Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

Linear Classified With Answers (Part 1 of 2)

Every question followed by Dr Eslam Ahmed's worked solution and final answer. Chapters: Chapter 1: Numbers & the Number System, Chapter 2: Equations, Formulae & Identities, Chapter 3: Sequences, Functions & Graphs.

815
topic placements

2020–2026
55 past papers

Questions with worked solutions
checked by Dr Eslam



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TOPIC

Number Toolkit

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 36

3 marks | Number Toolkit

36 Here is a list of numbers.

2 9 15 17 21 25 27

From the list, write down

(a) the two prime numbers,

.....

(1)

(b) a square number that is also odd and greater than 10,

.....

(1)

(c) a cube number.

.....

(1)

(Total for Question 36 is 3 marks)

WORKED SOLUTION

Question 36 · EP Specimen

1 **(a) Exactly two factors**

2 and 17 are prime ($15 = 3 \times 5$, $21 = 3 \times 7$).

2 **(b) Square numbers in the list**

$9 = 3^2$ and $25 = 5^2$; the odd one greater than 10 is 25.

3 **(c) Cube numbers**

$27 = 3^3$

✓ **FINAL ANSWER**

(a) $2, 17$ (b) 25 (c) 27

EP SPECIMEN

Question 37

2 marks | Number Toolkit

37 Use the information that $23 \times 47 = 1081$ to write down the value of

(a) 2.3×4.7

.....

(1)

(b) $1081 \div 4.7$

.....

(1)

(Total for Question 37 is 2 marks)

WORKED SOLUTION

Question 37 · EP Specimen

1 **(a) Each number is ten times smaller**

$2.3 \times 4.7 = 1081 \div 100 = 10.81$

2 **(b) Reverse the multiplication**

$1081 \div 47 = 23$, and 4.7 is ten times smaller than 47, so $1081 \div 4.7 = 230$.

✓ **FINAL ANSWER**

(a) 10.81 (b) 230

4MA1/2HR JANUARY 2022

Question 10

3 marks | Number Toolkit

10 The table gives information about the population and the total amount of money, in dollars, spent on healthcare for two countries in 2016

Country	Total population	Total spent on healthcare ($)
Austria	8.7×10^6	4.2×10^{10}
Luxembourg	6.3×10^5	3.7×10^9

Work out how much more was spent **per person** on healthcare in Luxembourg than in Austria.
Give your answer correct to the nearest whole number.

..... dollars

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2022

1 Austria spending per person

Austria spending per person:

$$\frac{4.2 \times 10^{10}}{8.7 \times 10^6} = 4827.586\ldots$$

2 Luxembourg spending per person

Luxembourg spending per person:

$$\frac{3.7 \times 10^9}{6.3 \times 10^5} = 5873.015\ldots$$

3 Difference

Difference:

$$5873.015\ldots - 4827.586\ldots = 1045.429\ldots$$

4 Convert standard form

To the nearest whole number, this is

$$1045$$

FINAL ANSWER

1045 dollars

4MA1/2H JUNE 2026

Question 1

3 marks | Number Toolkit

1 Write 4500 as a product of powers of its prime factors.
Show your working clearly.

.....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2026

1 Solution | Step 1

$$4500 = 45 \times 100 = (9 \times 5)(4 \times 25).$$

Split into factors that are easy to prime-factorise.

2 Solution | Step 2

$$4500 = (3^2 \times 5)(2^2 \times 5^2) = 2^2 \times 3^2 \times 5^3.$$

Express each factor as prime powers and combine powers of 5.

FINAL ANSWER

$2^2 \times 3^2 \times 5^3$

4MA1/1HR JUNE 2026

Question 2

3 marks | Number Toolkit

2 Write 504 as a product of powers of its prime factors.
Show your working clearly.

.....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2026

1 Solution | Step 1

$$504 = 2 \times 252 = 2^2 \times 126 = 2^3 \times 63.$$

Divide repeatedly by the prime 2.

2 Solution | Step 2

$$63 = 3 \times 21 = 3^2 \times 7, \qquad 504 = 2^3 \times 3^2 \times 7.$$

All remaining factors are prime; group repeated factors as powers.

FINAL ANSWER

$504 = 2^3 \times 3^2 \times 7$

TOPIC

Prime Factors, HCF & LCM

Every past-paper question on this topic, 2020 to 2026.

Question 2

4 marks | Prime Factors, HCF & LCM

2 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

.....

(2)

(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.

.....

(2)

(Total for Question 2 is 4 marks)

1 **Find the LCM**

$$56 = 2^3 \times 7, \qquad 84 = 2^2 \times 3 \times 7$$

2 **The common prime factors are**

The common prime factors are 2^2 and 7, so

$$\text{HCF} = 2^2 \times 7 = 28$$

3 **Find the LCM**

Also

$$60 = 2^2 \times 3 \times 5, \qquad 72 = 2^3 \times 3^2$$

4 **Find the LCM**

Use the highest powers of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

✓ **FINAL ANSWER**

HCF $= 28$, LCM $= 360$.

4MA1/2HR JANUARY 2020

Question 1

3 marks | Prime Factors, HCF & LCM

1 (a) Write $5^{17} \times 5^2$ as a single power of 5

.....

(1)

(b) Write 800 as a product of its prime factors.
Show your working clearly.

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2020

1 Use the index law

Use the index law $a^m \times a^n = a^{m+n}$:

$$5^{17} \times 5^2 = 5^{19}$$

2 Find the LCM

Now write 800 as a product of prime factors:

$$800 = 8 \times 100$$

$$800 = 2^3 \times 2^2 \times 5^2$$

$$800 = 2^5 \times 5^2$$

FINAL ANSWER

(a) 5^{19}, (b) $2^5 \times 5^2$

4MA1/1H JANUARY 2020

Question 2

2 marks | Prime Factors, HCF & LCM

2 Find the lowest common multiple (LCM) of 28 and 105

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2020

1 Write both numbers as products

Write both numbers as products of prime factors:

$$28 = 2^2 \times 7$$

$$105 = 3 \times 5 \times 7$$

2 Find the LCM

For the lowest common multiple, take the highest power of each prime:

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

FINAL ANSWER

420

4MA1/1H MAY/NOVEMBER 2020

Question 10

3 marks | Prime Factors, HCF & LCM

10 $A = 2 \times 3^{43}$
$B = 16 \times 3^{37}$

(a) Find the highest common factor (HCF) of A and B.

.....

(1)

(b) Express the number $A \times B$ as a product of powers of its prime factors.
Give your answer in its simplest form.

.....

(2)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May/November 2020

1 Find the LCM

$$A = 2 \times 3^{43}$$

$$B = 16 \times 3^{37} = 2^4 \times 3^{37}$$

2 Find the LCM

For the HCF, take the smaller powers:

$$\text{HCF} = 2 \times 3^{37}$$

3 Find the LCM

For $A \times B$,

$$A \times B = (2 \times 3^{43})(2^4 \times 3^{37})$$

$$A \times B = 2^5 \times 3^{80}$$

FINAL ANSWER

(a) 2×3^{37}, (b) $2^5 \times 3^{80}$

4MA1/1HR JANUARY 2021

Question 9

4 marks | Prime Factors, HCF & LCM

9 $A = 2^8 \times 3^5 \times 11^4$ $B = 2^6 \times 3 \times 11^8$

(a) Find the highest common factor (HCF) of A and B.

.....

(2)

(b) Find the lowest common multiple (LCM) of $2A$ and $3B$.
Give the LCM as a product of powers of its prime factors.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2021

1 Find the LCM

$$A = 2^8 \times 3^5 \times 11^4$$

$$B = 2^6 \times 3 \times 11^8$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 2^6 \times 3 \times 11^4$$

3 Find the LCM

Now

$$2A = 2^9 \times 3^5 \times 11^4$$

4 Find the LCM

and

$$3B = 2^6 \times 3^2 \times 11^8$$

5 Find the LCM

For the LCM of $2A$ and $3B$, take the larger power of each prime:

$$\text{LCM} = 2^9 \times 3^5 \times 11^8$$

FINAL ANSWER

(a) $2^6 \times 3 \times 11^4$, (b) $2^9 \times 3^5 \times 11^8$

4MA1/2H JANUARY 2021

Question 9

2 marks | Prime Factors, HCF & LCM

9 $A = 2^3 \times 3^2 \times 5^2 \times 11$
$B = 2^4 \times 3 \times 5^4 \times 13$

Find the lowest common multiple (LCM) of A and B.
Give your answer as a product of powers of prime numbers.

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2021

1 Find the LCM

$$A = 2^3 \times 3^2 \times 5^2 \times 11$$

$$B = 2^4 \times 3 \times 5^4 \times 13$$

2 Find the LCM

For the LCM, take the highest power of every prime that appears:

$$\text{LCM} = 2^4 \times 3^2 \times 5^4 \times 11 \times 13$$

FINAL ANSWER

$2^4 \times 3^2 \times 5^4 \times 11 \times 13$

4MA1/2H MAY 2021

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 600 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H May 2021

1 Find the LCM

$$600 = 6 \times 100$$

$$600 = (2 \times 3)(2^2 \times 5^2)$$

$$600 = 2^3 \times 3 \times 5^2$$

FINAL ANSWER

$2^3 \times 3 \times 5^2$

4MA1/1H NOVEMBER 2021

Question 7

3 marks | Prime Factors, HCF & LCM

7 Write 3.6×10^3 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2021

1 Find the LCM

$$3.6 \times 10^3 = 3600$$

$$3600 = 36 \times 100$$

$$36 = 2^2 \times 3^2, \qquad 100 = 2^2 \times 5^2$$

2 Find the LCM

So

$$3600 = 2^4 \times 3^2 \times 5^2$$

FINAL ANSWER

$2^4 \times 3^2 \times 5^2$

4MA1/1HR JANUARY 2022

Question 6

4 marks | Prime Factors, HCF & LCM

6 (a) Work out the lowest common multiple (LCM) of 36 and 120

.....

(2)

$A = 5^2 \times 7^4 \times 11^p$
$B = 5^m \times 7^{n-5} \times 11$

m, n and p are integers such that
$m > 2$
$n > 10$
$p > 1$

(b) Find the highest common factor (HCF) of A and B
Give your answer as a product of powers of its prime factors.

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2022

1 **Find the LCM**

$$36 = 2^2 \times 3^2$$

$$120 = 2^3 \times 3 \times 5$$

2 **Find the LCM**

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

3 **Find the LCM**

For part (b),

$$A = 5^2 \times 7^4 \times 11^p$$

$$B = 5^m \times 7^{n-5} \times 11$$

4 **Find the LCM**

Since $m > 2$, the smaller power of 5 is 5^2.

5 **Find the LCM**

Since $n > 10$, $n - 5 > 5$, so the smaller power of 7 is 7^4.

6 **Find the LCM**

Since $p > 1$, the smaller power of 11 is 11^1.

$$\text{HCF} = 5^2 \times 7^4 \times 11$$

✓ **FINAL ANSWER**

(a) 360, (b) $5^2 \times 7^4 \times 11$

4MA1/2H JANUARY 2022

Question 7

3 marks | Prime Factors, HCF & LCM

7 Find the lowest common multiple (LCM) of 28, 42 and 63
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2022

1 Find the LCM

$$28 = 2^2 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3^2 \times 7$$

2 Find the LCM

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^2 \times 3^2 \times 7 = 252$$

✓ **FINAL ANSWER**

252

4MA1/2HR JUNE 2022

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1200 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2022

1 Find the LCM

$$1200 = 12 \times 100$$

$$1200 = (2^2 \times 3)(2^2 \times 5^2)$$

$$1200 = 2^4 \times 3 \times 5^2$$

FINAL ANSWER

$2^4 \times 3 \times 5^2$

4MA1/1H JUNE 2022

Question 3

4 marks | Prime Factors, HCF & LCM

3 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

.....

(2)

(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2022

1 Find the LCM

For the HCF of 56 and 84:

$$56 = 2^3 \times 7$$

$$84 = 2^2 \times 3 \times 7$$

2 Take the smaller powers

Take the smaller powers:

$$\text{HCF} = 2^2 \times 7 = 28$$

3 Find the LCM

For the LCM of 60 and 72:

$$60 = 2^2 \times 3 \times 5$$

$$72 = 2^3 \times 3^2$$

4 Take the larger powers

Take the larger powers:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

FINAL ANSWER

(a) 28, (b) 360

4MA1/1HR JUNE 2022

Question 12

3 marks | Prime Factors, HCF & LCM

12 $P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(a) Write down the highest common factor (HCF) of P and Q

.....

(1)

$P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(b) Work out the value of $P^3 \times Q$
Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

.....

(2)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR June 2022

1 Find the LCM

$$P = 3^3 \times 5^2 \times 7$$

$$Q = 3^2 \times 5 \times 7^2$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 3^2 \times 5 \times 7$$

3 Find the LCM

For $P^3 \times Q$:

$$P^3 = (3^3 \times 5^2 \times 7)^3 = 3^9 \times 5^6 \times 7^3$$

4 **Find the LCM**

Now multiply by Q:

$$P^3 \times Q = (3^9 \times 5^6 \times 7^3)(3^2 \times 5 \times 7^2)$$

$$P^3 \times Q = 3^{11} \times 5^7 \times 7^5$$

✓ **FINAL ANSWER**

(a) $3^2 \times 5 \times 7$, (b) $3^{11} \times 5^7 \times 7^5$

4MA1/1HR MAY 2023

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 2250 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May 2023

1 Find the LCM

$$2250 = 225 \times 10$$

$$225 = 3^2 \times 5^2, \qquad 10 = 2 \times 5$$

2 Find the LCM

So

$$2250 = 3^2 \times 5^2 \times 2 \times 5$$

$$2250 = 2 \times 3^2 \times 5^3$$

✓ **FINAL ANSWER**

$2 \times 3^2 \times 5^3$

4MA1/2HR JUNE 2023

Question 2

4 marks | Prime Factors, HCF & LCM

2 Sandeep wants to buy some packets of pens and some boxes of pencils for his stationery shop.

Each packet of pens contains 9 pens.
Each box of pencils contains 12 pencils.

Each packet of pens costs £7.60
Each box of pencils costs £4.80

Sandeep can only buy full packets of pens and full boxes of pencils.
He wants to buy exactly the same number of pens as pencils.

Work out the minimum amount Sandeep needs to pay.

£.....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2023

1 Find the LCM

Pens come in packs of 9 and pencils come in boxes of 12.

2 Find the LCM

The smallest equal number of pens and pencils is

$$\text{LCM}(9, 12) = 36$$

3 Find the LCM

So Sandeep buys

$$\text{4 packs of pens}$$

4 Find the LCM

and

$$\text{3 boxes of pencils}$$

5 Cost

Cost:

$$4(7.60) + 3(4.80) = 30.40 + 14.40 = 44.80$$

FINAL ANSWER

\pounds44.80.

4MA1/1H MAY 2023

Question 8

4 marks | Prime Factors, HCF & LCM

8 (a) Write 300 as a product of its prime factors.
Show your working clearly.

.....

(2)

$A = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$B = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

(b) Find the lowest common multiple (LCM) of $5A$ and $7B$
Show your working clearly.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2023

1 **Find the LCM**

First write 300 as a product of prime factors:

$$300 = 3 \times 100 = 3 \times 2^2 \times 5^2$$

$$300 = 2^2 \times 3 \times 5^2$$

2 **Find the LCM**

Now

$$A = 2^3 \times 3^2 \times 5$$

3 **Find the LCM**

and

$$B = 2^2 \times 3^3 \times 5$$

4 **Find the LCM**

So

$$5A = 2^3 \times 3^2 \times 5^2$$

5 **Find the LCM**

and

$$7B = 2^2 \times 3^3 \times 5 \times 7$$

---

6 **Find the LCM**

For the LCM of $5A$ and $7B$, take the highest power of every prime:

$$\text{LCM} = 2^3 \times 3^3 \times 5^2 \times 7$$

✓ **FINAL ANSWER**

(a) $2^2 \times 3 \times 5^2$, (b) $2^3 \times 3^3 \times 5^2 \times 7$

4MA1/1H NOVEMBER 2023

Question 7

2 marks | Prime Factors, HCF & LCM

7 $A = 5^3 \times 7^3 \times 11^6$ and $B = 5^6 \times 7^2 \times 11^4$

Find the highest common factor (HCF) of A and B
Give your answer as a product of powers of its prime factors.

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2023

1 Find the LCM

$$A = 5^3 \times 7^3 \times 11^6$$

$$B = 5^6 \times 7^2 \times 11^4$$

2 The HCF takes the smaller

The HCF takes the smaller power of each common prime:

$$\text{HCF} = 5^3 \times 7^2 \times 11^4$$

FINAL ANSWER

$5^3 \times 7^2 \times 11^4$

4MA1/2HR JUNE 2024

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2024

1 Find the LCM

$$1400 = 14 \times 100$$

$$1400 = (2 \times 7)(2^2 \times 5^2)$$

$$1400 = 2^3 \times 5^2 \times 7$$

FINAL ANSWER

$2^3 \times 5^2 \times 7$

4MA1/1H MAY 2024

Question 3

2 marks | Prime Factors, HCF & LCM

3 Find the highest common factor (HCF) of 72 and 108
Show your working clearly.

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2024

1 **Find the LCM**

$$72 = 2^3 \times 3^2$$

$$108 = 2^2 \times 3^3$$

2 **Find the LCM**

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 2^2 \times 3^2$$

$$2^2 \times 3^2 = 4 \times 9 = 36$$

✓ **FINAL ANSWER**

36

4MA1/1HR MAY 2024

Question 11

4 marks | Prime Factors, HCF & LCM

11 $A = 2^5 \times 5 \times 7^2$

$B = 2^3 \times 5^3 \times 7^4$

(a) Write down the highest common factor (HCF) of $5A$ and $2B$
Give your answer as a product of prime factors.

.....

(2)

$A = 2^5 \times 5 \times 7^2$

$B = 2^3 \times 5^3 \times 7^4$

(b) Work out the value of $(AB)^2$
Give your answer as a product of prime factors.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR May 2024

1 Find the LCM

$$A = 2^5 \times 5 \times 7^2$$

$$B = 2^3 \times 5^3 \times 7^4$$

2 Find the LCM

For part (a),

$$5A = 2^5 \times 5^2 \times 7^2$$

$$2B = 2^4 \times 5^3 \times 7^4$$

3 The HCF takes the smaller

The HCF takes the smaller powers:

$$\text{HCF} = 2^4 \times 5^2 \times 7^2$$

4 **Find the LCM**

For part (b),

$$AB = 2^{5+3} \times 5^{1+3} \times 7^{2+4}$$

$$AB = 2^{8} \times 5^{4} \times 7^{6}$$

---

5 **Find the LCM**

Therefore

$$(AB)^2 = 2^{16} \times 5^{8} \times 7^{12}$$

✓ **FINAL ANSWER**

(a) $2^4 \times 5^2 \times 7^2$, (b) $2^{16} \times 5^8 \times 7^{12}$

4MA1/1H NOVEMBER 2024

Question 6

2 marks | Prime Factors, HCF & LCM

6 $A = 2^3 \times 5^4 \times 7 \times 11$

$B = 2^2 \times 5^2 \times 7^2$

$C = 2^2 \times 5^3 \times 7^4$

Find the highest common factor (HCF) of A, B and C
Write your answer as a product of prime factors.

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2024

1 Find the LCM

$$A = 2^3 \times 5^4 \times 7 \times 11$$

$$B = 2^2 \times 5^2 \times 7^2$$

$$C = 2^2 \times 5^3 \times 7^4$$

2 Find the LCM

The HCF of A, B, and C uses the smallest power of each prime common to all three.

$$2^2, \quad 5^2, \quad 7$$

3 Find the LCM

The prime 11 is not in all three numbers, so it is not included.

$$\text{HCF} = 2^2 \times 5^2 \times 7$$

✓ **FINAL ANSWER**

$2^2 \times 5^2 \times 7$

4MA1/1HR JUNE 2025

Question 1

2 marks | Prime Factors, HCF & LCM

1 Find the lowest common multiple (LCM) of 45 and 70

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2025

1 **Find the LCM**

$$45 = 3^2 \times 5$$

$$70 = 2 \times 5 \times 7$$

2 **Find the LCM**

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2 \times 3^2 \times 5 \times 7 = 630$$

✓ **FINAL ANSWER**

630

4MA1/2HR JUNE 2025

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2025

1 Find the LCM

$$1400 = 14 \times 100$$

$$1400 = (2 \times 7)(2^2 \times 5^2)$$

$$1400 = 2^3 \times 5^2 \times 7$$

FINAL ANSWER

$2^3 \times 5^2 \times 7$

4WM2/01R MAY 2025

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM2/01R May 2025

1 Required

Write 1400 as a product of powers of its prime factors.

2 Divide repeatedly by the smallest

Divide repeatedly by the smallest prime factor each time:

$$1400;\ xrightarrowdiv2;\ 700;\ xrightarrowdiv2;\ 350;\ xrightarrowdiv2;\ 175;\ xrightarrowdiv5;\ 35;\ xrightarrowdiv5;\ 7;\ xrightarrowdiv7;\ 1$$

3 Collect the prime factors

Collect the prime factors:

$$1400 = underbrace2times2times2_2^3timesunderbrace5times5_5^2times7$$

FINAL ANSWER

1400 = 2³ × 5² × 7

4WM2/01 MAY 2025

Question 17

4 marks | Prime Factors, HCF & LCM

17 $A = 3^2 \times 5^4$ $B = 2^5 \times 3^2$ $C = 2^3 \times 5^6$

(a) Work out the value of $(ABC)^3$

Give your answer as a product of powers of its prime factors.

.....

(2)

(b) Work out $(5 \times 10^{150}) \times (3 \times 10^{140})$

Give your answer as a product of powers of its prime factors.

.....

(2)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM2/01 May 2025

1 **(a)**

(a)

$$ABC = (3^2 \times 5^4)(2^5 \times 3^2)(2^3 \times 5^6)$$

$$= 2^8 \times 3^4 \times 5^{10}$$

$$(ABC)^3 = 2^{24} \times 3^{12} \times 5^{30}$$

2 **(b)**

(b)

$$(5 \times 10^{150})(3 \times 10^{140}) = 15 \times 10^{290}$$

$$= 3 \times 5 \times (2 \times 5)^{290}$$

$$= 2^{290} \times 3 \times 5^{291}$$

FINAL ANSWER

$2^{24} \times 3^{12} \times 5^{30}$; $2^{290} \times 3 \times 5^{291}$.

4MA1/2H JUNE 2026

Question 11

2 marks | Prime Factors, HCF & LCM

11 Find the highest prime number that is a factor of 3.6×10^{24}

.....

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2026

1 Solution | Step 1

$$3.6 \times 10^{24} = 36 \times 10^{23} = (2^2 \times 3^2)(2 \times 5)^{23} = 2^{25}3^2 5^{23}.$$

Rewrite with an integer coefficient, then decompose 36 and 10 into primes.

2 Solution | Step 2

The only prime factors are $2, 3, 5$, so the highest is 5.

A larger composite factor is not a prime factor.

✓ **FINAL ANSWER**

5

TOPIC

Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 2

3 marks | Fractions

2 Show that $5\frac{1}{3} - 2\frac{6}{7} = 2\frac{10}{21}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 Specimen

1 Convert to improper fractions

$$5\frac{1}{3} = \frac{16}{3}, \qquad 2\frac{6}{7} = \frac{20}{7}$$

2 Use a common denominator

$$\frac{16}{3} - \frac{20}{7} = \frac{112}{21} - \frac{60}{21} = \frac{52}{21}$$

3 Convert back to a mixed number

$$\frac{52}{21} = 2\frac{10}{21}$$

FINAL ANSWER

$2\frac{10}{21}$.

4MA1/1HR JANUARY 2020

Question 2

3 marks | Fractions

2 Show that $3\frac{1}{5} \times 2\frac{5}{8} = 8\frac{2}{5}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2020

1 Simplify fraction

Convert the mixed numbers to improper fractions:

$$3\frac{1}{5} = \frac{16}{5}$$

$$2\frac{5}{8} = \frac{21}{8}$$

2 Simplify fraction

Now multiply:

$$\frac{16}{5} \times \frac{21}{8} = \frac{2 \times 21}{5}$$

$$= \frac{42}{5} = 8\frac{2}{5}$$

FINAL ANSWER

$8\frac{2}{5}$

4MA1/2H JANUARY 2020

Question 3

3 marks | Fractions

3 Show that $4\frac{2}{3} + 3\frac{4}{5} = 8\frac{7}{15}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2020

1 Convert to improper fractions

Convert to improper fractions:

$$4\frac{2}{3} = \frac{14}{3}$$

$$3\frac{4}{5} = \frac{19}{5}$$

2 Use a common denominator

Use a common denominator:

$$\frac{14}{3} + \frac{19}{5} = \frac{70}{15} + \frac{57}{15}$$

$$= \frac{127}{15} = 8\frac{7}{15}$$

FINAL ANSWER

$8\frac{7}{15}$

4MA1/1HR MAY/NOVEMBER 2020

Question 1

3 marks | Fractions

1 Show that $3\frac{3}{4} \times \frac{7}{9} = 2\frac{11}{12}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May/November 2020

1 **Simplify fraction**

$$3\frac{3}{4} = \frac{15}{4}$$

2 **Simplify fraction**

Now multiply:

$$\frac{15}{4} \times \frac{7}{9}$$

3 **Simplify fraction**

Cancel 15 and 9:

$$\frac{5}{4} \times \frac{7}{3} = \frac{35}{12}$$

$$\frac{35}{12} = 2\frac{11}{12}$$

✓ **FINAL ANSWER**

$2\frac{11}{12}$

4MA1/1HR JANUARY 2021

Question 1

3 marks | Fractions

1 Show that $3\frac{1}{5} \times 1\frac{5}{6} = 5\frac{13}{15}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2021

1 Simplify fraction

Convert the mixed numbers to improper fractions:

$$3\frac{1}{5} = \frac{16}{5}$$

$$1\frac{5}{6} = \frac{11}{6}$$

2 Simplify fraction

Now multiply:

$$\frac{16}{5} \times \frac{11}{6} = \frac{176}{30}$$

$$= \frac{88}{15} = 5\frac{13}{15}$$

FINAL ANSWER

$5\frac{13}{15}$

4MA1/2H MAY 2021

Question 2

3 marks | Fractions

2 Show that $2\frac{4}{7} \div 1\frac{1}{8} = 2\frac{2}{7}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H May 2021

1 **Simplify fraction**

$$2\frac{4}{7} = \frac{18}{7}$$

$$1\frac{1}{8} = \frac{9}{8}$$

2 **Divide by multiplying by the**

Divide by multiplying by the reciprocal:

$$\frac{18}{7} \div \frac{9}{8} = \frac{18}{7} \times \frac{8}{9}$$

$$= \frac{16}{7} = 2\frac{2}{7}$$

✓ **FINAL ANSWER**

$2\frac{2}{7}$

4MA1/2H JANUARY 2022

Question 2

3 marks | Fractions

2 Show that $6\frac{3}{4} \div 2\frac{4}{7} = 2\frac{5}{8}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2022

1 Convert to improper fractions

Convert to improper fractions:

$$6\frac{3}{4} = \frac{27}{4}$$

$$2\frac{4}{7} = \frac{18}{7}$$

2 Divide by multiplying by the

Divide by multiplying by the reciprocal:

$$\frac{27}{4} \div \frac{18}{7} = \frac{27}{4} \times \frac{7}{18}$$

$$= \frac{21}{8} = 2\frac{5}{8}$$

FINAL ANSWER

$2\frac{5}{8}$

4MA1/2H JUNE 2022

Question 4

3 marks | Fractions

4 Show that $5\frac{1}{3} - 2\frac{6}{7} = 2\frac{10}{21}$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2022

1 Simplify fraction

$$5\frac{1}{3} = \frac{16}{3}$$

$$2\frac{6}{7} = \frac{20}{7}$$

2 Use a common denominator

Use a common denominator:

$$\frac{16}{3} - \frac{20}{7} = \frac{112}{21} - \frac{60}{21}$$

$$= \frac{52}{21} = 2\frac{10}{21}$$

FINAL ANSWER

$2\frac{10}{21}$

4MA1/1HR JUNE 2022

Question 6

3 marks | Fractions

6 Show that $2\frac{2}{3} + 3\frac{3}{4} = 6\frac{5}{12}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2022

1 Simplify fraction

$$2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{4} = \frac{15}{4}$$

2 Use a common denominator

Use a common denominator:

$$\frac{8}{3} + \frac{15}{4} = \frac{32}{12} + \frac{45}{12}$$

$$= \frac{77}{12} = 6\frac{5}{12}$$

FINAL ANSWER

$6\frac{5}{12}$

4MA1/2H JUNE 2024

Question 4

3 marks | Fractions

4 Show that $2\frac{4}{7} \times 3\frac{1}{9} = 8$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2024

1 Simplify fraction

$$2\frac{4}{7} = \frac{18}{7}$$

$$3\frac{1}{9} = \frac{28}{9}$$

2 Simplify fraction

Now multiply:

$$\frac{18}{7} \times \frac{28}{9}$$

3 Cancel first

Cancel first:

$$\frac{18}{9} \times \frac{28}{7} = 2 \times 4 = 8$$

FINAL ANSWER

8

4WM1/01 MAY 2025

Question 1

3 marks | Fractions

1 Without using a calculator, work out

$$3\frac{2}{3} \times 2\frac{4}{7}$$

Show each stage of your working.
Give your answer as a mixed number in its simplest form.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 May 2025

1 **Simplify fraction**

Convert the mixed numbers to improper fractions.

$$3\frac{2}{3} = \frac{11}{3}, \qquad 2\frac{4}{7} = \frac{18}{7}$$

2 **Simplify fraction**

Then multiply.

$$\frac{11}{3} \times \frac{18}{7} = \frac{198}{21} = \frac{66}{7}$$

3 **Convert to a mixed number**

Convert to a mixed number.

$$\frac{66}{7} = 9\frac{3}{7}$$

✓ **FINAL ANSWER**

$9\frac{3}{7}$

4WM1/01R MAY 2025

Question 2

3 marks | Fractions

2 Show that $2\frac{1}{4} \times 1\frac{5}{7} = 3\frac{6}{7}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01R May 2025

1 **Evaluate fraction**

$$2\frac{1}{4} = \frac{9}{4}, \qquad 1\frac{5}{7} = \frac{12}{7}$$

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{9}{4} \times \frac{12}{7}$$

$$= \frac{108}{28} = \frac{27}{7} = 3\frac{6}{7}$$

✓ **FINAL ANSWER**

$3\frac{6}{7}$.

4WM1/01 NOVEMBER 2025

Question 3

3 marks | Fractions

3 Show that $3\frac{5}{7} + 1\frac{2}{3} = 5\frac{8}{21}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM1/01 November 2025

1 Evaluate fraction

$$\frac{3}{7} + \frac{1}{3}$$

2 Use denominator 21

Use denominator 21:

$$\frac{3}{7} = \frac{9}{21}, \qquad \frac{1}{3} = \frac{7}{21}$$

$$\frac{9}{21} + \frac{7}{21} = \frac{16}{21}$$

FINAL ANSWER

$\frac{16}{21}$.

4MA1/2H NOVEMBER 2025

Question 3

4 marks | Fractions

3 A box contains only

9 red bricks
43 blue bricks
and some yellow bricks

$\frac{7}{20}$ of the bricks are yellow bricks.

Each brick weighs 35 grams.

Work out the total weight of the yellow bricks.

..... grams

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2025

1 There are red and blue

There are $9 + 43 = 52$ red and blue bricks.

2 Simplify fraction

Let the total number of bricks be T.

3 Simplify fraction

Since $\frac{7}{20}$ of the bricks are yellow, the remaining fraction is

$$1 - \frac{7}{20} = \frac{13}{20}$$

4 Simplify fraction

So

$$\frac{13}{20}T = 52$$

$$T = 52 \times \frac{20}{13} = 80$$

5 Yellow bricks

Yellow bricks:

$$\frac{7}{20} \times 80 = 28$$

6 Simplify fraction

Each brick weighs 35 grams, so total yellow-brick weight is

$$28 \times 35 = 980$$

FINAL ANSWER

980 grams

4MA1/1H JUNE 2026

Question 1

3 marks | Fractions

1 Show that $4\frac{1}{7} - 2\frac{3}{5} = 1\frac{19}{35}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2026

1 **Solution | Step 1**

$$4\frac{1}{7} = \frac{29}{7}, \qquad 2\frac{3}{5} = \frac{13}{5}.$$

Convert each mixed number to an improper fraction.

2 **Solution | Step 2**

$$\frac{29}{7} - \frac{13}{5} = \frac{145 - 91}{35} = \frac{54}{35} = 1\frac{19}{35}.$$

Use the common denominator 35, subtract, then convert back to a mixed number.

✓ **FINAL ANSWER**

$1\frac{19}{35}$

4MA1/1HR JUNE 2026

Question 1

3 marks | Fractions

1 Show that $2\frac{4}{5} \times 3\frac{3}{7} = 9\frac{3}{5}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2026

1 Solution | Step 1

$$2\frac{4}{5} = \frac{14}{5}, \qquad 3\frac{3}{7} = \frac{24}{7}.$$

Convert each mixed number into an improper fraction.

2 Solution | Step 2

$$\frac{14}{5} \times \frac{24}{7} = \frac{2 \times 24}{5} = \frac{48}{5} = 9\frac{3}{5}.$$

Cancel the common factor 7, multiply, then convert back to a mixed number.

✓ FINAL ANSWER

$2\frac{4}{5} \times 3\frac{3}{7} = 9\frac{3}{5}$

4WM1/01 MAY 2026

Question 1

3 marks | Fractions

1 Show that $4\frac{1}{7} - 2\frac{3}{5} = 1\frac{19}{35}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 May 2026

1 Improper fractions

$$4\tfrac{1}{7} = \frac{29}{7}, \qquad 2\tfrac{3}{5} = \frac{13}{5}$$

2 Common denominator 35

$$\frac{29}{7} - \frac{13}{5} = \frac{145}{35} - \frac{91}{35} = \frac{54}{35}$$

3 Back to a mixed number

$$\frac{54}{35} = 1\tfrac{19}{35} \quad \text{as required.}$$

✓ **FINAL ANSWER**

$\frac{54}{35} = 1\frac{19}{35}$ (shown)

4MA1/2HR JUNE 2026

Question 17

2 marks | Fractions

17 Use algebra to show that $0.\dot{8}1\dot{4} = \frac{22}{27}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2026

1 **Solution | Step 1**

Let $z = 0.\overline{814} = 0.814814814\ldots$. Then $1000z = 814.814814\ldots$.

Multiplication by 1000 aligns the repeating three-digit tails.

2 **Solution | Step 2**

$$1000z - z = 814 \quad \Rightarrow \quad 999z = 814 \quad \Rightarrow \quad z = \frac{814}{999} = \frac{22}{27}.$$

Subtract to cancel the recurring tails, then cancel the common factor 37.

✓ **FINAL ANSWER**

$0.\overline{814} = \frac{22}{27}$.

TOPIC

Fractions, Decimals & Percentages

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 15

2 marks | Fractions, Decimals & Percentages

15 Write these numbers in order of size. Start with the smallest number.

$0.62 \quad \frac{5}{8} \quad 61\% \quad \frac{2}{3} \quad \frac{3}{5}$

.....

(2)

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · EP Specimen

1 Change every number to a decimal

$0.62,\ \frac{5}{8} = 0.625,\ 61\% = 0.61,\ \frac{2}{3} = 0.666\ldots,\ \frac{3}{5} = 0.6$

2 Order them

$0.6,\ 0.61,\ 0.62,\ 0.625,\ 0.666\ldots$

✓ **FINAL ANSWER**

$\frac{3}{5},\ 61\%,\ 0.62,\ \frac{5}{8},\ \frac{2}{3}$

EP SPECIMEN

Question 16

2 marks | Fractions, Decimals & Percentages

16 (a) Write $\frac{7}{40}$ as a decimal.

.....

(1)

(b) Write $\frac{7}{40}$ as a percentage.

..... %

(1)

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · EP Specimen

1 **(a) Divide**

$$7 \div 40 = 0.175$$

2 **(b) Multiply by 100**

$$0.175 \times 100 = 17.5\%$$

✓ **FINAL ANSWER**

(a) 0.175 (b) 17.5%

EP SPECIMEN

Question 17

3 marks | Fractions, Decimals & Percentages

17 The students in a class travel to school by walking, by bicycle or by bus.

$\frac{3}{8}$ of the students walk. 0.35 of the students cycle. The rest travel by bus.

11 students travel by bus.

Work out the number of students in the class.

.....

(3)

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · EP Specimen

1 Proportion that walk or cycle

$$\frac{3}{8} + 0.35 = 0.375 + 0.35 = 0.725$$

2 Proportion that travel by bus

$$1 - 0.725 = 0.275$$

3 Number of students

$$0.275 \times n = 11 \Rightarrow n = \frac{11}{0.275} = 40$$

FINAL ANSWER

40

EP SPECIMEN

Question 18

2 marks | Fractions, Decimals & Percentages

18 Which is larger, $\frac{13}{20}$ or 0.66?

You must show your working.

.....

(2)

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · EP Specimen

1 Write both as decimals

$$\frac{13}{20} = 13 \div 20 = 0.65$$

2 Compare

$0.66 > 0.65$, so 0.66 is larger.

FINAL ANSWER

0.66

EP SPECIMEN

Question 19

2 marks | Fractions, Decimals & Percentages

19 Mia took two tests.
She scored 34 out of 40 in the first test.
She scored 87% in the second test.

In which test did Mia get the better score? You must show how you get your answer.

.....

(2)

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · EP Specimen

1 Change the first score to a percentage

$$\frac{34}{40} \times 100 = 85\%$$

2 Compare

$87\% > 85\%$, so the second test was better.

FINAL ANSWER

The second test (85% against 87%)

EP SPECIMEN

Question 20

2 marks | Fractions, Decimals & Percentages

20 Write 0.0375 as a fraction in its simplest form.

.....

(2)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · EP Specimen

1 **Write over a power of 10**

$$0.0375 = \frac{375}{10\,000}$$

2 **Simplify (divide by 125)**

$$\frac{375}{10\,000} = \frac{3}{80}$$

✓ **FINAL ANSWER**

$$\frac{3}{80}$$

EP SPECIMEN

Question 21

3 marks | Fractions, Decimals & Percentages

21 A water tank is $\frac{2}{5}$ full.

120 litres of water are added to the tank.
The tank is now 65% full.

Work out the capacity of the tank.

..... litres

(3)

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · EP Specimen

1 Change both amounts to percentages

$\frac{2}{5} = 40\%$, so the 120 litres fill $65\% - 40\% = 25\%$ of the tank.

2 Find 100%

$$25\% \to 120 \Rightarrow 100\% \to 120 \times 4 = 480 \text{ litres}$$

FINAL ANSWER

480 litres

TOPIC

Recurring Decimals

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY 2021

Question 16

2 marks | Recurring Decimals

16 Use algebra to show that the recurring decimal $0.28\dot{1}\dot{3} = \frac{557}{1980}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2021

1 Let

Let

$$x = 0.28131313\ldots$$

2 Calculate value

Then

$$100x = 28.131313\ldots$$

3 Calculate value

and

$$10000x = 2813.131313\ldots$$

4 Subtract

Subtract:

$$10000x - 100x = 2813.131313\ldots - 28.131313\ldots$$

$$9900x = 2785$$

$$x = \frac{2785}{9900} = \frac{557}{1980}$$

FINAL ANSWER

$0.28131313\ldots = \frac{557}{1980}$.

4MA1/1H NOVEMBER 2021

Question 18

3 marks | Recurring Decimals

18 $0.4\dot{x}$ is a recurring decimal.
x is a whole number such that $1 \leqslant x \leqslant 9$

Find, in terms of x, the recurring decimal $0.4\dot{x}$ as a fraction.
Give your fraction in its simplest form.
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2021

1 Let

Let

$$y = 0.4\dot{x}$$

2 Simplify fraction

So

$$y = 0.4xxx\ldots$$

3 Simplify fraction

Multiply by 10 and 100:

$$10y = 4.xxx\ldots$$

$$100y = 4x.xxx\ldots$$

4 Subtract

Subtract:

$$100y - 10y = (4x.xxx\ldots) - (4.xxx\ldots)$$

$$90y = x + 36$$

5 Simplify fraction

Therefore

$$y = \frac{x + 36}{90}$$

FINAL ANSWER

$$\frac{x+36}{90}$$

4WM1/01R MAY 2025

Question 14

2 marks | Recurring Decimals

14 Use algebra to show that the recurring decimal $0.6\dot{1}\dot{2} = \frac{101}{165}$

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4WM1/01R May 2025

1 Let

Let

$$x = 0.6121212\ldots$$

2 Calculate value

Then

$$10x = 6.121212\ldots$$

3 Calculate value

and

$$1000x = 612.121212\ldots$$

4 Subtract

Subtract:

$$1000x - 10x = 612.121212\ldots - 6.121212\ldots$$

$$990x = 606$$

$$x = \frac{606}{990} = \frac{101}{165}$$

✓ **FINAL ANSWER**

$0.6121212\ldots = \frac{101}{165}$.

4MA1/1HR JUNE 2025

Question 16

2 marks | Recurring Decimals

16 Use algebra to show that the recurring decimal $0.6\dot{1}\dot{2} = \frac{101}{165}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2025

1 Let

Let

$$x = 0.6121212\ldots$$

2 Calculate value

Then

$$10x = 6.121212\ldots$$

3 Calculate value

and

$$1000x = 612.121212\ldots$$

4 Subtract

Subtract:

$$1000x - 10x = 612.121212\ldots - 6.121212\ldots$$

$$990x = 606$$

$$x = \frac{606}{990} = \frac{101}{165}$$

FINAL ANSWER

$0.6121212\ldots = \frac{101}{165}$.

TOPIC

Percentages

Every past-paper question on this topic, 2020 to 2026.

Question 12

5 marks | Percentages

12 Zimo is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Zimo pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{2}{5}$ of the total cost
Payment 2	45% of the total cost
Payment 3	£405

Work out how much Zimo has to pay for Payment 2

£.....

(Total for Question 12 is 5 marks)

1 **Calculate percentage**

$$\frac{2}{5} = 40\%$$

2 **Calculate percentage**

So Payments 1 and 2 together are

$$40\% + 45\% = 85\%$$

3 **This means Payment 3 is**

This means Payment 3 is

$$100\% - 85\% = 15\%$$

4 **Calculate percentage**

So

$$15\% = 405$$

$$1\% = 27$$

5 **Payment 2 is**

Payment 2 is 45%:

$$45 \times 27 = 1215$$

FINAL ANSWER

$\$1215$.

4MA1/2H JANUARY 2021

Question 10

3 marks | Percentages

10 The people working for a company work in Team A or in Team B.

number of people in Team A : number of people in Team B = 3 : 4

$\frac{4}{5}$ of Team A work full time.

24% of Team B work full time.

Work out what fraction of the people working for the company work full time.
Give your fraction in its simplest form.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2021

1 The ratio of Team A

The ratio of Team A to Team B is $3 : 4$.

2 Use parts for Team A

Use 3 parts for Team A and 4 parts for Team B.

3 Full time part from Team

Full-time part from Team A:

$$\frac{4}{5} \times 3 = \frac{12}{5}$$

4 Full time part from Team

Full-time part from Team B:

$$24\% \times 4 = \frac{24}{100} \times 4 = \frac{24}{25}$$

5 Total full time parts

Total full-time parts:

$$\frac{12}{5} + \frac{24}{25} = \frac{60}{25} + \frac{24}{25} = \frac{84}{25}$$

6 **Total parts**

Total parts:

$$3+4=7$$

7 **Simplify fraction**

So the fraction who work full time is

$$\frac{\frac{84}{25}}{7}=\frac{84}{175}=\frac{12}{25}$$

✓ **FINAL ANSWER**

$$\frac{12}{25}$$

4MA1/1H NOVEMBER 2021

Question 8

3 marks | Percentages

8 In 2018, the population of Sydney was 5.48 million.
This was 22% of the total population of Australia.

Work out the total population of Australia in 2018
Give your answer correct to 3 significant figures.

..... million

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2021

1 Calculate percentage

Sydney's population was 22% of Australia's total population.

$$0.22 \times \text{total} = 5.48$$

$$\text{total} = \frac{5.48}{0.22} = 24.909\ldots$$

2 Calculate percentage

Correct to 3 significant figures,

$$24.909\ldots \approx 24.9$$

FINAL ANSWER

24.9 million

4MA1/1HR JUNE 2022

Question 4

4 marks | Percentages

4 The language department of a college has 180 students.
Each student studies exactly one of French, German, Italian or Spanish.

15 students study French.
45% of the students study German.

Express the percentage of students studying Italian or Spanish as a percentage of those studying French or German.

.....%

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2022

1 Total students

Total students:

$$180$$

2 French

French:

$$15$$

3 German

German:

$$45\% \times 180 = 81$$

4 French or German

French or German:

$$15 + 81 = 96$$

5 Italian or Spanish

Italian or Spanish:

$$180 - 96 = 84$$

6 **Convert standard form**

Write 84 as a percentage of 96:

$$\frac{84}{96} \times 100 = 87.5$$

✓ **FINAL ANSWER**

87.5%

4MA1/2H JUNE 2022

Question 8

5 marks | Percentages

8 Behnaz makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7:5:3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.
36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2022

1 The ratio has parts

The ratio $7 : 5 : 3$ has 15 parts.

2 Birthday cards

Birthday cards:

$$\frac{7}{15} \times 300 = 140$$

3 Anniversary cards

Anniversary cards:

$$\frac{5}{15} \times 300 = 100$$

4 Congratulations cards

Congratulations cards:

$$\frac{3}{15} \times 300 = 60$$

5 Birthday cards with numbers

Birthday cards with numbers:

$$\frac{2}{5} \times 140 = 56$$

6 Anniversary cards with numbers

Anniversary cards with numbers:

$$36\% \times 100 = 36$$

7 Congratulations cards with numbers

Congratulations cards with numbers:

$$0$$

8 Total with numbers

Total with numbers:

$$56 + 36 = 92$$

9 Fraction of all cards

Fraction of all cards:

$$\frac{92}{300} = \frac{23}{75}$$

FINAL ANSWER

$$\frac{23}{75}$$

4MA1/1H JUNE 2022

Question 13

5 marks | Percentages

13 Antoine is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Antoine pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{3}{8}$ of the total cost
Payment 2	45% of the total cost
Payment 3	\$406

Work out how much Antoine has to pay for Payment 2

\$ .....

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2022

1 **Payment 1 is**

Payment 1 is

$$\frac{3}{8}$$

2 **Payment 2 is**

Payment 2 is

$$45\% = \frac{45}{100} = \frac{9}{20}$$

3 **Together**

Together:

$$\frac{3}{8} + \frac{9}{20} = \frac{15}{40} + \frac{18}{40} = \frac{33}{40}$$

4 **Calculate percentage**

So Payment 3 is

$$1 - \frac{33}{40} = \frac{7}{40}$$

5 **Calculate percentage**

Payment 3 is $406, so

$$\frac{7}{40} \times \text{total} = 406$$

$$\text{total} = 406 \times \frac{40}{7} = 2320$$

6 **Payment 2 is**

Payment 2 is

$$45\% \times 2320 = 1044$$

✓ **FINAL ANSWER**

$1044

4MA1/2H JUNE 2024

Question 8

5 marks | Percentages

8 Ishir plants 600 bulbs in a garden.
He plants tulip bulbs, crocus bulbs and daffodil bulbs so that

number of tulip bulbs : number of crocus bulbs : number of daffodil bulbs = 9 : 4 : 2

45% of the tulip bulbs are for yellow flowers.

$\frac{5}{8}$ of the crocus bulbs are for yellow flowers.

All of the daffodil bulbs are for yellow flowers.

Work out the number of bulbs that are for yellow flowers.

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2024

1 The ratio parts are

The ratio parts are

$$9 + 4 + 2 = 15$$

$$\text{tulips} = 600 \times \frac{9}{15} = 360, \quad \text{crocuses} = 600 \times \frac{4}{15} = 160, \quad \text{daffodils} = 600 \times \frac{2}{15} = 80$$

2 Yellow tulips

Yellow tulips:

$$45\% \times 360 = 0.45 \times 360 = 162$$

3 Yellow crocuses

Yellow crocuses:

$$\frac{5}{8} \times 160 = 100$$

4 All the daffodil bulbs are

All the daffodil bulbs are yellow, so

$$162 + 100 + 80 = 342$$

FINAL ANSWER

342 bulbs

4MA1/1H NOVEMBER 2024

Question 7

4 marks | Percentages

7 Shop **A** and Shop **B** have offers for buying the same type of laptop.

The normal price of the laptop in Shop **A** is different to the normal price of the laptop in Shop **B**

Shop A	**Shop B**
Our normal price £475 Get 16% off our normal price	Get 15% off our normal price Only pay £408

Which shop gives more money off their normal price?
Show your working clearly.

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2024

1 Shop A discount

Shop A discount:

$$16\% \text{ of } 475 = 0.16 \times 475 = 76$$

2 Calculate percentage

Shop B: 408 euros is 85% of the normal price.

$$\text{normal price} = 408 \div 0.85 = 480$$

3 Shop B discount

Shop B discount:

$$480 - 408 = 72$$

4 Calculate percentage

Since

$$76 > 72$$

5

Shop A gives the greater

Shop A gives the greater discount.

✓ **FINAL ANSWER**

Shop A gives the greater discount.

4WM2/01 NOVEMBER 2025

Question 4

4 marks | Percentages

4 Pete plants a total of 960 trees in a park.
He plants only apple trees, cherry trees and pear trees such that

number of apple trees : number of cherry trees : number of pear trees = 6 : 7 : 3

65% of the cherry trees will grow morello cherries.

Work out the number of trees that will grow morello cherries.

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4WM2/01 November 2025

1 Split the ratio

Total parts = 6 + 7 + 3 = 16 parts

2 Evaluate fraction

$$1 textpart = frac96016 = 60 texttrees$$

3 Calculate value

Cherry trees = 7 parts × 60 = 420 trees

4 Calculate percentage

Morello trees = 0.65 × 420 = 273

FINAL ANSWER

273 Always read carefully what the final question asks. A common mistake is stopping at the total number of cherry trees (420) instead of calculating the 65% morello portion.

4MA1/1H NOVEMBER 2025

Question 9

3 marks | Percentages

9 Hari scored 140 points playing a game on Tuesday.
This was 12% more points than he scored playing the game on Monday.

Work out the number of points that Hari scored playing the game on Monday.

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2025

1 **Tuesday s score was more**

Tuesday's score was 12% more than Monday's score, so Tuesday was 112% of Monday.

$$1.12 \times \text{Monday} = 140$$

$$\text{Monday} = \frac{140}{1.12} = 125$$

FINAL ANSWER

125 points

4MA1/2HR JUNE 2026

Question 27

4 marks | Percentages

27 Bryan owns a house.

During 2024, the value of Bryan's house increased by $x\%$
During 2025, the value of Bryan's house increased by $2x\%$

Overall, during 2024 and 2025, the value of Bryan's house increased by 38.88%

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 27 is 4 marks)

WORKED SOLUTION

Question 27 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\left(1+\frac{x}{100}\right)\left(1+\frac{2x}{100}\right)=1.3888.$$

The second increase acts on the already-increased house value.

2 Solution | Step 2

$$1+\frac{3x}{100}+\frac{2x^2}{10000}=1.3888 \quad \Rightarrow \quad x^2+150x-1944=0.$$

Expand, subtract 1, and multiply by 5000.

3 Solution | Step 3

$$(x-12)(x+162)=0 \quad \Rightarrow \quad x=12 \text{ or } x=-162.$$

Factorize the quadratic.

4 Solution | Step 4

Reject $x=-162$, since both changes are increases, so $x>0$.
The positive root gives multipliers 1.12 and 1.24, whose product is 1.3888.

FINAL ANSWER

$x=12$.

TOPIC

Compound Interest & Depreciation

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 8

6 marks | Compound Interest & Depreciation

8 In 2021, the value of Asha's apartment was 634 400 euros.
The value of Asha's apartment had increased by 4% from its value in 2020

(a) Work out the value of Asha's apartment in 2020

..... euros
(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

..... %
(3)

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4WM2/01 Specimen

1 **Use compound interest**

For part (a), the 2021 value is 104% of the 2020 value.

$$\text{2020 value} = \frac{634400}{1.04} = 610000$$

2 **Use compound interest**

For part (b), after one year the multiplier is 0.85. After two years:

$$0.85^2 = 0.7225$$

3 **Use compound interest**

So 72.25% of the value remains, and the total depreciation is

$$100\% - 72.25\% = 27.75\%$$

✓ **FINAL ANSWER**

610000 euros; 27.75%.

4MA1/2HR JANUARY 2020

Question 8

3 marks | Compound Interest & Depreciation

8 Hamish buys a new car for $20 000
The car depreciates in value by 19% each year.

Work out the value of the car at the end of 3 years.
Give your answer to the nearest $.

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2020

1 A depreciation of gives a

A depreciation of 19% gives a multiplier of

$$1 - 0.19 = 0.81$$

2 Use compound interest

After 3 years,

$$20000(0.81)^3 = 10628.82$$

3 Use compound interest

Correct to the nearest dollar,

$$10628.82 \approx 10629$$

✓ **FINAL ANSWER**

$10629

4MA1/1H JANUARY 2020

Question 11

3 marks | Compound Interest & Depreciation

11 Max invests \$6000 in a savings account for 3 years.
The account pays compound interest at a rate of 1.5% per year for the first 2 years.

The compound interest rate changes for the third year.
At the end of 3 years, there is a total of \$6311.16 in the account.

Work out the compound interest rate for the third year.
Give your answer correct to 1 decimal place.

..... %

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H January 2020

1 **Use compound interest**

After the first 2 years,

$$6000(1.015)^2 = 6181.35$$

2 **Let the third year interest**

Let the third-year interest rate be $r\%$.

$$6181.35\left(1+\frac{r}{100}\right)=6311.16$$

$$1+\frac{r}{100}=\frac{6311.16}{6181.35}=1.021000\ldots$$

$$r=2.1000\ldots$$

✓ **FINAL ANSWER**

2.1%

4MA1/1HR MAY/NOVEMBER 2020

Question 8

3 marks | Compound Interest & Depreciation

8 Harold bought an antique clock for £1200
The clock increased in value by 8% per year.

Find the value of the clock exactly 3 years after Harold bought the clock.
Give your answer correct to the nearest £.

£.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May/November 2020

1 The multiplier for an increase

The multiplier for an 8% increase is

$$1.08$$

2 Value after 3 years

Value after 3 years:

$$1200(1.08)^3 = 1511.6544$$

3 Use compound interest

Correct to the nearest pound,

$$1511.6544 \approx 1512$$

FINAL ANSWER

1512 pounds

4MA1/1H MAY/NOVEMBER 2020

Question 13

3 marks | Compound Interest & Depreciation

13 Jan invests \$8000 in a savings account.
The account pays compound interest at a rate of $x\%$ per year.

At the end of 6 years, there is a total of \$8877.62 in the account.

Work out the value of x.
Give your answer correct to 2 decimal places.

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May/November 2020

1 Let the compound interest rate

Let the compound interest rate be $x\%$.

$$8000\left(1+\frac{x}{100}\right)^6 = 8877.62$$

$$1+\frac{x}{100} = \sqrt[6]{\frac{8877.62}{8000}} = 1.017500\ldots$$

$$x = 1.7500\ldots$$

2 Use compound interest

Correct to 2 decimal places,

$$x = 1.75$$

FINAL ANSWER

$x = 1.75$

4MA1/1H JANUARY 2021

Question 7

6 marks | Compound Interest & Depreciation

7 Chen invests 40 000 yuan in a fixed-term bond for 3 years.

The fixed-term bond pays compound interest at a rate of 3.5% each year.

(a) Work out the value of Chen's investment at the end of 3 years.
Give your answer to the nearest yuan.

..... yuan

(3)

Wang invested P yuan.
The value of his investment decreased by 6.5% each year.

At the end of the first year, the value of Wang's investment was 30 481 yuan.

(b) Work out the value of P.

$P =$ .....

(3)

(Total for Question 7 is 6 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2021

1 Use compound interest

For part (a),

$$40000(1.035)^3 = 44348.715$$

2 Use compound interest

Correct to the nearest yuan,

$$44348.715 \approx 44349$$

3 Use compound interest

For part (b), after one year the value is 93.5% of P.

$$0.935P = 30481$$

$$P = \frac{30481}{0.935} = 32600$$

FINAL ANSWER

(a) $\mathbf{44349}$ yuan, (b) $\boldsymbol{P = 32600}$

4MA1/1HR JANUARY 2021

Question 7

3 marks | Compound Interest & Depreciation

7 Kuro invests 50 000 yen for 3 years in a savings account.
She gets 2.4% per year compound interest.

Work out how much money Kuro will have in her savings account at the end of the 3 years.
Give your answer correct to the nearest yen.

..... yen

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2021

1 The multiplier for a increase

The multiplier for a 2.4% increase is

$$1.024$$

2 Use compound interest

After 3 years,

$$50000(1.024)^3 = 53687.0912$$

3 Use compound interest

Correct to the nearest yen,

$$53687.0912 \approx 53687$$

FINAL ANSWER

53687 yen

4MA1/2HR JANUARY 2021

Question 14

4 marks | Compound Interest & Depreciation

14 Simon bought a house at the beginning of 2018
The value of Simon's house had decreased by 15% by the end of 2018

The house increased in value during both 2019 and 2020
The percentage increases in the value of the house during 2019 and 2020 were the same.

The value of Simon's house at the end of 2020 was 2.85% greater than the amount he paid for his house at the beginning of 2018

Calculate the percentage increase in the value of the house during 2019

.....%

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR January 2021

1 **Let the original house value**

Let the original house value be V.

2 **Use compound interest**

After the 15% decrease in 2018:

$$0.85V$$

3 **Use compound interest**

Let the percentage increase in each of 2019 and 2020 be r.

4 **The final value is greater**

The final value is 2.85% greater than the original value:

$$1.0285V$$

5 **Use compound interest**

So

$$0.85V(1+r)^2 = 1.0285V$$

$$(1+r)^2 = \frac{1.0285}{0.85} = 1.21$$

$$1+r = \sqrt{1.21} = 1.1$$

$$r = 0.1 = 10\%$$

✓ **FINAL ANSWER**

10%

4MA1/2H MAY 2021

Question 11

6 marks | Compound Interest & Depreciation

11 Chengbo sold a house for 180 000 yuan.

The amount for which he sold the house is 24% more than the amount he paid for the house.

(a) Work out how much Chengbo paid for the house.
Give your answer correct to 3 significant figures.

..... yuan
(3)

Zhi bought a house on 1st January 2017
When she bought the house, its value was 120 000 yuan.

The value of the house increased by 1.8% per year.

(b) Work out the value of Zhi's house on 1st January 2020
Give your answer correct to 3 significant figures.

..... yuan
(3)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H May 2021

1 **Use compound interest**

For Chengbo's house, 180000 yuan is 124% of the price paid.

$$\text{price paid} = 180000 \div 1.24 = 145161.29\ldots$$

2 **To 3 significant figures**

To 3 significant figures:

$$145000$$

3 **Use compound interest**

For Zhi's house, the value increases by 1.8% each year for 3 years.

$$120000 \times 1.018^3 = 126598.2\ldots$$

4 **To 3 significant figures**

To 3 significant figures:

$$127000$$

✓ **FINAL ANSWER**

Chengbo paid 145000 yuan. Zhi's house was worth 127000 yuan.

4MA1/2H NOVEMBER 2021

Question 7

4 marks | Compound Interest & Depreciation

7 Ali and Badia each have 25 000 dollars to invest.

Cyclone Bank	Tornado Bank
Invest 25 000 dollars 4.5% compound interest per year for 3 years	Invest 25 000 dollars Receive 1150 dollars interest each year for 3 years

Ali invests in the Cyclone Bank for 3 years.
Badia invests in the Tornado Bank for 3 years.

By the end of the 3 years, Ali will have received more interest than Badia.

How much more?
Show your working clearly.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2021

1 Ali s interest from Cyclone

Ali's interest from Cyclone Bank is

$$25000(1.045)^3 - 25000 = 3529.153\ldots$$

2 Badia s interest from Tornado

Badia's interest from Tornado Bank is

$$1150 \times 3 = 3450$$

3 Difference

Difference:

$$3529.153\ldots - 3450 = 79.153\ldots$$

4 Use compound interest

Correct to the nearest dollar,

$$79.153\ldots \approx 79$$

FINAL ANSWER

$79

4MA1/2HR JANUARY 2022

Question 5

3 marks | Compound Interest & Depreciation

5 Shane invests 7200 dollars for 3 years in a savings account.
He gets 2.5% per year compound interest.

How much money will Shane have in his savings account at the end of 3 years?
Give your answer to the nearest dollar.

..... dollars

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2022

1 The multiplier for compound interest

The multiplier for 2.5% compound interest is

$$1.025$$

2 Use compound interest

After 3 years,

$$7200(1.025)^3 = 7753.6125$$

3 Use compound interest

Correct to the nearest dollar,

$$7753.6125 \approx 7754$$

✓ **FINAL ANSWER**

7754 dollars

4MA1/1H JANUARY 2022

Question 11

3 marks | Compound Interest & Depreciation

11 Himari invests 200000 yen for 3 years in a savings account paying compound interest.

The rate of interest is 1.8% for the first year and x% for each of the second year and the third year.

The value of the investment at the end of the third year is 209754 yen.

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H January 2022

1 The first year multiplier is

The first-year multiplier is

$$1.018$$

2 The second and third years

The second and third years both have multiplier $1 + \frac{x}{100}$, so

$$200000(1.018)\left(1 + \frac{x}{100}\right)^2 = 209754$$

$$\left(1 + \frac{x}{100}\right)^2 = \frac{209754}{200000(1.018)}$$

$$1 + \frac{x}{100} = 1.015000\ldots$$

$$x = 1.5000\ldots$$

FINAL ANSWER

$x = 1.5$

4MA1/1H JUNE 2022

Question 6

6 marks | Compound Interest & Depreciation

6 Asha bought an apartment.

The table gives information about the value of apartments, in euros, and the annual service charge band.

Value (x euros)	Service charge band
$x \geqslant 700\,000$	A
$600\,000 \leqslant x < 700\,000$	B
$500\,000 \leqslant x < 600\,000$	C
$400\,000 \leqslant x < 500\,000$	D
$0 < x < 400\,000$	E

In 2021, the value of Asha's apartment was 634 400 euros.

The value of Asha's apartment had increased by 4% from its value in 2020

(a) Has the annual service charge band changed for Asha's apartment?
Show your working clearly.

(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

..... %

(3)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2022

1 Use compound interest

For part (a), the value in 2021 was 4% more than in 2020.

$$\text{2020 value} = \frac{634400}{1.04} = 610000$$

2 **Use compound interest**

In 2020, $600000 < x < 700000$, so the service charge band was B.

3 **Use compound interest**

In 2021, 634400 is also between 600000 and 700000, so the band was still B.

4 **Use compound interest**

For part (b), a depreciation of 15% has multiplier 0.85.

5 **Use compound interest**

After 2 years, the multiplier is

$$0.85^2 = 0.7225$$

6 **Use compound interest**

So the value left is 72.25%, meaning the total depreciation is

$$100\% - 72.25\% = 27.75\%$$

✓ **FINAL ANSWER**

(a) No, it stayed in band B. (b) 27.75%

4MA1/1HR JUNE 2022

Question 8

3 marks | Compound Interest & Depreciation

8 Jane bought a new car for $18 000
The car depreciates in value by 15% each year.

Work out the value of the car at the end of 4 years.
Give your answer correct to the nearest $

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2022

1 A depreciation of gives multiplier

A depreciation of 15% gives multiplier

$$0.85$$

2 Use compound interest

After 4 years,

$$18000(0.85)^4 = 9396.1125$$

3 Use compound interest

Correct to the nearest dollar,

$$9396.1125 \approx 9396$$

✓ **FINAL ANSWER**

$9396

4MA1/2H JUNE 2022

Question 9

3 marks | Compound Interest & Depreciation

9 Pasha invests 50 000 dollars in a savings account for 4 years.
He gets 1.3% per year compound interest.

Work out how much money Pasha will have in his savings account at the end of 4 years.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2022

1 The multiplier for compound interest

The multiplier for 1.3% compound interest is

$$1.013$$

2 Use compound interest

After 4 years,

$$50000(1.013)^4 = 52651.1408\ldots$$

3 Use compound interest

Correct to the nearest dollar,

$$52651.1408\ldots \approx 52651$$

FINAL ANSWER

52651 dollars

4MA1/2H JUNE 2023

Question 8

3 marks | Compound Interest & Depreciation

8 Charlotte buys a painting for $680
The value of the painting increases by 4% each year.

Work out the value of the painting at the end of 3 years.
Give your answer correct to the nearest $

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2023

1 A increase each year gives

A 4% increase each year gives a multiplier of

$$1.04$$

2 Use compound interest

After 3 years, the value is

$$680(1.04)^3$$

$$= 680(1.124864)$$

$$= 764.90752$$

3 Correct to the nearest dollar

Correct to the nearest dollar:

$$765$$

✓ **FINAL ANSWER**

$765

4MA1/1HR MAY 2023

Question 9

3 marks | Compound Interest & Depreciation

9 Kazi buys a car for 700 000 taka.
The value of the car depreciates by 12% each year.

Work out the value of the car at the end of 3 years.
Give your answer correct to the nearest taka.

..... taka

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May 2023

1 A depreciation of gives multiplier

A depreciation of 12% gives multiplier

$$0.88$$

2 Use compound interest

After 3 years,

$$700000(0.88)^3 = 477030.4$$

3 Use compound interest

Correct to the nearest taka,

$$477030.4 \approx 477030$$

FINAL ANSWER

477030 taka

4MA1/2HR JUNE 2023

Question 13

3 marks | Compound Interest & Depreciation

13 Feruzi invests 80 000 Kenyan shillings (KES)
He invests the money for 3 years at $x\%$ compound interest each year.

At the end of 3 years, the total interest he receives is 6151.25 KES

Work out the value of x

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2023

1 The final value is

The final value is

$$80000 + 6151.25 = 86151.25$$

2 Let the annual compound interest

Let the annual compound interest rate be $x\%$.

$$80000\left(1 + \frac{x}{100}\right)^3 = 86151.25$$

$$1 + \frac{x}{100} = \sqrt[3]{\frac{86151.25}{80000}} = 1.025$$

$$x = 2.5$$

FINAL ANSWER

$x = 2.5$

4MA1/2H NOVEMBER 2023

Question 7

3 marks | Compound Interest & Depreciation

7 Hermione buys a boat for $26 800
The value of the boat depreciates by 8% each year.

Work out the value of the boat at the end of 3 years.
Give your answer correct to the nearest dollar.

$.....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2023

1 A depreciation of gives multiplier

A depreciation of 8% gives multiplier

$$0.92$$

2 Use compound interest

After 3 years,

$$26800(0.92)^3 = 20868.8384$$

3 Use compound interest

Correct to the nearest dollar,

$$20868.8384 \approx 20869$$

✓ **FINAL ANSWER**

$20869

4MA1/1HR MAY 2024

Question 5

3 marks | Compound Interest & Depreciation

5 Slavomir invests 5200 euros in a savings account for 4 years.
He gets 2.5% per year compound interest.

Work out how much money Slavomir will have in the savings account at the end of 4 years.
Give your answer correct to the nearest euro.

..... euros

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May 2024

1 The multiplier for compound interest

The multiplier for 2.5% compound interest is

$$1.025$$

2 Use compound interest

After 4 years,

$$5200(1.025)^4 = 5739.827\ldots$$

3 Use compound interest

Correct to the nearest euro,

$$5739.827\ldots \approx 5740$$

FINAL ANSWER

5740 euros

4MA1/2H JUNE 2024

Question 9

3 marks | Compound Interest & Depreciation

9 Giovanni invests 4500 koruna in a savings account for 4 years.
He gets 2.4% per year compound interest.

Work out how much money Giovanni will have in the savings account at the end of 4 years.
Give your answer correct to the nearest koruna.

..... koruna

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2024

1 The multiplier for compound interest

The multiplier for 2.4% compound interest is

$$1.024$$

2 Use compound interest

After 4 years,

$$4500(1.024)^4 = 4947.8023\ldots$$

3 Use compound interest

Correct to the nearest koruna,

$$4947.8023\ldots \approx 4948$$

FINAL ANSWER

4948 koruna

4MA1/2H NOVEMBER 2024

Question 9

3 marks | Compound Interest & Depreciation

9 Nisha invests 20 000 euros for 3 years in a savings account.
She gets 3.5% per year compound interest.

Work out how much money Nisha will have in her savings account at the end of the 3 years.
Give your answer correct to the nearest euro.

..... euros

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2024

1 The multiplier for compound interest

The multiplier for 3.5% compound interest is

$$1.035$$

2 Use compound interest

After 3 years,

$$20000(1.035)^3 = 22174.3575$$

3 Use compound interest

Correct to the nearest euro,

$$22174.3575 \approx 22174$$

FINAL ANSWER

22174 euros

4MA1/2H JUNE 2025

Question 7

3 marks | Compound Interest & Depreciation

7 Tim buys a bracelet for 4000 Swiss francs.
The value of the bracelet increases by 7% each year.

Work out the value of the bracelet at the end of 3 years.
Give your answer correct to the nearest Swiss franc.

..... Swiss francs

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2025

1 The multiplier for a increase

The multiplier for a 7% increase is

$$1.07$$

2 Use compound interest

After 3 years,

$$4000(1.07)^3 = 4900.172$$

3 Correct to the nearest Swiss

Correct to the nearest Swiss franc,

$$4900.172 \approx 4900$$

✓ **FINAL ANSWER**

4900 Swiss francs

4WM2/01 MAY 2025

Question 9

3 marks | Compound Interest & Depreciation

9 Tim buys a bracelet for 4000 Swiss francs.
The value of the bracelet increases by 7% each year.

Work out the value of the bracelet at the end of 3 years.
Give your answer correct to the nearest Swiss franc.

..... Swiss francs

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 May 2025

1 The multiplier for a increase

The multiplier for a 7% increase is

$$1.07$$

2 Use compound interest

After 3 years,

$$4000(1.07)^3 = 4900.172$$

3 Correct to the nearest Swiss

Correct to the nearest Swiss franc,

$$4900.172 \approx 4900$$

FINAL ANSWER

4900 Swiss francs

4MA1/1HR JUNE 2025

Question 11

5 marks | Compound Interest & Depreciation

11 Zhou invests some money for 2 years.
He invests \$2500 with Bank A and \$3000 with Bank B.

Bank A
Invests \$2500
amount of money invested : total amount of interest after 2 years = 20 : 3

Bank B
Invests \$3000
4% per year compound interest for 2 years

Zhou receives more **interest** from Bank A than from Bank B.

How much more?

\$ .....

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR June 2025

1 **Use compound interest**

For Bank A,

$$\text{amount invested} : \text{interest} = 20 : 3$$

2 **Use compound interest**

So the interest from Bank A is

$$2500 \times \frac{3}{20} = 375$$

3 Use compound interest

For Bank B, the compound interest after 2 years is

$$3000(1.04)^2 - 3000$$

$$= 3000(1.0816) - 3000 = 244.80$$

4 Difference

Difference:

$$375 - 244.80 = 130.20$$

FINAL ANSWER

$130.20

4MA1/2HR JUNE 2025

Question 12

3 marks | Compound Interest & Depreciation

12 Osman buys a car for \$16 000

The car depreciates at a rate of 12% each year for the first 2 years.
In the third year, the car depreciates at a rate of $x\%$

At the end of 3 years, the value of the car is \$11 461.12

Work out the value of x

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2025

1 Use compound interest

After the first 2 years,

$$16000(0.88)^2 = 12390.40$$

2 Use compound interest

In the third year, the value becomes 11461.12, so

$$12390.40\left(1 - \frac{x}{100}\right) = 11461.12$$

$$1 - \frac{x}{100} = \frac{11461.12}{12390.40} = 0.925$$

$$x = 7.5$$

FINAL ANSWER

$x = 7.5$

4WM2/01R MAY 2025

Question 12

3 marks | Compound Interest & Depreciation

12 Osman buys a car for $16 000

The car depreciates at a rate of 12% each year for the first 2 years.
In the third year, the car depreciates at a rate of $x\%$

At the end of 3 years, the value of the car is $11 461.12

Work out the value of x

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01R May 2025

1 Required

Car bought for $16\,000; depreciates 12 11\,461.12$. Find *x*.

2 Value after 2 years at

Value after 2 years at 12% depreciation:

*V*₂ = 16000 × (0.88)² = 16000 × 0.7744 = $12 390.40

3 Use compound interest

Apply year-3 depreciation and set equal to final value:

$$12390.40 times ! left(1 - frac x 100 right) = 11461.12$$

4 Solve

Solve:

$$1 - frac x 100 = frac 11461.12 12390.40 = 0.925 implies frac x 100 = 0.075 implies x = 7.5$$

✓ FINAL ANSWER

x = 7.5

4MA1/1H JUNE 2025

Question 13

3 marks | Compound Interest & Depreciation

13 Charlotte bought an apartment for \$750 000

In the first year, the value of the apartment decreased by 4%
In the second year, the value of the apartment decreased by 6.5%

In the third year, the value of the apartment increased by $x\%$

At the end of the third year, the value of Charlotte's apartment was \$698 445

Work out the value of x

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2025

1 **Use compound interest**

After the first two years, the value is

$$750000(0.96)(0.935) = 673200$$

2 **Use compound interest**

Then the value increases by $x\%$ to 698445, so

$$673200\left(1+\frac{x}{100}\right) = 698445$$

$$1+\frac{x}{100} = \frac{698445}{673200} = 1.0375$$

$$x = 3.75$$

FINAL ANSWER

$x = 3.75$

4WM2/01 MAY 2025

Question 15

3 marks | Compound Interest & Depreciation

15 Fred buys a painting.

The value of the painting increases by 12% during the first year.
The value of the painting then decreases by 15% during the second year.

Work out the overall percentage decrease in the value of the painting during this two-year period.

.....%

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM2/01 May 2025

1 Use multipliers

Use multipliers:

$$1.12 \times 0.85 = 0.952$$

2 Calculate percentage

So the final value is 95.2% of the original.

$$100 - 95.2 = 4.8$$

FINAL ANSWER

4.8% decrease.

4MA1/2H NOVEMBER 2025

Question 5

6 marks | Compound Interest & Depreciation

5 Otis sells ice creams.

On Friday, Otis sells 75 ice creams.
On Saturday, Otis sells 87 ice creams.

(a) Work out the percentage increase in the number of ice creams Otis sells from Friday to Saturday.

.....%
(3)

Claudia buys an ice cream machine for 960 Swiss francs.

The value of the ice cream machine depreciates by 20% each year.

(b) Work out the value of the ice cream machine at the end of 3 years.

..... Swiss francs
(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2025

1 Use compound interest

For part (a), the increase is

$$87 - 75 = 12$$

2 Percentage increase

Percentage increase:

$$\frac{12}{75} \times 100 = 16$$

3 Use compound interest

For part (b), a depreciation of 20% gives multiplier

$$0.80$$

4 **Use compound interest**

After 3 years,

$$960(0.80)^3 = 491.52$$

✓ **FINAL ANSWER**

(a) 16%, (b) 491.52 Swiss francs

4WM2/01 NOVEMBER 2025

Question 6

6 marks | Compound Interest & Depreciation

6 Otis sells ice creams.

On Friday, Otis sells 75 ice creams.
On Saturday, Otis sells 87 ice creams.

(a) Work out the percentage increase in the number of ice creams Otis sells from Friday to Saturday.

..... %
(3)

Claudia buys an ice cream machine for 960 Swiss francs.

The value of the ice cream machine depreciates by 20% each year.

(b) Work out the value of the ice cream machine at the end of 3 years.

..... Swiss francs
(3)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 November 2025

1 Calculate value

Increase = 87 − 75 = 12

2 Convert standard form

Extra open brace or missing close brace

3 Answer

16%

Step 1: Determine the depreciation multiplier. A 20% decrease means the multiplier is 1 − 0.20 = 0.80.

4 Solve inequality

Final Value = 960 × (0.80)³

Final Value = 960 × 0.512 = 491.52

FINAL ANSWER

491.52 Swiss francs

4WM2/01 NOVEMBER 2025

Question 12

3 marks | Compound Interest & Depreciation

12 The value of a house

- decreased by 5% from the end of 2021 to the end of 2022
- increased by 20% from the end of 2022 to the end of 2023
- increased by 6.5% from the end of 2023 to the end of 2024

Calculate the percentage by which the value of the house increased from the end of 2021 to the end of 2024

..... %

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01 November 2025

1 Calculate percentage

Decrease by 5% $\Longrightarrow$ Multiplier = 1 − 0.05 = 0.95

Increase by 20% $\Longrightarrow$ Multiplier = 1 + 0.20 = 1.20

Increase by 6.5% $\Longrightarrow$ Multiplier = 1 + 0.065 = 1.065

2 Calculate value

Overall Multiplier = 0.95 × 1.20 × 1.065 = 1.2141

3 Calculate percentage

1.2141 − 1 = 0.2141 = 21.41%

✓ **FINAL ANSWER**

21.41%

4MA1/1HR JUNE 2026

Question 10

3 marks | Compound Interest & Depreciation

10 Arwa invests 200 000 yen in a savings account for 3 years.
She gets 1.4% per year compound interest.

Work out how much money Arwa will have in her savings account at the end of 3 years.
Give your answer correct to the nearest yen.

..... yen

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2026

1 Solution | Step 1

$$\text{annual multiplier} = 1 + \frac{1.4}{100} = 1.014.$$

An increase of 1.4% retains the original amount and adds 1.4%.

2 Solution | Step 2

$$A = 200000(1.014)^3 = 208518.1488 \text{ yen}.$$

Apply the multiplier once for each of the 3 years.

3 Solution | Step 3

$$A \approx 208518 \text{ yen}.$$

Round to the nearest whole yen.

FINAL ANSWER

208518 yen

4MA1/2H JUNE 2026

Question 10

4 marks | Compound Interest & Depreciation

10 Johan owns a car.

The current value of the car is 32 000 euros.
The value of the car will now depreciate by 15% per year.

Lena is going to save up and buy Johan's car in 3 years' time.
She has already saved 2516 euros.

She will save an equal amount of money each month for the next 3 years.

Work out how much money Lena needs to save each month.

..... euros

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2026

1 Solution | Step 1

The annual multiplier is $1 - 0.15 = 0.85$. After 3 years the value is

$$32000(0.85)^3 = 19652 \text{ euros.}$$

Apply the decrease to each successive year, giving a power of 3.

2 Solution | Step 2

Lena still needs $19652 - 2516 = 17136$ euros. There are $3 \times 12 = 36$ months.

Subtract her existing savings and convert years to months.

3 Solution | Step 3

$$\text{Monthly saving} = \frac{17136}{36} = 476 \text{ euros.}$$

Divide the remaining target equally over the months.

✓ **FINAL ANSWER**

476 euros per month

4MA1/1H JUNE 2026

Question 14

3 marks | Compound Interest & Depreciation

14 In September, the sales in a shop increased by 20%
In October, the sales in the same shop decreased by x%

Over these two months, the sales in the shop decreased by 10%

Work out the value of x

$x =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2026

1 **Solution | Step 1**

Let the original sales be 100 units. After September they are $100 \times 1.20 = 120$; after both months they are $100 \times 0.90 = 90$.
Both overall percentages refer to the original sales.

2 **Solution | Step 2**

$$x = \frac{120 - 90}{120} \times 100 = 25.$$

The October decrease is measured relative to the September sales of 120.

✓ **FINAL ANSWER**

$x = 25$

TOPIC

Reverse Percentages

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 7

3 marks | Reverse Percentages

7 In a sale, the normal price of a hat is reduced by 15%
The sale price of the hat is 20.40 euros.

Work out the normal price of the hat.

..... euros

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2020

1 The hat is reduced by

The hat is reduced by 15%, so the sale price is

$$100\% - 15\% = 85\%$$

2 of the normal price

of the normal price.

$$0.85 \times \text{normal price} = 20.40$$

$$\text{normal price} = 20.40 \div 0.85 = 24$$

FINAL ANSWER

24 euros.

4MA1/2H JANUARY 2020

Question 8

7 marks | Reverse Percentages

8 The table shows the cost, in euros, of Brigitte's car insurance in each of the years 2016, 2017 and 2018

Year	2016	2017	2018
Cost of insurance (euros)	500	545	592

Brigitte says,

"The percentage increase in the cost of my car insurance from 2017 to 2018 is more than the percentage increase in the cost of my car insurance from 2016 to 2017"

(a) Is Brigitte correct?
You must show how you get your answer.

(4)

Henri wants to insure his car.

He gets a discount of 15% off the normal price.
Henri pays 952 euros for his car insurance after the discount.

(b) Work out the discount that Henri gets.

..... euros

(3)

(Total for Question 8 is 7 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2020

1 Convert standard form

From 2016 to 2017, the increase is

$$545 - 500 = 45$$

$$\frac{45}{500} \times 100 = 9\%$$

2 **Convert standard form**

From 2017 to 2018, the increase is

$$592 - 545 = 47$$

$$\frac{47}{545} \times 100 \approx 8.62\%$$

3 **Calculate percentage**

So Brigitte is not correct because the second percentage increase is smaller.

4 **Calculate percentage**

For Henri, 952 is 85% of the original price because there is a 15% discount.

$$\text{original price} = 952 \div 0.85 = 1120$$

5 **The discount is**

The discount is

$$1120 - 952 = 168$$

✓ **FINAL ANSWER**

Brigitte is not correct; the discount is 168 euros.

4MA1/1HR MAY/NOVEMBER 2020

Question 5

6 marks | Reverse Percentages

5 In 2017, the population of a village was 7500
In 2019, the population of the village was 8265

(a) Work out the percentage increase in the population of the village from 2017 to 2019

..... %

(3)

In a sale, normal prices are reduced by 30%
The sale price of a T-shirt was 31.50 euros.

(b) Work out the normal price of the T-shirt.

..... euros

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May/November 2020

1 The population increase is

The population increase is

$$8265 - 7500 = 765$$

2 Percentage increase

Percentage increase:

$$\frac{765}{7500} \times 100 = 10.2\%$$

3 The T shirt sale price

The T-shirt sale price is after a 30% reduction, so it is 70% of the normal price.

$$0.70 \times \text{normal price} = 31.50$$

$$\text{normal price} = 31.50 \div 0.70 = 45$$

FINAL ANSWER

(a) 10.2%, (b) 45 euros.

4MA1/1H MAY/NOVEMBER 2020

Question 9

3 marks | Reverse Percentages

9 In a sale, normal prices are reduced by 20%

A bag costs 1080 rupees in the sale.

Work out the normal price of the bag.

..... rupees

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May/November 2020

1 A reduction of means the

A reduction of 20% means the sale price is 80% of the normal price.

$$0.80 \times \text{normal price} = 1080$$

$$\text{normal price} = \frac{1080}{0.80} = 1350$$

FINAL ANSWER

1350 rupees

4MA1/1HR MAY/NOVEMBER 2020

Question 10

5 marks | Reverse Percentages

10 Alex makes 80 cakes to sell.
He makes chocolate cakes, lemon cakes and fruit cakes where

$$\text{number of chocolate cakes} : \text{number of lemon cakes} : \text{number of fruit cakes} = 3 : 2 : 5$$

Alex sells

all of the chocolate cakes

$\frac{3}{4}$ of the lemon cakes

$\frac{7}{8}$ of the fruit cakes

The profit he makes on each cake he sells is shown in the table.

Type of cake	Profit per cake he sells
chocolate	£2.00
lemon	£1.70
fruit	£2.40

Work out the total profit that Alex makes from the cakes he sells.

£.....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR May/November 2020

1 The ratio parts are

The ratio parts are

$$3 + 2 + 5 = 10$$

2 Numbers of cakes made

Numbers of cakes made:

$$\text{chocolate} = 80 \times \frac{3}{10} = 24$$

$$\text{lemon} = 80 \times \frac{2}{10} = 16$$

$$\text{fruit} = 80 \times \frac{5}{10} = 40$$

3 Cakes sold

Cakes sold:

$$\text{chocolate} = 24, \quad \text{lemon} = \frac{3}{4} \times 16 = 12, \quad \text{fruit} = \frac{7}{8} \times 40 = 35$$

4 Total profit

Total profit:

$$24(2.00) + 12(1.70) + 35(2.40) = 48 + 20.40 + 84 = 152.40$$

FINAL ANSWER

152.40 pounds

4MA1/2H JANUARY 2021

Question 3

6 marks | Reverse Percentages

3 Gladys buys a table for \$465 to sell in her shop.

She sells the table for \$520

(a) Work out the percentage profit that Gladys makes from the sale of the table.
Give your answer correct to 3 significant figures.

.....%

(3)

Gladys has a sale in her shop.

She decreases all the normal prices by 12%
The normal price of an armchair was \$550

(b) Work out the sale price of the armchair.

\$.....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2021

1 Profit

Profit:

$$520 - 465 = 55$$

2 Percentage profit

Percentage profit:

$$\frac{55}{465} \times 100 = 11.827\ldots$$

3 Correct to 3 significant figures

Correct to 3 significant figures:

$$11.8\%$$

4 **Calculate statistic**

For the sale price, a 12% decrease means 88% of the original price:

$$550 \times 0.88 = 484$$

✓ **FINAL ANSWER**

(a) 11.8%, (b) $\$484$

4MA1/2HR JANUARY 2021

Question 6

3 marks | Reverse Percentages

6 On Wednesday, the price of 1 litre of petrol was £1.26
The price of petrol on Wednesday was 5% more than the price of petrol on the previous Monday.

Calculate the price of 30 litres of petrol on the previous Monday.

£.....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR January 2021

1 Wednesday s price was more

Wednesday's price was 5% more than Monday's price.

2 Calculate percentage

So

$$1.26 = 1.05 \times \text{Monday price}$$

$$\text{Monday price} = \frac{1.26}{1.05} = 1.20$$

3 The cost of litres on

The cost of 30 litres on Monday was

$$30 \times 1.20 = 36$$

FINAL ANSWER

$\pounds 36$

4MA1/2H MAY 2021

Question 6

4 marks | Reverse Percentages

6 Mariana sells bags of bird food.

The bags that Mariana sold last week each contained 12 kg of seeds.

The bags that she is going to sell next week will each contain a mixture of nuts and seeds where for each bag

$$\text{weight of nuts} : \text{weight of seeds} = 4 : 5$$

The total weight of the nuts and the seeds in each bag will be 19.35 kg

The weight of seeds in each bag that Mariana sells next week will be less than the weight of seeds in each bag that Mariana sold last week.

Work out this decrease as a percentage of the weight of seeds in each bag that Mariana sold last week.
Give your answer correct to one decimal place.

..... %

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H May 2021

1 Split the ratio

In each new bag, the ratio

$$\text{nuts} : \text{seeds} = 4 : 5$$

2 has total parts

has 9 total parts.

3 Weight of seeds in each

Weight of seeds in each new bag:

$$\frac{5}{9} \times 19.35 = 10.75 \text{ kg}$$

4 Decrease in seed weight

Decrease in seed weight:

$$12 - 10.75 = 1.25 \text{ kg}$$

5 **Percentage decrease**

Percentage decrease:

$$\frac{1.25}{12} \times 100 = 10.416\ldots\%$$

✓ **FINAL ANSWER**

10.4%

4MA1/2H NOVEMBER 2021

Question 6

3 marks | Reverse Percentages

6 Victor buys 12 bottles of apple juice for a total cost of $21
Victor sells all 12 bottles at $2.45 each bottle.

Work out Victor's percentage profit.

.....%

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2021

1 Total selling price

Total selling price:

$$12 \times 2.45 = 29.40$$

2 Profit

Profit:

$$29.40 - 21 = 8.40$$

3 Percentage profit

Percentage profit:

$$\frac{8.40}{21} \times 100 = 40$$

FINAL ANSWER

40%

Question 2

7 marks | Reverse Percentages

2 Mary saves for a holiday each year.

In 2020 she saved a total of $720
In 2021, each month she saved $78

The total amount Mary saved in 2021 was P% more than the total she saved in 2020

(a) Work out the value of P

.....

(4)

Roberto is going to go on holiday.

He has two coupons that will save him money on his holiday.

Coupon A	**Coupon B**
18% off the cost of the accommodation	12.5% off the total cost of the accommodation **and** the flights

For Roberto's holiday
the cost of the accommodation is $1600
the cost of the flights is $800

Roberto can only use one of the coupons.
He wants to save as much money as he can.

(b) Which of the two coupons, **A** or **B**, should he use?
Show your working clearly.

(3)

(Total for Question 2 is 7 marks)

1 Mary saved in 2021

Mary saved in 2021:

$$78 \times 12 = 936$$

2 Increase from 2020

Increase from 2020:

$$936 - 720 = 216$$

3 Percentage increase

Percentage increase:

$$\frac{216}{720} \times 100 = 30$$

4 Calculate percentage

So $P = 30$.

5 Coupon A saves

Coupon A saves

$$18\% \times 1600 = 0.18 \times 1600 = 288$$

6 Coupon B saves of the

Coupon B saves 12.5% of the accommodation and flights together:

$$1600 + 800 = 2400$$

$$12.5\% \times 2400 = 0.125 \times 2400 = 300$$

7 Coupon B saves more money

Coupon B saves more money.

FINAL ANSWER

(a) $P = 30$, (b) Coupon B

4MA1/1H JANUARY 2022

Question 4

5 marks | Reverse Percentages

4 Sarah makes and sells mugs.

One day she makes 150 mugs.
Her total cost for making these mugs is £1140

Of these mugs

$\frac{2}{5}$ are small mugs

32% are medium mugs

and the rest are large mugs

Here is Sarah's price list for selling each mug.

MUGS	
Small	**£8.50**
Medium	**£11.20**
Large	**£14.20**

Sarah sells all 150 mugs.

Work out her percentage profit.
Give your answer correct to the nearest whole number.

..... %

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2022

1 **Small mugs**

Small mugs:

$$\frac{2}{5} \times 150 = 60$$

2 **Medium mugs**

Medium mugs:

$$32\% \times 150 = 48$$

3 **Large mugs**

Large mugs:

$$150 - 60 - 48 = 42$$

---

4 **Revenue**

Revenue:

$$60(8.50) + 48(11.20) + 42(14.20)$$

$$= 510 + 537.60 + 596.40 = 1644$$

---

5 **Profit**

Profit:

$$1644 - 1140 = 504$$

---

6 **Percentage profit**

Percentage profit:

$$\frac{504}{1140} \times 100 = 44.21\ldots$$

---

7 **Correct to the nearest whole**

Correct to the nearest whole number:

$$44\%$$

✓ **FINAL ANSWER**

44%

4MA1/2H JANUARY 2022

Question 8

5 marks | Reverse Percentages

8 The table gives information about the average house price in England in 2018 and in 2019

Year	2017	2018	2019
Average house price (£)		228 314	231 776

(a) Work out the percentage increase in the average house price from 2018 to 2019
Give your answer correct to one decimal place.

..... %

(2)

The average house price in 2019 was 7.7% greater than the average house price in 2017

(b) Work out the average house price in 2017
Give your answer correct to 3 significant figures.

£ .....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2022

1 Increase from 2018 to 2019

Increase from 2018 to 2019:

$$231776 - 228314 = 3462$$

2 Percentage increase

Percentage increase:

$$\frac{3462}{228314} \times 100 = 1.516\ldots$$

3 Correct to one decimal place

Correct to one decimal place:

$$1.5\%$$

4 **Calculate percentage**

For part (b), the 2019 price was 7.7% greater than the 2017 price.

$$231776 = 1.077 \times 2017\ \text{price}$$

$$2017\ \text{price} = \frac{231776}{1.077} = 215205.19\ldots$$

5 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$215000$$

FINAL ANSWER

(a) 1.5%, (b) $\pounds 215000$

4MA1/2HR JUNE 2022

Question 8

3 marks | Reverse Percentages

8 A cinema increased the cost of an adult ticket by 12%

After the increase, the cost of an adult ticket was £18.20

Work out the cost of an adult ticket before the increase.

£.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2022

1 Calculate percentage

After a 12% increase, the ticket price is 112% of the original price.

$$18.20 = 1.12 \times \text{original price}$$

$$\text{original price} = \frac{18.20}{1.12} = 16.25$$

FINAL ANSWER

$\pounds 16.25$

4MA1/1H MAY 2023

Question 3

4 marks | Reverse Percentages

3 Nanette buys 60 notebooks for a total cost of 780 dirhams.

Nanette sells 70% of the notebooks for 22 dirhams each.
She sells the remaining notebooks for 19 dirhams each.

Work out Nanette's percentage profit.
Give your answer correct to 3 significant figures.

.....%

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2023

1 Number sold for 22 dirhams

Number sold for 22 dirhams:

$$70\% \times 60 = 42$$

2 Number remaining

Number remaining:

$$60 - 42 = 18$$

3 Total selling price

Total selling price:

$$42 \times 22 + 18 \times 19 = 924 + 342 = 1266$$

4 Profit

Profit:

$$1266 - 780 = 486$$

5 Percentage profit

Percentage profit:

$$\frac{486}{780} \times 100 = 62.307\ldots\%$$

FINAL ANSWER

62.3%

4MA1/2HR JUNE 2023

Question 6

7 marks | Reverse Percentages

6 In his previous job, Pierre was paid 400 euros in total for working a 5-day week.

In his new job, Pierre is paid 14 euros per hour.
In his new job, Pierre works for 7 hours each day for a 5-day week.

(a) Work out the percentage increase in the amount that Pierre is paid for a 5-day week.

..... %
(4)

Marie changes her job.

Her salary decreases by 6%
Her new salary is 23 030 euros.

(b) Work out Marie's salary before she changes her job.

..... euros
(3)

(Total for Question 6 is 7 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2023

1 **Pierre s old weekly pay**

Pierre's old weekly pay is 400 euros.

2 **His new weekly pay is**

His new weekly pay is

$$14 \times 7 \times 5 = 490$$

3 **The increase is**

The increase is

$$490 - 400 = 90$$

4 **Percentage increase**

Percentage increase:

$$\frac{90}{400} \times 100 = 22.5\%$$

5 **Marie s new salary is**

Marie's new salary is after a 6% decrease, so it is 94% of her old salary.

$$0.94 \times \text{old salary} = 23030$$

$$\text{old salary} = 23030 \div 0.94 = 24500$$

FINAL ANSWER

Pierre's pay increased by 22.5%. Marie's old salary was 24500 euros.

4MA1/1H NOVEMBER 2023

Question 3

4 marks | Reverse Percentages

3 Joshua buys a car for $12 500

He sells the car to Nina.

Nina pays

- a deposit of $1500
- followed by 36 monthly payments of $450

Work out Joshua's percentage profit.

.....%

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2023

1 Nina pays

Nina pays

$$1500 + 36(450) = 1500 + 16200 = 17700$$

2 Joshua s profit is

Joshua's profit is

$$17700 - 12500 = 5200$$

3 Percentage profit

Percentage profit:

$$\frac{5200}{12500} \times 100 = 41.6$$

FINAL ANSWER

41.6%

4MA1/1H NOVEMBER 2023

Question 6

4 marks | Reverse Percentages

6 Juan wants to buy a ticket to fly from Madrid to Berlin.

He finds two different types of ticket he can buy in a sale, ticket **A** and ticket **B**

ticket **A**	ticket **B**
$\frac{1}{6}$ off normal price	20% off normal price

The sale price of ticket **A** is 140 euros.
The sale price of ticket **B** is 136 euros.

Work out the difference between the normal price of ticket **A** and the normal price of ticket **B**

..... euros

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2023

1 Calculate percentage

Ticket A is sold for $\frac{5}{6}$ of its normal price.

$$\text{normal price of A} = 140 \times \frac{6}{5} = 168$$

2 Calculate percentage

Ticket B has a 20% discount, so its sale price is 80% of the normal price.

$$\text{normal price of B} = 136 \div 0.80 = 170$$

3 The difference is

The difference is

$$170 - 168 = 2$$

FINAL ANSWER

The difference in normal prices is 2 euros.

4MA1/1HR MAY 2024

Question 3

5 marks | Reverse Percentages

3 Norberto sells white loaves of bread and brown loaves of bread.

He sells a total of 200 loaves such that

the number of white loaves sold : the number of brown loaves sold = 3 : 2

Norberto sells the white loaves for £1.50 each.
He sells the brown loaves for £1.75 each.

40% of the price of a white loaf is profit.
60% of the price of a brown loaf is profit.

Work out Norberto's total profit when he sells all 200 loaves.

£.....

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May 2024

1 The ratio parts are

The ratio parts are

$$3 + 2 = 5$$

2 White loaves

White loaves:

$$200 \times \frac{3}{5} = 120$$

3 Brown loaves

Brown loaves:

$$200 \times \frac{2}{5} = 80$$

4 Profit on each white loaf

Profit on each white loaf:

$$40\% \times 1.50 = 0.60$$

5 **Profit on each brown loaf**

Profit on each brown loaf:

$$60\% \times 1.75 = 1.05$$

6 **Total profit**

Total profit:

$$120 \times 0.60 + 80 \times 1.05 = 72 + 84 = 156$$

✓ **FINAL ANSWER**

156 pounds

4MA1/1H MAY 2024

Question 4

3 marks | Reverse Percentages

4 Ava records the number of kilometres she drives each month.

In April, Ava drove 943 kilometres.
This is 15% more than the number of kilometres she drove in March.

Work out the number of kilometres Ava drove in March.

..... kilometres

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May 2024

1 Calculate percentage

April is 15% more than March, so April is 115% of March.

$$1.15 \times \text{March} = 943$$

$$\text{March} = \frac{943}{1.15} = 820$$

FINAL ANSWER

820 kilometres

4MA1/2HR JUNE 2024

Question 8

3 marks | Reverse Percentages

8 In a sale, all normal prices are reduced by 17%

The sale price of a fridge is 6225 rupees.

Work out the normal price of the fridge.

..... rupees

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2024

1 **A reduction of means the**

A reduction of 17% means the sale price is 83% of the normal price.

$$0.83 \times \text{normal price} = 6225$$

$$\text{normal price} = \frac{6225}{0.83} = 7500$$

FINAL ANSWER

7500 rupees

4MA1/1H MAY 2024

Question 14

5 marks | Reverse Percentages

14 The combined savings of Abel and Bahira are 15 435 dinars.

The savings of Bahira are 45% more than the savings of Abel.

The savings of Bahira are $\frac{3}{2}$ times the savings of Chanda.

Work out the savings of Chanda.

..... dinars

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2024

1 Let Abel s savings be

Let Abel's savings be A.

2 Calculate percentage

Bahira has 45% more than Abel, so

$$B = 1.45A$$

3 Their combined savings are

Their combined savings are

$$A + 1.45A = 15435$$

$$2.45A = 15435$$

$$A = 6300$$

4 Calculate percentage

So

$$B = 1.45 \times 6300 = 9135$$

5 **Bahira s savings are times**

Bahira's savings are $\frac{3}{2}$ times Chanda's savings, so

$$9135 = \frac{3}{2}C$$

$$C = 9135 \times \frac{2}{3} = 6090$$

✓ **FINAL ANSWER**

6090 dinars

4MA1/2H NOVEMBER 2024

Question 4

5 marks | Reverse Percentages

4 N is a number.
17% of N is 357

(a) Work out the value of N

$N =$ .....

(2)

In 2019, the population of a village was 650
In 2020, the population of the village was 806

(b) Work out the percentage increase in the population.

..... %

(3)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2024

1 **Calculate percentage**

For part (a),

$$17\% \text{ of } N = 357$$

$$0.17N = 357$$

$$N = \frac{357}{0.17} = 2100$$

2 **Calculate percentage**

For part (b), the increase in population is

$$806 - 650 = 156$$

3 **Percentage increase**

Percentage increase:

$$\frac{156}{650} \times 100 = 24$$

FINAL ANSWER

(a) $N = 2100$, (b) 24%

4MA1/1H JUNE 2025

Question 2

6 marks | Reverse Percentages

2 Anna makes cups.
Each cup costs 6 Swiss francs to make.

Anna puts the cups into boxes to sell.
Each box contains 4 cups.

Anna sells 80 boxes of cups for a total of 2160 Swiss francs.

(a) Work out the percentage profit Anna makes.
Show your working clearly.

.....%
(4)

The height of each cup is 9 cm, correct to the nearest cm

(b) Write down the lower bound of the height.

.....cm
(1)

The weight of each cup is 120 g, correct to the nearest 10 g

(c) Write down the upper bound of the weight.

.....g
(1)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2025

1 Number of cups

Number of cups:

$$80 \times 4 = 320$$

2 Cost to make the cups

Cost to make the cups:

$$320 \times 6 = 1920$$

3 **Profit**

Profit:

$$2160 - 1920 = 240$$

4 **Percentage profit**

Percentage profit:

$$\frac{240}{1920} \times 100 = 12.5$$

5 **Find lower bound**

The height 9 cm correct to the nearest cm has lower bound

$$8.5 \text{ cm}$$

6 **Find upper bound**

The weight 120 g correct to the nearest 10 g has upper bound

$$125 \text{ g}$$

✓ **FINAL ANSWER**

(a) 12.5%, (b) 8.5 cm, (c) 125 g

4WM2/01 MAY 2025

Question 3

3 marks | Reverse Percentages

3 Bernice bought a car for $8000
She sold the car for $5600

Work out Bernice's percentage loss when she sold the car.

.....%

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 May 2025

1 Convert standard form

$$\text{loss} = 8000 - 5600 = 2400$$

$$\text{percentage loss} = \frac{2400}{8000} \times 100 = 30\%$$

FINAL ANSWER

30%.

4MA1/2HR JUNE 2025

Question 4

5 marks | Reverse Percentages

4 Joshua is going to cover a floor with tiles for a customer.
The area of the floor is 45 m^2

Joshua buys one box of tiles for each 1.5 m^2 of floor area.
Each box of tiles costs £64

Joshua also buys 5 bags of tile adhesive.
Each bag of tile adhesive costs £12

Joshua charges the customer £3000

Work out his percentage profit.
Give your answer correct to one decimal place.

..... %

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2025

1 Number of boxes of tiles

Number of boxes of tiles:

$$\frac{45}{1.5} = 30$$

2 Cost of tiles

Cost of tiles:

$$30 \times 64 = 1920$$

3 Cost of adhesive

Cost of adhesive:

$$5 \times 12 = 60$$

4 Total cost

Total cost:

$$1920 + 60 = 1980$$

5 **Profit**

Profit:

$$3000 - 1980 = 1020$$

6 **Percentage profit**

Percentage profit:

$$\frac{1020}{1980} \times 100 = 51.515\ldots\% $$

✓ **FINAL ANSWER**

51.5%

4MA1/1H JUNE 2025

Question 5

3 marks | Reverse Percentages

5 In a sale, normal prices are reduced by 28%
The sale price of a watch is 198 euros.

Work out the normal price of the watch.

..... euros

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2025

1 The watch is reduced by

The watch is reduced by 28%, so the sale price is

$$100\% - 28\% = 72\%$$

2 of the normal price

of the normal price.

$$0.72 \times \text{normal price} = 198$$

$$\text{normal price} = 198 \div 0.72 = 275$$

FINAL ANSWER

275 euros.

4WM2/01R MAY 2025

Question 5

5 marks | Reverse Percentages

5 Joshua is going to cover a floor with tiles for a customer.
The area of the floor is 45 m^2

Joshua buys one box of tiles for each 1.5 m^2 of floor area.
Each box of tiles costs £64

Joshua also buys 5 bags of tile adhesive.
Each bag of tile adhesive costs £12

Joshua charges the customer £3000

Work out his percentage profit.
Give your answer correct to one decimal place.

.....%

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4WM2/01R May 2025

1 Required

Calculate Joshua's percentage profit on the flooring job. Give answer to 1 d.p.

2 Number of boxes of tiles

Number of boxes of tiles needed:

$$frac45, textm^2 1.5, textm^2 / textbox = 30 textboxes$$

3 Cost of tiles 30 64

Cost of tiles:

30 × £64 = £1920

4 Cost of adhesive 5 12

Cost of adhesive:

5 × £12 = £60

5 Total cost (cost price) 1920

Total cost (cost price):

£1920 + £60 = £1980

6 **Profit and percentage profit Profit**

Profit and percentage profit:

Profit = £3000 − £1980 = £1020

7 **Note**

Always divide profit by the **cost price** (£1980), not the selling price (£3000). Dividing by £3000 gives 34.0% — a common but wrong answer.

✓ **FINAL ANSWER**

Percentage profit = 51.5%

4WM2/01R MAY 2025

Question 8

3 marks | Reverse Percentages

8 In a sale, normal prices are reduced by 15%

The sale price of a dishwasher is 612 Swiss francs.

Work out the normal price of the dishwasher.

..... Swiss francs

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM2/01R May 2025

1 Required

Sale price (after 15% reduction) = 612 Swiss francs. Find the normal price.

2 A 15% reduction means the

A 15% reduction means the sale price is 85% of the normal price:

612 = 0.85 × Normal price

3 Solve for the normal price

Solve for the normal price:

$$textNormalprice = frac6120.85 = 720$$

4 Note

Never calculate 15% of 612 and add it on — that gives 703.80 (wrong!). Always divide by the decimal multiplier.

✓ **FINAL ANSWER**

Normal price = 720 Swiss francs

4MA1/2HR JUNE 2025

Question 9

3 marks | Reverse Percentages

9 In a sale, normal prices are reduced by 15%

The sale price of a dishwasher is 612 Swiss francs.

Work out the normal price of the dishwasher.

..... Swiss francs

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2025

1 **A reduction of means the**

A reduction of 15% means the sale price is 85% of the normal price.

$$0.85 \times \text{normal price} = 612$$

$$\text{normal price} = \frac{612}{0.85} = 720$$

FINAL ANSWER

720 Swiss francs

4WM2/01 NOVEMBER 2025

Question 8

4 marks | Reverse Percentages

8 Shop A and shop B have offers for buying the same type of chair.

Shop A	Shop B
25% off the normal price	$\frac{2}{9}$ off the normal price
Sale price 240 euros	Sale price 245 euros

The normal price of the chair in shop A is more than the normal price of the chair in shop B

How much more?
Show your working clearly.

..... euros

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4WM2/01 November 2025

1 Calculate normal price

Step 1: Calculate Normal Price for Shop A. A 25% discount means the sale price is 75% of the normal price.

0.75 × Normal_{*A*} = 240

$$\text{Normal}_A = \frac{240}{0.75} = 320\text{ euros}$$

2 Calculate normal price

Step 2: Calculate Normal Price for Shop B. A discount of $\frac{2}{9}$ means the sale price is $1 - \frac{2}{9} = \frac{7}{9}$ of the normal price.

$$\frac{7}{9} \times \text{Normal}_B = 245$$

$$\text{Normal}_B = 245 \times \frac{9}{7} = 35 \times 9 = 315\text{ euros}$$

3 Calculate value

Difference = 320 − 315 = 5 euros

FINAL ANSWER

5 euros

4MA1/2H JUNE 2026

Question 4

3 marks | Reverse Percentages

4 Sara sells water bottles.

In July, she sold 450 water bottles.
In August, she sold 612 water bottles.

Work out the percentage increase in the number of water bottles she sold from July to August.

.....%

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2026

1 Solution | Step 1

$$\text{Increase} = 612 - 450 = 162.$$

Find the change from the original number.

2 Solution | Step 2

$$\text{Percentage increase} = \frac{162}{450} \times 100 = 36\%.$$

Express the increase as a percentage of the July value, not the August value.

FINAL ANSWER

36%

4MA1/2H JUNE 2026

Question 8

3 marks | Reverse Percentages

8 A golf club increases the cost of membership by 17%
After the increase, the cost of membership is 3276 zloty.

Work out the cost of membership before the increase.

..... zloty

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2026

1 Solution | Step 1

Let the original membership cost be C zloty. After a 17% increase, $1.17C = 3276$.

The new value is 117% of the original.

2 Solution | Step 2

$$C = \frac{3276}{1.17} = 2800 \text{ zloty.}$$

Divide by the increase multiplier.

FINAL ANSWER

2800 zloty

4MA1/2HR JUNE 2026

Question 8

3 marks | Reverse Percentages

8 Lauren's salary increased by 4%
After the increase, her salary was 36 400 euros.

Work out Lauren's salary before the increase.

..... euros

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2026

1 Solution | Step 1

Let the old salary be s euros. Then

$$1.04s = 36400.$$

The increased salary is 104% of the original.

2 Solution | Step 2

$$s = \frac{36400}{1.04} = 35000.$$

Divide by the percentage multiplier.

FINAL ANSWER

35000 euros.

TOPIC

Rounding, Estimation & Bounds

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 1

1 marks | Rounding, Estimation & Bounds

1 The weight of a cake is 2.75 kg, correct to 2 decimal places.

Write down the lower bound of the weight of the cake.

..... kg

(Total for Question 1 is 1 mark)

WORKED SOLUTION

Question 1 · 4WM1/01 Specimen

1 Find half the rounding interval

Correct to 2 decimal places means the nearest 0.01 kg.

$$\frac{0.01}{2} = 0.005$$

2 Subtract from the rounded value

$$2.75 - 0.005 = 2.745$$

FINAL ANSWER

2.745 kg.

Question 15

4 marks | Rounding, Estimation & Bounds

15 Martin and Lucia went on two different car journeys.

For Martin's journey

distance = 80 km correct to the nearest 5 km
time = 2.7 hours correct to 1 decimal place

For Lucia's journey

distance = 33 km correct to 2 significant figures
time = 1 hour correct to the nearest 0.1 hour

Martin says,

"My average speed could have been greater than Lucia's average speed."

By considering bounds, show that Martin is correct.
Show your working clearly.

(Total for Question 15 is 4 marks)

1 Use Martin's greatest possible speed

Martin's greatest possible distance is 82.5 km and his least possible time is 2.65 hours.

$$\frac{82.5}{2.65} = 31.132\ldots$$

2 Use Lucia's least possible speed

Lucia's least possible distance is 32.5 km and her greatest possible time is 1.05 hours.

$$\frac{32.5}{1.05} = 30.952\ldots$$

3 Compare the possible speeds

$$31.132\ldots > 30.952\ldots$$

So Martin's speed could have been greater than Lucia's speed.

FINAL ANSWER

Martin could have been faster because $31.132\ldots > 30.952\ldots$ km/h.

4MA1/1HR JANUARY 2020

Question 11

4 marks | Rounding, Estimation & Bounds

11 The weight of a cat is 4.3 kg correct to 2 significant figures.

(a) Write down the upper bound of the weight of the cat.

..... kg

(1)

(b) Write down the lower bound of the weight of the cat.

..... kg

(1)

$G = e - f$

e = 17 correct to the nearest integer
f = 9.4 correct to one decimal place

(c) Work out the upper bound for the value of G.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2020

1 **Estimate the value**

The weight 4.3 kg correct to 2 significant figures has bounds

$$4.25 \leq w < 4.35$$

2 **Find upper bound**

So the upper bound is 4.35 kg and the lower bound is 4.25 kg.

3 **Estimate the value**

For

$$G = e - f$$

4 **the upper bound uses the**

the upper bound uses the upper bound of e and the lower bound of f.

$$e < 17.5, \quad f \geq 9.35$$

$$G_{\text{upper}} = 17.5 - 9.35 = 8.15$$

✓ **FINAL ANSWER**

(a) 4.35 kg, (b) 4.25 kg, (c) 8.15

4MA1/1H JANUARY 2020

Question 16

3 marks | Rounding, Estimation & Bounds

16 $P = \dfrac{2a - c}{d}$

$a = 58.4$ correct to 3 significant figures.
$c = 20$ correct to 2 significant figures.
$d = 3.6$ correct to 2 significant figures.

Work out the upper bound for the value of P.
Show your working clearly.
Give your answer correct to 2 decimal places.

.....

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2020

1 **Estimate the value**

$$P = \frac{2a - c}{d}$$

2 **Find upper bound**

For the upper bound of P, make the numerator as large as possible and the denominator as small as possible.

$$a < 58.45, \quad c \geq 19.5, \quad d \geq 3.55$$

$$P_{\text{upper}} = \frac{2(58.45) - 19.5}{3.55} = 27.4366\ldots$$

✓ **FINAL ANSWER**

27.44

4MA1/1H MAY/NOVEMBER 2020

Question 17

4 marks | Rounding, Estimation & Bounds

17 A metal block has a mass of 5 kg, correct to the nearest 50 grams.
The block has a volume of $(1.84 \times 10^{-3})\,m^3$, correct to 3 significant figures.

Work out the upper bound for the density of the block.
Give your answer in kg/m^3 correct to 1 decimal place.
Show your working clearly.

..... kg/m^3

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May/November 2020

1 Density is

Density is

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2 To get the upper bound

To get the upper bound for density, use the upper bound for mass and the lower bound for volume.

3 The mass is kg correct

The mass is 5 kg correct to the nearest 50 g.

$$50\text{ g} = 0.05\text{ kg}$$

4 Find upper bound

Half of this is 0.025 kg, so

$$\text{upper bound for mass} = 5 + 0.025 = 5.025$$

5 Find upper bound

The volume is $1.84 \times 10^{-3} = 0.00184\ m^3$, correct to 3 significant figures.

6 The lower bound is

The lower bound is

$$0.00184 - 0.000005 = 0.001835$$

7 **Find upper bound**

So the upper bound for the density is

$$\frac{5.025}{0.001835} = 2738.419\ldots$$

8 **Find upper bound**

Correct to 1 decimal place,

$$2738.4$$

✓ **FINAL ANSWER**

$2738.4\ \mathrm{kg/m^3}$

4MA1/1HR MAY/NOVEMBER 2020

Question 19

3 marks | Rounding, Estimation & Bounds

19 The diagram shows rectangle $ABCD$ with rectangle $EFGH$ cut out to form the shaded region.

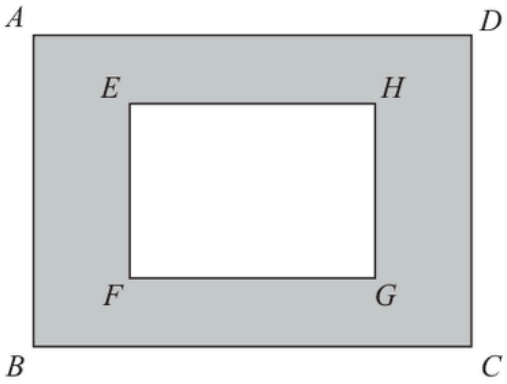


Diagram **NOT** accurately drawn

$AD = 8.3$ cm correct to one decimal place
$DC = 7.2$ cm correct to one decimal place
$EH = 6.2$ cm correct to one decimal place
$HG = 5.3$ cm correct to one decimal place

Work out the upper bound of the area of the shaded region.
Show your working clearly.

..... cm²

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May/November 2020

1 **Find upper bound**

For the upper bound of the shaded area, use the upper bound of the outer rectangle and subtract the lower bound of the cut-out rectangle.

2 **Outer rectangle**

Outer rectangle:

$$AD < 8.35, \quad DC < 7.25$$

3 Cut out rectangle

Cut-out rectangle:

$$EH \geq 6.15, \quad HG \geq 5.25$$

$$\text{upper shaded area} = 8.35(7.25) - 6.15(5.25)$$

$$= 60.5375 - 32.2875 = 28.25$$

FINAL ANSWER

$28.25\ \text{cm}^2$

4MA1/1H JANUARY 2021

Question 15

3 marks | Rounding, Estimation & Bounds

15

$$a = \frac{v - u}{t}$$

$v = 9.6$ correct to 1 decimal place
$u = 3.8$ correct to 1 decimal place
$t = 1.84$ correct to 2 decimal places

Calculate the upper bound for the value of a.
Give your answer as a decimal correct to 2 decimal places.
Show your working clearly.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2021

1 Estimate the value

$$a = \frac{v - u}{t}$$

2 Find upper bound

For the upper bound, use the upper bound of v, the lower bound of u, and the lower bound of t.

$$v < 9.65, \quad u \geq 3.75, \quad t \geq 1.835$$

$$a_{\text{upper}} = \frac{9.65 - 3.75}{1.835} = 3.2152\ldots$$

FINAL ANSWER

3.22

4MA1/1HR JANUARY 2021

Question 19

3 marks | Rounding, Estimation & Bounds

19 $k = \dfrac{t}{a-h}$

$t = 14$ correct to 2 significant figures
$a = 7.8$ correct to 2 significant figures
$h = 3.4$ correct to 2 significant figures

Work out the lower bound for the value of k.
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2021

1 Estimate the value

$$k = \frac{t}{a-h}$$

2 Find lower bound

For the lower bound of k, use the lower bound of t and make the denominator as large as possible.

$$t \geq 13.5$$

$$a < 7.85, \quad h \geq 3.35$$

$$(a-h)_{\text{upper}} = 7.85 - 3.35 = 4.5$$

$$k_{\text{lower}} = \frac{13.5}{4.5} = 3$$

FINAL ANSWER

3

4MA1/1H MAY 2021

Question 20

3 marks | Rounding, Estimation & Bounds

20 $P = \dfrac{t - w}{y}$

t = 9.7 correct to 1 decimal place

w = 5.9 correct to 1 decimal place

y = 3 correct to 1 significant figure

Calculate the upper bound for the value of P.
Show your working clearly.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2021

1 Estimate the value

$$P = \frac{t - w}{y}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$t < 9.75, \quad w \geq 5.85, \quad y \geq 2.5$$

$$P_{\text{upper}} = \frac{9.75 - 5.85}{2.5} = 1.56$$

FINAL ANSWER

1.56

4MA1/1H NOVEMBER 2021

Question 13

3 marks | Rounding, Estimation & Bounds

13 Here is a triangle XYZ.

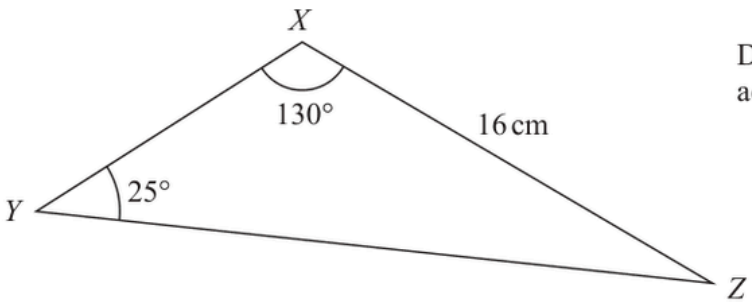


Diagram **NOT** accurately drawn

The length XZ and the angles YXZ and XYZ are each given correct to 2 significant figures.

Calculate the upper bound for the length YZ.
Give your answer correct to one decimal place.
Show your working clearly.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2021

1 Use trigonometry

Using the sine rule,

$$\frac{YZ}{\sin X} = \frac{XZ}{\sin Y}$$

2 Find upper bound

For the upper bound of YZ:

$$XZ < 16.5, \quad X < 25.5°, \quad Y < 135°$$

$$YZ_{\text{upper}} = \frac{16.5 \sin 25.5°}{\sin 135°} = 10.0457\ldots$$

3 Use trigonometry

Correct to one decimal place,

$$10.0457\ldots \approx 10.0$$

FINAL ANSWER

10.0 cm

4MA1/2H JANUARY 2022

Question 4

2 marks | Rounding, Estimation & Bounds

4 Each side of a regular octagon has a length of 18 mm, correct to the nearest 0.5 mm

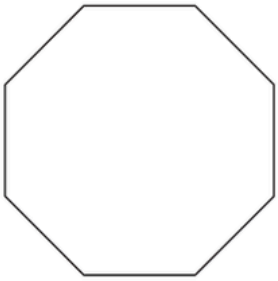


Diagram **NOT** accurately drawn

(a) Write down the lower bound of the length of each side of the octagon.

..... mm

(1)

(b) Write down the upper bound of the length of each side of the octagon.

..... mm

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2022

1 The side length is mm

The side length is 18 mm correct to the nearest 0.5 mm.

2 Half of mm is mm

Half of 0.5 mm is 0.25 mm.

$$18 - 0.25 = 17.75$$

$$18 + 0.25 = 18.25$$

FINAL ANSWER

(a) 17.75 mm, (b) 18.25 mm

4MA1/1H JANUARY 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $A = w - \dfrac{x^2}{y}$

$w = 3.45$ correct to 2 decimal places.
$x = 1.9$ correct to 1 decimal place.
$y = 5$ correct to the nearest whole number.

Work out the lower bound of the value of A
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2022

1 Estimate the value

$$A = w - \frac{x^2}{y}$$

2 Find lower bound

For the lower bound, use the lower bound of w, and make $\frac{x^2}{y}$ as large as possible.

$$w \geq 3.445, \quad x < 1.95, \quad y \geq 4.5$$

$$A_{\text{lower}} = 3.445 - \frac{1.95^2}{4.5}$$

$$A_{\text{lower}} = 2.6$$

FINAL ANSWER

2.6

4MA1/1HR JANUARY 2022

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows triangle ABC

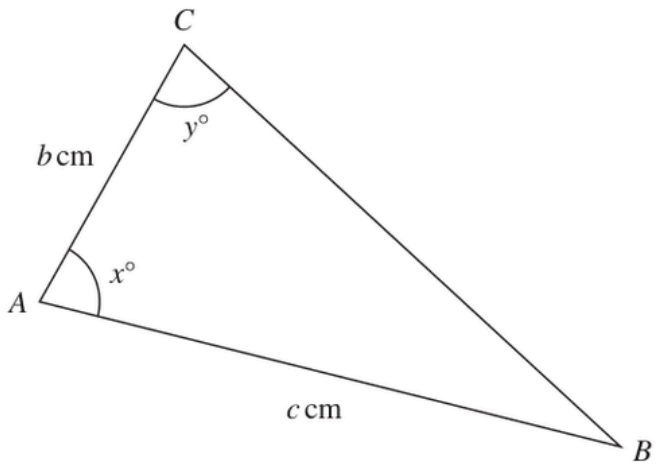


Diagram **NOT** accurately drawn

$c = 11.5$ correct to one decimal place
$x = 80$ correct to the nearest whole number
$y = 75$ correct to the nearest whole number

Calculate the upper bound for the value of b
Show your working clearly.
Give your answer correct to 3 significant figures.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2022

1 Find upper bound

For an upper bound for b, use the upper bound of c:

$c = 11.55$

2 Use sine rule

Use the lower bounds of x and y, because this makes angle B as large as possible and makes $\sin y$ as small as possible.

$x = 79.5°, \quad y = 74.5°$

3 Use trigonometry

So

$\angle B = 180° - 79.5° - 74.5° = 26°$

4 **Use trigonometry**

Using the sine rule,

$$\frac{b}{\sin 26^\circ} = \frac{11.55}{\sin 74.5^\circ}$$

$$b = \frac{11.55 \sin 26^\circ}{\sin 74.5^\circ} = 5.254...$$

✓ **FINAL ANSWER**

5.25 cm to 3 significant figures.

4MA1/2H JUNE 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $X = \dfrac{2a - b}{f}$

$a = 7.5$ correct to 1 decimal place.
$b = 3.42$ correct to 2 decimal places.
$f = 2$ correct to the nearest whole number.

Work out the upper bound of the value of X
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2022

1 Estimate the value

$$X = \frac{2a - b}{f}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$a < 7.55, \quad b \geq 3.415, \quad f \geq 1.5$$

$$X_{\text{upper}} = \frac{2(7.55) - 3.415}{1.5} = 7.79$$

FINAL ANSWER

7.79

4MA1/2HR JUNE 2022

Question 18

4 marks | Rounding, Estimation & Bounds

18 Kaidan and Sonja went on two different car journeys.

For Kaidan's journey
distance = 80 km correct to the nearest 5 km
time = 2.7 hours correct to 1 decimal place

For Sonja's journey
distance = 33 km correct to 2 significant figures
time = 1 hour correct to the nearest 0.1 hour

Kaidan says,
"My average speed could have been greater than Sonja's average speed."

By considering bounds, show that Kaidan is correct.
Show your working clearly.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2022

1 To show Kaidan s average

To show Kaidan's average speed could be greater, compare Kaidan's greatest possible speed with Sonja's smallest possible speed.

2 Estimate the value

For Kaidan:

$$d < 82.5, \quad t \geq 2.65$$

$$\text{greatest speed} = \frac{82.5}{2.65} = 31.13\ldots \text{ km/h}$$

3 Estimate the value

For Sonja:

$$d \geq 32.5, \quad t < 1.05$$

$$\text{smallest speed} = \frac{32.5}{1.05} = 30.95\ldots \text{ km/h}$$

4 **Estimate the value**

Since

$$31.13\ldots > 30.95\ldots$$

5 **Kaidan s average speed could**

Kaidan's average speed could have been greater.

✓ **FINAL ANSWER**

Kaidan is correct.

4MA1/1HR MAY 2023

Question 18

4 marks | Rounding, Estimation & Bounds

18 Diego builds a fence using fence panels.

The total length of the fence is 50 metres, correct to the nearest 5 metres.
The length of each fence panel is 1.3 metres, correct to the nearest 10 cm.

The cost of each fence panel is £8.65

Diego may only buy complete fence panels.
Diego only pays for the number of panels he needs to build the fence.

Work out the greatest difference in the possible amounts that Diego could pay to build the fence.
Show your working clearly.

£.....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May 2023

1 The fence length m correct

The fence length 50 m correct to the nearest 5 m has bounds

$$47.5 \leq L < 52.5$$

2 The panel length m correct

The panel length 1.3 m correct to the nearest 10 cm has bounds

$$1.25 \leq p < 1.35$$

3 Smallest possible number of panels

Smallest possible number of panels:

$$\left\lceil \frac{47.5}{1.35} \right\rceil = 36$$

4 Greatest possible number of panels

Greatest possible number of panels:

$$\left\lceil \frac{52.5}{1.25} \right\rceil = 42$$

5 Difference in number of panels

Difference in number of panels:

$$42 - 36 = 6$$

6 Greatest difference in cost

Greatest difference in cost:

$$6 \times 8.65 = 51.90$$

FINAL ANSWER

51.90 pounds

4MA1/2H JUNE 2023

Question 25

4 marks | Rounding, Estimation & Bounds

25 A solid sphere has a radius of 2.8 centimetres, correct to 1 decimal place.
The sphere has a mass of $M\pi$ grams, where $M = 260$ correct to 2 significant figures.

Work out the upper bound for the density of the sphere.
Give your answer in g/cm^3 correct to 2 decimal places.
Show your working clearly.

..... g/cm^3

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2023

1 Find upper bound

For the upper bound of density, use the upper bound of the mass and the lower bound of the volume.

2 The radius is cm correct

The radius is 2.8 cm correct to 1 decimal place, so the lower bound is
$r = 2.75$

3 Find upper bound

$M = 260$ correct to 2 significant figures, so the upper bound is
$M = 265$

4 The mass is and the

The mass is $M\pi$ and the volume of the sphere is $\frac{4}{3}\pi r^3$.

5 Rearrange formula

So
$$\text{density} = \frac{M\pi}{\frac{4}{3}\pi r^3} = \frac{3M}{4r^3}$$

6 Upper bound

Upper bound:
$$\frac{3(265)}{4(2.75)^3} = 9.5567\ldots$$

FINAL ANSWER

$9.56\ \mathrm{g/cm^3}$ to 2 decimal places.

4MA1/2H NOVEMBER 2023

Question 17

3 marks | Rounding, Estimation & Bounds

17 $P = a(c + y)$

$a =$ 8.3 correct to 2 significant figures
$c =$ 2 correct to 1 significant figure
$y =$ 15 correct to the nearest 5

Work out the upper bound for the value of P
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2023

1 Estimate the value

$$P = a(c + y)$$

2 Find upper bound

For the upper bound, use the upper bounds of all three values.

$$a < 8.35, \quad c < 2.5, \quad y < 17.5$$

$$P_{\text{upper}} = 8.35(2.5 + 17.5) = 167$$

FINAL ANSWER

167

4MA1/2H JUNE 2024

Question 19

3 marks | Rounding, Estimation & Bounds

19 $G = \dfrac{c}{2f - 3h}$

$c = 8$ correct to the nearest whole number
$f = 6.62$ correct to 2 decimal places
$h = 1.2$ correct to 1 decimal place

Work out the lower bound for the value of G
Give your answer correct to 3 decimal places.
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2024

1 Estimate the value

$$G = \frac{c}{2f - 3h}$$

2 Find lower bound

For the lower bound, use the lower bound of c and make the denominator as large as possible.

$$c \geq 7.5$$

$$f < 6.625, \quad h \geq 1.15$$

$$(2f - 3h)_{\text{upper}} = 2(6.625) - 3(1.15) = 9.8$$

$$G_{\text{lower}} = \frac{7.5}{9.8} = 0.765306\ldots$$

FINAL ANSWER

0.765

4MA1/2HR JUNE 2024

Question 19

3 marks | Rounding, Estimation & Bounds

19 Aviv goes on a cycle journey.

For the cycle journey

average speed = 19 km/h correct to the nearest whole number

time = 1.5 hours correct to one decimal place

Work out the upper bound for the distance Aviv travels.
Give your answer correct to 3 significant figures.

..... km

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2024

1 Distance is found using

Distance is found using

$$\text{distance} = \text{speed} \times \text{time}$$

2 Find upper bound

For the upper bound,

$$\text{speed} < 19.5, \quad \text{time} < 1.55$$

$$\text{distance}_{\text{upper}} = 19.5 \times 1.55 = 30.225$$

3 Estimate the value

Correct to 3 significant figures,

$$30.225 \approx 30.2$$

FINAL ANSWER

30.2 km

4MA1/2H NOVEMBER 2024

Question 2

2 marks | Rounding, Estimation & Bounds

2 The length of a table is measured as 1.4 metres correct to one decimal place.

(a) Write down the upper bound of the length of the table.

..... metres

(1)

(b) Write down the lower bound of the length of the table.

..... metres

(1)

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2024

1 The length is m correct

The length is 1.4 m correct to one decimal place.

2 Half of is

Half of 0.1 is 0.05.

$$1.4 + 0.05 = 1.45$$

$$1.4 - 0.05 = 1.35$$

FINAL ANSWER

(a) 1.45 m, (b) 1.35 m

4MA1/1H NOVEMBER 2024

Question 21

3 marks | Rounding, Estimation & Bounds

21 $T = \dfrac{x^2 + y^2}{w}$

$x = 28.4$ correct to 1 decimal place.

$y = 17$ correct to 2 significant figures.

$w = 90$ correct to the nearest 5

Calculate the upper bound for the value of T

Give your answer correct to 3 significant figures.
Show your working clearly.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2024

1 Estimate the value

$$T = \frac{x^2 + y^2}{w}$$

2 Find upper bound

For the upper bound, use the upper bounds of x and y, and the lower bound of w.

$$x < 28.45, \quad y < 17.5, \quad w \geq 87.5$$

$$T_{\text{upper}} = \frac{28.45^2 + 17.5^2}{87.5} = 12.7503\ldots$$

3 Estimate the value

Correct to 3 significant figures,

$$12.7503\ldots \approx 12.8$$

FINAL ANSWER

12.8

4WM1/01R MAY 2025

Question 1

2 marks | Rounding, Estimation & Bounds

1 The length of a ship is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the ship.

..... m

(1)

(ii) Write down the upper bound of the length of the ship.

..... m

(1)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4WM1/01R May 2025

1 **Calculate value**

The value is correct to 1 decimal place, so the half-unit is 0.05.

$$142.8 - 0.05 = 142.75, \qquad 142.8 + 0.05 = 142.85$$

✓ **FINAL ANSWER**

lower bound 142.75 m, upper bound 142.85 m.

4MA1/1HR JUNE 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 The length of a ship is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the ship.

..... m

(1)

(ii) Write down the upper bound of the length of the ship.

..... m

(1)

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2025

1 The length is m correct

The length is 142.8 m correct to 1 decimal place.

2 Half of is

Half of 0.1 is 0.05.

$$142.8 - 0.05 = 142.75$$

$$142.8 + 0.05 = 142.85$$

FINAL ANSWER

(i) 142.75 m, (ii) 142.85 m

4WM1/01 MAY 2025

Question 2

4 marks | Rounding, Estimation & Bounds

2 The height of a tree is 17.8 metres correct to one decimal place.

(a) Write down the lower bound for the height of the tree.

..... metres

(1)

The weight of an apple is 145 grams correct to the nearest 5 grams.

(b) Write down the upper bound for the weight of the apple.

..... grams

(1)

$$A = \frac{4.766 \times 815}{0.399}$$

(c) By rounding each number to one significant figure, work out an estimate for the value of A
Show your working clearly.

.....

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 May 2025

1 Find lower bound

For 17.8 correct to 1 decimal place, the lower bound is

$$17.8 - 0.05 = 17.75$$

2 Find upper bound

For 145 correct to the nearest 5 grams, the upper bound is

$$145 + 2.5 = 147.5$$

3 Estimate the value

For the estimate,

$$A = \frac{4.766 \times 815}{0.399}$$

4 **Round to 1 significant figure**

Round to 1 significant figure.

$$4.766 \approx 5, \quad 815 \approx 800, \quad 0.399 \approx 0.4$$

$$A \approx \frac{5 \times 800}{0.4} = 10000$$

✓ **FINAL ANSWER**

17.75 m, 147.5 g, and 10000.

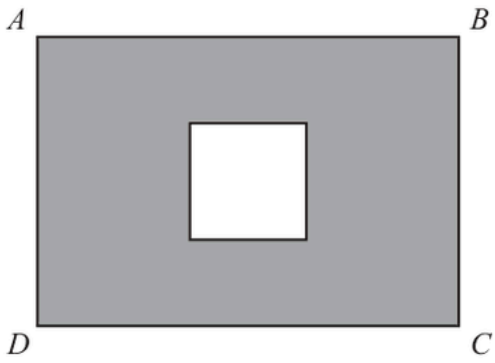
4MA1/2H JUNE 2025

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows a square inside rectangle $ABCD$

Diagram **NOT** accurately drawn



The total area of the region shown shaded in the diagram is $X\text{cm}^2$

$AB = 11.5$ cm correct to the nearest 0.5 cm
$BC = 9.2$ cm correct to 2 significant figures
side of square $= 4.1$ cm correct to 2 significant figures

By considering bounds, work out the value of X to a suitable degree of accuracy.
Show your working clearly.

$X =$ .....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2025

1 The shaded area is

The shaded area is

$$X = \text{area of rectangle} - \text{area of square}$$

2 Bounds

Bounds:

$$11.25 \leq AB < 11.75$$

$$9.15 \leq BC < 9.25$$

$$4.05 \leq \text{square side} < 4.15$$

3 **Lower bound**

Lower bound:

$$11.25(9.15) - 4.15^2 = 85.715$$

4 **Upper bound**

Upper bound:

$$11.75(9.25) - 4.05^2 = 92.285$$

5 **Estimate the value**

So

$$85.715 < X < 92.285$$

6 **Estimate the value**

Both bounds round to 90 to the nearest 10.

✓ **FINAL ANSWER**

$X = 90\ \text{cm}^2$ to a suitable degree of accuracy

4WM1/01 NOVEMBER 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 By rounding each number to one significant figure, work out an estimate for the value of

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}}$$

Show your working clearly.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 November 2025

1 Round each number to one

Round each number to one significant figure:

$$2.11 \approx 2, \quad 58.9 \approx 60, \quad 8.859 \approx 9$$

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}} \approx \frac{2^2 \times 60}{\sqrt{9}}$$

$$= \frac{4 \times 60}{3} = 80$$

FINAL ANSWER

80

4MA1/1H NOVEMBER 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 By rounding each number to one significant figure, work out an estimate for the value of

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}}$$

Show your working clearly.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2025

1 Round each number to one

Round each number to one significant figure:

$$2.11 \approx 2, \quad 58.9 \approx 60, \quad 8.859 \approx 9$$

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}} \approx \frac{2^2 \times 60}{\sqrt{9}}$$

$$= \frac{4 \times 60}{3} = 80$$

FINAL ANSWER

80

4WM1/01 NOVEMBER 2025

Question 6

2 marks | Rounding, Estimation & Bounds

6 The weight of a bag of apples is 475 g correct to the nearest g

(a) Write down the lower bound of the weight.

..... g

(1)

The height of a box is 120 cm correct to the nearest 10 cm

(b) Write down the upper bound for the height.

..... cm

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 November 2025

1 Find lower bound

The weight 475 g correct to the nearest g has lower bound

$$475 - 0.5 = 474.5$$

2 Find upper bound

The height 120 cm correct to the nearest 10 cm has upper bound

$$120 + 5 = 125$$

FINAL ANSWER

(a) 474.5 g, (b) 125 cm

4MA1/1H NOVEMBER 2025

Question 7

2 marks | Rounding, Estimation & Bounds

7 The weight of a bag of apples is 475 g correct to the nearest g

(a) Write down the lower bound of the weight.

..... g

(1)

The height of a box is 120 cm correct to the nearest 10 cm

(b) Write down the upper bound for the height.

..... cm

(1)

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2025

1 Find lower bound

The weight 475 g correct to the nearest g has lower bound

$$475 - 0.5 = 474.5$$

2 Find upper bound

The height 120 cm correct to the nearest 10 cm has upper bound

$$120 + 5 = 125$$

FINAL ANSWER

(a) 474.5 g, (b) 125 cm

4MA1/2H NOVEMBER 2025

Question 24

5 marks | Rounding, Estimation & Bounds

24 $D = \dfrac{n}{p - q}$

$n = 10.3$ correct to 1 decimal place
$p = 7.24$ correct to 2 decimal places
$q = 4.39$ correct to 2 decimal places

By considering bounds, work out the value of D to a suitable degree of accuracy.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2025

1 Estimate the value

$$D = \frac{n}{p - q}$$

2 Bounds

Bounds:

$$10.25 \leq n < 10.35$$

$$7.235 \leq p < 7.245$$

$$4.385 \leq q < 4.395$$

3 Lower bound

Lower bound:

$$D_{\text{lower}} = \frac{10.25}{7.245 - 4.385} = 3.5839\ldots$$

4 Upper bound

Upper bound:

$$D_{\text{upper}} = \frac{10.35}{7.235 - 4.395} = 3.6443\ldots$$

5 **Estimate the value**

So

$$3.5839\ldots < D < 3.6443\ldots$$

---

6 **Estimate the value**

Both bounds round to $\mathbf{3.6}$ to 1 decimal place.

✓ **FINAL ANSWER**

$D = \mathbf{3.6}$ to a suitable degree of accuracy

4MA1/1HR JUNE 2026

Question 4

2 marks | Rounding, Estimation & Bounds

4 The weight of a giraffe is 1178 kg, correct to the nearest kg

(i) Write down the lower bound of the weight of the giraffe.

..... kg

(1)

(ii) Write down the upper bound of the weight of the giraffe.

..... kg

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2026

1 Part (i) | Step 1

$$1178 - 0.5 = 1177.5 \text{ kg}.$$

Rounding to the nearest kilogram allows half a kilogram below the stated value.

2 Part (ii) | Step 1

$$1178 + 0.5 = 1178.5 \text{ kg}.$$

The rounding interval is $1177.5 \leq w < 1178.5$; the upper endpoint is the upper bound, not an included weight.

FINAL ANSWER

(i) 1177.5 kg (ii) 1178.5 kg

Question 18

5 marks | Rounding, Estimation & Bounds

18 The final velocity, v m/s, of an object is given by

$$v = 2\left(\frac{s}{t}\right) - u$$

where

$s = 100$ correct to the nearest integer
$t = 1.68$ correct to 2 decimal places
$u = 0.5$ correct to 1 decimal place

By considering bounds, work out the value of v to a suitable degree of accuracy.
Show your working clearly.

.....

(Total for Question 18 is 5 marks)

1 **Bounds**

$99.5 \leq s < 100.5$, $1.675 \leq t < 1.685$, $0.45 \leq u < 0.55$

2 **Upper bound of v**

Big s, small t, small u:

$$v_{\max} = 2\left(\frac{100.5}{1.675}\right) - 0.45 = 119.55$$

3 **Lower bound of v**

Small s, big t, big u:

$$v_{\min} = 2\left(\frac{99.5}{1.685}\right) - 0.55 = 117.55\ldots$$

4 **Suitable accuracy**

Both bounds round to 120 to 2 s.f. (they differ to 3 s.f.), so $v = 120$.

✓ **FINAL ANSWER**

$v = 120$ m/s (2 s.f.)

4MA1/1H JUNE 2026

Question 20

5 marks | Rounding, Estimation & Bounds

20 The final velocity, v m/s, of an object is given by

$$v = 2\left(\frac{s}{t}\right) - u$$

where

$s = 100$ correct to the nearest integer
$t = 1.68$ correct to 2 decimal places
$u = 0.5$ correct to 1 decimal place

By considering bounds, work out the value of v to a suitable degree of accuracy.
Show your working clearly.

.....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H June 2026

1 Solution | Step 1

$$99.5 \leq s < 100.5, \quad 1.675 \leq t < 1.685, \quad 0.45 \leq u < 0.55.$$

Use half a rounding unit on each side of the stated value.

2 Solution | Step 2

For $v = 2s/t - u$, the lower limiting value is

$$v_L = \frac{2(99.5)}{1.685} - 0.55 = 117.550890\ldots \text{ m/s}.$$

Decrease the numerator, increase the positive denominator and subtract the largest u.

3 Solution | Step 3

The upper limiting value is

$$v_U = \frac{2(100.5)}{1.675} - 0.45 = 119.55 \text{ m/s}.$$

Increase the numerator, decrease the denominator and subtract the smallest u.

4

Solution | Step 4

Both limiting values round to $120\ \mathrm{m/s}$ to 2 significant figures. They do not round to the same integer, so

$$v = 120\ \mathrm{m/s} \quad (2\ \mathrm{s.f.}).$$

This is the greatest common significant-figure accuracy justified throughout the interval.

✓ **FINAL ANSWER**

$120\ \mathrm{m/s}$ to 2 significant figures.

4MA1/2HR JUNE 2026

Question 21

3 marks | Rounding, Estimation & Bounds

21 $p = \dfrac{d-e}{f}$

$d = 6.6$ correct to 1 decimal place
$e = 2.8$ correct to 1 decimal place
$f = 0.47$ correct to 2 decimal places

Calculate the lower bound for the value of p
Give your answer correct to 3 significant figures.
Show your working clearly.

.....

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2026

1 Solution | Step 1

$$6.55 \le d < 6.65, \quad 2.75 \le e < 2.85, \quad 0.465 \le f < 0.475.$$

Use half of each rounding unit to obtain the bounds.

2 Solution | Step 2

$$p_{\text{lower}} = \frac{6.55 - 2.85}{0.475} = \frac{3.70}{0.475} = 7.789473684\ldots.$$

Minimize the positive numerator by using the lower bound of d and upper bound of e; maximize the positive denominator.

3 Solution | Step 3

$$p_{\text{lower}} = 7.79 \quad (\text{3 significant figures}).$$

Round the boundary value to the requested accuracy.

FINAL ANSWER

Lower bound $= \mathbf{7.79}$ (3 significant figures).

TOPIC

Powers & Roots

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 33

3 marks | Powers & Roots

33 Find the value of

(a) 5^0

.....

(1)

(b) $8^{2/3}$

.....

(1)

(c) $16^{-1/2}$

.....

(1)

(Total for Question 33 is 3 marks)

WORKED SOLUTION

Question 33 · EP Specimen

1 **(a) Any non-zero number to the power 0**

$5^0 = 1$

2 **(b) Root first, then power**

$$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

3 **(c) Negative power means reciprocal**

$$16^{-1/2} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

✓ **FINAL ANSWER**

(a) 1 (b) 4 (c) $\frac{1}{4}$

EP SPECIMEN

Question 34

2 marks | Powers & Roots

34 Write $\frac{3^5 \times 3^{-2}}{3^4}$ as a single power of 3

.....

(2)

(Total for Question 34 is 2 marks)

WORKED SOLUTION

Question 34 · EP Specimen

1 **Multiply: add the powers**

$3^5 \times 3^{-2} = 3^3$

2 **Divide: subtract the powers**

$$\frac{3^3}{3^4} = 3^{-1}$$

✓ **FINAL ANSWER**

3^{-1}

EP SPECIMEN

Question 35

3 marks | Powers & Roots

35 $2^n = 8^4 \div 4^3$

Find the value of n.

$n =$ .....

(3)

(Total for Question 35 is 3 marks)

WORKED SOLUTION

Question 35 · EP Specimen

1 Write everything as a power of 2

$8 = 2^3$, so $8^4 = 2^{12}$. $4 = 2^2$, so $4^3 = 2^6$.

2 Divide

$$\frac{2^{12}}{2^6} = 2^6 \Rightarrow n = 6$$

FINAL ANSWER

$n = 6$

Question 3

5 marks | Powers & Roots

3 $20^{70} \times 20^{8} = 20^{n}$

(a) Find the value of n

$n =$ .....

(1)

$36^{80} \div 36^{5} = 36^{m}$

(b) Find the value of m

$m =$ .....

(1)

$\left(52^{4}\right)^{16} = 52^{p}$

(c) Find the value of p

$p =$ .....

(1)

In a race, Dani ran a distance of 100 metres, correct to the nearest metre.

(d) Write down the lower bound for the distance.

..... metres

(1)

Dani ran this race in a time of 16.3 seconds, correct to one decimal place.

(e) Write down the upper bound for the time.

..... seconds

(1)

(Total for Question 3 is 5 marks)

1 **(a) Multiply: add powers**

$$20^{70} \times 20^{8} = 20^{78} \Rightarrow n = 78$$

2 **(b) Divide: subtract powers**

$$36^{80} \div 36^{5} = 36^{75} \Rightarrow m = 75$$

3 **(c) Power of a power: multiply**

$$(52^{4})^{16} = 52^{64} \Rightarrow p = 64$$

4 **(d) Lower bound**

Nearest metre means ± 0.5 m: lower bound $= 99.5$ m.

5 **(e) Upper bound**

One decimal place means ± 0.05 s: upper bound $= 16.35$ s.

✓ **FINAL ANSWER**

(a) 78 (b) 75 (c) 64 (d) 99.5 m (e) 16.35 s

TOPIC

Standard Form

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 9

2 marks | Standard Form

9 (a) Write 0.000 089 in standard form.

.....

(1)

(b) Write 8.34×10^4 as an ordinary number.

.....

(1)

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 Specimen

1 Convert standard form

$$0.000089 = 8.9 \times 10^{-5}$$

2 Convert standard form

Also

$$8.34 \times 10^4 = 83400$$

FINAL ANSWER

8.9×10^{-5}, 83400.

4MA1/1H JANUARY 2020

Question 6

3 marks | Standard Form

6 (a) Write 7.8×10^{-4} as an ordinary number.

.....

(1)

(b) Work out $\dfrac{5.6 \times 10^4 + 7 \times 10^3}{2.8 \times 10^{-3}}$

Give your answer in standard form.

.....

(2)

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2020

1 Convert standard form

$$7.8 \times 10^{-4} = 0.00078$$

2 Convert standard form

For part (b),

$$5.6 \times 10^4 + 7 \times 10^3 = 56000 + 7000 = 63000$$

3 Convert standard form

Then

$$\frac{63000}{2.8 \times 10^{-3}} = \frac{63000}{0.0028}$$

$$= 22500000 = 2.25 \times 10^7$$

FINAL ANSWER

(a) 0.00078, (b) 2.25×10^7

4MA1/2HR JANUARY 2020

Question 10

3 marks | Standard Form

10 The table shows information about the surface area of each of the world's oceans.

Ocean	Surface area in square kilometres
Pacific	1.56×10^8
Indian	6.86×10^7
Southern	2.03×10^7
Arctic	1.41×10^7
Atlantic	1.06×10^8

(a) Work out the difference, in square kilometres, between the surface area of the Atlantic Ocean and the surface area of the Indian Ocean.
Give your answer in standard form.

..... square kilometres
(2)

The surface area of the Pacific Ocean is k times the surface area of the Arctic Ocean.

(b) Work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....
(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2020

1 Atlantic surface area

Atlantic surface area:

$$1.06 \times 10^8 = 106000000$$

2 Indian surface area

Indian surface area:

$$6.86 \times 10^7 = 68600000$$

3 **Difference**

Difference:

$$106000000 - 68600000 = 37400000$$

$$37400000 = 3.74 \times 10^{7}$$

---

4 **Convert standard form**

For part (b),

$$k = \frac{1.56 \times 10^{8}}{1.41 \times 10^{7}}$$

$$k = 11.063\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 3.74×10^{7}, (b) 11

4MA1/1H MAY/NOVEMBER 2020

Question 5

4 marks | Standard Form

5 (a) Write 5.7×10^{-3} as an ordinary number.

.....

(1)

(b) Write 800 000 in standard form.

.....

(1)

(c) Work out $\dfrac{3 \times 10^5 - 2.7 \times 10^4}{6 \times 10^{-2}}$

.....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May/November 2020

1 **Convert standard form**

$$5.7 \times 10^{-3} = 0.0057$$

$$800000 = 8 \times 10^5$$

2 **Convert standard form**

For part (c), first simplify the numerator:

$$3 \times 10^5 - 2.7 \times 10^4 = 300000 - 27000 = 273000$$

3 **Convert standard form**

Then

$$\frac{273000}{6 \times 10^{-2}} = \frac{273000}{0.06} = 4550000$$

$$4550000 = 4.55 \times 10^6$$

FINAL ANSWER

(a) 0.0057, (b) 8×10^5, (c) 4.55×10^6

4MA1/1H JANUARY 2021

Question 6

2 marks | Standard Form

6 (a) Write 2 840 000 000 in standard form.

.....

(1)

(b) Write 2.5×10^{-4} as an ordinary number.

.....

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2021

1 Convert standard form

$$2840000000 = 2.84 \times 10^{9}$$

2 Convert standard form

Also,

$$2.5 \times 10^{-4} = 0.00025$$

FINAL ANSWER

(a) 2.84×10^{9}, (b) 0.00025

4MA1/2H JANUARY 2021

Question 6

4 marks | Standard Form

6 (a) Write down the value of y^0

.....

(1)

(b) Work out $\dfrac{9.6 \times 10^{141} + 6.4 \times 10^{140}}{3.2 \times 10^{16}}$

Give your answer in standard form.

.....

(3)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2021

1 Convert standard form

Any non-zero value to the power 0 is 1, so

$$y^0 = 1$$

2 Convert standard form

For part (b),

$$9.6 \times 10^{141} + 6.4 \times 10^{140} = 9.6 \times 10^{141} + 0.64 \times 10^{141}$$

$$= 10.24 \times 10^{141} = 1.024 \times 10^{142}$$

3 Convert standard form

Now divide:

$$\frac{1.024 \times 10^{142}}{3.2 \times 10^{16}} = 0.32 \times 10^{126}$$

$$= 3.2 \times 10^{125}$$

✓ **FINAL ANSWER**

(a) 1, (b) 3.2×10^{125}

4MA1/2HR JANUARY 2021

Question 8

3 marks | Standard Form

8 The table gives the length of the coastline, in kilometres, of each of five oceans.

Ocean	Length of coastline (km)
Arctic	4.539×10^4
Atlantic	1.119×10^5
Pacific	1.357×10^5
Indian	6.653×10^4
Southern	1.797×10^4

(a) Which ocean has the greatest length of coastline?

.....

(1)

(b) Calculate the difference between the length of the Atlantic Ocean's coastline and the length of the Southern Ocean's coastline.
Give your answer in standard form.

..... km

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2021

1 **The greatest coastline is the**

The greatest coastline is the largest number in standard form.

$$1.357 \times 10^5$$

2 **Convert standard form**

is the largest, so the answer is the Pacific Ocean.

3 Convert standard form

For part (b),

$$1.119 \times 10^5 = 111900$$

$$1.797 \times 10^4 = 17970$$

4 Difference

Difference:

$$111900 - 17970 = 93930$$

$$93930 = 9.393 \times 10^4$$

FINAL ANSWER

(a) Pacific Ocean, (b) 9.393×10^4 km

4MA1/1H MAY 2021

Question 8

4 marks | Standard Form

8 The table shows the populations of five countries.

Country	Population
China	1.4×10^9
Germany	8.2×10^7
Sweden	9.9×10^6
Fiji	9.1×10^5
Malta	4.3×10^5

(a) Work out the difference between the population of China and the population of Germany.
Give your answer in standard form.

.....

(2)

Given that

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

(b) work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2021

1 **Convert standard form**

For part (a),

$$1.4 \times 10^9 = 1400000000$$

$$8.2 \times 10^7 = 82000000$$

2 **Difference**

Difference:

$$1400000000 - 82000000 = 1318000000$$

$$1318000000 = 1.318 \times 10^{9}$$

---

3 **Convert standard form**

For part (b),

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

---

4 **Convert standard form**

So

$$k = \frac{\text{population of Sweden}}{\text{population of Fiji}}$$

$$k = \frac{9.9 \times 10^{6}}{9.1 \times 10^{5}} = 10.879\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 1.318×10^{9}, (b) 11

4MA1/2H NOVEMBER 2021

Question 9

5 marks | Standard Form

9 A rainwater tank contains 2.4×10^7 raindrops.
The rainwater tank also contains 1.75×10^6 bacteria.

(a) Work out the number of bacteria per raindrop in the tank.
Give your answer in standard form correct to 2 significant figures.

.....

(3)

A drop of rainwater contains 5.01×10^{21} atoms.

In a drop of rainwater the number of atoms is 3 times the number of molecules.

(b) Work out the number of molecules in the rainwater tank.
Give your answer in standard form correct to one significant figure.

..... molecules

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2021

1 Convert standard form

For part (a), divide the number of bacteria by the number of raindrops:

$$\frac{1.75 \times 10^6}{2.4 \times 10^7} = \frac{1.75}{2.4} \times 10^{-1}$$

$$= 0.072916\ldots$$

2 Correct to 2 significant figures

Correct to 2 significant figures:

$$7.3 \times 10^{-2}$$

3 Convert standard form

For part (b), one drop contains 5.01×10^{21} atoms.

4 **Convert standard form**

The number of molecules in one drop is

$$\frac{5.01 \times 10^{21}}{3} = 1.67 \times 10^{21}$$

5 **Convert standard form**

There are 2.4×10^{7} drops, so the number of molecules in the tank is

$$(1.67 \times 10^{21})(2.4 \times 10^{7}) = 4.008 \times 10^{28}$$

6 **Correct to one significant figure**

Correct to one significant figure:

$$4 \times 10^{28}$$

✓ **FINAL ANSWER**

(a) 7.3×10^{-2}, (b) 4×10^{28} molecules

4MA1/1H JANUARY 2022

Question 8

2 marks | Standard Form

8 $a = 4.2 \times 10^{-24}$ $b = 3 \times 10^{145}$

Work out the value of $a \times b$
Give your answer in standard form.

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2022

1 Convert standard form

$$a = 4.2 \times 10^{-24}$$

$$b = 3 \times 10^{145}$$

2 Convert standard form

So

$$ab = (4.2 \times 3) \times 10^{-24+145}$$

$$ab = 12.6 \times 10^{121}$$

3 Write this in standard form

Write this in standard form:

$$12.6 \times 10^{121} = 1.26 \times 10^{122}$$

FINAL ANSWER

1.26×10^{122}

4MA1/1HR JANUARY 2022

Question 8

4 marks | Standard Form

8 (a) Write 5×10^4 as an ordinary number.

.....

(1)

(b) Write 0.000 06 in standard form.

.....

(1)

(c) Work out $(4 \times 10^{512}) \div (1.6 \times 10^{700})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2022

1 Convert standard form

$$5 \times 10^4 = 50000$$

$$0.00006 = 6 \times 10^{-5}$$

2 Convert standard form

For part (c),

$$\frac{4 \times 10^{512}}{1.6 \times 10^{700}} = \frac{4}{1.6} \times 10^{512-700}$$

$$= 2.5 \times 10^{-188}$$

FINAL ANSWER

(a) 50000, (b) 6×10^{-5}, (c) 2.5×10^{-188}

4MA1/2H JANUARY 2022

Question 12

3 marks | Standard Form

12 $a = 6 \times 10^{40}$

Work out the value of a^3
Give your answer in standard form.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2022

1 Convert standard form

$$a = 6 \times 10^{40}$$

$$a^3 = (6 \times 10^{40})^3$$

$$a^3 = 6^3 \times 10^{120}$$

$$a^3 = 216 \times 10^{120}$$

2 Write this in standard form

Write this in standard form:

$$216 \times 10^{120} = 2.16 \times 10^{122}$$

FINAL ANSWER

2.16×10^{122}

4MA1/1H JUNE 2022

Question 8

2 marks | Standard Form

8 (a) Write $0.000\,089$ in standard form.

.....

(1)

(b) Write 8.34×10^4 as an ordinary number.

.....

(1)

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2022

1 Convert standard form

$$0.000089 = 8.9 \times 10^{-5}$$

$$8.34 \times 10^4 = 83400$$

FINAL ANSWER

(a) 8.9×10^{-5}, (b) 83400

4MA1/2HR JUNE 2022

Question 9

4 marks | Standard Form

9 The table gives information about the population, correct to 2 significant figures, of each of five cities in 2018

City	Population (2018)
Ahmedabad	7.7×10^6
Barcelona	5.5×10^6
Chicago	8.8×10^6
Lagos	1.3×10^7
Tokyo	3.7×10^7

(a) Write 8.8×10^6 as an ordinary number.

.....

(1)

(b) Which of these cities had the least population in 2018?

.....

(1)

(c) Work out the difference between the population of Tokyo and the population of Ahmedabad in 2018
Give your answer in standard form correct to 2 significant figures.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2022

1 Convert standard form

$$8.8 \times 10^6 = 8800000$$

2 The least population in the

The least population in the table is Barcelona, with

$$5.5 \times 10^6$$

3 Convert standard form

For the difference between Tokyo and Ahmedabad:

$$3.7 \times 10^7 - 7.7 \times 10^6$$

$$37000000 - 7700000 = 29300000$$

$$29300000 = 2.93 \times 10^7$$

---

4 Correct to 2 significant figures

Correct to 2 significant figures:

$$2.9 \times 10^7$$

FINAL ANSWER

(a) 8800000, (b) Barcelona, (c) 2.9×10^7

4MA1/2H JUNE 2023

Question 6

2 marks | Standard Form

6 (a) Write $76\,000\,000$ in standard form.

.....

(1)

(b) Write 5.4×10^{-4} as an ordinary number.

.....

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2023

1 Convert standard form

$$76000000 = 7.6 \times 10^{7}$$

$$5.4 \times 10^{-4} = 0.00054$$

FINAL ANSWER

(a) 7.6×10^{7}, (b) 0.00054

4MA1/1HR MAY 2023

Question 8

3 marks | Standard Form

8 (a) Write 5.6×10^{-3} as an ordinary number.

.....

(1)

(b) Work out $\dfrac{6 \times 10^{3}}{2.1 \times 10^{-4} + 9 \times 10^{-5}}$

Give your answer in standard form.

.....

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May 2023

1 Convert standard form

$$5.6 \times 10^{-3} = 0.0056$$

2 Convert standard form

For part (b), simplify the denominator:

$$2.1 \times 10^{-4} + 9 \times 10^{-5} = 0.00021 + 0.00009$$

$$= 0.00030 = 3 \times 10^{-4}$$

3 Convert standard form

Then

$$\frac{6 \times 10^{3}}{3 \times 10^{-4}} = 2 \times 10^{3-(-4)}$$

$$= 2 \times 10^{7}$$

FINAL ANSWER

(a) 0.0056, (b) 2×10^{7}

4MA1/1H MAY 2023

Question 15

6 marks | Standard Form

15 (a) $\sqrt{2} \div \dfrac{8^3}{16^{\frac{3}{2}}} = 2^n$

Work out the value of n
Show your working clearly.

$n =$ .....

(3)

(b) Find 4% of 4.5×10^{157}
Give your answer in standard form.

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2023

1 **Convert standard form**

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$8^3 = (2^3)^3 = 2^9$$

$$16^{\frac{3}{2}} = (2^4)^{\frac{3}{2}} = 2^6$$

2 **Convert standard form**

So

$$\frac{8^3}{16^{\frac{3}{2}}} = \frac{2^9}{2^6} = 2^3$$

3 **Convert standard form**

Therefore

$$\sqrt{2} \div \frac{8^3}{16^{\frac{3}{2}}} = 2^{\frac{1}{2}} \div 2^3$$

$$= 2^{\frac{1}{2}-3} = 2^{-\frac{5}{2}}$$

4 **Convert standard form**

So

$$n = -\frac{5}{2}$$

---

5 **Convert standard form**

For part (b),

$$4\% = 0.04 = 4 \times 10^{-2}$$

$$0.04(4.5 \times 10^{157}) = 18 \times 10^{155}$$

$$= 1.8 \times 10^{156}$$

✓ **FINAL ANSWER**

(a) $n = -\frac{5}{2}$, (b) 1.8×10^{156}

4MA1/1H NOVEMBER 2023

Question 9

4 marks | Standard Form

9 (a) Write 5.87×10^{-4} as an ordinary number.

.....

(1)

(b) Write 84 000 000 in standard form.

.....

(1)

The number of neurons in a human brain is 8.5×10^{10}
The number of neurons in a monkey brain is 1.47×10^{9}

The number of neurons in a human brain is K × the number of neurons in a monkey brain.

(c) Work out the value of K
Give your answer correct to one decimal place.

$K =$ .....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2023

1 Convert standard form

$$5.87 \times 10^{-4} = 0.000587$$

$$84000000 = 8.4 \times 10^{7}$$

2 Convert standard form

For part (c),

$$K = \frac{8.5 \times 10^{10}}{1.47 \times 10^{9}}$$

$$K = \frac{8.5}{1.47} \times 10$$

$$K = 57.823\ldots$$

3 **Convert standard form**

Correct to one decimal place,

$$K = 57.8$$

✓ **FINAL ANSWER**

(a) 0.000587, (b) 8.4×10^{7}, (c) 57.8

4MA1/2HR JUNE 2024

Question 9

4 marks | Standard Form

9 (a) Write 6.04×10^5 as an ordinary number.

.....

(1)

(b) Write 0.000 07 in standard form.

.....

(1)

(c) Work out $\dfrac{7.6 \times 10^{10}}{4 \times 10^5 - 2 \times 10^4}$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2024

1 Convert standard form

$$6.04 \times 10^5 = 604000$$

$$0.00007 = 7 \times 10^{-5}$$

2 Convert standard form

For part (c), first simplify the denominator:

$$4 \times 10^5 - 2 \times 10^4 = 400000 - 20000 = 380000$$

$$380000 = 3.8 \times 10^5$$

3 Convert standard form

Then

$$\frac{7.6 \times 10^{10}}{3.8 \times 10^5} = 2 \times 10^5$$

FINAL ANSWER

(a) 604000, (b) 7×10^{-5}, (c) 2×10^{5}

4MA1/1H MAY 2024

Question 9

3 marks | Standard Form

9 The table gives the amount of rice produced by each of two countries in 2020

Country	Amount of rice (tonnes)
Indonesia	3.5×10^7
Argentina	8.2×10^5

(a) Write 3.5×10^7 as an ordinary number.

.....

(1)

In 2020, Japan produced 6 780 000 more tonnes of rice than Argentina.

(b) Work out the amount of rice Japan produced in 2020
Give your answer in standard form.

..... tonnes

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2024

1 Convert standard form

$$3.5 \times 10^7 = 35000000$$

2 Argentina produced

Argentina produced

$$8.2 \times 10^5 = 820000$$

3 tonnes of rice

tonnes of rice.

4 Japan produced more tonnes than

Japan produced 6780000 more tonnes than Argentina, so

$$820000 + 6780000 = 7600000$$

5 **Convert standard form**

In standard form,

$$7600000 = 7.6 \times 10^6$$

✓ **FINAL ANSWER**

(a) 35000000, (b) 7.6×10^6 tonnes

4MA1/1H NOVEMBER 2024

Question 9

3 marks | Standard Form

9 (a) Write 8.4×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(6.5 \times 10^{-40}) \times (8 \times 10^{185})$
Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2024

1 Convert standard form

$$8.4 \times 10^{-5} = 0.000084$$

2 Convert standard form

For part (b),

$$(6.5 \times 10^{-40})(8 \times 10^{185}) = 52 \times 10^{145}$$

3 Write this in standard form

Write this in standard form:

$$52 \times 10^{145} = 5.2 \times 10^{146}$$

✓ **FINAL ANSWER**

(a) 0.000084, (b) 5.2×10^{146}

4WM2/01R MAY 2025

Question 6

3 marks | Standard Form

6 (a) Write $82\,000\,000$ in standard form.

.....

(1)

(b) Write 5.8×10^{-6} as an ordinary number.

.....

(1)

(c) Write 160×10^{300} in standard form.

.....

(1)

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM2/01R May 2025

1 Required

a Write 82 000 000 in standard form. (b) Write 5.8 × 10^{−6} as an ordinary number. (c) Write 160 × 10³⁰⁰ in standard form.

2 Part (a)

82 000 000 = 8.2 × 10⁷

3 Answer

8.2 × 10⁷

4 Part (b)

Move the decimal 6 places **left**:

5.8 × 10^{−6} = 0.0000058

5 Answer

0.0000058

6 Part (c)

160 × 10³⁰⁰ = 1.6 × 10¹ × 10³⁰⁰ = 1.6 × 10³⁰¹

FINAL ANSWER

1.6 × 10³⁰¹

4MA1/1H JUNE 2025

Question 8

4 marks | Standard Form

8 (a) Write 520 million in standard form.

.....

(1)

(b) Write 8.79×10^{-5} as an ordinary number.

.....

(1)

(c) Work out $(5 \times 10^{42}) \times (7 \times 10^{-180})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2025

1 Convert standard form

$$520 \text{ million} = 520000000 = 5.2 \times 10^{8}$$

$$8.79 \times 10^{-5} = 0.0000879$$

2 Convert standard form

For part (c),

$$(5 \times 10^{42})(7 \times 10^{-180}) = 35 \times 10^{-138}$$

3 Write this in standard form

Write this in standard form:

$$35 \times 10^{-138} = 3.5 \times 10^{-137}$$

FINAL ANSWER

(a) 5.2×10^{8}, (b) 0.0000879, (c) 3.5×10^{-137}

4WM2/01 MAY 2025

Question 12

3 marks | Standard Form

12 (a) Write 8.6×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(3 \times 10^{143}) \div (6 \times 10^{137})$

Give your answer in standard form.

.....

(2)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01 May 2025

1 **(a)**

(a)

$$8.6 \times 10^{-5} = 0.000086$$

2 **(b)**

(b)

$$(3 \times 10^{43}) \div (6 \times 10^{37})$$

$$= 0.5 \times 10^{6} = 5 \times 10^{5}$$

✓ **FINAL ANSWER**

0.000086, 5×10^{5}.

4WM2/01R MAY 2025

Question 13

3 marks | Standard Form

13 The distance from Earth to Andromeda Galaxy is 1.6×10^{11} astronomical units.

- 1 astronomical unit $= 1.5 \times 10^{8}$ km
- 1 light year $= 9.5 \times 10^{12}$ km

Work out the distance, in light years, from Earth to Andromeda Galaxy.
Give your answer in standard form, correct to 2 significant figures.

..... light years

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4WM2/01R May 2025

1 Required

Distance to Andromeda: 1.6 × 10¹¹ AU. Convert to light years (2 s.f., standard form).

2 Convert AU km 1 6

Convert AU → km:
1.6 × 10¹¹ × 1.5 × 10⁸ = (1.6×1.5) × 10¹⁹ = 2.4 × 10¹⁹ km

3 Convert km light years

Convert km → light years:

$$frac2.4times10^{19}9.5times10^{12} = frac2.49.5times10^{7} = 0.2526ldotstimes10^{7}$$

4 Convert standard form

Write in standard form to 2 s.f.:
= 2.526… × 10⁶ ≈ 2.5 × 10⁶

✓ **FINAL ANSWER**

Distance = 2.5 × 10⁶ light years

4MA1/1HR JUNE 2025

Question 14

2 marks | Standard Form

14 Work out $6.7 \times 10^{135} + 3 \times 10^{134}$

Give your answer in standard form.

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR June 2025

1 Convert standard form

$$6.7 \times 10^{135} + 3 \times 10^{134}$$

2 Write both terms using

Write both terms using 10^{135}:

$$3 \times 10^{134} = 0.3 \times 10^{135}$$

3 Convert standard form

So

$$6.7 \times 10^{135} + 0.3 \times 10^{135} = 7.0 \times 10^{135}$$

$$= 7 \times 10^{135}$$

FINAL ANSWER

7×10^{135}

4MA1/2H NOVEMBER 2025

Question 8

3 marks | Standard Form

8 (a) Write 5.76×10^4 as an ordinary number.

.....

(1)

(b) Work out $\dfrac{3 \times 10^5 + 8 \times 10^3}{4 \times 10^{-2}}$

Give your answer in standard form.

.....

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2025

1 Convert standard form

$$5.76 \times 10^4 = 57600$$

2 Convert standard form

For part (b), simplify the numerator:

$$3 \times 10^5 + 8 \times 10^3 = 300000 + 8000 = 308000$$

3 Convert standard form

Then

$$\frac{308000}{4 \times 10^{-2}} = \frac{308000}{0.04} = 7700000$$

4 In standard form

In standard form:

$$7700000 = 7.7 \times 10^6$$

FINAL ANSWER

(a) 57600, (b) 7.7×10^6

4WM2/01 NOVEMBER 2025

Question 9

3 marks | Standard Form

9 (a) Write 5.76×10^4 as an ordinary number.

.....

(1)

(b) Work out $\dfrac{3 \times 10^5 + 8 \times 10^3}{4 \times 10^{-2}}$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 November 2025

1 Move the decimal point 4

Move the decimal point 4 places to the right:

5.76 × 10⁴ = 57600

2 Answer

57600

3 Expand brackets

3 × 10⁵ = 300,000

8 × 10³ = 8,000

Numerator = 300,000 + 8,000 = 308,000 = 3.08 × 10⁵

4 Convert standard form

$$frac3.08times10^54times10^{-2} = left(frac3.084right)times10^{5-(-2)}$$

= 0.77 × 10⁷

Step 3: Convert to strict standard form (1 ≤ *A* \< 10).

0.77 × 10⁷ = 7.7 × 10⁶

FINAL ANSWER

7.7 × 10⁶

4WM2/01 NOVEMBER 2025

Question 13

2 marks | Standard Form

13 Work out $\left(9 \times 10^{80}\right)^3$

Give your answer in standard form.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4WM2/01 November 2025

1 Use index laws

(9×10⁸⁰)³ = 9³ × (10⁸⁰)³

2 Solve inequality

9³ = 729

(10⁸⁰)³ = 10^{80 × 3} = 10²⁴⁰

3 Convert standard form

729 × 10²⁴⁰ = 7.29 × 10² × 10²⁴⁰ = 7.29 × 10²⁴²

✓ **FINAL ANSWER**

7.29 × 10²⁴²

4MA1/2HR JUNE 2026

Question 7

3 marks | Standard Form

7 (a) Write 4×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(6 \times 10^{158}) \times (2 \times 10^{72})$
Give your answer in standard form.

.....

(2)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2026

1 Part (a) | Step 1

$$4 \times 10^{-5} = \frac{4}{100000} = 0.00004.$$

Move the decimal point five places left.

2 Part (b) | Step 1

$$(6 \times 10^{158})(2 \times 10^{72}) = 12 \times 10^{230} = 1.2 \times 10^{231}.$$

Multiply the coefficients and add the exponents; normalize the coefficient into [1,10).

FINAL ANSWER

(a) 0.00004. (b) 1.2×10^{231}.

4MA1/1H JUNE 2026

Question 9

3 marks | Standard Form

9 (a) Write 0.0651 in standard form.

.....

(1)

(b) Work out $\left(4 \times 10^{56}\right) \times \left(8 \times 10^{74}\right)$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2026

1 Part (a) | Step 1

$$0.0651 = 6.51 \times 10^{-2}.$$

The coefficient must be at least 1 and less than 10.

2 Part (b) | Step 1

$$(4 \times 10^{56})(8 \times 10^{74}) = 32 \times 10^{130} = 3.2 \times 10^{131}.$$

Multiply the coefficients, add the exponents, and adjust the coefficient to standard form.

FINAL ANSWER

(a) 6.51×10^{-2} (b) 3.2×10^{131}

TOPIC

Surds

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 17

4 marks | Surds

17 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show clearly each stage of your working.

.....

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 Specimen

1 Simplify the surds

$$\sqrt{8}=2\sqrt{2}$$

$$(\sqrt{2}-1)^2=2-2\sqrt{2}+1=3-2\sqrt{2}$$

2 Use the conjugate

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

The denominator is

$$9-(2\sqrt{2})^2=9-8=1$$

3 **Expand the numerator**

$$(3+2\sqrt{2})^2 = 9 + 12\sqrt{2} + 8 = 17 + 12\sqrt{2}$$

4 **Write in the requested form**

$$12\sqrt{2} = \sqrt{144 \times 2} = \sqrt{288}$$

✓ **FINAL ANSWER**

$17 + \sqrt{288}$.

4MA1/2H JANUARY 2020

Question 20

3 marks | Surds

20 The area of a rectangle is $18\,\text{cm}^2$

The length of the rectangle is $\left(\sqrt{7}+1\right)$ cm.

Without using a calculator and showing each stage of your working,

find the width of the rectangle.
Give your answer in the form $a\sqrt{b}+c$ where a, b and c are integers.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2020

1 Simplify surd

$$\text{width} = \frac{18}{\sqrt{7}+1}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{18}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{18(\sqrt{7}-1)}{7-1}$$

$$= \frac{18(\sqrt{7}-1)}{6} = 3(\sqrt{7}-1)$$

$$= 3\sqrt{7}-3$$

FINAL ANSWER

$3\sqrt{7}-3$ cm

4MA1/1H MAY/NOVEMBER 2020

Question 19

3 marks | Surds

19 Without using a calculator, rationalise the denominator of $\frac{6}{3-\sqrt{7}}$

Simplify your answer.
You must show each stage of your working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May/November 2020

1 Simplify surd

$$\frac{6}{3-\sqrt{7}}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{6}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{6(3+\sqrt{7})}{9-7}$$

$$= \frac{6(3+\sqrt{7})}{2} = 3(3+\sqrt{7})$$

$$= 9+3\sqrt{7}$$

FINAL ANSWER

$9+3\sqrt{7}$

4MA1/1H JANUARY 2021

Question 18

3 marks | Surds

18 Given that $(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$ where x is a prime number and y is an integer,

find the value of x and the value of y.
Show each stage of your working clearly.

$x =$ .....

$y =$ .....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2021

1 Simplify surd

$$(8 - \sqrt{x})(5 + \sqrt{x}) = 40 + 8\sqrt{x} - 5\sqrt{x} - x$$

$$= 40 + 3\sqrt{x} - x$$

2 Simplify surd

This is equal to

$$y\sqrt{x} + 21$$

3 Compare the rational parts

Compare the rational parts:

$$40 - x = 21$$

$$x = 19$$

4 Compare the surd coefficients

Compare the surd coefficients:

$$y = 3$$

FINAL ANSWER

$x = 19, y = 3$

4MA1/2H NOVEMBER 2021

Question 17

3 marks | Surds

17 Express $\frac{8}{\sqrt{5}-1}$ in the form $\sqrt{a}+b$ where a and b are integers.

Show each stage of your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2021

1 Simplify surd

$$\frac{8}{\sqrt{5}-1}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{8}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{8(\sqrt{5}+1)}{5-1}$$

$$= 2(\sqrt{5}+1) = 2\sqrt{5}+2$$

3 Simplify surd

In the form $\sqrt{a}+b$,

$$2\sqrt{5}+2 = \sqrt{20}+2$$

FINAL ANSWER

$\sqrt{20}+2$

4MA1/1H JUNE 2022

Question 16

3 marks | Surds

16 Without using a calculator, show that $\frac{12}{\sqrt{2}-1} - \left(\sqrt{2}\right)^5 = 2\sqrt{32} + 12$

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2022

1 **Proof**

Rationalise the denominator:

$$\frac{12}{\sqrt{2}-1} = \frac{12(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= 12(\sqrt{2}+1) = 12\sqrt{2}+12$$

Also,

$$(\sqrt{2})^5 = (\sqrt{2})^4\sqrt{2} = 4\sqrt{2}$$

So the left side is

$$12\sqrt{2}+12-4\sqrt{2} = 8\sqrt{2}+12$$

And

$$2\sqrt{32}+12 = 2(4\sqrt{2})+12 = 8\sqrt{2}+12$$

So both sides are equal.

FINAL ANSWER

Shown.

4MA1/1HR JUNE 2022

Question 21

4 marks | Surds

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2022

1 **Simplify surd**

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)^2}$$

2 **Simplify surd**

Since $\sqrt{8}=2\sqrt{2}$,

$$\frac{3+2\sqrt{2}}{(\sqrt{2}-1)^2}$$

3 **Simplify surd**

Also

$$(\sqrt{2}-1)^2=3-2\sqrt{2}$$

4 **Simplify surd**

So

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{(3+2\sqrt{2})^2}{9-8}$$

$$=(3+2\sqrt{2})^2=9+12\sqrt{2}+8$$

$$=17+12\sqrt{2}=17+\sqrt{288}$$

FINAL ANSWER

$17+\sqrt{288}$

4MA1/2H JUNE 2023

Question 23

4 marks | Surds

23 The diagram shows a cuboid with a square cross section.

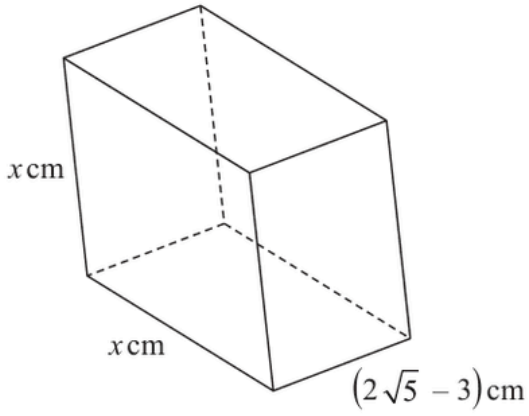


Diagram **NOT** accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of x
Give your answer in the form $a + \sqrt{b}$ where a and b are integers.
Show your working clearly.

$x =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2023

1 The volume is

The volume is

$$x^2(2\sqrt{5} - 3) = 13 + 6\sqrt{5}$$

2 Simplify surd

So

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3}$$

3 **Rationalise**

Rationalise:

$$x^2 = \frac{(13+6\sqrt{5})(2\sqrt{5}+3)}{(2\sqrt{5})^2 - 3^2}$$

$$= \frac{26\sqrt{5}+39+60+18\sqrt{5}}{20-9}$$

$$= \frac{99+44\sqrt{5}}{11} = 9+4\sqrt{5}$$

---

4 **Simplify surd**

Since

$$(\sqrt{5}+2)^2 = 5+4+4\sqrt{5} = 9+4\sqrt{5}$$

$$x = \sqrt{5}+2$$

✓ **FINAL ANSWER**

$2+\sqrt{5}$

4WM1/01R MAY 2025

Question 16

4 marks | Surds

16 $\sqrt{96} = 2\sqrt{k}$

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{8}{3-\sqrt{5}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers.

Show each stage of your working.

(3)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4WM1/01R May 2025

1 **(a)**

(a)

$$\sqrt{96} = 4\sqrt{6} = 2\sqrt{24}$$

2 **Calculate value**

So $k = 24$.

3 **(b)**

(b)

$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{8(3+\sqrt{5})}{9-5} = 2(3+\sqrt{5})$$

$$= 6 + 2\sqrt{5}$$

FINAL ANSWER

$k = 24, 6 + 2\sqrt{5}$.

4WM1/01 MAY 2025

Question 17

4 marks | Surds

17 $(\sqrt{3})^5 = k\sqrt{3}$ where k is an integer.

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{21}{3-\sqrt{2}}$ can be written in the form $c + \sqrt{d}$

where c and d are integers.
Show each stage of your working clearly.

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 May 2025

1 **Simplify surd**

$$(\sqrt{3})^5 = (\sqrt{3})^4\sqrt{3} = 3^2\sqrt{3} = 9\sqrt{3}$$

2 **Simplify surd**

So $k = 9$.

3 **Split the ratio**

For part (b), rationalise the denominator.

$$\frac{21}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{21(3+\sqrt{2})}{9-2}$$

$$= 3(3+\sqrt{2}) = 9+3\sqrt{2}$$

4 **Simplify surd**

Since $3\sqrt{2} = \sqrt{18}$,

$$9+3\sqrt{2} = 9+\sqrt{18}$$

FINAL ANSWER

$k = 9$, and $9 + \sqrt{18}$.

4WM1/01 NOVEMBER 2025

Question 16

3 marks | Surds

16 Show that $\dfrac{2\sqrt{7}+2}{\sqrt{7}-3}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM1/01 November 2025

1 **Simplify surd**

$$\frac{2\sqrt{7}+2}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}$$

$$=\frac{(2\sqrt{7}+2)(\sqrt{7}+3)}{7-9}$$

$$=\frac{20+8\sqrt{7}}{-2}$$

$$=-10-4\sqrt{7}=-10-\sqrt{112}$$

FINAL ANSWER

$-10-\sqrt{112}$.

Question 16

6 marks | Surds

16 (a) Solve $x^2 - \left(2\sqrt{3}\right)x - 13 = 0$

Give your solutions in the form $\sqrt{a} \pm b$ where a and b are integers.
Show your working clearly.

.....

(3)

(b) Show that $\dfrac{\sqrt{20}}{\sqrt{5}+2}$ can be written in the form $c - \sqrt{d}$ where c and d are integers.

(3)

(Total for Question 16 is 6 marks)

1 **(a) Quadratic formula**

$$x = \frac{2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 + 4(13)}}{2} = \frac{2\sqrt{3} \pm \sqrt{64}}{2}$$

2 **Simplify**

$$x = \frac{2\sqrt{3} \pm 8}{2} = \sqrt{3} \pm 4$$

3 **(b) Rationalise**

$$\frac{\sqrt{20}}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{2\sqrt{5}(\sqrt{5}-2)}{5-4}$$

4 **Simplify**

$$= 10 - 4\sqrt{5} = 10 - \sqrt{80} \quad (c = 10,\ d = 80)$$

✓ **FINAL ANSWER**

(a) $\sqrt{3} \pm 4$ (b) $10 - \sqrt{80}$

4MA1/1H JUNE 2026

Question 17

3 marks | Surds

17 Show that $\left(4-\sqrt{20}\right)\left(5-2\sqrt{5}\right)$ can be written in the form $a-\sqrt{b}$

where a and b are integers.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2026

1 Solution | Step 1

$$\sqrt{20}=2\sqrt{5}, \qquad (4-\sqrt{20})(5-2\sqrt{5})=(4-2\sqrt{5})(5-2\sqrt{5}).$$

Simplify the surd first.

2 Solution | Step 2

$$20-8\sqrt{5}-10\sqrt{5}+20=40-18\sqrt{5}.$$

Expand; the product of the two surd terms is positive 20.

3 Solution | Step 3

$$18\sqrt{5}=\sqrt{18^2\times 5}=\sqrt{1620}, \quad (4-\sqrt{20})(5-2\sqrt{5})=40-\sqrt{1620}.$$

Thus the required integer values are a=40 and b=1620.

FINAL ANSWER

$40-\sqrt{1620}$, so $a=40,\ b=1620$.

TOPIC

Using a Calculator

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 1

2 marks | Using a Calculator

1 Work out $\dfrac{5.7^2 - \sqrt{30.25}}{1.8 \times 0.65}$

Write down all the figures on your calculator display.

.....

(2)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · EP Specimen

1 **Work out the top and the bottom**

$$5.7^2 - \sqrt{30.25} = 32.49 - 5.5 = 26.99, \qquad 1.8 \times 0.65 = 1.17$$

2 **Divide**

$$\frac{26.99}{1.17} = 23.06837607\ldots$$

✓ **FINAL ANSWER**

23.06837607

EP SPECIMEN

Question 2

3 marks | Using a Calculator

2 (a) Work out $4.3^3 - \frac{1}{0.16}$

Write down all the figures on your calculator display.

.....

(2)

(b) Give your answer to part (a) correct to 3 significant figures.

.....

(1)

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · EP Specimen

1 **(a) Use the calculator**

$$4.3^3 - \frac{1}{0.16} = 79.507 - 6.25 = 73.257$$

2 **(b) Round to 3 significant figures**

The fourth figure is 5, so round up: 73.3.

FINAL ANSWER

(a) 73.257 (b) 73.3

EP SPECIMEN

Question 3

2 marks | Using a Calculator

3 Work out $\dfrac{8.4 \times \tan 35°}{\cos 62°}$

Give your answer correct to 3 significant figures.

.....

(2)

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · EP Specimen

1 **Check the calculator is in degrees**

Then

$$\frac{8.4 \times \tan 35°}{\cos 62°} = \frac{5.8817\ldots}{0.46947\ldots} = 12.528\ldots$$

2 **Round**

12.5 (3 significant figures)

FINAL ANSWER

12.5

EP SPECIMEN

Question 4

3 marks | Using a Calculator

4 Work out $\dfrac{3.7 \times 10^5 + 2.9 \times 10^4}{8.1 \times 10^{-3}}$

Give your answer in standard form correct to 3 significant figures.

.....

(3)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · EP Specimen

1 Add the top

$$3.7 \times 10^5 + 2.9 \times 10^4 = 370\,000 + 29\,000 = 399\,000$$

2 Divide

$$\frac{399\,000}{8.1 \times 10^{-3}} = 49\,259\,259.26\ldots$$

3 Standard form, 3 significant figures

4.93×10^7

✓ **FINAL ANSWER**

4.93×10^7

EP SPECIMEN

Question 5

2 marks | Using a Calculator

5 $p = 2.6 \quad q = -1.8$

Work out the value of $\dfrac{p^2 - 3q}{\sqrt{(p + q^2)}}$

Give your answer correct to 3 decimal places.

.....

(2)

(Total for Question 5 is 2 marks)

WORKED SOLUTION

Question 5 · EP Specimen

1 Substitute (brackets round the negative)

$$p^2 - 3q = 2.6^2 - 3(-1.8) = 6.76 + 5.4 = 12.16$$

$$\sqrt{p + q^2} = \sqrt{2.6 + (-1.8)^2} = \sqrt{5.84} = 2.4166\ldots$$

2 Divide and round

$$\frac{12.16}{2.4166\ldots} = 5.03184\ldots \approx 5.032$$

✓ **FINAL ANSWER**

5.032

EP SPECIMEN

Question 6

2 marks | Using a Calculator

6 Work out the value of $4.8^{-0.5} + 1.7^3$

Give your answer correct to 4 significant figures.

.....

(2)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · EP Specimen

1 Evaluate each power

$$4.8^{-0.5} = \frac{1}{\sqrt{4.8}} = 0.45643\ldots, \qquad 1.7^3 = 4.913$$

2 Add and round

$$0.45643\ldots + 4.913 = 5.36943\ldots \approx 5.369$$

✓ FINAL ANSWER

5.369

EP SPECIMEN

Question 7

3 marks | Using a Calculator

7 Here is a calculation: $\frac{19.6 \times 0.48}{4.1}$

(a) By rounding each number to 1 significant figure, work out an estimate for the calculation.

.....

(2)

(b) Use your calculator to work out the calculation. Give your answer correct to 3 significant figures.

.....

(1)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · EP Specimen

1 **(a) Round each number to 1 significant figure**

$$\frac{20 \times 0.5}{4} = \frac{10}{4} = 2.5$$

2 **(b) Calculator value**

$$\frac{19.6 \times 0.48}{4.1} = 2.2946\ldots \approx 2.29$$

✓ **FINAL ANSWER**

(a) 2.5 (b) 2.29

TOPIC

Ratio Toolkit

Every past-paper question on this topic, 2020 to 2026.

Question 10

5 marks | Ratio Toolkit

10 Payel makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7 : 5 : 3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.

36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

.....

(Total for Question 10 is 5 marks)

1 **The total number of parts**

The total number of parts is

$$7 + 5 + 3 = 15$$

2 **Each part is**

Each part is

$$300 \div 15 = 20$$

3 **Split the ratio**

So the numbers of cards are:

$$140, \quad 100, \quad 60$$

4 **The birthday cards with numbers**

The birthday cards with numbers are

$$\frac{2}{5} \times 140 = 56$$

5 **The anniversary cards with numbers**

The anniversary cards with numbers are

$$36\% \times 100 = 36$$

6 **Split the ratio**

So the total with numbers is

$$56 + 36 = 92$$

7 **The fraction of all cards**

The fraction of all cards is

$$\frac{92}{300} = \frac{23}{75}$$

✓ **FINAL ANSWER**

$\frac{23}{75}$.

4MA1/1HR JANUARY 2020

Question 1

3 marks | Ratio Toolkit

1 Brendon, Asha and Julie share some money in the ratios 3 : 2 : 6
The **total** amount of money that Asha and Julie receive is \$36

Work out the amount of money that Brendon receives.

\$.....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2020

1 The ratio is

The ratio is

$$\text{Brendon} : \text{Asha} : \text{Julie} = 3 : 2 : 6$$

2 Asha and Julie together have

Asha and Julie together have

$$2 + 6 = 8 \text{ parts}$$

3 These 8 parts are worth

These 8 parts are worth $\$36$, so one part is

$$36 \div 8 = 4.50$$

4 Brendon has 3 parts

Brendon has 3 parts:

$$3 \times 4.50 = 13.50$$

FINAL ANSWER

Brendon receives $\$13.50$.

4MA1/1H JANUARY 2020

Question 5

6 marks | Ratio Toolkit

5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3 : 5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2020

1 Split the ratio

Girls : boys $= 3 : 5$, so there are 8 parts.

$$\text{girls} = 120 \times \frac{3}{8} = 45$$

$$\text{boys} = 120 \times \frac{5}{8} = 75$$

2 Split the ratio

Boys climbing:

$$\frac{16}{25} \times 75 = 48$$

3 Split the ratio

Boys sailing:

$$75 - 48 = 27$$

4 Split the ratio

For the girls, twice as many go sailing as climbing, so the girls are split in the ratio

$$\text{sailing} : \text{climbing} = 2 : 1$$

5 **Split the ratio**

Girls sailing:

$$\frac{2}{3} \times 45 = 30$$

6 **Total sailing**

Total sailing:

$$27 + 30 = 57$$

✓ **FINAL ANSWER**

57

4MA1/2H JANUARY 2020

Question 12

4 marks | Ratio Toolkit

12 Astrid wants to buy some oil.
She can buy the oil from either Dane Oil or Arctic Oil.

Here is information about the price that each company will charge Astrid.

Dane Oil	Arctic Oil
(4.2×10^5) litres for 2 500 000 Krone	(8.6×10^5) litres for 770 000 Dollars

Astrid wants to get the better value for money for the oil.

1 Dollar = 6.57 Krone

From which company should she buy her oil, Dane Oil or Arctic Oil?
You must show your working.

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2020

1 Convert standard form

Dane Oil costs 2500000 Krone for

$$4.2 \times 10^5 = 420000$$

2 litres

litres.

$$\text{Dane cost per litre} = \frac{2500000}{420000} = 5.952\ldots$$

3 Convert standard form

So Dane Oil costs about 5.95 Krone per litre.

4 Arctic Oil costs dollars. Convert

Arctic Oil costs 770000 dollars. Convert this to Krone:

$$770000 \times 6.57 = 5058900$$

5 **Arctic Oil gives**

Arctic Oil gives

$$8.6 \times 10^5 = 860000$$

6 **litres**

litres.

$$\text{Arctic cost per litre} = \frac{5058900}{860000} = 5.882\ldots$$

7 **Arctic Oil has the lower**

Arctic Oil has the lower cost per litre.

✓ **FINAL ANSWER**

Arctic Oil

4MA1/1H JANUARY 2021

Question 1

4 marks | Ratio Toolkit

1 Pieter owns a currency conversion shop.

Last Monday, Pieter changed a total of 20 160 rand into a number of different currencies.

He changed $\frac{3}{10}$ of the 20 160 rand into euros.

He changed the rest of the rands into dollars, rupees and francs in the ratios $9:5:2$

Pieter changed more rands into dollars than he changed into francs.

Work out how many more.

..... rand

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2021

1 **of the money is changed**

$\frac{3}{10}$ of the money is changed to euros, so the rest is

$$\frac{7}{10} \times 20160 = 14112$$

2 **Split the ratio**

This rest is divided in the ratio

$$\text{dollars} : \text{rupees} : \text{francs} = 9 : 5 : 2$$

3 **There are**

There are

$$9 + 5 + 2 = 16 \text{ parts}$$

4 **Split the ratio**

The difference between dollars and francs is

$$9 - 2 = 7 \text{ parts}$$

5 **Split the ratio**

So the difference is

$$\frac{7}{16} \times 14112 = 6174$$

✓ **FINAL ANSWER**

6174 rand more is changed into dollars than francs.

4MA1/1H MAY 2021

Question 4

3 marks | Ratio Toolkit

4 On a farm there are chickens, ducks and pigs.

The ratio of the number of chickens to the number of ducks is $7:2$
The ratio of the number of ducks to the number of pigs is $5:9$
There are 36 pigs on the farm.

Work out the number of chickens on the farm.

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May 2021

1 Ducks pigs

Ducks : pigs $= \mathbf{5 : 9}$.

2 Split the ratio

There are $\mathbf{36}$ pigs, so

$$9 \text{ parts} = 36$$

$$1 \text{ part} = 4$$

$$\text{ducks} = 5 \times 4 = 20$$

3 Chickens ducks

Chickens : ducks $= \mathbf{7 : 2}$.

$$2 \text{ parts} = 20$$

$$1 \text{ part} = 10$$

$$\text{chickens} = 7 \times 10 = 70$$

FINAL ANSWER

70

4MA1/1H JANUARY 2022

Question 2

3 marks | Ratio Toolkit

2 Danil, Gabriel and Hadley share some money in the ratios $3:5:9$
The difference between the amount of money that Gabriel receives and the amount of money that Hadley receives is 196 euros.

Work out the amount of money that Danil receives.

..... euros

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2022

1 The ratio is

The ratio is

$$\text{Danil} : \text{Gabriel} : \text{Hadley} = 3 : 5 : 9$$

2 Split the ratio

The difference between Gabriel and Hadley is

$$9 - 5 = 4 \text{ parts}$$

3 Split the ratio

These 4 parts are 196 euros, so one part is

$$196 \div 4 = 49$$

4 Danil has 3 parts

Danil has 3 parts:

$$3 \times 49 = 147$$

FINAL ANSWER

Danil receives 147 euros.

4MA1/1H MAY 2023

Question 1

3 marks | Ratio Toolkit

1 Last season, the number of goals scored by Arjun, by Simon and by Kath for their football team were in the ratios $2:5:8$

Simon scored 12 more goals than Arjun.

Work out the number of goals scored by Kath.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2023

1 The ratio is

The ratio is

$$\text{Arjun} : \text{Simon} : \text{Kath} = 2 : 5 : 8$$

2 Simon scored more than Arjun

Simon scored 12 more than Arjun.

$$5 - 2 = 3 \text{ parts}$$

$$3 \text{ parts} = 12$$

$$1 \text{ part} = 4$$

3 Kath scored

Kath scored

$$8 \times 4 = 32$$

FINAL ANSWER

32

4MA1/2H JUNE 2023

Question 5

4 marks | Ratio Toolkit

5 Nancy has some coins with a total value of 85 pence.
She has only 2 pence coins and 5 pence coins.
The ratio

$$\text{number of 2 pence coins} : \text{number of 5 pence coins} = 1 : 3$$

Nancy has more 5 pence coins than 2 pence coins.

How many more?

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2023

1 Let the number of pence

Let the number of 2 pence coins be x.

2 Split the ratio

Then the number of 5 pence coins is $3x$.

3 Total value

Total value:

$$2x + 5(3x) = 85$$

$$17x = 85$$

$$x = 5$$

4 Split the ratio

So there are 5 two-pence coins and 15 five-pence coins.

$$15 - 5 = 10$$

FINAL ANSWER

10 more 5 pence coins

4MA1/2H NOVEMBER 2023

Question 5

3 marks | Ratio Toolkit

5 Avril bakes a cake.

She uses flour, butter and sugar such that

weight of flour : weight of butter = 6 : 5
weight of butter : weight of sugar = 3 : 2

Avril uses 120 grams of sugar.

Work out the weight of flour Avril uses.

..... grams

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2023

1 Butter sugar

Butter : sugar $= \mathbf{3 : 2}$.

2 Split the ratio

Sugar is 120 g, so

$$2 \text{ parts} = 120$$

$$1 \text{ part} = 60$$

$$\text{butter} = 3 \times 60 = 180$$

3 Flour butter

Flour : butter $= \mathbf{6 : 5}$.

$$5 \text{ parts} = 180$$

$$1 \text{ part} = 36$$

$$\text{flour} = 6 \times 36 = 216$$

FINAL ANSWER

216 grams

4MA1/2H JUNE 2025

Question 6

4 marks | Ratio Toolkit

6 Pau, Sam and Tia share £240 in the ratios $3:4:5$

Sam and Tia each give £10 of their share to Pau.

Work out the ratios of the amounts of money that Pau, Sam and Tia now have.
Give your answer in its simplest form.

..... : ..... : .....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2025

1 The original ratio is

The original ratio is

$$\text{Pau} : \text{Sam} : \text{Tia} = 3 : 4 : 5$$

2 There are

There are

$$3 + 4 + 5 = 12 \text{ parts}$$

3 Each part is

Each part is

$$240 \div 12 = 20$$

4 Split the ratio

So the original amounts are

$$60,\ 80,\ 100$$

5 Sam gives 10 to Pau

Sam gives 10 to Pau and Tia gives 10 to Pau, so the new amounts are

$$80,\ 70,\ 90$$

6 **Divide by 10**

Divide by 10:

$$80 : 70 : 90 = 8 : 7 : 9$$

✓ **FINAL ANSWER**

The new ratio is $8 : 7 : 9$.

4WM2/01 MAY 2025

Question 7

4 marks | Ratio Toolkit

7 Pau, Sam and Tia share £240 in the ratios $3 : 4 : 5$

Sam and Tia each give £10 of their share to Pau.

Work out the ratios of the amounts of money that Pau, Sam and Tia now have.
Give your answer in its simplest form.

..... : ..... : .....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4WM2/01 May 2025

1 The original ratio is

The original ratio is

$$\text{Pau} : \text{Sam} : \text{Tia} = 3 : 4 : 5$$

2 There are

There are

$$3 + 4 + 5 = 12 \text{ parts}$$

3 Each part is

Each part is

$$240 \div 12 = 20$$

4 Split the ratio

So the original amounts are

$$60,\ 80,\ 100$$

5 Sam gives 10 to Pau

Sam gives 10 to Pau and Tia gives 10 to Pau, so the new amounts are

$$80,\ 70,\ 90$$

6 Divide by 10

Divide by 10:

$$80 : 70 : 90 = 8 : 7 : 9$$

FINAL ANSWER

The new ratio is $8 : 7 : 9$.

4MA1/1H NOVEMBER 2025

Question 6

4 marks | Ratio Toolkit

6 Eli and Peta share \$275 in the ratio 2 : 3

Eli gives $\frac{3}{11}$ of his share to charity.

Peta gives 0.32 of her share to charity.

Work out the total amount that Eli and Peta give to charity.

\$.....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2025

1 The ratio is

The ratio is

$$\text{Eli} : \text{Peta} = 2 : 3$$

2 Split the ratio

There are 5 parts, so one part is

$$275 \div 5 = 55$$

3 Eli gets

Eli gets

$$2 \times 55 = 110$$

4 Peta gets

Peta gets

$$3 \times 55 = 165$$

5 Eli gives

Eli gives

$$\frac{3}{11} \times 110 = 30$$

6 **Peta gives**

Peta gives

$$0.32 \times 165 = 52.8$$

7 **Total given to charity**

Total given to charity:

$$30 + 52.8 = 82.8$$

✓ **FINAL ANSWER**

82.8.

4MA1/2HR JUNE 2026

Question 5

5 marks | Ratio Toolkit

5 Sarah and Dej each give some money to charity **A** and to charity **B**

Sarah shares \$400 between charity **A** and charity **B** in the ratio 2 : 3

Dej shares \$375 between the two charities.
He gives twice as much money to charity **B** as he gives to charity **A**

Dej gives more money to charity **B** than Sarah gives to charity **B**

How much more?

\$.....

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\text{Sarah to B} = 400 \times \frac{3}{2+3} = 240 \text{ dollars.}$$

Charity B receives 3 of the 5 equal shares.

2 Solution | Step 2

Dej gives to A and B in the ratio $1 : 2$, so

$$\text{Dej to B} = 375 \times \frac{2}{3} = 250 \text{ dollars.}$$

Twice as much to B means that B receives 2 of the 3 shares.

3 Solution | Step 3

$$250 - 240 = 10 \text{ dollars.}$$

Subtract the two donations to charity B.

FINAL ANSWER

\$10 more.

TOPIC

Standard & Compound Units

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 5

3 marks | Standard & Compound Units

5 A plane flies from New York City to Los Angeles.

The plane flies a distance of 3980 kilometres in 5 hours 24 minutes.

Work out the average speed of the plane.
Give your answer in kilometres per hour correct to the nearest whole number.

..... kilometres per hour

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 Specimen

1 Convert the time

$$24 \text{ minutes} = \frac{24}{60} = 0.4 \text{ hours}$$

$$5 \text{ hours } 24 \text{ minutes} = 5.4 \text{ hours}$$

2 Calculate the speed

$$\text{speed} = \frac{3980}{5.4} = 737.037\ldots$$

3 Round the answer

$$737.037\ldots \approx 737$$

FINAL ANSWER

737 kilometres per hour.

Question 7

4 marks | Standard & Compound Units

7 A cylinder is placed on the ground.

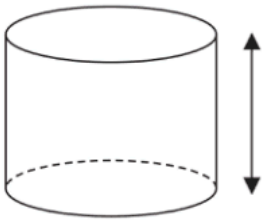


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 7 is 4 marks)

1 **Calculate area**

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

2 **Calculate area**

So the area of the base is

$$\text{area} = \frac{72}{1.4} = 51.428571\ldots$$

3 **The volume is**

The volume is

$$51.428571\ldots \times 18 = 925.714285\ldots$$

4 **Calculate volume**

Correct to 3 significant figures,

$$925.714\ldots \approx 926$$

✓ **FINAL ANSWER**

$926\ \text{cm}^3$.

4WM1/01 SPECIMEN

Question 8

3 marks | Standard & Compound Units

(Total for Question 7 is 4 marks)

8 Change a speed of 81 kilometres per hour to a speed in metres per second.

..... metres per second

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 Specimen

1 Convert kilometres to metres

$$81 \text{ km} = 81000 \text{ m}$$

2 Convert hours to seconds

$$1 \text{ hour} = 3600 \text{ seconds}$$

3 Divide to find metres per second

$$\frac{81000}{3600} = 22.5$$

FINAL ANSWER

22.5 metres per second.

4MA1/2H JANUARY 2020

Question 2

2 marks | Standard & Compound Units

2 Change 32.4 m^3 into cm^3

..... cm^3

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2020

1 Calculate value

$$1\text{ m} = 100\text{ cm}$$

2 Calculate value

So

$$1\text{ m}^3 = 100^3\text{ cm}^3 = 1,000,000\text{ cm}^3$$

$$32.4\text{ m}^3 = 32.4 \times 1,000,000$$

$$= 32,400,000\text{ cm}^3$$

FINAL ANSWER

$32,400,000\text{ cm}^3$.

4MA1/2HR JANUARY 2020

Question 5

3 marks | Standard & Compound Units

5 A plane takes 3 hours 36 minutes to fly from the Cayman Islands to New York.
The plane flies a distance of 2470 km.

Work out the average speed of the plane in km/h.
Give your answer correct to the nearest whole number.

..... km/h

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2020

1 The time is

The time is

$$3\text{ h }36\text{ min} = 3.6\text{ h}$$

2 Average speed

Average speed:

$$2470 \div 3.6 = 686.111\ldots$$

FINAL ANSWER

686 km/h.

4MA1/1HR JANUARY 2020

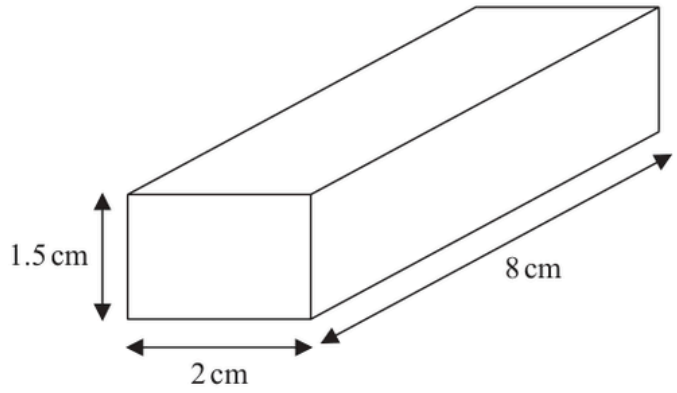
Question 9

5 marks | Standard & Compound Units

9 Pablo made a solid gold statue.

He melted down some gold blocks and used the gold to make the statue.
Each block of gold was a cuboid, as shown below.

Diagram **NOT** accurately drawn



The mass of the statue is 5.73 kg.
The density of gold is 19.32 g/cm^3

Work out the least number of gold blocks Pablo melted down in order to make the statue.
Show your working clearly.

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2020

1 Convert the mass to grams

Convert the mass to grams:

$$5.73 \text{ kg} = 5730 \text{ g}$$

2 Calculate volume

Using

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume of gold} = \frac{5730}{19.32} = 296.58\ldots \text{ cm}^3$$

3 **Volume of one cuboid block**

Volume of one cuboid block is

$$1.5 \times 2 \times 8 = 24 \text{ cm}^3$$

4 **Number of blocks needed is**

Number of blocks needed is

$$\frac{296.58\ldots}{24} = 12.35\ldots$$

5 **Pablo must have a whole**

Pablo must have a whole number of blocks, so he needs at least 13 blocks.

✓ **FINAL ANSWER**

13 blocks.

4MA1/2H JANUARY 2020

Question 9

2 marks | Standard & Compound Units

9 The density of gold is 19.3 g/cm^3
A gold bar has volume 150 cm^3

Work out the mass of the gold bar.

..... g

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2020

1 Calculate volume

$$\text{mass} = \text{density} \times \text{volume}$$

$$\text{mass} = 19.3 \times 150$$

$$\text{mass} = 2895$$

FINAL ANSWER

2895 g.

4MA1/2HR JANUARY 2020

Question 9

3 marks | Standard & Compound Units

9 The diagram shows a box in the shape of a cuboid.

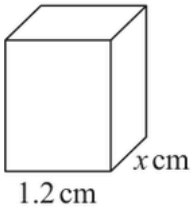


Diagram **NOT** accurately drawn

The box is put on a table.
The face of the box in contact with the table has length 1.2 metres and width x metres.

The force exerted by the box on the table is 27 newtons.
The pressure on the table due to the box is 30 newtons/m^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the value of x.

$x =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2020

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$30 = \frac{27}{\text{area}}$$

$$\text{area} = \frac{27}{30} = 0.9 \text{ m}^2$$

2 **The base area is**

The base area is

$$1.2x = 0.9$$

$$x = \frac{0.9}{1.2} = 0.75$$

✓ **FINAL ANSWER**

$x = 0.75\ \text{m}$.

4MA1/2H JANUARY 2020

Question 10

3 marks | Standard & Compound Units

10 Change a speed of 50 metres per second to a speed in kilometres per hour.

..... kilometres per hour

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2020

1 **Convert m/s to km/h multiply**

To convert m/s to km/h, multiply by $\mathbf{3.6}$.

$$50 \times 3.6 = 180$$

FINAL ANSWER

$180\ \mathrm{km/h}$.

4MA1/1H JANUARY 2020

Question 13

4 marks | Standard & Compound Units

13 The diagram shows a solid cube.
The cube is placed on a table so that the whole of one face of the cube is in contact with the table.

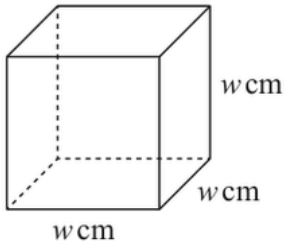


Diagram **NOT** accurately drawn

The cube exerts a force of 56 newtons on the table.
The pressure on the table due to the cube is 0.14 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cube.

..... cm^3

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2020

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$0.14 = \frac{56}{\text{area}}$$

$$\text{area} = \frac{56}{0.14} = 400 \text{ cm}^2$$

2 **The cube rests on one**

The cube rests on one square face, so

$$w^2 = 400$$

$$w = 20$$

3 **Volume of the cube is**

Volume of the cube is

$$20^3 = 8000$$

FINAL ANSWER

$8000\ \text{cm}^3$.

4MA1/1H MAY/NOVEMBER 2020

Question 6

3 marks | Standard & Compound Units

6 A rocket travelled 100 km at an average speed of 28 440 km/h.

Work out how long it took the rocket to travel the 100 km.
Give your answer in seconds, correct to the nearest second.

..... seconds

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May/November 2020

1 Use

Use

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{100}{28440} \text{ hours}$$

2 Convert hours to seconds

Convert hours to seconds:

$$\frac{100}{28440} \times 3600 = 12.658\ldots$$

3 To the nearest second

To the nearest second,

$$12.658\ldots \approx 13$$

FINAL ANSWER

13 seconds.

4MA1/1HR MAY/NOVEMBER 2020

Question 7

4 marks | Standard & Compound Units

7 Pedro drove from Toulouse to Montpellier in 2 hours 42 minutes.
He drove at an average speed of 90 km/hour.

Janine drove from Toulouse to Montpellier along the same route as Pedro.
The journey took her 3 hours.

Work out Janine's average speed for the journey.

..... km/hour

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May/November 2020

1 Pedro s time was

Pedro's time was

$$2\text{ h }42\text{ min} = 2.7\text{ h}$$

2 Distance from Toulouse to Montpellier

Distance from Toulouse to Montpellier:

$$90 \times 2.7 = 243\text{ km}$$

3 Calculate value

Janine took 3 hours, so her average speed was

$$243 \div 3 = 81$$

FINAL ANSWER

81 km/hour.

4MA1/1HR MAY/NOVEMBER 2020

Question 9

3 marks | Standard & Compound Units

9 A box is put on a horizontal table.

The face of the box in contact with the table is a square of side 1.5 metres.
The pressure on the table due to the box is 34.8 newtons/m^2

Work out the force exerted by the box on the table.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

..... newtons

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May/November 2020

1 The area of the square

The area of the square face is

$$1.5 \times 1.5 = 2.25 \text{ m}^2$$

2 Use

Use

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

3 Calculate area

So

$$\text{force} = \text{pressure} \times \text{area}$$

$$34.8 \times 2.25 = 78.3$$

FINAL ANSWER

78.3 newtons.

4MA1/2H JANUARY 2021

Question 1

3 marks | Standard & Compound Units

1 A train takes 6 hours 39 minutes to travel from New Delhi to Kanpur.
The train travels a distance of 429 km.

Work out the average speed of the train.
Give your answer in km/h correct to one decimal place.

..... km/h

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2021

1 The time is

The time is

$$6\text{ h }39\text{ min} = 6.65\text{ h}$$

2 Average speed

Average speed:

$$429 \div 6.65 = 64.511\ldots$$

FINAL ANSWER

64.5 km/h.

4MA1/1HR JANUARY 2021

Question 4

3 marks | Standard & Compound Units

4 A train journey from Paris to Amsterdam took 3 hours 24 minutes.
The total distance the train travelled was 433.5 km.

Work out the average speed of the train.
Give your answer in kilometres per hour.

..... km/h

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR January 2021

1 The time is

The time is

$$3\text{ h }24\text{ min} = 3.4\text{ h}$$

2 Average speed

Average speed:

$$433.5 \div 3.4 = 127.5$$

✓ **FINAL ANSWER**

127.5 km/h.

4MA1/1H MAY 2021

Question 1

3 marks | Standard & Compound Units

1 A plane flew from Madrid to Dubai.

The distance the plane flew was 5658 km.
The flight time was 8 hours 12 minutes.

Work out the average speed of the plane.

..... km/h

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2021

1 Evaluate fraction

$$8\text{ h }12\text{ min} = 8 + \frac{12}{60} = 8.2\text{ hours}$$

$$\text{average speed} = \frac{5658}{8.2} = 690$$

FINAL ANSWER

690 km/h.

4MA1/1H NOVEMBER 2021

Question 10

5 marks | Standard & Compound Units

10 A solid aluminium cylinder has radius 10 cm and height h cm.

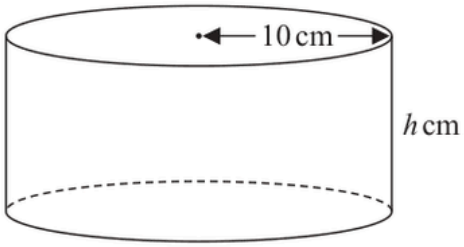


Diagram **NOT** accurately drawn

The mass of the cylinder is 5.4 kg.
The density of aluminium is 0.0027 kg/cm^3

Calculate the value of h.
Give your answer correct to one decimal place.

$h =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2021

1 Calculate volume

The mass is $5.4\ \text{kg}$, and the density is $0.0027\ \text{kg/cm}^3$.

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{5.4}{0.0027} = 2000\ \text{cm}^3$$

2 Evaluate fraction

For the cylinder,

$$V = \pi r^2 h$$

$$2000 = \pi(10)^2 h$$

$$h = \frac{2000}{100\pi} = 6.366\ldots$$

FINAL ANSWER

$6.4\ \text{cm}$.

4MA1/1HR JANUARY 2022

Question 7

4 marks | Standard & Compound Units

7 Milly went on a car journey.
She travelled from Anesey to Breigh to Clando and then to Duckbridge.

For Anesey to Breigh, Milly drove the 245 km in 2.5 hours.
For Breigh to Clando, Milly drove the 220 km at an average speed of 80 km/h
For Clando to Duckbridge, Milly drove at an average speed of 72 km/h in 50 minutes.

Work out Milly's average speed, in km/h, for the journey from Anesey to Duckbridge.
Give your answer correct to one decimal place.

..... km/h

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2022

1 Calculate value

First part:

$$\text{distance} = 245 \text{ km}, \qquad \text{time} = 2.5 \text{ h}$$

2 Second part

Second part:

$$\text{time} = \frac{220}{80} = 2.75 \text{ h}$$

3 Third part

Third part:

$$50 \text{ minutes} = \frac{5}{6} \text{ h}$$

$$\text{distance} = 72 \times \frac{5}{6} = 60 \text{ km}$$

4 Total distance

Total distance:

$$245 + 220 + 60 = 525 \text{ km}$$

5 **Total time**

Total time:

$$2.5 + 2.75 + \frac{5}{6} = 6.0833\ldots\ \text{h}$$

6 **Average speed**

Average speed:

$$\frac{525}{6.0833\ldots} = 86.301\ldots$$

✓ **FINAL ANSWER**

$86.3\ \text{km/h}$.

Question 21

6 marks | Standard & Compound Units

21 The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

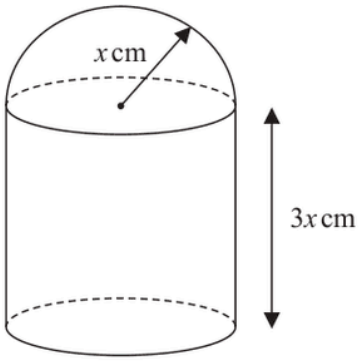


Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is 81π cm^2
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm^3)
Aluminium	2.7
Nickel	8.9
Gold	19.3
Silver	10.5

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

(Total for Question 21 is 6 marks)

1 **The outside surface area is**

The outside surface area is:

2 **curved surface area of cylinder**

curved surface area of cylinder:

$$2\pi x(3x) = 6\pi x^2$$

3 **bottom of cylinder**

bottom of cylinder:

$$\pi x^2$$

4 **curved surface area of hemisphere**

curved surface area of hemisphere:

$$2\pi x^2$$

5 **Calculate value**

So

$$6\pi x^2 + \pi x^2 + 2\pi x^2 = 81\pi$$

$$9\pi x^2 = 81\pi$$

$$x^2 = 9$$

$$x = 3$$

6 **Volume of the cylinder**

Volume of the cylinder:

$$\pi x^2(3x) = 3\pi x^3$$

7 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

8 **Total volume**

Total volume:

$$3\pi(3)^3 + \frac{2}{3}\pi(3)^3$$

$$= 81\pi + 18\pi = 99\pi$$

---

9 **Density**

Density:

$$\frac{840}{99\pi} = 2.70\ldots$$

---

10 **Compare density**

This matches aluminium, whose density is $2.7\ \text{g/cm}^3$.

✓ **FINAL ANSWER**

Aluminium

4MA1/2H JUNE 2022

Question 3

3 marks | Standard & Compound Units

3 An aeroplane travelled from New York City to Los Angeles.

The aeroplane travelled a distance of 3980 kilometres in 5 hours 24 minutes.

Work out the average speed of the aeroplane.
Give your answer in kilometres per hour correct to the nearest whole number.

..... kilometres per hour

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2022

1 The time is

The time is

$$5\text{ h }24\text{ min} = 5.4\text{ h}$$

2 Average speed

Average speed:

$$3980 \div 5.4 = 737.037\ldots$$

✓ FINAL ANSWER

737 kilometres per hour.

4MA1/1H JUNE 2022

Question 7

4 marks | Standard & Compound Units

7 A cylinder is placed on the ground.

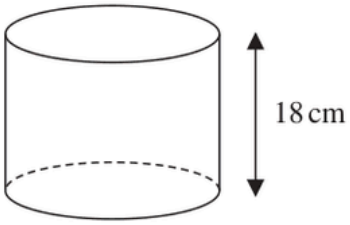


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm.

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2022

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$1.4 = \frac{72}{\text{area}}$$

$$\text{area} = \frac{72}{1.4} = 51.428\ldots \text{ cm}^2$$

2 The base area of the

The base area of the cylinder is $51.428\ldots$ cm^2.

3 **Volume is**

Volume is

$$51.428\ldots \times 18 = 925.714\ldots$$

✓ **FINAL ANSWER**

$926\ \text{cm}^3$ to 3 significant figures.

4MA1/1HR JUNE 2022

Question 7

3 marks | Standard & Compound Units

7 The diagram shows a solid cylinder made from iron.

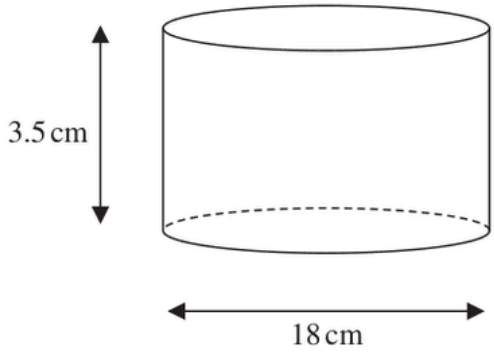


Diagram **NOT** accurately drawn

The cylinder has diameter 18 cm and height 3.5 cm
The mass of the cylinder is 7.04 kg

Work out the density of the iron.
Give your answer in g/cm^3 correct to 2 significant figures.

..... g/cm^3

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR June 2022

1 Radius of the cylinder is

Radius of the cylinder is

$$\frac{18}{2} = 9 \text{ cm}$$

2 Volume is

Volume is

$$\pi r^2 h = \pi(9^2)(3.5) = 283.5\pi$$

$$= 890.64\ldots \text{ cm}^3$$

3 Mass is

Mass is

$$7.04 \text{ kg} = 7040 \text{ g}$$

4 **Density is**

Density is

$$\frac{7040}{890.64\ldots} = 7.904\ldots$$

✓ **FINAL ANSWER**

$7.9\ \mathrm{g/cm^3}$ to 2 significant figures.

4MA1/2H JUNE 2022

Question 7

3 marks | Standard & Compound Units

7 Change a speed of 81 kilometres per hour to a speed in metres per second.

..... metres per second

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2022

1 Convert km/h to m/s divide

To convert km/h to m/s, divide by 3.6.

$$81 \div 3.6 = 22.5$$

FINAL ANSWER

$22.5\ \text{m/s}$.

4MA1/2HR JUNE 2023

Question 3

3 marks | Standard & Compound Units

3 Anjali travels on the Eurostar train from Paris to Amsterdam.

The distance the train travels between Paris and Amsterdam is 515 km.
The time taken by the train to travel between Paris and Amsterdam is 3 hours 18 minutes.

Work out the average speed of the train.
Give your answer in km/h correct to the nearest whole number.

..... km/h

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2023

1 Time hours minutes

Time $=$ 3 hours 18 minutes:

$$3 + \frac{18}{60} = 3.3 \text{ hours}$$

$$\text{average speed} = \frac{515}{3.3} = 156.06\ldots$$

FINAL ANSWER

156 km/h.

4MA1/2H JUNE 2023

Question 9

3 marks | Standard & Compound Units

9 Change a speed of 27 kilometres per hour to a speed in metres per second.

..... m/s

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2023

1 Convert km/h to m/s divide

To convert km/h to m/s, divide by 3.6.

$$27 \div 3.6 = 7.5$$

FINAL ANSWER

7.5 m/s.

Question 11

3 marks | Standard & Compound Units

11 The diagram shows a block of iron in the shape of a cuboid.

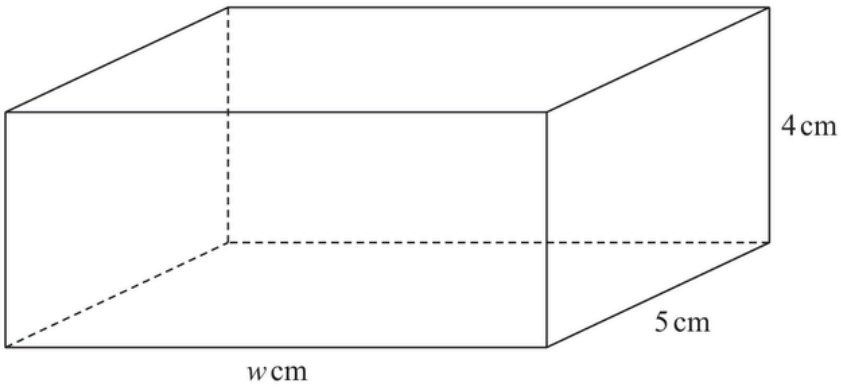


Diagram **NOT** accurately drawn

The block has length w cm, width 5 cm and height 4 cm

The density of iron is 7.8 g/cm^3
The mass of the block is 1950 g

Work out the value of w

$w =$ .....

(Total for Question 11 is 3 marks)

1 Calculate volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$7.8 = \frac{1950}{\text{volume}}$$

$$\text{volume} = \frac{1950}{7.8} = 250 \text{ cm}^3$$

2 Calculate value

For the cuboid,

$$w \times 5 \times 4 = 250$$

$$20w = 250$$

$$w = 12.5$$

FINAL ANSWER

$w = 12.5$ cm.

4MA1/2H JUNE 2024

Question 3

3 marks | Standard & Compound Units

3 A plane takes 9 hours 36 minutes to fly from New Delhi to Perth.

The plane flies at an average speed of 820 km/h.

Work out the total distance the plane flies.

..... km

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2024

1 Evaluate fraction

$$9\text{ h }36\text{ min} = 9 + \frac{36}{60} = 9.6\text{ hours}$$

$$\text{distance} = 820 \times 9.6 = 7872$$

FINAL ANSWER

7872 km

4MA1/2H JUNE 2024

Question 6

3 marks | Standard & Compound Units

6 Change a speed of w metres per second to a speed in kilometres per hour.
Give your answer in terms of w in its simplest form.

..... kilometres per hour

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2024

1 Convert m/s to km/h multiply

To convert m/s to km/h, multiply by 3.6.

$$w \text{ m/s} = 3.6w \text{ km/h}$$

$$3.6w = \frac{18w}{5}$$

FINAL ANSWER

$\frac{18w}{5}$ km/h.

4MA1/1HR MAY 2024

Question 6

4 marks | Standard & Compound Units

6 The diagram shows a solid wooden cylinder.

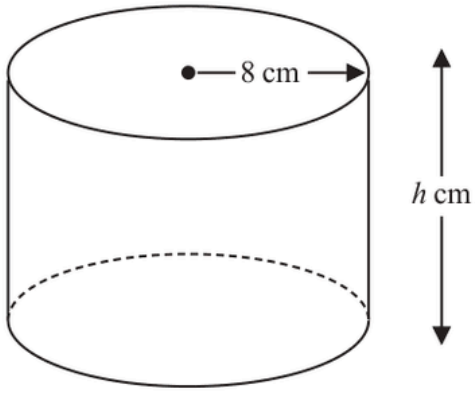


Diagram **NOT** accurately drawn

The cylinder has radius 8 cm and height h cm.
The volume of the cylinder is 1208 cm^3

(a) Work out the value of h
Give your answer correct to the nearest whole number.

$h =$ .....

(2)

The density of the wood is 1.25 g/cm^3

(b) Work out the mass of the cylinder.
Give your answer in kilograms.

..... kilograms

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May 2024

1 **Evaluate fraction**

For the cylinder,

$$V = \pi r^2 h$$

$$1208 = \pi(8)^2 h$$

$$h = \frac{1208}{64\pi} = 6.006\ldots$$

2 Calculate value

So, to the nearest whole number,

$$h = 6$$

3 Calculate volume

For the mass, use the given volume:

$$\text{mass} = \text{density} \times \text{volume}$$

$$1.25 \times 1208 = 1510 \text{ g}$$

$$1510 \text{ g} = 1.51 \text{ kg}$$

FINAL ANSWER

$h = 6$ cm, and the mass is 1.51 kg.

4MA1/1H MAY 2024

Question 8

3 marks | Standard & Compound Units

8 A cylinder is placed on a table.

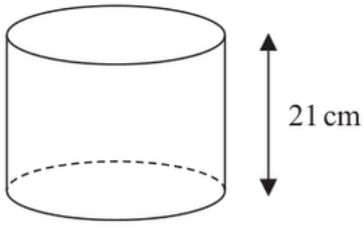


Diagram **NOT** accurately drawn

The volume of the cylinder is $1575\,\text{cm}^3$
The force exerted by the cylinder on the table is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the table due to the cylinder.

..... newtons/cm^2

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2024

1 **The volume of a cylinder**

The volume of a cylinder is

$$\text{volume} = \text{area of base} \times \text{height}$$

2 **Calculate area**

So the area in contact with the table is

$$1575 \div 21 = 75\ \text{cm}^2$$

3 **Use**

Use

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{pressure} = \frac{84}{75} = 1.12$$

FINAL ANSWER

$1.12 \text{ newtons/cm}^2$.

4WM1/01 MAY 2025

Question 6

3 marks | Standard & Compound Units

6 The diagram shows a block in the shape of a cuboid.

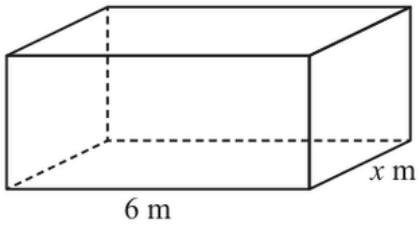


Diagram **NOT** accurately drawn

The block has a horizontal base with length 6 m and width x m

The block is placed on a table so that the whole of its horizontal base is in contact with the table.

The block exerts a force of 702 newtons on the table.
The pressure on the table due to the block is 65 newtons/m²

Work out the value of x

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 May 2025

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$65 = \frac{702}{\text{area}}$$

$$\text{area} = \frac{702}{65} = 10.8$$

2 **Calculate area**

The base area is $6x$, so

$$6x = 10.8$$

$$x = 1.8$$

✓ **FINAL ANSWER**

$x = 1.8$

4MA1/1HR JUNE 2025

Question 7

3 marks | Standard & Compound Units

7 Change a speed of $50x$ metres per second to a speed in kilometres per hour.

..... km/h

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR June 2025

1 Convert m/s to km/h multiply

To convert m/s to km/h, multiply by $\mathbf{3.6}$.

$$50x \times 3.6 = 180x$$

FINAL ANSWER

$180x$ km/h.

4MA1/2HR JUNE 2025

Question 7

2 marks | Standard & Compound Units

7 The mass of a silver coin is 48.3 g
The density of silver is 10.5 g/cm^3

Work out the volume of the silver coin.

..... cm^3

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2025

1 Use

Use

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2 Calculate volume

So

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{48.3}{10.5} = 4.6$$

FINAL ANSWER

$4.6\ \text{cm}^3$.

4WM1/01R MAY 2025

Question 7

3 marks | Standard & Compound Units

7 Change a speed of 40 metres per second to a speed in kilometres per hour.
Show your working clearly.

..... kilometres per hour

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4WM1/01R May 2025

1 Evaluate fraction

$$40\ \text{m/s} = 40 \times \frac{3600}{1000}\ \text{km/h}$$

$$= 144\ \text{km/h}$$

FINAL ANSWER

144 kilometres per hour.

4WM1/01R MAY 2025

Question 10

5 marks | Standard & Compound Units

10 The diagram shows a rectangular sheet of metal.

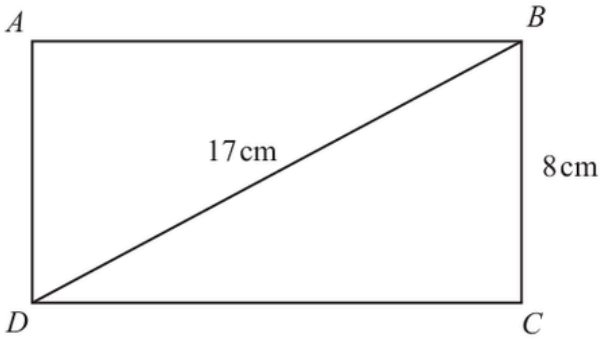


Diagram **NOT** accurately drawn

$BC = 8$ cm $\quad DB = 17$ cm

The sheet of metal lies flat on a table.
The force exerted by the sheet of metal on the table is 150 newtons.

Work out the pressure on the table due to the sheet of metal.
Show your working clearly.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

..... newtons/cm^2

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM1/01R May 2025

1 **Use Pythagoras to find the**

Use Pythagoras to find the other side of the rectangle.

$$DC = \sqrt{17^2 - 8^2} = 15$$

$$\text{area} = 15 \times 8 = 120 \text{ cm}^2$$

$$\text{pressure} = \frac{150}{120} = 1.25$$

FINAL ANSWER

1.25 newtons/cm^2.

4WM1/01 NOVEMBER 2025

Question 10

3 marks | Standard & Compound Units

10 A parcel in the shape of a cube is placed on a table so that the whole of one face of the parcel is in contact with the table.

The parcel has sides of length 7 cm

The pressure on the table due to the parcel is 62 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the force exerted by the parcel on the table.

..... newtons

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM1/01 November 2025

1 **The contact area is one**

The contact area is one square face:

$$7 \times 7 = 49 \text{ cm}^2$$

$$\text{force} = \text{pressure} \times \text{area} = 62 \times 49 = 3038$$

✓ **FINAL ANSWER**

3038 newtons.

4MA1/2H NOVEMBER 2025

Question 26

6 marks | Standard & Compound Units

26 A solid cone is joined to a solid hemisphere to make the solid shown below.

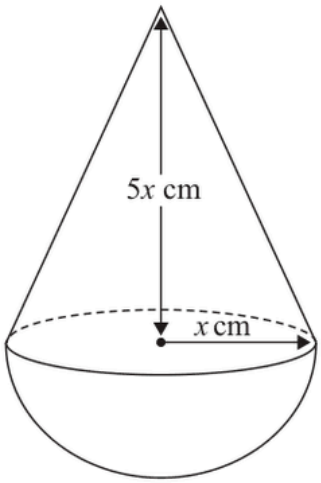


Diagram **NOT** accurately drawn

The cone is made from copper.
The density of copper is $9\,\text{g/cm}^3$

The hemisphere is made from a different metal.

The total mass of the solid is 4752π grams
The total volume of the solid is $504\pi\,\text{cm}^3$

Work out the density of the hemisphere.
Show your working clearly.

..... g/cm^3

(Total for Question 26 is 6 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H November 2025

1 **Volume of cone**

Volume of cone:

$$\frac{1}{3}\pi x^2(5x) = \frac{5}{3}\pi x^3$$

2 **Volume of hemisphere**

Volume of hemisphere:

$$\frac{2}{3}\pi x^3$$

3 **Total volume**

Total volume:

$$\frac{5}{3}\pi x^3 + \frac{2}{3}\pi x^3 = 504\pi$$

$$\frac{7}{3}\pi x^3 = 504\pi$$

$$x^3 = 216$$

$$x = 6$$

4 **Cone volume**

Cone volume:

$$\frac{5}{3}\pi (6)^3 = 360\pi$$

5 **Mass of copper cone**

Mass of copper cone:

$$360\pi \times 9 = 3240\pi$$

6 **Mass of hemisphere**

Mass of hemisphere:

$$4752\pi - 3240\pi = 1512\pi$$

7 **Hemisphere volume**

Hemisphere volume:

$$\frac{2}{3}\pi (6)^3 = 144\pi$$

8 **Density of the hemisphere**

Density of the hemisphere:

$$\frac{1512\pi}{144\pi} = 10.5$$

FINAL ANSWER

$10.5\ \text{g/cm}^3$

4MA1/1H JUNE 2026

Question 4

4 marks | Standard & Compound Units

4 Henri competes in a race.

For the first part of the race Henri cycles 60 km at an average speed of 24 km/h
For the remaining part of the race Henri runs for 1 hour and 15 minutes at an average speed of 12 km/h

Work out Henri's average speed for the whole race.
Give your answer in km/h
Show your working clearly.

..... km/h

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2026

1 Solution | Step 1

Cycling time: $60 \div 24 = 2.5$ hours. Running time: $1 + 15/60 = 1.25$ hours.
Time equals distance divided by speed.

2 Solution | Step 2

Running distance: $12 \times 1.25 = 15$ km. Total distance: $60 + 15 = 75$ km. Total time: $2.5 + 1.25 = 3.75$ hours.
Combine the actual distances and times.

3 Solution | Step 3

$$\text{Average speed} = \frac{75}{3.75} = 20 \text{ km/h}.$$

Average speed is total distance divided by total time, not the mean of the two speeds.

FINAL ANSWER

20 km/h

4MA1/1H JUNE 2026

Question 8

3 marks | Standard & Compound Units

8 Change a speed of 70 metres per second to a speed in kilometres per hour.
Show your working clearly.

..... km/h

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2026

1 Solution | Step 1

$$70\ \text{m/s} = 70 \times 3600 = 252000\ \text{m/h}.$$

There are 3600 seconds in one hour.

2 Solution | Step 2

$$252000 \div 1000 = 252\ \text{km/h}.$$

There are 1000 metres in one kilometre.

FINAL ANSWER

$252\ \text{km/h}$

4WM1/01 MAY 2026

Question 8

3 marks | Standard & Compound Units

8 A block of wood has a mass of 480 kg
The wood has a density of 640 kg/m^3

Work out the volume of the block of wood.

..... m^3

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 May 2026

1 **Volume = mass ÷ density**

$$V = \frac{480}{640} = 0.75 \text{ m}^3$$

FINAL ANSWER

0.75 m^3

4MA1/2HR JUNE 2026

Question 9

3 marks | Standard & Compound Units

9 Change a speed of 198 kilometres per hour to a speed in metres per second.
Show your working clearly.

..... m/s

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2026

1 Solution | Step 1

$198\ \text{km} = 198000\ \text{m}$ and $1\ \text{hour} = 3600\ \text{s}$.

Convert both the distance and the time in the rate.

2 Solution | Step 2

$$198\ \text{km/h} = \frac{198000}{3600} = 55\ \text{m/s}.$$

Metres per second means metres divided by seconds.

✓ FINAL ANSWER

$55\ \text{m/s}$.

4MA1/2HR JUNE 2026

Question 10

3 marks | Standard & Compound Units

10 The diagram shows a solid cylinder.

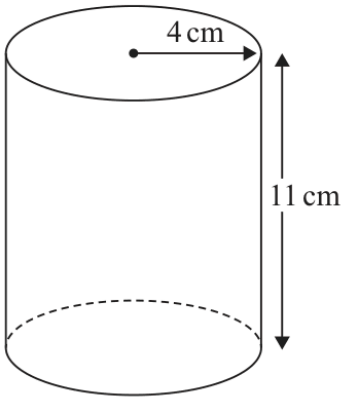


Diagram **NOT** accurately drawn

The cylinder has radius 4 cm and height 11 cm
The cylinder is made from wood with a density of 0.6 g/cm^3

Work out the mass of the cylinder.
Give your answer correct to 3 significant figures.

..... g

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$$V = \pi r^2 h = \pi(4)^2(11) = 176\pi \text{ cm}^3.$$

Use the cylinder volume formula.

2 **Solution | Step 2**

$$m = \rho V = 0.6(176\pi) = 105.6\pi = 331.7521842\ldots \text{ g}.$$

Mass equals density times volume; keep full precision until the end.

3 **Solution | Step 3**

$$m = 332 \text{ g} \quad (3 \text{ significant figures}).$$

Round the final mass as requested.

FINAL ANSWER

332 g (3 significant figures).

TOPIC

Algebra Toolkit

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 22

2 marks | Algebra Toolkit

22 (a) Simplify $5a + 3b - 2a + 7b$

.....

(1)

(b) Simplify $4x \times 3y$

.....

(1)

(Total for Question 22 is 2 marks)

WORKED SOLUTION

Question 22 · EP Specimen

1 **(a) Collect like terms**

$$5a - 2a = 3a, \qquad 3b + 7b = 10b \Rightarrow 3a + 10b$$

2 **(b) Multiply the numbers, then the letters**

$$4 \times 3 = 12 \Rightarrow 12xy$$

✓ **FINAL ANSWER**

(a) $3a + 10b$ (b) $12xy$

EP SPECIMEN

Question 23

2 marks | Algebra Toolkit

23 Work out the value of $3p^2 - 2q$ when $p = -4$ and $q = 5$

.....

(2)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · EP Specimen

1 **Substitute, with brackets round the negative**

$$3(-4)^2 - 2(5) = 3 \times 16 - 10$$

2 **Work it out**

$$48 - 10 = 38$$

✓ **FINAL ANSWER**

38

EP SPECIMEN

Question 24

2 marks | Algebra Toolkit

24 $v = u + at$

Work out the value of v when $u = 12$, $a = -3.5$ and $t = 6$

$v =$ .....

(2)

(Total for Question 24 is 2 marks)

WORKED SOLUTION

Question 24 · EP Specimen

1 Substitute

$$v = 12 + (-3.5) \times 6$$

2 Work it out

$$v = 12 - 21 = -9$$

FINAL ANSWER

$v = -9$

EP SPECIMEN

Question 25

2 marks | Algebra Toolkit

25 Pens cost 35p each. Pencils cost 18p each.

Write down an expression, in terms of x and y, for the total cost, in pence, of x pens and y pencils.

.....

(2)

(Total for Question 25 is 2 marks)

WORKED SOLUTION

Question 25 · EP Specimen

1 **Cost of each item × number bought**

x pens cost $35x$ pence; y pencils cost $18y$ pence.

2 **Add**

$35x + 18y$

✓ **FINAL ANSWER**

$35x + 18y$

EP SPECIMEN

Question 26

2 marks | Algebra Toolkit

26 $T = \dfrac{a^2 + b}{2c}$

Work out the value of T when $a = -3$, $b = 7$ and $c = 0.5$

$T =$ .....

(2)

(Total for Question 26 is 2 marks)

WORKED SOLUTION

Question 26 · EP Specimen

1 Substitute

$$T = \frac{(-3)^2 + 7}{2 \times 0.5} = \frac{9 + 7}{1}$$

2 Work it out

$T = 16$

FINAL ANSWER

$T = 16$

EP SPECIMEN

Question 27

2 marks | Algebra Toolkit

27 Simplify $6x^2y - 2xy^2 + 3x^2y + 5xy^2$

.....

(2)

(Total for Question 27 is 2 marks)

WORKED SOLUTION

Question 27 · EP Specimen

1 Like terms have exactly the same letters and powers

x^2y terms: $6 + 3 = 9$. xy^2 terms: $-2 + 5 = 3$.

2 Write the answer

$9x^2y + 3xy^2$

FINAL ANSWER

$9x^2y + 3xy^2$

EP SPECIMEN

Question 28

2 marks | Algebra Toolkit

28 A rectangle has length $(3x + 2)$ cm and width $(x - 1)$ cm.

Show that the perimeter, in cm, of the rectangle is $8x + 2$

.....

(2)

(Total for Question 28 is 2 marks)

WORKED SOLUTION

Question 28 · EP Specimen

1 **Perimeter = 2 lengths + 2 widths**

$$2(3x + 2) + 2(x - 1)$$

2 **Expand and collect**

$$6x + 4 + 2x - 2 = 8x + 2$$

✓ **FINAL ANSWER**

Shown: $8x + 2$

EP SPECIMEN

Question 29

2 marks | Algebra Toolkit

29 Here are five expressions.

$3n$ $\quad n+3$ $\quad n^3$ $\quad 3n^2$ $\quad -n$

When $n = -2$, which expression has the greatest value?
You must show your working.

.....

(2)

(Total for Question 29 is 2 marks)

WORKED SOLUTION

Question 29 · EP Specimen

1 Substitute n = −2 into each expression

$$3n = -6, \quad n+3 = 1, \quad n^3 = -8, \quad 3n^2 = 3 \times 4 = 12, \quad -n = 2$$

2 Pick the greatest

$3n^2 = 12$ is the greatest.

FINAL ANSWER

$3n^2$ (value 12)

TOPIC

Expanding Brackets

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

Question 14

3 marks | Expanding Brackets

14 Expand and simplify $(4x + 1)(x - 3)(5x + 6)$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR May/November 2020

1 Expand brackets

$$(4x + 1)(x - 3)(5x + 6)$$

$$(x - 3)(5x + 6) = 5x^2 - 9x - 18$$

$$(4x + 1)(5x^2 - 9x - 18) = 20x^3 - 31x^2 - 81x - 18$$

FINAL ANSWER

$20x^3 - 31x^2 - 81x - 18$

4MA1/2HR JANUARY 2021

Question 1

8 marks | Expanding Brackets

1 $w = 5y^2 - y^3$

(a) Work out the value of w when $y = -2$

$w =$ .....

(2)

(b) Factorise fully $8p^2 - 2p$

.....

(2)

(c) Expand $4t(3t - 2)$

.....

(2)

(d) Expand and simplify $(5x - 2)(x + 4)$

.....

(2)

(Total for Question 1 is 8 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2021

1 **Expand brackets**

For $w = 5y^2 - y^3$ and $y = -2$:

$$w = 5(-2)^2 - (-2)^3 = 20 + 8 = 28$$

$$8p^2 - 2p = 2p(4p - 1)$$

$$4t(3t - 2) = 12t^2 - 8t$$

$$(5x - 2)(x + 4) = 5x^2 + 18x - 8$$

FINAL ANSWER

$28,\ 2p(4p - 1),\ 12t^2 - 8t,\ 5x^2 + 18x - 8$

4MA1/1H JANUARY 2021

Question 12

6 marks | Expanding Brackets

12 (a) Express $\frac{4}{x-2} - \frac{3}{x+1}$ as a single fraction.
Give your answer in its simplest form.

.....

(3)

(b) Expand and simplify $2x(x-5)(x-3)$

.....

(3)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H January 2021

1 **(a)**

(a)

$$\frac{4}{x-2} - \frac{3}{x+1} = \frac{4(x+1) - 3(x-2)}{(x-2)(x+1)}$$

$$= \frac{4x + 4 - 3x + 6}{(x-2)(x+1)} = \frac{x+10}{(x-2)(x+1)}$$

2 **(b)**

(b)

$$2x(x-5)(x-3) = 2x(x^2 - 8x + 15)$$

$$= 2x^3 - 16x^2 + 30x$$

✓ **FINAL ANSWER**

(a) $\frac{x+10}{(x-2)(x+1)}$, (b) $2x^3 - 16x^2 + 30x$

4MA1/1HR JANUARY 2021

Question 16

5 marks | Expanding Brackets

16 (a) Expand and simplify $(x + 4)(x - 2)(x + 1)$

.....

(3)

(b) Express $x^2 - 10x + 40$ in the form $(x + a)^2 + b$, where a and b are integers.

.....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$(x+4)(x-2)(x+1)$$

$$(x-2)(x+1) = x^2 - x - 2$$

$$(x+4)(x^2 - x - 2) = x^3 + 3x^2 - 6x - 8$$

2 **(b)**

(b)

$$x^2 - 10x + 40 = (x-5)^2 + 15$$

✓ **FINAL ANSWER**

(a) $x^3 + 3x^2 - 6x - 8$, (b) $(x-5)^2 + 15$

4MA1/1H MAY 2021

Question 13

3 marks | Expanding Brackets

13 Expand and simplify $4x(3x + 1)(2x - 3)$
Show your working clearly.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2021

1 Expand brackets

$$4x(3x+1)(2x-3) = 4x(6x^2 - 7x - 3)$$

$$= 24x^3 - 28x^2 - 12x$$

FINAL ANSWER

$24x^3 - 28x^2 - 12x$

4MA1/1H NOVEMBER 2021

Question 1

3 marks | Expanding Brackets

1 (a) Simplify $e^8 \div e^2$

.....

(1)

(b) Expand and simplify $(x - 3)(x + 1)$

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2021

1 **(a)**

(a) $e^8 \div e^2 = e^6$.

2 **(b)**

(b) $(x - 3)(x + 1) = x^2 - 2x - 3$.

✓ **FINAL ANSWER**

(a) e^6, (b) $x^2 - 2x - 3$

4MA1/2H NOVEMBER 2021

Question 12

7 marks | Expanding Brackets

12 (a) Simplify $(64p^9q^{12})^{\frac{2}{3}}$

.....

(2)

(b) Write as a single fraction $\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x}$

Give your answer in its simplest form.

.....

(2)

(c) Expand and simplify $4x(x - 5)(2x + 3)$

Show your working clearly.

.....

(3)

(Total for Question 12 is 7 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2021

1 **(a)**

(a)

$$(64p^9q^{12})^{2/3} = 16p^6q^8$$

2 **(b)**

(b)

$$\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x} = \frac{20 + 24 - 27}{30x} = \frac{17}{30x}$$

3 **(c)**

(c)

$$4x(x-5)(2x+3) = 4x(2x^2 - 7x - 15)$$

$$= 8x^3 - 28x^2 - 60x$$

✓ **FINAL ANSWER**

(a) $16p^6q^8$, (b) $\frac{17}{30x}$, (c) $8x^3 - 28x^2 - 60x$

4MA1/2H JANUARY 2022

Question 1

4 marks | Expanding Brackets

1 (a) Expand and simplify $(y + 4)(2 - y)$

.....

(2)

(b) Factorise fully $15b^5c - 35b^3c^9$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2022

1 (a)

(a)

$$(y+4)(2-y) = 2y - y^2 + 8 - 4y = -y^2 - 2y + 8$$

2 (b)

(b)

$$15b^5c - 35b^3c^9 = 5b^3c(3b^2 - 7c^8)$$

FINAL ANSWER

(a) $-y^2 - 2y + 8$, (b) $5b^3c(3b^2 - 7c^8)$

4MA1/1H JANUARY 2022

Question 13

6 marks | Expanding Brackets

13 (a) Expand and simplify $5x(x + 2)(3x - 4)$

.....

(3)

(b) Simplify completely $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

.....

(3)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x(x+2)(3x-4) = 5x(3x^2+2x-8)$$

$$= 15x^3 + 10x^2 - 40x$$

2 **(b)**

(b)

$$\left(\frac{16w^8}{y^{20}}\right)^{-3/4} = 16^{-3/4}w^{-6}y^{15}$$

3 **Expand brackets**

Since $16^{3/4} = 8$,

$$= \frac{y^{15}}{8w^6}$$

✓ **FINAL ANSWER**

(a) $15x^3 + 10x^2 - 40x$, (b) $\frac{y^{15}}{8w^6}$

4MA1/1HR JUNE 2022

Question 16

6 marks | Expanding Brackets

16 (a) Expand and simplify $(3x - 1)(x + 2)(3x + 1)$

.....

(3)

(b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

.....

(3)

(Total for Question 16 is 6 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$(3x-1)(x+2)(3x+1) = (9x^2-1)(x+2)$$

$$= 9x^3 + 18x^2 - x - 2$$

2 **(b)**

(b)

$$\left(\frac{2x^5}{8xy^2}\right)^{-2} = \left(\frac{x^4}{4y^2}\right)^{-2}$$

$$= \left(\frac{4y^2}{x^4}\right)^{2} = \frac{16y^4}{x^8}$$

✓ **FINAL ANSWER**

(a) $9x^3 + 18x^2 - x - 2$, (b) $\frac{16y^4}{x^8}$

4MA1/2HR JUNE 2023

Question 1

4 marks | Expanding Brackets

1 $P = m^2 - 4c$

(a) Work out the value of P when $m = -5$ and $c = 3$

$P =$ .....

(2)

(b) Expand and simplify $(x + 5)(x - 7)$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2023

1 **Expand brackets**

$$P = m^2 - 4c = (-5)^2 - 4(3) = 25 - 12 = 13$$

$$(x + 5)(x - 7) = x^2 - 7x + 5x - 35 = x^2 - 2x - 35$$

✓ **FINAL ANSWER**

$P = 13$, and $x^2 - 2x - 35$

4MA1/2H NOVEMBER 2023

Question 16

3 marks | Expanding Brackets

16 Expand and simplify $(2x + 3)(x - 5)(x + 4)$

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2023

1 Expand brackets

$$(x - 5)(x + 4) = x^2 - x - 20$$

$$(2x + 3)(x^2 - x - 20) = 2x^3 + x^2 - 43x - 60$$

FINAL ANSWER

$2x^3 + x^2 - 43x - 60$

4MA1/1HR MAY 2024

Question 14

6 marks | Expanding Brackets

14 (a) Expand and simplify $(3x+1)(2-x)(4+x)$

.....

(3)

(b) Simplify fully $\left(\frac{a^3b}{a^9b^5}\right)^{-\frac{1}{2}}$

.....

(3)

(Total for Question 14 is 6 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR May 2024

1 **(a)**

(a)

$$(2-x)(4+x) = 8 - 2x - x^2$$

$$(3x+1)(8-2x-x^2) = -3x^3 - 7x^2 + 22x + 8$$

2 **(b)**

(b)

$$\left(\frac{a^3b}{a^9b^5}\right)^{-1/2} = \left(\frac{1}{a^6b^4}\right)^{-1/2} = a^3b^2$$

✓ **FINAL ANSWER**

(a) $-3x^3 - 7x^2 + 22x + 8$, (b) a^3b^2

4MA1/1H NOVEMBER 2024

Question 15

3 marks | Expanding Brackets

15 Write $3x(2x-1)(5x+4)$ in the form ax^3+bx^2+cx where a, b and c are integers.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2024

1 Expand brackets

$$3x(2x-1)(5x+4)$$

2 Expand brackets

First expand the two brackets:

$$(2x-1)(5x+4) = 10x^2+8x-5x-4$$

$$= 10x^2+3x-4$$

3 Expand brackets

Now multiply by $3x$:

$$3x(10x^2+3x-4) = 30x^3+9x^2-12x$$

FINAL ANSWER

$30x^3+9x^2-12x$

4WM1/01 MAY 2025

Question 11

6 marks | Expanding Brackets

11 (a) Expand and simplify $5x(5x + 4)(2x - 3)$

.....

(3)

(b) Express $\frac{9}{4} + \frac{y - 7}{5y}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4WM1/01 May 2025

1 **Expand brackets**

$$5x(5x + 4)(2x - 3)$$

2 **Expand brackets**

First,

$$(5x + 4)(2x - 3) = 10x^2 - 15x + 8x - 12 = 10x^2 - 7x - 12$$

3 **Expand brackets**

Then multiply by $5x$.

$$5x(10x^2 - 7x - 12) = 50x^3 - 35x^2 - 60x$$

4 **Expand brackets**

For part (b), use denominator $20y$.

$$\frac{9}{4} + \frac{y - 7}{5y} = \frac{45y}{20y} + \frac{4(y - 7)}{20y}$$

$$= \frac{49y - 28}{20y}$$

FINAL ANSWER

$50x^3 - 35x^2 - 60x$ and $\frac{49y - 28}{20y}$.

4MA1/2H JUNE 2025

Question 12

3 marks | Expanding Brackets

12 Expand and simplify $3x(2x+5)(7x-4)$

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2025

1 Expand brackets

$$3x(2x+5)(7x-4) = 3x(14x^2+27x-20)$$

$$= 42x^3+81x^2-60x$$

FINAL ANSWER

$42x^3+81x^2-60x$

4MA1/2HR JUNE 2025

Question 13

5 marks | Expanding Brackets

13 (a) Factorise $4x^2 - 25y^2$

.....

(2)

(b) Show that $4x(x + 3)(2x - 5)$ can be written in the form $ax^3 + bx^2 + cx$ where a, b and c are integers to be found.

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2025

1 **(a)**

(a)

$$4x^2 - 25y^2 = (2x - 5y)(2x + 5y)$$

2 **(b)**

(b)

$$4x(x + 3)(2x - 5)$$

3 **Factorise expression**

First,

$$(x + 3)(2x - 5) = 2x^2 + x - 15$$

4 **Factorise expression**

Then

$$4x(2x^2 + x - 15) = 8x^3 + 4x^2 - 60x$$

5 **Factorise expression**

So $a = 8$, $b = 4$, and $c = -60$.

FINAL ANSWER

(a) $(2x - 5y)(2x + 5y)$, (b) $8x^3 + 4x^2 - 60x$

4WM1/01R MAY 2025

Question 15

6 marks | Expanding Brackets

15 (a) Solve $3x^2 + 4x - 18 = 0$
Show your working clearly.
Give your solutions correct to 3 significant figures.

.....

(3)

(b) Expand and simplify $(2x - 3)(x + 4)(x - 1)$

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4WM1/01R May 2025

1 **Simplify surd**

(a) For $3x^2 + 4x - 18 = 0$,

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-18)}}{2(3)}$$

$$x \approx 1.87 \quad \text{or} \quad x \approx -3.21$$

2 **(b)**

(b)

$$(2x - 3)(x + 4)(x - 1)$$

$$= (2x^2 + 5x - 12)(x - 1)$$

$$= 2x^3 + 3x^2 - 17x + 12$$

✓ **FINAL ANSWER**

$x = 1.87$ or $x = -3.21$; $2x^3 + 3x^2 - 17x + 12$.

4WM1/01 NOVEMBER 2025

Question 13

6 marks | Expanding Brackets

13 (a) Expand and simplify $7x(3x + 2)(2x - 5)$

.....

(3)

(b) Solve $\frac{9}{2y} + \frac{5}{7} = 5$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4WM1/01 November 2025

1 **(a)**

(a)

$$7x(3x + 2)(2x - 5)$$

$$= 7x(6x^2 - 11x - 10)$$

$$= 42x^3 - 77x^2 - 70x$$

2 **(b)**

(b)

$$\frac{9}{2y} + \frac{5}{7} = 5$$

$$\frac{9}{2y} = \frac{30}{7}$$

$$63 = 60y$$

$$y = \frac{21}{20}$$

FINAL ANSWER

$42x^3 - 77x^2 - 70x$, $y = \frac{21}{20}$.

4MA1/1H NOVEMBER 2025

Question 14

6 marks | Expanding Brackets

14 (a) Expand and simplify $7x(3x + 2)(2x - 5)$

.....

(3)

(b) Solve $\frac{9}{2y} + \frac{5}{7} = 5$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 14 is 6 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2025

1 **(a)**

(a)

$$7x(3x+2)(2x-5) = 7x(6x^2 - 11x - 10)$$

$$= 42x^3 - 77x^2 - 70x$$

2 **(b)**

(b)

$$\frac{9}{2y} + \frac{5}{7} = 5$$

$$\frac{9}{2y} = \frac{30}{7}$$

$$63 = 60y$$

FINAL ANSWER

(a) $42x^3 - 77x^2 - 70x$, (b) $y = \frac{21}{20}$

4MA1/1H JUNE 2026

Question 12

6 marks | Expanding Brackets

12 $\dfrac{2^3}{8^x} = 64$

(a) Find the value of x
Show clear algebraic working.

$x =$ .....

(3)

(b) Expand and simplify $4t(t + 5)(2t - 3)$

.....

(3)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$\frac{2^{x/3}}{8^x} = \frac{2^{x/3}}{2^{3x}} = 2^{x/3-3x} = 2^{-8x/3}.$$

Write 8 as a power of 2 and subtract exponents when dividing.

2 **Part (a) | Step 2**

$$2^{-8x/3} = 64 = 2^6 \implies -\frac{8x}{3} = 6 \implies x = -\frac{9}{4}.$$

Equal powers of base 2 have equal exponents.

3 **Part (b) | Step 1**

$$(t+5)(2t-3) = 2t^2 - 3t + 10t - 15 = 2t^2 + 7t - 15.$$

Expand the two brackets and collect like terms.

4 **Part (b) | Step 2**

$$4t(t+5)(2t-3) = 4t(2t^2 + 7t - 15) = 8t^3 + 28t^2 - 60t.$$

Multiply every term by 4t.

FINAL ANSWER

(a) $x = -\frac{9}{4}$ (b) $8t^3 + 28t^2 - 60t$

TOPIC

Factorising

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY/NOVEMBER 2020

Question 7

8 marks | Factorising

7 (a) Solve $5(4 - x) = 7 - 3x$
Show clear algebraic working.

$x =$ .....

(3)

(b) Factorise fully $16m^3g^3 + 24m^2g^5$

.....

(2)

(c) (i) Factorise $y^2 - 2y - 48$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 48 = 0$

.....

(1)

(Total for Question 7 is 8 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May/November 2020

1 **(a)**

(a)

$$5(4 - x) = 7 - 3x$$

$$20 - 5x = 7 - 3x$$

$$13 = 2x$$

$$x = \frac{13}{2}$$

2 **(b)**

(b)

$$16m^3g^3 + 24m^2g^5 = 8m^2g^3(2m + 3g^2)$$

3 **(c)(i)**

(c)(i)

$$y^2 - 2y - 48 = (y - 8)(y + 6)$$

4 **Factorise expression**

(c)(ii) $y = 8$ or $y = -6$.

FINAL ANSWER

$x = \frac{13}{2}$, $8m^2g^3(2m + 3g^2)$, $(y - 8)(y + 6)$, $y = 8$ or $y = -6$

4MA1/1H JANUARY 2021

Question 9

5 marks | Factorising

9 (a) Simplify $(2x^3y^5)^4$

.....

(2)

(b) (i) Factorise $x^2 + 5x - 36$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 36 = 0$

.....

(1)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2021

1 **(a)**

(a)

$$(2x^3y^5)^4 = 16x^{12}y^{20}$$

2 **(b)(i)**

(b)(i)

$$x^2 + 5x - 36 = (x + 9)(x - 4)$$

3 **Factorise expression**

(b)(ii) $x = -9$ or $x = 4$.

✓ **FINAL ANSWER**

(a) $16x^{12}y^{20}$, (b)(i) $(x + 9)(x - 4)$, (b)(ii) $x = -9$ or $x = 4$

4MA1/1H MAY 2021

Question 9

5 marks | Factorising

9 (a) Factorise fully $25a^4c^7d + 45a^9c^3h$

.....

(2)

(b) Solve $(2x + 5)^2 = (2x + 3)(2x - 1)$

$x =$ .....

(3)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2021

1 **(a)**

(a)

$$25a^4c^7d + 45a^9c^3h = 5a^4c^3(5c^4d + 9a^5h)$$

2 **(b)**

(b)

$$(2x + 5)^2 = (2x + 3)(2x - 1)$$

$$4x^2 + 20x + 25 = 4x^2 + 4x - 3$$

$$16x = -28$$

$$x = -\frac{7}{4}$$

✓ **FINAL ANSWER**

(a) $5a^4c^3(5c^4d + 9a^5h)$, (b) $x = -\frac{7}{4}$

4MA1/2H MAY 2021

Question 9

3 marks | Factorising

9 (i) Factorise $x^2 + 2x - 24$

.....

(2)

(ii) Hence solve $x^2 + 2x - 24 = 0$

.....

(1)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H May 2021

1 Factorise expression

$$x^2 + 2x - 24 = (x + 6)(x - 4)$$

2 Factorise expression

So

$$(x + 6)(x - 4) = 0$$

FINAL ANSWER

$x = -6$ or $x = 4$

4MA1/2H NOVEMBER 2021

Question 8

4 marks | Factorising

8 (a) Simplify $(3x^2y)^0$

.....

(1)

(b) (i) Factorise $x^2 - 5x - 36$

.....

(2)

(ii) Hence solve $x^2 - 5x - 36 = 0$

.....

(1)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2021

1 (a)

(a) $(3x^2y)^0 = 1$.

2 (b)(i)

(b)(i)

$$x^2 - 5x - 36 = (x - 9)(x + 4)$$

3 Factorise expression

(b)(ii) $x = 9$ or $x = -4$.

FINAL ANSWER

1, $(x - 9)(x + 4)$, $x = 9$ or $x = -4$

4MA1/1HR JANUARY 2022

Question 9

5 marks | Factorising

9 (a) Simplify $x^4 \times x^5$

.....

(1)

(b) Simplify $(4y^2)^3$

.....

(2)

(c) Factorise $n^2 - 7n + 12$

.....

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2022

1 **(a)**

(a) $x^4 \times x^5 = x^9$.

2 **(b)**

(b) $(4y^2)^3 = 64y^6$.

3 **(c)**

(c)

$$n^2 - 7n + 12 = (n - 3)(n - 4)$$

✓ **FINAL ANSWER**

(a) x^9, (b) $64y^6$, (c) $(n - 3)(n - 4)$

4MA1/1HR JUNE 2022

Question 5

5 marks | Factorising

5 (a) Expand $3c^3(c + 4)$

.....

(2)

(b) (i) Factorise $x^2 + 8x - 9$

.....

(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

.....

(1)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR June 2022

1 (a)

(a) $3c^3(c+4) = 3c^4 + 12c^3$.

2 (b)(i)

(b)(i)

$$x^2 + 8x - 9 = (x+9)(x-1)$$

3 (b)(ii)

(b)(ii)

$$(x+9)(x-1) = 0$$

FINAL ANSWER

(a) $3c^4 + 12c^3$, (b)(i) $(x+9)(x-1)$, (b)(ii) $x = -9$ or $x = 1$

4MA1/1H JUNE 2022

Question 11

5 marks | Factorising

11 (a) Factorise $9x^2 - 4y^2$

.....

(2)

(b) Express $\frac{7}{8} - \frac{x+3}{4x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2022

1 **(a)**

(a)

$$9x^2 - 4y^2 = (3x)^2 - (2y)^2 = (3x - 2y)(3x + 2y)$$

2 **(b)**

(b)

$$\frac{7}{8} - \frac{x+3}{4x} = \frac{7x}{8x} - \frac{2(x+3)}{8x}$$

$$= \frac{7x - 2x - 6}{8x} = \frac{5x - 6}{8x}$$

FINAL ANSWER

(a) $(3x - 2y)(3x + 2y)$, (b) $\frac{5x-6}{8x}$

4MA1/2H JUNE 2022

Question 11

3 marks | Factorising

11 (i) Factorise $x^2 + 5x - 24$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

.....

(1)

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2022

1 **Factorise expression**

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

2 **Factorise expression**

So

$$(x + 8)(x - 3) = 0$$

✓ **FINAL ANSWER**

$x = -8$ or $x = 3$

4MA1/1H MAY 2023

Question 6

8 marks | Factorising

6 (a) Simplify $(2c^4d^7)^3$

.....

(2)

(b) Find the value of $5y^0$ where $y > 0$

.....

(1)

(c) Factorise fully $16a^2b^3 + 20a^3b$

.....

(2)

(d) (i) Factorise $x^2 + 9x - 22$

.....

(2)

(ii) Hence solve $x^2 + 9x - 22 = 0$

.....

(1)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2023

1 **(a)**

(a)

$$(2c^4d^7)^3 = 8c^{12}d^{21}$$

2 **Factorise expression**

(b) $5y^0 = 5$, since $y^0 = 1$.

3 **(c)**

(c)

$$16a^2b^3 + 20a^3b = 4a^2b(4b^2 + 5a)$$

4 **(d)(i)**

(d)(i)

$$x^2 + 9x - 22 = (x + 11)(x - 2)$$

5 **Factorise expression**

(d)(ii) $x = -11$ or $x = 2$.

✓ **FINAL ANSWER**

$8c^{12}d^{21}$, 5, $4a^2b(4b^2 + 5a)$, $(x + 11)(x - 2)$, $x = -11$ or $x = 2$

4MA1/1HR MAY 2023

Question 7

3 marks | Factorising

7 (a) Expand $4x(x-5)$

.....

(1)

(b) Factorise $y^2 - 9y + 20$

.....

(2)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May 2023

1 (a)

(a)

$$4x(x-5) = 4x^2 - 20x$$

2 (b)

(b)

$$y^2 - 9y + 20 = (y-4)(y-5)$$

FINAL ANSWER

(a) $4x^2 - 20x$, (b) $(y-4)(y-5)$

4MA1/1H MAY 2023

Question 14

3 marks | Factorising

14 Factorise fully $50g^2 - 18$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2023

1 Factorise expression

$$50g^2 - 18 = 2(25g^2 - 9)$$

$$= 2(5g - 3)(5g + 3)$$

FINAL ANSWER

$2(5g - 3)(5g + 3)$

4MA1/2H JUNE 2024

Question 11

3 marks | Factorising

11 (i) Factorise $x^2 + 9x - 22$

.....

(2)

(ii) Hence, solve $x^2 + 9x - 22 = 0$

.....

(1)

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2024

1 **Factorise expression**

$$x^2 + 9x - 22 = (x + 11)(x - 2)$$

2 **Factorise expression**

So

$$(x + 11)(x - 2) = 0$$

FINAL ANSWER

$x = -11$ or $x = 2$

4MA1/1H JUNE 2025

Question 6

6 marks | Factorising

6 (a) Solve $x - 4 = \frac{3 + 2x}{6}$

Show clear algebraic working.

$x =$ .....

(3)

(b) (i) Factorise $y^2 - 11y + 30$

.....

(2)

(ii) Hence solve $y^2 - 11y + 30 = 0$

.....

(1)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2025

1 **(a)**

(a)

$$x - 4 = \frac{3 + 2x}{6}$$

$$6x - 24 = 3 + 2x$$

$$4x = 27$$

$$x = \frac{27}{4}$$

2 **(b)(i)**

(b)(i)

$$y^2 - 11y + 30 = (y - 5)(y - 6)$$

3 **(b)(ii)**

(b)(ii)

$$y = 5 \quad \text{or} \quad y = 6$$

✓ **FINAL ANSWER**

$x = \frac{27}{4}$, $(y-5)(y-6)$, $y = 5$ or $y = 6$

4WM1/01 MAY 2025

Question 7

6 marks | Factorising

7 (a) Solve $x - 4 = \dfrac{3 + 2x}{6}$

Show clear algebraic working.

$x =$ .....

(3)

(b) (i) Factorise $y^2 - 11y + 30$

.....

(2)

(ii) Hence solve $y^2 - 11y + 30 = 0$

.....

(1)

(Total for Question 7 is 6 marks)

WORKED SOLUTION

Question 7 · 4WM1/01 May 2025

1 **Factorise expression**

For part (a),

$$x - 4 = \frac{3 + 2x}{6}$$

2 **Multiply by**

Multiply by 6.

$$6x - 24 = 3 + 2x$$

$$4x = 27$$

$$x = \frac{27}{4} = 6.75$$

3 **Factorise expression**

For part (b),

$$y^2 - 11y + 30 = (y - 5)(y - 6)$$

4 **Factorise expression**

Hence,

$$(y-5)(y-6)=0$$

$$y=5 \quad \text{or} \quad y=6$$

✓ **FINAL ANSWER**

$x=6.75$, $(y-5)(y-6)$, and $y=5$ or 6.

4MA1/1HR JUNE 2025

Question 8

5 marks | Factorising

8 (a) Simplify $a^6 \times a^{10}$

.....

(1)

(b) Simplify $c^{30} \div c^{12}$

.....

(1)

(c) (i) Factorise $y^2 - 10y + 21$

.....

(2)

(ii) Hence, solve $y^2 - 10y + 21 = 0$

.....

(1)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2025

1 **(a)**

(a) $a^6 \times a^{10} = a^{16}$.

2 **(b)**

(b) $c^{30} \div c^{12} = c^{18}$.

3 **(c)(i)**

(c)(i)

$$y^2 - 10y + 21 = (y - 3)(y - 7)$$

4 **(c)(ii)**

(c)(ii)

$$y = 3 \quad \text{or} \quad y = 7$$

✓ **FINAL ANSWER**

a^{16}, c^{18}, $(y - 3)(y - 7)$, $y = 3$ or $y = 7$

4WM1/01R MAY 2025

Question 9

7 marks | Factorising

9 (a) Simplify $\left(5k^3n^4\right)^2$

.....

(2)

(b) Factorise fully $12m^2y^4 + 18m^3y$

.....

(2)

(c) (i) Factorise $x^2 - 13x + 42$

.....

(2)

(ii) Hence, solve $x^2 - 13x + 42 = 0$

.....

(1)

(Total for Question 9 is 7 marks)

WORKED SOLUTION

Question 9 · 4WM1/01R May 2025

1 **(a)**

(a)

$$(5k^3n^4)^2 = 25k^6n^8$$

2 **(b)**

(b)

$$12m^2y^4 + 18m^3y^3 = 6m^2y^3(2y + 3m)$$

3 **(c)**

(c)

$$x^2 - 13x + 42 = (x - 6)(x - 7)$$

4 **Calculate value**

So

$$x = 6 \quad \text{or} \quad x = 7$$

FINAL ANSWER

$25k^6n^8$, $6m^2y^3(2y+3m)$, $(x-6)(x-7)$, $x=6,7$.

4MA1/2H NOVEMBER 2025

Question 7

3 marks | Factorising

7 (i) Factorise $x^2 + 2x - 48$

.....

(2)

(ii) Hence, solve $x^2 + 2x - 48 = 0$

.....

(1)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2025

1 Factorise expression

$$x^2 + 2x - 48 = (x + 8)(x - 6)$$

2 Factorise expression

So

$$(x + 8)(x - 6) = 0$$

FINAL ANSWER

$x = -8$ or $x = 6$

4WM1/01 NOVEMBER 2025

Question 8

9 marks | Factorising

8 (a) Expand $5w(3w - 4w^2)$

.....

(2)

(b) Simplify fully $\left(4a^3c^5\right)^3$

.....

(2)

(c) Factorise fully $14m^3p^2 + 21m^2p^4$

.....

(2)

(d) (i) Factorise $x^2 - 10x + 24$

.....

(2)

(ii) Hence solve $x^2 - 10x + 24 = 0$

.....

(1)

(Total for Question 8 is 9 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 November 2025

1 **(a)**

(a)

$$5w(3w - 4w^2) = 15w^2 - 20w^3$$

2 **(b)**

(b)

$$(4a^3c^5)^3 = 64a^9c^{15}$$

3 **(c)**

(c)

$$14m^3p^2 + 21m^2p^4 = 7m^2p^2(2m + 3p^2)$$

4 **(d)**

(d)

$$x^2 - 10x + 24 = (x - 4)(x - 6)$$

---

5 **Calculate value**

So

$$x = 4 \quad \text{or} \quad x = 6$$

FINAL ANSWER

$15w^2 - 20w^3$, $64a^9c^{15}$, $7m^2p^2(2m + 3p^2)$, $(x - 4)(x - 6)$, $x = 4, 6$.

Question 10

7 marks | Factorising

10 (a) Simplify $(6m)^0$

.....

(1)

(b) Solve $5y + 20 < 7y + 1$

.....

(2)

(c) Factorise fully $15w^2x^5 + 25w^3x^2$

.....

(2)

(d) Simplify fully $\dfrac{36a^5c^7}{12a^2c^3}$

.....

(2)

(Total for Question 10 is 7 marks)

1 **(a)**

(a) $(6m)^0 = 1$.

2 **(b)**

(b)

$$5y + 20 < 7y + 1$$

$$19 < 2y$$

$$y > \frac{19}{2}$$

3 **(c)**

(c)

$$15w^2x^5 + 25w^3x^2 = 5w^2x^2(3x^3 + 5w)$$

4 **(d)**

(d)

$$\frac{36a^5c^7}{12a^2c^3} = 3a^3c^4$$

FINAL ANSWER

1, $y > \frac{19}{2}$, $5w^2x^2(3x^3 + 5w)$, $3a^3c^4$

4MA1/1HR JUNE 2026

Question 8

2 marks | Factorising

8 Factorise $x^2 - 12x + 32$

.....

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2026

1 Solution | Step 1

$$(-4)(-8) = 32, \qquad -4 + (-8) = -12.$$

Choose two numbers with product 32 and sum -12.

2 Solution | Step 2

$$x^2 - 12x + 32 = (x - 4)(x - 8).$$

The chosen numbers give the two constant terms in the factors.

FINAL ANSWER

$(x - 4)(x - 8)$

4MA1/2H JUNE 2026

Question 9

7 marks | Factorising

9 (a) Find the value of $7n^0$ where $n > 0$

.....

(1)

(b) Simplify fully $\left(2m^5p^4\right)^3$

.....

(2)

(c) Factorise fully $18c^5w^2 + 30cw^3$

.....

(2)

(d) Factorise $x^2 + 6x - 27$

.....

(2)

(Total for Question 9 is 7 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

Since $n > 0$, $n^0 = 1$. Hence $7n^0 = 7(1) = 7$.

A nonzero number raised to power zero equals 1; only n is raised to zero.

2 **Part (b) | Step 1**

$$(2m^5p^4)^3 = 2^3m^{5\times3}p^{4\times3} = 8m^{15}p^{12}.$$

Cube the coefficient and multiply each exponent by 3.

3 **Part (c) | Step 1**

The highest common factor of $18c^5w^2$ and $30cw^3$ is $6cw^2$.

Take the numerical HCF and the smallest power of each shared variable.

4 **Part (c) | Step 2**

$$18c^5w^2 + 30cw^3 = 6cw^2(3c^4 + 5w).$$

Divide each original term by the common factor.

5

Part (d) | Step 1

Two numbers with product -27 and sum 6 are 9 and -3.

For a monic quadratic, find a factor pair with the required sum.

6

Part (d) | Step 2

$$x^2 + 6x - 27 = (x + 9)(x - 3).$$

The cross terms are -3x+9x=6x.

✓ **FINAL ANSWER**

(a) 7 (b) $8m^{15}p^{12}$ (c) $6cw^2(3c^4 + 5w)$ (d) $(x + 9)(x - 3)$

Question 9

5 marks | Factorising

9 (a) Factorise $15g^4h - 20gh^3$

.....

(2)

(b) (i) Factorise $x^2 + 3x - 28$

.....

(2)

(ii) Hence, solve $x^2 + 3x - 28 = 0$

.....

(1)

(Total for Question 9 is 5 marks)

1 **(a) Take out the HCF**

HCF of $15g^4h$ and $20gh^3$ is $5gh$:

$$15g^4h - 20gh^3 = 5gh(3g^3 - 4h^2)$$

2 **(b)(i) Two numbers: product −28, sum 3**

7 and -4:

$$x^2 + 3x - 28 = (x + 7)(x - 4)$$

3 **(b)(ii) Each bracket = 0**

$$x = -7 \quad \text{or} \quad x = 4$$

✓ **FINAL ANSWER**

(a) $5gh(3g^3 - 4h^2)$ (b)(i) $(x + 7)(x - 4)$ (ii) $x = -7,\ 4$

TOPIC

Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 12

5 marks | Algebraic Fractions

12 (a) Factorise $9x^2 - 4y^2$

.....

(2)

(b) Express $\frac{7}{8} - \frac{x+3}{4x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 Specimen

1 **Use difference of two squares**

$$9x^2 - 4y^2 = (3x)^2 - (2y)^2$$

$$= (3x - 2y)(3x + 2y)$$

2 **Use a common denominator**

$$\frac{7}{8} - \frac{x+3}{4x} = \frac{7x}{8x} - \frac{2(x+3)}{8x}$$

3 **Simplify the numerator**

$$\frac{7x - 2x - 6}{8x} = \frac{5x - 6}{8x}$$

✓ **FINAL ANSWER**

$(3x - 2y)(3x + 2y)$, and $\frac{5x-6}{8x}$.

4WM1/01 SPECIMEN

Question 21

3 marks | Algebraic Fractions

21 Express $\left(\frac{20}{x^2 - 36} - \frac{2}{x - 6}\right) \times \frac{1}{4 - x}$ as a single fraction in its simplest form.

Show clear algebraic working.

1 Factorise the denominator

$$x^2 - 36 = (x - 6)(x + 6)$$

2 Simplify the bracket

$$\frac{20}{(x-6)(x+6)} - \frac{2}{x-6} = \frac{20 - 2(x+6)}{(x-6)(x+6)}$$

$$= \frac{8 - 2x}{(x-6)(x+6)} = \frac{-2(x-4)}{(x-6)(x+6)}$$

3 Cancel the common factor

Since $4 - x = -(x - 4)$,

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \frac{1}{4-x} = \frac{2}{(x-6)(x+6)}$$

FINAL ANSWER

$\frac{2}{x^2-36}$.

Question 21

3 marks | Algebraic Fractions

21 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 21 is 3 marks)

1 **Simplify fraction**

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 **Simplify fraction**

Substitute for x:

$$a = \frac{14}{3\left(\frac{7}{4y-3}\right) - 7}$$

3 **Simplify the denominator**

Simplify the denominator:

$$3\left(\frac{7}{4y - 3}\right) - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

4 **Simplify fraction**

So

$$a = \frac{14}{\frac{42-28y}{4y-3}} = \frac{14(4y - 3)}{42 - 28y}$$

$$a = \frac{4y - 3}{3 - 2y}$$

✓ **FINAL ANSWER**

$a = \dfrac{4y - 3}{3 - 2y}$.

4MA1/2H JANUARY 2020

Question 24

4 marks | Algebraic Fractions

24 Express

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2020

1 Use trigonometry

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

2 Use trigonometry

First simplify the bracket:

$$\frac{4}{2x-5}-\frac{3}{2x-3}=\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$

$$=\frac{8x-12-6x+15}{(2x-5)(2x-3)}$$

$$=\frac{2x+3}{(2x-5)(2x-3)}$$

3 Use trigonometry

Now factor the second fraction:

$$9x-4x^3=-x(2x-3)(2x+3)$$

$$6x^2-17x+5=(2x-5)(3x-1)$$

4 **Dividing by a fraction means**

Dividing by a fraction means multiplying by its reciprocal:

$$\frac{2x+3}{(2x-5)(2x-3)} \times \frac{(2x-5)(3x-1)}{-x(2x-3)(2x+3)}$$

5 **Cancel common factors**

Cancel common factors:

$$= -\frac{3x-1}{x(2x-3)^2}$$

$$= \frac{1-3x}{x(2x-3)^2}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{1-3x}{x(2x-3)^2}}$$

4MA1/1H MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Fractions

21 Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May/November 2020

1 Use trigonometry

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

2 Factor

Factor:

$$9x^2-4 = (3x-2)(3x+2)$$

$$3x^2-13x-10 = (3x+2)(x-5)$$

3 Use trigonometry

So the first product becomes

$$\frac{1}{3x-2} \times \frac{(3x-2)(3x+2)}{(3x+2)(x-5)}$$

4 Cancel common factors

Cancel common factors:

$$\frac{1}{x-5}$$

5 Use trigonometry

Now subtract:

$$\frac{1}{x-5} - \frac{7}{x-1}$$

6 **Use the common denominator**

Use the common denominator $(x-5)(x-1)$:

$$\frac{x-1-7(x-5)}{(x-5)(x-1)}$$

$$= \frac{x-1-7x+35}{(x-5)(x-1)}$$

$$= \frac{34-6x}{(x-5)(x-1)}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{34-6x}{(x-5)(x-1)}}$$

4MA1/2HR JANUARY 2021

Question 18

5 marks | Algebraic Fractions

18 Solve the equation

$$\frac{5}{x+2}+\frac{3}{x^2+2x}=2$$

Show clear algebraic working.

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2021

1 **Solve equation**

$$\frac{5}{x+2}+\frac{3}{x^2+2x}=2$$

2 **Solve equation**

Since

$$x^2+2x=x(x+2)$$

3 **multiply by**

multiply by $x(x+2)$:

$$5x+3=2x(x+2)$$

$$5x+3=2x^2+4x$$

$$2x^2-x-3=0$$

4 **Factorise**

Factorise:

$$(2x-3)(x+1)=0$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-1$$

5 **Both values are allowed because**

Both values are allowed because $x \neq 0$ and $x \neq -2$.

✓ **FINAL ANSWER**

$x = \frac{3}{2}$ or $x = -1$.

4MA1/2H JANUARY 2021

Question 21

4 marks | Algebraic Fractions

21 Write $\dfrac{25x^2 - 64}{5x^2 - 13x - 6} \times \dfrac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

as a single fraction in its simplest form.
Show clear algebraic working.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2021

1 Use trigonometry

$$\frac{25x^2 - 64}{5x^2 - 13x - 6} \times \frac{x^2 - 8x + 15}{5x + 8} - (x - 7)$$

2 Factor

Factor:

$$25x^2 - 64 = (5x - 8)(5x + 8)$$

$$5x^2 - 13x - 6 = (5x + 2)(x - 3)$$

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

3 Use trigonometry

So

$$\frac{(5x - 8)(5x + 8)}{(5x + 2)(x - 3)} \times \frac{(x - 3)(x - 5)}{5x + 8} - (x - 7)$$

4 Cancel common factors

Cancel common factors:

$$\frac{(5x - 8)(x - 5)}{5x + 2} - (x - 7)$$

5 **Use the common denominator**

Use the common denominator $5x + 2$:

$$\frac{(5x - 8)(x - 5) - (x - 7)(5x + 2)}{5x + 2}$$

6 **Expand**

Expand:

$$(5x - 8)(x - 5) = 5x^2 - 33x + 40$$

$$(x - 7)(5x + 2) = 5x^2 - 33x - 14$$

7 **Use trigonometry**

So the numerator is

$$5x^2 - 33x + 40 - (5x^2 - 33x - 14) = 54$$

✓ **FINAL ANSWER**

$$\boxed{\frac{54}{5x + 2}}$$

4MA1/1H MAY 2021

Question 21

4 marks | Algebraic Fractions

21 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2021

1 Given

Given

$$x = \frac{5}{9y + 5}$$

2 Use trigonometry

and

$$y = \frac{5}{5a - 2}$$

3 Use cosine rule

Substitute $y = \frac{5}{5a-2}$ into the expression for x:

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5}$$

$$x = \frac{5}{\frac{45}{5a-2} + 5}$$

4 **Write the denominator as a**

Write the denominator as a single fraction:

$$\frac{45}{5a-2}+5=\frac{45+5(5a-2)}{5a-2}$$

$$=\frac{45+25a-10}{5a-2}$$

$$=\frac{25a+35}{5a-2}$$

---

5 **Use trigonometry**

So

$$x=5\div\frac{25a+35}{5a-2}$$

$$x=5\times\frac{5a-2}{25a+35}$$

$$x=\frac{5(5a-2)}{5(5a+7)}$$

$$x=\frac{5a-2}{5a+7}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5a-2}{5a+7}}$$

4MA1/1HR JANUARY 2022

Question 12

5 marks | Algebraic Fractions

12 (a) Expand and simplify $n(n-4)(3n+5)$

.....

(2)

(b) Express

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$n(n-4)(3n+5) = n(3n^2-7n-20) = 3n^3-7n^2-20n$$

2 **(b)**

(b)

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

3 **Use denominator**

Use denominator $4x$:

$$\frac{12}{4x}+\frac{2(x+2)}{4x}+\frac{x}{4x}=\frac{12+2x+4+x}{4x}$$

$$=\frac{3x+16}{4x}$$

FINAL ANSWER

(a) $3n^3-7n^2-20n$, (b) $\frac{3x+16}{4x}$

4MA1/2H JUNE 2022

Question 19

3 marks | Algebraic Fractions

19 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2022

1 Simplify fraction

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 Simplify fraction

Substitute for x:

$$3x - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

3 Simplify fraction

So

$$a = \frac{14}{\frac{42 - 28y}{4y - 3}} = \frac{4y - 3}{3 - 2y}$$

FINAL ANSWER

$a = \frac{4y - 3}{3 - 2y}$

4MA1/2HR JUNE 2022

Question 23

3 marks | Algebraic Fractions

23 Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2022

1 Use trigonometry

$$\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$$

2 Factor

Factor:

$$x^2-36=(x-6)(x+6)$$

3 Use trigonometry

So

$$\frac{20}{(x-6)(x+6)}-\frac{2}{x-6}$$

4 Use the common denominator

Use the common denominator $(x-6)(x+6)$:

$$\frac{20-2(x+6)}{(x-6)(x+6)}$$

$$=\frac{20-2x-12}{(x-6)(x+6)}$$

$$=\frac{8-2x}{(x-6)(x+6)}$$

$$=\frac{-2(x-4)}{(x-6)(x+6)}$$

5 **Use trigonometry**

Now multiply by

$$\frac{1}{4-x} = -\frac{1}{x-4}$$

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \left(-\frac{1}{x-4}\right)$$

---

6 **Cancel**

Cancel $x-4$:

$$\frac{2}{(x-6)(x+6)}$$

$$= \frac{2}{x^2-36}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{2}{x^2-36}}$$

4MA1/1HR MAY 2023

Question 23

4 marks | Algebraic Fractions

23 Simplify $(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$

Give your answer in the form $\frac{ax^2}{bx + c}$ where a, b and c are integers.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2023

1 Simplify surd

$$(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$$

2 Change division into multiplication by

Change division into multiplication by the reciprocal:

$$(x^2 - 4) \times \frac{x}{4x^2 - 7x - 2} - 2x$$

3 Factorise

Factorise:

$$x^2 - 4 = (x - 2)(x + 2)$$

$$4x^2 - 7x - 2 = (4x + 1)(x - 2)$$

4 Simplify surd

So

$$\frac{x(x - 2)(x + 2)}{(4x + 1)(x - 2)} - 2x = \frac{x(x + 2)}{4x + 1} - 2x$$

$$\frac{x(x + 2) - 2x(4x + 1)}{4x + 1} = \frac{x^2 + 2x - 8x^2 - 2x}{4x + 1} = \frac{-7x^2}{4x + 1}$$

✓ **FINAL ANSWER**

$$\frac{-7x^2}{4x + 1}$$

4MA1/1H NOVEMBER 2023

Question 11

3 marks | Algebraic Fractions

11 Express $\frac{3}{4}+\frac{5-x}{6x}$ as a single fraction in its simplest terms.

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H November 2023

1 Simplify fraction

$$\frac{3}{4}+\frac{5-x}{6x}$$

2 The common denominator is

The common denominator is $12x$:

$$\frac{3}{4}=\frac{9x}{12x}$$

$$\frac{5-x}{6x}=\frac{10-2x}{12x}$$

$$\frac{9x+10-2x}{12x}=\frac{7x+10}{12x}$$

FINAL ANSWER

$\frac{7x+10}{12x}$.

4MA1/2H NOVEMBER 2023

Question 24

5 marks | Algebraic Fractions

24 Solve $\frac{45x^3-80x}{3x^2+x-4}\times\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$

Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2023

1 Solve quadratic equation

$$\frac{45x^3-80x}{3x^2+x-4}\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$$

2 Factorise

Factorise:

$$45x^3-80x=5x(3x-4)(3x+4)$$

$$3x^2+x-4=(3x+4)(x-1)$$

3 Solve quadratic equation

Also,

$$\frac{1}{3x-4}+\frac{1}{x}=\frac{x+3x-4}{x(3x-4)}=\frac{4(x-1)}{x(3x-4)}$$

4 Solve quadratic equation

So the left side becomes

$$\frac{5x(3x-4)(3x+4)}{(3x+4)(x-1)}\times\frac{4(x-1)}{x(3x-4)}=20$$

5 **Solve quadratic equation**

Therefore,

$$20 = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4(x+2)$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{7}{4}$$

✓ **FINAL ANSWER**

$$x = \frac{7}{4}$$

4MA1/1H MAY 2024

Question 12

5 marks | Algebraic Fractions

12 (a) Factorise fully $6y^2 - 5y - 4$

.....

(2)

(b) Express $\frac{2x+1}{4x} + \frac{7-5x}{3x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May 2024

1 **(a)**

(a)

$$6y^2 - 5y - 4 = (3y - 4)(2y + 1)$$

2 **(b)**

(b)

$$\frac{2x+1}{4x} + \frac{7-5x}{3x}$$

3 **Use denominator**

Use denominator $12x$:

$$\frac{3(2x+1) + 4(7-5x)}{12x}$$

$$= \frac{6x + 3 + 28 - 20x}{12x} = \frac{31 - 14x}{12x}$$

FINAL ANSWER

(a) $(3y - 4)(2y + 1)$, (b) $\frac{31-14x}{12x}$

4MA1/2H JUNE 2024

Question 20

3 marks | Algebraic Fractions

20 Given that $k = x - y$ and $x = \dfrac{1}{4y}$

express $\dfrac{5k}{x+2}$ in the form $\dfrac{a - by^2}{c + dy}$ where a, b, c and d are integers.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2024

1 **Simplify fraction**

$$k = x - y, \quad x = \frac{1}{4y}$$

2 **Simplify fraction**

So

$$k = \frac{1}{4y} - y = \frac{1 - 4y^2}{4y}$$

3 **Simplify fraction**

Also

$$x + 2 = \frac{1}{4y} + 2 = \frac{1 + 8y}{4y}$$

4 **Simplify fraction**

Therefore

$$\frac{5k}{x+2} = \frac{5\left(\frac{1-4y^2}{4y}\right)}{\frac{1+8y}{4y}}$$

5 **Cancel**

Cancel $4y$:

$$\frac{5(1-4y^2)}{1+8y}$$

$$= \frac{5-20y^2}{1+8y}$$

✓ **FINAL ANSWER**

$$\frac{5-20y^2}{1+8y}$$

4MA1/2HR JUNE 2024

Question 26

4 marks | Algebraic Fractions

26 Write $4 - \left[(3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}\right]$ as a single fraction in its simplest form.

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2024

1 Use trigonometry

$$4 - \left[(3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}\right]$$

2 Factor the quadratic

Factor the quadratic:

$$3x^2 + x - 10 = (3x - 5)(x + 2)$$

3 Use trigonometry

So the expression inside the square brackets is

$$(3x - 5) \div \frac{(3x - 5)(x + 2)}{4x - 1}$$

4 Dividing by a fraction means

Dividing by a fraction means multiplying by its reciprocal:

$$(3x - 5) \times \frac{4x - 1}{(3x - 5)(x + 2)}$$

5 Cancel

Cancel $3x - 5$:

$$\frac{4x - 1}{x + 2}$$

6 Use trigonometry

Therefore the full expression is

$$4 - \frac{4x - 1}{x + 2}$$

7 Use the common denominator

Use the common denominator $x + 2$:

$$\frac{4(x+2) - (4x-1)}{x+2}$$

$$= \frac{4x + 8 - 4x + 1}{x+2}$$

$$= \frac{9}{x+2}$$

FINAL ANSWER

$$\boxed{\frac{9}{x+2}}$$

4MA1/1H JUNE 2025

Question 12

3 marks | Algebraic Fractions

12 Express $\frac{5}{4} + \frac{x-3}{6x}$ as a single fraction in its simplest form.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2025

1 Simplify fraction

$$\frac{5}{4} + \frac{x-3}{6x}$$

2 The common denominator is

The common denominator is $12x$:

$$\frac{5}{4} = \frac{15x}{12x}$$

$$\frac{x-3}{6x} = \frac{2x-6}{12x}$$

$$\frac{15x}{12x} + \frac{2x-6}{12x} = \frac{17x-6}{12x}$$

FINAL ANSWER

$\frac{17x-6}{12x}$.

4WM1/01 MAY 2025

Question 19

3 marks | Algebraic Fractions

19 Simplify fully $\dfrac{3x^2 - 3y^2}{5x + 5y} \div \dfrac{xy^2 - x^2y}{10xy}$

Show clear algebraic working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM1/01 May 2025

1 **Simplify fraction**

$$\frac{3x^2 - 3y^2}{5x + 5y} \div \frac{xy^2 - x^2y}{10xy}$$

2 **Factorise**

Factorise.

$$= \frac{3(x - y)(x + y)}{5(x + y)} \div \frac{xy(y - x)}{10xy}$$

$$= \frac{3(x - y)}{5} \div \frac{y - x}{10}$$

$$= \frac{3(x - y)}{5} \times \frac{10}{y - x}$$

3 **Simplify fraction**

Since $y - x = -(x - y)$, the expression is

$$-6$$

FINAL ANSWER

-6

4WM1/01 MAY 2025

Question 20

3 marks | Algebraic Fractions

20 Given that $k = m + n$ and $m = \dfrac{3}{4n}$

express $\dfrac{7k}{5-m}$ in the form $\dfrac{a+bn^2}{cn-d}$ where a, b and c are integers and d is prime.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01 May 2025

1 Simplify fraction

$$k = m + n, \qquad m = \frac{3}{4n}$$

2 Simplify fraction

So

$$k = \frac{3}{4n} + n = \frac{3 + 4n^2}{4n}$$

3 Simplify fraction

Then

$$7k = \frac{21 + 28n^2}{4n}$$

4 Simplify fraction

and

$$5 - m = 5 - \frac{3}{4n} = \frac{20n - 3}{4n}$$

5 Simplify fraction

Therefore

$$\frac{7k}{5-m} = \frac{\frac{21+28n^2}{4n}}{\frac{20n-3}{4n}}$$

$$= \frac{21 + 28n^2}{20n - 3}$$

FINAL ANSWER

$$\frac{21+28n^2}{20n-3}$$

4MA1/2H JUNE 2025

Question 23

4 marks | Algebraic Fractions

23 $\dfrac{4x^2-4x-120}{5x^2-180} \div \dfrac{x^2+5x}{10x^2+60x} = p$ where p is an integer.

Find the value of p

Show clear algebraic working.

$p =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2025

1 **Split the ratio**

$$\frac{4x^2-4x-120}{5x^2-180} \div \frac{x^2+5x}{10x^2+60x}$$

2 **Factorise**

Factorise:

$$4x^2-4x-120 = 4(x-6)(x+5)$$

$$5x^2-180 = 5(x-6)(x+6)$$

$$x^2+5x = x(x+5)$$

$$10x^2+60x = 10x(x+6)$$

3 **Split the ratio**

So

$$\frac{4(x-6)(x+5)}{5(x-6)(x+6)} \div \frac{x(x+5)}{10x(x+6)}$$

$$= \frac{4(x+5)}{5(x+6)} \times \frac{10(x+6)}{x+5} = 8$$

FINAL ANSWER

$p = 8$

4WM1/01R MAY 2025

Question 24

4 marks | Algebraic Fractions

24 Write $(2x-1)+\dfrac{2x^2-5x-12}{x^2-16}\times\dfrac{x^2+4x}{6x^2+11x+3}$ as a single fraction in its simplest form.

Show clear algebraic working.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM1/01R May 2025

1 Factor the fractional part first

Factor the fractional part first:

$$\frac{2x^2-5x-12}{x^2-16}\times\frac{x^2+4x}{6x^2+11x+3}$$

$$=\frac{(2x+3)(x-4)}{(x-4)(x+4)}\times\frac{x(x+4)}{(3x+1)(2x+3)}$$

$$=\frac{x}{3x+1}$$

2 Evaluate fraction

So

$$(2x-1)+\frac{x}{3x+1}=\frac{(2x-1)(3x+1)+x}{3x+1}$$

$$=\frac{6x^2-1}{3x+1}$$

FINAL ANSWER

$$\frac{6x^2-1}{3x+1}.$$

4MA1/1H NOVEMBER 2025

Question 23

3 marks | Algebraic Fractions

23 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 25

3 marks | Algebraic Fractions

25 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 28

4 marks | Algebraic Fractions

28 Solve $\dfrac{6x^3+5x^2+x}{4x^2-1} \div \dfrac{15x^2-x-2}{8x-4} = 2x$

Show clear algebraic working.

$x =$ .....

(Total for Question 28 is 4 marks)

WORKED SOLUTION

Question 28 · 4WM1/01 November 2025

1 Factor each expression

Factor each expression:

$$\frac{x(3x+1)(2x+1)}{(2x-1)(2x+1)} \div \frac{(5x-2)(3x+1)}{4(2x-1)} = 2x$$

$$\frac{x(3x+1)}{2x-1} \times \frac{4(2x-1)}{(5x-2)(3x+1)} = 2x$$

$$\frac{4x}{5x-2} = 2x$$

$$4x = 2x(5x-2)$$

$$10x^2 - 8x = 0$$

$$2x(5x-4) = 0$$

2 The original fractions do not

The original fractions do not allow $x = 0$, so

$$x = \frac{4}{5}$$

FINAL ANSWER

$x = \frac{4}{5}$.

4MA1/1HR JUNE 2026

Question 16

6 marks | Algebraic Fractions

16 (a) Express $\frac{5}{8} - \frac{x+2}{4x}$ as a single fraction in its simplest form.

.....

(3)

(b) Expand and simplify $3x(2x - 1)(x + 4)$

.....

(3)

(Total for Question 16 is 6 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\frac{5}{8} - \frac{x+2}{4x} = \frac{5x}{8x} - \frac{2(x+2)}{8x}, \qquad x \neq 0.$$

Use 8x as a common denominator.

2 **Part (a) | Step 2**

$$\frac{5x - 2(x+2)}{8x} = \frac{5x - 2x - 4}{8x} = \frac{3x - 4}{8x}.$$

Subtract the entire numerator of the second fraction and collect like terms.

3 **Part (b) | Step 1**

$$(2x - 1)(x + 4) = 2x^2 + 8x - x - 4 = 2x^2 + 7x - 4.$$

Multiply each term in the first bracket by each term in the second.

4 **Part (b) | Step 2**

$$3x(2x^2 + 7x - 4) = 6x^3 + 21x^2 - 12x.$$

Multiply every term by 3x.

FINAL ANSWER

(a) $\frac{3x-4}{8x}$ (b) $6x^3 + 21x^2 - 12x$

4MA1/1H JUNE 2026

Question 18

3 marks | Algebraic Fractions

18 Simplify fully $\dfrac{4x^2 - 9}{2x^2 - 5x - 12}$

Show clear algebraic working.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2026

1 **Solution | Step 1**

$$4x^2 - 9 = (2x - 3)(2x + 3), \qquad 2x^2 - 5x - 12 = (2x + 3)(x - 4).$$

Use difference of squares for the numerator and factorise the denominator.

2 **Solution | Step 2**

$$\frac{4x^2 - 9}{2x^2 - 5x - 12} = \frac{(2x - 3)(2x + 3)}{(2x + 3)(x - 4)} = \frac{2x - 3}{x - 4}.$$

Cancel the common factor, not individual terms.

3 **Solution | Step 3**

The original fraction requires $x \neq -\frac{3}{2}$ and $x \neq 4$.
Cancellation does not restore excluded values.

✓ **FINAL ANSWER**

$\frac{2x-3}{x-4}$, $x \neq -\frac{3}{2}, 4$.

Question 20

4 marks | Algebraic Fractions

20 Express $\left(\frac{5}{4x-7}-\frac{3}{3x-5}\right)\div\frac{9x^2-16}{6x^2-7x-5}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question 20 is 4 marks)

1 **Subtract the fractions**

$$\frac{5(3x-5)-3(4x-7)}{(4x-7)(3x-5)} = \frac{3x-4}{(4x-7)(3x-5)}$$

2 **Factorise the second fraction**

$$9x^2-16=(3x-4)(3x+4), \qquad 6x^2-7x-5=(3x-5)(2x+1)$$

3 **Divide = multiply by the reciprocal**

$$\frac{3x-4}{(4x-7)(3x-5)} \times \frac{(3x-5)(2x+1)}{(3x-4)(3x+4)}$$

4 **Cancel**

$$= \frac{2x+1}{(4x-7)(3x+4)}$$

✓ **FINAL ANSWER**

$$\frac{2x+1}{(4x-7)(3x+4)}$$

TOPIC

Algebraic Roots & Indices

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 3

2 marks | Algebraic Roots & Indices

3 $\frac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 Specimen

1 **Use index laws**

$$4^n = (2^2)^n = 2^{2n}$$

2 **Use index laws**

So

$$\frac{2^k}{4^n} = \frac{2^k}{2^{2n}} = 2^{k-2n}$$

3 **Use index laws**

Since this equals 2^x,

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

4WM1/01 SPECIMEN

Question 7

4 marks | Algebraic Roots & Indices

7 (a) Simplify $8 \times (4t)^0$

.....

(1)

$x^6 \div x^{-5} = x^p$

(b) Find the value of p

$p =$ .....

(1)

(c) Simplify fully $(2k^2m^4)^3$

.....

(2)

WORKED SOLUTION

Question 7 · 4WM1/01 Specimen

1 Use the zero power

$$(4t)^0 = 1$$

$$8(4t)^0 = 8$$

2 Subtract the indices

$$x^6 \div x^{-5} = x^{6-(-5)} = x^{11}$$

So $p = 11$.

3 **Raise each factor to the power**

$$(2k^2m^4)^3 = 2^3k^{2\times3}m^{4\times3}$$

$$= 8k^6m^{12}$$

✓ **FINAL ANSWER**

8, $p = 11$, and $8k^6m^{12}$.

Question 13

6 marks | Algebraic Roots & Indices

13 (a) Expand and simplify $(3x-1)(x+2)(3x+1)$

.....

(3)

(b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

.....

(3)

(Total for Question 13 is 6 marks)

1 **Multiply the conjugate factors**

$$(3x-1)(3x+1) = 9x^2 - 1$$

2 **Expand the remaining bracket**

$$(9x^2-1)(x+2) = 9x^3 + 18x^2 - x - 2$$

3 **Simplify inside the bracket**

$$\frac{2x^5}{8xy^2} = \frac{x^4}{4y^2}$$

4 **Apply the negative power**

$$\left(\frac{x^4}{4y^2}\right)^{-2} = \left(\frac{4y^2}{x^4}\right)^{2} = \frac{16y^4}{x^8}$$

✓ **FINAL ANSWER**

$9x^3 + 18x^2 - x - 2$, and $\frac{16y^4}{x^8}$.

Question 22

3 marks | Algebraic Roots & Indices

22 $\dfrac{16 \times (\sqrt{27})^{}}{6 \times 9^{2n+8}} = 3^x$

Express x in terms of n

Show your working clearly and simplify your expression.

$x =$ .....

1 Write everything as powers of 3

$$\sqrt{27} = 3^{3/2}, \qquad 9 = 3^2, \qquad \frac{18}{6} = 3$$

2 Simplify the powers

$$\frac{18(\sqrt{27})^{4n+6}}{6(9)^{2n+8}} = 3 \times \frac{\left(3^{3/2}\right)^{4n+6}}{(3^2)^{2n+8}}$$

$$= 3^1 \times \frac{3^{6n+9}}{3^{4n+16}}$$

3 Subtract the indices

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

So

$$x = 2n - 6$$

FINAL ANSWER

$x = 2n - 6$.

4MA1/2H JANUARY 2020

Question 1

3 marks | Algebraic Roots & Indices

1 (a) Simplify $\dfrac{x^9}{x^2}$

.....

(1)

(b) Write $\dfrac{7^8 \times 7^4}{7^3}$ as a single power of 7

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2020

1 **(a)**

(a)

$$\frac{x^9}{x^2} = x^{9-2} = x^7$$

2 **(b)**

(b)

$$\frac{7^8 \times 7^4}{7^3} = 7^{8+4-3} = 7^9$$

✓ **FINAL ANSWER**

(a) x^7, (b) 7^9.

4MA1/1H JANUARY 2020

Question 17

6 marks | Algebraic Roots & Indices

17 (a) Show that $\left(6+2\sqrt{12}\right)^2 = 12\left(7+4\sqrt{3}\right)$

Show each stage of your working.

(3)

(b) Simplify fully $\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}}$

.....

(3)

(Total for Question 17 is 6 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H January 2020

1 **Simplify surd**

First,

$$\sqrt{12} = 2\sqrt{3}$$

2 **Simplify surd**

So

$$6 + 2\sqrt{12} = 6 + 4\sqrt{3}$$

3 **Simplify surd**

Now square:

$$(6 + 4\sqrt{3})^2 = 36 + 48\sqrt{3} + 48$$

$$= 84 + 48\sqrt{3}$$

4 **Factor out**

Factor out 12:

$$84 + 48\sqrt{3} = 12(7 + 4\sqrt{3})$$

5 **Simplify surd**

For part (b),

$$\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}} = \left(\frac{t^{15}}{27a^{12}}\right)^{\frac{2}{3}}$$

$$= \frac{t^{10}}{9a^{8}}$$

✓ **FINAL ANSWER**

(a) shown, (b) $\frac{t^{10}}{9a^{8}}$

4MA1/2HR JANUARY 2020

Question 21

7 marks | Algebraic Roots & Indices

21 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

.....

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

.....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2020

1 Factorise expression

For part (a), factorise the numerator and denominator:

$$10x^2 + 23x + 12 = (5x + 4)(2x + 3)$$

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

2 Use index laws

So

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} = \frac{(5x + 4)(2x + 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{5x + 4}{2x - 3}$$

3 Use index laws

For part (b),

$$2^{2y} \times 2^{3y+2} = 2^{5y+2}$$

4 **Use index laws**

Also,

$$\frac{8^{5y}}{4^n} = \frac{(2^3)^{5y}}{(2^2)^n} = 2^{15y-2n}$$

5 **Equate powers of 2**

Equate powers of 2:

$$5y + 2 = 15y - 2n$$

$$2n = 10y - 2$$

$$n = 5y - 1$$

✓ **FINAL ANSWER**

(a) $\frac{5x+4}{2x-3}$, (b) $n = 5y - 1$.

4MA1/1HR MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Roots & Indices

21 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of n for which $M = 2$

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May/November 2020

1 **Use index laws**

$$18^{4n} = (2 \times 3^2)^{4n} = 2^{4n}3^{8n}$$

$$12^2 = (2^2 \times 3)^2 = 2^4 3^2$$

2 **Use index laws**

So

$$M = \frac{2^{4n}3^{8n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{2^4 3^2}$$

3 **Combine powers of 2**

Combine powers of 2:

$$2^{4n+3n^2-18n-4} = 2^{3n^2-14n-4}$$

4 **Combine powers of 3**

Combine powers of 3:

$$3^{8n+2-8n-2} = 3^0 = 1$$

5 **Use index laws**

So

$$M = 2^{3n^2-14n-4}$$

6 **Use index laws**

Given $M = 2$,

$$3n^2 - 14n - 4 = 1$$

$$3n^2 - 14n - 5 = 0$$

$$(3n + 1)(n - 5) = 0$$

FINAL ANSWER

$n = -\frac{1}{3}$ or $n = 5$

4MA1/2HR JANUARY 2021

Question 4

4 marks | Algebraic Roots & Indices

4 (a) Simplify $(3k^2)^4$

.....

(2)

(b) Simplify $(21m^4n) \div (3n^{-5})$

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2021

1 Use index laws

For part (a),

$$(3k^2)^4 = 3^4(k^2)^4$$

$$= 81k^8$$

2 Use index laws

For part (b),

$$(21m^4n) \div (3n^{-5}) = \frac{21m^4n}{3n^{-5}}$$

$$= 7m^4n^{1-(-5)}$$

$$= 7m^4n^6$$

FINAL ANSWER

(a) $81k^8$, (b) $7m^4n^6$.

4MA1/2H JANUARY 2021

Question 11

3 marks | Algebraic Roots & Indices

11 Simplify fully $\left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2021

1 Use index laws

$$\left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$$

2 Simplify inside the brackets

Simplify inside the brackets:

$$\frac{9t^4w^9}{18t^6w^{10}} = \frac{1}{2}t^{-2}w^{-1} = \frac{1}{2t^2w}$$

3 Use index laws

Now apply the power -2:

$$\left(\frac{1}{2t^2w}\right)^{-2} = (2t^2w)^2$$

$$= 4t^4w^2$$

✓ **FINAL ANSWER**

$4t^4w^2$.

4MA1/1HR JANUARY 2021

Question 12

9 marks | Algebraic Roots & Indices

12 (a) Simplify $(16e^{10}f^{6})^{\frac{1}{2}}$

.....

(2)

(b) Write $\frac{2x+1}{4}+\frac{x-2}{3}$ as a single fraction in its simplest form.

.....

(3)

Given that $4^{k+3}=16\times 2^{k}$

(c) find the value of k.
Show your working clearly.

$k=$ .....

(4)

(Total for Question 12 is 9 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$(16e^{10}f^{6})^{\frac{1}{2}}=4e^{5}f^{3}$$

2 **(b)**

(b)

$$\frac{2x+1}{4}+\frac{x-2}{3}=\frac{3(2x+1)+4(x-2)}{12}$$

$$=\frac{6x+3+4x-8}{12}=\frac{10x-5}{12}=\frac{5(2x-1)}{12}$$

3 **(c)**

(c)

$$4^{k+3} = 16 \times 2^k$$

$$(2^2)^{k+3} = 2^4 \times 2^k$$

$$2^{2k+6} = 2^{k+4}$$

$$2k + 6 = k + 4$$

$$k = -2$$

FINAL ANSWER

(a) $4e^5f^3$, (b) $\frac{5(2x-1)}{12}$, (c) $k = -2$.

4MA1/2HR JANUARY 2021

Question 19

7 marks | Algebraic Roots & Indices

19 (a) Simplify $8^2 \times \sqrt[3]{4^6}$

Give your answer in the form 2^a where a is an integer.

Show each stage of your working clearly.

.....

(3)

Given that $n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$ where $n > 0$

(b) find the value of n.

$n =$ .....

(4)

(Total for Question 19 is 7 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2021

1 **Simplify surd**

$$8^2 = (2^3)^2 = 2^6$$

$$\sqrt[3]{4^6} = \sqrt[3]{(2^2)^6} = \sqrt[3]{2^{12}} = 2^4$$

2 **Simplify surd**

So

$$8^2 \times \sqrt[3]{4^6} = 2^6 \times 2^4 = 2^{10}$$

3 **Use index laws**

For part (b),

$$n^{-4/5} = \left(\frac{1}{2}\right)^4 = 2^{-4}$$

4 **Use index laws**

So

$$n^{4/5} = 2^4$$

5 **Raise both sides to the**

Raise both sides to the power $\frac{5}{4}$:

$$n = (2^4)^{5/4} = 2^5 = 32$$

FINAL ANSWER

(a) 2^{10}, (b) $n = 32$.

4MA1/1H MAY 2021

Question 11

4 marks | Algebraic Roots & Indices

11 $\sqrt{2} \times 16 = 2^x$

(a) Find the value of x.
Show your working clearly.

$x =$ .....

(2)

$$\frac{(11^{-6})^5}{11^4} = 11^n$$

(b) Find the value of n.
Show your working clearly.

$n =$ .....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2021

1 **Simplify surd**

$$\sqrt{2} \times 16 = 2^x$$

2 **Write both factors as powers**

Write both factors as powers of 2:

$$\sqrt{2} = 2^{1/2}, \quad 16 = 2^4$$

3 **Use index laws**

So

$$2^{1/2} \times 2^4 = 2^{9/2}$$

4 **Use index laws**

Therefore

$$x = \frac{9}{2}$$

5 **Use index laws**

For part (b),

$$\frac{(11^{-6})^5}{11^4} = \frac{11^{-30}}{11^4} = 11^{-30-4} = 11^{-34}$$

6 **Use index laws**

So

$$n = -34$$

FINAL ANSWER

(a) $x = \frac{9}{2}$, (b) $n = -34$.

4MA1/2H MAY 2021

Question 14

3 marks | Algebraic Roots & Indices

14 Simplify fully $\left(\frac{9x^4}{16y^{10}}\right)^{-\frac{1}{2}}$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H May 2021

1 Use index laws

$$\left(\frac{9x^4}{16y^{10}}\right)^{-1/2}$$

2 A negative power takes the

A negative power takes the reciprocal:

$$\left(\frac{16y^{10}}{9x^4}\right)^{1/2}$$

3 Simplify surd

Now take the square root of each factor:

$$\frac{\sqrt{16y^{10}}}{\sqrt{9x^4}} = \frac{4y^5}{3x^2}$$

FINAL ANSWER

$\frac{4y^5}{3x^2}$.

4MA1/2H NOVEMBER 2021

Question 1

4 marks | Algebraic Roots & Indices

1 (a) Write down the value of m, given that $3^4 \times 3^5 = 3^m$

$m =$ .....

(1)

(b) Write down the value of n, given that $(5^3)^7 = 5^n$

$n =$ .....

(1)

(c) Find the value of p, given that $\dfrac{7^8 \times 7^2}{7^p} = 7^6$

$p =$ .....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2021

1 **Use index laws**

(a) $3^4 \times 3^5 = 3^9$, so $m = 9$.

2 **Use index laws**

(b) $(5^3)^7 = 5^{21}$, so $n = 21$.

3 **(c)**

(c)

$$\frac{7^8 \times 7^2}{7^p} = 7^{10-p} = 7^6$$

4 **Use index laws**

So $10 - p = 6$, hence $p = 4$.

✓ **FINAL ANSWER**

$m = 9$, $n = 21$, $p = 4$

4MA1/1H JANUARY 2022

Question 1

6 marks | Algebraic Roots & Indices

1 (a) Simplify $a^7 \times a^4$

.....

(1)

(b) Simplify $w^{15} \div w^3$

.....

(1)

(c) Simplify $(8x^5y^3)^2$

.....

(2)

(d) Make t the subject of $c = t^3 - 8v$

.....

(2)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2022

1 **(a)**

(a)

$$a^7 \times a^4 = a^{11}$$

2 **(b)**

(b)

$$w^{15} \div w^3 = w^{12}$$

3 **(c)**

(c)

$$(8x^5y^3)^2 = 64x^{10}y^6$$

4 **(d)**

(d)

$$c = t^3 - 8v$$

$$t^3 = c + 8v$$

FINAL ANSWER

a^{11}, w^{12}, $64x^{10}y^6$, $t = \sqrt[3]{c + 8v}$

4MA1/2HR JANUARY 2022

Question 6

4 marks | Algebraic Roots & Indices

6 (a) Write down the value of x^0

.....

(1)

Given that $2^{-3} \times 2^9 = 2^n$

(b) find the value of n

$n =$ .....

(1)

Given that $\dfrac{7^{206} \times 7^m}{7^{214}} = 7^{-3}$

(c) find the value of m

$m =$ .....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR January 2022

1 **Use index laws**

$$x^0 = 1$$

2 **Use index laws**

For part (b),

$$2^{-3} \times 2^9 = 2^{-3+9} = 2^6$$

3 **Use index laws**

So

$$n = 6$$

4 **Use index laws**

For part (c),

$$\frac{7^{206} \times 7^m}{7^{214}} = 7^{206+m-214}$$

5 **Use index laws**

This equals 7^{-3}, so

$$206 + m - 214 = -3$$

$$m = 5$$

FINAL ANSWER

(a) 1, (b) $n = 6$, (c) $m = 5$.

4MA1/2HR JUNE 2022

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2022

1 Use index laws

$$\frac{2^k}{4^n} = 2^x$$

2 Use index laws

Since $4 = 2^2$,

$$4^n = (2^2)^n = 2^{2n}$$

3 Use index laws

So

$$\frac{2^k}{2^{2n}} = 2^{k-2n}$$

4 Use index laws

Therefore

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

4MA1/1H JUNE 2022

Question 9

4 marks | Algebraic Roots & Indices

9 (a) Simplify $8 \times (4t)^0$

.....

(1)

$x^6 \div x^{-5} = x^p$

(b) Find the value of p

$p =$ .....

(1)

(c) Simplify fully $(2k^2m^4)^3$

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2022

1 **Use index laws**

$$8 \times (4t)^0 = 8 \times 1 = 8$$

2 **Use index laws**

For part (b),

$$x^6 \div x^{-5} = x^{6-(-5)} = x^{11}$$

3 **Use index laws**

So

$$p = 11$$

4 **Use index laws**

For part (c),

$$(2k^2m^4)^3 = 2^3k^{2\times3}m^{4\times3}$$

$$= 8k^6m^{12}$$

FINAL ANSWER

(a) 8, (b) $p = 11$, (c) $8k^6m^{12}$.

4MA1/2H JUNE 2022

Question 24

3 marks | Algebraic Roots & Indices

24 $$\frac{18 \times (\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2022

1 Simplify surd

$$\frac{18(\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

2 Use index laws

First,

$$\frac{18}{6} = 3 = 3^1$$

3 Simplify surd

Also,

$$\sqrt{27} = \sqrt{3^3} = 3^{3/2}$$

4 Simplify surd

So

$$(\sqrt{27})^{4n+6} = 3^{\frac{3}{2}(4n+6)} = 3^{6n+9}$$

5 Use index laws

And

$$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$$

6 Use index laws

Therefore the whole expression is

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

7 **Use index laws**

So

$$x = 2n - 6$$

✓ **FINAL ANSWER**

$x = 2n - 6$.

4MA1/1HR MAY 2023

Question 6

4 marks | Algebraic Roots & Indices

6 (a) Simplify $m^{10} \div m^3$

.....

(1)

$k^n \times k^4 = k^{12}$

(b) Write down the value of n

$n =$ .....

(1)

(c) Simplify $(3x^6y^8)^2$

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May 2023

1 **(a)**

(a) $m^{10} \div m^3 = m^{10-3} = m^7$.

2 **Use index laws**

(b) $k^n \times k^4 = k^{n+4} = k^{12}$, so $n + 4 = 12$ and $n = 8$.

3 **(c)**

(c) $(3x^6y^8)^2 = 3^2x^{12}y^{16} = 9x^{12}y^{16}$.

✓ **FINAL ANSWER**

(a) m^7, (b) 8, (c) $9x^{12}y^{16}$

4MA1/2HR JUNE 2023

Question 7

2 marks | Algebraic Roots & Indices

7 (a) Simplify $(4^{-2})^0$

.....

(1)

$3^{-14} \times 3^8 = 3^m$

(b) Find the value of m

$m =$ .....

(1)

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2023

1 Use index laws

Any non-zero expression to the power 0 is 1:

$$(4^{-2})^0 = 1$$

2 Use index laws

For part (b),

$$3^{-14} \times 3^8 = 3^{-14+8} = 3^{-6}$$

3 Use index laws

So

$$m = -6$$

FINAL ANSWER

(a) 1, (b) $m = -6$.

4MA1/2HR JUNE 2023

Question 11

2 marks | Algebraic Roots & Indices

11 Express $\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}}$ in the form $\frac{m^ak^b}{c}$ where a, b and c are integers to be found.

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2023

1 Convert standard form

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{(m^6)^{\frac{3}{2}}(k^{10})^{\frac{3}{2}}}{25^{\frac{3}{2}}}$$

$$(m^6)^{\frac{3}{2}} = m^9$$

$$(k^{10})^{\frac{3}{2}} = k^{15}$$

2 Convert standard form

Also,

$$25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$$

3 Convert standard form

So

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{m^9k^{15}}{125}$$

✓ **FINAL ANSWER**

$$\frac{m^9k^{15}}{125}$$

4MA1/2H JUNE 2023

Question 15

7 marks | Algebraic Roots & Indices

15 (a) Simplify fully $\left(32a^{15}\right)^{\frac{3}{5}}$

.....

(2)

(b) Express $\left(\frac{1}{10x}\right)^{-3}$ in the form px^n where p and n are integers.

.....

(2)

(c) Solve $\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2023

1 **Solve equation**

For part (a),

$$(32a^{15})^{3/5} = 32^{3/5}a^{15\times 3/5}$$

$$= 8a^9$$

2 **Solve equation**

For part (b),

$$\left(\frac{1}{10x}\right)^{-3} = (10x)^3 = 1000x^3$$

3 **Solve equation**

For part (c),

$$\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$$

4 **Multiply by**

Multiply by 30:

$$10(1-2y) = 24 - 15(2y-1)$$

$$10 - 20y = 39 - 30y$$

$$10y = 29$$

$$y = \frac{29}{10}$$

✓ **FINAL ANSWER**

(a) $8a^9$, (b) $1000x^3$, (c) $y = \frac{29}{10}$

4MA1/1HR MAY 2023

Question 24

5 marks | Algebraic Roots & Indices

24 Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for x in terms of n
State any restrictions on n

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR May 2023

1 **Use index laws**

$$16^x = 2^{4x}, \qquad 8 = 2^3$$

2 **Use index laws**

So

$$2^n = 2^{x^2} \times 2^{4x} \times 2^3 = 2^{x^2+4x+3}$$

3 **Hence**

Hence

$$n = x^2 + 4x + 3 = (x+2)^2 - 1$$

$$(x+2)^2 = n + 1$$

4 **Simplify surd**

Since $x > 0$,

$$x + 2 = \sqrt{n+1}$$

$$x = -2 + \sqrt{n+1}$$

5 **Simplify surd**

For $x > 0$, $\sqrt{n+1} > 2$, so $n > 3$.

FINAL ANSWER

$x = -2 + \sqrt{n+1}, n > 3$

4MA1/2H NOVEMBER 2023

Question 4

8 marks | Algebraic Roots & Indices

4 (a) Solve $\frac{2x+5}{6} = 2x - 5$

Show clear algebraic working.

$x =$ .....

(3)

(b) Simplify $h^{15} \div h^3$

.....

(1)

(c) Simplify fully $\left(2g^3k^5\right)^4$

.....

(2)

(d) Given that $\frac{y^5 \times y^n}{y^7} = y^{12}$

work out the value of n

$n =$ .....

(2)

(Total for Question 4 is 8 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2023

1 **(a)**

(a)

$$\frac{2x+5}{6} = 2x - 5$$

$$2x + 5 = 12x - 30$$

$$35 = 10x$$

$$x = \frac{7}{2}$$

2 **(b)**

(b)

$$h^{15} \div h^3 = h^{15-3} = h^{12}$$

---

3 **(c)**

(c)

$$(2g^3k^5)^4 = 2^4 g^{12} k^{20} = 16g^{12}k^{20}$$

---

4 **(d)**

(d)

$$\frac{y^5 \times y^n}{y^7} = y^{5+n-7} = y^{n-2}$$

---

5 **Use index laws**

Given that this equals y^{12},

$$n - 2 = 12$$

$$n = 14$$

✓ **FINAL ANSWER**

$x = \frac{7}{2}$, h^{12}, $16g^{12}k^{20}$, and $n = 14$.

4MA1/2H NOVEMBER 2023

Question 12

4 marks | Algebraic Roots & Indices

12 $2^7 \times 4^5 = 4^x$

(a) Calculate the value of x

$x =$ .....

(2)

(b) Simplify fully $\left(125p^6y^{24}\right)^{\frac{2}{3}}$

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2023

1 **(a)**

(a)

$$2^7 \times 4^5 = 2^7 \times (2^2)^5 = 2^{17}$$

2 **Use index laws**

Also

$$4^x = (2^2)^x = 2^{2x}$$

3 **Use index laws**

So

$$2x = 17$$

$$x = \frac{17}{2}$$

4 **(b)**

(b)

$$(125p^6y^{24})^{2/3} = 125^{2/3}p^4y^{16}$$

$$= 25p^4y^{16}$$

FINAL ANSWER

(a) $x = \frac{17}{2}$, (b) $25p^4y^{16}$

4MA1/2HR JUNE 2024

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{3^{-2}\times 3^{5}}{3^{10}} = 3^{n}$

Find the value of n

$n =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2024

1 Use index laws

$$\frac{3^{-2}\times 3^{5}}{3^{10}} = 3^{n}$$

2 Use index laws

Use index laws:

$$\frac{3^{-2+5}}{3^{10}} = 3^{3-10} = 3^{-7}$$

3 Use index laws

So

$$n = -7$$

FINAL ANSWER

$n = -7$.

4MA1/1H MAY 2024

Question 10

4 marks | Algebraic Roots & Indices

10 (a) Simplify $(2p)^0$ where $p > 0$

.....

(1)

$y^9 \times y^{-3} = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4c^2)^3$

.....

(2)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2024

1 **(a)**

(a) $(2p)^0 = 1$.

2 **Use index laws**

(b) $y^9 \times y^{-3} = y^{9-3} = y^6$, so $n = 6$.

3 **(c)**

(c) $(5a^4c^2)^3 = 5^3a^{12}c^6 = 125a^{12}c^6$.

✓ **FINAL ANSWER**

(a) 1, (b) 6, (c) $125a^{12}c^6$

4MA1/1HR MAY 2024

Question 17

4 marks | Algebraic Roots & Indices

17 (a) $\left(\sqrt[4]{k^{12}}\right)^5 = k^n$

Find the value of n

$n =$ .....

(1)

(b) Express $\dfrac{7}{2-\sqrt{3}}$ in the form $\sqrt{c} + d$ where c and d are integers.

Show your working clearly.

.....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May 2024

1 **(a) so**

(a)

$$\left(\sqrt[4]{k^{12}}\right)^5 = (k^3)^5 = k^{15}$$

so $n = 15$.

2 **(b)**

(b)

$$\frac{7}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{7(2+\sqrt{3})}{4-3} = 14 + 7\sqrt{3}$$

$$7\sqrt{3} = \sqrt{49 \times 3} = \sqrt{147}$$

FINAL ANSWER

(a) 15, (b) $\sqrt{147} + 14$

4MA1/2H JUNE 2024

Question 23

3 marks | Algebraic Roots & Indices

23 Simplify $\dfrac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$

Give your answer in the form 5^w where w is an expression in terms of x
Show each stage of your working clearly.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2024

1 Simplify surd

$$\frac{30 \times 25^{2x+7}}{\sqrt{180} \times (\sqrt{5})^{4x+9}}$$

2 Simplify the number part

Simplify the number part:

$$\sqrt{180} = 6\sqrt{5}$$

$$\frac{30}{6\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5} = 5^{1/2}$$

3 Simplify surd

Now write the powers with base 5:

$$25^{2x+7} = (5^2)^{2x+7} = 5^{4x+14}$$

$$(\sqrt{5})^{4x+9} = 5^{(4x+9)/2} = 5^{2x+9/2}$$

4 Use index laws

So the full expression is

$$5^{1/2+4x+14-(2x+9/2)}$$

$$= 5^{2x+10}$$

FINAL ANSWER

5^{2x+10}.

4MA1/2H NOVEMBER 2024

Question 14

3 marks | Algebraic Roots & Indices

14 Given that $\frac{3^{2n+3}}{3^4} = 3^3 \times 3^{1-2n}$

find the value of n
Show your working clearly.

$n =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H November 2024

1 Use index laws

$$\frac{3^{2n+3}}{3^4} = 3^{2n-1}$$

$$3^3 \times 3^{1-2n} = 3^{4-2n}$$

2 Use index laws

So

$$2n - 1 = 4 - 2n$$

$$4n = 5$$

✓ **FINAL ANSWER**

$n = \frac{5}{4}$

4MA1/1H JUNE 2025

Question 4

4 marks | Algebraic Roots & Indices

4 $x^5 \times x^7 = x^m$

(a) Find the value of m

$m =$ .....

(1)

$y^8 \div y^3 = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4r^2)^3$

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2025

1 **Use index laws**

For part (a),

$$x^5 \times x^7 = x^{12}$$

2 **Use index laws**

So $m = 12$.

3 **Use index laws**

For part (b),

$$y^8 \div y^3 = y^{8-3} = y^5$$

4 **Use index laws**

So $n = 5$.

5 **Use index laws**

For part (c),

$$(5a^4r^2)^3 = 5^3a^{4\times3}r^{2\times3}$$

$$= 125a^{12}r^6$$

✓ **FINAL ANSWER**

(a) $m = 12$, (b) $n = 5$, (c) $125a^{12}r^6$.

4WM1/01 MAY 2025

Question 5

4 marks | Algebraic Roots & Indices

5 $x^5 \times x^7 = x^m$

(a) Find the value of m

$m =$ .....

(1)

$y^8 \div y^3 = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4r^2)^3$

.....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 May 2025

1 **Use index laws**

$$x^5 \times x^7 = x^{5+7} = x^{12}$$

2 **Use index laws**

So $m = 12$.

$$y^8 \div y^3 = y^{8-3} = y^5$$

3 **Use index laws**

So $n = 5$.

$$(5a^4r^2)^3 = 5^3(a^4)^3(r^2)^3 = 125a^{12}r^6$$

✓ **FINAL ANSWER**

$m = 12$, $n = 5$, and $125a^{12}r^6$.

Question 5

4 marks | Algebraic Roots & Indices

5 $3^8 \times 3^4 = 3^m$

(a) Find the value of m

$m =$ .....

(1)

$4^{12} \div 4^7 = 4^n$

(b) Find the value of n

$n =$ .....

(1)

$\left(6^2\right)^{10} = 6^p$

(c) Find the value of p

$p =$ .....

(1)

$5^r = 1$

(d) Write down the value of r

$r =$ .....

(1)

(Total for Question 5 is 4 marks)

1 Evaluate fraction

$$3^8 \times 3^4 = 3^{8+4} = 3^{12}$$

$$\frac{4^{12}}{4^7} = 4^{12-7} = 4^5$$

$$(6^2)^{10} = 6^{20}$$

$$5^0 = 1$$

FINAL ANSWER

$m = 12, n = 5, p = 20, r = 0.$

4MA1/2HR JUNE 2025

Question 6

5 marks | Algebraic Roots & Indices

6 (a) Write down the value of 5^0

.....

(1)

$$\frac{5^9 \times 5^{-3}}{5^{-2}} = 5^k$$

(b) Find the value of k

$k =$ .....

(2)

(c) Simplify fully $\left(2d^4e^5\right)^3$

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2025

1 **Use index laws**

$$5^0 = 1$$

2 **Use index laws**

For part (b),

$$\frac{5^9 \times 5^{-3}}{5^{-2}} = 5^{9+(-3)-(-2)} = 5^8$$

3 **Use index laws**

So

$$k = 8$$

4 **Use index laws**

For part (c),

$$(2d^4e^5)^3 = 2^3 d^{4\times 3} e^{5\times 3}$$

$$= 8d^{12}e^{15}$$

FINAL ANSWER

(a) $\mathbf{1}$, (b) $\boldsymbol{k = 8}$, (c) $\boldsymbol{8d^{12}e^{15}}$.

4WM1/01 MAY 2025

Question 9

3 marks | Algebraic Roots & Indices

9 $7^x = 1$

(a) Write down the value of x

$x =$ .....

(1)

$$\frac{8^{-4} \times 8^{11}}{8^{12}} = 8^n$$

(b) Work out the value of n

$n =$ .....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM1/01 May 2025

1 Use index laws

$$7^x = 1$$

2 Use index laws

Since $7^0 = 1$,

$$x = 0$$

3 Use index laws

For part (b),

$$\frac{8^{-4} \times 8^{11}}{8^{12}} = 8^n$$

$$8^{-4} \times 8^{11} = 8^7$$

$$\frac{8^7}{8^{12}} = 8^{7-12} = 8^{-5}$$

4 Use index laws

So $n = -5$.

FINAL ANSWER

$x = 0, n = -5.$

4MA1/2H JUNE 2025

Question 15

3 marks | Algebraic Roots & Indices

15 Simplify fully $\left(\frac{2a^3}{10a^7c^2}\right)^{-3}$

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2025

1 **Use index laws**

$$\left(\frac{2a^3}{10a^7c^2}\right)^{-3}$$

2 **Use index laws**

First simplify inside the bracket:

$$\frac{2a^3}{10a^7c^2} = \frac{1}{5a^4c^2}$$

3 **A negative power means take**

A negative power means take the reciprocal:

$$\left(\frac{1}{5a^4c^2}\right)^{-3} = (5a^4c^2)^3$$

$$= 125a^{12}c^6$$

✓ **FINAL ANSWER**

$125a^{12}c^6$.

4WM1/01 MAY 2025

Question 16

3 marks | Algebraic Roots & Indices

16 Solve the equation $8^{3y+4} \times 4^{3y} = 2^{5y}$
Show your working clearly.

$y =$ .....

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM1/01 May 2025

1 Solve equation

$$8^{3y+4} \times 4^{3y} = 2^{5y}$$

2 Solve equation

Write 8 and 4 as powers of 2.

$$(2^3)^{3y+4}(2^2)^{3y} = 2^{5y}$$

$$2^{9y+12} \times 2^{6y} = 2^{5y}$$

$$2^{15y+12} = 2^{5y}$$

3 Solve equation

So

$$15y + 12 = 5y$$

$$y = -\frac{6}{5}$$

✓ **FINAL ANSWER**

$y = -\frac{6}{5}$

4WM1/01 NOVEMBER 2025

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{8^{-2} \times 8^{9}}{8^{10}} = 8^{n}$

Work out the value of n

$n =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4WM1/01 November 2025

1 **Use index laws**

$$\frac{8^{-2} \times 8^{9}}{8^{10}} = 8^{-2+9-10} = 8^{-3}$$

✓ **FINAL ANSWER**

$n = -3$

4MA1/1H NOVEMBER 2025

Question 8

2 marks | Algebraic Roots & Indices

8 $\dfrac{8^{-2} \times 8^{9}}{8^{10}} = 8^{n}$

Work out the value of n

$n =$ .....

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2025

1 Use index laws

$$\frac{8^{-2} \times 8^{9}}{8^{10}} = 8^{-2+9-10} = 8^{-3}$$

FINAL ANSWER

$n = -3$

4MA1/2H NOVEMBER 2025

Question 13

4 marks | Algebraic Roots & Indices

13 (a) Simplify fully $\left(\frac{125x^6}{y^{15}}\right)^3$

.....

(2)

Given that $4^n = \frac{4^m}{64^p}$

(b) Express n in terms of m and p

$n =$ .....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2025

1 **(a)**

(a)

$$\left(\frac{125x^6}{y^{15}}\right)^{1/3} = \frac{5x^2}{y^5}$$

2 **(b)**

(b)

$$64^p = (4^3)^p = 4^{3p}$$

$$4^n = \frac{4^m}{64^p} = \frac{4^m}{4^{3p}} = 4^{m-3p}$$

FINAL ANSWER

(a) $\frac{5x^2}{y^5}$, (b) $n = m - 3p$

4WM1/01 NOVEMBER 2025

Question 15

3 marks | Algebraic Roots & Indices

15 $\dfrac{16^{x-3} \times 8^{x+1}}{32^x} = 2^n$

Express n in terms of x
Give your answer in its simplest form.

$n =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM1/01 November 2025

1 Evaluate fraction

$$\frac{16^{x-3} \times 8^{x+1}}{32^x} = \frac{2^{4x-12} \times 2^{3x+3}}{2^{5x}}$$

$$= 2^{4x-12+3x+3-5x} = 2^{2x-9}$$

FINAL ANSWER

$n = 2x - 9$.

4MA1/1HR JUNE 2026

Question 3

2 marks | Algebraic Roots & Indices

3 (a) Simplify $e^7 \times e^3$

.....

(1)

(b) Simplify $g^5 \div g^2$

.....

(1)

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$e^7 \times e^3 = e^{7+3} = e^{10}.$$

For equal bases, add the indices when multiplying.

2 **Part (b) | Step 1**

$$g^5 \div g^2 = g^{5-2} = g^3 \quad (g \neq 0).$$

For equal bases, subtract the indices when dividing.

✓ **FINAL ANSWER**

(a) e^{10} (b) g^3

4MA1/1H JUNE 2026

Question 6

2 marks | Algebraic Roots & Indices

6 Simplify $\left(4x^5y^6\right)^3$

.....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2026

1 Solution | Step 1

$$(4x^5y^6)^3 = 4^3x^{5\times3}y^{6\times3} = 64x^{15}y^{18}.$$

Raise each factor to the third power and multiply exponents for a power of a power.

FINAL ANSWER

$64x^{15}y^{18}$

4WM1/01 MAY 2026

Question 6

2 marks | Algebraic Roots & Indices

6 Simplify $\left(4x^5y^6\right)^3$

.....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 May 2026

1 Cube every part

$$(4x^5y^6)^3 = 4^3x^{15}y^{18} = 64x^{15}y^{18}$$

✓ **FINAL ANSWER**

$64x^{15}y^{18}$

Question 12

6 marks | Algebraic Roots & Indices

12 $\dfrac{2^{\frac{x}{3}}}{8^x} = 64$

(a) Find the value of x
Show clear algebraic working.

$x =$ .....

(3)

(b) Expand and simplify $4t(t + 5)(2t - 3)$

.....

(3)

(Total for Question 12 is 6 marks)

1 **(a) Write everything as powers of 2**

$$8^x = 2^{3x}, \quad 64 = 2^6 \Rightarrow \frac{2^{x/3}}{2^{3x}} = 2^6$$

2 **Compare powers**

$$\frac{x}{3} - 3x = 6 \Rightarrow -\frac{8x}{3} = 6 \Rightarrow x = -\frac{9}{4}$$

3 **(b) Expand two brackets first**

$$(t+5)(2t-3) = 2t^2 + 7t - 15$$

4 **Multiply by 4t**

$$4t(2t^2 + 7t - 15) = 8t^3 + 28t^2 - 60t$$

✓ **FINAL ANSWER**

(a) $x = -\frac{9}{4}$ (b) $8t^3 + 28t^2 - 60t$

4MA1/2HR JUNE 2026

Question 16

2 marks | Algebraic Roots & Indices

16 $\left(\frac{1}{\sqrt{a}}\right)^6 = a^n$

Work out the value of n

$n =$ .....

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\frac{1}{\sqrt{a}} = a^{-1/2}, \qquad \left(a^{-1/2}\right)^6 = a^{-3}.$$

A reciprocal changes the sign of the exponent and an outer power multiplies exponents.

FINAL ANSWER

$n = -3$.

4MA1/1HR JUNE 2026

Question 26

3 marks | Algebraic Roots & Indices

26

$$\frac{27^x \times 3^{n+6}}{9^{4-n}} = \frac{6^x}{2^x}$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/1HR June 2026

1 Solution | Step 1

$$27 = 3^3, \qquad 9 = 3^2, \qquad \frac{6^x}{2^x} = \left(\frac{6}{2}\right)^x = 3^x.$$

Write both sides in a common base.

2 Solution | Step 2

$$\frac{27^x \times 3^{n+6}}{9^{4-n}} = 3^{3x+n+6-2(4-n)} = 3^{3x+3n-2}.$$

Multiply powers by adding indices and divide powers by subtracting indices.

3 Solution | Step 3

$$3^{3x+3n-2} = 3^x \quad \Longrightarrow \quad 3x + 3n - 2 = x.$$

The base-3 exponential is one-to-one, so equal powers have equal indices.

4 Solution | Step 4

$$2x = 2 - 3n \quad \Longrightarrow \quad x = 1 - \frac{3n}{2}.$$

Collect the x terms and divide by 2.

FINAL ANSWER

$x = 1 - \frac{3n}{2}$

TOPIC

Linear Equations

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 4

5 marks | Linear Equations

4 (a) Expand and simplify $(n - 6)(n + 4)$

.....

(2)

(b) Solve $2x - 3 = \frac{3x - 5}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 4 is 5 marks)

1 **Expand the brackets**

$$(n-6)(n+4) = n^2 + 4n - 6n - 24$$

$$= n^2 - 2n - 24$$

2 **Clear the denominator**

$$2x - 3 = \frac{3x - 5}{4}$$

$$4(2x - 3) = 3x - 5$$

3 **Solve the equation**

$$8x - 12 = 3x - 5$$

$$5x = 7$$

$$x = \frac{7}{5}$$

✓ **FINAL ANSWER**

$n^2 - 2n - 24$, and $x = \frac{7}{5}$.

4MA1/1H JANUARY 2020

Question 7

9 marks | Linear Equations

7 (a) Expand and simplify $(m - 8)(m + 5)$

.....

(2)

(b) Factorise fully $5y + 20y^2$

.....

(2)

(c) Simplify $(p^2 + 3)^0$

.....

(1)

(d) Solve $3(2x - 5) = \frac{9 - x}{2}$

Show clear algebraic working.

$x =$ .....

(4)

(Total for Question 7 is 9 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2020

1 **(a)**

(a) $(m-8)(m+5) = m^2 - 3m - 40$.

2 **(b)**

(b) $5y + 20y^2 = 5y(1 + 4y)$.

3 **(c)**

(c) $(p^2+3)^0 = 1$.

4 **(d)**

(d)

$$3(2x-5)=\frac{9-x}{2}$$

$$12x-30=9-x$$

$$13x=39$$

FINAL ANSWER

(a) $m^2-3m-40$, (b) $5y(1+4y)$, (c) 1, (d) $x=3$

4MA1/1HR MAY/NOVEMBER 2020

Question 3

6 marks | Linear Equations

3 (a) Simplify $h^7 \times h^2$

.....

(1)

$G = c^2 - 4c$

(b) Find the value of G when $c = -5$

$G =$ .....

(2)

(c) Solve $\dfrac{5x - 3}{4} = 2x + 3$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May/November 2020

1 **(a)**

(a)

$$h^7 \times h^2 = h^{7+2} = h^9$$

2 **(b)**

(b)

$$G = c^2 - 4c$$

3 **Solve quadratic equation**

When $c = -5$:

$$G = (-5)^2 - 4(-5) = 25 + 20 = 45$$

4 **(c)**

(c)

$$\frac{5x-3}{4}=2x+3$$

---

5 **Multiply by 4**

Multiply by 4:

$$5x-3=8x+12$$

$$-3x=15$$

$$x=-5$$

FINAL ANSWER

h^9, $G=45$, and $x=-5$.

4MA1/1H JANUARY 2021

Question 5

5 marks | Linear Equations

5 (a) Factorise fully $15y^4 + 20uy^3$

.....

(2)

(b) Solve $4 - 3x = \dfrac{5 - 8x}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2021

1 **(a)**

(a)

$$15y^4 + 20uy^3 = 5y^3(3y + 4u)$$

2 **(b)**

(b)

$$4 - 3x = \frac{5 - 8x}{4}$$

$$16 - 12x = 5 - 8x$$

$$11 = 4x$$

FINAL ANSWER

(a) $5y^3(3y + 4u)$, (b) $x = \frac{11}{4}$

4MA1/2HR JANUARY 2022

Question 3

5 marks | Linear Equations

3 (a) Solve $4y + 5 > 12$

.....

(2)

(b) Solve $6x - 5 = \frac{4x - 7}{2}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2022

1 **(a)**

(a)

$$4y + 5 > 12$$

$$4y > 7$$

$$y > \frac{7}{4}$$

2 **(b)**

(b)

$$6x - 5 = \frac{4x - 7}{2}$$

3 Multiply by 2

Multiply by 2:

$$12x - 10 = 4x - 7$$

$$8x = 3$$

$$x = \frac{3}{8}$$

FINAL ANSWER

$y > \frac{7}{4}$, and $x = \frac{3}{8}$.

4MA1/1H JUNE 2022

Question 5

5 marks | Linear Equations

5 (a) Expand and simplify $(n-6)(n+4)$

.....

(2)

(b) Solve $2x-3=\dfrac{3x-5}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2022

1 **(a)**

(a)

$$(n-6)(n+4)=n^2-2n-24$$

2 **(b)**

(b)

$$2x-3=\frac{3x-5}{4}$$

$$8x-12=3x-5$$

$$5x=7$$

✓ **FINAL ANSWER**

(a) $n^2-2n-24$, (b) $x=\frac{7}{5}$

4MA1/2HR JUNE 2022

Question 6

8 marks | Linear Equations

6 (a) Solve $p = \dfrac{3p - 5}{10}$

Show clear algebraic working.

$p =$ .....

(3)

(b) Simplify a^0 where $a > 0$

.....

(1)

(c) Simplify fully $\dfrac{3xy^3}{6x^2y}$

.....

(2)

(d) Factorise fully $10c^3d^2 + 15cd^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2022

1 **(a)**

(a)

$$p = \frac{3p - 5}{10}$$

$$10p = 3p - 5$$

$$7p = -5$$

$$p = -\frac{5}{7}$$

2 **(b)**

(b) $a^0 = 1$.

---

3 **(c)**

(c)

$$\frac{3xy^3}{6x^2y} = \frac{y^2}{2x}$$

---

4 **(d)**

(d)

$$10c^3d^2 + 15cd^4 = 5cd^2(2c^2 + 3d^2)$$

✓ **FINAL ANSWER**

(a) $p = -\frac{5}{7}$, (b) 1, (c) $\frac{y^2}{2x}$, (d) $5cd^2(2c^2 + 3d^2)$

4MA1/2HR JUNE 2024

Question 6

8 marks | Linear Equations

6 (a) Solve $\frac{2f}{3} = 4f - 17$

Show clear algebraic working.

$f =$ .....

(3)

(b) Simplify $(e + 12)^0$ where $e > 0$

.....

(1)

(c) Simplify fully $\frac{12a^4h^6}{4ah^2}$

.....

(2)

(d) Factorise fully $20x^5y + 12x^3y^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2024

1 (a)

(a)

$$\frac{2f}{3} = 4f - 17$$

$$2f = 12f - 51$$

$$10f = 51$$

$$f = \frac{51}{10}$$

2 **(b)**

(b) $(e+12)^0 = 1$.

---

3 **(c)**

(c)

$$\frac{12a^4h^6}{4ah^2} = 3a^3h^4$$

---

4 **(d)**

(d)

$$20x^5y + 12x^3y^4 = 4x^3y(5x^2 + 3y^3)$$

✓ **FINAL ANSWER**

$f = \frac{51}{10}$, 1, $3a^3h^4$, $4x^3y(5x^2 + 3y^3)$

4MA1/1H MAY 2024

Question 6

5 marks | Linear Equations

6 (a) Expand and simplify $(m + 5)(m - 8)$

.....

(2)

(b) Solve $3n - 4 = \frac{5n + 6}{3}$

Show clear algebraic working.

$n =$ .....

(3)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2024

1 (a)

(a)

$$(m+5)(m-8) = m^2 - 3m - 40$$

2 (b)

(b)

$$3n - 4 = \frac{5n+6}{3}$$

$$9n - 12 = 5n + 6$$

$$4n = 18$$

FINAL ANSWER

(a) $m^2 - 3m - 40$, (b) $n = \frac{9}{2}$

4MA1/1H NOVEMBER 2024

Question 3

6 marks | Linear Equations

3 (a) Simplify $\left(p^3\right)^5$

.....

(1)

(b) Expand and simplify $2n(4n+3)+n(n-4)$

.....

(2)

(c) Solve $\frac{2x+5}{3} = 4-x$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2024

1 **(a)**

(a) $(p^3)^5 = p^{15}$.

2 **(b)**

(b)

$$2n(4n+3)+n(n-4) = 8n^2+6n+n^2-4n = 9n^2+2n$$

3 **(c)**

(c)

$$\frac{2x+5}{3} = 4-x$$

$$2x+5 = 12-3x$$

$$5x = 7$$

FINAL ANSWER

(a) p^{15}, (b) $9n^2 + 2n$, (c) $x = \frac{7}{5}$

4MA1/2HR JUNE 2025

Question 1

5 marks | Linear Equations

1 (a) Factorise fully $18c - 45cd$

.....

(2)

(b) Solve $\frac{5 - 2x}{6} = 3x - 4$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2025

1 **(a)**

(a)

$$18c - 45cd = 9c(2 - 5d)$$

2 **(b)**

(b)

$$\frac{5 - 2x}{6} = 3x - 4$$

$$5 - 2x = 18x - 24$$

$$29 = 20x$$

$$x = \frac{29}{20}$$

FINAL ANSWER

(a) $9c(2 - 5d)$, (b) $x = \frac{29}{20}$

4WM1/01R MAY 2025

Question 12

6 marks | Linear Equations

12 (a) Solve $\frac{5a+8}{3} - \frac{2a+5}{4} = 23$

Show clear algebraic working.

$a =$ .....

(4)

(b) Express $\left(\frac{\sqrt{y}}{3}\right)^{-1}$ in the form cy^n where c and n are numbers to be found.

.....

(2)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4WM1/01R May 2025

1 Solve equation

For part (a),

$$\frac{5a+8}{3} - \frac{2a+5}{4} = 23$$

2 Multiply by

Multiply by 12:

$$4(5a+8) - 3(2a+5) = 276$$

$$20a + 32 - 6a - 15 = 276$$

$$14a + 17 = 276$$

$$14a = 259$$

$$a = \frac{259}{14} = \frac{37}{2}$$

3 **Solve equation**

For part (b),

$$\left(\frac{\sqrt{y}}{3}\right)^{-1} = \frac{3}{\sqrt{y}}$$

$$= 3y^{-1/2}$$

✓ **FINAL ANSWER**

(a) $a = \frac{37}{2}$, (b) $3y^{-1/2}$

4MA1/1HR JUNE 2025

Question 15

6 marks | Linear Equations

15 (a) Solve $\frac{5a+8}{3} - \frac{2a+5}{4} = 23$

Show clear algebraic working.

$a =$ .....

(4)

(b) Express $\left(\frac{\sqrt{y}}{3}\right)^{-1}$ in the form cy^n where c and n are numbers to be found.

.....

(2)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR June 2025

1 Solve equation

For part (a),

$$\frac{5a+8}{3} - \frac{2a+5}{4} = 23$$

2 Multiply by

Multiply by 12:

$$4(5a+8) - 3(2a+5) = 276$$

$$20a + 32 - 6a - 15 = 276$$

$$14a + 17 = 276$$

$$14a = 259$$

$$a = \frac{259}{14} = \frac{37}{2}$$

3 **Solve equation**

For part (b),

$$\left(\frac{\sqrt{y}}{3}\right)^{-1} = \frac{3}{\sqrt{y}}$$

$$= 3y^{-1/2}$$

✓ **FINAL ANSWER**

(a) $a = \frac{37}{2}$, (b) $3y^{-1/2}$

4MA1/2HR JUNE 2026

Question 4

8 marks | Linear Equations

4 (a) Solve $3(x + 2) = 1 - 5x$
Show clear algebraic working.

$x =$ .....
(3)

$P = c^2 - 5d$

(b) Work out the value of P when $c = -7$ and $d = 3$

$P =$ .....
(2)

(c) Simplify m^0

.....
(1)

(d) Simplify $(4r^2t)^3$

.....
(2)

(Total for Question 4 is 8 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2026

1 Part (a) | Step 1

$$3x + 6 = 1 - 5x.$$

Expand the bracket.

2 Part (a) | Step 2

$$8x = -5 \quad \Rightarrow \quad x = -\frac{5}{8}.$$

Add 5x and subtract 6 on both sides, then divide by 8.

3 Part (b) | Step 1

$$P = (-7)^2 - 5(3) = 49 - 15 = 34.$$

The square applies to the whole negative number, so (-7) squared is positive.

4 **Part (c) | Step 1**

$m^0 = 1$ for $m \neq 0$.

Any nonzero base to the zero power equals 1.

---

5 **Part (d) | Step 1**

$$(4r^2t)^3 = 4^3(r^2)^3t^3 = 64r^6t^3.$$

Cube each factor and multiply the exponents in the power of a power.

✓ **FINAL ANSWER**

(a) $x = -\frac{5}{8}$. (b) $P = 34$. (c) 1. (d) $64r^6t^3$.

TOPIC

Forming & Solving Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

Question 3

4 marks | Forming & Solving Equations

3 Three tins, A, B and C, each contain buttons.

Tin A contains x buttons.
Tin B contains 4 times the number of buttons that tin A contains.
Tin C contains 7 fewer buttons than tin A.

The total number of buttons in the three tins is 137

Work out the number of buttons in tin C.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2020

1 Write each tin in terms of x

Tin A has x buttons, tin B has $4x$, tin C has $x - 7$.

2 Form and solve the equation

$$x + 4x + (x - 7) = 137 \Rightarrow 6x - 7 = 137 \Rightarrow 6x = 144 \Rightarrow x = 24$$

3 Buttons in tin C

$$24 - 7 = 17$$

✓ **FINAL ANSWER**

17 buttons

4MA1/2H NOVEMBER 2021

Question 2

4 marks | Forming & Solving Equations

2 Here are two rectangles, rectangle A and rectangle B.

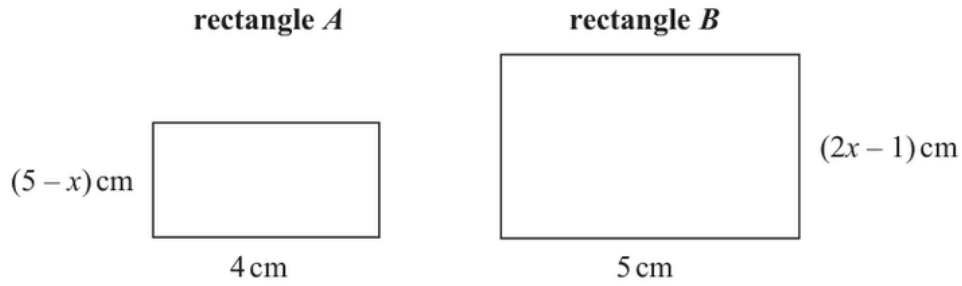


Diagram **NOT** accurately drawn

The area of rectangle B is twice the area of rectangle A.

Work out the value of x.
Show your working clearly.

$x =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2021

1 Area of rectangle A

Area of rectangle A:

$$4(5 - x)$$

2 Area of rectangle B

Area of rectangle B:

$$5(2x - 1)$$

3 **Rectangle B has twice the**

Rectangle B has twice the area of rectangle A:

$$5(2x - 1) = 2 \cdot 4(5 - x)$$

$$10x - 5 = 40 - 8x$$

$$18x = 45$$

$$x = 2.5$$

FINAL ANSWER

$x = 2.5$.

4MA1/1H NOVEMBER 2021

Question 5

5 marks | Forming & Solving Equations

5 Yasmin has some identical rectangular tiles.
Each tile is L cm by W cm.

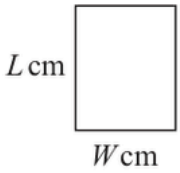


Diagram **NOT** accurately drawn

Using 9 of her tiles, Yasmin makes rectangle $ABCD$, shown in the diagram below.

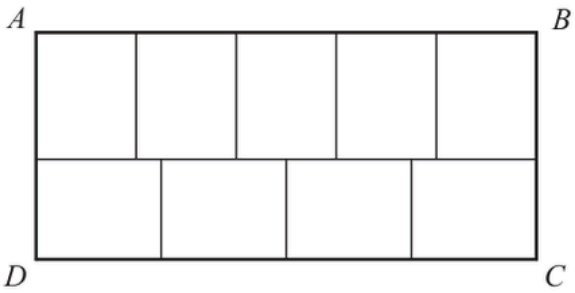


Diagram **NOT** accurately drawn

The area of $ABCD$ is $1620\,\text{cm}^2$

Work out the value of L and the value of W.

$L =$ ..... $W =$ .....

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2021

1 The top row has 5

The top row has 5 tile widths and the bottom row has 4 tile lengths, so

$$5W = 4L$$

$$L = \frac{5}{4}W$$

2 Use trigonometry

The rectangle has width $5W$ and height $L + W$:

$$5W(L + W) = 1620$$

3 **Substitute**

Substitute $L = \frac{5}{4}W$:

$$5W\left(\frac{5}{4}W + W\right) = 1620$$

$$5W\left(\frac{9}{4}W\right) = 1620$$

$$\frac{45}{4}W^2 = 1620$$

$$W^2 = 144$$

$$W = 12, \qquad L = \frac{5}{4}(12) = 15$$

✓ **FINAL ANSWER**

$L = 15\ \text{cm}$, $W = 12\ \text{cm}$.

4MA1/1H NOVEMBER 2023

Question 2

6 marks | Forming & Solving Equations

2 (a) Factorise fully $12pq - 18p$

.....

(2)

There are 56 metal bars in a box.
Each metal bar is gold or silver or zinc.

y metal bars are gold.
$(3y + 7)$ metal bars are silver.
$(2y - 5)$ metal bars are zinc.

(b) Work out the number of metal bars that are zinc.
Show clear algebraic working.

.....

(4)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2023

1 **(a)**

(a)

$$12pq - 18p = 6p(2q - 3)$$

2 **Solve equation**

(b) The total number of bars is 56, so

$$y + (3y + 7) + (2y - 5) = 56$$

$$6y + 2 = 56$$

$$y = 9$$

3 **The number of zinc bars**

The number of zinc bars is

$$2y - 5 = 2(9) - 5 = 13$$

FINAL ANSWER

(a) $6p(2q - 3)$, (b) 13 zinc bars

Question 6

3 marks | Forming & Solving Equations

$\left(6^2\right)^{10} = 6^p$

(c) Find the value of p

$p =$ .....

(1)

$5^r = 1$

(d) Write down the value of r

$r =$ .....

(1)

(Total for Question 5 is 4 marks)

6 The diagram shows triangle ABC

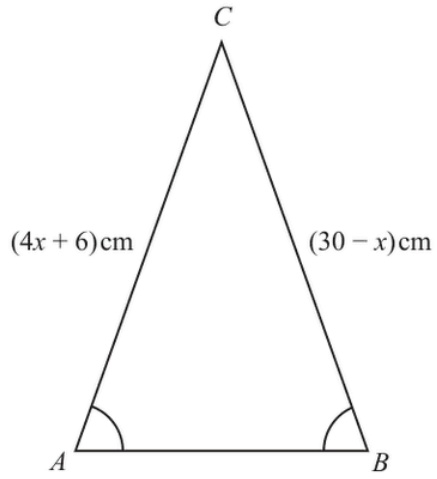


Diagram **NOT** accurately drawn

$CA = (4x + 6)$ cm $\quad CB = (30 - x)$ cm $\quad$ angle CAB = angle CBA

Work out the value of x

$x =$ .....

(Total for Question 6 is 3 marks)

1 Use trigonometry

The base angles are equal, so the opposite sides are equal.

$$4x + 6 = 30 - x$$

$$5x = 24$$

$$x = 4.8$$

FINAL ANSWER

$x = 4.8$.

4MA1/2H NOVEMBER 2025

Question 20

3 marks | Forming & Solving Equations

20 Here is a quadratic equation.

$$ax^2 + 4x + c = 0$$

The solutions of this equation are given by $x = \dfrac{-4 \pm 2\sqrt{39}}{10}$

Find the value of a and the value of c
Show your working clearly.

$a =$ .....

$c =$ .....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2025

1 Calculate value

For

$$ax^2 + 4x + c = 0$$

2 the quadratic formula gives

the quadratic formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4ac}}{2a}$$

3 The given solutions are

The given solutions are

$$x = \frac{-4 \pm 2\sqrt{39}}{10}$$

4 Compare denominators

Compare denominators:

$$2a = 10$$

$$a = 5$$

5 **Compare the square root parts**

Compare the square root parts:

$$\sqrt{16 - 4ac} = 2\sqrt{39}$$

6 **Square both sides**

Square both sides:

$$16 - 4ac = 156$$

7 **Use**

Use $a = 5$:

$$16 - 20c = 156$$

$$-20c = 140$$

$$c = -7$$

✓ **FINAL ANSWER**

$a = 5,\ c = -7$

TOPIC

Rearranging Formulae

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 3

8 marks | Rearranging Formulae

3 (a) Make a the subject of $d = g + 2ac$

.....

(2)

(b) Factorise fully $9ef - 12f$

.....

(2)

(c) Expand and simplify $(x + 2)(x - 5)$

.....

(2)

(d) Simplify fully $\dfrac{n^4 \times n^7}{n^5}$

.....

(2)

(Total for Question 3 is 8 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2020

1 **(a)**

(a)

$$d = g + 2ac$$

$$d - g = 2ac$$

$$a = \frac{d - g}{2c}$$

2 **(b)**

(b)

$$9ef - 12f = 3f(3e - 4)$$

3 **(c)**

(c)

$$(x+2)(x-5) = x^2 - 3x - 10$$

---

4 **(d)**

(d)

$$\frac{n^4 \times n^7}{n^5} = n^{4+7-5} = n^6$$

✓ **FINAL ANSWER**

$a = \frac{d-g}{2c}$, $3f(3e-4)$, $x^2 - 3x - 10$, n^6

4MA1/2H JANUARY 2020

Question 16

4 marks | Rearranging Formulae

16 Make x the subject of $y = \sqrt{\dfrac{x+1}{x-4}}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H January 2020

1 Rearrange formula

$$y = \sqrt{\frac{x+1}{x-4}}$$

2 Square both sides

Square both sides:

$$y^2 = \frac{x+1}{x-4}$$

$$y^2(x-4) = x+1$$

$$y^2x - 4y^2 = x+1$$

$$x(y^2-1) = 4y^2+1$$

FINAL ANSWER

$$x = \frac{4y^2+1}{y^2-1}$$

4MA1/1HR JANUARY 2021

Question 6

5 marks | Rearranging Formulae

6 (a) Make c the subject of $A = \frac{c}{y} - 5z$

.....

(2)

(b) Write down the value of g^0

.....

(1)

(c) Factorise $x^2 - 11x + 24$

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$A = \frac{c}{y} - 5z$$

$$A + 5z = \frac{c}{y}$$

$$c = y(A + 5z)$$

2 **(b)**

(b) $g^0 = 1$.

3 **(c)**

(c)

$$x^2 - 11x + 24 = (x - 3)(x - 8)$$

✓ **FINAL ANSWER**

$c = y(A + 5z)$, 1, $(x - 3)(x - 8)$

4MA1/2H JANUARY 2021

Question 14

7 marks | Rearranging Formulae

14 (a) Solve $\frac{9a-7}{5} - \frac{3a-7}{4} = 4.55$

Show clear algebraic working.

$a =$ .....

(3)

(b) Make c the subject of the formula $p = \sqrt{\frac{ac+8}{3+c}}$

.....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2021

1 **(a)**

(a)

$$\frac{9a-7}{5} - \frac{3a-7}{4} = 4.55$$

2 **Multiply by 20**

Multiply by 20:

$$4(9a-7) - 5(3a-7) = 91$$

$$36a - 28 - 15a + 35 = 91$$

$$21a + 7 = 91$$

$$a = 4$$

3 **(b)**

(b)

$$p = \sqrt{\frac{ac+8}{3+c}}$$

$$p^2 = \frac{ac+8}{3+c}$$

$$p^2(3+c) = ac+8$$

$$c(p^2 - a) = 8 - 3p^2$$

FINAL ANSWER

(a) $a = 4$, (b) $c = \frac{8-3p^2}{p^2-a}$

4MA1/1H MAY 2021

Question 5

6 marks | Rearranging Formulae

5 (a) Expand and simplify $3x(2x + 3) - x(3x + 5)$

.....

(2)

(b) Make t the subject of the formula $p = at - d$

.....

(2)

Given that $\frac{w^5 \times w^n}{w^3} = w^{10}$

(c) work out the value of n.

$n =$ .....

(2)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2021

1 **(a)**

(a)

$$3x(2x+3) - x(3x+5) = 6x^2 + 9x - 3x^2 - 5x$$

$$= 3x^2 + 4x$$

2 **(b)**

(b)

$$p = at - d$$

$$p + d = at$$

$$t = \frac{p+d}{a}$$

3 **(c)**

(c)

$$\frac{w^5 \times w^n}{w^3} = w^{n+2} = w^{10}$$

4 **Rearrange formula**

So $n + 2 = 10$, hence $n = 8$.

FINAL ANSWER

(a) $3x^2 + 4x$, (b) $t = \frac{p+d}{a}$, (c) $n = 8$

4MA1/1H NOVEMBER 2021

Question 17

8 marks | Rearranging Formulae

17 (a) Factorise $6y^2 - y - 5$

.....

(2)

(b) Make f the subject of $w = \dfrac{2f + 3}{8 - f}$

.....

(3)

(c) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

(Total for Question 17 is 8 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2021

1 **(a)**

(a)

$$6y^2 - y - 5 = (6y + 5)(y - 1)$$

2 **(b)**

(b)

$$w = \frac{2f + 3}{8 - f}$$

$$w(8 - f) = 2f + 3$$

$$8w - 3 = f(w + 2)$$

$$f = \frac{8w - 3}{w + 2}$$

3 **(c)**

(c)

$$4x^2 - 8x + 7 = 4(x^2 - 2x) + 7$$

$$= 4\left((x-1)^2 - 1\right) + 7$$

$$= 4(x-1)^2 + 3$$

FINAL ANSWER

$(6y+5)(y-1)$, $f = \frac{8w-3}{w+2}$, $4(x-1)^2 + 3$

4MA1/2HR JANUARY 2022

Question 15

4 marks | Rearranging Formulae

15 Make t the subject of $n^2 = \dfrac{4d + t^3}{t^3}$

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR January 2022

1 Rearrange formula

$$n^2 = \frac{4d + t^3}{t^3}$$

$$n^2 t^3 = 4d + t^3$$

$$t^3(n^2 - 1) = 4d$$

$$t^3 = \frac{4d}{n^2 - 1}$$

FINAL ANSWER

$$t = \sqrt[3]{\frac{4d}{n^2 - 1}}$$

4MA1/2H JUNE 2022

Question 14

7 marks | Rearranging Formulae

14 (a) Expand and simplify $(5 - x)(2x + 3)(x + 4)$
Show your working clearly.

.....

(3)

(b) Make c the subject of $g = \dfrac{c + 3}{4 + c} - 7$

.....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2022

1 **(a)**

(a)

$$(5 - x)(2x + 3)(x + 4)$$

$$(2x + 3)(x + 4) = 2x^2 + 11x + 12$$

$$(5 - x)(2x^2 + 11x + 12) = -2x^3 - x^2 + 43x + 60$$

2 **(b)**

(b)

$$g = \frac{c + 3}{4 + c} - 7$$

$$g + 7 = \frac{c + 3}{c + 4}$$

$$(g + 7)(c + 4) = c + 3$$

$$c(g + 6) = -4g - 25$$

FINAL ANSWER

(a) $-2x^3 - x^2 + 43x + 60$, (b) $c = \frac{-4g-25}{g+6}$

4MA1/2H JUNE 2023

Question 17

7 marks | Rearranging Formulae

17 (a) Expand and simplify $(x + 6)(3x - 2)(x + 6)$

.....

(3)

(b) Make e the subject of $w = \sqrt{\dfrac{e+g}{ef-d}}$

.....

(4)

(Total for Question 17 is 7 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2023

1 **(a)**

(a)

$$(x+6)(3x-2)(x+6) = (x+6)^2(3x-2)$$

$$= (x^2+12x+36)(3x-2)$$

$$= 3x^3+34x^2+84x-72$$

2 **(b)**

(b)

$$w = \sqrt{\frac{e+g}{ef-d}}$$

$$w^2 = \frac{e+g}{ef-d}$$

$$w^2(ef-d) = e+g$$

$$e(fw^2-1) = g+dw^2$$

FINAL ANSWER

(a) $3x^3 + 34x^2 + 84x - 72$, (b) $e = \frac{g+dw^2}{fw^2-1}$

4MA1/2HR JUNE 2023

Question 17

4 marks | Rearranging Formulae

17 Make x the subject of $y = \sqrt[3]{\dfrac{6+5x}{x+4}}$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2023

1 Rearrange formula

$$y = \sqrt[3]{\frac{6+5x}{x+4}}$$

$$y^3 = \frac{6+5x}{x+4}$$

$$y^3(x+4) = 6+5x$$

$$x(y^3-5) = 6-4y^3$$

FINAL ANSWER

$$x = \frac{6-4y^3}{y^3-5}$$

4MA1/1H NOVEMBER 2023

Question 15

3 marks | Rearranging Formulae

15 Make n the subject of the formula $x = \dfrac{3p+n}{3n-4}$

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2023

1 Rearrange formula

$$x = \frac{3p+n}{3n-4}$$

$$x(3n-4) = 3p+n$$

$$3xn - 4x = 3p+n$$

$$n(3x-1) = 3p+4x$$

FINAL ANSWER

$n = \frac{3p+4x}{3x-1}$

4MA1/2H JUNE 2024

Question 15

7 marks | Rearranging Formulae

15 (a) Make g the subject of $e = \sqrt{\dfrac{7g + 5}{11 + 2g}}$

.....

(4)

(b) Solve the inequality $3y^2 + 4y - 32 > 0$
Show your working clearly.

.....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2024

1 **(a)**

(a)

$$e = \sqrt{\frac{7g + 5}{11 + 2g}}$$

$$e^2 = \frac{7g + 5}{11 + 2g}$$

$$e^2(11 + 2g) = 7g + 5$$

$$g(2e^2 - 7) = 5 - 11e^2$$

$$g = \frac{5 - 11e^2}{2e^2 - 7}$$

2 **(b)**

(b)

$$3y^2 + 4y - 32 = (3y - 8)(y + 4)$$

3 Solve quadratic equation

The critical values are $y = -4$ and $y = \frac{8}{3}$. Since the quadratic is positive outside the roots,

$$y < -4 \quad \text{or} \quad y > \frac{8}{3}$$

FINAL ANSWER

(a) $g = \frac{5-11e^2}{2e^2-7}$, (b) $y < -4$ or $y > \frac{8}{3}$

4MA1/2H NOVEMBER 2024

Question 12

7 marks | Rearranging Formulae

12 (a) Solve $\frac{3x+2}{5} - \frac{2x+1}{3} = x$

Show clear algebraic working.

$x =$ .....

(3)

(b) Make c the subject of the formula $f = \sqrt{\frac{a+bc}{c-d}}$

.....

(4)

(Total for Question 12 is 7 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2024

1 **(a)**

(a)

$$\frac{3x+2}{5} - \frac{2x+1}{3} = x$$

2 **Multiply by 15**

Multiply by 15:

$$3(3x+2) - 5(2x+1) = 15x$$

$$9x + 6 - 10x - 5 = 15x$$

$$1 = 16x$$

$$x = \frac{1}{16}$$

3 **(b)**

(b)

$$f = \sqrt{\frac{a+bc}{c-d}}$$

$$f^2 = \frac{a+bc}{c-d}$$

$$f^2(c-d) = a+bc$$

$$c(f^2-b) = a+df^2$$

FINAL ANSWER

(a) $x = \frac{1}{16}$, (b) $c = \frac{a+df^2}{f^2-b}$

4WM2/01R MAY 2025

Question 14

4 marks | Rearranging Formulae

14 Make k the subject of $p = \dfrac{8k^2 + 5}{7 - 3k^2}$

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4WM2/01R May 2025

1 Required

Make *k* the subject of $displaystylep = dfrac8k^2 + 57 - 3k^2$.

2 Multiply both sides by (7

Multiply both sides by (7−3*k*²):

p(7−3*k*²) = 8*k*² + 5

3 Expand 7 p 3 p

Expand:

7*p* − 3*p***k*² = 8*k*² + 5

4 Collect all k 2 terms

Collect all *k*² terms on one side:

7*p* − 5 = 8*k*² + 3*p***k*²

5 Factorise and solve for k

Factorise and solve for *k*:

$$7p - 5 = k^2(3p + 8) impliesk^2 = frac7p - 53p + 8$$

$$k = pmsqrtdfrac7p - 53p + 8$$

6 Note

Include both ± roots unless the context specifies the sign of *k*.

FINAL ANSWER

$k = pmsqrtdfrac7p - 53p + 8$

4MA1/2H JUNE 2025

Question 16

4 marks | Rearranging Formulae

16 Make t the subject of the formula $c = \dfrac{t^2 + 3}{7 - 8t^2}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2025

1 Rearrange formula

$$c = \frac{t^2 + 3}{7 - 8t^2}$$

$$c(7 - 8t^2) = t^2 + 3$$

$$7c - 3 = t^2(8c + 1)$$

$$t^2 = \frac{7c - 3}{8c + 1}$$

FINAL ANSWER

$$t = \pm\sqrt{\frac{7c-3}{8c+1}}$$

4MA1/2HR JUNE 2025

Question 17

4 marks | Rearranging Formulae

17 Make k the subject of $p = \dfrac{8k^2 + 5}{7 - 3k^2}$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2025

1 Rearrange formula

$$p = \frac{8k^2 + 5}{7 - 3k^2}$$

$$p(7 - 3k^2) = 8k^2 + 5$$

$$7p - 5 = k^2(3p + 8)$$

$$k^2 = \frac{7p - 5}{3p + 8}$$

FINAL ANSWER

$$k = \pm\sqrt{\frac{7p-5}{3p+8}}$$

4WM2/01 MAY 2025

Question 19

4 marks | Rearranging Formulae

19 Make t the subject of the formula $c = \frac{t^2 + 3}{7 - 8t^2}$

.....

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4WM2/01 May 2025

1 Evaluate fraction

$$c = \frac{t^2 + 3}{7 - 8t^2}$$

$$c(7 - 8t^2) = t^2 + 3$$

$$7c - 3 = t^2(8c + 1)$$

$$t^2 = \frac{7c - 3}{8c + 1}$$

FINAL ANSWER

$$t = \pm\sqrt{\frac{7c - 3}{8c + 1}}.$$

4WM2/01 NOVEMBER 2025

Question 18

4 marks | Rearranging Formulae

18 Make w the subject of the formula $p = \sqrt{\dfrac{7w + y}{cw + k}}$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 November 2025

1 Multiply both sides denominator

$$p^2 = frac7w + ycw + k$$

Step 2: Multiply both sides by the denominator (*cw*+*k*).**

*p*²(*c**w*+*k*) = 7*w* + *y*

2 Expand brackets

*p*²*c**w* + *p*²*k* = 7*w* + *y*

Step 4: Collect all terms involving *w* on one side.

*p*²*c**w* − 7*w* = *y* − *p*²*k*

Step 5: Factor out *w*.

w(*p*²*c*−7) = *y* − *p*²*k*

Step 6: Divide to isolate *w*.

$$w = fracy - p^2kp^2c - 7$$

FINAL ANSWER

$$w = fracy - p^2kp^2c - 7$$

4MA1/1H NOVEMBER 2025

Question 18

4 marks | Rearranging Formulae

18 Make y the subject of $p = \sqrt{\dfrac{y+w}{3y-t}}$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2025

1 Rearrange formula

$$p = \sqrt{\frac{y+w}{3y-t}}$$

$$p^2 = \frac{y+w}{3y-t}$$

$$p^2(3y-t) = y+w$$

$$3p^2y - p^2t = y+w$$

$$y(3p^2-1) = w+p^2t$$

FINAL ANSWER

$y = \frac{w+p^2t}{3p^2-1}$

4MA1/2H JUNE 2026

Question 18

4 marks | Rearranging Formulae

18 Make x the subject of the formula $a = \dfrac{3w - x^2}{x^2 + 6}$

.....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2026

1 Solution | Step 1

$$a = \frac{3w - x^2}{x^2 + 6} \quad \Rightarrow \quad a(x^2 + 6) = 3w - x^2.$$

Multiply by x^2+6, which is positive for real x.

2 Solution | Step 2

$$ax^2 + 6a = 3w - x^2 \quad \Rightarrow \quad ax^2 + x^2 = 3w - 6a.$$

Collect both terms involving x^2 on one side.

3 Solution | Step 3

$$(a + 1)x^2 = 3w - 6a \quad \Rightarrow \quad x^2 = \frac{3w - 6a}{a + 1}.$$

Factor out x^2 and divide by a+1, assuming a is not -1.

4 Solution | Step 4

$$x = \pm\sqrt{\frac{3w - 6a}{a + 1}} \quad (a \neq -1).$$

For real solutions the radicand must be nonnegative. If $a = -1$, the original equation instead requires $w = -2$; then every real x satisfies it.

Taking square roots gives both signs; handle separately the value excluded by division.

FINAL ANSWER

$x = \pm\sqrt{\dfrac{3w - 6a}{a + 1}}$, for $a \neq -1$ and nonnegative radicand.

4WM2/01R JUNE 2026

Question 19

3 marks | Rearranging Formulae

19 Make x the subject of $y = \frac{3x+5}{4-2x}$

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM2/01R June 2026

1 Multiply out the fraction

$$y(4-2x) = 3x+5 \Rightarrow 4y-2xy = 3x+5$$

2 Collect x terms on one side

$$4y-5 = 3x+2xy = x(3+2y)$$

3 Divide

$$x = \frac{4y-5}{2y+3}$$

✓ **FINAL ANSWER**

$$x = \frac{4y-5}{2y+3}$$

4MA1/2HR JUNE 2026

Question 20

3 marks | Rearranging Formulae

20 Make x the subject of $y = \dfrac{3x+5}{4-2x}$

.....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2026

1 Solution | Step 1

$$y(4-2x) = 3x+5 \quad \Rightarrow \quad 4y-2xy = 3x+5.$$

Multiply by the denominator, which is nonzero in the original expression.

2 Solution | Step 2

$$4y-5 = 3x+2xy = x(3+2y).$$

Collect both x-terms together and factor out x.

3 Solution | Step 3

$$x = \frac{4y-5}{2y+3}.$$

Divide by 2y+3. The inverse formula requires y not equal to -3/2; the original requires x not equal to 2.

FINAL ANSWER

$x = \frac{4y-5}{2y+3}$.

TOPIC

Simultaneous Equations

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 11

3 marks | Simultaneous Equations

11 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4WM2/01 Specimen

1 **Solve simultaneous equations**

From

$$3x - y = 7$$

$$y = 3x - 7$$

2 **Solve simultaneous equations**

Substitute into $7x + 3y = 3$:

$$7x + 3(3x - 7) = 3$$

$$16x - 21 = 3$$

$$16x = 24$$

$$x = 1.5$$

---

3 **Solve simultaneous equations**

Then

$$y = 3(1.5) - 7 = -2.5$$

✓ **FINAL ANSWER**

$x = 1.5,\ y = -2.5$.

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

1 **Solve simultaneous equations**

From

$$x - 2y = 3$$

$$x = 2y + 3$$

2 **Solve simultaneous equations**

Substitute into $x^2 - y^2 + 2x = 10$:

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

3 **Solve simultaneous equations**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

4 **Solve simultaneous equations**

If $y = -\frac{1}{3}$,

$$x = 2\left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

5 **Solve simultaneous equations**

If $y = -5$,

$$x = 2(-5) + 3 = -7$$

✓ **FINAL ANSWER**

$\left(\frac{7}{3}, -\frac{1}{3}\right)$ and $(-7, -5)$.

4MA1/2HR JANUARY 2020

Question 7

3 marks | Simultaneous Equations

7 Solve the simultaneous equations

$$3x + 5y = 6$$
$$7x - 5y = -11$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2020

1 Solve simultaneous equations

$$3x + 5y = 6$$

$$7x - 5y = -11$$

2 Add the equations

Add the equations:

$$10x = -5$$

$$x = -\frac{1}{2}$$

3 **Solve simultaneous equations**

Substitute into $3x + 5y = 6$:

$$3\left(-\frac{1}{2}\right) + 5y = 6$$

$$-\frac{3}{2} + 5y = 6$$

$$5y = \frac{15}{2}$$

$$y = \frac{3}{2}$$

✓ **FINAL ANSWER**

$x = -\frac{1}{2}$, $y = \frac{3}{2}$.

4MA1/1H MAY/NOVEMBER 2020

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$7x - 2y = 34$$
$$3x + 5y = -3$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May/November 2020

1 **Solve simultaneous equations**

$$7x - 2y = 34$$

$$3x + 5y = -3$$

2 **Multiply the first equation by**

Multiply the first equation by 5:

$$35x - 10y = 170$$

3 **Multiply the second equation by**

Multiply the second equation by 2:

$$6x + 10y = -6$$

4 **Add**

Add:

$$41x = 164$$

$$x = 4$$

5 **Solve simultaneous equations**

Substitute into $3x + 5y = -3$:

$$12 + 5y = -3$$

$$y = -3$$

FINAL ANSWER

$x = 4$, $y = -3$.

4MA1/1HR MAY/NOVEMBER 2020

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$7x + 2y = 5.5$$
$$3x - 5y = 17$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR May/November 2020

1 Solve simultaneous equations

$$7x + 2y = 5.5$$

$$3x - 5y = 17$$

2 Multiply the first equation by

Multiply the first equation by 5:

$$35x + 10y = 27.5$$

3 Multiply the second equation by

Multiply the second equation by 2:

$$6x - 10y = 34$$

4 Add

Add:

$$41x = 61.5$$

$$x = 1.5$$

5 **Solve simultaneous equations**

Substitute into $7x + 2y = 5.5$:

$$10.5 + 2y = 5.5$$

$$y = -2.5$$

FINAL ANSWER

$x = 1.5$, $y = -2.5$.

4MA1/2HR JANUARY 2021

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations $2x + 7y = 17$
$5x + 3y = -1$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2021

1 **Solve simultaneous equations**

$$2x + 7y = 17$$

$$5x + 3y = -1$$

2 **Multiply the first equation by**

Multiply the first equation by 5:

$$10x + 35y = 85$$

3 **Multiply the second equation by**

Multiply the second equation by 2:

$$10x + 6y = -2$$

4 **Subtract**

Subtract:

$$29y = 87$$

$$y = 3$$

5 **Solve simultaneous equations**

Substitute into $2x + 7y = 17$:

$$2x + 21 = 17$$

$$2x = -4$$

$$x = -2$$

✓ **FINAL ANSWER**

$x = -2$, $y = 3$.

4MA1/1HR JANUARY 2021

Question 17

5 marks | Simultaneous Equations

17 Solve the simultaneous equations

$$x - 6y = 5$$
$$xy - 2y^2 = 6$$

Show clear algebraic working.

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR January 2021

1 Solve simultaneous equations

$$x - 6y = 5$$

$$xy - 2y^2 = 6$$

2 From the first equation

From the first equation:

$$x = 6y + 5$$

3 Substitute into the second equation

Substitute into the second equation:

$$y(6y + 5) - 2y^2 = 6$$

$$4y^2 + 5y - 6 = 0$$

4 Factorise

Factorise:

$$(4y - 3)(y + 2) = 0$$

5 Solve simultaneous equations

So

$$y = \frac{3}{4} \quad \text{or} \quad y = -2$$

6 **Solve simultaneous equations**

If $y = \frac{3}{4}$:

$$x = 6\left(\frac{3}{4}\right) + 5 = \frac{19}{2}$$

---

7 **Solve simultaneous equations**

If $y = -2$:

$$x = 6(-2) + 5 = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{19}{2}, \frac{3}{4}\right)$ or $(-7, -2)$.

4MA1/1H JANUARY 2021

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$
$$x + 2y - 1 = 0$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2021

1 Solve simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$

$$x + 2y - 1 = 0$$

2 From the second equation

From the second equation:

$$x = 1 - 2y$$

3 Substitute into the first equation

Substitute into the first equation:

$$(1 - 2y)^2 - 9y - (1 - 2y) = 2y^2 - 12$$

$$4y^2 - 11y = 2y^2 - 12$$

$$2y^2 - 11y + 12 = 0$$

4 Factorise

Factorise:

$$(2y - 3)(y - 4) = 0$$

5 **Solve simultaneous equations**

So

$$y = \frac{3}{2} \quad \text{or} \quad y = 4$$

---

6 **Solve simultaneous equations**

If $y = \frac{3}{2}$:

$$x = 1 - 2\left(\frac{3}{2}\right) = -2$$

---

7 **Solve simultaneous equations**

If $y = 4$:

$$x = 1 - 8 = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(-2, \frac{3}{2}\right)$ or $(-7, 4)$.

4MA1/2H MAY 2021

Question 8

3 marks | Simultaneous Equations

8 Solve the simultaneous equations

$$5a + 2c = 10$$
$$2a - 4c = 7$$

Show clear algebraic working.

$a =$ .....

$c =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H May 2021

1 Solve simultaneous equations

$$5a + 2c = 10$$

$$2a - 4c = 7$$

2 Double the first equation

Double the first equation:

$$10a + 4c = 20$$

3 Add this to the second

Add this to the second equation:

$$12a = 27$$

$$a = \frac{9}{4}$$

4 **Solve simultaneous equations**

Substitute into $5a + 2c = 10$:

$$5\left(\frac{9}{4}\right) + 2c = 10$$

$$\frac{45}{4} + 2c = \frac{40}{4}$$

$$2c = -\frac{5}{4}$$

$$c = -\frac{5}{8}$$

✓ **FINAL ANSWER**

$a = \frac{9}{4}$, $c = -\frac{5}{8}$.

4MA1/2H MAY 2021

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$y = 3 - 2x$$
$$x^2 + y^2 = 18$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H May 2021

1 **Solve simultaneous equations**

$$y = 3 - 2x$$

$$x^2 + y^2 = 18$$

2 **Substitute**

Substitute $y = 3 - 2x$:

$$x^2 + (3 - 2x)^2 = 18$$

$$x^2 + 9 - 12x + 4x^2 = 18$$

$$5x^2 - 12x - 9 = 0$$

3 **Factorise**

Factorise:

$$(5x + 3)(x - 3) = 0$$

4 **Solve simultaneous equations**

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = 3$$

5 **Solve simultaneous equations**

If $x = -\frac{3}{5}$, then

$$y = 3 - 2\left(-\frac{3}{5}\right) = \frac{21}{5}$$

---

6 **Solve simultaneous equations**

If $x = 3$, then

$$y = 3 - 6 = -3$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, \frac{21}{5}\right)$ or $(3, -3)$.

4MA1/1H NOVEMBER 2021

Question 6

5 marks | Simultaneous Equations

6 Alison buys 5 apples and 3 pears for a total cost of $1.96
Greg buys 3 apples and 2 pears for a total cost of $1.22

Michael buys 10 apples and 10 pears.

Work out how much Michael pays for his 10 apples and 10 pears.
Show your working clearly.

$.....

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2021

1 Solve simultaneous equations

Let a be the cost of one apple and p be the cost of one pear.

$$5a + 3p = 1.96$$

$$3a + 2p = 1.22$$

2 Multiply the first equation by

Multiply the first equation by 2:

$$10a + 6p = 3.92$$

3 Multiply the second equation by

Multiply the second equation by 3:

$$9a + 6p = 3.66$$

4 Subtract

Subtract:

$$a = 0.26$$

5 **Solve simultaneous equations**

Then

$$3(0.26) + 2p = 1.22$$

$$2p = 0.44$$

$$p = 0.22$$

---

6 **Solve simultaneous equations**

Michael buys 10 apples and 10 pears:

$$10(0.26) + 10(0.22) = 4.80$$

✓ **FINAL ANSWER**

$\$4.80$.

4MA1/2H JANUARY 2022

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$3x + 5y = 3.1$$
$$6x + 3y = 3.75$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2022

1 Solve simultaneous equations

$$3x + 5y = 3.1$$

$$6x + 3y = 3.75$$

2 Double the first equation

Double the first equation:

$$6x + 10y = 6.2$$

3 Subtract the second equation

Subtract the second equation:

$$7y = 2.45$$

$$y = 0.35$$

4 **Solve simultaneous equations**

Substitute into $3x + 5y = 3.1$:

$$3x + 5(0.35) = 3.1$$

$$3x + 1.75 = 3.1$$

$$3x = 1.35$$

$$x = 0.45$$

✓ **FINAL ANSWER**

$x = 0.45$, $y = 0.35$.

4MA1/1H JANUARY 2022

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$3x^2 + y^2 - xy = 5$$
$$y = 2x - 3$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2022

1 **Solve simultaneous equations**

$$3x^2 + y^2 - xy = 5$$

$$y = 2x - 3$$

2 **Substitute**

Substitute $y = 2x - 3$:

$$3x^2 + (2x - 3)^2 - x(2x - 3) = 5$$

$$3x^2 + 4x^2 - 12x + 9 - 2x^2 + 3x = 5$$

$$5x^2 - 9x + 9 = 5$$

$$5x^2 - 9x + 4 = 0$$

3 **Factorise**

Factorise:

$$(5x - 4)(x - 1) = 0$$

4 **Solve simultaneous equations**

So

$$x = \frac{4}{5} \quad \text{or} \quad x = 1$$

5 **Solve simultaneous equations**

If $x = \frac{4}{5}$:

$$y = 2\left(\frac{4}{5}\right) - 3 = -\frac{7}{5}$$

---

6 **Solve simultaneous equations**

If $x = 1$:

$$y = 2(1) - 3 = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{4}{5}, -\frac{7}{5}\right)$ or $(1, -1)$.

4MA1/2H JUNE 2022

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2022

1 Solve simultaneous equations

$$7x + 3y = 3$$

$$3x - y = 7$$

2 From the second equation

From the second equation:

$$y = 3x - 7$$

3 Substitute into the first equation

Substitute into the first equation:

$$7x + 3(3x - 7) = 3$$

$$7x + 9x - 21 = 3$$

$$16x = 24$$

$$x = \frac{3}{2}$$

4 **Solve simultaneous equations**

Then

$$y = 3\left(\frac{3}{2}\right) - 7 = \frac{9}{2} - 7 = -\frac{5}{2}$$

✓ **FINAL ANSWER**

$x = \frac{3}{2}, y = -\frac{5}{2}$.

4MA1/1HR JUNE 2022

Question 14

4 marks | Simultaneous Equations

14 Solve the simultaneous equations

$$3x - 5y = 25$$
$$4x + 3y = 14$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR June 2022

1 Solve simultaneous equations

$$3x - 5y = 25$$

$$4x + 3y = 14$$

2 Multiply the first equation by

Multiply the first equation by 3:

$$9x - 15y = 75$$

3 Multiply the second equation by

Multiply the second equation by 5:

$$20x + 15y = 70$$

4 Add

Add:

$$29x = 145$$

$$x = 5$$

5 **Solve simultaneous equations**

Substitute into $3x - 5y = 25$:

$$15 - 5y = 25$$

$$-5y = 10$$

$$y = -2$$

✓ **FINAL ANSWER**

$x = 5$, $y = -2$.

4MA1/2HR JUNE 2022

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2022

1 **Calculate value**

$$x - 2y = 3$$

2 **Calculate value**

So

$$x = 2y + 3$$

3 **Calculate value**

Substitute into

$$x^2 - y^2 + 2x = 10$$

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

4 **Evaluate fraction**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

5 **Find**

Find x:

$$y = -\frac{1}{3} \Rightarrow x = \frac{7}{3}$$

$$y = -5 \Rightarrow x = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{7}{3}, -\frac{1}{3}\right)$ or $(-7, -5)$

4MA1/1H MAY 2023

Question 9

3 marks | Simultaneous Equations

9 Solve the simultaneous equations

$$2x + 9y = 14.5$$
$$7x + 3y = 8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2023

1 Solve simultaneous equations

$$2x + 9y = 14.5$$

$$7x + 3y = 8$$

2 Multiply the second equation by

Multiply the second equation by 3:

$$21x + 9y = 24$$

3 Subtract the first equation

Subtract the first equation:

$$19x = 9.5$$

$$x = 0.5$$

4 **Solve simultaneous equations**

Substitute into $7x + 3y = 8$:

$$7(0.5) + 3y = 8$$

$$3.5 + 3y = 8$$

$$y = 1.5$$

✓ **FINAL ANSWER**

$x = 0.5$, $y = 1.5$.

4MA1/2H JUNE 2023

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2023

1 Calculate value

$$2x^2 + 3y^2 = 11$$

$$x = 3y - 1$$

2 Substitute

Substitute:

$$2(3y - 1)^2 + 3y^2 = 11$$

$$18y^2 - 12y + 2 + 3y^2 = 11$$

$$21y^2 - 12y - 9 = 0$$

$$7y^2 - 4y - 3 = 0$$

$$(7y + 3)(y - 1) = 0$$

3 Evaluate fraction

So

$$y = -\frac{3}{7} \quad \text{or} \quad y = 1$$

4 **Find**

Find x:

$$y = -\frac{3}{7} \Rightarrow x = -\frac{16}{7}$$

$$y = 1 \Rightarrow x = 2$$

✓ **FINAL ANSWER**

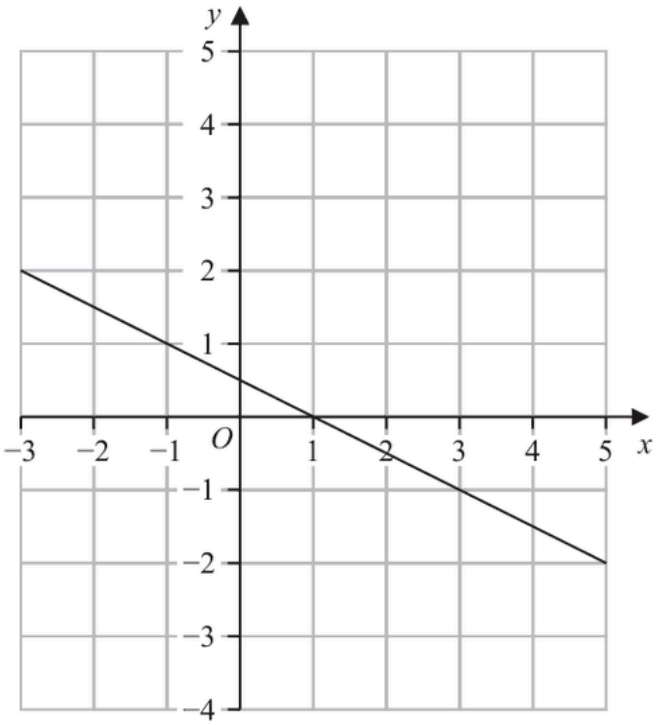
$(x, y) = \left(-\frac{16}{7}, -\frac{3}{7}\right)$ or $(2, 1)$

4MA1/2H NOVEMBER 2023

Question 14

3 marks | Simultaneous Equations

14 Here is the graph of the equation $2y + x = 1$ drawn on a grid.



By drawing another straight line on the grid, solve the simultaneous equations

$$y - x - 2 = 0$$
$$2y + x = 1$$

$x =$ .....

$y =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H November 2023

1 **The equations are**

The equations are

$$y - x - 2 = 0$$

$$2y + x = 1$$

2 **Solve simultaneous equations**

From the first equation,

$$y = x + 2$$

3 **Substitute into the second equation**

Substitute into the second equation:

$$2(x + 2) + x = 1$$

$$3x + 4 = 1$$

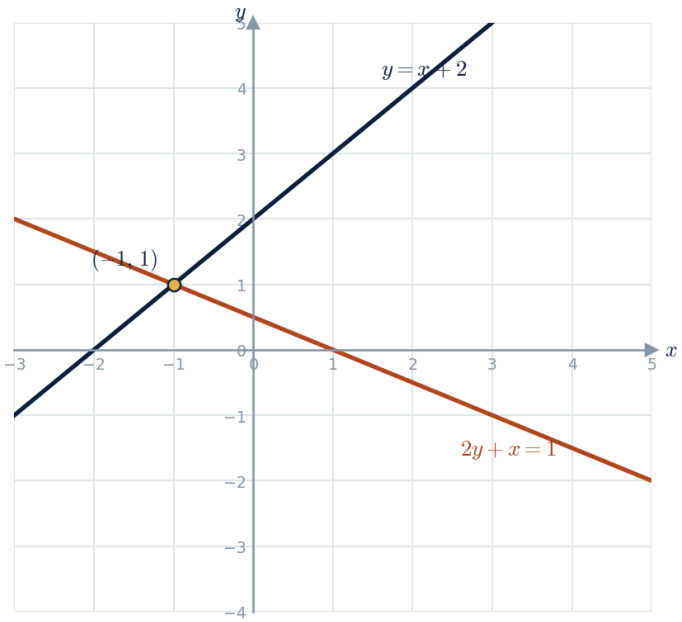
$$3x = -3$$

$$x = -1$$

4 **Solve simultaneous equations**

Then

$$y = -1 + 2 = 1$$



FINAL ANSWER

$x = -1, y = 1.$

4MA1/2H JUNE 2024

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$6x + 4y = 1$$
$$3x + 5y = 8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2024

1 Solve simultaneous equations

$$6x + 4y = 1$$

$$3x + 5y = 8$$

2 Double the second equation

Double the second equation:

$$6x + 10y = 16$$

3 Subtract the first equation

Subtract the first equation:

$$6y = 15$$

$$y = \frac{5}{2}$$

4 **Solve simultaneous equations**

Substitute into $3x + 5y = 8$:

$$3x + 5\left(\frac{5}{2}\right) = 8$$

$$3x + \frac{25}{2} = 8$$

$$3x = -\frac{9}{2}$$

$$x = -\frac{3}{2}$$

✓ **FINAL ANSWER**

$x = -\frac{3}{2}$, $y = \frac{5}{2}$.

4MA1/1HR MAY 2024

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$4x + 3y = 9.6$$
$$6x + 5y = 16.8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR May 2024

1 Solve simultaneous equations

$$4x + 3y = 9.6$$

$$6x + 5y = 16.8$$

2 Multiply the first equation by

Multiply the first equation by 5:

$$20x + 15y = 48$$

3 Multiply the second equation by

Multiply the second equation by 3:

$$18x + 15y = 50.4$$

4 Subtract

Subtract:

$$2x = -2.4$$

$$x = -1.2$$

5 **Solve simultaneous equations**

Substitute into $4x + 3y = 9.6$:

$$4(-1.2) + 3y = 9.6$$

$$3y = 14.4$$

$$y = 4.8$$

✓ **FINAL ANSWER**

$x = -1.2$, $y = 4.8$.

4MA1/2HR JUNE 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = y + 11$$
$$y = 3x - 1$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2024

1 Calculate value

$$x^2 + y^2 = y + 11$$

$$y = 3x - 1$$

2 Substitute

Substitute:

$$x^2 + (3x - 1)^2 = 3x - 1 + 11$$

$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

$$10x^2 - 9x - 9 = 0$$

$$(5x + 3)(2x - 3) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{3}{2}$$

4 **Find**

Find y:

$$x = -\frac{3}{5} \Rightarrow y = -\frac{14}{5}$$

$$x = \frac{3}{2} \Rightarrow y = \frac{7}{2}$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, -\frac{14}{5}\right)$ or $\left(\frac{3}{2}, \frac{7}{2}\right)$

4MA1/2H NOVEMBER 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 + y = 3$$

$$x + 2 = y$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2024

1 Calculate value

$$x^2 + y^2 + y = 3$$

$$y = x + 2$$

2 Substitute

Substitute:

$$x^2 + (x + 2)^2 + (x + 2) = 3$$

$$x^2 + x^2 + 4x + 4 + x + 2 = 3$$

$$2x^2 + 5x + 3 = 0$$

$$(2x + 3)(x + 1) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

4 **Find**

Find y:

$$x = -\frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$x = -1 \Rightarrow y = 1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{2}, \frac{1}{2}\right)$ or $(-1, 1)$

4MA1/2HR JUNE 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$3x + 2y = 10$$
$$3x - 4y = 16$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2025

1 Solve simultaneous equations

$$3x + 2y = 10$$

$$3x - 4y = 16$$

2 Solve simultaneous equations

Subtract the second equation from the first:

$$6y = -6$$

$$y = -1$$

3 Solve simultaneous equations

Substitute into $3x + 2y = 10$:

$$3x + 2(-1) = 10$$

$$3x = 12$$

$$x = 4$$

FINAL ANSWER

$x = 4, y = -1.$

4WM2/01R MAY 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$3x + 2y = 10$$
$$3x - 4y = 16$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01R May 2025

1 Required

Solve: 3*x* + 2*y* = 10 and 3*x* − 4*y* = 16 algebraically.

2 Label the equations

Label the equations:

Misplaced &

3 Subtract (2) from (1) eliminate

Subtract (2) from (1) to eliminate *x*:

$$(1) - (2) : quad 6y = -6 implies boxed{y = -1}$$

4 Substitute y 1 into (1)

Substitute *y* = − 1 into (1):

$$3x + 2(-1) = 10 implies 3x = 12 implies boxed{x = 4}$$

5 Check in equation (2) 3(4)

Check in equation (2): 3(4) − 4(−1) = 12 + 4 = 16

FINAL ANSWER

x = 4, *y* = − 1

4MA1/2H JUNE 2025

Question 8

3 marks | Simultaneous Equations

8 Solve the simultaneous equations

$$3x + 5y = 8$$
$$4x + y = -3.5$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2025

1 Solve simultaneous equations

$$3x + 5y = 8$$

$$4x + y = -3.5$$

2 Solve simultaneous equations

From the second equation,

$$y = -3.5 - 4x$$

3 Substitute into the first equation

Substitute into the first equation:

$$3x + 5(-3.5 - 4x) = 8$$

$$3x - 17.5 - 20x = 8$$

$$-17x = 25.5$$

$$x = -1.5$$

4 **Solve simultaneous equations**

Then

$$y = -3.5 - 4(-1.5) = 2.5$$

✓ **FINAL ANSWER**

$x = -1.5$, $y = 2.5$.

4WM2/01 MAY 2025

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$3x + 5y = 8$$
$$4x + y = -3.5$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM2/01 May 2025

1 Calculate value

From

$$4x + y = -3.5$$

$$y = -3.5 - 4x$$

2 Substitute into equation

Substitute into $3x + 5y = 8$:

$$3x + 5(-3.5 - 4x) = 8$$

$$-17x = 25.5$$

$$x = -1.5$$

3 Calculate value

Then

$$y = -3.5 - 4(-1.5) = 2.5$$

✓ **FINAL ANSWER**

$x = -1.5$, $y = 2.5$.

4MA1/1H JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + 3y + y^2 = 7$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2025

1 Calculate value

$$x^2 + y^2 + 3y = 7$$

$$y = x + 2$$

2 Substitute

Substitute:

$$x^2 + (x+2)^2 + 3(x+2) = 7$$

$$x^2 + x^2 + 4x + 4 + 3x + 6 = 7$$

$$2x^2 + 7x + 3 = 0$$

$$(2x+1)(x+3) = 0$$

3 Evaluate fraction

So

$$x = -\frac{1}{2} \quad \text{or} \quad x = -3$$

4 **Find**

Find y:

$$x = -\frac{1}{2} \Rightarrow y = \frac{3}{2}$$

$$x = -3 \Rightarrow y = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{1}{2}, \frac{3}{2}\right)$ or $(-3, -1)$

4MA1/1HR JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = 41$$
$$2x + y = 3$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR June 2025

1 **Calculate value**

$$x^2 + y^2 = 41$$

$$2x + y = 3$$

2 **Calculate value**

So

$$y = 3 - 2x$$

3 **Substitute**

Substitute:

$$x^2 + (3 - 2x)^2 = 41$$

$$x^2 + 9 - 12x + 4x^2 = 41$$

$$5x^2 - 12x - 32 = 0$$

$$(5x + 8)(x - 4) = 0$$

4 **Evaluate fraction**

So

$$x = -\frac{8}{5} \quad \text{or} \quad x = 4$$

5 **Find**

Find y:

$$x = -\frac{8}{5} \Rightarrow y = \frac{31}{5}$$

$$x = 4 \Rightarrow y = -5$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{8}{5}, \frac{31}{5}\right)$ or $(4, -5)$

4WM2/01R MAY 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 + 5y + x^2 = 12$$
$$x = y + 5$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01R May 2025

1 Required

Solve *y*² + 5*y* + *x*² = 12 and *x* = *y* + 5.

2 Substitute x y + 5

Substitute *x* = *y* + 5 into the first equation:

*y*² + 5*y* + (*y*+5)² = 12

3 Expand (y +5) 2

Expand (*y*+5)² = *y*² + 10*y* + 25:

*y*² + 5*y* + *y*² + 10*y* + 25 = 12 ⟹ 2*y*² + 15*y* + 13 = 0

4 Factorise

Factorise:

$$(2y + 13)(y + 1) = 0 implies y = -tfrac132; textor; y = -1$$

5 Find corresponding x values using

Find corresponding *x* values using *x* = *y* + 5:

✓ FINAL ANSWER

(*x*,*y*) = (4, −1)and$left(-dfrac32, , -dfrac132right)$

4MA1/2H NOVEMBER 2025

Question 2

3 marks | Simultaneous Equations

2 Solve the simultaneous equations

$$5x + y = 11$$
$$3x - y = 9$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2025

1 Solve simultaneous equations

$$5x + y = 11$$

$$3x - y = 9$$

2 Add the equations

Add the equations:

$$8x = 20$$

$$x = 2.5$$

3 Solve simultaneous equations

Substitute into $5x + y = 11$:

$$5(2.5) + y = 11$$

$$y = -1.5$$

FINAL ANSWER

$x = 2.5$, $y = -1.5$.

4WM2/01 NOVEMBER 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$5x + y = 11$$
$$3x - y = 9$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 November 2025

1 Label the equations

Label the equations:

Misplaced &

2 Add (1) and (2) eliminate

Step 1: Add (1) and (2) to eliminate *y*.

Misplaced &

3 Solve equation

Step 2: Substitute *x* = 2.5 into equation (1) to find *y*.

Misplaced &

FINAL ANSWER

x = 2.5, *y* = − 1.5

4WM2/01 NOVEMBER 2025

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x^2 + y^2 = 3xy - 11$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 November 2025

1 Substitute into equation

Step 1: Substitute *y* = (*x*+2) into the quadratic equation.

*x*² + (*x*+2)² = 3*x*(*x*+2) − 11

2 Expand brackets

*x*² + (*x*²+4*x*+4) = 3*x*² + 6*x* − 11

2*x*² + 4*x* + 4 = 3*x*² + 6*x* − 11

3 Solve equation

0 = 3*x*² − 2*x*² + 6*x* − 4*x* − 11 − 4

*x*² + 2*x* − 15 = 0

4 Solve quadratic equation

(*x*+5)(*x*−3) = 0

So, *x* = − 5 or *x* = 3.

Step 5: Find corresponding *y* values using *y* = *x* + 2.

If *x* = − 5 ⟹ *y* = − 5 + 2 = − 3

If *x* = 3 ⟹ *y* = 3 + 2 = 5

FINAL ANSWER

(−5,−3) and (3,5)

4MA1/1H JUNE 2026

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$4x + 3y = 9$$
$$2x - 2y = -13$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H June 2026

1 Solution | Step 1

$$4x + 3y = 9, \qquad 2x - 2y = -13.$$

Use both equations.

2 Solution | Step 2

Double the second equation: $4x - 4y = -26$. Subtract this from the first: $7y = 35$, so $y = 5$.

Equal x coefficients allow elimination.

3 Solution | Step 3

$$2x - 2(5) = -13 \implies 2x = -3 \implies x = -\frac{3}{2}.$$

Substitute back to find the other unknown.

FINAL ANSWER

$x = -\frac{3}{2}$, $y = 5$

4MA1/1HR JUNE 2026

Question 12

3 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$4x + 4y = 5$$
$$x + 2y = 1$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR June 2026

1 Solution | Step 1

$$4x + 4y = 5, \qquad x + 2y = 1.$$

Use the two given equations.

2 Solution | Step 2

$$4(x + 2y) = 4 \implies 4x + 8y = 4.$$

Multiply the second equation by 4 to match the coefficients of x.

3 Solution | Step 3

$$(4x + 8y) - (4x + 4y) = 4 - 5 \implies 4y = -1, \quad y = -\frac{1}{4}.$$

Subtract the first equation to eliminate x.

4 Solution | Step 4

$$x + 2\left(-\frac{1}{4}\right) = 1 \implies x = \frac{3}{2}.$$

Substitute into the second original equation.

FINAL ANSWER

$x = \frac{3}{2},\ y = -\frac{1}{4}$

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

1 **Substitute y = 3x − 5**

$$(3x-5)^2 = x^2 + 7 \Rightarrow 9x^2 - 30x + 25 = x^2 + 7$$

2 **Simplify and factorise**

$$8x^2 - 30x + 18 = 0 \Rightarrow 4x^2 - 15x + 9 = 0 \Rightarrow (x-3)(4x-3) = 0$$

3 **Find y for each x**

$x = 3 \Rightarrow y = 4$;\quad $x = \frac{3}{4} \Rightarrow y = \frac{9}{4} - 5 = -\frac{11}{4}$

✓ **FINAL ANSWER**

$x = 3,\ y = 4$ and $x = \frac{3}{4},\ y = -\frac{11}{4}$

4MA1/2HR JUNE 2026

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

.....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2026

1 Solution | Step 1

$$(3x - 5)^2 = x^2 + 7 \quad \Rightarrow \quad 9x^2 - 30x + 25 = x^2 + 7.$$

Substitute the linear expression for y into the nonlinear equation.

2 Solution | Step 2

$$8x^2 - 30x + 18 = 0 \quad \Rightarrow \quad 4x^2 - 15x + 9 = 0.$$

Collect all terms on one side and divide by 2.

3 Solution | Step 3

$$(4x - 3)(x - 3) = 0 \quad \Rightarrow \quad x = \frac{3}{4} \text{ or } x = 3.$$

Factorize the quadratic and set each factor to zero.

4 Solution | Step 4

For $x = \frac{3}{4}$, $y = 3(\frac{3}{4}) - 5 = -\frac{11}{4}$. For $x = 3$, $y = 9 - 5 = 4$.

Find the corresponding y-value for each x, preserving the pairs.

FINAL ANSWER

$(x, y) = (\frac{3}{4}, -\frac{11}{4})$ or $(3, 4)$.

4MA1/2H JUNE 2026

Question 23

5 marks | Simultaneous Equations

23 The straight line **L** has equation $y = 3x - 2$

The curve **C** has equation $x^2 + y^2 - 2y = 12$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , ...) and (..... , ...)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2026

1 Solution | Step 1

Substitute $y = 3x - 2$ into $x^2 + y^2 - 2y = 12$:

$$x^2 + (3x - 2)^2 - 2(3x - 2) = 12.$$

Both equations must hold at an intersection.

2 Solution | Step 2

$$x^2 + 9x^2 - 12x + 4 - 6x + 4 = 12$$

$$10x^2 - 18x - 4 = 0 \quad \Rightarrow \quad 5x^2 - 9x - 2 = 0.$$

Expand carefully, collect like terms, and divide by 2.

3 Solution | Step 3

$$(5x + 1)(x - 2) = 0 \quad \Rightarrow \quad x = -\frac{1}{5} \text{ or } x = 2.$$

Use factorisation and the zero-product property.

4 Solution | Step 4

When $x = -\frac{1}{5}$, $y = 3(-\frac{1}{5}) - 2 = -\frac{13}{5}$. When $x = 2$, $y = 3(2) - 2 = 4$.
Substitute each root into the line, preserving the coordinate pairings.

FINAL ANSWER

$\left(-\frac{1}{5}, -\frac{13}{5}\right)$ and $(2, 4)$.

Dr Eslam Ahmed · eliteigcse.com

TOPIC

Inequalities (Solving & Graphing)

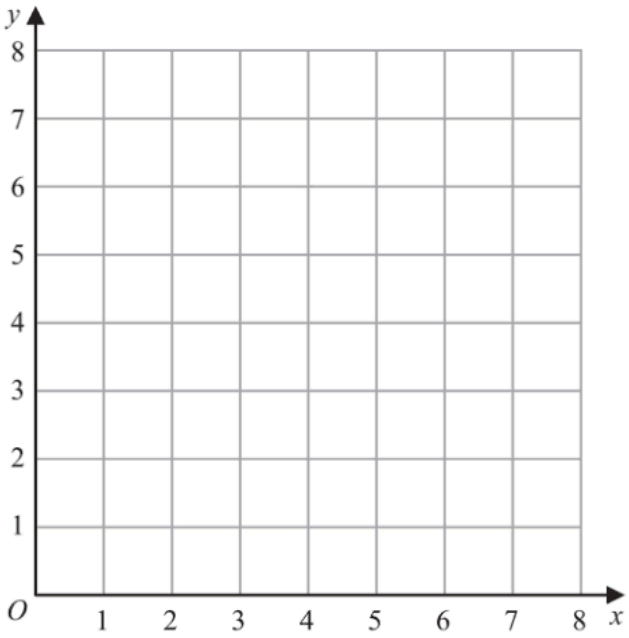
Every past-paper question on this topic, 2020 to 2026.

Question 5

4 marks | Inequalities (Solving & Graphing)

5 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**

(1)

(Total for Question 5 is 4 marks)

1 Draw the three boundary lines

Draw the three boundary lines:

$$y = 1, \qquad x = 2, \qquad x + y = 7$$

2 Read the graph

For $x + y = 7$, use points such as $(0, 7)$ and $(7, 0)$.

3 The required region is

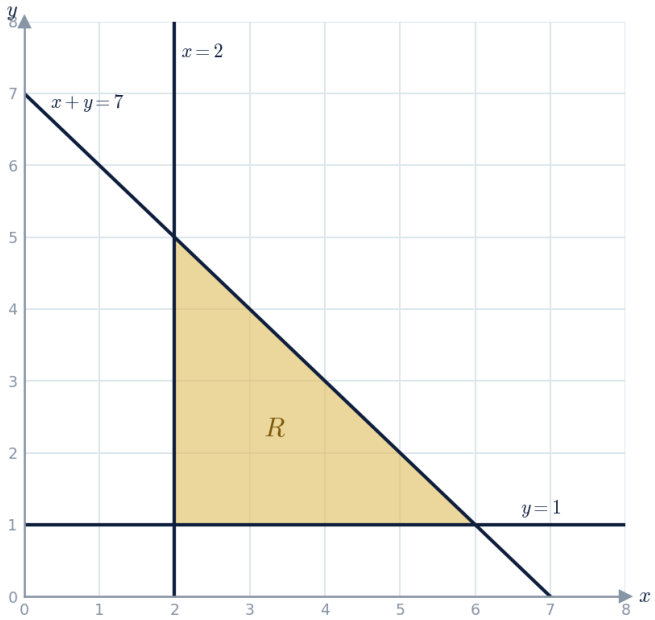
The required region is:

$$y \geq 1, \qquad x \geq 2, \qquad x + y \leq 7$$

4 Read the graph

So shade the triangular region above $y = 1$, to the right of $x = 2$, and below the line $x + y = 7$. Its vertices are

$$(2, 1), \quad (2, 5), \quad (6, 1)$$



FINAL ANSWER

the shaded region R is the triangle with vertices $(2, 1)$, $(2, 5)$, and $(6, 1)$.

4WM2/01 SPECIMEN

Question 18

3 marks | Inequalities (Solving & Graphing)

18 Solve the inequality $2y^2 - 7y - 30 \leqslant 0$
Show your working clearly.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 Specimen

1 **Factorise**

Factorise:

$$2y^2 - 7y - 30 = (2y + 5)(y - 6)$$

2 **Solve inequality**

So

$$(2y + 5)(y - 6) \leq 0$$

3 **The roots are**

The roots are

$$y = -\frac{5}{2}, \qquad y = 6$$

4 **Solve quadratic equation**

The quadratic opens upwards, so it is less than or equal to zero between the roots.

✓ **FINAL ANSWER**

$-\frac{5}{2} \leq y \leq 6$.

4MA1/2H JANUARY 2020

Question 7

5 marks | Inequalities (Solving & Graphing)

7 (a) Solve the inequality $2x + 7 > 4$

.....

(2)

(b) Solve $x^2 - 3x - 40 = 0$
Show clear algebraic working.

.....

(3)

(Total for Question 7 is 5 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2020

1 **(a)**

(a)

$$2x + 7 > 4$$

$$2x > -3$$

$$x > -\frac{3}{2}$$

2 **(b)**

(b)

$$x^2 - 3x - 40 = 0$$

3 **Factorise**

Factorise:

$$(x - 8)(x + 5) = 0$$

4 **Solve quadratic equation**

So

$$x = 8 \quad \text{or} \quad x = -5$$

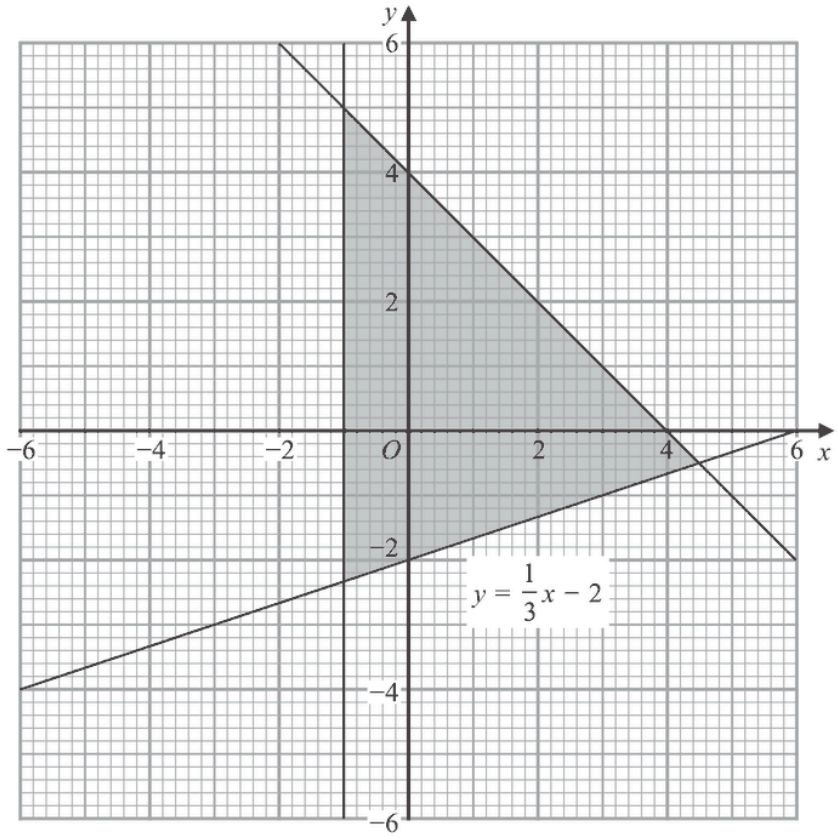
FINAL ANSWER

$x > -\frac{3}{2}$, and $x = 8$ or $x = -5$.

Question 10

3 marks | Inequalities (Solving & Graphing)

10 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down the three inequalities that define the shaded region.

.....

.....

.....

(Total for Question 10 is 3 marks)

1 **The shaded region is left**

The shaded region is left of the y-axis:

$$x \leq 0$$

2 **It is below the line**

It is below the line through $(0, 4)$ and $(4, 0)$:

$$y \leq -x + 4$$

3 **It is above the line**

It is above the line

$$y = \frac{1}{3}x - 2$$

4 **Evaluate fraction**

So

$$y \geq \frac{1}{3}x - 2$$

✓ **FINAL ANSWER**

$x \leq 0$, $y \leq -x + 4$, $y \geq \frac{1}{3}x - 2$.

4MA1/2HR JANUARY 2020

Question 11

4 marks | Inequalities (Solving & Graphing)

11 (a) Write down the integer values of x that satisfy the inequality $-2 < x \leqslant 4$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.

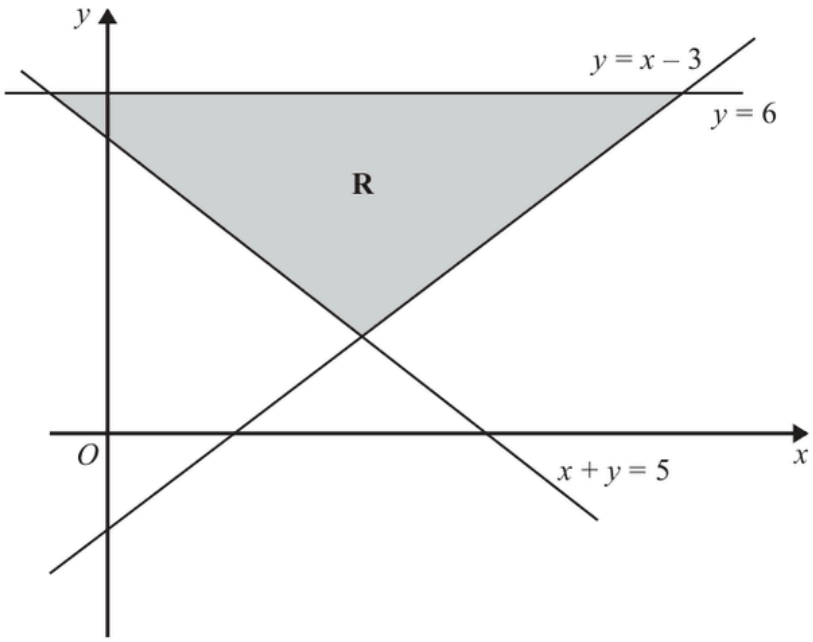


Diagram **NOT** accurately drawn

(b) Write down the three inequalities that define the region **R**.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR January 2020

1 **Solve inequality**

For part (a),

$$2 < x \leq 4$$

2 **The integer values are**

The integer values are

$$3, 4$$

3 **Calculate value**

For the region R, the boundaries are

$$y = x - 3, \qquad x + y = 5, \qquad y = 6$$

4 **The shaded region is above**

The shaded region is above the two sloping lines and below the horizontal line:

$$y \geq x - 3$$

$$x + y \geq 5$$

$$y \leq 6$$

✓ **FINAL ANSWER**

(a) $3, 4$. (b) $y \geq x - 3$, $x + y \geq 5$, $y \leq 6$.

4MA1/2HR JANUARY 2020

Question 19

6 marks | Inequalities (Solving & Graphing)

19 (a) Solve $\frac{4-3x}{5} - \frac{3x-5}{2} = -3$

Show clear algebraic working.

$x =$ .....

(3)

(b) Solve the inequality $5y^2 - 17y \leqslant 40$

.....

(3)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2020

1 **(a)**

(a)

$$\frac{4-3x}{5} - \frac{3x-5}{2} = -3$$

2 **Multiply by 10**

Multiply by 10:

$$2(4-3x) - 5(3x-5) = -30$$

$$8 - 6x - 15x + 25 = -30$$

$$33 - 21x = -30$$

$$-21x = -63$$

$$x = 3$$

3 **(b)**

(b)

$$5y^2 - 17y \leq 40$$

$$5y^2 - 17y - 40 \leq 0$$

---

4 **Factorise**

Factorise:

$$(5y + 8)(y - 5) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{8}{5}$ and 5. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

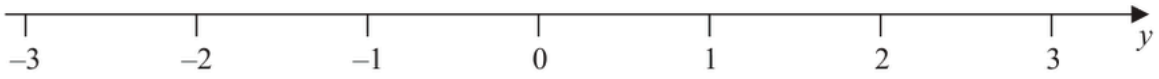
$x = 3$, and $-\frac{8}{5} \leq y \leq 5$.

4MA1/1HR JANUARY 2021

Question 3

4 marks | Inequalities (Solving & Graphing)

3 (a) On the number line, show the inequality $-2 \leqslant y < 1$



(2)

n is an integer.

(b) Write down all the values of n that satisfy $-3.4 < n \leqslant 2$

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2021

1 Solve inequality

(a) For

$$-2 \leq y < 1$$

2 use a closed circle at

use a closed circle at -2, an open circle at 1, and shade the line between them.

3 (b)

(b)

$$-3.4 < n \leq 2$$

4 The integer values are

The integer values are

$$-3,\ -2,\ -1,\ 0,\ 1,\ 2$$

FINAL ANSWER

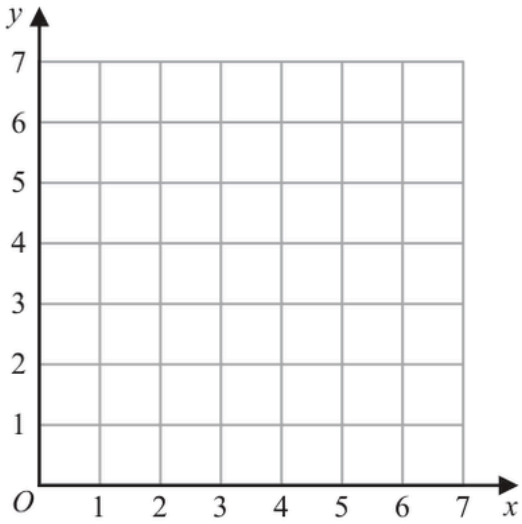
Closed at -2, open at 1; integers $-3, -2, -1, 0, 1, 2$.

4MA1/2H JANUARY 2021

Question 4

4 marks | Inequalities (Solving & Graphing)

4



(a) On the grid, draw and **label** the straight line with equation

(i) $x = 1.5$

(ii) $y = x$

(iii) $x + y = 6$

(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$$x \geqslant 1.5 \qquad y \geqslant x \qquad x + y \leqslant 6$$

Label the region **R**.

(1)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2021

1 **Draw the three boundary lines**

Draw the three boundary lines:

$$x = 1.5, \qquad y = x, \qquad x + y = 6$$

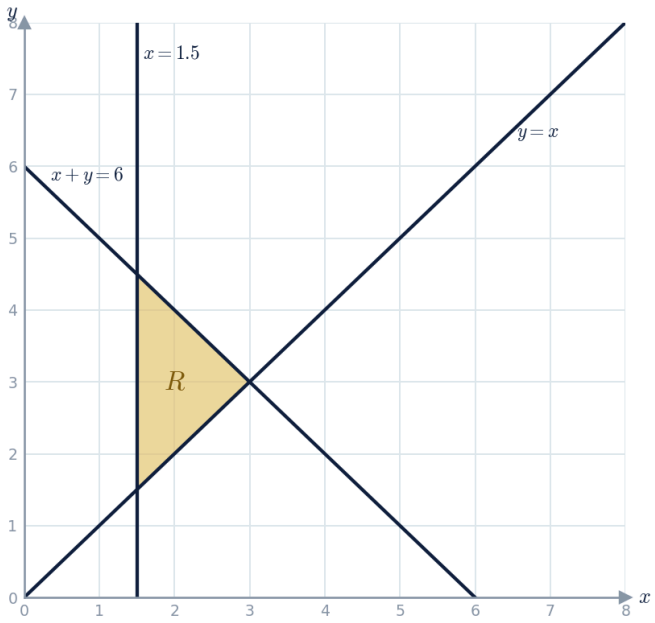
2 **The required region satisfies**

The required region satisfies

$$x > 1.5$$

$$y \geq x$$

$$x + y \leq 6$$



FINAL ANSWER

Shade the region $x > 1.5$, $y \geq x$, $x + y \leq 6$.

4MA1/2H JANUARY 2021

Question 5

7 marks | Inequalities (Solving & Graphing)

5 (a) Expand and simplify $4x(2x + 5) - 3x(2x - 3)$

.....

(2)

Given that $\dfrac{y^5 \times y^n}{y^6} = y^{13}$

(b) work out the value of n.

$n =$ .....

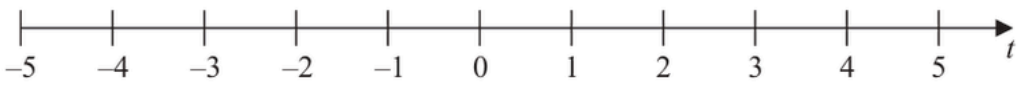
(2)

(c) (i) Solve the inequality $7t - 8 < 2t + 7$

.....

(2)

(ii) On the number line below, represent the solution set of the inequality solved in part (c)(i)



(1)

(Total for Question 5 is 7 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H January 2021

1 **(a)**

(a)

$$4x(2x+5) - 3x(2x-3) = 8x^2 + 20x - 6x^2 + 9x = 2x^2 + 29x$$

2 **Expand brackets**

(b)

$$\frac{y^5 \times y^n}{y^6} = y^{n-1} = y^{13}$$

so $n - 1 = 13$, hence $n = 14$.

3 **(c)**

(c)

$$7t - 8 < 2t + 7$$

$$5t < 15$$

$$t < 3$$

---

4 **Expand brackets**

On the number line, use an open circle at 3 and shade to the left.

✓ **FINAL ANSWER**

(a) $2x^2 + 29x$, (b) $n = 14$, (c) $t < 3$

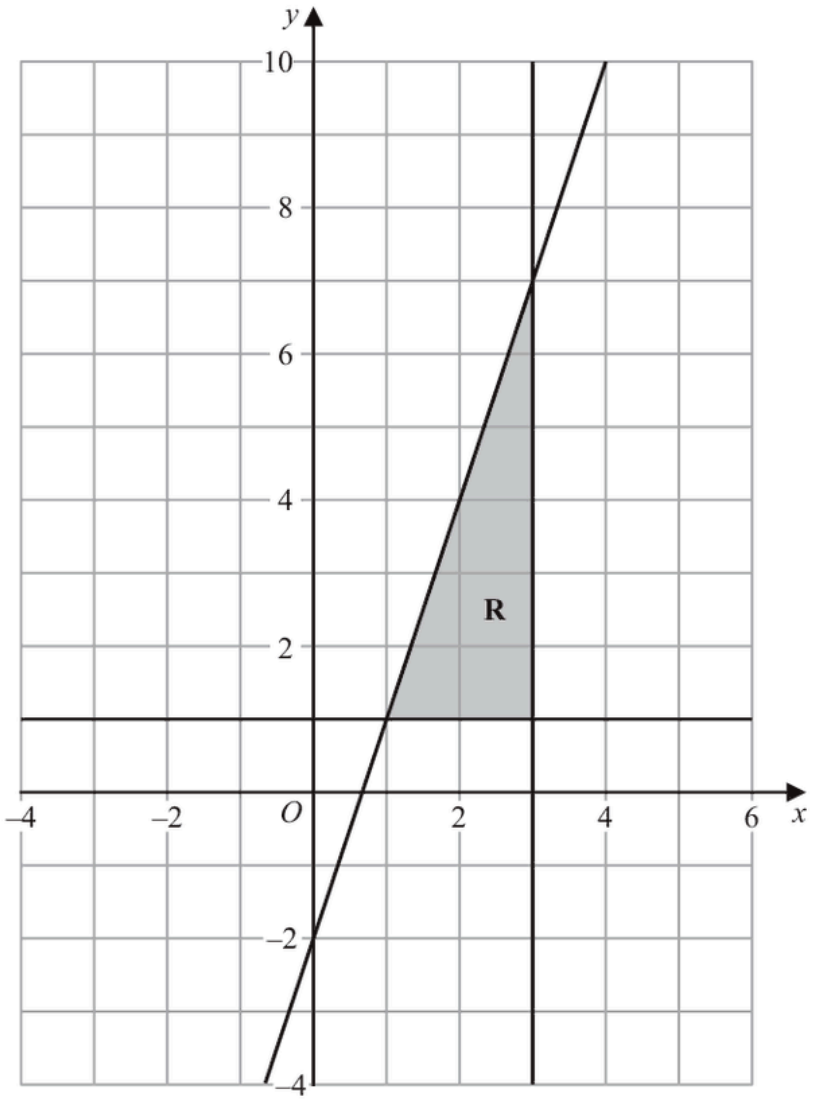
4MA1/2HR JANUARY 2021

Question 7

3 marks | Inequalities (Solving & Graphing)

7 The shaded region **R**, shown in the diagram below, is bounded by the straight line with equation $y = 3x - 2$ and by two other straight lines.

Write down the three inequalities that define region **R**.



(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2021

1 The sloping boundary is

The sloping boundary is

$$y = 3x - 2$$

2 **The shaded region is below**

The shaded region is below this line:

$$y \leq 3x - 2$$

3 **The vertical boundary is so**

The vertical boundary is

$$x = 3$$

so the region has

$$x \leq 3$$

4 **The lower boundary is the**

The lower boundary is the x-axis, so

$$y \geq 0$$

✓ **FINAL ANSWER**

$y \leq 3x - 2$, $x \leq 3$, $y \geq 0$.

4MA1/1H MAY 2021

Question 7

4 marks | Inequalities (Solving & Graphing)

7 $-4 \leqslant 2y < 6$

y is an integer.

(a) Write down all the possible values of y.

.....

(2)

(b) Solve the inequality $7t - 3 \leqslant 2t + 31$

Show your working clearly.

.....

(2)

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2021

1 **(a)**

(a)

$$-4 \leq 2y < 6$$

2 **Divide by 2**

Divide by 2:

$$-2 \leq y < 3$$

3 **Calculate value**

Since y is an integer,

$$y = -2, -1, 0, 1, 2$$

4 **(b)**

(b)

$$7t - 3 \leq 2t + 31$$

$$5t \leq 34$$

$$t \leq \frac{34}{5}$$

FINAL ANSWER

$y = -2, -1, 0, 1, 2$, and $t \leq \frac{34}{5}$.

4MA1/1H NOVEMBER 2021

Question 9

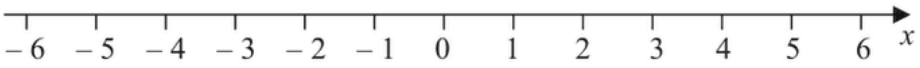
5 marks | Inequalities (Solving & Graphing)

9 (i) Solve the inequalities $-7 \leqslant 2x - 3 < 5$

.....

(3)

(ii) On the number line, represent the solution set to part (i)



(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2021

1 Solve inequality

$$-7 \leq 2x - 3 < 5$$

2 Add 3 to each part

Add 3 to each part:

$$-4 \leq 2x < 8$$

3 Divide by 2

Divide by 2:

$$-2 \leq x < 4$$

4 Use a closed circle

On the number line, use a closed circle at -2, an open circle at 4, and shade between them.

✓ FINAL ANSWER

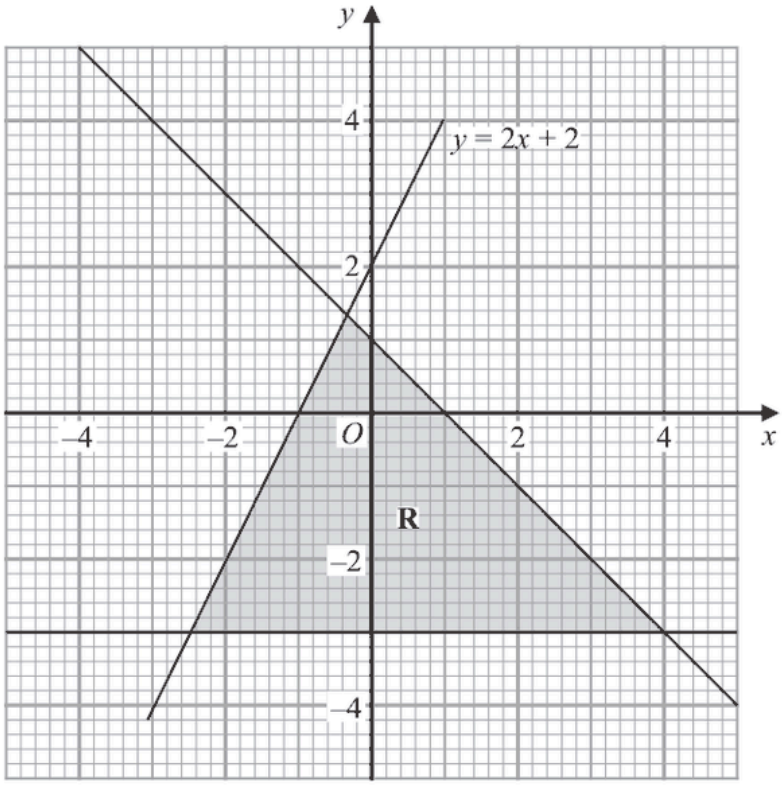
$-2 \leq x < 4$.

4MA1/2H NOVEMBER 2021

Question 13

3 marks | Inequalities (Solving & Graphing)

13



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Write down the three inequalities that define **R**.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2021

1 **The bottom boundary is so**

The bottom boundary is

$$y = -3$$

so

$$y \geq -3$$

2 **The rising boundary is so**

The rising boundary is

$$y = 2x + 2$$

so the region has

$$y \leq 2x + 2$$

---

3 **Calculate value**

The falling boundary goes through $(0, 1)$ and $(1, 0)$, so

$$x + y = 1$$

---

4 **The region is below this**

The region is below this line:

$$x + y \leq 1$$

✓ **FINAL ANSWER**

$y \geq -3$, $y \leq 2x + 2$, $x + y \leq 1$.

4MA1/1HR JANUARY 2022

Question 1

3 marks | Inequalities (Solving & Graphing)

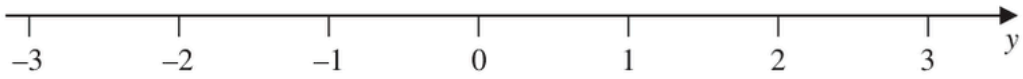
1 n is an integer.

(a) Write down all the values of n such that $-2 \leqslant n < 3$

.....

(2)

(b) On the number line, represent the inequality $y \leqslant 1$



(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$-2 \leq n < 3$$

2 **Calculate value**

Since n is an integer,

$$n = -2, -1, 0, 1, 2$$

3 **Solve inequality**

(b) For

$$y \leq 1$$

4 **use a closed circle at**

use a closed circle at 1 and shade to the left.

✓ **FINAL ANSWER**

$n = -2, -1, 0, 1, 2$; closed at 1, shaded left.

4MA1/1H JANUARY 2022

Question 6

5 marks | Inequalities (Solving & Graphing)

6 (a) Solve the inequality $5x - 7 \leqslant 2$

.....

(2)

(b) (i) Factorise $y^2 - 2y - 35$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 35 = 0$

.....

(1)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x - 7 < 2$$

$$5x < 9$$

$$x < \frac{9}{5}$$

2 **(b)(i)**

(b)(i)

$$y^2 - 2y - 35 = (y - 7)(y + 5)$$

3 **Factorise expression**

(b)(ii) $y = 7$ or $y = -5$.

FINAL ANSWER

(a) $x < \frac{9}{5}$, (b)(i) $(y - 7)(y + 5)$, (b)(ii) $y = 7$ or $y = -5$

4MA1/2HR JANUARY 2022

Question 7

5 marks | Inequalities (Solving & Graphing)

7 (a) Write down an equation of the straight line with gradient -3 and which passes through the point with coordinates $(0, 5)$

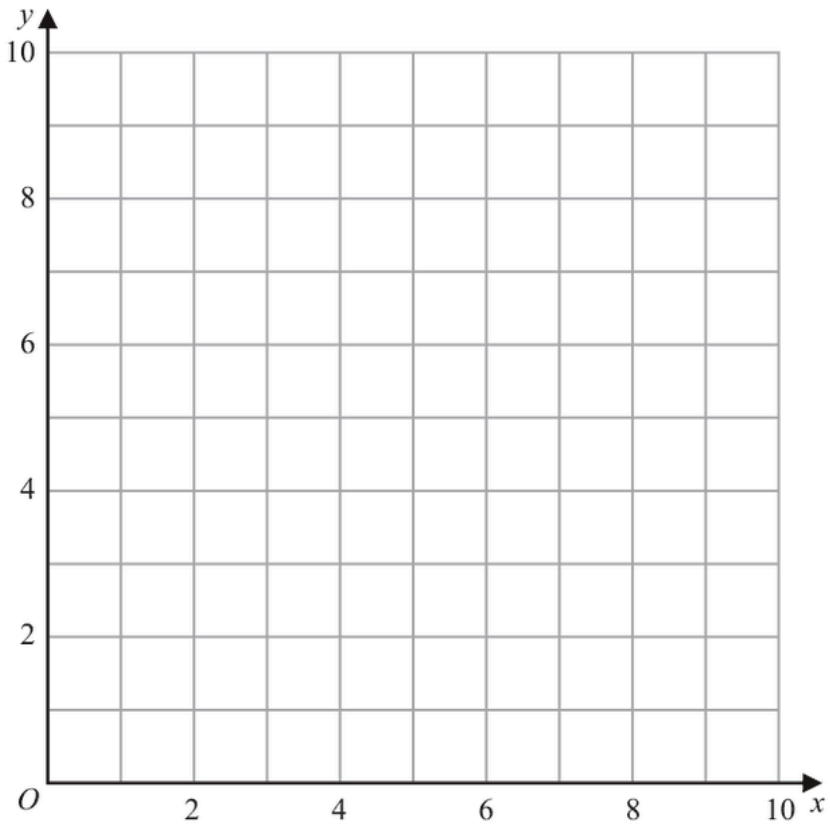
.....

(2)

(b) Show, by shading on the grid, the region defined by **all three** of the inequalities

$$x \leqslant 6 \qquad y \geqslant 2 \qquad y \leqslant x + 1$$

Label the region **R**



(3)

(Total for Question 7 is 5 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2022

1 **A straight line with gradient**

A straight line with gradient -3 passing through $(0, 5)$ has equation

$$y = -3x + 5$$

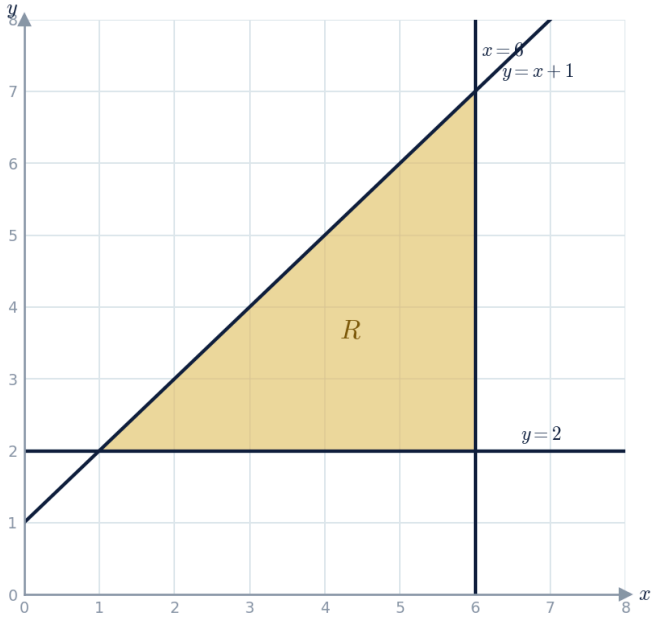
2 Use the inequalities

For the region, use the inequalities:

$$x < 6$$

$$y > 2$$

$$y \leq x + 1$$



FINAL ANSWER

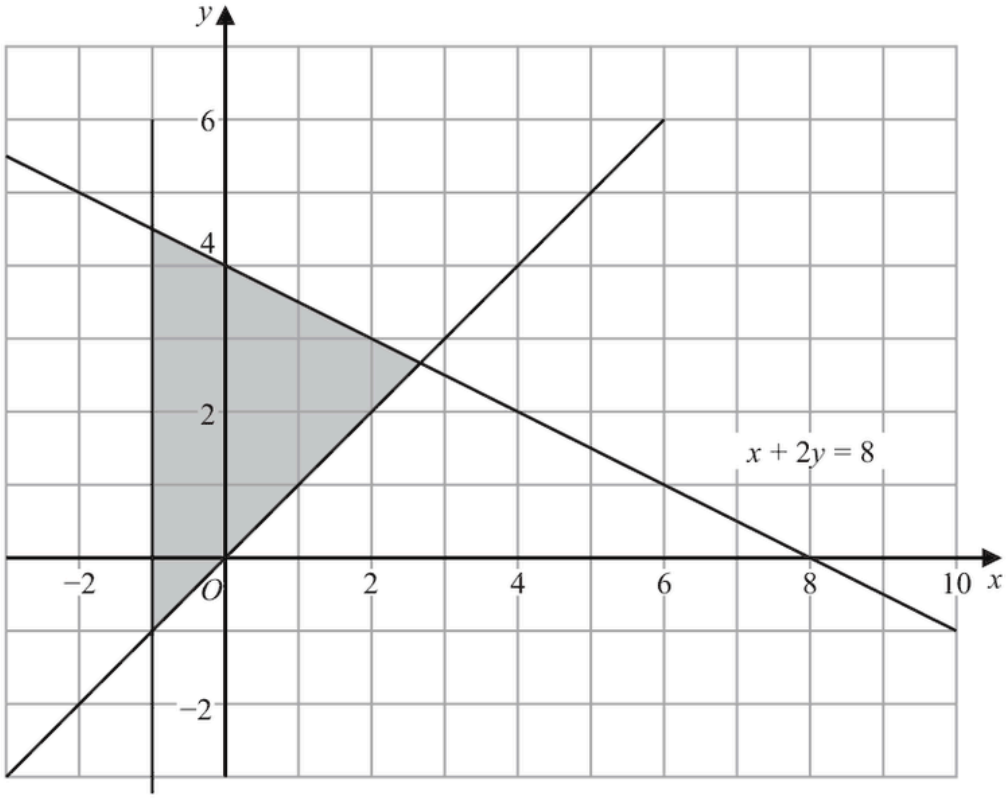
(a) $y = -3x + 5$. (b) Shade $x < 6$, $y > 2$, $y \leq x + 1$.

4MA1/2H JANUARY 2022

Question 13

3 marks | Inequalities (Solving & Graphing)

13 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down three inequalities that define the shaded region.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2022

1 **The region is to the**

The region is to the right of the y-axis:

$$x \geq 0$$

2 **It is above the line**

It is above the line $y = x$:

$$y \geq x$$

3 It is below the line

It is below the line

$$x + 2y = 8$$

so

$$x + 2y \leq 8$$

FINAL ANSWER

$x \geq 0$, $y \geq x$, $x + 2y \leq 8$.

4MA1/2H JANUARY 2022

Question 22

5 marks | Inequalities (Solving & Graphing)

22 Here is a rectangle.

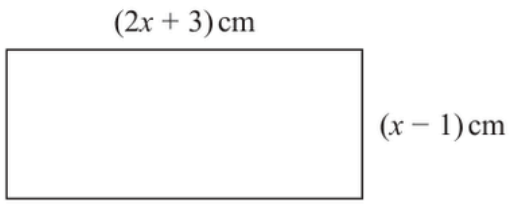


Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than $75\,\text{cm}^2$

find the range of possible values of x

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2022

1 The rectangle has side lengths

The rectangle has side lengths

$$2x + 3$$

2 Use Functions Solving Inequalities

and

$$x - 1$$

3 Solve inequality

For the side lengths to be possible,

$$x > 1$$

4 The area is less than

The area is less than 75:

$$(2x + 3)(x - 1) < 75$$

$$2x^2 + x - 3 < 75$$

$$2x^2 + x - 78 < 0$$

5 **Factorise**

Factorise:

$$2x^2 + x - 78 = (2x + 13)(x - 6)$$

---

6 **Evaluate fraction**

So

$$-\frac{13}{2} < x < 6$$

---

7 **Solve inequality**

Combining with $x > 1$:

$$1 < x < 6$$

✓ **FINAL ANSWER**

$1 < x < 6$

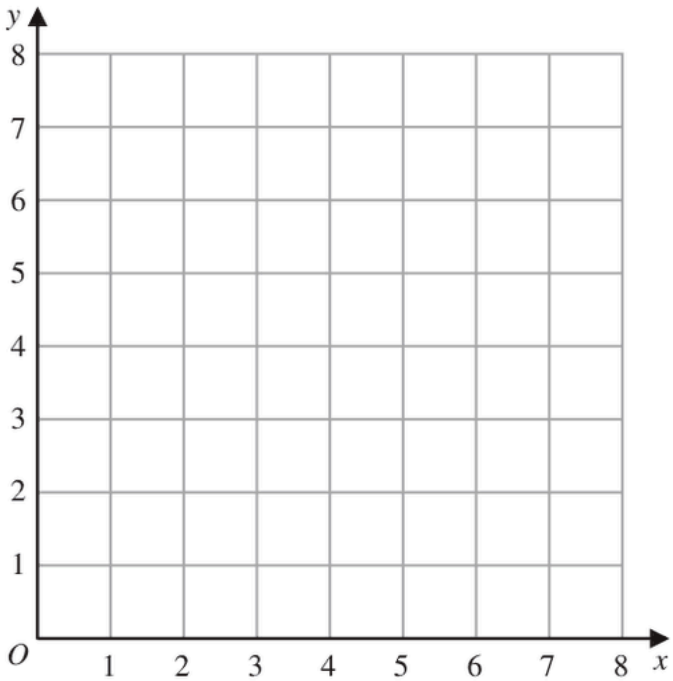
4MA1/2H JUNE 2022

Question 2

4 marks | Inequalities (Solving & Graphing)

2 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**.

(1)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2022

1 **Draw the boundary lines**

Draw the boundary lines:

$$y = 1, \quad x = 2, \quad x + y = 7$$

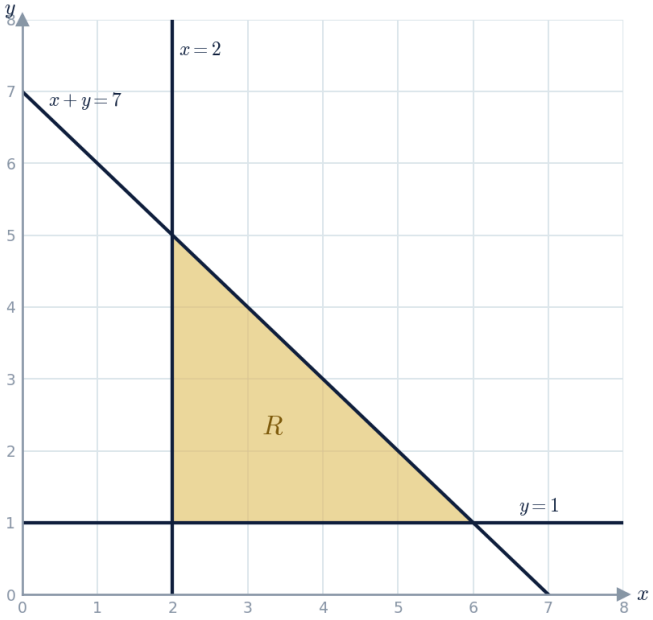
2 **The required region is**

The required region is

$$y \geq 1$$

$$x \geq 2$$

$$x + y \leq 7$$



FINAL ANSWER

Shade $y \geq 1$, $x \geq 2$, $x + y \leq 7$.

4MA1/1HR JUNE 2022

Question 9

2 marks | Inequalities (Solving & Graphing)

9 Solve the inequality $3 - 4x \leqslant 11$

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2022

1 Solve inequality

$$3 - 4x \leq 11$$

$$-4x \leq 8$$

2 Solve inequality

Divide by -4, reversing the inequality:

$$x \geq -2$$

FINAL ANSWER

$x \geq -2$.

4MA1/2H JUNE 2022

Question 15

7 marks | Inequalities (Solving & Graphing)

15 (a) Solve $\frac{4x+5}{3} - \frac{3-2x}{2} = 13$

Show clear algebraic working.

$x =$ .....

(4)

(b) Solve the inequality $2y^2 - 7y - 30 \leqslant 0$

Show your working clearly.

.....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2022

1 (a)

(a)

$$\frac{4x+5}{3} - \frac{3-2x}{2} = 13$$

2 Multiply by 6

Multiply by 6:

$$2(4x+5) - 3(3-2x) = 78$$

$$8x + 10 - 9 + 6x = 78$$

$$14x + 1 = 78$$

$$14x = 77$$

$$x = \frac{77}{14} = \frac{11}{2}$$

3 **(b)**

(b)

$$2y^2 - 7y - 30 \leq 0$$

---

4 **Factorise**

Factorise:

$$(2y + 5)(y - 6) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{5}{2}$ and 6. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

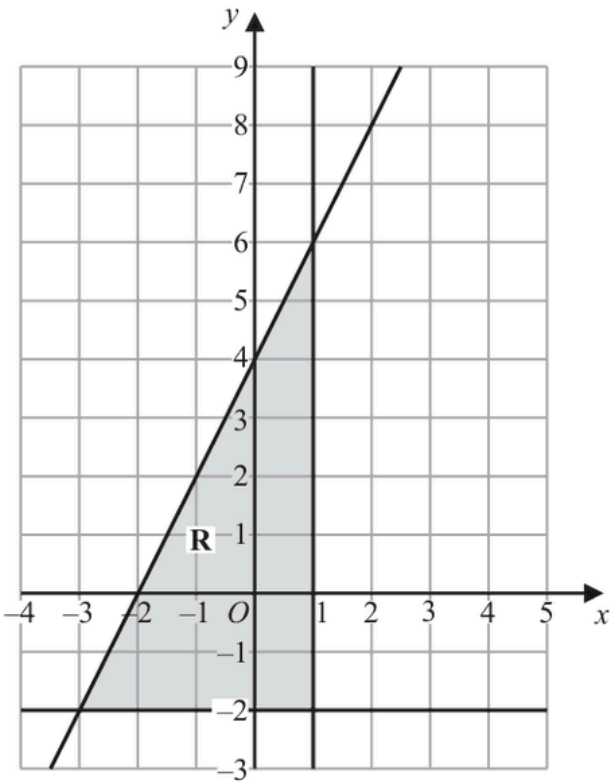
$x = \frac{11}{2}$, and $-\frac{5}{2} \leq y \leq 6$.

4MA1/1H MAY 2023

Question 7

4 marks | Inequalities (Solving & Graphing)

7



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Find the inequalities that define **R**

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2023

1 **The vertical boundary is so**

The vertical boundary is

$$x = 1$$

so the region has

$$x \leq 1$$

2 **The horizontal boundary is so**

The horizontal boundary is

$$y = -2$$

so

$$y \geq -2$$

---

3 **Find the gradient**

The sloping line passes through $(0, 4)$ and has gradient 2, so

$$y = 2x + 4$$

---

4 **The shaded region is below**

The shaded region is below this line:

$$y \leq 2x + 4$$

✓ **FINAL ANSWER**

$x \leq 1$, $y \geq -2$, $y \leq 2x + 4$.

4MA1/2HR JUNE 2023

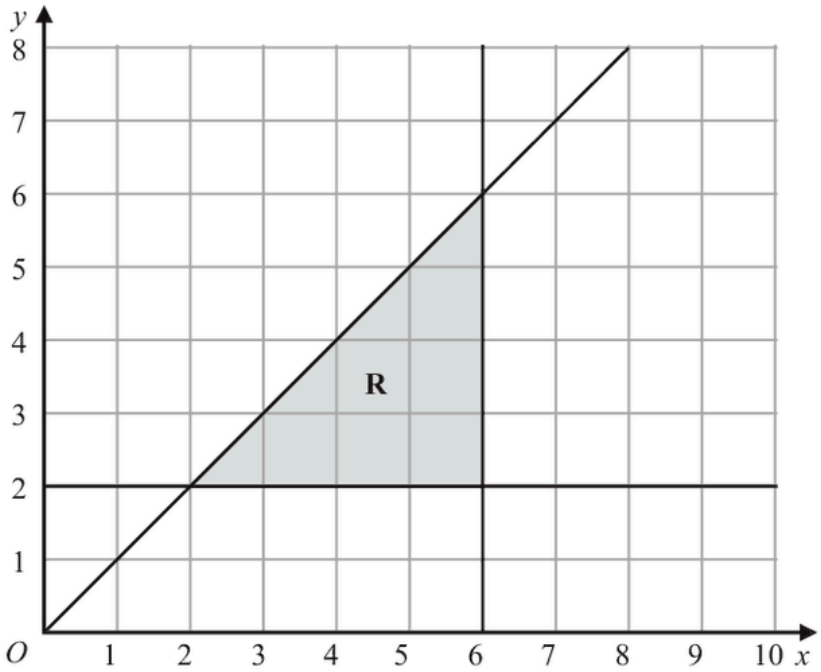
Question 8

5 marks | Inequalities (Solving & Graphing)

8 (a) Solve $9 - 4x > 17$

.....

(2)



(b) Write down the three inequalities that represent the shaded region **R**

.....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2023

1 **Read the graph**

For part (a),

$$9 - 4x > 17$$

$$-4x > 8$$

$$x < -2$$

2 **Read the graph**

For part (b), the region is above $y = 2$, left of $x = 6$, and below $y = x$:

$$y \geq 2$$

$$x \leq 6$$

$$y \leq x$$

✓ **FINAL ANSWER**

(a) $x < -2$. (b) $y \geq 2$, $x \leq 6$, $y \leq x$.

Question 8

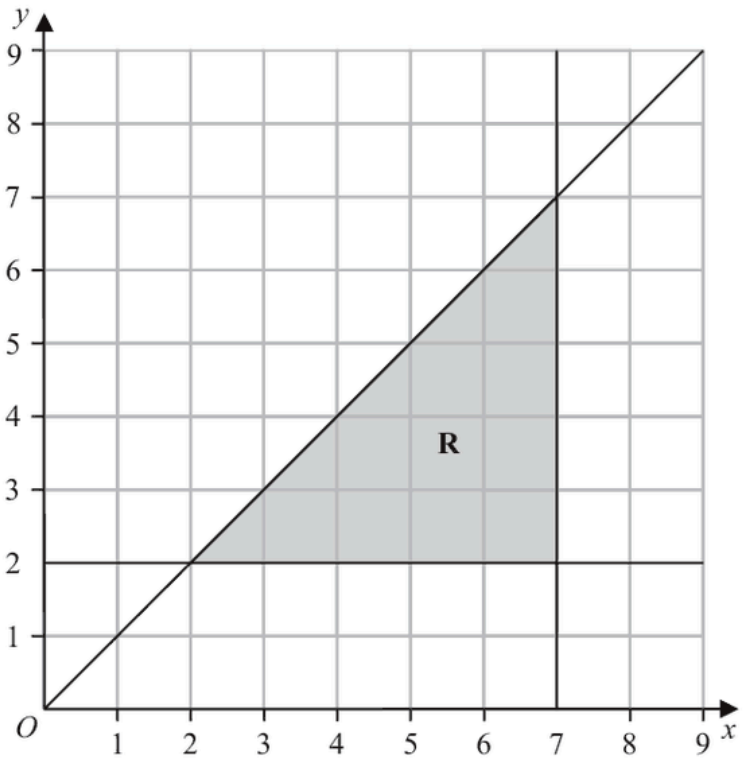
5 marks | Inequalities (Solving & Graphing)

8 (a) Solve the inequality $8x - 4 \geqslant 3x - 10$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 8 is 5 marks)

1 Evaluate fraction

For part (a),

$$8x - 4 \geq 3x - 10$$

$$5x \geq -6$$

$$x \geq -\frac{6}{5}$$

2 Solve inequality

For part (b), the region is above $y = 2$, left of $x = 7$, and below $y = x$:

$$y \geq 2$$

$$x \leq 7$$

$$y \leq x$$

FINAL ANSWER

(a) $x \geq -\frac{6}{5}$. (b) $y \geq 2$, $x \leq 7$, $y \leq x$.

4MA1/2H JUNE 2024

Question 2

4 marks | Inequalities (Solving & Graphing)

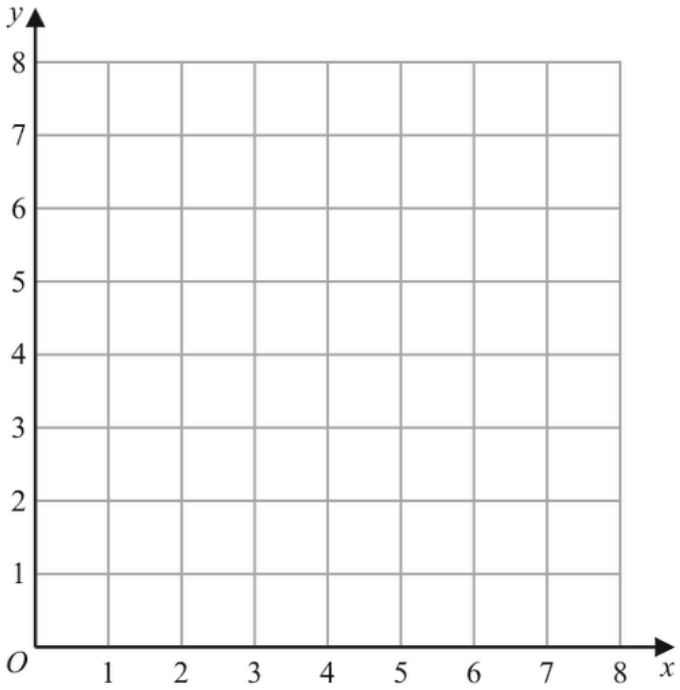
2 (a) On the grid, draw the straight line with equation

(i) $y = 2$

(ii) $x = 6$

(iii) $y = x + 1$

Label each line with its equation.



(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$y \geqslant 2$ $x \leqslant 6$ $y \leqslant x + 1$

Label the region **R**

(1)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2024

1 **Draw the boundary lines**

Draw the boundary lines:

$$y = 2, \quad x = 6, \quad y = x + 1$$

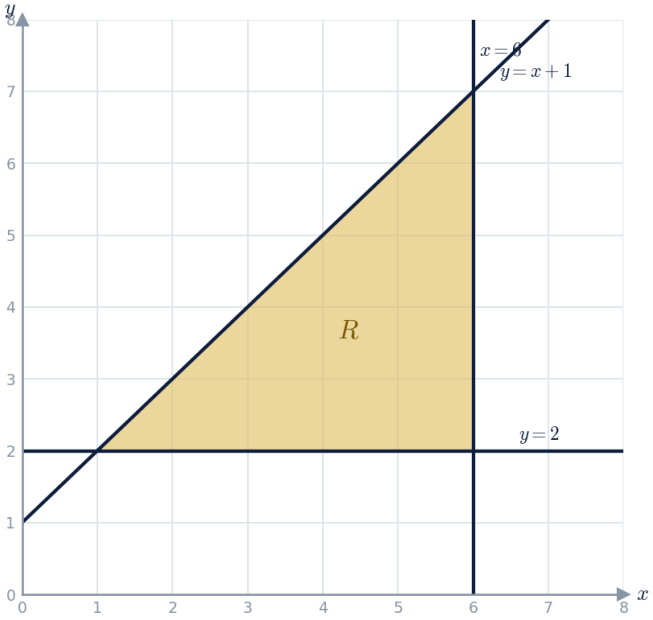
2 **The required region is**

The required region is

$$y \geq 2$$

$$x \leq 6$$

$$y \leq x + 1$$



FINAL ANSWER

Shade $y \geq 2$, $x \leq 6$, $y \leq x + 1$.

4MA1/1HR MAY 2024

Question 7

9 marks | Inequalities (Solving & Graphing)

7 (a) Simplify $g^9 \div g^2$

.....

(1)

(b) Expand $5k^2(k^3 + 4)$

.....

(2)

(c) (i) Factorise $x^2 - 2x - 63$

.....

(2)

(ii) Hence, solve $x^2 - 2x - 63 = 0$

.....

(1)

(d) Solve the inequality $7 - 2y < 3y - 12$

.....

(3)

(Total for Question 7 is 9 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May 2024

1 **(a)**

(a) $g^9 \div g^2 = g^7$.

2 **(b)**

(b) $5k^2(k^3 + 4) = 5k^5 + 20k^2$.

3 **(c)(i)**

(c)(i)

$$x^2 - 2x - 63 = (x - 9)(x + 7)$$

4 **Solve quadratic equation**

(c)(ii) $x = 9$ or $x = -7$.

5 **(d)**

(d)

$$7 - 2y < 3y - 12$$

$$19 < 5y$$

FINAL ANSWER

(a) g^7, (b) $5k^5 + 20k^2$, (c) $(x - 9)(x + 7)$, $x = 9$ or $x = -7$, (d) $y > \frac{19}{5}$

4MA1/2HR JUNE 2024

Question 20

4 marks | Inequalities (Solving & Graphing)

20 Solve $6x^2 - 7x - 20 > 0$
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2024

1 Solve quadratic equation

$$6x^2 - 7x - 20 > 0$$

2 Factorise

Factorise:

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{4}{3} \quad \text{and} \quad x = \frac{5}{2}$$

4 Solve quadratic equation

Since the coefficient of x^2 is positive, the quadratic is greater than 0 outside the roots.

✓ **FINAL ANSWER**

$$x < -\frac{4}{3} \text{ or } x > \frac{5}{2}$$

4MA1/2H NOVEMBER 2024

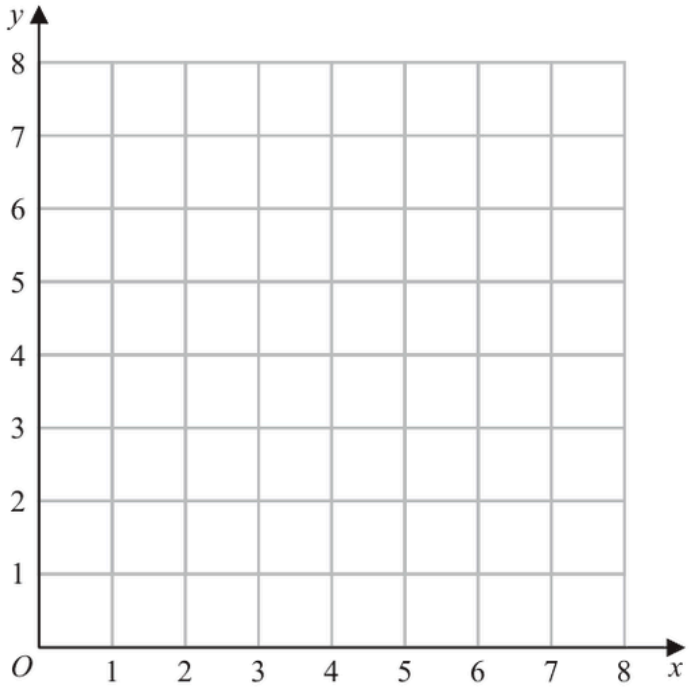
Question 7

4 marks | Inequalities (Solving & Graphing)

7 (a) On the grid, draw the straight line with equation

(i) $x = 3$ (ii) $y = 1$ (iii) $x + y = 7$

Label each line with its equation.



(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$x \geqslant 3$ $y \geqslant 1$ $x + y \leqslant 7$

Label the region **R**

(1)

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2024

1 Draw the boundary lines

Draw the boundary lines:

$$x = 3, \quad y = 1, \quad x + y = 7$$

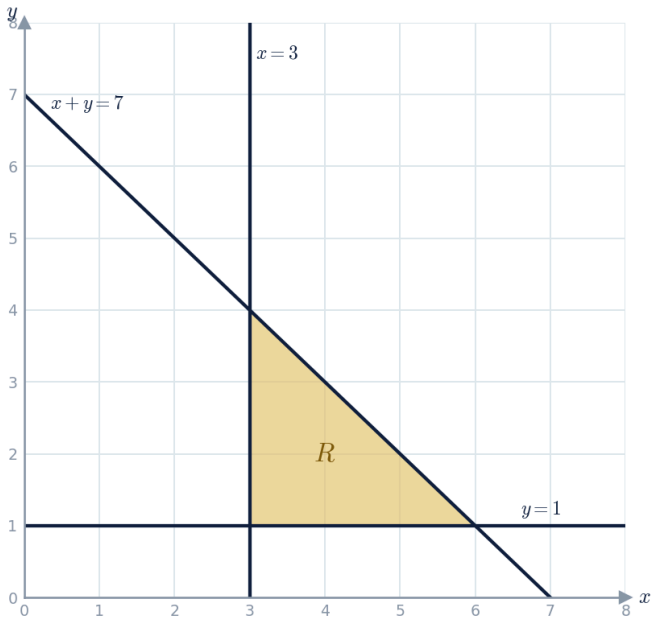
2 **The required region is**

The required region is

$$x \geq 3$$

$$y \geq 1$$

$$x + y \leq 7$$



FINAL ANSWER

Shade $x \geq 3$, $y \geq 1$, $x + y \leq 7$.

4MA1/1H NOVEMBER 2024

Question 8

6 marks | Inequalities (Solving & Graphing)

8 (a) (i) Factorise $x^2 + 5x - 24$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

.....

(1)

(b) Solve the inequality $3y + 5 > 7y - 10$

Show clear algebraic working.

.....

(3)

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2024

1 **(a)(i)**

(a)(i)

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

2 **Solve quadratic equation**

(a)(ii) $x = -8$ or $x = 3$.

3 **(b)**

(b)

$$3y + 5 > 7y - 10$$

$$15 > 4y$$

$$y < \frac{15}{4}$$

✓ **FINAL ANSWER**

$(x + 8)(x - 3)$, $x = -8$ or $x = 3$, $y < \frac{15}{4}$

4MA1/2H NOVEMBER 2024

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $10x^2 + 11x - 21 < 0$
Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2024

1 Solve inequality

$$10x^2 + 11x - 21 < 0$$

2 Factorise

Factorise:

$$10x^2 + 11x - 21 = (10x + 21)(x - 1)$$

3 The critical values are

The critical values are

$$x = -\frac{21}{10} \quad \text{and} \quad x = 1$$

4 The quadratic is less than

The quadratic is less than 0 between the roots.

FINAL ANSWER

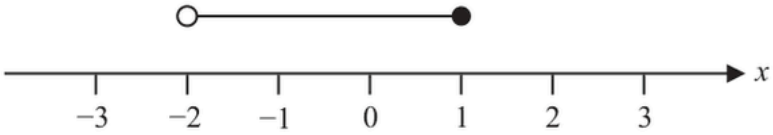
$$-\frac{21}{10} < x < 1$$

4MA1/1HR JUNE 2025

Question 6

4 marks | Inequalities (Solving & Graphing)

6



(a) Write down the inequality shown on the number line.

.....

(2)

(b) Solve the inequality $7a - 5 \leqslant 3a + 28$
Show clear algebraic working.

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2025

1 **(a) The number line has**

(a) The number line has an open circle at -2, a closed circle at 1, and shading between them.

$$-2 < x \leq 1$$

2 **(b)**

(b)

$$7a - 5 \leq 3a + 28$$

$$4a \leq 33$$

$$a \leq \frac{33}{4}$$

FINAL ANSWER

$-2 < x \leq 1$, and $a \leq \frac{33}{4}$.

Question 9

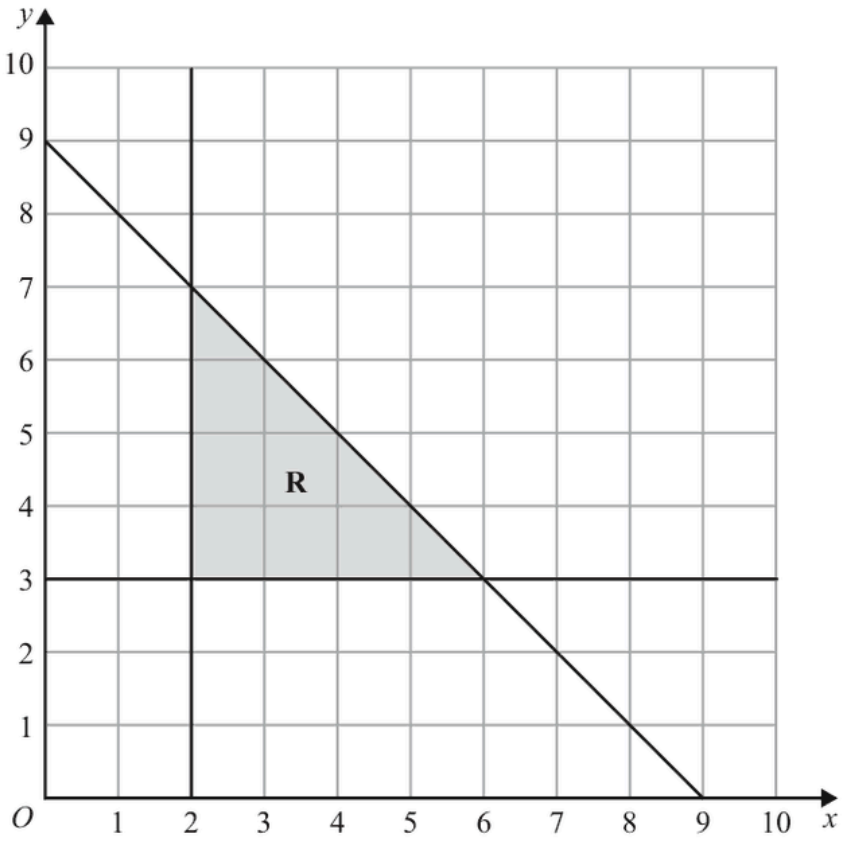
5 marks | Inequalities (Solving & Graphing)

9 (a) Solve the inequality $7 - 3t < 2t + 15$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down three inequalities that define the region **R**

.....

(3)

(Total for Question 9 is 5 marks)

1 Evaluate fraction

For part (a),

$$7 - 3x < 2x + 15$$

$$-8 < 5x$$

$$x > -\frac{8}{5}$$

2 Calculate value

For part (b), the boundaries are $x = 2$, $y = 3$, and $x + y = 9$.

3 The shaded region is

The shaded region is

$$x \geq 2$$

$$y \geq 3$$

$$x + y \leq 9$$

FINAL ANSWER

(a) $x > -\frac{8}{5}$. (b) $x \geq 2$, $y \geq 3$, $x + y \leq 9$.

4WM2/01R MAY 2025

Question 9

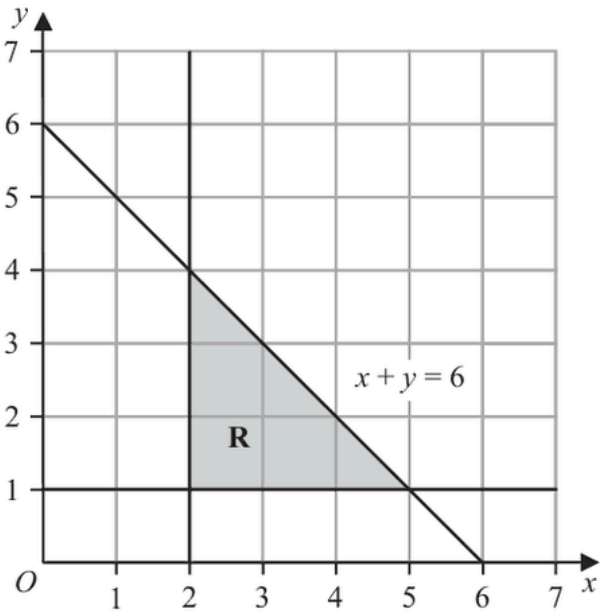
6 marks | Inequalities (Solving & Graphing)

9 (a) Solve the inequality $3x - 22 > 8x - 40$
Show clear algebraic working.

.....

(3)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 9 is 6 marks)

WORKED SOLUTION

Question 9 · 4WM2/01R May 2025

1 **Required**

a Solve 3*x* − 22 \> 8*x* − 40 algebraically. (b) Write the three inequalities defining region R.

2 **Find the nth term**

Move all *x* terms to the right and constants to the left:

3*x* − 22 \> 8*x* − 40 ⟹ − 22 + 40 \> 8*x* − 3*x* ⟹ 18 \> 5*x*

3

Divide both sides by 5

Divide both sides by 5:

$$x \< frac185 = 3.6$$

4

Answer

x \< 3.6

5

Identify the three boundary lines

Identify the three boundary lines and their regions using the graph. The shaded region R is bounded by *x* = 2, *y* = 1, and *x* + *y* = 6.

✓ **FINAL ANSWER**

x ≥ 2,*y* ≥ 1,*x* + *y* ≤ 6

Question 11

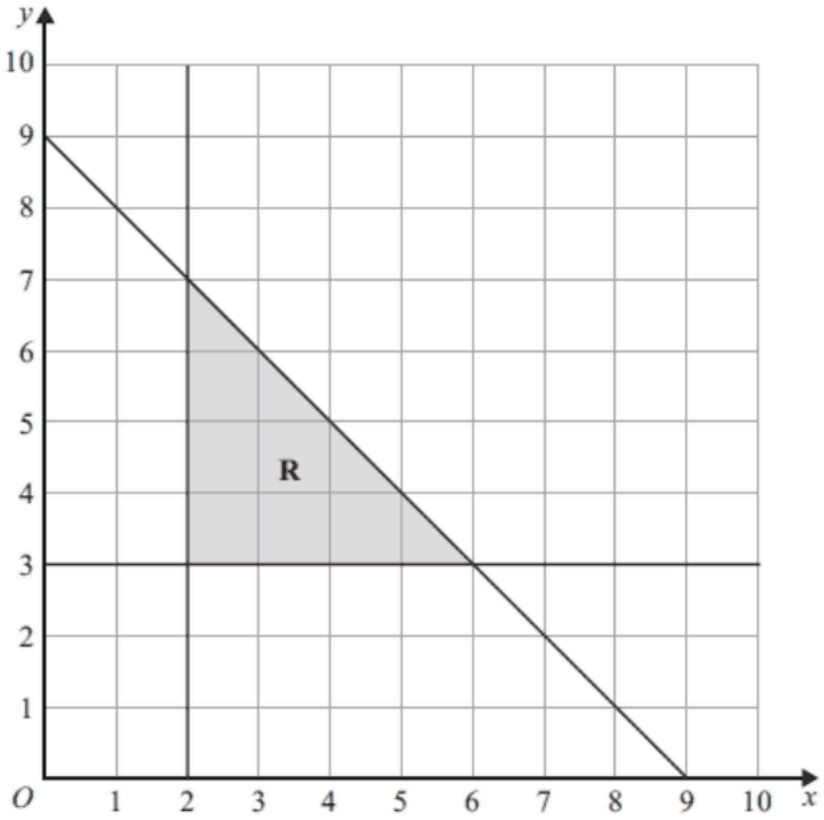
5 marks | Inequalities (Solving & Graphing)

11 (a) Solve the inequality $7 - 3t < 2t + 15$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down three inequalities that define the region **R**

.....

.....

.....

(3)

(Total for Question 11 is 5 marks)

1 (a)

(a)

$$7 - 3t < 2t + 15$$

$$-8 < 5t$$

$$t > -1.6$$

2 (b) The three boundary lines

(b) The three boundary lines are $x = 2$, $y = 3$, and $x + y = 9$. The shaded side gives

$$x \geq 2, \qquad y \geq 3, \qquad x + y \leq 9$$

FINAL ANSWER

$t > -1.6$; $x \geq 2$, $y \geq 3$, $x + y \leq 9$.

4MA1/2H JUNE 2025

Question 19

3 marks | Inequalities (Solving & Graphing)

19 Solve the inequality $4x^2 + 4x - 15 < 0$
Show clear algebraic working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2025

1 Solve inequality

$$4x^2 + 4x - 15 < 0$$

2 Factorise

Factorise:

$$(2x + 5)(2x - 3) < 0$$

3 The roots are

The roots are

$$x = -\frac{5}{2}, \qquad x = \frac{3}{2}$$

4 The quadratic is negative between

The quadratic is negative between the roots.

✓ FINAL ANSWER

$-\frac{5}{2} < x < \frac{3}{2}$.

4WM2/01R MAY 2025

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01R May 2025

1 Required

Solve 2*x*² − 7*x* − 15 \> 0. Show clear algebraic working.

2 Factorise the quadratic 2 x

Factorise the quadratic:

2*x*² − 7*x* − 15 = (2*x*+3)(*x*−5)

Check: (2*x*+3)(*x*−5) = 2*x*² − 10*x* + 3*x* − 15 = 2*x*² − 7*x* − 15

3 Set each factor zero

Roots: set each factor to zero:

$$2x + 3 = 0 implies x = -tfrac32; qquad x - 5 = 0 implies x = 5$$

4 Solve inequality

Since the coefficient of *x*² is + 2 \> 0, the parabola opens **upward**. The expression is \> 0 *outside* the roots:

FINAL ANSWER

$x \< - dfrac32$or*x* \> 5

4MA1/2HR JUNE 2025

Question 21

3 marks | Inequalities (Solving & Graphing)

21 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2025

1 Solve inequality

$$2x^2 - 7x - 15 > 0$$

2 Factorise

Factorise:

$$2x^2 - 7x - 15 = (2x + 3)(x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{3}{2} \quad \text{and} \quad x = 5$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

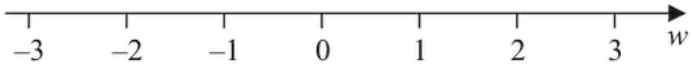
$x < -\frac{3}{2}$ or $x > 5$

4MA1/2H NOVEMBER 2025

Question 10

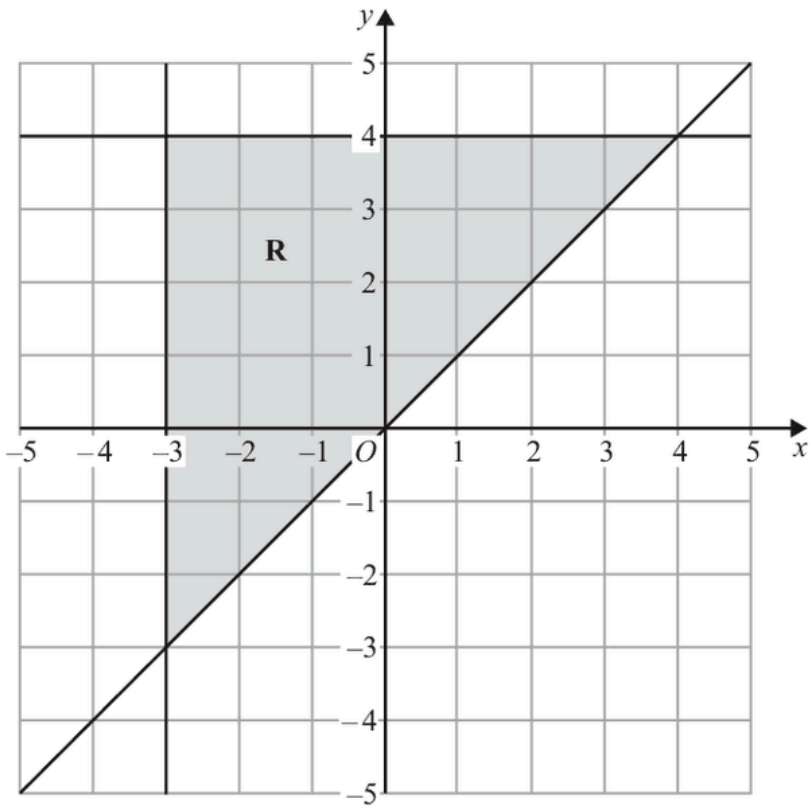
4 marks | Inequalities (Solving & Graphing)

10 (a) On the number line, represent the inequality $w < 1$



(1)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2025

1 **Read the graph**

For part (a), $w < 1$ is shown with an open circle at 1 and an arrow to the left.

2 **Read the graph**

For part (b), the region is right of $x = -3$, below $y = 4$, and above $y = x$:

$$x \geq -3$$

$$y \leq 4$$

$$y \geq x$$

FINAL ANSWER

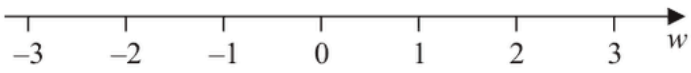
(a) Open circle at 1, shade left. (b) $x \geq -3$, $y \leq 4$, $y \geq x$.

4WM2/01 NOVEMBER 2025

Question 11

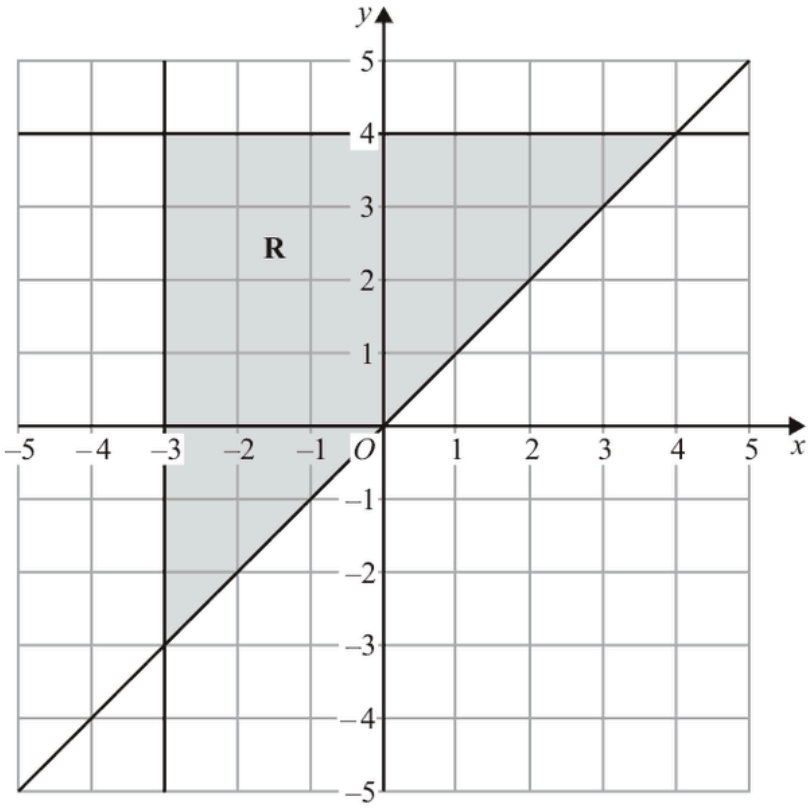
4 marks | Inequalities (Solving & Graphing)

11 (a) On the number line, represent the inequality $w < 1$



(1)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4WM2/01 November 2025

1 **Draw an open circle at**

Draw an open circle at 1 and an arrow pointing to the left.

2 **Identify vertical boundary**

Step 1: Identify the vertical boundary. The region is to the right of the vertical line at *x* = − 3. Inequality 1: *x* ≥ − 3

3 **Solve inequality**

Step 2: Identify the horizontal boundary. The region is below the horizontal line at *y* = 4. Inequality 2: *y* ≤ 4

4 **Calculate statistic**

Step 3: Identify the diagonal boundary. The line passes through (0,0), (1,1), (−1,−1), etc., meaning its equation is *y* = *x*. The shaded region is above this line. Inequality 3: *y* ≥ *x*

✓ **FINAL ANSWER**

x ≥ − 3, *y* ≤ 4, *y* ≥ *x* Always pick a test point inside the shaded region, like (0,2), to verify your inequalities. For *y* ≥ *x*, 2 ≥ 0 is true!

4MA1/1H NOVEMBER 2025

Question 25

3 marks | Inequalities (Solving & Graphing)

25 Solve the inequality $2x^2 + x - 28 > 0$
Show clear algebraic working.

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2025

1 Solve inequality

$$2x^2 + x - 28 > 0$$

2 Factorise

Factorise:

$$2x^2 + x - 28 = (2x - 7)(x + 4)$$

3 The critical values are

The critical values are

$$x = \frac{7}{2} \quad \text{and} \quad x = -4$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

$x < -4$ or $x > \frac{7}{2}$

4MA1/1H JUNE 2026

Question 5

7 marks | Inequalities (Solving & Graphing)

5 (a) Expand $4e(2e-3)$

.....

(1)

(b) Expand and simplify $(f+3)(f-6)$

.....

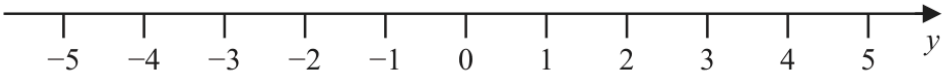
(2)

(c) Make x the subject of $t = ax - 2n$

.....

(2)

(d) On the number line, represent the inequality $-1 < y \leqslant 3$



(2)

(Total for Question 5 is 7 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$4e(2e-3) = 8e^2 - 12e.$$

Multiply 4e by both terms in the bracket.

2 **Part (b) | Step 1**

$$(f+3)(f-6) = f^2 - 6f + 3f - 18 = f^2 - 3f - 18.$$

Multiply each term in the first bracket by each term in the second and collect like terms.

3 **Part (c) | Step 1**

From $t = ax - 2n$, add $2n$ to give $t + 2n = ax$.

Use the same operation on both sides.

4 **Part (c) | Step 2**

$$x = \frac{t + 2n}{a} \quad (a \neq 0).$$

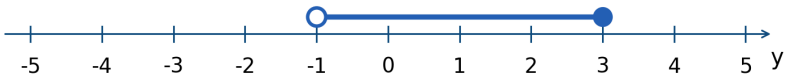
Divide by the coefficient of x.

---

5 **Part (d) | Step 1**

Draw an open circle at $y = -1$, a filled circle at $y = 3$, and a straight segment joining them. The represented interval is $(-1, 3]$.

The strict lower inequality excludes -1; the non-strict upper inequality includes 3.



✓ **FINAL ANSWER**

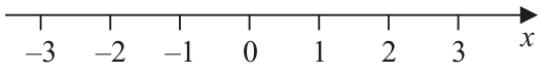
(a) $8e^2 - 12e$ (b) $f^2 - 3f - 18$ (c) $x = (t + 2n)/a$ (d) $(-1, 3]$

4MA1/1HR JUNE 2026

Question 7

5 marks | Inequalities (Solving & Graphing)

7



(a) Represent the inequality $x < 1$ on the number line above.

(1)

$2 \leqslant y + 3 < 7$
y is an integer.

(b) Write down all the possible values of y

.....

(2)

(c) Solve the inequality $8 - 5w > -10$

.....

(2)

(Total for Question 7 is 5 marks)

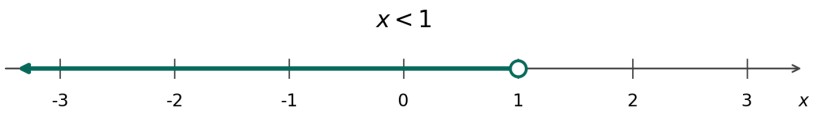
WORKED SOLUTION

Question 7 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

Place an open circle at 1 and draw a horizontal ray from it to the left, with a left-pointing arrow. Do not fill the circle.

The strict inequality excludes 1 and includes every real number less than 1.



2 **Part (b) | Step 1**

$$2 \leq y + 3 < 7 \implies -1 \leq y < 4.$$

Subtract 3 from all three expressions.

3 **Part (b) | Step 2**

The integer values in this interval are $-1, 0, 1, 2, 3$.

Include the lower endpoint but exclude 4.

---

4 **Part (c) | Step 1**

$$8 - 5w > -10 \quad \Longrightarrow \quad -5w > -18.$$

Subtract 8 from both sides.

---

5 **Part (c) | Step 2**

$$w < \frac{18}{5} = 3.6.$$

Division by a negative number reverses the inequality sign.

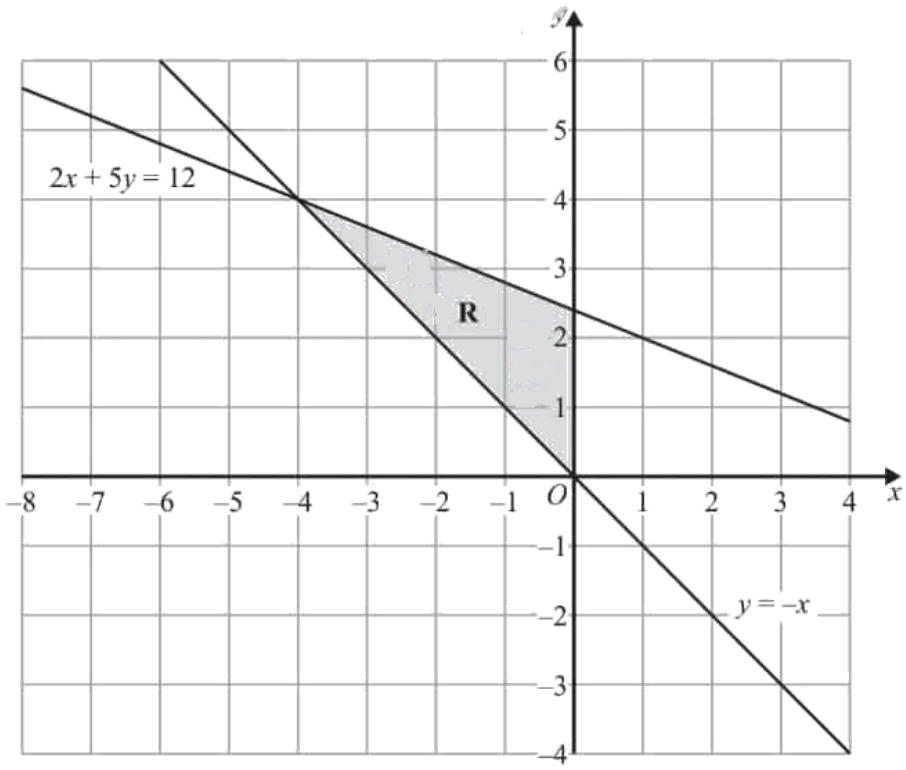
✓ **FINAL ANSWER**

(a) Open circle at 1, ray to the left: $(-\infty, 1)$. (b) $y = -1, 0, 1, 2, 3$ (c) $w < 3.6$

Question 12

3 marks | Inequalities (Solving & Graphing)

12



The region **R**, shown shaded in the diagram, is bounded by the y-axis and the straight lines with equations

$$y = -x$$

$$2x + 5y = 12$$

Write down the three inequalities that define **R**

(Total for Question 12 is 3 marks)

1 **Boundary lines**

R lies to the left of the y-axis, above $y = -x$ and below $2x + 5y = 12$.

2 **Inequalities**

$$x \leq 0, \qquad y \geq -x, \qquad 2x + 5y \leq 12$$

3 **Check a point inside R**

$(-1, 2)$: $-1 \leq 0$, $2 \geq 1$, $-2 + 10 = 8 \leq 12$ ✓

✓ **FINAL ANSWER**

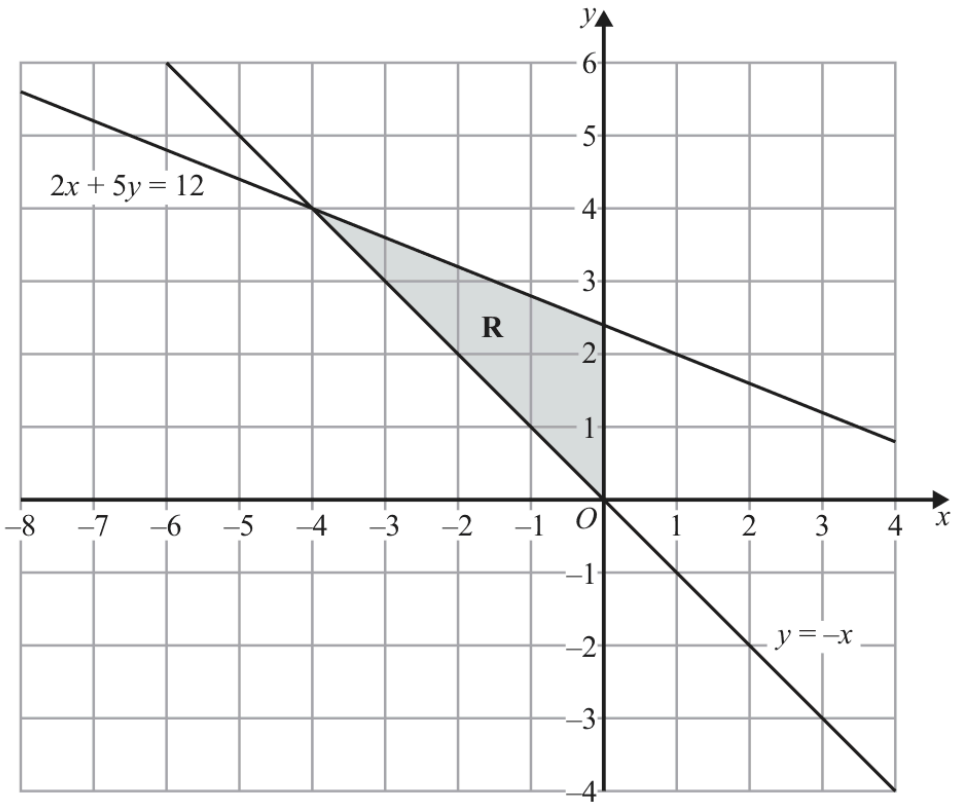
$x \leq 0$, $y \geq -x$, $2x + 5y \leq 12$

4MA1/2HR JUNE 2026

Question 12

3 marks | Inequalities (Solving & Graphing)

12



The region **R**, shown shaded in the diagram, is bounded by the y-axis and the straight lines with equations

$$y = -x$$

$$2x + 5y = 12$$

Write down the three inequalities that define **R**

.....

.....

.....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$x \leq 0$.

R lies to the left of the y-axis.

2 **Solution | Step 2**

$y \geq -x$.

R lies above the line y=-x.

3 **Solution | Step 3**

$2x + 5y \leq 12$.

R lies below the other line. All three boundaries are solid, so equality is included.

✓ **FINAL ANSWER**

$x \leq 0, \quad y \geq -x, \quad 2x + 5y \leq 12$.

TOPIC

Completing the Square

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 20

4 marks | Completing the Square

20 Find the values of a, b and c so that

$$7 + 12x - 2x^2$$

is written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 20 is 4 marks)

1 **Factor out -2 from the quadratic terms**

$$7 + 12x - 2x^2 = -2(x^2 - 6x) + 7$$

2 **Complete the square**

$$x^2 - 6x = (x - 3)^2 - 9$$

3 **Simplify the expression**

$$-2((x - 3)^2 - 9) + 7 = -2(x - 3)^2 + 18 + 7$$

$$= 25 - 2(x - 3)^2$$

4 **Read the values**

$$a = 25, \qquad b = 2, \qquad c = 3$$

✓ **FINAL ANSWER**

$a = 25,\ b = 2,\ c = 3.$

4MA1/2HR JANUARY 2020

Question 23

3 marks | Completing the Square

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2020

1 Solve inequality

$$7 - 12x - 2x^2 = -2x^2 - 12x + 7$$

$$= -2(x^2 + 6x) + 7$$

$$= -2\left((x + 3)^2 - 9\right) + 7$$

$$= -2(x + 3)^2 + 18 + 7$$

$$= 25 - 2(x + 3)^2$$

FINAL ANSWER

$25 - 2(x + 3)^2$

4MA1/2H MAY 2021

Question 22

4 marks | Completing the Square

22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a, b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H May 2021

1 Solve inequality

$$g(x) = 2x^2 - 12x + 13$$

$$= 2(x^2 - 6x) + 13$$

$$= 2\left((x - 3)^2 - 9\right) + 13$$

$$= 2(x - 3)^2 - 18 + 13$$

$$= 2(x - 3)^2 - 5$$

2 Calculate value

Starting with $S : y = x^2$:

3 Translate 3 units to the

- Translate 3 units to the right to get $y = (x - 3)^2$.
- Stretch parallel to the y-axis by scale factor 2 to get $y = 2(x - 3)^2$.
- Translate 5 units down to get $y = 2(x - 3)^2 - 5$.

✓ **FINAL ANSWER**

$g(x) = 2(x - 3)^2 - 5$. Transformations: right 3, stretch parallel to the y-axis by scale factor 2, then down 5.

4MA1/1HR JANUARY 2022

Question 19

3 marks | Completing the Square

19 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^m$

(a) find the value of m

$m =$ .....

(1)

Given that a, b and c are integers,

(b) express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$

.....

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$\sqrt[3]{\frac{1}{x}} = x^{-1/3}$$

2 **Evaluate fraction**

So $m = -\frac{1}{3}$.

3 **(b)**

(b)

$$3x^2 + 12x + 19 = 3(x^2 + 4x) + 19$$

$$= 3\left((x + 2)^2 - 4\right) + 19$$

$$= 3(x + 2)^2 + 7$$

FINAL ANSWER

$m = -\frac{1}{3}$, and $3(x + 2)^2 + 7$

4MA1/1H JANUARY 2022

Question 20

4 marks | Completing the Square

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

.....

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$
The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(..... , ...)

(1)

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2022

1 **(a)**

(a)

$$7 + 12x - 3x^2 = -3x^2 + 12x + 7$$

$$= -3(x^2 - 4x) + 7$$

$$= -3\left((x - 2)^2 - 4\right) + 7$$

$$= 19 - 3(x - 2)^2$$

2 **This is the same as**

This is the same as

$$19 - 3(x + (-2))^2$$

3 **(b) The maximum point is**

(b) The maximum point is read from the completed square form.

$$y = 19 - 3(x - 2)^2$$

4 **Calculate value**

The maximum occurs when $x = 2$, giving $y = 19$.

✓ **FINAL ANSWER**

(a) $19 - 3(x - 2)^2$, (b) $A = (2, 19)$

4MA1/1HR JUNE 2022

Question 23

5 marks | Completing the Square

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

(..... , ...)

(2)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$2x^2 - 12x + 3 = 2(x^2 - 6x) + 3$$

$$= 2\left((x - 3)^2 - 9\right) + 3$$

$$= 2(x - 3)^2 - 18 + 3$$

$$= 2(x - 3)^2 - 15$$

2 **(b)**

(b)

$$y = 2(x + 4)^2 - 12(x + 4) + 3$$

3 **Let**

Let $u = x + 4$. From part (a),

$$2u^2 - 12u + 3 = 2(u - 3)^2 - 15$$

4 **Calculate value**

So

$$y = 2((x + 4) - 3)^2 - 15$$

$$y = 2(x + 1)^2 - 15$$

---

5 **The minimum point is when**

The minimum point is when $x = -1$, giving $y = -15$.

✓ **FINAL ANSWER**

(a) $2(x - 3)^2 - 15$, (b) $M = (-1, -15)$

4MA1/1H JUNE 2022

Question 24

4 marks | Completing the Square

24 Express each of a, b and c in terms of q so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2022

1 Evaluate fraction

$$q + 12x - qx^2 = -q\left(x^2 - \frac{12}{q}x\right) + q$$

2 Complete the square

Complete the square:

$$x^2 - \frac{12}{q}x = \left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}$$

3 Evaluate fraction

So

$$q + 12x - qx^2 = -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}\right] + q$$

$$= q + \frac{36}{q} - q\left(x - \frac{6}{q}\right)^2$$

4 Compare coefficients

Compare with

$$a - b(x - c)^2$$

FINAL ANSWER

$$a = q + \frac{36}{q},\ b = q,\ c = \frac{6}{q}$$

4MA1/2H JUNE 2023

Question 19

3 marks | Completing the Square

19 Express $3x^2 - 6x + 5$ in the form $a(x - b)^2 + c$

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2023

1 **Calculate value**

$$3x^2 - 6x + 5 = 3(x^2 - 2x) + 5$$

$$= 3\left((x - 1)^2 - 1\right) + 5$$

$$= 3(x - 1)^2 + 2$$

FINAL ANSWER

$3(x - 1)^2 + 2$

Question 20

5 marks | Completing the Square

20 (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where a, b and c are numbers to be found.

.....

(3)

The curve **C** has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point P is the minimum point on **C**

(b) Find the coordinates of P

(..... , ...)

(2)

(Total for Question 20 is 5 marks)

1 **Use Moved Coordinate Geometry Completing**

For part (a),

$$2x^2 - 11x + 9$$

2 **Factor out 2 from the**

Factor out 2 from the quadratic terms:

$$2\left(x^2 - \frac{11}{2}x\right) + 9$$

3 **Complete the square**

Complete the square:

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{72}{8}$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

4 **Evaluate fraction**

So

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

5 **Calculate value**

For part (b),

$$y = 2(x-3)^2 - 11(x-3) + 9$$

6 **Let**

Let

$$u = x - 3$$

7 **Evaluate fraction**

From part (a), the minimum occurs when

$$u = \frac{11}{4}$$

8 **Evaluate fraction**

So

$$x - 3 = \frac{11}{4}$$

$$x = \frac{23}{4}$$

9 **The minimum value of is**

The minimum value of y is

$$-\frac{49}{8}$$

✓ **FINAL ANSWER**

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8},\ P = \left(\frac{23}{4}, -\frac{49}{8}\right)$$

4MA1/1H MAY 2024

Question 25

4 marks | Completing the Square

25 $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where a, b and c are constants.

$f(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2024

1 Solve inequality

$$f(x) = 17 - 3x^2 + 12x$$

$$= -3x^2 + 12x + 17$$

$$= -3(x^2 - 4x) + 17$$

$$= -3\left((x-2)^2 - 4\right) + 17$$

$$= -3(x-2)^2 + 12 + 17$$

$$= 29 - 3(x-2)^2$$

FINAL ANSWER

$f(x) = 29 - 3(x-2)^2$

4WM1/01R MAY 2025

Question 18

4 marks | Completing the Square

18 (a) Express $2x^2 - 12x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.
Show clear algebraic working.

.....

(3)

(b) Hence, state the least value of $2x^2 - 12x + 23$

.....

(1)

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM1/01R May 2025

1 **Calculate value**

$$2x^2 - 12x + 23 = 2(x^2 - 6x) + 23$$

$$= 2\left((x - 3)^2 - 9\right) + 23$$

$$= 2(x - 3)^2 + 5$$

2 **The squared part is at**

The squared part is at least zero.

✓ **FINAL ANSWER**

$2(x - 3)^2 + 5$; least value 5.

Question 25

6 marks | Completing the Square

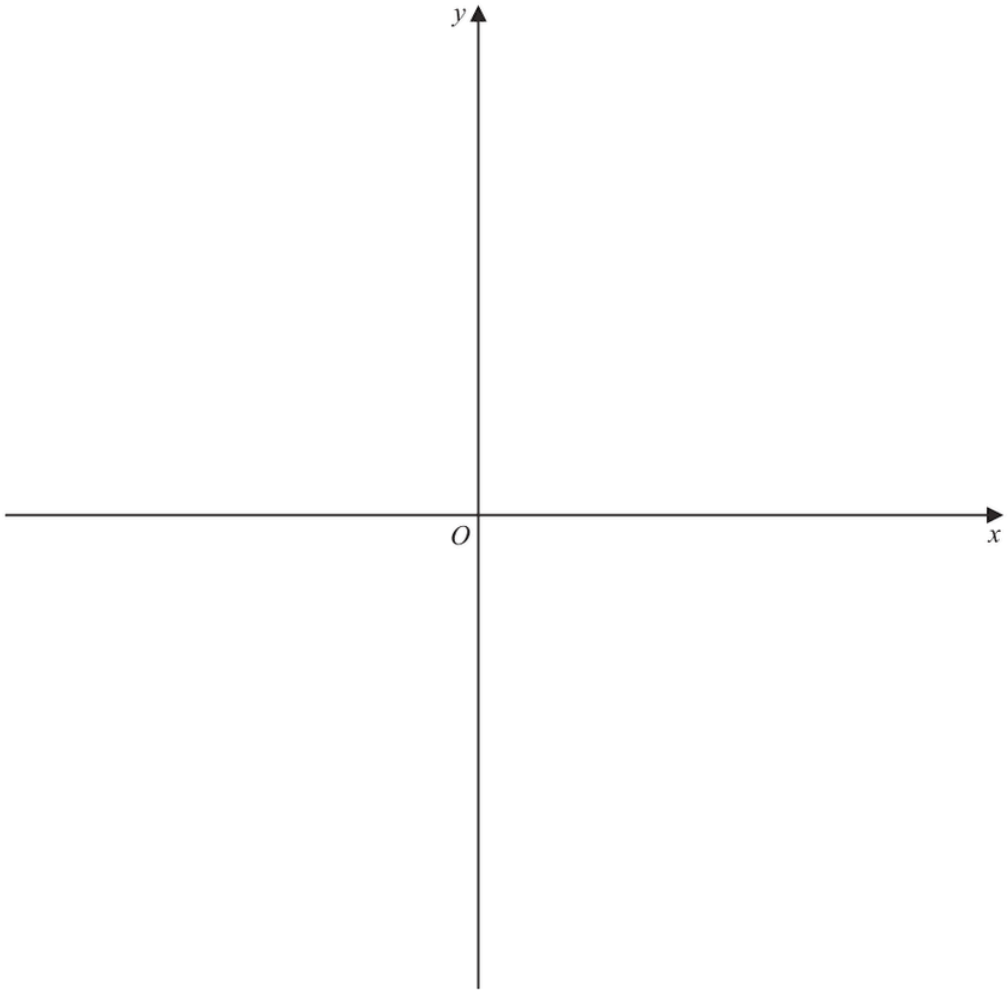
25 (a) Write $28+24x-6x^2$ in the form $a-b(x-c)^2$ where a, b and c are integers.

.....

(3)

(b) On the axes below, sketch the graph of $y=28+24x-6x^2$

Show clearly the coordinates of the turning point and the coordinates of the point of intersection of the graph with the y-axis.



(3)

(Total for Question 25 is 6 marks)

1 **(a)**

(a)

$$28 + 24x - 6x^2 = -6x^2 + 24x + 28$$

$$= -6(x^2 - 4x) + 28$$

$$= -6\left((x-2)^2 - 4\right) + 28$$

$$= 52 - 6(x-2)^2$$

2 **(b)**

(b)

$$y = 52 - 6(x-2)^2$$

3 **The graph is a downward**

The graph is a downward-opening parabola.

4 **The turning point is**

The turning point is

$$(2, 52)$$

5 **Calculate value**

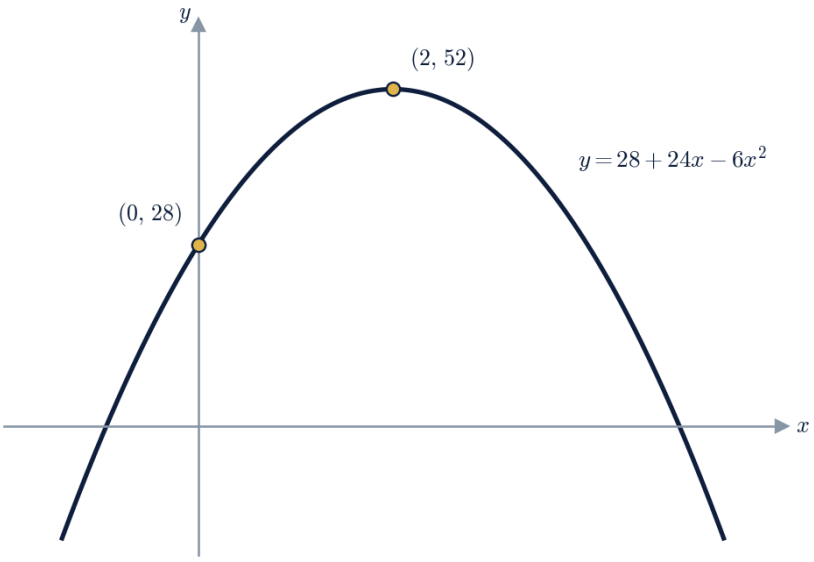
The point of intersection with the y-axis is found by putting $x = 0$:

$$y = 28$$

6 **Find intercepts**

So the y-intercept is

$$(0, 28)$$



FINAL ANSWER

(a) $52 - 6(x - 2)^2$. For the sketch, show a downward parabola with turning point $(2, 52)$ and y-intercept $(0, 28)$.

4WM1/01 MAY 2025

Question 25

5 marks | Completing the Square

25 (a) Express $8 - 12x - 4x^2$ in the form $a - b(x + c)^2$
where a, b and c are constants.

.....

(3)

(b) Hence solve $8 - 12x - 4x^2 = 0$
Show your working clearly.
Give your solutions in surd form.

.....

(2)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 May 2025

1 **Simplify surd**

$$8x^2 - 12x - 4 = 8\left(x^2 - \frac{3}{2}x\right) - 4$$

2 **Complete the square**

Complete the square.

$$x^2 - \frac{3}{2}x = \left(x - \frac{3}{4}\right)^2 - \frac{9}{16}$$

3 **Simplify surd**

So

$$8x^2 - 12x - 4 = 8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$$

4 **Simplify surd**

Now solve

$$8x^2 - 12x - 4 = 0$$

$$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2} = 0$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

✓ **FINAL ANSWER**

$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$, and $x = \frac{3 \pm \sqrt{17}}{4}$.

4MA1/1H NOVEMBER 2025

Question 21

3 marks | Completing the Square

21 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2025

1 Solve inequality

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 - 20 + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$

4WM1/01 NOVEMBER 2025

Question 22

2 marks | Completing the Square

22 $9x^2 - 12x + q$ can be written in the form $(px - r)^2$ where p, q and r are integers.

Find the value of q

$q =$ .....

(Total for Question 22 is 2 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 November 2025

1 **Calculate value**

$$9x^2 - 12x + q = (3x - 2)^2$$

$$(3x - 2)^2 = 9x^2 - 12x + 4$$

FINAL ANSWER

$q = 4$.

4WM1/01 NOVEMBER 2025

Question 24

3 marks | Completing the Square

24 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 November 2025

1 **Calculate value**

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$.

4MA1/1H JUNE 2026

Question 24

5 marks | Completing the Square

24 (a) Express $10 - 24x - 3x^2$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(3)

T is the curve with equation $y = 10 - 24(x - 1) - 3(x - 1)^2$
The point P is the maximum point on **T**

(b) Use your answer to part (a) to find the coordinates of P

(..... , ...)

(1)

(c) Write down the equation for the line of symmetry of **T**

.....

(1)

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$10 - 24x - 3x^2 = 10 - 3(x^2 + 8x).$$

Factor -3 from the terms involving x.

2 **Part (a) | Step 2**

$$x^2 + 8x = (x + 4)^2 - 16.$$

Half the coefficient of x is 4.

3 **Part (a) | Step 3**

$$10 - 3\big((x + 4)^2 - 16\big) = 58 - 3(x + 4)^2.$$

Thus a=58, b=3 and c=4.

4 **Part (b) | Step 1**

$$y = 58 - 3\big((x - 1) + 4\big)^2 = 58 - 3(x + 3)^2.$$

Use (a) with x replaced throughout by x-1.

5

Part (b) | Step 2

Since $(x+3)^2 \geq 0$, the greatest y is 58, attained at $x = -3$. Thus $P = (-3, 58)$.

The negative square term is largest when its square is zero.

6

Part (c) | Step 1

The parabola is symmetric about its maximum, so its line of symmetry is $x = -3$.

In 58-3(x+3)^2, equal positive and negative offsets from -3 give the same y.

✓ **FINAL ANSWER**

(a) $58 - 3(x+4)^2$ (b) $P = (-3, 58)$ (c) $x = -3$

4WM1/01 MAY 2026

Question 24

3 marks | Completing the Square

24 Express $22 - 3x^2 + 30x$ in the form $a - b(x - c)^2$ where a, b and c are integers.

.....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2026

1 Factor out −3 from the x terms

$$22 - 3x^2 + 30x = 22 - 3(x^2 - 10x)$$

2 Complete the square

$$x^2 - 10x = (x - 5)^2 - 25$$

3 Simplify

$$22 - 3\big((x - 5)^2 - 25\big) = 97 - 3(x - 5)^2$$

FINAL ANSWER

$97 - 3(x - 5)^2$

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TOPIC

Quadratic Formula

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 30

3 marks | Quadratic Formula

30 Solve $2x^2 - 5x - 4 = 0$

Show your working clearly.
Give your solutions correct to 3 significant figures.

.....

(3)

(Total for Question 30 is 3 marks)

WORKED SOLUTION

Question 30 · EP Specimen

1 Identify a, b and c

$a = 2,\ b = -5,\ c = -4$

2 Substitute into the formula

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)} = \frac{5 \pm \sqrt{57}}{4}$$

3 Work out both solutions

$$x = \frac{5 + 7.5498\ldots}{4} = 3.14, \qquad x = \frac{5 - 7.5498\ldots}{4} = -0.637$$

FINAL ANSWER

$x = 3.14$ or $x = -0.637$

EP SPECIMEN

Question 31

3 marks | Quadratic Formula

31 Solve $x^2 + 6x + 2 = 0$

Give your solutions in the form $a \pm \sqrt{b}$ where a and b are integers.

.....

(3)

(Total for Question 31 is 3 marks)

WORKED SOLUTION

Question 31 · EP Specimen

1 Use the formula

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2} = \frac{-6 \pm \sqrt{28}}{2}$$

2 Simplify the surd

$\sqrt{28} = 2\sqrt{7}$, so

$$x = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

✓ **FINAL ANSWER**

$x = -3 \pm \sqrt{7}$

EP SPECIMEN

Question 32

5 marks | Quadratic Formula

32 A rectangle has length $(x + 3)$ cm and width $(x - 1)$ cm.
The area of the rectangle is 20 cm^2.

(a) Show that $x^2 + 2x - 23 = 0$

.....

(2)

(b) Work out the length of the rectangle.
Give your answer correct to 3 significant figures.

..... cm

(3)

(Total for Question 32 is 5 marks)

WORKED SOLUTION

Question 32 · EP Specimen

1 **(a) Area = length × width**

$$(x+3)(x-1) = 20 \Rightarrow x^2 + 2x - 3 = 20 \Rightarrow x^2 + 2x - 23 = 0$$

2 **(b) Use the formula**

$$x = \frac{-2 \pm \sqrt{4 + 92}}{2} = \frac{-2 \pm \sqrt{96}}{2} = -1 \pm \sqrt{24}$$

The width $x - 1$ must be positive, so $x = -1 + \sqrt{24} = 3.898\ldots$

3 **(b) Length**

$$x + 3 = 6.898\ldots \approx 6.90 \text{ cm}$$

✓ **FINAL ANSWER**

(b) **6.90** cm

TOPIC

Quadratic Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 6

3 marks | Quadratic Equations

6 Solve $x^2 - 5x - 36 = 0$
Show clear algebraic working.

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2020

1 Solve quadratic equation

$$x^2 - 5x - 36 = 0$$

2 Factorise

Factorise:

$$(x - 9)(x + 4) = 0$$

3 Solve quadratic equation

So

$$x = 9 \quad \text{or} \quad x = -4$$

FINAL ANSWER

$x = 9$ or $x = -4$.

4MA1/2HR JANUARY 2021

Question 9

3 marks | Quadratic Equations

9 Solve $x^2 - 21x + 20 = 0$
Show your working clearly.

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2021

1 Solve quadratic equation

$$x^2 - 21x + 20 = 0$$

2 Factorise

Factorise:

$$(x - 20)(x - 1) = 0$$

3 Solve quadratic equation

So

$$x = 20 \quad \text{or} \quad x = 1$$

FINAL ANSWER

$x = 20$ or $x = 1$.

4WM1/01 MAY 2025

Question 15

3 marks | Quadratic Equations

15 Solve the equation $5x^2 + 9x - 17 = 0$
Show your working clearly.
Give your solutions correct to 3 significant figures.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM1/01 May 2025

1 **Solve quadratic equation**

$$5x^2 + 9x - 17 = 0$$

2 **Use quadratic formula**

Use the quadratic formula with $a = 5$, $b = 9$, $c = -17$.

$$x = \frac{-9 \pm \sqrt{9^2 - 4(5)(-17)}}{2(5)}$$

$$x = \frac{-9 \pm \sqrt{421}}{10}$$

$$x = 1.152\ldots \quad \text{or} \quad x = -2.952\ldots$$

3 **Solve quadratic equation**

To 3 significant figures,

$$x = 1.15 \quad \text{or} \quad x = -2.95$$

✓ **FINAL ANSWER**

$x = 1.15$ or $x = -2.95$.

TOPIC

Algebraic Proof

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 13

4 marks | Algebraic Proof

13 (a) Use algebra to show that $0.5\dot{7}\dot{2} = \frac{63}{110}$

(2)

Given that y is a prime number,

(b) express $\frac{3}{2-\sqrt{y}}$ in the form $\frac{a+b\sqrt{y}}{c-y}$ where a, b and c are integers.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR January 2020

1 **(a) Let**

(a) Let

$$x = 0.5727272\ldots$$

2 **Calculate value**

Then

$$1000x = 572.727272\ldots$$

3 **Calculate value**

and

$$10x = 5.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 567$$

$$x = \frac{567}{990} = \frac{63}{110}$$

5 **Evaluate fraction**

So

$$0.5\dot{7}\dot{2} = \frac{63}{110}$$

---

6 **(b)**

(b)

$$\frac{3}{2-\sqrt{y}} \times \frac{2+\sqrt{y}}{2+\sqrt{y}} = \frac{3(2+\sqrt{y})}{4-y}$$

$$= \frac{6+3\sqrt{y}}{4-y}$$

✓ **FINAL ANSWER**

(a) shown, (b) $\frac{6+3\sqrt{y}}{4-y}$

4MA1/2H JANUARY 2020

Question 17

3 marks | Algebraic Proof

17 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H January 2020

1 Let the two consecutive square

Let the two consecutive square numbers be n^2 and $(n+1)^2$.

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$$

2 is one more than an

$2n + 1$ is one more than an even number, so it is always odd.

FINAL ANSWER

The difference is always odd.

4MA1/2HR JANUARY 2020

Question 25

3 marks | Algebraic Proof

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR January 2020

1 Use trigonometry

Since N is a multiple of 5, let

$$N = 5k$$

2 where is an integer

where k is an integer.

3 Given

Given

$$A = N + 1, \qquad B = N - 1$$

4 Use trigonometry

So

$$A^2 - B^2 = (N+1)^2 - (N-1)^2$$

5 Expand

Expand:

$$(N+1)^2 = N^2 + 2N + 1$$

$$(N-1)^2 = N^2 - 2N + 1$$

6 **Use trigonometry**

Therefore

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$

$$= 4N$$

---

7 **Substitute**

Substitute $N = 5k$:

$$4N = 4(5k) = 20k$$

---

8 **Use trigonometry**

Since k is an integer, $20k$ is a multiple of 20.

✓ **FINAL ANSWER**

$A^2 - B^2$ is always a multiple of 20.

4MA1/1HR JANUARY 2021

Question 15

5 marks | Algebraic Proof

15 (a) Use algebra to show that $4.\dot{5}\dot{7} = 4\frac{19}{33}$

(2)

(b) Show that $\frac{2}{6-3\sqrt{2}}$ can be written in the form $\frac{a+\sqrt{a}}{b}$

where a and b are integers.

Show your working clearly.

(3)

(Total for Question 15 is 5 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2021

1 (a) Let

(a) Let

$$x = 4.\dot{5}\dot{7}$$

2 Calculate value

Then

$$100x = 457.\dot{5}\dot{7}$$

3 Subtract

Subtract:

$$100x - x = 457.\dot{5}\dot{7} - 4.\dot{5}\dot{7}$$

$$99x = 453$$

$$x = \frac{453}{99} = \frac{151}{33} = 4\frac{19}{33}$$

4 **(b)**

(b)

$$\frac{2}{6-3\sqrt{2}} \times \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = \frac{12+6\sqrt{2}}{36-18}$$

$$= \frac{12+6\sqrt{2}}{18} = \frac{2+\sqrt{2}}{3}$$

---

5 **Simplify surd**

So this is in the form $\frac{a+\sqrt{a}}{b}$, with $a = 2$ and $b = 3$.

FINAL ANSWER

$4.\dot{5}\dot{7} = 4\frac{19}{33}$ and $\frac{2}{6-3\sqrt{2}} = \frac{2+\sqrt{2}}{3}$.

4MA1/2HR JANUARY 2021

Question 15

4 marks | Algebraic Proof

15 Prove algebraically that the product of any two odd numbers is always an odd number.

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR January 2021

1 Let the two odd numbers

Let the two odd numbers be

$$2m+1 \quad \text{and} \quad 2n+1$$

2 Their product is

Their product is

$$(2m+1)(2n+1)$$

$$= 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1$$

3 This is of the form

This is of the form

$$2k+1$$

where k is an integer.

4 Use algebraic proof asks general

So the product is odd.

FINAL ANSWER

The product of any two odd numbers is always odd.

4MA1/1H MAY 2021

Question 17

3 marks | Algebraic Proof

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2021

1 Let the three consecutive even

Let the three consecutive even numbers be

$$2n-2, \quad 2n, \quad 2n+2$$

2 The middle number is

The middle number is $2n$.

3 The difference between the square

The difference between the square of the largest and the square of the smallest is

$$(2n+2)^2-(2n-2)^2$$

$$=(4n^2+8n+4)-(4n^2-8n+4)$$

$$=16n$$

4 Calculate value

Since $8 \times \text{middle number} = 8(2n) = 16n$, the result is 8 times the middle number.

FINAL ANSWER

The difference is always 8 times the middle number.

Question 19

6 marks | Algebraic Proof

19 *ABCED* is a five-sided shape.

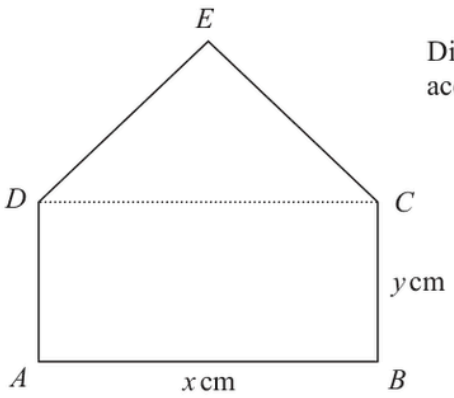


Diagram **NOT** accurately drawn

ABCD is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $BC = y$ cm

The perimeter of *ABCED* is 100 cm.
The area of *ABCED* is R cm^2

(a) Show that $R = \frac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

(3)

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\frac{p}{q - \sqrt{3}}$ where p and q are integers.

$x =$ .....

(2)

(ii) Explain why the maximum value of R is given by this value of x.

.....

(1)

(Total for Question 19 is 6 marks)

1 **Rearrange formula**

The perimeter is made from three sides of length x and two sides of length y:

$$3x + 2y = 100$$

2 **Rearrange formula**

So

$$y = \frac{100 - 3x}{2}$$

3 **The area is the area**

The area is the area of the rectangle plus the area of the equilateral triangle:

$$R = xy + \frac{\sqrt{3}}{4}x^2$$

4 **Substitute**

Substitute $y = \frac{100 - 3x}{2}$:

$$R = x\left(\frac{100 - 3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$

$$R = 50x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$

$$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$$

5 **Rearrange formula**

For the maximum, write

$$R = -\frac{6 - \sqrt{3}}{4}x^2 + 50x$$

6 **The maximum occurs at**

The maximum occurs at

$$x = \frac{-b}{2a} = \frac{-50}{2\left(-\frac{6-\sqrt{3}}{4}\right)}$$

$$x = \frac{100}{6 - \sqrt{3}}$$

7 **Solve quadratic equation**

The coefficient of x^2 is negative, so the quadratic graph opens downwards and this vertex gives a maximum.

✓ **FINAL ANSWER**

$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$, and $x = \frac{100}{6 - \sqrt{3}}$.

4MA1/2HR JANUARY 2022

Question 13

3 marks | Algebraic Proof

13 $a = \sqrt{8} + 4$

$b = \sqrt{8} - 4$

$(a - b)(a + b)$ can be written in the form $y\sqrt{4y}$

Find the value of y
Show your working clearly.

$y =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2022

1 Simplify surd

$$a = \sqrt{8} + 4, \qquad b = \sqrt{8} - 4$$

$$a - b = (\sqrt{8} + 4) - (\sqrt{8} - 4) = 8$$

$$a + b = (\sqrt{8} + 4) + (\sqrt{8} - 4) = 2\sqrt{8} = 4\sqrt{2}$$

2 Simplify surd

So

$$(a - b)(a + b) = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

3 The question says this equals

The question says this equals $y\sqrt{4y}$. Try $y = 8$:

$$8\sqrt{32} = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

FINAL ANSWER

$y = 8$.

4MA1/1HR JANUARY 2022

Question 15

3 marks | Algebraic Proof

15 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2022

1 Let the three consecutive whole

Let the three consecutive whole numbers be

$$n, \quad n+1, \quad n+2$$

2 The sum of the squares

The sum of the squares of the smallest and largest is

$$n^2 + (n+2)^2 = n^2 + n^2 + 4n + 4 = 2n^2 + 4n + 4$$

3 Twice the square of the

Twice the square of the middle number is

$$2(n+1)^2 = 2(n^2 + 2n + 1) = 2n^2 + 4n + 2$$

4 Calculate value

So

$$2n^2 + 4n + 4 = (2n^2 + 4n + 2) + 2$$

FINAL ANSWER

The sum is always 2 more than twice the square of the middle number.

4MA1/2H JANUARY 2022

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a+\sqrt{b}$ where a and b are integers.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H January 2022

1 Simplify surd

$$\frac{\sqrt{12}}{\sqrt{3}+2}=\frac{2\sqrt{3}}{\sqrt{3}+2}$$

2 Multiply by the conjugate

Multiply by the conjugate:

$$\frac{2\sqrt{3}}{\sqrt{3}+2}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2\sqrt{3}(2-\sqrt{3})}{4-3}$$

$$=4\sqrt{3}-6$$

3 Simplify surd

Since $4\sqrt{3}=\sqrt{48}$,

$$4\sqrt{3}-6=-6+\sqrt{48}$$

FINAL ANSWER

$-6+\sqrt{48}$, so $a=-6$ and $b=48$.

4MA1/2H JANUARY 2022

Question 18

3 marks | Algebraic Proof

18 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H January 2022

1 Let the two consecutive odd

Let the two consecutive odd numbers be

$$2n+1 \quad \text{and} \quad 2n+3$$

2 The sum of their squares

The sum of their squares is

$$(2n+1)^2+(2n+3)^2$$

$$=4n^2+4n+1+4n^2+12n+9$$

$$=8n^2+16n+10$$

$$=8(n^2+2n+1)+2=8(n+1)^2+2$$

3 Use Algebraic proof correct

This is a multiple of 8 plus 2, so the remainder is always 2.

FINAL ANSWER

The remainder is always 2.

4MA1/2HR JUNE 2022

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.3\dot{4}\dot{5} = \frac{19}{55}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2022

1 Let

Let

$$x = 0.3\dot{4}\dot{5}$$

2 Calculate value

Then

$$10x = 3.\dot{4}\dot{5}$$

3 Calculate value

and

$$1000x = 345.\dot{4}\dot{5}$$

4 Subtract

Subtract:

$$1000x - 10x = 345.\dot{4}\dot{5} - 3.\dot{4}\dot{5}$$

$$990x = 342$$

$$x = \frac{342}{990} = \frac{19}{55}$$

FINAL ANSWER

$0.3\dot{4}\dot{5} = \frac{19}{55}$.

4MA1/2H JUNE 2023

Question 1

3 marks | Algebraic Proof

1 Show that $4\frac{2}{3} \div 1\frac{1}{5} = 3\frac{8}{9}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2023

1 **Evaluate fraction**

$$4\frac{2}{3} = \frac{14}{3}, \qquad 1\frac{1}{5} = \frac{6}{5}$$

$$4\frac{2}{3} \div 1\frac{1}{5} = \frac{14}{3} \div \frac{6}{5}$$

$$= \frac{14}{3} \times \frac{5}{6} = \frac{70}{18} = \frac{35}{9}$$

$$\frac{35}{9} = 3\frac{8}{9}$$

FINAL ANSWER

$3\frac{8}{9}$.

4MA1/2HR JUNE 2023

Question 16

3 marks | Algebraic Proof

16 Show that $\dfrac{2\sqrt{3}}{\sqrt{3}-1}$ can be written in the form $a+\sqrt{a}$ where a is an integer.

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2023

1 Simplify surd

$$\frac{2\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{2\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$=\frac{6+2\sqrt{3}}{2}=3+\sqrt{3}$$

2 This is in the form

This is in the form $a+\sqrt{a}$, where $a=3$.

FINAL ANSWER

$3+\sqrt{3}$, so $a=3$.

4MA1/1HR MAY 2023

Question 16

2 marks | Algebraic Proof

16 Use algebra to show that $0.1\dot{7}\dot{6} = \frac{35}{198}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May 2023

1 Let

Let

$$x = 0.1767676\ldots$$

2 Calculate value

Then

$$10x = 1.767676\ldots$$

3 Calculate value

and

$$1000x = 176.767676\ldots$$

4 Subtract

Subtract:

$$990x = 175$$

$$x = \frac{175}{990} = \frac{35}{198}$$

FINAL ANSWER

$0.1767676\ldots = \frac{35}{198}$.

4MA1/2H NOVEMBER 2023

Question 6

3 marks | Algebraic Proof

6 Show that $3\frac{3}{7} \div 2\frac{2}{3} = 1\frac{2}{7}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2023

1 Evaluate fraction

$$3\frac{3}{7} = \frac{24}{7}, \qquad 2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{7} \div 2\frac{2}{3} = \frac{24}{7} \div \frac{8}{3}$$

$$= \frac{24}{7} \times \frac{3}{8} = \frac{9}{7} = 1\frac{2}{7}$$

FINAL ANSWER

$1\frac{2}{7}$.

4MA1/2H NOVEMBER 2023

Question 15

5 marks | Algebraic Proof

15 (a) Use algebra to show that $0.3\dot{7}\dot{2} = \frac{41}{110}$

(2)

(b) Express $\frac{\sqrt{125} + \sqrt{80}}{\sqrt{3}}$ in the form $\sqrt{n}$ where n is an integer.

Show your working clearly.

.....

(3)

(Total for Question 15 is 5 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2023

1 **(a) Let**

(a) Let

$$x = 0.3727272\ldots$$

2 **Calculate value**

Then

$$10x = 3.727272\ldots$$

3 **Calculate value**

and

$$1000x = 372.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 369$$

$$x = \frac{369}{990} = \frac{41}{110}$$

5 **(b)**

(b)

$$\frac{\sqrt{125}+\sqrt{80}}{\sqrt{3}} = \frac{5\sqrt{5}+4\sqrt{5}}{\sqrt{3}}$$

$$= \frac{9\sqrt{5}}{\sqrt{3}}$$

---

6 **Rationalise**

Rationalise:

$$\frac{9\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{15}}{3} = 3\sqrt{15}$$

$$3\sqrt{15} = \sqrt{135}$$

✓ **FINAL ANSWER**

$0.3727272\ldots = \frac{41}{110}$, and $n = 135$.

4MA1/1HR MAY 2024

Question 4

3 marks | Algebraic Proof

4 Show that $2\frac{1}{3} \div 5\frac{1}{4} = \frac{4}{9}$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May 2024

1 Evaluate fraction

$$2\frac{1}{3} = \frac{7}{3}, \qquad 5\frac{1}{4} = \frac{21}{4}$$

$$2\frac{1}{3} \div 5\frac{1}{4} = \frac{7}{3} \div \frac{21}{4}$$

$$= \frac{7}{3} \times \frac{4}{21} = \frac{28}{63} = \frac{4}{9}$$

✓ **FINAL ANSWER**

$\frac{4}{9}$.

4MA1/2H JUNE 2024

Question 14

7 marks | Algebraic Proof

14 (a) Show that $3y(2y + 5)(y + 7)$ can be written in the form $ay^3 + by^2 + cy$ where a, b and c are integers.

(3)

(b) Solve $\frac{2x + 3}{5} + \frac{6x - 5}{4} = \frac{163}{100}$

Show clear algebraic working.

$x =$ .....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2024

1 **(a)**

(a)

$$3y(2y+5)(y+7)$$

2 **Expand brackets**

First expand the two brackets:

$$(2y+5)(y+7) = 2y^2 + 14y + 5y + 35 = 2y^2 + 19y + 35$$

3 **Multiply by**

Now multiply by $3y$:

$$3y(2y^2 + 19y + 35) = 6y^3 + 57y^2 + 105y$$

4 **Calculate value**

So $a = 6$, $b = 57$, and $c = 105$.

5 **(b)**

(b)

$$\frac{2x+3}{5} + \frac{6x-5}{4} = \frac{163}{100}$$

6 Multiply by 100

Multiply by 100:

$$20(2x+3)+25(6x-5)=163$$

$$40x+60+150x-125=163$$

$$190x-65=163$$

$$190x=228$$

$$x=\frac{228}{190}=\frac{6}{5}$$

FINAL ANSWER

$6y^3+57y^2+105y$, and $x=\frac{6}{5}$.

4MA1/1H MAY 2024

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{1+\sqrt{5}}{3-\sqrt{5}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers.

Show each stage of your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2024

1 **Simplify surd**

$$\frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(1+\sqrt{5})(3+\sqrt{5})}{9-5}$$

$$= \frac{3+\sqrt{5}+3\sqrt{5}+5}{4}$$

$$= \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$$

FINAL ANSWER

$2+\sqrt{5}$, so $a=2$ and $b=5$.

4MA1/2HR JUNE 2024

Question 18

2 marks | Algebraic Proof

18 Use algebra to show that $0.\dot{3}0\dot{6} = \frac{34}{111}$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2024

1 Let

Let

$$x = 0.\dot{3}0\dot{6}$$

2 Calculate value

Then

$$1000x = 306.\dot{3}0\dot{6}$$

3 Subtract

Subtract:

$$1000x - x = 306$$

$$999x = 306$$

$$x = \frac{306}{999} = \frac{34}{111}$$

FINAL ANSWER

$0.\dot{3}0\dot{6} = \frac{34}{111}$.

4MA1/2H NOVEMBER 2024

Question 1

3 marks | Algebraic Proof

1 Show that $1\frac{5}{7} \times 2\frac{3}{16} = 3\frac{3}{4}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2024

1 **Evaluate fraction**

$$1\frac{5}{7} = \frac{12}{7}, \qquad 2\frac{3}{16} = \frac{35}{16}$$

$$1\frac{5}{7} \times 2\frac{3}{16} = \frac{12}{7} \times \frac{35}{16}$$

$$= \frac{420}{112} = \frac{15}{4} = 3\frac{3}{4}$$

✓ **FINAL ANSWER**

$3\frac{3}{4}$.

4MA1/2H NOVEMBER 2024

Question 15

2 marks | Algebraic Proof

15 Use algebra to show that $0.7\dot{6}\dot{3} = \frac{42}{55}$

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2024

1 Let

Let

$$x = 0.7636363\ldots$$

2 Calculate value

Then

$$10x = 7.636363\ldots$$

3 Calculate value

and

$$1000x = 763.636363\ldots$$

4 Subtract

Subtract:

$$990x = 756$$

$$x = \frac{756}{990} = \frac{42}{55}$$

FINAL ANSWER

$0.7636363\ldots = \frac{42}{55}$.

4MA1/1H NOVEMBER 2024

Question 17

4 marks | Algebraic Proof

17 (a) Express $\sqrt{675}$ in the form $n\sqrt{27}$ where n is a positive integer.

.....

(1)

(b) Show that $\dfrac{5-\sqrt{2}}{\sqrt{2}-1}$ can be written in the form $a+b\sqrt{2}$ where a and b are integers.

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2024

1 **(a)**

(a)

$$\sqrt{675} = \sqrt{25 \times 27} = 5\sqrt{27}$$

2 **Calculate value**

So $n = 5$.

3 **(b)**

(b)

$$\frac{5-\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(5-\sqrt{2})(\sqrt{2}+1)}{2-1}$$

$$= 5\sqrt{2} + 5 - 2 - \sqrt{2}$$

$$= 3 + 4\sqrt{2}$$

✓ **FINAL ANSWER**

$n = 5$, and $3 + 4\sqrt{2}$.

4MA1/2H NOVEMBER 2024

Question 23

4 marks | Algebraic Proof

23 Show that $\dfrac{16x^2 - 36}{x - 7} \div \dfrac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x) = n$

where n is an integer to be found.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2024

1 Evaluate fraction

$$\frac{16x^2 - 36}{x - 7} \div \frac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x)$$

2 Factorise

Factorise:

$$16x^2 - 36 = 4(2x - 3)(2x + 3)$$

$$2x^2 + 7x + 6 = (2x + 3)(x + 2)$$

$$x^2 - 5x - 14 = (x - 7)(x + 2)$$

3 Evaluate fraction

So

$$\frac{4(2x-3)(2x+3)}{x-7} \div \frac{(2x+3)(x+2)}{(x-7)(x+2)} - (7+8x)$$

$$= \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)} - (7+8x)$$

$$= 4(2x-3) - (7+8x)$$

$$= 8x - 12 - 7 - 8x$$

$$= -19$$

4 Calculate value

So $n = -19$, which is an integer.

FINAL ANSWER

$n = -19$

4MA1/1HR JUNE 2025

Question 3

3 marks | Algebraic Proof

3 Show that $2\frac{1}{4} \times 1\frac{5}{7} = 3\frac{6}{7}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2025

1 **Evaluate fraction**

$$2\frac{1}{4} = \frac{9}{4}, \qquad 1\frac{5}{7} = \frac{12}{7}$$

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{9}{4} \times \frac{12}{7}$$

$$= \frac{108}{28} = \frac{27}{7} = 3\frac{6}{7}$$

FINAL ANSWER

$3\frac{6}{7}$.

4MA1/2H JUNE 2025

Question 3

3 marks | Algebraic Proof

3 Show that $7\frac{1}{3} - 3\frac{4}{7} = 3\frac{16}{21}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2025

1 **Evaluate fraction**

$$7\frac{1}{3} = \frac{22}{3}, \qquad 5\frac{4}{7} = \frac{39}{7}$$

$$7\frac{1}{3} - 5\frac{4}{7} = \frac{22}{3} - \frac{39}{7}$$

$$= \frac{154}{21} - \frac{117}{21} = \frac{37}{21}$$

$$\frac{37}{21} = 1\frac{16}{21}$$

FINAL ANSWER

$1\frac{16}{21}$.

Question 15

4 marks | Algebraic Proof

15 $(\sqrt{3})^5 = k\sqrt{3}$ where k is an integer.

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{21}{3-\sqrt{2}}$ can be written in the form $c + \sqrt{d}$

where c and d are integers.
Show each stage of your working clearly.

(3)

(Total for Question 15 is 4 marks)

1 (a)

(a)

$$(\sqrt{3})^5 = (\sqrt{3})^4\sqrt{3} = 9\sqrt{3}$$

2 Calculate value

So $k = 9$.

3 (b)

(b)

$$\frac{21}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{21(3+\sqrt{2})}{9-2}$$

$$= 3(3+\sqrt{2}) = 9+3\sqrt{2}$$

4 Simplify surd

Since $3\sqrt{2} = \sqrt{18}$,

$$9+3\sqrt{2} = 9+\sqrt{18}$$

FINAL ANSWER

$k = 9$, and $\dfrac{21}{3-\sqrt{2}} = 9+\sqrt{18}$.

4MA1/2HR JUNE 2025

Question 16

3 marks | Algebraic Proof

16 Show that $\dfrac{4}{3\sqrt{5}+7}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show each stage of your working.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2025

1 **Simplify surd**

$$\frac{4}{3\sqrt{5}+7}\times\frac{7-3\sqrt{5}}{7-3\sqrt{5}}=\frac{4(7-3\sqrt{5})}{49-45}$$

$$=7-3\sqrt{5}$$

2 **Simplify surd**

Since $3\sqrt{5}=\sqrt{45}$,

$$7-3\sqrt{5}=7-\sqrt{45}$$

FINAL ANSWER

$7-\sqrt{45}$, so $a=7$ and $b=45$.

4MA1/1HR JUNE 2025

Question 17

3 marks | Algebraic Proof

17 Prove that, for any three numbers which are consecutive multiples of 4, the difference between the square of the largest number and the square of the smallest number is always a multiple of 64

Show clear algebraic working.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2025

1 Let the three consecutive multiples

Let the three consecutive multiples of 4 be

$$4n, \quad 4n+4, \quad 4n+8$$

2 The difference between the square

The difference between the square of the largest and the square of the smallest is

$$(4n+8)^2 - (4n)^2$$

$$= 16n^2 + 64n + 64 - 16n^2$$

$$= 64n + 64 = 64(n+1)$$

3 This is always a multiple

This is always a multiple of 64.

FINAL ANSWER

The difference is always a multiple of 64.

4MA1/2H JUNE 2025

Question 18

2 marks | Algebraic Proof

18 Use algebra to show that $0.\dot{7}0\dot{2} = \frac{26}{37}$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2025

1 Let

Let

$$x = 0.\dot{7}0\dot{2}$$

2 Calculate value

Then

$$1000x = 702.\dot{7}0\dot{2}$$

3 Subtract

Subtract:

$$999x = 702$$

$$x = \frac{702}{999} = \frac{26}{37}$$

FINAL ANSWER

$0.\dot{7}0\dot{2} = \frac{26}{37}$.

4WM2/01 MAY 2025

Question 20

3 marks | Algebraic Proof

20 a, b and c are three consecutive even numbers where $a < b < c$

Prove algebraically that $b^2 = ac + 4$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 May 2025

1 Let

Let

$$a = 2n, \qquad b = 2n + 2, \qquad c = 2n + 4$$

2 Calculate value

Then

$$ac + 4 = (2n)(2n + 4) + 4$$

$$= 4n^2 + 8n + 4$$

$$= (2n + 2)^2 = b^2$$

3 Calculate value

So $b^2 = ac + 4$.

FINAL ANSWER

So $b^2 = ac + 4$.

4MA1/1H NOVEMBER 2025

Question 5

3 marks | Algebraic Proof

5 Show that $3\frac{5}{7} + 1\frac{2}{3} = 5\frac{8}{21}$

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2025

1 Evaluate fraction

$$\frac{3}{7} + \frac{1}{3}$$

2 Use denominator 21

Use denominator 21:

$$\frac{3}{7} = \frac{9}{21}, \qquad \frac{1}{3} = \frac{7}{21}$$

$$\frac{9}{21} + \frac{7}{21} = \frac{16}{21}$$

FINAL ANSWER

$\frac{16}{21}$.

4MA1/2H NOVEMBER 2025

Question 16

2 marks | Algebraic Proof

16 Use algebra to show that $0.3\dot{7}\dot{8} = \frac{25}{66}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2025

1 Let

Let

$$x = 0.3787878\ldots$$

2 Calculate value

Then

$$10x = 3.787878\ldots$$

3 Calculate value

and

$$1000x = 378.787878\ldots$$

4 Subtract

Subtract:

$$990x = 375$$

$$x = \frac{375}{990} = \frac{25}{66}$$

FINAL ANSWER

$0.3787878\ldots = \frac{25}{66}$.

4WM1/01 NOVEMBER 2025

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.95\dot{4}\dot{5} = \frac{21}{22}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 November 2025

1 Let

Let

$$x = 0.95454545\ldots$$

2 Calculate value

Then

$$10x = 9.5454545\ldots$$

3 Calculate value

and

$$1000x = 954.5454545\ldots$$

4 Subtract

Subtract:

$$990x = 945$$

$$x = \frac{945}{990} = \frac{21}{22}$$

FINAL ANSWER

$0.954545\ldots = \frac{21}{22}$.

4MA1/1H NOVEMBER 2025

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{2\sqrt{7}+2}{\sqrt{7}-3}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2025

1 **Simplify surd**

$$\frac{2\sqrt{7}+2}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{(2\sqrt{7}+2)(\sqrt{7}+3)}{7-9}$$

$$=\frac{14+6\sqrt{7}+2\sqrt{7}+6}{-2}$$

$$=\frac{20+8\sqrt{7}}{-2}=-10-4\sqrt{7}$$

2 **Simplify surd**

Since $4\sqrt{7}=\sqrt{112}$,

$$-10-4\sqrt{7}=-10-\sqrt{112}$$

FINAL ANSWER

$-10-\sqrt{112}$, so $a=-10$ and $b=112$.

4MA1/2H JUNE 2026

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.5\dot{3}\dot{9} = \frac{89}{165}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2026

1 Solution | Step 1

Let $x = 0.5\overline{39} = 0.5393939\ldots$. Then $10x = 5.393939\ldots$ and $1000x = 539.393939\ldots$.

Shift past the non-recurring digit and then one complete two-digit recurring block.

2 Solution | Step 2

Subtract:

$$1000x - 10x = 539.393939\ldots - 5.393939\ldots = 534.$$

$$990x = 534 \quad \Rightarrow \quad x = \frac{534}{990} = \frac{89}{165}.$$

Identical infinite decimal tails cancel exactly; reduce the resulting fraction by 6.

FINAL ANSWER

$0.5\overline{39} = \frac{89}{165}$.

TOPIC

Sequences

Every past-paper question on this topic, 2020 to 2026.

Question 24

5 marks | Sequences

24 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.
The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

.....

(Total for Question 24 is 5 marks)

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

2 Find the nth term

So

$$S_{10} = 5(2a + 9d)$$

3 Find the nth term

and

$$S_5 = \frac{5}{2}(2a + 4d)$$

4 Given

Given $S_{10} = 4S_5$:

$$5(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$10a + 45d = 20a + 40d$$

$$5d = 10a$$

$$d = 2a$$

5 The 8th term is

The 8th term is 45:

$$a + 7d = 45$$

6 Substitute

Substitute $d = 2a$:

$$a + 14a = 45$$

$$15a = 45$$

$$a = 3$$

FINAL ANSWER

the first term is $\mathbf{3}$.

4MA1/2HR JANUARY 2020

Question 22

4 marks | Sequences

22 The first term of an arithmetic series S is -6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2020

1 Find the nth term

First term:

$$a = -6$$

$$S_{30} = 2865$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$2865 = \frac{30}{2}(2(-6) + 29d)$$

$$2865 = 15(-12 + 29d)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7$$

3 **The 9th term is**

The 9th term is

$$u_9 = a + 8d$$

$$u_9 = -6 + 8(7) = 50$$

✓ **FINAL ANSWER**

50

4MA1/2H JANUARY 2020

Question 25

3 marks | Sequences

25 Mario is going to save \$50 in the year 2021

He is going to continue to save, up to and including the year 2070, by increasing the amount he saves each year by \$$k$

Mario will save a total of \$33 125 from 2021 to 2070

Work out the value of k.

$k =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2020

1 Mario saves from 2021 up

Mario saves from 2021 up to and including 2070.

2 Number of years

Number of years:

$$2070 - 2021 + 1 = 50$$

3 This is an arithmetic sequence

This is an arithmetic sequence.

4 Use trigonometry

First term:

$$50$$

5 Common difference

Common difference:

$$k$$

6 Last term

Last term:

$$50 + 49k$$

7 Use the arithmetic series formula

Use the arithmetic series formula:

$$S_n = \frac{n}{2}(a + l)$$

$$33125 = \frac{50}{2}(50 + (50 + 49k))$$

$$33125 = 25(100 + 49k)$$

$$1325 = 100 + 49k$$

$$49k = 1225$$

$$k = 25$$

FINAL ANSWER

$$\boxed{k = 25}$$

4MA1/1HR MAY/NOVEMBER 2020

Question 18

4 marks | Sequences

18 In an arithmetic series, the 6th term is 39
In the same arithmetic series, the 19th term is 7.8

Work out the sum of the first 25 terms of the arithmetic series.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May/November 2020

1 Let the first term be

Let the first term be a and the common difference be d.

2 The 6th term is 39

The 6th term is 39:

$$a + 5d = 39$$

3 Find the nth term

The 19th term is 7.8:

$$a + 18d = 7.8$$

4 Subtract

Subtract:

$$13d = -31.2$$

$$d = -2.4$$

5 Find the nth term

Then

$$a + 5(-2.4) = 39$$

$$a = 51$$

6 **The sum of the first**

The sum of the first 25 terms is

$$S_{25} = \frac{25}{2}(2a + 24d)$$

$$S_{25} = \frac{25}{2}(102 - 57.6) = 555$$

✓ **FINAL ANSWER**

555.

4MA1/1H MAY/NOVEMBER 2020

Question 24

4 marks | Sequences

24 Here are the first five terms of an arithmetic sequence.

8 15 22 29 36

Work out the sum of all the terms from the 50th term to the 100th term inclusive.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May/November 2020

1 The arithmetic sequence is

The arithmetic sequence is

$$8,\ 15,\ 22,\ 29,\ 36,\ldots$$

2 Calculate value

So

$$a = 8, \quad d = 7$$

3 The th term is

The nth term is

$$u_n = 8 + 7(n-1) = 7n + 1$$

$$u_{50} = 7(50) + 1 = 351$$

$$u_{100} = 7(100) + 1 = 701$$

4 There are

There are

$$100 - 50 + 1 = 51$$

5 **Find the nth term**

terms from the 50th to the 100th inclusive.

$$\text{sum} = \frac{51}{2}(351 + 701)$$

$$= 26826$$

✓ **FINAL ANSWER**

26826

4MA1/1HR JANUARY 2021

Question 21

6 marks | Sequences

21 The nth term of an arithmetic series is u_n where $u_n > 0$ for all n
The sum to n terms of the series is S_n

Given that $u_4 = 6$ and that $S_{11} = (u_6)^2 + 18$

find the value of u_{20}

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2021

1 Let

Let $u_6 = v$.

2 Use series formula

For an arithmetic series,

$$S_{11} = \frac{11}{2}(u_1 + u_{11})$$

3 The average of and is

The average of u_1 and u_{11} is u_6, so

$$S_{11} = 11u_6 = 11v$$

4 **Given**

Given

$$S_{11} = (u_6)^2 + 18$$

$$11v = v^2 + 18$$

$$v^2 - 11v + 18 = 0$$

$$(v - 2)(v - 9) = 0$$

5 **Calculate value**

So

$$u_6 = 2 \quad \text{or} \quad u_6 = 9$$

6 **Find the nth term**

But $u_4 = 6$. If $u_6 = 2$, then the common difference is negative and later terms would not all be positive. Therefore,

$$u_6 = 9$$

$$d = \frac{u_6 - u_4}{2} = \frac{9 - 6}{2} = \frac{3}{2}$$

$$u_{20} = u_6 + 14d$$

$$u_{20} = 9 + 14\left(\frac{3}{2}\right) = 30$$

✓ **FINAL ANSWER**

$u_{20} = 30$

4MA1/1H JANUARY 2021

Question 24

4 marks | Sequences

24 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$a =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2021

1 Evaluate fraction

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{2n} = n(2a + (2n-1)d)$$

2 Given

Given

$$S_{2n} = 4S_n$$

$$n(2a + (2n-1)d) = 4 \cdot \frac{n}{2}(2a + (n-1)d)$$

3 Divide by

Divide by n:

$$2a + (2n-1)d = 2(2a + (n-1)d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$d = 2a$$

$$a = \frac{d}{2}$$

FINAL ANSWER

$$a = \frac{d}{2}$$

4MA1/1H MAY 2021

Question 2

2 marks | Sequences

2 Here are the first 4 terms of an arithmetic sequence.

85 79 73 67

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2021

1 The sequence is

The sequence is

$$85,\ 79,\ 73,\ 67$$

2 The common difference is

The common difference is -6.

$$\text{nth term} = 85 + (n-1)(-6)$$

$$= 85 - 6n + 6 = 91 - 6n$$

FINAL ANSWER

$91 - 6n$.

4MA1/1H MAY 2021

Question 23

5 marks | Sequences

23 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$N =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2021

1 **The 2nd term is**

The 2nd term is

$$a + d = 8.5$$

2 **The 5th term is**

The 5th term is

$$a + 4d = 13$$

3 **Subtract**

Subtract:

$$3d = 4.5$$

$$d = 1.5$$

$$a = 8.5 - 1.5 = 7$$

4 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$292 = \frac{N}{2}(14 + 1.5(N-1))$$

$$292 = \frac{N}{2}(1.5N + 12.5)$$

$$3N^2 + 25N - 1168 = 0$$

$$N = \frac{-25 + 121}{6} = 16$$

✓ **FINAL ANSWER**

$N = 16$

4MA1/2H NOVEMBER 2021

Question 20

5 marks | Sequences

20 Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2021

1 The first four terms are

The first four terms are

$$k, \quad \frac{3k}{4}, \quad \frac{k}{2}, \quad \frac{k}{4}$$

2 The common difference is

The common difference is

$$d = -\frac{k}{4}$$

3 The 15th term is

The 15th term is

$$u_{15} = k + 14\left(-\frac{k}{4}\right)$$

$$u_{15} = k - \frac{7k}{2} = -\frac{5k}{2}$$

4 **Given**

Given

$$-\frac{5k}{2} = 90 + 2k$$

$$-5k = 180 + 4k$$

$$k = -20$$

---

5 **Evaluate fraction**

So

$$a = -20, \quad d = 5$$

$$S_{30} = \frac{30}{2}(2(-20) + 29(5))$$

$$S_{30} = 15(105) = 1575$$

✓ **FINAL ANSWER**

1575

4MA1/1HR JANUARY 2022

Question 16

4 marks | Sequences

16 An arithmetic series has first term 1 and common difference 4

Find the sum of all terms of the series from the 41st term to the 100th term inclusive.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2022

1 Find the nth term

The first term is 1 and the common difference is 4.

2 The 41st term is

The 41st term is

$$1 + 40(4) = 161$$

3 The 100th term is

The 100th term is

$$1 + 99(4) = 397$$

4 There are

There are

$$100 - 41 + 1 = 60$$

5 Find the nth term

terms from the 41st to the 100th inclusive.

$$S = \frac{60}{2}(161 + 397)$$

$$S = 30 \times 558 = 16740$$

FINAL ANSWER

16740.

4MA1/2H JANUARY 2022

Question 26

5 marks | Sequences

26 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$d =$ .....

$m =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H January 2022

1 Calculate value

$$S_m = 39$$

$$S_{2m} = 320$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

3 Calculate value

Then

$$2S_m = m(2a + (m-1)d) = 78$$

4 Calculate value

and

$$S_{2m} = m(2a + (2m-1)d) = 320$$

5 **Subtract**

Subtract:

$$S_{2m} - 2S_m = m[(2m-1)d - (m-1)d]$$

$$320 - 78 = m(md)$$

$$242 = m^2 d$$

$$242 = 2 \times 11^2$$

---

6 **Square factor**

Since d is prime and m^2 is a square factor,

$$d = 2, \quad m^2 = 121$$

$$m = 11$$

✓ **FINAL ANSWER**

$d = 2,\ m = 11$

4MA1/1H JUNE 2022

Question 1

3 marks | Sequences

1 Here are the first five terms of an arithmetic sequence.

1 5 9 13 17

(a) Find an expression, in terms of n, for the nth term of this sequence.

.....

(2)

The nth term of another arithmetic sequence is $3n + 5$

(b) Find an expression, in terms of m, for the $(2m)$th term of this sequence.

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2022

1 The sequence is

The sequence is

$$1,\ 5,\ 9,\ 13,\ 17$$

2 Find the nth term

The common difference is 4, so the nth term is

$$4n - 3$$

3 Find the nth term

For the second sequence, the nth term is $3n + 5$.

4 The th term is

The $(2m)$th term is

$$3(2m) + 5 = 6m + 5$$

FINAL ANSWER

$4n - 3$, and $6m + 5$.

4MA1/1HR JUNE 2022

Question 2

4 marks | Sequences

2 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

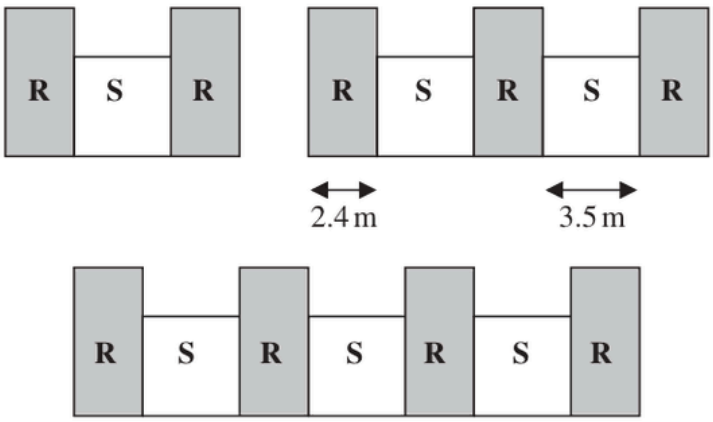


Diagram **NOT** accurately drawn

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

..... m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
Give your answer in its simplest form.

$W =$ .....

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2022

1 **Find the nth term**

For a shelving unit with 6 type R shelves, there are 5 type S shelves.

$$6(2.4) + 5(3.5) = 14.4 + 17.5 = 31.9$$

2 **Find the nth term**

For n type R shelves, there are $n-1$ type S shelves.

$$W = 2.4n + 3.5(n-1)$$

$$W = 2.4n + 3.5n - 3.5 = 5.9n - 3.5$$

FINAL ANSWER

$31.9\ \text{m}$, and $W = 5.9n - 3.5$.

4MA1/1HR JUNE 2022

Question 25

5 marks | Sequences

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2022

1 **Calculate value**

$$S_{10} = 4S_5$$

2 **Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

3 **Calculate value**

So

$$5(2a + 9d) = 4 \cdot 5(a + 2d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

4 **The 8th term is 45**

The 8th term is 45:

$$u_8 = a + 7d = 45$$

$$a + 7(2a) = 45$$

$$15a = 45$$

$$a = 3$$

✓ **FINAL ANSWER**

The first term is $\mathbf{3}$

4MA1/2HR JUNE 2023

Question 4

2 marks | Sequences

4 Here are the first four terms of an arithmetic sequence.

38 31 24 17

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2023

1 The sequence is

The sequence is

$$38,\ 31,\ 24,\ 17$$

2 The common difference is

The common difference is -7.

$$\text{nth term} = 38 + (n-1)(-7)$$

$$= 38 - 7n + 7 = 45 - 7n$$

FINAL ANSWER

$45 - 7n$.

4MA1/1H MAY 2023

Question 20

6 marks | Sequences

20 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

$X =$ .....

(Total for Question 20 is 6 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2023

1 Evaluate fraction

$$S_{80} = 470$$

$$40(2a + 79d) = 470$$

$$2a + 79d = \frac{47}{4}$$

2 The 75th term is

The 75th term is

$$a + 74d = 14.5 = \frac{29}{2}$$

3 Double it

Double it:

$$2a + 148d = 29$$

4 **Subtract**

Subtract:

$$69d = 29 - \frac{47}{4} = \frac{69}{4}$$

$$d = \frac{1}{4}$$

$$a + 74\left(\frac{1}{4}\right) = \frac{29}{2}$$

$$a = -4$$

---

5 **Evaluate fraction**

Now

$$S_X = 171$$

$$171 = \frac{X}{2}\left(2(-4) + (X-1)\frac{1}{4}\right)$$

$$171 = \frac{X(X-33)}{8}$$

$$X^2 - 33X - 1368 = 0$$

$$X = \frac{33 + 81}{2} = 57$$

✓ **FINAL ANSWER**

$X = 57$

4MA1/2HR JUNE 2023

Question 23

5 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$8p \qquad 7p - 3 \qquad 4p + 2$$

The sum of the first n terms of the sequence is -1914

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2023

1 The first three terms are

The first three terms are

$$8p, \quad 7p - 3, \quad 4p + 2$$

2 Find the nth term

For an arithmetic sequence, consecutive differences are equal:

$$(7p - 3) - 8p = (4p + 2) - (7p - 3)$$

$$-p - 3 = -3p + 5$$

$$2p = 8$$

$$p = 4$$

3 Find the nth term

So the first term is

$$8p = 32$$

4 **Find common difference**

and the common difference is

$$(7p-3)-8p=25-32=-7$$

5 **The sum of the first**

The sum of the first n terms is

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$-1914=\frac{n}{2}(64-7(n-1))$$

$$-1914=\frac{n}{2}(71-7n)$$

$$-3828=n(71-7n)$$

$$7n^2-71n-3828=0$$

$$n=\frac{71\pm 335}{14}$$

6 **Only the positive value is**

Only the positive value is possible:

$$n=29$$

✓ **FINAL ANSWER**

$n=29$

4MA1/1H NOVEMBER 2023

Question 19

6 marks | Sequences

19 Here are the first 4 terms in an arithmetic sequence.

3 7 11 15

The last term of the sequence is x
The sum of the terms of the sequence is 7260

Find the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2023

1 The sequence is

The sequence is

$$3,\ 7,\ 11,\ 15$$

2 Find the nth term

The common difference is 4, so the nth term is

$$4n - 1$$

3 Find the nth term

The last term is x, so

$$x = 4n - 1$$

$$n = \frac{x + 1}{4}$$

4 The sum is 7260

The sum is 7260:

$$\frac{n}{2}(3 + x) = 7260$$

5 **Substitute**

Substitute $n = \dfrac{x+1}{4}$:

$$\frac{x+1}{8}(x+3) = 7260$$

$$(x+1)(x+3) = 58080$$

$$x^2 + 4x - 58077 = 0$$

---

6 **Factorise**

Factorise:

$$(x-239)(x+243) = 0$$

---

7 **Find the nth term**

The last term is positive, so

$$x = 239$$

✓ **FINAL ANSWER**

$x = 239$.

4MA1/1H MAY 2024

Question 1

3 marks | Sequences

1 Here are the first four terms of an arithmetic sequence.

1 4 7 10

(a) Find an expression, in terms of n, for the nth term of this sequence.

.....

(2)

The nth term of a different arithmetic sequence is $5n + 17$

(b) Find the 12th term of this sequence.

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2024

1 The sequence is

The sequence is

$$1,\ 4,\ 7,\ 10$$

2 The common difference is

The common difference is 3.

$$\text{nth term} = 1 + 3(n-1) = 3n - 2$$

3 Find the nth term

For the other sequence,

$$5n + 17$$

4 The 12th term is

The 12th term is

$$5(12) + 17 = 77$$

FINAL ANSWER

$3n - 2$, and 77.

4MA1/1HR MAY 2024

Question 23

4 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$(4x-14) \quad , \quad (x+2) \quad , \quad (7x-9)$$

Find, as an integer, the sum of the first 40 terms of the sequence.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2024

1 **The first three terms are**

The first three terms are

$$4x - 14, \quad x + 2, \quad 7x - 9$$

2 **Equal differences give**

Equal differences give

$$(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$$

$$-3x + 16 = 6x - 11$$

$$9x = 27$$

$$x = 3$$

3 **Find the nth term**

So the sequence is

$$-2,\ 5,\ 12, \ldots$$

4 **with common difference**

with common difference 7.

$$S_{40} = \frac{40}{2}(2(-2) + 39(7))$$

$$S_{40} = 20(-4 + 273) = 5380$$

FINAL ANSWER

5380

4MA1/1H MAY 2024

Question 24

6 marks | Sequences

24 A polygon has n sides, where $n > 5$

The interior angles of the polygon form an arithmetic sequence.

The smallest angle of the polygon is 84°
The common difference of the sequence is 4°

Work out the sum of the interior angles of the polygon.
Show clear algebraic working.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2024

1 The interior angles form an

The interior angles form an arithmetic sequence with first term 84 and common difference 4.

2 Use trigonometry

So the sum of the angles is

$$\frac{n}{2}(2(84) + (n-1)4)$$

$$= \frac{n}{2}(168 + 4n - 4)$$

$$= 2n^2 + 82n$$

3 The sum of the interior

The sum of the interior angles of an n-sided polygon is

$$180(n-2)$$

4 **Calculate value**

So

$$2n^2 + 82n = 180(n - 2)$$

$$2n^2 - 98n + 360 = 0$$

$$n^2 - 49n + 180 = 0$$

$$(n - 45)(n - 4) = 0$$

---

5 **Solve inequality**

Since $n > 5$,

$$n = 45$$

---

6 **Use trigonometry**

Therefore the sum of the interior angles is

$$180(45 - 2) = 7740$$

✓ **FINAL ANSWER**

7740 degrees

4MA1/1H NOVEMBER 2024

Question 19

5 marks | Sequences

19 An arithmetic series has first term a and common difference d

The sum of the first 30 terms of the arithmetic series is 4395

The sum of the 10th term and the 20th term is 284

Work out the sum of the first 45 terms of the arithmetic series.
Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2024

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_{30} = 4395$$

$$\frac{30}{2}(2a + 29d) = 4395$$

$$2a + 29d = 293$$

2 The 8th term plus the

The 8th term plus the 20th term is 284:

$$(a + 7d) + (a + 19d) = 284$$

$$2a + 26d = 284$$

3 Subtract the equations

Subtract the equations:

$$3d = 9$$

$$d = 3$$

4 **Find the nth term**

Then

$$2a + 26(3) = 284$$

$$2a = 206$$

$$a = 103$$

---

5 **Find the nth term**

Now find S_{45}:

$$S_{45} = \frac{45}{2}(2a + 44d)$$

$$S_{45} = \frac{45}{2}(206 + 132) = 7605$$

✓ **FINAL ANSWER**

7605.

4WM2/01 MAY 2025

Question 5

3 marks | Sequences

5 Here are the first five terms of a number sequence.

11 15 19 23 27

(a) Find an expression in terms of n for the nth term of this sequence.

.....

(2)

The nth term of another number sequence is $5n + 4$

Salim says no term in this sequence is a prime number.

(b) Find a term in this sequence that shows Salim is wrong.

.....

(1)

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM2/01 May 2025

1 Find the nth term

The sequence increases by 4, so

$$u_n = 4n + 7$$

2 Find the nth term

For the sequence $5n + 4$, take $n = 3$:

$$5(3) + 4 = 19$$

3 Check prime condition

and 19 is prime.

FINAL ANSWER

$4n + 7$; a suitable counterexample is 19.

4MA1/1H JUNE 2025

Question 24

5 marks | Sequences

24 An arithmetic series has 30 terms.

The first term is a
The common difference is d

The 20th term is 123
The sum of the 30 terms is 2880

Work out the value of a and the value of d
Show clear algebraic working.

$a =$ .....

$d =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2025

1 Evaluate fraction

$$u_{20} = a + 19d = 123$$

$$S_{30} = \frac{30}{2}(2a + 29d) = 2880$$

$$2a + 29d = 192$$

2 Double the first equation

Double the first equation:

$$2a + 38d = 246$$

3 **Subtract**

Subtract:

$$9d = 54$$

$$d = 6$$

$$a + 19(6) = 123$$

$$a = 9$$

✓ **FINAL ANSWER**

$a = 9,\ d = 6$

4MA1/1HR JUNE 2025

Question 24

6 marks | Sequences

24 The first 3 terms of an arithmetic series are

$$(2x + 5) \qquad (3y - 4) \qquad (4x - 2)$$

where x and y are constants.

The sum of the first 9 terms of the series is 216

Find the value of x and the value of y
Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2025

1 The first 3 terms are

The first 3 terms are

$$2x + 5, \quad 3y - 4, \quad 4x - 2$$

2 Equal differences give

Equal differences give

$$(3y - 4) - (2x + 5) = (4x - 2) - (3y - 4)$$

$$6y - 6x = 11$$

3 Use series formula

For an arithmetic series,

$$S_9 = 9(a + 4d)$$

4 **Here**

Here

$$a = 2x + 5$$

$$d = (3y - 4) - (2x + 5) = 3y - 2x - 9$$

5 **Given**

Given $S_9 = 216$:

$$9[(2x + 5) + 4(3y - 2x - 9)] = 216$$

$$12y - 6x = 55$$

6 **Solve**

Now solve

$$6y - 6x = 11$$

$$12y - 6x = 55$$

7 **Subtracting gives**

Subtracting gives

$$6y = 44$$

$$y = \frac{22}{3}$$

8 **Evaluate fraction**

Then

$$6\left(\frac{22}{3}\right) - 6x = 11$$

$$x = \frac{11}{2}$$

✓ **FINAL ANSWER**

$$x = \frac{11}{2},\ y = \frac{22}{3}$$

4WM2/01 MAY 2025

Question 24

4 marks | Sequences

24 The 21st term of an arithmetic series is 109
The sum of the first 52 terms of the series is 4381

Work out the 5th term of the series.
Show clear algebraic working.

.....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM2/01 May 2025

1 Use series formula

For an arithmetic series,

$$a + 20d = 109$$

2 Evaluate fraction

and

$$\frac{52}{2}(2a + 51d) = 4381$$

$$26(2a + 51d) = 4381$$

3 Solving gives

Solving gives

$$d = -4.5, \qquad a = 199$$

4 The fifth term is

The fifth term is

$$a + 4d = 199 + 4(-4.5) = 181$$

✓ FINAL ANSWER

181.

4WM2/01R MAY 2025

Question 24

6 marks | Sequences

24 An arithmetic series P has first term a and common difference d

The sum of the first 40 terms of P is 2070

An arithmetic series Q has first term $2a$ and common difference $3d$

The 25th term of Q is 186

Work out the sum of the first 40 terms of Q
Show clear algebraic working.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4WM2/01R May 2025

1 Required

Series P: first term *a*, c.d. *d*; *S*₄₀(*P*) = 2070. Series Q: first term 2*a*, c.d. 3*d*; 25th term of Q = 186. Find *S*₄₀(*Q*).

2 Form equation from S 40

Form equation from *S*₄₀(*P*) = 2070:

$$S_40 = frac402\[2a + 39d\] = 20(2a + 39d) = 2070implies2a + 39d = 103.5tag1$$

3 Form equation from 25th term

Form equation from 25th term of Q = 186:

*T*₂₅(*Q*) = 2*a* + 24(3*d*) = 2*a* + 72*d* = 186

4 Solve simultaneous equations

Solve simultaneously — subtract (1) from (2):

33*d* = 82.5 ⟹ *d* = 2.5

5 Substitute d 2 5 (1)

Substitute *d* = 2.5 into (1):

2*a* + 39(2.5) = 103.5 ⟹ 2*a* = 6 ⟹ *a* = 3

6 Compute S 40 (Q

Compute *S*₄₀(*Q*), with first term = 2*a* = 6 and c.d. = 3*d* = 7.5:

$$S_40(Q) = frac402bigl\[2(6) + 39(7.5)bigr\] = 20bigl\[12 + 292.5bigr\] = 20times304.5 = 6090$$

FINAL ANSWER

*S*₄₀(*Q*) = 6090

4MA1/2H NOVEMBER 2025

Question 25

5 marks | Sequences

25 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2025

1 Calculate value

$$a = 10$$

$$u_{20} = 86$$

$$10 + 19d = 86$$

$$d = 4$$

2 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$5194 = \frac{N}{2}(20 + 4(N-1))$$

$$5194 = \frac{N}{2}(4N + 16)$$

$$5194 = 2N(N+4)$$

$$N^2 + 4N - 2597 = 0$$

$$N = \frac{-4 + 102}{2} = 49$$

FINAL ANSWER

$N = 49$

4WM2/01 NOVEMBER 2025

Question 26

5 marks | Sequences

26 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4WM2/01 November 2025

1 Find common difference

Step 1: Find the common difference *d*. The *n*th term is given by *u*_{*n*} = *a* + (*n*−1)*d*.

*u*₂₀ = 10 + 19*d* = 86

19*d* = 76

$$d = frac7619 = 4$$

2 Use sum formula

Step 2: Use the sum formula to form an equation in terms of *N*.

$$S_N = fracN2left\[2a + (N-1)dright\]$$

Substitute the known values *S*_{*N*} = 5194, *a* = 10, and *d* = 4:

$$5194 = fracN2left\[2(10) + (N-1)(4)right\]$$

3 Solve quadratic equation

Step 3: Simplify and solve the quadratic equation. Multiply by 2 to clear the fraction:

10388 = *N*

$$20 + 4 * N * -4$$

10388 = *N*(4*N*+16)

4*N*² + 16*N* − 10388 = 0

4 Divide the entire equation by

Divide the entire equation by 4 to make it simpler:

*N*² + 4*N* − 2597 = 0

5 Use quadratic formula

Step 4: Use the quadratic formula to find *N*.

$$N = frac-bpmsqrtb^2 - 4ac2a$$

$$N = frac-4pmsqrt16 - 4(1)(-2597)2$$

$$N = frac-4pmsqrt16 + 103882$$

$$N = frac-4pmsqrt104042$$

$$N = frac-4pm1022$$

6 Evaluate fraction

Since *N* must be a positive integer, we discard the negative root:

$$N = frac-4 + 1022 = frac982 = 49$$

✓ FINAL ANSWER

N = 49 Always double-check that your *N* (number of terms) evaluates to a positive whole integer. If it ends up as a decimal or fraction, recheck your arithmetic progression formulas.

4MA1/2H JUNE 2026

Question 2

5 marks | Sequences

2 The nth term of an arithmetic sequence is given by $8n - 5$

(a) Work out the ratio of the 40th term of the sequence to the 60th term of the sequence.
Give your ratio in its simplest form.

.....

(3)

Here are the first five terms of a different arithmetic sequence.

3 8 13 18 23

(b) Find an expression for the nth term of this sequence.

.....

(2)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

$$u_{40} = 8(40) - 5 = 315, \qquad u_{60} = 8(60) - 5 = 475.$$

Substitute the two term numbers into the given rule.

2 **Part (a) | Step 2**

$$315 : 475 = (315 \div 5) : (475 \div 5) = 63 : 95.$$

Divide both entries by their highest common factor, 5.

3 **Part (b) | Step 1**

The common difference is $8 - 3 = 5$, so the rule has the form $5n + c$.
A constant difference gives a linear nth term.

4 **Part (b) | Step 2**

At $n = 1$, $5 + c = 3$, giving $c = -2$. Thus $u_n = 5n - 2$.
Use the first term to find the constant.

✓ **FINAL ANSWER**

(a) $63 : 95$ (b) $5n - 2$

4MA1/2HR JUNE 2026

Question 2

2 marks | Sequences

2 Here are the first five terms of an arithmetic sequence.

14 8 2 –4 –10

Find an expression, in terms of n, for the nth term of this sequence.

.....

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2026

1 **Solution | Step 1**

The common difference is $8 - 14 = -6$.

Each successive term decreases by 6.

2 **Solution | Step 2**

$$u_n = 14 + (n-1)(-6) = 20 - 6n.$$

Use first term plus (n-1) common differences.

✓ **FINAL ANSWER**

$u_n = 20 - 6n.$

Question 25

6 marks | Sequences

25 Here are the first four terms of an arithmetic series.

$$(k + 2) \qquad (k + 5) \qquad (k + 8) \qquad (k + 11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first n terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

(Total for Question 25 is 6 marks)

1 First term and difference

$a = k + 2,\ d = 3.$

2 Use the sum formula

$$\frac{2n}{2}\left[2a + (2n-1)3\right] = 4 \times \frac{n}{2}\left[2a + (n-1)3\right]$$

3 Simplify (divide by n)

$$2a + 6n - 3 = 4a + 6n - 6 \Rightarrow 2a = 3 \Rightarrow a = 1.5,\ k = -\tfrac{1}{2}$$

4 100th term

$$a + 99d = 1.5 + 99(3) = 298.5$$

✓ **FINAL ANSWER**

298.5

4MA1/2H JUNE 2026

Question 25

5 marks | Sequences

25 An arithmetic series has first term 3 and common difference 8

The nth term of the series is U_n
The sum of the first n terms of the series is S_n

Given that $S_{5k} - S_{3k} = 25\,560$

find the value of U_{4k}
Show clear algebraic working.

.....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2026

1 Solution | Step 1

With first term 3 and difference 8,

$$S_n = \frac{n}{2}[2(3) + (n-1)8] = \frac{n}{2}(8n - 2) = 4n^2 - n.$$

Use the arithmetic-series sum formula with the given first term and common difference.

2 Solution | Step 2

$$S_{5k} = 100k^2 - 5k, \qquad S_{3k} = 36k^2 - 3k.$$

$$(100k^2 - 5k) - (36k^2 - 3k) = 25560 \quad \Rightarrow \quad 64k^2 - 2k - 25560 = 0.$$

Substitute 5k and 3k as the term counts, then subtract the complete expressions.

3 Solution | Step 3

$$32k^2 - k - 12780 = 0 \quad \Rightarrow \quad (k - 20)(32k + 639) = 0.$$

The roots are $k = 20$ and $k = -639/32$. Reject the negative root because the numbers of terms must be positive.
Factor the quadratic and apply the contextual restriction.

4 Solution | Step 4

$$U_n = 3 + (n-1)8 = 8n - 5, \qquad U_{4k} = U_{80} = 8(80) - 5 = 635.$$

Use the nth-term formula with 4k=80, not with k=20.

FINAL ANSWER

$U_{4k} = 635$

4MA1/2HR JUNE 2026

Question 26

6 marks | Sequences

26 Here are the first four terms of an arithmetic series.

$$(k+2) \qquad (k+5) \qquad (k+8) \qquad (k+11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first n terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

.....

(Total for Question 26 is 6 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2026

1 Solution | Step 1

The first term is $a = k + 2$ and the common difference is $d = 3$.

Successive listed terms differ by 3.

2 Solution | Step 2

$$S_n = \frac{n}{2}\{2(k+2) + 3(n-1)\} = \frac{n}{2}(2k + 1 + 3n).$$

Apply the arithmetic-series sum formula.

3 Solution | Step 3

$$S_{2n} = n\{2(k+2) + 3(2n-1)\} = n(2k + 1 + 6n).$$

Use 2n in place of n throughout the sum formula.

4 Solution | Step 4

Using $S_{2n} = 4S_n$ and $n > 0$,

$$2k + 1 + 6n = 2(2k + 1 + 3n) = 4k + 2 + 6n.$$

Divide the equation by n, then expand the right-hand side.

5 Solution | Step 5

$$2k + 1 = 4k + 2 \quad \Rightarrow \quad k = -\frac{1}{2}.$$

The n-terms cancel, leaving a linear equation for k.

6 Solution | Step 6

$$u_{100} = k + 2 + 99(3) = -\frac{1}{2} + 2 + 297 = 298.5.$$

The 100th term is the first term plus 99 common differences.

FINAL ANSWER

The 100th term is $\mathbf{298.5}$.

TOPIC

Direct & Inverse Proportion

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 16

4 marks | Direct & Inverse Proportion

16 M is directly proportional to h^3
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

$h =$ .....

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 Specimen

1 **Find inverse function**

Since M is directly proportional to h^3,

$$M = kh^3$$

2 **Use when**

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$4 = 0.125k$$

$$k = 32$$

---

3 **Use**

Now use $M = 500$:

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

✓ **FINAL ANSWER**

$h = 2.5$.

4MA1/1H MAY/NOVEMBER 2020

Question 14

3 marks | Direct & Inverse Proportion

14 F is inversely proportional to the square of v.

Given that $F = 6.5$ when $v = 4$

find a formula for F in terms of v.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May/November 2020

1 Find inverse function

F is inversely proportional to v^2, so

$$F = \frac{k}{v^2}$$

2 Use when

Use $F = 6.5$ when $v = 4$:

$$6.5 = \frac{k}{4^2}$$

$$k = 6.5 \times 16 = 104$$

3 Find inverse function

Therefore

$$F = \frac{104}{v^2}$$

FINAL ANSWER

$F = \frac{104}{v^2}$.

Question 16

6 marks | Direct & Inverse Proportion

16 A is inversely proportional to the square of r

$A = 5$ when $r = 0.3$

(a) Find a formula for A in terms of r

.....

(3)

(b) Find the value of A when $r = 7.5A$

$A =$ .....

(3)

(Total for Question 16 is 6 marks)

1 **Find inverse function**

A is inversely proportional to r^2, so

$$A = \frac{k}{r^2}$$

2 **Use when**

Use $A = 5$ when $r = 0.3$:

$$5 = \frac{k}{0.3^2}$$

$$k = 5 \times 0.09 = 0.45$$

3 **Find inverse function**

So

$$A = \frac{0.45}{r^2}$$

4 **Rearrange formula**

For part (b), $r = 7.5A$. Substitute into the formula:

$$A = \frac{0.45}{(7.5A)^2}$$

$$A = \frac{0.45}{56.25A^2}$$

$$56.25A^3 = 0.45$$

$$A^3 = 0.008$$

$$A = 0.2$$

✓ **FINAL ANSWER**

(a) $A = \frac{0.45}{r^2}$, (b) $A = 0.2$.

4MA1/1H JANUARY 2022

Question 15

3 marks | Direct & Inverse Proportion

15 A is inversely proportional to C^2

$A = 40$ when $C = 1.5$

Calculate the value of C when $A = 1000$

$C =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2022

1 Find inverse function

A is inversely proportional to C^2, so

$$A = \frac{k}{C^2}$$

2 Use when

Use $A = 40$ when $C = 1.5$:

$$40 = \frac{k}{1.5^2}$$

$$k = 40 \times 1.5^2 = 90$$

3 Find inverse function

So

$$A = \frac{90}{C^2}$$

4 Find inverse function

When $A = 1000$,

$$1000 = \frac{90}{C^2}$$

$$C^2 = 0.09$$

$$C = 0.3$$

FINAL ANSWER

$C = 0.3$.

4MA1/2HR JANUARY 2022

Question 20

4 marks | Direct & Inverse Proportion

20 y is inversely proportional to $\sqrt{x}$
x is directly proportional to T^3

Given that $y = 8$ when $T = 25$

find the exact value of T when $y = 27$

$T =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2022

1 Find inverse function

y is inversely proportional to $\sqrt{x}$, so

$$y \propto \frac{1}{\sqrt{x}}$$

2 Find inverse function

x is directly proportional to T^3, so $\sqrt{x} \propto T^{3/2}$.

3 Find inverse function

Therefore

$$y \propto \frac{1}{T^{3/2}}$$

4 Find inverse function

So

$$yT^{3/2} = k$$

5 Use when

Use $y = 8$ when $T = 25$:

$$k = 8 \times 25^{3/2} = 8 \times 125 = 1000$$

6 **Find inverse function**

When $y = 27$,

$$27T^{3/2} = 1000$$

$$T^{3/2} = \frac{1000}{27} = \left(\frac{10}{3}\right)^3$$

---

7 **Raise both sides to the**

Raise both sides to the power $\frac{2}{3}$:

$$T = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

✓ **FINAL ANSWER**

$T = \frac{100}{9}$.

4MA1/2H JUNE 2022

Question 17

4 marks | Direct & Inverse Proportion

17 M varies directly as the cube of h
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2022

1 Find inverse function

M varies directly as h^3, so

$$M = kh^3$$

2 Use when

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$k = \frac{4}{0.125} = 32$$

3 Find inverse function

So

$$M = 32h^3$$

4 Find inverse function

When $M = 500$,

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

FINAL ANSWER

$h = 2.5$.

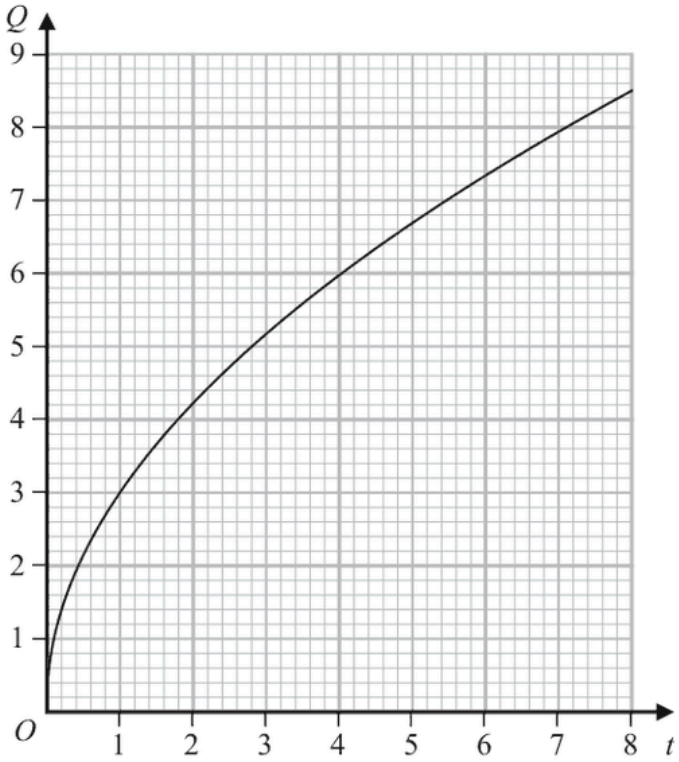
4MA1/2H JUNE 2023

Question 16

5 marks | Direct & Inverse Proportion

16 Q is directly proportional to $\sqrt{t}$

The graph shows the relationship between Q and t for $0 < t < 8$



(a) Find a formula for Q in terms of t

.....

(3)

Q is increased by 20%

(b) Find the percentage increase in t

.....%

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2023

1 **Find inverse function**

$$Q \propto \sqrt{t}$$

2 **Find inverse function**

So

$$Q = k\sqrt{t}$$

3 **Read the graph**

From the graph, when $t = 4$, $Q = 6$.

$$6 = k\sqrt{4}$$

$$6 = 2k$$

$$k = 3$$

4 **Rearrange formula**

So the formula is

$$Q = 3\sqrt{t}$$

5 **Find inverse function**

For part (b), Q is increased by 20%, so Q is multiplied by 1.2.

6 **Find inverse function**

Since $Q \propto \sqrt{t}$,

$$t \propto Q^2$$

7 **Find inverse function**

So t is multiplied by

$$1.2^2 = 1.44$$

8 **This is a increase**

This is a 44% increase.

✓ **FINAL ANSWER**

(a) $Q = 3\sqrt{t}$, (b) 44%

4MA1/1HR MAY 2023

Question 17

4 marks | Direct & Inverse Proportion

17 F is inversely proportional to the square of r

$F = 36$ when $r = 4$

(a) Find a formula for F in terms of r

.....

(3)

(b) Work out the value of F when $r = 48$

.....

(1)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May 2023

1 Find inverse function

F is inversely proportional to r^2, so

$$F = \frac{k}{r^2}$$

2 Use when

Use $F = 36$ when $r = 4$:

$$36 = \frac{k}{4^2}$$

$$k = 36 \times 16 = 576$$

3 Find inverse function

So

$$F = \frac{576}{r^2}$$

4 Find inverse function

When $r = 48$,

$$F = \frac{576}{48^2} = \frac{576}{2304} = 0.25$$

FINAL ANSWER

$F = \frac{576}{r^2}$, and $F = 0.25$ when $r = 48$.

4MA1/2H JUNE 2024

Question 17

3 marks | Direct & Inverse Proportion

17 Q is directly proportional to the square root of d

$Q = 4.5$ when $d = 324$

Find a formula for Q in terms of d

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2024

1 Find inverse function

Q is directly proportional to $\sqrt{d}$, so

$$Q = k\sqrt{d}$$

2 Use when

Use $Q = 4.5$ when $d = 324$:

$$4.5 = k\sqrt{324}$$

$$4.5 = 18k$$

$$k = 0.25$$

3 Find inverse function

Therefore

$$Q = \frac{1}{4}\sqrt{d}$$

FINAL ANSWER

$Q = \frac{1}{4}\sqrt{d}$.

4MA1/1H NOVEMBER 2024

Question 14

3 marks | Direct & Inverse Proportion

14 B is inversely proportional to the square of d

$B = 0.25$ when $d = 12$

Find a formula for B in terms of d

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2024

1 Find inverse function

B is inversely proportional to d^2, so

$$B = \frac{k}{d^2}$$

2 Use when

Use $B = 0.25$ when $d = 12$:

$$0.25 = \frac{k}{12^2}$$

$$k = 0.25 \times 144 = 36$$

3 Find inverse function

Therefore

$$B = \frac{36}{d^2}$$

FINAL ANSWER

$B = \frac{36}{d^2}$.

4MA1/1H JUNE 2025

Question 18

4 marks | Direct & Inverse Proportion

18 y is inversely proportional to x^3

$y = 4$ when $x = 3$

Work out the value of x when $y = 864$

$x =$ .....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2025

1 Find inverse function

y is inversely proportional to x^3, so

$$y = \frac{k}{x^3}$$

2 Use when

Use $y = 4$ when $x = 3$:

$$4 = \frac{k}{3^3}$$

$$k = 4 \times 27 = 108$$

3 Find inverse function

So

$$y = \frac{108}{x^3}$$

4 **Find inverse function**

When $y = 864$,

$$864 = \frac{108}{x^3}$$

$$x^3 = \frac{108}{864} = \frac{1}{8}$$

$$x = \frac{1}{2}$$

FINAL ANSWER

$x = \frac{1}{2}$.

4MA1/1HR JUNE 2025

Question 18

5 marks | Direct & Inverse Proportion

18 F is inversely proportional to the cube of r

$F = 6$ when $r = 2$

(a) Find a formula for F in terms of r

.....

(3)

(b) Find the value of r when $F = 3072$

$r =$ .....

(2)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2025

1 Find inverse function

F is inversely proportional to r^3, so

$$F = \frac{k}{r^3}$$

2 Use when

Use $F = 6$ when $r = 2$:

$$6 = \frac{k}{2^3}$$

$$k = 48$$

3 Find inverse function

So

$$F = \frac{48}{r^3}$$

4 **Find inverse function**

When $F = 3072$,

$$3072 = \frac{48}{r^3}$$

$$r^3 = \frac{48}{3072} = \frac{1}{64}$$

$$r = \frac{1}{4}$$

✓ **FINAL ANSWER**

$F = \frac{48}{r^3}$, and $r = \frac{1}{4}$.

4WM2/01 MAY 2025

Question 18

3 marks | Direct & Inverse Proportion

18 W is inversely proportional to y^2

$W = 50$ when $y = 4$

Find a formula for W in terms of y

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 May 2025

1 Find inverse function

Since W is inversely proportional to y^2,

$$W = \frac{k}{y^2}$$

2 Evaluate fraction

Using $W = 50$ when $y = 4$:

$$50 = \frac{k}{16}$$

$$k = 800$$

FINAL ANSWER

$$W = \frac{800}{y^2}.$$

4WM2/01 NOVEMBER 2025

Question 17

3 marks | Direct & Inverse Proportion

17 T is inversely proportional to $\sqrt{m}$

$T = 15$ when $m = 36$

Find a formula for T in terms of m

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4WM2/01 November 2025

1 Solve equation

$$T = fracksqrtm$$

Step 2: Substitute the given values to find constant *k*.

$$15 = fracksqrt36$$

$$15 = frack6$$

k = 15 × 6 = 90

2 Rearrange formula

$$T = frac90sqrtm$$

FINAL ANSWER

$$T = frac90sqrtm$$

4MA1/2H NOVEMBER 2025

Question 17

3 marks | Direct & Inverse Proportion

17 T is inversely proportional to $\sqrt{m}$

$T = 15$ when $m = 36$

Find a formula for T in terms of m

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2025

1 Find inverse function

T is inversely proportional to m, so

$$T = \frac{k}{m}$$

2 Use when

Use $T = 15$ when $m = 36$:

$$15 = \frac{k}{36}$$

$$k = 540$$

3 Find inverse function

Therefore

$$T = \frac{540}{m}$$

FINAL ANSWER

$T = \frac{540}{m}$.

4WM2/01R JUNE 2026

Question 16

3 marks | Direct & Inverse Proportion

16 m is directly proportional to r^2

$m = 180$ when $r = 6$

Find a formula for m in terms of r

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM2/01R June 2026

1 Write the proportion

$$m = kr^2$$

2 Find k

$$180 = k(6)^2 \Rightarrow k = 5$$

3 Formula

$$m = 5r^2$$

FINAL ANSWER

$m = 5r^2$

4MA1/1H JUNE 2026

Question 19

4 marks | Direct & Inverse Proportion

19 M is directly proportional to p^2

$M = 2$ when $p = 5$

(a) Find a formula for M in terms of p

.....

(3)

(b) Work out the value of M when $p = 15$

$M =$ .....

(1)

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$M = kp^2, \qquad 2 = k(5^2) \implies k = \frac{2}{25}.$$

Introduce a constant of proportionality and use the given pair.

2 **Part (a) | Step 2**

$$M = \frac{2p^2}{25}.$$

Substitute the constant into the general relationship.

3 **Part (b) | Step 1**

$$M = \frac{2(15)^2}{25} = 18.$$

Use the formula from (a).

✓ **FINAL ANSWER**

(a) $M = 2p^2/25$ (b) $M = 18$

4MA1/1HR JUNE 2026

Question 20

5 marks | Direct & Inverse Proportion

20 y is directly proportional to t^2
$y = 40$ when $t = 4$

t is directly proportional to the cube root of x
$t = 12$ when $x = 8$

Work out the value of y when $x = 27$

$y =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2026

1 Solution | Step 1

$$y = kt^2, \qquad 40 = k(4^2) \implies k = \frac{5}{2}.$$

Direct proportionality to t squared means a constant times t squared.

2 Solution | Step 2

$$t = c\sqrt[3]{x}, \qquad 12 = c\sqrt[3]{8} = 2c \implies c = 6.$$

Find the second proportionality constant using the given pair of values.

3 Solution | Step 3

$$x = 27 \implies t = 6\sqrt[3]{27} = 18.$$

Use the cube-root relationship first.

4 Solution | Step 4

$$y = \frac{5}{2}(18)^2 = 810.$$

Substitute the new value of t into the square relationship.

FINAL ANSWER

$y = 810$

TOPIC

Linear Graphs

Every past-paper question on this topic, 2020 to 2026.

Question 9

7 marks | Linear Graphs

9 (a) Simplify $3a^4b^5 \times 4a^7b^2$

.....

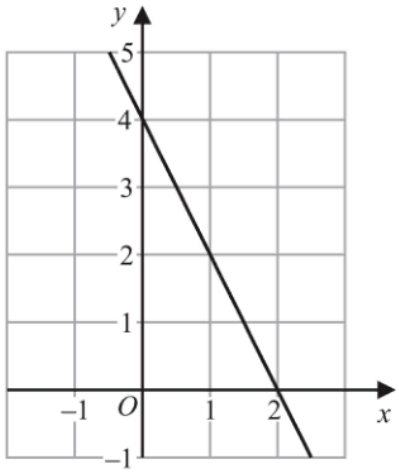
(2)

(b) Factorise fully $14x^2y^4 + 21x^3y^2$

.....

(2)

The diagram shows a straight line drawn on a grid.



(c) Find an equation of the line.

.....

(2)

A different straight line has equation $y = 3x - 5$

(d) Write down the coordinates of the point at which the line crosses the y-axis.

(....., ...)

(1)

(Total for Question 9 is 7 marks)

1 **Simplify the product**

$$3a^4b^5 \times 4a^7b^2 = 12a^{11}b^7$$

2 **Factorise fully**

$$14x^2y^4 + 21x^3y^2 = 7x^2y^2(2y^2 + 3x)$$

3 **Find the gradient**

The line passes through $(0, 4)$ and $(2, 0)$.

$$m = \frac{0 - 4}{2 - 0} = -2$$

4 **Write the equation**

The y-intercept is 4, so

$$y = -2x + 4$$

5 **Read the y-axis crossing**

For $y = 3x - 5$, put $x = 0$:

$$y = -5$$

So the crossing point is $(0, -5)$.

✓ **FINAL ANSWER**

$12a^{11}b^7$, $7x^2y^2(2y^2 + 3x)$, $y = -2x + 4$, and $(0, -5)$.

Question 23

6 marks | Linear Graphs

23 $ABCD$ is a kite.

$AB = AD$ and $CB = CD$

The point B has coordinates $(k, 1)$ where k is a negative constant.
The point D has coordinates $(8, 7)$

The straight line **L** passes through the points B and D and has a gradient of $\frac{3}{5}$

Find an equation of AC
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show your working clearly.

.....

(Total for Question 23 is 6 marks)

1 Find k from the gradient

The gradient of BD is

$$\frac{7-1}{8-k}=\frac{3}{5}$$

$$\frac{6}{8-k}=\frac{3}{5}$$

$$30=24-3k$$

$$k=-2$$

2 Find the midpoint of BD

So $B=(-2,1)$ and $D=(8,7)$.

$$\text{midpoint of } BD=\left(\frac{-2+8}{2},\frac{1+7}{2}\right)=(3,4)$$

3 Use the perpendicular gradient

In this kite, AC is the perpendicular bisector of BD.

The gradient of BD is $\frac{3}{5}$, so the gradient of AC is

$$-\frac{5}{3}$$

4 Form the equation

Using point $(3,4)$:

$$y-4=-\frac{5}{3}(x-3)$$

$$3y-12=-5x+15$$

$$5x+3y=27$$

FINAL ANSWER

$5x+3y=27$.

4MA1/1H JANUARY 2020

Question 1

4 marks | Linear Graphs

1 The point A has coordinates $(5, -4)$
The point B has coordinates $(13, 1)$

(a) Work out the coordinates of the midpoint of AB.

(..... , ...)

(2)

Line **L** has equation $y = 2 - 3x$

(b) Write down the gradient of line **L**.

.....

(1)

Line **L** has equation $y = 2 - 3x$

(c) Does the point with coordinates $(100, -302)$ lie on line **L**?
You must give a reason for your answer.

.....

(1)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2020

1 **Find the midpoint**

(a) The midpoint of $A(5, -4)$ and $B(13, 1)$ is

$$\left(\frac{5+13}{2}, \frac{-4+1}{2}\right)$$

$$= (9, -1.5)$$

2 **(b)**

(b)

$$y = 2 - 3x$$

3 **The gradient is**

The gradient is -3.

4 **Calculate value**

(c) For $x = 100$,

$$y = 2 - 3(100) = -298$$

5 **Calculate value**

The point has $y = -302$, not -298, so it does not lie on the line.

FINAL ANSWER

$(9, -1.5)$, gradient -3, and no.

4MA1/2H JANUARY 2020

Question 19

5 marks | Linear Graphs

19 (a) Write down an equation of a line that is parallel to the line with equation $y = 7 - 4x$

.....

(1)

The line **L** passes through the points with coordinates (–3, 1) and (2, –2)

(b) Find an equation of the line that is perpendicular to **L** and passes through the point with coordinates (–6, 4)
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

.....

(4)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2020

1 **(a)**

(a)

$$y = 7 - 4x$$

2 **A parallel line has the**

A parallel line has the same gradient, -4. One possible answer is

$$y = -4x$$

3 **(b) The gradient of line**

(b) The gradient of line L through $(-3, 1)$ and $(2, -2)$ is

$$\frac{-2-1}{2-(-3)} = -\frac{3}{5}$$

4 **A perpendicular line has gradient**

A perpendicular line has gradient $\frac{5}{3}$.

5 **Evaluate fraction**

Through $(-6, 4)$:

$$y - 4 = \frac{5}{3}(x + 6)$$

6 **Multiply by 3**

Multiply by 3:

$$3y - 12 = 5x + 30$$

$$5x - 3y + 42 = 0$$

✓ **FINAL ANSWER**

For example $y = -4x$, and $5x - 3y + 42 = 0$.

4MA1/1H JANUARY 2020

Question 22

6 marks | Linear Graphs

22 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.

$j =$ .....

$k =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2020

1 Find the midpoint

Since M is the midpoint of JK,

$$M = \left(\frac{j+6}{2}, \frac{15+k}{2}\right)$$

2 Find the gradient

The gradient of HM is 2, and $H = (-4, 1)$.

$$\frac{\frac{15+k}{2} - 1}{\frac{j+6}{2} + 4} = 2$$

$$\frac{k+13}{j+14} = 2$$

$$k = 2j + 15$$

3 **Simplify surd**

Also,

$$JK = \sqrt{80}$$

4 **Calculate value**

So

$$(6 - j)^2 + (k - 15)^2 = 80$$

5 **Substitute**

Substitute $k = 2j + 15$:

$$(6 - j)^2 + (2j)^2 = 80$$

$$j^2 - 12j + 36 + 4j^2 = 80$$

$$5j^2 - 12j - 44 = 0$$

$$(5j + 10)(j - 4.4) = 0$$

6 **Solve inequality**

Since $j < 0$,

$$j = -2$$

7 **Calculate value**

Then

$$k = 2(-2) + 15 = 11$$

✓ **FINAL ANSWER**

$j = -2,\ k = 11$

4MA1/2H JANUARY 2020

Question 22

5 marks | Linear Graphs

22 The line with equation $y = x + 2$ intersects the curve with equation $x^2 + y^2 - 2y = 24$ at the points A and B.

Find the coordinates of A and B.
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2020

1 **The line is**

The line is

$$y = x + 2$$

2 **Substitute this into the circle**

Substitute this into the circle equation:

$$x^2 + y^2 - 2y = 24$$

$$x^2 + (x+2)^2 - 2(x+2) = 24$$

$$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

3 **Calculate value**

So

$$x = -4 \quad \text{or} \quad x = 3$$

---

4 **Calculate value**

Using $y = x + 2$:

$$x = -4 \Rightarrow y = -2$$

$$x = 3 \Rightarrow y = 5$$

✓ **FINAL ANSWER**

$(-4, -2)$ and $(3, 5)$

4MA1/2HR JANUARY 2020

Question 24

5 marks | Linear Graphs

24 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the x-axis at the point A.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR January 2020

1 Evaluate fraction

For L_1,

$$5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

2 Find the gradient

So the gradient of L_1 is $-\frac{1}{2}$.

3 Find the gradient

Since L_2 is perpendicular to L_1, the gradient of L_2 is 2.

4 Calculate value

Using the point $(8, 6)$:

$$y - 6 = 2(x - 8)$$

$$y = 2x - 10$$

5 **Calculate value**

At A, the line crosses the x-axis, so $y = 0$:

$$0 = 2x - 10$$

$$x = 5$$

---

6 **Calculate value**

So $A = (5, 0)$.

---

7 **Calculate value**

At B, $x = -3$:

$$y = 2(-3) - 10 = -16$$

---

8 **Calculate value**

So $B = (-3, -16)$.

---

9 **Triangle has base and perpendicular**

Triangle AOB has base $OA = 5$ and perpendicular height 16.

$$\text{Area} = \frac{1}{2} \times 5 \times 16 = 40$$

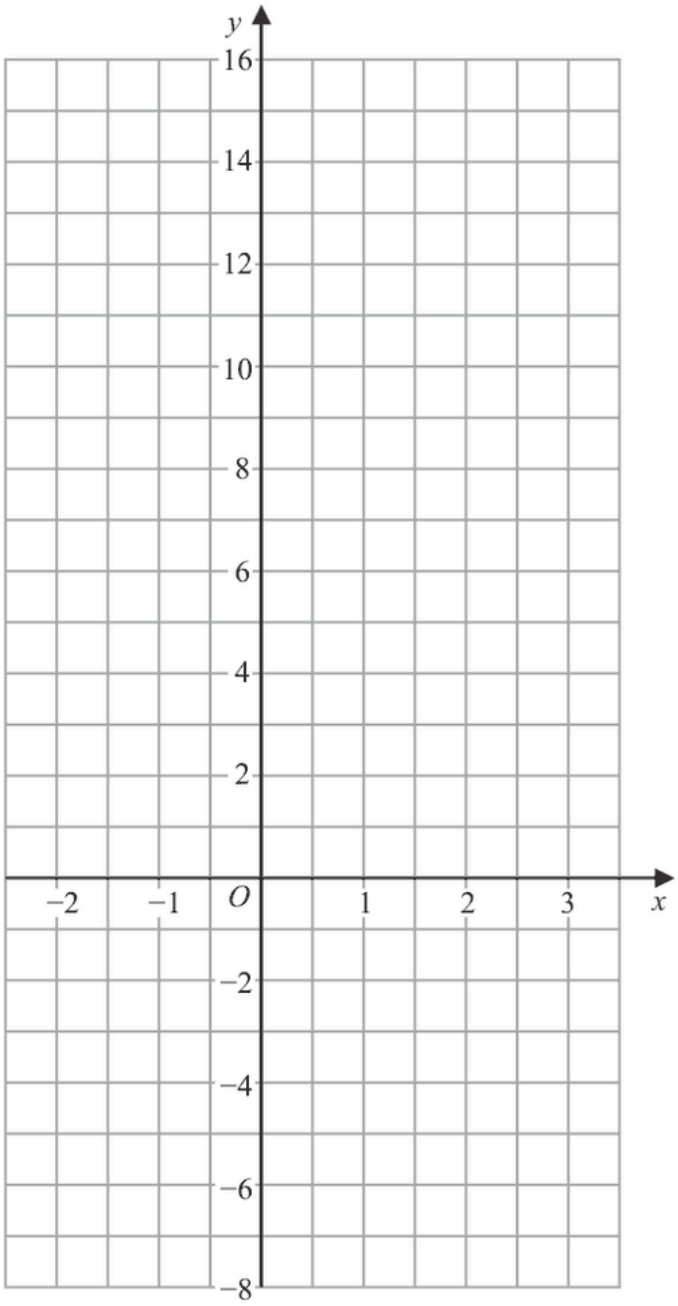
✓ **FINAL ANSWER**

40

Question 3

3 marks | Linear Graphs

3 On the grid, draw the graph of $y = 7 - 4x$ for values of x from -2 to 3



(Total for Question 3 is 3 marks)

1 Read the graph

$$y = 7 - 4x$$

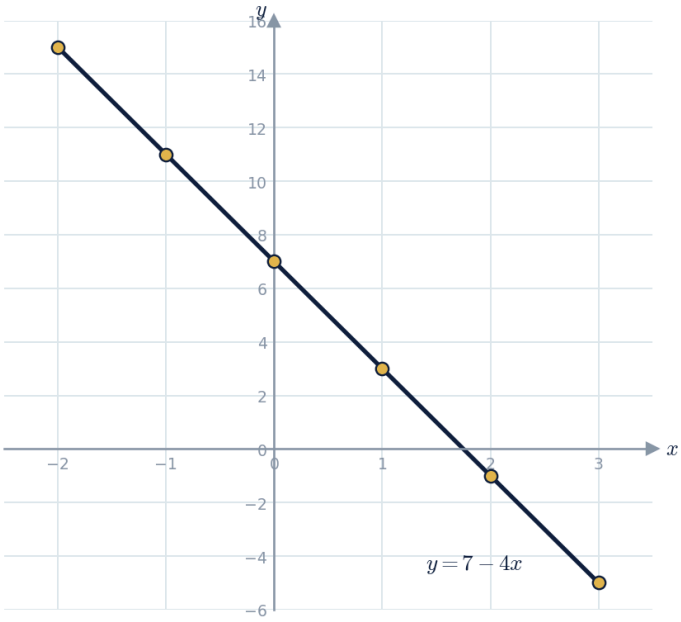
2 Useful points are

Useful points are:

$$(-2, 15), \quad (0, 7), \quad (1, 3), \quad (2, -1), \quad (3, -5)$$

3 Plot these points and join

Plot these points and join them with a straight line.



FINAL ANSWER

The graph is the straight line $y = 7 - 4x$.

4MA1/1HR MAY/NOVEMBER 2020

Question 6

2 marks | Linear Graphs

6 Point A has coordinates $(-3, 11)$
Point B has coordinates $(47, b)$
The midpoint of AB has coordinates $(a, -19)$

Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May/November 2020

1 Calculate value

$$A = (3, 11), \qquad B = (47, b)$$

2 The midpoint is

The midpoint is $(a, 19)$.

3 Evaluate fraction

For the x-coordinate:

$$a = \frac{3 + 47}{2} = 25$$

4 Evaluate fraction

For the y-coordinate:

$$19 = \frac{11 + b}{2}$$

$$38 = 11 + b$$

$$b = 27$$

FINAL ANSWER

$a = 25, b = 27$.

4MA1/1H MAY/NOVEMBER 2020

Question 22

4 marks | Linear Graphs

22 $ABCD$ is a rhombus.

The diagonals, AC and BD, intersect at the point M.
The coordinates of M are $(6, -11)$

The points A and C both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through B and D intersects the y-axis.

(..... , ...)

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May/November 2020

1 Find the gradient

In a rhombus, the diagonals are perpendicular and bisect each other.

2 The diagonal lies on

The diagonal AC lies on

$$2y + 7x = 20$$

$$2y = -7x + 20$$

$$y = -\frac{7}{2}x + 10$$

3 Find the gradient

So the gradient of AC is

$$-\frac{7}{2}$$

4 Find the gradient

Therefore the gradient of BD is

$$\frac{2}{7}$$

5 **Calculate value**

The line BD passes through

$$M = (6, -11)$$

6 **Evaluate fraction**

So

$$y + 11 = \frac{2}{7}(x - 6)$$

7 **To find where this line**

To find where this line intersects the y-axis, put $x = 0$:

$$y + 11 = \frac{2}{7}(0 - 6)$$

$$y + 11 = -\frac{12}{7}$$

$$y = -11 - \frac{12}{7}$$

$$y = -\frac{89}{7}$$

✓ **FINAL ANSWER**

$$\left(0, -\frac{89}{7}\right)$$

4MA1/1H JANUARY 2021

Question 13

3 marks | Linear Graphs

13 Point A has coordinates (5, 8)
Point B has coordinates (9, –4)

(a) Work out the gradient of AB.

.....

(2)

The straight line **L** has equation $y = -4x + 5$

(b) Write down the gradient of a straight line that is perpendicular to **L**.

.....

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2021

1 **Find the gradient**

(a) The gradient of AB, where $A(5, 8)$ and $B(9, -4)$, is

$$\frac{-4-8}{9-5} = \frac{-12}{4} = -3$$

2 **Find the gradient**

(b) A line perpendicular to a line with gradient -3 has gradient

$$\frac{1}{3}$$

FINAL ANSWER

Gradient of $AB = -3$, perpendicular gradient $\frac{1}{3}$.

4MA1/2H JANUARY 2021

Question 18

7 marks | Linear Graphs

18 A rectangle $ABCD$ is to be drawn on a centimetre grid such that

A has coordinates $(-4, -2)$
B has coordinates $(1, 10)$
C has coordinates $(19, a)$
D has coordinates (b, c)

(a) Work out the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(4)

(b) Calculate the perimeter, in centimetres, of rectangle $ABCD$.

..... cm

(3)

(Total for Question 18 is 7 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H January 2021

1 Calculate value

$$A = (-4, -2), \quad B = (1, 10), \quad C = (19, a), \quad D = (b, c)$$

2 The vector is

The vector AB is

$$(1 - (-4), 10 - (-2)) = (5, 12)$$

3 The vector is

The vector BC is

$$(19 - 1, a - 10) = (18, a - 10)$$

4 **Adjacent sides of a rectangle**

Adjacent sides of a rectangle are perpendicular, so their dot product is zero:

$$5(18) + 12(a - 10) = 0$$

$$90 + 12a - 120 = 0$$

$$a = \frac{5}{2}$$

---

5 **Calculate value**

Now

$$BC = (18, -7.5)$$

---

6 **Calculate value**

So

$$D = A + BC = (-4, -2) + (18, -7.5) = (14, -9.5)$$

$$b = 14, \qquad c = -9.5$$

---

7 **The side lengths are**

The side lengths are

$$AB = \sqrt{5^2 + 12^2} = 13$$

$$BC = \sqrt{18^2 + (-7.5)^2} = 19.5$$

---

8 **Perimeter**

Perimeter:

$$2(13 + 19.5) = 65$$

✓ **FINAL ANSWER**

$a = \frac{5}{2}$, $b = 14$, $c = -9.5$, and perimeter $65\ \text{cm}$.

4MA1/1HR JANUARY 2021

Question 22

5 marks | Linear Graphs

22 ABC is an isosceles triangle with $AB = AC$.

B is the point with coordinates $(-1, 5)$
C is the point with coordinates $(2, 10)$
M is the midpoint of BC.

Find an equation of the line through the points A and M.
Give your answer in the form $py + qx = r$ where p, q and r are integers.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2021

1 Find the gradient

Since $AB = AC$, point A lies on the perpendicular bisector of BC.

2 The midpoint of is

The midpoint of BC is

$$M = \left(\frac{-1+2}{2}, \frac{5+10}{2}\right)$$

$$M = \left(\frac{1}{2}, \frac{15}{2}\right)$$

3 The gradient of is

The gradient of BC is

$$\frac{10-5}{2-(-1)} = \frac{5}{3}$$

4 Find the gradient

So the gradient of the perpendicular bisector is

$$-\frac{3}{5}$$

5 **Find line equation**

Use the line through M:

$$y - \frac{15}{2} = -\frac{3}{5}\left(x - \frac{1}{2}\right)$$

6 **Multiply by 10**

Multiply by 10:

$$10y - 75 = -6x + 3$$

$$10y + 6x = 78$$

7 **Divide by 2**

Divide by 2:

$$5y + 3x = 39$$

✓ **FINAL ANSWER**

$5y + 3x = 39$

Question 17

4 marks | Linear Graphs

17 The straight line **L** passes through the points (4, −1) and (6, 4)

The straight line **M** is perpendicular to **L** and intersects the y-axis at the point (0, 8)

Find the coordinates of the point where **M** intersects the x-axis.

(..... , ...)

(Total for Question 17 is 4 marks)

1 **Use Coordinate geometry tag correct**

Line L passes through $(4, -1)$ and $(6, 4)$.

2 **Its gradient is**

Its gradient is

$$\frac{4-(-1)}{6-4} = \frac{5}{2}$$

3 **Find the gradient**

Line M is perpendicular to L, so its gradient is

$$-\frac{2}{5}$$

4 **Line intersects the y axis**

Line M intersects the y-axis at $(0, 8)$, so

$$y = -\frac{2}{5}x + 8$$

5 **Evaluate fraction**

At the x-axis, $y = 0$:

$$0 = -\frac{2}{5}x + 8$$

$$\frac{2}{5}x = 8$$

$$x = 20$$

✓ **FINAL ANSWER**

$(20, 0)$.

4MA1/2H NOVEMBER 2021

Question 19

6 marks | Linear Graphs

19 The straight line **L** has equation $x - y = 3$
The curve **C** has equation $3x^2 - y^2 + xy = 9$

L and **C** intersect at the points P and Q.

Find the coordinates of the midpoint of PQ.
Show clear algebraic working.

(..... , ...)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2021

1 **Line is**

Line L is

$$x - y = 3$$

2 **Calculate value**

So

$$y = x - 3$$

3 **The curve is**

The curve is

$$3x - y + y^2 = 9$$

4 **Substitute**

Substitute $y = x - 3$:

$$3x - (x - 3) + (x - 3)^2 = 9$$

$$2x + 3 + x^2 - 6x + 9 = 9$$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

5 **Calculate value**

So $x = 1$ or $x = 3$.

6 **Calculate value**

If $x = 1$, then $y = -2$. If $x = 3$, then $y = 0$.

7 **The midpoint of and is**

The midpoint of $(1, -2)$ and $(3, 0)$ is

$$\left(\frac{1+3}{2}, \frac{-2+0}{2}\right) = (2, -1)$$

FINAL ANSWER

$(2, -1)$.

4MA1/1H NOVEMBER 2021

Question 20

7 marks | Linear Graphs

20 The straight line **L** passes through point A $(-6, 2)$ and point B $(5, 3)$
The straight line **M** is perpendicular to **L** and passes through the midpoint of A and B.
The line **M** intersects the line $x = -1$ at point C.

Calculate the area of triangle ABC.

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2021

1 **Calculate value**

$$A = (-6, 2), \qquad B = (5, 3)$$

2 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

3 **The gradient of is**

The gradient of AB is

$$\frac{3-2}{5-(-6)} = \frac{1}{11}$$

4 **Find the gradient**

So the perpendicular gradient is

$$-11$$

5 **Evaluate fraction**

Line M passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$, so

$$y - \frac{5}{2} = -11\left(x + \frac{1}{2}\right)$$

6 **Evaluate fraction**

At point C, $x = -1$:

$$y - \frac{5}{2} = -11\left(-\frac{1}{2}\right)$$

$$y - \frac{5}{2} = \frac{11}{2}$$

$$y = 8$$

---

7 **Calculate value**

So

$$C = (-1, 8)$$

---

8 **Use vectors**

Now use vectors from A:

$$\vec{AB} = (11, 1), \qquad \vec{AC} = (5, 6)$$

---

9 **Area of triangle is**

Area of triangle ABC is

$$\frac{1}{2}|11(6) - 1(5)|$$

$$= \frac{1}{2}(61)$$

$$= 30.5$$

✓ **FINAL ANSWER**

30.5

4MA1/1H JANUARY 2022

Question 10

3 marks | Linear Graphs

10 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

.....

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

(..... , ...)

(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2022

1 **The line is**

The line is

$$2y + 7x = 10$$

$$2y = -7x + 10$$

$$y = -\frac{7}{2}x + 5$$

2 **Find the gradient**

So the gradient is

$$-\frac{7}{2}$$

3 **Evaluate fraction**

For the x-axis, $y = 0$:

$$2(0) + 7x = 10$$

$$x = \frac{10}{7}$$

FINAL ANSWER

Gradient $-\frac{7}{2}$, and x-axis intersection $\left(\frac{10}{7}, 0\right)$.

4MA1/2HR JANUARY 2022

Question 18

5 marks | Linear Graphs

18 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...) and (..... , ...)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2022

1 The line is

The line is

$$2y = x + 1$$

2 Calculate value

So

$$x = 2y - 1$$

3 Substitute into the curve

Substitute into the curve

$$3y^2 + 7y + 16 = x^2 - x$$

$$3y^2 + 7y + 16 = (2y - 1)^2 - (2y - 1)$$

$$3y^2 + 7y + 16 = 4y^2 - 6y + 2$$

$$y^2 - 13y - 14 = 0$$

$$(y - 14)(y + 1) = 0$$

4 **Calculate value**

So

$$y = 14 \quad \text{or} \quad y = -1$$

---

5 **Calculate value**

If $y = 14$, then $x = 27$.

---

6 **Calculate value**

If $y = -1$, then $x = -3$.

✓ **FINAL ANSWER**

$(27, 14)$ and $(-3, -1)$.

4MA1/2HR JANUARY 2022

Question 22

5 marks | Linear Graphs

22 ABC is a triangle in which angle $ABC = 90°$

p and q are integers such that

the coordinates of A are $(p, 10)$
the coordinates of B are $(-1, -5)$
the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$p =$ .....

$q =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2022

1 The gradient of is

The gradient of AC is $-\frac{6}{7}$.

$$\frac{q-10}{8-p} = -\frac{6}{7}$$

$$7(q-10) = -6(8-p)$$

$$7q - 70 = -48 + 6p$$

$$6p - 7q = -22$$

2 **Find the gradient**

Since $\angle ABC = 90°$, lines AB and BC are perpendicular.

$$\text{gradient of } AB = \frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$$

$$\text{gradient of } BC = \frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$$

---

3 **The product of perpendicular gradients**

The product of perpendicular gradients is -1:

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$5(q+5) = -3(p+1)$$

$$3p + 5q = -28$$

---

4 **Solve**

Now solve

$$6p - 7q = -22$$

$$3p + 5q = -28$$

---

5 **Double the second equation**

Double the second equation:

$$6p + 10q = -56$$

---

6 **Subtract**

Subtract:

$$17q = -34$$

$$q = -2$$

---

7 **Calculate value**

Then

$$3p + 5(-2) = -28$$

$$3p = -18$$

$$p = -6$$

FINAL ANSWER

$p = -6,\ q = -2$

4MA1/1H JUNE 2022

Question 10

4 marks | Linear Graphs

10 Two circles, C_1 and C_2, are drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis.

The centre of circle C_1 is at the point with coordinates $(-1, 3)$ and the radius of C_1 is 13 cm.

The centre of circle C_2 is at the point with coordinates $(7, 18)$ and the radius of C_2 is 6 cm.

(a) Work out the distance between the centre of C_1 and the centre of C_2

..... cm

(3)

(b) Explain why circle C_1 intersects circle C_2

.....

(1)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H June 2022

1 The centres are

The centres are

$$C_1 = (-1, 3), \qquad C_2 = (7, 18)$$

2 The distance between the centres

The distance between the centres is

$$\sqrt{(7-(-1))^2 + (18-3)^2}$$

$$= \sqrt{8^2 + 15^2} = \sqrt{289} = 17$$

3 Calculate value

The radii are 3 cm and 15 cm.

$$3 + 15 = 18$$

4 **Use Coordinate geometry tag correct**

Since the distance between the centres is 17 cm, which is less than the sum of the radii, the circles intersect.

✓ **FINAL ANSWER**

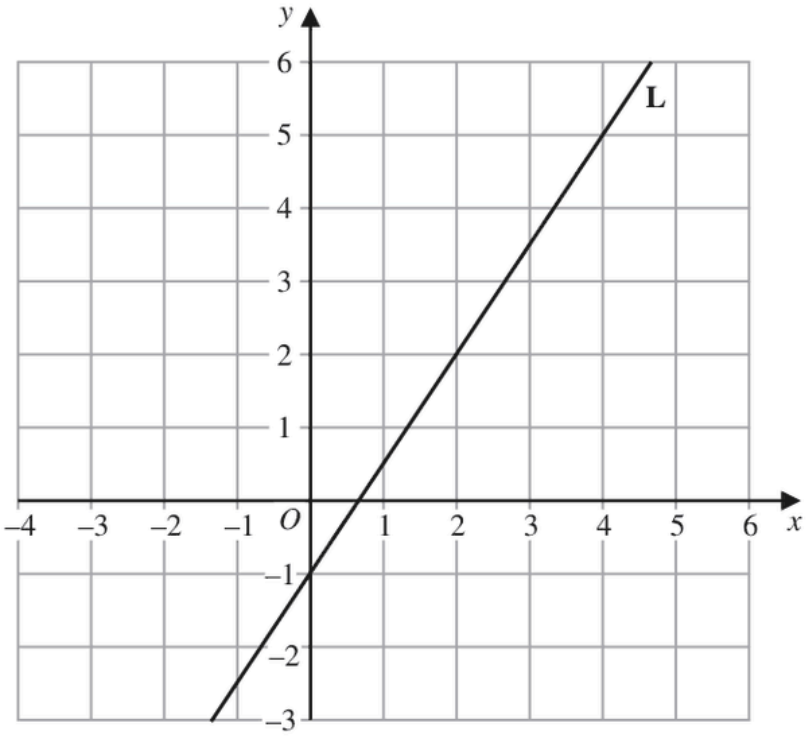
$17\ \text{cm}$, and the circles intersect because $17 < 18$.

4MA1/1HR JUNE 2022

Question 10

3 marks | Linear Graphs

10 Line **L** is drawn on the grid.



Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2022

1 **Solve equation**

From the graph, the line crosses the y-axis at -1, so

$$c = -1$$

2 **Find the gradient**

Using two points on the line, for example $(0, -1)$ and $(2, 2)$, the gradient is

$$\frac{2-(-1)}{2-0} = \frac{3}{2}$$

3 **Solve equation**

So the equation is

$$y = \frac{3}{2}x - 1$$

✓ **FINAL ANSWER**

$y = \frac{3}{2}x - 1$.

4MA1/2HR JUNE 2022

Question 20

4 marks | Linear Graphs

20 The centre O of a circle has coordinates $(4, 7)$

The point A, on the circle, has coordinates $(6, 11)$ and AOP is a diameter of the circle.

Find an equation of the tangent to the circle at the point P

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2022

1 The centre of the circle

The centre of the circle is

$$O = (4, 7)$$

2 The point is

The point A is

$$A = (6, 11)$$

3 Find the midpoint

Since AOP is a diameter, O is the midpoint of AP.

4 Let

Let $P = (x, y)$.

$$\left(\frac{6+x}{2}, \frac{11+y}{2}\right) = (4, 7)$$

$$6 + x = 8, \qquad 11 + y = 14$$

$$P = (2, 3)$$

5 The gradient of is

The gradient of OP is

$$\frac{3-7}{2-4} = 2$$

6 **Use circle theorem**

The tangent is perpendicular to the radius, so its gradient is

$$-\frac{1}{2}$$

---

7 **Evaluate fraction**

Using point $P = (2, 3)$,

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 4$$

✓ **FINAL ANSWER**

$y = -\frac{1}{2}x + 4$

4MA1/2H JUNE 2022

Question 22

7 marks | Linear Graphs

22 $ABCD$ is a kite, with diagonals AC and BD, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point M with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the x coordinate of B is positive,

find the coordinates of the points B and D.

(..... , ...)

(..... , ...)

(Total for Question 22 is 7 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2022

1 The midpoint of is

The midpoint of BD is $M = (0, 2)$.

2 Find the gradient

Since $A = (-3, 4)$, the gradient of AM is

$$\frac{2-4}{0-(-3)} = -\frac{2}{3}$$

3 Find the gradient

For the kite, diagonal AC is perpendicular to BD, so the gradient of BD is

$$\frac{3}{2}$$

4 Use direction vector along

Use direction vector $(2, 3)$ along BD.

5 **Let**

Let

$$B = (2t, 2 + 3t)$$

6 **Calculate value**

and

$$D = (-2t, 2 - 3t)$$

7 **Calculate value**

Since $AB = 6.5$,

$$(2t - (-3))^2 + (2 + 3t - 4)^2 = 6.5^2$$

$$(2t + 3)^2 + (3t - 2)^2 = 42.25$$

$$4t^2 + 12t + 9 + 9t^2 - 12t + 4 = 42.25$$

$$13t^2 + 13 = 42.25$$

$$13t^2 = 29.25$$

$$t^2 = 2.25$$

$$t = 1.5$$

8 **Use**

The x-coordinate of B is positive, so use $t = 1.5$.

$$B = (3, 6.5)$$

$$D = (-3, -2.5)$$

✓ **FINAL ANSWER**

$B = (3, 6.5),\ D = (-3, -2.5)$

4MA1/1H MAY 2023

Question 13

3 marks | Linear Graphs

13 A is the point with coordinates $(-5, 12)$

B is the point with coordinates $(19, -48)$

Find an equation of the straight line that passes through the points A and B

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2023

1 Calculate value

$$A = (-5, 12), \qquad B = (19, -48)$$

2 The gradient is

The gradient is

$$\frac{-48 - 12}{19 - (-5)} = \frac{-60}{24} = -\frac{5}{2}$$

3 Evaluate fraction

Using point $(-5, 12)$:

$$y - 12 = -\frac{5}{2}(x + 5)$$

4 Multiply by 2

Multiply by 2:

$$2y - 24 = -5x - 25$$

$$5x + 2y + 1 = 0$$

FINAL ANSWER

$5x + 2y + 1 = 0$.

4MA1/2HR JUNE 2023

Question 21

5 marks | Linear Graphs

21 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2023

1 The line is

The line is

$$y - 2x = 1$$

2 Calculate value

So

$$y = 2x + 1$$

3 Calculate value

Substitute into

$$y^2 + xy = 7$$

$$(2x + 1)^2 + x(2x + 1) = 7$$

$$4x^2 + 4x + 1 + 2x^2 + x = 7$$

$$6x^2 + 5x - 6 = 0$$

4 Factorise

Factorise:

$$(3x - 2)(2x + 3) = 0$$

5 **Evaluate fraction**

So

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

---

6 **Evaluate fraction**

Using $y = 2x + 1$:

$$x = \frac{2}{3} \Rightarrow y = \frac{7}{3}$$

$$x = -\frac{3}{2} \Rightarrow y = -2$$

✓ **FINAL ANSWER**

$\left(\frac{2}{3}, \frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -2\right)$

4MA1/2H JUNE 2023

Question 24

6 marks | Linear Graphs

24 $ABCD$ is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of D

(..... , ...)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2023

1 Calculate value

In the kite, $AB = AD$ and $CB = CD$, so AC is the line of symmetry.

2 Apply transformation

Therefore D is the reflection of B in the line AC.

$$A = (-2, 10), \qquad B = \left(-\frac{27}{5}, 4\right), \qquad C = (4, -5)$$

3 A direction vector for is

A direction vector for AC is

$$\begin{pmatrix} 4 - (-2) \\ -5 - 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$$

4 Use the simpler direction vector

Use the simpler direction vector

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

5 **Find the gradient**

Let H be the foot of the perpendicular from B to AC.

$$\vec{AB} = \begin{pmatrix} -\frac{17}{5} \\ -6 \end{pmatrix}$$

6 **The fraction along to is**

The fraction along AC to H is

$$\frac{\vec{AB} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}}{2^2 + (-5)^2} = \frac{-\frac{34}{5} + 30}{29}$$

$$= \frac{\frac{116}{5}}{29} = \frac{4}{5}$$

7 **Use matrices**

So

$$H = A + \frac{4}{5}\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$H = \left(-2 + \frac{8}{5}, 10 - 4\right)$$

$$H = \left(-\frac{2}{5}, 6\right)$$

8 **Find the midpoint**

Since H is the midpoint of BD,

$$D = 2H - B$$

$$D = \left(-\frac{4}{5}, 12\right) - \left(-\frac{27}{5}, 4\right)$$

$$D = \left(\frac{23}{5}, 8\right)$$

✓ **FINAL ANSWER**

$$D = \left(\frac{23}{5}, 8\right)$$

4MA1/2HR JUNE 2023

Question 25

6 marks | Linear Graphs

25 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

(..... , ...)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2023

1 The perpendicular bisector is

The perpendicular bisector is

$$y - 2x = 7$$

2 Calculate value

so

$$y = 2x + 7$$

3 The gradient of the perpendicular

The gradient of the perpendicular bisector is 2, so the gradient of AB is

$$-\frac{1}{2}$$

4 **Evaluate fraction**

Since $A = (j, 7)$ and $B = (6, k)$,

$$\frac{k-7}{6-j} = -\frac{1}{2}$$

$$2(k-7) = -(6-j)$$

$$j = 2k - 8$$

5 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$$

6 **Substitute**

Substitute $j = 2k - 8$:

$$\left(k-1, \frac{k+7}{2}\right)$$

7 **Find the midpoint**

The midpoint lies on $y = 2x + 7$, so

$$\frac{k+7}{2} = 2(k-1) + 7$$

$$k + 7 = 4k + 10$$

$$k = -1$$

8 **Calculate value**

Then

$$j = 2(-1) - 8 = -10$$

9 **Find the midpoint**

Therefore the midpoint is

$$\left(\frac{-10+6}{2}, \frac{7+(-1)}{2}\right) = (-2, 3)$$

FINAL ANSWER

$$\boxed{(-2, 3)}$$

4MA1/2H NOVEMBER 2023

Question 9

2 marks | Linear Graphs

9 A straight line passes through the points with coordinates (0, –3) and (2, 0)

Find an equation of the line.

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2023

1 Solve equation

The line passes through $(0, -3)$ and $(2, 0)$.

2 The gradient is

The gradient is

$$\frac{0-(-3)}{2-0} = \frac{3}{2}$$

3 Solve equation

The y-intercept is -3, so

$$y = \frac{3}{2}x - 3$$

FINAL ANSWER

$y = \frac{3}{2}x - 3$.

4MA1/2H NOVEMBER 2023

Question 21

5 marks | Linear Graphs

21 The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2023

1 The line is

The line is

$$x + 2y = 5$$

2 Calculate value

So

$$x = 5 - 2y$$

3 Calculate value

Substitute into

$$x^2 + 3y^2 = 13$$

$$(5 - 2y)^2 + 3y^2 = 13$$

$$25 - 20y + 4y^2 + 3y^2 = 13$$

$$7y^2 - 20y + 12 = 0$$

4 **Factorise**

Factorise:

$$(7y - 6)(y - 2) = 0$$

5 **Evaluate fraction**

So

$$y = \frac{6}{7} \quad \text{or} \quad y = 2$$

6 **Evaluate fraction**

Using $x = 5 - 2y$:

$$y = \frac{6}{7} \Rightarrow x = \frac{23}{7}$$

$$y = 2 \Rightarrow x = 1$$

✓ **FINAL ANSWER**

$\left(\frac{23}{7}, \frac{6}{7}\right)$ and $(1, 2)$

4MA1/1H NOVEMBER 2023

Question 22

6 marks | Linear Graphs

22 [In this question 1 cm = 1 unit on the x-axis and 1 cm = 1 unit on the y-axis]

P is a point on a circle with centre $(0, 0)$
The coordinates of P are $(8, -10)$

The line **L** is the tangent to the circle at the point P
L crosses the x-axis at the point Q and crosses the y-axis at the point R

Work out the length of RQ
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2023

1 The centre of the circle

The centre of the circle is $(0, 0)$, and

$$P = (8, -10)$$

2 The gradient of is

The gradient of OP is

$$\frac{-10}{8} = -\frac{5}{4}$$

3 Find the gradient

So the gradient of the tangent is

$$\frac{4}{5}$$

4 The tangent through is

The tangent through P is

$$y + 10 = \frac{4}{5}(x - 8)$$

$$y = \frac{4}{5}x - \frac{82}{5}$$

5 **Evaluate fraction**

At the x-axis, $y = 0$:

$$0 = \frac{4}{5}x - \frac{82}{5}$$

$$x = \frac{41}{2}$$

---

6 **Evaluate fraction**

So

$$Q = \left(\frac{41}{2}, 0\right)$$

---

7 **Evaluate fraction**

At the y-axis, $x = 0$:

$$R = \left(0, -\frac{82}{5}\right)$$

---

8 **Simplify surd**

Therefore

$$RQ = \sqrt{\left(\frac{41}{2}\right)^2 + \left(\frac{82}{5}\right)^2}$$

$$RQ = 26.252\ldots$$

✓ **FINAL ANSWER**

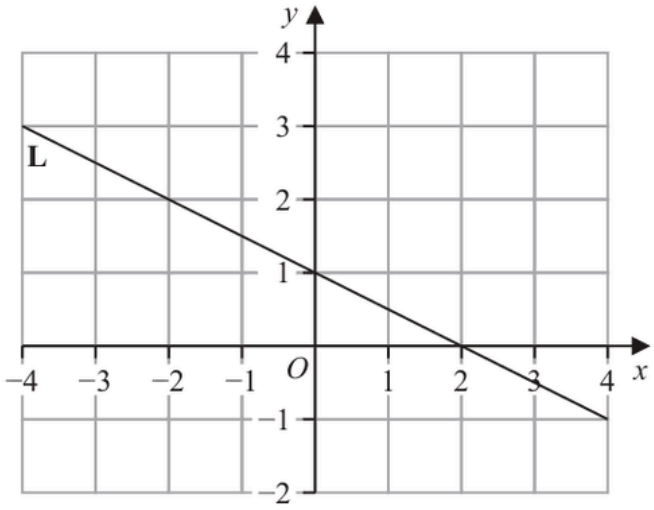
$26.3\ \text{cm}$

4MA1/1HR MAY 2024

Question 9

3 marks | Linear Graphs

9 Line **L** is drawn on the grid.



Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May 2024

1 Solve equation

From the graph, the line crosses the y-axis at 1, so

$$c = 1$$

2 It also crosses the x

It also crosses the x-axis at 2, so using $(0, 1)$ and $(2, 0)$,

$$m = \frac{0 - 1}{2 - 0} = -\frac{1}{2}$$

3 Solve equation

Therefore

$$y = -\frac{1}{2}x + 1$$

FINAL ANSWER

$y = -\frac{1}{2}x + 1$.

4MA1/2H JUNE 2024

Question 18

2 marks | Linear Graphs

18 The straight line **P** has equation $5y + 2x = 7$

Find the gradient of a straight line that is perpendicular to **P**

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2024

1 The line is

The line is

$$5y + 2x = 7$$

$$5y = -2x + 7$$

$$y = -\frac{2}{5}x + \frac{7}{5}$$

2 Find the gradient

So the gradient of line P is $-\frac{2}{5}$.

3 A line perpendicular to P

A line perpendicular to P has gradient

$$\frac{5}{2}$$

FINAL ANSWER

$\frac{5}{2}$.

4MA1/2HR JUNE 2024

Question 21

4 marks | Linear Graphs

21 $ABCD$ is a square.

The point A has coordinates $(-5, 2)$
The point B has coordinates $(3, 5)$

Find an equation of the line that passes through B and C
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2024

1 Calculate value

$$A = (-5, 2), \qquad B = (3, 5)$$

2 The gradient of is

The gradient of AB is

$$\frac{5-2}{3-(-5)} = \frac{3}{8}$$

3 Find the gradient

Since $ABCD$ is a square, BC is perpendicular to AB.

4 Find the gradient

So the gradient of BC is

$$-\frac{8}{3}$$

5 The line through is

The line through $B = (3, 5)$ is

$$y - 5 = -\frac{8}{3}(x - 3)$$

6

Multiply by 3

Multiply by 3:

$$3y - 15 = -8x + 24$$

$$8x + 3y - 39 = 0$$

✓ **FINAL ANSWER**

$8x + 3y - 39 = 0$

4MA1/2H JUNE 2024

Question 22

5 marks | Linear Graphs

22 The straight line **L** has equation $x + y = 5$

The curve **C** has equation $2x^2 + 3y^2 = 210$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , ...) (..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2024

1 The line is

The line is

$$x + y = 5$$

2 Calculate value

So

$$y = 5 - x$$

3 Substitute into the curve

Substitute into the curve:

$$2x^2 + 3y^2 = 210$$

$$2x^2 + 3(5 - x)^2 = 210$$

$$2x^2 + 3(x^2 - 10x + 25) = 210$$

$$5x^2 - 30x + 75 = 210$$

$$5x^2 - 30x - 135 = 0$$

4 **Divide by 5**

Divide by 5:

$$x^2 - 6x - 27 = 0$$

5 **Factorise**

Factorise:

$$(x - 9)(x + 3) = 0$$

6 **Calculate value**

So

$$x = 9 \quad \text{or} \quad x = -3$$

7 **Calculate value**

Using $y = 5 - x$:

$$x = 9 \Rightarrow y = -4$$

$$x = -3 \Rightarrow y = 8$$

✓ **FINAL ANSWER**

$(9, -4)$ and $(-3, 8)$

4MA1/2H NOVEMBER 2024

Question 18

4 marks | Linear Graphs

18 The straight line **L** is perpendicular to the straight line with equation $2x + y = 9$ and passes through the point with coordinates $(8, 11)$

Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2024

1 The given line is

The given line is

$$2x + y = 9$$

$$y = -2x + 9$$

2 Find the gradient

Its gradient is -2, so a perpendicular line has gradient $\frac{1}{2}$.

3 Find the gradient

Line L passes through $(8, 11)$:

$$y - 11 = \frac{1}{2}(x - 8)$$

$$y = \frac{1}{2}x + 7$$

FINAL ANSWER

$y = \frac{1}{2}x + 7$.

4MA1/2HR JUNE 2025

Question 10

2 marks | Linear Graphs

10 A straight line, **L**, is parallel to the line with equation $y = 2 - 5x$
The line **L** passes through the point $(0, 6)$

Find an equation of the line **L**

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2025

1 The line has gradient

The line

$$y = 2 - 5x$$

has gradient -5.

2 A parallel line has the

A parallel line has the same gradient, so line L has form

$$y = -5x + c$$

3 Find the gradient

Line L passes through $(0, 6)$, so

$$6 = c$$

FINAL ANSWER

$y = -5x + 6$.

4WM1/01R MAY 2025

Question 23

5 marks | Linear Graphs

23 A circle C has centre $(3q, 4q)$ where q is an integer.

The straight line with equation $y = -\frac{5}{4}x + \frac{21}{4}$ is the tangent to C at the point $(p, p+3)$ where p is an integer.

Work out the radius of C
Give your answer in the form $\sqrt{a}$ where a is an integer.
Show your working clearly.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4WM1/01R May 2025

1 The tangent point lies on

The tangent point lies on the tangent line, so

$$p + 3 = -\frac{5}{4}p + \frac{21}{4}$$

$$p = 1$$

2 Find the gradient

The tangent gradient is $-\frac{5}{4}$, so the radius gradient is $\frac{4}{5}$.

3 Use trigonometry

Using centre $(3q, 4q)$ and tangent point $(1, 4)$,

$$\frac{4 - 4q}{1 - 3q} = \frac{4}{5}$$

$$q = 2$$

4 **Simplify surd**

So the centre is $(6, 8)$. Therefore

$$r = \sqrt{(6-1)^2 + (8-4)^2} = \sqrt{41}$$

FINAL ANSWER

$\sqrt{41}$.

4MA1/2HR JUNE 2025

Question 24

6 marks | Linear Graphs

24 PQ is a straight line drawn on a square grid, with a scale of 1 cm for 1 unit on each axis.

P has coordinates $(-5,\ a)$ and Q has coordinates $(7,\ 3a)$ where $a > 0$

The length of PQ is $4\sqrt{10}$ cm

Find an equation of the perpendicular bisector of PQ
Give your answer in the form $y = mx + c$ where m and c are integers.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2025

1 Calculate value

$$P = (-5, a), \qquad Q = (7, 3a)$$

2 Simplify surd

The length of PQ is $4\sqrt{10}$, so

$$\sqrt{(7-(-5))^2 + (3a-a)^2} = 4\sqrt{10}$$

$$\sqrt{12^2 + (2a)^2} = 4\sqrt{10}$$

3 Square both sides

Square both sides:

$$144 + 4a^2 = 160$$

$$4a^2 = 16$$

$$a^2 = 4$$

4 Solve inequality

Since $a > 0$,

$$a = 2$$

5 **Calculate value**

So

$$P = (-5, 2), \qquad Q = (7, 6)$$

6 **The midpoint of is**

The midpoint of PQ is

$$\left(\frac{-5+7}{2}, \frac{2+6}{2}\right) = (1, 4)$$

7 **The gradient of is**

The gradient of PQ is

$$\frac{6-2}{7-(-5)} = \frac{4}{12} = \frac{1}{3}$$

8 **Find the gradient**

So the gradient of the perpendicular bisector is

$$-3$$

9 **Calculate value**

Using point $(1, 4)$:

$$y - 4 = -3(x - 1)$$

$$y = -3x + 7$$

FINAL ANSWER

$$\boxed{y = -3x + 7}$$

4WM1/01 MAY 2025

Question 24

5 marks | Linear Graphs

24 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2025

1 The diagonals of a square

The diagonals of a square bisect each other and are perpendicular.

2 Find the midpoint

Midpoint of $P(4, 7)$ and $R(8, -5)$:

$$\left(\frac{4+8}{2}, \frac{7-5}{2}\right) = (6, 1)$$

3 Find the gradient

Gradient of PR:

$$\frac{-5-7}{8-4} = -3$$

4 Find the gradient

So the perpendicular gradient is $\frac{1}{3}$.

5 **The line through and passes**

The line through Q and S passes through $(6, 1)$, so

$$y - 1 = \frac{1}{3}(x - 6)$$

$$3y - 3 = x - 6$$

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H JUNE 2025

Question 25

5 marks | Linear Graphs

25 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H June 2025

1 Calculate value

$$P = (4, 7), \qquad R = (8, -5)$$

2 The gradient of diagonal is

The gradient of diagonal PR is

$$\frac{-5 - 7}{8 - 4} = -3$$

3 The other diagonal is perpendicular

The other diagonal QS is perpendicular to PR, so its gradient is

$$\frac{1}{3}$$

4 The diagonals of a square

The diagonals of a square bisect each other.

5 The midpoint of is

The midpoint of PR is

$$\left(\frac{4 + 8}{2}, \frac{7 + (-5)}{2}\right) = (6, 1)$$

6 **Find the gradient**

So QS passes through $(6, 1)$ with gradient $\frac{1}{3}$.

$$y - 1 = \frac{1}{3}(x - 6)$$

$$y = \frac{1}{3}x - 1$$

---

7 **Multiply by 3**

Multiply by 3:

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/2H NOVEMBER 2025

Question 1

2 marks | Linear Graphs

1 The point P has coordinates (3, 4)
The point Q has coordinates (9, 16)

M is the midpoint of the line PQ

Find the coordinates of M

(..... , ...)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2025

1 Calculate value

$$P = (-3, 0), \qquad Q = (9, 16)$$

2 The midpoint is

The midpoint is

$$\left(\frac{-3+9}{2}, \frac{0+16}{2}\right)$$

$$= (3, 8)$$

FINAL ANSWER

$(3, 8)$.

4MA1/1H NOVEMBER 2025

Question 20

3 marks | Linear Graphs

20 The straight line **L** has equation $y = 4x + 7$

The straight line **M** is perpendicular to **L** and passes through the point with coordinates (8, 1)

Find an equation for **M**
Give your answer in the form $y = mx + c$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2025

1 Line has equation

Line L has equation

$$y = 4x + 7$$

2 Find the gradient

So its gradient is

$$4$$

3 Find the gradient

Line M is perpendicular to L, so its gradient is

$$-\frac{1}{4}$$

4 Evaluate fraction

Line M passes through $(8, 1)$.

$$y - 1 = -\frac{1}{4}(x - 8)$$

$$y - 1 = -\frac{1}{4}x + 2$$

$$y = -\frac{1}{4}x + 3$$

FINAL ANSWER

$$y = -\frac{1}{4}x + 3$$

4WM1/01 NOVEMBER 2025

Question 27

5 marks | Linear Graphs

27 $ABCD$ is a rhombus with $AB = BC = CD = DA$

A has coordinates $(16, -9)$

C has coordinates $(24, 15)$

Find an equation of the line BD
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.
Show your working clearly.

(Total for Question 27 is 5 marks)

WORKED SOLUTION

Question 27 · 4WM1/01 November 2025

1 Diagonals of a rhombus bisect

Diagonals of a rhombus bisect each other at right angles.

2 Find the midpoint

Midpoint of AC:

$$\left(\frac{16+24}{2}, \frac{-9+15}{2}\right) = (20, 3)$$

3 Find the gradient

Gradient of AC:

$$\frac{15-(-9)}{24-16} = 3$$

4 Find the gradient

So gradient of BD is $-\frac{1}{3}$.

$$y - 3 = -\frac{1}{3}(x - 20)$$

$$x + 3y - 29 = 0$$

FINAL ANSWER

$x + 3y - 29 = 0$.

4MA1/1H NOVEMBER 2025

Question 28

6 marks | Linear Graphs

28 The curve **C** has equation $x^2 + y^2 = d - 11x$ where d is an integer.
The line **L** has equation $y = 3x + e$ where e is an integer.

C and **L** intersect at the point A and at the point B

The coordinates of A are $(0.2, 2.6)$

Work out the coordinates of B
Show your working clearly.

(..... , ...)

(Total for Question 28 is 6 marks)

WORKED SOLUTION

Question 28 · 4MA1/1H November 2025

1 The line is

The line is

$$y = 3x + e$$

2 Point lies on the line

Point $A = (0.2, 2.6)$ lies on the line.

$$2.6 = 3(0.2) + e$$

$$2.6 = 0.6 + e$$

$$e = 2$$

3 Calculate value

So

$$y = 3x + 2$$

4 **The curve is**

The curve is

$$x^2 + y^2 = d - 11x$$

---

5 **Use point**

Use point $A = (0.2, 2.6)$:

$$0.2^2 + 2.6^2 = d - 11(0.2)$$

$$0.04 + 6.76 = d - 2.2$$

$$d = 9$$

---

6 **Substitute into the curve**

Now substitute $y = 3x + 2$ into the curve:

$$x^2 + (3x + 2)^2 = 9 - 11x$$

$$x^2 + 9x^2 + 12x + 4 = 9 - 11x$$

$$10x^2 + 23x - 5 = 0$$

---

7 **One root is**

One root is $0.2 = \frac{1}{5}$.

---

8 **The product of the roots**

The product of the roots is

$$\frac{-5}{10} = -\frac{1}{2}$$

---

9 **Evaluate fraction**

So the other root is

$$\frac{-\frac{1}{2}}{\frac{1}{5}} = -\frac{5}{2}$$

---

10 **Evaluate fraction**

Then

$$y = 3\left(-\frac{5}{2}\right) + 2$$

$$y = -\frac{11}{2}$$

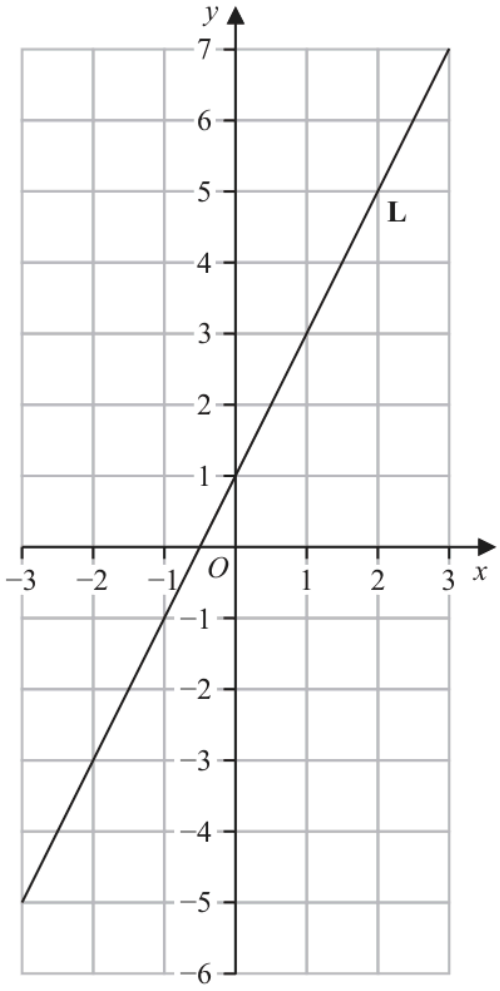
FINAL ANSWER

$$B = \left(-\frac{5}{2}, -\frac{11}{2}\right)$$

Question 10

3 marks | Linear Graphs

10 The line **L** is drawn on a grid.



Find an equation for **L**

.....

(Total for Question 10 is 3 marks)

1 **Read two points**

The line passes through $(0, 1)$ and $(2, 5)$.

2 **Gradient and intercept**

$$m = \frac{5 - 1}{2 - 0} = 2, \qquad c = 1$$

3 **Equation**

$$y = 2x + 1$$

✓ **FINAL ANSWER**

$y = 2x + 1$

4MA1/1HR JUNE 2026

Question 13

3 marks | Linear Graphs

13 The straight line **L** has equation $4y - 10x - 5 = 0$

(a) Find the gradient of **L**

.....

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

(..... , ...)

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$4y - 10x - 5 = 0 \implies 4y = 10x + 5 \implies y = \frac{5}{2}x + \frac{5}{4}.$$

Rearrange into the form y=mx+c.

2 **Part (a) | Step 2**

The gradient is $\frac{5}{2}$.

The coefficient of x is the gradient.

3 **Part (b) | Step 1**

$$x = 0 \implies 4y - 5 = 0 \implies y = \frac{5}{4}.$$

Every point on the y-axis has x-coordinate zero.

✓ **FINAL ANSWER**

(a) $\frac{5}{2}$ (b) $\left(0, \frac{5}{4}\right)$

4MA1/1H JUNE 2026

Question 15

2 marks | Linear Graphs

15 Point A has coordinates $(-3, 4)$
Point B has coordinates $(2, -6)$

Work out the gradient of the line AB

.....

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H June 2026

1 Solution | Step 1

$$m = \frac{-6 - 4}{2 - (-3)} = \frac{-10}{5} = -2.$$

Use change in y divided by the corresponding change in x.

FINAL ANSWER

-2

Question 17

4 marks | Linear Graphs

17 The line **L** is perpendicular to the line with equation $y = 7 + 4x$

L passes through the point A with coordinates $\left(0, \frac{19}{2}\right)$

L crosses the x-axis at the point B

Find the area of triangle AOB where O is the origin.
Show your working clearly.

.....

(Total for Question 17 is 4 marks)

1 **Perpendicular gradient**

$y = 7 + 4x$ has gradient 4, so $\mathbf{L}$ has gradient $-\frac{1}{4}$.

2 **Equation of L**

$$y = -\tfrac{1}{4}x + \tfrac{19}{2}$$

3 **Point B (y = 0)**

$$0 = -\tfrac{1}{4}x + \tfrac{19}{2} \Rightarrow x = 38,\ B = (38, 0)$$

4 **Area of triangle AOB**

$$\tfrac{1}{2} \times 38 \times \tfrac{19}{2} = 180.5$$

✓ **FINAL ANSWER**

180.5

TOPIC

Graphs of Functions

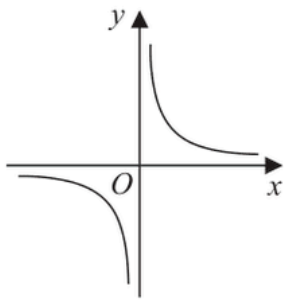
Every past-paper question on this topic, 2020 to 2026.

Question 15

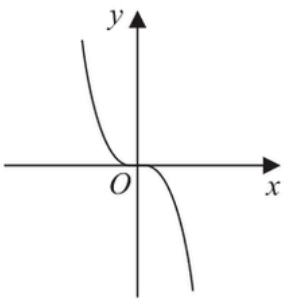
3 marks | Graphs of Functions

15 Here are six graphs.

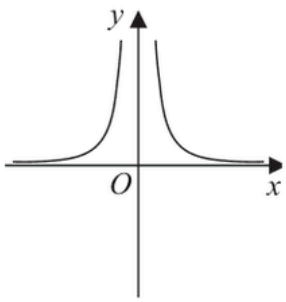
Graph A



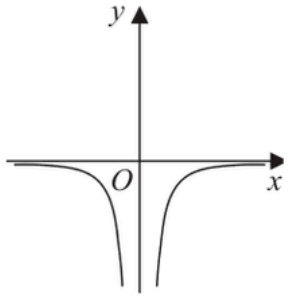
Graph B



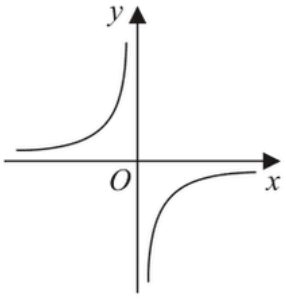
Graph C



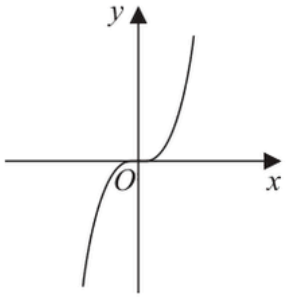
Graph D



Graph E



Graph F



Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \frac{2}{x^2}$	.....
$y = -\frac{1}{2}x^3$	.....
$y = -\frac{5}{x}$	.....

(Total for Question 15 is 3 marks)

1 **Solve equation**

For

$$y = -\frac{2}{x^2}$$

the graph is negative on both sides of the y-axis and is symmetrical in the y-axis.

2 **This is Graph D**

This is Graph D.

3 **Solve equation**

For

$$y = -\frac{1}{2}x^3$$

the graph is a decreasing cubic through the origin.

4 **This is Graph B**

This is Graph B.

5 **Solve equation**

For

$$y = \frac{5}{x}$$

the graph is a positive reciprocal graph in quadrants 1 and 3.

6 **This is Graph A**

This is Graph A.

✓ **FINAL ANSWER**

D, B, A.

Question 15

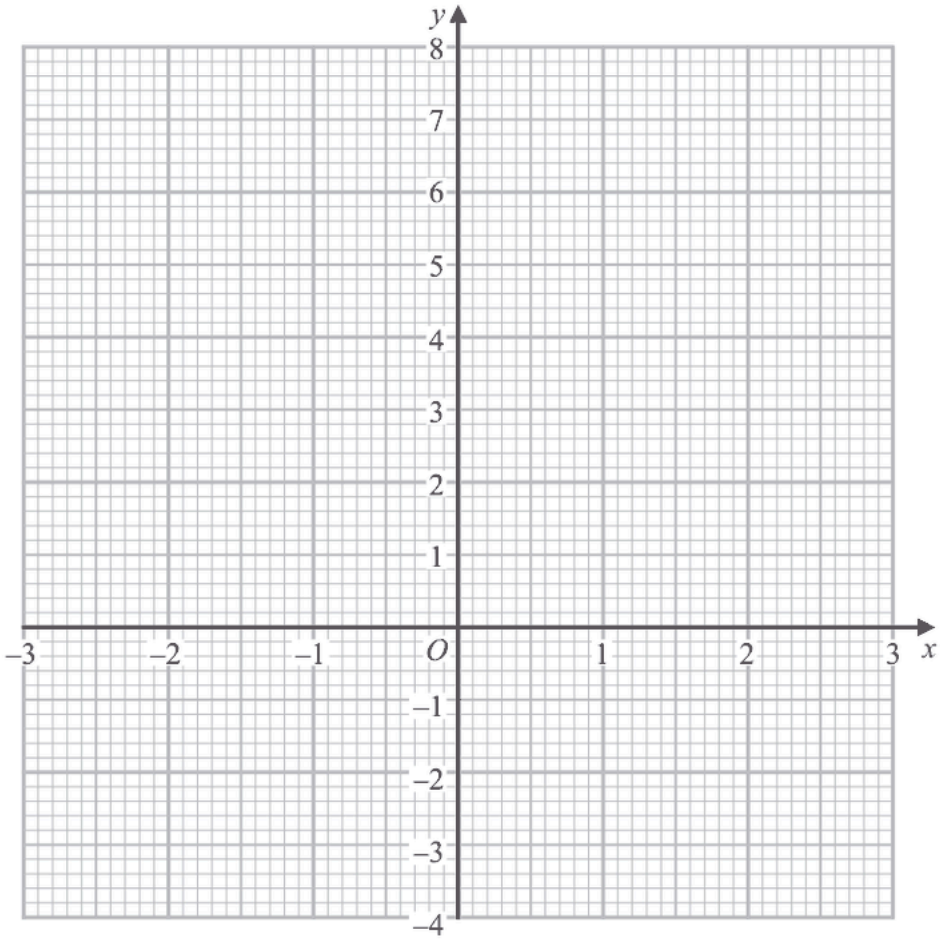
4 marks | Graphs of Functions

15 (a) Complete the table of values for $y = x^2 - \frac{x}{2} - 3$

x	–3	–2	–1	0	1	2	3
y	7.5				–2.5		4.5

(2)

(b) On the grid, draw the graph of $y = x^2 - \frac{x}{2} - 3$ for values of x from –3 to 3



(2)

(Total for Question 15 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - \frac{x}{2} - 3$$

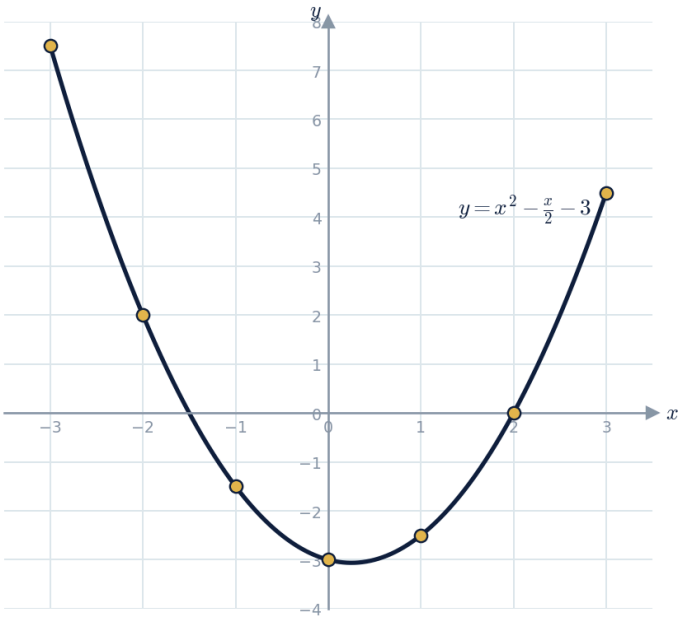
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	7.5	2	-1.5	-3	-2.5	0	4.5

3 Plot these points and join

Plot these points and join them with a smooth U-shaped curve.



FINAL ANSWER

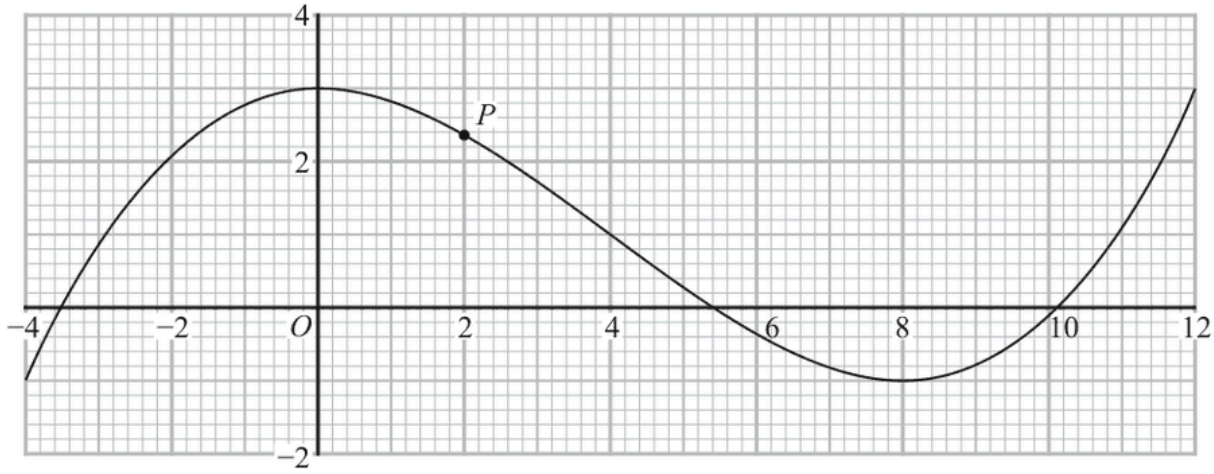
Missing values are $2, -1.5, -3, 0$.

4MA1/1HR JANUARY 2020

Question 18

7 marks | Graphs of Functions

18 The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$



The point P on the curve has x coordinate 2

(a) (i) Use the graph to find an estimate for the gradient of the curve at P.

.....

(3)

(ii) Hence find an equation of the tangent to the curve at P.
Give your answer in the form $y = mx + c$

.....

(2)

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

(b) Use the graph to find the two possible values of k.

..... , ...

(2)

(Total for Question 18 is 7 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2020

1 **Draw a tangent to the**

Draw a tangent to the curve at P.

2 **A reasonable tangent gives an**

A reasonable tangent gives an estimated gradient of about

$$m \approx -0.6$$

3 **The point is approximately**

The point P is approximately

$$(2, 2.4)$$

4 **Find the gradient**

Using $y = mx + c$,

$$2.4 \approx -0.6(2) + c$$

$$c \approx 3.6$$

5 **Find the gradient**

So an approximate tangent equation is

$$y \approx -0.6x + 3.6$$

6 **The equation has exactly two**

The equation $f(x) = k$ has exactly two different solutions when the horizontal line touches one turning level.

7 **Find the gradient**

From the graph, these levels are approximately

$$k = 3 \quad \text{and} \quad k = -1$$

✓ **FINAL ANSWER**

Gradient ≈ -0.6, tangent $y \approx -0.6x + 3.6$, and $k \approx 3, -1$.

4MA1/1H JANUARY 2020

Question 20

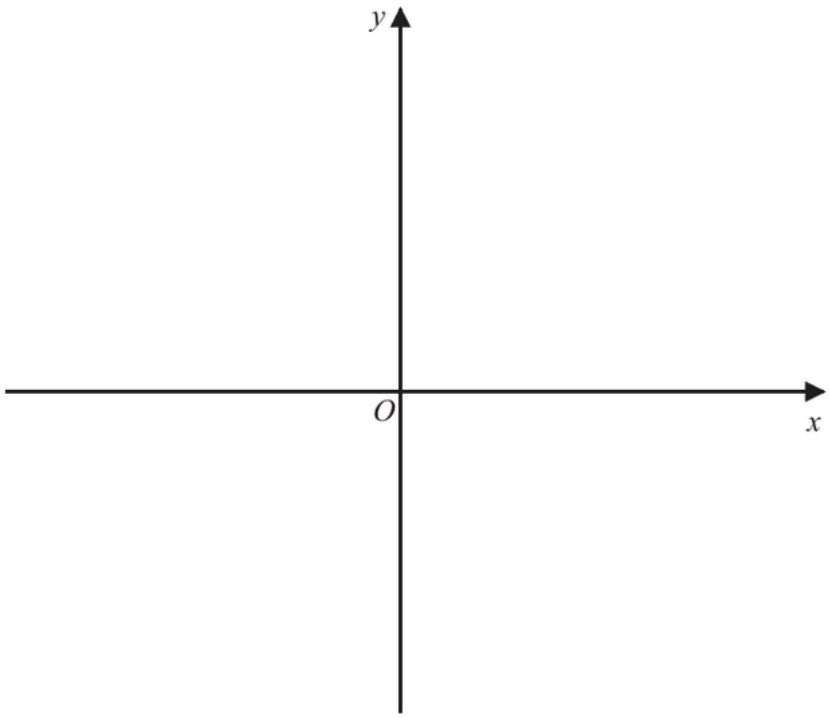
4 marks | Graphs of Functions

20 The curve **C** has equation $y = 4(x - 1)^2 - a$ where $a > 4$

Using the axes below, sketch the curve **C**.
On your sketch show clearly, in terms of a,

(i) the coordinates of any points of intersection of **C** with the coordinate axes,

(ii) the coordinates of the turning point.



(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2020

1 **Calculate value**

$$y = 4(x - 1)^2 - a$$

2 **The turning point is read**

The turning point is read directly from the completed square form:

$$(1, -a)$$

3 **Find intercepts**

For the y-intercept, put $x = 0$:

$$y = 4(0-1)^2 - a$$

$$y = 4 - a$$

4 **Find intercepts**

So the y-intercept is

$$(0, 4-a)$$

5 **Find intercepts**

For the x-intercepts, put $y = 0$:

$$4(x-1)^2 - a = 0$$

$$4(x-1)^2 = a$$

$$(x-1)^2 = \frac{a}{4}$$

$$x - 1 = \pm\frac{\sqrt{a}}{2}$$

$$x = 1 \pm \frac{\sqrt{a}}{2}$$

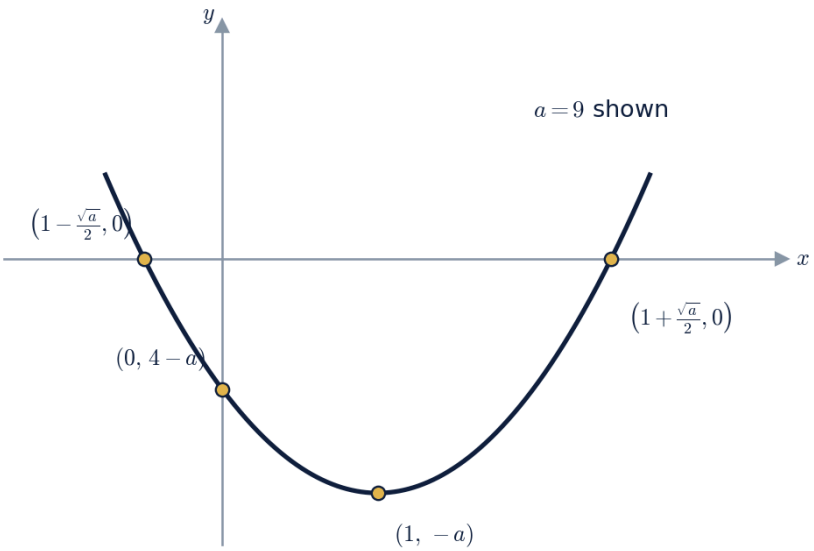
6 **Find intercepts**

So the x-intercepts are

$$\left(1 - \frac{\sqrt{a}}{2}, 0\right) \quad \text{and} \quad \left(1 + \frac{\sqrt{a}}{2}, 0\right)$$

7 **Solve inequality**

Since $a > 4$, the turning point is below the x-axis and the curve crosses the x-axis twice.



FINAL ANSWER

turning point $(1, -a)$, y-intercept $(0, 4-a)$, x-intercepts $\left(1-\frac{\sqrt{a}}{2}, 0\right)$ and $\left(1+\frac{\sqrt{a}}{2}, 0\right)$

Question 15

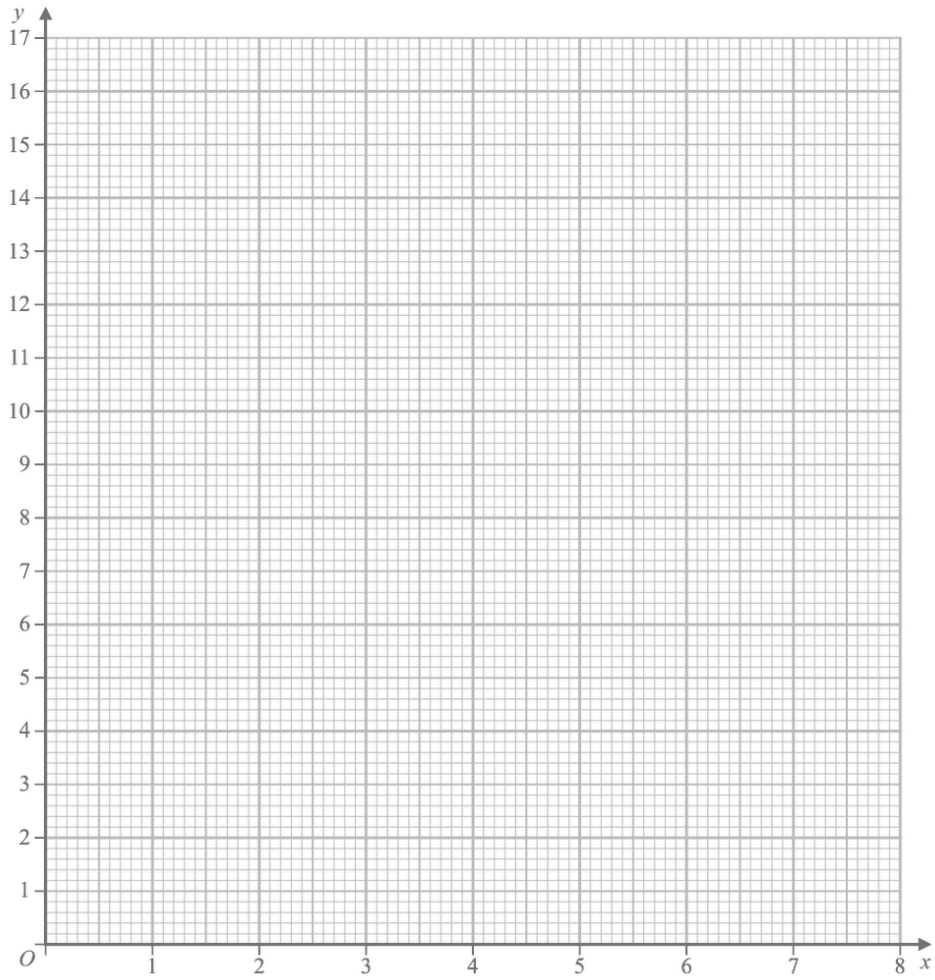
4 marks | Graphs of Functions

15 (a) Complete the table of values for $y = \frac{1}{x}(x^2 + 4)$

x	0.25	0.5	1	2	4	8
y	16.25					8.5

(2)

(b) On the grid, draw the graph of $y = \frac{1}{x}(x^2 + 4)$ for $0.25 \leqslant x \leqslant 8$



(2)

(Total for Question 15 is 4 marks)

1 Read the graph

$$y = \frac{1}{x}(x^2 + 4) = x + \frac{4}{x}$$

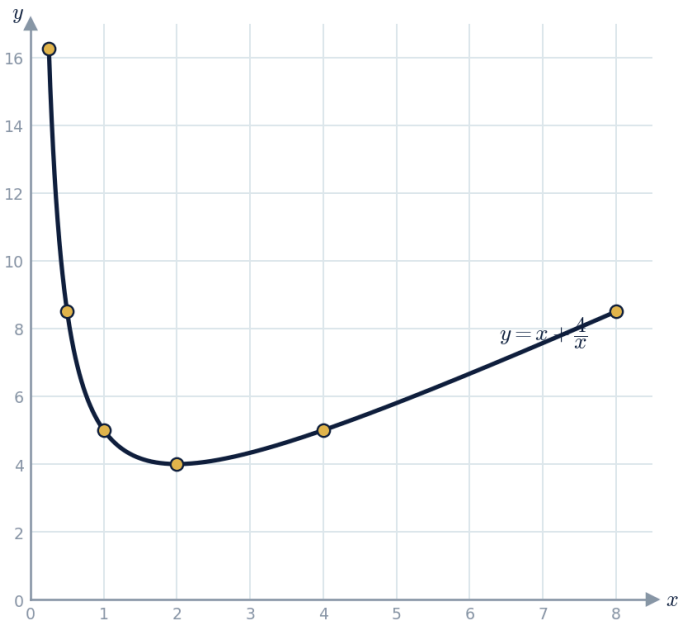
2 The completed table is

The completed table is:

x	0.25	0.5	1	2	4	8
y	16.25	8.5	5	4	5	8.5

3 Plot the points and join

Plot the points and join them with a smooth curve.



FINAL ANSWER

Missing values are $8.5, 5, 4, 5$.

Question 4

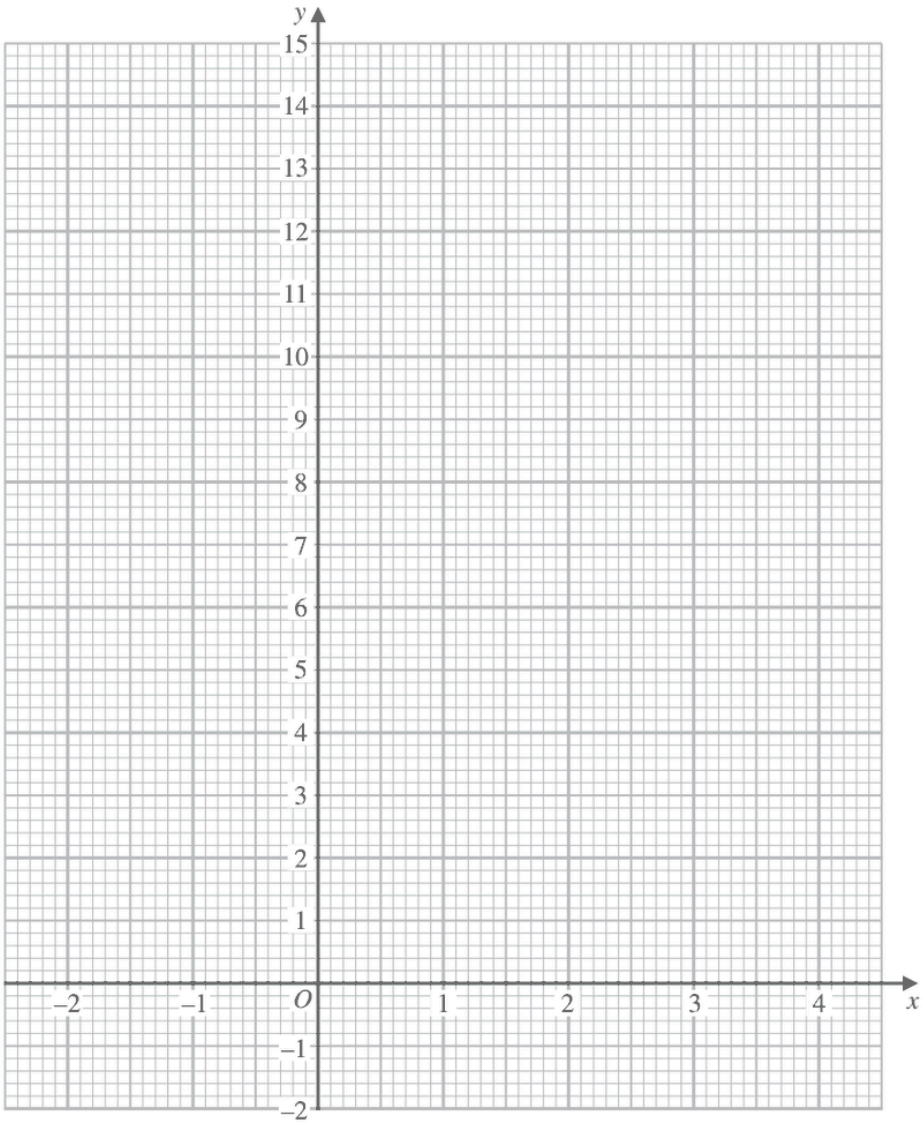
4 marks | Graphs of Functions

4 (a) Complete the table of values for $y = x^2 - 4x + 3$

x	–2	–1	0	1	2	3	4
y		8	3			0	

(2)

(b) On the grid, draw the graph of $y = x^2 - 4x + 3$ for values of x from –2 to 4



(2)

(Total for Question 4 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - 4x + 3$$

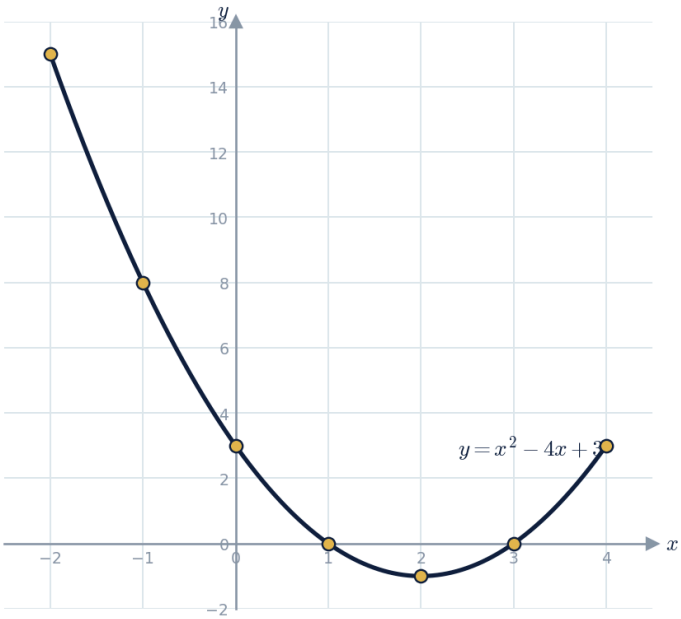
2 The completed table is

The completed table is:

x	-2	-1	0	1	2	3	4
y	15	8	3	0	-1	0	3

3 Plot the points and join

Plot the points and join them with a smooth U-shaped curve.



FINAL ANSWER

Missing values are $15, 0, -1, 3$.

Question 11

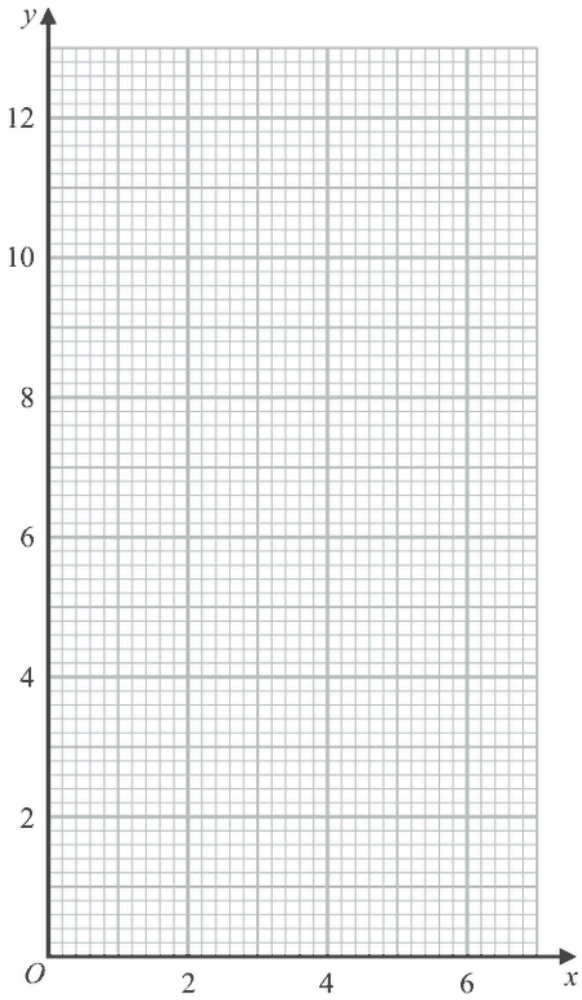
4 marks | Graphs of Functions

11 (a) Complete the table of values for $y = \frac{6}{x}$

x	0.5	1	2	3	4	5	6
y		6		2			1

(2)

(b) On the grid, draw the graph of $y = \frac{6}{x}$ for $0.5 \leqslant x \leqslant 6$



(2)

(Total for Question 11 is 4 marks)

1 Read the graph

$$y = \frac{6}{x}$$

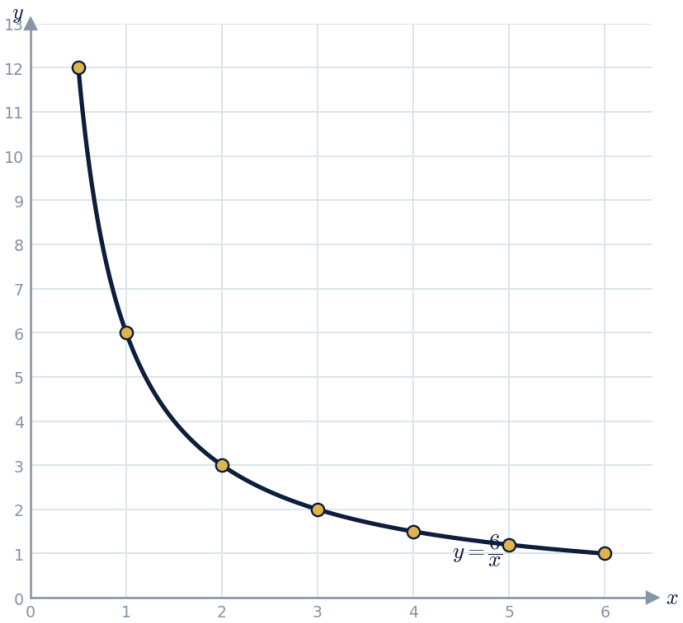
2 The completed table is

The completed table is:

x	0.5	1	2	3	4	5	6
y	12	6	3	2	1.5	1.2	1

3 Plot the points and join

Plot the points and join them with a smooth decreasing reciprocal curve.



FINAL ANSWER

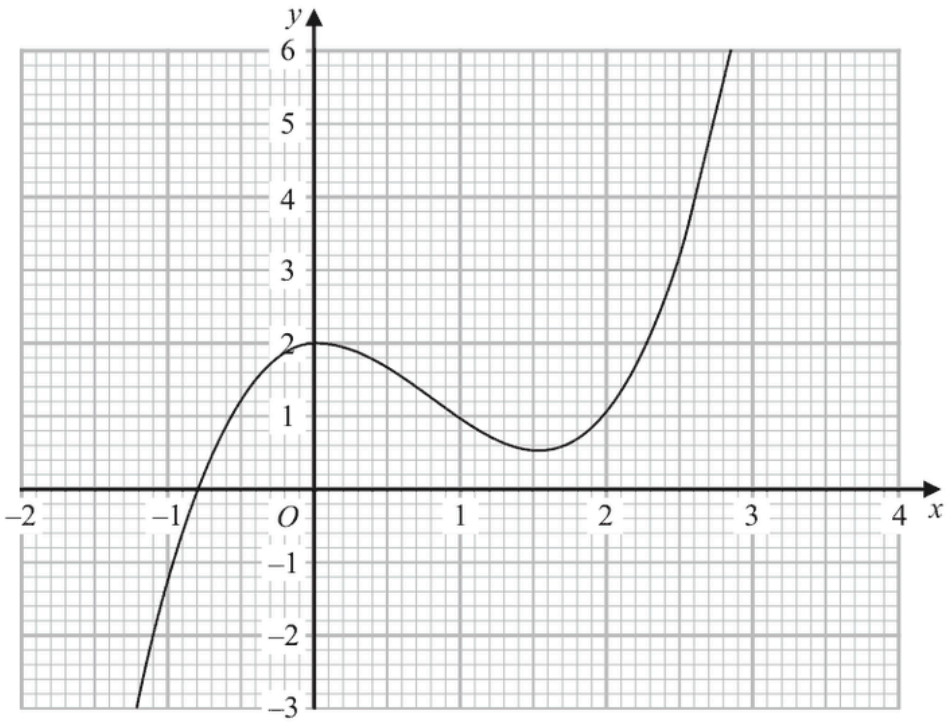
Missing values are $12, 3, 1.5, 1.2$.

4MA1/2HR JANUARY 2022

Question 17

3 marks | Graphs of Functions

17 Part of the curve with equation $y = f(x)$ is shown on the grid.



Find an estimate for the gradient of the curve at the point where $x = 2$
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR January 2022

1 **Draw a tangent to the**

Draw a tangent to the curve at the point where

$$x = 2$$

2 **Find the gradient**

Using two clear points on the tangent, for example approximately $(1.5, 0.6)$ and $(2.3, 2.2)$,

$$\text{gradient} \approx \frac{2.2 - 0.6}{2.3 - 1.5}$$

$$\text{gradient} \approx 2.0$$

FINAL ANSWER

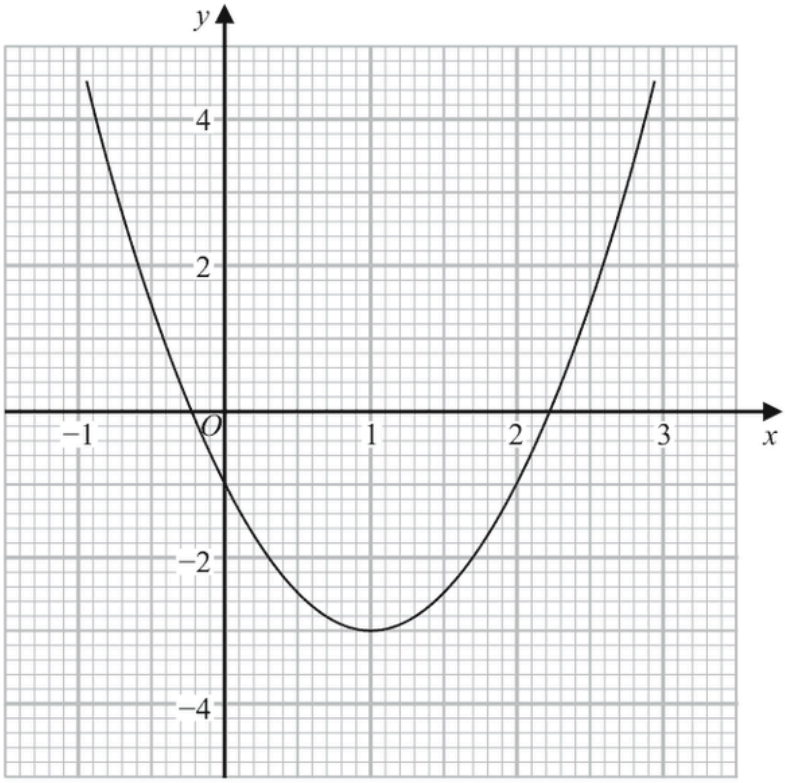
The gradient is approximately $\mathbf{2}$.

4MA1/2H JANUARY 2022

Question 21

5 marks | Graphs of Functions

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

.....

(2)

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

.....

(3)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2022

1 **Read the graph**

For part (a), solve

$$2x^2 - 4x - 1 = 0$$

2 **Read the graph**

Using the graph gives approximately

$$x = -0.2, \quad x = 2.2$$

3 **Read the graph**

For part (b), we need to solve

$$x^2 - x - 1 = 0$$

4 **Multiply by 2**

Multiply by 2:

$$2x^2 - 2x - 2 = 0$$

5 **The given graph is**

The given graph is

$$y = 2x^2 - 4x - 1$$

6 **Read the graph**

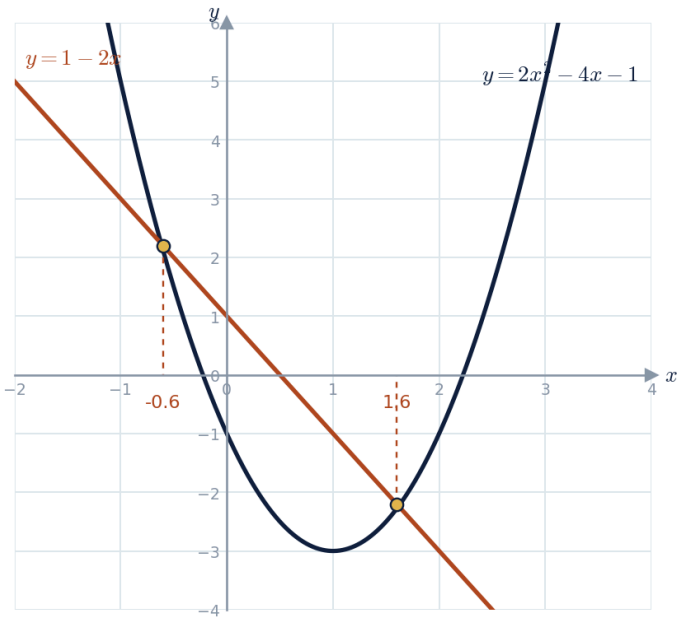
Now

$$2x^2 - 2x - 2 = (2x^2 - 4x - 1) + 2x - 1$$

7 **Read the graph**

So draw the straight line

$$y = 1 - 2x$$



8 **The intersections of this line**

The intersections of this line with the curve give approximately

$$x = -0.6, \quad x = 1.6$$

✓ **FINAL ANSWER**

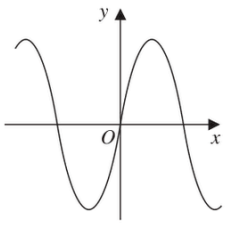
(a) $-0.2,\ 2.2$; (b) draw $y = 1 - 2x$, giving $-0.6,\ 1.6$

Question 16

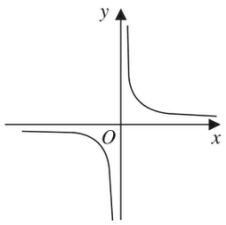
3 marks | Graphs of Functions

16 Here are nine graphs.

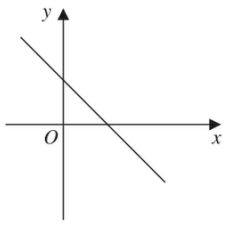
Graph A



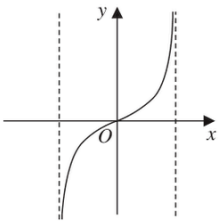
Graph B



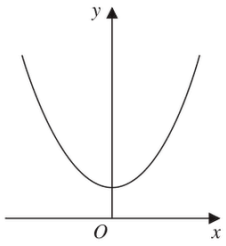
Graph C



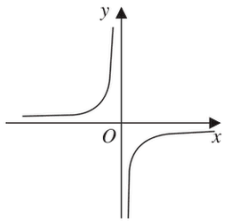
Graph D



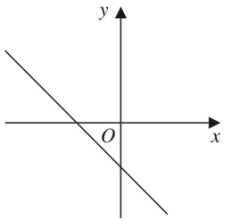
Graph E



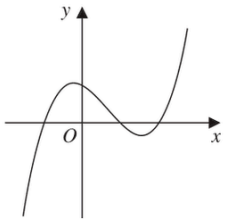
Graph F



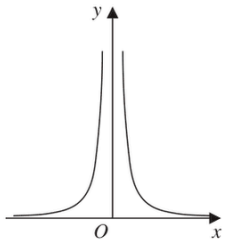
Graph G



Graph H



Graph I



Complete the table below with the letter of the graph that could represent each given equation.
Write each answer on the dotted line.

Equation	Graph
$y = -2x + 3$	.....
$y = -\frac{1}{x}$	.....
$y = \tan x°$	.....
$y = (x + 1)(x - 1)(x - 2)$	.....

(Total for Question 16 is 3 marks)

1 **is a negative straight line**

$$y = -2x + 3$$

is a negative straight line with positive y-intercept, so it is **Graph C**.

2 **Read the graph**

$$y = \frac{1}{x}$$

is the reciprocal graph in quadrants I and III, so it is **Graph B**.

3 **has repeating branches with vertical**

$$y = \tan x$$

has repeating branches with vertical asymptotes and passes through the origin, so it is **Graph D**.

4 **Read the graph**

$$y = (x + 1)(x - 1)(x - 2)$$

is a positive cubic with three roots, so it is **Graph H**.

✓ **FINAL ANSWER**

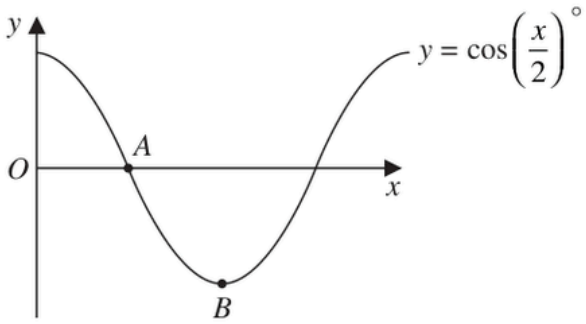
C, B, D, H.

4MA1/2H JUNE 2022

Question 23

2 marks | Graphs of Functions

23 The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$



(i) Find the coordinates of the point A

(.....,.....)

(1)

(ii) Find the coordinates of the point B

(.....,.....)

(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2022

1 **The graph is**

The graph is

$$y = \cos\left(\frac{x}{2}\right)^\circ$$

2 **Point is the first positive**

Point A is the first positive x-intercept, so $y = 0$.

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180$$

3 **Read the graph**

So

$$A = (180, 0)$$

---

4 **Point is the minimum point**

Point B is the minimum point.

---

5 **The minimum value of cosine**

The minimum value of cosine is -1, which occurs when

$$\frac{x}{2} = 180°$$

$$x = 360$$

---

6 **Read the graph**

So

$$B = (360, -1)$$

✓ **FINAL ANSWER**

$A = (180, 0),\ B = (360, -1)$

Question 4

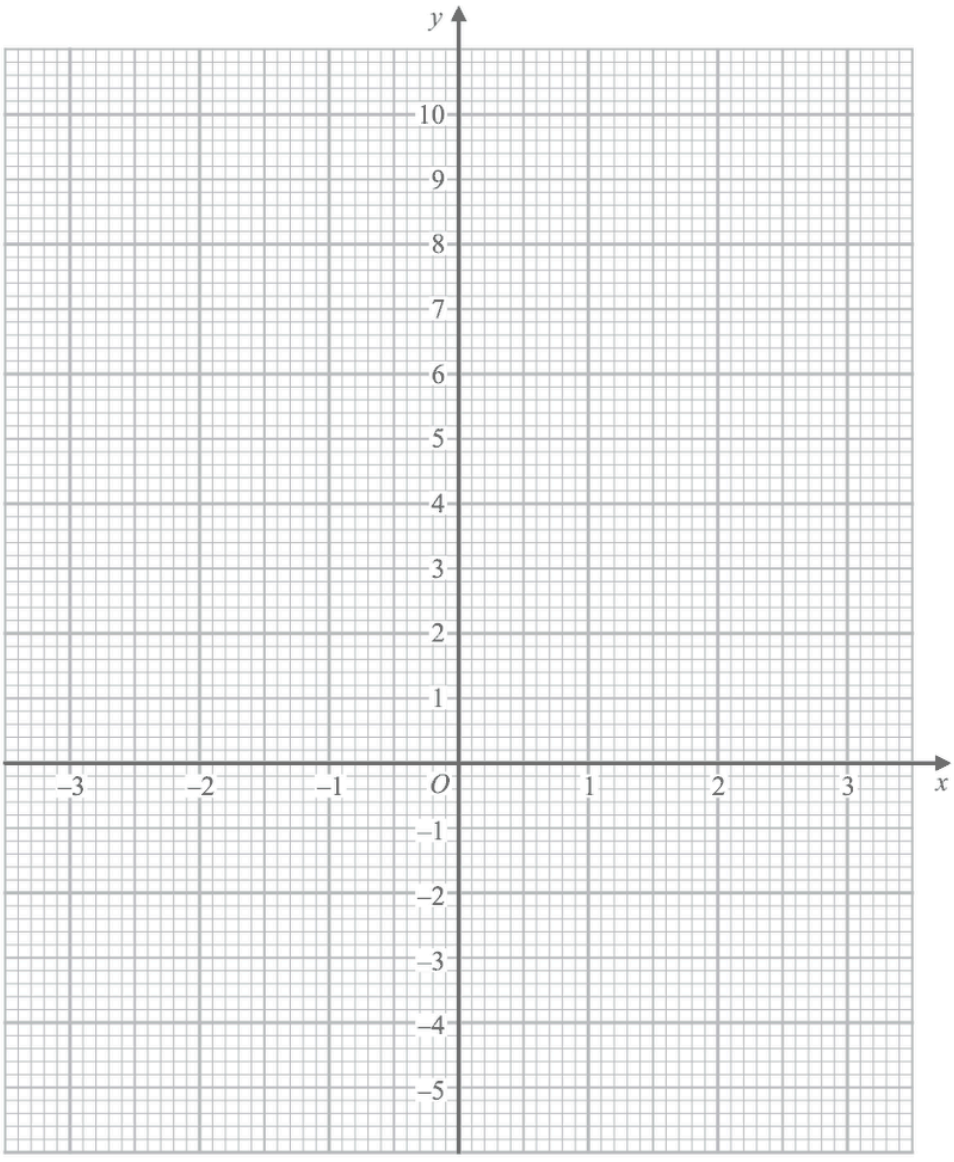
4 marks | Graphs of Functions

4 (a) Complete the table of values for $y = x^2 - x - 4$

x	–3	–2	–1	0	1	2	3
y		2			–4		

(2)

(b) On the grid below, draw the graph of $y = x^2 - x - 4$ for values of x from –3 to 3



(2)

(Total for Question 4 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - x - 4$$

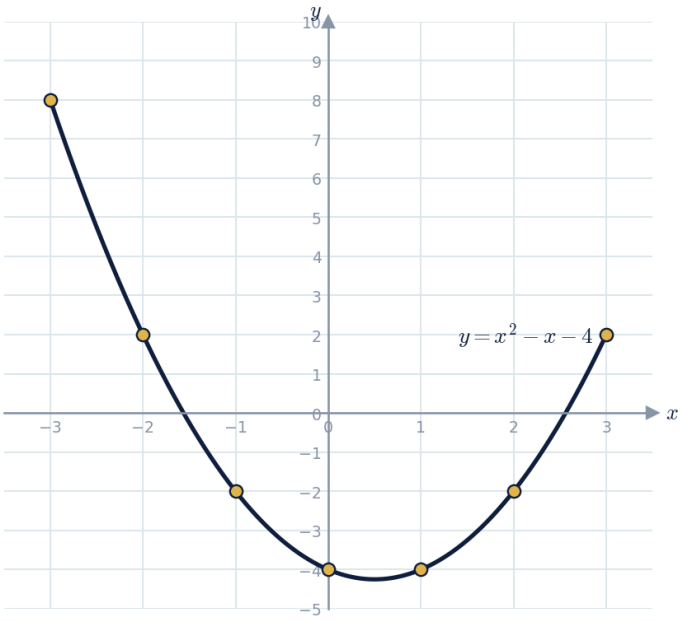
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2

3 Plot the points and join

Plot the points and join them with a smooth U-shaped curve.



FINAL ANSWER

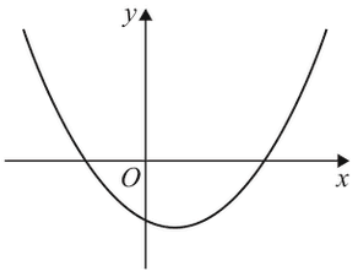
Missing values are $8, -2, -4, -2, 2$.

Question 12

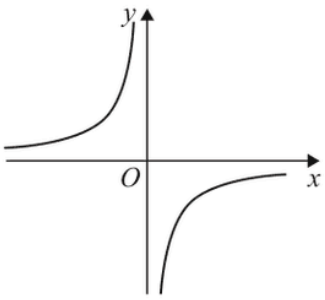
3 marks | Graphs of Functions

12 Here are six graphs.

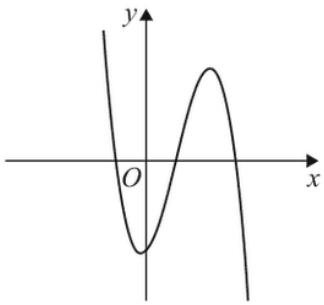
Graph **A**



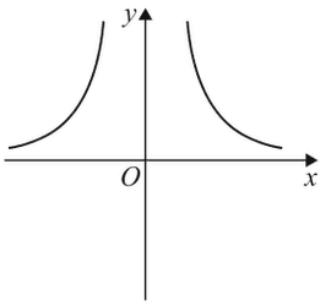
Graph **B**



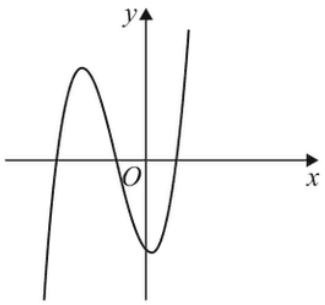
Graph **C**



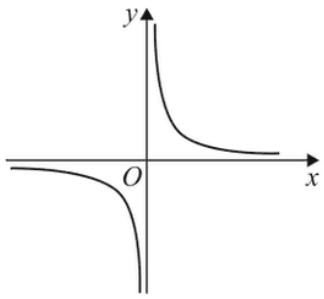
Graph **D**



Graph **E**



Graph **F**



Write down the letter of the graph of

(a) $y = \frac{10}{x^2}$

.....

(1)

(b) $y = x - 3 + 3x^2 - x^3$

.....

(1)

(c) $y = -\frac{3}{x}$

.....

(1)

(Total for Question 12 is 3 marks)

1 **Solve equation**

For

$$y = \frac{10}{x^2}$$

the graph is positive on both sides and symmetrical in the y-axis.

2 **This is Graph D**

This is Graph D.

3 **Solve equation**

For

$$y = x - 3 + 3x^2 - x^3$$

the leading term is $-x^3$, so it is a cubic falling to the right with two turning points.

4 **This is Graph C**

This is Graph C.

5 **Solve equation**

For

$$y = -\frac{3}{x}$$

the graph is a negative reciprocal graph in quadrants 2 and 4.

6 **This is Graph B**

This is Graph B.

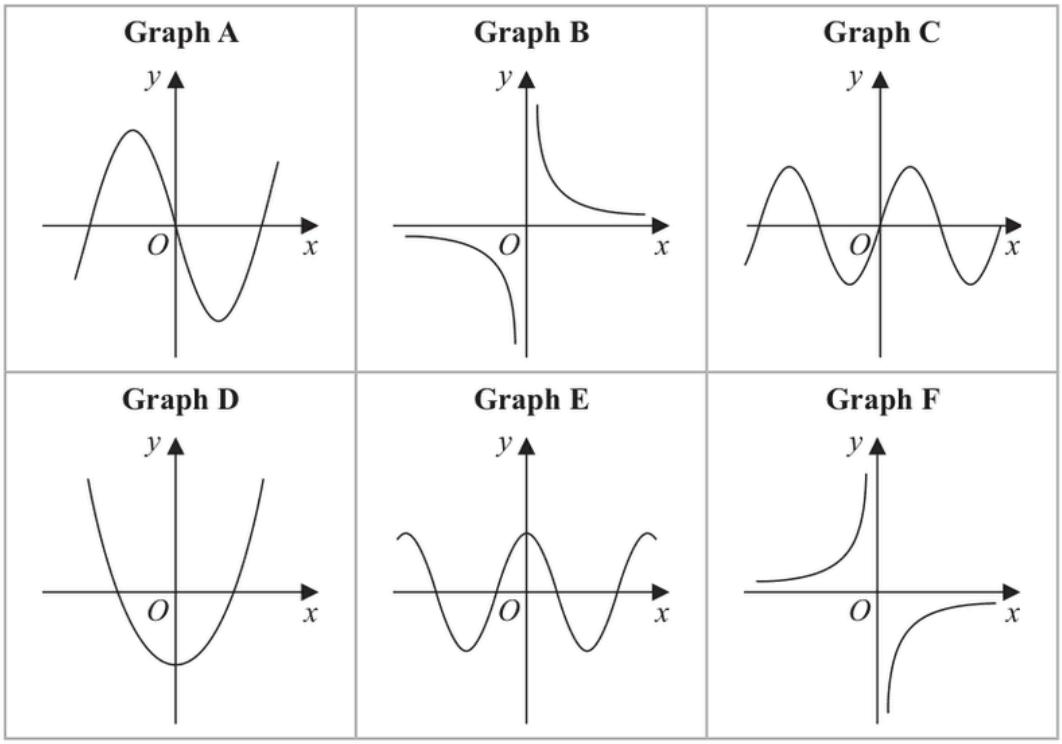
✓ **FINAL ANSWER**

D, C, B.

Question 18

3 marks | Graphs of Functions

18 Here are 6 graphs.



Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \sin x$	.....
$y = -\frac{3}{x}$	.....
$y = 4x^3 - 5x$	.....

(Total for Question 18 is 3 marks)

1 **is the periodic sine shaped**

$$y = \sin x$$

is the periodic sine-shaped graph, so it is **Graph C**.

2 **Read the graph**

$$y = \frac{3}{x}$$

is the positive reciprocal graph, so it is **Graph B**.

3 **Find turning point**

$$y = 4x^3 - 5x$$

is a cubic graph with two turning points, so it is **Graph A**.

✓ **FINAL ANSWER**

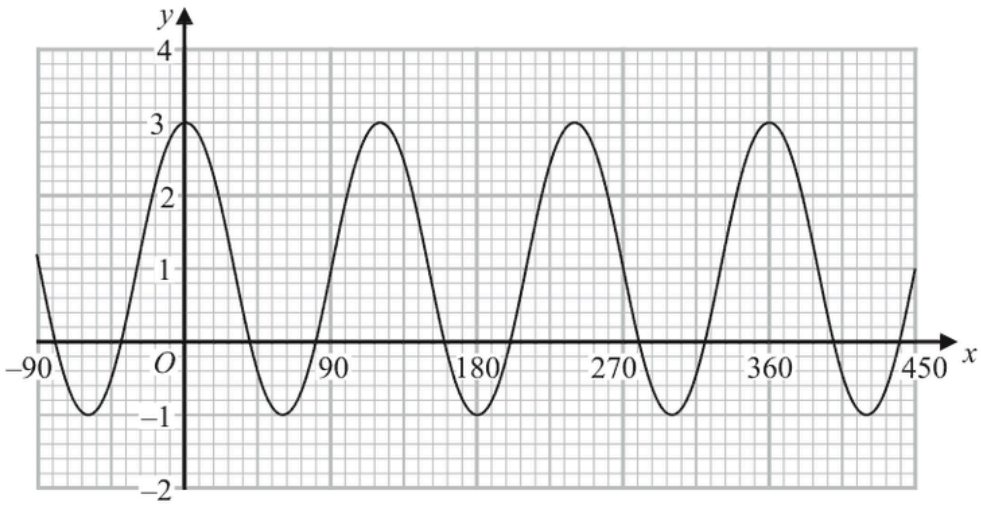
C, B, A.

4MA1/2HR JUNE 2023

Question 26

3 marks | Graphs of Functions

26 Here is a sketch of the curve with equation $y = a\cos bx° + c$ where $-90 \leqslant x \leqslant 450$



Find the value of a, the value of b and the value of c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2023

1 The curve is

The curve is

$$y = a\cos bx° + c$$

2 Use trigonometry

From the graph, the maximum value is

$$3$$

3 Use trigonometry

and the minimum value is

$$-1$$

4 **The amplitude is half the**

The amplitude is half the difference:

$$a = \frac{3 - (-1)}{2} = 2$$

5 **The midline is the average**

The midline is the average:

$$c = \frac{3 + (-1)}{2} = 1$$

6 **The distance between consecutive maxima**

The distance between consecutive maxima is $120°$, so the period is

$$120°$$

7 **Use trigonometry**

For $y = a\cos bx° + c$, the period is

$$\frac{360°}{b}$$

8 **Use trigonometry**

So

$$\frac{360}{b} = 120$$

$$b = 3$$

✓ **FINAL ANSWER**

$$\boxed{a = 2,\ b = 3,\ c = 1}$$

Question 12

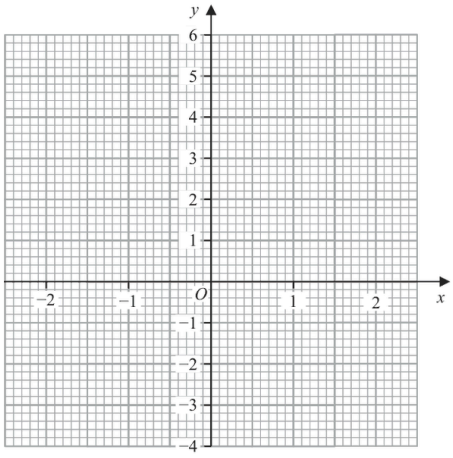
5 marks | Graphs of Functions

12 (a) Complete the table of values for $y = x^3 - 3x + 1$

x	−2	−1	0	1	2
y				−1	

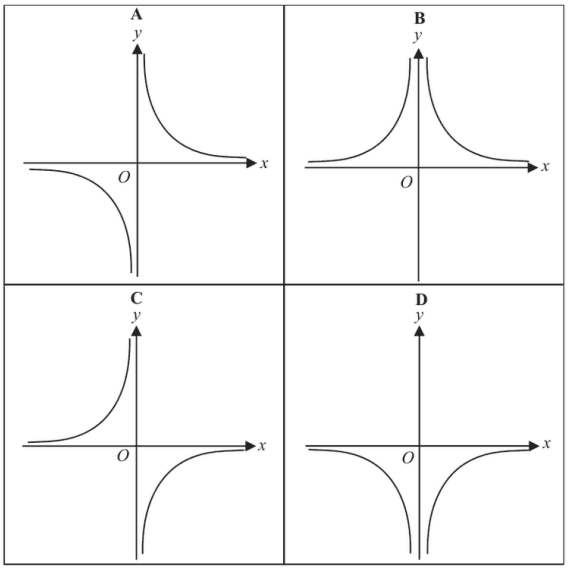
(2)

(b) On the grid, draw the graph of $y = x^3 - 3x + 1$ for values of x from − 2 to 2



(2)

Here are four graphs.



(c) Write down the letter of the graph that could have the equation $y = -\frac{1}{x^2}$

.....

(1)

(Total for Question 12 is 5 marks)

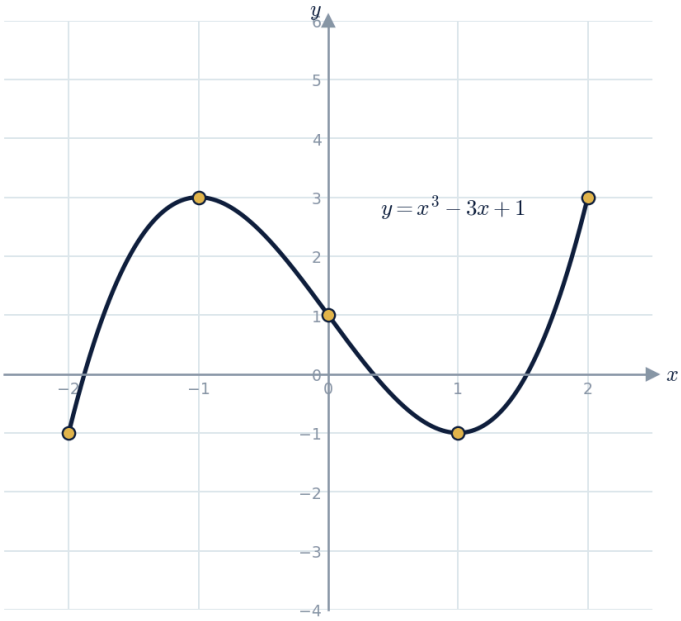
1 Read the graph

$$y = x^3 - 3x + 1$$

2 The completed table is

The completed table is:

x	-2	-1	0	1	2
y	-1	3	1	-1	3



3 The graph of is below

The graph of

$$y = -\frac{1}{x^2}$$

is below the x-axis on both sides and is symmetrical in the y-axis.

FINAL ANSWER

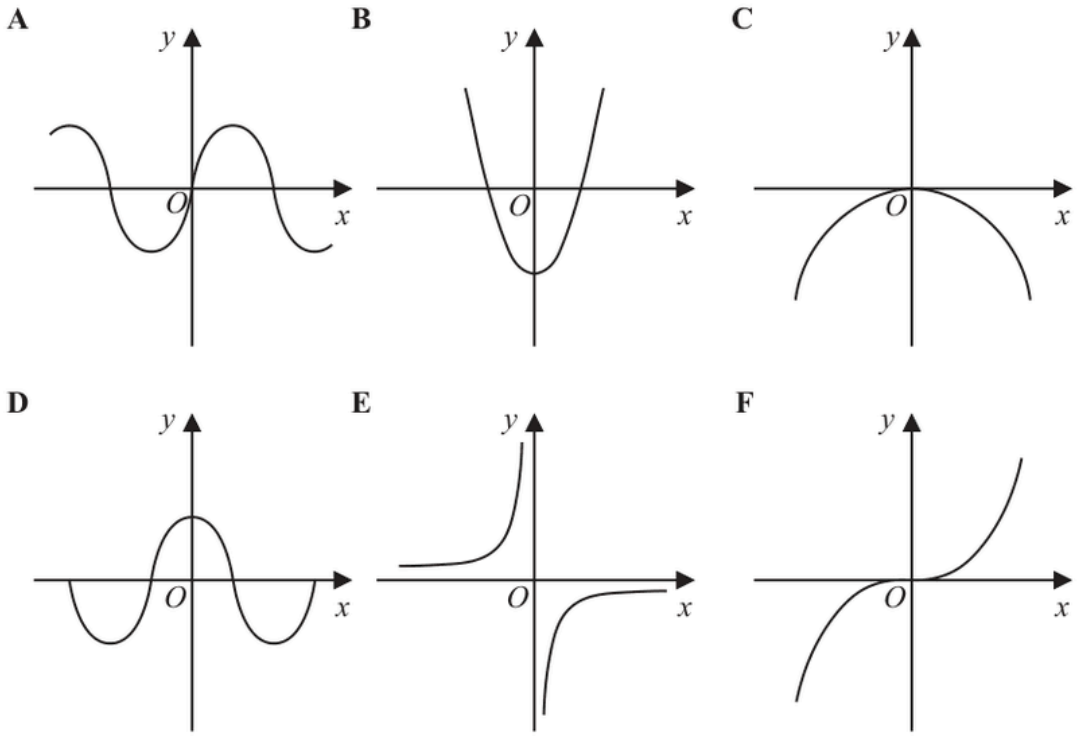
The graph letter is D.

4MA1/2HR JUNE 2024

Question 16

2 marks | Graphs of Functions

16 Here are six graphs.



Write down the letter of the graph that could have the equation

(i) $y = -\frac{1}{x}$

.....

(1)

(ii) $y = \sin x°$

.....

(1)

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2024

1 **(i) has branches in quadrants**

(i)

$$y = -\frac{1}{x}$$

has branches in quadrants II and IV, so it is **Graph E**.

2 **(ii) is the sine shaped**

(ii)

$$y = \sin x^\circ$$

is the sine-shaped graph through the origin, so it is **Graph A**.

✓ **FINAL ANSWER**

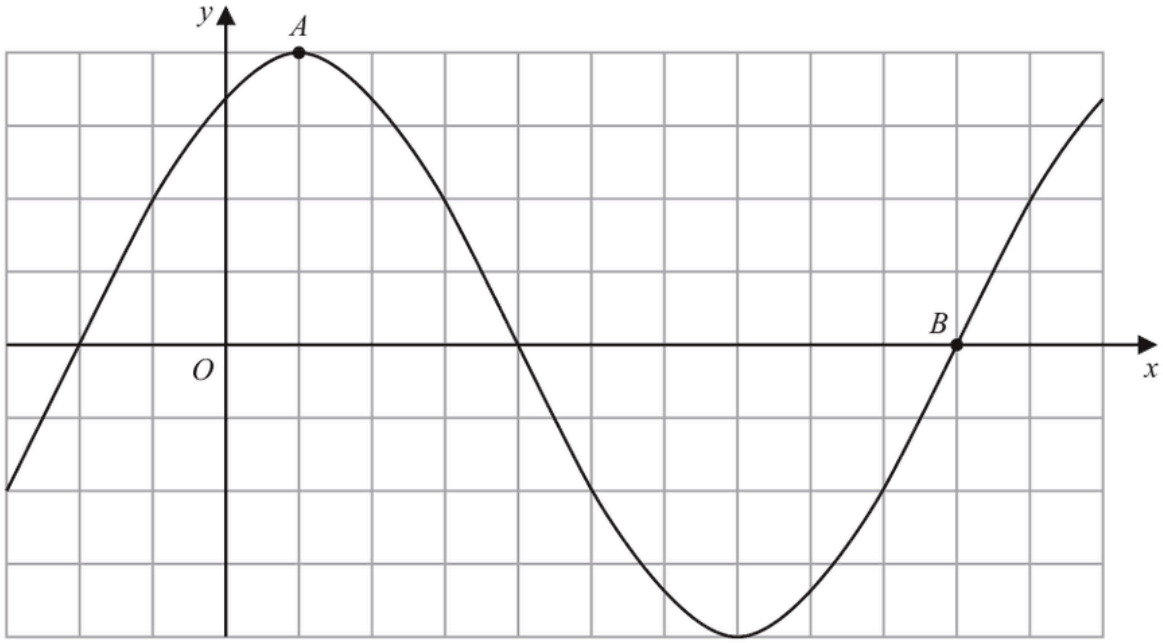
(i) E, (ii) A.

4MA1/2H JUNE 2024

Question 25

2 marks | Graphs of Functions

25 The diagram shows a sketch of the graph of $y = 2\sin(x + 60)°$



(i) Find the coordinates of the point A

(..... , ...)

(1)

(ii) Find the coordinates of the point B

(..... , ...)

(1)

(Total for Question 25 is 2 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2024

1 **Use trigonometry**

$$y = 2\sin(x + 60)°$$

2 **Point is the maximum point**

Point A is the maximum point.

3 **The maximum value is**

The maximum value is

$$y = 2$$

4 **This happens when**

This happens when

$$x + 60 = 90$$

$$x = 30$$

5 **Read the graph**

So

$$A = (30, 2)$$

6 **Point is the positive intercept**

Point B is the positive x-intercept after the minimum.

$$2\sin(x + 60)° = 0$$

$$\sin(x + 60)° = 0$$

7 **Read the graph**

For this point,

$$x + 60 = 360$$

$$x = 300$$

8 **Read the graph**

So

$$B = (300, 0)$$

✓ **FINAL ANSWER**

$A = (30, 2),\ B = (300, 0)$

Question 11

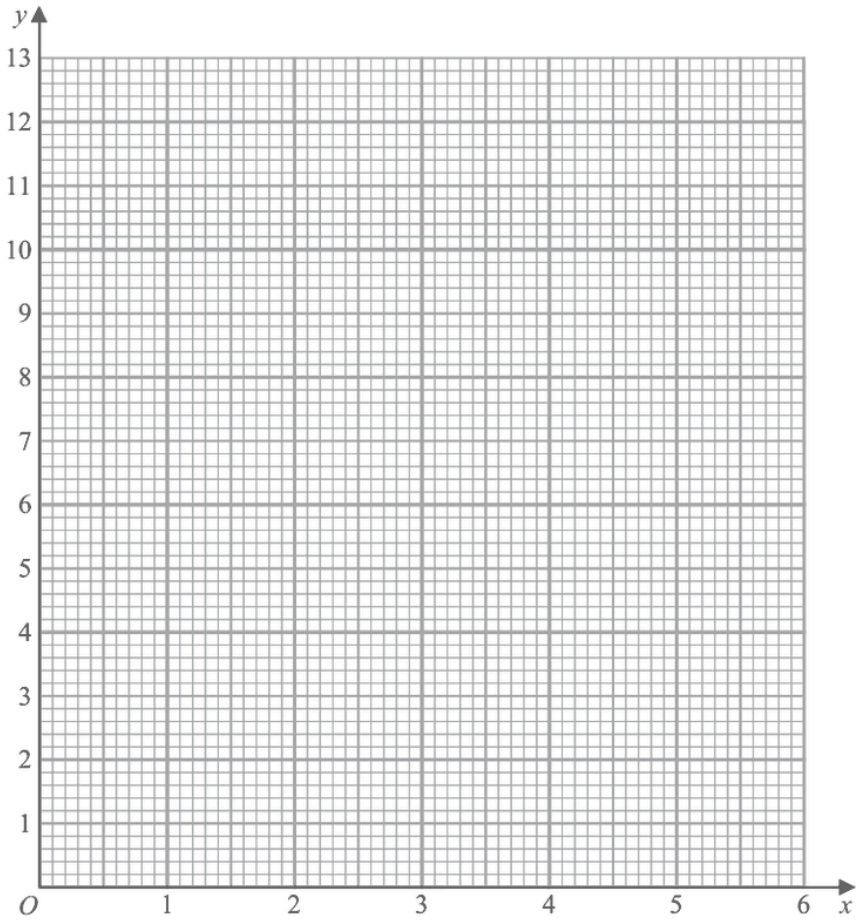
4 marks | Graphs of Functions

11 (a) Complete the table of values for $y = 2\left(x + \frac{1}{x}\right)$

x	0.5	1	2	3	4	5	6
y			5	6.7			12.3

(2)

(b) On the grid, draw the graph of $y = 2\left(x + \frac{1}{x}\right)$ for $0.5 \leqslant x \leqslant 6$



(2)

(Total for Question 11 is 4 marks)

1 Read the graph

$$y = 2\left(x + \frac{1}{x}\right)$$

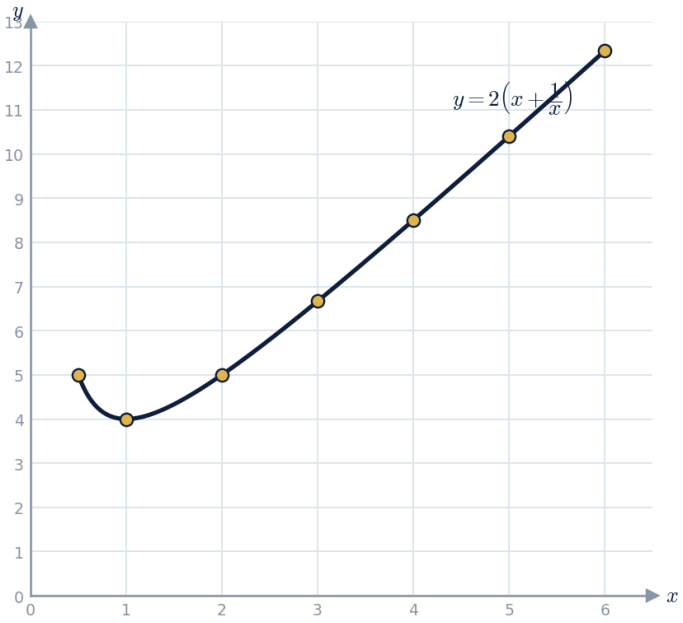
2 The completed table is

The completed table is:

x	0.5	1	2	3	4	5	6
y	5	4	5	6.7	8.5	10.4	12.3

3 Plot the points and join

Plot the points and join them with a smooth curve.



FINAL ANSWER

Missing values are $5, 4, 8.5, 10.4$.

4MA1/1H NOVEMBER 2024

Question 23

5 marks | Graphs of Functions

23 The curve **C** has equation $y = x^2 - 8x - 9$
The straight line **L** has equation $y = k$ where k is an integer.

C and **L** intersect at the points A and B

The coordinates of point A are (p, k)
The coordinates of point B are (q, k)

Given that $p - q = 14$

find the value of k
Show clear algebraic working.

$k =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2024

1 **The curve is**

The curve is

$$y = x^2 - 8x - 9$$

2 **The line is**

The line is

$$y = k$$

3 **Read the graph**

At the intersection points,

$$x^2 - 8x - 9 = k$$

$$x^2 - 8x - (9 + k) = 0$$

4 **Read the graph**

The two roots are p and q.

5 **Solve quadratic equation**

For this quadratic,

$$p + q = 8$$

6 **We are also given**

We are also given

$$p - q = 14$$

7 **Solve the two equations**

Solve the two equations:

$$2p = 22$$

$$p = 11$$

$$q = -3$$

8 **Read the graph**

Use one root in $y = x^2 - 8x - 9$:

$$k = 11^2 - 8(11) - 9$$

$$k = 121 - 88 - 9$$

$$k = 24$$

✓ **FINAL ANSWER**

$k = 24$

Question 10

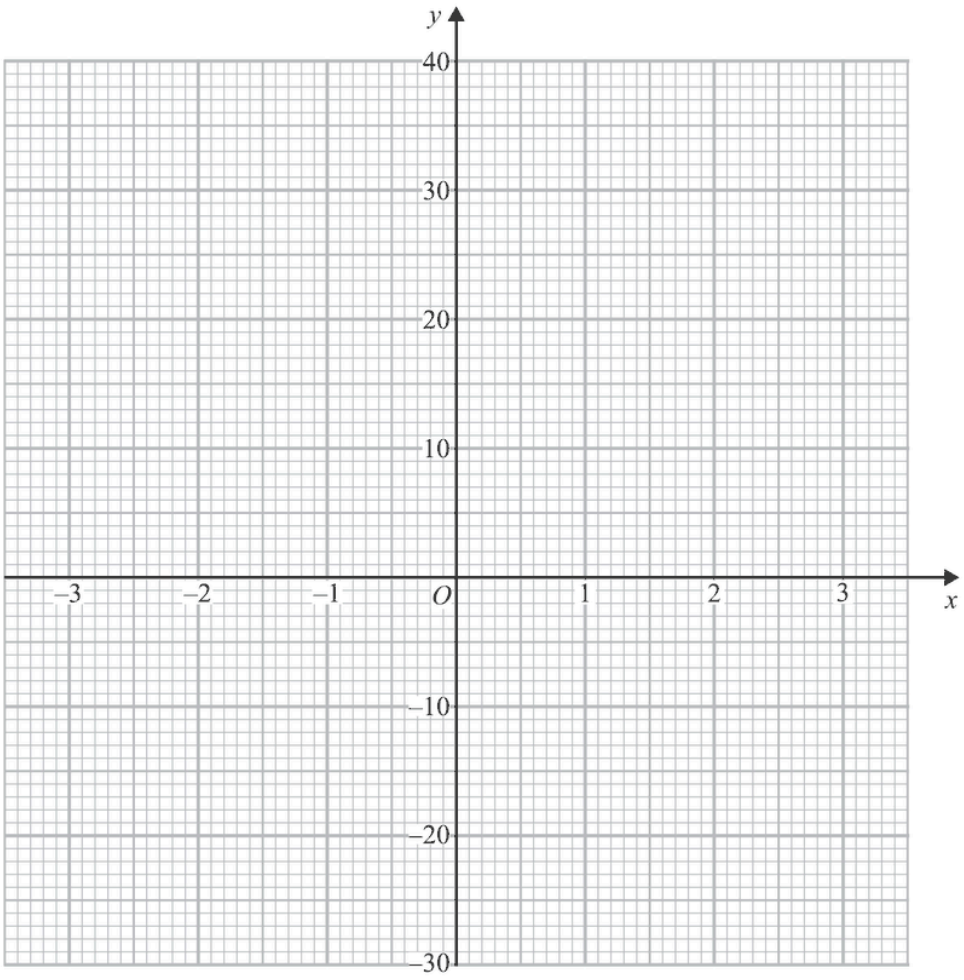
3 marks | Graphs of Functions

10 (a) Complete the table of values for $y = x^3 + 2x + 3$

x	–3	–2	–1	0	1	2	3
y		–9	0	3	6		36

(1)

(b) On the grid, draw the graph of $y = x^3 + 2x + 3$ for $-3 \leqslant x \leqslant 3$



(2)

(Total for Question 10 is 3 marks)

1 Read the graph

$$y = x^3 + 2x + 3$$

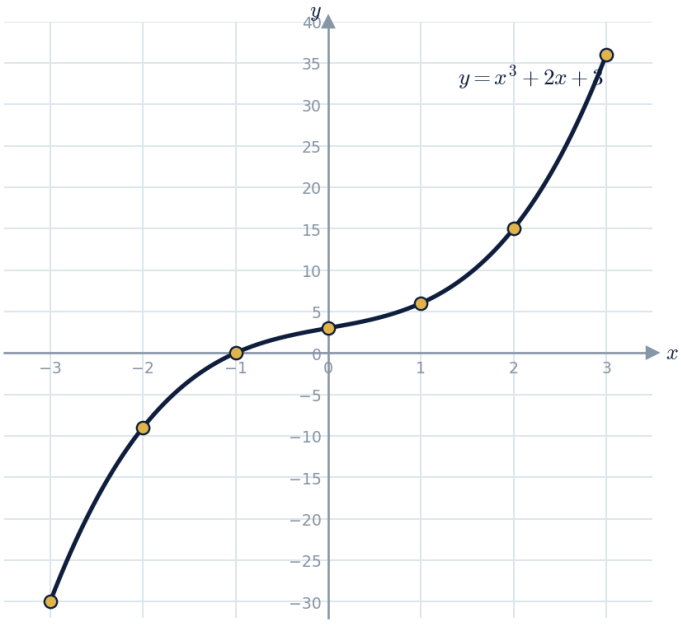
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	-30	-9	0	3	6	15	36

3 Plot the points and join

Plot the points and join them with a smooth cubic curve.



FINAL ANSWER

Missing values are $-30, 15$.

Question 13

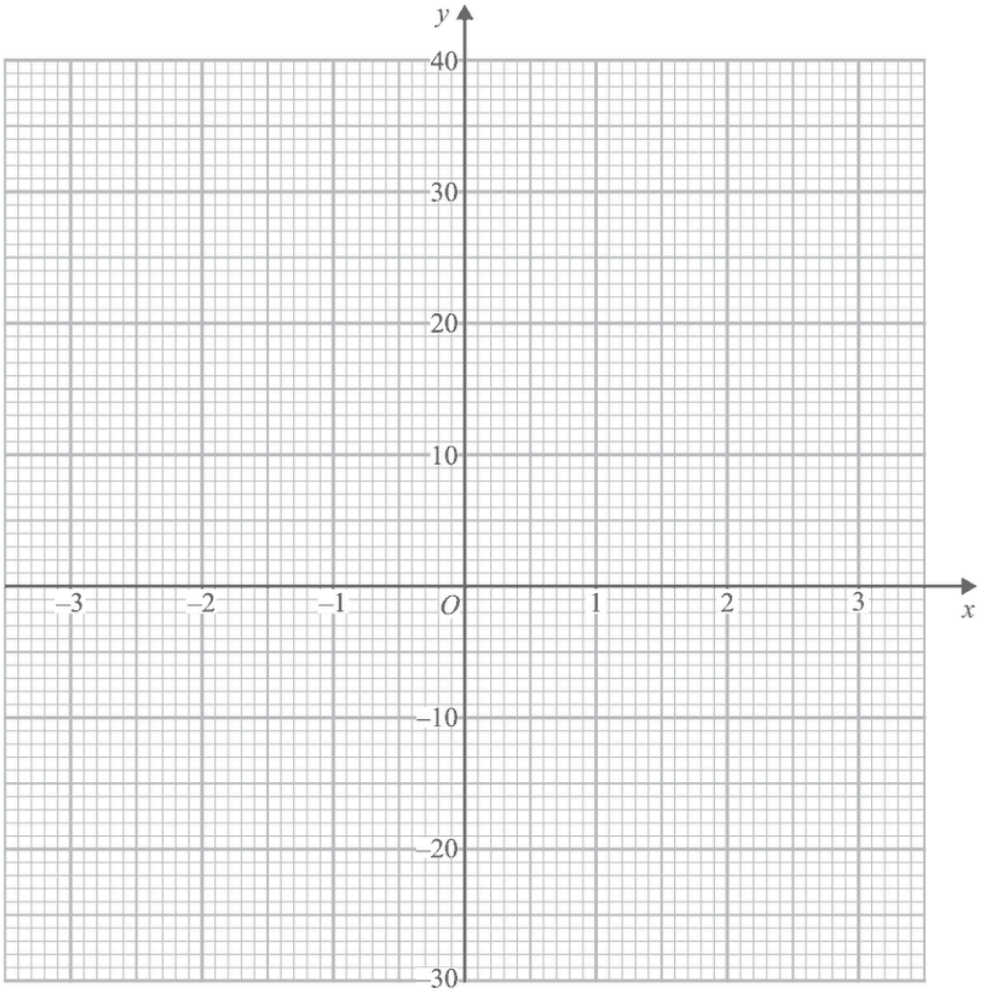
3 marks | Graphs of Functions

13 (a) Complete the table of values for $y = x^3 + 2x + 3$

x	–3	–2	–1	0	1	2	3
y		–9	0	3	6		36

(1)

(b) On the grid, draw the graph of $y = x^3 + 2x + 3$ for $-3 \leqslant x \leqslant 3$



(2)

(Total for Question 13 is 3 marks)

1 **Use**

Use $y = x^3 + 2x + 3$.

2 **Read the graph**

For $x = -3$,

$$y = (-3)^3 + 2(-3) + 3 = -30$$

3 **Read the graph**

For $x = 2$,

$$y = 2^3 + 2(2) + 3 = 15$$

4 **Read the graph**

So the table is

x	-3	-2	-1	0	1	2	3
y	-30	-9	0	3	6	15	36

5 **Plot these points and draw**

Plot these points and draw a smooth cubic curve through them.



✓ **FINAL ANSWER**

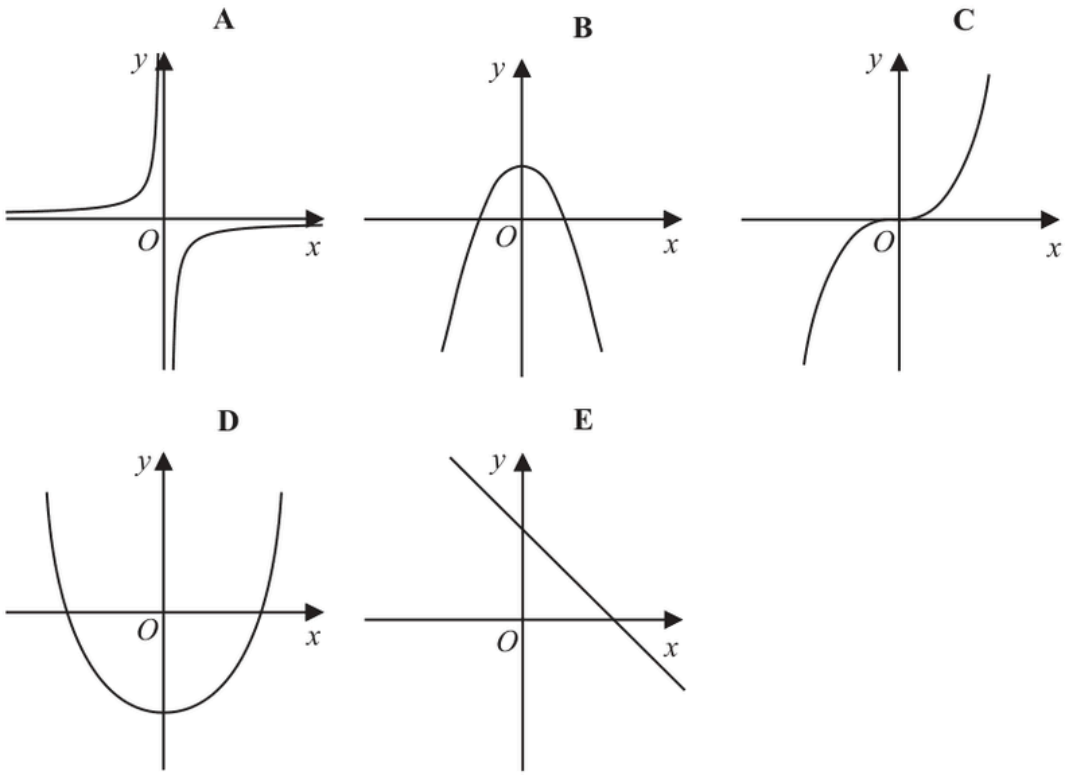
Missing values -30 and 15; draw the smooth curve through the table points.

4MA1/2H JUNE 2025

Question 14

2 marks | Graphs of Functions

14 Here are five graphs.



(a) Write down the letter of the graph that could have the equation $y = 5 - x^2$

.....

(1)

(b) Write down the letter of the graph that could have the equation $y = 2x^3$

.....

(1)

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2025

1 **(a)**

(a)

$$y = 5 - x^2$$

2 **This is a quadratic graph**

This is a quadratic graph with a negative coefficient of x^2, so it is a downward-opening parabola.

3 **That is graph B**

That is graph **B**.

4 **(b)**

(b)

$$y = 2x^3$$

5 **This is an increasing cubic**

This is an increasing cubic graph passing through the origin.

6 **That is graph C**

That is graph **C**.

✓ **FINAL ANSWER**

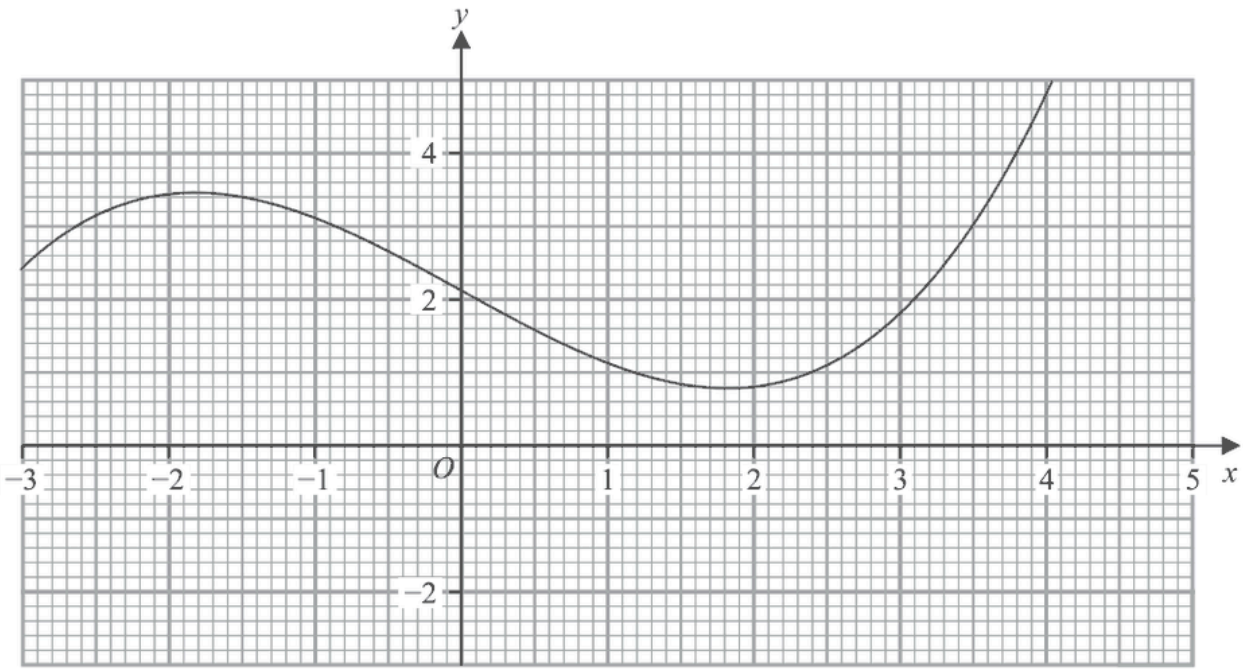
(a) B, (b) C.

4WM2/01R MAY 2025

Question 19

3 marks | Graphs of Functions

19 The diagram shows the graph of $y = f(x)$



Find an estimate for the gradient of the graph at the point where $x = 3$
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM2/01R May 2025

1 **Required**

Estimate the gradient of *y* = *f*(*x*) at the point where *x* = 3.

2 **Draw a tangent to the**

Draw a tangent to the curve at *x* = 3. From the graph, a suitable tangent passes through approximately (1, −2) and (5, 6):

3 **Calculate gradient of the tangent**

Calculate gradient of the tangent:

$$textGradient = fracDeltayDeltax = frac6 - (-2)5 - 1 = frac84 = 2$$

4 **Note**

Answers in the range 1 to 3 are typically accepted. Draw the tangent triangle as large as possible for the most accurate estimate.

✓ **FINAL ANSWER**

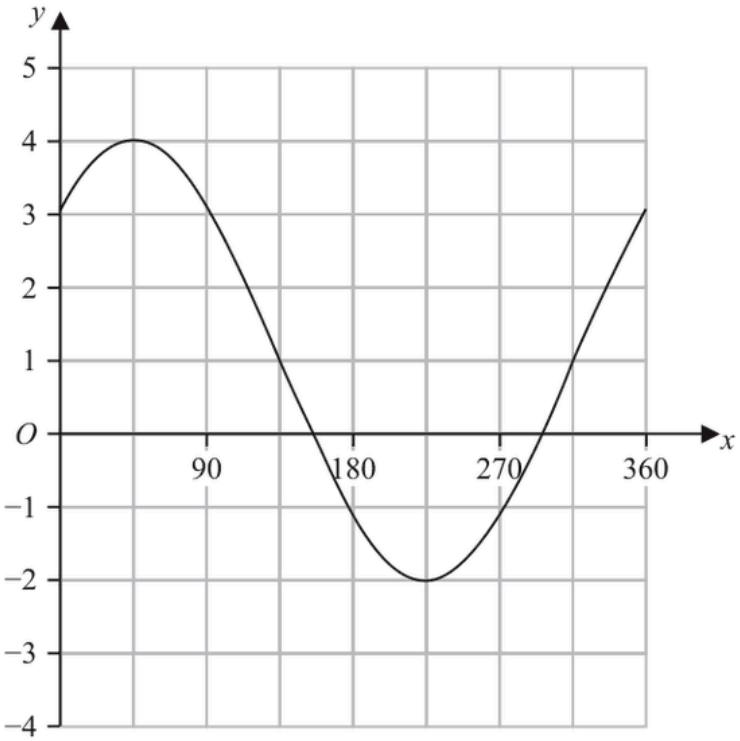
Gradient ≈ 2 at *x* = 3

4WM2/01R MAY 2025

Question 23

3 marks | Graphs of Functions

23 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01R May 2025

1 **Required**

The graph of *y* = *a*sin(*x*+*b*)[∘] + *c* for 0 ≤ *x* ≤ 360 is shown. Find suitable values for *a*, *b*, and *c*.

2 **Read the maximum and minimum**

Read the maximum and minimum values from the graph to find the **amplitude** *a* and **vertical shift** *c*:

$$a = \frac{\text{max} - \text{min}}{2} = \frac{4 - (-2)}{2} = 3 \quad\quad c = \frac{\text{max} + \text{min}}{2} = \frac{4 + (-2)}{2} = 1$$

3 Find the phase shift b

Find the **phase shift** *b*. The standard sin has its peak at *x* = 90°. From the graph, the maximum occurs at *x* ≈ 45°:

x + *b* = 90° ⟹ 45° + *b* = 90° ⟹ *b* = 45°

---

4 Verify at x 0 consistent

Verify at *x* = 0: $y = 4sin(0 + 45)^\circ = 4times fracsqrt22approx2.83$ — consistent with graph

---

5 Note

The period of the graph is still 360° (unchanged), so no horizontal stretch is applied.

✓ **FINAL ANSWER**

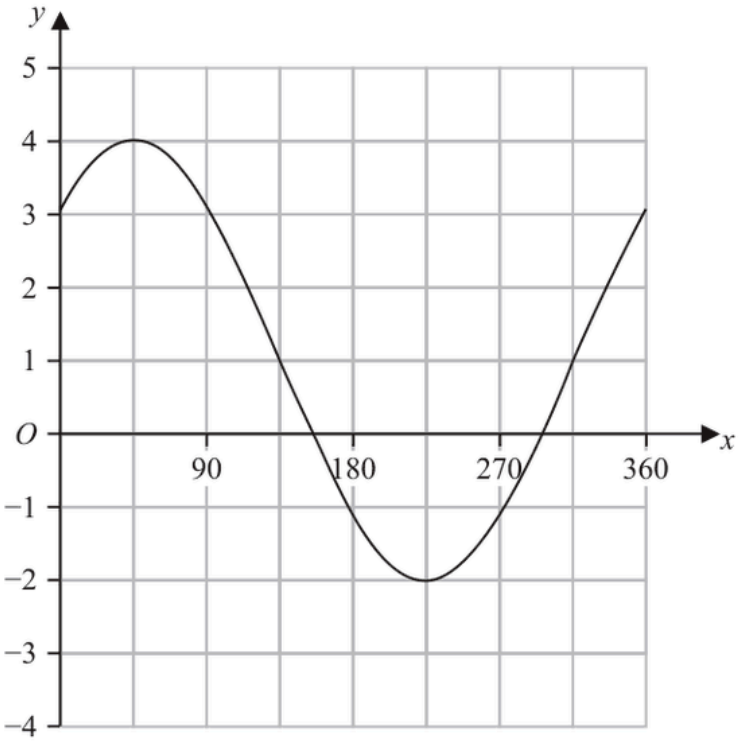
a = 3,*b* = 45,*c* = 1

4MA1/2HR JUNE 2025

Question 25

3 marks | Graphs of Functions

25 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2025

1 **Read the graph**

From the graph,

$$\text{maximum} = 4$$

$$\text{minimum} = -2$$

2 **The amplitude is**

The amplitude is

$$a = \frac{4 - (-2)}{2} = 3$$

3 **The centre line is**

The centre line is

$$c = \frac{4 + (-2)}{2} = 1$$

4 **Use trigonometry**

The maximum is about $45°$. For a sine graph, a maximum happens when the angle inside the sine is $90°$, so

$$45 + b = 90$$

$$b = 45$$

✓ **FINAL ANSWER**

$a = 3,\ b = 45,\ c = 1$

Question 12

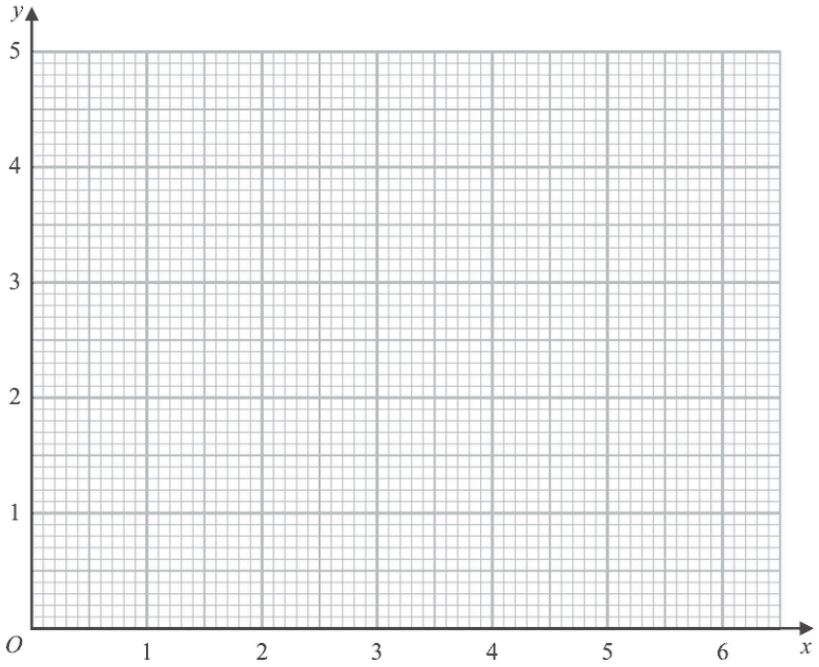
5 marks | Graphs of Functions

12 (a) Complete the table of values for $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$

x	0.5	1	2	3	4	5	6
y	4.25			2.17	2.5	2.9	3.33

(1)

(b) Draw the graph of $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$ for $0.5 \leqslant x \leqslant 6$



(2)

(c) By drawing a suitable line on the grid, find estimates for the solutions of the equation $x + \frac{4}{x} = 6$

Give your answers correct to one decimal place.

.....

(2)

(Total for Question 12 is 5 marks)

1 **Read the graph**

$$y = \frac{1}{2}\left(x + \frac{4}{x}\right)$$

2 **The completed table is**

The completed table is:

x	0.5	1	2	3	4	5	6
y	4.25	2.5	2	2.17	2.5	2.9	3.33

3 **Read the graph**

For

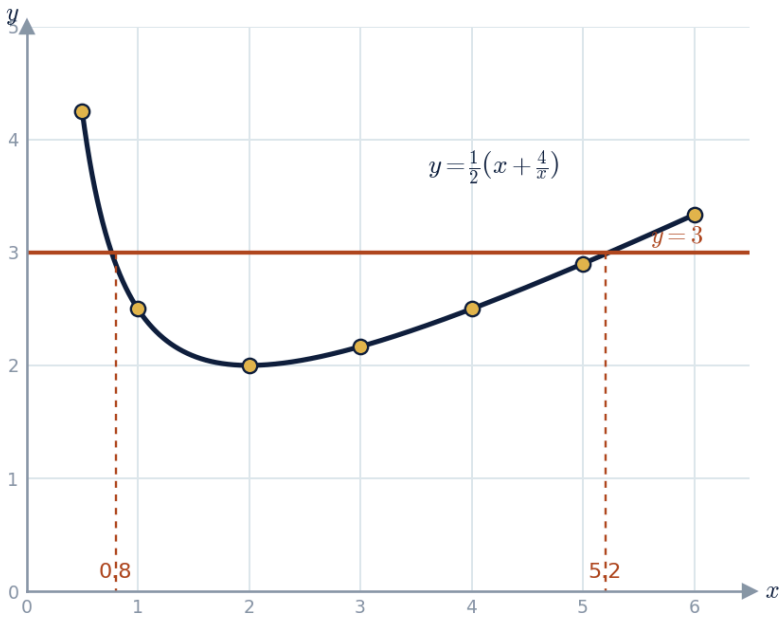
$$x + \frac{4}{x} = 6$$

we need

$$\frac{1}{2}\left(x + \frac{4}{x}\right) = 3$$

4 **Read the graph**

So draw the horizontal line $y = 3$ on the graph.



5 **The intersections give approximately**

The intersections give approximately

$$x = 0.8 \quad \text{and} \quad x = 5.2$$

FINAL ANSWER

Table missing values $2.5, 2$. Estimates 0.8 and 5.2.

Question 25

3 marks | Graphs of Functions

25 A curve has equation $y = f(x)$

There is one minimum point on this curve.

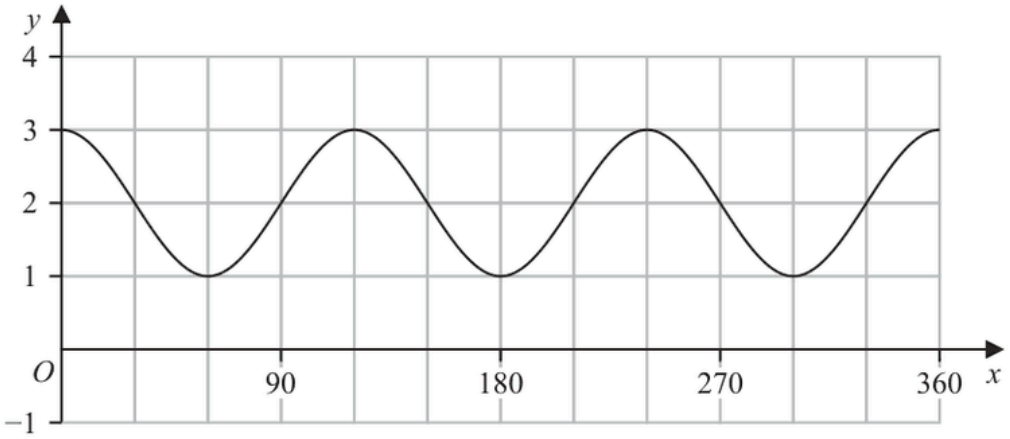
The coordinates of this minimum point are $(8, 1)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(4x)$

(..... , ...)

(1)

The diagram shows the graph of $y = \cos(ax) + b$ for $0 \leqslant x \leqslant 360$



(b) (i) Find the value of a

$a =$ .....

(1)

(ii) Find the value of b

$b =$ .....

(1)

(Total for Question 25 is 3 marks)

1 The transformation y f (4

The transformation *y* = *f*(4*x*) represents a horizontal stretch by a scale factor of $frac14$. The x-coordinate is multiplied by $frac14$ while the y-coordinate stays the same.

$$textNewminimum = leftbigl(8timesfrac14, 1rightbigr) = (2, 1)$$

2 Answer

(2,1)

Step 1: Find the vertical shift *b*. The standard *y* = cos (*x*) graph fluctuates between − 1 and 1, centered at *y* = 0. The given graph fluctuates between a minimum of *y* = 1 and a maximum of *y* = 3. The center line is $frac3 + 12 = 2$. Therefore, *b* = 2.

Step 1: Find the frequency multiplier *a*. The period of the given graph completes one full cycle between *x* = 0 and *x* = 90[∘] (it starts at 3, dips to 1, and returns to 3 at 90[∘]). The standard cosine period is 360[∘].

Missing open brace for superscript

Missing open brace for superscript

FINAL ANSWER

a = 4, *b* = 2

Question 13

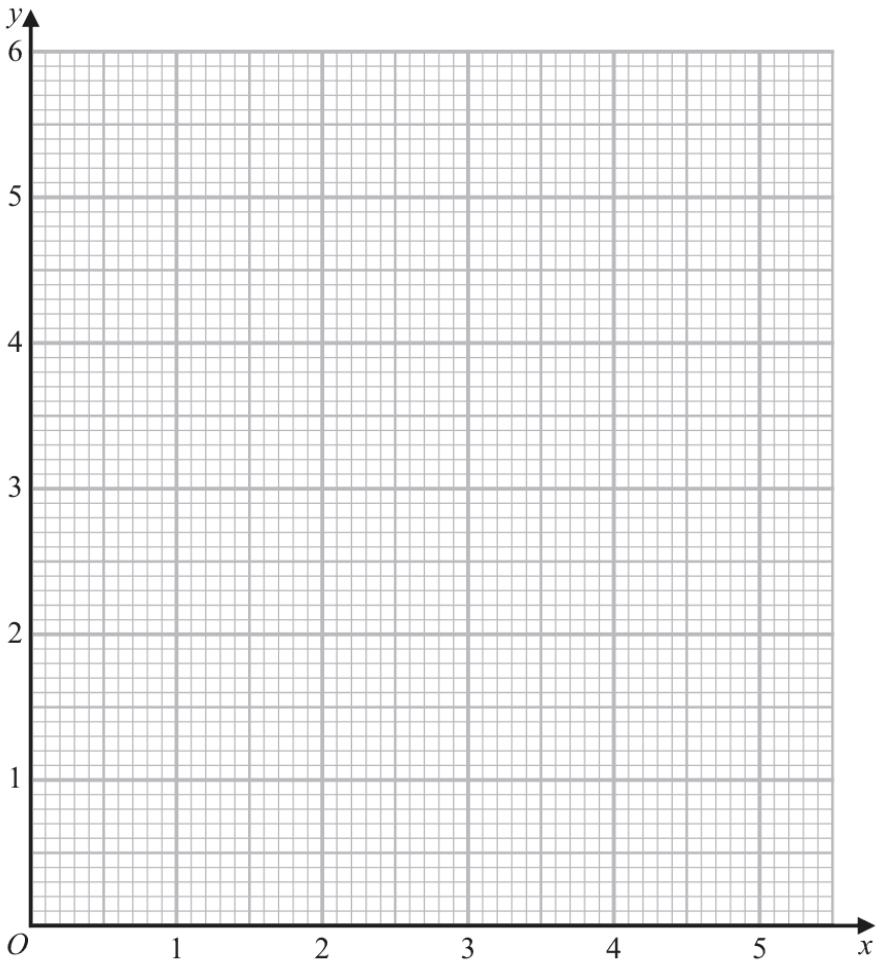
4 marks | Graphs of Functions

13 (a) Complete the table of values for $y = x + \frac{1}{x}$

x	0.25	0.5	1	2	3	4	5
y	4.25			2.5	3.33		

(2)

(b) On the grid, draw the graph of $y = x + \frac{1}{x}$ for $0.25 \leqslant x \leqslant 5$



(2)

(Total for Question 13 is 4 marks)

1 Part (a) | Step 1

Use $y = x + 1/x$:

$$y(0.5) = 0.5 + 2 = 2.5, \quad y(1) = 1 + 1 = 2, \quad y(4) = 4 + 0.25 = 4.25, \quad y(5) = 5 + 0.2 = 5.2.$$

Substitute each required x-value into the formula.

2 Part (b) | Step 1

Plot

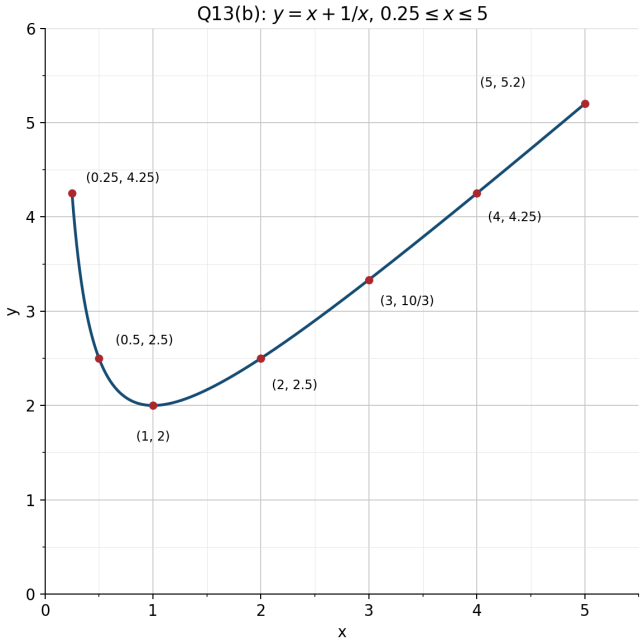
$$(0.25, 4.25),\ (0.5, 2.5),\ (1, 2),\ (2, 2.5),\ (3, 10/3),\ (4, 4.25),\ (5, 5.2).$$

Use x horizontally and y vertically on the supplied grid; 10/3 is approximately 3.33.

3 Part (b) | Step 2

Join the points with one smooth curve for $0.25 \le x \le 5$, descending to its minimum $(1, 2)$ and then ascending.

The graph is curved, not a set of straight segments, and includes both endpoints.



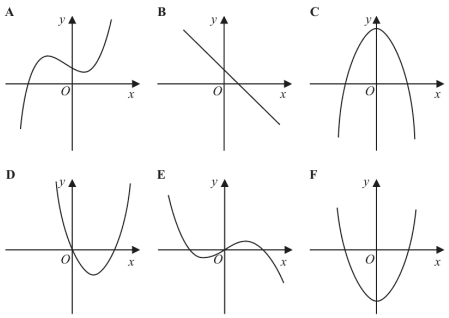
FINAL ANSWER

(a) Table entries, in order: $2.5, 2, 4.25, 5.2$. (b) The smooth curve $y = x + 1/x$ through the stated coordinates on $[0.25, 5]$.

Question 14

6 marks | Graphs of Functions

14 Here are six graphs.



(a) Write down the letter of the graph that could have the equation

(i) $y = x^2 - 3x$

.....

(1)

(ii) $y = -x^3 + 9x$

.....

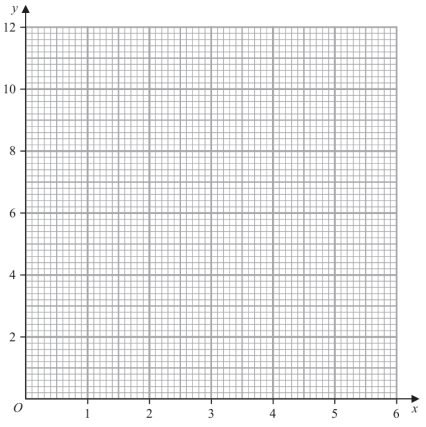
(1)

(b) Complete the table of values for $y = \frac{12}{x^2}$

x	1	1.5	2	3	4	5	6
y		5.33		1.33			0.33

(2)

(c) On the grid, draw the graph of $y = \frac{12}{x^2}$ for $1 \leqslant x \leqslant 6$



(2)

(Total for Question 14 is 6 marks)

1 **(a)(i) Positive quadratic**

$y = x^2 - 3x = x(x-3)$: U-shape through O and $x = 3$ → graph D.

2 **(a)(ii) Negative cubic**

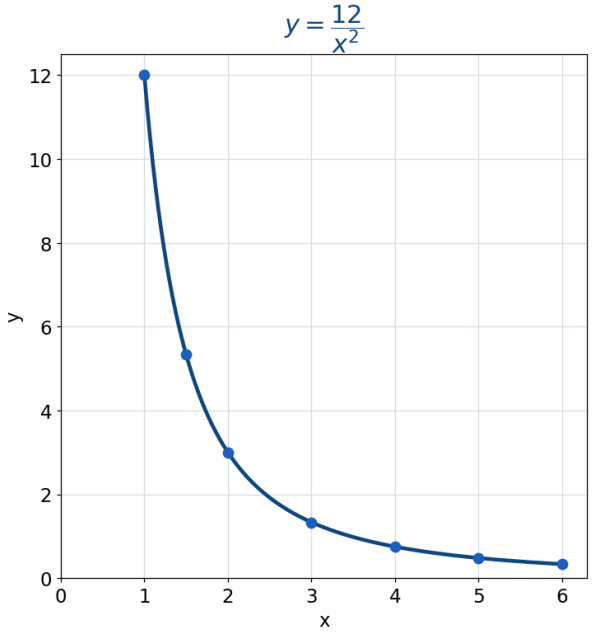
$y = -x^3 + 9x$: negative cubic through the origin → graph E.

3 **(b) Substitute**

$$x = 1 : \ 12, \quad x = 2 : \ 3, \quad x = 4 : \ 0.75, \quad x = 5 : \ 0.48$$

4 **(c) Plot and join with a smooth curve**

Plot all 7 points and draw a smooth curve (no straight segments).



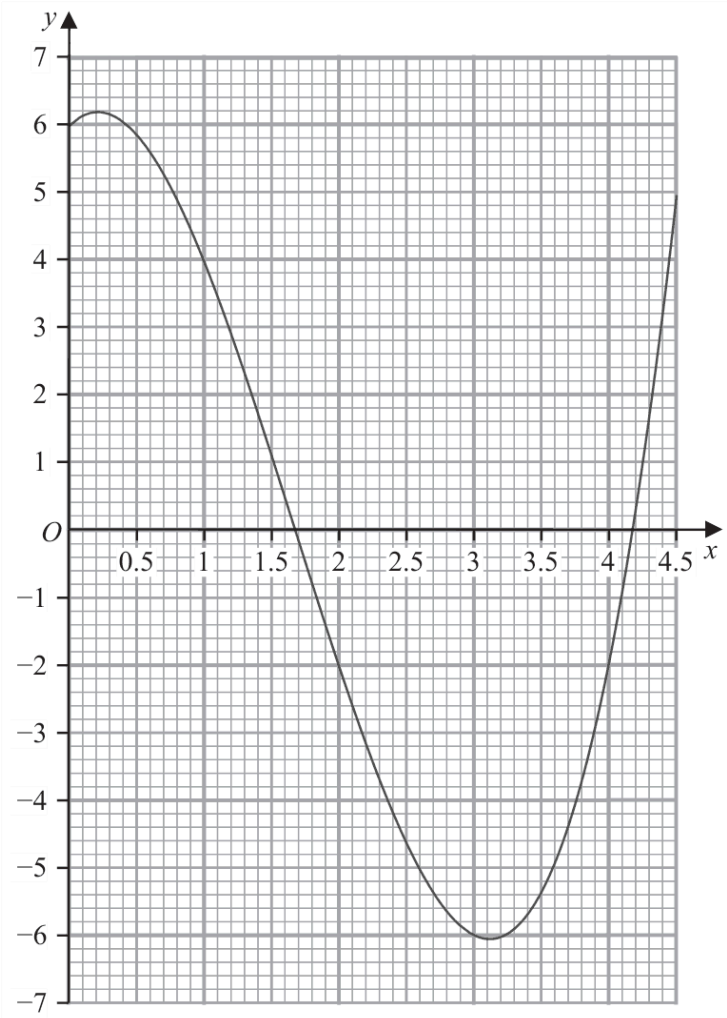
FINAL ANSWER

(a)(i) D (ii) E (b) $12,\ 3,\ 0.75,\ 0.48$ (c) smooth curve through the points

Question 21

5 marks | Graphs of Functions

21 The curve with equation $y = f(x)$ is drawn on a grid.



(a) By drawing a suitable straight line (at least 3 cm long) on the grid, work out an estimate for the gradient of the curve at the point where $x = 1$
Show your working clearly.

.....

(2)

(b) By drawing a suitable straight line on the grid, find the solutions of the equation $x = 3 + \frac{1}{2}f(x)$
Show your working clearly.

.....

(3)

(Total for Question 21 is 5 marks)

1 **Part (a) | Step 1**

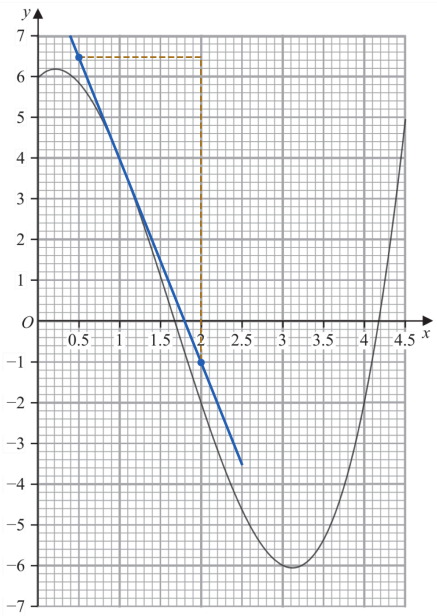
At $x = 1$, the point is approximately $(1, 4)$. Draw a tangent there, extending at least $3\ \text{cm}$. A suitable approximate tangent is $y = -5x + 9$, through $(0.5, 6.5)$ and $(2, -1)$.

The gradient at a point is estimated by the gradient of its tangent; use two well-separated points on the line.

2 **Part (a) | Step 2**

$$\text{Gradient} \approx \frac{-1 - 6.5}{2 - 0.5} = \frac{-7.5}{1.5} = -5.$$

Calculate change in y divided by change in x using the axis values.



3 **Part (b) | Step 1**

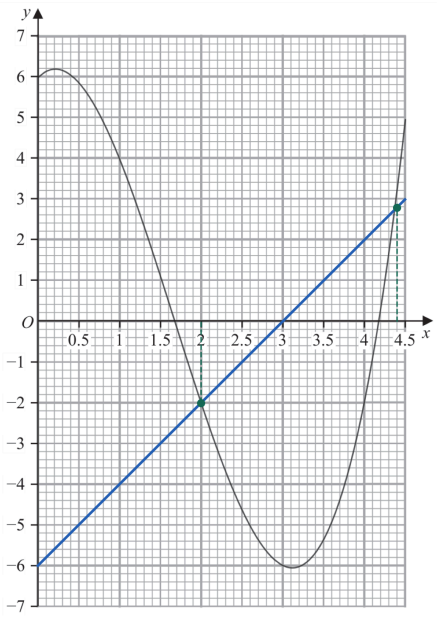
$$x = 3 + \tfrac{1}{2} f(x) \quad \Rightarrow \quad f(x) = 2x - 6.$$

Rearrange so the given curve y=f(x) can be intersected by a straight line.

4 **Part (b) | Step 2**

Draw $y = 2x - 6$ through $(0, -6), (3, 0), (4.5, 3)$ on the same grid. Read the x-coordinates of its two intersections with the curve: approximately 2.0 and 4.4.

Each intersection has equal y-values on the line and curve and therefore solves the original equation.



FINAL ANSWER

(a) Gradient ≈ -5. (b) $x \approx 2.0$ or $x \approx 4.4$.

TOPIC

Functions

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 13

4 marks | Functions

13 The function f is defined as

$$f : x \mapsto \frac{2x}{x-6}$$

(a) Find f(10)

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1} : x \mapsto \ldots$

$f^{-1} : x \mapsto$ .....

(3)

WORKED SOLUTION

Question 13 · 4WM2/01 Specimen

1 **Evaluate fraction**

$$f(10) = \frac{2(10)}{10-6} = \frac{20}{4} = 5$$

2 **Let**

For the inverse, let

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$xy - 6y = 2x$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

3 **Evaluate fraction**

So

$$f^{-1} : x \mapsto \frac{6x}{x-2}$$

FINAL ANSWER

5, and $f^{-1} : x \mapsto \frac{6x}{x-2}$.

4MA1/1HR JANUARY 2020

Question 17

4 marks | Functions

17 The function f is such that $f(x) = (x - 4)^2$ for all values of x.

(a) Find f(1)

.....

(1)

(b) State the range of the function f.

.....

(1)

The function g is such that $g(x) = \dfrac{4}{x+3}$ $x \neq -3$

(c) Work out fg(2)

.....

(2)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR January 2020

1 **Calculate value**

$$f(x) = (x - 4)^2$$

2 **(a)**

(a)

$$f(1) = (1 - 4)^2 = 9$$

3 **(b) A square is always**

(b) A square is always greater than or equal to zero, so the range is

$$f(x) \geq 0$$

4 **(c)**

(c)

$$g(x) = \frac{4}{x+3}$$

$$g(2) = \frac{4}{2+3} = \frac{4}{5}$$

FINAL ANSWER

$f(1) = 9$, range $f(x) \geq 0$, and $g(2) = \frac{4}{5}$.

4MA1/1H JANUARY 2020

Question 21

5 marks | Functions

21 The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2020

1 **Calculate value**

$$f(x) = x^2 - 2x$$

$$g(x) = x + 3$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = (x+3)^2 - 2(x+3)$$

$$h(x) = x^2 + 4x + 3$$

$$h(x) = (x+2)^2 - 1$$

2 **Let**

Let

$$y = (x+2)^2 - 1$$

3 Find inverse function

Since the domain is $x \geq -2$, take the positive square root:

$$x + 2 = \sqrt{y + 1}$$

$$x = \sqrt{y + 1} - 2$$

FINAL ANSWER

$$h^{-1}(x) = \sqrt{x + 1} - 2$$

4MA1/1HR MAY/NOVEMBER 2020

Question 16

9 marks | Functions

16 The functions f and g are defined as

$$f: x \mapsto 5x - 7$$

$$g: x \mapsto \frac{5x}{x+4}$$

(a) Write down the value of x that must be excluded from any domain of g

.....

(1)

(b) Find gf(2.6)

.....

(2)

(c) Solve $fg(x) = 2$

$x =$ .....

(3)

(d) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto ...$

$g^{-1}: x \mapsto$ .....

(3)

(Total for Question 16 is 9 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May/November 2020

1 **Evaluate fraction**

$$f(x) = 5x - 7, \qquad g(x) = \frac{5x}{x+4}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero, so

$$x = -4$$

3 **(b)**

(b)

$$f(2.6) = 5(2.6) - 7 = 6$$

$$gf(2.6) = g(6) = \frac{5(6)}{6+4} = 3$$

---

4 **(c)**

(c)

$$fg(x) = 2$$

$$f(g(x)) = 2$$

$$5g(x) - 7 = 2$$

$$g(x) = \frac{9}{5}$$

$$\frac{5x}{x+4} = \frac{9}{5}$$

$$25x = 9x + 36$$

$$16x = 36$$

$$x = \frac{9}{4}$$

---

5 **(d) Let**

(d) Let

$$y = \frac{5x}{x+4}$$

$$y(x+4) = 5x$$

$$yx + 4y = 5x$$

$$x(y-5) = -4y$$

$$x = \frac{4y}{5-y}$$

6 **Evaluate fraction**

So

$$g^{-1}(x) = \frac{4x}{5-x}$$

FINAL ANSWER

-4, 3, $x = \frac{9}{4}$, and $g^{-1}(x) = \frac{4x}{5-x}$.

4MA1/2HR JANUARY 2021

Question 17

9 marks | Functions

17 The functions f and g are defined as

$f(x) = x^2 + 6$
$g(x) = x - 10$

(a) Find fg(3)

.....

(2)

(b) Solve the equation $fg(x) = f(x)$
Show clear algebraic working.

.....

(3)

The function h is defined as

$$h(x) = \frac{2x - 4}{x}$$

(c) State the value of x that cannot be included in the domain of h

.....

(1)

(d) Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(3)

(Total for Question 17 is 9 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR January 2021

1 **Solve quadratic equation**

$$f(x) = x^2 + 6, \qquad g(x) = x - 10$$

2 **(a)**

(a)

$$g(3) = 3 - 10 = -7$$

$$fg(3) = f(-7) = (-7)^2 + 6 = 55$$

3 **(b)**

(b)

$$fg(x) = f(g(x)) = (x-10)^2 + 6$$

4 **Solve**

Solve $fg(x) = f(x)$:

$$(x-10)^2 + 6 = x^2 + 6$$

$$x^2 - 20x + 100 = x^2$$

$$-20x + 100 = 0$$

$$x = 5$$

5 **(c)**

(c)

$$h(x) = \frac{2x-4}{x}$$

6 **Solve quadratic equation**

The denominator cannot be zero, so $x = 0$ cannot be included.

7 **(d) Let**

(d) Let

$$y = \frac{2x-4}{x}$$

$$y = 2 - \frac{4}{x}$$

$$y - 2 = -\frac{4}{x}$$

$$x = \frac{4}{2-y}$$

8 **Solve quadratic equation**

Therefore

$$h^{-1}(x) = \frac{4}{2-x}$$

FINAL ANSWER

$fg(3) = 55$, $x = 5$, excluded value 0, and $h^{-1}(x) = \dfrac{4}{2 - x}$.

4MA1/1H JANUARY 2021

Question 22

3 marks | Functions

22 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2021

1 Calculate value

$$f(x) = x^2 - 8x + 5$$

$$f(x) = (x-4)^2 - 11$$

2 Let

Let

$$y = (x-4)^2 - 11$$

$$y + 11 = (x-4)^2$$

3 Find inverse function

Since the domain is $x < 4$, take the negative square root:

$$x - 4 = -\sqrt{y+11}$$

$$x = 4 - \sqrt{y+11}$$

FINAL ANSWER

$$f^{-1}(x) = 4 - \sqrt{x+11}$$

4MA1/1H MAY 2021

Question 24

6 marks | Functions

24 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2021

1 **(a)**

(a)

$$g(2) = 7(2) - 6 = 8$$

$$fg(2) = f(8)$$

$$f(8) = 5(8)^2 - 10(8) + 7$$

$$f(8) = 247$$

2 **(b)**

(b)

$$f(x) = 5x^2 - 10x + 7$$

$$f(x) = 5(x - 1)^2 + 2$$

3 **Let**

Let

$$y = 5(x-1)^2 + 2$$

$$(x-1)^2 = \frac{y-2}{5}$$

---

4 **Find inverse function**

Since the domain is $x \geq 1$, take the positive square root:

$$x = 1 + \sqrt{\frac{y-2}{5}}$$

✓ **FINAL ANSWER**

(a) 247, (b) $f^{-1}(x) = 1 + \sqrt{\frac{x-2}{5}}$

4MA1/2H NOVEMBER 2021

Question 15

6 marks | Functions

15 The functions f and g are such that

$$f(x) = 2x - 3$$

$$g(x) = \frac{x}{3x + 1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find gf(x)
Simplify your answer.

gf(x) = ...

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2021

1 **Evaluate fraction**

$$f(x) = 2x - 3, \qquad g(x) = \frac{x}{3x + 1}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{2x-3}{3(2x-3)+1}$$

$$gf(x) = \frac{2x-3}{6x-8}$$

---

4 **(c) Let**

(c) Let

$$y = \frac{x}{3x+1}$$

$$y(3x+1) = x$$

$$3xy + y = x$$

$$x(3y-1) = -y$$

$$x = \frac{y}{1-3y}$$

---

5 **Evaluate fraction**

So

$$g^{-1}(x) = \frac{x}{1-3x}$$

✓ **FINAL ANSWER**

Excluded value $-\frac{1}{3}$, $gf(x) = \frac{2x-3}{6x-8}$, and $g^{-1}(x) = \frac{x}{1-3x}$.

4MA1/2HR JANUARY 2022

Question 23

4 marks | Functions

23 The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function h^{-1} in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2022

1 **Calculate value**

$$f(x) = x + 25$$

$$g(x) = x^2 - 12x$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = x^2 - 12x + 25$$

$$h(x) = (x - 6)^2 - 11$$

2 **Let**

Let

$$y = (x - 6)^2 - 11$$

$$y + 11 = (x - 6)^2$$

3 Find inverse function

Since the domain of h is $x < 6$, take the negative square root:

$$x - 6 = -\sqrt{y + 11}$$

$$x = 6 - \sqrt{y + 11}$$

FINAL ANSWER

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$

4MA1/2H JANUARY 2022

Question 25

5 marks | Functions

25 The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain} \quad \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

$g^{-1}: x \mapsto$ .....

(4)

(b) State the domain of g^{-1}

.....

(1)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2022

1 Calculate value

$$g(x) = 5 + 6x - x^2$$

2 Complete the square

Complete the square:

$$g(x) = 14 - (x - 3)^2$$

3 Let

Let

$$y = 14 - (x - 3)^2$$

$$(x - 3)^2 = 14 - y$$

4 Find inverse function

The domain of g is $x \geq 3$, so take the positive square root:

$$x - 3 = \sqrt{14 - y}$$

$$x = 3 + \sqrt{14 - y}$$

5 Simplify surd

Therefore

$$g^{-1} : x \mapsto 3 + \sqrt{14 - x}$$

6 Calculate statistic

The range of g is $x \leq 14$, so this is the domain of g^{-1}.

FINAL ANSWER

(a) $g^{-1} : x \mapsto 3 + \sqrt{14 - x}$, (b) domain $x \leq 14$

4MA1/1H JUNE 2022

Question 14

4 marks | Functions

14 The function f is defined as

$$f: x \mapsto \frac{2x}{x-6} \qquad x \neq 6$$

(a) Find $f(10)$

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}: x \mapsto \ldots$

$f^{-1}: x \mapsto$ .....

(3)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2022

1 **Evaluate fraction**

$$f(x) = \frac{2x}{x-6}$$

2 **(a)**

(a)

$$f(10) = \frac{2(10)}{10-6} = \frac{20}{4} = 5$$

3 **(b) Let**

(b) Let

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$yx - 6y = 2x$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

---

4 **Evaluate fraction**

So

$$f^{-1}(x) = \frac{6x}{x-2}$$

✓ **FINAL ANSWER**

$f(10) = 5$, and $f^{-1}(x) = \frac{6x}{x-2}$.

4MA1/2HR JUNE 2022

Question 19

4 marks | Functions

19 $f(x) = x^2 - 4$

$g(x) = 2x + 1$

Solve $fg(x) > 0$

Show clear algebraic working.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2022

1 Solve quadratic equation

$$f(x) = x^2 - 4, \qquad g(x) = 2x + 1$$

$$fg(x) = f(g(x)) = (2x + 1)^2 - 4$$

$$= 4x^2 + 4x + 1 - 4 = 4x^2 + 4x - 3$$

2 Solve

Solve

$$4x^2 + 4x - 3 > 0$$

3 Factorise

Factorise:

$$(2x + 3)(2x - 1) > 0$$

4 The critical values are

The critical values are

$$x = -\frac{3}{2}, \qquad x = \frac{1}{2}$$

5 The product is positive outside

The product is positive outside the roots.

FINAL ANSWER

$x < -\frac{3}{2}$ or $x > \frac{1}{2}$.

4MA1/1H MAY 2023

Question 17

4 marks | Functions

17 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(a) What value of x must be excluded from any domain of g?

.....

(1)

(b) Solve $gh(x) = 1$

.....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2023

1 Find inverse function

$$g(x) = \frac{11}{2x - 5}, \qquad h(x) = x^2 + 4, \quad x \geq 0$$

2 (a) The denominator of cannot

(a) The denominator of $g(x)$ cannot be zero.

$$2x - 5 = 0$$

$$x = \frac{5}{2}$$

3 Find inverse function

So $x = \frac{5}{2}$ must be excluded.

4 **(b)**

(b)

$$gh(x) = g(h(x)) = \frac{11}{2(x^2+4)-5}$$

$$= \frac{11}{2x^2+3}$$

---

5 **Solve**

Solve

$$\frac{11}{2x^2+3} = 1$$

$$2x^2 + 3 = 11$$

$$2x^2 = 8$$

$$x^2 = 4$$

---

6 **Solve inequality**

Since $x \geq 0$,

$$x = 2$$

✓ **FINAL ANSWER**

Exclude $x = \frac{5}{2}$, and $x = 2$.

4MA1/2H NOVEMBER 2023

Question 19

7 marks | Functions

19 The functions f and g are such that

$$f: x \mapsto 5x + 7$$

$$g: x \mapsto \frac{5}{2x - 9}$$

(a) State which value of x cannot be included in any domain of g

.....

(1)

(b) Find fg(4)

.....

(2)

The function h is such that

$$h: x \mapsto 3x^2 - 12x + 8 \quad \text{where } x > 2$$

(c) Express the inverse function h^{-1} in the form $h^{-1}: x \mapsto ...$

$h^{-1}: x \mapsto$ .....

(4)

(Total for Question 19 is 7 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2023

1 **Evaluate fraction**

$$f(x) = 5x + 7, \qquad g(x) = \frac{5}{2x - 9}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$2x - 9 = 0$$

$$x = \frac{9}{2}$$

3 **(b)**

(b)

$$g(4) = \frac{5}{8-9} = -5$$

$$fg(4) = f(-5) = 5(-5) + 7 = -18$$

---

4 **(c)**

(c)

$$h(x) = 3x^2 - 12x + 8, \qquad x > 2$$

---

5 **Complete the square**

Complete the square:

$$h(x) = 3(x-2)^2 - 4$$

---

6 **Let**

Let

$$y = 3(x-2)^2 - 4$$

$$y + 4 = 3(x-2)^2$$

$$x - 2 = \sqrt{\frac{y+4}{3}}$$

---

7 **Simplify surd**

Since $x > 2$, take the positive root:

$$x = 2 + \sqrt{\frac{y+4}{3}}$$

---

8 **Simplify surd**

So

$$h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$$

✓ **FINAL ANSWER**

Excluded value $\frac{9}{2}$, $fg(4) = -18$, and $h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$.

4MA1/1H MAY 2024

Question 15

4 marks | Functions

15 The function f is defined as

$$f: x \mapsto \frac{3x+1}{x-2}$$

(a) State the value of x that cannot be included in any domain of the function f

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(3)

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2024

1 **Evaluate fraction**

$$f(x) = \frac{3x+1}{x-2}$$

2 **(a) The denominator cannot be**

(a) The denominator cannot be zero, so

$$x - 2 = 0$$

$$x = 2$$

3 **(b) Let**

(b) Let

$$y = \frac{3x+1}{x-2}$$

$$y(x-2) = 3x+1$$

$$yx - 2y = 3x + 1$$

$$x(y-3) = 2y+1$$

$$x = \frac{2y+1}{y-3}$$

---

4 **Evaluate fraction**

So

$$f^{-1}(x) = \frac{2x+1}{x-3}$$

✓ **FINAL ANSWER**

Excluded value 2, and $f^{-1}(x) = \frac{2x+1}{x-3}$.

4MA1/2HR JUNE 2024

Question 17

3 marks | Functions

17 $f(x) = \dfrac{x}{2x-4}$ $\quad g(x) = 3x + 1$

Given that $fg(k) = 2$

work out the value of k

$k =$ .....

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2024

1 Evaluate fraction

$$f(x) = \frac{k}{2x-4}, \qquad g(x) = 3x + 1$$

2 Given

Given

$$fg(k) = 2$$

3 This means

This means

$$f(g(k)) = 2$$

4 Calculate value

Now

$$g(k) = 3k + 1$$

5 **Evaluate fraction**

So

$$f(g(k)) = \frac{k}{2(3k+1)-4} = \frac{k}{6k-2}$$

$$\frac{k}{6k-2} = 2$$

$$k = 12k - 4$$

$$11k = 4$$

$$k = \frac{4}{11}$$

FINAL ANSWER

$k = \frac{4}{11}$.

4MA1/1H NOVEMBER 2024

Question 25

4 marks | Functions

25 The function f is such that $f(x) = 2x^2 - 24x + 7$ where $x \geqslant 6$

Find the inverse function $f^{-1}(x)$

$f^{-1}(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2024

1 Calculate value

$$f(x) = 2x^2 - 24x + 7$$

2 Complete the square

Complete the square:

$$f(x) = 2(x^2 - 12x) + 7$$

$$f(x) = 2[(x - 6)^2 - 36] + 7$$

$$f(x) = 2(x - 6)^2 - 65$$

3 Let

Let $y = f(x)$.

$$y = 2(x - 6)^2 - 65$$

$$y + 65 = 2(x - 6)^2$$

$$\frac{y + 65}{2} = (x - 6)^2$$

4 **Simplify surd**

Since $x \geq 6$, take the positive square root:

$$x - 6 = \sqrt{\frac{y+65}{2}}$$

$$x = 6 + \sqrt{\frac{y+65}{2}}$$

---

5 **Replace by**

Replace y by x.

✓ **FINAL ANSWER**

$$f^{-1}(x) = 6 + \sqrt{\frac{x+65}{2}}$$

4MA1/1H JUNE 2025

Question 16

4 marks | Functions

16 $f(x) = 9 - \sqrt{x}$ where $x \geqslant 0$

$g(x) = 4x^2$

(a) Find $f(9)$

.....

(1)

(b) Solve $fg(x) < 0$

Show clear algebraic working.

.....

(3)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2025

1 **Solve quadratic equation**

$$f(x) = 9 - \sqrt{x}, \qquad g(x) = 4x^2$$

2 **(a)**

(a)

$$f(9) = 9 - \sqrt{9} = 6$$

3 **(b)**

(b)

$$fg(x) = f(g(x)) = 9 - \sqrt{4x^2}$$

$$= 9 - 2|x|$$

4 **Solve**

Solve

$$9 - 2|x| < 0$$

$$2|x| > 9$$

$$|x| > \frac{9}{2}$$

---

5 **Solve quadratic equation**

So

$$x < -\frac{9}{2} \quad \text{or} \quad x > \frac{9}{2}$$

---

6 **Solve quadratic equation**

If the input is restricted to $x \geq 0$, this becomes $x > \frac{9}{2}$.

✓ **FINAL ANSWER**

$f(9) = 6$, and $x < -\frac{9}{2}$ or $x > \frac{9}{2}$.

4WM2/01R MAY 2025

Question 17

4 marks | Functions

17 The function g is defined as

$$g(x) = \frac{3x}{x-2}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots$

$g^{-1}(x) =$ .....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM2/01R May 2025

1 Required

$g(x) = dfrac3xx - 2$. (a) Value excluded from domain. (b) Find *g*^{−1}(*x*).

2 Part (a)

The denominator must not equal zero: *x* − 2 ≠ 0.

3 Answer

x = 2 cannot be in any domain of *g*

4 Set y g (x

Set *y* = *g*(*x*) and swap *x* ↔ *y*:

$$x = frac3yy - 2$$

5 Multiply both sides by (

Multiply both sides by (*y*−2):

x(*y*−2) = 3*y* ⟹ *x**y* − 2*x* = 3*y*

6 Collect y terms and factorise

Collect *y* terms and factorise:

*x**y* − 3*y* = 2*x* ⟹ *y*(*x*−3) = 2*x*

7 **Solve for y**

Solve for *y*:

$$y = frac2xx - 3$$

8 **Note**

g^{−1}(*x*) is undefined at *x* = 3, which is the range restriction of *g*(*x*) (horizontal asymptote).

✓ **FINAL ANSWER**

$$g^{-1}(x) = dfrac2xx - 3$$

4MA1/2HR JUNE 2025

Question 19

3 marks | Functions

19 The functions f and g are such that

$$f(x) = 3x - 4 \text{ where } x > 2$$

$$g(x) = \frac{x}{2x+1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find $gf(x)$
Give your answer in its simplest form.

$gf(x) =$ .....

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2025

1 **Evaluate fraction**

$$f(x) = 3x - 4, \qquad g(x) = \frac{x}{2x+1}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$2x + 1 = 0$$

$$x = -\frac{1}{2}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{3x-4}{2(3x-4)+1}$$

$$gf(x) = \frac{3x-4}{6x-7}$$

FINAL ANSWER

Excluded value $-\frac{1}{2}$, and $gf(x) = \dfrac{3x-4}{6x-7}$.

4WM2/01 MAY 2025

Question 21

7 marks | Functions

21 The function g is defined as

$$g(x) = \frac{7x + 20}{2x} \qquad x \neq 0$$

(a) Solve $gg(x) = 6$
Show clear algebraic working.

$x =$ .....

(3)

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The function h is defined as

$$h(x) = 5x^2 + 30x - 7 \qquad x \geqslant -3$$

(b) Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 May 2025

1 **Evaluate fraction**

(a) If $g(g(x)) = 6$, then

$$\frac{7g(x) + 20}{2g(x)} = 6$$

$$7g(x) + 20 = 12g(x)$$

$$g(x) = 4$$

2 **Evaluate fraction**

So

$$\frac{7x+20}{2x}=4$$

$$7x+20=8x$$

$$x=20$$

---

3 **(b)**

(b)

$$h(x)=5x^2+30x-7=5(x+3)^2-52$$

---

4 **Simplify surd**

Let $y=5(x+3)^2-52$. Since $x\geq -3$, use the positive square root:

$$x+3=\sqrt{\frac{y+52}{5}}$$

$$h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$$

✓ **FINAL ANSWER**

$x=20$; $h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$.

4MA1/2H NOVEMBER 2025

Question 19

3 marks | Functions

19 The functions f and g are such that

$$f(x) = 3x - 2 \qquad g(x) = \frac{x}{2x - 1}$$

(a) Find g(3)

.....

(1)

(b) Find gf(x)
Give your answer in its simplest form.

gf(x) = ...

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2025

1 **Evaluate fraction**

$$f(x) = 3x - 2, \qquad g(x) = \frac{x}{x - 1}$$

2 **(a)**

(a)

$$g(3) = \frac{3}{3 - 1} = \frac{3}{2}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{3x - 2}{(3x - 2) - 1}$$

$$gf(x) = \frac{3x - 2}{3x - 3}$$

FINAL ANSWER

$g(3) = \frac{3}{2}$, and $gf(x) = \dfrac{3x-2}{3x-3}$.

4WM2/01 NOVEMBER 2025

Question 21

7 marks | Functions

21 The functions f and g are defined as

$$f(x) = 5x - 3$$

$$g(x) = 4x^2 + 16x - 9 \qquad \text{where} \quad x \geqslant -2$$

(a) Find g(–1)

.....

(1)

(b) Find fg(x)

Give your answer in its simplest form.

fg(x) = ...

(2)

$g(x) = 4x^2 + 16x - 9$ where $x \geqslant -2$

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 November 2025

1 **(a) Find g (1)**

(a) Find *g*(−1):

g(−1) = 4(−1)² + 16(−1) − 9 = 4 − 16 − 9 = − 21

2 **Answer**

a − 21

(b) Find *fg*(*x*):

*f**g*(*x*) = *f*(*g*(*x*)) = 5(4*x*²+16*x*−9) − 3

= 20*x*² + 80*x* − 45 − 3 = 20*x*² + 80*x* − 48

---

3 **Answer**

b 20*x*² + 80*x* − 48

Let *y* = 4*x*² + 16*x* − 9. We must rearrange to make *x* the subject.

Step 1: Complete the square for the *x* terms.

y = 4(*x*²+4*x*) − 9

y = 4((*x*+2)²−4) − 9

y = 4(*x*+2)² − 16 − 9

y = 4(*x*+2)² − 25

Step 2: Rearrange for *x*.

y + 25 = 4(*x*+2)²

$$fracy + 254 = (x + 2)^2$$

$$x + 2 = pmsqrtfracy + 254$$

Since *x* ≥ − 2, *x* + 2 is positive, so we take the positive root.

$$x + 2 = fracsqrty + 252$$

$$x = fracsqrty + 252 - 2$$

Step 3: Swap *y* for *x* to write the inverse function.

✓ **FINAL ANSWER**

$g^{-1}(x) = fracsqrtx + 252 - 2$ When finding the inverse of a quadratic, always complete the square. The restricted domain (*x* ≥ − 2) tells you whether to take the positive or negative square root!

4MA1/2H JUNE 2026

Question 16

5 marks | Functions

16 The functions f and g are such that

$$f(x) = 4 + x \qquad g(x) = 3x^2 + 2$$

(a) Find $g(4)$

.....

(1)

(b) Find $f^{-1}(7)$

.....

(2)

(c) Express $gf(x)$ in the form $ax^2 + bx + c$ where a, b and c are integers.

$gf(x) =$ .....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

$$g(4) = 3(4)^2 + 2 = 48 + 2 = 50.$$

Substitute 4 for the function input.

2 **Part (b) | Step 1**

If $y = 4 + x$, then $x = y - 4$, so $f^{-1}(x) = x - 4$.

Reverse the input-output relationship.

3 **Part (b) | Step 2**

$$f^{-1}(7) = 7 - 4 = 3.$$

Apply the inverse function to 7.

4 **Part (c) | Step 1**

$$gf(x) = g(f(x)) = g(x + 4) = 3(x + 4)^2 + 2.$$

Apply f first and use its whole output as the input of g.

5 **Part (c) | Step 2**

$$3(x^2 + 8x + 16) + 2 = 3x^2 + 24x + 50.$$

Expand and collect the constant terms.

✓ **FINAL ANSWER**

(a) 50 (b) 3 (c) $gf(x) = 3x^2 + 24x + 50$

Question 17

4 marks | Functions

17 The functions f and g are such that

$$f(x) = x^2 - 5$$
$$g(x) = 2x + 1$$

Solve $fg(x) = 2gf(x)$
Show clear algebraic working.

$x =$ .....

(Total for Question 17 is 4 marks)

1 **fg(x): put g into f**

$$\mathrm{fg}(x) = (2x+1)^2 - 5 = 4x^2 + 4x - 4$$

2 **gf(x): put f into g**

$$\mathrm{gf}(x) = 2(x^2-5) + 1 = 2x^2 - 9, \qquad 2\,\mathrm{gf}(x) = 4x^2 - 18$$

3 **Solve**

$$4x^2 + 4x - 4 = 4x^2 - 18 \Rightarrow 4x = -14 \Rightarrow x = -3.5$$

✓ **FINAL ANSWER**

$x = -3.5$

4MA1/1HR JUNE 2026

Question 23

7 marks | Functions

23 The function f is such that $f(x) = \frac{1}{x}$

(a) Write down the value of x that cannot be included in any domain of f

.....

(1)

The function g is such that $g(x) = 4x^2 - 8x + 15$
The domain of g is $x \geqslant 1$

(b) Work out $gf(0.5)$

.....

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x) =$ .....

(4)

(Total for Question 23 is 7 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

For $f(x) = \frac{1}{x}$, the denominator is zero when $x = 0$.
Division by zero is undefined, so this input must be excluded.

2 **Part (b) | Step 1**

$$f(0.5) = \frac{1}{0.5} = 2.$$

In gf, apply f first.

3 **Part (b) | Step 2**

$$gf(0.5) = g(2) = 4(2)^2 - 8(2) + 15 = 16 - 16 + 15 = 15.$$

The input 2 lies in the stated domain x>=1 of g.

4 **Part (c) | Step 1**

$$y = 4x^2 - 8x + 15 = 4(x - 1)^2 + 11.$$

Complete the square to isolate the expression containing x.

5 **Part (c) | Step 2**

$$(x-1)^2 = \frac{y-11}{4}, \qquad x-1 = \frac{\sqrt{y-11}}{2}.$$

Choose the nonnegative root because the domain of g is x>=1.

6 **Part (c) | Step 3**

$$x = 1 + \frac{\sqrt{y-11}}{2}.$$

Rearrange to make the original input the subject.

7 **Part (c) | Step 4**

$$g^{-1}(x) = 1 + \frac{\sqrt{x-11}}{2}, \qquad x \geq 11.$$

Exchange the input and output labels; the range of g becomes the domain of the inverse.

✓ **FINAL ANSWER**

(a) $x = 0$ must be excluded. (b) $gf(0.5) = 15$ (c) $g^{-1}(x) = 1 + \frac{\sqrt{x-11}}{2}, \quad x \geq 11$

TOPIC

Differentiation & Turning Points

Every past-paper question on this topic, 2020 to 2026.

Question 17

5 marks | Differentiation & Turning Points

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

1 **Differentiate**

$$s = 4t^2 + \frac{125}{t}$$

2 **Differentiate to find velocity**

Differentiate to find velocity:

$$v = \frac{ds}{dt} = 8t - \frac{125}{t^2}$$

3 **Set**

Set $v = 0$:

$$8t - \frac{125}{t^2} = 0$$

$$8t^3 = 125$$

$$t^3 = \frac{125}{8}$$

$$t = 2.5$$

4 **Differentiate**

Now find s:

$$s = 4(2.5)^2 + \frac{125}{2.5}$$

$$s = 25 + 50 = 75$$

✓ **FINAL ANSWER**

75 m.

4MA1/1H JANUARY 2020

Question 15

8 marks | Differentiation & Turning Points

15

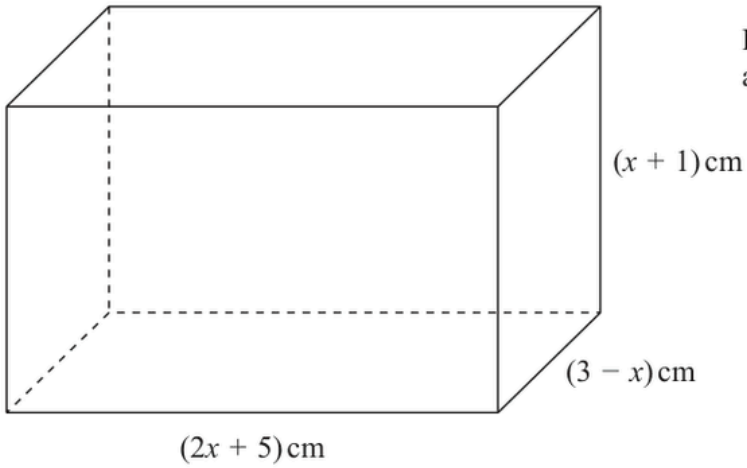


Diagram **NOT** accurately drawn

The diagram shows a cuboid of volume $V\,\text{cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

(3)

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x.
Show your working clearly.
Give your answer correct to 3 significant figures.

$x =$ .....

(5)

(Total for Question 15 is 8 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2020

1 The volume is

The volume is

$$V = (2x + 5)(x + 1)(3 - x)$$

2 **Expand brackets**

First expand:

$$(2x+5)(x+1) = 2x^2 + 7x + 5$$

$$V = (2x^2 + 7x + 5)(3 - x)$$

$$V = 15 + 16x - x^2 - 2x^3$$

---

3 **Calculate volume**

For the maximum volume,

$$\frac{dV}{dx} = 0$$

$$\frac{dV}{dx} = 16 - 2x - 6x^2$$

$$16 - 2x - 6x^2 = 0$$

$$3x^2 + x - 8 = 0$$

---

4 **Solve quadratic equation**

Using the quadratic formula,

$$x = \frac{-1 \pm \sqrt{97}}{6}$$

---

5 **The valid positive value is**

The valid positive value is

$$x = 1.4748\ldots$$

✓ **FINAL ANSWER**

$x = 1.47$ to 3 significant figures.

4MA1/1HR JANUARY 2020

Question 23

6 marks | Differentiation & Turning Points

23 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$T =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2020

1 Calculate value

$$s = t^3 + 4t^2 - 5t + 7$$

2 Evaluate fraction

Velocity is the derivative of displacement:

$$v = \frac{ds}{dt} = 3t^2 + 8t - 5$$

3 Calculate value

At time T, the velocity is V, so

$$V = 3T^2 + 8T - 5$$

$$3T^2 + 8T - (V + 5) = 0$$

4 **Use the quadratic formula**

Use the quadratic formula:

$$T = \frac{-8 \pm \sqrt{8^2 - 4(3)(-(V+5))}}{2(3)}$$

$$T = \frac{-8 \pm \sqrt{64 + 12V + 60}}{6}$$

$$T = \frac{-8 \pm \sqrt{124 + 12V}}{6}$$

$$T = \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

---

5 **Simplify surd**

Since $T \geq 0$, take the positive root:

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

✓ **FINAL ANSWER**

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

4MA1/1HR MAY/NOVEMBER 2020

Question 13

6 marks | Differentiation & Turning Points

13 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

.....

(4)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR May/November 2020

1 **Find the gradient**

$$y = 5x^3 - x^2 - 6x + 4$$

$$\frac{dy}{dx} = 15x^2 - 2x - 6$$

2 **Find the gradient**

The gradient is 2, so

$$15x^2 - 2x - 6 = 2$$

$$15x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(15)(-8)}}{30}$$

$$x = \frac{2 \pm 22}{30}$$

$$x = \frac{4}{5} \quad \text{or} \quad x = -\frac{2}{3}$$

FINAL ANSWER

$x = -\frac{2}{3}$ and $x = \frac{4}{5}$.

4MA1/1H MAY/NOVEMBER 2020

Question 23

4 marks | Differentiation & Turning Points

23 Curve **C** has equation $y = px^3 - mx$ where p and m are positive integers.

Find the range of values of x, in terms of p and m, for which the gradient of **C** is negative.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May/November 2020

1 Differentiate

$$y = px^3 - mx$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3px^2 - m$$

3 The gradient is negative when

The gradient is negative when

$$3px^2 - m < 0$$

$$3px^2 < m$$

$$x^2 < \frac{m}{3p}$$

4 Simplify surd

So

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

FINAL ANSWER

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

Question 17

5 marks | Differentiation & Turning Points

17 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

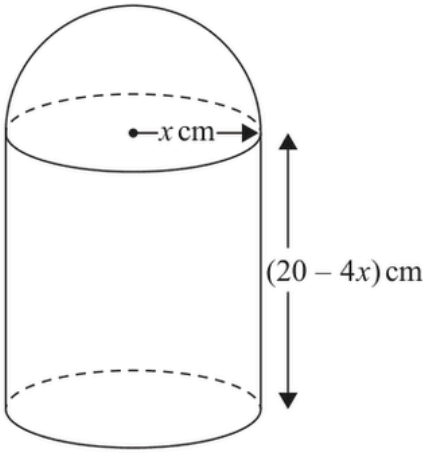


Diagram **NOT** accurately drawn

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is $V\,\text{cm}^3$ where $V = \frac{1}{3}\pi y$

Find the maximum value of y.
Show clear algebraic working.

(Total for Question 17 is 5 marks)

1 **Volume of the solid is**

Volume of the solid is cylinder plus hemisphere:

$$V = \pi x^2(20 - 4x) + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - 4\pi x^3 + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - \frac{10}{3}\pi x^3$$

2 **The question gives**

The question gives

$$V = \frac{1}{3}\pi y$$

3 **Differentiate**

So

$$y = 60x^2 - 10x^3$$

4 **Differentiate**

Differentiate:

$$\frac{dy}{dx} = 120x - 30x^2$$

5 **Differentiate**

For a maximum,

$$120x - 30x^2 = 0$$

$$30x(4 - x) = 0$$

6 **Differentiate**

So the valid value is

$$x = 4$$

$$y = 60(4^2) - 10(4^3) = 960 - 640 = 320$$

FINAL ANSWER

The maximum value of y is 320.

4MA1/2H JANUARY 2021

Question 19

5 marks | Differentiation & Turning Points

19 A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds where $t \geqslant 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 5t^2 - 8t + 10$$

Find the displacement of P from O when P is instantaneously at rest.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

..... metres

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2021

1 Calculate statistic

$$s = t^3 + 5t^2 - 8t + 10$$

2 Velocity is

Velocity is

$$\frac{ds}{dt} = 3t^2 + 10t - 8$$

3 Calculate statistic

At rest,

$$3t^2 + 10t - 8 = 0$$

4 Factorise

Factorise:

$$(3t - 2)(t + 4) = 0$$

5 Calculate statistic

So

$$t = \frac{2}{3} \quad \text{or} \quad t = -4$$

6 **Calculate statistic**

Since $t \geq 0$,

$$t = \frac{2}{3}$$

---

7 **Calculate statistic**

Now find s:

$$s = \left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) + 10$$

$$s = \frac{8}{27} + \frac{20}{9} - \frac{16}{3} + 10$$

$$s = \frac{194}{27}$$

✓ **FINAL ANSWER**

$\frac{194}{27}$ metres.

4MA1/1HR JANUARY 2021

Question 20

5 marks | Differentiation & Turning Points

20 A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

..... m/s

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2021

1 **Read the graph**

$$s = t^3 - 9t^2 + 33t - 6$$

2 **Read the graph**

Velocity is the rate of change of displacement:

$$v = \frac{ds}{dt}$$

$$v = 3t^2 - 18t + 33$$

3 **Complete the square**

Complete the square:

$$v = 3(t^2 - 6t + 11)$$

$$v = 3[(t-3)^2 + 2]$$

$$v = 3(t-3)^2 + 6$$

4 **Read the graph**

Since $(t-3)^2 \geq 0$, the minimum value of v is

$$6$$

5 **Read the graph**

The velocity is always positive, so the minimum speed is also 6.

✓ **FINAL ANSWER**

$6\ \text{m/s}$

4MA1/2H MAY 2021

Question 21

5 marks | Differentiation & Turning Points

21 The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a.
Show your working clearly.

$a =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H May 2021

1 Evaluate fraction

$$y = kx^2 + \frac{16}{x}$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 2kx - \frac{16}{x^2}$$

3 Find the gradient

At the stationary point, $x = \frac{2}{3}$, so the gradient is 0:

$$2k\left(\frac{2}{3}\right) - \frac{16}{\left(\frac{2}{3}\right)^2} = 0$$

$$\frac{4k}{3} - \frac{16}{\frac{4}{9}} = 0$$

$$\frac{4k}{3} - 36 = 0$$

$$\frac{4k}{3} = 36$$

$$k = 27$$

4 Find the -coordinate

Now find a, the y-coordinate when $x = \frac{2}{3}$:

$$a = 27\left(\frac{2}{3}\right)^2 + \frac{16}{\frac{2}{3}}$$

$$a = 27 \cdot \frac{4}{9} + 24$$

$$a = 12 + 24 = 36$$

FINAL ANSWER

$a = 36$

4MA1/1HR JANUARY 2022

Question 23

6 marks | Differentiation & Turning Points

23 Two particles, P and Q, move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2022

1 **Particle**

Particle P:

$$s = t^3 - 4t^2 + 5t$$

$$\frac{ds}{dt} = 3t^2 - 8t + 5$$

$$\frac{ds}{dt} = (3t - 5)(t - 1)$$

2 **Particle**

Particle Q:

$$x = t^2 - 4t + 4$$

$$\frac{dx}{dt} = 2t - 4 = 2(t - 2)$$

3 **Differentiate**

For $t > 1$:

---

4 **moves in the negative direction**

- P moves in the negative direction when $1 < t < \frac{5}{3}$.
- P moves in the positive direction when $t > \frac{5}{3}$.
- Q moves in the negative direction when $1 < t < 2$.
- Q moves in the positive direction when $t > 2$.

---

5 **Differentiate**

So both particles move in the same direction when both velocities have the same sign.

✓ **FINAL ANSWER**

$1 < t < \frac{5}{3}$ or $t > 2$

4MA1/1H JANUARY 2022

Question 24

6 marks | Differentiation & Turning Points

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2022

1 Calculate value

$$y = ax^3 + bx^2 - 12x + 6$$

2 Point lies on the curve

Point $A = (2, -6)$ lies on the curve:

$$-6 = 8a + 4b - 24 + 6$$

$$8a + 4b = 12$$

$$2a + b = 3$$

3 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3ax^2 + 2bx - 12$$

4 **The gradient at is 16**

The gradient at $x = 2$ is 16:

$$16 = 12a + 4b - 12$$

$$12a + 4b = 28$$

$$3a + b = 7$$

---

5 **Solve**

Now solve

$$2a + b = 3$$

$$3a + b = 7$$

---

6 **Subtracting gives**

Subtracting gives

$$a = 4$$

---

7 **Calculate value**

Then

$$2(4) + b = 3$$

$$b = -5$$

---

8 **Calculate value**

For $x = 3$,

$$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$$

$$y = 108 - 45 - 36 + 6$$

$$y = 33$$

✓ **FINAL ANSWER**

$y = 33$

4MA1/2H JANUARY 2022

Question 24

5 marks | Differentiation & Turning Points

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a\ \text{m/s}^2$

Find the value of a

$a =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2022

1 Split the ratio

$$x = 4t^3 - 27t + 8$$

2 Velocity is

Velocity is

$$\frac{dx}{dt} = 12t^2 - 27$$

3 The direction of motion reverses

The direction of motion reverses when velocity is zero:

$$12t^2 - 27 = 0$$

$$12t^2 = 27$$

$$t^2 = \frac{9}{4}$$

4 Split the ratio

Since $t \geq 0$, $t = \frac{3}{2}$.

5 **Acceleration is**

Acceleration is

$$\frac{d^2x}{dt^2} = 24t$$

6 **Split the ratio**

At $t = \frac{3}{2}$,

$$a = 24\left(\frac{3}{2}\right) = 36$$

✓ **FINAL ANSWER**

$a = 36$

4MA1/1H JUNE 2022

Question 17

5 marks | Differentiation & Turning Points

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2022

1 Differentiate

$$s = 4t^2 + \frac{125}{t}$$

2 Velocity is

Velocity is

$$v = \frac{ds}{dt} = 8t - \frac{125}{t^2}$$

3 Differentiate

When $v = 0$,

$$8t - \frac{125}{t^2} = 0$$

$$8t^3 = 125$$

$$t^3 = 15.625$$

$$t = 2.5$$

4 **Differentiate**

Now find s:

$$s = 4(2.5)^2 + \frac{125}{2.5}$$

$$s = 25 + 50 = 75$$

✓ **FINAL ANSWER**

$75\ \text{m}$.

Question 22

7 marks | Differentiation & Turning Points

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

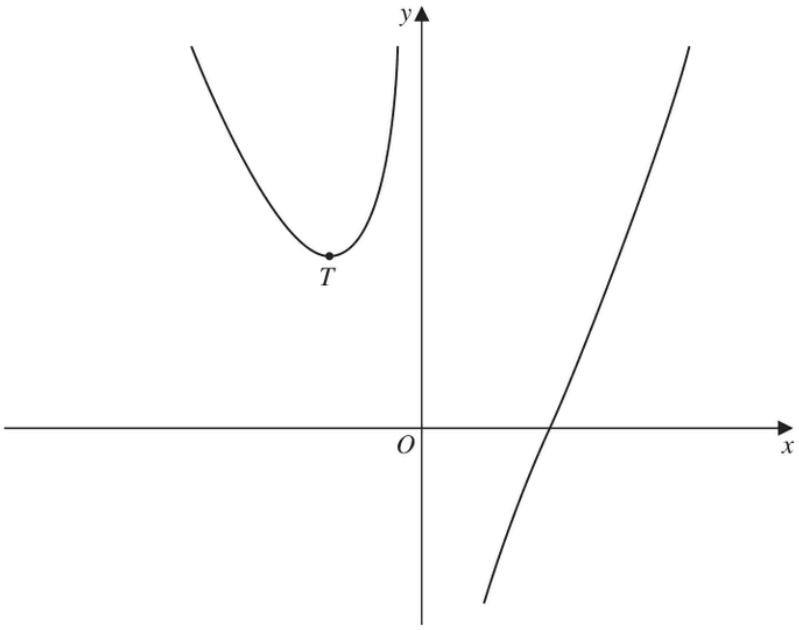


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

$p =$ .....

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$k =$ .....

(3)

(Total for Question 22 is 7 marks)

1 **Find the gradient**

For part (a),

$$y = x^2 - \frac{p}{x}$$

$$\frac{dy}{dx} = 2x + \frac{p}{x^2}$$

2 **Find the gradient**

At the minimum point, $x = -3$ and the gradient is 0.

$$2(-3) + \frac{p}{(-3)^2} = 0$$

$$-6 + \frac{p}{9} = 0$$

$$p = 54$$

3 **Find the gradient**

For part (b),

$$y = x^2 - \frac{16}{x}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2}$$

4 **Find the gradient**

At a horizontal tangent,

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$x^3 = -8$$

$$x = -2$$

5 **Find the gradient**

Now substitute $x = -2$ into the curve:

$$k = (-2)^2 - \frac{16}{-2}$$

$$k = 4 + 8 = 12$$

✓ **FINAL ANSWER**

$p = 54,\ k = 12$

4MA1/1H MAY 2023

Question 11

6 marks | Differentiation & Turning Points

11 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the x coordinates of the points on **C** where the gradient is 4
Show clear algebraic working.

.....

(4)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2023

1 **Find the gradient**

$$y = 4x^3 + x^2 - 20x$$

2 **(a)**

(a)

$$\frac{dy}{dx} = 12x^2 + 2x - 20$$

3 **Find the gradient**

(b) The gradient is 4, so

$$12x^2 + 2x - 20 = 4$$

$$12x^2 + 2x - 24 = 0$$

$$6x^2 + x - 12 = 0$$

4 **Factorise**

Factorise:

$$(3x - 4)(2x + 3) = 0$$

---

5 **Find the gradient**

So

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

✓ **FINAL ANSWER**

$\frac{dy}{dx} = 12x^2 + 2x - 20$, and $x = \frac{4}{3}$ or $x = -\frac{3}{2}$.

4MA1/1HR MAY 2023

Question 21

6 marks | Differentiation & Turning Points

21 The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the range of values of x for which **T** has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

(4)

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2023

1 **(a)**

(a)

$$y = x^3 - 2x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 4x - 9$$

2 **(b) The curve has a**

(b) The curve has a positive gradient when

$$3x^2 - 4x - 9 > 0$$

3 **Solve**

Solve

$$3x^2 - 4x - 9 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 108}}{6}$$

$$x = \frac{2 \pm \sqrt{31}}{3}$$

---

4 **Differentiate**

To 3 significant figures, the roots are

$$x = -1.19, \quad x = 2.52$$

---

5 **Solve quadratic equation**

Since the quadratic opens upwards, it is positive outside the roots.

✓ **FINAL ANSWER**

(a) $\frac{dy}{dx} = 3x^2 - 4x - 9$, (b) $x < -1.19$ or $x > 2.52$

4MA1/1H NOVEMBER 2023

Question 16

4 marks | Differentiation & Turning Points

16 A curve has equation $y = 4x^3 - 8x + 5$

Find the x coordinates of the two points on the curve where the gradient is $\frac{1}{3}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2023

1 Find the gradient

$$y = 4x^3 - 8x + 5$$

$$\frac{dy}{dx} = 12x^2 - 8$$

2 Find the gradient

The gradient is $\frac{1}{3}$, so

$$12x^2 - 8 = \frac{1}{3}$$

$$12x^2 = \frac{25}{3}$$

$$x^2 = \frac{25}{36}$$

$$x = \frac{5}{6} \quad \text{or} \quad x = -\frac{5}{6}$$

FINAL ANSWER

$x = \frac{5}{6}$ or $x = -\frac{5}{6}$.

4MA1/2H NOVEMBER 2023

Question 18

4 marks | Differentiation & Turning Points

18 A particle is moving along a straight line that passes through the fixed point O
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 2t^3 - 5t^2 + 6t - 5$$

Find the value of t when the acceleration of the particle is 5 m/s²

$t =$ .....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2023

1 Split the ratio

$$s = 2t^3 - 5t^2 + 6t - 5$$

2 Velocity is

Velocity is

$$\frac{ds}{dt} = 6t^2 - 10t + 6$$

3 Acceleration is

Acceleration is

$$\frac{d^2s}{dt^2} = 12t - 10$$

4 Split the ratio

The acceleration is 5, so

$$12t - 10 = 5$$

$$12t = 15$$

$$t = \frac{15}{12} = \frac{5}{4}$$

FINAL ANSWER

$t = \frac{5}{4}$ seconds

4MA1/1H MAY 2024

Question 18

5 marks | Differentiation & Turning Points

18 A curve C has equation $y = x^3 - 40x + 1$

Find the coordinates of both the points on C at which the gradient is 8

(.....,.....)

(.....,.....)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H May 2024

1 Differentiate

$$y = x^3 - 40x + 1$$

$$\frac{dy}{dx} = 3x^2 - 40$$

2 Find the gradient

The gradient is 8, so

$$3x^2 - 40 = 8$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = 4 \quad \text{or} \quad x = -4$$

3 Differentiate

When $x = 4$:

$$y = 4^3 - 40(4) + 1 = -95$$

4 Differentiate

When $x = -4$:

$$y = (-4)^3 - 40(-4) + 1 = 97$$

FINAL ANSWER

$(-4, 97)$ and $(4, -95)$.

4MA1/1HR MAY 2024

Question 19

5 marks | Differentiation & Turning Points

19 A curve C has equation $y = x^3 - 8x^2 - 12x + 5$

Curve C has exactly two stationary points, one at point A and one at point B such that

x coordinate of point $A > x$ coordinate of point B

Find the coordinates of point A
Show clear algebraic working.

(..... , ...)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May 2024

1 Differentiate

$$y = x^3 - 8x^2 - 12x + 5$$

$$\frac{dy}{dx} = 3x^2 - 16x - 12$$

2 Stationary points have

Stationary points have

$$3x^2 - 16x - 12 = 0$$

$$x = \frac{16 \pm \sqrt{(-16)^2 - 4(3)(-12)}}{6}$$

$$x = \frac{16 \pm 20}{6}$$

$$x = 6 \quad \text{or} \quad x = -\frac{2}{3}$$

3 **Differentiate**

Point A has the greater x-coordinate, so $x = 6$.

$$y = 6^3 - 8(6^2) - 12(6) + 5$$

$$y = 216 - 288 - 72 + 5 = -139$$

✓ **FINAL ANSWER**

$A = (6, -139)$.

4MA1/1H NOVEMBER 2024

Question 20

4 marks | Differentiation & Turning Points

20 The curve with equation $y = 2x^4 - 64x$ has a minimum point.

Find an equation of the tangent to the curve at the minimum point.
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2024

1 Find the gradient

$$y = 2x^4 - 64x$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 8x^3 - 64$$

3 Find the gradient

At the minimum point, the gradient is 0.

$$8x^3 - 64 = 0$$

$$8x^3 = 64$$

$$x^3 = 8$$

$$x = 2$$

4 Find the coordinate

Find the y-coordinate:

$$y = 2(2)^4 - 64(2)$$

$$y = 32 - 128 = -96$$

5 **The tangent at a minimum**

The tangent at a minimum point is horizontal, so its equation is

$$y = -96$$

✓ **FINAL ANSWER**

$y = -96$

Question 17

6 marks | Differentiation & Turning Points

17 $y = 4x^3 + 5x^2 + 2x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the coordinates of the turning points on the graph with equation $y = 4x^3 + 5x^2 + 2x$
Show clear algebraic working.

.....

(4)

(Total for Question 17 is 6 marks)

1 **Differentiate**

$$y = 4x^3 + 5x^2 + 2x$$

$$\frac{dy}{dx} = 12x^2 + 10x + 2$$

2 **Differentiate**

At a turning point,

$$12x^2 + 10x + 2 = 0$$

$$2(6x^2 + 5x + 1) = 0$$

$$(3x + 1)(2x + 1) = 0$$

$$x = -\frac{1}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

3 **Differentiate**

When $x = -\frac{1}{3}$,

$$y = -\frac{7}{27}$$

4 **Differentiate**

When $x = -\frac{1}{2}$,

$$y = -\frac{1}{4}$$

FINAL ANSWER

$\left(-\frac{1}{2}, -\frac{1}{4}\right)$ and $\left(-\frac{1}{3}, -\frac{7}{27}\right)$.

4MA1/2HR JUNE 2025

Question 22

5 marks | Differentiation & Turning Points

22 The diagram shows a solid cuboid.

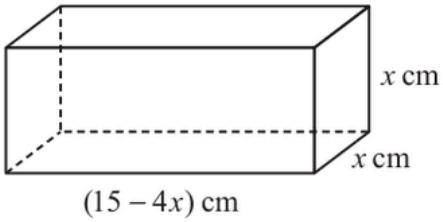


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2025

1 **The cuboid has dimensions**

The cuboid has dimensions

$$x, \quad x, \quad 15 - 4x$$

2 **Calculate value**

So

$$V = x^2(15 - 4x)$$

$$V = 15x^2 - 4x^3$$

3 **Differentiate**

Differentiate:

$$\frac{dV}{dx} = 30x - 12x^2$$

4 **Calculate value**

At a maximum,

$$30x - 12x^2 = 0$$

$$6x(5 - 2x) = 0$$

5 **The useful value is**

The useful value is

$$x = 2.5$$

6 **Calculate volume**

Now substitute into the volume:

$$V = (2.5)^2(15 - 4(2.5))$$

$$V = 6.25 \times 5$$

$$V = 31.25$$

✓ **FINAL ANSWER**

$31.25\ \text{cm}^3$

4WM2/01 MAY 2025

Question 22

4 marks | Differentiation & Turning Points

22 Work out the range of values of x for which the graph of $y = x^3 + \frac{2}{2}x^2 - 10x + 5$ has a positive gradient.
Show clear algebraic working.

.....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 May 2025

1 Evaluate fraction

$$\frac{dy}{dx} = 3x^2 + 2x - 10$$

2 Find the gradient

For a positive gradient,

$$3x^2 + 2x - 10 > 0$$

3 The roots are

The roots are

$$x = \frac{-1 \pm \sqrt{31}}{3}$$

4 Solve quadratic equation

Since the quadratic opens upwards,

$$x < \frac{-1 - \sqrt{31}}{3} \quad \text{or} \quad x > \frac{-1 + \sqrt{31}}{3}$$

FINAL ANSWER

$x < \frac{-1 - \sqrt{31}}{3}$ or $x > \frac{-1 + \sqrt{31}}{3}$.

4WM2/01 NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 November 2025

1 **Find the gradient**

$$fracdydx = 3x^2 - 16$$

2 **Find the gradient**

3*x*² − 16 = 11

3*x*² = 27

*x*² = 9 ⟹ *x* = 3 or *x* = − 3

3 **Solve inequality**

For *x* = 3: *y* = (3)³ − 16(3) + 7 = 27 − 48 + 7 = − 14

4 **Solve inequality**

For *x* = − 3: *y* = (−3)³ − 16(−3) + 7 = − 27 + 48 + 7 = 28

5 **Use trigonometry**

Step 4: Find the y-intercept (*b*) for each tangent (*y* = 11*x* + *b*).

Using (3,−14):

− 14 = 11(3) + *b* ⟹ − 14 = 33 + *b* ⟹ *b* = − 47

Using (−3,28):

28 = 11(−3) + *b* ⟹ 28 = − 33 + *b* ⟹ *b* = 61

✓ **FINAL ANSWER**

b = − 47 and 61

4MA1/2H NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2025

1 Differentiate

$$y = x^3 - 16x + 7$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3x^2 - 16$$

3 The gradient of the tangent

The gradient of the tangent is 11, so

$$3x^2 - 16 = 11$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = 3 \quad \text{or} \quad x = -3$$

4 Differentiate

When $x = 3$:

$$y = 3^3 - 16(3) + 7 = 27 - 48 + 7 = -14$$

5 **Use trigonometry**

The tangent has equation $y = 11x + b$, so

$$-14 = 11(3) + b$$

$$b = -47$$

---

6 **Differentiate**

When $x = -3$:

$$y = (-3)^3 - 16(-3) + 7 = -27 + 48 + 7 = 28$$

$$28 = 11(-3) + b$$

$$b = 61$$

✓ **FINAL ANSWER**

$\boxed{b = -47 \text{ or } b = 61}$

4MA1/1HR JUNE 2026

Question 17

5 marks | Differentiation & Turning Points

17 $y = x^3 + 5x^2 - 8x$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} =$ .....

(2)

The curve with equation $y = x^3 + 5x^2 - 8x$ has two stationary points, P and Q
The x-coordinate of P is less than the x-coordinate of Q

(b) Work out the coordinates of P
Show clear algebraic working.

(..... , ...)

(3)

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\frac{dy}{dx} = 3x^2 + 10x - 8.$$

Apply the power rule to each term of y=x^3+5x^2-8x.

2 **Part (b) | Step 1**

$$3x^2 + 10x - 8 = 0 \quad \Longrightarrow \quad (3x - 2)(x + 4) = 0.$$

At a stationary point the derivative is zero.

3 **Part (b) | Step 2**

$$x = \frac{2}{3} \quad \text{or} \quad x = -4.$$

Set each factor equal to zero.

4

Part (b) | Step 3

P has the smaller x-coordinate, so $x_P = -4$. Then

$$y_P = (-4)^3 + 5(-4)^2 - 8(-4) = -64 + 80 + 32 = 48.$$

Choose the left-hand stationary point and substitute into the original curve equation.

FINAL ANSWER

(a) $\frac{dy}{dx} = 3x^2 + 10x - 8$ (b) $P = (-4, 48)$

4MA1/1H JUNE 2026

Question 21

5 marks | Differentiation & Turning Points

21 The diagram shows a solid cuboid.

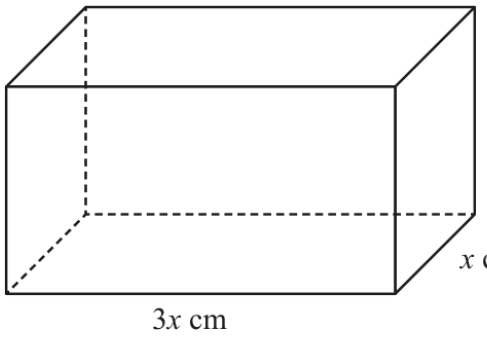


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V
Show clear algebraic working.

$V =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2026

1 Solution | Step 1

$$V = (3x)(x)(13.5 - 2x) = 40.5x^2 - 6x^3, \qquad 0 < x < 6.75.$$

Multiply the three dimensions and require them all to be positive.

2 Solution | Step 2

$$\frac{dV}{dx} = 81x - 18x^2 = 9x(9 - 2x).$$

Differentiate each power of x.

3 Solution | Step 3

At a stationary point, $9x(9 - 2x) = 0$, giving $x = 0$ or $x = 4.5$. Only $x = 4.5$ is inside the domain.
A stationary volume has zero derivative.

4 Solution | Step 4

The derivative is positive for $0 < x < 4.5$ and negative for $4.5 < x < 6.75$. Hence $x = 4.5$ gives the maximum.
The volume increases then decreases; its limiting endpoint volumes are zero.

5 Solution | Step 5

$$V_{\max} = 3(4.5)^2(13.5 - 9) = 273.375 \text{ cm}^3.$$

Substitute the maximising value into the original volume.

FINAL ANSWER

273.375 cm^3

Question 23

5 marks | Differentiation & Turning Points

23 A particle is moving along a straight line that passes through the fixed point O

The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 15t^2 - t^3 + 12$$

Find the maximum velocity of the particle where $0 < t < 11$
Show clear algebraic working.

..... m/s

(Total for Question 23 is 5 marks)

1 **Velocity**

$$v = \frac{ds}{dt} = 30t - 3t^2$$

2 **Maximum when dv/dt = 0**

$$\frac{dv}{dt} = 30 - 6t = 0 \Rightarrow t = 5 \quad \left(\frac{d^2v}{dt^2} = -6 < 0, \text{ a maximum}\right)$$

3 **Maximum velocity**

$$v = 30(5) - 3(5)^2 = 75$$

✓ **FINAL ANSWER**

75 m/s

TOPIC

Transformations of Graphs

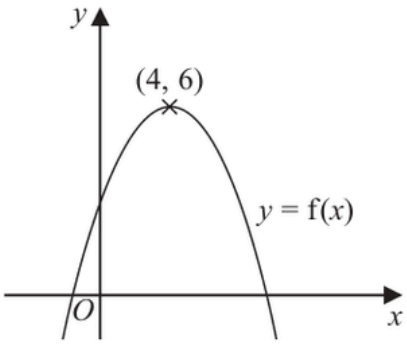
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 21

4 marks | Transformations of Graphs

21 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (4, 6)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 4)$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(2)

The equation of a curve **C** is $y = x^2 + 3x + 4$

The curve **C** is transformed to curve **S** under the translation $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) Find an equation of curve **S**.

You do not need to simplify the equation.

.....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2020

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(4, 6)$.

2 **Calculate value**

For

$$y = f(x + 4)$$

3 **the graph moves 4 units**

the graph moves 4 units left:

$$(4, 6) \to (0, 6)$$

4 **Calculate value**

For

$$y = f(2x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 6) \to (2, 6)$$

6 **Calculate value**

For part (b),

$$y = x^2 + 3x + 4$$

7 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

8 **Read the graph**

So the graph moves 4 units right and 6 units up.

9 **Add 6**

Replace x by $x - 4$, then add 6:

$$y = (x - 4)^2 + 3(x - 4) + 4 + 6$$

$$y = (x - 4)^2 + 3(x - 4) + 10$$

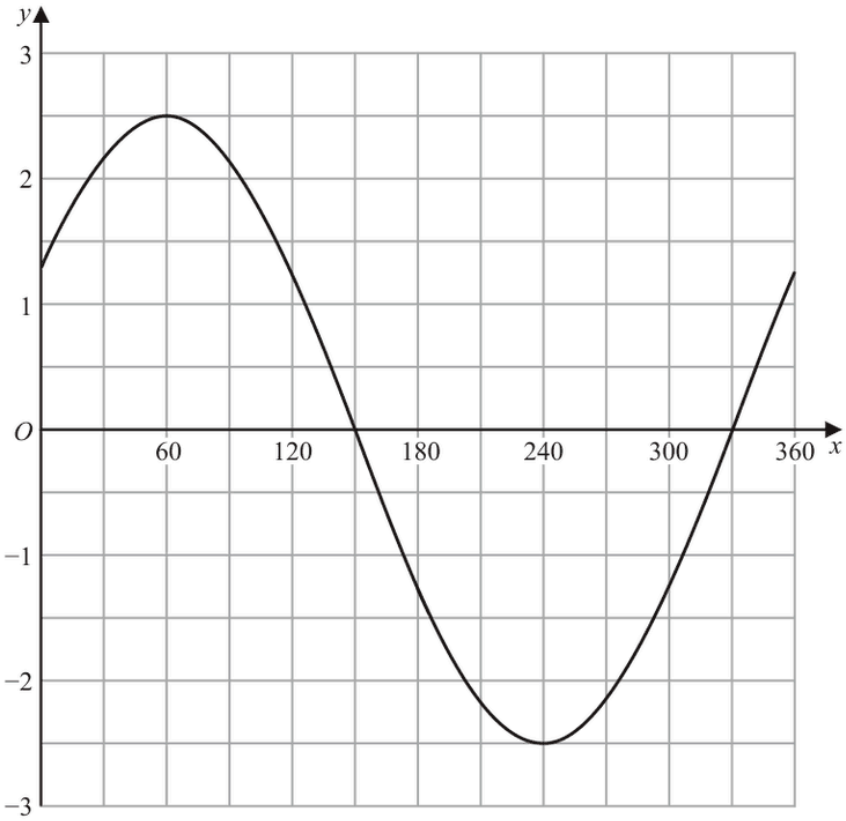
✓ **FINAL ANSWER**

$(0, 6)$, $(2, 6)$, and $y = (x - 4)^2 + 3(x - 4) + 10$

Question 22

4 marks | Transformations of Graphs

22 The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.



(a) Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

Another curve C has equation $y = f(x)$
The coordinates of the minimum point of C are (4, 5)

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

(..... , ...)

(ii) $y = f(x) - 7$

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Read the graph**

For part (a), the graph has maximum 2.5 and minimum -2.5, so

$$a = 2.5$$

2 **Use trigonometry**

The maximum occurs at $x = 60°$. For $y = a\cos(x + b)°$, a maximum occurs when

$$x + b = 360$$

3 **Calculate value**

So

$$60 + b = 360$$

$$b = 300$$

4 **Calculate value**

For part (b), the minimum point of $y = f(x)$ is $(4, 5)$.

5 **Calculate value**

For

$$y = f(2x)$$

6 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 5) \to (2, 5)$$

7 **Calculate value**

For

$$y = f(x) - 7$$

8 **the coordinate decreases by 7**

the y-coordinate decreases by 7:

$$(4, 5) \to (4, -2)$$

✓ **FINAL ANSWER**

$a = 2.5,\ b = 300;\ (2, 5)$ and $(4, -2)$

4MA1/1HR MAY/NOVEMBER 2020

Question 20

2 marks | Transformations of Graphs

20 A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are $(-3, 4)$

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May/November 2020

1 The maximum point of is

The maximum point of $y = f(x)$ is $(-3, 4)$.

2 Calculate value

For

$$y = f(x) - 6$$

3 the graph moves 6 units

the graph moves 6 units down:

$$(-3, 4) \rightarrow (-3, -2)$$

4 Calculate value

For

$$y = f(2x)$$

5 the coordinate is divided by

the x-coordinate is divided by 2:

$$(-3, 4) \rightarrow \left(-\frac{3}{2}, 4\right)$$

FINAL ANSWER

$(-3, -2)$ and $\left(-\frac{3}{2}, 4\right)$

Question 25

3 marks | Transformations of Graphs

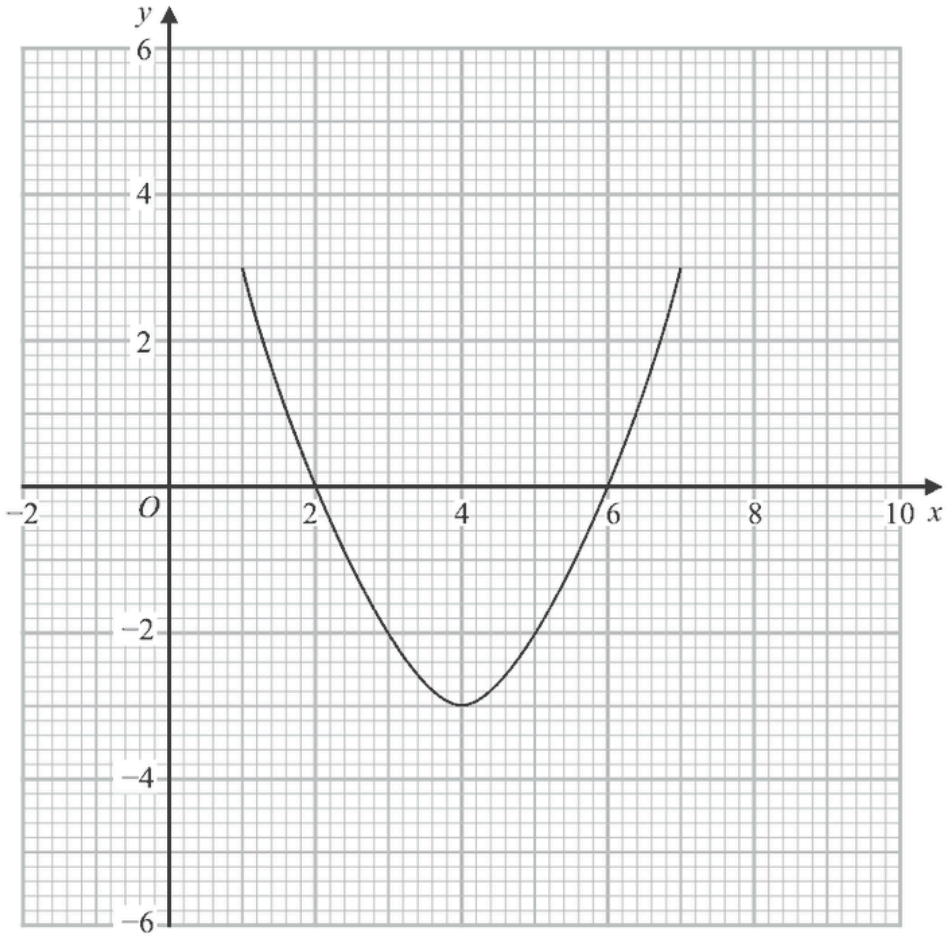
25 The curve with equation $y = g(x)$ is transformed to the curve with equation $y = -g(x)$ by the single transformation **T**.

(a) Describe fully the transformation **T**.

.....

(1)

The diagram shows the graph of $y = f(x)$



(b) On the grid, draw the graph of $y = 2f(x - 1)$

(2)

(Total for Question 25 is 3 marks)

1 **Apply transformation**

For part (a),

$$y = g(x) \rightarrow y = -g(x)$$

2 **The coordinates stay the same**

The x-coordinates stay the same and the y-coordinates change sign.

3 **Apply transformation**

So the transformation is a reflection in the x-axis.

4 **Calculate value**

For part (b),

$$y = 2f(x - 1)$$

5 **Read the graph**

First, $x - 1$ moves the graph 1 unit right. Then the factor 2 stretches the graph vertically by scale factor 2.

6 **Use points from the original**

Use points from the original graph:

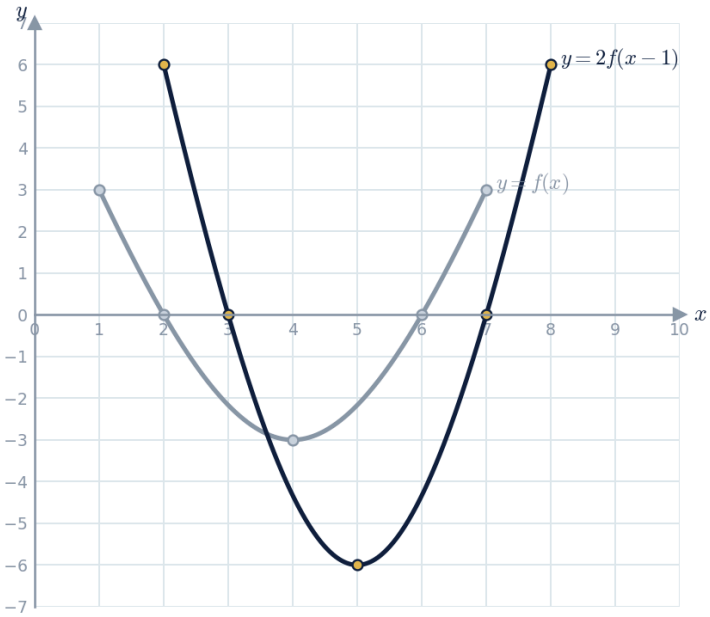
$$(1,3) \to (2,6)$$

$$(2,0) \to (3,0)$$

$$(4,-3) \to (5,-6)$$

$$(6,0) \to (7,0)$$

$$(7,3) \to (8,6)$$



7 **Apply transformation**

Draw the transformed parabola through these points.

✓ **FINAL ANSWER**

reflection in the x-axis; transformed graph through $(2,6)$, $(3,0)$, $(5,-6)$, $(7,0)$, $(8,6)$

4MA1/1H JANUARY 2021

Question 21

4 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

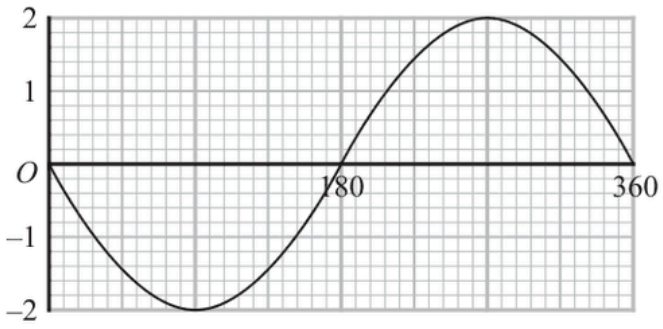
(..... , ...)

(ii) $y = \frac{1}{3} f(x)$

(..... , ...)

(2)

The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2021

1 **The minimum point of is**

The minimum point of $y = f(x)$ is $(-9, 15)$.

2 **Calculate value**

For

$$y = f(x + 3)$$

3 **the graph moves 3 units**

the graph moves 3 units left:

$$(-9, 15) \to (-12, 15)$$

4 **Evaluate fraction**

For

$$y = \frac{1}{3} f(x)$$

5 **the coordinate is divided by**

the y-coordinate is divided by 3:

$$(-9, 15) \to (-9, 5)$$

6 **Use trigonometry**

For part (b), the cosine graph has maximum 2 and minimum -2, so

$$a = 2$$

7 **The graph has a minimum**

The graph has a minimum at $x = 90°$. For $a\cos(x + b)°$, a minimum occurs when

$$x + b = 180$$

8 **Calculate value**

So

$$90 + b = 180$$

$$b = 90$$

✓ **FINAL ANSWER**

$(-12, 15)$, $(-9, 5)$, $a = 2,\ b = 90$

Question 21

6 marks | Transformations of Graphs

21 The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point A is the maximum point on **C**.

(a) Write down the coordinates of A.

(..... , ...)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

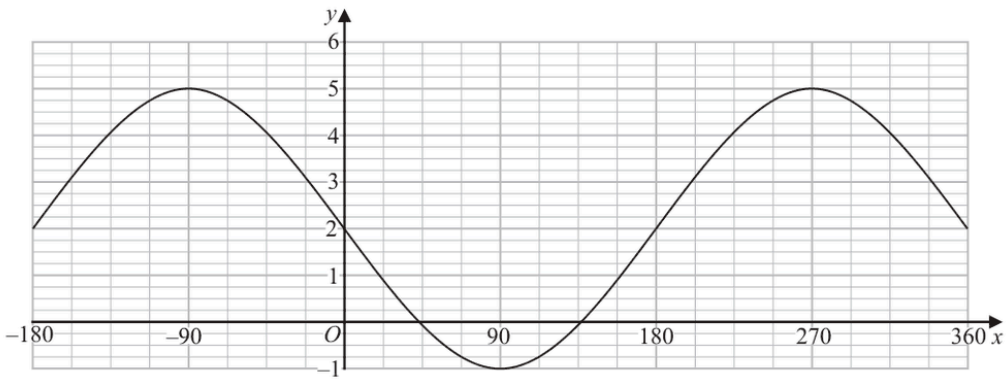
The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)

The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

(Total for Question 21 is 6 marks)

1 **Calculate value**

$$f(x) = 9 - 3(x+2)^2$$

2 **This is in completed square**

This is in completed square form, so the maximum point is

$$(-2, 9)$$

3 **Use matrices**

For the translation

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

4 **the graph moves 4 units**

the graph moves 4 units right. Replace x by $x - 4$:

$$y = 9 - 3((x-4)+2)^2$$

$$y = 9 - 3(x-2)^2$$

5 **The curve has equation**

The curve T has equation

$$y = 3(x+2)^2 - 9$$

6 **This is**

This is

$$y = -\left(9 - 3(x+2)^2\right)$$

7 **Apply transformation**

So C is reflected in the x-axis.

8 **Use trigonometry**

For the cosine graph, the maximum is 5 and the minimum is -1.

$$a = \frac{5 - (-1)}{2} = 3$$

$$c = \frac{5 + (-1)}{2} = 2$$

9 **Use trigonometry**

For

$$y = a\cos(x - b)^\circ + c$$

10 **Read the graph**

a maximum occurs when $x - b = 0$. The graph has a maximum at $x = 270°$, so

$$b = 270$$

✓ **FINAL ANSWER**

$A = (-2, 9)$, $y = 9 - 3(x - 2)^2$, reflection in the x-axis, $a = 3,\ b = 270,\ c = 2$

Question 20

6 marks | Transformations of Graphs

20 The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

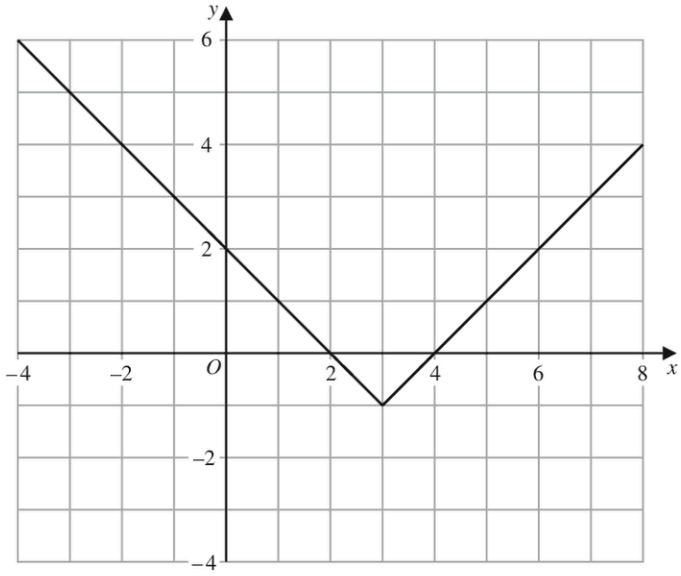
(.....,.....)

(ii) $y = f(3x)$

(.....,.....)

(2)

The graph of $y = g(x)$ is shown on the grid below.



(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

(2)

The graph of $y = h(x)$ intersects the x-axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where a is a constant.

(c) Find the two possible values of a

....., ...

(2)

(Total for Question 20 is 6 marks)

1 **The turning point of is**

The turning point of $y = f(x)$ is $(-6, -4)$.

2 **Calculate value**

For

$$y = f(x) + 5$$

3 **the graph moves 5 units**

the graph moves 5 units up:

$$(-6, -4) \rightarrow (-6, 1)$$

4 **Calculate value**

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(-6, -4) \rightarrow (-2, -4)$$

6 **Read the graph**

For the sketch of $y = 2g(x)$, keep every x-coordinate the same and double every y-coordinate.

7 **Useful points are**

Useful points are

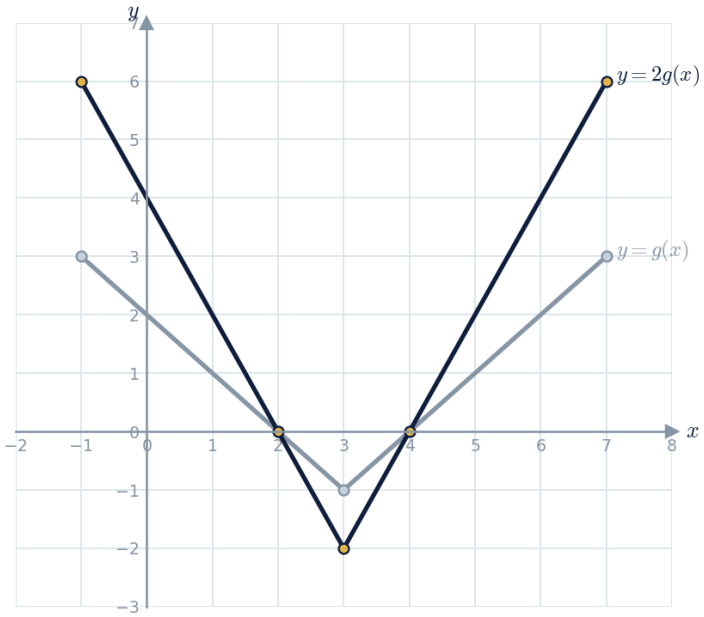
$$(-1,3) \to (-1,6)$$

$$(2,0) \to (2,0)$$

$$(3,-1) \to (3,-2)$$

$$(4,0) \to (4,0)$$

$$(7,3) \to (7,6)$$



8 **Solve inequality**

Draw straight line segments through these points for $-1 \leq x \leq 7$.

9 **Calculate value**

For part (c), $h(x)$ has roots at $x = -1$ and $x = 6$.

10 **Calculate value**

For $h(x + a)$ to pass through $(2,0)$,

$$2 + a = -1 \quad \text{or} \quad 2 + a = 6$$

$$a = -3 \quad \text{or} \quad a = 4$$

FINAL ANSWER

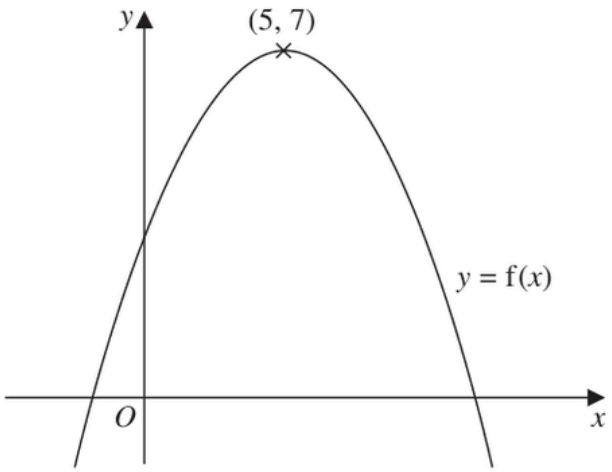
$(-6, 1)$, $(-2, -4)$, sketch through the doubled points, and $a = -3$ or $a = 4$

4MA1/1H JANUARY 2022

Question 23

2 marks | Transformations of Graphs

23 The diagram shows a sketch of the curve with equation $y = f(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 9)$

(..... , ...)

(ii) $y = f(x) + 3$

(..... , ...)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2022

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(5, 7)$.

2 **Calculate value**

For

$$y = f(x + 9)$$

3 **the graph moves 9 units**

the graph moves 9 units left:

$$(5, 7) \rightarrow (-4, 7)$$

4 **Calculate value**

For

$$y = f(x) + 3$$

5 **the graph moves 3 units**

the graph moves 3 units up:

$$(5, 7) \to (5, 10)$$

✓ **FINAL ANSWER**

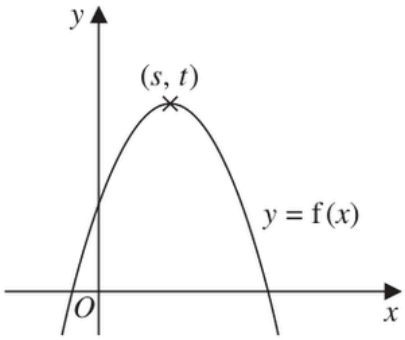
$(-4, 7)$ and $(5, 10)$

4MA1/1H JUNE 2022

Question 20

2 marks | Transformations of Graphs

20 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (s, t)

Find, in terms of s and t, the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

(.....,.....)
(1)

(ii) $y = 3f(x)$

(.....,.....)
(1)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H June 2022

1 The maximum point of is

The maximum point of $y = f(x)$ is (s, t).

2 Calculate value

For

$$y = f(x - 2)$$

3 the graph moves 2 units

the graph moves 2 units right:

$$(s, t) \to (s + 2, t)$$

4 **Calculate value**

For

$$y = 3f(x)$$

5 **the coordinate is multiplied by**

the y-coordinate is multiplied by 3:

$$(s, t) \to (s, 3t)$$

✓ **FINAL ANSWER**

$(s + 2, t)$ and $(s, 3t)$

Question 22

4 marks | Transformations of Graphs

22 The point A with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point A on the straight line with equation

(i) $y = f(x) - 3$

(..... , ...)

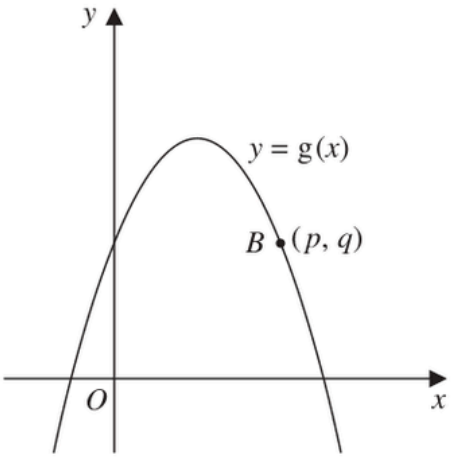
(1)

(ii) $y = f\left(\frac{x}{2}\right)$

(..... , ...)

(1)

Here is a sketch of part of the curve with equation $y = g(x)$



The point B with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of the point B on the curve with equation

$$y = -g(x - c)$$

where c is a constant.

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Point lies on**

Point $A = (-3, 2)$ lies on $y = f(x)$.

2 **Calculate value**

For

$$y = f(x) - 3$$

3 **the graph moves 3 units**

the graph moves 3 units down:

$$(-3, 2) \to (-3, -1)$$

4 **Evaluate fraction**

For

$$y = f\left(\frac{x}{2}\right)$$

5 **the coordinate is multiplied by**

the x-coordinate is multiplied by 2:

$$(-3, 2) \to (-6, 2)$$

6 **Calculate value**

For part (b), point $B = (p, q)$ lies on $y = g(x)$.

7 **Calculate value**

For

$$y = -g(x - c)$$

8 **Read the graph**

$x - c$ moves the graph c units right, and the minus sign reflects the graph in the x-axis.

9 **Apply transformation**

So

$$(p, q) \to (p + c, -q)$$

✓ **FINAL ANSWER**

$(-3, -1)$, $(-6, 2)$, and $(p + c, -q)$

4MA1/1HR MAY 2023

Question 20

7 marks | Transformations of Graphs

20 Curve **C** has equation $y = f(x)$
The graph of curve **C** has one maximum point.
The coordinates of this maximum point are (3, 5)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = 2f(x)$

(..... , ...)
(1)

(ii) $y = f(x) - 7$

(..... , ...)
(1)

(iii) $y = f(-x)$

(..... , ...)
(1)

Curve **L** has equation $y = x^2 + 7x + 20$

Curve **L** is transformed to curve **S** under the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(b) Find an equation for **S**
Give your answer in the form $y = ax^2 + bx + c$

$y =$ .....
(4)

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May 2023

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(3, 5)$.

2 **Calculate value**

For

$$y = 2f(x)$$

3 **the coordinate is multiplied by**

the y-coordinate is multiplied by 2:

$$(3, 5) \to (3, 10)$$

4 **Calculate value**

For

$$y = f(x) - 7$$

5 **the graph moves 7 units**

the graph moves 7 units down:

$$(3, 5) \to (3, -2)$$

6 **Calculate value**

For

$$y = f(-x)$$

7 **the graph reflects in the**

the graph reflects in the y-axis:

$$(3, 5) \to (-3, 5)$$

8 **Calculate value**

For part (b),

$$y = x^2 + 7x + 20$$

9 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

10 **Read the graph**

So the graph moves 2 units right.

11 **Replace by**

Replace x by $x - 2$:

$$y = (x - 2)^2 + 7(x - 2) + 20$$

$$y = x^2 + 3x + 10$$

FINAL ANSWER

$(3, 10)$, $(3, -2)$, $(-3, 5)$, and $y = x^2 + 3x + 10$

4MA1/1H MAY 2023

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(..... , ...)

(1)

(b) $y = f(3x)$

(..... , ...)

(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2023

1 The turning point of is

The turning point of $y = f(x)$ is $(6, 5)$.

2 Calculate value

For

$$y = f(x - 4)$$

3 the graph moves 4 units

the graph moves 4 units right:

$$(6, 5) \to (10, 5)$$

4 Calculate value

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(6, 5) \rightarrow (2, 5)$$

✓ **FINAL ANSWER**

$(10, 5)$ and $(2, 5)$

Question 24

4 marks | Transformations of Graphs

24 The curve with equation $f(x) = 5x^2 + 9x + 2$ is transformed to the curve with equation

$g(x) = 5(x+4)^2 + 9(x+4) + 8$ by the translation $\begin{pmatrix} a \\ b \end{pmatrix}$

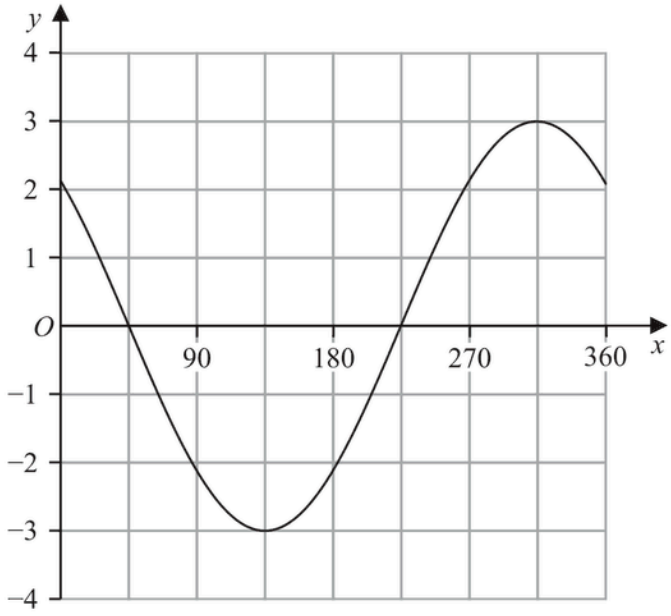
(a) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

The graph of $y = p\cos(x+q)^\circ$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $p > 0$ and $0 < q < 360$

(b) find the value of p and the value of q

$p =$ .....

$q =$ .....

(2)

(Total for Question 24 is 4 marks)

1 **Read the graph**

For part (a),

$$f(x) = 5x^2 + 9x + 2$$

$$g(x) = 5(x+4)^2 + 9(x+4) + 8$$

2 **Read the graph**

Since

$$f(x+4) = 5(x+4)^2 + 9(x+4) + 2$$

$$g(x) = f(x+4) + 6$$

3 **Read the graph**

So the graph has moved 4 units left and 6 units up.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

4 **Read the graph**

For part (b), the graph has maximum value 3 and minimum value -3, so

$$p = 3$$

5 **The minimum occurs when**

The minimum occurs when

$$x + q = 180$$

6 **Read the graph**

From the graph, the minimum is at $x = 135$, so

$$135 + q = 180$$

$$q = 45$$

✓ **FINAL ANSWER**

$a = -4,\ b = 6,\ p = 3,\ q = 45$

4MA1/1H MAY 2024

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is one minimum point on this curve.
The coordinates of this minimum point are $(5, -4)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 7)$

(.....,.....)
(1)

(ii) $y = f(x) - 6$

(.....,.....)
(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(5, -4)$.

2 Calculate value

For

$$y = f(x + 7)$$

3 the graph moves 7 units

the graph moves 7 units left:

$$(5, -4) \rightarrow (-2, -4)$$

4 Calculate value

For

$$y = f(x) - 6$$

5 **the graph moves 6 units**

the graph moves 6 units down:

$$(5, -4) \rightarrow (5, -10)$$

✓ **FINAL ANSWER**

$(-2, -4)$ and $(5, -10)$

4MA1/2HR JUNE 2024

Question 23

2 marks | Transformations of Graphs

23 A curve has equation $y = f(x)$
The coordinates of the minimum point on this curve are $(6, -3)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 10$

(.....,.....)
(1)

(ii) $y = f(3x)$

(.....,.....)
(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(6, -3)$.

2 Calculate value

For

$$y = f(x) + 10$$

3 the graph moves 10 units

the graph moves 10 units up:

$$(6, -3) \to (6, 7)$$

4 Calculate value

For

$$y = f(3x)$$

5 the coordinate is divided by

the x-coordinate is divided by 3:

$$(6, -3) \to (2, -3)$$

FINAL ANSWER

$(6, 7)$ and $(2, -3)$

4MA1/2H NOVEMBER 2024

Question 19

3 marks | Transformations of Graphs

19 A curve has equation $y = f(x)$

There is only one minimum point on the curve.
The coordinates of this minimum point are (5, 4)

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 5)$

(..... , ...)
(1)

(ii) $y = 3f(x)$

(..... , ...)
(1)

(iii) $y = f(x) - 7$

(..... , ...)
(1)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is

$$(5, 4)$$

2 Read the graph

(i) For

$$y = f(x + 5)$$

3 the graph moves 5 units

the graph moves 5 units left, so the minimum point becomes

$$(0, 4)$$

4 **Read the graph**

(ii) For

$$y = 3f(x)$$

5 **the y coordinate is multiplied**

the y-coordinate is multiplied by 3, so the minimum point becomes

$$(5, 12)$$

6 **Read the graph**

(iii) For

$$y = f(x) - 7$$

7 **the graph moves 7 units**

the graph moves 7 units down, so the minimum point becomes

$$(5, -3)$$

✓ **FINAL ANSWER**

$(0, 4)$, $(5, 12)$, and $(5, -3)$.

4MA1/1H JUNE 2025

Question 19

2 marks | Transformations of Graphs

19 The equation of a curve is $y = f(x)$

There is only one minimum point on the curve.
The coordinates of this minimum point are $(8, -12)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 3$

(..... , ...)

(1)

(ii) $y = f(2x)$

(..... , ...)

(1)

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2025

1 **The minimum point of is**

The minimum point of $y = f(x)$ is

$$(8, 12)$$

2 **Read the graph**

For

$$y = f(x) + 5$$

the graph moves up by 5:

$$(8, 12) \rightarrow (8, 17)$$

3 **Read the graph**

For

$$y = f(2x)$$

the graph is horizontally compressed by scale factor $\frac{1}{2}$, so the x-coordinate is halved:

$$(8, 12) \to (4, 12)$$

✓ **FINAL ANSWER**

(i) $(8, 17)$ and (ii) $(4, 12)$.

4MA1/1HR JUNE 2025

Question 21

3 marks | Transformations of Graphs

21 The curve with equation $y = f(x)$ has one turning point.
The coordinates of this turning point are $(-6, 9)$

(a) Write down the coordinates of the turning point on the curve with equation $y = f(3x)$

(..... , ...)

(1)

The curve **C** with equation $y = g(x)$ is transformed to give the curve **S** with equation $y = g(x + a) + b$

The point $(4, -5)$ on **C** is mapped to the point $(1, -16)$ on **S**

(b) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(-6, 9)$.

2 Calculate value

For

$$y = f(3x)$$

3 the coordinate is divided by

the x-coordinate is divided by 3:

$$(-6, 9) \rightarrow (-2, 9)$$

4 Calculate value

For part (b), a point $(4, -5)$ on $y = g(x)$ maps to $(1, -16)$ on

$$y = g(x + a) + b$$

5 **Calculate value**

For the x-coordinate:

$$1 + a = 4$$

$$a = 3$$

---

6 **Calculate value**

For the y-coordinate:

$$-5 + b = -16$$

$$b = -11$$

✓ **FINAL ANSWER**

$(-2, 9)$, $a = 3$, $b = -11$

Question 27

4 marks | Transformations of Graphs

27 A curve has equation $y = f(x)$

There is one maximum point on this curve.
The coordinates of this maximum point are $(6, -5)$

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 4)$

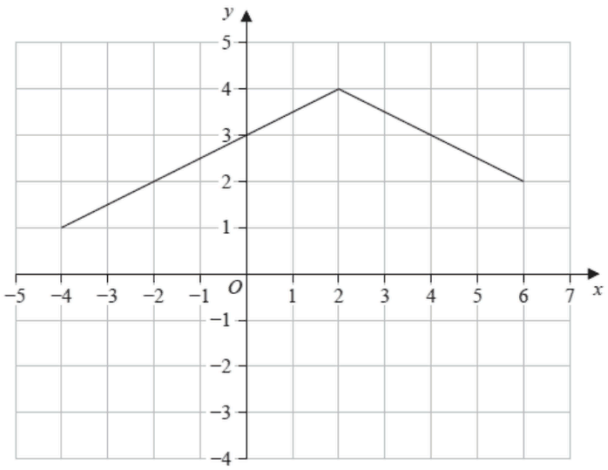
(.....)
(1)

(ii) $y = f(3x)$

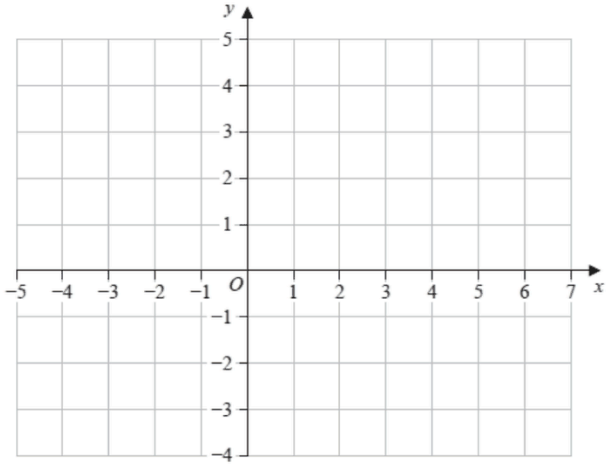
(.....)
(1)

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Here is the graph of $y = g(x)$



(b) On the grid below, draw the graph of $y = g(2x) - 3$



(2)

(Total for Question 27 is 4 marks)

1 **(a)**

(a)

2 **Read the graph**

For $y = f(x - 4)$, move the graph 4 units right:

$$(6, -5) \to (10, -5)$$

3 **Divide the -coordinate**

For $y = f(3x)$, divide the x-coordinate by 3:

$$(6, -5) \to (2, -5)$$

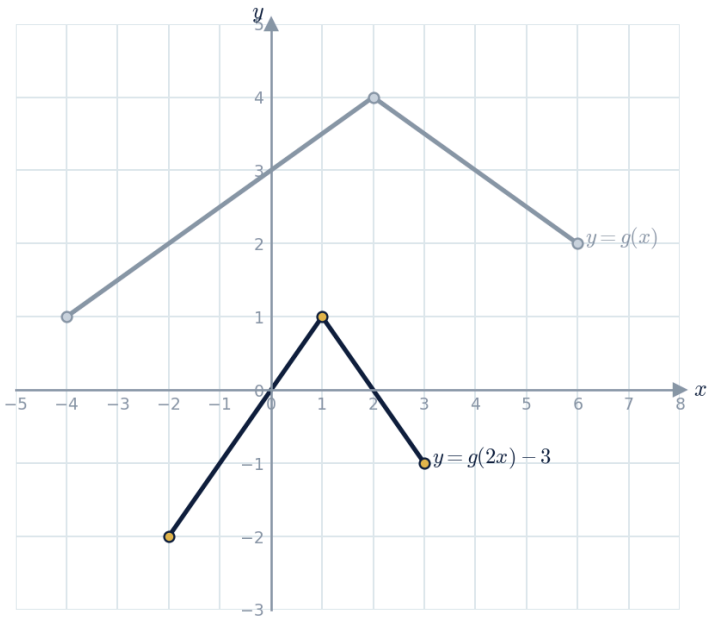
4 **Subtract from each -coordinate**

(b) For $y = g(2x) - 3$, halve each x-coordinate and subtract 3 from each y-coordinate.

5 **Useful transformed points are**

Useful transformed points are

$$(-4, 1) \to (-2, -2), \quad (2, 4) \to (1, 1), \quad (6, 2) \to (3, -1)$$



6 **Join these points with straight**

Join these points with straight line segments.

FINAL ANSWER

$(10, -5)$, $(2, -5)$, graph through $(-2, -2), (1, 1), (3, -1)$.

4MA1/1H NOVEMBER 2025

Question 24

3 marks | Transformations of Graphs

24 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (4, 3)

Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x + 5)$

(..... , ...)
(1)

(ii) $y = f(x) + 7$

(..... , ...)
(1)

(iii) $y = f(2x)$

(..... , ...)
(1)

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H November 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(4, 3)$.

2 Calculate value

For

$$y = f(x + 5)$$

3 the graph moves 5 units

the graph moves 5 units left:

$$(4, 3) \to (-1, 3)$$

4 **Calculate value**

For

$$y = f(x) + 7$$

5 **the graph moves 7 units**

the graph moves 7 units up:

$$(4, 3) \to (4, 10)$$

6 **Calculate value**

For

$$y = f(2x)$$

7 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 3) \to (2, 3)$$

✓ **FINAL ANSWER**

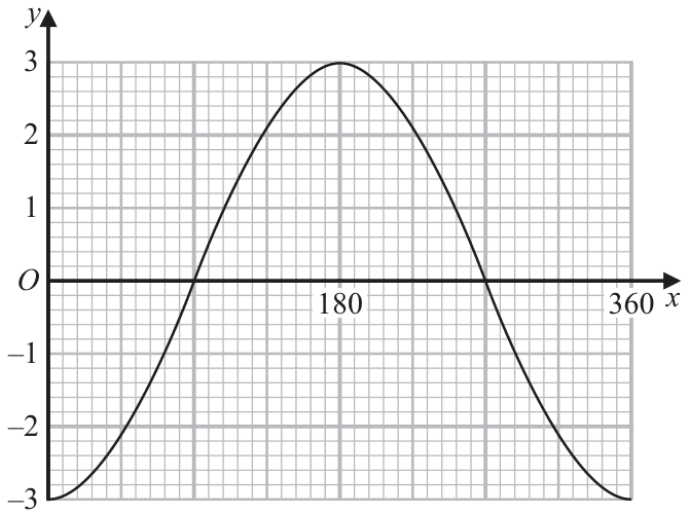
$(-1, 3)$, $(4, 10)$, and $(2, 3)$

4MA1/2HR JUNE 2026

Question 25

4 marks | Transformations of Graphs

25 The graph of $y = a\sin(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a < 0$ and that $0 < b < 360$

(a) find the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

A curve has equation $y = f(x)$
The point T with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of T on the curve with equation

$$y = f(2x) + 5$$

Give your coordinates in terms of p and q

(..... , ...)

(2)

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2026

1 **Part (a) | Step 1**

The graph has maximum 3 and minimum -3, so $|a| = 3$. Since $a < 0$, $a = -3$.

The amplitude is half the vertical range.

2 **Part (a) | Step 2**

At $x = 0$, $y = -3$, so $-3 \sin b° = -3$ and $\sin b° = 1$. Hence $b = 90$ for $0 < b < 360$.

The minimum at x=0 fixes the phase when the amplitude is negative.

---

3 **Part (b) | Step 1**

To use the original input p, set $2x = p$, giving $x = p/2$.

The replacement of x by 2x halves horizontal coordinates.

---

4 **Part (b) | Step 2**

The corresponding output is $f(p) + 5 = q + 5$.

Adding 5 translates every output upwards by 5.

✓ **FINAL ANSWER**

(a) $a = -3, \qquad b = 90$. (b) $\left(\frac{p}{2}, q + 5\right)$.

Question 26

5 marks | Transformations of Graphs

26 The curve **P** has equation $y = f(x)$
The coordinates of the minimum point on **P** are $(-5, 4)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = 3f(x)$

(..... , ...)

(1)

The curve ***Q*** has equation $y = 2 - (x + 3)^2$

The curve ***R*** has equation $y = 2 - (3 - x)^2$

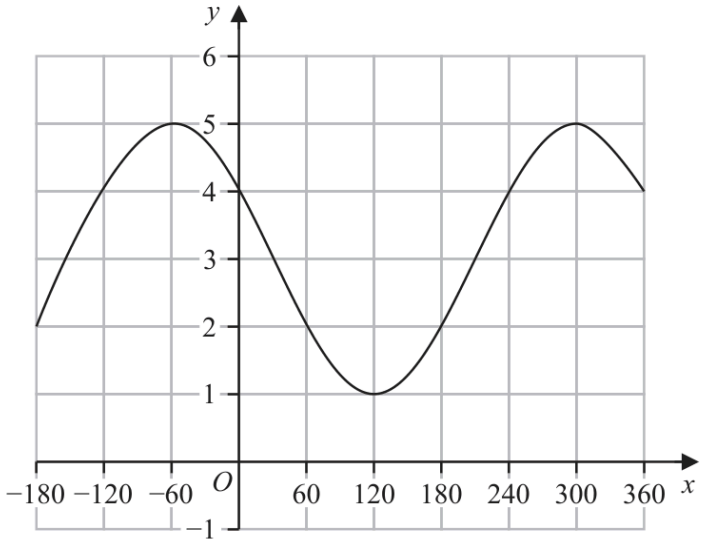
(b) Describe fully the single transformation that maps ***Q*** to ***R***

.....

.....

(1)

The graph of $y = a\cos(x + b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid.



Given that $a > 0$ and $0 < b < 360$

(c) find the value of a, the value of b and the value of c

a = ...

b = ...

c = ...

(3)

(Total for Question 26 is 5 marks)

1 **Part (a) | Step 1**

For $y = 3f(x)$, x-coordinates are unchanged and y-coordinates are multiplied by 3. Thus $(-5, 4) \mapsto (-5, 12)$.

The vertical scale factor is positive, so a minimum remains a minimum.

2 **Part (b) | Step 1**

For $Q(x) = 2 - (x + 3)^2$,

$$Q(-x) = 2 - (-x + 3)^2 = 2 - (3 - x)^2 = R(x).$$

Replacing x by $-x$ reflects the graph in the y-axis.

The point mapping is (x,y) to (-x,y).

3 **Part (c) | Step 1**

The maximum is 5 and the minimum is 1, so

$$a = \frac{5 - 1}{2} = 2, \qquad c = \frac{5 + 1}{2} = 3.$$

The positive amplitude is half the range; the vertical shift is the midline.

4 **Part (c) | Step 2**

A maximum occurs at $x = -60°$. For positive a, cosine is maximal when $x + b = 0°$ modulo $360°$, giving $b = 60$ in the stated interval. Hence $y = 2\cos(x + 60)° + 3$.

Read the phase from a maximum and apply 0<b<360.

✓ **FINAL ANSWER**

(a) $(-5, 12)$ (b) Reflection in the y-axis. (c) $a = 2, \quad b = 60, \quad c = 3$.

TOPIC

Kinematic Graphs

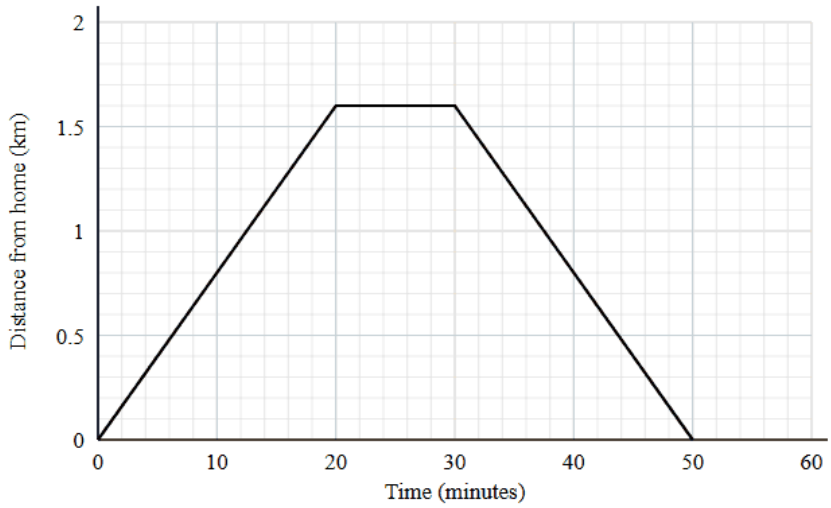
Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 8

4 marks | Kinematic Graphs

8 Ali walked from his home to a shop. He stayed at the shop and then walked back home.
Here is the distance-time graph for his journey.



(a) How far was Ali from home after 10 minutes?

..... km

(1)

(b) How many minutes did Ali stay at the shop?

..... minutes

(1)

(c) Work out Ali's speed, in kilometres per hour, on his walk back home.

..... km/h

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · EP Specimen

1 **(a) Read the graph at 10 minutes**

The line from $(0, 0)$ to $(20, 1.6)$ is halfway up at 10 minutes: 0.8 km.

2 **(b) The flat section**

From 20 to 30 minutes: 10 minutes.

3 **(c) Speed = distance ÷ time**

He walks 1.6 km in $50 - 30 = 20$ minutes $= \frac{1}{3}$ hour.

$$1.6 \div \frac{1}{3} = 4.8 \text{ km/h}$$

FINAL ANSWER

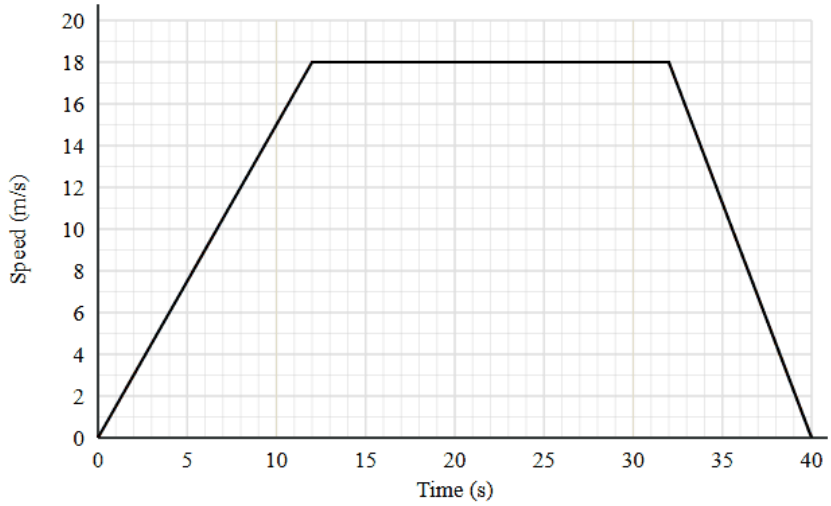
(a) 0.8 km (b) 10 minutes (c) 4.8 km/h

EP SPECIMEN

Question 9

5 marks | Kinematic Graphs

9 Here is the speed-time graph for the first 40 seconds of a car's journey.



(a) Work out the acceleration of the car during the first 12 seconds.

..... m/s 2

(2)

(b) Work out the total distance the car travels in the 40 seconds.

..... m

(3)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · EP Specimen

1 **(a) Acceleration = gradient**

$$\frac{18-0}{12} = 1.5\ \text{m/s}^2$$

2 **(b) Distance = area under the graph**

Split into a triangle, a rectangle and a triangle:

$$\tfrac{1}{2} \times 12 \times 18 + 20 \times 18 + \tfrac{1}{2} \times 8 \times 18 = 108 + 360 + 72 = 540\ \text{m}$$

✓ **FINAL ANSWER**

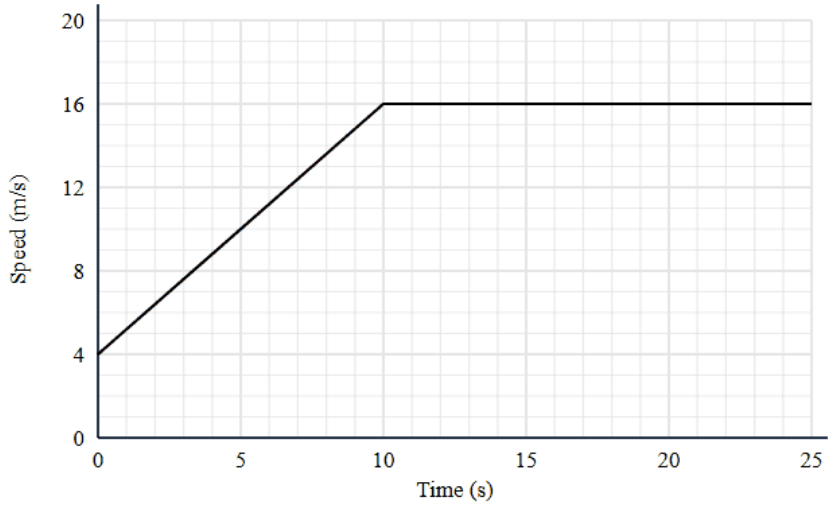
(a) 1.5 m/s^2 (b) 540 m

EP SPECIMEN

Question 10

4 marks | Kinematic Graphs

10 Here is the speed-time graph for 25 seconds of a cyclist's journey.



(a) Work out the distance the cyclist travels in the 25 seconds.

..... m

(3)

(b) Work out the average speed of the cyclist for the 25 seconds.

..... m/s

(1)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · EP Specimen

1 **(a) Area of the trapezium (first 10 s)**

$$\frac{4+16}{2} \times 10 = 100 \text{ m}$$

2 **(a) Area of the rectangle (last 15 s)**

$$16 \times 15 = 240 \text{ m}$$

Total: $100 + 240 = 340$ m.

3 **(b) Average speed = total distance ÷ total time**

$$340 \div 25 = 13.6 \text{ m/s}$$

FINAL ANSWER

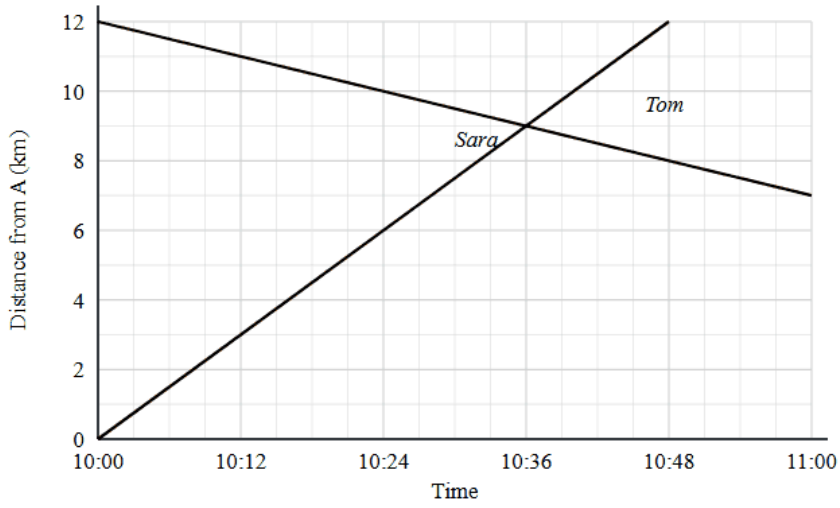
(a) $\mathbf{340}$ m (b) $\mathbf{13.6}$ m/s

EP SPECIMEN

Question 11

4 marks | Kinematic Graphs

11 At 10:00 Sara leaves town A and cycles towards town B.
At 10:00 Tom leaves town B and walks towards town A.
The graph shows their distances from town A.



(a) At what time do Sara and Tom pass each other?

.....

(1)

(b) How far from town A are they when they pass each other?

..... km

(1)

(c) Work out Tom's speed in kilometres per hour.

..... km/h

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · EP Specimen

1 **(a) Where the lines cross**

The lines cross at 36 minutes after 10:00, so at $10{:}36$.

2 **(b) Read across**

At the crossing point the distance from A is 9 km.

3 (c) Tom's speed

Tom goes from 12 km to 7 km from A in 1 hour:

$$\frac{12-7}{1} = 5\ \text{km/h}$$

FINAL ANSWER

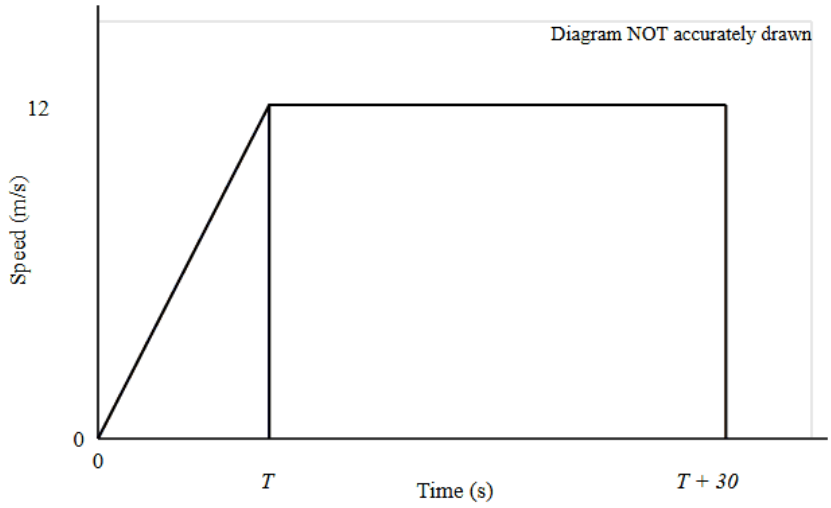
(a) $10{:}36$ (b) 9 km (c) 5 km/h

EP SPECIMEN

Question 12

3 marks | Kinematic Graphs

12 A particle starts from rest. It speeds up at a constant rate for T seconds until its speed is 12 m/s.
It then travels at 12 m/s for 30 seconds.
The total distance it travels is 450 m.



Work out the value of T.

$T =$ .....

(3)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · EP Specimen

1 **Distance = area under the graph**

Triangle: $\frac{1}{2} \times T \times 12 = 6T$. Rectangle: $30 \times 12 = 360$.

2 **Form and solve the equation**

$$6T + 360 = 450 \Rightarrow 6T = 90 \Rightarrow T = 15$$

✓ **FINAL ANSWER**

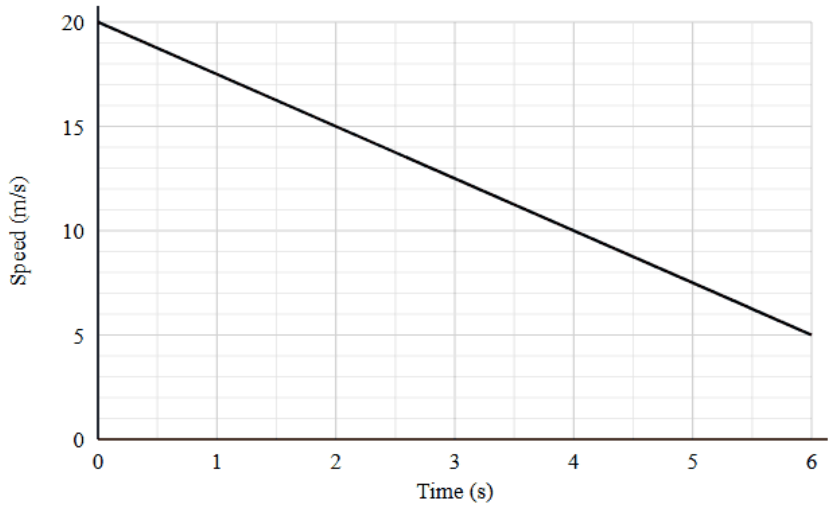
$T = 15$

EP SPECIMEN

Question 13

4 marks | Kinematic Graphs

13 A car is travelling at 20 m/s. The driver brakes and the car slows down at a constant rate. Here is the speed-time graph for the next 6 seconds.



(a) Work out the deceleration of the car.

..... m/s 2

(2)

(b) Work out the distance the car travels in the 6 seconds.

..... m

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · EP Specimen

1 **(a) Gradient of the line**

$$\frac{5-20}{6} = -2.5$$

So the deceleration is $\mathbf{2.5}$ m/s^2.

2 **(b) Area of the trapezium**

$$\frac{20+5}{2} \times 6 = 75 \text{ m}$$

FINAL ANSWER

(a) 2.5 m/s^2 (b) 75 m

EP SPECIMEN

Question 14

4 marks | Kinematic Graphs

14 A train starts from rest and speeds up at a constant rate for 50 seconds until its speed is 30 m/s.
It then travels at 30 m/s for t seconds.
It then slows down at a constant rate and stops 40 seconds later.
The total distance the train travels is 6 km.

Work out the value of t.

$t =$ .....

(4)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · EP Specimen

1 Sketch the speed-time graph

A trapezium: up for 50 s, flat at 30 m/s for t s, down for 40 s.

2 Distance = area (in metres)

$$\tfrac{1}{2} \times 50 \times 30 + 30t + \tfrac{1}{2} \times 40 \times 30 = 750 + 30t + 600$$

3 Form and solve the equation

6 km $= 6000$ m:

$$1350 + 30t = 6000 \Rightarrow 30t = 4650 \Rightarrow t = 155$$

FINAL ANSWER

$t = 155$