} - \mathbf{a}, \quad \overrightarrow{OD} = -\mathbf{a}, \quad \overrightarrow{OE} = -\mathbf{b}, \quad \overrightarrow{OF} = \mathbf{a} - \mathbf{b}$$

4 **The outer hexagon has scale**

The outer hexagon has scale factor $\mathbf{2}$, so

$$\overrightarrow{OI} = 2(\mathbf{b} - \mathbf{a}), \quad \overrightarrow{OK} = -2\mathbf{b}, \quad \overrightarrow{OL} = 2(\mathbf{a} - \mathbf{b})$$

5 **(ii)**

(ii)

$$\overrightarrow{KI} = \overrightarrow{OI} - \overrightarrow{OK} = 2(\mathbf{b} - \mathbf{a}) - (-2\mathbf{b}) = 4\mathbf{b} - 2\mathbf{a}$$

6 **(iii)**

(iii)

$$\overrightarrow{LD} = \overrightarrow{OD} - \overrightarrow{OL} = -\mathbf{a} - 2(\mathbf{a} - \mathbf{b}) = 2\mathbf{b} - 3\mathbf{a}$$

7 **(b) The inner hexagon is**

(b) The inner hexagon is made from 6 equal triangles like OAB:

$$\text{inner area} = 6 \times 5 = 30$$

8 **The outer hexagon is an**

The outer hexagon is an enlargement with scale factor $\mathbf{2}$, so area scale factor is $\mathbf{2^2 = 4}$:

$$\text{outer area} = 4 \times 30 = 120$$

$$\text{shaded area} = 120 - 30 = 90$$

FINAL ANSWER

(i) $\mathbf{b} - \mathbf{a}$, (ii) $4\mathbf{b} - 2\mathbf{a}$, (iii) $2\mathbf{b} - 3\mathbf{a}$, (b) $90\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 22

5 marks | Vectors

22 The diagram shows triangle OAB

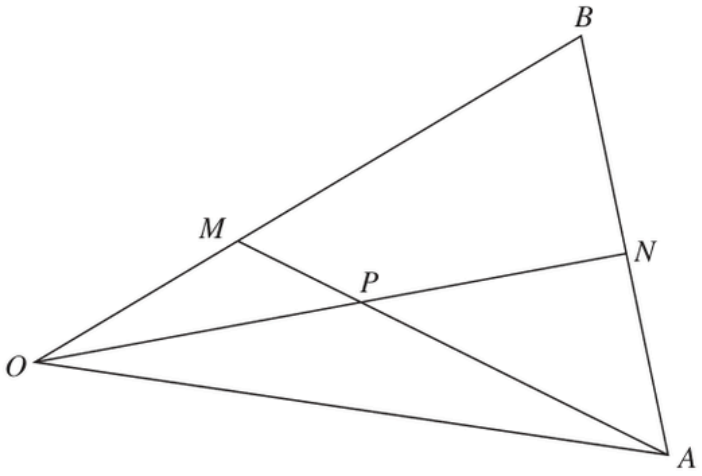


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

M is the point on OB such that $OM : MB = 1 : 2$
N is the midpoint of AB
P is the point of intersection of ON and AM

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2022

1 **Use vectors**

$$\vec{OA} = 8\mathbf{a}, \qquad \vec{OB} = 6\mathbf{b}$$

2 **Use vectors**

Since $OM : MB = 1 : 2$,

$$\vec{OM} = \frac{1}{3}\vec{OB}$$

$$\vec{OM} = 2\mathbf{b}$$

---

3 **Find the midpoint**

Since N is the midpoint of AB,

$$\vec{ON} = \frac{8\mathbf{a} + 6\mathbf{b}}{2}$$

$$\vec{ON} = 4\mathbf{a} + 3\mathbf{b}$$

---

4 **Use vectors**

Point P lies on ON, so

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AM, so

$$\vec{OP} = 8\mathbf{a} + \mu(2\mathbf{b} - 8\mathbf{a})$$

$$\vec{OP} = (8 - 8\mu)\mathbf{a} + 2\mu\mathbf{b}$$

---

6 **Equate coefficients**

Equate coefficients:

$$4\lambda = 8 - 8\mu$$

$$3\lambda = 2\mu$$

---

7 **Use vectors**

From $3\lambda = 2\mu$,

$$\mu = \frac{3}{2}\lambda$$

---

8 **Substitute**

Substitute:

$$4\lambda = 8 - 8\left(\frac{3}{2}\lambda\right)$$

$$4\lambda = 8 - 12\lambda$$

$$16\lambda = 8$$

$$\lambda = \frac{1}{2}$$

---

9 **Use vectors**

Therefore

$$\vec{OP} = \frac{1}{2}(4\mathbf{a} + 3\mathbf{b})$$

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

4MA1/1HR JANUARY 2022

Question 24

5 marks | Vectors

24 $OXYZ$ is a trapezium.

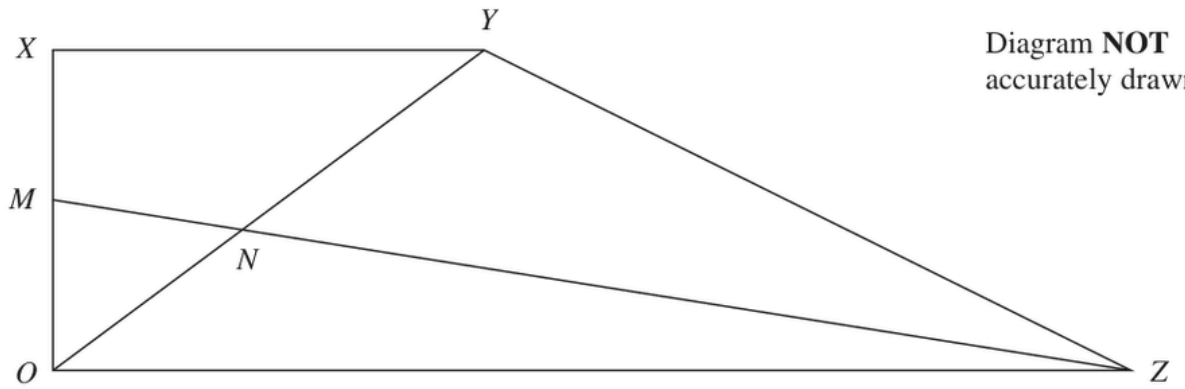


Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

M is the midpoint of OX
N is the point such that MNZ and ONY are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of λ

$\lambda =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR January 2022

1 **Take as the origin**

Take O as the origin.

$$\vec{OX} = \mathbf{a}, \qquad \vec{XY} = \mathbf{b}, \qquad \vec{OZ} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\vec{OY} = \mathbf{a} + \mathbf{b}$$

---

3 **Find the midpoint**

Since M is the midpoint of OX,

$$\vec{OM} = \frac{1}{2}\mathbf{a}$$

---

4 **Use vectors**

Given $ON : OY = \lambda : 1$,

$$\vec{ON} = \lambda(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point N also lies on MZ, so

$$\vec{ON} = \frac{1}{2}\mathbf{a} + t\left(3\mathbf{b} - \frac{1}{2}\mathbf{a}\right)$$

$$\vec{ON} = \left(\frac{1}{2} - \frac{t}{2}\right)\mathbf{a} + 3t\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients with $\lambda\mathbf{a} + \lambda\mathbf{b}$:

$$\lambda = \frac{1}{2} - \frac{t}{2}$$

$$\lambda = 3t$$

---

7 **Use vectors**

So

$$t = \frac{\lambda}{3}$$

---

8 **Substitute**

Substitute:

$$\lambda = \frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{3}\right)$$

$$\lambda = \frac{1}{2} - \frac{\lambda}{6}$$

$$\frac{7}{6}\lambda = \frac{1}{2}$$

$$\lambda = \frac{3}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}$$

4MA1/2H JUNE 2022

Question 21

5 marks | Vectors

21

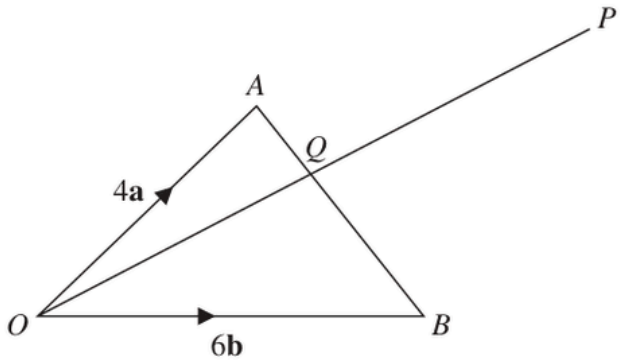


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2022

1 Split the ratio

$$\overrightarrow{OA} = 4a, \quad \overrightarrow{OB} = 6b, \quad \overrightarrow{AP} = 2a + 8b$$

2 Split the ratio

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4a + (2a + 8b) = 6a + 8b$$

3 Split the ratio

Since Q lies on OP, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Split the ratio**

Since Q lies on AB, let

$$\overrightarrow{OQ} = \overrightarrow{OA} + \mu\overrightarrow{AB}$$

$$\overrightarrow{AB} = 6b - 4a$$

5 **Split the ratio**

So

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a) = (4 - 4\mu)a + 6\mu b$$

6 **Equating coefficients**

Equating coefficients:

$$6\lambda = 4 - 4\mu, \quad 8\lambda = 6\mu$$

7 **Split the ratio**

From $8\lambda = 6\mu$, $\mu = \frac{4}{3}\lambda$.

$$6\lambda = 4 - \frac{16}{3}\lambda$$

$$34\lambda = 12, \quad \lambda = \frac{6}{17}$$

$$\mu = \frac{8}{17}$$

8 **Split the ratio**

So $AQ : QB = \frac{8}{17} : \frac{9}{17}$.

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$

4MA1/2HR JUNE 2022

Question 25

5 marks | Vectors

25 $ABCD$ is a parallelogram and ADM is a straight line.

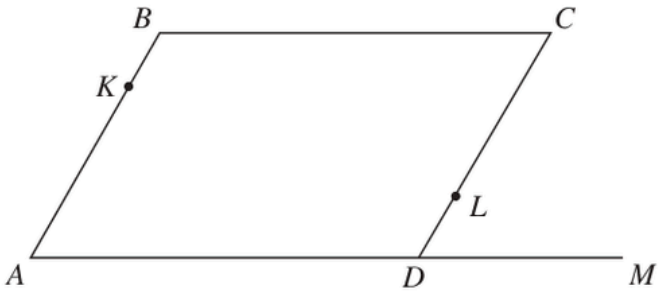


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$ $\overrightarrow{DM} = \frac{1}{2}\mathbf{b}$

K is the point on AB such that $AK : AB = \lambda : 1$
L is the point on CD such that $CL : CD = \mu : 1$
KLM is a straight line.

Given that $\lambda : \mu = 1 : 2$

use a vector method to find the value of λ and the value of μ

$\lambda =$ .....

$\mu =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2022

1 Let be the origin

Let A be the origin.

$$\vec{AB} = \mathbf{a}, \qquad \vec{BC} = \mathbf{b}$$

2 Read the graph

So

$$B = \mathbf{a}, \qquad D = \mathbf{b}, \qquad C = \mathbf{a} + \mathbf{b}$$

3 **Read the graph**

Also

$$\vec{DM} = \frac{1}{2}\mathbf{b}$$

---

4 **Read the graph**

So

$$M = \frac{3}{2}\mathbf{b}$$

---

5 **Point is on**

Point K is on AB:

$$K = \lambda\mathbf{a}$$

---

6 **Read the graph**

Point L is on CD, and $\vec{CD} = -\mathbf{a}$:

$$L = C + \mu(D - C)$$

$$L = \mathbf{a} + \mathbf{b} + \mu(-\mathbf{a})$$

$$L = (1 - \mu)\mathbf{a} + \mathbf{b}$$

---

7 **Given**

Given

$$\lambda : \mu = 1 : 2$$

$$\mu = 2\lambda$$

---

8 **Read the graph**

Since K, L, M are collinear, $\vec{KM}$ is a multiple of $\vec{KL}$.

$$\vec{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$$

$$\vec{KL} = (1 - \mu - \lambda)\mathbf{a} + \mathbf{b}$$

$$\vec{KL} = (1 - 3\lambda)\mathbf{a} + \mathbf{b}$$

---

9 **Compare coefficients using**

Compare coefficients using

$$\vec{KM} = \frac{3}{2}\vec{KL}$$

$$-\lambda = \frac{3}{2}(1 - 3\lambda)$$

$$-2\lambda = 3 - 9\lambda$$

$$7\lambda = 3$$

$$\lambda = \frac{3}{7}$$

---

10 **Read the graph**

Then

$$\mu = 2\lambda = \frac{6}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}, \quad \mu = \frac{6}{7}$$

4MA1/1H MAY 2023

Question 19

5 marks | Vectors

19

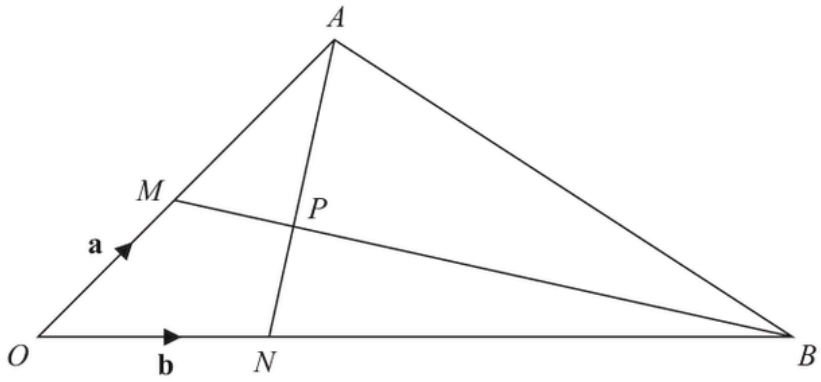


Diagram **NOT** accurately drawn

OMA, *ONB*, *MPB* and *NPA* are straight lines.

M is the midpoint of *OA*

$ON : NB = 1 : 5$

$\overrightarrow{OM} = \mathbf{a}$ $\overrightarrow{ON} = \mathbf{b}$

(a) Find in terms of **a** and **b** the vector $\overrightarrow{AN}$

.....

(1)

(b) Use a vector method to find the ratio $AP : PN$

$AP : PN =$ .....

(4)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2023

1 **Split the ratio**

Since M is the midpoint of OA,

$$\overrightarrow{OA} = 2a$$

2 **Split the ratio**

Also,

$$\overrightarrow{ON} = b$$

---

3 **Split the ratio**

So

$$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA} = b - 2a$$

---

4 **Let**

Let

$$\overrightarrow{AP} = \lambda \overrightarrow{AN}$$

---

5 **Split the ratio**

Then

$$\overrightarrow{OP} = 2a + \lambda(b - 2a) = (2 - 2\lambda)a + \lambda b$$

---

6 **Split the ratio**

Since $ON : NB = 1 : 5$,

$$\overrightarrow{OB} = 6b$$

---

7 **Split the ratio**

Point P is also on MB, so

$$\overrightarrow{OP} = a + \mu(6b - a) = (1 - \mu)a + 6\mu b$$

---

8 **Compare coefficients**

Compare coefficients:

$$2 - 2\lambda = 1 - \mu$$

$$\lambda = 6\mu$$

---

9 **Split the ratio**

From $\mu = 2\lambda - 1$,

$$\lambda = 6(2\lambda - 1)$$

$$11\lambda = 6$$

$$\lambda = \frac{6}{11}$$

---

10 **Split the ratio**

So

$$AP : PN = \frac{6}{11} : \frac{5}{11} = 6 : 5$$

✓ **FINAL ANSWER**

(a) $b - 2a$, (b) $6 : 5$

4MA1/1HR MAY 2023

Question 19

5 marks | Vectors

19 The diagram shows a triangle ABC where A, B and C represent the positions of three towns.

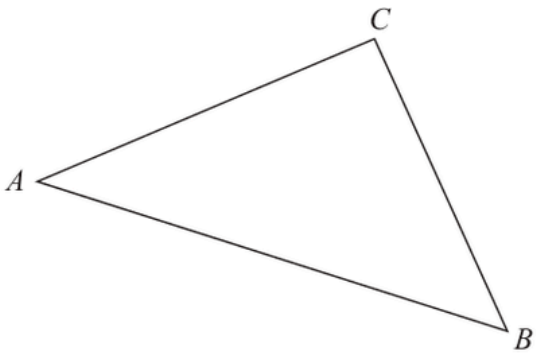


Diagram **NOT** accurately drawn

$$\overrightarrow{AB} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

Pru travels directly from A to B and then directly from B to C

Yang travels directly from A to C

Given that the values for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are in kilometres,

work out how much further Pru travels than Yang travels.
Give your answer in km, correct to one decimal place.

..... km

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May 2023

1 **Pru travels**

Pru travels

$$|AB| + |BC| = \sqrt{7^2 + (-2)^2} + \sqrt{(-3)^2 + 5^2}$$

$$= \sqrt{53} + \sqrt{34} = 13.111\ldots$$

2 **Simplify surd**

Yang travels from A to C.

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$|AC| = \sqrt{4^2 + 3^2} = 5$$

---

3 **Difference**

Difference:

$$13.111\ldots - 5 = 8.111\ldots$$

FINAL ANSWER

Pru travels $8.1\ \text{km}$ further.

Question 22

5 marks | Vectors

22

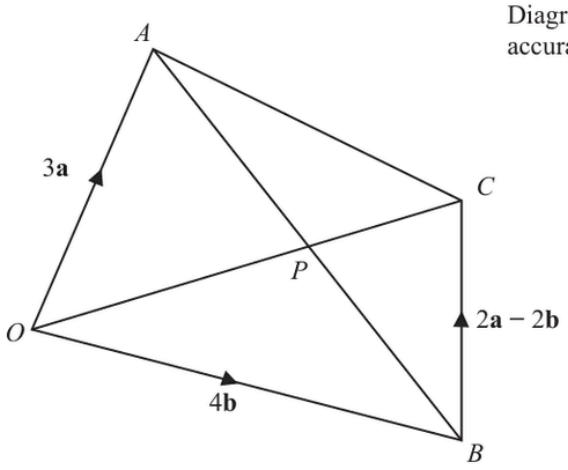


Diagram **NOT** accurately drawn

$OACB$ is a quadrilateral.

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{4b}$ $\overrightarrow{BC} = \mathbf{2a} - \mathbf{2b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$\overrightarrow{OC} =$ .....

(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .....

(1)

The point P lies on AB and on OC

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

.....

(3)

(Total for Question 22 is 5 marks)

1 **Split the ratio**

$$\overrightarrow{OA} = 3a, \quad \overrightarrow{OB} = 4b, \quad \overrightarrow{BC} = 2a - 2b$$

2 **Part (a)**

Part (a):

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 4b + (2a - 2b) = 2a + 2b$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4b - 3a$$

3 **Split the ratio**

For part (b), let P divide AB so that

$$\overrightarrow{AP} = \mu\overrightarrow{AB}$$

4 **Split the ratio**

Then

$$\overrightarrow{OP} = 3a + \mu(4b - 3a) = (3 - 3\mu)a + 4\mu b$$

5 **Split the ratio**

Since P lies on OC, also let

$$\overrightarrow{OP} = \lambda(2a + 2b)$$

6 **Equating coefficients**

Equating coefficients:

$$3 - 3\mu = 2\lambda, \quad 4\mu = 2\lambda$$

7 **Split the ratio**

From $4\mu = 2\lambda$, $\lambda = 2\mu$.

$$3 - 3\mu = 4\mu$$

$$\mu = \frac{3}{7}$$

8 **Split the ratio**

So $AP : PB = \frac{3}{7} : \frac{4}{7}$.

FINAL ANSWER

$\overrightarrow{OC} = 2\boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{AB} = 4\boldsymbol{b} - 3\boldsymbol{a}$, and $AP : PB = 3 : 4$

4MA1/2H JUNE 2024

Question 24

6 marks | Vectors

24

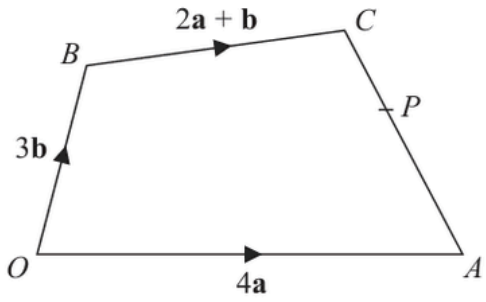


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OACB$ in which

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 3\mathbf{b}$ $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{AC} =$ .....

(2)

The point P lies on AC such that $AP : PC = 3 : 2$

The point Q is such that OPQ and BCQ are straight lines.

(b) Using a vector method, find $\overrightarrow{OQ}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OQ} =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2024

1 **Read the graph**

$$\vec{OA} = 4\mathbf{a}, \quad \vec{OB} = 3\mathbf{b}, \quad \vec{BC} = 2\mathbf{a} + \mathbf{b}$$

2 **Read the graph**

So

$$\vec{OC} = \vec{OB} + \vec{BC}$$

$$\vec{OC} = 2\mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\vec{AC} = (2\mathbf{a} + 4\mathbf{b}) - 4\mathbf{a}$$

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$$

---

4 **Read the graph**

For part (b), $AP : PC = 3 : 2$, so

$$\vec{OP} = \vec{OA} + \frac{3}{5}\vec{AC}$$

$$\vec{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a})$$

$$\vec{OP} = \frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

---

5 **Read the graph**

Since O, P, Q are collinear,

$$\vec{OQ} = t\vec{OP}$$

$$\vec{OQ} = t\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

---

6 **Read the graph**

Since B, C, Q are collinear,

$$\vec{OQ} = \vec{OB} + s\vec{BC}$$

$$\vec{OQ} = 3\mathbf{b} + s(2\mathbf{a} + \mathbf{b})$$

$$\vec{OQ} = 2s\mathbf{a} + (3 + s)\mathbf{b}$$

---

7 **Equate coefficients**

Equate coefficients:

$$\frac{14}{5}t = 2s$$

$$\frac{12}{5}t = 3 + s$$

---

8 **Solve equation**

From the first equation,

$$s = \frac{7}{5}t$$

---

9 **Substitute into the second**

Substitute into the second:

$$\frac{12}{5}t = 3 + \frac{7}{5}t$$

$$t = 3$$

---

10 **Read the graph**

Therefore

$$\vec{OQ} = 3\vec{OP}$$

$$\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$, $\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$

Question 25

6 marks | Vectors

25 $OPQR$ is a parallelogram.

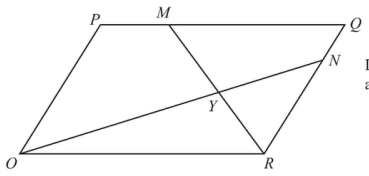


Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point M lies on PQ such that $PM = \frac{1}{4}PQ$

The point N lies on RQ such that $RN = \frac{4}{5}RQ$

(a) Find, in terms of **a** and **b**, giving your answers in simplest form

(i) $\overrightarrow{ON}$

.....

(1)

(ii) $\overrightarrow{MR}$

.....

(1)

MR and ON intersect at the point Y

Given that

$$OY = k \times ON$$

(b) use a vector method to find the value of k

$k =$ .....

(4)

(Total for Question 25 is 6 marks)

1 **Calculate value**

$$\overrightarrow{OP} = 2a, \qquad \overrightarrow{OR} = 3b$$

2 **Calculate value**

So

$$\overrightarrow{OQ} = 2a + 3b$$

3 **Evaluate fraction**

Since $PM : PQ = 1 : 4$,

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{4}(3b) = \frac{3}{4}b$$

$$\overrightarrow{OM} = 2a + \frac{3}{4}b$$

4 **Evaluate fraction**

Since $RN : RQ = 4 : 5$,

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ}$$

$$\overrightarrow{RN} = \frac{4}{5}(2a) = \frac{8}{5}a$$

$$\overrightarrow{ON} = 3b + \frac{8}{5}a$$

5 **Evaluate fraction**

So

$$\overrightarrow{MR} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{MR} = 3b - \left(2a + \frac{3}{4}b\right)$$

$$\overrightarrow{MR} = -2a + \frac{9}{4}b$$

---

6 **Let be on**

Let Y be on ON, so

$$\overrightarrow{OY} = k\overrightarrow{ON} = k\left(\frac{8}{5}a + 3b\right)$$

---

7 **Evaluate fraction**

Also Y is on MR, so

$$\overrightarrow{OY} = \overrightarrow{OM} + t\overrightarrow{MR}$$

$$\overrightarrow{OY} = 2a + \frac{3}{4}b + t\left(-2a + \frac{9}{4}b\right)$$

---

8 **Evaluate fraction**

Compare coefficients of a and b:

$$\frac{8}{5}k = 2 - 2t$$

$$3k = \frac{3}{4} + \frac{9}{4}t$$

---

9 **Solve equation**

From the first equation,

$$t = 1 - \frac{4}{5}k$$

---

10 **Substitute into the second equation**

Substitute into the second equation:

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = 3 - \frac{9}{5}k$$

$$\frac{24}{5}k = 3$$

$$k = \frac{5}{8}$$

✓ **FINAL ANSWER**

$\overrightarrow{ON} = \frac{8}{5}a + 3b$, $\overrightarrow{MR} = -2a + \frac{9}{4}b$, and $k = \frac{5}{8}$

4MA1/2H NOVEMBER 2024

Question 17

3 marks | Vectors

17 Here are two vectors.

$$\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad \overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$$

Calculate the magnitude of the vector $\overrightarrow{HF}$

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2024

1 Use vectors

$$\overrightarrow{GF} = -\overrightarrow{FG} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\overrightarrow{HF} = \overrightarrow{HG} + \overrightarrow{GF} = \begin{pmatrix} 4 \\ 14 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

2 Magnitude

Magnitude:

$$|\overrightarrow{HF}| = \sqrt{9^2 + 12^2} = 15$$

FINAL ANSWER

15.

Question 24

6 marks | Vectors

24 OAB is a triangle.

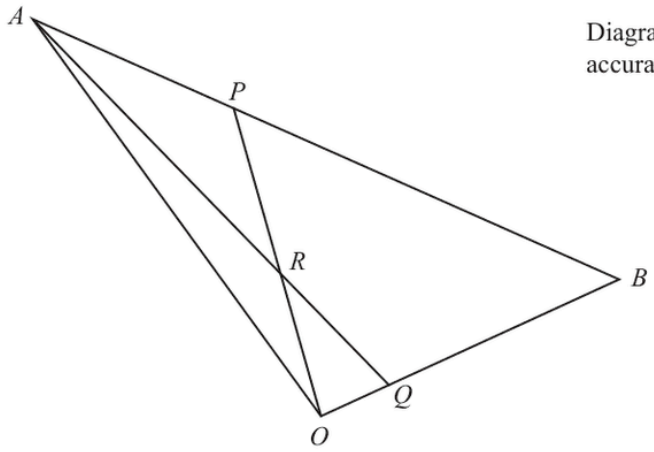


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 10\mathbf{a}$ $\overrightarrow{OB} = 10\mathbf{b}$

ARQ and ORP are straight lines.

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

Write the following vectors in terms of **a** and **b**
Simplify your answers.

(i) $\overrightarrow{AQ}$

.....

(1)

(ii) $\overrightarrow{OP}$

.....

(1)

(iii) $\overrightarrow{OR}$

.....

(4)

(Total for Question 24 is 6 marks)

1 **Read the graph**

$$\vec{OA} = 10\mathbf{a}, \qquad \vec{OB} = 10\mathbf{b}$$

2 **Read the graph**

Also,

$$\vec{AP} = \frac{1}{4}\vec{AB}$$

3 **Read the graph**

and

$$\vec{OQ} = \frac{1}{5}\vec{OB}$$

4 **Read the graph**

For part (i),

$$\vec{OQ} = 2\mathbf{b}$$

$$\vec{AQ} = \vec{OQ} - \vec{OA}$$

$$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$$

5 **Read the graph**

For part (ii),

$$\vec{AB} = 10\mathbf{b} - 10\mathbf{a}$$

$$\vec{AP} = \frac{1}{4}(10\mathbf{b} - 10\mathbf{a})$$

$$\vec{AP} = \frac{5}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

6 **Read the graph**

For part (iii), R lies on OP, so

$$\vec{OR} = t\vec{OP}$$

---

7 **Read the graph**

It also lies on AQ, so

$$\vec{OR} = \vec{OA} + s\vec{AQ}$$

$$\vec{OR} = 10\mathbf{a} + s(2\mathbf{b} - 10\mathbf{a})$$

$$\vec{OR} = (10 - 10s)\mathbf{a} + 2s\mathbf{b}$$

---

8 **Read the graph**

Equate with

$$t\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$$

$$\frac{15}{2}t = 10 - 10s$$

$$\frac{5}{2}t = 2s$$

---

9 **Solve equation**

From the second equation,

$$s = \frac{5}{4}t$$

---

10 **Substitute**

Substitute:

$$\frac{15}{2}t = 10 - \frac{25}{2}t$$

$$20t = 10$$

$$t = \frac{1}{2}$$

---

11 **Read the graph**

So

$$\vec{OR} = \frac{1}{2}\vec{OP}$$

$$\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$$

FINAL ANSWER

$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$, $\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$, $\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$

4WM2/01R MAY 2025

Question 16

3 marks | Vectors

16 OAB is a triangle.

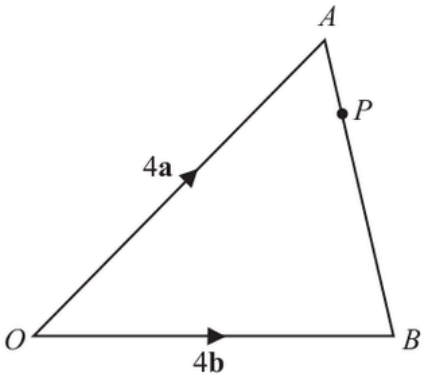


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{4a}$

$\overrightarrow{OB} = \mathbf{4b}$

P is the point on AB such that $AP : PB = 1 : 3$

(a) Write down $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

(b) Express $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM2/01R May 2025

1 Required

$overrightarrowOA = 4mathbfa$, $overrightarrowOB = 4mathbfb$, AP:PB = 1:3. (a) Find $overrightarrowAB$. (b) Express $overrightarrowOP$ in simplest form.

2 Part (a)

$$overrightarrowAB = overrightarrowAO + overrightarrowOB = -4mathbfa + 4mathbfb$$

3 Answer

$$overrightarrowAB = 4mathbfb - 4mathbfa$$

4 **AP PB 1 3 means**

AP:PB = 1:3 means $overrightarrowAP = tfrac14overrightarrowAB$:

$$overrightarrowAP = tfrac14(4mathbfb - 4mathbfa) = mathbfb - mathbfa$$

---

5 **Use**

Use $overrightarrowOP = overrightarrowOA + overrightarrowAP$:

$$overrightarrowOP = 4mathbfa + (mathbfb - mathbfa) = 3mathbfa + mathbfb$$

✓ **FINAL ANSWER**

$$overrightarrowOP = 3mathbfa + mathbfb$$

4MA1/2HR JUNE 2025

Question 18

3 marks | Vectors

18 OAB is a triangle.

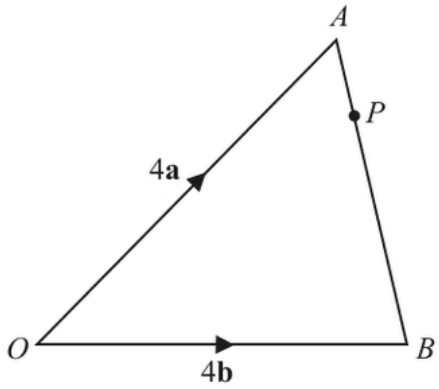


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 4\mathbf{a}$

$\overrightarrow{OB} = 4\mathbf{b}$

P is the point on AB such that $AP : PB = 1 : 3$

(a) Write down $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

(b) Express $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2025

1 **Use vectors**

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4\mathbf{b} - 4\mathbf{a}$$

2 **Use vectors**

Since $AP : PB = 1 : 3$, point P is one quarter of the way from A to B.

$$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}(4\mathbf{b} - 4\mathbf{a}) = \mathbf{b} - \mathbf{a}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4\mathbf{a} + \mathbf{b} - \mathbf{a} = 3\mathbf{a} + \mathbf{b}$$

✓ **FINAL ANSWER**

$\overrightarrow{AB} = 4\mathbf{b} - 4\mathbf{a}$, and $\overrightarrow{OP} = 3\mathbf{a} + \mathbf{b}$.

Question 23

5 marks | Vectors

23

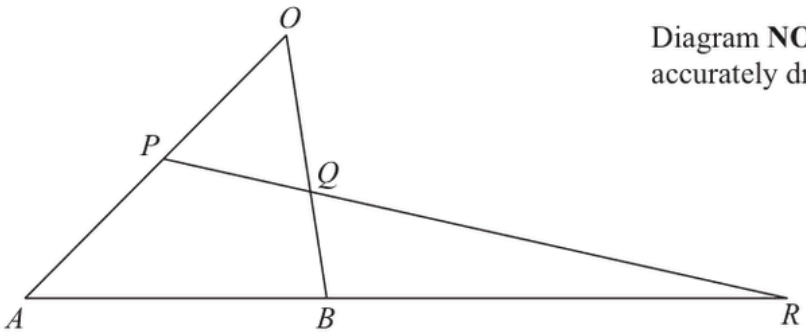


Diagram **NOT** accurately drawn

OAB is a triangle.
P is the midpoint of OA
Q is a point on OB

ABR and PQR are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\overrightarrow{OB} = 8\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

$AB : BR = 1 : 2$ $\overrightarrow{OQ} = n\mathbf{b}$

(b) Use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 23 is 5 marks)

1 **Use vectors**

For part (a),

$$\vec{OA} = 12\mathbf{a}, \qquad \vec{OB} = 8\mathbf{b}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}$$

2 **Find the midpoint**

For part (b), P is the midpoint of OA, so

$$\vec{OP} = 6\mathbf{a}$$

3 **Given**

Given

$$AB : BR = 1 : 2$$

$$\vec{BR} = 2\vec{AB}$$

$$\vec{BR} = 16\mathbf{b} - 24\mathbf{a}$$

4 **Use vectors**

Therefore

$$\vec{OR} = \vec{OB} + \vec{BR}$$

$$\vec{OR} = 8\mathbf{b} + 16\mathbf{b} - 24\mathbf{a}$$

$$\vec{OR} = -24\mathbf{a} + 24\mathbf{b}$$

5 **Use vectors**

Since P, Q, R are collinear, write Q on PR:

$$\vec{OQ} = \vec{OP} + t(\vec{OR} - \vec{OP})$$

$$\vec{OQ} = 6\mathbf{a} + t(-30\mathbf{a} + 24\mathbf{b})$$

$$\vec{OQ} = (6 - 30t)\mathbf{a} + 24t\mathbf{b}$$

---

6 **Use vectors**

But Q lies on OB, so there is no $\mathbf{a}$ component:

$$6 - 30t = 0$$

$$t = \frac{1}{5}$$

---

7 **Use vectors**

Then

$$\vec{OQ} = 24\left(\frac{1}{5}\right)\mathbf{b}$$

$$\vec{OQ} = \frac{24}{5}\mathbf{b}$$

---

8 **Use vectors**

Since $\vec{OQ} = n\mathbf{b}$,

$$n = \frac{24}{5}$$

✓ **FINAL ANSWER**

$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}, \quad n = \frac{24}{5}$

Question 26

5 marks | Vectors

26

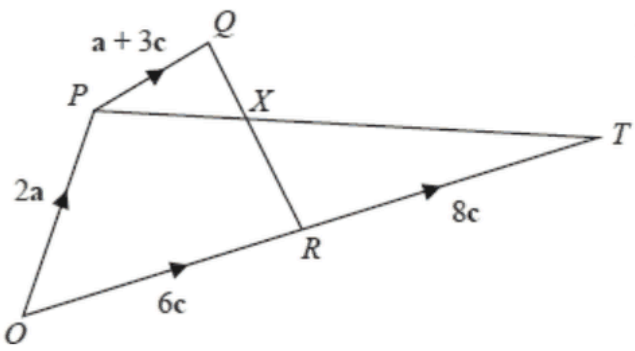


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OPQR$ in which

$\overrightarrow{OP} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{c}$ $\overrightarrow{PQ} = \mathbf{a} + 3\mathbf{c}$

(a) Find in terms of **a** and **c**

$\overrightarrow{QR}$ = ...

(1)

The point T is such that $\overrightarrow{RT} = 8\mathbf{c}$
The point X lies on QR such that PXT is a straight line.

(b) Using a vector method, find the ratio $QX : XR$
Give your answer in the form $m : n$ where m and n are integers.
Show your working clearly.

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$QX : XR$ = ...

(4)

(Total for Question 26 is 5 marks)

1 (a)

(a)

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$\overrightarrow{OQ} = 2a + (a + 3c) = 3a + 3c$$

$$\overrightarrow{QR} = 6c - (3a + 3c) = 3c - 3a$$

2 (b) Let divide so that

(b) Let X divide QR so that

$$\overrightarrow{QX} = \lambda\overrightarrow{QR}$$

3 Calculate value

Then

$$\overrightarrow{OX} = 3a + 3c + \lambda(3c - 3a)$$

4 Calculate value

Also X lies on PT, where $\overrightarrow{OT} = 14c$, so

$$\overrightarrow{OX} = 2a + \mu(14c - 2a)$$

5 Equating coefficients gives . Therefore

Equating coefficients gives $\lambda = \frac{5}{9}$. Therefore

$$QX : XR = \frac{5}{9} : \frac{4}{9} = 5 : 4$$

FINAL ANSWER

$\overrightarrow{QR} = 3c - 3a$; $QX : XR = 5 : 4$.

Question 24

6 marks | Vectors

24 The diagram shows a quadrilateral $OABC$

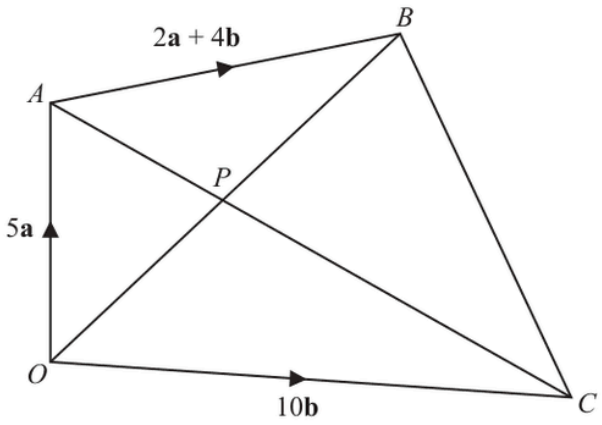


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 5\mathbf{a}$ $\overrightarrow{OC} = 10\mathbf{b}$ $\overrightarrow{AB} = 2\mathbf{a} + 4\mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**

$\overrightarrow{AC} =$ .....

(1)

(b) Find $\overrightarrow{OB}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{OB} =$ .....

(1)

The point P is such that APC and OPB are straight lines.

(c) Using a vector method, find $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OP} =$ .....

(4)

(Total for Question 24 is 6 marks)

1

(a) Find $overrightarrow{AC}$

$$overrightarrow{AC} = overrightarrow{AO} + overrightarrow{OC} = -overrightarrow{OA} + overrightarrow{OC} = -5mathbf{a} + 10mathbf{b}$$

2

Answer

a − 5**a** + 10**b**

3

(b) Find $overrightarrow{OB}$

$$overrightarrow{OB} = overrightarrow{OA} + overrightarrow{AB} = 5mathbf{a} + (2mathbf{a} + 4mathbf{b}) = 7mathbf{a} + 4mathbf{b}$$

4

Answer

b 7**a** + 4**b**

Step 1: Set up vector equations for $overrightarrow{OP}$. Since *O**P**B* is a straight line, $overrightarrow{OP}$ is a scalar multiple of $overrightarrow{OB}$. Let this be *λ*.

$$overrightarrow{OP} = lambda(7mathbf{a} + 4mathbf{b}) = 7lambdamathbf{a} + 4lambdamathbf{b}$$

Since *P* lies on the straight line *A**C*, $overrightarrow{AP} = muoverrightarrow{AC}$ for some scalar *μ*.

$$overrightarrow{OP} = overrightarrow{OA} + overrightarrow{AP} = 5mathbf{a} + mu(-5mathbf{a} + 10mathbf{b})$$

$$overrightarrow{OP} = (5 - 5mu)mathbf{a} + 10mumathbf{b}$$

Step 2: Equate the coefficients of a **and** b**.**

7*λ* = 5 − 5*μ* — (1)

$$4lambda = 10mu implies mu = frac{4}{10}lambda = 0.4lambda quad text{---(2)}$$

Step 3: Solve for *λ*. Substitute (2) into (1):

7*λ* = 5 − 5(0.4*λ*)

7*λ* = 5 − 2*λ*

$$9lambda = 5 implies lambda = frac{5}{9}$$

5

Evaluate fraction

$$overrightarrow{OP} = frac{5}{9}(7mathbf{a} + 4mathbf{b}) = frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

✓ **FINAL ANSWER**

$$frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

4MA1/1H NOVEMBER 2025

Question 27

6 marks | Vectors

27

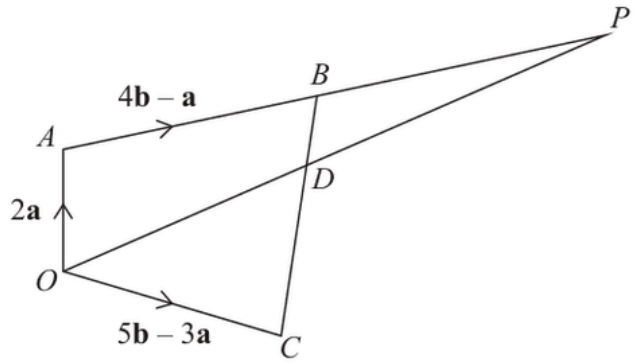


Diagram **NOT** accurately drawn

$OABC$ is a quadrilateral.
ABP and ODP are straight lines.

$\overrightarrow{OA} = \mathbf{2a}$ $\overrightarrow{AB} = \mathbf{4b - a}$ $\overrightarrow{OC} = \mathbf{5b - 3a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

.....

(2)

The point D lies on BC such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 27 is 6 marks)

WORKED SOLUTION

Question 27 · 4MA1/1H November 2025

1 Read the graph

$$\vec{OA} = \mathbf{2a}, \qquad \vec{AB} = \mathbf{4b - a}, \qquad \vec{OC} = \mathbf{5b - 3a}$$

2 **Read the graph**

First find $\vec{OB}$:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = 2\mathbf{a} + 4\mathbf{b} - \mathbf{a}$$

$$\vec{OB} = \mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$\vec{BC} = (5\mathbf{b} - 3\mathbf{a}) - (\mathbf{a} + 4\mathbf{b})$$

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}$$

---

4 **Read the graph**

For part (b), $BD : DC = 1 : 3$, so

$$\vec{OD} = \vec{OB} + \frac{1}{4}\vec{BC}$$

$$\vec{OD} = \mathbf{a} + 4\mathbf{b} + \frac{1}{4}(\mathbf{b} - 4\mathbf{a})$$

$$\vec{OD} = \frac{17}{4}\mathbf{b}$$

---

5 **Read the graph**

Since A, B, P are collinear, let

$$\vec{OP} = \vec{OA} + s\vec{AB}$$

$$\vec{OP} = 2\mathbf{a} + s(4\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = (2 - s)\mathbf{a} + 4s\mathbf{b}$$

---

6 **Read the graph**

But O, D, P are collinear, and $\vec{OD}$ has only a $\mathbf{b}$ component.

---

7 **Read the graph**

So

$$2 - s = 0$$

$$s = 2$$

---

8 **Read the graph**

Then

$$\vec{OP} = 8\mathbf{b}$$

---

9 **Given**

Given

$$\vec{OP} = n\vec{OD}$$

$$8\mathbf{b} = n\left(\frac{17}{4}\mathbf{b}\right)$$

$$n = \frac{32}{17}$$

✓ **FINAL ANSWER**

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}, \quad n = \frac{32}{17}$$

Question 18

5 marks | Vectors

18 Here are two vectors.

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \qquad \overrightarrow{BA} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$$

(a) Find $2\overrightarrow{AO}$ as a column vector.

$2\overrightarrow{AO} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$

(1)

(b) Find $\overrightarrow{OB}$ as a column vector.

$\overrightarrow{OB} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$

(2)

(c) Calculate the magnitude of the vector $\overrightarrow{BA}$

.....

(2)

(Total for Question 18 is 5 marks)

1 **(a) Reverse OA and double**

$$\overrightarrow{AO} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \qquad 2\overrightarrow{AO} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

2 **(b) Route O → A → B**

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 15 \\ -8 \end{pmatrix} = \begin{pmatrix} 19 \\ -11 \end{pmatrix}$$

3 **(c) Magnitude**

$$|\overrightarrow{BA}| = \sqrt{(-15)^2 + 8^2} = \sqrt{289} = 17$$

✓ **FINAL ANSWER**

(a) $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ (b) $\begin{pmatrix} 19 \\ -11 \end{pmatrix}$ (c) 17

4MA1/2HR JUNE 2026

Question 18

2 marks | Vectors

18 Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$$

Find $\overrightarrow{AC}$ as a column vector.

$$\overrightarrow{AC} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AB} - \overrightarrow{CB}.$$

Reverse CB to travel from B to C.

2 Solution | Step 2

$$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}.$$

Subtract corresponding components.

FINAL ANSWER

$$\overrightarrow{AC} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}.$$

4MA1/2H JUNE 2026

Question 22

4 marks | Vectors

22 The diagram shows a quadrilateral $OABC$ where O, A, B and C represent the positions of four houses.

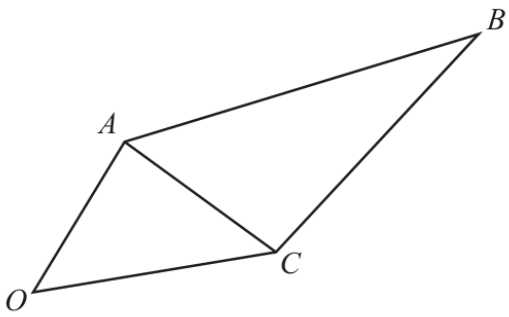


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$

Ria walks directly from A to B
Anna walks directly from A to C and then walks directly from C to B

$\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are given in metres.

Work out how much further Anna walks than Ria walks.
Give your answer correct to one decimal place.

..... metres

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2026

1 **Solution | Step 1**

$$\overrightarrow{AB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 19 \\ 6 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 12 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix},$$

$$\overrightarrow{CB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix} - \begin{pmatrix} 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \end{pmatrix}.$$

Subtract the starting position vector from the finishing position vector.

2 **Solution | Step 2**

$$AB = \sqrt{19^2 + 6^2} = \sqrt{397}, \quad AC = \sqrt{10^2 + (-5)^2} = \sqrt{125}, \quad CB = \sqrt{9^2 + 11^2} = \sqrt{202}.$$

The modulus of a two-dimensional vector is the square root of the sum of squares of its components.

3

Solution | Step 3

$$\text{Extra distance} = AC + CB - AB = \sqrt{125} + \sqrt{202} - \sqrt{397} = 5.468151\ldots\ \text{m}.$$

Compare Anna's two-stage route with Ria's direct route; do not round the intermediate lengths.

FINAL ANSWER

Anna walks $5.5\ \text{m}$ further (1 decimal place).

4MA1/1HR JUNE 2026

Question 25

6 marks | Vectors

25

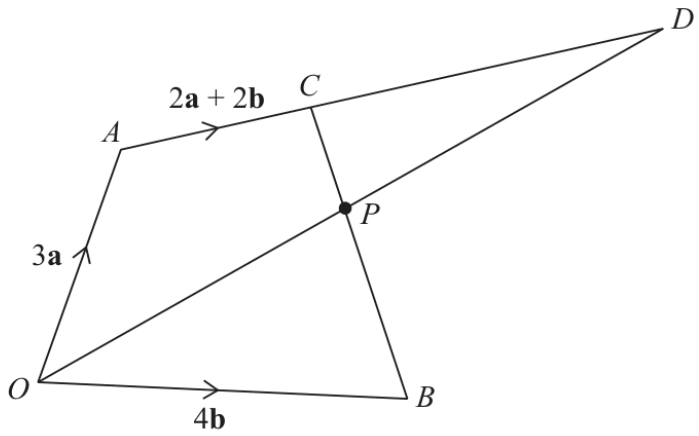


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = 4\mathbf{b}$ $\overrightarrow{AC} = 2\mathbf{a} + 2\mathbf{b}$ $\overrightarrow{CD} = \frac{3}{2} \times \overrightarrow{AC}$

P is the point on BC such that OPD is a straight line.

(a) Find $\overrightarrow{OD}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(b) Find the ratio $OP : PD$
Show your working clearly.

.....

(4)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\overrightarrow{CD} = \frac{3}{2}(2\mathbf{a} + 2\mathbf{b}) = 3\mathbf{a} + 3\mathbf{b}.$$

Use the stated relationship between CD and AC.

2 **Part (a) | Step 2**

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD} = 3\mathbf{a} + (2\mathbf{a} + 2\mathbf{b}) + (3\mathbf{a} + 3\mathbf{b}) = 8\mathbf{a} + 5\mathbf{b}.$$

Add consecutive vectors along the route O-A-C-D.

3 **Part (b) | Step 1**

$$\overrightarrow{OC} = 3\mathbf{a} + (2\mathbf{a} + 2\mathbf{b}) = 5\mathbf{a} + 2\mathbf{b}, \qquad \overrightarrow{BC} = 5\mathbf{a} - 2\mathbf{b}.$$

Use OC=OA+AC and BC=OC-OB.

4 **Part (b) | Step 2**

Write $\overrightarrow{OP} = t\overrightarrow{OD} = 8t\mathbf{a} + 5t\mathbf{b}$. Also, because P lies on BC,

$$\overrightarrow{OP} = \overrightarrow{OB} + s\overrightarrow{BC} = 5s\mathbf{a} + (4 - 2s)\mathbf{b}.$$

Use one scalar parameter on each of the two intersecting lines.

5 **Part (b) | Step 3**

$$8t = 5s, \qquad 5t = 4 - 2s, \qquad s = \frac{8}{5}t.$$

Equate the coefficients of the two nonparallel base vectors.

6 **Part (b) | Step 4**

$$5t = 4 - \frac{16}{5}t \quad \Longrightarrow \quad 41t = 20 \quad \Longrightarrow \quad t = \frac{20}{41}.$$

Substitute the first relation into the second and solve.

7 **Part (b) | Step 5**

$$OP : PD = t : (1 - t) = \frac{20}{41} : \frac{21}{41} = 20 : 21.$$

OD is divided into the fractions t and 1-t at P.

✓ **FINAL ANSWER**

(a) $\overrightarrow{OD} = 8\mathbf{a} + 5\mathbf{b}$ (b) $OP : PD = 20 : 21$

TOPIC

Statistics Toolkit

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 1

3 marks | Statistics Toolkit

1 Here are five cards, where x represents a number.

15	7	−2	23	x

The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM2/01 Specimen

1 **Calculate statistic**

The mean is 12, so the total of the five numbers is

$$5 \times 12 = 60$$

2 **Calculate value**

The known numbers add to

$$15 + 7 - 2 + 23 = 43$$

3 **Calculate value**

Therefore

$$43 + x = 60$$

$$x = 17$$

FINAL ANSWER

$x = 17$.

4WM2/01 SPECIMEN

Question 6

3 marks | Statistics Toolkit

6 Here are some integers where $a < b < c < d$

$$a \quad b \quad c \quad d \quad d \quad d$$

The mode of the integers is 9
The range of the integers is 4
The median of the integers is 8

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 Specimen

1 **Calculate statistic**

The mode is 9, and d appears three times, so

$$d = 9$$

2 **Calculate statistic**

The range is 4, so

$$d - a = 4$$

$$9 - a = 4$$

$$a = 5$$

3 Calculate statistic

There are six values, so the median is the mean of the 3rd and 4th values:

$$\frac{c+d}{2} = 8$$

$$\frac{c+9}{2} = 8$$

$$c = 7$$

4 Solve inequality

Since $a < b < c < d$, the remaining integer is $b = 6$.

FINAL ANSWER

$a = 5,\ b = 6,\ c = 7,\ d = 9$.

4MA1/2HR JANUARY 2020

Question 2

3 marks | Statistics Toolkit

2 The table gives information about the amount of money, in £, that Fiona spent in a grocery store each week during 2019

Amount spent (£x)	Frequency
$0 \leqslant x < 20$	5
$20 \leqslant x < 40$	11
$40 \leqslant x < 60$	8
$60 \leqslant x < 80$	19
$80 \leqslant x < 100$	9

Work out an estimate for the total amount of money that Fiona spent in the grocery store during 2019

£.....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR January 2020

1 Use midpoints

Use midpoints $10, 30, 50, 70, 90$.

$$10(5) + 30(11) + 50(8) + 70(19) + 90(9)$$

$$= 50 + 330 + 400 + 1330 + 810 = 2920$$

FINAL ANSWER

$\pounds 2920$.

4MA1/1HR JANUARY 2020

Question 8

3 marks | Statistics Toolkit

8 5 children are playing on a trampoline.
The mean weight of the 5 children is 28 kg.

2 of the children get off the trampoline.
The mean weight of these 2 children is 26.5 kg.

Work out the mean weight of the 3 children who remain on the trampoline.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2020

1 Total weight of 5 children

Total weight of 5 children:

$$5 \times 28 = 140 \text{ kg}$$

2 Total weight of the 2

Total weight of the 2 children who get off:

$$2 \times 26.5 = 53 \text{ kg}$$

3 Total weight of the 3

Total weight of the 3 children left:

$$140 - 53 = 87 \text{ kg}$$

4 Mean weight

Mean weight:

$$\frac{87}{3} = 29$$

FINAL ANSWER

29 kg.

4MA1/2HR JANUARY 2020

Question 14

3 marks | Statistics Toolkit

14 Max kept a record of the marks he scored in each of the 11 spelling tests he took one term.

Here are his marks.

18 5 7 12 11 18 15 16 17 13 14

Find the interquartile range of the marks.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR January 2020

1 **Put the marks in order**

Put the marks in order:

$$5,\ 7,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 18$$

$$Q_1 = 11, \qquad Q_3 = 17$$

$$\text{IQR} = 17 - 11 = 6$$

FINAL ANSWER

6.

4MA1/1H MAY/NOVEMBER 2020

Question 4

4 marks | Statistics Toolkit

4 Here is a list of six numbers written in order of size.

$$4 \quad 7 \quad x \quad 10 \quad y \quad y$$

The numbers have

a median of 9
a mean of 11

Find the value of x and the value of y.

$x =$ .....

$y =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May/November 2020

1 The median is

The median is 9:

$$\frac{x + 10}{2} = 9$$

$$x = 8$$

2 Calculate statistic

The mean is 11, so the total is

$$6 \times 11 = 66$$

$$4 + 7 + 8 + 10 + y + y = 66$$

$$29 + 2y = 66$$

$$2y = 37$$

$$y = 18.5$$

FINAL ANSWER

$x = 8,\ y = 18.5$.

4MA1/1HR JANUARY 2021

Question 2

4 marks | Statistics Toolkit

2 Given that $a < b < c$

the four whole numbers a, a, b and c have

a mode of 7
a median of 8.5
a mean of 9

Work out the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2021

1 **Calculate statistic**

The mode is 7, and the numbers are a, a, b, c, so

$$a = 7$$

2 **The median of four numbers**

The median of four numbers is

$$\frac{a+b}{2}$$

3 **Evaluate fraction**

So

$$\frac{7+b}{2} = 8.5$$

$$7 + b = 17$$

$$b = 10$$

4 **Calculate statistic**

The mean is 9, so the total is

$$4 \times 9 = 36$$

$$7 + 7 + 10 + c = 36$$

$$c = 12$$

✓ **FINAL ANSWER**

$a = 7,\ b = 10,\ c = 12$.

4MA1/2H JANUARY 2021

Question 2

3 marks | Statistics Toolkit

2 Ava writes down five whole numbers.

For these five numbers

the median is 7

the mode is 8

the range is 5

Find a possible value for each of the five numbers that Ava writes down.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2021

1 Calculate statistic

The median is 7, so the middle number is 7.

2 Calculate statistic

The mode is 8, so use two 8s.

3 Calculate statistic

The range is 5, so choose the smallest and largest numbers to differ by 5.

One possible set is

$$3,\ 6,\ 7,\ 8,\ 8$$

4 Range

Range:

$$8 - 3 = 5$$

5 Mode

Mode:

$$8$$

6 **Median**

Median:

$$7$$

✓ **FINAL ANSWER**

$3,\ 6,\ 7,\ 8,\ 8.$

4MA1/2H JANUARY 2021

Question 7

3 marks | Statistics Toolkit

7 There are 5 cocoa pods in a bag.
The mean weight of the 5 cocoa pods is 398 grams.

A sixth cocoa pod is put into the bag.
The mean weight of the 6 cocoa pods is 401 grams.

Work out the weight of the sixth cocoa pod that is put into the bag.

..... grams

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2021

1 Total weight of 5 cocoa

Total weight of 5 cocoa pods:

$$5 \times 398 = 1990 \text{ g}$$

2 Total weight of 6 cocoa

Total weight of 6 cocoa pods:

$$6 \times 401 = 2406 \text{ g}$$

3 Weight of the sixth cocoa

Weight of the sixth cocoa pod:

$$2406 - 1990 = 416$$

FINAL ANSWER

416 grams.

4MA1/2H JANUARY 2021

Question 12

2 marks | Statistics Toolkit

12 15 people were asked how long, in minutes, they had been waiting for a bus.

Here are the results.

2 3 3 4 5 6 6 8 9 10 11 13 14 15 18

Find the interquartile range of these times.

..... minutes

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2021

1 Use Statistics Toolkit correct

The data is already in order:

$$2, 3, 3, 4, 5, 6, 6, 8, 9, 10, 11, 13, 14, 15, 18$$

2 Calculate value

For 15 values,

$$Q_1 = 4, \qquad Q_3 = 13$$

$$\text{IQR} = 13 - 4 = 9$$

FINAL ANSWER

9 minutes.

4MA1/1H MAY 2021

Question 10

3 marks | Statistics Toolkit

10 Jethro has sat 5 tests.
Each test was marked out of 100 and Jethro's mean mark for the 5 tests is 74

Jethro has to sit one more test that is also to be marked out of 100

Jethro wants his mean mark for all 6 tests to be at least 77

Work out the least mark that Jethro needs to get for the last test.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2021

1 **The total for the first**

The total for the first 5 tests is

$$5 \times 74 = 370$$

2 **Calculate value**

For an average of at least 77 after 6 tests, the total must be at least

$$6 \times 77 = 462$$

3 **Calculate value**

So the last test score must be at least

$$462 - 370 = 92$$

✓ **FINAL ANSWER**

at least 92.

4MA1/1H MAY 2021

Question 14

2 marks | Statistics Toolkit

14 Here is the number of goals that Henri's team scored one summer in each water polo match.

5 8 9 11 13 13 14 15 16 17 20

Find the interquartile range of the numbers of goals.
Show your working clearly.

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2021

1 **The goals in order are**

The goals in order are

$$5,\ 8,\ 9,\ 11,\ 13,\ 13,\ 14,\ 15,\ 16,\ 17,\ 20$$

$$Q_1 = 9, \qquad Q_3 = 16$$

$$\text{IQR} = 16 - 9 = 7$$

FINAL ANSWER

7.

4MA1/1H JANUARY 2022

Question 5

6 marks | Statistics Toolkit

5 Jenny has six cards.

Each card has a whole number written on it so that

the smallest number is 5
the largest number is 24
the median of the six numbers is 14
the mode of the six numbers is 8

Jenny arranges her cards so that the numbers are in order of size.



(a) For the remaining four cards, write on each dotted line a number that could be on the card.

(3)

A basketball team plays 6 games.
After playing 5 games, the team has a mean score of 21 points per game.
After playing 6 games, the team has a mean score of 23 points per game.

(b) Work out the number of points the team scored in its 6th game.

.....

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2022

1 **One possible ordered set is**

One possible ordered set is

$$5,\ 8,\ 8,\ 20,\ 21,\ 24$$

2 **Check**

Check:

$$\text{median} = \frac{8 + 20}{2} = 14$$

3 **Calculate statistic**

The mode is 8, the smallest number is 5, and the largest number is 24.

4 **Calculate value**

For the basketball scores:

$$5 \times 21 = 105$$

$$6 \times 23 = 138$$

$$138 - 105 = 33$$

✓ **FINAL ANSWER**

$5, 8, 8, 20, 21, 24$ and 33.

4MA1/1HR JANUARY 2022

Question 5

4 marks | Statistics Toolkit

5 Yusuf sat 8 examinations.

Here are his marks for 5 of the examinations.

68 72 75 77 80

For his results in all 8 examinations

the mode of his marks is 80
the median of his marks is 74
the range of his marks is 16

Find Yusuf's marks for each of the other 3 examinations.

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2022

1 The range is . Since

The range is 16. Since the greatest known mark is 80,

$$80 - 16 = 64$$

2 Use Statistics Toolkit tag correct

So one missing mark is 64.

3 Calculate statistic

The mode is 80, so another missing mark is 80.

4 Use Statistics Toolkit tag correct

Now the marks in order are

$$64,\ 68,\ 72,\ x,\ 75,\ 77,\ 80,\ 80$$

5 Calculate statistic

The median is 74, so

$$\frac{x + 75}{2} = 74$$

$$x = 73$$

FINAL ANSWER

64, 73, 80.

4MA1/2HR JANUARY 2022

Question 8

3 marks | Statistics Toolkit

8 A scientist is investigating the weight of 50 tigers.

Here is some information about these tigers.

	Type of tiger	
	Siberian	**Bengal**
Number of tigers	22	28
Mean weight of tigers (kg)	260	

The mean weight of all 50 tigers is 218 kg

Work out the mean weight of the Bengal tigers.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2022

1 Total mass of all 50

Total mass of all 50 tigers:

$$50 \times 218 = 10900$$

2 Total mass of the 22

Total mass of the 22 Siberian tigers:

$$22 \times 260 = 5720$$

3 Total mass of the 28

Total mass of the 28 Bengal tigers:

$$10900 - 5720 = 5180$$

4 Mean mass of the Bengal

Mean mass of the Bengal tigers:

$$\frac{5180}{28} = 185$$

FINAL ANSWER

185 kg.

4MA1/2H JANUARY 2022

Question 15

3 marks | Statistics Toolkit

15 Diyar recorded the distance, in kilometres, that he cycled each day for 11 days.
Here are his results.

8 10 12 13 5 23 21 7 5 16 14

Find the interquartile range of his results.

..... km

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H January 2022

1 Put the distances in order

Put the distances in order:

$$5,\ 5,\ 7,\ 8,\ 10,\ 12,\ 13,\ 14,\ 16,\ 21,\ 23$$

2 Calculate statistic

There are 11 values, so the median is the 6th value.

3 The lower half is

The lower half is

$$5,\ 5,\ 7,\ 8,\ 10$$

4 Calculate value

so

$$Q_1 = 7$$

5 The upper half is

The upper half is

$$13,\ 14,\ 16,\ 21,\ 23$$

6 **Calculate value**

so

$$Q_3 = 16$$

$$\text{IQR} = 16 - 7 = 9$$

✓ **FINAL ANSWER**

$9\ \text{km}$.

4MA1/2H JUNE 2022

Question 1

3 marks | Statistics Toolkit

1 Here are some integers where $a < b < c < d$

$$a \quad b \quad c \quad d \quad d \quad d$$

The mode of the integers is 9
The median of the integers is 8
The range of the integers is 4

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2022

1 **Calculate statistic**

The mode is $\mathbf{9}$, so

$$d = 9$$

2 **The range is**

The range is $\mathbf{4}$:

$$9 - a = 4$$

$$a = 5$$

3 **Calculate statistic**

The median is 8, so

$$\frac{c+d}{2}=8$$

$$\frac{c+9}{2}=8$$

$$c=7$$

4 **The integers are in order**

The integers are in order and $a<b<c$, so

$$b=6$$

FINAL ANSWER

$a=5,\ b=6,\ c=7,\ d=9$.

4MA1/1HR JUNE 2022

Question 3

3 marks | Statistics Toolkit

3 Here are five cards.

Each card has a number written on it.



The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2022

1 The mean of the five

The mean of the five cards is 12, so the total is

$$5 \times 12 = 60$$

2 The known total is

The known total is

$$15 + 7 - 2 + 23 = 43$$

$$x = 60 - 43 = 17$$

FINAL ANSWER

$x = 17$.

4MA1/2HR JUNE 2022

Question 3

3 marks | Statistics Toolkit

3 Alberto, Bill, Candela and Diana are four friends.

Here is some information about the height of each of these friends.

Alberto's height is 158 cm.
Bill's height is 175 cm.
Candela's height is greater than Diana's height.

The median height of these four friends is 160 cm.
The range of the heights of these four friends is 21 cm.

Work out Candela's height and Diana's height.

Candela ..... cm

Diana ..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2022

1 The median is . Since

The median is 160. Since the ordered heights are

$$\text{Diana},\ 158,\ \text{Candela},\ 175$$

$$\frac{158 + \text{Candela}}{2} = 160$$

$$\text{Candela} = 162$$

2 The range is

The range is 21:

$$175 - \text{Diana} = 21$$

$$\text{Diana} = 154$$

FINAL ANSWER

Candela $= 162$ cm, Diana $= 154$ cm.

4MA1/2H JUNE 2022

Question 12

3 marks | Statistics Toolkit

12 Larry is a delivery man.

He has 7 parcels to deliver.
The mean weight of the 7 parcels is 2.7 kg

Larry delivers 3 of the parcels.
Each of these 3 parcels has a weight of W kg

The mean weight of the other 4 parcels is 3.3 kg

Work out the value of W

$W =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2022

1 Total mass of the 7

Total mass of the 7 parcels:

$$7 \times 2.7 = 18.9$$

2 Total mass of the 4

Total mass of the 4 remaining parcels:

$$4 \times 3.3 = 13.2$$

3 Total mass of the 3

Total mass of the 3 delivered parcels:

$$18.9 - 13.2 = 5.7$$

4 Each delivered parcel has mass

Each delivered parcel has mass W:

$$3W = 5.7$$

$$W = 1.9$$

FINAL ANSWER

1.9 kg.

4MA1/1HR JUNE 2022

Question 13

2 marks | Statistics Toolkit

13 Here is the number of runs scored by a baseball team in each of its 15 games this season.

The number of runs have been arranged in order of size.

0 1 1 3 5 6 7 7 8 9 9 12 12 15 16

Work out the interquartile range of the number of runs.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2022

1 Use Statistics Toolkit correct

The runs are already in order:

$$0,\ 1,\ 1,\ 3,\ 5,\ 6,\ 7,\ 7,\ 8,\ 9,\ 9,\ 12,\ 12,\ 15,\ 16$$

2 Calculate statistic

There are 15 values, so ignore the median when finding the quartiles.

$$Q_1 = 3, \qquad Q_3 = 12$$

$$\text{IQR} = 12 - 3 = 9$$

FINAL ANSWER

9.

4MA1/1H MAY 2023

Question 2

3 marks | Statistics Toolkit

2 The table gives information about the number of minutes that Abby spent walking each day in September.

Number of minutes (M)	Frequency
$0 < M \leqslant 30$	5
$30 < M \leqslant 60$	6
$60 < M \leqslant 90$	8
$90 < M \leqslant 120$	9
$120 < M \leqslant 150$	2

Work out an estimate for the total number of minutes that Abby spent walking in September.

..... minutes

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2023

1 **Use midpoints**

Use midpoints $15, 45, 75, 105, 135$.

$$15(5) + 45(6) + 75(8) + 105(9) + 135(2) = 2160$$

✓ **FINAL ANSWER**

2160 minutes.

4MA1/2H JUNE 2023

Question 10

4 marks | Statistics Toolkit

10 Team **A** and Team **B** take part in a quiz league.

After 11 rounds, Team **A** has a mean score per round of 17
After 9 rounds, Team **B** has a mean score per round of 18

Both teams take part in a further round.
After this round, both teams have a mean score per round of 18.5

In the further round, Team **A** scored more points than Team **B**.

How many more?

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2023

1 **Team A**

Team A:

$$11 \times 17 = 187$$

2 **Estimate the value**

After the next round:

$$12 \times 18.5 = 222$$

$$\text{next score} = 222 - 187 = 35$$

3 **Team B**

Team B:

$$9 \times 18 = 162$$

4 **Estimate the value**

After the next round:

$$10 \times 18.5 = 185$$

$$\text{next score} = 185 - 162 = 23$$

5 **Difference**

Difference:

$$35 - 23 = 12$$

✓ **FINAL ANSWER**

12.

4MA1/2HR JUNE 2023

Question 10

3 marks | Statistics Toolkit

10 Gemara works as a taxi driver.
Last week, he recorded the following information about the distances he drove.

For the 5 days from Monday to Friday, the mean number of kilometres he drove was 104
For the 7 days from Monday to Sunday, the mean number of kilometres he drove was 127

On Saturday, Gemara drove 132 kilometres.

Work out the number of kilometres he drove on Sunday.

..... kilometres

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2023

1 Total for Monday to Friday

Total for Monday to Friday:

$$5 \times 104 = 520$$

2 Total for Monday to Sunday

Total for Monday to Sunday:

$$7 \times 127 = 889$$

3 Calculate value

So the total for Saturday and Sunday is

$$889 - 520 = 369$$

4 Calculate value

Saturday was $132\ \text{km}$, so

$$369 - 132 = 237$$

✓ **FINAL ANSWER**

$237\ \text{km}$.

4MA1/1H MAY 2023

Question 10

2 marks | Statistics Toolkit

10 Here are the test marks of 15 students.

7 10 14 15 16 17 18 19 20 20 23 25 30 39 40

Find the interquartile range of these marks.

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2023

1 The marks in order are

The marks in order are

$$7,\ 10,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20,\ 20,\ 23,\ 25,\ 30,\ 39,\ 40$$

$$Q_1 = 15, \qquad Q_3 = 25$$

$$\text{IQR} = 25 - 15 = 10$$

FINAL ANSWER

10.

4MA1/2HR JUNE 2023

Question 14

4 marks | Statistics Toolkit

14 Akari played a computer game eleven times.
Here are her scores.

25 20 28 27 26 22 23 29 20 29 26

(a) Find the interquartile range of her scores.

.....

(3)

Machi played the same computer game eleven times.

The interquartile range for Machi's scores was 9

(b) Who had the more consistent scores, Akari or Machi?
Give a reason for your answer.

.....

(1)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2023

1 **Akari s scores in order**

Akari's scores in order are

$$20,\ 20,\ 22,\ 23,\ 25,\ 26,\ 26,\ 27,\ 28,\ 29,\ 29$$

$$Q_1 = 22, \qquad Q_3 = 28$$

$$\text{IQR} = 28 - 22 = 6$$

2 **Machi s interquartile range is**

Machi's interquartile range is $\mathbf{9}$.

3 **Akari is more consistent because**

Akari is more consistent because her interquartile range is smaller.

FINAL ANSWER

Akari's IQR is $\mathbf{6}$, and Akari is more consistent.

4MA1/2H NOVEMBER 2023

Question 8

3 marks | Statistics Toolkit

8 The mean number of goals scored by a hockey team in 8 matches is 6
The team plays 2 more matches and scores k goals in each match.
The mean number of goals scored by the hockey team in the 10 matches is 7

Work out the value of k

$k =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2023

1 The total for the first

The total for the first 8 matches is

$$8 \times 6 = 48$$

2 The total for all 10

The total for all 10 matches is

$$10 \times 7 = 70$$

3 Compare density

So the last two matches have total

$$70 - 48 = 22$$

4 The last two scores are

The last two scores are k and $k + 10$:

$$k + (k + 10) = 22$$

$$2k + 10 = 22$$

$$k = 6$$

FINAL ANSWER

$k = 6$.

4MA1/2H NOVEMBER 2023

Question 13

2 marks | Statistics Toolkit

13 Robert asked 11 people how many meetings they attended last week.

Here are the results in numerical order.

1 2 4 6 6 8 11 12 13 14 17

Find the interquartile range of the number of meetings.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2023

1 The results in order are

The results in order are

$$1,\ 2,\ 4,\ 6,\ 6,\ 8,\ 11,\ 12,\ 13,\ 14,\ 17$$

2 There are 11 values

There are 11 values.

$$Q_1 = 4, \qquad Q_3 = 13$$

$$\text{IQR} = 13 - 4 = 9$$

FINAL ANSWER

9.

4MA1/2H JUNE 2024

Question 1

3 marks | Statistics Toolkit

1 Here are eight numbers written in order of size

$$h \quad 6 \quad 7 \quad 8 \quad j \quad 16 \quad k \quad k$$

where h, j and k are integers.

The median of the eight numbers is 10

The mode of the eight numbers is 18

The range of the eight numbers is 13

Work out the value of h, the value of j and the value of k

$h =$ .....

$j =$ .....

$k =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2024

1 **The median is**

The median is 10:

$$\frac{8+j}{2} = 10$$

$$j = 12$$

2 **Calculate statistic**

The mode is 18, so

$$k = 18$$

3 **The range is**

The range is 13:

$$18 - h = 13$$

$$h = 5$$

✓ **FINAL ANSWER**

$h = 5,\ j = 12,\ k = 18.$

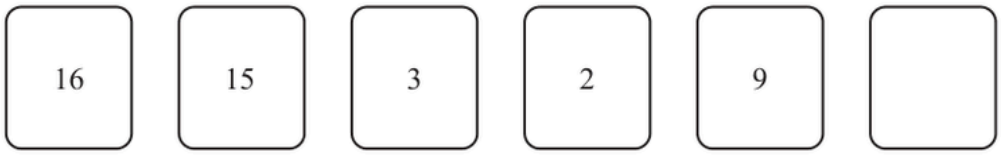
4MA1/1HR MAY 2024

Question 1

3 marks | Statistics Toolkit

1 Here are six cards.

Five of the cards have a number written on them.



Work out the number that should be written on the last card so that the mean of the six numbers will be 11

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May 2024

1 The total of the 6

The total of the 6 cards is

$$6 \times 11 = 66$$

2 The known total is

The known total is

$$16 + 15 + 3 + 2 + 9 = 45$$

$$x = 66 - 45 = 21$$

FINAL ANSWER

$x = 21$.

4MA1/2HR JUNE 2024

Question 3

2 marks | Statistics Toolkit

3 Here is a list of four numbers written in ascending order of size

$$x \quad x \quad y \quad 15$$

where x and y are integers.

The numbers have

a median of 12.5
a range of 4

Find the value of x and the value of y

$x =$ .....

$y =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2024

1 The range is

The range is 4:

$$15 - x = 4$$

$$x = 11$$

2 The median is

The median is 12.5:

$$\frac{x + y}{2} = 12.5$$

$$x + y = 25$$

$$y = 25 - 11 = 14$$

FINAL ANSWER

$x = 11,\ y = 14.$

4MA1/1HR MAY 2024

Question 10

2 marks | Statistics Toolkit

10 Here are the numbers of goals scored by a hockey team in its 11 games this season.

0 1 2 2 3 4 4 6 7 9 11

Work out the interquartile range of the numbers of goals.

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR May 2024

1 The goals in order are

The goals in order are

$$0,\ 1,\ 2,\ 2,\ 3,\ 4,\ 4,\ 6,\ 7,\ 9,\ 11$$

$$Q_1 = 2, \qquad Q_3 = 7$$

$$\text{IQR} = 7 - 2 = 5$$

FINAL ANSWER

5.

4MA1/2H JUNE 2024

Question 12

3 marks | Statistics Toolkit

12 Ali uses a fitness tracker to count the number of steps he walks each day for 7 days.

For the first 4 days, his mean number of steps is 11 800
For the next 3 days, his mean number of steps is 13 207

Work out his mean number of steps for the 7 days.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2024

1 Total for the first 4

Total for the first 4 days:

$$4 \times 11800 = 47200$$

2 Total for the next 3

Total for the next 3 days:

$$3 \times 13207 = 39621$$

3 Total for all 7 days

Total for all 7 days:

$$47200 + 39621 = 86821$$

4 Mean for the 7 days

Mean for the 7 days:

$$\frac{86821}{7} = 12403$$

FINAL ANSWER

12403.

4MA1/2H NOVEMBER 2024

Question 8

3 marks | Statistics Toolkit

8 Kim puts 4 bananas in a bag.
The mean weight of the 4 bananas in the bag is 145 grams.

Andy puts one more banana into the bag.
The mean weight of the 5 bananas in the bag is 142 grams.

Work out the weight of the banana that Andy puts into the bag.

..... grams

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2024

1 The total weight of the

The total weight of the first 4 bananas is

$$4 \times 145 = 580$$

2 The total weight of the

The total weight of the 5 bananas is

$$5 \times 142 = 710$$

3 Weight of the banana Andy

Weight of the banana Andy added:

$$710 - 580 = 130$$

✓ **FINAL ANSWER**

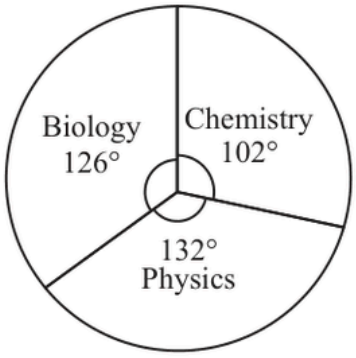
130 g.

Question 10

5 marks | Statistics Toolkit

10 All the students in year 10 and all the students in year 11 named their favourite science subject from Biology, Chemistry and Physics.

The pie chart shows information about the results for the year 10 students.
The table shows information about the results for the year 11 students.



Pie chart for year 10

science subject	number of students
Biology	$3x + 6$
Chemistry	$5x + 8$
Physics	$7x - 9$

Table for year 11

There are 300 students in year 10
There are 320 students in year 11

More students in year 10 than in year 11 said Biology was their favourite science subject.

How many more?

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2024

1 **Evaluate fraction**

For year 10, Biology is $126°$ out of $360°$.

$$\frac{126}{360} \times 300 = 105$$

2 **Calculate value**

For year 11:

$$(3x + 6) + (5x + 8) + (7x - 9) = 320$$

$$15x + 5 = 320$$

$$x = 21$$

---

3 **Year 11 Biology**

Year 11 Biology:

$$3(21) + 6 = 69$$

---

4 **Difference**

Difference:

$$105 - 69 = 36$$

✓ **FINAL ANSWER**

36 more students.

4MA1/2H JUNE 2025

Question 1

3 marks | Statistics Toolkit

1 Rio has six cards.
He writes a number on each card so that

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

Rio arranges the cards so that the numbers are in order of size.

		6		10	14

Three of the numbers are hidden.

Complete the cards above to show the three numbers that are hidden.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2025

1 **Calculate statistic**

The range is 10, and the greatest card is 14:

$$14 - \text{smallest} = 10$$

2 **Use Statistics Toolkit tag correct**

So the smallest card is 4.

3 **Calculate statistic**

The median is 7.5. For 6 cards, the median is the mean of the 3rd and 4th cards:

$$\frac{6 + x}{2} = 7.5$$

$$x = 9$$

4 **Calculate statistic**

The mode is 6, so the other missing card must be 6.

FINAL ANSWER

$4,\ 6,\ 9$.

4WM2/01 MAY 2025

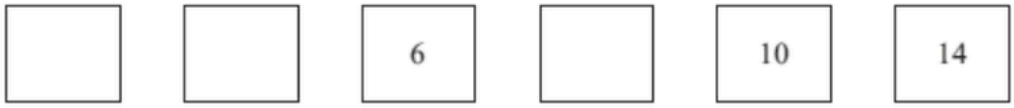
Question 1

3 marks | Statistics Toolkit

1 Rio has six cards.
He writes a number on each card so that

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

Rio arranges the cards so that the numbers are in order of size.



Three of the numbers are hidden.
Complete the cards above to show the three numbers that are hidden.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM2/01 May 2025

1 **Calculate statistic**

The range is 10, so the first number is

$$14 - 10 = 4$$

2 **Calculate statistic**

For six numbers, the median is the mean of the third and fourth values.

$$\frac{6 + \text{fourth value}}{2} = 7.5$$

3 **Calculate statistic**

So the fourth value is 9. To make 6 the mode, the second value must also be 6.

✓ **FINAL ANSWER**

the hidden numbers are $4, 6, 9$.

4WM2/01R MAY 2025

Question 7

3 marks | Statistics Toolkit

7 The mean of 7 numbers is 60

The mean of 3 of the numbers is 46

Work out the mean of the other 4 numbers.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4WM2/01R May 2025

1 Required

Mean of 7 numbers = 60; mean of 3 of them = 46. Find the mean of the other 4.

2 Sum of all 7 numbers

Sum of all 7 numbers:

$7 \times 60 = 420$

3 Sum of the 3 numbers

Sum of the 3 numbers:

$3 \times 46 = 138$

4 Sum of the remaining 4

Sum of the remaining 4:

$420 - 138 = 282$

5 Mean of the 4 numbers

Mean of the 4 numbers:

$$frac2824 = 70.5$$

✓ FINAL ANSWER

Mean of the other 4 numbers = 70.5

4MA1/2HR JUNE 2025

Question 8

3 marks | Statistics Toolkit

8 The mean of 7 numbers is 60

The mean of 3 of the numbers is 46

Work out the mean of the other 4 numbers.

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2025

1 The total of the 7

The total of the 7 numbers is

$$7 \times 60 = 420$$

2 The total of the 3

The total of the 3 numbers is

$$3 \times 46 = 138$$

3 The total of the other

The total of the other 4 numbers is

$$420 - 138 = 282$$

4 Their mean is

Their mean is

$$\frac{282}{4} = 70.5$$

FINAL ANSWER

70.5.

4MA1/1HR JUNE 2025

Question 13

2 marks | Statistics Toolkit

13 Here are the numbers of eggs laid by some hens on each of 11 days in June.

9 10 12 15 17 18 19 19 20 25 26

Work out the interquartile range of the numbers of eggs.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2025

1 Put the numbers in order

Put the numbers in order:

$$9,\ 10,\ 12,\ 15,\ 17,\ 18,\ 19,\ 19,\ 20,\ 25,\ 26$$

$$Q_1 = 12, \qquad Q_3 = 20$$

$$\text{IQR} = 20 - 12 = 8$$

FINAL ANSWER

8.

4WM2/01 MAY 2025

Question 16

2 marks | Statistics Toolkit

16 Here are the numbers of goals scored by a netball team in its 11 games this season.

7 8 10 11 13 14 16 17 19 21 24

Work out the interquartile range for the number of goals.

.....

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 May 2025

1 There are values. Excluding the

There are 11 values. Excluding the median 14:

2 Calculate value

Lower half: $7, 8, 10, 11, 13$, so $Q_1 = 10$.

3 Calculate value

Upper half: $16, 17, 19, 21, 24$, so $Q_3 = 19$.

$$\text{IQR} = 19 - 10 = 9$$

✓ **FINAL ANSWER**

9.

4MA1/1H NOVEMBER 2025

Question 4

3 marks | Statistics Toolkit

4 e, f and g represent integers such that

$$e \quad f \quad 7 \quad g \quad 12 \quad 17$$

is a list of integers written in order of size.

The integers have

a range of 15
a median of 8.5
a mean of 9

Work out the value of e, the value of f and the value of g

$e =$ .....

$f =$ .....

$g =$ .....

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2025

1 **The range is**

The range is 15:

$$17 - e = 15$$

$$e = 2$$

2 **Calculate statistic**

The mean is 9, so the total is

$$6 \times 9 = 54$$

$$2 + f + 7 + g + 12 + 17 = 54$$

$$f + g = 16$$

3 **The median is**

The median is 8.5:

$$\frac{7+g}{2} = 8.5$$

$$g = 10$$

$$f = 16 - 10 = 6$$

✓ **FINAL ANSWER**

$e = 2,\ f = 6,\ g = 10$.

4WM2/01 NOVEMBER 2025

Question 10

3 marks | Statistics Toolkit

10 There are 5 potatoes in a bag.
The mean weight of the 5 potatoes is 217 grams.

One more potato, of weight 175 grams, is put into the bag.

Work out the mean weight of the 6 potatoes in the bag.

..... grams

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM2/01 November 2025

1 Calculate statistic

Total₅ = Mean × Number = 217 × 5 = 1085 grams

2 Solve inequality

Total₆ = 1085 + 175 = 1260 grams

3 Calculate statistic

$$textNewMean = frac12606 = 210textgrams$$

FINAL ANSWER

210

4WM2/01 NOVEMBER 2025

Question 16

2 marks | Statistics Toolkit

16 11 people were asked how long, in minutes, they stayed in a coffee shop last Friday.

Here are the results.

14 15 16 18 19 20 22 25 28 40 50

Work out the interquartile range of these times.

..... minutes

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 November 2025

1 Calculate value

Data (sorted): 14, 15, 16, 18, 19, 20, 22, 25, 28, 40, 50.

Number of data points *n* = 11.

2 Find median

Step 1: Find the Median (Q2). The median is the $left(frac11 + 12right) = 6^{textth}$ value.

Median = 20

3 Find median

Step 2: Find Lower Quartile (Q1) and Upper Quartile (Q3). Q1 is the median of the lower 5 values (14, 15, **16**, 18, 19).

*Q*1 = 16

Q3 is the median of the upper 5 values (22, 25, **28**, 40, 50).

*Q*3 = 28

4 Calculate statistic

IQR = *Q*3 − *Q*1 = 28 − 16 = 12

✓ **FINAL ANSWER**

12 minutes

4MA1/1H NOVEMBER 2025

Question 16

2 marks | Statistics Toolkit

16 Ivan asked 15 people how many books they read last year.

Here are his results.

1 1 3 4 6 7 8 9 10 12 15 25 30 37 50

Work out the interquartile range of the number of books read.

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2025

1 Use Statistics Toolkit correct

The data are already in order:

$$1,\ 1,\ 3,\ 4,\ 6,\ 7,\ 8,\ 9,\ 10,\ 12,\ 15,\ 25,\ 30,\ 37,\ 50$$

2 There are 15 values

There are 15 values.

$$Q_1 = 4, \qquad Q_3 = 25$$

$$\text{IQR} = 25 - 4 = 21$$

FINAL ANSWER

21.

4MA1/2HR JUNE 2026

Question 6

4 marks | Statistics Toolkit

6 Mika has six cards.
There is a number on each card.
The cards are arranged with the numbers in order of size.
Three of the numbers are hidden.



The mode of the six numbers is 8
The range of the six numbers is 7
The sum of the six numbers is 31

Work out the median of the six numbers.

.....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2026

1 **Solution | Step 1**

The largest card is 8, so the smallest is $8 - 7 = 1$.

Range is largest minus smallest.

2 **Solution | Step 2**

The fifth card must be 8, since 8 is the mode.

Only the fifth position can supply another 8 in the ordered cards.

3 **Solution | Step 3**

If the third card is z, then

$$1 + 3 + z + 7 + 8 + 8 = 31 \quad \Rightarrow \quad z = 4.$$

Use the total of all six cards.

4 **Solution | Step 4**

The ordered cards are $1, 3, 4, 7, 8, 8$, so

$$\text{median} = \frac{4 + 7}{2} = 5.5.$$

With six values, average the third and fourth.

FINAL ANSWER

Median $= \mathbf{5.5}$.

4MA1/1HR JUNE 2026

Question 9

3 marks | Statistics Toolkit

9 Caleb owns 11 sheep.

The mean weight of all 11 sheep is 77 kg
The mean weight of 8 of the sheep is 75.2 kg

Work out the mean weight of the other 3 sheep.

..... kg

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2026

1 Solution | Step 1

$$W_{11} = 11(77) = 847 \text{ kg}, \qquad W_8 = 8(75.2) = 601.6 \text{ kg}.$$

Total weight equals number of sheep multiplied by their mean weight.

2 Solution | Step 2

$$W_3 = 847 - 601.6 = 245.4 \text{ kg}, \qquad \overline{w}_3 = \frac{245.4}{3} = 81.8 \text{ kg}.$$

Subtract the known subgroup total and divide the remaining total by 3.

FINAL ANSWER

81.8 kg

4MA1/2H JUNE 2026

Question 13

4 marks | Statistics Toolkit

13 Leo collects apples from three trees.
The incomplete table gives some information about the apples he collects from these trees.

Tree	Number of apples	Mean weight of apples (grams)	Total weight of apples (grams)
A	12	150	
B	20		2700
C		145	2610

Work out the mean weight of all the apples Leo collects from these three trees.

..... grams

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H June 2026

1 Solution | Step 1

Tree A contributes $12 \times 150 = 1800$ grams.

Total weight equals count times mean weight.

2 Solution | Step 2

Tree C has $2610 \div 145 = 18$ apples.

Number of items equals total divided by mean.

3 Solution | Step 3

The total weight is $1800 + 2700 + 2610 = 7110$ grams and the total number of apples is $12 + 20 + 18 = 50$.

Add weights and counts separately; do not average the three group means.

4 Solution | Step 4

$$\text{Combined mean} = \frac{7110}{50} = 142.2 \text{ grams.}$$

Divide the total weight by the total count.

FINAL ANSWER

142.2 grams

4WM2/01R JUNE 2026

Question 15

2 marks | Statistics Toolkit

15 Mina recorded the times taken, in minutes, by 11 people to complete a puzzle.
Here are her results.

2 2 3 6 7 8 9 12 13 17 20

Find the interquartile range of these times.

..... minutes

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4WM2/01R June 2026

1 Positions of the quartiles

$n = 11$: LQ is the $\frac{11+1}{4} = 3$rd value, UQ is the 9th value.

2 Read from the ordered list

$2, 2, \mathbf{3}, 6, 7, 8, 9, 12, \mathbf{13}, 17, 20$: LQ $= \mathbf{3}$, UQ $= \mathbf{13}$.

3 Interquartile range

$$13 - 3 = 10$$

✓ **FINAL ANSWER**

10 minutes

4MA1/2HR JUNE 2026

Question 15

2 marks | Statistics Toolkit

15 Mina recorded the times taken, in minutes, by 11 people to complete a puzzle.
Here are her results.

2 2 3 6 7 9 9 12 13 17 20

Find the interquartile range of these times.

..... minutes

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2026

1 Solution | Step 1

The overall median is the sixth value, 9. The lower five values are $2, 2, 3, 6, 7$, and the upper five are $9, 12, 13, 17, 20$.

Exclude the overall median when forming the two halves.

2 Solution | Step 2

$$Q_1 = 3, \quad Q_3 = 13, \quad \text{IQR} = 13 - 3 = 10 \text{ minutes}.$$

Take each half-median, then subtract the lower quartile from the upper.

✓ FINAL ANSWER

10 minutes.

TOPIC

Averages from Frequency Tables

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 4

7 marks | Averages from Frequency Tables

4 The table shows information about the weights, in kilograms, of 40 babies.

Weight (w kg)	Frequency
$2 < w \leqslant 3$	12
$3 < w \leqslant 4$	16
$4 < w \leqslant 5$	9
$5 < w \leqslant 6$	2
$6 < w \leqslant 7$	1

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean weight of the 40 babies.

..... kg

(4)

One of the 40 babies is going to be chosen at random.

(c) Find the probability that this baby has a weight of more than 5 kg.

.....

(2)

(Total for Question 4 is 7 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2020

1 **(a) The greatest frequency is**

(a) The greatest frequency is 16, so the modal class is

$$3 < w \leq 4$$

2 **(b) Use class midpoints**

(b) Use class midpoints:

$$\frac{2.5(12) + 3.5(16) + 4.5(9) + 5.5(2) + 6.5(1)}{40} = 3.6$$

3 (c) Weights greater than kg

(c) Weights greater than 4 kg:

$$9 + 2 + 1 = 12$$

$$P(w > 4) = \frac{12}{40} = \frac{3}{10}$$

FINAL ANSWER

(a) $3 < w \leq 4$, (b) $3.6\ \text{kg}$, (c) $\frac{3}{10}$.

4MA1/1HR MAY/NOVEMBER 2020

Question 4

5 marks | Averages from Frequency Tables

4 The table gives information about the length of time, in minutes, that each of 60 students took to travel to school on Monday.

Length of time (t minutes)	Frequency
$0 < t \leqslant 10$	4
$10 < t \leqslant 20$	10
$20 < t \leqslant 30$	15
$30 < t \leqslant 40$	25
$40 < t \leqslant 50$	6

(a) Write down the modal class interval.

.....

(1)

(b) Work out an estimate for the mean length of time taken by these 60 students to travel to school on Monday.
Give your answer correct to one decimal place.

..... minutes

(4)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May/November 2020

1 The modal class is the

The modal class is the class with the greatest frequency:

$$30 < t \leq 40$$

2 **Use midpoints**

Use midpoints $5, 15, 25, 35, 45$.

$$5(4) + 15(10) + 25(15) + 35(25) + 45(6) = 1690$$

$$\text{estimated mean} = \frac{1690}{60} = 28.166\ldots$$

✓ **FINAL ANSWER**

modal class $30 < t \leq 40$, estimated mean 28.2 minutes.

4MA1/1H JANUARY 2021

Question 2

5 marks | Averages from Frequency Tables

2 The table gives information about the speeds, in kilometres per hour, of 80 motorbikes as each pass under a bridge.

Speed (s kilometres per hour)	Frequency
$40 < s \leqslant 50$	10
$50 < s \leqslant 60$	16
$60 < s \leqslant 70$	19
$70 < s \leqslant 80$	23
$80 < s \leqslant 90$	12

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean speed of the motorbikes as they pass under the bridge. Give your answer correct to 3 significant figures.

..... kilometres per hour

(4)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2021

1 **(a) The greatest frequency is**

(a) The greatest frequency is $\mathbf{23}$, so the modal class is

$$70 < s \leq 80$$

2 **(b) Use midpoints**

(b) Use midpoints $45, 55, 65, 75, 85$:

$$\frac{45(10) + 55(16) + 65(19) + 75(23) + 85(12)}{80} = 66.375$$

3 **To 3 significant figures**

To 3 significant figures:

$$66.4$$

✓ **FINAL ANSWER**

(a) $70 < s \leq 80$, (b) $66.4\ \text{km/h}$.

4MA1/2HR JANUARY 2021

Question 10

4 marks | Averages from Frequency Tables

10 A mathematics teacher at a school asked a group of students how far, in kilometres, each student had travelled to get to school that day.

The table gives information about their answers.

Distance travelled (d km)	Number of students
$0 < d \leqslant 2$	x
$2 < d \leqslant 4$	11
$4 < d \leqslant 6$	8
$6 < d \leqslant 8$	6
$8 < d \leqslant 10$	5

The teacher calculated that an estimate for the mean distance travelled by the whole group of students was 4.25 km.

Work out the value of x.
Show your working clearly.

$x =$.....

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2021

1 Use class midpoints

Use class midpoints $1, 3, 5, 7, 9$.

2 The total frequency is

The total frequency is

$$x + 11 + 8 + 6 + 5 = x + 30$$

3 The total estimated distance is

The total estimated distance is

$$1x + 3(11) + 5(8) + 7(6) + 9(5) = x + 160$$

4 **The estimated mean is**

The estimated mean is 4.25:

$$\frac{x+160}{x+30}=4.25$$

$$x+160=4.25x+127.5$$

$$32.5=3.25x$$

$$x=10$$

✓ **FINAL ANSWER**

$x=10$.

4MA1/2H NOVEMBER 2021

Question 3

5 marks | Averages from Frequency Tables

3 The table gives information about the amounts of money, in euros, that 70 of Anjali's friends spent last Saturday.

Money spent (S euros)	Frequency
$0 < S \leqslant 8$	6
$8 < S \leqslant 16$	14
$16 < S \leqslant 24$	19
$24 < S \leqslant 32$	25
$32 < S \leqslant 40$	6

One of Anjali's 70 friends is going to be chosen at random.

(a) Find the probability that this friend spent more than 24 euros last Saturday.

.....

(1)

(b) Work out an estimate for the mean amount of money spent by Anjali's friends last Saturday. Give your answer correct to 2 decimal places.

..... euros

(4)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2021

1 **More than euros**

More than 24 euros:

$$25 + 6 = 31$$

2 **Total frequency**

Total frequency:

$$6 + 14 + 19 + 25 + 6 = 70$$

$$P(S > 24) = \frac{31}{70}$$

3 **Use midpoints**

Use midpoints $4, 12, 20, 28, 36$.

$$4(6) + 12(14) + 20(19) + 28(25) + 36(6) = 1488$$

$$\text{estimated mean} = \frac{1488}{70} = 21.257\ldots$$

✓ **FINAL ANSWER**

$\frac{31}{70}$, estimated mean 21.3 euros.

4MA1/2H JANUARY 2022

Question 9

4 marks | Averages from Frequency Tables

9 The frequency table gives information about the number of points scored by a player.

Number of points	Frequency
0	13
1	17
2	8
3	x
4	11

The mean number of points scored is 2

Work out the value of x

$x =$ .....

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2022

1 **The total number of points**

The total number of points is

$$0(13) + 1(17) + 2(8) + 3x + 4(11) = 77 + 3x$$

2 **The total frequency is**

The total frequency is

$$13 + 17 + 8 + x + 11 = 49 + x$$

3 **Calculate statistic**

The mean is 2, so

$$\frac{77+3x}{49+x}=2$$

$$77+3x=98+2x$$

$$x=21$$

✓ **FINAL ANSWER**

$x=21$.

4MA1/2H NOVEMBER 2023

Question 1

4 marks | Averages from Frequency Tables

1 The table shows information about the lengths, in minutes, of 50 telephone calls.

Length of telephone call (m minutes)	Frequency
$0 < m \leqslant 5$	8
$5 < m \leqslant 10$	2
$10 < m \leqslant 15$	6
$15 < m \leqslant 20$	4
$20 < m \leqslant 25$	12
$25 < m \leqslant 30$	18

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the total length, in minutes, of these telephone calls.

..... minutes

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2023

1 The modal class is the

The modal class is the class with the greatest frequency:

$$25 < m \leq 30$$

2 Use midpoints

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5, 27.5$.

$$2.5(8) + 7.5(2) + 12.5(6) + 17.5(4) + 22.5(12) + 27.5(18) = 945$$

FINAL ANSWER

modal class $25 < m \leq 30$, estimated total 945 minutes.

4MA1/1H NOVEMBER 2024

Question 1

5 marks | Averages from Frequency Tables

1 The table shows some information about the hourly rates of pay of 60 workers.

Hourly rate of pay (p dollars)	Frequency
$10 < p \leqslant 15$	18
$15 < p \leqslant 20$	16
$20 < p \leqslant 25$	14
$25 < p \leqslant 30$	8
$30 < p \leqslant 35$	4

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean hourly rate of pay of the 60 workers.

..... dollars

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2024

1 **The modal class is the**

The modal class is the class with the greatest frequency:

$$10 < p \leq 15$$

2 **Use midpoints**

Use midpoints $12.5, 17.5, 22.5, 27.5, 32.5$.

$$12.5(18) + 17.5(16) + 22.5(14) + 27.5(8) + 32.5(4) = 1170$$

$$\text{estimated mean} = \frac{1170}{60} = 19.5$$

FINAL ANSWER

modal class $10 < p \leq 15$, estimated mean 19.5 dollars.

4MA1/1H JUNE 2025

Question 1

5 marks | Averages from Frequency Tables

1 The table gives information about the distances 100 adults travel to work.

Distance (d km)	Frequency
$0 < d \leqslant 5$	26
$5 < d \leqslant 10$	40
$10 < d \leqslant 15$	16
$15 < d \leqslant 20$	10
$20 < d \leqslant 25$	8

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean distance.

..... km

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2025

1 The modal class is the

The modal class is the class with the greatest frequency.

2 Modal class

Modal class: $5 < d \leq 10$.

3 Use midpoints

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5$.

$$2.5(26) + 7.5(40) + 12.5(16) + 17.5(10) + 22.5(8) = 920$$

$$\text{estimated mean} = \frac{920}{100} = 9.2$$

FINAL ANSWER

modal class $5 < d \leq 10$, estimated mean $9.2\ \text{km}$.

4WM2/01R MAY 2025

Question 1

5 marks | Averages from Frequency Tables

1 The table shows information about the weights, in kilograms, of 50 dogs.

Weight (w kg)	Frequency
$4 < w \leqslant 8$	5
$8 < w \leqslant 12$	17
$12 < w \leqslant 16$	15
$16 < w \leqslant 20$	9
$20 < w \leqslant 24$	4

(a) Write down the modal class.

..... kg

(1)

(b) Work out an estimate for the mean weight of the 50 dogs.

..... kg

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4WM2/01R May 2025

1 Required

a Write down the modal class. (b) Estimate the mean weight of the 50 dogs.

2 Part (a)

The **modal class** is the interval with the highest frequency. From the table, the highest frequency is **17**, which belongs to:

3 Answer

Modal class: 8 \< *w* ≤ 12 kg

4 **Find the midpoint of each**

Find the midpoint of each class interval:

5 **Apply the estimated mean formula**

Apply the estimated mean formula:

$$barx = fracsumf!xsumf = frac66050 = 13.2textkg$$

✓ **FINAL ANSWER**

Estimated mean weight = 13.2 kg

Question 8

4 marks | Averages from Frequency Tables

8 The table gives information about the lengths of time, in minutes, that some people were in a coffee shop.

Length of time (m minutes)	Frequency
$0 < m \leqslant 10$	x
$10 < m \leqslant 20$	3
$20 < m \leqslant 30$	2
$30 < m \leqslant 40$	4
$40 < m \leqslant 50$	2

The information in the table is used to calculate an estimate for the mean length of time these people were in the coffee shop.

The estimate for the mean length of time is 20

Work out the value of x

x = ...

(Total for Question 8 is 4 marks)

1 Use class midpoints

Use class midpoints $5, 15, 25, 35, 45$.

$$\frac{5x + 15(3) + 25(2) + 35(4) + 45(2)}{x + 3 + 2 + 4 + 2} = 20$$

$$\frac{5x + 325}{x + 11} = 20$$

$$5x + 325 = 20x + 220$$

$$105 = 15x$$

$$x = 7$$

FINAL ANSWER

$x = 7$.

4MA1/1H NOVEMBER 2025

Question 1

4 marks | Averages from Frequency Tables

1 The table gives information about the number of kilometres that Ted cycled on each of the 30 days in April.

Number of kilometres (K)	Frequency
$0 \leqslant K < 5$	8
$5 \leqslant K < 10$	7
$10 \leqslant K < 15$	3
$15 \leqslant K < 20$	10
$20 \leqslant K < 25$	2

Calculate an estimate for the mean number of kilometres that Ted cycled on each day in April.

..... kilometres

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2025

1 **Use midpoints**

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5$.

$$2.5(8) + 7.5(7) + 12.5(3) + 17.5(10) + 22.5(2) = 330$$

$$\text{estimated mean} = \frac{330}{30} = 11$$

FINAL ANSWER

11 km.

4WM2/01 NOVEMBER 2025

Question 2

4 marks | Averages from Frequency Tables

2 The table gives information about the heights, in cm, of 60 flowers in Abby's garden.

Height (h cm)	Frequency
$0 < h \leqslant 10$	17
$10 < h \leqslant 20$	9
$20 < h \leqslant 30$	16
$30 < h \leqslant 40$	14
$40 < h \leqslant 50$	4

Work out an estimate for the mean height of these flowers.

..... cm

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4WM2/01 November 2025

1 Find class midpoints

Step 1: Find the midpoints (*x*) for each class interval.

2 0 h 10 x 5

0 \< *h* ≤ 10 ⟹ *x* = 5

3 10 h 20 x 15

10 \< *h* ≤ 20 ⟹ *x* = 15

4 20 h 30 x 25

20 \< *h* ≤ 30 ⟹ *x* = 25

5 30 h 40 x 35

30 \< *h* ≤ 40 ⟹ *x* = 35

6 40 h 50 x 45

40 \< *h* ≤ 50 ⟹ *x* = 45

7 **Find the midpoint**

Step 2: Multiply frequency (*f*) by midpoint (*x*) to find *f*x*.

8 **Calculate statistic**

$$textMean = fracsumfxsumf = frac129060 = 21.5$$

✓ **FINAL ANSWER**

21.5 cm

4MA1/2H JUNE 2026

Question 3

5 marks | Averages from Frequency Tables

3 The table shows information about the number of hours each of 40 candles burned.

Number of hours (h)	Frequency
$15 < h \leqslant 19$	2
$19 < h \leqslant 23$	5
$23 < h \leqslant 27$	12
$27 < h \leqslant 31$	15
$31 < h \leqslant 35$	6

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean number of hours.

..... hours

(4)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

The greatest frequency is 15, in the interval $27 < h \leq 31$.

The modal class is the class with the highest frequency.

2 **Part (b) | Step 1**

The class midpoints are $17, 21, 25, 29, 33$ hours.

Use the midpoint of each class as a representative value.

3 **Part (b) | Step 2**

$$\sum fm = 2(17) + 5(21) + 12(25) + 15(29) + 6(33)$$

$$= 34 + 105 + 300 + 435 + 198 = 1072.$$

Multiply each midpoint by its frequency and add.

4 **Part (b) | Step 3**

$$\text{Estimated mean} = \frac{1072}{2+5+12+15+6} = \frac{1072}{40} = 26.8 \text{ hours}.$$

Divide the estimated total by the number of candles.

FINAL ANSWER

(a) $27 < h \leq 31$ (b) 26.8 hours

TOPIC

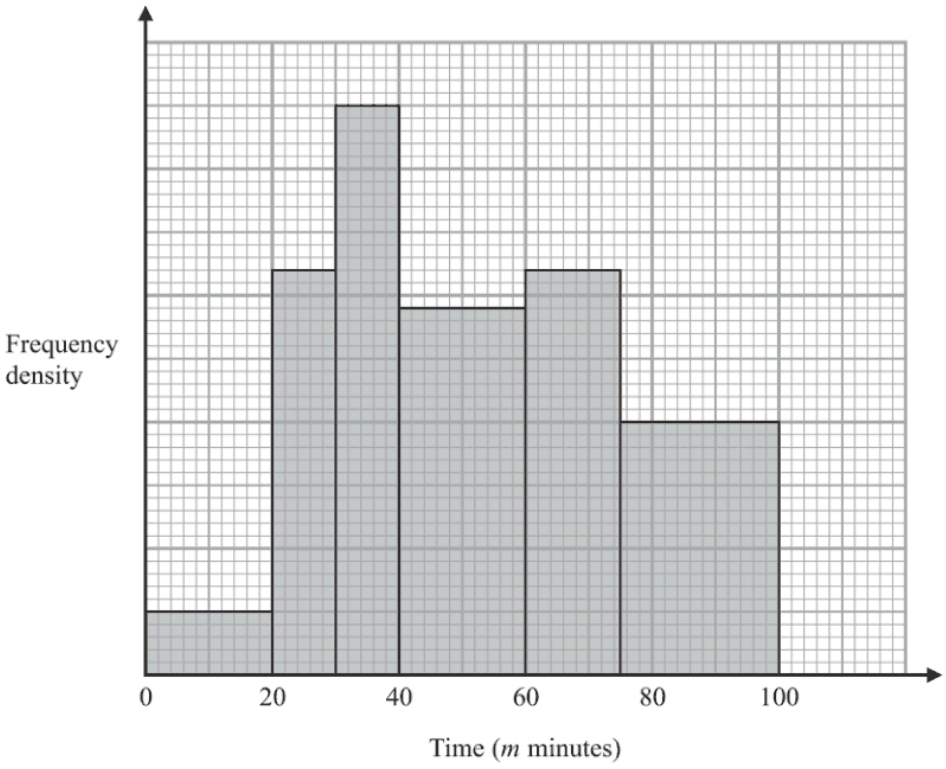
Histograms

Every past-paper question on this topic, 2020 to 2026.

Question 16

3 marks | Histograms

16 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$
Work out an estimate for the number of children for whom $50 < m \leqslant 80$

.....

(Total for Question 16 is 3 marks)

1 Set the frequency-density scale

For $0 < m \leq 20$, the frequency is 10, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

2 Read the needed bar heights

Using the same scale from the histogram:

$$50 < m \leq 60 : \ 10 \times 2.9 = 29$$

$$60 < m \leq 75 : \ 15 \times 3.2 = 48$$

$$75 < m \leq 80 : \ 5 \times 2 = 10$$

3 Add the estimates

$$19 + 36 + 32 = 87$$

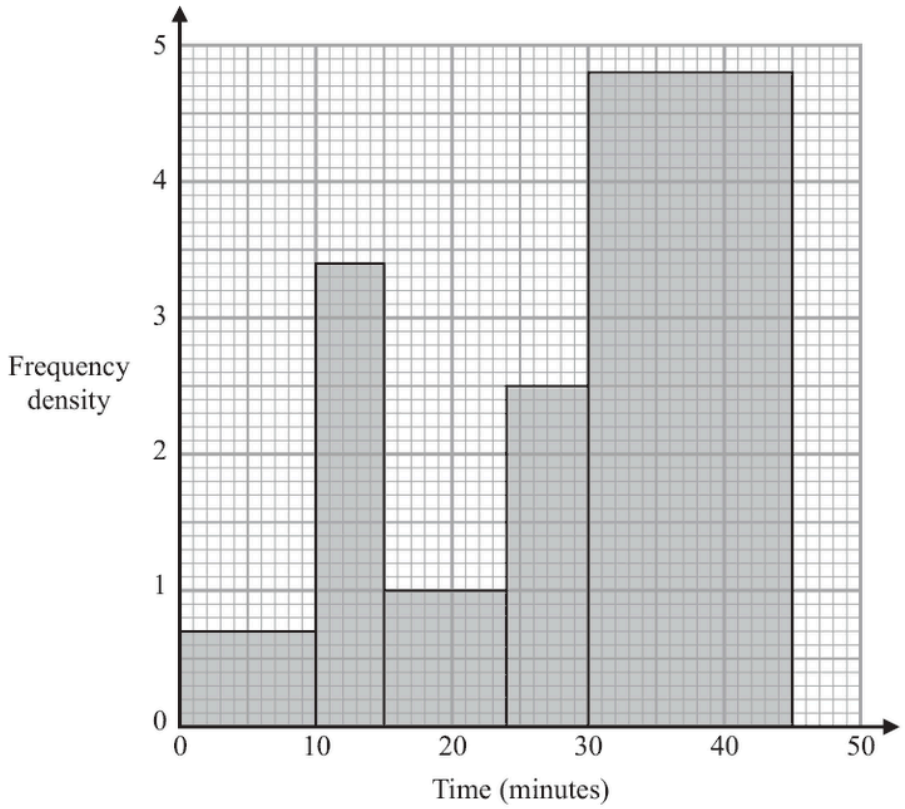
FINAL ANSWER

87.

Question 18

5 marks | Histograms

18 The histogram gives information about the times, in minutes, that some customers spent in a supermarket.



(a) Work out an estimate for the proportion of these customers who spent between 17 minutes and 35 minutes in the supermarket.

.....

(3)

One of the customers is selected at random.

Given that this customer had spent more than 30 minutes in the supermarket,

(b) find the probability that this customer spent more than 36 minutes in the supermarket.

.....

(2)

(Total for Question 18 is 5 marks)

1 Total frequency

Total frequency:

$$10(0.7)+5(3.4)+9(1)+6(2.5)+15(4.8)=120$$

2 Between 17 and 35 minutes

Between 17 and 35 minutes:

$$7(1)+6(2.5)+5(4.8)=46$$

$$\text{proportion}=\frac{46}{120}=\frac{23}{60}$$

3 Given the customer spent more

Given the customer spent more than 30 minutes, they are in the 30 to 45 minute class.

4 More than 36 minutes is

More than 36 minutes is 9 minutes out of the 15-minute class:

$$\frac{9}{15}=\frac{3}{5}$$

FINAL ANSWER

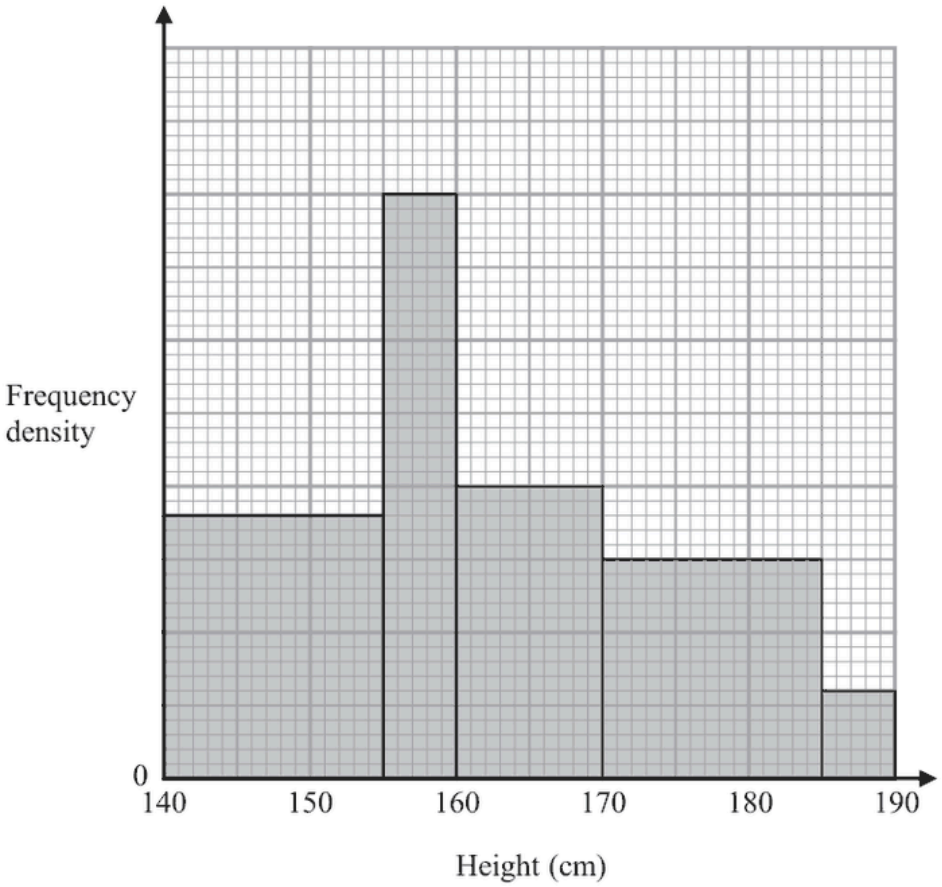
$\frac{23}{60}$, then $\frac{3}{5}$.

4MA1/1HR JANUARY 2020

Question 19

4 marks | Histograms

19



The histogram gives information about the heights of all the Year 11 students at a school.

There are 160 students in Year 11 with a height between 155 cm and 170 cm.

Work out the total number of students in Year 11 at the school.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2020

1 Use the relative histogram heights

Use the relative histogram heights.

2 Calculate area

For $155 < h \leq 170$, the relative area is

$$5(40) + 10(20) = 400$$

3 Calculate area

This represents 160 students, so each relative area unit represents

$$\frac{160}{400} = 0.4$$

4 Total relative area

Total relative area:

$$15(18) + 5(40) + 10(20) + 15(15) + 5(6) = 925$$

5 Estimated total

Estimated total:

$$925 \times 0.4 = 370$$

FINAL ANSWER

370 students.

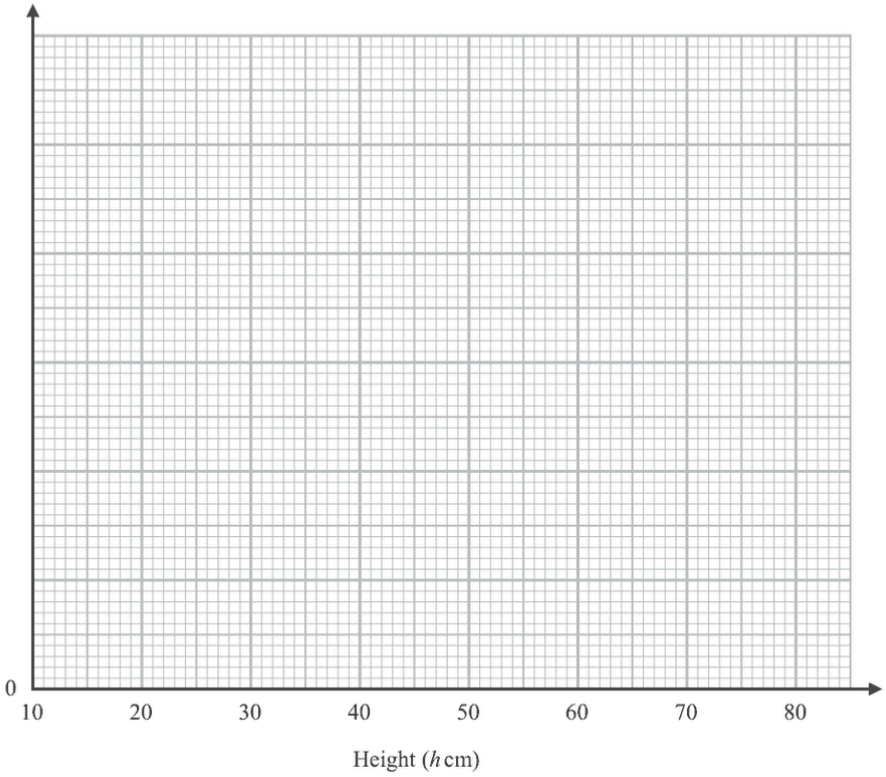
Question 18

5 marks | Histograms

18 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$10 < h \leqslant 20$	35
$20 < h \leqslant 35$	45
$35 < h \leqslant 50$	75
$50 < h \leqslant 70$	40
$70 < h \leqslant 80$	8

(a) On the grid, draw a histogram for this information.



(3)

(b) Work out an estimate for the number of these plants with a height greater than 40 cm.

.....

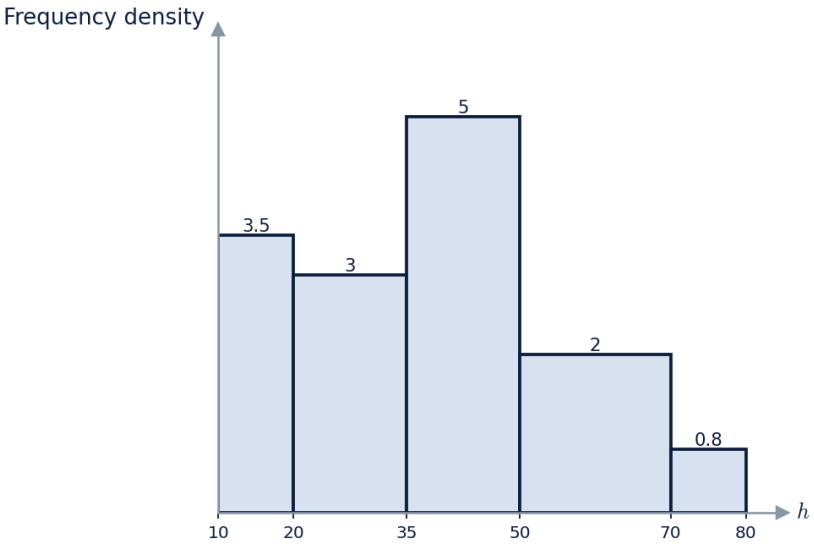
(2)

(Total for Question 18 is 5 marks)

1 **The histogram densities are**

The histogram densities are

$$\frac{35}{10} = 3.5, \quad \frac{45}{15} = 3, \quad \frac{75}{15} = 5, \quad \frac{40}{20} = 2, \quad \frac{8}{10} = 0.8$$



2 **Use histogram**

For plants taller than 40 cm:

3 **Use histogram**

From $40 < h \leq 50$:

$$10(5) = 50$$

4 **Use histogram**

From $50 < h \leq 70$:

$$40$$

5 **Use histogram**

From $70 < h \leq 80$:

$$8$$

$$50 + 40 + 8 = 98$$

FINAL ANSWER

draw bars with densities $3.5, 3, 5, 2, 0.8$, and 98 plants.

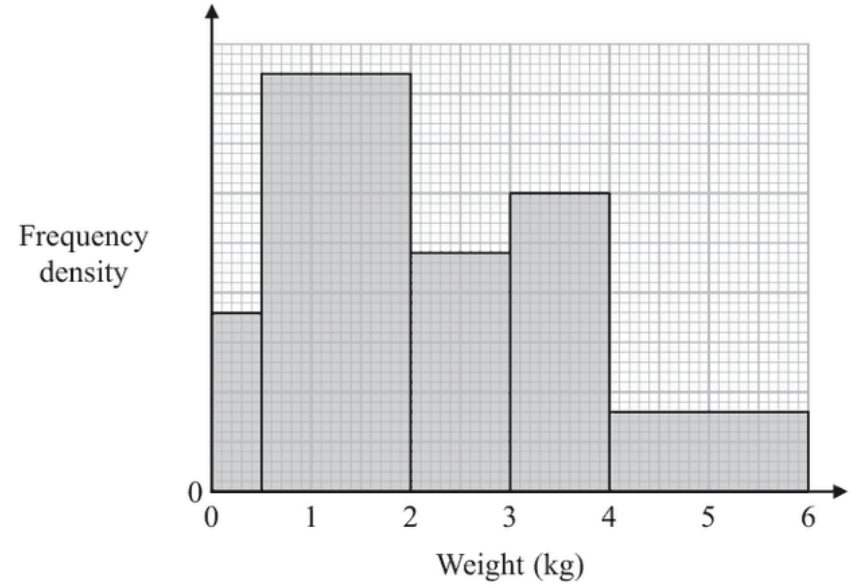
4MA1/2H JANUARY 2021

Question 15

6 marks | Histograms

15 A postman records the weight of each parcel that he delivers.

The histogram shows information about the weights of all the parcels that the postman delivered last Monday. No parcels weighed more than 6 kg.



63 of the parcels that the postman delivered last Monday each had a weight between 0.5 kg and 2 kg.

(a) Work out the total number of parcels the postman delivered last Monday.

.....

(3)

The postman picks at random two of the records of the parcels he delivered last Monday.

(b) Work out an estimate for the probability that each parcel weighed more than 2.25 kg.

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H January 2021

1 **Use histogram**

For $0.5 < w \leq 2$, the frequency is 63, so the density is

$$\frac{63}{1.5} = 42$$

2 **Use histogram**

Using the histogram heights, the class frequencies are

$$9,\ 63,\ 24,\ 30,\ 16$$

---

3 **Use histogram**

So the total number of parcels is

$$9 + 63 + 24 + 30 + 16 = 142$$

---

4 **Use histogram**

For weight greater than $2.25\ \text{kg}$:

$$0.75(24) + 30 + 16 = 64$$

---

5 **The probability that both randomly**

The probability that both randomly chosen records are greater than $2.25\ \text{kg}$ is

$$\frac{64}{142} \times \frac{63}{141} = 0.201\ldots$$

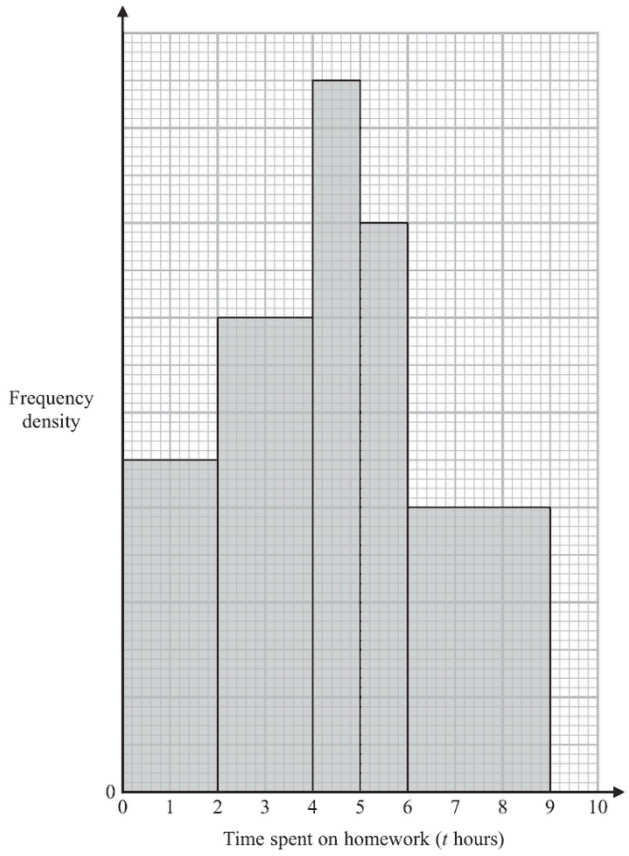
✓ **FINAL ANSWER**

142, then 0.201 approximately.

Question 18

5 marks | Histograms

18 The histogram and the table give some information about the amounts of time, in hours, that Year 11 students at Bergdesh Academy spent, in total, on their homework last week.
No student in Year 11 spent longer than 9 hours on their homework.



Time spent on homework (t hours)	Frequency
$0 < t \leqslant 2$	28
$2 < t \leqslant 4$	
$4 < t \leqslant 5$	
$5 < t \leqslant 6$	
$6 < t \leqslant 9$	

Using the information in the histogram and in the table, work out an estimate for the mean amount of time the Year 11 students spent on their homework last week.
Give your answer in hours correct to 3 significant figures.

..... hours

(Total for Question 18 is 5 marks)

1 **Use histogram**

The $0 < t \leq 2$ class has frequency 28. From the histogram, the relative heights are

$$1,\ 4,\ 9,\ 6,\ 1$$

2 **Use histogram**

So the frequencies are

$$28,\ 112,\ 126,\ 84,\ 42$$

3 **Use midpoints**

Use midpoints $1, 3, 4.5, 5.5, 7.5$.

$$\text{total frequency} = 28 + 112 + 126 + 84 + 42 = 392$$

$$\text{estimated total} = 1(28) + 3(112) + 4.5(126) + 5.5(84) + 7.5(42) = 1708$$

$$\text{mean} = \frac{1708}{392} = 4.357\ldots$$

FINAL ANSWER

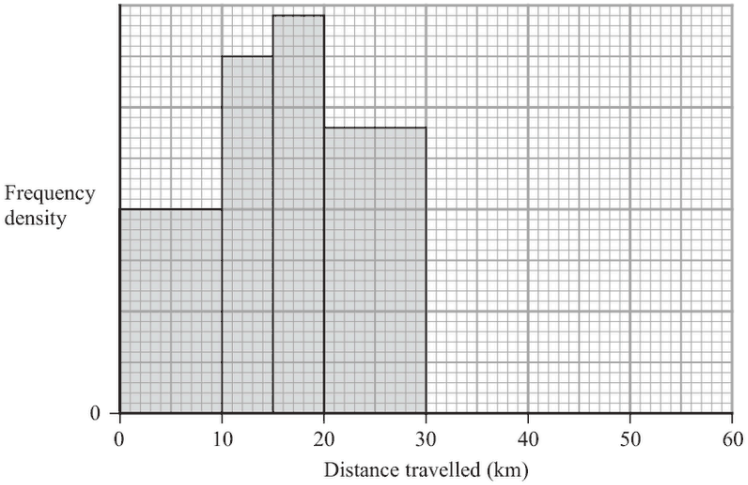
4.36 hours.

Question 18

5 marks | Histograms

18 The table and histogram give information about the distance travelled, in order to get to work, by each person working in a large store.

Distance (d km)	Frequency
$0 \leqslant d < 10$	40
$10 \leqslant d < 15$	
$15 \leqslant d < 20$	
$20 \leqslant d < 30$	
$30 \leqslant d < 60$	30



Using the information in the table and in the histogram,

(a) complete the table,

(2)

(b) complete the histogram.

(1)

One of the people working in the store is chosen at random.

(c) Work out an estimate for the probability that the distance travelled by this person, in order to get to work, was greater than 25 km.

.....

(2)

(Total for Question 18 is 5 marks)

1 **Use histogram**

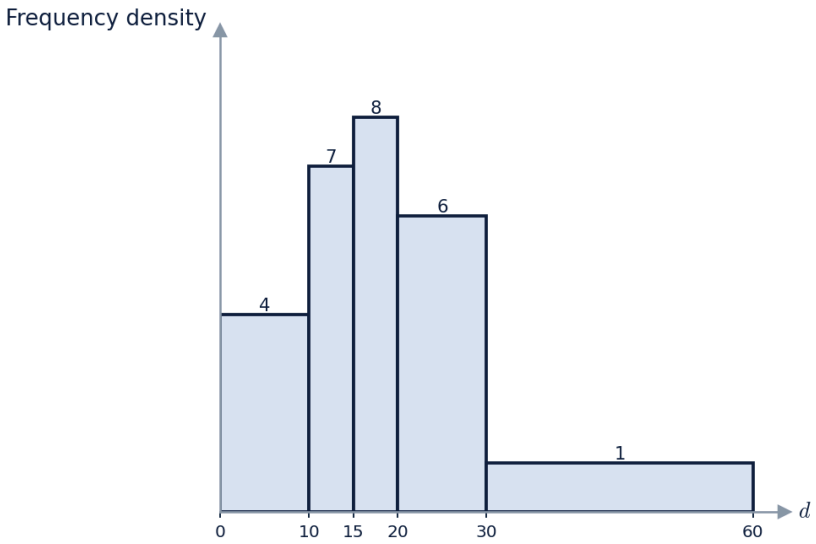
The $0 \leq d < 10$ class has frequency 40, so its density is

$$\frac{40}{10} = 4$$

2 **Use histogram**

From the histogram, the missing frequencies are

$$10 \leq d < 15 : \ 35, \qquad 15 \leq d < 20 : \ 40, \qquad 20 \leq d < 30 : \ 60$$



3 **Together with the table**

Together with the table:

$$40 + 35 + 40 + 60 + 30 = 205$$

4 **Use histogram**

For $d > 25$:

$$5(6) + 30 = 60$$

$$P(d > 25) = \frac{60}{205} = \frac{12}{41}$$

FINAL ANSWER

completed frequencies $35, 40, 60$, missing histogram density 1, and probability $\frac{12}{41}$.

Question 14

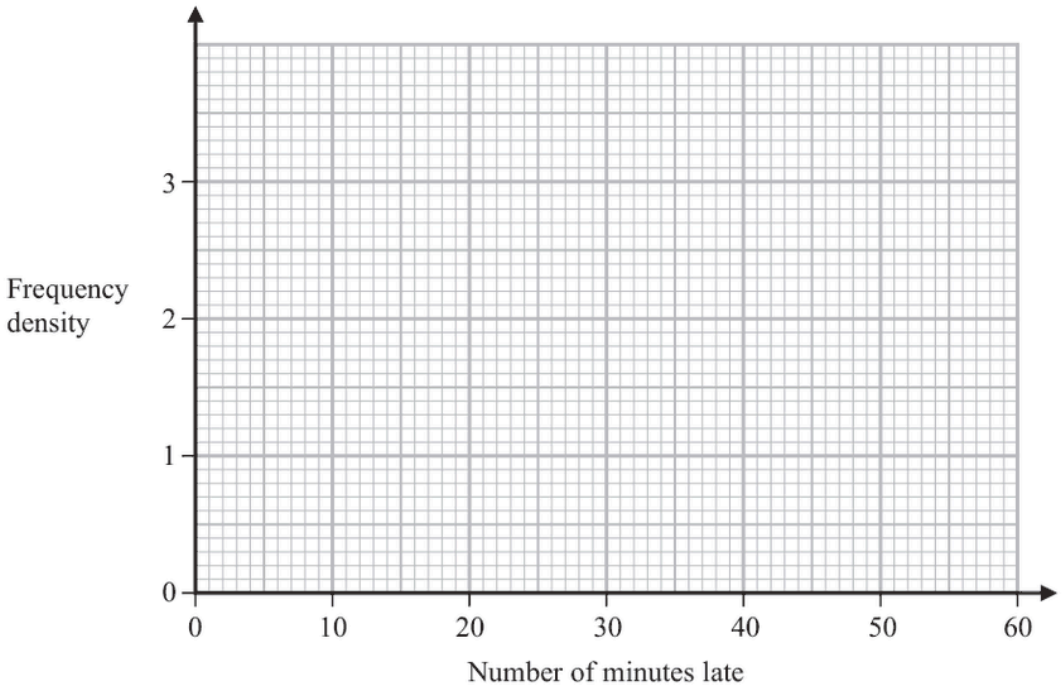
5 marks | Histograms

14 Manuel collected information about the flights that arrived late at an airport last month.

The table gives information about the number of minutes that these flights were late.

Minutes late (L minutes)	Frequency
$0 < L \leqslant 10$	8
$10 < L \leqslant 15$	13
$15 < L \leqslant 25$	19
$25 < L \leqslant 40$	24
$40 < L \leqslant 60$	6

(a) On the grid, draw a histogram for this information.



(3)

Manuel selected at random a flight that was late by 25 minutes or less from his results.

(b) Work out an estimate for the probability that this flight was late by 5 minutes or less.

.....

(2)

(Total for Question 14 is 5 marks)

1 **The histogram densities are**

The histogram densities are

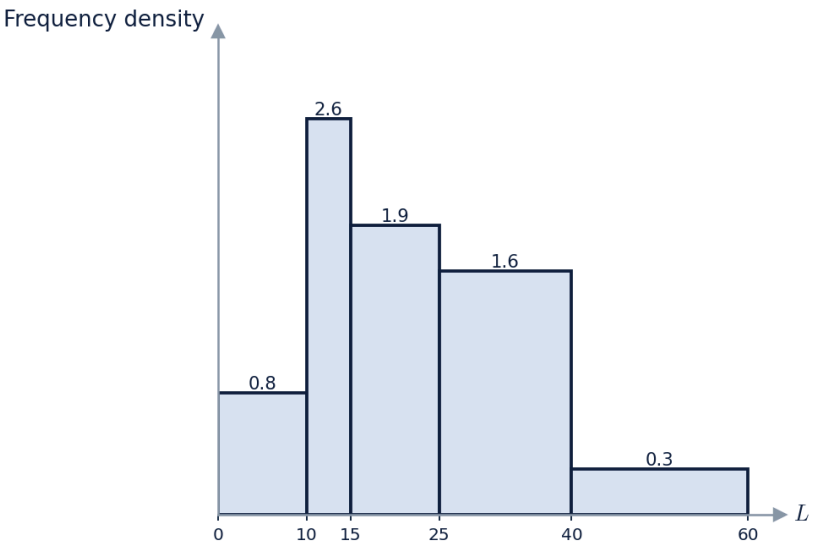
$$0 < L \leq 10 : \frac{8}{10} = 0.8$$

$$10 < L \leq 15 : \frac{13}{5} = 2.6$$

$$15 < L \leq 25 : \frac{19}{10} = 1.9$$

$$25 < L \leq 40 : \frac{24}{15} = 1.6$$

$$40 < L \leq 60 : \frac{6}{20} = 0.3$$



2 **Use histogram**

Given $L \leq 25$, the total frequency is

$$8 + 13 + 19 = 40$$

3 **Use histogram**

For $L \leq 5$, estimate half of the first class:

$$5(0.8) = 4$$

$$P(L \leq 5 \mid L \leq 25) = \frac{4}{40} = \frac{1}{10}$$

FINAL ANSWER

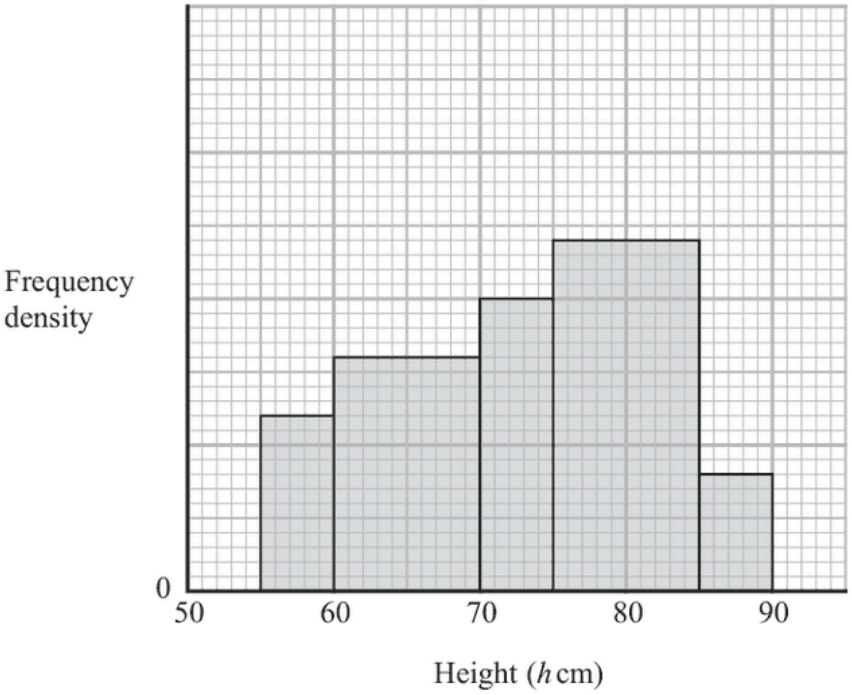
histogram densities $0.8, 2.6, 1.9, 1.6, 0.3$, and probability $\frac{1}{10}$.

4MA1/2H JANUARY 2022

Question 20

4 marks | Histograms

20 The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $75 < h \leq 85$, the frequency is 12 and the class width is 10, so

$$\text{frequency density} = \frac{12}{10} = 1.2$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$0.6,\ 0.8,\ 1.0,\ 1.2,\ 0.4$$

4 The total number of tomato

The total number of tomato plants is

$$(5)(0.6) + (10)(0.8) + (5)(1.0) + (10)(1.2) + (5)(0.4)$$

$$= 3 + 8 + 5 + 12 + 2 = 30$$

5 Solve inequality

For $h > 82.5$:

$$(2.5)(1.2) + (5)(0.4) = 3 + 2 = 5$$

6 Calculate probability

So the probability is

$$\frac{5}{30} = \frac{1}{6}$$

FINAL ANSWER

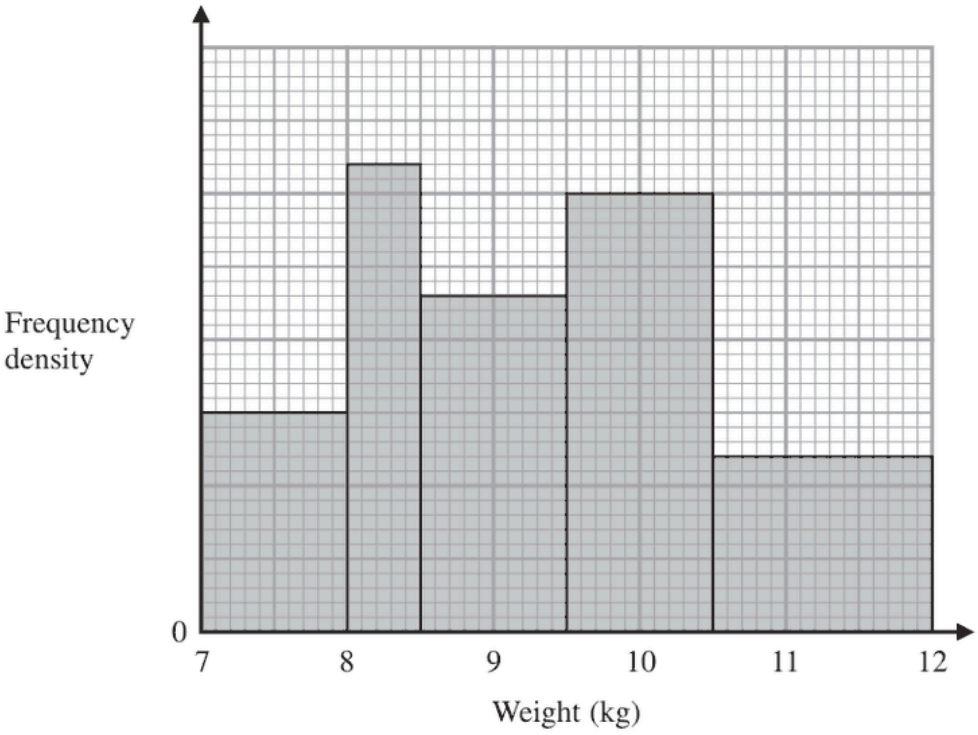
$\frac{1}{6}$

4MA1/1HR JANUARY 2022

Question 21

3 marks | Histograms

21



The histogram gives information about the weights, in kg, of all the watermelons in a field.

There are 16 watermelons with a weight between 8 kg and 8.5 kg

Work out the total number of watermelons in the field.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2022

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $8 \leq w < 8.5$, the frequency is 16 and the class width is 0.5, so

$$\text{frequency density} = \frac{16}{0.5} = 32$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$15,\ 32,\ 23,\ 30,\ 12$$

4 The total frequency is

The total frequency is

$$(1)(15) + (0.5)(32) + (1)(23) + (1)(30) + (1.5)(12)$$

$$= 15 + 16 + 23 + 30 + 18$$

$$= 102$$

FINAL ANSWER

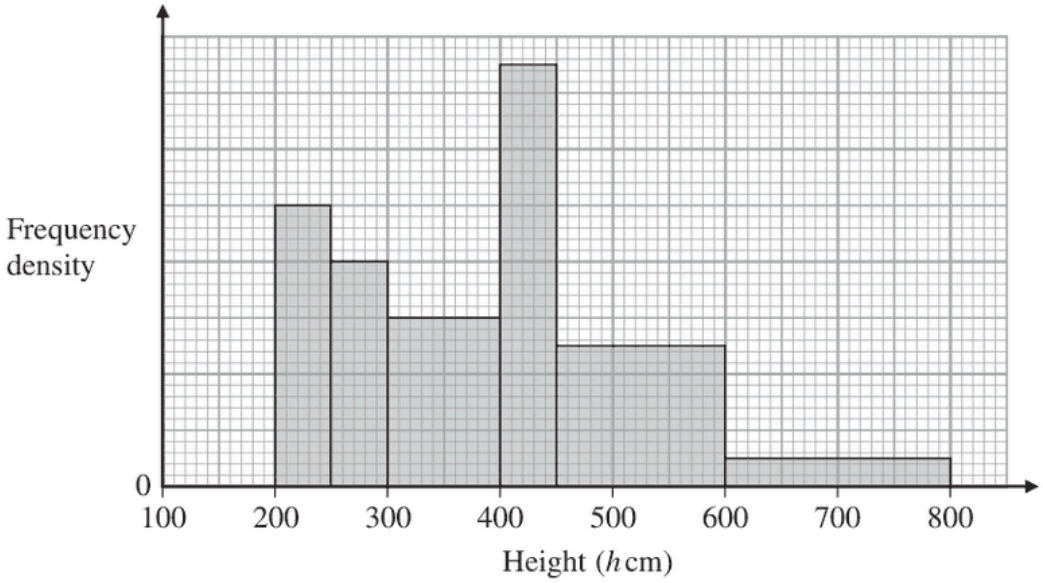
102 watermelons

4MA1/1HR JUNE 2022

Question 19

3 marks | Histograms

19 The histogram gives information about the height, h cm, of each tree in part of a forest.



There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2022

1 **Use relative densities from the**

Use relative densities from the histogram:

$$10,\ 8,\ 6,\ 15,\ 5,\ 1$$

2 **Use histogram**

The number of trees with $300 < h \leq 400$ is

$$100(6s) = 600s$$

3 **Use histogram**

The number of trees with $400 < h \leq 500$ is

$$50(15s) + 50(5s) = 1000s$$

4 **The difference is**

The difference is 8:

$$1000s - 600s = 8$$

$$400s = 8$$

$$s = 0.02$$

5 **Use histogram**

For height greater than 500:

$$100(5s) + 200(1s) = 700s$$

$$700(0.02) = 14$$

✓ **FINAL ANSWER**

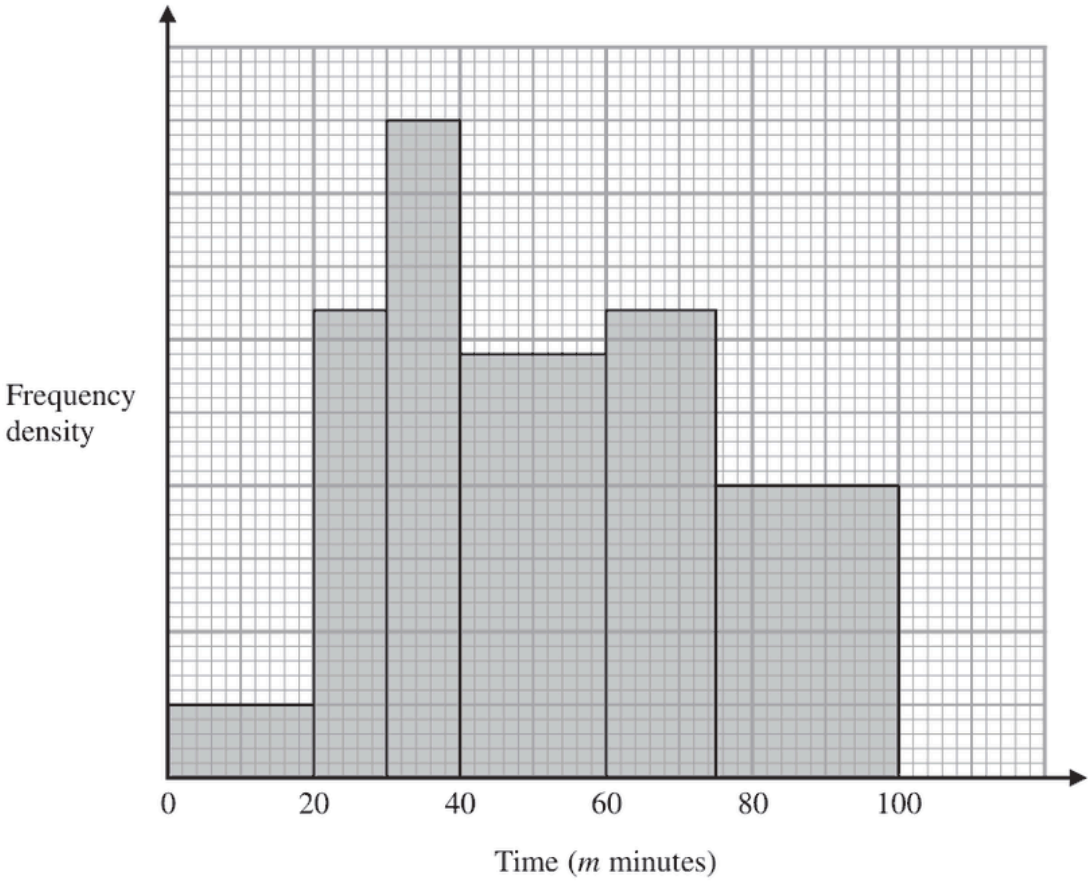
14 trees.

4MA1/1H JUNE 2022

Question 21

3 marks | Histograms

21 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$

Work out an estimate for the number of children for whom $50 < m \leqslant 80$

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2022

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $m \leq 20$, the frequency is 10 and the class width is 20, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

---

3 **Use histogram**

Using this scale from the histogram:

$$40 < m \leq 60 \quad \text{has frequency density } 2.9$$

$$60 < m \leq 75 \quad \text{has frequency density } 3.2$$

$$75 < m \leq 100 \quad \text{has frequency density } 2.0$$

---

4 **Use part of each relevant**

For $50 < m \leq 80$, use part of each relevant bar:

$$(10)(2.9) + (15)(3.2) + (5)(2.0)$$

$$= 29 + 48 + 10$$

$$= 87$$

✓ **FINAL ANSWER**

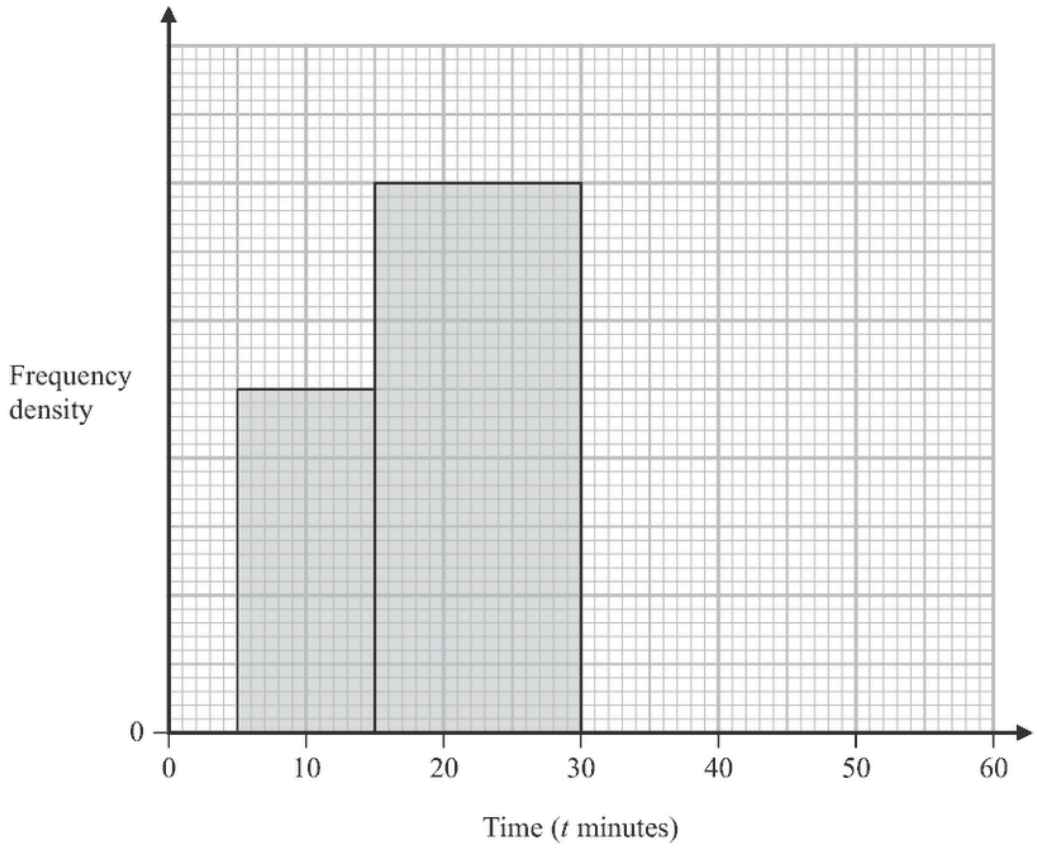
87 children

Question 18

4 marks | Histograms

18 The incomplete table and incomplete histogram give information about the times, in minutes, that 140 people waited at a station for a train.

Time (t minutes)	Frequency
$0 < t \leqslant 5$	23
$5 < t \leqslant 15$	
$15 < t \leqslant 30$	
$30 < t \leqslant 40$	18
$40 < t \leqslant 60$	14



Complete the table and the histogram.

(Total for Question 18 is 4 marks)

1 **The total frequency is**

The total frequency is 140.

2 **The known frequencies are**

The known frequencies are

$$23 + 18 + 14 = 55$$

3 **Use histogram**

So the two missing frequencies have total

$$140 - 55 = 85$$

4 **Use histogram**

From the histogram, the missing frequencies are

$$5 < t \leq 15 : \ 25, \qquad 15 < t \leq 30 : \ 60$$

5 **The missing histogram densities are**

The missing histogram densities are

$$0 < t \leq 5 : \frac{23}{5} = 4.6$$

$$30 < t \leq 40 : \frac{18}{10} = 1.8$$

$$40 < t \leq 60 : \frac{14}{20} = 0.7$$

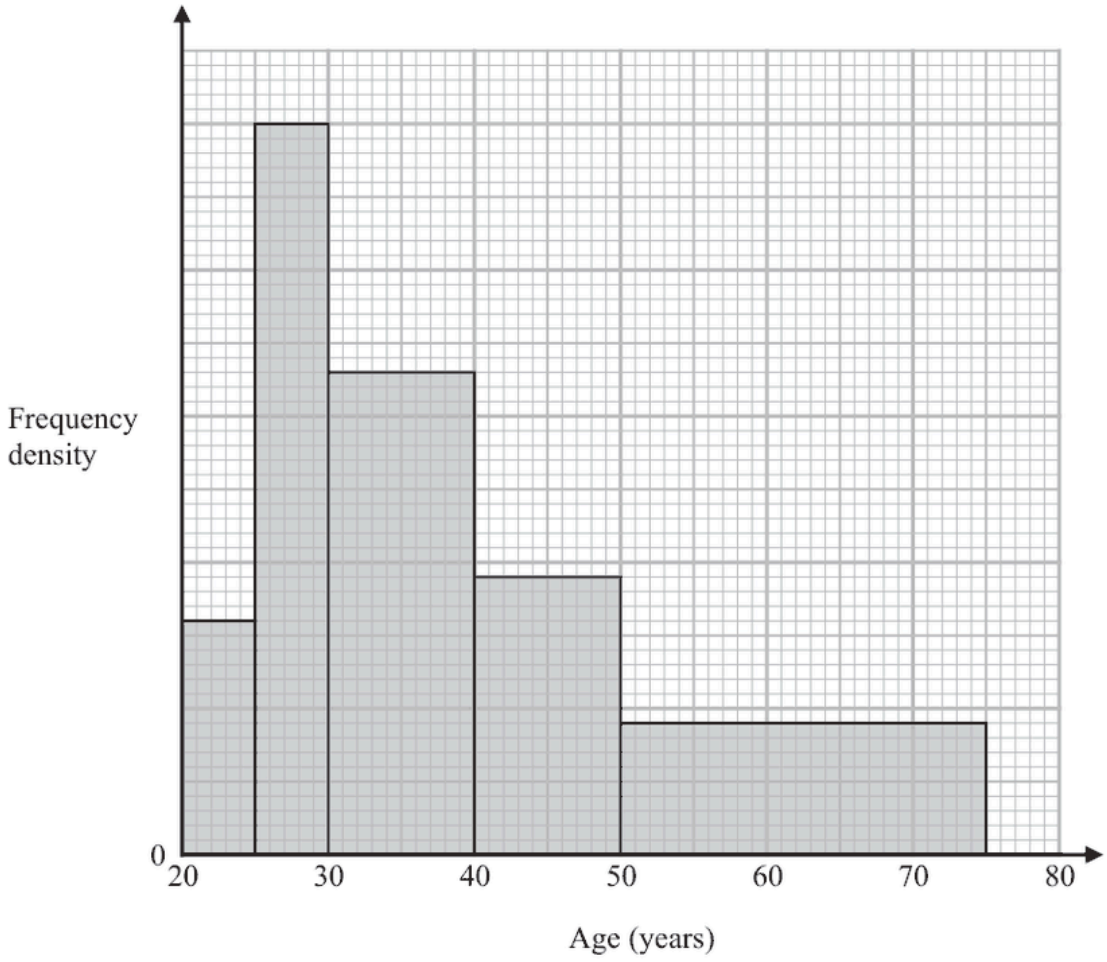
FINAL ANSWER

missing frequencies $25, 60$, and missing densities $4.6, 1.8, 0.7$.

Question 22

4 marks | Histograms

22 Some people attend a concert.
The histogram shows information about the ages of these people.



Work out an estimate for the percentage of these people who are aged more than 55 years.
Give your answer correct to one decimal place.

.....%

(Total for Question 22 is 4 marks)

1 The scale of the frequency

The scale of the frequency density axis is not needed because the percentage is found from areas.

2 Calculate area

Using the bar heights from the histogram, the relative areas are:

$$(5)(277) + (5)(866) + (10)(571) + (10)(329) + (25)(155)$$

$$= 1385 + 4330 + 5710 + 3290 + 3875$$

$$= 18590$$

3 Calculate value

For ages more than 55, use the part of the last bar from 55 to 75:

$$(20)(155) = 3100$$

4 Calculate percentage

So the estimated percentage is

$$\frac{3100}{18590} \times 100 = 16.675\ldots$$

FINAL ANSWER

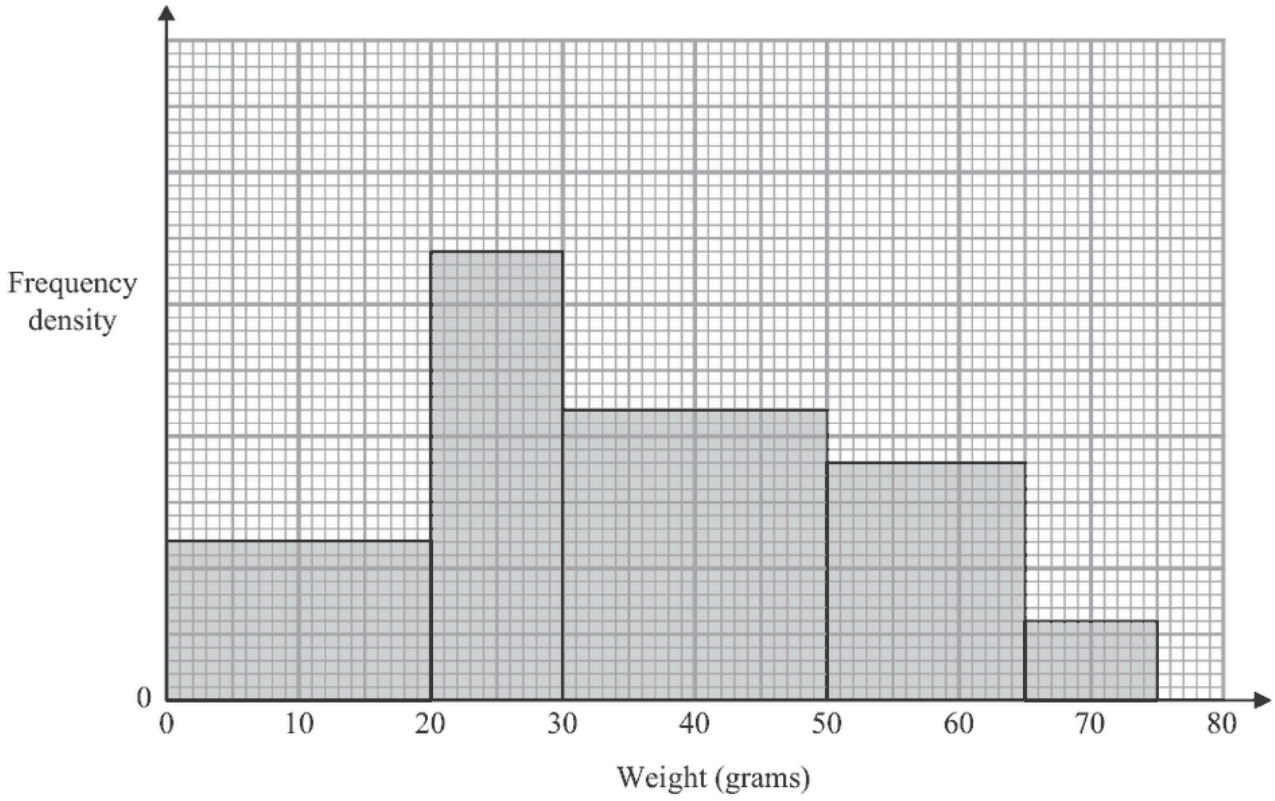
16.7%

4MA1/1H NOVEMBER 2023

Question 17

3 marks | Histograms

17 The histogram gives information about the weights, in grams, of some oranges in a box.



24 of these oranges weigh less than 20 grams.

Medium oranges weigh between 35 grams and 55 grams.

Work out an estimate for the number of medium oranges in the box.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2023

1 **The to gram class has**

The 0 to 20 gram class has frequency 24, so its density is

$$\frac{24}{20} = 1.2$$

2 **Use histogram**

From the histogram scale, the density for $30 < w \leq 50$ is about 2.2, and the density for $50 < w \leq 65$ is about 1.8.

3 **Medium oranges weigh between g**

Medium oranges weigh between 35 g and 55 g:

$$15(2.2) + 5(1.8) = 33 + 9 = 42$$

✓ **FINAL ANSWER**

about 42 medium oranges.

4MA1/1HR MAY 2024

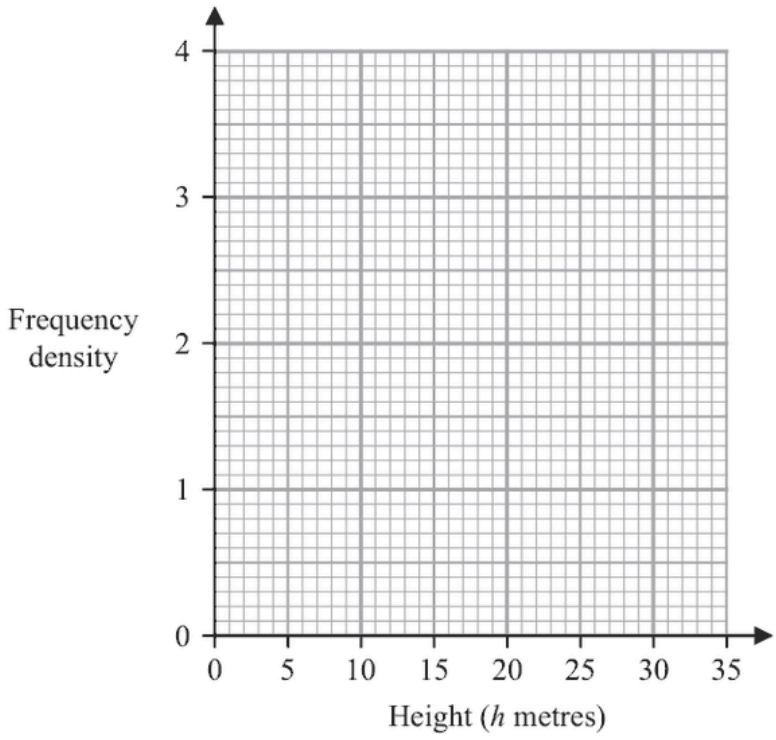
Question 16

3 marks | Histograms

16 The table gives information about the heights, in metres, of the trees in a park.

Height (h metres)	Frequency
$0 < h \leqslant 2$	5
$2 < h \leqslant 5$	12
$5 < h \leqslant 10$	18
$10 < h \leqslant 20$	14
$20 < h \leqslant 35$	9

On the grid, draw a histogram for this information.



(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May 2024

1 **Use histogram**

For a histogram,

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

2 **The bar heights are**

The bar heights are:

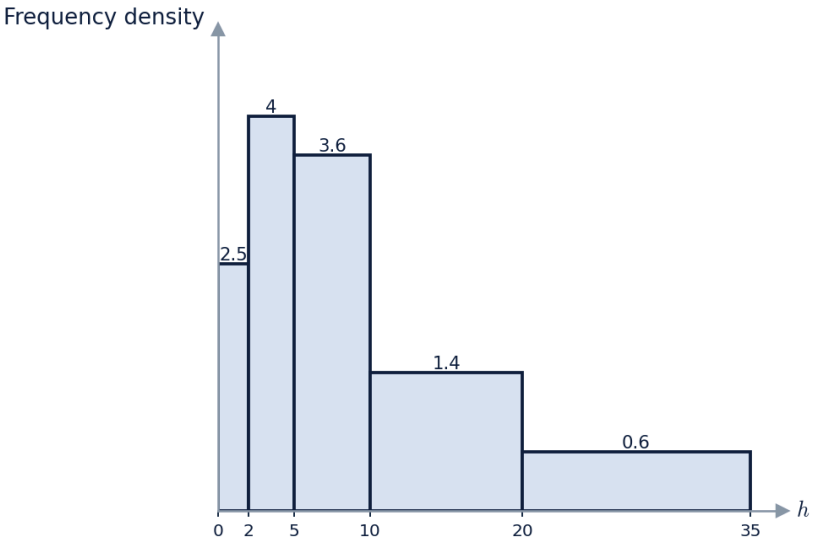
$$0 < h \leq 2 : \frac{5}{2} = 2.5$$

$$2 < h \leq 5 : \frac{12}{3} = 4$$

$$5 < h \leq 10 : \frac{18}{5} = 3.6$$

$$10 < h \leq 20 : \frac{14}{10} = 1.4$$

$$20 < h \leq 35 : \frac{9}{15} = 0.6$$



FINAL ANSWER

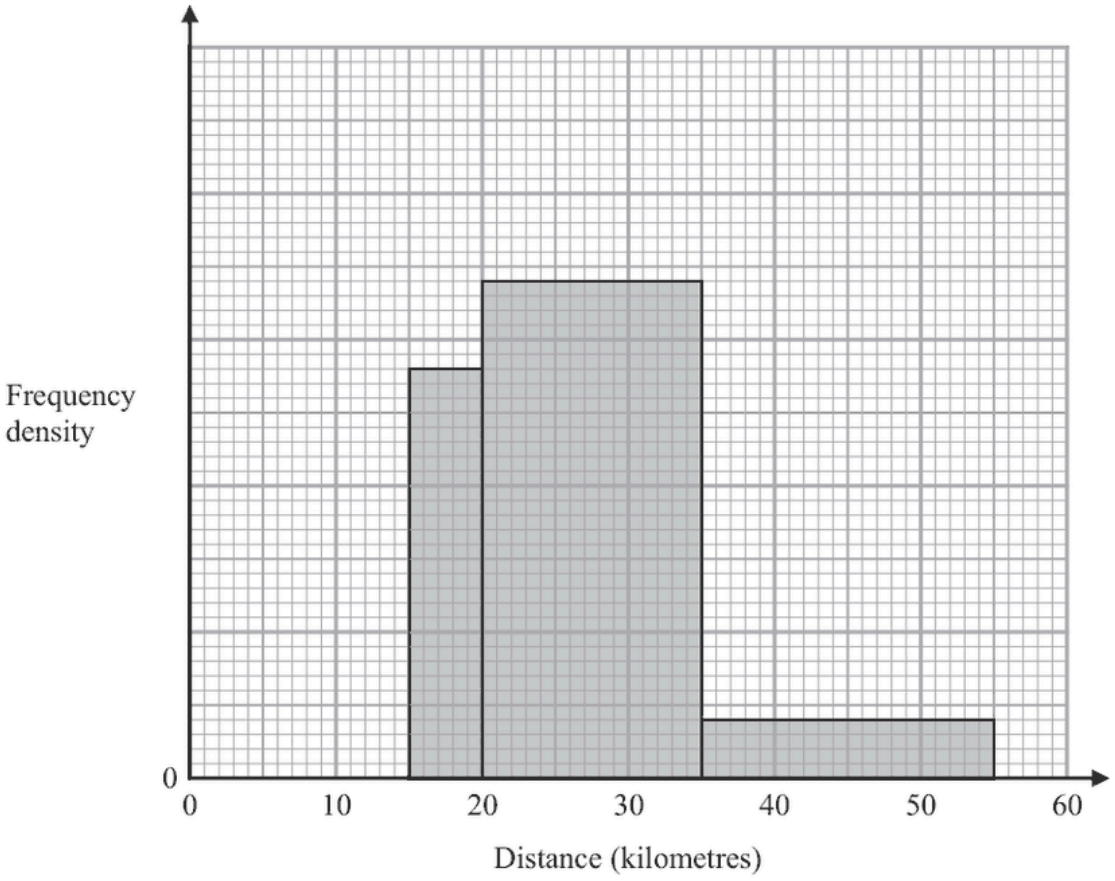
draw bars with heights $2.5, 4, 3.6, 1.4, 0.6$.

4MA1/1H MAY 2024

Question 22

3 marks | Histograms

22 The incomplete histogram shows some information about the distances, in kilometres, that 100 adults ran last week.



All of the adults ran at least 5 kilometres.
None of the adults ran more than 55 kilometres.
14 adults ran between 15 kilometres and 20 kilometres.

Complete the histogram.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2024

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $15 \leq d < 20$, the frequency is 14 and the class width is 5, so

$$\text{frequency density} = \frac{14}{5} = 2.8$$

3 **Use histogram**

Using this scale from the histogram:

$$20 \leq d < 35 \quad \text{has frequency density } 3.4$$

$$35 \leq d \leq 55 \quad \text{has frequency density } 0.4$$

4 **Calculate value**

So the known frequencies are

$$14 + (15)(3.4) + (20)(0.4)$$

$$= 14 + 51 + 8 = 73$$

5 **Use frequency table**

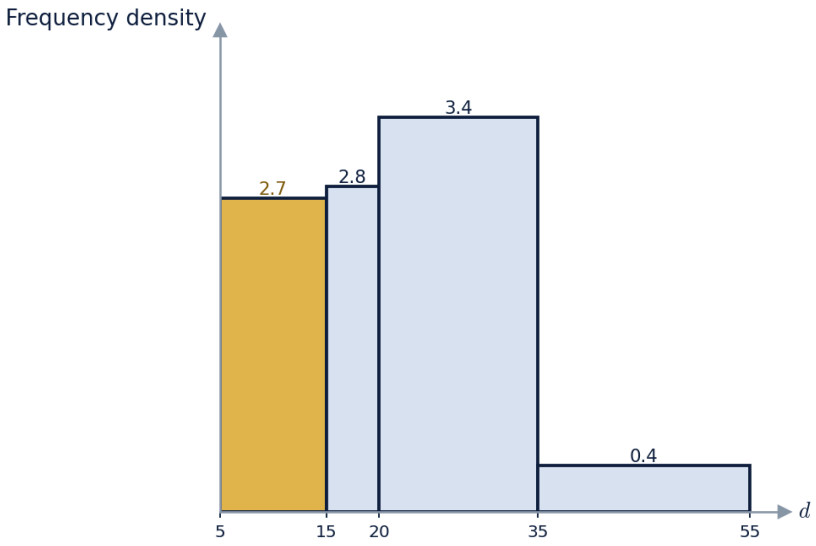
There are 100 adults in total, so the missing frequency from 5 km to 15 km is

$$100 - 73 = 27$$

6 **The missing class width is**

The missing class width is 10, so its frequency density is

$$\frac{27}{10} = 2.7$$



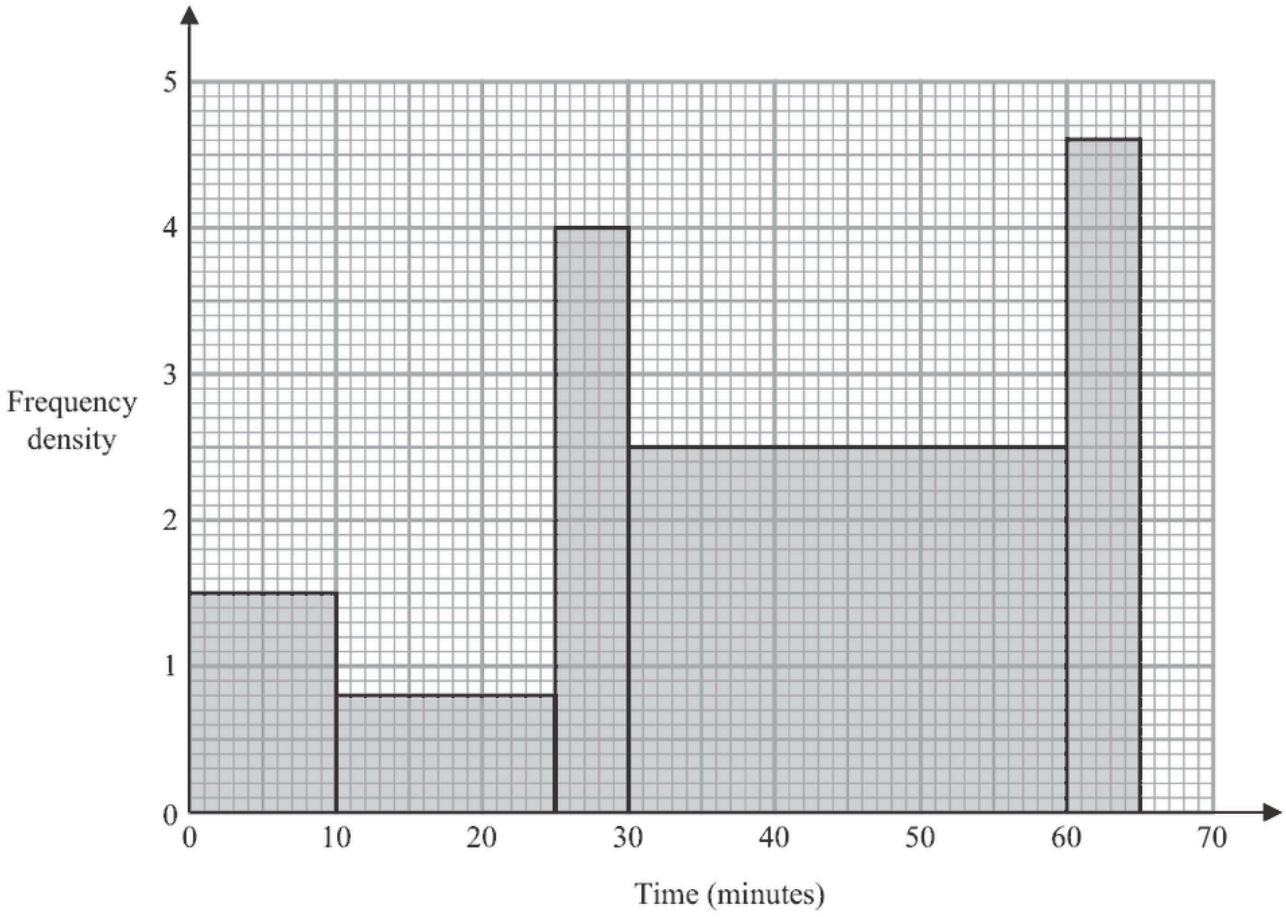
FINAL ANSWER

draw the missing bar from 5 to 15 with frequency density 2.7

Question 18

4 marks | Histograms

18 The histogram shows information about the times some students took to complete a puzzle.



Work out an estimate for the fraction of these students who took between 20 minutes and 60 minutes to complete the puzzle.

(Total for Question 18 is 4 marks)

1 Estimate the frequencies from each

Estimate the frequencies from each bar:

$$0 \text{ to } 10: \ 10(1.5) = 15$$

$$10 \text{ to } 25: \ 15(0.8) = 12$$

$$25 \text{ to } 30: \ 5(4) = 20$$

$$30 \text{ to } 60: \ 30(2.5) = 75$$

$$60 \text{ to } 65: \ 5(4.6) = 23$$

2 Total frequency

Total frequency:

$$15 + 12 + 20 + 75 + 23 = 145$$

3 Between and minutes

Between 20 and 60 minutes:

$$5(0.8) + 20 + 75 = 99$$

$$\text{fraction} = \frac{99}{145}$$

FINAL ANSWER

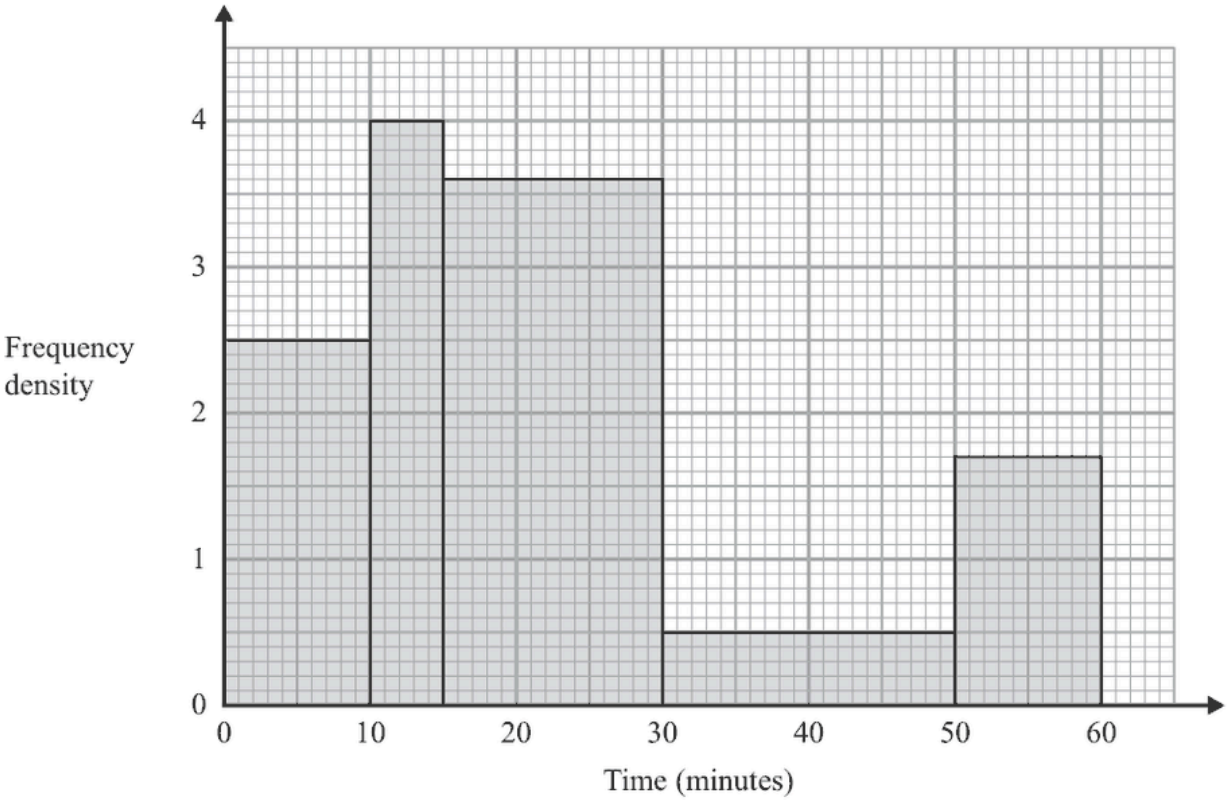
$\frac{99}{145}$.

4MA1/1H JUNE 2025

Question 17

3 marks | Histograms

17 The histogram shows information about the times, in minutes, that some people were in a shop.



Work out an estimate for the proportion of these people who were in the shop for more than 40 minutes.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2025

1 Total estimated frequency

Total estimated frequency:

$$10(2.5) + 5(4) + 15(3.6) + 20(0.5) + 10(1.7) = 126$$

2 More than minutes

More than 40 minutes:

$$10(0.5) + 10(1.7) = 22$$

3 Use histogram

So the proportion is

$$\frac{22}{126} = \frac{11}{63}$$

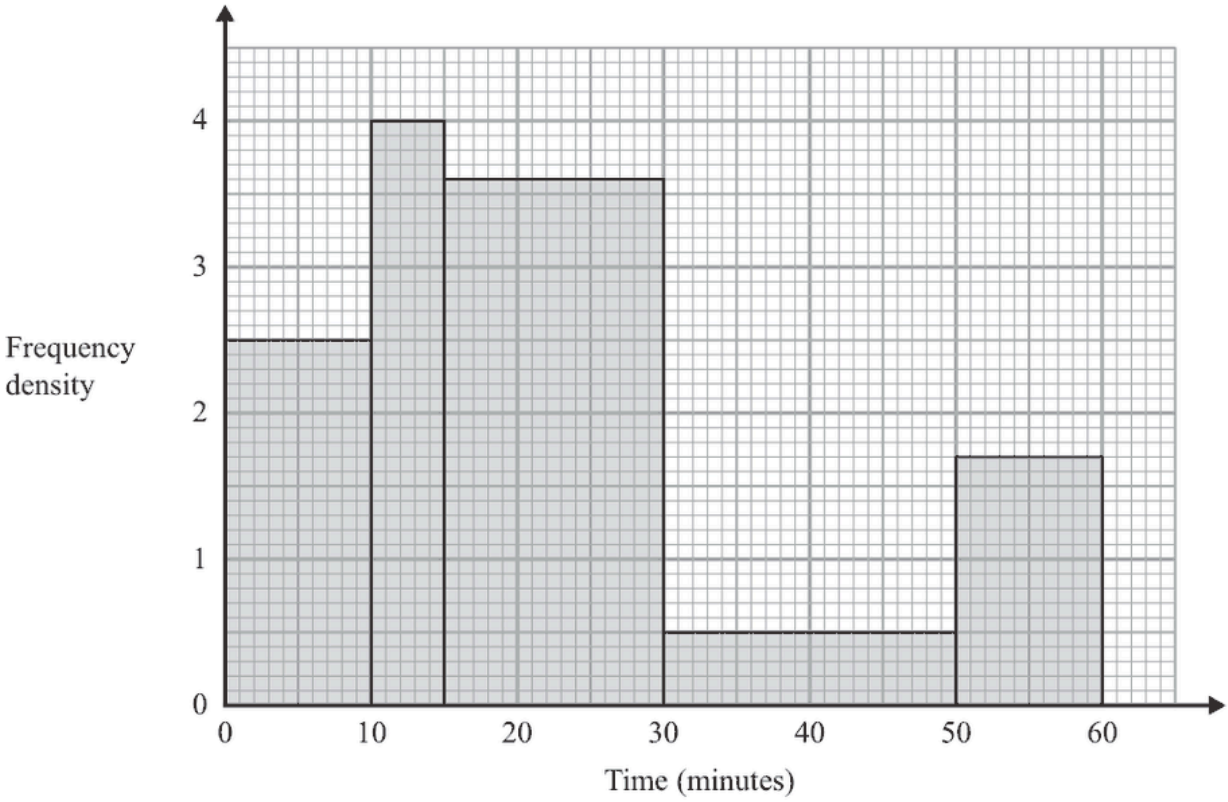
FINAL ANSWER

$\frac{11}{63}$.

Question 18

3 marks | Histograms

18 The histogram shows information about the times, in minutes, that some people were in a shop.



Work out an estimate for the proportion of these people who were in the shop for more than 40 minutes.

(Total for Question 18 is 3 marks)

1 **Frequency is class width multiplied**

Frequency is class width multiplied by frequency density.

$$0\text{--}10 : 10 \times 2.5 = 25$$

$$10\text{--}15 : 5 \times 4 = 20$$

$$15\text{--}30 : 15 \times 3.6 = 54$$

$$30\text{--}50 : 20 \times 0.5 = 10$$

$$50\text{--}60 : 10 \times 1.7 = 17$$

2 **Total**

Total:

$$25 + 20 + 54 + 10 + 17 = 126$$

3 **More than minutes is the**

More than 40 minutes is the part from 40 to 50, plus 50 to 60.

$$10 \times 0.5 + 17 = 22$$

4 **Use histogram**

So the proportion is

$$\frac{22}{126} = \frac{11}{63}$$

✓ **FINAL ANSWER**

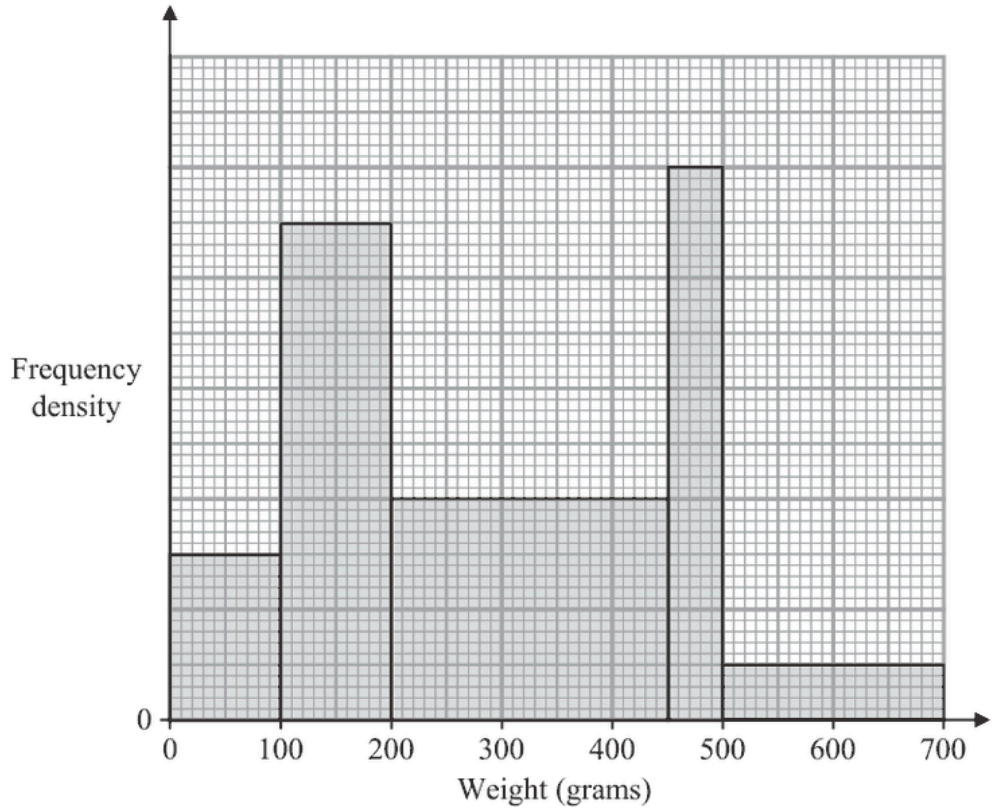
$\frac{11}{63}$

4MA1/2HR JUNE 2025

Question 20

4 marks | Histograms

20 The histogram gives some information about the weights, in grams, of some books.



75 books weigh less than 100 grams.

A book is chosen at random.

Find an estimate for the probability that this book weighs between 400 grams and 600 grams.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For books weighing less than 100 g, the frequency is 75 and the class width is 100, so

$$\text{frequency density} = \frac{75}{100} = 0.75$$

3 **Use histogram**

Using this scale from the histogram, the frequency densities are:

$$0.75,\ 2.25,\ 1.0,\ 2.5,\ 0.25$$

4 **The total number of books**

The total number of books is

$$(100)(0.75) + (100)(2.25) + (250)(1.0) + (50)(2.5) + (200)(0.25)$$

$$= 75 + 225 + 250 + 125 + 50 = 725$$

5 **Calculate value**

For weights between 400 g and 600 g:

$$(50)(1.0) + (50)(2.5) + (100)(0.25)$$

$$= 50 + 125 + 25 = 200$$

6 **Calculate probability**

So the probability is

$$\frac{200}{725} = \frac{8}{29}$$

✓ **FINAL ANSWER**

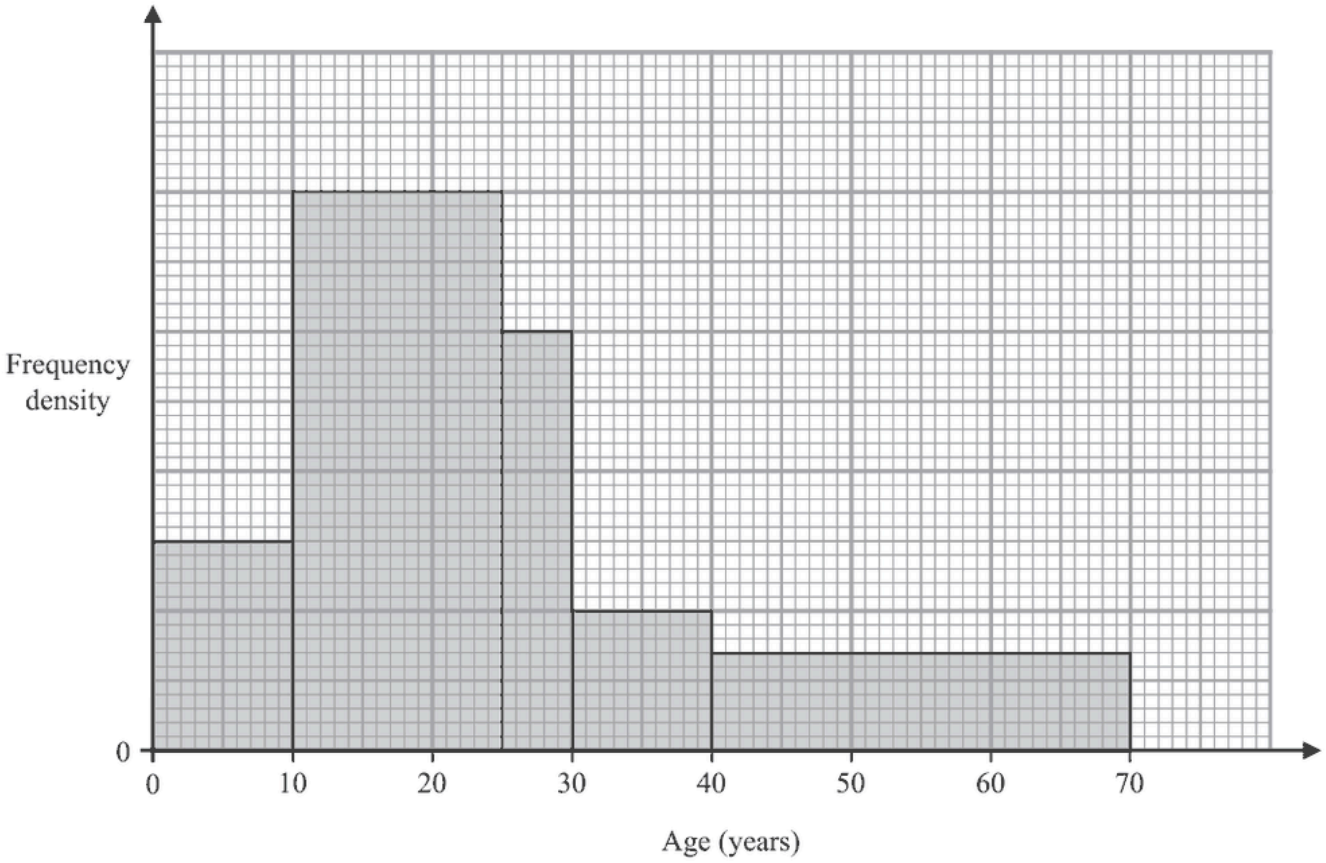
$\frac{8}{29}$ or 0.276 to 3 significant figures

4WM1/01R MAY 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the ages of the members of a chess club.



There are 60 members aged between 10 and 25
There are no members older than 70

Work out an estimate for the number of members who are over 35 years of age.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4WM1/01R May 2025

1 **The interval to has width**

The interval 10 to 25 has width 15 and frequency 60, so its frequency density is

$$\frac{60}{15} = 4$$

2 **Use histogram**

Using that scale from the histogram:

$$35 \text{ to } 40 : 5 \times 1 = 5$$

$$40 \text{ to } 70 : 30 \times 0.75 = 22.5$$

$$5 + 22.5 = 27.5$$

✓ **FINAL ANSWER**

about 28 members.

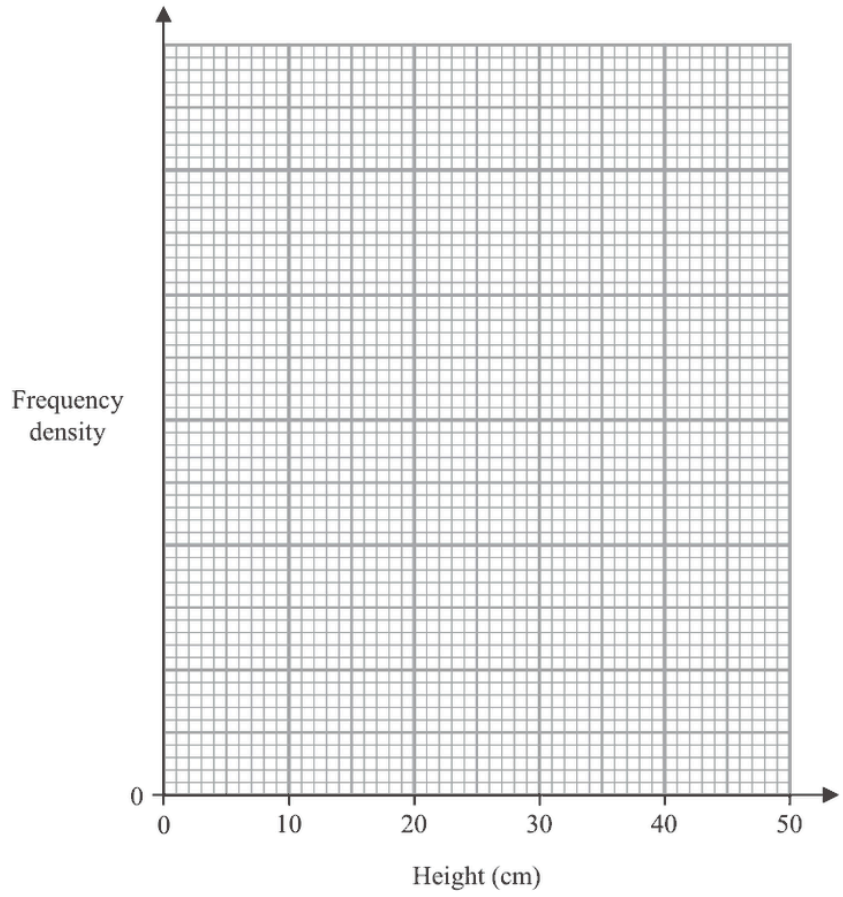
Question 20

3 marks | Histograms

20 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$0 < h \leqslant 5$	12
$5 < h \leqslant 15$	34
$15 < h \leqslant 30$	45
$30 < h \leqslant 50$	28

On the grid, draw a histogram for this information.
You must include a scale on the vertical axis.



(Total for Question 20 is 3 marks)

1 Frequency density

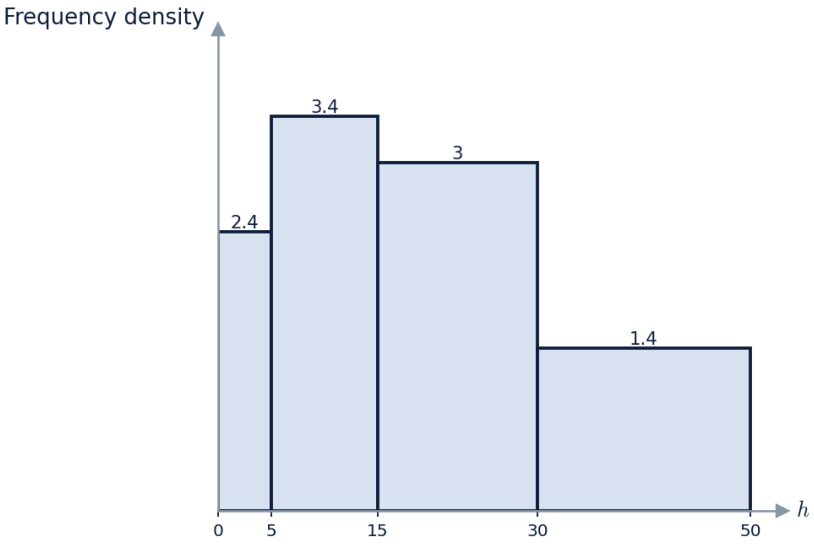
Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$.

$$0 < h \leq 5 : \frac{12}{5} = 2.4$$

$$5 < h \leq 15 : \frac{34}{10} = 3.4$$

$$15 < h \leq 30 : \frac{45}{15} = 3$$

$$30 < h \leq 50 : \frac{28}{20} = 1.4$$



FINAL ANSWER

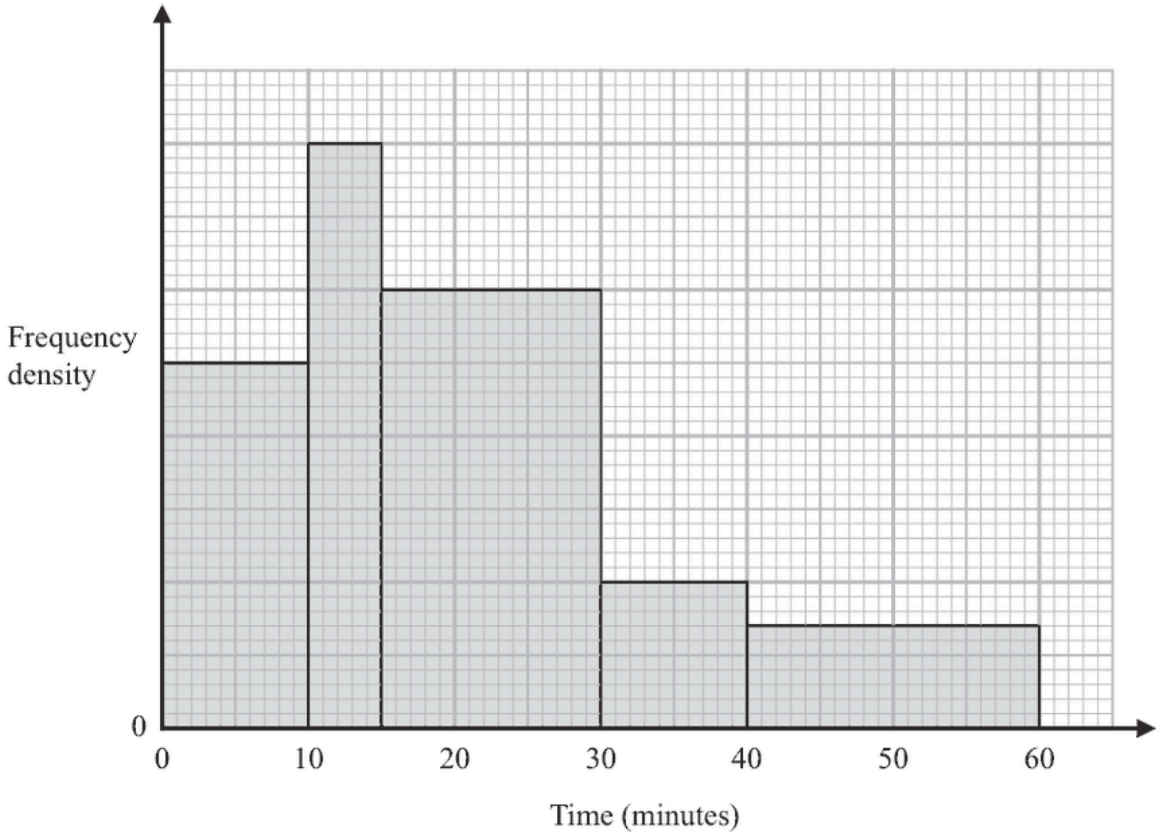
draw bars with densities $2.4, 3.4, 3, 1.4$.

4MA1/2H NOVEMBER 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the times, in minutes, that some trains arrived late at a station one day.



20 of these trains arrived between 10 minutes late and 15 minutes late.

No trains arrived more than 60 minutes late.

Work out an estimate for the number of these trains that arrived at least 25 minutes late.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $10 < t \leq 15$, the frequency is 20 and the class width is 5, so

$$\text{frequency density} = \frac{20}{5} = 4$$

3 **Use histogram**

Using this scale from the histogram:

$$15 < t \leq 30 \quad \text{has frequency density } 3$$

$$30 < t \leq 40 \quad \text{has frequency density } 1$$

$$40 < t \leq 60 \quad \text{has frequency density } 0.7$$

4 **Calculate value**

For trains at least 25 minutes late:

$$(5)(3) + (10)(1) + (20)(0.7)$$

$$= 15 + 10 + 14$$

$$= 39$$

✓ **FINAL ANSWER**

39 trains

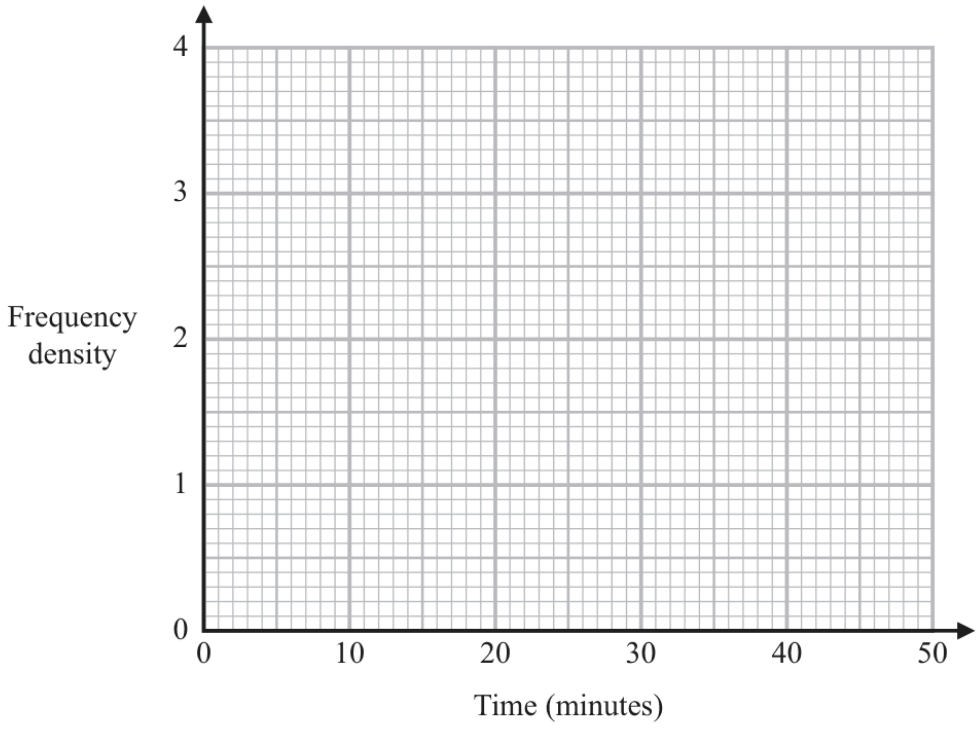
Question 19

3 marks | Histograms

19 The table gives information about the times, in minutes, some people spent waiting to go on a rollercoaster.

Time (t minutes)	Frequency
$0 < t \leqslant 20$	14
$20 < t \leqslant 25$	12
$25 < t \leqslant 35$	32
$35 < t \leqslant 50$	30

On the grid, draw a histogram for this information.



(Total for Question 19 is 3 marks)

1 Solution | Step 1

Use $\textbf{frequency density} = \textbf{frequency}/\textbf{class width}$:

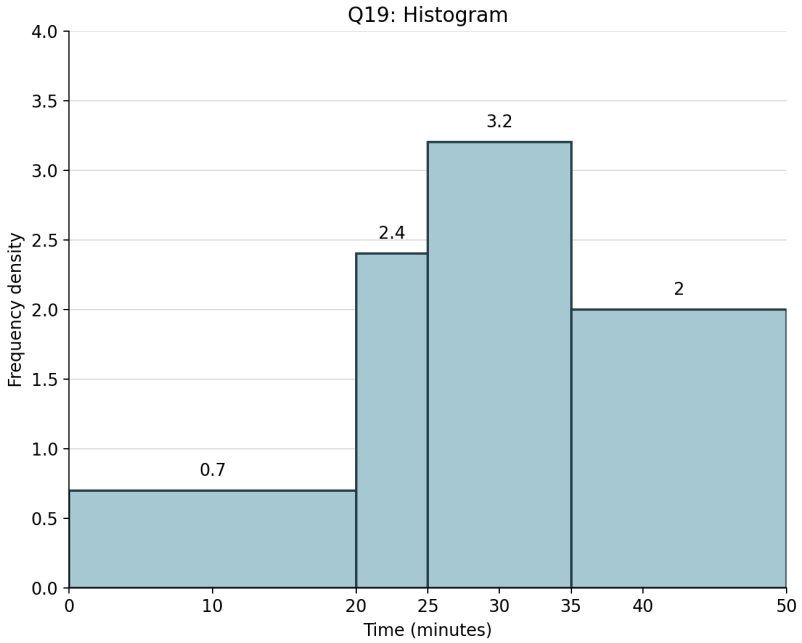
$$14/20 = 0.7, \quad 12/5 = 2.4, \quad 32/10 = 3.2, \quad 30/15 = 2.$$

Histogram areas, rather than heights alone, represent frequencies.

2 Solution | Step 2

Draw four adjoining rectangles on the supplied time and frequency-density axes: $[0, 20]$ at height 0.7, $[20, 25]$ at height 2.4, $[25, 35]$ at height 3.2, and $[35, 50]$ at height 2.

Each rectangle starts at density 0 and has the stated class boundaries as its vertical edges; leave no gaps between classes.



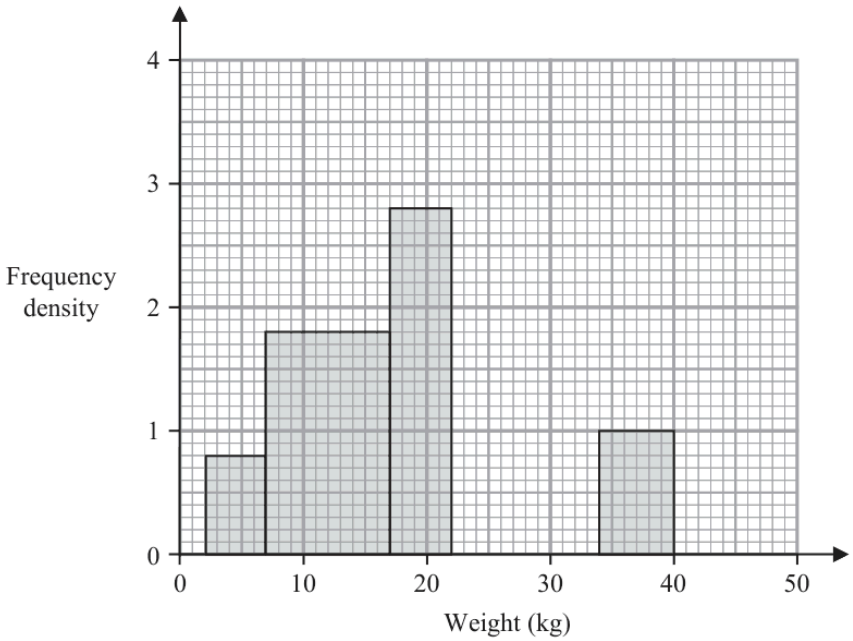
FINAL ANSWER

Bar intervals/heights: $[0, 20] : 0.7;\ [20, 25] : 2.4;\ [25, 35] : 3.2;\ [35, 50] : 2$.

Question 21

3 marks | Histograms

21 The incomplete histogram shows information about the weights, in kg, of some parcels.



No parcels weigh less than 2 kg
No parcels weigh more than 40 kg
Half of the parcels weigh between 22 kg and 34 kg

Complete the histogram by drawing a bar to represent the parcels that weigh between 22 kg and 34 kg

(Total for Question 21 is 3 marks)

1 **Frequency = width × height**

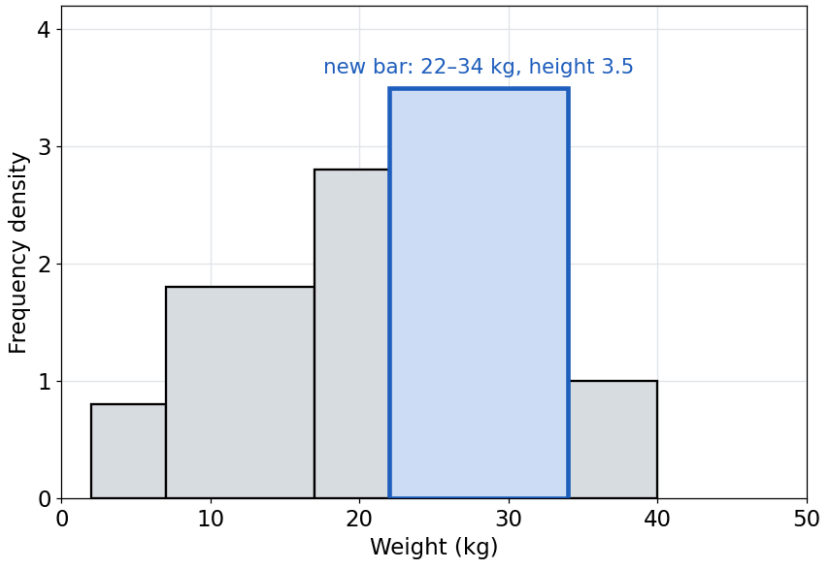
$$5(0.8) + 10(1.8) + 5(2.8) + 6(1) = 4 + 18 + 14 + 6 = 42$$

2 **Half are in 22–34**

So the other half is also 42 parcels.

3 **Height of the new bar**

$$\text{f.d.} = \frac{42}{34 - 22} = 3.5$$



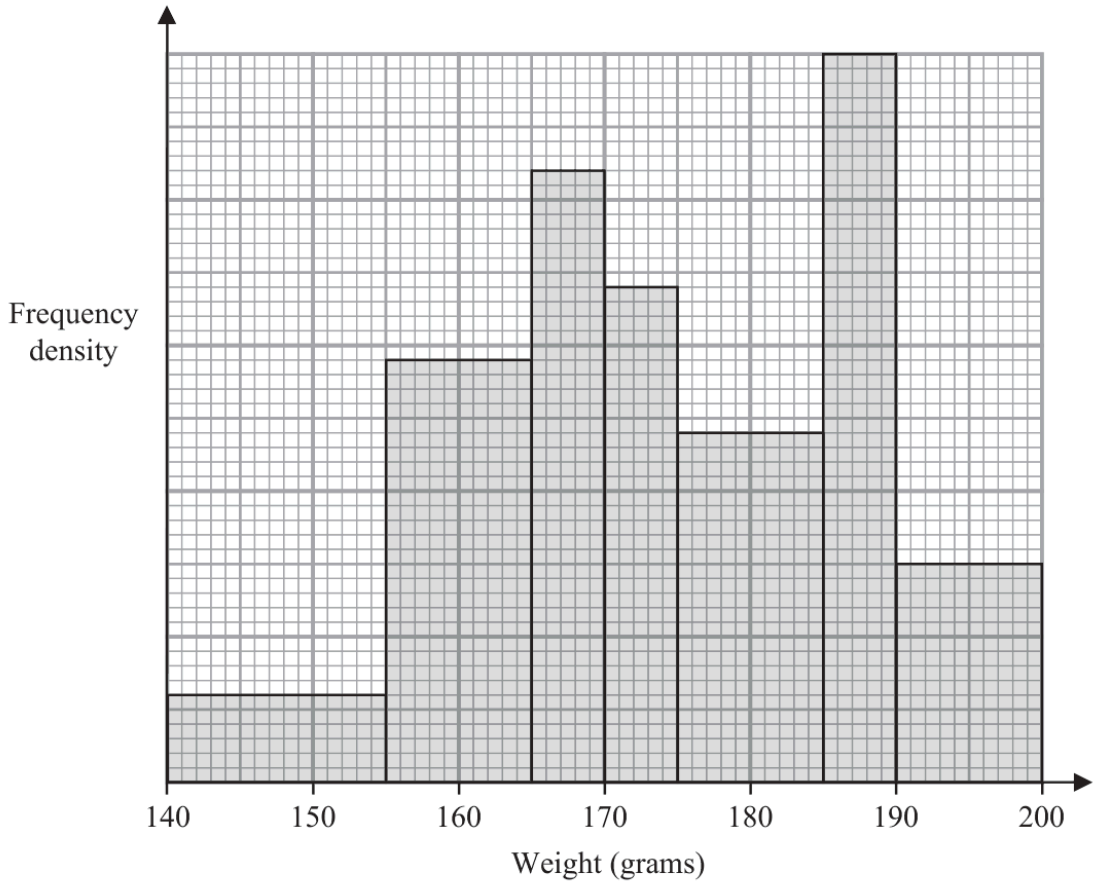
FINAL ANSWER

Bar from 22 to 34 kg, height 3.5

Question 22

5 marks | Histograms

22 The histogram gives information about the weights, in grams, of all the apples in a shop.



There are 9 apples with a weight between 140 grams and 155 grams.

(a) Work out the number of apples with a weight less than 175 grams.

.....

(3)

An apple is classified as a large apple if it has a weight of more than 180 grams.
One of the large apples is chosen at random.

(b) Work out the probability that it has a weight of more than 190 grams.

.....

(2)

(Total for Question 22 is 5 marks)

1 **Part (a) | Step 1**

The first class has width 15 g and frequency 9, so its density is $9/15 = 0.6$. Its height is 6 small grid squares, so each vertical square represents density 0.1.

Frequency is class width multiplied by frequency density.

2 **Part (a) | Step 2**

The classes below 175 g have densities $0.6, 2.9, 4.2, 3.4$ and widths $15, 10, 5, 5$, respectively.

Read the bar heights using the calibrated vertical scale.

3 **Part (a) | Step 3**

$$N = 15(0.6) + 10(2.9) + 5(4.2) + 5(3.4) = 9 + 29 + 21 + 17 = 76.$$

Add the frequencies, not the heights.

4 **Part (b) | Step 1**

Above 180 g, the frequency from 180 to 185 is $5(2.4) = 12$, from 185 to 190 is $5(5) = 25$, and from 190 to 200 is $10(1.5) = 15$.

Use only half of the 175-185 bar and all of the two heavier classes.

5 **Part (b) | Step 2**

$$P(W > 190 \mid W > 180) = \frac{15}{12 + 25 + 15} = \frac{15}{52}.$$

Restrict the denominator to the large apples.

✓ **FINAL ANSWER**

(a) 76 apples (b) $15/52$

TOPIC

Cumulative Frequency Diagrams

Every past-paper question on this topic, 2020 to 2026.

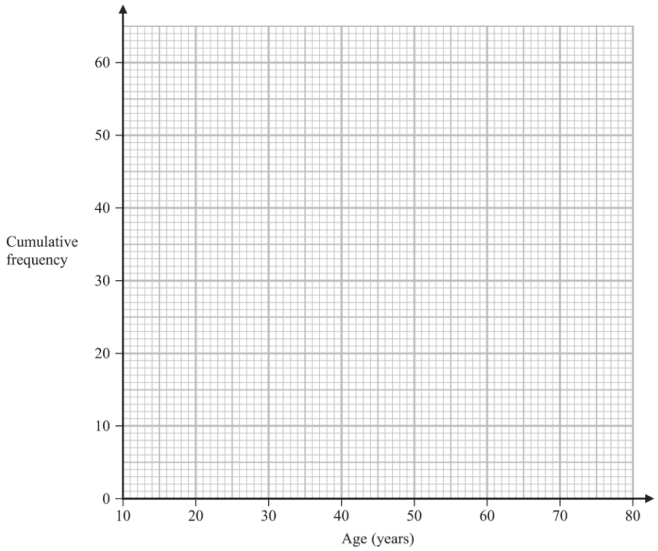
Question 15

7 marks | Cumulative Frequency Diagrams

15 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

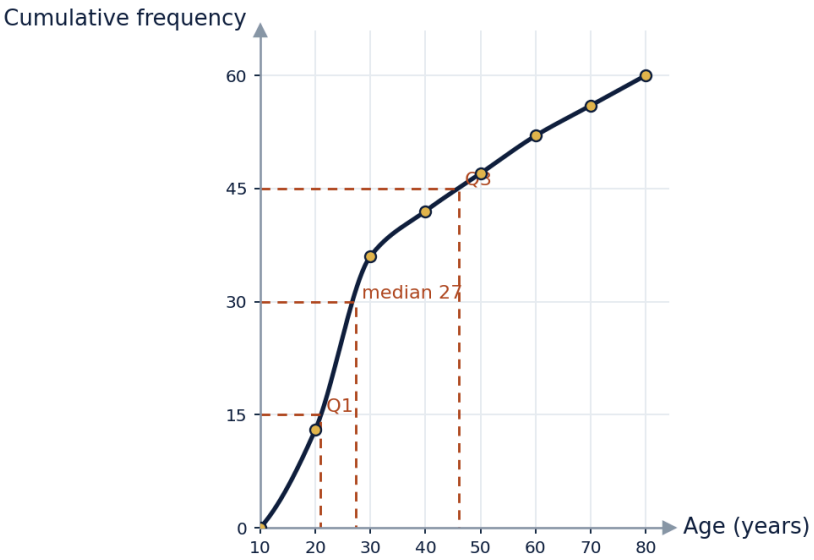
(2)

(Total for Question 15 is 7 marks)

1 **Plot the cumulative frequency points**

Plot the cumulative frequency points and join with a smooth increasing curve:

$$(10, 0),\ (20, 13),\ (30, 36),\ (40, 42),\ (50, 47),\ (60, 52),\ (70, 56),\ (80, 60)$$



2 **Use cumulative frequency**

There are 60 people, so the median is read at cumulative frequency 30.

3 **Convert standard form**

Using linear interpolation between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 \approx 27.4$$

4 **Use cumulative frequency**

So the median is about 27 years.

5 **Use cumulative frequency**

For the interquartile range, read Q_1 at cumulative frequency 15 and Q_3 at cumulative frequency 45:

$$Q_1 \approx 20.9, \qquad Q_3 \approx 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 **Use cumulative frequency**

So the IQR is about 25 years.

7 Use cumulative frequency

At age 55, the cumulative frequency is about 49.5, so the number older than 55 is

$$60 - 49.5 \approx 10.5$$

8 This gives about people

This gives about 11 people.

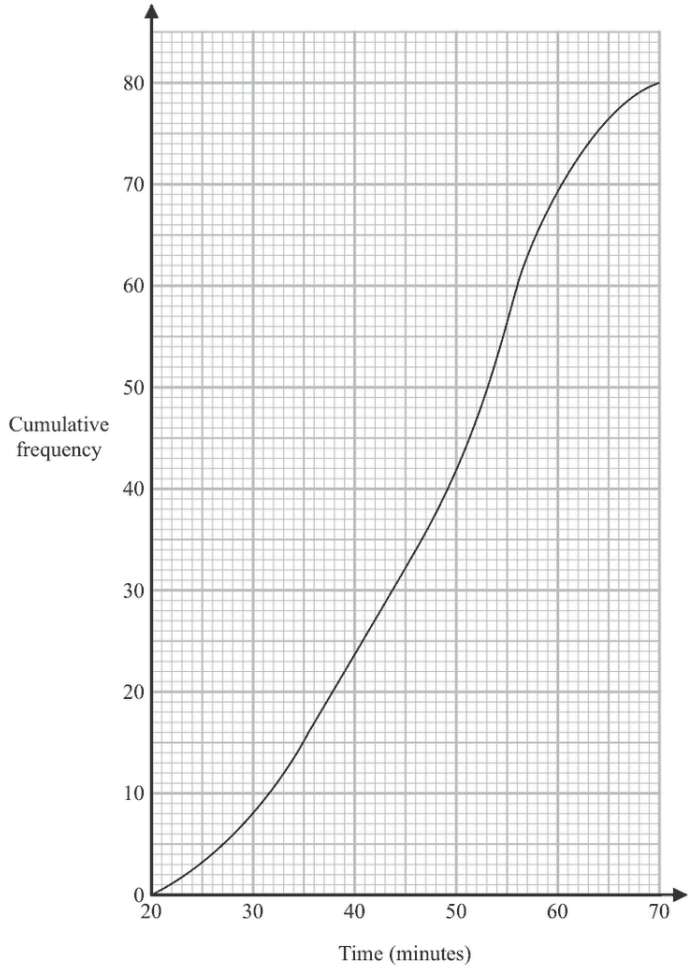
FINAL ANSWER

median about 27 years, IQR about 25 years, about 11 people older than 55.

Question 12

5 marks | Cumulative Frequency Diagrams

12 A total of 80 men and women took part in a race.
The cumulative frequency graph gives information about the times, in minutes, they took for the race.



(a) Use the graph to find an estimate for the interquartile range.

..... minutes
(2)

60% of the men took 50 minutes or less for the race.
No women took 50 minutes or less for the race.

(b) Work out an estimate for the number of men who took part in the race.

.....
(3)

(Total for Question 12 is 5 marks)

1 **There are people**

There are 80 people.

2 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at cumulative frequency } 20$$

$$Q_3 \text{ is at cumulative frequency } 60$$

3 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 38, \qquad Q_3 \approx 56$$

$$\text{IQR} \approx 56 - 38 = 18$$

4 **Use cumulative frequency**

At 50 minutes, the cumulative frequency is about 42.

5 **Use cumulative frequency**

No women took 50 minutes or less, so these 42 people are men.

$$0.6 \times \text{number of men} = 42$$

$$\text{number of men} = \frac{42}{0.6} = 70$$

FINAL ANSWER

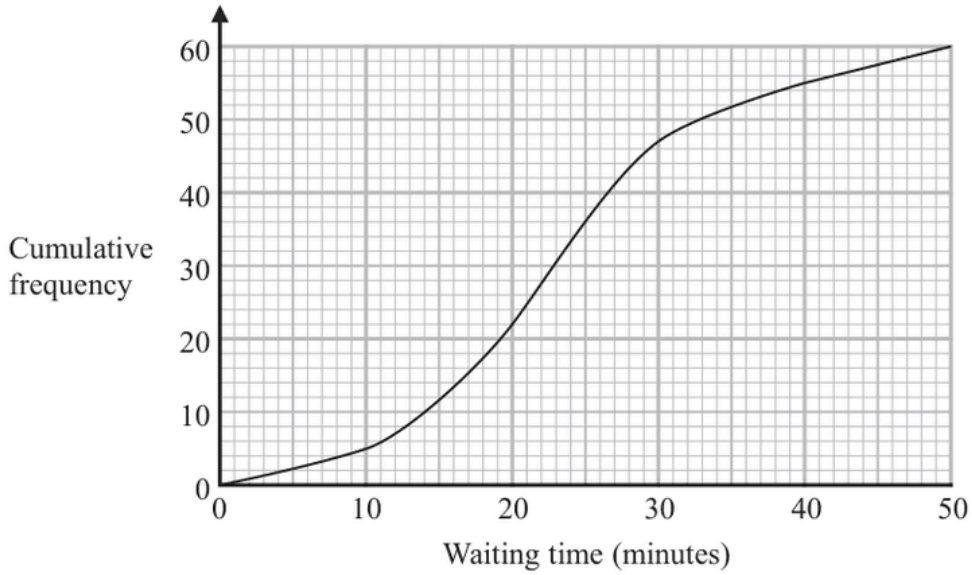
IQR about 18 minutes, and about 70 men.

4MA1/1HR JANUARY 2020

Question 12

5 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph gives information about the waiting times, in minutes, of people with appointments at Hospital A.



(a) Use the graph to find an estimate of the median waiting time at Hospital A.

..... minutes

(1)

(b) Use the graph to find an estimate of the interquartile range of the waiting times at Hospital A.

..... minutes

(2)

At a different hospital, Hospital B, the median waiting time is 28 minutes and the interquartile range of the waiting times is 19 minutes.

(c) Compare the waiting times at Hospital A with the waiting times at Hospital B.

.....

(2)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2020

1 **There are people**

There are **60** people.

2 **The median is at cumulative**

The median is at cumulative frequency 30.

3 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 22.5 \text{ minutes}$$

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 16.5, \qquad Q_3 \approx 28.5$$

$$\text{IQR} \approx 28.5 - 16.5 = 12$$

6 **Hospital B has median minutes**

Hospital B has median 28 minutes and IQR 19 minutes.

7 **Use cumulative frequency**

So Hospital A has a shorter median waiting time and a smaller spread.

✓ **FINAL ANSWER**

median about 22.5 minutes, IQR about 12 minutes. Hospital A is quicker and more consistent.

Question 11

5 marks | Cumulative Frequency Diagrams

11 The frequency table gives information about the ages of the 80 people in a train carriage.

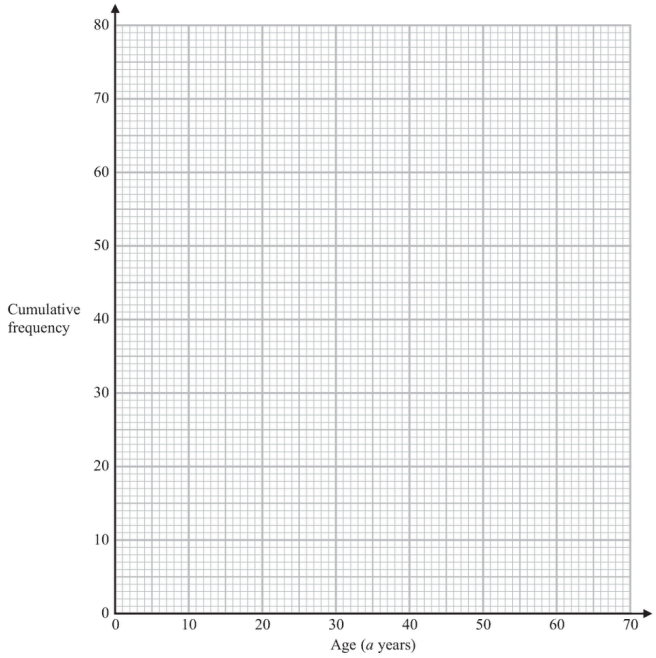
Age (a years)	Frequency
$0 < a \leqslant 20$	9
$20 < a \leqslant 30$	19
$30 < a \leqslant 40$	17
$40 < a \leqslant 50$	18
$50 < a \leqslant 60$	13
$60 < a \leqslant 70$	4

(a) Complete the cumulative frequency table.

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	
$0 < a \leqslant 30$	
$0 < a \leqslant 40$	
$0 < a \leqslant 50$	
$0 < a \leqslant 60$	
$0 < a \leqslant 70$	

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median age of the people in the train carriage.

..... years

(2)

(Total for Question 11 is 5 marks)

1

(a) Running totals

$$9,\ 28,\ 45,\ 63,\ 76,\ 80$$

2

(b) Plot at the upper class boundaries

Plot $(20, 9), (30, 28), (40, 45), (50, 63), (60, 76), (70, 80)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Median

Read across from $\frac{80}{2} = 40$: about 37 years.

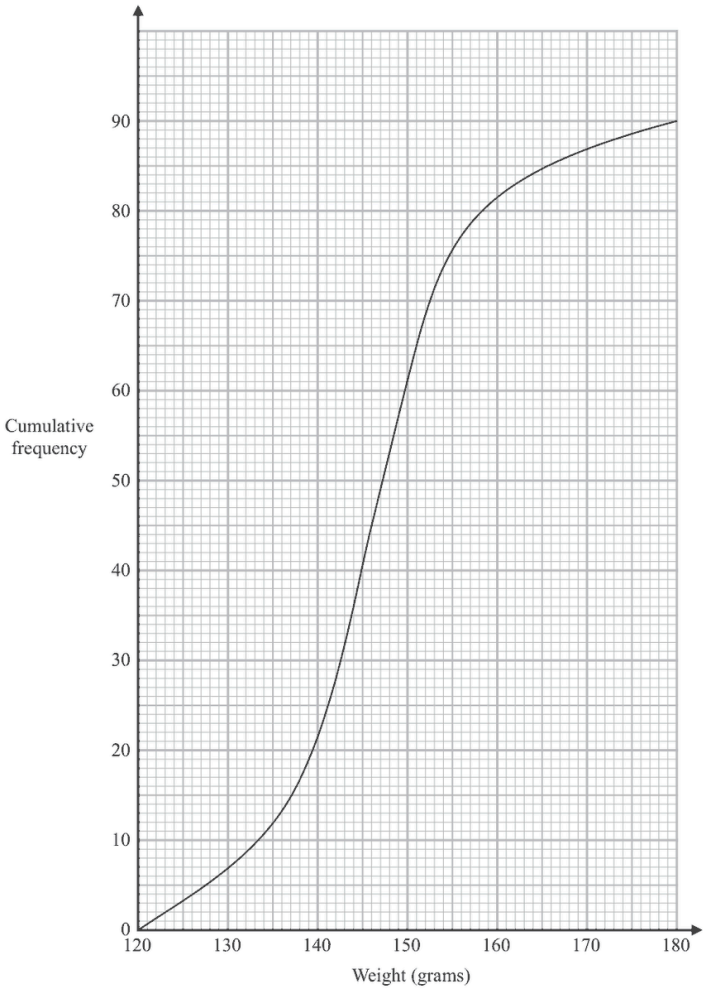
✓ **FINAL ANSWER**

(a) $9, 28, 45, 63, 76, 80$ (c) about 37 years

Question 11

5 marks | Cumulative Frequency Diagrams

11 The cumulative frequency graph gives information about the weights, in grams, of 90 bags of sweets.



(a) Find an estimate for the median of the weights of these bags of sweets.

..... grams

(2)

Roberto sells the bags of sweets to raise money for charity.
Bags with a weight greater than d grams are labelled large bags and sold for 3.75 euros each bag.

The total amount of money he receives by selling all the large bags is 93.75 euros.

(b) Find the value of d.

$d =$ .....

(3)

(Total for Question 11 is 5 marks)

1 **Use cumulative frequency**

There are 90 bags, so the median is at cumulative frequency 45.

2 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 146 \text{ g}$$

3 **Large bags sold**

Large bags sold:

$$\frac{93.75}{3.75} = 25$$

4 **Use cumulative frequency**

So 25 bags have weight greater than d.

5 **That means**

That means

$$90 - 25 = 65$$

6 **Use cumulative frequency**

bags have weight d grams or less.

7 **Use cumulative frequency**

From the graph, cumulative frequency 65 gives

$$d \approx 150$$

✓ **FINAL ANSWER**

median about 146 g, and $d \approx 150$.

Question 11

6 marks | Cumulative Frequency Diagrams

11 The manager of a call centre asked the 120 people, who rang the call centre last week, how long they each waited before their call was answered.

The table gives information about their replies.

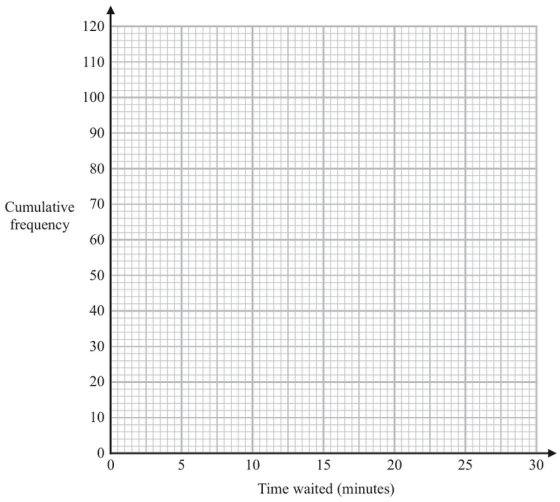
Time waited (t minutes)	Frequency
$0 < t \leqslant 5$	8
$5 < t \leqslant 10$	15
$10 < t \leqslant 15$	17
$15 < t \leqslant 20$	28
$20 < t \leqslant 25$	33
$25 < t \leqslant 30$	19

(a) Complete the cumulative frequency table.

Time waited (t minutes)	Cumulative frequency
$0 < t \leqslant 5$	
$0 < t \leqslant 10$	
$0 < t \leqslant 15$	
$0 < t \leqslant 20$	
$0 < t \leqslant 25$	
$0 < t \leqslant 30$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median of the times waited.

..... minutes

(1)

(d) Using your graph, find an estimate for the percentage of the 120 people who said that they waited longer than 23 minutes before their call was answered.
Show your working clearly.

.....%

(2)

(Total for Question 11 is 6 marks)

1 **Complete the cumulative frequencies**

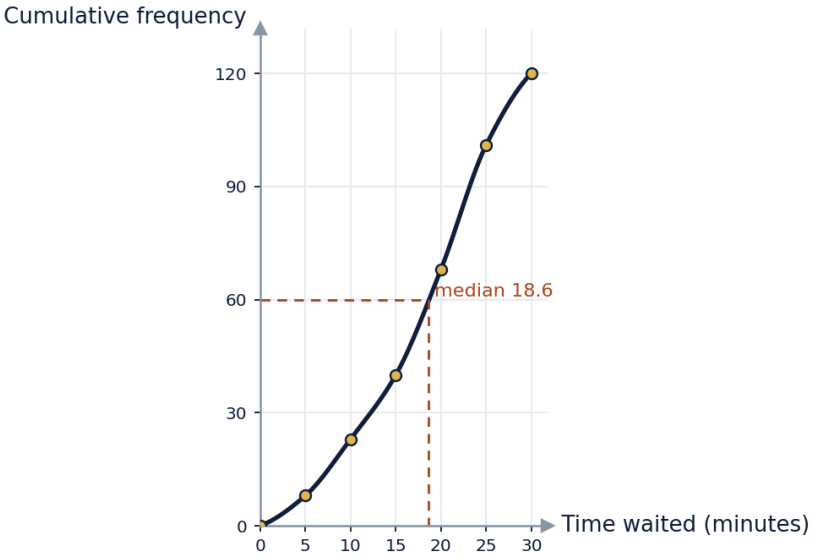
Complete the cumulative frequencies:

$$8,\ 23,\ 40,\ 68,\ 101,\ 120$$

2 **Plot the points**

Plot the points

$$(0, 0), (5, 8), (10, 23), (15, 40), (20, 68), (25, 101), (30, 120)$$



3 **The median is at cumulative**

The median is at cumulative frequency 60.

4 **Use cumulative frequency**

This lies between $(15, 40)$ and $(20, 68)$:

$$15 + \frac{60 - 40}{68 - 40} \times 5 = 18.57\ldots$$

5 **Use cumulative frequency**

So the median is about 18.6 minutes.

6 **At minutes**

At 23 minutes:

$$68 + \frac{3}{5}(101 - 68) = 87.8$$

7 Number who waited longer than

Number who waited longer than 23 minutes:

$$120 - 87.8 = 32.2$$

$$\frac{32.2}{120} \times 100 = 26.8\%$$

FINAL ANSWER

cumulative frequencies $8, 23, 40, 68, 101, 120$, median about 18.6 minutes, percentage about 27%.

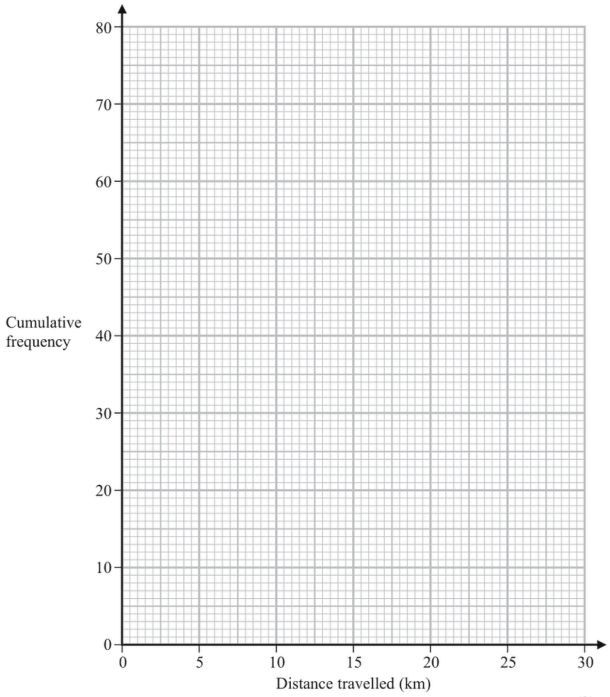
Question 12

7 marks | Cumulative Frequency Diagrams

12 The cumulative frequency table gives information about the distance, in kilometres, that each of 80 workers travel from home to work at Office *A*.

Distance travelled (d km)	Cumulative frequency
$0 < d \leqslant 5$	17
$0 < d \leqslant 10$	32
$0 < d \leqslant 15$	57
$0 < d \leqslant 20$	70
$0 < d \leqslant 25$	76
$0 < d \leqslant 30$	80

(a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the median distance travelled.

..... km

(1)

(c) Use your graph to find an estimate for the interquartile range of the distances travelled.

..... km

(2)

For Office *B*, the median distance workers travel from home to work is 15 km and the interquartile range is 5 km.

(d) Use the information above to compare the distances that workers at Office *A* and workers at Office *B* travel from home to work.
Write down **two** comparisons.

1 .....

2 .....

.....

(2)

(Total for Question 12 is 7 marks)

1 **(a) Plot the table**

Plot $(5, 17), (10, 32), (15, 57), (20, 70), (25, 76), (30, 80)$, start at $(0, 0)$, and join with a smooth curve.

2 **(b) Median**

Read across from 40: about 11.6 km.

3 **(c) Interquartile range**

Lower quartile at 20: about 6 km. Upper quartile at 60: about 16.2 km.

$$\text{IQR} \approx 16.2 - 6 = 10.2 \text{ km}$$

4 **(d) Two comparisons**

1. Office A has a lower median (about 11.6 km against 15 km), so on average workers at Office A travel a shorter distance.
2. Office A has a larger IQR (about 10 km against 5 km), so the distances at Office A are more varied (less consistent).

✓ **FINAL ANSWER**

(b) about 11.6 km (c) about 10 km (d) A shorter on average; A more spread out

Question 11

5 marks | Cumulative Frequency Diagrams

11 The table gives information about the times taken by 90 runners to complete a 10 km race.

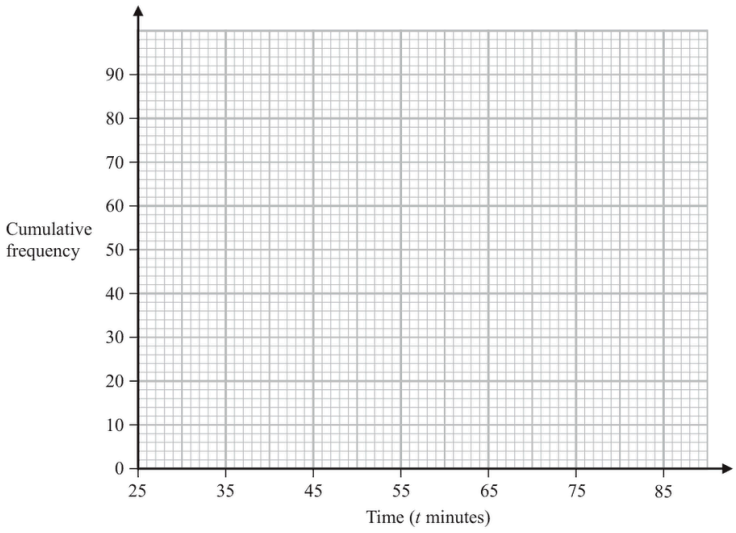
Time (t minutes)	Frequency
$25 < t \leqslant 35$	12
$35 < t \leqslant 45$	24
$45 < t \leqslant 55$	28
$55 < t \leqslant 65$	12
$65 < t \leqslant 75$	10
$75 < t \leqslant 85$	4

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$25 < t \leqslant 35$	12
$25 < t \leqslant 45$	
$25 < t \leqslant 55$	
$25 < t \leqslant 65$	
$25 < t \leqslant 75$	
$25 < t \leqslant 85$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

Any runner who completed the race in a time T minutes such that $42 < T \leqslant 52$ minutes was awarded a silver medal.

(c) Use your graph to find an estimate for the number of runners who were awarded a silver medal.

..... runners

(2)

(Total for Question 11 is 5 marks)

1 **(a) Running totals**

$$12,\ 36,\ 64,\ 76,\ 86,\ 90$$

2 **(b) Plot at the upper class boundaries**

Plot $(35, 12), (45, 36), (55, 64), (65, 76), (75, 86), (85, 90)$, start at $(25, 0)$, and join with a smooth curve.

3 **(c) Read the curve at 52 and 42 minutes**

At 52 minutes: about 56. At 42 minutes: about 29.

4 **(c) Silver medals**

$$56 - 29 \approx 27 \text{ runners}$$

✓ **FINAL ANSWER**

(a) $12, 36, 64, 76, 86, 90$ (c) about 27

Question 12

6 marks | Cumulative Frequency Diagrams

12 The table gives information about the times, in minutes, taken by 80 customers to do their shopping in a supermarket.

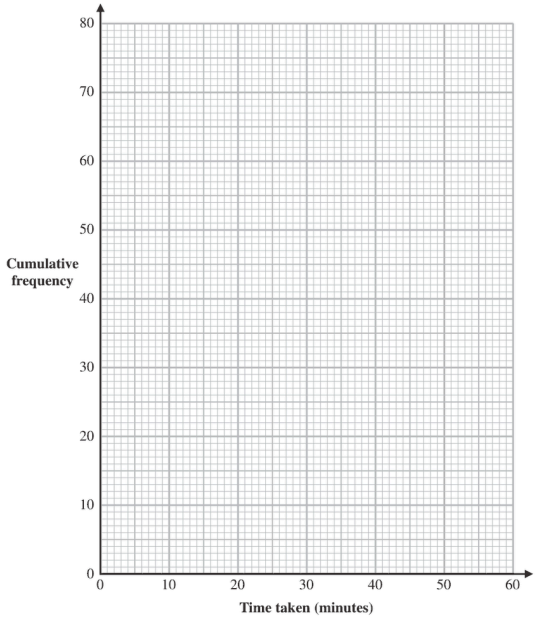
Time taken (t minutes)	Frequency
$0 < t \leqslant 10$	7
$10 < t \leqslant 20$	26
$20 < t \leqslant 30$	24
$30 < t \leqslant 40$	14
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	2

(a) Complete the cumulative frequency table.

Time taken (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	
$0 < t \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time taken.

..... minutes

(1)

One of the 80 customers is chosen at random.

(d) Use your graph to find an estimate for the probability that the time taken by this customer was more than 42 minutes.

.....

(2)

(Total for Question 12 is 6 marks)

1 Complete the cumulative frequencies

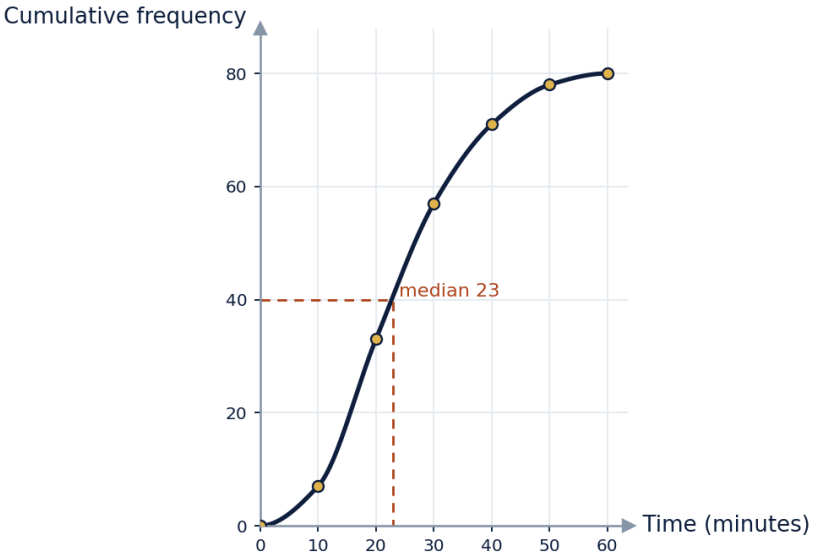
Complete the cumulative frequencies:

$$7,\ 33,\ 57,\ 71,\ 78,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (10, 7), (20, 33), (30, 57), (40, 71), (50, 78), (60, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(20, 33)$ and $(30, 57)$:

$$20 + \frac{40 - 33}{57 - 33} \times 10 = 22.9$$

5 Use cumulative frequency

So the median is about 23 minutes.

6 At minutes

At 42 minutes:

$$71 + \frac{2}{10}(78 - 71) = 72.4$$

7 Number taking more than minutes

Number taking more than 42 minutes:

$$80 - 72.4 = 7.6$$

$$P(t > 42) \approx \frac{7.6}{80} = 0.095$$

FINAL ANSWER

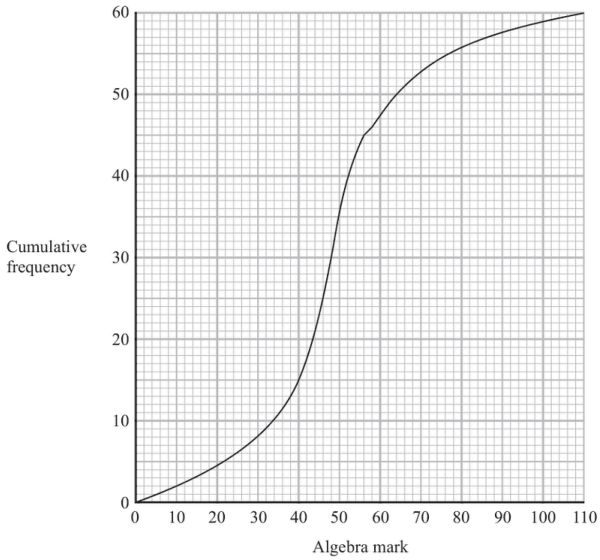
cumulative frequencies $7, 33, 57, 71, 78, 80$, median about 23 minutes, probability about 0.095.

Question 14

8 marks | Cumulative Frequency Diagrams

14 A group of 60 students each sat an algebra test and a geometry test.
Each test was marked out of 110

The cumulative frequency graph gives information about the marks gained by the 60 students in the algebra test.



(a) Use the graph to find an estimate for the median mark in the algebra test.

.....

(1)

(b) Use the graph to find an estimate for the number of students who gained 58 marks or less in the algebra test.

.....

(1)

(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

.....

(2)

The interquartile range of the marks gained in the geometry test is 9

Luis says

"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Luis correct?
Give a reason for your answer.

.....

(1)

To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64

Two students are to be selected at random from the 60 students in the group.

(e) Use the graph to find an estimate for the probability that both of these students were awarded a grade A in the algebra test.

.....

(3)

(Total for Question 14 is 8 marks)

1 **There are students**

There are 60 students.

2 **The median is at cumulative**

The median is at cumulative frequency 30. From the graph,

$$\text{median} \approx 49$$

3 **Use cumulative frequency**

At 58 marks, the cumulative frequency is about 45, so about 45 students gained 58 marks or less.

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 40, \qquad Q_3 \approx 57$$

$$\text{IQR} \approx 57 - 40 = 17$$

6 **Use cumulative frequency**

The geometry IQR is 9, so Luis is correct because the algebra IQR is larger.

7 **Use cumulative frequency**

For grade A, use mark 64. From the graph, about 50 students gained 64 or less, so about

$$60 - 50 = 10$$

8 **students gained more than**

students gained more than 64.

9 **Probability both selected students were**

Probability both selected students were grade A:

$$\frac{10}{60} \times \frac{9}{59} = 0.0254\ldots$$

FINAL ANSWER

median about 49, about 45 students, IQR about 17, Luis is correct, probability about 0.025.

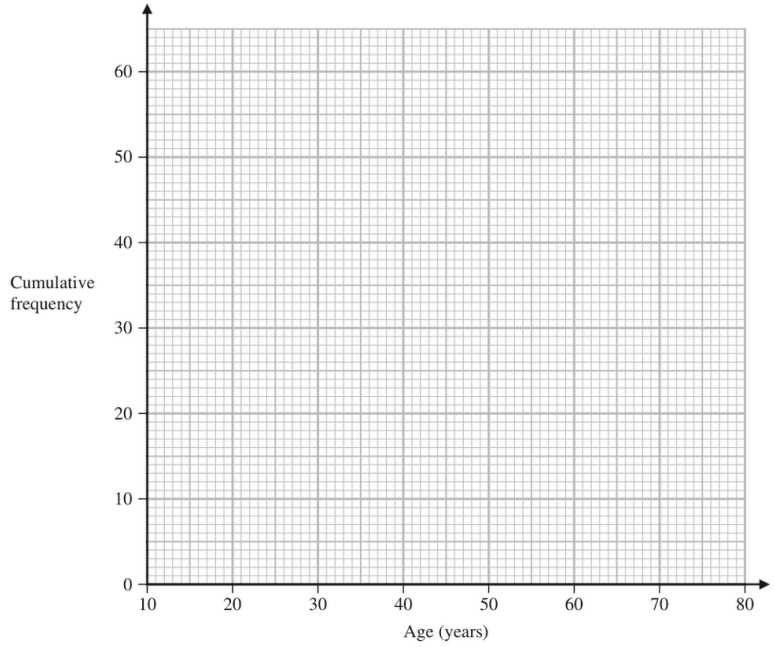
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

Question 11 continued

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

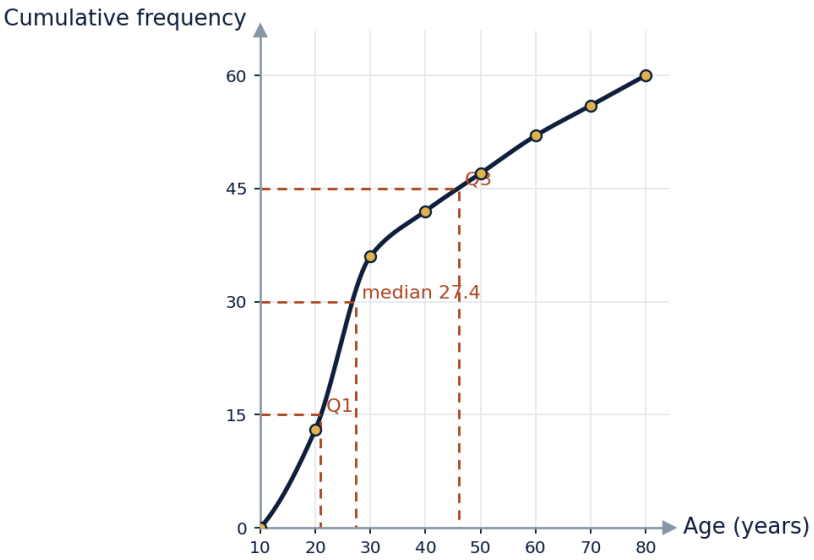
(2)

(Total for Question 11 is 7 marks)

1 Plot the points

Plot the points

$$(10, 0), (20, 13), (30, 36), (40, 42), (50, 47), (60, 52), (70, 56), (80, 60)$$



2 The median is at cumulative

The median is at cumulative frequency 30.

3 Convert standard form

This is between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 = 27.39\ldots$$

4 Median age is about years

Median age is about 27.4 years.

5 Convert standard form

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

$$Q_1 \approx 20 + \frac{15 - 13}{36 - 13} \times 10 = 20.9$$

$$Q_3 \approx 40 + \frac{45 - 42}{47 - 42} \times 10 = 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 Use cumulative frequency

For people older than 55:

$$F(55) \approx 47 + \frac{5}{10}(52 - 47) = 49.5$$

$$60 - 49.5 = 10.5$$

FINAL ANSWER

median about 27.4 years, IQR about 25.1 years, about 11 people older than 55.

Question 13

7 marks | Cumulative Frequency Diagrams

13 The table gives information about the ages, in years, of 80 people in a train carriage.

Age (a years)	Frequency
$0 < a \leqslant 20$	7
$20 < a \leqslant 30$	25
$30 < a \leqslant 40$	20
$40 < a \leqslant 50$	14
$50 < a \leqslant 60$	8
$60 < a \leqslant 70$	6

(a) Complete the cumulative frequency table.

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	
$0 < a \leqslant 30$	
$0 < a \leqslant 40$	
$0 < a \leqslant 50$	
$0 < a \leqslant 60$	
$0 < a \leqslant 70$	

(1)

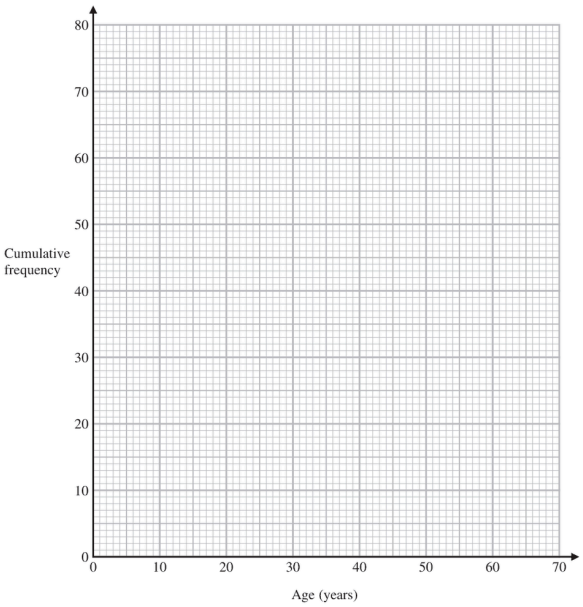
(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)

(c) Use your graph to find an estimate for the median age of the 80 people.

..... years

(1)



Of the people in the train carriage, 60% of those who are aged between 18 and 65 are going to work. None of the other people in the train carriage are going to work.

(d) Use your graph to find an estimate for the number of people in the train carriage who are going to work.

.....

(3)

(Total for Question 13 is 7 marks)

1 Complete the cumulative frequencies

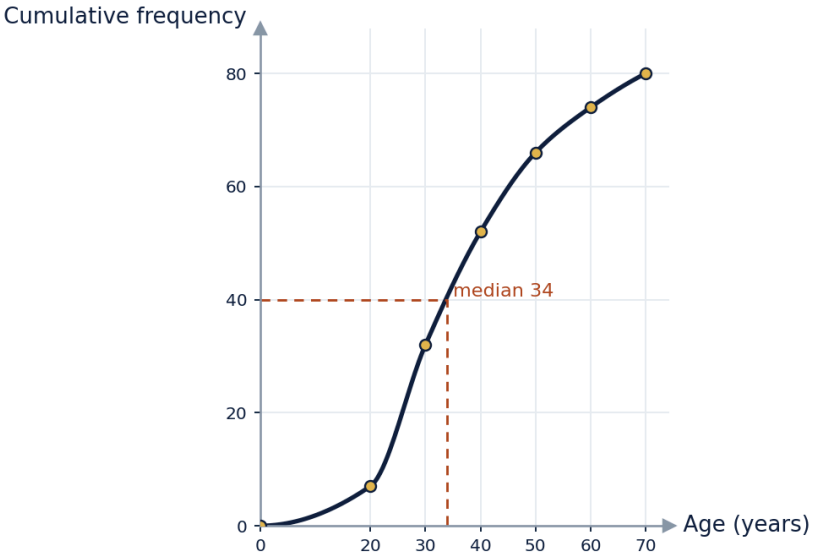
Complete the cumulative frequencies:

$$7,\ 32,\ 52,\ 66,\ 74,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (20, 7), (30, 32), (40, 52), (50, 66), (60, 74), (70, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(30, 32)$ and $(40, 52)$:

$$30 + \frac{40 - 32}{52 - 32} \times 10 = 34$$

5 Use cumulative frequency

So the median age is about 34 years.

6 Estimate the number aged between

Estimate the number aged between 18 and 65:

$$F(18) \approx \frac{18}{20} \times 7 = 6.3$$

$$F(65) \approx 74 + \frac{5}{10}(80 - 74) = 77$$

$$77 - 6.3 = 70.7$$

---

7 60% are going to work

60% are going to work:

$$0.6(70.7) = 42.42$$

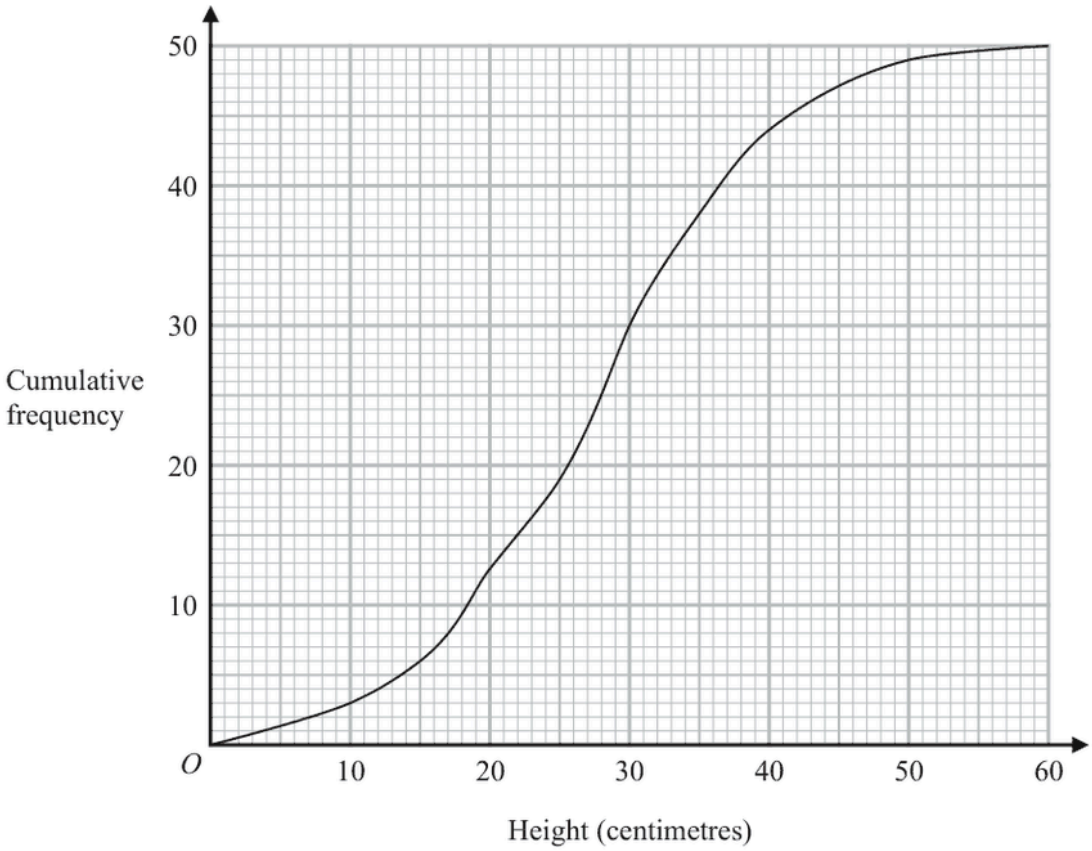
FINAL ANSWER

cumulative frequencies $7, 32, 52, 66, 74, 80$, median about 34 years, about 42 people going to work.

Question 12

4 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph shows information about the heights, in centimetres, of 50 plants in a flowerbed.



(a) Use the graph to find an estimate for the median height of these plants.

..... centimetres

(1)

(b) Use the graph to find the frequency for the class interval $30 < \text{Height} \leqslant 40$

.....

(1)

(c) Use the graph to find an estimate for the number of plants with a height greater than 35 centimetres.

.....

(2)

(Total for Question 12 is 4 marks)

1

(a) Median

There are 50 plants, so read across from 25: the curve reaches 25 at about 28.5 cm.

2

(b) Frequency for 30 < height ≤ 40

Read the curve at both ends of the class: about 44 at 40 cm and 30 at 30 cm.

$$44 - 30 = 14$$

3

(c) Taller than 35 cm

At 35 cm the curve reads about 37, so about $50 - 37 = 13$ plants.

✓ **FINAL ANSWER**

(a) about 28.5 cm (b) 14 (c) about 13

Question 14

6 marks | Cumulative Frequency Diagrams

14 The table gives information about the times taken by 80 people to run a race.

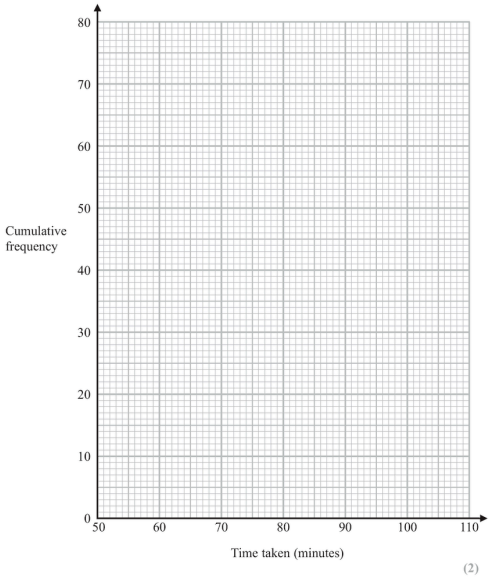
Time taken (t minutes)	Frequency
$50 < t \leqslant 60$	15
$60 < t \leqslant 70$	16
$70 < t \leqslant 80$	21
$80 < t \leqslant 90$	14
$90 < t \leqslant 100$	8
$100 < t \leqslant 110$	6

(a) Complete the cumulative frequency table.

Time taken (t minutes)	Cumulative frequency
$50 < t \leqslant 60$	
$50 < t \leqslant 70$	
$50 < t \leqslant 80$	
$50 < t \leqslant 90$	
$50 < t \leqslant 100$	
$50 < t \leqslant 110$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time taken.

..... minutes

(1)

(d) Use your graph to find an estimate for the interquartile range of the times taken.

..... minutes

(2)

(Total for Question 14 is 6 marks)

1 **(a) Running totals**

$$15,\ 31,\ 52,\ 66,\ 74,\ 80$$

2 **(b) Plot at the upper class boundaries**

Plot $(60, 15), (70, 31), (80, 52), (90, 66), (100, 74), (110, 80)$, start at $(50, 0)$, and join with a smooth curve.

3 **(c) Median**

Read across from 40: about 74 minutes.

4 **(d) Interquartile range**

Lower quartile at 20: about 63 minutes. Upper quartile at 60: about 86 minutes.

$$\text{IQR} \approx 86 - 63 = 23 \text{ minutes}$$

✓ **FINAL ANSWER**

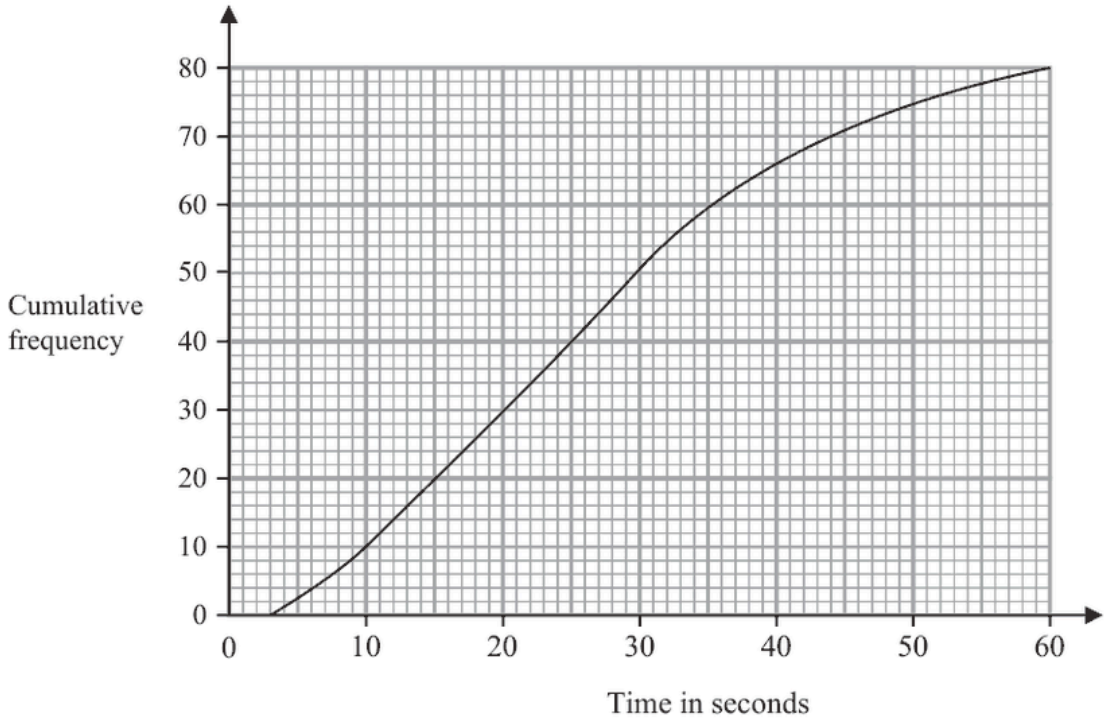
(a) $15, 31, 52, 66, 74, 80$ (c) about 74 minutes (d) about 23 minutes

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Question 13

4 marks | Cumulative Frequency Diagrams

13 The cumulative frequency graph gives information about the times, in seconds, that 80 adults took to log in to an online bank.



(a) Find an estimate for the median time.

..... seconds

(1)

(b) Work out the percentage of these adults that took longer than 50 seconds to log in.
Show your working clearly.

.....%

(3)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2023

1 **(a) Median**

Read across from $\frac{80}{2} = 40$: about 25 seconds.

2 **(b) Adults who took longer than 50 seconds**

At 50 seconds the curve reads about 75, so $80 - 75 = 5$ adults.

3 **(b) Percentage**

$$\frac{5}{80} \times 100 = 6.25\%$$

✓ **FINAL ANSWER**

(a) about 25 seconds (b) about 6.25%

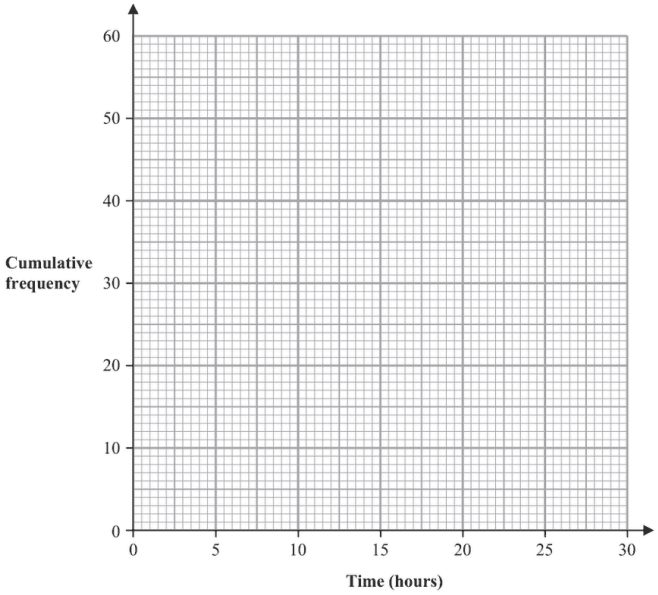
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table gives information about the time, in hours, that each of 60 workers spent working from home in one week.

Time (t hours)	Cumulative frequency
$0 < t \leqslant 5$	6
$0 < t \leqslant 10$	17
$0 < t \leqslant 15$	27
$0 < t \leqslant 20$	42
$0 < t \leqslant 25$	53
$0 < t \leqslant 30$	60

(a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the interquartile range of the times.

..... hours

(2)

25 workers spent more than W hours working from home.

(c) Use your graph to find an estimate for the value of W

$W =$ .....

(2)

One of the 60 workers is chosen at random.
This worker spent H hours working from home.

(d) Find the probability that $5 < H \leqslant 10$

.....

(1)

(Total for Question 11 is 7 marks)

1

(a) Plot the table

Plot $(5, 6), (10, 17), (15, 27), (20, 42), (25, 53), (30, 60)$, start at $(0, 0)$, and join with a smooth curve.

2

(b) Read the quartiles

Lower quartile at $\frac{60}{4} = 15$: about 9 hours. Upper quartile at 45: about 21.4 hours.

$$\text{IQR} \approx 21.4 - 9.1 \approx 12.3 \text{ hours}$$

(Answers from about 11 to 14 are accepted, depending on the curve.)

3

(c) 25 workers above W

So $60 - 25 = 35$ workers spent W hours or less. Reading across from 35: $W \approx 17.7$.

4

(d) Between 5 and 10 hours

$17 - 6 = 11$ workers, so

$$P(5 < H \leq 10) = \frac{11}{60}$$

✓ **FINAL ANSWER**

(b) about 12.3 hours (c) $W \approx 17.7$ (d) $\frac{11}{60}$

Question 13

7 marks | Cumulative Frequency Diagrams

13 The table gives information about the distances, in km, that 70 teachers travel to school.

Distance (d km)	Frequency
$0 < d \leqslant 10$	7
$10 < d \leqslant 20$	17
$20 < d \leqslant 30$	18
$30 < d \leqslant 40$	14
$40 < d \leqslant 50$	10
$50 < d \leqslant 60$	4

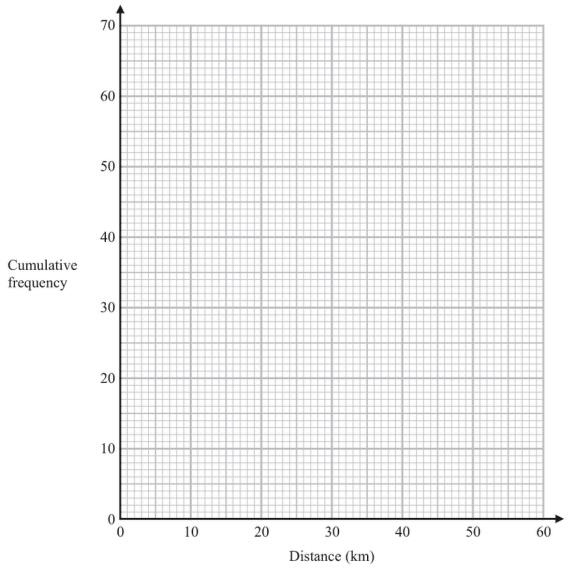
(a) Complete the cumulative frequency table.

Distance (d km)	Cumulative frequency
$0 < d \leqslant 10$	
$0 < d \leqslant 20$	
$0 < d \leqslant 30$	
$0 < d \leqslant 40$	
$0 < d \leqslant 50$	
$0 < d \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)



(c) Use your graph to find an estimate for the interquartile range of the distances.

..... km

(2)

(d) Use your graph to find an estimate for the number of teachers who travel more than 46 km.

.....

(2)

(Total for Question 13 is 7 marks)

1

(a) Running totals

$$7,\ 24,\ 42,\ 56,\ 66,\ 70$$

2

(b) Plot at the upper class boundaries

Plot $(10, 7), (20, 24), (30, 42), (40, 56), (50, 66), (60, 70)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Read the quartiles

Lower quartile at $\frac{70}{4} = 17.5$: about 16 km. Upper quartile at $\frac{3}{4} \times 70 = 52.5$: about 37.5 km.

$$\text{IQR} \approx 37.5 - 16 = 21.5 \text{ km}$$

(Answers from about 19 to 23 km are accepted, depending on the curve.)

4

(d) More than 46 km

At 46 km the curve reads about 62, so

$$70 - 62 = 8 \text{ teachers}$$

✓ **FINAL ANSWER**

(a) $7, 24, 42, 56, 66, 70$ (c) about 21 km (d) about 8

Question 13

6 marks | Cumulative Frequency Diagrams

13 The frequency table gives information about the weights, in kilograms, of 60 parcels in a delivery van.

Weight (w kilograms)	Frequency
$0 < w \leqslant 1$	4
$1 < w \leqslant 2$	15
$2 < w \leqslant 3$	20
$3 < w \leqslant 4$	11
$4 < w \leqslant 5$	6
$5 < w \leqslant 6$	4

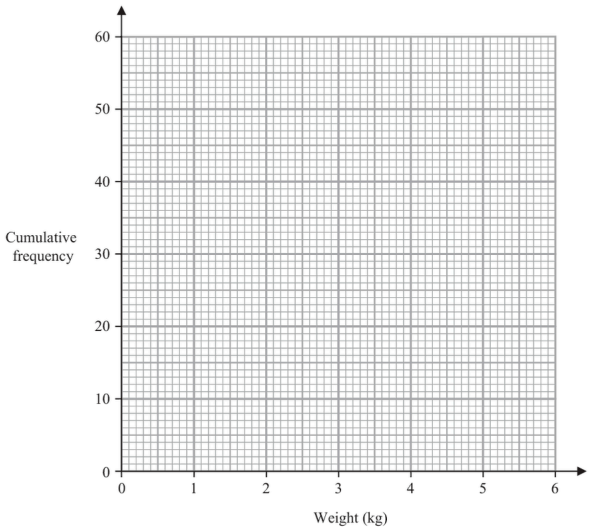
(a) Complete the cumulative frequency table.

Weight (w kilograms)	Cumulative frequency
$0 < w \leqslant 1$	
$0 < w \leqslant 2$	
$0 < w \leqslant 3$	
$0 < w \leqslant 4$	
$0 < w \leqslant 5$	
$0 < w \leqslant 6$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)



(c) Use your graph to find an estimate for the median weight of the 60 parcels.

..... kilograms

(1)

(d) Use your graph to find an estimate for the number of these parcels that weigh more than 3.7 kilograms.

.....

(2)

(Total for Question 13 is 6 marks)

1

(a) Running totals

$$4,\ 19,\ 39,\ 50,\ 56,\ 60$$

2

(b) Plot at the upper class boundaries

Plot $(1, 4), (2, 19), (3, 39), (4, 50), (5, 56), (6, 60)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Median

Read across from $\frac{60}{2} = 30$: about 2.55 kg.

4

(d) Heavier than 3.7 kg

At 3.7 kg the curve reads about 47, so about $60 - 47 = 13$ parcels.

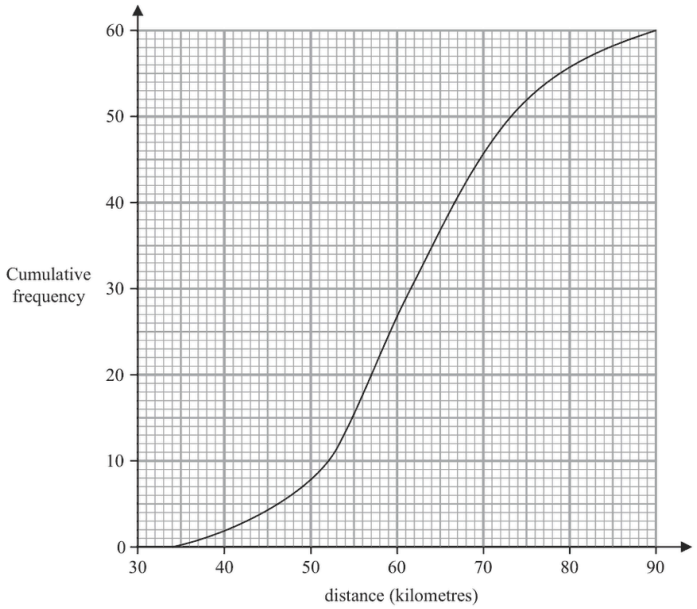
✓ **FINAL ANSWER**

(a) $4, 19, 39, 50, 56, 60$ (c) about 2.55 kg (d) about 13

Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency graph shows information about the distance, in kilometres, Ehrick drives each day for 60 days.



(a) Use the graph to find an estimate for the median of the distances.

..... kilometres
(1)

(b) Use the graph to find an estimate for the interquartile range of the distances.

..... kilometres
(2)

(c) Work out the percentage of days that Ehrick drives for more than 75 kilometres.
Give your answer correct to one decimal place.
Show your working clearly.

..... %
(3)

Tomas and Alison record the number of kilometres that they drive each day for 60 days.
The table gives information about their results.

	Interquartile range
Tomas	28 kilometres
Alison	15 kilometres

(d) Which of Tomas or Alison is more consistent in the number of kilometres that they drive.
Give a reason for your answer.

.....
(1)

(Total for Question 11 is 7 marks)

1 **Required**

a Median(b) IQR(c) % days \> 75 km(d) More consistent?

2 **Median at the th value**

Median at the $frac602 = 30$th value — read from graph at cf = 30:

3 **Answer**

Median ≈ 62 km

4 **Read the graph**

*Q*₁ at cf = 15, *Q*₃ at cf = 45. Read from graph:

*Q*₁ ≈ 56 km, *Q*₃ ≈ 74 km

IQR = *Q*₃ − *Q*₁ = 69 − 55 = 14 km

5 **Answer**

IQR ≈ 14 km

6 **Note**

Graph-reading answers are accepted within ± 2 km of the model answer.

7 **Use cumulative frequency**

From the graph, read the cumulative frequency at *d* = 75 km:

cf at 75 km ≈ 52

8 **Number of days with distance**

Number of days with distance \> 75 km:

60 − 52 = 8 days

9 **Percentage**

Percentage:

$$frac860times100 = 13.3$$

10 **Answer**

13.3% of days Ehrick drives more than 75 km

FINAL ANSWER

Alison is more consistent — her IQR (15 km) is smaller than Tomas's (28 km), meaning less variability in daily distances.

Question 12

6 marks | Cumulative Frequency Diagrams

12 The table gives information about the ages of 80 people in a cinema.

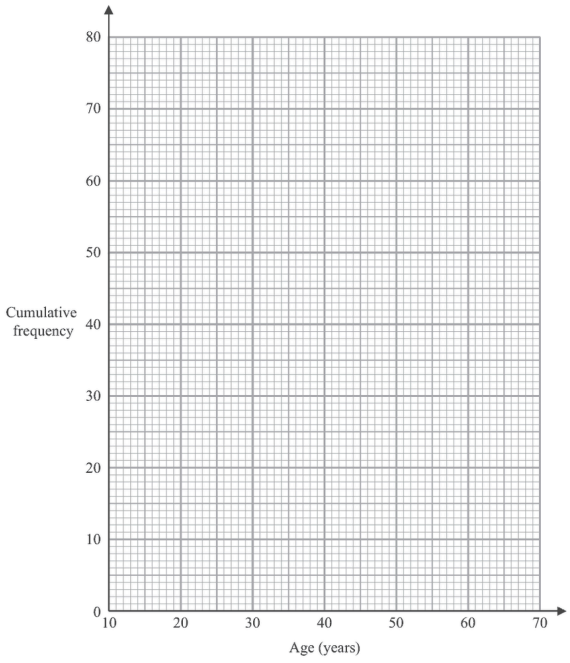
Age (n years)	Frequency
$10 < n \leqslant 20$	12
$20 < n \leqslant 30$	15
$30 < n \leqslant 40$	20
$40 < n \leqslant 50$	18
$50 < n \leqslant 60$	9
$60 < n \leqslant 70$	6

(a) Complete the cumulative frequency table.

Age (n years)	Cumulative frequency
$10 < n \leqslant 20$	
$10 < n \leqslant 30$	
$10 < n \leqslant 40$	
$10 < n \leqslant 50$	
$10 < n \leqslant 60$	
$10 < n \leqslant 70$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the percentage of the 80 people who are more than 46 years of age.
Give your answer correct to the nearest whole number.

.....%

(3)

(Total for Question 12 is 6 marks)

1 **(a) Running totals**

$$12,\ 27,\ 47,\ 65,\ 74,\ 80$$

2 **(b) Plot at the upper class boundaries**

Plot $(20, 12), (30, 27), (40, 47), (50, 65), (60, 74), (70, 80)$, start at $(10, 0)$, and join with a smooth curve.

3 **(c) People aged 46 or under**

Read up from 46: about 58 people.

4 **(c) Percentage older than 46**

$80 - 58 = 22$ people.

$$\frac{22}{80} \times 100 \approx 28\%$$

(About 26% to 30% is accepted, depending on the curve.)

✓ **FINAL ANSWER**

(a) $12, 27, 47, 65, 74, 80$ (c) about 28%

Question 13

6 marks | Cumulative Frequency Diagrams

13 The frequency table gives information about the times, in minutes, that 60 people took to complete a puzzle.

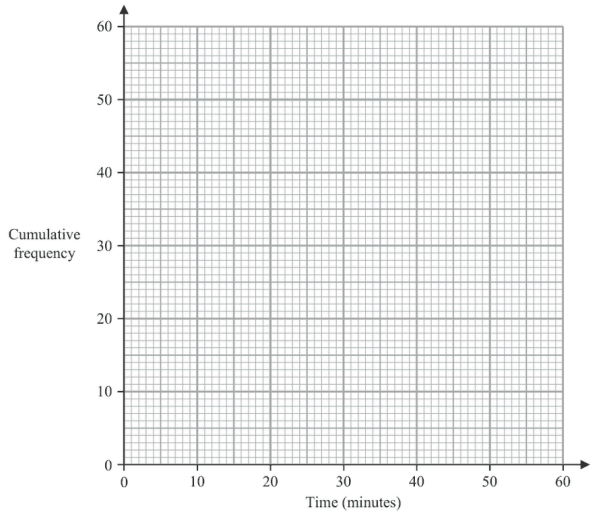
Time (t minutes)	Frequency
$0 < t \leqslant 10$	8
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	17
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	
$0 < t \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time.

..... minutes

(1)

(d) Use your graph to find an estimate for the number of these people who took more than 45 minutes to complete the puzzle.

.....

(2)

(Total for Question 13 is 6 marks)

1

(a) Running totals

$$8,\ 21,\ 33,\ 50,\ 57,\ 60$$

2

(b) Plot at the upper class boundaries

Plot $(10, 8), (20, 21), (30, 33), (40, 50), (50, 57), (60, 60)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Median

Read across from $\frac{60}{2} = 30$: about 27.5 minutes.

4

(d) More than 45 minutes

At 45 minutes the curve reads about 54, so about $60 - 54 = 6$ people. (Answers from about 5 to 8 are accepted, depending on the curve.)

✓ **FINAL ANSWER**

(a) $8, 21, 33, 50, 57, 60$ (c) about 27.5 minutes (d) about 6

Question 14

6 marks | Cumulative Frequency Diagrams

14 The frequency table gives information about the times, in minutes, that 60 people took to complete a puzzle.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	8
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	17
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

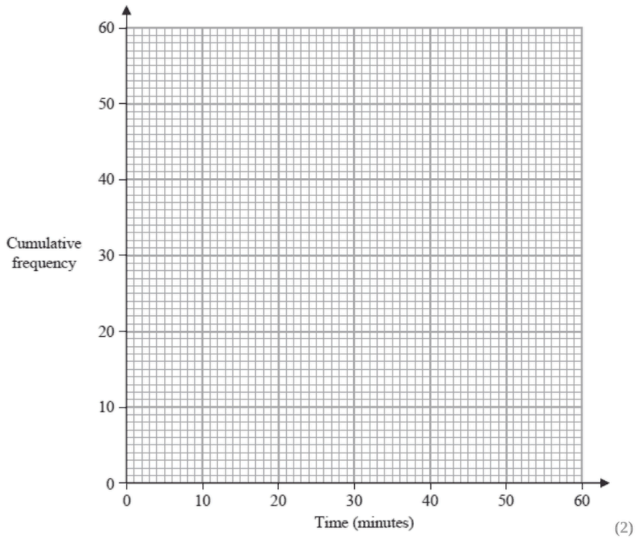
(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$10 < t \leqslant 20$	
$20 < t \leqslant 30$	
$30 < t \leqslant 40$	
$40 < t \leqslant 50$	
$50 < t \leqslant 60$	

(1)

(b) On the grid on the next page, draw a cumulative frequency graph for your table.

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(2)

(c) Use your graph to find an estimate for the median time.

..... minutes

(1)

(d) Use your graph to find an estimate for the number of these people who took more than 45 minutes to complete the puzzle.

.....

(2)

(Total for Question 14 is 6 marks)

1 Cumulative frequencies

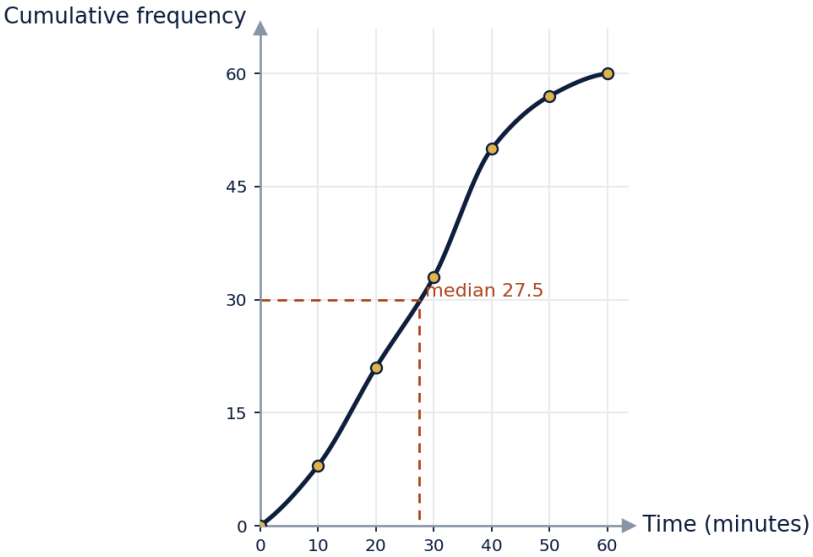
Cumulative frequencies:

$$8,\ 21,\ 33,\ 50,\ 57,\ 60$$

2 Plot the points

Plot the points

$$(10, 8), (20, 21), (30, 33), (40, 50), (50, 57), (60, 60)$$



3 The median is the th

The median is the 30th value. Interpolating in the 20 to 30 interval gives about 27.5 minutes.

4 Estimate the value

For more than 45 minutes, the estimate is about 7 people.

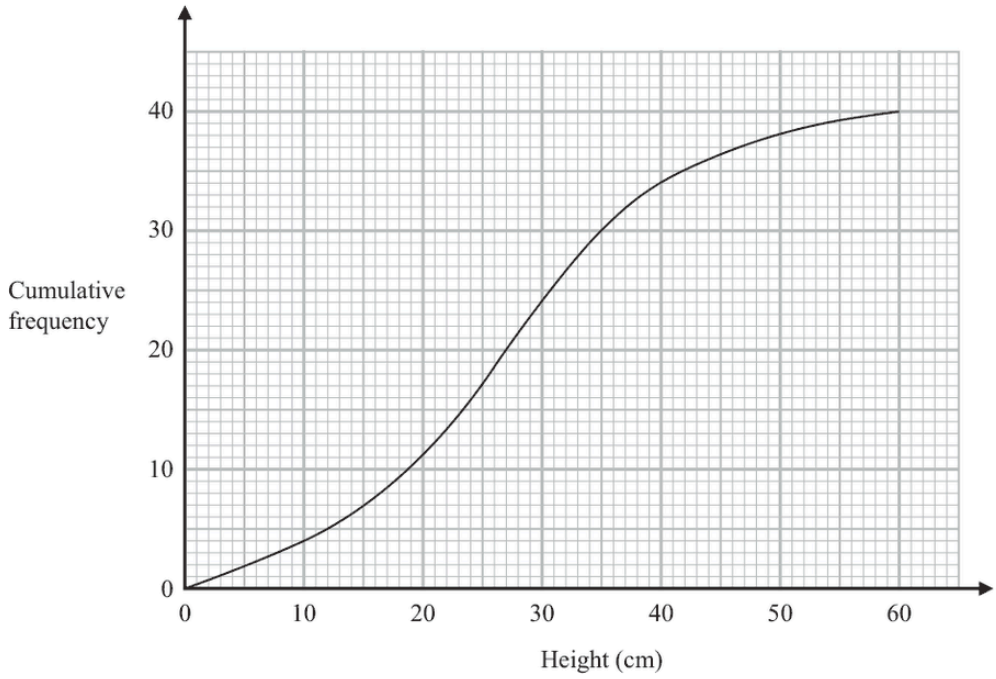
FINAL ANSWER

cumulative frequencies $8, 21, 33, 50, 57, 60$; median about 27.5 minutes; more than 45 minutes about 7 people.

Question 14

5 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph shows information about the heights of 40 plants that Greta has grown.



(a) Use the graph to find an estimate for the median height.

..... cm

(1)

(b) Use the graph to find an estimate for the interquartile range of the heights.

..... cm

(2)

Plants with a height greater than 40 cm are premium plants.
Greta sells all the premium plants for 30 euros each.

(c) Work out the total amount of money Greta receives for the premium plants.

..... euros

(2)

(Total for Question 14 is 5 marks)

1 **Part (a) Median Total plants**

Part (a) Median: Total plants = 40. The median position is at cumulative frequency = 40/2 = 20. Reading across from CF = 20 to the curve and down to the x-axis gives:

2 **Answer**

a 30 cm

3 **Part (b) Interquartile Range (IQR)**

Lower Quartile (Q1) is at CF = 10. Reading the graph ⟹ Height = 20 cm.

Upper Quartile (Q3) is at CF = 30. Reading the graph ⟹ Height = 40 cm.

IQR = *Q*3 − *Q*1 = 40 − 20 = 20 cm

4 **Answer**

b 20 cm

Part (c) Premium plants revenue: Premium plants have height \> 40 cm. From the graph, cumulative frequency at 40 cm is 30. This means 30 plants are ≤ 40 cm.

Number of premium plants = 40 − 30 = 10 plants

Total Money = 10 plants × 30 euros = 300 euros

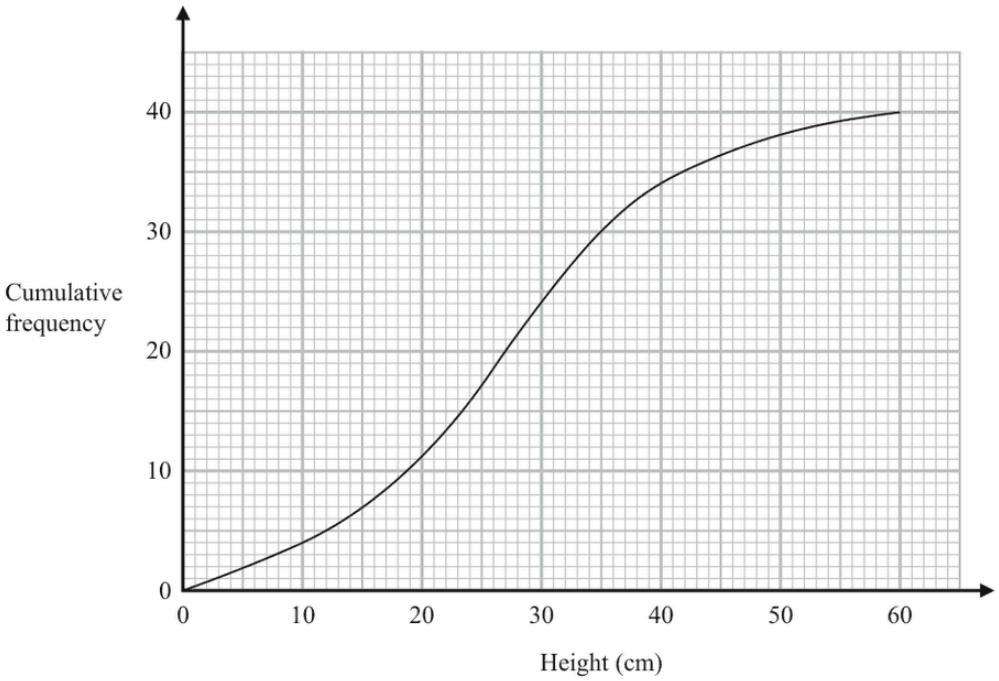
✓ **FINAL ANSWER**

c 300 euros

Question 14

5 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph shows information about the heights of 40 plants that Greta has grown.



(a) Use the graph to find an estimate for the median height.

..... cm

(1)

(b) Use the graph to find an estimate for the interquartile range of the heights.

..... cm

(2)

Plants with a height greater than 40 cm are premium plants.
Greta sells all the premium plants for 30 euros each.

(c) Work out the total amount of money Greta receives for the premium plants.

..... euros

(2)

(Total for Question 14 is 5 marks)

1

(a) Median

Read across from $\frac{40}{2} = 20$: about 27 cm.

2

(b) Interquartile range

Lower quartile at 10: about 19 cm. Upper quartile at 30: about 35 cm.

$$\text{IQR} \approx 35 - 19 = 16 \text{ cm}$$

3

(c) Premium plants

At 40 cm the curve reads about 34, so $40 - 34 = 6$ plants are taller than 40 cm.

$$6 \times 30 = 180 \text{ euros}$$

FINAL ANSWER

(a) about 27 cm (b) about 16 cm (c) 180 euros

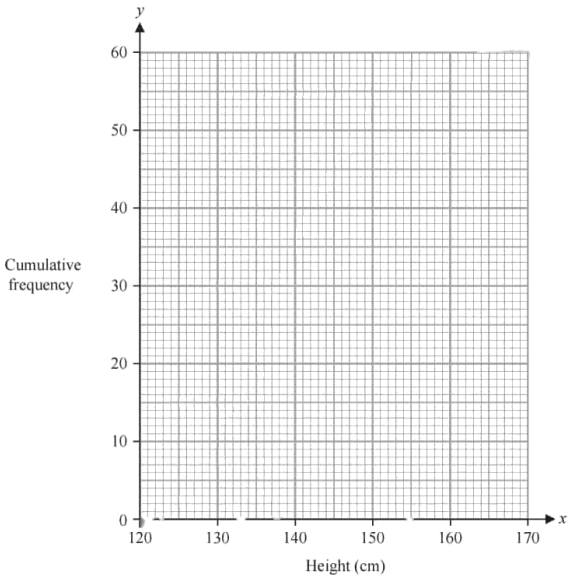
Question 13

6 marks | Cumulative Frequency Diagrams

13 The cumulative frequency table shows information about the heights, in centimetres, of 60 plants.

Height (h cm)	Cumulative frequency
$120 \leqslant h < 130$	8
$120 \leqslant h < 140$	35
$120 \leqslant h < 150$	52
$120 \leqslant h < 160$	58
$120 \leqslant h < 170$	60

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use your graph to find an estimate for the median.

..... cm

(1)

(c) Use your graph to find an estimate for the lower quartile.

..... cm

(1)

(d) Use your graph to find an estimate for the number of plants with a height greater than 155 cm

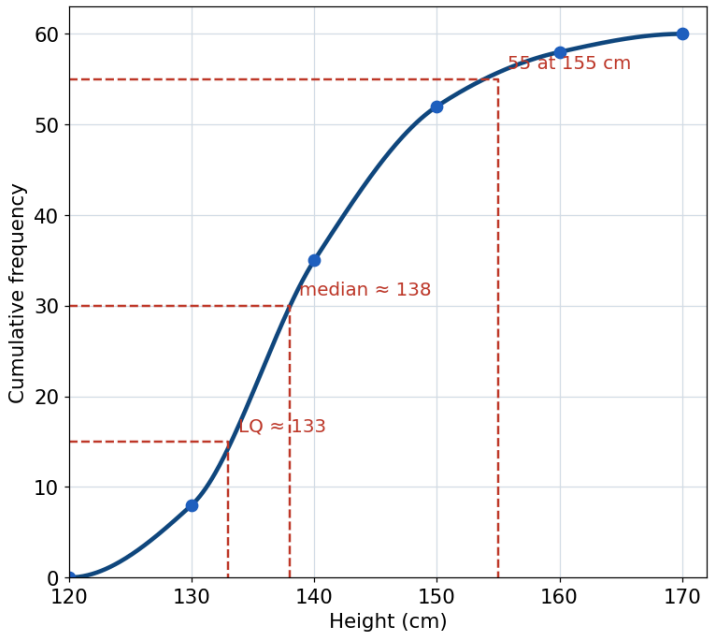
.....

(2)

(Total for Question 13 is 6 marks)

1 (a) Plot at the upper class boundaries

Plot $(130, 8), (140, 35), (150, 52), (160, 58), (170, 60)$, start at $(120, 0)$ and join with a smooth curve.



2 (b) Median

$\frac{1}{2} \times 60 = 30$: read across from 30 → about 138 cm.

3 (c) Lower quartile

$\frac{1}{4} \times 60 = 15$: read across from 15 → about 133 cm.

4 (d) Taller than 155 cm

Up from 155 cm the curve reads about 55, so $60 - 55 = 5$ plants.

FINAL ANSWER

(b) 138 cm (c) 133 cm (d) about 5 plants

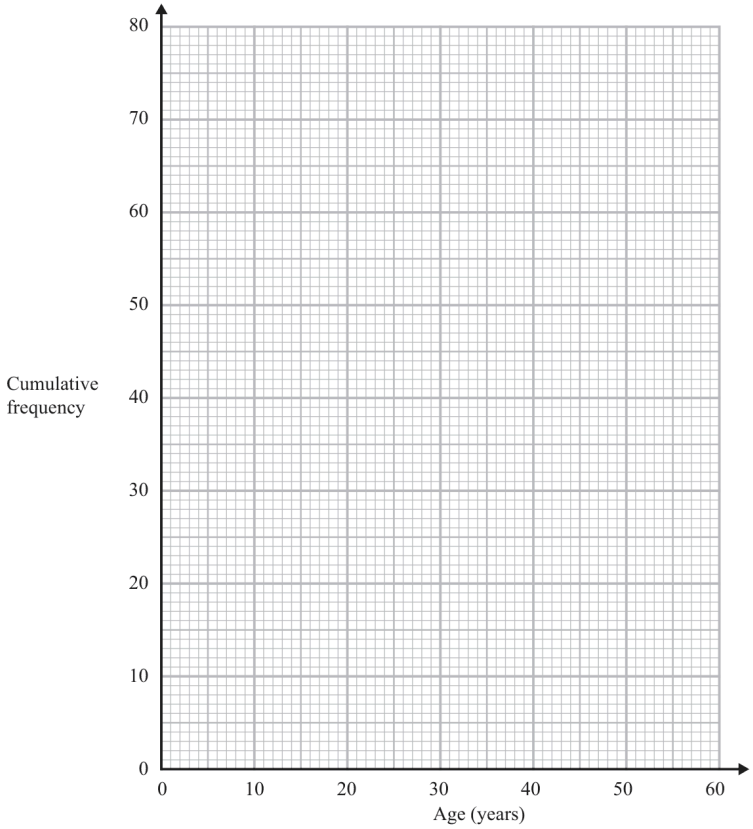
Question 14

4 marks | Cumulative Frequency Diagrams

14 The cumulative frequency table gives information about the ages, in years, of 80 people who went to a cinema on Saturday.

Age (a years)	Cumulative frequency
$0 < a \leqslant 10$	19
$0 < a \leqslant 20$	37
$0 < a \leqslant 30$	52
$0 < a \leqslant 40$	63
$0 < a \leqslant 50$	73
$0 < a \leqslant 60$	80

(a) On the grid below, draw a cumulative frequency graph for this information.



(2)

(b) Use your graph to find an estimate for the median age of the 80 people.

..... years

(1)

(c) Use your graph to find an estimate for the number of the 80 people aged 16 and under.

.....

(1)

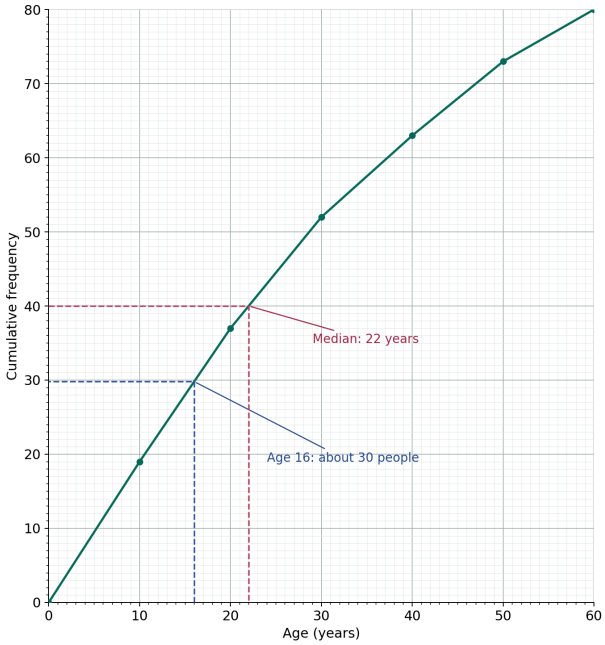
(Total for Question 14 is 4 marks)

1 **Part (a) | Step 1**

Use age (years) on the horizontal axis from 0 to 60, and cumulative frequency on the vertical axis from 0 to 80. Plot $(0, 0), (10, 19), (20, 37), (30, 52), (40, 63), (50, 73), (60, 80)$.
Each cumulative total is plotted at its upper age boundary.

2 **Part (a) | Step 2**

Join the points in order with an increasing curve or straight line segments. Do not draw bars or a staircase.
This represents the accumulated number up to each age boundary.



3 **Part (b) | Step 1**

$$\text{median position} = \frac{80}{2} = 40.$$

Read horizontally from cumulative frequency 40 to the graph, then vertically down to the age axis.

4 **Part (b) | Step 2**

Using straight segments,

$$20 + \frac{40 - 37}{52 - 37} \times 10 = 22 \text{ years}.$$

The median is estimated between ages 20 and 30; a smooth graph gives a nearby estimate.

5 **Part (c) | Step 1**

Read vertically from age 16 to the graph, then horizontally to the cumulative-frequency axis.
The ordinate is the estimated number aged 16 and under.

6

Part (c) | Step 2

Using straight segments,

$$19 + \frac{16 - 10}{20 - 10}(37 - 19) = 29.8 \approx 30.$$

Round the estimated count to a whole number of people.

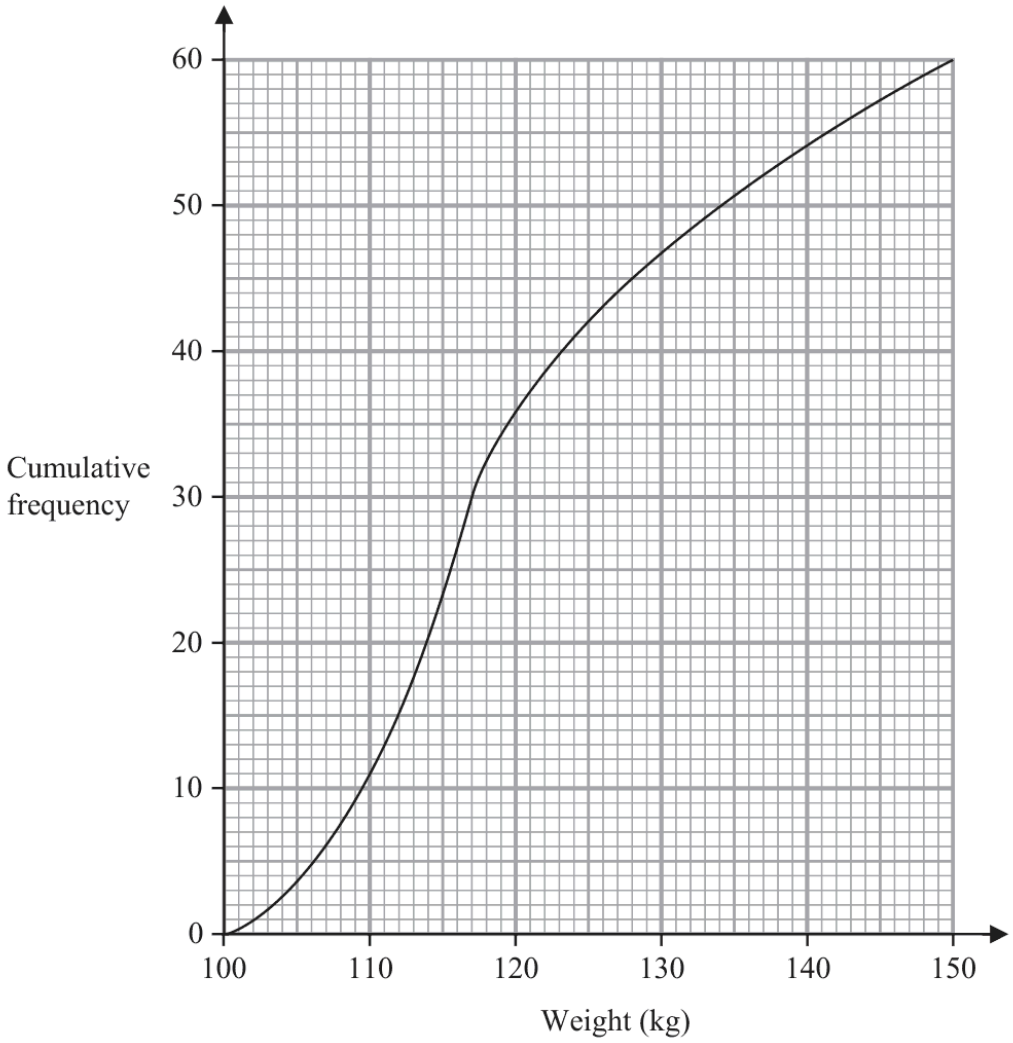
FINAL ANSWER

(a) Increasing cumulative-frequency graph through $(0, 0), (10, 19), (20, 37), (30, 52), (40, 63), (50, 73), (60, 80)$. (b) Approximately 22 years. (c) Approximately 30 people.

Question 14

3 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph gives information about the weights, in kg, of 60 crates.



(a) Use the graph to find an estimate for the median weight.

..... kg

(1)

(b) Use the graph to find an estimate for the interquartile range.

..... kg

(2)

(Total for Question 14 is 3 marks)

1

Part (a) | Step 1

The median is the $60/2 = 30$th value. Draw horizontally from cumulative frequency 30 to the curve, then vertically to the weight axis: about $117\ \text{kg}$.

The median splits the 60 observations in half.

2

Part (b) | Step 1

Read the lower quartile at cumulative frequency $60/4 = 15$, giving $Q_1 \approx 112\ \text{kg}$. Read the upper quartile at cumulative frequency $3(60)/4 = 45$, giving $Q_3 \approx 128\ \text{kg}$.

Quartiles occur at one-quarter and three-quarters of the total frequency.

3

Part (b) | Step 2

$$\text{IQR} = Q_3 - Q_1 \approx 128 - 112 = 16\ \text{kg}.$$

Interquartile range is upper quartile minus lower quartile.

✓ **FINAL ANSWER**

(a) $117\ \text{kg}$ approximately. (b) $16\ \text{kg}$ approximately.

TOPIC

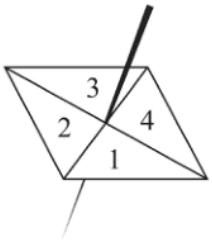
Probability Toolkit

Every past-paper question on this topic, 2020 to 2026.

Question 3

4 marks | Probability Toolkit

3 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

Number	1	2	3	4
Probability	0.26		0.18	

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4

Priya is going to spin the spinner 250 times.

Work out an estimate for the number of times the spinner will land on a 1 or a 2

.....

(Total for Question 3 is 4 marks)

1 **Find the remaining probability**

$$1 - 0.26 - 0.18 = 0.56$$

2 **Split the equal probabilities**

The probabilities for 2 and 4 are equal, so

$$P(2) = P(4) = \frac{0.56}{2} = 0.28$$

3 **Find the probability of 1 or 2**

$$P(1 \text{ or } 2) = 0.26 + 0.28 = 0.54$$

4 **Calculate the estimate**

$$0.54 \times 250 = 135$$

FINAL ANSWER

135.

4MA1/2H JANUARY 2020

Question 6

4 marks | Probability Toolkit

6 A bag contains only red beads, blue beads, green beads and yellow beads.

The table gives the probabilities that, when a bead is taken at random from the bag, the bead will be blue or the bead will be yellow.

Colour	red	blue	green	yellow
Probability		0.24		0.31

The probability that the bead will be green is twice the probability that the bead will be red.

Sofia takes at random a bead from the bag.
She writes down the colour of the bead and puts the bead back into the bag.

She does this 180 times.

Work out an estimate for the number of times she takes a red bead from the bag.

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2020

1 Use the fact that probabilities add to 1

Let $P(\text{red}) = r$, so $P(\text{green}) = 2r$.

$$r + 0.24 + 2r + 0.31 = 1 \Rightarrow 3r = 0.45 \Rightarrow r = 0.15$$

2 Expected number of red beads

$$180 \times 0.15 = 27$$

FINAL ANSWER

27

4MA1/1H JANUARY 2020

Question 18

4 marks | Probability Toolkit

18 There are 16 sweets in a bowl.

4 of the sweets are blackcurrant.
5 of the sweets are lemon.
7 of the sweets are orange.

Anna, Ravi and Sam each take at random one sweet from the bowl.

Work out the probability that the 5 lemon sweets are still in the bowl.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2020

1 Count the sweets that are not lemon

There are 16 sweets and 5 are lemon, so 11 are not lemon.

2 Three picks without replacement, none lemon

All 5 lemon sweets stay in the bowl only if Anna, Ravi and Sam each take a non-lemon sweet:

$$\frac{11}{16} \times \frac{10}{15} \times \frac{9}{14} = \frac{990}{3360} = \frac{33}{112}$$

FINAL ANSWER

$$\frac{33}{112}$$

4MA1/1H MAY/NOVEMBER 2020

Question 2

4 marks | Probability Toolkit

2 Toy cars are made in a factory.
The toy cars are made for 15 hours each day.
5 toy cars are made every 12 seconds.

For the toy cars made each day, the probability of a toy car being faulty is 0.002

Work out an estimate of the number of faulty toy cars that are made each day.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May/November 2020

1 Seconds in a working day

$$15 \times 60 \times 60 = 54\,000 \text{ seconds}$$

2 Toy cars made each day

$$\frac{54\,000}{12} \times 5 = 22\,500$$

3 Expected number of faulty cars

$$22\,500 \times 0.002 = 45$$

✓ **FINAL ANSWER**

45

4MA1/1HR MAY/NOVEMBER 2020

Question 23

4 marks | Probability Toolkit

23 In a bag, there are only

3 blue beads
4 white beads
and x orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May/November 2020

1 The total number of beads

The total number of beads is

$$x + 7$$

2 The probability of choosing two

The probability of choosing two beads of the same colour is

$$\frac{\binom{3}{2} + \binom{4}{2} + \binom{x}{2}}{\binom{x+7}{2}}$$

3 **Evaluate fraction**

This is equal to $\frac{3}{8}$, so

$$\frac{3+6+\frac{x(x-1)}{2}}{\frac{(x+7)(x+6)}{2}}=\frac{3}{8}$$

$$\frac{x^2-x+18}{(x+7)(x+6)}=\frac{3}{8}$$

$$8(x^2-x+18)=3(x+7)(x+6)$$

$$8x^2-8x+144=3x^2+39x+126$$

$$5x^2-47x+18=0$$

$$(x-9)(5x-2)=0$$

4 **Calculate value**

Since x must be a whole number,

$$x=9$$

5 **Calculate value**

So the total number of beads is

$$9+7=16$$

FINAL ANSWER

16 beads

4MA1/2HR JANUARY 2021

Question 21

6 marks | Probability Toolkit

21 A bag contains n beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of n.

$n =$ .....

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2021

1 There are beads altogether

There are n beads altogether.

2 Red beads

Red beads:

$$6$$

3 Blue beads

Blue beads:

$$n - 6$$

4 The probability of taking two

The probability of taking two beads of the same colour is

$$P(RR) + P(BB) = \frac{9}{17}$$

5 **Use trigonometry**

So

$$\frac{6}{n} \cdot \frac{5}{n-1} + \frac{n-6}{n} \cdot \frac{n-7}{n-1} = \frac{9}{17}$$

$$\frac{30 + (n-6)(n-7)}{n(n-1)} = \frac{9}{17}$$

6 **Expand**

Expand:

$$(n-6)(n-7) = n^2 - 13n + 42$$

7 **Use trigonometry**

So

$$\frac{n^2 - 13n + 72}{n(n-1)} = \frac{9}{17}$$

8 **Cross multiply**

Cross multiply:

$$17(n^2 - 13n + 72) = 9n(n-1)$$

$$17n^2 - 221n + 1224 = 9n^2 - 9n$$

$$8n^2 - 212n + 1224 = 0$$

9 **Divide by 4**

Divide by 4:

$$2n^2 - 53n + 306 = 0$$

10 **Factor**

Factor:

$$(2n - 17)(n - 18) = 0$$

11 **Use trigonometry**

So

$$n = \frac{17}{2} \quad \text{or} \quad n = 18$$

12 **The number of beads must**

The number of beads must be a whole number, so

$$n = 18$$

✓ **FINAL ANSWER**

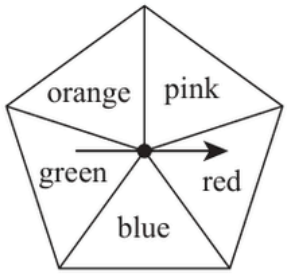
$\boxed{18}$

4MA1/1H MAY 2021

Question 6

5 marks | Probability Toolkit

6 Grace has a biased 5-sided spinner.



Grace is going to spin the arrow on the spinner once.

The table below gives the probabilities that the spinner will land on red or on blue or on green.

Colour	Red	Blue	Green	Orange	Pink
Probability	0.20	0.12	0.08		

The probability that the spinner will land on orange is 3 times the probability that the spinner will land on pink.

(a) Work out the probability that the spinner will land on orange.

.....

(3)

Grace spins the arrow on the spinner 150 times.

(b) Work out an estimate for the number of times the spinner lands on blue.

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2021

1 **(a) Set up with the missing probabilities**

Let $P(\text{pink}) = x$, so $P(\text{orange}) = 3x$.

$$0.20 + 0.12 + 0.08 + 3x + x = 1 \Rightarrow 4x = 0.60 \Rightarrow x = 0.15$$

2 **(a) Probability of orange**

$$3x = 0.45$$

3 (b) Estimate for blue

$$150 \times 0.12 = 18$$

FINAL ANSWER

(a) 0.45 (b) 18

4MA1/2H MAY 2021

Question 23

5 marks | Probability Toolkit

23 Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H May 2021

1 Let the number of red

Let the number of red pens be r.

2 List probability cases

Then the number of black pens is

$$r + 3$$

3 Calculate value

and

$$N = 2r + 3$$

4 Evaluate fraction

So

$$r = \frac{N - 3}{2}$$

5 **Evaluate fraction**

and

$$r+3=\frac{N+3}{2}$$

6 **The probability of black followed**

The probability of black followed by red is

$$\frac{(N+3)/2}{N}\cdot\frac{(N-3)/2}{N-1}=\frac{9}{35}$$

$$\frac{N^2-9}{4N(N-1)}=\frac{9}{35}$$

$$35(N^2-9)=36N(N-1)$$

$$35N^2-315=36N^2-36N$$

$$N^2-36N+315=0$$

$$(N-15)(N-21)=0$$

✓ **FINAL ANSWER**

$N=15$ or $N=21$

4MA1/1H NOVEMBER 2021

Question 3

4 marks | Probability Toolkit

3 There are 54 fish in a tank.
Some of the fish are white and the rest of the fish are red.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is $\frac{4}{9}$

(a) Work out the number of white fish originally in the tank.

.....

(2)

Jeevan puts the fish he took out, back into the tank.
He puts some more white fish into the tank.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is now $\frac{1}{2}$

(b) Work out the number of white fish Jeevan put into the tank.

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2021

1 Calculate probability

(a) Let the number of white fish be w.

$$\frac{w}{54} = \frac{4}{9}$$

$$w = 54 \times \frac{4}{9} = 24$$

2 (b) There are still 54

(b) There are still 54 fish, and now

$$\frac{\text{white fish}}{54} = \frac{1}{2}$$

$$\text{white fish} = 27$$

3 **Jeevan added**

Jeevan added

$$27 - 24 = 3$$

FINAL ANSWER

(a) 24 white fish, (b) 3 white fish.

4MA1/2HR JANUARY 2022

Question 1

4 marks | Probability Toolkit

1 A tin contains tea bags with a choice of four different flavours of tea.
The four flavours of tea are Assam or Darjeeling or Nilgiri or Rize.

Sara takes at random a tea bag from the tin.

The table shows each of the probabilities that the flavour of the tea Sara takes is Assam or Darjeeling or Rize.

Flavour of tea	Assam	Darjeeling	Nilgiri	Rize
Probability	0.38	0.24		0.16

(a) Work out the probability that the flavour of the tea Sara takes is Nilgiri.

.....

(2)

(b) Work out the probability that the flavour of the tea Sara takes is either Darjeeling or Rize.

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2022

1 **(a) Probabilities add to 1**

$$P(\text{Nilgiri}) = 1 - (0.38 + 0.24 + 0.16) = 1 - 0.78 = 0.22$$

2 **(b) Darjeeling or Rize**

$$0.24 + 0.16 = 0.40$$

✓ **FINAL ANSWER**

(a) 0.22 (b) 0.4

4MA1/1HR JANUARY 2022

Question 2

2 marks | Probability Toolkit

2 Each time John plays a game, the probability that he wins the game is 0.65

John is going to play the game 300 times.

Work out an estimate for the number of games that John wins.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2022

1 **Expected number = probability × number of games**

$$0.65 \times 300 = 195$$

FINAL ANSWER

195

4MA1/2H JANUARY 2022

Question 6

5 marks | Probability Toolkit

6 A bag contains only pink sweets, white sweets, green sweets and red sweets.

The table gives each of the probabilities that, when a sweet is taken at random from the bag, the sweet will be green or the sweet will be red.

Sweet	pink	white	green	red
Probability			0.2	0.35

The ratio
number of pink sweets : number of white sweets = 2 : 1

There are 28 red sweets in the bag.

Work out the number of white sweets in the bag.

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2022

1 **Split the ratio**

The probabilities for green and red are

$$0.20 + 0.35 = 0.55$$

2 **Split the ratio**

So

$$P(\text{pink or white}) = 1 - 0.55 = 0.45$$

3 **Split the ratio**

Pink : white $= 2 : 1$, so white is $\frac{1}{3}$ of 0.45.

$$P(\text{white}) = \frac{1}{3} \times 0.45 = 0.15$$

4 **Split the ratio**

The red probability is 0.35, and there are 28 red sweets.

$$\text{total sweets} = \frac{28}{0.35} = 80$$

5 **Number of white sweets**

Number of white sweets:

$$0.15 \times 80 = 12$$

✓ **FINAL ANSWER**

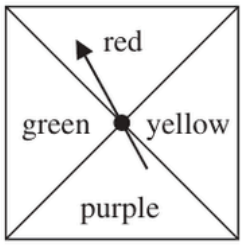
12

4MA1/1HR JUNE 2022

Question 1

4 marks | Probability Toolkit

1 Here is a biased spinner.



When the spinner is spun once, the probabilities that it lands on red or on yellow or on green are given in the table.

Colour	red	yellow	purple	green
Probability	0.25	0.2		0.2

(a) Work out the probability that the spinner lands on red or on yellow.

.....

(1)

Yang is going to spin the spinner 300 times.

(b) Work out an estimate for the number of times the spinner will land on purple.

.....

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2022

1 **(a) Red or yellow**

$$0.25 + 0.2 = 0.45$$

2 **(b) Probability of purple**

$$1 - (0.25 + 0.2 + 0.2) = 0.35$$

3 **(b) Estimate for 300 spins**

$$300 \times 0.35 = 105$$

FINAL ANSWER

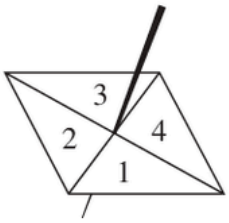
(a) 0.45 (b) 105

4MA1/1H JUNE 2022

Question 2

4 marks | Probability Toolkit

2 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

Number	1	2	3	4
Probability	0.26		0.18	

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4

Ravina is going to spin the spinner a number of times.

Ravina works out that an estimate for the number of times the spinner will land on 3 is 45

Work out an estimate for the number of times the spinner will land on 4

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2022

1 **Probabilities of landing on 2 and on 4**

The probabilities add to 1, and $P(2) = P(4)$:

$$1 - (0.26 + 0.18) = 0.56, \qquad P(4) = 0.56 \div 2 = 0.28$$

2 **Number of spins**

An estimate of 45 for the number 3 means

$$0.18 \times n = 45 \Rightarrow n = 250$$

3 **Estimate for landing on 4**

$$250 \times 0.28 = 70$$

FINAL ANSWER

70

4MA1/1H JUNE 2022

Question 15

4 marks | Probability Toolkit

15 Abraham is going to play a computer game.
Abraham can win the game, draw the game or lose the game.

For any game that Abraham plays

the probability that he wins the game is 0.3
the probability that he draws the game is 0.5
the probability that he loses the game is 0.2

When Abraham wins a game, he scores +10 points.
When Abraham draws a game, he scores 0 points.
When Abraham loses a game, he scores −5 points.

Abraham plays 3 games and the points he scores in each of the 3 games are added together to get his total score.

Work out the probability that when he has played 3 games his total score is 0 points.

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H June 2022

1 Ways to score a total of 0

Three draws ($0+0+0$), or one win and two losses ($10-5-5=0$) in any order.

2 Three draws

$$0.5^3 = 0.125$$

3 One win and two losses (3 orders)

$$3 \times 0.3 \times 0.2 \times 0.2 = 0.036$$

4 Add the cases

$$0.125 + 0.036 = 0.161$$

FINAL ANSWER

0.161

4MA1/1HR JUNE 2022

Question 24

5 marks | Probability Toolkit

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2022

1 There are counters. Initially 5

There are x counters. Initially 5 show red, so $x - 5$ show green.

2 List probability cases

After two turns, there are still 5 red faces if:

3 the same counter is picked

- the same counter is picked twice, or
- one red counter and one green counter are picked.

4 The probability of picking the

The probability of picking the same counter twice is

$$\frac{1}{x}$$

5 **The probability of picking one**

The probability of picking one red and one green in either order is

$$\frac{5}{x} \cdot \frac{x-5}{x} + \frac{x-5}{x} \cdot \frac{5}{x}$$

$$= \frac{10(x-5)}{x^2}$$

---

6 **Evaluate fraction**

So

$$\frac{1}{x} + \frac{10(x-5)}{x^2} = \frac{19}{32}$$

$$\frac{11x-50}{x^2} = \frac{19}{32}$$

$$32(11x-50) = 19x^2$$

$$19x^2 - 352x + 1600 = 0$$

$$(x-8)(19x-200) = 0$$

---

7 **Calculate value**

Since x must be a whole number,

$$x = 8$$

✓ **FINAL ANSWER**

$x = 8$

4MA1/2H JUNE 2023

Question 2

3 marks | Probability Toolkit

2 A biased spinner can land on green or on yellow or on brown or on pink.

The table gives the probabilities that, when the spinner is spun, it will land on green or on yellow or on brown.

Colour	green	yellow	brown	pink
Probability	0.32	0.13	0.28	

Timucin spins the spinner 200 times.

Work out an estimate for the number of times the spinner lands on pink.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2023

1 Probability of pink

$$1 - (0.32 + 0.13 + 0.28) = 1 - 0.73 = 0.27$$

2 Estimate for 200 spins

$$200 \times 0.27 = 54$$

FINAL ANSWER

54

4MA1/1HR MAY 2023

Question 5

2 marks | Probability Toolkit

5 Each time Evie plays a game against her computer, she will win or lose.

For each game, the probability that Evie will win is 0.74
Evie is going to play 300 games against her computer.

Work out an estimate for the number of games that Evie will lose.

(Total for Question 5 is 2 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May 2023

1 Probability of losing

$$1 - 0.74 = 0.26$$

2 Estimate for 300 games

$$300 \times 0.26 = 78$$

FINAL ANSWER

78

4MA1/2HR JUNE 2023

Question 15

3 marks | Probability Toolkit

15 Osvaldo has a biased coin.
He spins the coin three times.

The probability that the coin lands on a head three times is $\frac{27}{64}$

Work out the probability that the coin will land on a tail three times.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2023

1 Find the probability of a head

$$P(H)^3 = \frac{27}{64} \Rightarrow P(H) = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

2 Three tails

$P(T) = 1 - \frac{3}{4} = \frac{1}{4}$, so

$$P(TTT) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

FINAL ANSWER

$$\frac{1}{64}$$

4MA1/2HR JUNE 2023

Question 19

4 marks | Probability Toolkit

19 30 adults booked to stay in a hotel.

19 adults booked breakfast
15 adults booked dinner
4 adults did not book breakfast or dinner

Some adults booked breakfast **and** dinner.

Meihui chooses at random two of the 30 adults.

Work out the probability that these two adults each booked breakfast **and** dinner.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2023

1 **Adults who booked breakfast and dinner**

$30 - 4 = 26$ adults booked at least one meal.

$$n(B \cap D) = 19 + 15 - 26 = 8$$

2 **Choose two of them (without replacement)**

$$\frac{8}{30} \times \frac{7}{29} = \frac{56}{870} = \frac{28}{435}$$

✓ **FINAL ANSWER**

$$\frac{28}{435}$$

4MA1/1H NOVEMBER 2023

Question 4

4 marks | Probability Toolkit

4 A biased spinner has three sections each of a different colour.

The table shows the probability that, when the spinner is spun once, it will land on blue or on orange or on white.

Colour	blue	orange	white
Probability	0.58	$2x$	x

(a) Work out the value of x

$x =$ .....

(2)

The spinner is spun 250 times.

(b) Work out an estimate for the number of times the spinner will land on blue.

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2023

1 **(a) Probabilities add to 1**

$$0.58 + 2x + x = 1 \Rightarrow 3x = 0.42 \Rightarrow x = 0.14$$

2 **(b) Estimate for blue in 250 spins**

$$250 \times 0.58 = 145$$

✓ **FINAL ANSWER**

(a) $x = 0.14$ (b) 145

4MA1/1H NOVEMBER 2023

Question 20

4 marks | Probability Toolkit

20 A bag contains only 10 cent coins and 20 cent coins.

Josip takes at random a coin from the bag, records its value and replaces it in the bag. He then takes at random a second coin from the bag, records its value and replaces it in the bag.

Josip finds the mean value of the two coins.

The probability that the two coins have a mean value of 10 cents is $\frac{49}{121}$

Work out the probability that the two coins have a mean value of 15 cents.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2023

1 Let the probability of taking

Let the probability of taking a 10 cent coin be p.

2 The mean is 10 cents

The mean is 10 cents only when both coins are 10 cent coins.

$$p^2 = \frac{49}{121}$$

$$p = \frac{7}{11}$$

3 Calculate probability

So the probability of taking a 20 cent coin is

$$1 - \frac{7}{11} = \frac{4}{11}$$

4 The mean is 15 cents

The mean is 15 cents when one coin is 10 cents and the other is 20 cents.

5 **There are two possible orders**

There are two possible orders:

$$P(10, 20) + P(20, 10)$$

$$= \frac{7}{11} \cdot \frac{4}{11} + \frac{4}{11} \cdot \frac{7}{11}$$

$$= \frac{56}{121}$$

✓ **FINAL ANSWER**

$\frac{56}{121}$

4MA1/1H MAY 2024

Question 2

4 marks | Probability Toolkit

2 450 students were asked how they travelled to school on Monday.
Each student walked or travelled by bus or travelled by car or travelled by bicycle.
Each student used just one method of travel.

One of these students is chosen at random.
The table shows information about the probability of each method of travel.

Method of travel	walk	bus	car	bicycle
Probability	0.20	x	$2x$	0.26

Work out how many of the 450 students travelled by car.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2024

1 Find x

$$0.20 + x + 2x + 0.26 = 1 \Rightarrow 3x = 0.54 \Rightarrow x = 0.18$$

2 Probability of travelling by car

$$2x = 0.36$$

3 Number of students

$$450 \times 0.36 = 162$$

✓ **FINAL ANSWER**

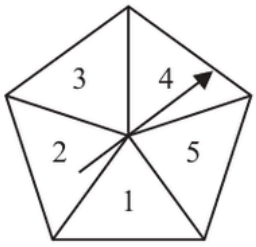
162

4MA1/1HR MAY 2024

Question 2

4 marks | Probability Toolkit

2 Here is a biased spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4	5
Probability	$2x$	0.27	0.04	x	0.12

Alexis is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on an odd number.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR May 2024

1 Find x

$$2x + 0.27 + 0.04 + x + 0.12 = 1 \Rightarrow 3x = 0.57 \Rightarrow x = 0.19$$

2 Probability of an odd number (1, 3 or 5)

$$2x + 0.04 + 0.12 = 0.38 + 0.04 + 0.12 = 0.54$$

3 Estimate for 400 spins

$$400 \times 0.54 = 216$$

FINAL ANSWER

216

4MA1/2HR JUNE 2024

Question 14

3 marks | Probability Toolkit

14 Adriana is going to roll a biased dice and spin a biased coin.

The probability that the coin will land on Heads is 0.8
The probability that the dice will land on 6 and the coin will land on Heads is 0.24

Work out the probability that the dice will land on 6 and the coin will land on Tails.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2024

1 Find the probability of a 6

The dice and the coin are independent:

$$P(6) \times 0.8 = 0.24 \Rightarrow P(6) = 0.3$$

2 6 and Tails

$P(\text{Tails}) = 1 - 0.8 = 0.2$, so

$$0.3 \times 0.2 = 0.06$$

FINAL ANSWER

0.06

4MA1/1HR MAY 2024

Question 21

5 marks | Probability Toolkit

21 There are 25 counters in a bag such that

6 counters are blue
x counters are orange, where $x > 9$
the rest of the counters are pink

Maalam takes at random two of the counters from the bag.

The probability that Maalam takes one orange counter and one pink counter is $\frac{22}{75}$

Calculate the probability that Maalam takes 2 pink counters from the bag.
Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2024

1 **There are 25 counters**

There are 25 counters.

2 **Blue counters**

Blue counters:

$$6$$

3 **Orange counters**

Orange counters:

$$x$$

4 **Pink counters**

Pink counters:

$$25 - 6 - x = 19 - x$$

5 **The probability of taking one**

The probability of taking one orange and one pink in either order is

$$\frac{x}{25} \cdot \frac{19-x}{24} + \frac{19-x}{25} \cdot \frac{x}{24}$$

$$= \frac{2x(19-x)}{25 \cdot 24}$$

---

6 **Evaluate fraction**

This is equal to $\frac{22}{75}$, so

$$\frac{2x(19-x)}{600} = \frac{22}{75}$$

$$\frac{x(19-x)}{300} = \frac{22}{75}$$

$$x(19-x) = 88$$

$$19x - x^2 = 88$$

$$x^2 - 19x + 88 = 0$$

$$(x-8)(x-11) = 0$$

---

7 **Solve inequality**

Since $x > 9$,

$$x = 11$$

---

8 **Calculate value**

So the number of pink counters is

$$19 - 11 = 8$$

---

9 **The probability of taking 2**

The probability of taking 2 pink counters is

$$\frac{8}{25} \cdot \frac{7}{24} = \frac{56}{600}$$

$$= \frac{7}{75}$$

FINAL ANSWER

$$\frac{7}{75}$$

4MA1/2H NOVEMBER 2024

Question 5

4 marks | Probability Toolkit

5 Cody has a biased 5-sided spinner, numbered 1, 2, 3, 4, 5

The table gives the probabilities that when the spinner is spun it will land on 2 or on 3 or on 5

Number	1	2	3	4	5
Probability		0.14	0.17		0.21

The probability that the spinner will land on 1 is the same as the probability that the spinner will land on 4

Cody is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on 4

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2024

1 Probabilities of 1 and 4 together

$$1 - (0.14 + 0.17 + 0.21) = 1 - 0.52 = 0.48$$

2 They are equal

$$P(4) = 0.48 \div 2 = 0.24$$

3 Estimate for 400 spins

$$400 \times 0.24 = 96$$

✓ **FINAL ANSWER**

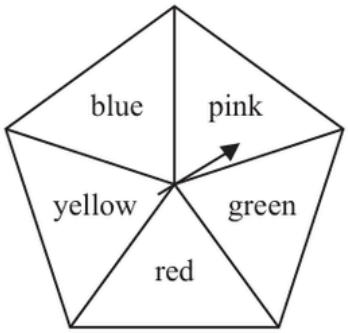
96

4WM1/01 MAY 2025

Question 3

4 marks | Probability Toolkit

3 Here is a biased spinner.
The spinner can land on pink or on green or on red or on yellow or on blue.



The table gives information about the probability that, when the spinner is spun, it will land on each colour.

Colour	pink	green	red	yellow	blue
Probability	0.14	0.23	$2x$	0.18	$7x$

Jim is going to spin the spinner 500 times.

Work out the expected number of times the spinner will land on blue.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4WM1/01 May 2025

1 Calculate probability

The probabilities add to 1.

$$0.14 + 0.23 + 2x + 0.18 + 7x = 1$$

$$0.55 + 9x = 1$$

$$x = 0.05$$

2 The probability of blue is

The probability of blue is

$$7x = 7(0.05) = 0.35$$

3 Expected number of blue results

Expected number of blue results:

$$500 \times 0.35 = 175$$

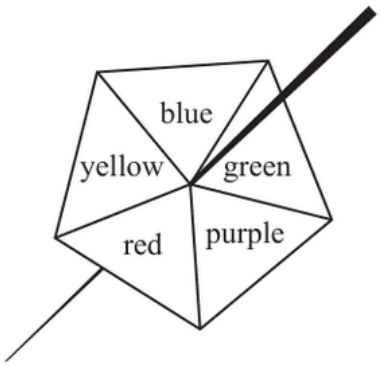
FINAL ANSWER

175

Question 3

5 marks | Probability Toolkit

3 Here is a biased 5-sided spinner.
When the spinner is spun, it can land on blue or on green or on purple or on red or on yellow.



The table gives information about the probability of the spinner landing on each colour.

Colour	blue	green	purple	red	yellow
Probability	0.12	0.20	0.38	$4x$	x

Sophie spins the spinner once.

(a) Work out the probability that the spinner lands on blue or on green or on purple.

.....

(1)

Max spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on red.

.....

(4)

(Total for Question 3 is 5 marks)

1 Let the yellow probability be

Let the yellow probability be x. Then the red probability is $4x$.

$$0.12 + 0.20 + 0.30 + 4x + x = 1$$

$$5x = 0.38$$

$$x = 0.076$$

2 Calculate probability

So

$$P(\text{red}) = 4x = 0.304$$

$$P(\text{blue, green or purple}) = 0.12 + 0.20 + 0.30 = 0.62$$

3 Expected red

Expected red:

$$350 \times 0.304 = 106.4$$

FINAL ANSWER

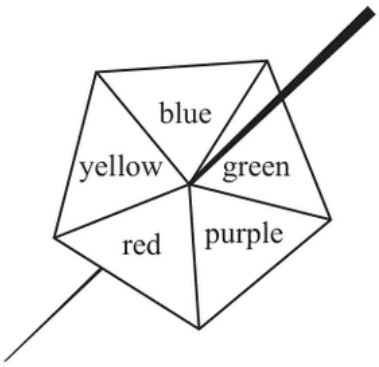
0.62, and about 106.

4MA1/1HR JUNE 2025

Question 4

5 marks | Probability Toolkit

4 Here is a biased 5-sided spinner.
When the spinner is spun, it can land on blue or on green or on purple or on red or on yellow.



The table gives information about the probability of the spinner landing on each colour.

Colour	blue	green	purple	red	yellow
Probability	0.12	0.20	0.38	$4x$	x

Sophie spins the spinner once.

(a) Work out the probability that the spinner lands on blue or on green or on purple.

.....

(1)

Max spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on red.

.....

(4)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2025

1 **(a) Blue, green or purple**

$$0.12 + 0.20 + 0.38 = 0.70$$

2 **(b) Find x**

Red and yellow share the rest: $4x + x = 1 - 0.70 = 0.30$, so $5x = 0.30$ and $x = 0.06$.

3 (b) Probability of red

$$4x = 4 \times 0.06 = 0.24$$

4 (b) Estimate for 350 spins

$$350 \times 0.24 = 84$$

FINAL ANSWER

(a) 0.7 (b) 84

4MA1/2H JUNE 2025

Question 11

3 marks | Probability Toolkit

11 Khalid has a box of counters.

17 of the counters are red
28 of the counters are blue
the rest of the counters are orange

Khalid is going to take at random a counter from the box.

The probability that Khalid will take an orange counter is $\frac{4}{9}$

Work out the number of orange counters that are in the box.

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2025

1 Counters that are not orange

$17 + 28 = 45$ counters are not orange, and $P(\text{not orange}) = 1 - \frac{4}{9} = \frac{5}{9}$.

2 Total number of counters

$$\frac{5}{9} \times T = 45 \Rightarrow T = 81$$

3 Orange counters

$$81 - 45 = 36$$

FINAL ANSWER

36

4WM1/01R MAY 2025

Question 17

3 marks | Probability Toolkit

17 Kannika has 9 counters.
There is a number on each counter.



Kannika puts the 9 counters in a bag.
She takes at random a counter from the bag and does not replace the counter.
She then takes at random a second counter from the bag.

Work out the probability that the sum of the numbers on the two counters is less than 5

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4WM1/01R May 2025

1 The counters are

The counters are

$$1,\ 1,\ 2,\ 3,\ 3,\ 3,\ 5,\ 6,\ 6$$

2 There are

There are

$$\binom{9}{2} = 36$$

3 ways to choose two counters

ways to choose two counters.

4 Calculate probability

For a sum less than 5, the pairs are:

$$(1,1),\quad (1,2),\quad (1,3)$$

5 Counting the repeated counters gives

Counting the repeated counters gives

$$1+2+6=9$$

6 Calculate probability

So

$$P(\text{sum} < 5) = \frac{9}{36} = \frac{1}{4}$$

FINAL ANSWER

$\frac{1}{4}$.

4MA1/1HR JUNE 2025

Question 19

3 marks | Probability Toolkit

19 Kannika has 9 counters.
There is a number on each counter.



Kannika puts the 9 counters in a bag.
She takes at random a counter from the bag and does not replace the counter.
She then takes at random a second counter from the bag.

Work out the probability that the sum of the numbers on the two counters is less than 5

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2025

1 Pairs with a sum less than 5

The counters are $1, 1, 2, 3, 3, 3, 5, 6, 6$. The only possible sums under 5 are $1+1$, $1+2$ and $1+3$.

2 First counter, then second (no replacement)

$$P(1,1) = \frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$$

$$P(1 \text{ and } 2) = 2 \times \frac{2}{9} \times \frac{1}{8} = \frac{4}{72}, \qquad P(1 \text{ and } 3) = 2 \times \frac{2}{9} \times \frac{3}{8} = \frac{12}{72}$$

3 Add the cases

$$\frac{2+4+12}{72} = \frac{18}{72} = \frac{1}{4}$$

✓ **FINAL ANSWER**

$$\frac{1}{4}$$

4MA1/1H JUNE 2025

Question 21

3 marks | Probability Toolkit

21 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2025

1 **There are**

There are

$$\binom{20}{3} = 1140$$

2 **ways to choose 3 counters**

ways to choose 3 counters.

3 **Exactly two counters are the**

Exactly two counters are the same colour:

$$\binom{9}{2}(11) + \binom{7}{2}(13) + \binom{4}{2}(16)$$

$$= 36(11) + 21(13) + 6(16)$$

$$= 396 + 273 + 96 = 765$$

4 **Calculate probability**

So the probability is

$$\frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$$\frac{51}{76}$$

4WM1/01 MAY 2025

Question 22

3 marks | Probability Toolkit

22 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 May 2025

1 Total ways to choose counters

Total ways to choose 3 counters:

$$\binom{20}{3} = 1140$$

2 Exactly two red

Exactly two red:

$$\binom{9}{2}\binom{11}{1} = 396$$

3 Exactly two yellow

Exactly two yellow:

$$\binom{7}{2}\binom{13}{1} = 273$$

4 Exactly two green

Exactly two green:

$$\binom{4}{2}\binom{16}{1} = 96$$

5 Favourable outcomes

Favourable outcomes:

$$396 + 273 + 96 = 765$$

$$P = \frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$\frac{51}{76}$

4WM1/01 NOVEMBER 2025

Question 5

5 marks | Probability Toolkit

5 All students at a school are asked which sport they prefer from football, tennis, rugby and cricket.

The table gives information about the probabilities that, when a student is chosen at random, the student prefers football or tennis or rugby or cricket.

Favourite sport	football	tennis	rugby	cricket
Probability	$2x$	0.2	x	0.35

One of the students at the school is chosen at random.

(a) Work out the probability that this student prefers tennis or cricket.

.....

(1)

There are 700 students at the school.

(b) Work out how many students prefer football.

.....

(4)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 November 2025

1 **Calculate value**

$$P(\text{tennis or cricket}) = 0.2 + 0.35 = 0.55$$

2 **Calculate value**

Also,

$$2x + 0.2 + x + 0.35 = 1$$

$$3x = 0.45$$

$$x = 0.15$$

3 Calculate value

So

$$P(\text{football}) = 2x = 0.30$$

$$700 \times 0.30 = 210$$

FINAL ANSWER

0.55, 210 students.

Question 9

3 marks | Probability Toolkit

9 Mark has three bags, bag A, bag B and bag C

Bag A contains 120 counters.
The probability of taking at random a red counter from bag A is 0.45

Bag B contains 80 counters.
The probability of taking at random a red counter from bag B is 0.3

Bag C is empty.

Mark puts all the counters from bag A and bag B into bag C

Mark takes at random a counter from bag C

Work out the probability that he takes a red counter.

.....

(Total for Question 9 is 3 marks)

1 Red counters in each bag

Bag A: $120 \times 0.45 = 54$. Bag B: $80 \times 0.3 = 24$.

2 Probability of red from bag C

Bag C has $54 + 24 = 78$ red counters out of $120 + 80 = 200$.

$$\frac{78}{200} = 0.39$$

FINAL ANSWER

0.39

4MA1/1HR JUNE 2026

Question 5

4 marks | Probability Toolkit

5 A bag contains only blue counters, green counters, red counters and yellow counters.

Danny takes a counter at random from the bag.

The table gives information about the probabilities that he takes a blue counter or a green counter or a red counter or a yellow counter.

Colour	blue	green	red	yellow
Probability	0.45	0.31	$5x$	x

There are 400 counters in the bag.

Work out the number of red counters in the bag.

.....

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR June 2026

1 Solution | Step 1

$$0.45 + 0.31 + 5x + x = 1.$$

The four colours are mutually exclusive and exhaust all counters, so their probabilities sum to 1.

2 Solution | Step 2

$$6x = 1 - 0.76 = 0.24, \qquad x = 0.04.$$

Collect the unknown probabilities and solve the linear equation.

3 Solution | Step 3

$$P(\text{red}) = 5(0.04) = 0.20, \qquad N_{\text{red}} = 400(0.20) = 80.$$

Multiply the red proportion by the total number of counters.

✓ **FINAL ANSWER**

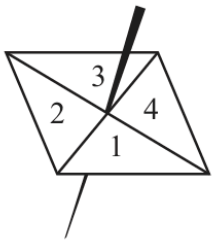
80 red counters

4MA1/1H JUNE 2026

Question 7

4 marks | Probability Toolkit

7 Here is a biased 4-sided spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4
Probability	0.12	x	$2x$	0.16

Nevan spins the spinner 500 times.

Work out the expected number of times that the spinner lands on an even number.

.....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2026

1 Solution | Step 1

$$0.12 + x + 2x + 0.16 = 1 \implies 3x = 0.72 \implies x = 0.24.$$

The four exhaustive outcome probabilities sum to 1.

2 Solution | Step 2

$$P(\text{even}) = P(2) + P(4) = 0.24 + 0.16 = 0.40.$$

The events 2 and 4 are mutually exclusive.

3 Solution | Step 3

$$\text{Expected number} = 500 \times 0.40 = 200.$$

Multiply the number of spins by the probability of an even result.

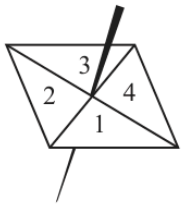
FINAL ANSWER

200

Question 7

4 marks | Probability Toolkit

7 Here is a biased 4-sided spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4
Probability	0.12	x	$2x$	0.16

Nevan spins the spinner 500 times.

Work out the expected number of times that the spinner lands on an even number.

.....

(Total for Question 7 is 4 marks)

1 **Probabilities add to 1**

$$0.12 + x + 2x + 0.16 = 1 \Rightarrow 3x = 0.72 \Rightarrow x = 0.24$$

2 **Even numbers are 2 and 4**

$$P(\text{even}) = 0.24 + 0.16 = 0.40$$

3 **Expected number**

$$500 \times 0.40 = 200$$

✓ **FINAL ANSWER**

200

Question 11

3 marks | Probability Toolkit

11 There are 40 counters in a bag.

25 of the counters are green
7 of the counters are blue
8 of the counters are red

Ben puts some extra blue counters and some extra red counters into the bag.
The number of blue counters added to the bag is twice as many as the number of red counters added to the bag.

Ben takes at random a counter from the bag.

The probability that he takes a green counter is $\frac{5}{11}$

How many of the extra counters that Ben put into the bag were blue?

.....

(Total for Question 11 is 3 marks)

1 **Let r red be added**

Then $2r$ blue are added and the total is $40 + 3r$.

2 **Use P(green)**

$$\frac{25}{40+3r} = \frac{5}{11} \Rightarrow 275 = 200 + 15r \Rightarrow r = 5$$

3 **Blue added**

$$2r = 10$$

✓ **FINAL ANSWER**

10

4MA1/2H JUNE 2026

Question 20

3 marks | Probability Toolkit

20 A box only contains green counters and red counters.

Nuno chooses at random a counter from the box.
He puts the counter back into the box.

Nuno then chooses at random a second counter from the box.

The probability that Nuno chooses at least one red counter is $\frac{39}{64}$

Work out the probability that Nuno chooses exactly one red counter.

.....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2026

1 Solution | Step 1

Let $g = P(G)$. With replacement, the draws are independent.

$$P(GG) = 1 - P(\text{at least one red}) = 1 - \frac{39}{64} = \frac{25}{64}.$$

The complementary event to at least one red is two greens.

2 Solution | Step 2

$$g^2 = \frac{25}{64} \quad \Rightarrow \quad g = \frac{5}{8}, \qquad P(R) = 1 - \frac{5}{8} = \frac{3}{8}.$$

Multiply identical independent green probabilities and take the nonnegative root.

3 Solution | Step 3

$$P(\text{exactly one red}) = P(RG) + P(GR) = \frac{3}{8}\frac{5}{8} + \frac{5}{8}\frac{3}{8} = \frac{30}{64} = \frac{15}{32}.$$

The two possible orders are mutually exclusive and must both be included.

✓ **FINAL ANSWER**

$$\frac{15}{32}$$

TOPIC

Tree Diagrams & Conditional Probability

Every past-paper question on this topic, 2020 to 2026.

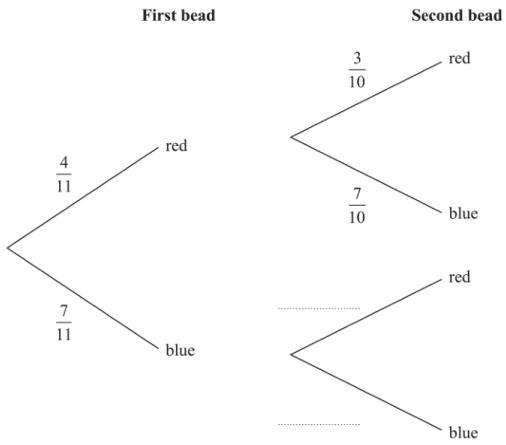
Question 11

6 marks | Tree Diagrams & Conditional Probability

11 Tess has a bag containing 11 beads.
4 of the beads are red.
7 of the beads are blue.

Tess takes at random a bead from the bag and keeps it.
Tess takes at random another bead from the bag and keeps it.

(a) Complete the probability tree diagram.



(1)

(b) Work out the probability that both of the beads are red.

.....

(2)

(c) Work out the probability that the two beads are of different colours.

.....

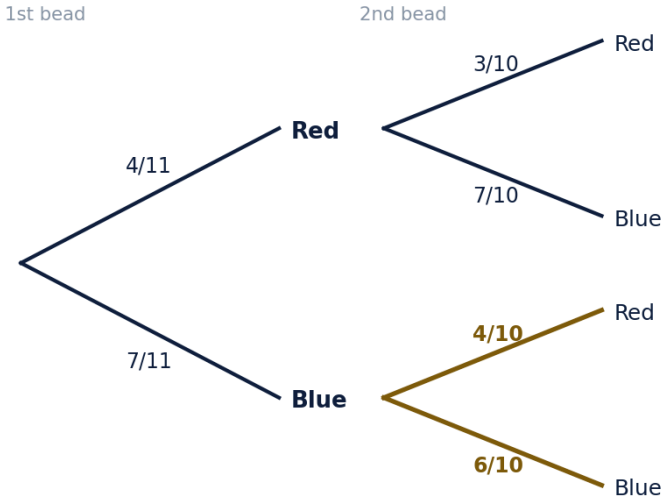
(3)

(Total for Question 11 is 6 marks)

1 Complete the blue-first branch

After a blue bead is taken first, there are 4 red and 6 blue beads left out of 10.

$$P(\text{red second after blue}) = \frac{4}{10}, \qquad P(\text{blue second after blue}) = \frac{6}{10}$$



2 Find both red

$$P(RR) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110} = \frac{6}{55}$$

3 Find different colours

$$P(RB) = \frac{4}{11} \times \frac{7}{10}$$

$$P(BR) = \frac{7}{11} \times \frac{4}{10}$$

$$P(\text{different}) = \frac{28}{110} + \frac{28}{110} = \frac{28}{55}$$

FINAL ANSWER

Missing probabilities $\frac{4}{10}$ and $\frac{6}{10}$; $P(RR) = \frac{6}{55}$; $P(\text{different}) = \frac{28}{55}$.

4MA1/2H JANUARY 2020

Question 14

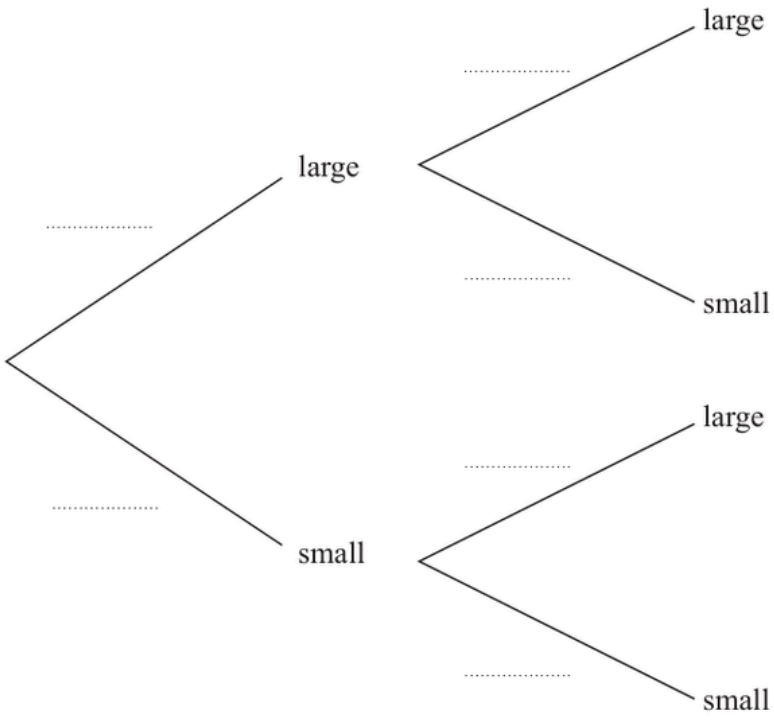
4 marks | Tree Diagrams & Conditional Probability

14 There are 20 glasses in a cupboard.

13 of the glasses are large
7 of the glasses are small

Roberto takes at random two glasses from the cupboard.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Roberto takes two small glasses.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2020

1 **(a) First glass**

$$P(\text{large}) = \frac{13}{20}, \qquad P(\text{small}) = \frac{7}{20}$$

2 **(a) Second glass (one fewer glass left)**

After a large glass: large $\frac{12}{19}$, small $\frac{7}{19}$.
After a small glass: large $\frac{13}{19}$, small $\frac{6}{19}$.

---

3 **(b) Two small glasses**

$$\frac{7}{20} \times \frac{6}{19} = \frac{42}{380} = \frac{21}{190}$$

✓ **FINAL ANSWER**

(a) $\frac{13}{20}, \frac{7}{20}$; $\frac{12}{19}, \frac{7}{19}$; $\frac{13}{19}, \frac{6}{19}$ (b) $\frac{21}{190}$

4MA1/1HR JANUARY 2020

Question 16

4 marks | Tree Diagrams & Conditional Probability

16 Steffi is going to play one game of tennis and one game of chess.

The probability that she will win the game of tennis is 0.6
The probability that she will win **both** games is 0.42

Work out the probability that she will **not** win either game.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2020

1 Find the probability of winning chess

Treat the two games as independent:

$$0.6 \times P(\text{win chess}) = 0.42 \Rightarrow P(\text{win chess}) = 0.42 \div 0.6 = 0.7$$

2 Probabilities of losing

$$P(\text{not win tennis}) = 1 - 0.6 = 0.4, \qquad P(\text{not win chess}) = 1 - 0.7 = 0.3$$

3 Win neither game

$$0.4 \times 0.3 = 0.12$$

FINAL ANSWER

0.12

Question 16

8 marks | Tree Diagrams & Conditional Probability

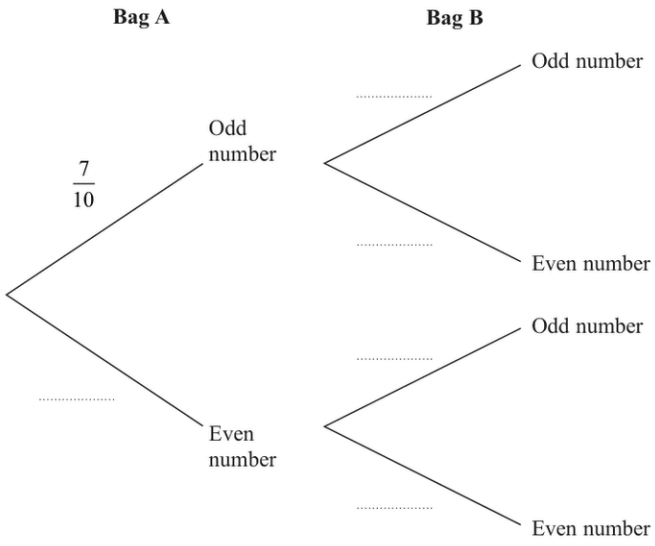
16 Cody has two bags of counters, bag **A** and bag **B**.

Each of the counters has either an odd number or an even number written on it.

There are 10 counters in bag **A** and 7 of these counters have an **odd** number written on them.
There are 12 counters in bag **B** and 7 of these counters have an **odd** number written on them.

Cody is going to take at random a counter from bag **A** and a counter from bag **B**.

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that the total of the numbers on the two counters will be an odd number.

.....

(3)

Harriet also has a bag of counters.
Each of her counters also has either an odd number or an even number written on it.

Harriet is going to take at random a counter from her bag of counters.

The probability that the number on each of Cody's two counters **and** the number on Harriet's counter will all be even is $\frac{3}{100}$

(c) Find the least number of counters that Harriet has in her bag.
Show your working clearly.

.....

(3)

(Total for Question 16 is 8 marks)

1 **(a) Complete the tree**

Bag A: odd $\frac{7}{10}$, even $\frac{3}{10}$.

Bag B (on both branches): odd $\frac{7}{12}$, even $\frac{5}{12}$.

2 **(b) Odd total = odd + even or even + odd**

$$\frac{7}{10} \times \frac{5}{12} + \frac{3}{10} \times \frac{7}{12} = \frac{35}{120} + \frac{21}{120} = \frac{56}{120} = \frac{7}{15}$$

3 **(c) Probability Harriet takes an even counter**

Cody takes two even counters with probability $\frac{3}{10} \times \frac{5}{12} = \frac{1}{8}$.

$$\frac{1}{8} \times p = \frac{3}{100} \Rightarrow p = \frac{24}{100} = \frac{6}{25}$$

4 **(c) Least number of counters**

$\frac{6}{25}$ is in its simplest form, so the number of counters must be a multiple of 25. The least number is 25.

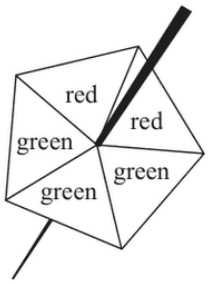
✓ **FINAL ANSWER**

(a) $\frac{3}{10}$; $\frac{7}{12}$, $\frac{5}{12}$ on both branches (b) $\frac{7}{15}$ (c) 25

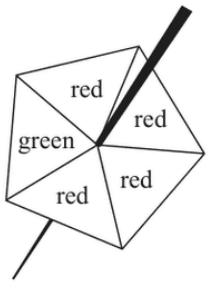
Question 15

5 marks | Tree Diagrams & Conditional Probability

15 Harry has two fair 5-sided spinners.



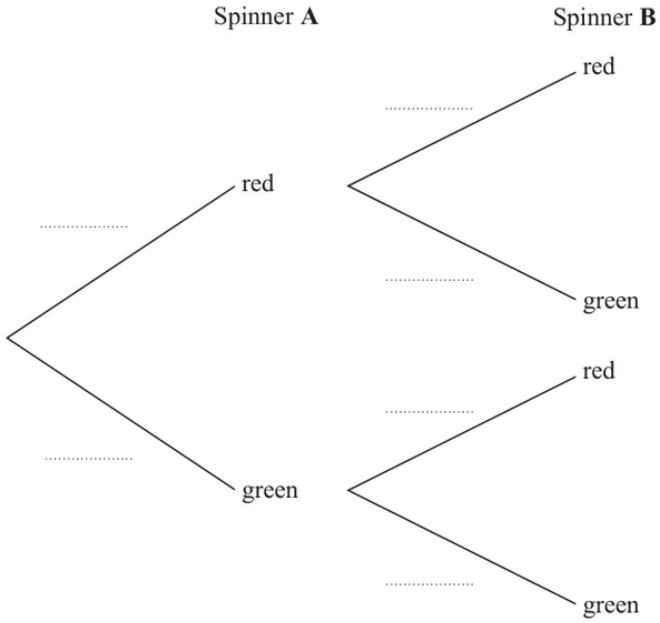
Spinner **A**



Spinner **B**

Harry is going to spin each spinner once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that at least one of the spinners will land on green.

.....

(3)

(Total for Question 15 is 5 marks)

1 (a) Complete the tree

Spinner A has 2 red and 3 green sections: red $\frac{2}{5}$, green $\frac{3}{5}$.

Spinner B has 4 red and 1 green (on both branches): red $\frac{4}{5}$, green $\frac{1}{5}$.

2 (b) Use the complement

"At least one green" is the opposite of "both red":

$$1 - \frac{2}{5} \times \frac{4}{5} = 1 - \frac{8}{25} = \frac{17}{25}$$

FINAL ANSWER

(a) $\frac{2}{5}, \frac{3}{5}$; $\frac{4}{5}, \frac{1}{5}$ on both branches (b) $\frac{17}{25}$

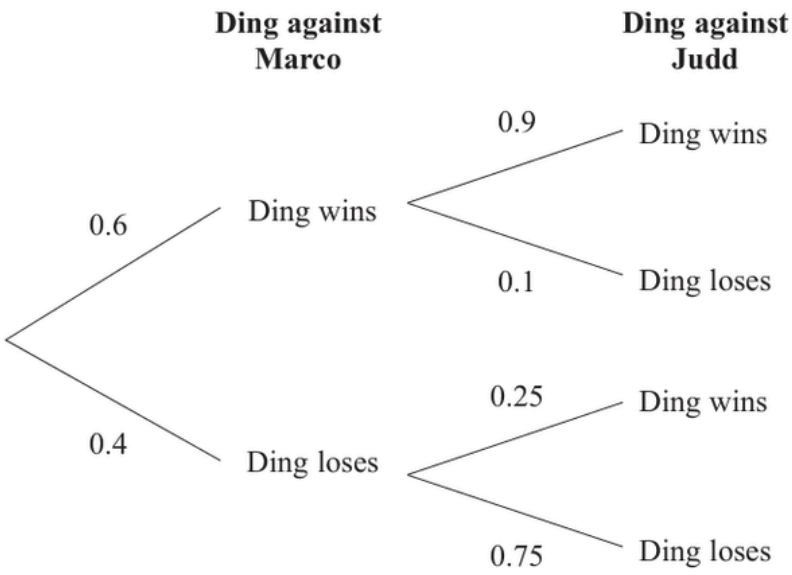
4MA1/1H JANUARY 2021

Question 14

5 marks | Tree Diagrams & Conditional Probability

14 Ding is going to play one game of snooker against each of two of his friends, Marco and Judd.

The probability tree diagram gives information about the probabilities that Ding will win or lose each of these two games.



(a) Work out the probability that Ding will win both games.

.....

(2)

(b) Work out the probability that Ding will win exactly one of the games.

.....

(3)

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2021

1 (a) Win both games

Multiply along the "win, win" branch:

$$0.6 \times 0.9 = 0.54$$

2 **(b) Win exactly one game**

Win then lose, or lose then win:

$$0.6 \times 0.1 + 0.4 \times 0.25 = 0.06 + 0.10 = 0.16$$

✓ **FINAL ANSWER**

(a) 0.54 (b) 0.16

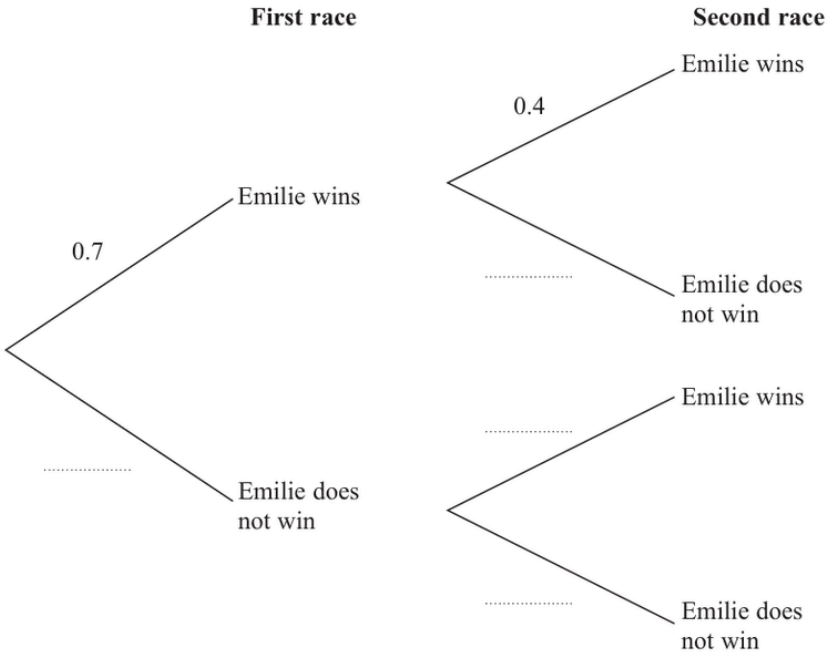
Question 13

8 marks | Tree Diagrams & Conditional Probability

13 Emilie takes part in two races.

The probability that she wins the first race is 0.7
The probability that she wins the second race is 0.4
The outcomes of the two races are independent.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Emilie wins exactly one of the two races.

.....

(3)

Emilie is going to take part in a third race.

If she wins both of the first two races, the probability that she will win the third race is 0.6

If she wins exactly one of the first two races, the probability that she will win the third race is 0.3

(c) Work out the probability that Emilie will win exactly two of the three races.

.....

(3)

(Total for Question 13 is 8 marks)

1 **(a) Complete the tree**

First race: does not win 0.3.

Second race: after a win, does not win 0.6; after not winning, wins 0.4 and does not win 0.6.

2 **(b) Exactly one win**

$$0.7 \times 0.6 + 0.3 \times 0.4 = 0.42 + 0.12 = 0.54$$

3 **(c) Win the first two, lose the third**

$P(\text{win both}) = 0.7 \times 0.4 = 0.28$, then lose the third with $1 - 0.6 = 0.4$:

$$0.28 \times 0.4 = 0.112$$

4 **(c) Exactly one of the first two, then win the third**

$$0.54 \times 0.3 = 0.162$$

5 **(c) Add the cases**

$$0.112 + 0.162 = 0.274$$

✓ **FINAL ANSWER**

(a) 0.3; 0.6; $0.4, 0.6$ (b) 0.54 (c) 0.274

Question 15

8 marks | Tree Diagrams & Conditional Probability

15 Magnus and Garry play 2 games of chess against each other.

The probability that Magnus beats Garry in any game is $\frac{2}{9}$

The probability that any game between Magnus and Garry is drawn is $\frac{4}{9}$

The result of any game is independent of the result of any other game.

(a) Complete the probability tree diagram.



(2)

For each game of chess,

the winner gets 2 points and the loser gets 0 points,
when the game is drawn, each player gets 1 point.

(b) Work out the probability that, after 2 games, Magnus and Garry have the same number of points.

.....

(3)

Magnus and Garry now play a third game of chess.

(c) Work out the probability that, after 3 games, Magnus and Garry have the same number of points.

.....

(3)

(Total for Question 15 is 8 marks)

1 **(a) Complete the tree**

Magnus loses: $1 - \frac{2}{9} - \frac{4}{9} = \frac{3}{9}$.

Every second-game branch repeats $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ (the games are independent).

2 **(b) Equal points after 2 games**

Two draws, or one win and one loss (either order):

$$\left(\frac{4}{9}\right)^2 + 2 \times \frac{2}{9} \times \frac{3}{9} = \frac{16}{81} + \frac{12}{81} = \frac{28}{81}$$

3 **(c) Equal points after 3 games**

Each player needs 3 of the 6 points: three draws, or one win, one draw and one loss ($3! = 6$ orders).

4 **(c) Add the cases**

$$\left(\frac{4}{9}\right)^3 + 6 \times \frac{2}{9} \times \frac{4}{9} \times \frac{3}{9} = \frac{64}{729} + \frac{144}{729} = \frac{208}{729}$$

✓ **FINAL ANSWER**

(a) $\frac{3}{9}$; $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ on every branch (b) $\frac{28}{81}$ (c) $\frac{208}{729}$

4MA1/2H NOVEMBER 2021

Question 16

3 marks | Tree Diagrams & Conditional Probability

16 A box contains 15 counters.

There are 4 red counters, 5 green counters and the rest are yellow counters.

Niklas takes at random a counter from the box and writes down the colour of his counter.
He then puts the counter back into the box.

Sasha then takes at random a counter from the box and writes down the colour of her counter.

Work out the probability that the counters taken by Niklas and Sasha both have the same colour.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2021

1 Count the colours

4 red, 5 green and $15 - 9 = 6$ yellow. The counter is replaced, so the second pick has the same probabilities.

2 Both the same colour

$$\left(\frac{4}{15}\right)^2 + \left(\frac{5}{15}\right)^2 + \left(\frac{6}{15}\right)^2 = \frac{16 + 25 + 36}{225} = \frac{77}{225}$$

FINAL ANSWER

$$\frac{77}{225}$$

Question 13

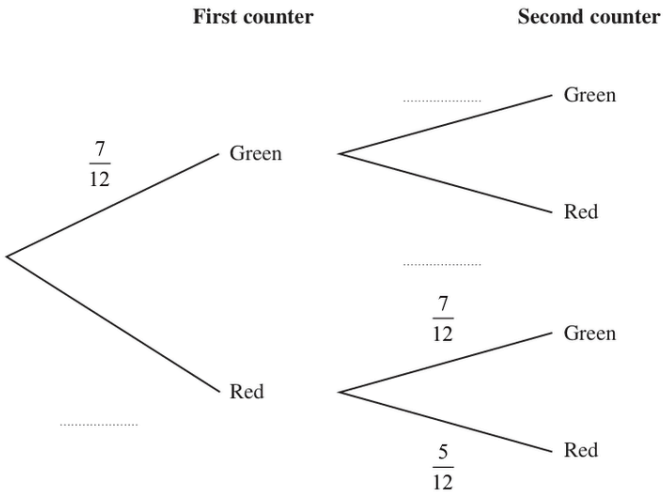
7 marks | Tree Diagrams & Conditional Probability

13 Hector has a bag that contains 12 counters.
There are 7 green counters and 5 red counters in the bag.

Hector takes at random a counter from the bag.
He looks at the counter and puts the counter back into the bag.

Hector then takes at random a second counter from the bag.
He looks at the counter and puts the counter back into the bag.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters are red.

.....

(2)

Meghan has a jar containing 15 counters.
There are only blue counters, green counters and red counters in the jar.

Hector is going to take at random one of the counters from his bag of 12 counters.
He will look at the counter and put the counter back into the bag.

Hector is then going to take at random a second counter from his bag.
He will look at the counter and put the counter back into the bag.

Meghan is then going to take at random one of the counters from her jar of counters.
She will look at the counter and put the counter back into the jar.

The probability that the 3 counters each have a different colour is $\frac{7}{24}$

(c) Work out how many blue counters there are in the jar.

.....

(3)

(Total for Question 13 is 7 marks)

1 (a) Complete the tree (counter is replaced)

First counter red: $\frac{5}{12}$.

After green, second counter: green $\frac{7}{12}$, red $\frac{5}{12}$.

2 (b) Both red

$$\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$$

3 (c) What "three different colours" needs

Hector only has green and red, so he must take one green and one red (either order), and Meghan must take blue.

$$P(\text{one green, one red}) = 2 \times \frac{7}{12} \times \frac{5}{12} = \frac{35}{72}$$

4 (c) Solve for the number of blue counters

Let b be the number of blue counters out of 15.

$$\frac{35}{72} \times \frac{b}{15} = \frac{7}{24} \Rightarrow \frac{b}{15} = \frac{7}{24} \times \frac{72}{35} = \frac{3}{5} \Rightarrow b = 9$$

FINAL ANSWER

(a) $\frac{5}{12}$; $\frac{7}{12}$, $\frac{5}{12}$ (b) $\frac{25}{144}$ (c) 9

4MA1/1H JANUARY 2022

Question 14

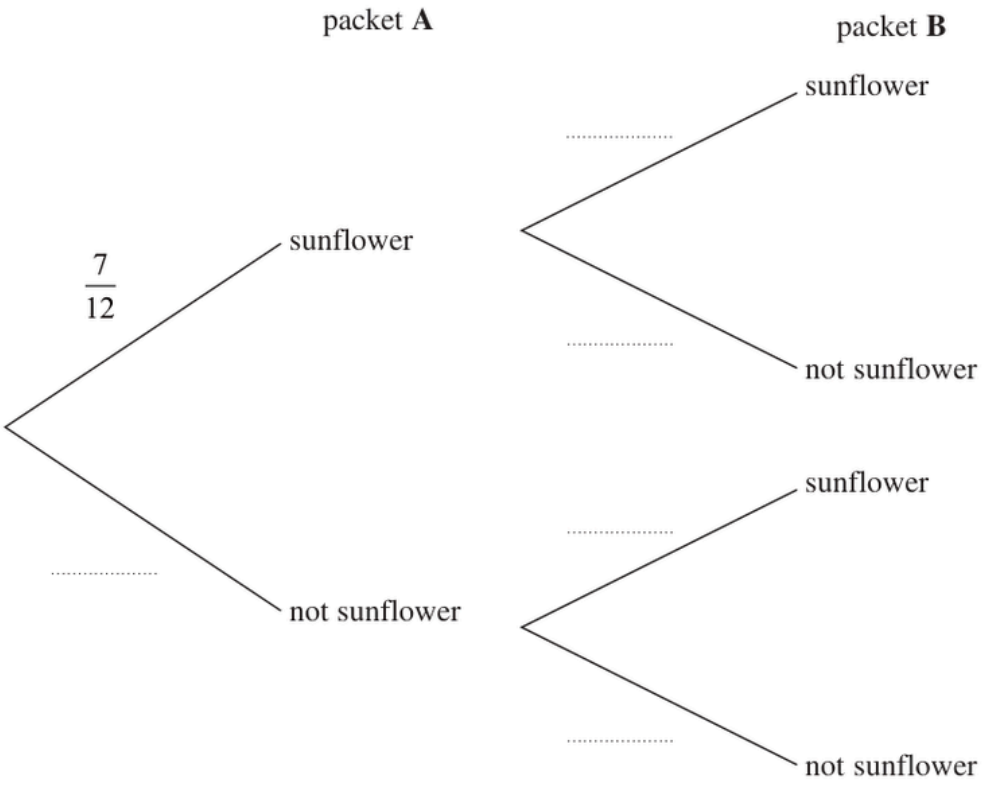
4 marks | Tree Diagrams & Conditional Probability

14 Aika has 2 packets of seeds, packet **A** and packet **B**

There are 12 seeds in packet **A** and 7 of these are sunflower seeds.
There are 15 seeds in packet **B** and 8 of these are sunflower seeds.

Aika is going to take at random a seed from packet **A** and a seed from packet **B**

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that Aika will take two sunflower seeds.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2022

1 **(a) Complete the tree**

Packet A: not sunflower $1 - \frac{7}{12} = \frac{5}{12}$.

Packet B (on both branches): sunflower $\frac{8}{15}$, not sunflower $\frac{7}{15}$.

2 **(b) Two sunflower seeds**

$$\frac{7}{12} \times \frac{8}{15} = \frac{56}{180} = \frac{14}{45}$$

✓ **FINAL ANSWER**

(a) $\frac{5}{12}$; $\frac{8}{15}$, $\frac{7}{15}$ on both branches (b) $\frac{14}{45}$

4MA1/1H JUNE 2022

Question 12

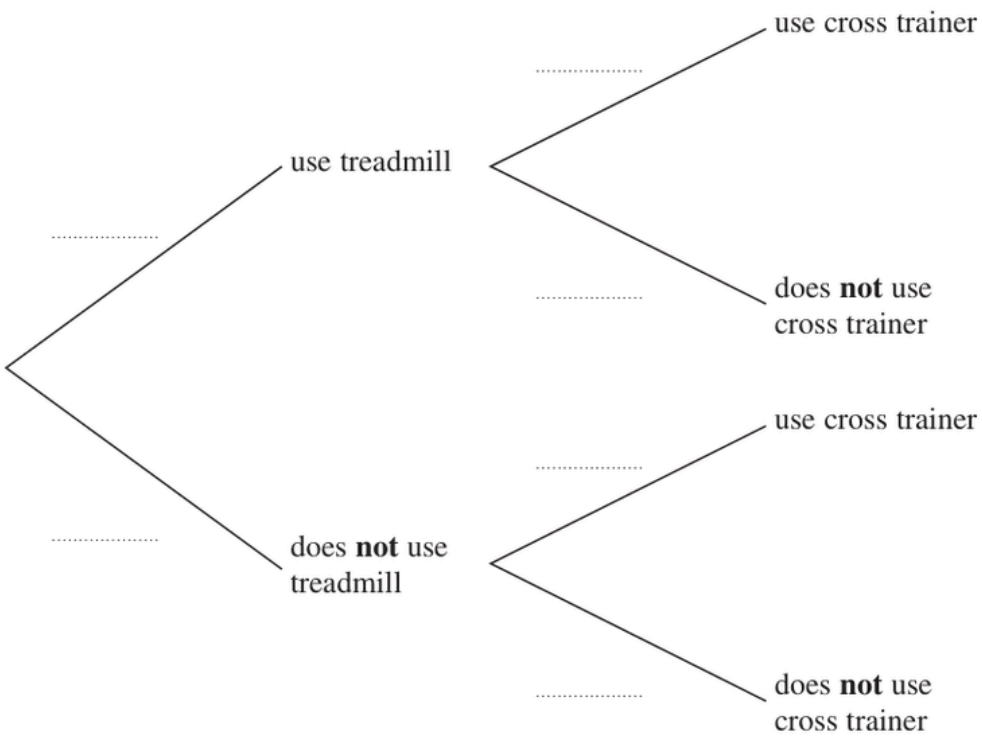
4 marks | Tree Diagrams & Conditional Probability

12 Rudolf goes to the gym.

The probability that he will use the treadmill is 0.8

When he uses the treadmill, the probability that he will use the cross trainer is 0.3
When he does **not** use the treadmill, the probability that he will use the cross trainer is 0.6

(a) Complete the probability tree diagram for this information.



(2)

(b) Work out the probability that Rudolf uses both the treadmill and the cross trainer.

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2022

1 **(a) Complete the tree**

Treadmill: use 0.8, not use 0.2.

After using the treadmill: cross trainer 0.3, not 0.7.

After not using the treadmill: cross trainer 0.6, not 0.4.

2 (b) Treadmill and cross trainer

$$0.8 \times 0.3 = 0.24$$

FINAL ANSWER

(a) $0.8, 0.2$; $0.3, 0.7$; $0.6, 0.4$ (b) 0.24

4MA1/2HR JUNE 2022

Question 14

3 marks | Tree Diagrams & Conditional Probability

14 Ciara throws **four** fair six-sided dice.

The faces of each dice are labelled with the numbers 1, 2, 3, 4, 5, 6

Work out the probability that at least one of the dice lands on an even number.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2022

1 Use the complement

"At least one even" is the opposite of "all four dice are odd". Each die is odd with probability $\frac{1}{2}$.

2 All four dice odd

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

3 At least one even

$$1 - \frac{1}{16} = \frac{15}{16}$$

FINAL ANSWER

$$\frac{15}{16}$$

Question 12

5 marks | Tree Diagrams & Conditional Probability

12 Moeen has a biased 6-sided dice.

The table gives information about the probability that, when the dice is thrown, it will land on each number.

Number	1	2	3	4	5	6
Probability	x	0.15	0.5	y	0.13	0.03

(a) Show that $x + y = 0.19$

(2)

Given that $3x - y = 0.09$

and $x + y = 0.19$

(b) work out the value of x and the value of y
Show clear algebraic working.

$x =$ .....

$y =$ .....

(3)

(Total for Question 12 is 5 marks)

1 **(a) Probabilities add to 1**

$$x + 0.15 + 0.5 + y + 0.13 + 0.03 = 1 \Rightarrow x + y + 0.81 = 1 \Rightarrow x + y = 0.19$$

2 **(b) Add the two equations**

$$(3x - y) + (x + y) = 0.09 + 0.19 \Rightarrow 4x = 0.28 \Rightarrow x = 0.07$$

3 **(b) Find y**

$$y = 0.19 - 0.07 = 0.12$$

✓ **FINAL ANSWER**

(b) $x = 0.07,\ y = 0.12$

4MA1/2H JUNE 2023

Question 13

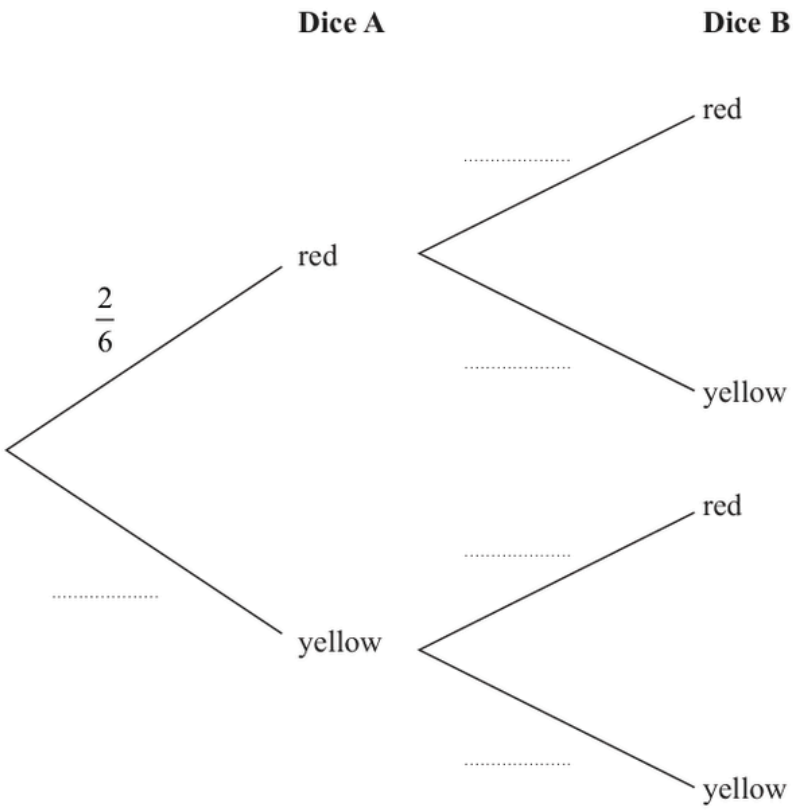
4 marks | Tree Diagrams & Conditional Probability

13 Narin has two fair 6-sided dice.

Dice **A** has 2 red faces and 4 yellow faces.
Dice **B** has 1 red face and 5 yellow faces.

Narin is going to throw each dice once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both dice land on yellow.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H June 2023

1 **(a) Complete the tree**

Dice A: yellow $\frac{4}{6}$.

Dice B (on both branches): red $\frac{1}{6}$, yellow $\frac{5}{6}$.

2 **(b) Both yellow**

$$\frac{4}{6} \times \frac{5}{6} = \frac{20}{36} = \frac{5}{9}$$

✓ **FINAL ANSWER**

(a) $\frac{4}{6}$; $\frac{1}{6}$, $\frac{5}{6}$ on both branches (b) $\frac{5}{9}$

Question 13

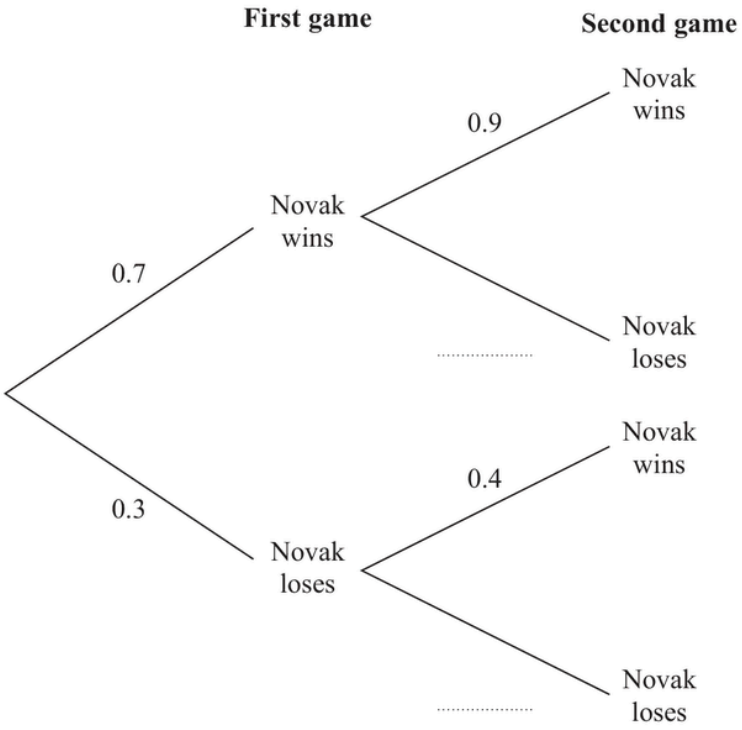
3 marks | Tree Diagrams & Conditional Probability

13 Novak is going to play two games of tennis.

The probability that he wins the first game is 0.7

If he wins the first game, the probability that he wins the second game is 0.9
If he loses the first game, the probability that he wins the second game is 0.4

(a) Complete the probability tree diagram.



(1)

(b) Work out the probability that Novak wins both games of tennis.

.....

(2)

(Total for Question 13 is 3 marks)

1 (a) Complete the tree

After a win: loses $1 - 0.9 = 0.1$. After a loss: loses $1 - 0.4 = 0.6$.

2 (b) Wins both games

$$0.7 \times 0.9 = 0.63$$

FINAL ANSWER

(a) 0.1, 0.6 (b) 0.63

4MA1/2H JUNE 2023

Question 20

3 marks | Tree Diagrams & Conditional Probability

20 There are 12 counters in a bag.

3 of the counters are red
9 of the counters are green

Ameya, Jack and Ella each take at random one counter from the bag.

Work out the probability that at least one red counter is still in the bag.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2023

1 There are 3 red counters

There are 3 red counters.

2 At least one red counter

At least one red counter is still in the bag unless all 3 red counters are taken.

3 Use the complement

So use the complement:

$$P(\text{all 3 red are taken}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$$

$$= \frac{6}{1320}$$

$$= \frac{1}{220}$$

4 Evaluate fraction

Therefore,

$$P(\text{at least one red is still in the bag}) = 1 - \frac{1}{220}$$

$$= \frac{219}{220}$$

FINAL ANSWER

$$\frac{219}{220}$$

4MA1/2H NOVEMBER 2023

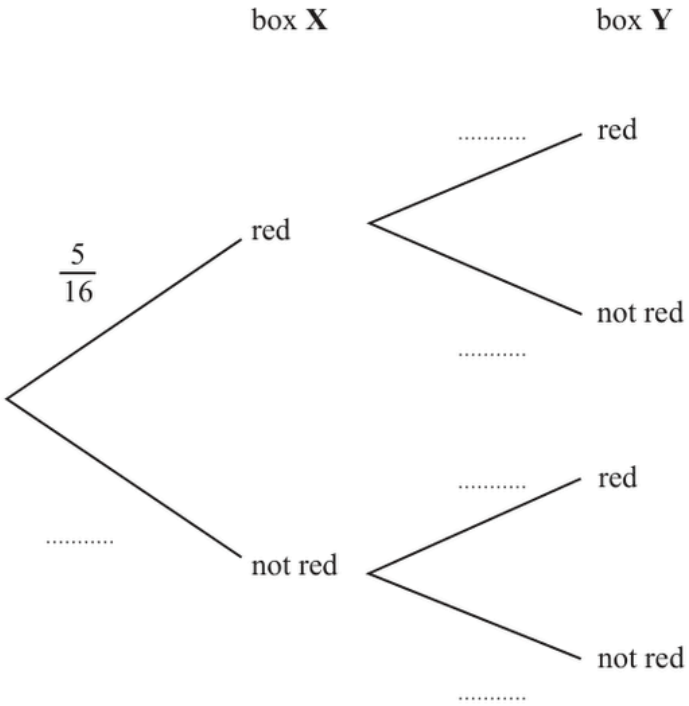
Question 11

5 marks | Tree Diagrams & Conditional Probability

11 Sid has 2 boxes of crayons, box **X** and box **Y**

5 of the 16 crayons in box **X** are red.
7 of the 20 crayons in box **Y** are red.
Sid takes at random one crayon from box **X** and one crayon from box **Y**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Sid takes two crayons that are red or two crayons that are not red.

.....

(3)

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2023

1 **(a) Complete the tree**

Box X: not red $\frac{11}{16}$.

Box Y (on both branches): red $\frac{7}{20}$, not red $\frac{13}{20}$.

2 **(b) Both red or both not red**

$$\frac{5}{16} \times \frac{7}{20} + \frac{11}{16} \times \frac{13}{20} = \frac{35}{320} + \frac{143}{320} = \frac{178}{320} = \frac{89}{160}$$

✓ **FINAL ANSWER**

(a) $\frac{11}{16}$; $\frac{7}{20}$, $\frac{13}{20}$ on both branches (b) $\frac{89}{160}$

4MA1/1H MAY 2024

Question 13

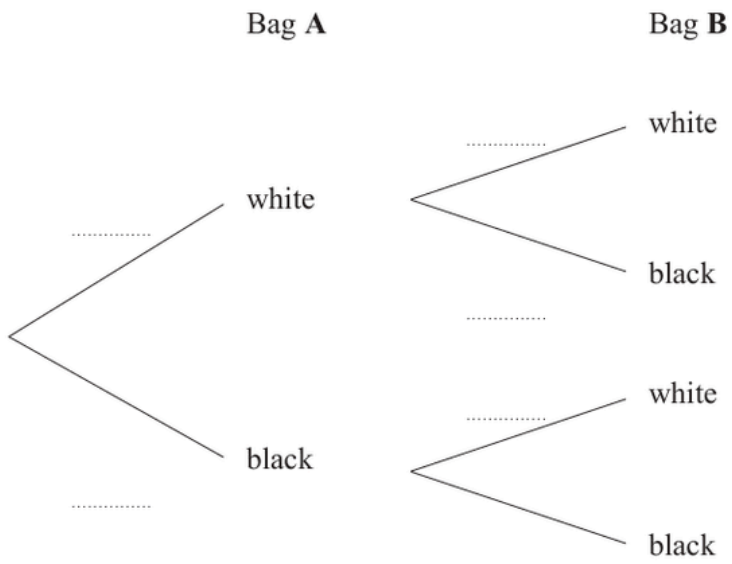
5 marks | Tree Diagrams & Conditional Probability

13 Harman has two bags of beads.

In bag **A**, there are 3 white beads and 7 black beads.
In bag **B**, there are 5 white beads and 4 black beads.

Harman takes at random a bead from bag **A** and a bead from bag **B**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Harman takes two beads of the same colour.

.....

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2024

1 **(a) Complete the tree**

Bag A: white $\frac{3}{10}$, black $\frac{7}{10}$.
Bag B (on both branches): white $\frac{5}{9}$, black $\frac{4}{9}$.

2 **(b) Same colour = both white or both black**

$$\frac{3}{10} \times \frac{5}{9} + \frac{7}{10} \times \frac{4}{9} = \frac{15}{90} + \frac{28}{90} = \frac{43}{90}$$

FINAL ANSWER

(a) $\frac{3}{10}, \frac{7}{10}$; $\frac{5}{9}, \frac{4}{9}$ on both branches (b) $\frac{43}{90}$

4MA1/1H MAY 2024

Question 16

4 marks | Tree Diagrams & Conditional Probability

16 There are 20 sweets in a box.

15 of the sweets are red
5 of the sweets are yellow

Fred takes at random 3 sweets from the box.

Work out the probability that Fred takes at least one sweet of each colour from the box.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2024

1 Use the complement

"At least one of each colour" fails only if all three sweets are red or all three are yellow.

2 All red, all yellow (no replacement)

$$P(RRR) = \frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} = \frac{2730}{6840}, \qquad P(YYY) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{60}{6840}$$

3 At least one of each

$$1 - \frac{2730 + 60}{6840} = \frac{4050}{6840} = \frac{45}{76}$$

✓ **FINAL ANSWER**

$$\frac{45}{76}$$

4MA1/1H NOVEMBER 2024

Question 13

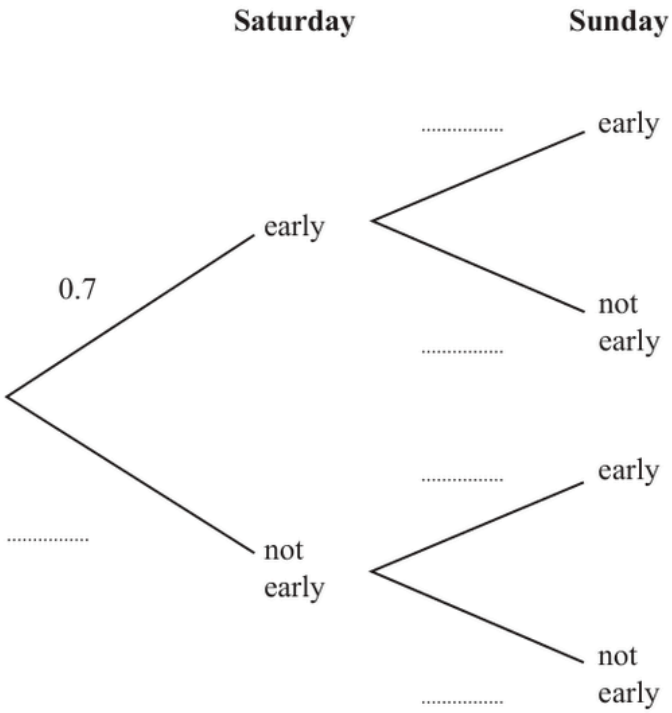
4 marks | Tree Diagrams & Conditional Probability

13 The probability that Thomas gets to work early on Saturday is 0.7

If Thomas gets to work early on Saturday, the probability that he will get to work early on Sunday is 0.9

If Thomas does **not** get to work early on Saturday, the probability that he will get to work early on Sunday is 0.6

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that Thomas gets to work early on both Saturday and Sunday.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2024

1 **(a) Complete the tree**

Saturday: not early 0.3.

After early: Sunday early 0.9, not early 0.1. After not early: Sunday early 0.6, not early 0.4.

2 (b) Early on both days

$$0.7 \times 0.9 = 0.63$$

FINAL ANSWER

(a) 0.3; $0.9, 0.1$; $0.6, 0.4$ (b) 0.63

Question 11

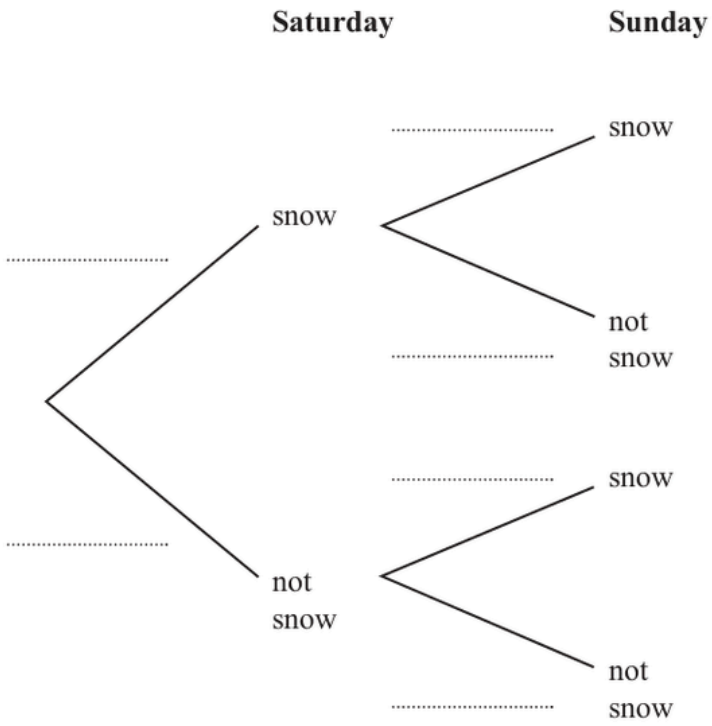
4 marks | Tree Diagrams & Conditional Probability

11 The probability that it will snow on Saturday is 0.2

If it snows on Saturday, the probability that it will snow on Sunday is 0.35

If it does not snow on Saturday, the probability that it will snow on Sunday is 0.4

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that it will snow on both Saturday and Sunday.

.....

(2)

(Total for Question 11 is 4 marks)

1 **The missing branch probabilities are**

The missing branch probabilities are complements.

$$P(\text{not snow on Saturday}) = 0.8$$

$$P(\text{not snow on Sunday} \mid \text{snow Saturday}) = 0.65$$

$$P(\text{not snow on Sunday} \mid \text{not snow Saturday}) = 0.6$$

2 **Calculate value**

For snow on both days,

$$0.2 \times 0.35 = 0.07$$

✓ **FINAL ANSWER**

tree labels $0.2, 0.8, 0.35, 0.65, 0.4, 0.6$; probability 0.07.

4WM1/01 MAY 2025

Question 12

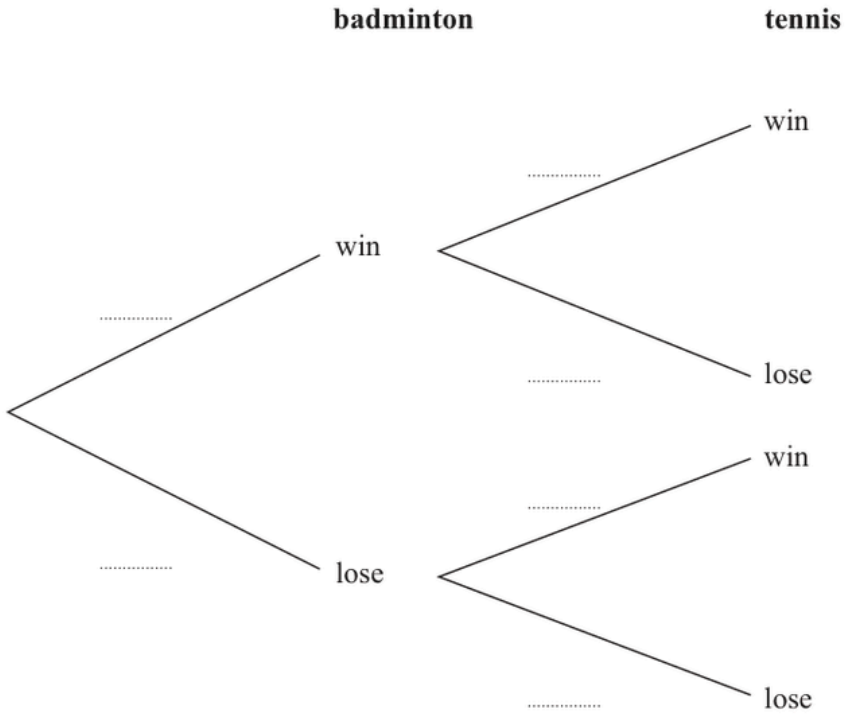
4 marks | Tree Diagrams & Conditional Probability

12 Ricardo is going to play one game of badminton and one game of tennis.

He can either win or lose each game.

The probability that he will win the game of badminton is 0.7
The probability that he will win the game of tennis is 0.4

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Ricardo loses both games.

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 May 2025

1 **Calculate probability**

For badminton,

$$P(\text{win}) = 0.7, \qquad P(\text{lose}) = 0.3$$

2 **Calculate probability**

For tennis,

$$P(\text{win}) = 0.4, \qquad P(\text{lose}) = 0.6$$

3 **The probability of losing both**

The probability of losing both games is

$$0.3 \times 0.6 = 0.18$$

✓ **FINAL ANSWER**

Tennis branches are 0.4 and 0.6 on each branch; probability of losing both is 0.18.

4MA1/1H JUNE 2025

Question 14

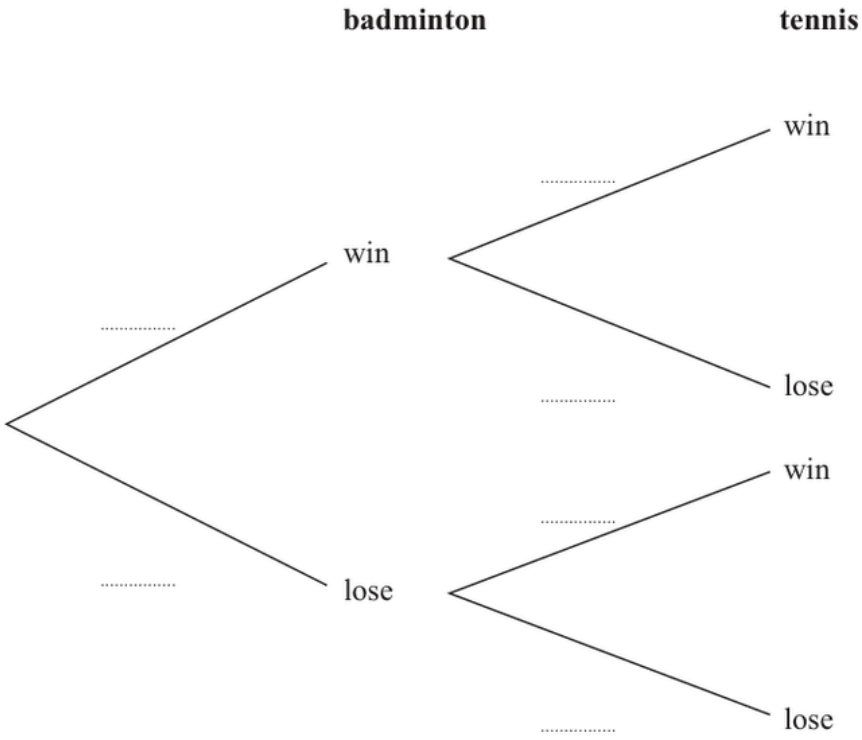
4 marks | Tree Diagrams & Conditional Probability

14 Ricardo is going to play one game of badminton and one game of tennis.

He can either win or lose each game.

The probability that he will win the game of badminton is 0.7
The probability that he will win the game of tennis is 0.4

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Ricardo loses both games.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2025

1 **(a) Complete the tree**

Badminton: win 0.7, lose 0.3.

Tennis (on both branches): win 0.4, lose 0.6.

2 **(b) Lose both**

$$0.3 \times 0.6 = 0.18$$

✓ **FINAL ANSWER**

(a) $0.7, 0.3$; $0.4, 0.6$ on both branches (b) 0.18

Question 14

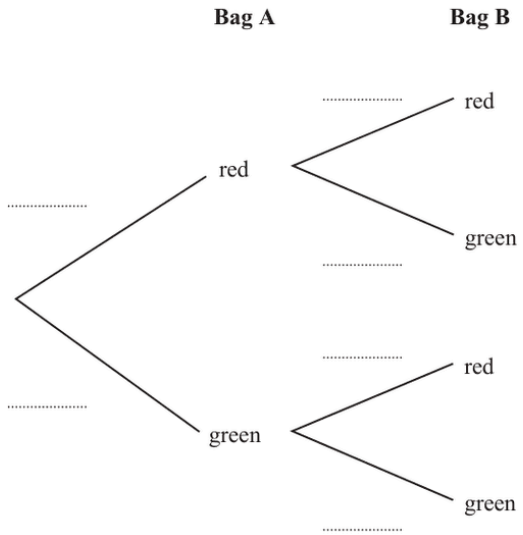
7 marks | Tree Diagrams & Conditional Probability

14 Sara has two bags, **A** and **B**

In bag **A**, there are only 5 red beads and 4 green beads.
In bag **B**, there are only 7 red beads and 3 green beads.

Sara takes at random a bead from bag **A**
She then takes at random a bead from bag **B**

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that Sara takes two red beads.

.....

(2)

Sara puts the beads back into their original bags.

Sara also has a box of beads.
In the box, there are only red beads and green beads.

When a bead is taken at random from the box, the probability that it is

a green bead is $\frac{2}{11}$

Sara takes at random a bead from bag **A**
She then takes at random a bead from bag **B**
She then takes at random a bead from the box.

(c) Work out the probability that Sara takes more red beads than green beads.

.....

(3)

(Total for Question 14 is 7 marks)

1 **(a) Complete the tree**

Bag A: red $\frac{5}{9}$, green $\frac{4}{9}$.

Bag B (on both branches): red $\frac{7}{10}$, green $\frac{3}{10}$.

2 **(b) Two red beads**

$$\frac{5}{9} \times \frac{7}{10} = \frac{35}{90} = \frac{7}{18}$$

3 **(c) More red than green with three beads**

She needs at least two reds: RRR, RRG, RGR or GRR. The box gives red $\frac{9}{11}$, green $\frac{2}{11}$.

4 **(c) Add the four cases**

$$\frac{5\cdot7\cdot9 + 5\cdot7\cdot2 + 5\cdot3\cdot9 + 4\cdot7\cdot9}{9\cdot10\cdot11} = \frac{315+70+135+252}{990} = \frac{772}{990} = \frac{386}{495}$$

✓ **FINAL ANSWER**

(a) $\frac{5}{9}, \frac{4}{9}$; $\frac{7}{10}, \frac{3}{10}$ on both branches (b) $\frac{7}{18}$ (c) $\frac{386}{495}$

4WM1/01 NOVEMBER 2025

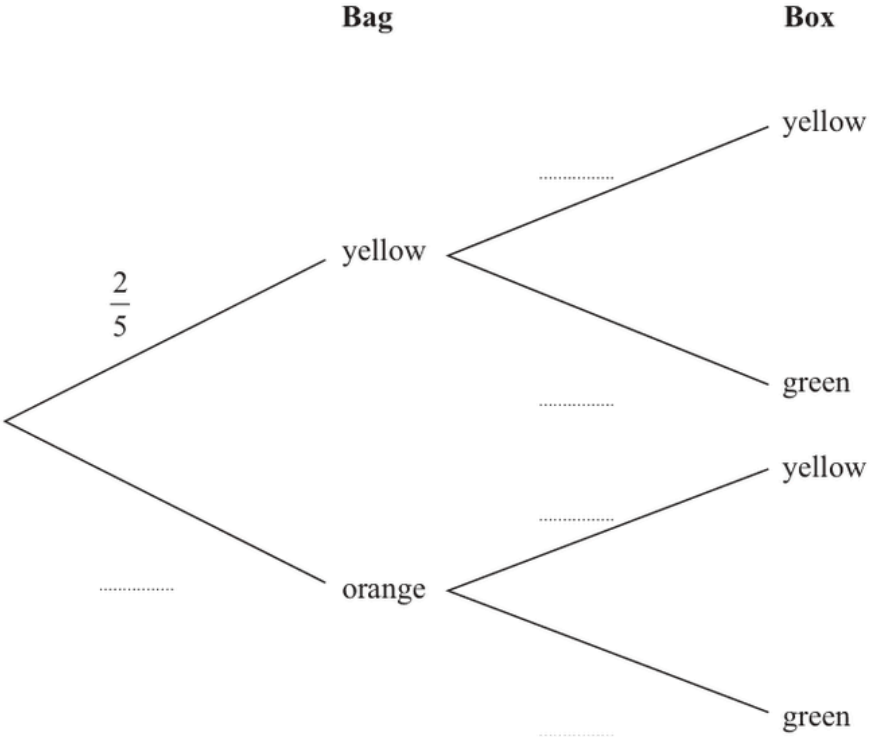
Question 12

5 marks | Tree Diagrams & Conditional Probability

12 In a bag, there are only 2 yellow beads and 3 orange beads.
In a box, there are only 3 yellow beads and 5 green beads.

Chi takes at random a bead from the bag and a bead from the box.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Chi takes two beads of different colours.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 November 2025

1 **Bag 1**

Bag 1:

$$P(Y) = \frac{2}{5}, \qquad P(O) = \frac{3}{5}$$

2 **Bag 2**

Bag 2:

$$P(Y) = \frac{3}{8}, \qquad P(G) = \frac{5}{8}$$

---

3 **The only way to get**

The only way to get the same colour is yellow then yellow.

$$P(\text{different}) = 1 - \frac{2}{5} \times \frac{3}{8}$$

$$= 1 - \frac{3}{20} = \frac{17}{20}$$

✓ **FINAL ANSWER**

$\frac{17}{20}$.

4MA1/1H NOVEMBER 2025

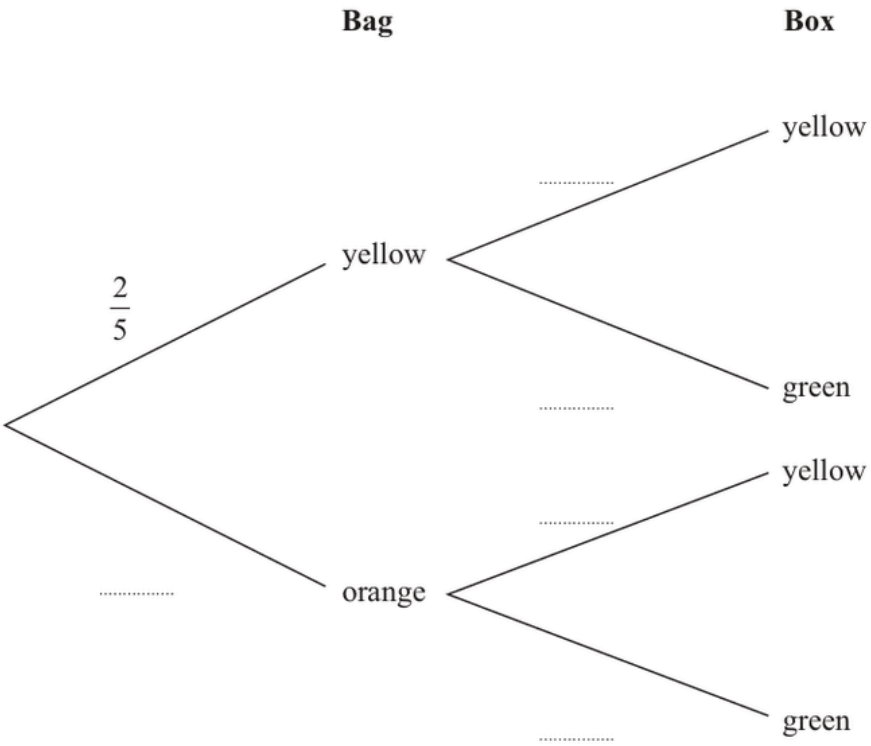
Question 13

5 marks | Tree Diagrams & Conditional Probability

13 In a bag, there are only 2 yellow beads and 3 orange beads.
In a box, there are only 3 yellow beads and 5 green beads.

Chi takes at random a bead from the bag and a bead from the box.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Chi takes two beads of different colours.

.....

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2025

1 **(a) Complete the tree**

Bag: orange $\frac{3}{5}$.

Box (on both branches): yellow $\frac{3}{8}$, green $\frac{5}{8}$.

2 (b) Use the complement

The only way to get the same colour is yellow from both, so

$$P(\text{different}) = 1 - \frac{2}{5} \times \frac{3}{8} = 1 - \frac{6}{40} = \frac{17}{20}$$

FINAL ANSWER

(a) $\frac{3}{5}$; $\frac{3}{8}, \frac{5}{8}$ on both branches (b) $\frac{17}{20}$

4MA1/1H NOVEMBER 2025

Question 22

3 marks | Tree Diagrams & Conditional Probability

22 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

4WM1/01 NOVEMBER 2025

Question 23

3 marks | Tree Diagrams & Conditional Probability

23 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

4MA1/1H JUNE 2026

Question 13

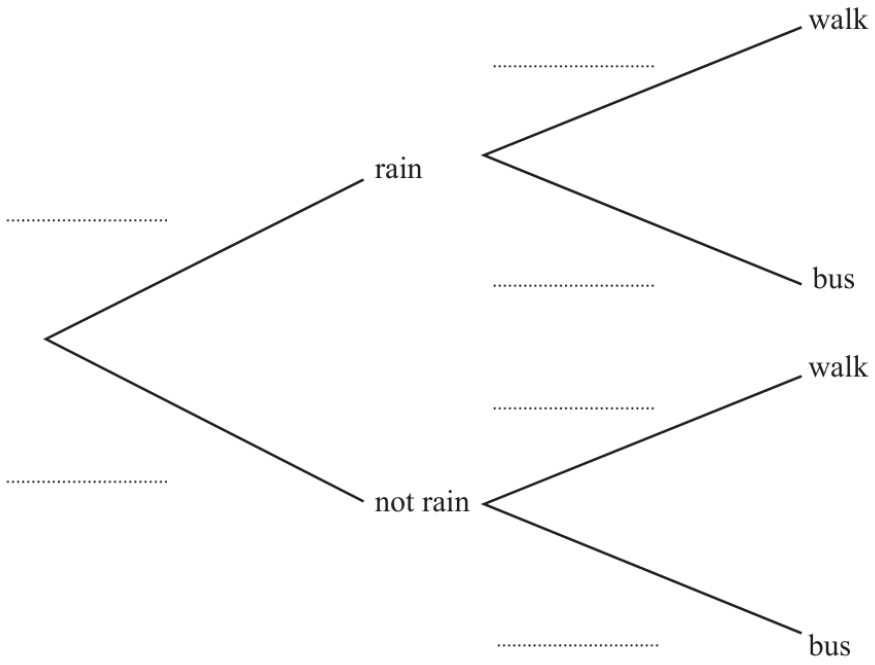
4 marks | Tree Diagrams & Conditional Probability

13 On any school day, Paulo either walks to school or travels by bus to school.

The probability that it rains on any school day is 0.15

If it rains on a school day, the probability that Paulo walks to school is 0.2
If it does not rain on a school day, the probability that Paulo walks to school is 0.75

(a) Complete the probability tree diagram for this information.



(2)

(b) Work out the probability that on a school day, it rains and Paulo walks to school.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

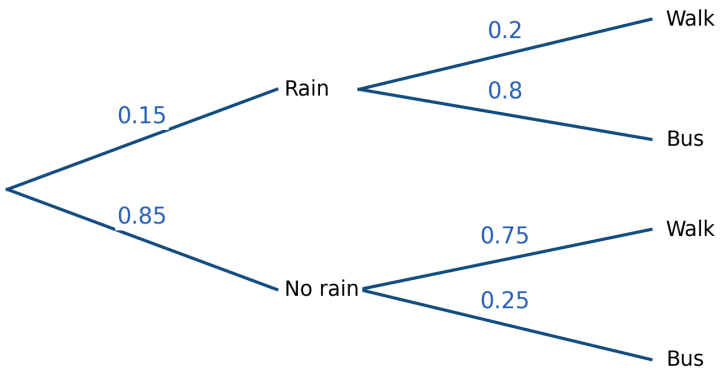
First split: rain 0.15, not rain 0.85.

Complementary branches sum to 1.

2 Part (a) | Step 2

After rain: walk 0.20, bus 0.80. After not rain: walk 0.75, bus 0.25.

Use the stated conditional probabilities and their complements on the corresponding branches.



3 Part (b) | Step 1

$$P(\text{rain and walk}) = P(\text{rain})P(\text{walk} \mid \text{rain}) = 0.15 \times 0.2 = 0.03.$$

Multiply the probabilities along the rain-then-walk path; independence is not assumed.

FINAL ANSWER

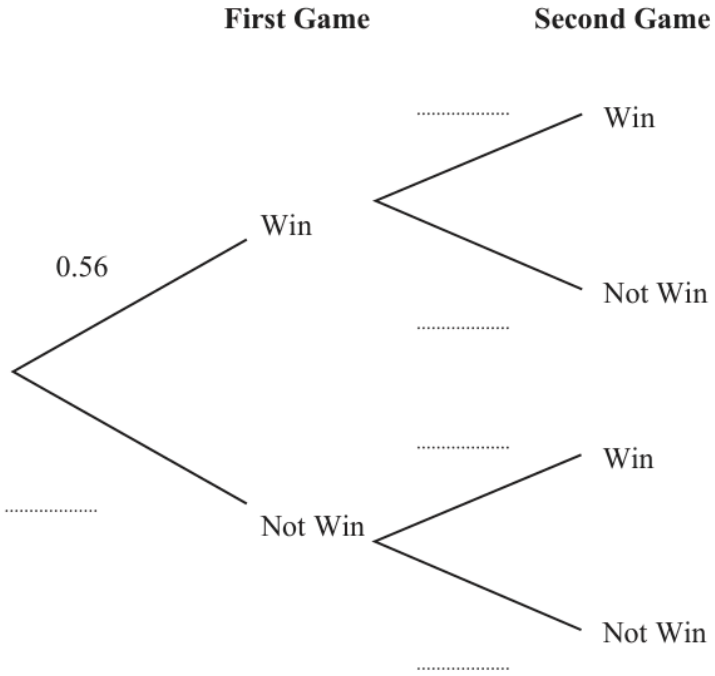
(a) Rain/not rain: $0.15, 0.85$; walk/bus after rain: $0.2, 0.8$; walk/bus after no rain: $0.75, 0.25$. (b) 0.03

Question 13

4 marks | Tree Diagrams & Conditional Probability

13 A team plays two games of football.
For each game, the probability that the team wins is 0.56

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that the team wins both games.

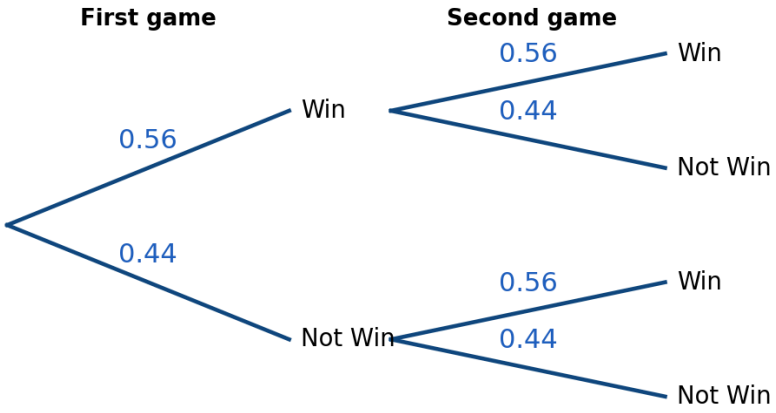
.....

(2)

(Total for Question 13 is 4 marks)

1 (a) Branches add to 1

Not win $= 1 - 0.56 = 0.44$ on every pair of branches.



2 (b) Win and win

$$0.56 \times 0.56 = 0.3136$$

FINAL ANSWER

(a) see diagram (b) 0.3136

Question 14

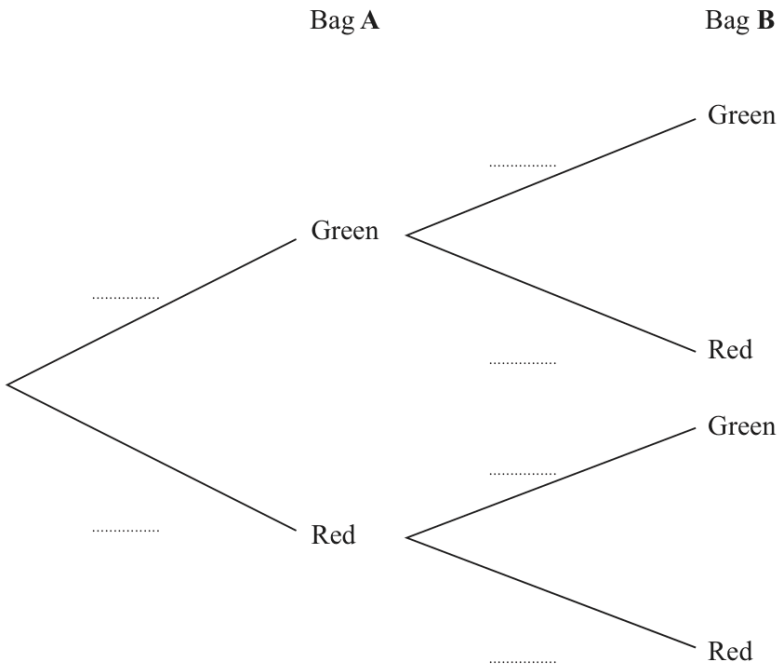
8 marks | Tree Diagrams & Conditional Probability

14 Pras has 2 bags of counters, bag **A** and bag **B**

In bag **A**, there are 4 green counters and 3 red counters.
In bag **B**, there are 5 green counters and 1 red counter.

Pras is going to take at random a counter from bag **A** and a counter from bag **B**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Pras will take two green counters.

.....

(2)

Pras also has a third bag of counters, bag **C**
In bag **C**, there are only green counters and red counters.
All the original counters are back in bag **A** and bag **B**

Pras is going to take at random a counter from bag **A**, a counter from bag **B** and a counter from bag **C**

The probability that Pras will take three red counters is $\frac{3}{70}$

(c) Work out the probability that Pras will take 1 green counter and 2 red counters.

.....

(4)

(Total for Question 14 is 8 marks)

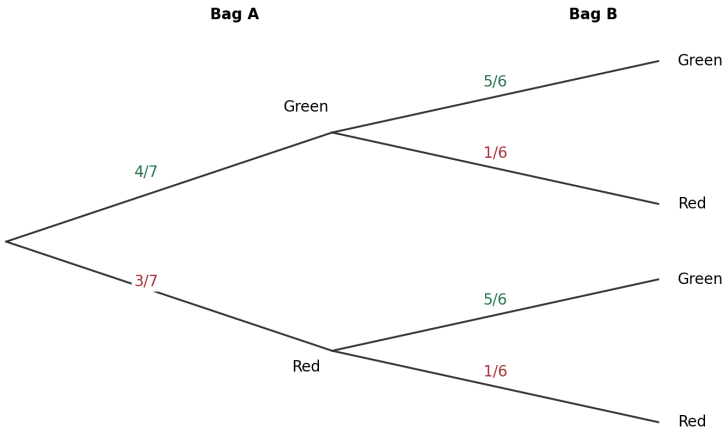
1 **Part (a) | Step 1**

From the start, label the Bag A branches $P(G_A) = 4/7$ and $P(R_A) = 3/7$.
Bag A has 7 counters in total.

2 **Part (a) | Step 2**

From each Bag A outcome, label the Bag B green branch $5/6$ and red branch $1/6$.
Bag B has 6 counters and is unaffected by the result from bag A.

Q14(a): Completed probability tree



3 **Part (b) | Step 1**

$$P(G_A G_B) = \frac{4}{7} \times \frac{5}{6} = \frac{20}{42} = \frac{10}{21}.$$

Both events must occur, so multiply along the green-green path.

4 **Part (c) | Step 1**

Let $r = P(R_C)$. Then

$$\frac{3}{7} \times \frac{1}{6} \times r = \frac{3}{70} \quad \Rightarrow \quad r = \frac{3}{5}, \qquad P(G_C) = \frac{2}{5}.$$

Use the given probability of three reds to find the bag C probabilities.

5 **Part (c) | Step 2**

Exactly one green means the disjoint outcomes $G_A R_B R_C$, $R_A G_B R_C$, or $R_A R_B G_C$.
The single green counter can come from A, B or C.

6 **Part (c) | Step 3**

$$P = \frac{4}{7}\frac{1}{6}\frac{3}{5} + \frac{3}{7}\frac{5}{6}\frac{3}{5} + \frac{3}{7}\frac{1}{6}\frac{2}{5} = \frac{2}{35} + \frac{3}{14} + \frac{1}{35} = \frac{3}{10}.$$

Multiply within each outcome and add the three mutually exclusive probabilities.

FINAL ANSWER

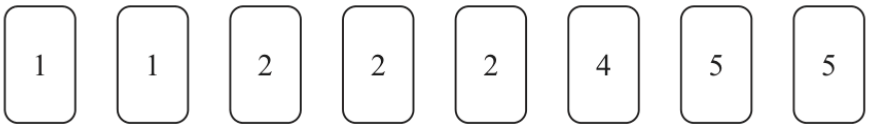
(a) Branch labels from top to bottom: $4/7, 3/7$ for A; $5/6, 1/6$ for B after either A result. (b) $\frac{10}{21}$. (c) $\frac{3}{10}$.

4MA1/1HR JUNE 2026

Question 18

3 marks | Tree Diagrams & Conditional Probability

18 Here are 8 cards.
There is a number on each card.



Gordon puts the 8 cards in a bag.
He takes at random a card from the bag and does not replace it.
Gordon then takes at random a second card from the bag.

Calculate the probability that the number on the second card is 3 more than the number on the first card.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2026

1 Solution | Step 1

The only possible ordered number pairs are $(1, 4)$ and $(2, 5)$.
The second number must be exactly 3 more than the first, not merely differ by 3.

2 Solution | Step 2

$$P(1 \text{ then } 4) = \frac{2}{8} \times \frac{1}{7}, \qquad P(2 \text{ then } 5) = \frac{3}{8} \times \frac{2}{7}.$$

There are 7 cards remaining for the second draw; there are two 1s, three 2s, one 4 and two 5s.

3 Solution | Step 3

$$P = \frac{2}{56} + \frac{6}{56} = \frac{8}{56} = \frac{1}{7}.$$

The two ordered events are mutually exclusive, so add their probabilities.

FINAL ANSWER

$\frac{1}{7}$

Question 22

5 marks | Tree Diagrams & Conditional Probability

22 A bag contains x marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 22 is 5 marks)

1 **Pink then yellow, or yellow then pink**

$$2 \times \frac{12}{x} \times \frac{x-12}{x-1} = \frac{18}{47}$$

2 **Cross-multiply**

$$1128(x-12) = 18x(x-1) \Rightarrow 18x^2 - 1146x + 13536 = 0$$

3 **Simplify (÷6) and solve**

$$3x^2 - 191x + 2256 = 0 \Rightarrow (x-48)(3x-47) = 0$$

4 **x must be a whole number**

$$x = 48$$

✓ **FINAL ANSWER**

$x = 48$

4MA1/1H JUNE 2026

Question 23

5 marks | Tree Diagrams & Conditional Probability

23 A bag contains x marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H June 2026

1 Solution | Step 1

$$P(\text{one pink, one yellow}) = \frac{12}{x}\frac{x-12}{x-1} + \frac{x-12}{x}\frac{12}{x-1} = \frac{24(x-12)}{x(x-1)}.$$

Include pink-then-yellow and yellow-then-pink, without replacement.

2 Solution | Step 2

$$\frac{24(x-12)}{x(x-1)} = \frac{18}{47} \implies 1128(x-12) = 18x(x-1).$$

Equate to the given probability and cross-multiply.

3 Solution | Step 3

$$18x^2 - 1146x + 13536 = 0 \implies 3x^2 - 191x + 2256 = 0.$$

Rearrange and divide by 6.

4 Solution | Step 4

$$(x-48)(3x-47) = 0 \implies x = 48 \text{ or } x = \frac{47}{3}.$$

Factorise the quadratic.

5 **Solution | Step 5**

The number of marbles must be an integer, so $x = 48$.

Reject the non-integer root.

✓ **FINAL ANSWER**

$x = 48$

TOPIC

Set Notation & Venn Diagrams

Every past-paper question on this topic, 2020 to 2026.

Question 14

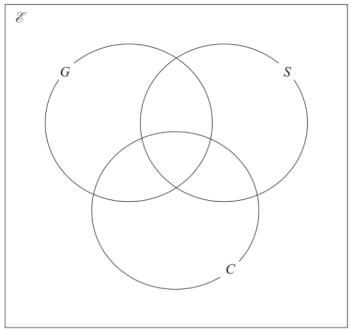
8 marks | Set Notation & Venn Diagrams

14 100 farmers are asked if they have goats (G), sheep (S) or chickens (C) on their farms.

Of these farmers

31 have sheep
53 have chickens
6 have goats, sheep and chickens
11 have sheep and goats
17 have sheep and chickens
18 have goats and chickens
20 do not have any goats, sheep or chickens

(a) Using this information, complete the Venn diagram to show the number of farmers in each appropriate subset.



(3)

(b) Find

(i) $n(G)$

.....

(1)

(ii) $n([G \cup S]')$

.....

(1)

(iii) $n(G' \cap C)$

.....

(1)

One of the farmers who has chickens is chosen at random.

(c) Find the probability that this farmer also has goats.

.....

(2)

(Total for Question 14 is 8 marks)

1 **Place the centre value**

The number with goats, sheep and chickens is 6.

2 **Fill the pair-only regions**

$$G \cap S \text{ only} = 11 - 6 = 5$$

$$S \cap C \text{ only} = 17 - 6 = 11$$

$$G \cap C \text{ only} = 18 - 6 = 12$$

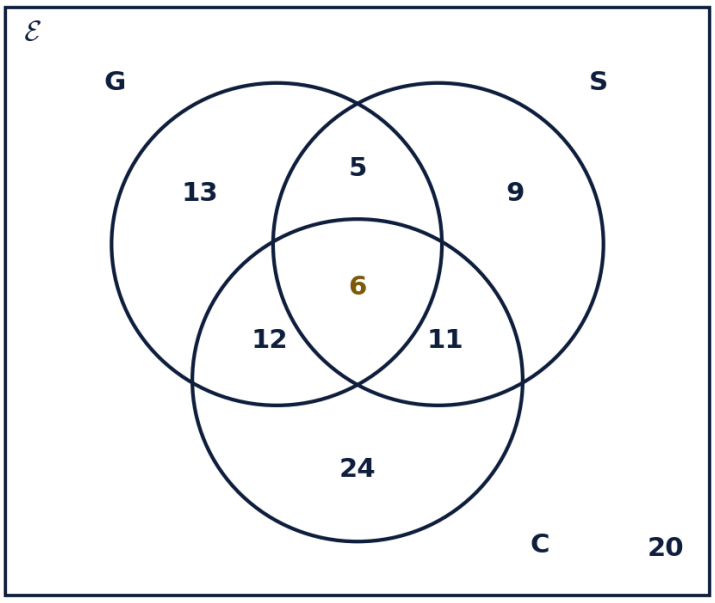
3 **Fill the single-set regions**

$$S \text{ only} = 31 - 5 - 11 - 6 = 9$$

$$C \text{ only} = 53 - 11 - 12 - 6 = 24$$

Since 20 have none, the total inside the circles is 80.

$$G \text{ only} = 80 - (5 + 11 + 12 + 6 + 9 + 24) = 13$$



4 Read the set values

$$n(G) = 13 + 5 + 12 + 6 = 36$$

$$n((G \cup S)') = 24 + 20 = 44$$

$$n(G' \cap C) = 24 + 11 = 35$$

5 Find the conditional probability

Out of the 53 farmers with chickens, 18 also have goats.

$$P(G \mid C) = \frac{18}{53}$$

FINAL ANSWER

Venn regions: G only 13, S only 9, C only 24, $G \cap S$ only 5, $S \cap C$ only 11, $G \cap C$ only 12, all three 6, none 20. Also 36, 44, 35, and $\frac{18}{53}$.

4MA1/1HR JANUARY 2020

Question 4

3 marks | Set Notation & Venn Diagrams

4 $B = \{b, l, u, e\}$

$G = \{g, r, e, y\}$

$W = \{w, h, i, t, e\}$

(a) List all the members of the set

(i) $B \cup G$

(ii) $W \cap G'$

.....

(2)

Serena writes down the statement $B \cap G \cap W = \varnothing$

(b) Is Serena's statement correct?
You must give a reason for your answer.

.....

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR January 2020

1 Calculate value

$$B = \{b, l, u, e\}, \quad G = \{g, r, e, y\}, \quad W = \{w, h, i, t, e\}$$

2 means all the letters that

$B \cup G$ means all the letters that are in B or in G.

$$B \cup G = \{b, l, u, e, g, r, y\}$$

3 Calculate statistic

$W \cap G'$ means letters that are in W, but not in G. Since e is in G, it is not included.

$$W \cap G' = \{w, h, i, t\}$$

4 **Serena is not correct because**

Serena is not correct because e is in all three sets.

$$B \cap G \cap W = \{e\}$$

✓ **FINAL ANSWER**

(a)(i) $\{b, l, u, e, g, r, y\}$, (a)(ii) $\{w, h, i, t\}$. Serena is incorrect.

Question 17

6 marks | Set Notation & Venn Diagrams

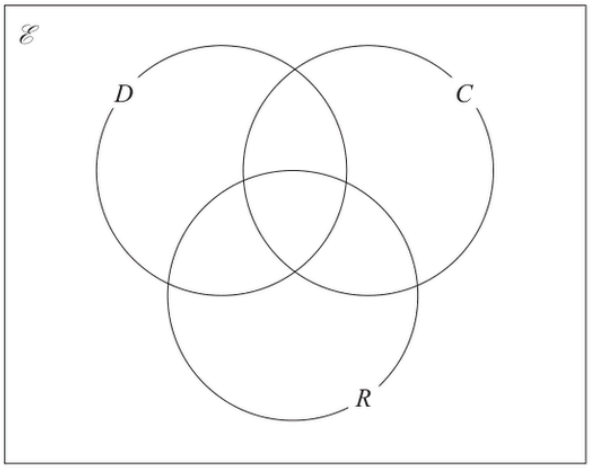
17 Some students in a school were asked the following question.

"Do you have a dog (D), a cat (C) or a rabbit (R)?"

Of these students

28 have a dog
18 have a cat
20 have a rabbit
8 have both a cat and a rabbit
9 have both a dog and a rabbit
x have both a dog and a cat
6 have a dog, a cat and a rabbit
5 have not got a dog or a cat or a rabbit

(a) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.
Give the numbers in terms of x where necessary.



(3)

Given that a total of 50 students answered the question,

(b) work out the value of x.

$x =$ .....

(2)

(c) Find $n(C' \cap D')$

.....

(1)

(Total for Question 17 is 6 marks)

1 **Let the sets dogs**

Let the sets be D for dogs, C for cats, and R for rabbits.

2 **The number in all three**

The number in all three sets is 6.

$$D \cap R \text{ only} = 9 - 6 = 3$$

$$C \cap R \text{ only} = 8 - 6 = 2$$

$$D \cap C \text{ only} = x - 6$$

3 **Calculate value**

So the remaining one-set regions are

$$D \text{ only} = 25 - x, \quad C \text{ only} = 16 - x, \quad R \text{ only} = 9$$

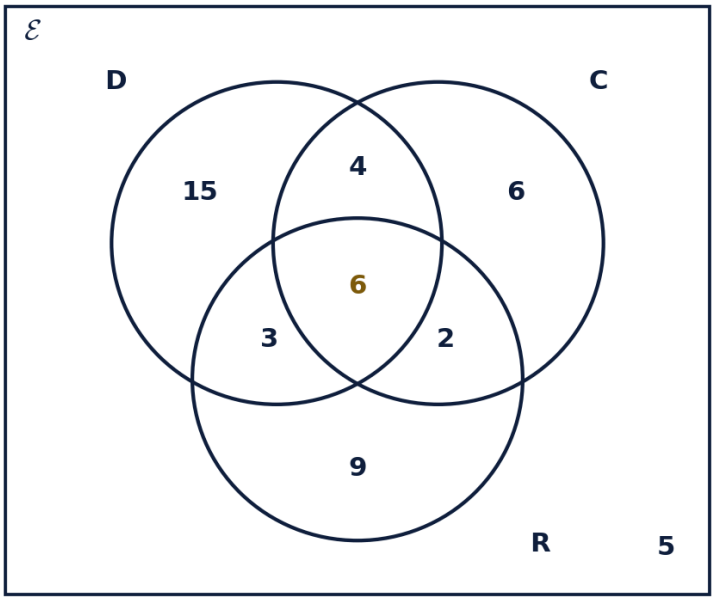
4 **There are students outside all**

There are 5 students outside all three sets. The total number of students is 50, so

$$(25 - x) + (16 - x) + 9 + (x - 6) + 3 + 2 + 6 + 5 = 50$$

$$60 - x = 50$$

$$x = 10$$



5

Calculate value

For $C' \cap D'$, we need students who are not in C and not in D. These are the rabbit-only students and the students outside all three sets.

$$9 + 5 = 14$$

✓ **FINAL ANSWER**

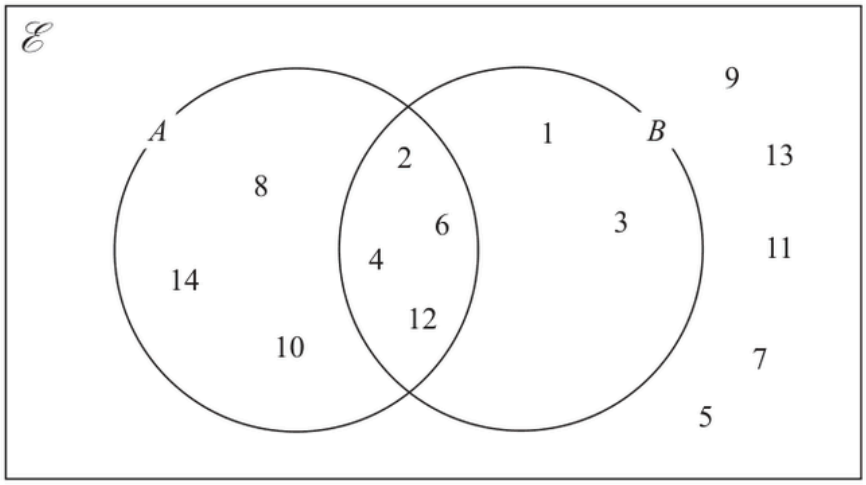
$x = 10$ and $n(C' \cap D') = 14$

4MA1/1H MAY/NOVEMBER 2020

Question 1

4 marks | Set Notation & Venn Diagrams

1 The numbers from 1 to 14 are shown in the Venn diagram.



(a) List the members of the set $A \cap B$

.....

(1)

(b) List the members of the set B'

.....

(1)

A number is picked at random from the numbers in the Venn diagram.

(c) Find the probability that this number is in set A but is **not** in set B.

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May/November 2020

1 **(a) In both A and B**

$A \cap B = \{2, 4, 6, 12\}$

2 **(b) Not in B**

Everything outside circle B: $B' = \{5, 7, 8, 9, 10, 11, 13, 14\}$

3 **(c) In A but not in B**

$\{8, 10, 14\}$: 3 of the 14 numbers.

$$\frac{3}{14}$$

✓ **FINAL ANSWER**

(a) $2, 4, 6, 12$ (b) $5, 7, 8, 9, 10, 11, 13, 14$ (c) $\frac{3}{14}$

4MA1/2HR JANUARY 2021

Question 3

3 marks | Set Notation & Venn Diagrams

3 $\mathscr{E} = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$
$A = \{22, 24, 26, 28, 30\}$
$B = \{21, 24, 27, 30\}$

(a) List the members of the set

(i) $A \cap B$

(ii) A'

.....

(2)

$C = \{23, 25, 29\}$

(b) Using set notation, find an expression for C in terms of A and B.

.....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2021

1 Calculate value

$$A \cap B = \{24, 30\}$$

2 Set is

The complement of A, using the given universal set, is

$$A' = \{21, 23, 25, 27, 29\}$$

3 The set

The set

$$C = \{23, 25, 29\}$$

4 Calculate value

is everything not in A and not in B, so

$$C = A' \cap B'$$

5 **Calculate value**

Equivalently,

$$C = (A \cup B)'$$

FINAL ANSWER

$A \cap B = \{24, 30\}$, $A' = \{21, 23, 25, 27, 29\}$, and $C = A' \cap B'$.

4MA1/1H JANUARY 2021

Question 4

2 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$A = \{\text{odd numbers}\}$
$B = \{\text{multiples of 3}\}$

List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2021

1 Calculate value

$$\mathcal{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$$

2 The odd numbers are

The odd numbers are

$$A = \{21, 23, 25, 27, 29\}$$

3 The multiples of are

The multiples of 3 are

$$B = \{21, 24, 27\}$$

4 The numbers in both sets

The numbers in both sets are

$$A \cap B = \{21, 27\}$$

5 The numbers in either set

The numbers in either set are

$$A \cup B = \{21, 23, 24, 25, 27, 29\}$$

FINAL ANSWER

(i) $\{21, 27\}$, (ii) $\{21, 23, 24, 25, 27, 29\}$

Question 16

6 marks | Set Notation & Venn Diagrams

16 Some students were asked the following question.

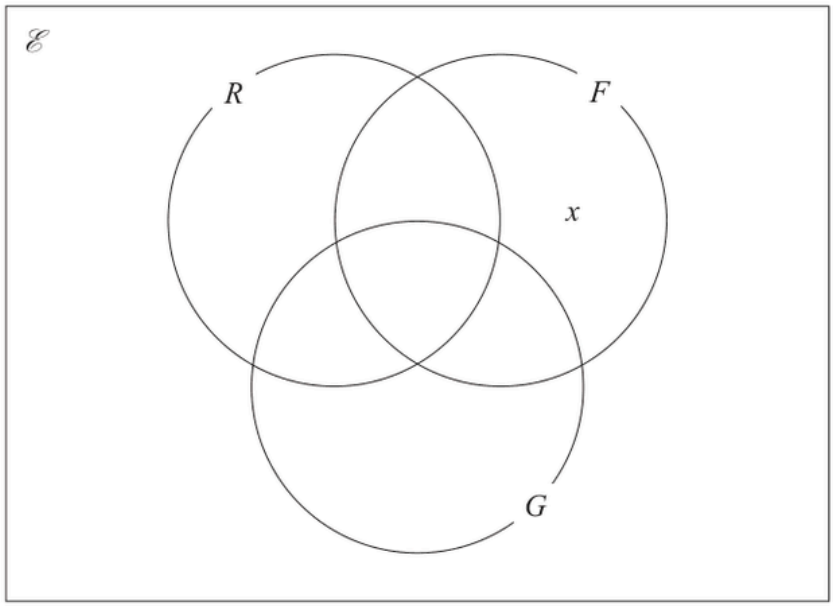
"Which of the subjects Russian (R), French (F) and German (G) do you study?"

Of these students

4 study all three of Russian, French and German
10 study Russian and French
13 study French and German
6 study Russian and German
24 study German
11 study none of the three subjects
the number who study Russian only is twice the number who study French only.

Let x be the number of students who study French only.

(a) Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of x where necessary.



(3)

Given that the number of students who were asked the question was 80

(b) work out the number of these students that study Russian.

.....

(3)

(Total for Question 16 is 6 marks)

1 **The number studying all three**

The number studying all three languages is 4.

2 **Calculate value**

So the two-language-only regions are

$$R \cap F \text{ only} = 10 - 4 = 6$$

$$F \cap G \text{ only} = 13 - 4 = 9$$

$$R \cap G \text{ only} = 6 - 4 = 2$$

3 **The German only region is**

The German-only region is

$$24 - 9 - 2 - 4 = 9$$

4 **The French only region is**

The French-only region is x and the Russian-only region is $2x$. There are 11 students outside all three sets.

5 **Calculate value**

Using the total 80,

$$2x + x + 9 + 6 + 9 + 2 + 4 + 11 = 80$$

$$3x + 41 = 80$$

$$x = 13$$

6 **The number studying Russian is**

The number studying Russian is

$$2x + 6 + 2 + 4 = 26 + 6 + 2 + 4 = 38$$

✓ **FINAL ANSWER**

38

4MA1/2HR JANUARY 2021

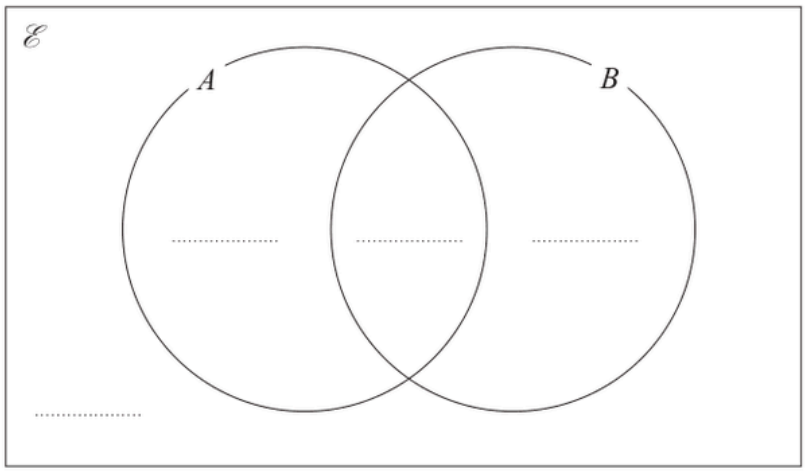
Question 16

5 marks | Set Notation & Venn Diagrams

16 Two events A and B are such that $n(A) = 62$ $n(B) = 30$ and $n(A \cup B) = 68$

Given that $n(\mathscr{E}) = 80$

(a) complete the Venn diagram to show the number of elements in each region.



(2)

An element is chosen at random from $\mathscr{E}$.

(b) Using the Venn diagram, find the probability that this element is in

(i) $A \cap B$

.....

(1)

(ii) $A \cup B'$

.....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR January 2021

1 **(a) Find the intersection**

$$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 62 + 30 - 68 = 24$$

2 **(a) Fill every region**

A only $= 62 - 24 = 38$, B only $= 30 - 24 = 6$, outside $= 80 - 68 = 12$.

3 **(b)(i) In both A and B**

$$P(A \cap B) = \frac{24}{80} = \frac{3}{10}$$

---

4 **(b)(ii) In A or not in B**

$A \cup B'$ is everything except "B only": $80 - 6 = 74$.

$$P(A \cup B') = \frac{74}{80} = \frac{37}{40}$$

✓ **FINAL ANSWER**

(a) $38, 24, 6$, outside 12 (b)(i) $\frac{3}{10}$ (ii) $\frac{37}{40}$

4MA1/2H MAY 2021

Question 4

3 marks | Set Notation & Venn Diagrams

4 $\mathscr{E}$ = {letters of the alphabet}
B = {b, r, a, z, i, l}
I = {i, r, e, l, a, n, d}

(a) List the members of the set

(i) $B \cup I$

(ii) $B \cap I'$

.....

(2)

K = {k, e, n, y, a}

Cody writes down the statement $B \cap K = \varnothing$
Cody's statement is wrong.

(b) Explain why.

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H May 2021

1 Calculate value

$$B = \{b, r, a, z, i, l\}$$

$$I = \{i, r, e, l, a, n, d\}$$

2 The union contains every letter

The union contains every letter that is in B or I:

$$B \cup I = \{a, b, d, e, i, l, n, r, z\}$$

3 Calculate statistic

$B \cap I'$ means letters in B, but not in I:

$$B \cap I' = \{b, z\}$$

4 **Calculate value**

For $K = \{k, e, n, y, a\}$, Cody is not correct because the letter a is in both B and K.

$$B \cap K = \{a\}$$

✓ **FINAL ANSWER**

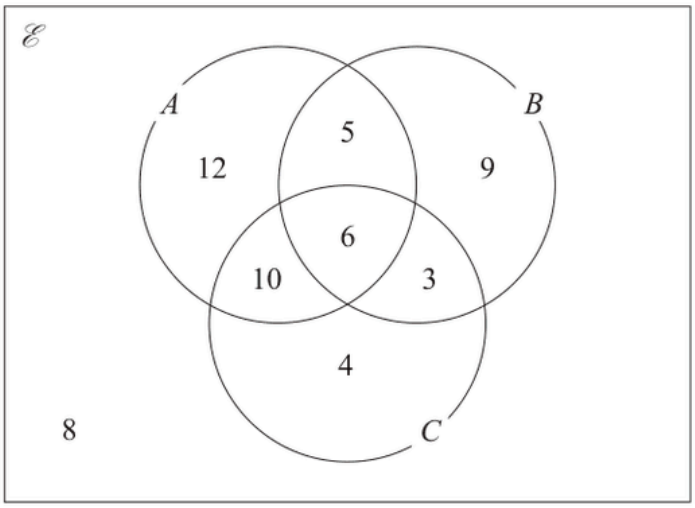
(a)(i) $\{a, b, d, e, i, l, n, r, z\}$, (a)(ii) $\{b, z\}$. Cody is incorrect.

4MA1/1H MAY 2021

Question 19

3 marks | Set Notation & Venn Diagrams

19 The Venn diagram shows a universal set, $\mathscr{E}$ and sets A, B and C.



12, 5, 9, 10, 6, 3, 4 and 8 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A' \cap B')$

.....

(1)

(iii) $n([A \cap B] \cup C)$

.....

(1)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2021

1 **From the Venn diagram**

From the Venn diagram:

2 **only is only is only**

- A only is 12
- $A \cap B$ only is 5
- B only is 9
- $A \cap C$ only is 10
- all three is 6
- $B \cap C$ only is 3
- C only is 4
- outside is 8

---

3 **Add every region inside**

For $A \cup B$, add every region inside A or B:

$$12 + 5 + 9 + 10 + 6 + 3 = 45$$

---

4 **Add the regions outside both**

For $A' \cap B'$, add the regions outside both A and B:

$$4 + 8 = 12$$

---

5 **Add and all**

For $(A \cap B) \cup C$, add $A \cap B$ and all of C:

$$5 + 6 + 10 + 3 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 45, (ii) 12, (iii) 28

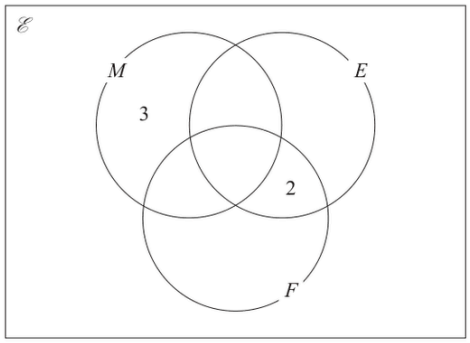
Question 16

5 marks | Set Notation & Venn Diagrams

16 There are 32 students in a class.
In one term these 32 students each took a test in Maths (M), in English (E) and in French (F).

25 students passed the test in Maths.
20 students passed the test in English.
14 students passed the test in French.
18 students passed the tests in Maths and English.
11 students passed the tests in Maths and French.
4 students failed all three tests.
x students passed all three tests.

The incomplete Venn diagram gives some more information about the results of the 32 students.

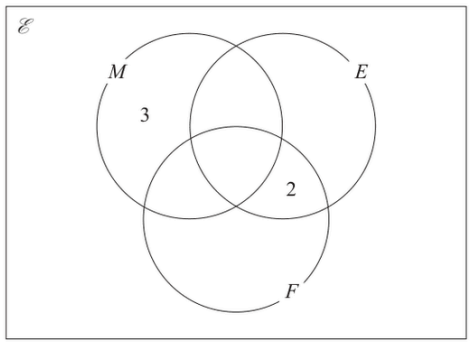


(a) Use all the given information about the results of students who passed the test in Maths to find the value of x.

$x =$ .....

(2)

(b) Use your value of x to complete the Venn diagram to show the number of students in each subset.



(2)

A student who passed the test in Maths is chosen at random.

(c) Find the probability that this student failed the test in French.

.....

(1)

(Total for Question 16 is 5 marks)

1 **(a) Use the Maths circle**

Maths only $= 3$, Maths and English only $= 18 - x$, Maths and French only $= 11 - x$, all three $= x$.

$$3 + (18 - x) + (11 - x) + x = 25 \;\Rightarrow\; 32 - x = 25 \;\Rightarrow\; x = 7$$

2 **(b) Fill every region**

$M \cap E$ only $= 11$, $M \cap F$ only $= 4$, centre $= 7$, $E \cap F$ only $= 2$.

$$E \text{ only} = 20 - 11 - 7 - 2 = 0, \qquad F \text{ only} = 14 - 4 - 7 - 2 = 1$$

Outside: 4. (Check: $3 + 11 + 4 + 7 + 0 + 2 + 1 + 4 = 32$.)

3 **(c) Passed Maths, failed French**

Maths only and Maths-and-English only: $3 + 11 = 14$ of the 25.

$$\frac{14}{25}$$

✓ **FINAL ANSWER**

(a) $x = 7$ (b) M only 3, $M \cap E$ only 11, $M \cap F$ only 4, centre 7, $E \cap F$ only 2, E only 0, F only 1, outside 4 (c) $\frac{14}{25}$

4MA1/1H JANUARY 2022

Question 7

3 marks | Set Notation & Venn Diagrams

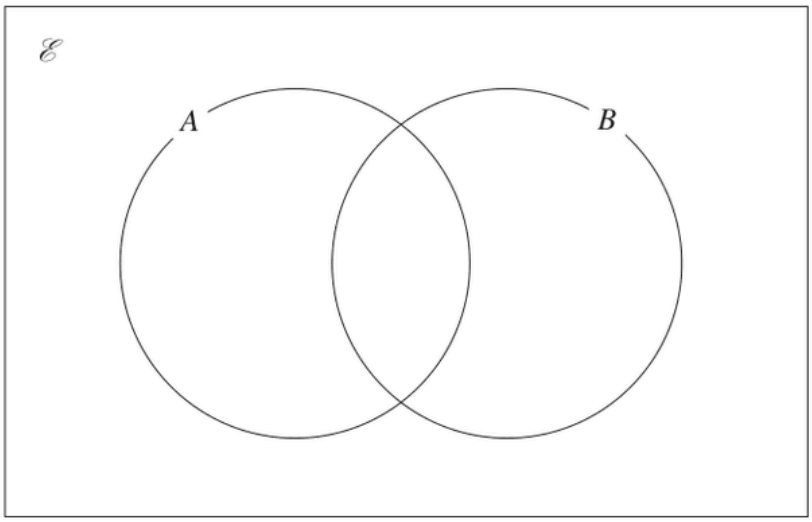
7 $\mathscr{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A \cap B = \{5, 10, 15\}$

$B' = \{7, 8, 9, 11, 12, 13, 14\}$

$A' = \{4, 6, 7, 8, 14\}$

Complete the Venn diagram for this information.



(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2022

1 Calculate value

$$\mathcal{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

2 We are given

We are given

$$A \cap B = \{5, 10, 15\}$$

3 Calculate value

Also,

$$B' = \{7, 8, 9, 11, 12, 13, 14\}$$

4 **Calculate value**

so

$$B = \{4, 5, 6, 10, 15\}$$

5 **Calculate value**

And

$$A' = \{4, 6, 7, 8, 14\}$$

6 **Calculate value**

so

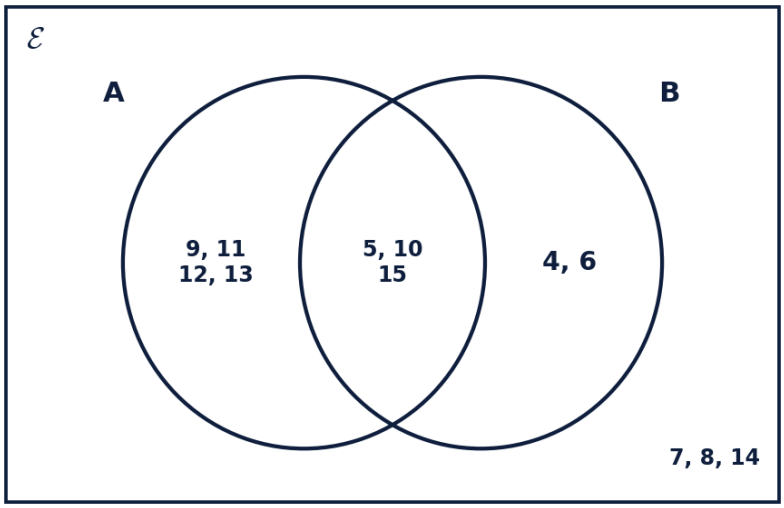
$$A = \{5, 9, 10, 11, 12, 13, 15\}$$

7 **Use Set notation Venn diagrams**

Therefore the Venn diagram regions are:

8 **only only outside both sets**

- $A \cap B$: $\{5, 10, 15\}$
- A only: $\{9, 11, 12, 13\}$
- B only: $\{4, 6\}$
- outside both sets: $\{7, 8, 14\}$



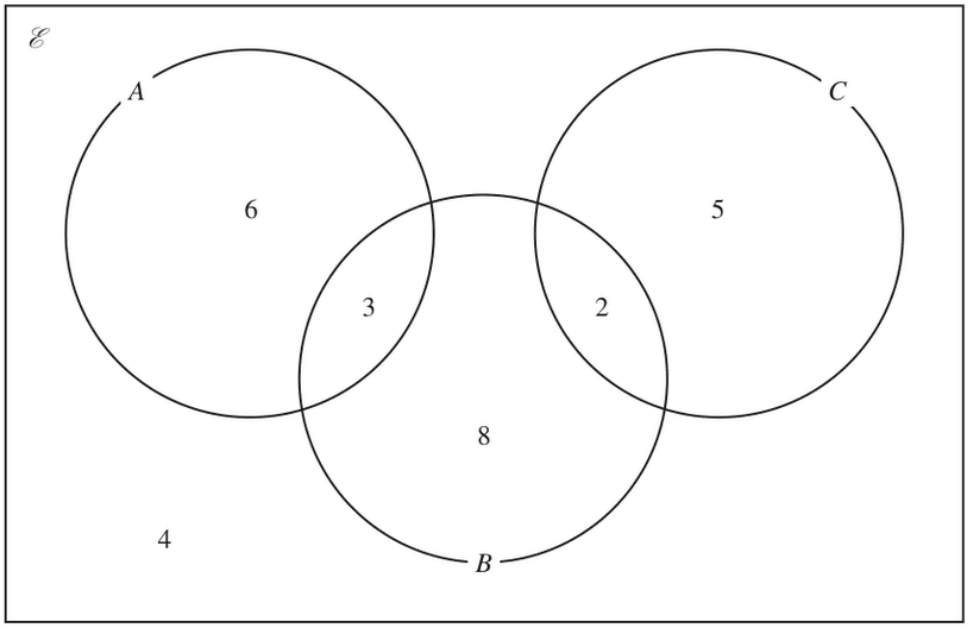
FINAL ANSWER

Put $\{5, 10, 15\}$ in the overlap, $\{9, 11, 12, 13\}$ in A only, $\{4, 6\}$ in B only, and $\{7, 8, 14\}$ outside.

Question 17

4 marks | Set Notation & Venn Diagrams

17 The Venn diagram shows a universal set $\mathscr{E}$ and three sets A, B and C.



6, 3, 8, 2, 5 and 4 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A \cap C)$

.....

(1)

(iii) $n(B \cap C')$

.....

(1)

(iv) $n(A' \cup B' \cup C')$

.....

(1)

(Total for Question 17 is 4 marks)

1 **From the Venn diagram**

From the Venn diagram:

$$A \text{ only} = 6, \quad A \cap B = 3, \quad B \text{ only} = 8, \quad B \cap C = 2, \quad C \text{ only} = 5, \quad \text{outside} = 4$$

2 **Calculate value**

There is no region in both A and C, and there is no triple intersection.

$$n(A \cup B) = 6 + 3 + 8 + 2 = 19$$

$$n(A \cap C) = 0$$

$$n(B \cap C') = 3 + 8 = 11$$

3 **Use**

For $A' \cup B' \cup C'$, use

$$A' \cup B' \cup C' = (A \cap B \cap C)'$$

4 **Set**

Since $A \cap B \cap C$ is empty, this includes everyone in the universal set.

$$6 + 3 + 8 + 2 + 5 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 19, (ii) 0, (iii) 11, (iv) 28

4MA1/2HR JUNE 2022

Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
$A = \{\text{multiples of } 3\}$
$B = \{\text{odd numbers}\}$

(a) List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(b) Is it true that $24 \in A$?

Tick one of the boxes below.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

Set C has 4 members such that $C \cap B' = \{10, 18\}$

(c) List the members of one possible set C

.....

(2)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2022

1 **Calculate value**

$$\mathcal{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

2 **The multiples of in are**

The multiples of 3 in $\mathcal{E}$ are

$$A = \{9, 12, 15, 18\}$$

3 **The odd numbers in are**

The odd numbers in $\mathcal{E}$ are

$$B = \{9, 11, 13, 15, 17, 19\}$$

4 **Calculate value**

So

$$A \cap B = \{9, 15\}$$

5 **Calculate value**

and

$$A \cup B = \{9, 11, 12, 13, 15, 17, 18, 19\}$$

6 **The statement is false because**

The statement $24 \in A$ is false because 24 is not in the universal set $\mathcal{E}$.

7 **Calculate value**

For C, we need $C \cap B' = \{10, 18\}$ and C has 4 members. One valid choice is

$$C = \{9, 10, 11, 18\}$$

✓ **FINAL ANSWER**

(a)(i) $\{9, 15\}$, (a)(ii) $\{9, 11, 12, 13, 15, 17, 18, 19\}$, (b) No, (c) one possible set is $\{9, 10, 11, 18\}$

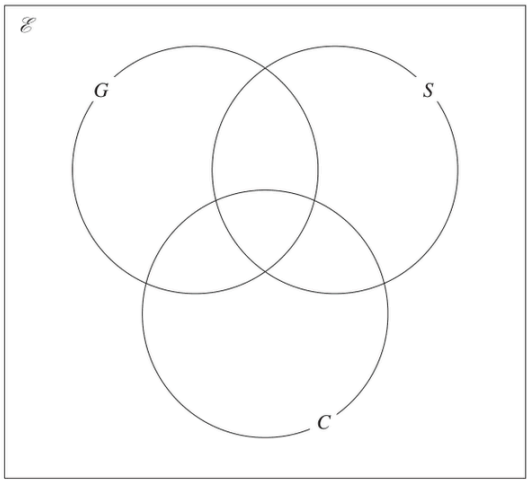
Question 16

8 marks | Set Notation & Venn Diagrams

16 100 farmers are asked if they have goats (G), sheep (S) or chickens (C) on their farms.

Of these farmers
- 31 have sheep
- 53 have chickens
- 6 have goats, sheep and chickens
- 11 have sheep and goats
- 17 have sheep and chickens
- 18 have goats and chickens
- 20 do not have any goats, sheep or chickens

(a) Using this information, complete the Venn diagram to show the number of farmers in each appropriate subset.



(3)

(b) Find

(i) n(G)

.....

(1)

(ii) n($[G \cup S]'$)

.....

(1)

(iii) n($G' \cap C$)

.....

(1)

One of the farmers who has chickens is chosen at random.

(c) Find the probability that this farmer also has goats.

.....

(2)

(Total for Question 16 is 8 marks)

1 **(a) Start from the centre**

All three: 6. Then the "two only" regions:

$$G \cap S \text{ only} = 11 - 6 = 5, \quad S \cap C \text{ only} = 17 - 6 = 11, \quad G \cap C \text{ only} = 18 - 6 = 12$$

2 **(a) Single-animal regions**

$$S \text{ only} = 31 - 5 - 11 - 6 = 9, \qquad C \text{ only} = 53 - 11 - 12 - 6 = 24$$

80 farmers are inside the circles ($100 - 20$), so

$$G \text{ only} = 80 - (9 + 24 + 5 + 11 + 12 + 6) = 13$$

Outside the circles: 20.

3 **(b)(i) n(G)**

$$13 + 5 + 12 + 6 = 36$$

4 **(b)(ii) Not in G or S**

Only "C only" and the outside region: $24 + 20 = 44$.

5 **(b)(iii) In C but not in G**

"C only" and "$S \cap C$ only": $24 + 11 = 35$.

6 **(c) Chickens farmers who also have goats**

53 farmers have chickens; $12 + 6 = 18$ of them also have goats.

$$\frac{18}{53}$$

FINAL ANSWER

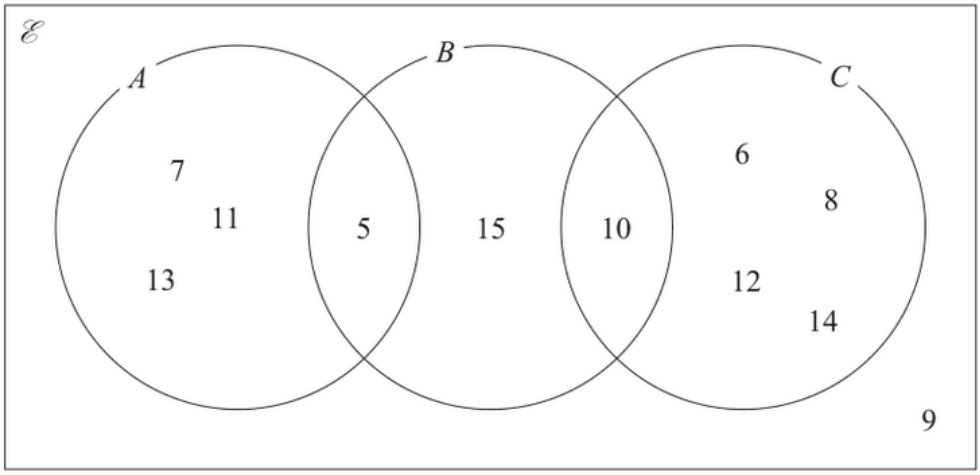
(a) G only 13, S only 9, C only 24, $G \cap S$ only 5, $S \cap C$ only 11, $G \cap C$ only 12, centre 6, outside 20
(b)(i) 36 (ii) 44 (iii) 35 (c) $\frac{18}{53}$

4MA1/1HR MAY 2023

Question 2

3 marks | Set Notation & Venn Diagrams

2 Here is a Venn diagram.



(a) Write down the numbers that are in the set

(i) A

.....

(1)

(ii) $B \cup C$

.....

(1)

Dominic writes down $9 \notin C$

(b) Explain why Dominic is correct.

.....

.....

(1)

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR May 2023

1 **The numbers inside are the**

The numbers inside A are the numbers in the A-only region and the overlap with B:

$$A = \{5, 7, 11, 13\}$$

2 **Calculate statistic**

$B \cup C$ means every number inside B or C:

$$B \cup C = \{5, 6, 8, 10, 12, 14, 15\}$$

---

3 **Dominic writes . This is**

Dominic writes $9 \notin C$. This is correct because 9 is outside circle C in the Venn diagram.

✓ **FINAL ANSWER**

(a)(i) $\{5, 7, 11, 13\}$, (a)(ii) $\{5, 6, 8, 10, 12, 14, 15\}$. Dominic is correct because 9 is not in C.

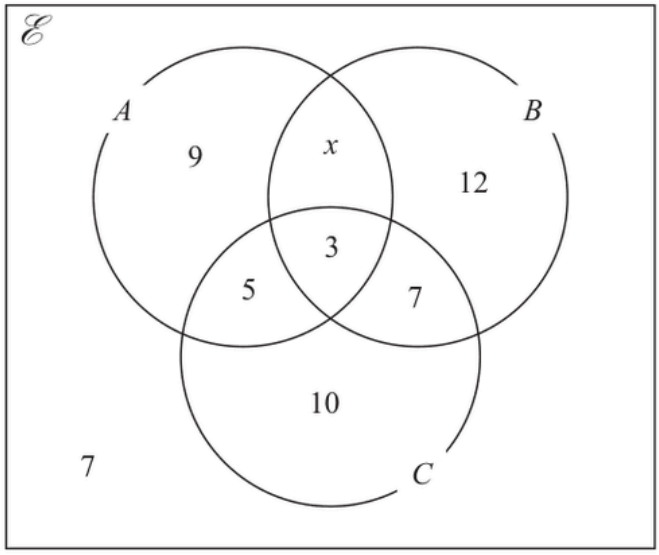
4MA1/1H MAY 2023

Question 16

3 marks | Set Notation & Venn Diagrams

16 The Venn diagram shows a universal set $\mathscr{E}$ and sets A, B and C

The numbers and the letter x represent **numbers** of elements.



Given that $n(A \cup B) = 42$

(a) find the value of x

$x =$ .....

(1)

(b) Find $n(A')$

.....

(1)

(c) Find $n(B' \cap C)$

.....

(1)

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2023

1 **The regions inside are**

The regions inside $A \cup B$ are

$$9 + x + 12 + 5 + 3 + 7$$

2 **We are told that**

We are told that

$$n(A \cup B) = 42$$

3 **Calculate value**

So

$$36 + x = 42$$

$$x = 6$$

4 **Add every region not inside**

For A', add every region not inside A:

$$12 + 7 + 10 + 7 = 36$$

5 **Add the regions**

For $B' \cap C$, add the regions that are in C but not in B:

$$5 + 10 = 15$$

✓ **FINAL ANSWER**

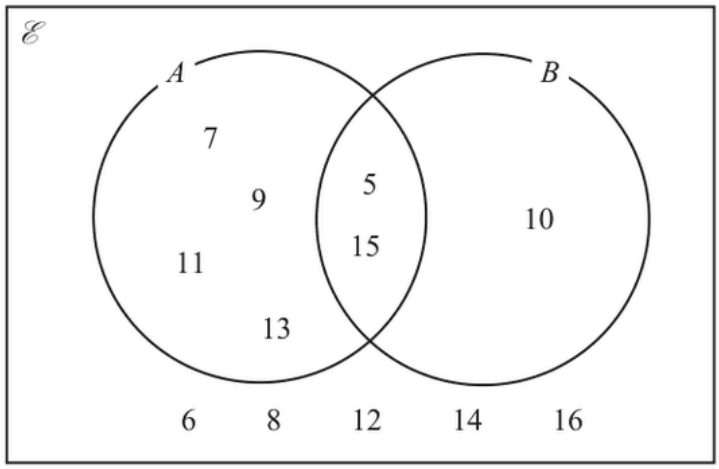
(a) $x = 6$, (b) 36, (c) 15

4MA1/1H NOVEMBER 2023

Question 1

3 marks | Set Notation & Venn Diagrams

1 Here is a Venn diagram.



List the members of the set

(a) A

.....

(1)

(b) $A \cap B$

.....

(1)

(c) $(A \cup B)'$

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2023

1 **The members of are all**

The members of A are all the numbers inside circle A, including the overlap:

$$A = \{5, 7, 9, 11, 13, 15\}$$

2 **The members of are the**

The members of $A \cap B$ are the numbers in both circles:

$$A \cap B = \{5, 15\}$$

3 **means outside both circles**

$(A \cup B)'$ means outside both circles:

$$(A \cup B)' = \{6, 8, 12, 14, 16\}$$

✓ **FINAL ANSWER**

(a) $\{5, 7, 9, 11, 13, 15\}$, (b) $\{5, 15\}$, (c) $\{6, 8, 12, 14, 16\}$

Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{factors of } 6\}$
$B = \{\text{prime numbers}\}$

(a) List the members of the set

(i) $A \cup B$

.....

(1)

(ii) A'

.....

(1)

Harpreet states that $A \cap B = \varnothing$

Harpreet is incorrect.

(b) Explain why.

.....

(1)

C is a set with 4 members such that

the set $A \cap C$ has 2 members
the set $B \cap C$ has 2 members

Set $A \cap C$ and set $B \cap C$ have no members in common.

(c) List the 4 members of set C

.....

(2)

(Total for Question 4 is 5 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 6\}$$

$$B = \{2, 3, 5, 7\}$$

2 **Calculate value**

So

$$A \cup B = \{1, 2, 3, 5, 6, 7\}$$

3 **Calculate value**

and

$$A' = \{4, 5, 7, 8, 9, 10\}$$

4 **Harpreet is not correct because**

Harpreet is not correct because 2 and 3 are in both A and B.

$$A \cap B = \{2, 3\}$$

5 **Calculate value**

For C, it has 4 members, $n(A \cap C) = 2$, $n(B \cap C) = 2$, and no member is in both of those overlaps. So choose the two elements of A only and the two elements of B only:

$$C = \{1, 5, 6, 7\}$$

✓ **FINAL ANSWER**

(a)(i) $\{1, 2, 3, 5, 6, 7\}$, (a)(ii) $\{4, 5, 7, 8, 9, 10\}$, (b) Harpreet is incorrect, (c) $C = \{1, 5, 6, 7\}$

Question 7

5 marks | Set Notation & Venn Diagrams

7 $\mathscr{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$
$A = \{\text{even numbers}\}$
$B = \{23, 29, 31\}$
$C = \{\text{multiples of 3}\}$

(a) List the members of the set

(i) $B \cup C$

.....

(1)

(ii) $A' \cap C$

.....

(1)

(b) Is it true that $B \cap C = \varnothing$?

Tick (✓) one of the boxes below.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

The set D has 4 members and is such that $D \cap (A \cup C) = \varnothing$

(c) List the members of set D

.....

(2)

(Total for Question 7 is 5 marks)

1 **Calculate value**

$$\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$$

$$A = \{24, 26, 28, 30, 32, 34\}$$

$$B = \{23, 29, 31\}$$

$$C = \{24, 27, 30, 33\}$$

2 **Calculate value**

So

$$B \cup C = \{23, 24, 27, 29, 30, 31, 33\}$$

3 **Set of odd numbers**

Also A' is the set of odd numbers in $\mathcal{E}$, so

$$A' \cap C = \{27, 33\}$$

4 **The statement is true because**

The statement $B \cap C = \emptyset$ is true because none of $23, 29, 31$ is a multiple of 3.

5 **Calculate value**

For D, we need $D \cap (A \cup C) = \emptyset$. So D must contain the four numbers outside $A \cup C$:

$$D = \{23, 25, 29, 31\}$$

✓ **FINAL ANSWER**

(a)(i) $\{23, 24, 27, 29, 30, 31, 33\}$, (a)(ii) $\{27, 33\}$, (b) Yes, (c) $\{23, 25, 29, 31\}$

Question 16

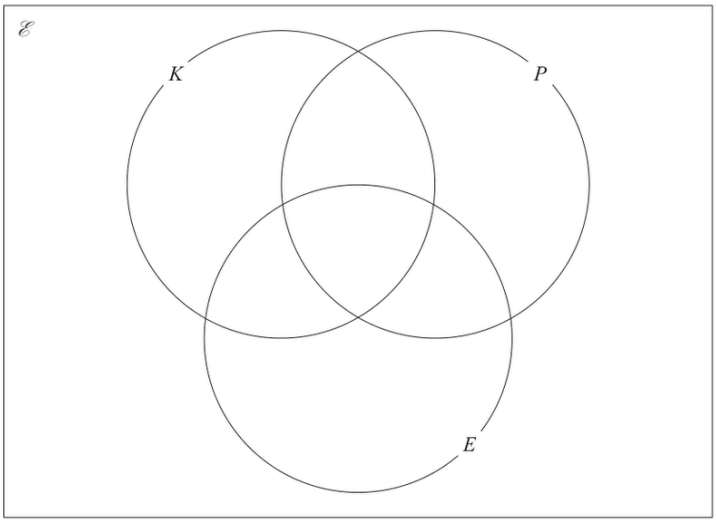
7 marks | Set Notation & Venn Diagrams

16 60 art students were asked if they would like to attend workshops for knitting (K), for photography (P) or for embroidery (E)

Of these students

9 chose knitting, photography and embroidery
17 chose knitting and photography
16 chose photography and embroidery
20 chose knitting and embroidery
28 chose photography
39 chose embroidery
2 chose none of the workshops

(a) Using this information, complete the Venn diagram to show the numbers of students in each subset.



(3)

One of the students is chosen at random.

Given that this student chose photography,

(b) find the probability that this student also chose knitting.

(2)

(c) Find $n(P \cap K')$

(1)

(d) Find $n([P \cup E] \cap K)$

(1)

(Total for Question 16 is 7 marks)

1 **(a) Start from the centre**

All three: 9.

$$K \cap P \text{ only} = 17 - 9 = 8, \quad P \cap E \text{ only} = 16 - 9 = 7, \quad K \cap E \text{ only} = 20 - 9 = 11$$

2 **(a) Single regions**

$$P \text{ only} = 28 - 8 - 7 - 9 = 4, \qquad E \text{ only} = 39 - 11 - 7 - 9 = 12$$

58 students are inside the circles ($60 - 2$):

$$K \text{ only} = 58 - (8 + 7 + 11 + 9 + 4 + 12) = 7$$

Outside: 2.

3 **(b) Knitting, given photography**

28 chose photography; $8 + 9 = 17$ of them also chose knitting.

$$\frac{17}{28}$$

4 **(c) Photography but not knitting**

P only and $P \cap E$ only: $4 + 7 = 11$.

5 **(d) Knitting and (photography or embroidery)**

$8 + 9 + 11 = 28$

FINAL ANSWER

(a) K only 7, P only 4, E only 12, $K \cap P$ only 8, $P \cap E$ only 7, $K \cap E$ only 11, centre 9, outside 2

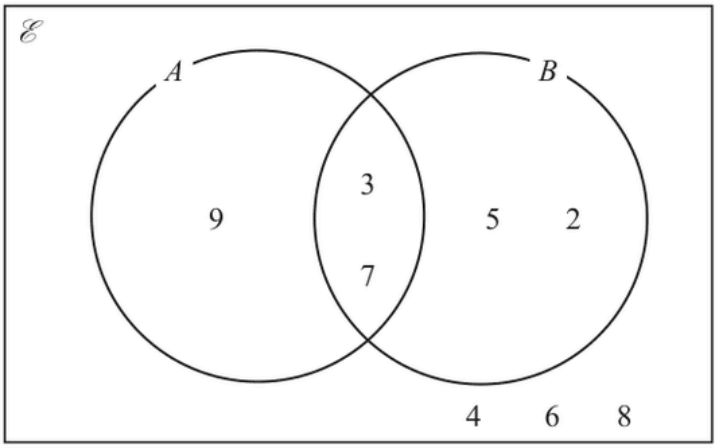
(b) $\frac{17}{28}$ (c) 11 (d) 28

4MA1/1H NOVEMBER 2024

Question 4

3 marks | Set Notation & Venn Diagrams

4 Here is a Venn diagram.



(a) List the members of the set B

.....

(1)

(b) List the members of the set $A \cap B$

.....

(1)

(c) List the members of the set A'

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2024

1 **The members of are all**

The members of B are all the numbers inside circle B, including the overlap:

$$B = \{2, 3, 5, 7\}$$

2 **The members of are the**

The members of $A \cap B$ are the numbers in both circles:

$$A \cap B = \{3, 7\}$$

3 Calculate statistic

A' means all the numbers in the universal set that are not in A:

$$A' = \{2, 4, 5, 6, 8\}$$

FINAL ANSWER

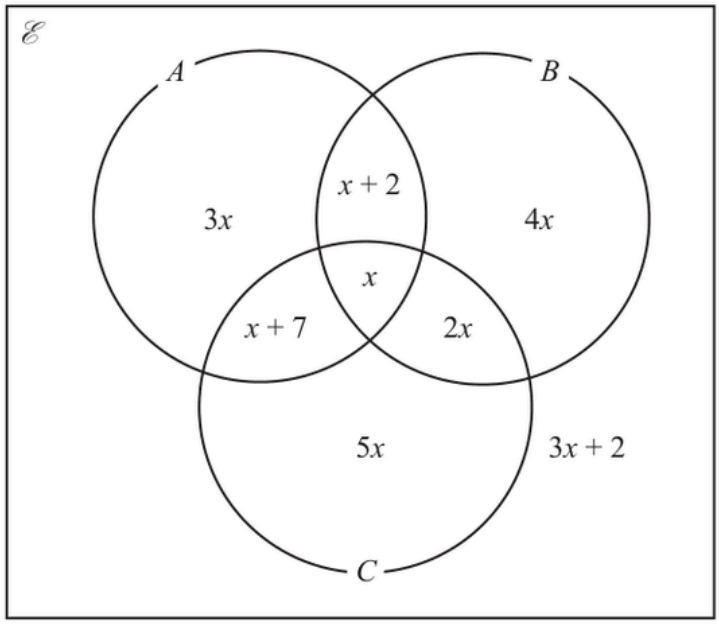
(a) $\{2, 3, 5, 7\}$, (b) $\{3, 7\}$, (c) $\{2, 4, 5, 6, 8\}$

4MA1/2H NOVEMBER 2024

Question 21

4 marks | Set Notation & Venn Diagrams

21 The Venn diagram shows information about the numbers of items in set A, set B and set C, where x is an integer.



Given that $\text{n}(A \cup B)' = 26$

find $\text{n}(A' \cap C)$

$\text{n}(A' \cap C) =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2024

1 The complement of contains the

The complement of $A \cup B$ contains the regions outside both A and B. These are $5x$ and $3x + 2$.

2 Calculate value

So $5x + (3x + 2) = 26$, hence $8x + 2 = 26$ and $x = 3$.

3 Calculate statistic

Now $A' \cap C$ means inside C, but not inside A. These are the regions $5x$ and $2x$.

4 Calculate value

So $n(A' \cap C) = 5x + 2x = 7x = 7(3) = 21$.

FINAL ANSWER

21

Question 4

4 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

(a) List the members of the set

(i) $A \cup B$

.....

(ii) B'

.....

(2)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

$\mathscr{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

(b) Write a symbol from the box on each dotted line to make each of the following a true statement.

(i) $A \cap C =$ .....

(ii) 13 ..... $\mathscr{E}$

(2)

(Total for Question 4 is 4 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 4, 6, 8, 10, 12\}$$

$$B = \{3, 6, 9, 12\}$$

$$C = \{1, 3, 5, 7, 9, 11\}$$

2 **Calculate value**

Therefore

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$$

3 **Calculate value**

and

$$B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$$

4 **No number is both even**

No number is both even and odd, so

$$A \cap C = \emptyset$$

5 **Set**

Also $13 \notin \mathcal{E}$ because 13 is not in the universal set.

✓ **FINAL ANSWER**

(a)(i) $\{2, 3, 4, 6, 8, 9, 10, 12\}$, (a)(ii) $\{1, 2, 4, 5, 7, 8, 10, 11\}$, (b)(i) $\emptyset$, (b)(ii) $13 \notin \mathcal{E}$

Question 5

4 marks | Set Notation & Venn Diagrams

5 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

(a) List the members of the set

(i) $A \cup B$

.....

(ii) B'

.....

(2)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

$\mathscr{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

(b) Write a symbol from the box on each dotted line to make each of the following a true statement.

(i) $A \cap C =$ .....

(ii) 13 ..... $\mathscr{E}$

(2)

(Total for Question 5 is 4 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 4, 6, 8, 10, 12\}$$

$$B = \{3, 6, 9, 12\}$$

$$C = \{1, 3, 5, 7, 9, 11\}$$

2 **Calculate value**

Therefore

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$$

3 **Calculate value**

and

$$B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$$

4 **No number is both even**

No number is both even and odd, so

$$A \cap C = \emptyset$$

5 **Set**

Also $13 \notin \mathcal{E}$ because 13 is not in the universal set.

✓ **FINAL ANSWER**

(a)(i) $\{2, 3, 4, 6, 8, 9, 10, 12\}$, (a)(ii) $\{1, 2, 4, 5, 7, 8, 10, 11\}$, (b)(i) $\emptyset$, (b)(ii) $13 \notin \mathcal{E}$

4WM1/01 MAY 2025

Question 8

5 marks | Set Notation & Venn Diagrams

8 $\mathscr{E} = \{18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{\text{even numbers}\}$

$B = \{\text{multiples of 3}\}$

$C = \{\text{multiples of 5}\}$

(a) List the members of the set $(A \cup B)'$

.....

(2)

Sophie writes down the statement $B \cap C = \emptyset$

(b) Explain why Sophie's statement is wrong.

.....

(1)

D is a set such that $A \cap D = \{18, 26\}$
The set D has exactly 5 members.

(c) List the members of one possible set D

.....

(2)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 May 2025

1 **Calculate value**

$$A = \{18, 20, 22, 24, 26, 28, 30\}$$

$$B = \{18, 21, 24, 27, 30\}$$

$$A \cup B = \{18, 20, 21, 22, 24, 26, 27, 28, 30\}$$

2 **Calculate value**

So

$$(A \cup B)' = \{19, 23, 25, 29\}$$

3 **Sophie is wrong because is**

Sophie is wrong because 30 is both a multiple of 3 and a multiple of 5, so $30 \in B \cap C$.

4 **Set is**

For D, include 18 and 26, but no other even numbers. One possible set is

$$D = \{18, 19, 23, 25, 26\}$$

✓ **FINAL ANSWER**

$\{19, 23, 25, 29\}$; Sophie is wrong because $30 \in B \cap C$; one possible D is $\{18, 19, 23, 25, 26\}$.

Question 20

6 marks | Set Notation & Venn Diagrams

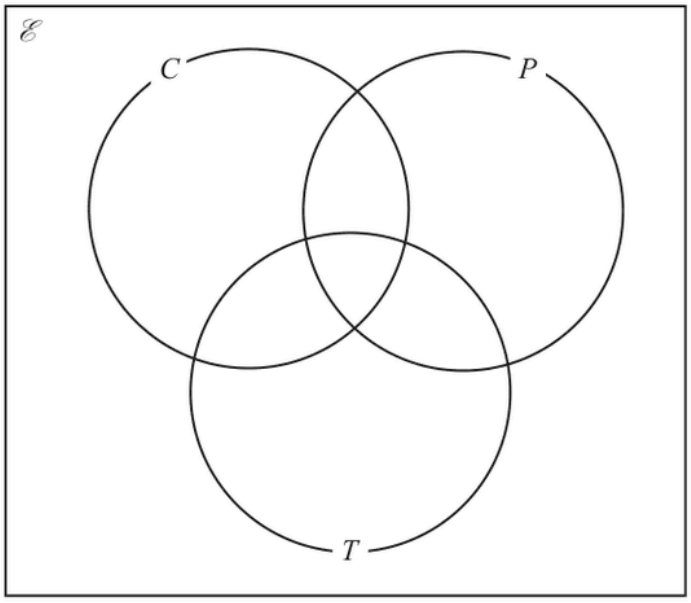
20 120 gardeners were asked if they grow carrots (C) or potatoes (P) or tomatoes (T)

Of these gardeners

43 grow carrots
12 grow carrots and potatoes and tomatoes
18 grow carrots and potatoes
27 grow carrots and tomatoes
32 grow potatoes and tomatoes
29 do not grow carrots or potatoes or tomatoes

The number of these gardeners who grow only potatoes is equal to the number of these gardeners who grow only tomatoes.

(a) Complete the Venn diagram to show this information.



(3)

(b) Find $n(T' \cap C)$

.....

(1)

One of the gardeners who grows carrots is chosen at random.

(c) Calculate the probability that this gardener also grows potatoes.

.....

(2)

(Total for Question 20 is 6 marks)

1 **Calculate value**

There are 120 gardeners in total and 29 are outside all three sets, so

$$n(C \cup P \cup T) = 120 - 29 = 91$$

2 **The triple intersection is**

The triple intersection is

$$n(C \cap P \cap T) = 12$$

3 **Calculate value**

So the pair-only regions are:

$$C \cap P \text{ only} = 18 - 12 = 6$$

$$C \cap T \text{ only} = 27 - 12 = 15$$

$$P \cap T \text{ only} = 32 - 12 = 20$$

4 **Calculate value**

Since 43 grow carrots,

$$C \text{ only} = 43 - 6 - 15 - 12 = 10$$

5 **Let the number who grow**

Let the number who grow only potatoes be y. The number who grow only tomatoes is also y.

6 **Add the regions union**

Now add the regions in the union:

$$10 + y + y + 6 + 15 + 20 + 12 = 91$$

$$63 + 2y = 91$$

$$2y = 28$$

$$y = 14$$

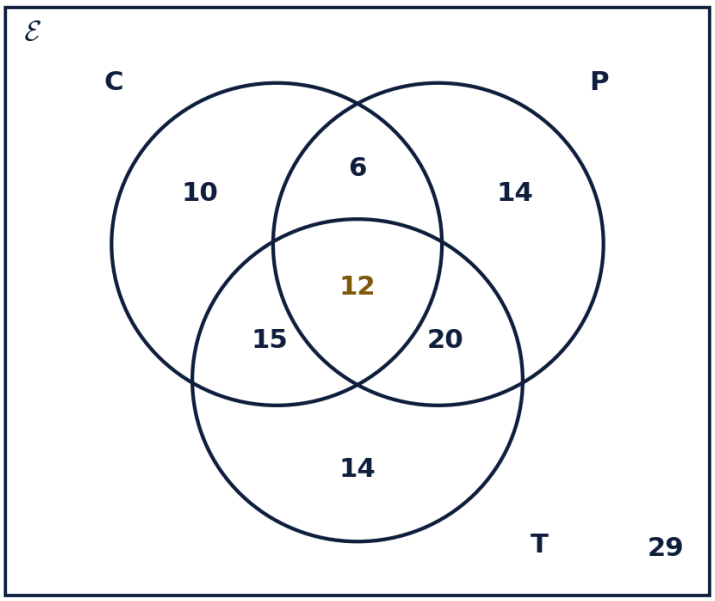
7 **Calculate value**

So the Venn regions are:

$$C \text{ only} = 10, \quad P \text{ only} = 14, \quad T \text{ only} = 14$$

$$C \cap P \text{ only} = 6, \quad C \cap T \text{ only} = 15, \quad P \cap T \text{ only} = 20$$

$$C \cap P \cap T = 12, \quad \text{outside} = 29$$



8 **Calculate value**

For part (b),

$$n(T' \cap C) = C \text{ only} + C \cap P \text{ only}$$

$$= 10 + 6 = 16$$

9 **Evaluate fraction**

For part (c), one gardener who grows carrots is chosen.

$$P(\text{also grows potatoes}) = \frac{n(C \cap P)}{n(C)}$$

$$= \frac{18}{43}$$

✓ **FINAL ANSWER**

(b) 16, (c) $\frac{18}{43}$

Question 4

4 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$
$A = \{\text{odd numbers}\}$
$B = \{\text{multiples of 3}\}$
$C = \{\text{prime numbers}\}$

(a) List the members of the set $A \cup C$

.....

(1)

(b) List the members of the set $(A \cap B)'$

.....

(1)

(c) Is it true that $15 \in (B \cap C)$?

Tick (✓) the correct box.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

D is a set such that $D \cup C = A$
D has 2 members.

(d) List the members of D

.....

(1)

(Total for Question 4 is 4 marks)

1 **Calculate value**

$$A = \{9, 11, 13, 15, 17\}, \quad B = \{9, 12, 15, 18\}, \quad C = \{11, 13, 17\}$$

$$A \cup C = \{9, 11, 13, 15, 17\}$$

$$A \cap B' = \{11, 13, 17\}$$

2 **Check prime condition**

$15 \notin B \cap C$ because 15 is not prime.

3 **Calculate value**

Since $D \cup C = A$ and D has two members,

$$D = \{9, 15\}$$

FINAL ANSWER

$\{9, 11, 13, 15, 17\}$, $\{11, 13, 17\}$, No, $D = \{9, 15\}$.

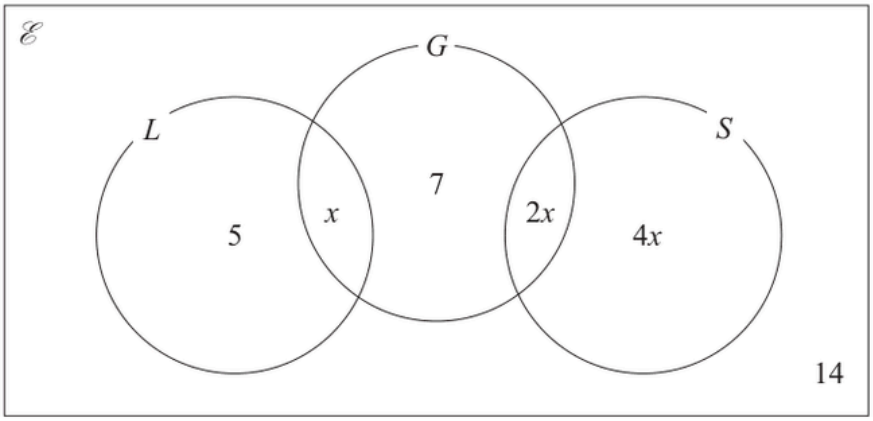
4MA1/2H NOVEMBER 2025

Question 18

3 marks | Set Notation & Venn Diagrams

18 Bianca asked 40 students which of the languages Latin (L), Greek (G) and Sanskrit (S) they study.

The Venn diagram gives some information about her results.
It shows the number of students in each subset.



One of these students is selected at random.
Work out the probability that this student studies Latin or Sanskrit.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2025

1 Find x (all regions add to 40)

$$5 + x + 7 + 2x + 4x + 14 = 40 \Rightarrow 26 + 7x = 40 \Rightarrow x = 2$$

2 Latin or Sanskrit

Regions in L or S: $5 + x + 2x + 4x = 5 + 2 + 4 + 8 = 19$.

$$\frac{19}{40}$$

✓ **FINAL ANSWER**

$$\frac{19}{40}$$

4MA1/1H JUNE 2026

Question 3

3 marks | Set Notation & Venn Diagrams

3 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

$A = \{\text{factors of } 20\}$

$B = \{\text{multiples of } 5\}$

(a) List the members of the set

(i) $A \cup B$

.....

(1)

(ii) $A \cap B'$

.....

(1)

$C = \{\text{prime numbers}\}$

Ana writes down the statement $A \cap C = \varnothing$
Ana's statement is wrong.

(b) Explain why.

.....

.....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2026

1 **Part (a(i)) | Step 1**

$A = \{1, 2, 4, 5, 10\}$ and $B = \{5, 10, 15\}$, so $A \cup B = \{1, 2, 4, 5, 10, 15\}$.

Include every member of either set once; 20 lies outside the universal set.

2 **Part (a(ii)) | Step 1**

$A \cap B' = \{1, 2, 4\}$.

Retain the factors of 20 in the universal set which are not multiples of 5.

3 **Part (b) | Step 1**

Both 2 and 5 are prime factors of 20, so $A \cap C = \{2, 5\} \neq \varnothing$.

A shared member is enough to disprove the claim that the intersection is empty.

FINAL ANSWER

(a(i)) $\{1, 2, 4, 5, 10, 15\}$ (a(ii)) $\{1, 2, 4\}$ (b) $A \cap C = \{2, 5\}$, not the empty set.

4MA1/2HR JUNE 2026

Question 3

1 marks | Set Notation & Venn Diagrams

3 $\mathscr{E}$ = {integers from 1 to 12}
A = {factors of 12}

List the members of the set A'

.....

(Total for Question 3 is 1 mark)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$A = \{1, 2, 3, 4, 6, 12\}$. Remove these factors from the integers 1 to 12.
The complement contains precisely the universal-set elements not in A.

FINAL ANSWER

$A' = \{5, 7, 8, 9, 10, 11\}$.

4WM1/01 MAY 2026

Question 5

4 marks | Set Notation & Venn Diagrams

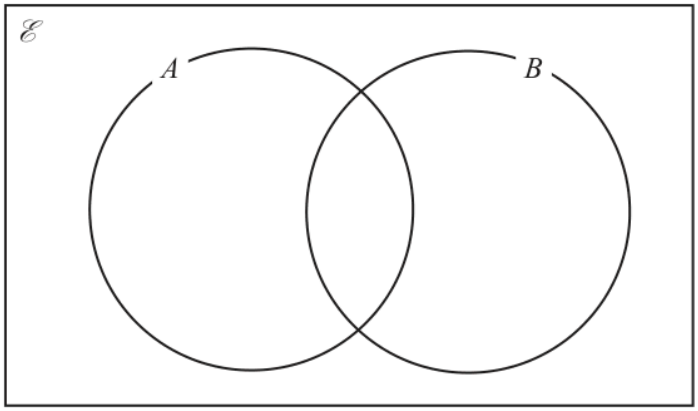
5 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 3, 5, 7, 11\}$

$A \cap B = \{2, 5\}$

$B' = \{3, 4, 6, 7, 9, 11\}$

(a) Complete the Venn diagram for this information.



(3)

A number is chosen at random from the universal set $\mathscr{E}$

(b) Write down the probability that this number is in the set $(A \cup B)'$

.....

(1)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

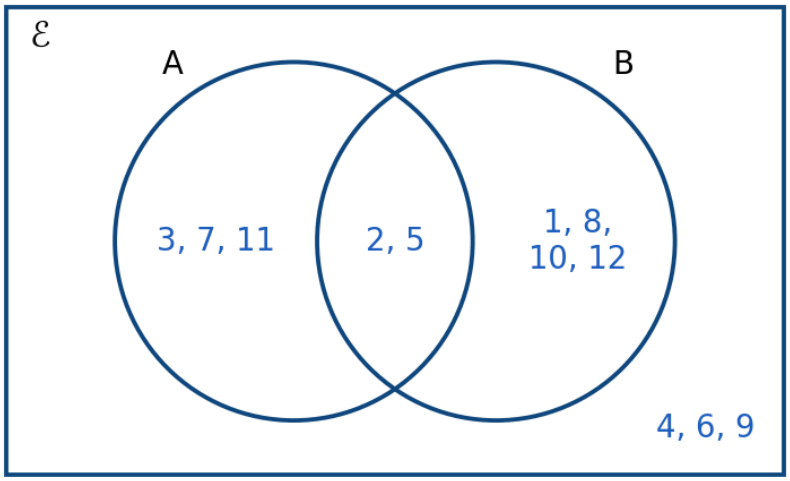
Question 5 · 4WM1/01 May 2026

1 **Find set B**

$B' = \{3, 4, 6, 7, 9, 11\}$, so $B = \{1, 2, 5, 8, 10, 12\}$.

2 **Fill each region**

$A \cap B = \{2, 5\}$; A only $= \{3, 7, 11\}$; B only $= \{1, 8, 10, 12\}$; outside both $= \{4, 6, 9\}$.



3 **(b) Outside both circles**

$$P\big((A \cup B)'\big) = \frac{3}{12} = \frac{1}{4}$$

✓ **FINAL ANSWER**

(a) see diagram (b) $\frac{3}{12} = \frac{1}{4}$

4MA1/2H JUNE 2026

Question 12

2 marks | Set Notation & Venn Diagrams

12 $\mathscr{E} = \{\text{integers } x \text{ such that } 20 \leqslant x \leqslant 32\}$

$A = \{x \text{ such that } x < 27\}$

$B = \{x \text{ such that } 25 \leqslant x \leqslant 30\}$

(a) List the members of the set $A \cap B$

.....

(1)

(b) List the members of the set $(A \cup B)'$

.....

(1)

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

Within the universal set, $A = \{20, 21, 22, 23, 24, 25, 26\}$ and $B = \{25, 26, 27, 28, 29, 30\}$. Their common members are 25 and 26.

Intersection consists of elements in both sets.

2 **Part (b) | Step 1**

$A \cup B = \{20, 21, \ldots, 30\}$. The members of $\mathcal{E} = \{20, 21, \ldots, 32\}$ outside this union are 31 and 32.

Take the complement relative to the stated universal set, not all integers.

✓ **FINAL ANSWER**

(a) $A \cap B = \{25, 26\}$ (b) $(A \cup B)' = \{31, 32\}$

4MA1/1HR JUNE 2026

Question 21

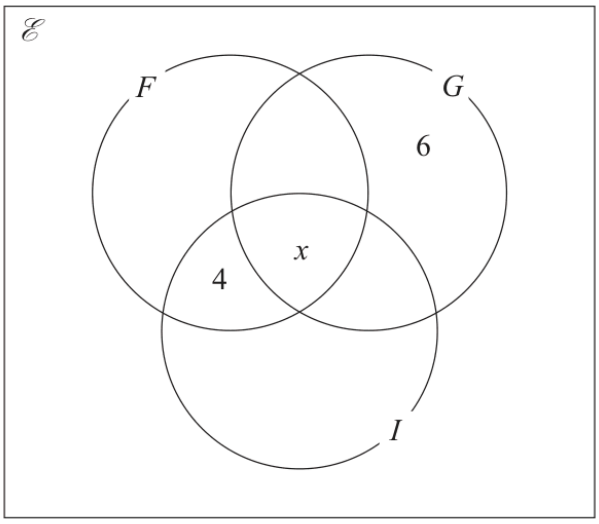
5 marks | Set Notation & Venn Diagrams

21 60 students were asked if they study French (F), German (G) or Italian (I)

Of these students

32 students study French
24 students study German
27 students study Italian
16 students study French and German
15 students study German and Italian

The incomplete Venn diagram gives some more information about the languages the 60 students study.



(a) Work out the value of x

$x =$ .....

(3)

(b) Work out how many of the 60 students study none of French or German or Italian.

.....

(2)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

The French-and-German-only region is $16 - x$, and the German-and-Italian-only region is $15 - x$. The totals 16 and 15 each include the triple intersection x.

2 **Part (a) | Step 2**

$$6 + (16 - x) + (15 - x) + x = 24.$$

The four disjoint regions in the German set must total 24.

---

3 **Part (a) | Step 3**

$$37 - x = 24 \quad \Longrightarrow \quad x = 13.$$

Solve for the common triple-intersection count.

---

4 **Part (b) | Step 1**

$$FG \text{ only} = 16 - 13 = 3, \qquad GI \text{ only} = 15 - 13 = 2.$$

Use the triple-intersection value from part (a).

---

5 **Part (b) | Step 2**

$$F \text{ only} = 32 - (3 + 4 + 13) = 12, \qquad I \text{ only} = 27 - (4 + 2 + 13) = 8.$$

The French-and-Italian-only count is the given 4.

---

6 **Part (b) | Step 3**

$$n(F \cup G \cup I) = 12 + 6 + 8 + 3 + 4 + 2 + 13 = 48.$$

Sum all seven mutually exclusive interior regions.

---

7 **Part (b) | Step 4**

$$n(\text{none}) = 60 - 48 = 12.$$

Subtract the union from the total number of students.

✓ **FINAL ANSWER**

(a) $x = 13$ (b) 12 students.