**Elite Mathematics**
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

# Linear Expertise With Answers

Q20+ questions with Dr Eslam Ahmed's worked solutions.

| 273 | 2020–2026 | Questions with worked solutions |
| --- | --- | --- |
| topic placements | 55 past papers | checked by Dr Eslam |



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TOPIC

# Rounding, Estimation & Bounds

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY 2021

# Question 20

3 marks | Rounding, Estimation & Bounds

**20** $P = \dfrac{t - w}{y}$

$t$ = 9.7 correct to 1 decimal place

$w$ = 5.9 correct to 1 decimal place

$y$ = 3 correct to 1 significant figure

Calculate the upper bound for the value of $P$.
Show your working clearly.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H May 2021

**1 Estimate the value**

$$P = \frac{t - w}{y}$$

**2 Find upper bound**

For the upper bound of $P$, make the numerator as large as possible and the denominator as small as possible.

$t = 9.75, \quad w = 5.85, \quad y = 2.5$

$$P = \frac{9.75 - 5.85}{2.5}$$

$$P = \frac{3.90}{2.5} = 1.56$$

✓ **FINAL ANSWER**

$1.56$

4MA1/1HR JANUARY 2022

# Question 22

4 marks | Rounding, Estimation & Bounds

**22** The diagram shows triangle $ABC$

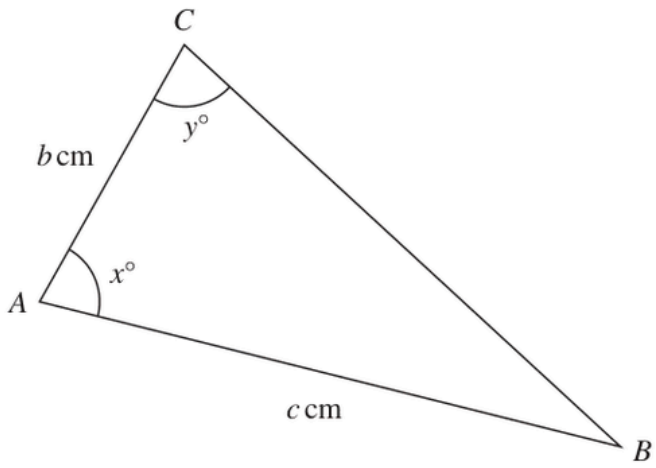



Diagram **NOT** accurately drawn

$c = 11.5$ correct to one decimal place
$x = 80$ correct to the nearest whole number
$y = 75$ correct to the nearest whole number

Calculate the upper bound for the value of $b$
Show your working clearly.
Give your answer correct to 3 significant figures.

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1HR January 2022

### 1 Find upper bound

For an upper bound for $b$, use the upper bound of $c$:

$c = 11.55$

### 2 Use sine rule

Use the lower bounds of $x$ and $y$, because this makes angle $B$ as large as possible and makes $\sin y$ as small as possible.

$x = 79.5°, \quad y = 74.5°$

### 3 Use trigonometry

So

$\angle B = 180° - 79.5° - 74.5° = 26°$

**4** **Use trigonometry**

Using the sine rule,

$$\frac{b}{\sin 26^\circ} = \frac{11.55}{\sin 74.5^\circ}$$

$$b = \frac{11.55 \sin 26^\circ}{\sin 74.5^\circ} = 5.254...$$

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✓ **FINAL ANSWER**

$5.25$ cm to 3 significant figures.

4MA1/2H JUNE 2023

# Question 25

4 marks | Rounding, Estimation & Bounds

25 A solid sphere has a radius of 2.8 centimetres, correct to 1 decimal place.
The sphere has a mass of $M\pi$ grams, where $M = 260$ correct to 2 significant figures.

Work out the upper bound for the density of the sphere.
Give your answer in $g/cm^3$ correct to 2 decimal places.
Show your working clearly.

..... $g/cm^3$

**(Total for Question 25 is 4 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H June 2023

**1 Find upper bound**

For the upper bound of density, use the upper bound of the mass and the lower bound of the volume.

**2 The radius is cm correct**

The radius is $2.8$ cm correct to 1 decimal place, so the lower bound is
$r = 2.75$

**3 Find upper bound**

$M = 260$ correct to 2 significant figures, so the upper bound is
$M = 265$

**4 The mass is given as**

The mass is given as $M\pi$ grams, where $M = 265$ for the upper bound.

**5 The volume of the sphere**

The volume of the sphere is

$$\frac{4}{3}\pi r^3$$

**6 Rearrange formula**

So

$$\text{density} = \frac{M\pi}{\frac{4}{3}\pi r^3}$$

**7** **The cancels**

The $\pi$ cancels:

$$\text{density} = \frac{3M}{4r^3}$$

**8** **Upper bound**

Upper bound:

$$\frac{3(265)}{4(2.75)^3} = 9.5567\ldots$$

✓ **FINAL ANSWER**

$9.56\ \text{g/cm}^3$ to 2 decimal places.

4MA1/1H NOVEMBER 2024

# Question 21

3 marks | Rounding, Estimation & Bounds

**21** $T = \dfrac{x^2 + y^2}{w}$

$x = 28.4$ correct to 1 decimal place.

$y = 17$ correct to 2 significant figures.

$w = 90$ correct to the nearest 5

Calculate the upper bound for the value of $T$

Give your answer correct to 3 significant figures.
Show your working clearly.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H November 2024

**1 Estimate the value**

$$T = \frac{x^2 + y^2}{w}$$

**2 Find upper bound**

For the upper bound of $T$, use the upper bounds of $x$ and $y$, and the lower bound of $w$.

$x = 28.45, \quad y = 17.5, \quad w = 87.5$

$$T = \frac{28.45^2 + 17.5^2}{87.5}$$

$$T = 12.7503\ldots$$

**FINAL ANSWER**

$12.8$ to 3 significant figures.

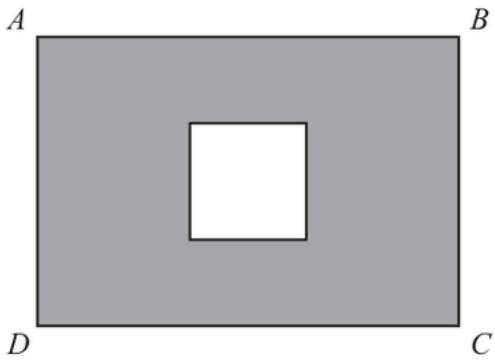
4MA1/2H JUNE 2025

# Question 22

4 marks | Rounding, Estimation & Bounds

**22** The diagram shows a square inside rectangle $ABCD$

Diagram **NOT** accurately drawn




The total area of the region shown shaded in the diagram is $X$ cm²

$AB = 11.5$ cm correct to the nearest 0.5 cm
$BC = 9.2$ cm correct to 2 significant figures
side of square = 4.1 cm correct to 2 significant figures

By considering bounds, work out the value of $X$ to a suitable degree of accuracy.
Show your working clearly.

$X =$ .....

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H June 2025

### 1 The shaded area is

The shaded area is

$$X = \text{area of rectangle} - \text{area of square}$$

### 2 Bounds

Bounds:

$$11.25 \leq AB < 11.75$$
$$9.15 \leq BC < 9.25$$
$$4.05 \leq \text{side of square} < 4.15$$

### 3 Find lower bound

Lower bound for $X$:

$$11.25(9.15) - 4.15^2 = 85.715$$

**4** **Find upper bound**

Upper bound for $X$:

$11.75(9.25) - 4.05^2 = 92.285$

**5** **Estimate the value**

So $85.715 \leq X < 92.285$. Both bounds round to $90$ to the nearest 10.

✓ **FINAL ANSWER**

$X = 90\ \text{cm}^2$ to a suitable degree of accuracy.

4MA1/2H NOVEMBER 2025

# Question 24

5 marks | Rounding, Estimation & Bounds

**24** $D = \dfrac{n}{p - q}$

$n = 10.3$ correct to 1 decimal place
$p = 7.24$ correct to 2 decimal places
$q = 4.39$ correct to 2 decimal places

By considering bounds, work out the value of $D$ to a suitable degree of accuracy.
Show your working clearly.

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H November 2025

**1 Estimate the value**

$$D = \frac{n}{p - q}$$

**2 Bounds**

Bounds:

$10.25 \leq n < 10.35$

$7.235 \leq p < 7.245$

$4.385 \leq q < 4.395$

**3 Lower bound**

Lower bound:

$$\frac{10.25}{7.245 - 4.385} = 3.5839...$$

**4 Upper bound**

Upper bound:

$$\frac{10.35}{7.235 - 4.395} = 3.6443...$$

**5 Estimate the value**

So $3.5839... \leq D < 3.6443...$. Both bounds round to $3.6$ to 1 decimal place.

**FINAL ANSWER**

$D = 3.6$ to a suitable degree of accuracy.



TOPIC

# Surds

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

# Question 20

3 marks | Surds

**20** The area of a rectangle is $18\,\text{cm}^2$

The length of the rectangle is $\left(\sqrt{7}+1\right)$ cm.

Without using a calculator and showing each stage of your working,

find the width of the rectangle.
Give your answer in the form $a\sqrt{b}+c$ where $a$, $b$ and $c$ are integers.

..... cm

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H January 2020

**1** **Area of rectangle**

Area of rectangle:

$$18\ \text{cm}^2$$

**2** **Length**

Length:

$$\sqrt{7}+1$$

**3** **Width**

Width:

$$\frac{18}{\sqrt{7}+1}$$

## 4 Rationalise the denominator

Rationalise the denominator:

$$\frac{18}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1}$$

$$= \frac{18(\sqrt{7}-1)}{7-1}$$

$$= \frac{18(\sqrt{7}-1)}{6}$$

$$= 3(\sqrt{7}-1)$$

$$= 3\sqrt{7}-3$$

**FINAL ANSWER**

$$\boxed{3\sqrt{7}-3 \text{ cm}}$$

4MA1/1HR JUNE 2022

# Question 21

4 marks | Surds

**21** Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where $p$ and $q$ are integers.

Show each stage of your working clearly.

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR June 2022

**1 Simplify surd**

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)^2}$$

**2 Simplify surd**

First simplify:

$$\sqrt{8}=2\sqrt{2}$$

**3 Simplify surd**

and

$$(\sqrt{2}-1)^2=2-2\sqrt{2}+1=3-2\sqrt{2}$$

**4 Simplify surd**

So

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

**5 Rationalise the denominator**

Rationalise the denominator:

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{(3+2\sqrt{2})^2}{9-8}$$

$$(3+2\sqrt{2})^2=9+12\sqrt{2}+8=17+12\sqrt{2}$$

**FINAL ANSWER**

$17 + 12\sqrt{2}$

4MA1/2H JUNE 2023

# Question 23

4 marks | Surds

**23** The diagram shows a cuboid with a square cross section.

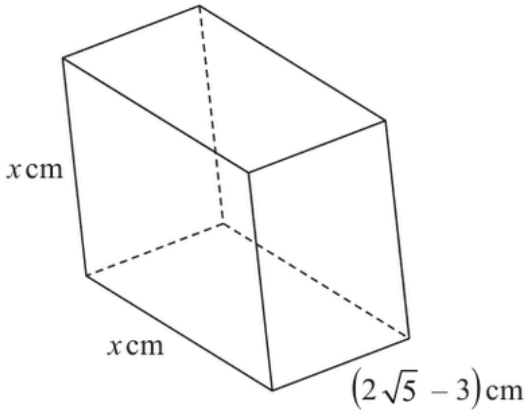



Diagram **NOT** accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of $x$
Give your answer in the form $a + \sqrt{b}$ where $a$ and $b$ are integers.
Show your working clearly.

$x =$ .....

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H June 2023

**1** **The cuboid has square cross**

The cuboid has square cross section, so its volume is

$$x^2(2\sqrt{5} - 3)$$

**2** **Given**

Given

$$x^2(2\sqrt{5} - 3) = 13 + 6\sqrt{5}$$

**3** **Use trigonometry**

So

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3}$$

**4** **Rationalise the denominator**

Rationalise the denominator:

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3} \cdot \frac{2\sqrt{5} + 3}{2\sqrt{5} + 3}$$

$$x^2 = \frac{(13 + 6\sqrt{5})(2\sqrt{5} + 3)}{20 - 9}$$

$$x^2 = \frac{99 + 44\sqrt{5}}{11}$$

$$x^2 = 9 + 4\sqrt{5}$$

**5** **Use trigonometry**

Now

$$(2 + \sqrt{5})^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

**6** **Use trigonometry**

Therefore

$$x = 2 + \sqrt{5}$$

✓ **FINAL ANSWER**

$$\boxed{2 + \sqrt{5}}$$

TOPIC

# Standard & Compound Units

Every past-paper question on this topic, 2020 to 2026.

# Question 21

6 marks | Standard & Compound Units

**21** The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

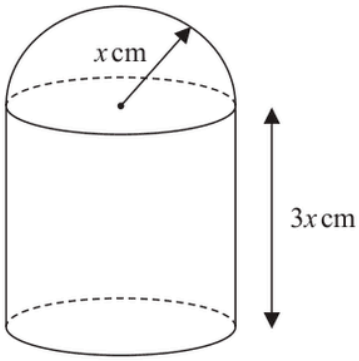



Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both $x$ cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is $81\pi$ cm$^2$
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

| Metal | Density (g/cm$^3$) |
|---|---|
| Aluminium | 2.7 |
| Nickel | 8.9 |
| Gold | 19.3 |
| Silver | 10.5 |

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

**(Total for Question 21 is 6 marks)**

**1** **The outside surface area is**

The outside surface area is:

**2** **curved surface area of cylinder**

curved surface area of cylinder:

$$2\pi x(3x) = 6\pi x^2$$

**3** **bottom of cylinder**

bottom of cylinder:

$$\pi x^2$$

**4** **curved surface area of hemisphere**

curved surface area of hemisphere:

$$2\pi x^2$$

**5** **Calculate value**

So

$$6\pi x^2 + \pi x^2 + 2\pi x^2 = 81\pi$$

$$9\pi x^2 = 81\pi$$

$$x^2 = 9$$

$$x = 3$$

**6** **Volume of the cylinder**

Volume of the cylinder:

$$\pi x^2(3x) = 3\pi x^3$$

**7** **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

**8** **Total volume**

Total volume:

$$3\pi(3)^3 + \frac{2}{3}\pi(3)^3$$

$$= 81\pi + 18\pi = 99\pi$$

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**9** **Density**

Density:

$$\frac{840}{99\pi} = 2.70\ldots$$

---

**10** **Compare density**

This matches aluminium, whose density is $2.7\ \text{g/cm}^3$.

---

✓ **FINAL ANSWER**

Aluminium

4MA1/2H NOVEMBER 2025

# Question 26

6 marks | Standard & Compound Units

**26** A solid cone is joined to a solid hemisphere to make the solid shown below.

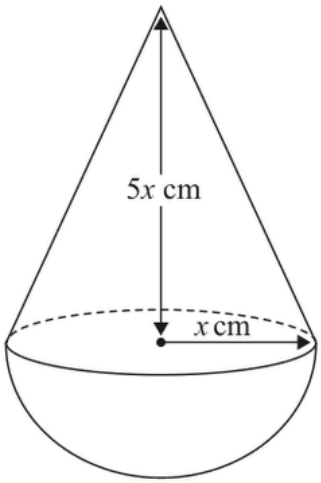



Diagram **NOT** accurately drawn

The cone is made from copper.
The density of copper is $9\,\text{g/cm}^3$

The hemisphere is made from a different metal.

The total mass of the solid is $4752\pi$ grams
The total volume of the solid is $504\pi\,\text{cm}^3$

Work out the density of the hemisphere.
Show your working clearly.

..... g/cm$^3$

**(Total for Question 26 is 6 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2H November 2025

**1** **Volume of cone**

Volume of cone:

$$\frac{1}{3}\pi x^2(5x) = \frac{5}{3}\pi x^3$$

**2** **Volume of hemisphere**

Volume of hemisphere:

$$\frac{2}{3}\pi x^3$$

**3** **Total volume**

Total volume:

$$\frac{5}{3}\pi x^3 + \frac{2}{3}\pi x^3 = 504\pi$$

$$\frac{7}{3}\pi x^3 = 504\pi$$

$$x^3 = 216$$

$$x = 6$$

**4** **Cone volume**

Cone volume:

$$\frac{5}{3}\pi (6)^3 = 360\pi$$

**5** **Mass of copper cone**

Mass of copper cone:

$$360\pi \times 9 = 3240\pi$$

**6** **Mass of hemisphere**

Mass of hemisphere:

$$4752\pi - 3240\pi = 1512\pi$$

**7** **Hemisphere volume**

Hemisphere volume:

$$\frac{2}{3}\pi (6)^3 = 144\pi$$

**8** **Density of the hemisphere**

Density of the hemisphere:

$$\frac{1512\pi}{144\pi} = 10.5$$

**FINAL ANSWER**

$10.5\ \text{g/cm}^3$

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TOPIC

# Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

# Question 21

3 marks | Algebraic Fractions

**21** Express $\left(\frac{20}{x^2 - 36} - \frac{2}{x - 6}\right) \times \frac{1}{4 - x}$ as a single fraction in its simplest form.

Show clear algebraic working.

## 1 Factorise the denominator

$$x^2 - 36 = (x - 6)(x + 6)$$

## 2 Simplify the bracket

$$\frac{20}{(x-6)(x+6)} - \frac{2}{x-6} = \frac{20 - 2(x+6)}{(x-6)(x+6)}$$

$$= \frac{8 - 2x}{(x-6)(x+6)} = \frac{-2(x-4)}{(x-6)(x+6)}$$

## 3 Cancel the common factor

Since $4 - x = -(x - 4)$,

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \frac{1}{4-x} = \frac{2}{(x-6)(x+6)}$$

**FINAL ANSWER**

$\frac{2}{x^2-36}$.

# Question 21

3 marks | Algebraic Fractions

**21** $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express $a$ in the form $\dfrac{py + q}{ry + s}$ where $p$, $q$, $r$ and $s$ are integers.

Give your answer in its simplest form.

$a =$ .....

**(Total for Question 21 is 3 marks)**

**1** **Simplify fraction**

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

**2** **Simplify fraction**

Substitute for $x$:

$$a = \frac{14}{3\left(\frac{7}{4y-3}\right) - 7}$$

**3** **Simplify the denominator**

Simplify the denominator:

$$3\left(\frac{7}{4y - 3}\right) - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

**4** **Simplify fraction**

So

$$a = \frac{14}{\frac{42-28y}{4y-3}} = \frac{14(4y - 3)}{42 - 28y}$$

$$a = \frac{4y - 3}{3 - 2y}$$

✓ **FINAL ANSWER**

$a = \dfrac{4y - 3}{3 - 2y}$.

4MA1/2H JANUARY 2020

# Question 24

4 marks | Algebraic Fractions

24 Express

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H January 2020

**1 Use trigonometry**

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

**2 Use trigonometry**

First simplify the bracket:

$$\frac{4}{2x-5}-\frac{3}{2x-3}=\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$

$$=\frac{8x-12-6x+15}{(2x-5)(2x-3)}$$

$$=\frac{2x+3}{(2x-5)(2x-3)}$$

**3 Use trigonometry**

Now factor the second fraction:

$$9x-4x^3=-x(2x-3)(2x+3)$$

$$6x^2-17x+5=(2x-5)(3x-1)$$

**4** **Dividing by a fraction means**

Dividing by a fraction means multiplying by its reciprocal:

$$\frac{2x+3}{(2x-5)(2x-3)} \times \frac{(2x-5)(3x-1)}{-x(2x-3)(2x+3)}$$

**5** **Cancel common factors**

Cancel common factors:

$$= -\frac{3x-1}{x(2x-3)^2}$$

$$= \frac{1-3x}{x(2x-3)^2}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{1-3x}{x(2x-3)^2}}$$

4MA1/1H MAY/NOVEMBER 2020

# Question 21

5 marks | Algebraic Fractions

**21** Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H May/November 2020

**1 Use trigonometry**

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

**2 Factor**

Factor:

$$9x^2-4 = (3x-2)(3x+2)$$

$$3x^2-13x-10 = (3x+2)(x-5)$$

**3 Use trigonometry**

So the first product becomes

$$\frac{1}{3x-2} \times \frac{(3x-2)(3x+2)}{(3x+2)(x-5)}$$

**4 Cancel common factors**

Cancel common factors:

$$\frac{1}{x-5}$$

**5 Use trigonometry**

Now subtract:

$$\frac{1}{x-5} - \frac{7}{x-1}$$

### 6 Use the common denominator

Use the common denominator $(x-5)(x-1)$:

$$\frac{x-1-7(x-5)}{(x-5)(x-1)}$$

$$= \frac{x-1-7x+35}{(x-5)(x-1)}$$

$$= \frac{34-6x}{(x-5)(x-1)}$$

---

**FINAL ANSWER**

$$\boxed{\frac{34-6x}{(x-5)(x-1)}}$$

4MA1/2H JANUARY 2021

# Question 21

4 marks | Algebraic Fractions

**21** Write $\dfrac{25x^2 - 64}{5x^2 - 13x - 6} \times \dfrac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

as a single fraction in its simplest form.
Show clear algebraic working.

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H January 2021

**1 Use trigonometry**

$$\frac{25x^2 - 64}{5x^2 - 13x - 6} \times \frac{x^2 - 8x + 15}{5x + 8} - (x - 7)$$

**2 Factor**

Factor:

$$25x^2 - 64 = (5x - 8)(5x + 8)$$

$$5x^2 - 13x - 6 = (5x + 2)(x - 3)$$

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

**3 Use trigonometry**

So

$$\frac{(5x - 8)(5x + 8)}{(5x + 2)(x - 3)} \times \frac{(x - 3)(x - 5)}{5x + 8} - (x - 7)$$

**4 Cancel common factors**

Cancel common factors:

$$\frac{(5x - 8)(x - 5)}{5x + 2} - (x - 7)$$

**5** **Use the common denominator**

Use the common denominator $5x + 2$:

$$\frac{(5x - 8)(x - 5) - (x - 7)(5x + 2)}{5x + 2}$$

**6** **Expand**

Expand:

$$(5x - 8)(x - 5) = 5x^2 - 33x + 40$$

$$(x - 7)(5x + 2) = 5x^2 - 33x - 14$$

**7** **Use trigonometry**

So the numerator is

$$5x^2 - 33x + 40 - (5x^2 - 33x - 14) = 54$$

✓ **FINAL ANSWER**

$$\boxed{\frac{54}{5x + 2}}$$

4MA1/1H MAY 2021

# Question 21

4 marks | Algebraic Fractions

**21** Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for $x$ in terms of $a$.
Give your expression as a single fraction in its simplest form.

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H May 2021

**1 Given**

Given

$$x = \frac{5}{9y + 5}$$

**2 Use trigonometry**

and

$$y = \frac{5}{5a - 2}$$

**3 Use cosine rule**

Substitute $y = \frac{5}{5a-2}$ into the expression for $x$:

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5}$$

$$x = \frac{5}{\frac{45}{5a-2} + 5}$$

**4** **Write the denominator as a**

Write the denominator as a single fraction:

$$\frac{45}{5a-2}+5=\frac{45+5(5a-2)}{5a-2}$$

$$=\frac{45+25a-10}{5a-2}$$

$$=\frac{25a+35}{5a-2}$$

---

**5** **Use trigonometry**

So

$$x=5\div\frac{25a+35}{5a-2}$$

$$x=5\times\frac{5a-2}{25a+35}$$

$$x=\frac{5(5a-2)}{5(5a+7)}$$

$$x=\frac{5a-2}{5a+7}$$

---

✓ **FINAL ANSWER**

$$\boxed{\frac{5a-2}{5a+7}}$$

4MA1/2HR JUNE 2022

# Question 23

3 marks | Algebraic Fractions

**23** Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR June 2022

**1 Use trigonometry**

$$\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$$

**2 Factor**

Factor:

$$x^2-36=(x-6)(x+6)$$

**3 Use trigonometry**

So

$$\frac{20}{(x-6)(x+6)}-\frac{2}{x-6}$$

**4 Use the common denominator**

Use the common denominator $(x-6)(x+6)$:

$$\frac{20-2(x+6)}{(x-6)(x+6)}$$

$$=\frac{20-2x-12}{(x-6)(x+6)}$$

$$=\frac{8-2x}{(x-6)(x+6)}$$

$$=\frac{-2(x-4)}{(x-6)(x+6)}$$

**5** **Use trigonometry**

Now multiply by

$$\frac{1}{4-x} = -\frac{1}{x-4}$$

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \left(-\frac{1}{x-4}\right)$$

---

**6** **Cancel**

Cancel $x-4$:

$$\frac{2}{(x-6)(x+6)}$$

$$= \frac{2}{x^2-36}$$

---

✓ **FINAL ANSWER**

$$\boxed{\frac{2}{x^2-36}}$$

4MA1/1HR MAY 2023

# Question 23

4 marks | Algebraic Fractions

23 Simplify $(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$

Give your answer in the form $\frac{ax^2}{bx + c}$ where $a$, $b$ and $c$ are integers.

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR May 2023

**1 Simplify surd**

$$(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$$

**2 Change division into multiplication by**

Change division into multiplication by the reciprocal:

$$(x^2 - 4) \times \frac{x}{4x^2 - 7x - 2} - 2x$$

**3 Factorise**

Factorise:

$$x^2 - 4 = (x - 2)(x + 2)$$

$$4x^2 - 7x - 2 = (4x + 1)(x - 2)$$

**4 Simplify surd**

So

$$\frac{x(x - 2)(x + 2)}{(4x + 1)(x - 2)} - 2x = \frac{x(x + 2)}{4x + 1} - 2x$$

$$\frac{x(x + 2) - 2x(4x + 1)}{4x + 1} = \frac{x^2 + 2x - 8x^2 - 2x}{4x + 1} = \frac{-7x^2}{4x + 1}$$

✓ **FINAL ANSWER**

$$\frac{-7x^2}{4x + 1}$$

4MA1/2H NOVEMBER 2023

# Question 24

5 marks | Algebraic Fractions

**24** Solve $\frac{45x^3-80x}{3x^2+x-4}\times\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$

Show clear algebraic working.

$x =$ .....

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H November 2023

**1 Solve quadratic equation**

$$\frac{45x^3-80x}{3x^2+x-4}\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$$

**2 Factorise**

Factorise:

$$45x^3-80x=5x(3x-4)(3x+4)$$

$$3x^2+x-4=(3x+4)(x-1)$$

**3 Solve quadratic equation**

Also,

$$\frac{1}{3x-4}+\frac{1}{x}=\frac{x+3x-4}{x(3x-4)}=\frac{4(x-1)}{x(3x-4)}$$

**4 Solve quadratic equation**

So the left side becomes

$$\frac{5x(3x-4)(3x+4)}{(3x+4)(x-1)}\times\frac{4(x-1)}{x(3x-4)}=20$$

**5** **Solve quadratic equation**

Therefore,

$$20 = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4(x+2)$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{7}{4}$$

---

✓ **FINAL ANSWER**

$x = \frac{7}{4}$

4MA1/2H JUNE 2024

# Question 20

3 marks | Algebraic Fractions

**20** Given that $k = x - y$ and $x = \dfrac{1}{4y}$

express $\dfrac{5k}{x+2}$ in the form $\dfrac{a - by^2}{c + dy}$ where $a$, $b$, $c$ and $d$ are integers.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H June 2024

**1** **Simplify fraction**

$$k = x - y, \quad x = \frac{1}{4y}$$

**2** **Simplify fraction**

So

$$k = \frac{1}{4y} - y = \frac{1 - 4y^2}{4y}$$

**3** **Simplify fraction**

Also

$$x + 2 = \frac{1}{4y} + 2 = \frac{1 + 8y}{4y}$$

**4** **Simplify fraction**

Therefore

$$\frac{5k}{x+2} = \frac{5\left(\frac{1-4y^2}{4y}\right)}{\frac{1+8y}{4y}}$$

**5** **Cancel**

Cancel $4y$:

$$\frac{5(1-4y^2)}{1+8y}$$

$$= \frac{5-20y^2}{1+8y}$$

---

✓ **FINAL ANSWER**

$$\frac{5-20y^2}{1+8y}$$

4MA1/2HR JUNE 2024

# Question 26

4 marks | Algebraic Fractions

**26** Write $4 - \left[(3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}\right]$ as a single fraction in its simplest form.

**(Total for Question 26 is 4 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2HR June 2024

**1 Use trigonometry**

$$4 - \left[(3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}\right]$$

**2 Factor the quadratic**

Factor the quadratic:

$$3x^2 + x - 10 = (3x - 5)(x + 2)$$

**3 Use trigonometry**

So the expression inside the square brackets is

$$(3x - 5) \div \frac{(3x - 5)(x + 2)}{4x - 1}$$

**4 Dividing by a fraction means**

Dividing by a fraction means multiplying by its reciprocal:

$$(3x - 5) \times \frac{4x - 1}{(3x - 5)(x + 2)}$$

**5 Cancel**

Cancel $3x - 5$:

$$\frac{4x - 1}{x + 2}$$

**6 Use trigonometry**

Therefore the full expression is

$$4 - \frac{4x - 1}{x + 2}$$

### 7 Use the common denominator

Use the common denominator $x+2$:

$$\frac{4(x+2)-(4x-1)}{x+2}$$

$$=\frac{4x+8-4x+1}{x+2}$$

$$=\frac{9}{x+2}$$

---

**FINAL ANSWER**

$$\boxed{\frac{9}{x+2}}$$

4WM1/01 MAY 2025

# Question 20

3 marks | Algebraic Fractions

**20** Given that $k = m + n$ and $m = \dfrac{3}{4n}$

express $\dfrac{7k}{5-m}$ in the form $\dfrac{a+bn^2}{cn-d}$ where $a$, $b$ and $c$ are integers and $d$ is prime.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4WM1/01 May 2025

**1 Simplify fraction**

$$k = m + n, \qquad m = \frac{3}{4n}$$

**2 Simplify fraction**

So

$$k = \frac{3}{4n} + n = \frac{3 + 4n^2}{4n}$$

**3 Simplify fraction**

Then

$$7k = \frac{21 + 28n^2}{4n}$$

**4 Simplify fraction**

and

$$5 - m = 5 - \frac{3}{4n} = \frac{20n - 3}{4n}$$

**5 Simplify fraction**

Therefore

$$\frac{7k}{5-m} = \frac{\frac{21+28n^2}{4n}}{\frac{20n-3}{4n}}$$

$$= \frac{21 + 28n^2}{20n - 3}$$

**FINAL ANSWER**

$$\frac{21+28n^2}{20n-3}$$

4MA1/2H JUNE 2025

# Question 23

4 marks | Algebraic Fractions

**23** $\dfrac{4x^2-4x-120}{5x^2-180} \div \dfrac{x^2+5x}{10x^2+60x} = p$ where $p$ is an integer.

Find the value of $p$

Show clear algebraic working.

$p =$ .....

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H June 2025

**1 Split the ratio**

$$\frac{4x^2-4x-120}{5x^2-180} \div \frac{x^2+5x}{10x^2+60x}$$

**2 Factorise**

Factorise:

$$4x^2-4x-120 = 4(x-6)(x+5)$$

$$5x^2-180 = 5(x-6)(x+6)$$

$$x^2+5x = x(x+5)$$

$$10x^2+60x = 10x(x+6)$$

**3 Split the ratio**

So

$$\frac{4(x-6)(x+5)}{5(x-6)(x+6)} \div \frac{x(x+5)}{10x(x+6)}$$

$$= \frac{4(x+5)}{5(x+6)} \times \frac{10(x+6)}{x+5} = 8$$

**FINAL ANSWER**

$p = 8$

4WM1/01R MAY 2025

# Question 24

4 marks | Algebraic Fractions

24 Write $(2x-1)+\dfrac{2x^2-5x-12}{x^2-16}\times\dfrac{x^2+4x}{6x^2+11x+3}$ as a single fraction in its simplest form.

Show clear algebraic working.

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4WM1/01R May 2025

### 1 Factor the fractional part first

Factor the fractional part first:

$$\frac{2x^2-5x-12}{x^2-16}\times\frac{x^2+4x}{6x^2+11x+3}$$

$$=\frac{(2x+3)(x-4)}{(x-4)(x+4)}\times\frac{x(x+4)}{(3x+1)(2x+3)}$$

$$=\frac{x}{3x+1}$$

### 2 Evaluate fraction

So

$$(2x-1)+\frac{x}{3x+1}=\frac{(2x-1)(3x+1)+x}{3x+1}$$

$$=\frac{6x^2-1}{3x+1}$$

**FINAL ANSWER**

$$\frac{6x^2-1}{3x+1}.$$

4MA1/1H NOVEMBER 2025

# Question 23

3 marks | Algebraic Fractions

23 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write $a$ in the form $\dfrac{m+ny}{p(y-1)}$ where $m$, $n$ and $p$ are integers.

Show your working clearly.

$a =$ .....

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H November 2025

**1 Split the ratio**

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

**2 Split the ratio**

Substitute for $x$.

**3 Numerator**

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

**4 Denominator**

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

**5** **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

---

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

# Question 25

3 marks | Algebraic Fractions

**25** $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write $a$ in the form $\dfrac{m+ny}{p(y-1)}$ where $m$, $n$ and $p$ are integers.

Show your working clearly.

$a =$ .....

**(Total for Question 25 is 3 marks)**

## WORKED SOLUTION

Question 25 · 4WM1/01 November 2025

**1 Split the ratio**

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

**2 Split the ratio**

Substitute for $x$.

**3 Numerator**

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

**4 Denominator**

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

**5** **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

---

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

# Question 28

4 marks | Algebraic Fractions

**28** Solve $\dfrac{6x^3+5x^2+x}{4x^2-1} \div \dfrac{15x^2-x-2}{8x-4} = 2x$

Show clear algebraic working.

$x =$ .....

**(Total for Question 28 is 4 marks)**

## WORKED SOLUTION

Question 28 · 4WM1/01 November 2025

### 1 Factor each expression

Factor each expression:

$$\frac{x(3x+1)(2x+1)}{(2x-1)(2x+1)} \div \frac{(5x-2)(3x+1)}{4(2x-1)} = 2x$$

$$\frac{x(3x+1)}{2x-1} \times \frac{4(2x-1)}{(5x-2)(3x+1)} = 2x$$

$$\frac{4x}{5x-2} = 2x$$

$$4x = 2x(5x-2)$$

$$10x^2 - 8x = 0$$

$$2x(5x-4) = 0$$

### 2 The original fractions do not

The original fractions do not allow $x = 0$, so

$$x = \frac{4}{5}$$

**FINAL ANSWER**

$x = \frac{4}{5}$.

# Question 20

4 marks | Algebraic Fractions

**20** Express $\left(\frac{5}{4x-7}-\frac{3}{3x-5}\right)\div\frac{9x^2-16}{6x^2-7x-5}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

**(Total for Question 20 is 4 marks)**

**1** **Subtract the fractions**

$$\frac{5(3x-5)-3(4x-7)}{(4x-7)(3x-5)} = \frac{3x-4}{(4x-7)(3x-5)}$$

**2** **Factorise the second fraction**

$$9x^2-16=(3x-4)(3x+4), \qquad 6x^2-7x-5=(3x-5)(2x+1)$$

**3** **Divide = multiply by the reciprocal**

$$\frac{3x-4}{(4x-7)(3x-5)} \times \frac{(3x-5)(2x+1)}{(3x-4)(3x+4)}$$

**4** **Cancel**

$$= \frac{2x+1}{(4x-7)(3x+4)}$$

✓ **FINAL ANSWER**

$$\frac{2x+1}{(4x-7)(3x+4)}$$



TOPIC

# Algebraic Roots & Indices

Every past-paper question on this topic, 2020 to 2026.

# Question 22

3 marks | Algebraic Roots & Indices

**22** $\dfrac{18 \times (\sqrt{27})^{}}{6 \times 9^{2n+8}} = 3^x$

Express $x$ in terms of $n$

Show your working clearly and simplify your expression.

$x =$ .....

## 1 Write everything as powers of 3

$$\sqrt{27} = 3^{3/2}, \qquad 9 = 3^2, \qquad \frac{18}{6} = 3$$

## 2 Simplify the powers

$$\frac{18(\sqrt{27})^{4n+6}}{6(9)^{2n+8}} = 3 \times \frac{\left(3^{3/2}\right)^{4n+6}}{(3^2)^{2n+8}}$$

$$= 3^1 \times \frac{3^{6n+9}}{3^{4n+16}}$$

## 3 Subtract the indices

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

So

$$x = 2n - 6$$

**FINAL ANSWER**

$x = 2n - 6$.

4MA1/2HR JANUARY 2020

# Question 21

7 marks | Algebraic Roots & Indices

**21** (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

.....

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for $n$ in terms of $y$.
Show clear algebraic working and simplify your expression.

.....

(4)

**(Total for Question 21 is 7 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR January 2020

**1** **(a)**

(a)

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} = \frac{(5x + 4)(2x + 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{5x + 4}{2x - 3}$$

**2** **(b)**

(b)

$$2^{2y} \times 2^{3y+2} = 2^{5y+2}$$

**3** **Evaluate fraction**

Also,

$$\frac{8^{5y}}{4^n} = \frac{(2^3)^{5y}}{(2^2)^n} = 2^{15y-2n}$$

**4** **Equate powers**

Equate powers:

$$5y + 2 = 15y - 2n$$

$$2n = 10y - 2$$

$$n = 5y - 1$$

---

✓ **FINAL ANSWER**

(a) $\dfrac{5x+4}{2x-3}$, (b) $n = 5y - 1$

4MA1/1HR MAY/NOVEMBER 2020

# Question 21

5 marks | Algebraic Roots & Indices

21 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of $n$ for which $M = 2$

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR May/November 2020

### 1 Evaluate fraction

$$M = \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$$

### 2 Use index laws

Write everything in powers of 2 and 3:

$$18^{4n} = (2 \cdot 3^2)^{4n} = 2^{4n}3^{8n}$$

$$2^{3(n^2-6n)} = 2^{3n^2-18n}$$

$$3^{2(1-4n)} = 3^{2-8n}$$

$$12^2 = (2^2 \cdot 3)^2 = 2^4 3^2$$

### 3 Calculate value

So

$$M = 2^{4n+3n^2-18n-4} \times 3^{8n+2-8n-2}$$

$$M = 2^{3n^2-14n-4}$$

**4** **Calculate value**

Given $M = 2$,

$$2^{3n^2-14n-4} = 2^1$$

$$3n^2 - 14n - 4 = 1$$

$$3n^2 - 14n - 5 = 0$$

$$(3n + 1)(n - 5) = 0$$

---

✓ **FINAL ANSWER**

$n = 5$ or $n = -\frac{1}{3}$

4MA1/2H JUNE 2022

# Question 24

3 marks | Algebraic Roots & Indices

24 $$\frac{18 \times (\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express $x$ in terms of $n$
Show your working clearly and simplify your expression.

$x =$ .....

**(Total for Question 24 is 3 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H June 2022

### 1 Simplify surd

$$\frac{18(\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

### 2 Use powers of 3

Use powers of 3:

$$18 \div 6 = 3$$

$$\sqrt{27} = (3^3)^{1/2} = 3^{3/2}$$

$$(\sqrt{27})^{4n+6} = 3^{6n+9}$$

$$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$$

### 3 Evaluate fraction

So

$$\frac{3 \cdot 3^{6n+9}}{3^{4n+16}} = 3^{1+6n+9-4n-16} = 3^{2n-6}$$

### 4 Calculate value

Therefore,

$$x = 2n - 6$$

**FINAL ANSWER**

$x = 2n - 6$

4MA1/1HR MAY 2023

# Question 24

5 marks | Algebraic Roots & Indices

**24** Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for $x$ in terms of $n$
State any restrictions on $n$

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1HR May 2023

**1** **Calculate value**

$$2^n = 2^{x^2} \times 16^x \times 8$$

$$16^x = (2^4)^x = 2^{4x}$$

$$8 = 2^3$$

**2** **Calculate value**

So

$$2^n = 2^{x^2+4x+3}$$

$$n = x^2 + 4x + 3$$

**3** **Complete the square**

Complete the square:

$$n = (x+2)^2 - 1$$

$$(x+2)^2 = n + 1$$

**4** **Simplify surd**

Since $x > 0$, take the positive root:

$$x + 2 = \sqrt{n+1}$$

$$x = \sqrt{n+1} - 2$$

---

**5** **Simplify surd**

For $x > 0$,

$$\sqrt{n+1} > 2$$

$$n > 3$$

---

✓ **FINAL ANSWER**

$x = \sqrt{n+1} - 2$, with $n > 3$

4MA1/2H JUNE 2024

# Question 23

3 marks | Algebraic Roots & Indices

**23** Simplify $\dfrac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$

Give your answer in the form $5^w$ where $w$ is an expression in terms of $x$
Show each stage of your working clearly.

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H June 2024

### 1 Simplify surd

$$\frac{30 \times 25^{2x+7}}{\sqrt{180} \times (\sqrt{5})^{4x+9}}$$

$$\frac{30}{\sqrt{180}} = \frac{30}{6\sqrt{5}} = \sqrt{5} = 5^{1/2}$$

$$25^{2x+7} = (5^2)^{2x+7} = 5^{4x+14}$$

$$(\sqrt{5})^{4x+9} = 5^{2x+\frac{9}{2}}$$

### 2 Evaluate fraction

So

$$5^{1/2} \times 5^{4x+14} \div 5^{2x+\frac{9}{2}} = 5^{\frac{1}{2}+4x+14-2x-\frac{9}{2}}$$

$$= 5^{2x+10}$$

**FINAL ANSWER**

$5^{2x+10}$, so $w = 2x + 10$

TOPIC

# Forming & Solving Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2025

# Question 20

3 marks | Forming & Solving Equations

**20** Here is a quadratic equation.

$$ax^2 + 4x + c = 0$$

The solutions of this equation are given by $x = \dfrac{-4 \pm 2\sqrt{39}}{10}$

Find the value of $a$ and the value of $c$
Show your working clearly.

$a =$ .....

$c =$ .....

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H November 2025

**1 Calculate value**

For

$$ax^2 + 4x + c = 0$$

**2 the quadratic formula gives**

the quadratic formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4ac}}{2a}$$

**3 The given solutions are**

The given solutions are

$$x = \frac{-4 \pm 2\sqrt{39}}{10}$$

**4 Compare denominators**

Compare denominators:

$$2a = 10$$

$$a = 5$$

**5** **Compare the square root parts**

Compare the square root parts:

$$\sqrt{16 - 4ac} = 2\sqrt{39}$$

**6** **Square both sides**

Square both sides:

$$16 - 4ac = 156$$

**7** **Use**

Use $a = 5$:

$$16 - 20c = 156$$

$$-20c = 140$$

$$c = -7$$

✓ **FINAL ANSWER**

$a = 5,\ c = -7$



TOPIC

# Simultaneous Equations

Every past-paper question on this topic, 2020 to 2026.

# Question 20

5 marks | Simultaneous Equations

**20** Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

**(Total for Question 20 is 5 marks)**

**1** **Solve simultaneous equations**

From

$$x - 2y = 3$$

$$x = 2y + 3$$

**2** **Solve simultaneous equations**

Substitute into $x^2 - y^2 + 2x = 10$:

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

**3** **Solve simultaneous equations**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

**4** **Solve simultaneous equations**

If $y = -\frac{1}{3}$,

$$x = 2\left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

**5** **Solve simultaneous equations**

If $y = -5$,

$$x = 2(-5) + 3 = -7$$

✓ **FINAL ANSWER**

$\left(\frac{7}{3}, -\frac{1}{3}\right)$ and $(-7, -5)$.

4MA1/2HR JUNE 2022

# Question 21

5 marks | Simultaneous Equations

**21** Solve the simultaneous equations

$$\begin{aligned} x - 2y &= 3 \\ x^2 - y^2 + 2x &= 10 \end{aligned}$$

Show clear algebraic working.

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR June 2022

**1** **Calculate value**

$$x - 2y = 3$$

**2** **Calculate value**

So

$$x = 2y + 3$$

**3** **Calculate value**

Substitute into

$$x^2 - y^2 + 2x = 10$$

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

**4** **Evaluate fraction**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

**5** **Find**

Find $x$:

$$y = -\frac{1}{3} \Rightarrow x = \frac{7}{3}$$

$$y = -5 \Rightarrow x = -7$$

---

**FINAL ANSWER**

$(x, y) = \left(\frac{7}{3}, -\frac{1}{3}\right)$ or $(-7, -5)$

4MA1/2H JUNE 2023

# Question 21

5 marks | Simultaneous Equations

**21** Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H June 2023

### 1 Calculate value

$$2x^2 + 3y^2 = 11$$

$$x = 3y - 1$$

### 2 Substitute

Substitute:

$$2(3y - 1)^2 + 3y^2 = 11$$

$$18y^2 - 12y + 2 + 3y^2 = 11$$

$$21y^2 - 12y - 9 = 0$$

$$7y^2 - 4y - 3 = 0$$

$$(7y + 3)(y - 1) = 0$$

### 3 Evaluate fraction

So

$$y = -\frac{3}{7} \quad \text{or} \quad y = 1$$

**4** **Find**

Find $x$:

$$y = -\frac{3}{7} \Rightarrow x = -\frac{16}{7}$$

$$y = 1 \Rightarrow x = 2$$

---

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{16}{7}, -\frac{3}{7}\right)$ or $(2, 1)$

4MA1/2HR JUNE 2024

# Question 22

5 marks | Simultaneous Equations

**22** Solve the simultaneous equations

$$x^2 + y^2 = y + 11$$
$$y = 3x - 1$$

Show clear algebraic working.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2HR June 2024

### 1 Calculate value

$$x^2 + y^2 = y + 11$$

$$y = 3x - 1$$

### 2 Substitute

Substitute:

$$x^2 + (3x - 1)^2 = 3x - 1 + 11$$

$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

$$10x^2 - 9x - 9 = 0$$

$$(5x + 3)(2x - 3) = 0$$

### 3 Evaluate fraction

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{3}{2}$$

**4** **Find**

Find $y$:

$$x = -\frac{3}{5} \Rightarrow y = -\frac{14}{5}$$

$$x = \frac{3}{2} \Rightarrow y = \frac{7}{2}$$

---

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, -\frac{14}{5}\right)$ or $\left(\frac{3}{2}, \frac{7}{2}\right)$

4MA1/2H NOVEMBER 2024

# Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 + y = 3$$

$$x + 2 = y$$

Show clear algebraic working.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H November 2024

### 1 Calculate value

$$x^2 + y^2 + y = 3$$

$$y = x + 2$$

### 2 Substitute

Substitute:

$$x^2 + (x+2)^2 + (x+2) = 3$$

$$x^2 + x^2 + 4x + 4 + x + 2 = 3$$

$$2x^2 + 5x + 3 = 0$$

$$(2x+3)(x+1) = 0$$

### 3 Evaluate fraction

So

$$x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

**4** **Find**

Find $y$:

$$x = -\frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$x = -1 \Rightarrow y = 1$$

---

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{2}, \frac{1}{2}\right)$ or $(-1, 1)$

4MA1/1H JUNE 2025

# Question 22

5 marks | Simultaneous Equations

**22** Solve the simultaneous equations

$$x^2 + 3y + y^2 = 7$$
$$y = x + 2$$

Show clear algebraic working.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H June 2025

**1** **Calculate value**

$$x^2 + y^2 + 3y = 7$$

$$y = x + 2$$

**2** **Substitute**

Substitute:

$$x^2 + (x+2)^2 + 3(x+2) = 7$$

$$x^2 + x^2 + 4x + 4 + 3x + 6 = 7$$

$$2x^2 + 7x + 3 = 0$$

$$(2x+1)(x+3) = 0$$

**3** **Evaluate fraction**

So

$$x = -\frac{1}{2} \quad \text{or} \quad x = -3$$

**4** **Find**

Find $y$:

$$x = -\frac{1}{2} \Rightarrow y = \frac{3}{2}$$

$$x = -3 \Rightarrow y = -1$$

---

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{1}{2}, \frac{3}{2}\right)$ or $(-3, -1)$

4MA1/1HR JUNE 2025

# Question 22

5 marks | Simultaneous Equations

**22** Solve the simultaneous equations

$$x^2 + y^2 = 41$$
$$2x + y = 3$$

Show clear algebraic working.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1HR June 2025

**1** **Calculate value**

$$x^2 + y^2 = 41$$

$$2x + y = 3$$

**2** **Calculate value**

So

$$y = 3 - 2x$$

**3** **Substitute**

Substitute:

$$x^2 + (3 - 2x)^2 = 41$$

$$x^2 + 9 - 12x + 4x^2 = 41$$

$$5x^2 - 12x - 32 = 0$$

$$(5x + 8)(x - 4) = 0$$

**4** **Evaluate fraction**

So

$$x = -\frac{8}{5} \quad \text{or} \quad x = 4$$

**5** **Find**

Find $y$:

$$x = -\frac{8}{5} \Rightarrow y = \frac{31}{5}$$

$$x = 4 \Rightarrow y = -5$$

---

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{8}{5}, \frac{31}{5}\right)$ or $(4, -5)$

4WM2/01R MAY 2025

# Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 + 5y + x^2 = 12$$
$$x = y + 5$$

Show clear algebraic working.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4WM2/01R May 2025

**1 Required**

Solve *y*<sup>2</sup> + 5*y* + *x*<sup>2</sup> = 12 and *x* = *y* + 5.

**2 Substitute x y + 5**

Substitute *x* = *y* + 5 into the first equation:

*y*<sup>2</sup> + 5*y* + (*y*+5)<sup>2</sup> = 12

**3 Expand ( y +5) 2**

Expand (*y*+5)<sup>2</sup> = *y*<sup>2</sup> + 10*y* + 25:

*y*<sup>2</sup> + 5*y* + *y*<sup>2</sup> + 10*y* + 25 = 12 ⟹ 2*y*<sup>2</sup> + 15*y* + 13 = 0

**4 Factorise**

Factorise:

$$(2y + 13)(y + 1) = 0 implies y = -tfrac132; textor; y = -1$$

**5 Find corresponding x values using**

Find corresponding *x* values using *x* = *y* + 5:

**✓ FINAL ANSWER**

(*x*,*y*) = (4, −1)and$left(-dfrac32, , -dfrac132right)$

4WM2/01 NOVEMBER 2025

# Question 20

5 marks | Simultaneous Equations

**20** Solve the simultaneous equations

$$x^2 + y^2 = 3xy - 11$$
$$y = x + 2$$

Show clear algebraic working.

**(Total for Question 20 is 5 marks)**

## WORKED SOLUTION

Question 20 · 4WM2/01 November 2025

### 1 Substitute into equation

**Step 1: Substitute *y* = (*x*+2) into the quadratic equation.**

*x*<sup>2</sup> + (*x*+2)<sup>2</sup> = 3*x*(*x*+2) − 11

### 2 Expand brackets

*x*<sup>2</sup> + (*x*<sup>2</sup>+4*x*+4) = 3*x*<sup>2</sup> + 6*x* − 11

2*x*<sup>2</sup> + 4*x* + 4 = 3*x*<sup>2</sup> + 6*x* − 11

### 3 Solve equation

0 = 3*x*<sup>2</sup> − 2*x*<sup>2</sup> + 6*x* − 4*x* − 11 − 4

*x*<sup>2</sup> + 2*x* − 15 = 0

### 4 Solve quadratic equation

(*x*+5)(*x*−3) = 0

So, *x* = − 5 or *x* = 3.

**Step 5: Find corresponding *y* values using *y* = *x* + 2.**

If *x* = − 5 ⟹ *y* = − 5 + 2 = − 3

If *x* = 3 ⟹ *y* = 3 + 2 = 5

**FINAL ANSWER**

(−5,−3) and (3,5)

# Question 21

5 marks | Simultaneous Equations

**21** Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

**(Total for Question 21 is 5 marks)**

**1** **Substitute y = 3x − 5**

$$(3x-5)^2 = x^2 + 7 \Rightarrow 9x^2 - 30x + 25 = x^2 + 7$$

**2** **Simplify and factorise**

$$8x^2 - 30x + 18 = 0 \Rightarrow 4x^2 - 15x + 9 = 0 \Rightarrow (x-3)(4x-3) = 0$$

**3** **Find y for each x**

$x = 3 \Rightarrow y = 4$;\quad $x = \frac{3}{4} \Rightarrow y = \frac{9}{4} - 5 = -\frac{11}{4}$

✓ **FINAL ANSWER**

$x = 3,\ y = 4$ and $x = \frac{3}{4},\ y = -\frac{11}{4}$



TOPIC

# Inequalities (Solving & Graphing)

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2022

# Question 22

5 marks | Inequalities (Solving & Graphing)

**22** Here is a rectangle.

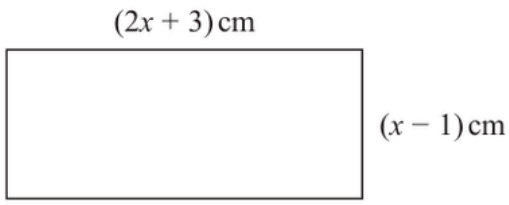



Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than $75\,\text{cm}^2$

find the range of possible values of $x$

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H January 2022

**1** **The rectangle has side lengths**

The rectangle has side lengths

$$2x + 3$$

**2** **Use Functions Solving Inequalities**

and

$$x - 1$$

**3** **Solve inequality**

For the side lengths to be possible,

$$x > 1$$

**4** **The area is less than**

The area is less than $75$:

$$(2x + 3)(x - 1) < 75$$

$$2x^2 + x - 3 < 75$$

$$2x^2 + x - 78 < 0$$

**5** **Factorise**

Factorise:

$$2x^2 + x - 78 = (2x + 13)(x - 6)$$

---

**6** **Evaluate fraction**

So

$$-\frac{13}{2} < x < 6$$

---

**7** **Solve inequality**

Combining with $x > 1$:

$$1 < x < 6$$

---

✓ **FINAL ANSWER**

$1 < x < 6$

4MA1/2HR JUNE 2024

# Question 20

4 marks | Inequalities (Solving & Graphing)

**20** Solve $6x^2 - 7x - 20 > 0$
Show clear algebraic working.

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR June 2024

**1 Solve quadratic equation**

$$6x^2 - 7x - 20 > 0$$

**2 Factorise**

Factorise:

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

**3 The critical values are**

The critical values are

$$x = -\frac{4}{3} \quad \text{and} \quad x = \frac{5}{2}$$

**4 Solve quadratic equation**

Since the coefficient of $x^2$ is positive, the quadratic is greater than 0 outside the roots.

✓ **FINAL ANSWER**

$$x < -\frac{4}{3} \text{ or } x > \frac{5}{2}$$

4MA1/2H NOVEMBER 2024

# Question 20

3 marks | Inequalities (Solving & Graphing)

**20** Solve the inequality $10x^2 + 11x - 21 < 0$
Show clear algebraic working.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H November 2024

**1 Solve inequality**

$$10x^2 + 11x - 21 < 0$$

**2 Factorise**

Factorise:

$$10x^2 + 11x - 21 = (10x + 21)(x - 1)$$

**3 The critical values are**

The critical values are

$$x = -\frac{21}{10} \quad \text{and} \quad x = 1$$

**4 The quadratic is less than**

The quadratic is less than 0 between the roots.

**✓ FINAL ANSWER**

$$-\frac{21}{10} < x < 1$$

4WM2/01R MAY 2025

# Question 20

3 marks | Inequalities (Solving & Graphing)

**20** Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4WM2/01R May 2025

### 1 Required

Solve 2*x*<sup>2</sup> − 7*x* − 15 \> 0. Show clear algebraic working.

### 2 Factorise the quadratic 2 x

Factorise the quadratic:

2*x*<sup>2</sup> − 7*x* − 15 = (2*x*+3)(*x*−5)

*Check:* (2*x*+3)(*x*−5) = 2*x*<sup>2</sup> − 10*x* + 3*x* − 15 = 2*x*<sup>2</sup> − 7*x* − 15

### 3 Set each factor zero

Roots: set each factor to zero:

$$2x + 3 = 0 implies x = -tfrac32; qquad x - 5 = 0 implies x = 5$$

### 4 Solve inequality

Since the coefficient of *x*<sup>2</sup> is + 2 \> 0, the parabola opens **upward**. The expression is \> 0 *outside* the roots:

**FINAL ANSWER**

$x \< - dfrac32$ or *x* \> 5

4MA1/2HR JUNE 2025

# Question 21

3 marks | Inequalities (Solving & Graphing)

**21** Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR June 2025

**1 Solve inequality**

$$2x^2 - 7x - 15 > 0$$

**2 Factorise**

Factorise:

$$2x^2 - 7x - 15 = (2x + 3)(x - 5)$$

**3 The critical values are**

The critical values are

$$x = -\frac{3}{2} \quad \text{and} \quad x = 5$$

**4 The quadratic is greater than**

The quadratic is greater than 0 outside the roots.

**FINAL ANSWER**

$x < -\frac{3}{2}$ or $x > 5$

4MA1/1H NOVEMBER 2025

# Question 25

3 marks | Inequalities (Solving & Graphing)

**25** Solve the inequality $2x^2 + x - 28 > 0$
Show clear algebraic working.

**(Total for Question 25 is 3 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1H November 2025

**1 Solve inequality**

$$2x^2 + x - 28 > 0$$

**2 Factorise**

Factorise:

$$2x^2 + x - 28 = (2x - 7)(x + 4)$$

**3 The critical values are**

The critical values are

$$x = \frac{7}{2} \quad \text{and} \quad x = -4$$

**4 The quadratic is greater than**

The quadratic is greater than 0 outside the roots.

**FINAL ANSWER**

$x < -4$ or $x > \frac{7}{2}$

TOPIC

# Completing the Square

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

# Question 20

4 marks | Completing the Square

**20** Find the values of $a$, $b$ and $c$ so that

$$7 + 12x - 2x^2$$

is written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

**(Total for Question 20 is 4 marks)**

**1** **Factor out -2 from the quadratic terms**

$$7 + 12x - 2x^2 = -2(x^2 - 6x) + 7$$

**2** **Complete the square**

$$x^2 - 6x = (x - 3)^2 - 9$$

**3** **Simplify the expression**

$$-2((x - 3)^2 - 9) + 7 = -2(x - 3)^2 + 18 + 7$$

$$= 25 - 2(x - 3)^2$$

**4** **Read the values**

$$a = 25, \qquad b = 2, \qquad c = 3$$

✓ **FINAL ANSWER**

$a = 25,\ b = 2,\ c = 3.$

4MA1/2HR JANUARY 2020

# Question 23

3 marks | Completing the Square

**23** Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where $a$, $b$ and $c$ are integers.

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR January 2020

### 1 Solve inequality

$$7 - 12x - 2x^2 = -2x^2 - 12x + 7$$

$$= -2(x^2 + 6x) + 7$$

$$= -2\left((x + 3)^2 - 9\right) + 7$$

$$= -2(x + 3)^2 + 18 + 7$$

$$= 25 - 2(x + 3)^2$$

**FINAL ANSWER**

$25 - 2(x + 3)^2$

4MA1/2H MAY 2021

# Question 22

4 marks | Completing the Square

22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where $a$, $b$ and $c$ are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H May 2021

**1 Solve inequality**

$$g(x) = 2x^2 - 12x + 13$$

$$= 2(x^2 - 6x) + 13$$

$$= 2\left((x - 3)^2 - 9\right) + 13$$

$$= 2(x - 3)^2 - 18 + 13$$

$$= 2(x - 3)^2 - 5$$

**2 Calculate value**

Starting with $S : y = x^2$:

**3 Translate 3 units to the**

- Translate 3 units to the right to get $y = (x - 3)^2$.
- Stretch parallel to the $y$-axis by scale factor 2 to get $y = 2(x - 3)^2$.
- Translate 5 units down to get $y = 2(x - 3)^2 - 5$.

✓ **FINAL ANSWER**

$g(x) = 2(x - 3)^2 - 5$. Transformations: right 3, stretch parallel to the $y$-axis by scale factor 2, then down 5.

4MA1/1H JANUARY 2022

# Question 20

4 marks | Completing the Square

**20** (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where $a$, $b$ and $c$ are integers.

.....

(3)

**C** is the curve with equation $y = 7 + 12x - 3x^2$
The point $A$ is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of $A$

(..... , .....)

(1)

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H January 2022

**1** **(a)**

(a)

$$7 + 12x - 3x^2 = -3x^2 + 12x + 7$$

$$= -3(x^2 - 4x) + 7$$

$$= -3\left((x - 2)^2 - 4\right) + 7$$

$$= 19 - 3(x - 2)^2$$

**2** **This is the same as**

This is the same as

$$19 - 3(x + (-2))^2$$

**3** **(b) The maximum point is**

(b) The maximum point is read from the completed square form.

$$y = 19 - 3(x - 2)^2$$

**4** **Calculate value**

The maximum occurs when $x = 2$, giving $y = 19$.

---

✓ **FINAL ANSWER**

(a) $19 - 3(x - 2)^2$, (b) $A = (2, 19)$

4MA1/1HR JUNE 2022

# Question 23

5 marks | Completing the Square

**23** (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where $a$, $b$ and $c$ are integers.

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point $M$ is the minimum point on **C**

(b) Find the coordinates of $M$

(..... , .....)

(2)

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR June 2022

**1** **(a)**

(a)

$$2x^2 - 12x + 3 = 2(x^2 - 6x) + 3$$

$$= 2\left((x - 3)^2 - 9\right) + 3$$

$$= 2(x - 3)^2 - 18 + 3$$

$$= 2(x - 3)^2 - 15$$

**2** **(b)**

(b)

$$y = 2(x + 4)^2 - 12(x + 4) + 3$$

**3** **Let**

Let $u = x + 4$. From part (a),

$$2u^2 - 12u + 3 = 2(u - 3)^2 - 15$$

**4** **Calculate value**

So

$$y = 2((x + 4) - 3)^2 - 15$$

$$y = 2(x + 1)^2 - 15$$

**5** **The minimum point is when**

The minimum point is when $x = -1$, giving $y = -15$.

✓ **FINAL ANSWER**

(a) $2(x - 3)^2 - 15$, (b) $M = (-1, -15)$

4MA1/1H JUNE 2022

# Question 24

4 marks | Completing the Square

**24** Express each of $a$, $b$ and $c$ in terms of $q$ so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H June 2022

**1 Evaluate fraction**

$$q + 12x - qx^2 = -q\left(x^2 - \frac{12}{q}x\right) + q$$

**2 Complete the square**

Complete the square:

$$x^2 - \frac{12}{q}x = \left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}$$

**3 Evaluate fraction**

So

$$q + 12x - qx^2 = -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}\right] + q$$

$$= q + \frac{36}{q} - q\left(x - \frac{6}{q}\right)^2$$

**4 Compare coefficients**

Compare with

$$a - b(x - c)^2$$

**FINAL ANSWER**

$$a = q + \frac{36}{q},\ b = q,\ c = \frac{6}{q}$$

# Question 20

5 marks | Completing the Square

**20** (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where $a$, $b$ and $c$ are numbers to be found.

.....

(3)

The curve **C** has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point $P$ is the minimum point on **C**

(b) Find the coordinates of $P$

(..... , .....)

(2)

**(Total for Question 20 is 5 marks)**

**1** **Use Moved Coordinate Geometry Completing**

For part (a),

$$2x^2 - 11x + 9$$

**2** **Factor out 2 from the**

Factor out 2 from the quadratic terms:

$$2\left(x^2 - \frac{11}{2}x\right) + 9$$

**3** **Complete the square**

Complete the square:

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{72}{8}$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

**4** **Evaluate fraction**

So

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

**5** **Calculate value**

For part (b),

$$y = 2(x-3)^2 - 11(x-3) + 9$$

**6** **Let**

Let

$$u = x - 3$$

**7** **Evaluate fraction**

From part (a), the minimum occurs when

$$u = \frac{11}{4}$$

**8** **Evaluate fraction**

So

$$x - 3 = \frac{11}{4}$$

$$x = \frac{23}{4}$$

**9** **The minimum value of is**

The minimum value of $y$ is

$$-\frac{49}{8}$$

✓ **FINAL ANSWER**

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8},\ P = \left(\frac{23}{4}, -\frac{49}{8}\right)$$

4MA1/1H MAY 2024

# Question 25

4 marks | Completing the Square

**25** $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where $a$, $b$ and $c$ are constants.

$f(x) =$ .....

**(Total for Question 25 is 4 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1H May 2024

**1 Solve inequality**

$$f(x) = 17 - 3x^2 + 12x$$

$$= -3x^2 + 12x + 17$$

$$= -3(x^2 - 4x) + 17$$

$$= -3\left((x - 2)^2 - 4\right) + 17$$

$$= -3(x - 2)^2 + 12 + 17$$

$$= 29 - 3(x - 2)^2$$

**FINAL ANSWER**

$f(x) = 29 - 3(x - 2)^2$

# Question 25

6 marks | Completing the Square

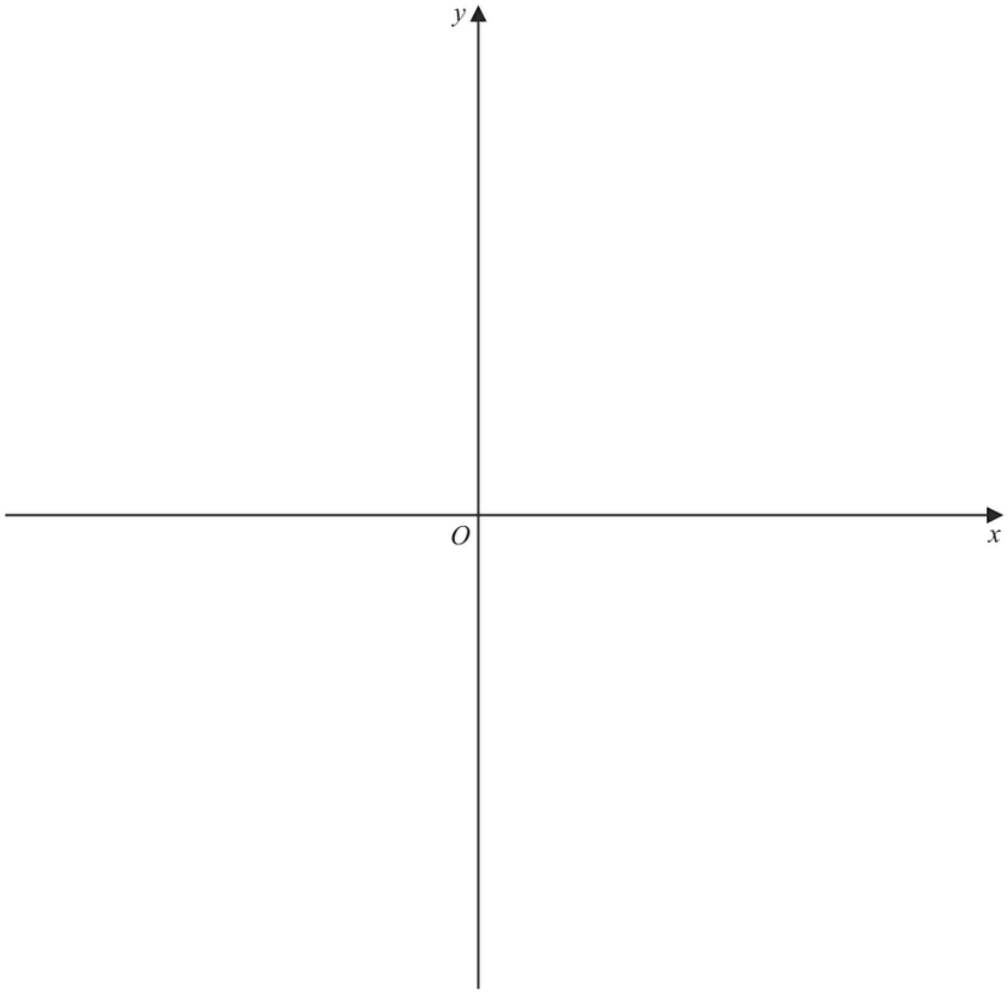
**25** (a) Write $28+24x-6x^2$ in the form $a-b(x-c)^2$ where $a$, $b$ and $c$ are integers.

.....

(3)

(b) On the axes below, sketch the graph of $y=28+24x-6x^2$

Show clearly the coordinates of the turning point and the coordinates of the point of intersection of the graph with the $y$-axis.




(3)

**(Total for Question 25 is 6 marks)**

1 **(a)**

(a)

$$28 + 24x - 6x^2 = -6x^2 + 24x + 28$$

$$= -6(x^2 - 4x) + 28$$

$$= -6\left((x-2)^2 - 4\right) + 28$$

$$= 52 - 6(x-2)^2$$

2 **(b)**

(b)

$$y = 52 - 6(x-2)^2$$

3 **The graph is a downward**

The graph is a downward-opening parabola.

4 **The turning point is**

The turning point is

$$(2, 52)$$

5 **Calculate value**

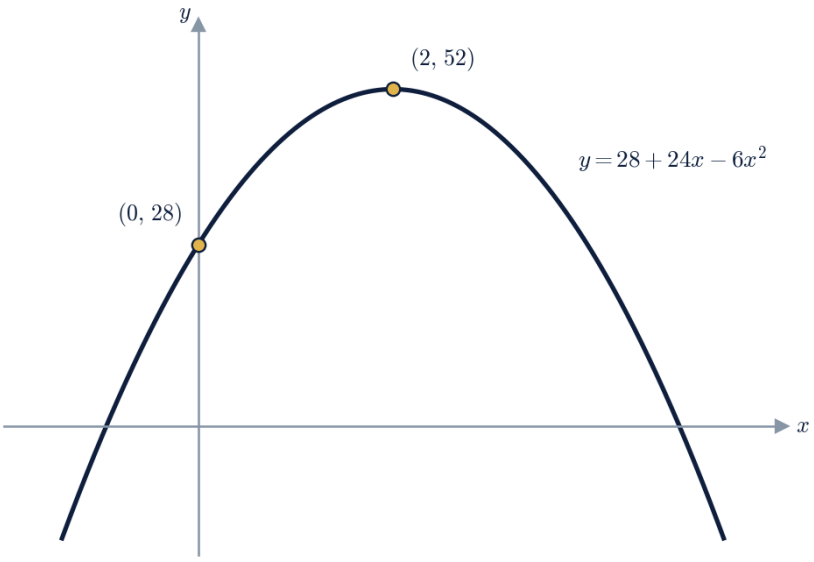
The point of intersection with the $y$-axis is found by putting $x = 0$:

$$y = 28$$

**6** **Find intercepts**

So the $y$-intercept is

$$(0, 28)$$




**FINAL ANSWER**

(a) $52 - 6(x - 2)^2$. For the sketch, show a downward parabola with turning point $(2, 52)$ and $y$-intercept $(0, 28)$.

4WM1/01 MAY 2025

# Question 25

5 marks | Completing the Square

**25** (a) Express $8 - 12x - 4x^2$ in the form $a - b(x + c)^2$ where $a$, $b$ and $c$ are constants.

.....

**(3)**

(b) Hence solve $8 - 12x - 4x^2 = 0$
Show your working clearly.
Give your solutions in surd form.

.....

**(2)**

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4WM1/01 May 2025

**1** **Simplify surd**

$$8x^2 - 12x - 4 = 8\left(x^2 - \frac{3}{2}x\right) - 4$$

**2** **Complete the square**

Complete the square.

$$x^2 - \frac{3}{2}x = \left(x - \frac{3}{4}\right)^2 - \frac{9}{16}$$

**3** **Simplify surd**

So

$$8x^2 - 12x - 4 = 8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$$

**4** **Simplify surd**

Now solve

$$8x^2 - 12x - 4 = 0$$

$$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2} = 0$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

---

✓ **FINAL ANSWER**

$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$, and $x = \frac{3 \pm \sqrt{17}}{4}$.

4MA1/1H NOVEMBER 2025

# Question 21

3 marks | Completing the Square

**21** Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where $a$, $b$ and $c$ are integers.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H November 2025

### 1 Solve inequality

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 - 20 + 23$$

$$= 5(x - 2)^2 + 3$$

**FINAL ANSWER**

$5(x - 2)^2 + 3$

4WM1/01 NOVEMBER 2025

# Question 22

2 marks | Completing the Square

**22** $9x^2 - 12x + q$ can be written in the form $(px - r)^2$ where $p$, $q$ and $r$ are integers.

Find the value of $q$

$q =$ .....

**(Total for Question 22 is 2 marks)**

## WORKED SOLUTION

Question 22 · 4WM1/01 November 2025

**1** **Calculate value**

$$9x^2 - 12x + q = (3x - 2)^2$$

$$(3x - 2)^2 = 9x^2 - 12x + 4$$

**FINAL ANSWER**

$q = 4$.

4WM1/01 NOVEMBER 2025

# Question 24

3 marks | Completing the Square

**24** Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where $a$, $b$ and $c$ are integers.

**(Total for Question 24 is 3 marks)**

## WORKED SOLUTION

Question 24 · 4WM1/01 November 2025

**1** **Calculate value**

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 + 3$$

**FINAL ANSWER**

$5(x - 2)^2 + 3$.

4WM1/01 MAY 2026

# Question 24

3 marks | Completing the Square

**24** Express $22 - 3x^2 + 30x$ in the form $a - b(x - c)^2$ where $a$, $b$ and $c$ are integers.

.....

**(Total for Question 24 is 3 marks)**

## WORKED SOLUTION

Question 24 · 4WM1/01 May 2026

**1 Factor out −3 from the x terms**

$$22 - 3x^2 + 30x = 22 - 3(x^2 - 10x)$$

**2 Complete the square**

$$x^2 - 10x = (x - 5)^2 - 25$$

**3 Simplify**

$$22 - 3\big((x - 5)^2 - 25\big) = 97 - 3(x - 5)^2$$

**FINAL ANSWER**

$97 - 3(x - 5)^2$

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TOPIC

# Algebraic Proof

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

# Question 25

3 marks | Algebraic Proof

**25** $N$ is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

**(Total for Question 25 is 3 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2HR January 2020

**1 Use trigonometry**

Since $N$ is a multiple of 5, let

$$N = 5k$$

**2 where is an integer**

where $k$ is an integer.

**3 Given**

Given

$$A = N + 1, \qquad B = N - 1$$

**4 Use trigonometry**

So

$$A^2 - B^2 = (N+1)^2 - (N-1)^2$$

**5 Expand**

Expand:

$$(N+1)^2 = N^2 + 2N + 1$$

$$(N-1)^2 = N^2 - 2N + 1$$

**6** **Use trigonometry**

Therefore

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$

$$= 4N$$

---

**7** **Substitute**

Substitute $N = 5k$:

$$4N = 4(5k) = 20k$$

---

**8** **Use trigonometry**

Since $k$ is an integer, $20k$ is a multiple of 20.

---

✓ **FINAL ANSWER**

$A^2 - B^2$ is always a multiple of $20$.

4MA1/2H NOVEMBER 2024

# Question 23

4 marks | Algebraic Proof

**23** Show that $\dfrac{16x^2 - 36}{x - 7} \div \dfrac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x) = n$

where $n$ is an integer to be found.
Show clear algebraic working.

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H November 2024

### 1 Evaluate fraction

$$\frac{16x^2 - 36}{x - 7} \div \frac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x)$$

### 2 Factorise

Factorise:

$$16x^2 - 36 = 4(2x - 3)(2x + 3)$$

$$2x^2 + 7x + 6 = (2x + 3)(x + 2)$$

$$x^2 - 5x - 14 = (x - 7)(x + 2)$$

**3** **Evaluate fraction**

So

$$\frac{4(2x-3)(2x+3)}{x-7} \div \frac{(2x+3)(x+2)}{(x-7)(x+2)} - (7+8x)$$

$$= \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)} - (7+8x)$$

$$= 4(2x-3) - (7+8x)$$

$$= 8x - 12 - 7 - 8x$$

$$= -19$$

---

**4** **Calculate value**

So $n = -19$, which is an integer.

---

✓ **FINAL ANSWER**

$n = -19$

4WM2/01 MAY 2025

# Question 20

3 marks | Algebraic Proof

**20** $a$, $b$ and $c$ are three consecutive even numbers where $a < b < c$

Prove algebraically that $b^2 = ac + 4$

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4WM2/01 May 2025

**1 Let**

Let

$$a = 2n, \qquad b = 2n + 2, \qquad c = 2n + 4$$

**2 Calculate value**

Then

$$ac + 4 = (2n)(2n + 4) + 4$$

$$= 4n^2 + 8n + 4$$

$$= (2n + 2)^2 = b^2$$

**3 Calculate value**

So $b^2 = ac + 4$.

**FINAL ANSWER**

So $b^2 = ac + 4$.

TOPIC

# Sequences

Every past-paper question on this topic, 2020 to 2026.

# Question 24

5 marks | Sequences

**24** The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.
The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

.....

**(Total for Question 24 is 5 marks)**

**1 Let the first term be**

Let the first term be $a$ and the common difference be $d$.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

**2 Find the nth term**

So

$$S_{10} = 5(2a + 9d)$$

**3 Find the nth term**

and

$$S_5 = \frac{5}{2}(2a + 4d)$$

**4 Given**

Given $S_{10} = 4S_5$:

$$5(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$10a + 45d = 20a + 40d$$

$$5d = 10a$$

$$d = 2a$$

**5 The 8th term is**

The 8th term is $45$:

$$a + 7d = 45$$

**6 Substitute**

Substitute $d = 2a$:

$$a + 14a = 45$$

$$15a = 45$$

$$a = 3$$

**FINAL ANSWER**

the first term is $\mathbf{3}$.

4MA1/2HR JANUARY 2020

# Question 22

4 marks | Sequences

22 The first term of an arithmetic series $S$ is $-6$
The sum of the first 30 terms of $S$ is 2865

Find the 9th term of $S$.

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2HR January 2020

**1 Find the nth term**

First term:

$$a = -6$$

$$S_{30} = 2865$$

**2 Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$2865 = \frac{30}{2}(2(-6) + 29d)$$

$$2865 = 15(-12 + 29d)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7$$

**3** **The 9th term is**

The 9th term is

$$u_9 = a + 8d$$

$$u_9 = -6 + 8(7) = 50$$

---

✓ **FINAL ANSWER**

$50$

4MA1/2H JANUARY 2020

# Question 25

3 marks | Sequences

**25** Mario is going to save \$50 in the year 2021

He is going to continue to save, up to and including the year 2070, by increasing the amount he saves each year by \$$k$

Mario will save a total of \$33 125 from 2021 to 2070

Work out the value of $k$.

$k =$ .....

**(Total for Question 25 is 3 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H January 2020

**1 Mario saves from 2021 up**

Mario saves from 2021 up to and including 2070.

**2 Number of years**

Number of years:

$$2070 - 2021 + 1 = 50$$

**3 This is an arithmetic sequence**

This is an arithmetic sequence.

**4 Use trigonometry**

First term:

$$50$$

**5 Common difference**

Common difference:

$$k$$

**6 Last term**

Last term:

$$50 + 49k$$

**7** **Use the arithmetic series formula**

Use the arithmetic series formula:

$$S_n = \frac{n}{2}(a + l)$$

$$33125 = \frac{50}{2}(50 + (50 + 49k))$$

$$33125 = 25(100 + 49k)$$

$$1325 = 100 + 49k$$

$$49k = 1225$$

$$k = 25$$

---

✓ **FINAL ANSWER**

$\boxed{k = 25}$

4MA1/1H MAY/NOVEMBER 2020

# Question 24

4 marks | Sequences

**24** Here are the first five terms of an arithmetic sequence.

8 15 22 29 36

Work out the sum of all the terms from the 50th term to the 100th term inclusive.

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H May/November 2020

**1 The arithmetic sequence is**

The arithmetic sequence is

$$8,\ 15,\ 22,\ 29,\ 36, \ldots$$

**2 Calculate value**

So

$$a = 8, \quad d = 7$$

**3 The th term is**

The $n$th term is

$$u_n = 8 + 7(n-1) = 7n + 1$$

$$u_{50} = 7(50) + 1 = 351$$

$$u_{100} = 7(100) + 1 = 701$$

**4 There are**

There are

$$100 - 50 + 1 = 51$$

**5** **Find the nth term**

terms from the 50th to the 100th inclusive.

$$\text{sum} = \frac{51}{2}(351 + 701)$$

$$= 26826$$

---

✓ **FINAL ANSWER**

$26826$

4MA1/1HR JANUARY 2021

# Question 21

6 marks | Sequences

**21** The $n$th term of an arithmetic series is $u_n$ where $u_n > 0$ for all $n$
The sum to $n$ terms of the series is $S_n$

Given that $u_4 = 6$ and that $S_{11} = (u_6)^2 + 18$

find the value of $u_{20}$

**(Total for Question 21 is 6 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR January 2021

**1 Let**

Let $u_6 = v$.

**2 Use series formula**

For an arithmetic series,

$$S_{11} = \frac{11}{2}(u_1 + u_{11})$$

**3 The average of and is**

The average of $u_1$ and $u_{11}$ is $u_6$, so

$$S_{11} = 11u_6 = 11v$$

**4** **Given**

Given

$$S_{11} = (u_6)^2 + 18$$

$$11v = v^2 + 18$$

$$v^2 - 11v + 18 = 0$$

$$(v - 2)(v - 9) = 0$$

---

**5** **Calculate value**

So

$$u_6 = 2 \quad \text{or} \quad u_6 = 9$$

---

**6** **Find the nth term**

But $u_4 = 6$. If $u_6 = 2$, then the common difference is negative and later terms would not all be positive. Therefore,

$$u_6 = 9$$

$$d = \frac{u_6 - u_4}{2} = \frac{9 - 6}{2} = \frac{3}{2}$$

$$u_{20} = u_6 + 14d$$

$$u_{20} = 9 + 14\left(\frac{3}{2}\right) = 30$$

---

✓ **FINAL ANSWER**

$u_{20} = 30$

4MA1/1H JANUARY 2021

# Question 24

4 marks | Sequences

**24** An arithmetic series has first term $a$ and common difference $d$.

The sum of the first $2n$ terms of the series is four times the sum of the first $n$ terms of the series.

Find an expression for $a$ in terms of $d$.
Show your working clearly.

$a =$ .....

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H January 2021

**1 Evaluate fraction**

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{2n} = n(2a + (2n-1)d)$$

**2 Given**

Given

$$S_{2n} = 4S_n$$

$$n(2a + (2n-1)d) = 4 \cdot \frac{n}{2}(2a + (n-1)d)$$

**3 Divide by**

Divide by $n$:

$$2a + (2n-1)d = 2(2a + (n-1)d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$d = 2a$$

$$a = \frac{d}{2}$$

**FINAL ANSWER**

$$a = \frac{d}{2}$$

4MA1/1H MAY 2021

# Question 23

5 marks | Sequences

**23** The sum of the first $N$ terms of an arithmetic series, $S$, is 292
The 2nd term of $S$ is 8.5
The 5th term of $S$ is 13

Find the value of $N$.
Show clear algebraic working.

$N =$ .....

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H May 2021

**1** **The 2nd term is**

The 2nd term is

$$a + d = 8.5$$

**2** **The 5th term is**

The 5th term is

$$a + 4d = 13$$

**3** **Subtract**

Subtract:

$$3d = 4.5$$

$$d = 1.5$$

$$a = 8.5 - 1.5 = 7$$

**4** **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$292 = \frac{N}{2}(14 + 1.5(N-1))$$

$$292 = \frac{N}{2}(1.5N + 12.5)$$

$$3N^2 + 25N - 1168 = 0$$

$$N = \frac{-25 + 121}{6} = 16$$

---

✓ **FINAL ANSWER**

$N = 16$

4MA1/2H NOVEMBER 2021

# Question 20

5 marks | Sequences

**20** Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

**(Total for Question 20 is 5 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H November 2021

**1 The first four terms are**

The first four terms are

$$k, \quad \frac{3k}{4}, \quad \frac{k}{2}, \quad \frac{k}{4}$$

**2 The common difference is**

The common difference is

$$d = -\frac{k}{4}$$

**3 The 15th term is**

The 15th term is

$$u_{15} = k + 14\left(-\frac{k}{4}\right)$$

$$u_{15} = k - \frac{7k}{2} = -\frac{5k}{2}$$

**4** **Given**

Given

$$-\frac{5k}{2} = 90 + 2k$$

$$-5k = 180 + 4k$$

$$k = -20$$

---

**5** **Evaluate fraction**

So

$$a = -20, \quad d = 5$$

$$S_{30} = \frac{30}{2}(2(-20) + 29(5))$$

$$S_{30} = 15(105) = 1575$$

---

✓ **FINAL ANSWER**

$1575$

4MA1/2H JANUARY 2022

# Question 26

5 marks | Sequences

**26** An arithmetic series has first term $a$ and common difference $d$, where $d$ is a prime number.

The sum of the first $n$ terms of the series is $S_n$ and

$S_m = 39$

$S_{2m} = 320$

Find the value of $d$ and the value of $m$
Show clear algebraic working.

$d =$ .....

$m =$ .....

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2H January 2022

**1 Calculate value**

$$S_m = 39$$

$$S_{2m} = 320$$

**2 Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

**3 Calculate value**

Then

$$2S_m = m(2a + (m-1)d) = 78$$

**4 Calculate value**

and

$$S_{2m} = m(2a + (2m-1)d) = 320$$

**5** **Subtract**

Subtract:

$$S_{2m} - 2S_m = m[(2m-1)d - (m-1)d]$$

$$320 - 78 = m(md)$$

$$242 = m^2 d$$

$$242 = 2 \times 11^2$$

---

**6** **Square factor**

Since $d$ is prime and $m^2$ is a square factor,

$$d = 2, \quad m^2 = 121$$

$$m = 11$$

---

✓ **FINAL ANSWER**

$d = 2,\ m = 11$

4MA1/1HR JUNE 2022

# Question 25

5 marks | Sequences

**25** The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1HR June 2022

**1** **Calculate value**

$$S_{10} = 4S_5$$

**2** **Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

**3** **Calculate value**

So

$$5(2a + 9d) = 4 \cdot 5(a + 2d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

**4** **The 8th term is 45**

The 8th term is 45:

$$u_8 = a + 7d = 45$$

$$a + 7(2a) = 45$$

$$15a = 45$$

$$a = 3$$

---

✓ **FINAL ANSWER**

The first term is $\mathbf{3}$

4MA1/1H MAY 2023

# Question 20

6 marks | Sequences

**20** The sum of the first 80 terms of an arithmetic series, $S$, is 470

The 75th term of $S$ is 14.5

The sum of the first $X$ terms of $S$ is 171

Work out the value of $X$
Show your working clearly.

$X =$ .....

**(Total for Question 20 is 6 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H May 2023

**1 Evaluate fraction**

$$S_{80} = 470$$

$$40(2a + 79d) = 470$$

$$2a + 79d = \frac{47}{4}$$

**2 The 75th term is**

The 75th term is

$$a + 74d = 14.5 = \frac{29}{2}$$

**3 Double it**

Double it:

$$2a + 148d = 29$$

**4** **Subtract**

Subtract:

$$69d = 29 - \frac{47}{4} = \frac{69}{4}$$

$$d = \frac{1}{4}$$

$$a + 74\left(\frac{1}{4}\right) = \frac{29}{2}$$

$$a = -4$$

---

**5** **Evaluate fraction**

Now

$$S_X = 171$$

$$171 = \frac{X}{2}\left(2(-4) + (X-1)\frac{1}{4}\right)$$

$$171 = \frac{X(X-33)}{8}$$

$$X^2 - 33X - 1368 = 0$$

$$X = \frac{33 + 81}{2} = 57$$

---

✓ **FINAL ANSWER**

$X = 57$

4MA1/2HR JUNE 2023

# Question 23

5 marks | Sequences

**23** Here are the first three terms of an arithmetic sequence.

$$8p \qquad 7p - 3 \qquad 4p + 2$$

The sum of the first $n$ terms of the sequence is $-1914$

Work out the value of $n$
Show your working clearly.

$n =$ .....

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR June 2023

**1 The first three terms are**

The first three terms are

$$8p, \quad 7p - 3, \quad 4p + 2$$

**2 Find the nth term**

For an arithmetic sequence, consecutive differences are equal:

$$(7p - 3) - 8p = (4p + 2) - (7p - 3)$$

$$-p - 3 = -3p + 5$$

$$2p = 8$$

$$p = 4$$

**3 Find the nth term**

So the first term is

$$8p = 32$$

**4** **Find common difference**

and the common difference is

$$(7p-3)-8p=25-32=-7$$

**5** **The sum of the first**

The sum of the first $n$ terms is

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$-1914=\frac{n}{2}(64-7(n-1))$$

$$-1914=\frac{n}{2}(71-7n)$$

$$-3828=n(71-7n)$$

$$7n^2-71n-3828=0$$

$$n=\frac{71\pm 335}{14}$$

**6** **Only the positive value is**

Only the positive value is possible:

$$n=29$$

✓ **FINAL ANSWER**

$n=29$

4MA1/1HR MAY 2024

# Question 23

4 marks | Sequences

**23** Here are the first three terms of an arithmetic sequence.

$$(4x-14) \quad , \quad (x+2) \quad , \quad (7x-9)$$

Find, as an integer, the sum of the first 40 terms of the sequence.
Show clear algebraic working.

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR May 2024

**1** **The first three terms are**

The first three terms are

$$4x - 14, \quad x + 2, \quad 7x - 9$$

**2** **Equal differences give**

Equal differences give

$$(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$$

$$-3x + 16 = 6x - 11$$

$$9x = 27$$

$$x = 3$$

**3** **Find the nth term**

So the sequence is

$$-2,\ 5,\ 12, \ldots$$

**4** **with common difference**

with common difference $7$.

$$S_{40} = \frac{40}{2}(2(-2) + 39(7))$$

$$S_{40} = 20(-4 + 273) = 5380$$

---

✓ **FINAL ANSWER**

$5380$

4MA1/1H MAY 2024

# Question 24

6 marks | Sequences

**24** A polygon has $n$ sides, where $n > 5$

The interior angles of the polygon form an arithmetic sequence.

The smallest angle of the polygon is 84°
The common difference of the sequence is 4°

Work out the sum of the interior angles of the polygon.
Show clear algebraic working.

.....°

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H May 2024

**1** **The interior angles form an**

The interior angles form an arithmetic sequence with first term $84$ and common difference $4$.

**2** **Use trigonometry**

So the sum of the angles is

$$\frac{n}{2}(2(84) + (n-1)4)$$

$$= \frac{n}{2}(168 + 4n - 4)$$

$$= 2n^2 + 82n$$

**3** **The sum of the interior**

The sum of the interior angles of an $n$-sided polygon is

$$180(n-2)$$

**4** **Calculate value**

So

$$2n^2 + 82n = 180(n - 2)$$

$$2n^2 - 98n + 360 = 0$$

$$n^2 - 49n + 180 = 0$$

$$(n - 45)(n - 4) = 0$$

---

**5** **Solve inequality**

Since $n > 5$,

$$n = 45$$

---

**6** **Use trigonometry**

Therefore the sum of the interior angles is

$$180(45 - 2) = 7740$$

---

✓ **FINAL ANSWER**

$7740$ degrees

4MA1/1H JUNE 2025

# Question 24

5 marks | Sequences

**24** An arithmetic series has 30 terms.

The first term is $a$
The common difference is $d$

The 20th term is 123
The sum of the 30 terms is 2880

Work out the value of $a$ and the value of $d$
Show clear algebraic working.

$a =$ .....

$d =$ .....

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H June 2025

### 1 Evaluate fraction

$$u_{20} = a + 19d = 123$$

$$S_{30} = \frac{30}{2}(2a + 29d) = 2880$$

$$2a + 29d = 192$$

### 2 Double the first equation

Double the first equation:

$$2a + 38d = 246$$

**3** **Subtract**

Subtract:

$$9d = 54$$

$$d = 6$$

$$a + 19(6) = 123$$

$$a = 9$$

---

✓ **FINAL ANSWER**

$a = 9,\ d = 6$

4MA1/1HR JUNE 2025

# Question 24

6 marks | Sequences

**24** The first 3 terms of an arithmetic series are

$$(2x + 5) \qquad (3y - 4) \qquad (4x - 2)$$

where $x$ and $y$ are constants.

The sum of the first 9 terms of the series is 216

Find the value of $x$ and the value of $y$
Show clear algebraic working.

$x =$ .....

$y =$ .....

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1HR June 2025

**1 The first 3 terms are**

The first 3 terms are

$$2x + 5, \quad 3y - 4, \quad 4x - 2$$

**2 Equal differences give**

Equal differences give

$$(3y - 4) - (2x + 5) = (4x - 2) - (3y - 4)$$

$$6y - 6x = 11$$

**3 Use series formula**

For an arithmetic series,

$$S_9 = 9(a + 4d)$$

**4** **Here**

Here

$$a = 2x + 5$$

$$d = (3y - 4) - (2x + 5) = 3y - 2x - 9$$

**5** **Given**

Given $S_9 = 216$:

$$9[(2x + 5) + 4(3y - 2x - 9)] = 216$$

$$12y - 6x = 55$$

**6** **Solve**

Now solve

$$6y - 6x = 11$$

$$12y - 6x = 55$$

**7** **Subtracting gives**

Subtracting gives

$$6y = 44$$

$$y = \frac{22}{3}$$

**8** **Evaluate fraction**

Then

$$6\left(\frac{22}{3}\right) - 6x = 11$$

$$x = \frac{11}{2}$$

✓ **FINAL ANSWER**

$$x = \frac{11}{2},\ y = \frac{22}{3}$$

4WM2/01 MAY 2025

# Question 24

4 marks | Sequences

24 The 21st term of an arithmetic series is 109
The sum of the first 52 terms of the series is 4381

Work out the 5th term of the series.
Show clear algebraic working.

.....

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4WM2/01 May 2025

**1 Use series formula**

For an arithmetic series,

$$a + 20d = 109$$

**2 Evaluate fraction**

and

$$\frac{52}{2}(2a + 51d) = 4381$$

$$26(2a + 51d) = 4381$$

**3 Solving gives**

Solving gives

$$d = -4.5, \qquad a = 199$$

**4 The fifth term is**

The fifth term is

$$a + 4d = 199 + 4(-4.5) = 181$$

✓ **FINAL ANSWER**

181.

4WM2/01R MAY 2025

# Question 24

6 marks | Sequences

**24** An arithmetic series $P$ has first term $a$ and common difference $d$

The sum of the first 40 terms of $P$ is 2070

An arithmetic series $Q$ has first term $2a$ and common difference $3d$

The 25th term of $Q$ is 186

Work out the sum of the first 40 terms of $Q$
Show clear algebraic working.

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4WM2/01R May 2025

### 1 Required

Series P: first term *a*, c.d. *d*; *S*<sub>40</sub>(*P*) = 2070. Series Q: first term 2*a*, c.d. 3*d*; 25th term of Q = 186. Find *S*<sub>40</sub>(*Q*).

### 2 Form equation from S 40

Form equation from *S*<sub>40</sub>(*P*) = 2070:

$$S_40 = frac402\[2a + 39d\] = 20(2a + 39d) = 2070implies2a + 39d = 103.5tag1$$

### 3 Form equation from 25th term

Form equation from 25th term of Q = 186:

*T*<sub>25</sub>(*Q*) = 2*a* + 24(3*d*) = 2*a* + 72*d* = 186

### 4 Solve simultaneous equations

Solve simultaneously — subtract (1) from (2):

33*d* = 82.5 ⟹ *d* = 2.5

### 5 Substitute d 2 5 (1)

Substitute *d* = 2.5 into (1):

2*a* + 39(2.5) = 103.5 ⟹ 2*a* = 6 ⟹ *a* = 3

### 6 Compute S 40 ( Q

Compute *S*<sub>40</sub>(*Q*), with first term = 2*a* = 6 and c.d. = 3*d* = 7.5:

$$S_40(Q) = frac402bigl\[2(6) + 39(7.5)bigr\] = 20bigl\[12 + 292.5bigr\] = 20times304.5 = 6090$$

**FINAL ANSWER**

*S*<sub>40</sub>(*Q*) = 6090

4MA1/2H NOVEMBER 2025

# Question 25

5 marks | Sequences

**25** The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first $N$ terms of the series is 5194

Work out the value of $N$
Show your working clearly.

$N =$ .....

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H November 2025

### 1 Calculate value

$$a = 10$$

$$u_{20} = 86$$

$$10 + 19d = 86$$

$$d = 4$$

**2** **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$5194 = \frac{N}{2}(20 + 4(N-1))$$

$$5194 = \frac{N}{2}(4N + 16)$$

$$5194 = 2N(N+4)$$

$$N^2 + 4N - 2597 = 0$$

$$N = \frac{-4 + 102}{2} = 49$$

---

**FINAL ANSWER**

$N = 49$

4WM2/01 NOVEMBER 2025

# Question 26

5 marks | Sequences

**26** The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first $N$ terms of the series is 5194

Work out the value of $N$
Show your working clearly.

$N =$ .....

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4WM2/01 November 2025

### 1 Find common difference

**Step 1: Find the common difference *d*.** The *n*<sup>th</sup> term is given by *u*<sub>*n*</sub> = *a* + (*n*−1)*d*.

*u*<sub>20</sub> = 10 + 19*d* = 86

19*d* = 76

$$d = frac7619 = 4$$

### 2 Use sum formula

**Step 2: Use the sum formula to form an equation in terms of *N*.**

$$S_N = fracN2left\[2a + (N-1)dright\]$$

Substitute the known values *S*<sub>*N*</sub> = 5194, *a* = 10, and *d* = 4:

$$5194 = fracN2left\[2(10) + (N-1)(4)right\]$$

### 3 Solve quadratic equation

**Step 3: Simplify and solve the quadratic equation.** Multiply by 2 to clear the fraction:

10388 = *N*

$$20 + 4 * N * -4$$

10388 = *N*(4*N*+16)

4*N*<sup>2</sup> + 16*N* − 10388 = 0

**4** **Divide the entire equation by**

Divide the entire equation by 4 to make it simpler:

*N*<sup>2</sup> + 4*N* − 2597 = 0

**5** **Use quadratic formula**

**Step 4: Use the quadratic formula to find *N*.**

$$N = frac-bpmsqrtb^2 - 4ac2a$$

$$N = frac-4pmsqrt16 - 4(1)(-2597)2$$

$$N = frac-4pmsqrt16 + 103882$$

$$N = frac-4pmsqrt104042$$

$$N = frac-4pm1022$$

**6** **Evaluate fraction**

Since *N* must be a positive integer, we discard the negative root:

$$N = frac-4 + 1022 = frac982 = 49$$

✓ **FINAL ANSWER**

*N* = 49 Always double-check that your *N* (number of terms) evaluates to a positive whole integer. If it ends up as a decimal or fraction, recheck your arithmetic progression formulas.

# Question 25

6 marks | Sequences

**25** Here are the first four terms of an arithmetic series.

$$(k + 2) \qquad (k + 5) \qquad (k + 8) \qquad (k + 11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first $n$ terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

**(Total for Question 25 is 6 marks)**

**1** **First term and difference**

$a = k + 2$, $d = 3$.

**2** **Use the sum formula**

$$\frac{2n}{2}\left[2a + (2n-1)3\right] = 4 \times \frac{n}{2}\left[2a + (n-1)3\right]$$

**3** **Simplify (divide by n)**

$$2a + 6n - 3 = 4a + 6n - 6 \Rightarrow 2a = 3 \Rightarrow a = 1.5,\ k = -\tfrac{1}{2}$$

**4** **100th term**

$$a + 99d = 1.5 + 99(3) = 298.5$$

✓ **FINAL ANSWER**

298.5



TOPIC

# Direct & Inverse Proportion

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2022

# Question 20

4 marks | Direct & Inverse Proportion

**20** $y$ is inversely proportional to $\sqrt{x}$
$x$ is directly proportional to $T^3$

Given that $y = 8$ when $T = 25$

find the exact value of $T$ when $y = 27$

$T =$ .....

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR January 2022

**1** **Find inverse function**

$$y \propto \frac{1}{\sqrt{x}}$$

**2** **Use Direct Inverse Proportion correct**

and

$$x \propto T^3$$

**3** **Find inverse function**

So

$$\sqrt{x} \propto T^{3/2}$$

**4** **Find inverse function**

Therefore

$$y \propto \frac{1}{T^{3/2}}$$

**5** **Use Direct Inverse Proportion correct**

So $yT^{3/2}$ is constant.

**6** **Find inverse function**

When $y = 8$ and $T = 25$,

$$8(25)^{3/2} = 8(125) = 1000$$

**7** **Find inverse function**

When $y = 27$,

$$27T^{3/2} = 1000$$

$$T^{3/2} = \frac{1000}{27} = \left(\frac{10}{3}\right)^3$$

$$T = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

---

✓ **FINAL ANSWER**

$$\frac{100}{9}$$



TOPIC

# Linear Graphs

Every past-paper question on this topic, 2020 to 2026.

# Question 23

6 marks | Linear Graphs

**23** $ABCD$ is a kite.

$AB = AD$ and $CB = CD$

The point $B$ has coordinates $(k, 1)$ where $k$ is a negative constant.
The point $D$ has coordinates $(8, 7)$

The straight line **L** passes through the points $B$ and $D$ and has a gradient of $\frac{3}{5}$

Find an equation of $AC$
Give your answer in the form $px + qy = r$ where $p$, $q$ and $r$ are integers.
Show your working clearly.

.....

**(Total for Question 23 is 6 marks)**

## 1 Find k from the gradient

The gradient of $BD$ is

$$\frac{7-1}{8-k}=\frac{3}{5}$$

$$\frac{6}{8-k}=\frac{3}{5}$$

$$30=24-3k$$

$$k=-2$$

## 2 Find the midpoint of BD

So $B=(-2,1)$ and $D=(8,7)$.

$$\text{midpoint of } BD=\left(\frac{-2+8}{2},\frac{1+7}{2}\right)=(3,4)$$

## 3 Use the perpendicular gradient

In this kite, $AC$ is the perpendicular bisector of $BD$.

The gradient of $BD$ is $\frac{3}{5}$, so the gradient of $AC$ is

$$-\frac{5}{3}$$

## 4 Form the equation

Using point $(3,4)$:

$$y-4=-\frac{5}{3}(x-3)$$

$$3y-12=-5x+15$$

$$5x+3y=27$$

**FINAL ANSWER**

$5x+3y=27$.

4MA1/1H JANUARY 2020

# Question 22

6 marks | Linear Graphs

**22** Triangle $HJK$ is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

$H$ is the point with coordinates $(-4, 1)$
$J$ is the point with coordinates $(j, 15)$ where $j < 0$
$K$ is the point with coordinates $(6, k)$

$M$ is the midpoint of $JK$.
The gradient of $HM$ is 2

Find the value of $j$ and the value of $k$.

$j =$ .....

$k =$ .....

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H January 2020

### 1 Find the midpoint

Since $M$ is the midpoint of $JK$,

$$M = \left(\frac{j+6}{2}, \frac{15+k}{2}\right)$$

### 2 Find the gradient

The gradient of $HM$ is 2, and $H = (-4, 1)$.

$$\frac{\frac{15+k}{2} - 1}{\frac{j+6}{2} + 4} = 2$$

$$\frac{k+13}{j+14} = 2$$

$$k = 2j + 15$$

**3** **Simplify surd**

Also,

$$JK = \sqrt{80}$$

**4** **Calculate value**

So

$$(6 - j)^2 + (k - 15)^2 = 80$$

**5** **Substitute**

Substitute $k = 2j + 15$:

$$(6 - j)^2 + (2j)^2 = 80$$

$$j^2 - 12j + 36 + 4j^2 = 80$$

$$5j^2 - 12j - 44 = 0$$

$$(5j + 10)(j - 4.4) = 0$$

**6** **Solve inequality**

Since $j < 0$,

$$j = -2$$

**7** **Calculate value**

Then

$$k = 2(-2) + 15 = 11$$

✓ **FINAL ANSWER**

$j = -2,\ k = 11$

4MA1/2H JANUARY 2020

# Question 22

5 marks | Linear Graphs

**22** The line with equation $y = x + 2$ intersects the curve with equation $x^2 + y^2 - 2y = 24$ at the points $A$ and $B$.

Find the coordinates of $A$ and $B$.
Show clear algebraic working.

(..... , .....)

(..... , .....)

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H January 2020

**1 The line is**

The line is

$$y = x + 2$$

**2 Substitute this into the circle**

Substitute this into the circle equation:

$$x^2 + y^2 - 2y = 24$$

$$x^2 + (x+2)^2 - 2(x+2) = 24$$

$$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

**3** **Calculate value**

So

$$x = -4 \quad \text{or} \quad x = 3$$

---

**4** **Calculate value**

Using $y = x + 2$:

$$x = -4 \Rightarrow y = -2$$

$$x = 3 \Rightarrow y = 5$$

---

✓ **FINAL ANSWER**

$(-4, -2)$ and $(3, 5)$

4MA1/2HR JANUARY 2020

# Question 24

5 marks | Linear Graphs

24 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is $O$.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the $x$-axis at the point $A$.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point $B$.

Find the area of triangle $AOB$.
Show your working clearly.

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2HR January 2020

**1 Evaluate fraction**

For $L_1$,

$$5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

**2 Find the gradient**

So the gradient of $L_1$ is $-\frac{1}{2}$.

**3 Find the gradient**

Since $L_2$ is perpendicular to $L_1$, the gradient of $L_2$ is $2$.

**4 Calculate value**

Using the point $(8, 6)$:

$$y - 6 = 2(x - 8)$$

$$y = 2x - 10$$

**5** **Calculate value**

At $A$, the line crosses the $x$-axis, so $y = 0$:

$$0 = 2x - 10$$

$$x = 5$$

---

**6** **Calculate value**

So $A = (5, 0)$.

---

**7** **Calculate value**

At $B$, $x = -3$:

$$y = 2(-3) - 10 = -16$$

---

**8** **Calculate value**

So $B = (-3, -16)$.

---

**9** **Triangle has base and perpendicular**

Triangle $AOB$ has base $OA = 5$ and perpendicular height $16$.

$$\text{Area} = \frac{1}{2} \times 5 \times 16 = 40$$

---

✓ **FINAL ANSWER**

$40$

4MA1/1H MAY/NOVEMBER 2020

# Question 22

4 marks | Linear Graphs

**22** $ABCD$ is a rhombus.

The diagonals, $AC$ and $BD$, intersect at the point $M$.
The coordinates of $M$ are $(6, -11)$

The points $A$ and $C$ both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through $B$ and $D$ intersects the $y$-axis.

(..... , .....)

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H May/November 2020

**1 Find the gradient**

In a rhombus, the diagonals are perpendicular and bisect each other.

**2 The diagonal lies on**

The diagonal $AC$ lies on

$$2y + 7x = 20$$

$$2y = -7x + 20$$

$$y = -\frac{7}{2}x + 10$$

**3 Find the gradient**

So the gradient of $AC$ is

$$-\frac{7}{2}$$

**4 Find the gradient**

Therefore the gradient of $BD$ is

$$\frac{2}{7}$$

**5** **Calculate value**

The line $BD$ passes through

$$M = (6, -11)$$

**6** **Evaluate fraction**

So

$$y + 11 = \frac{2}{7}(x - 6)$$

**7** **To find where this line**

To find where this line intersects the $y$-axis, put $x = 0$:

$$y + 11 = \frac{2}{7}(0 - 6)$$

$$y + 11 = -\frac{12}{7}$$

$$y = -11 - \frac{12}{7}$$

$$y = -\frac{89}{7}$$

✓ **FINAL ANSWER**

$$\left(0, -\frac{89}{7}\right)$$

4MA1/1HR JANUARY 2021

# Question 22

5 marks | Linear Graphs

**22** $ABC$ is an isosceles triangle with $AB = AC$.

$B$ is the point with coordinates $(-1, 5)$
$C$ is the point with coordinates $(2, 10)$
$M$ is the midpoint of $BC$.

Find an equation of the line through the points $A$ and $M$.
Give your answer in the form $py + qx = r$ where $p$, $q$ and $r$ are integers.

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1HR January 2021

**1 Find the gradient**

Since $AB = AC$, point $A$ lies on the perpendicular bisector of $BC$.

**2 The midpoint of is**

The midpoint of $BC$ is

$$M = \left(\frac{-1+2}{2}, \frac{5+10}{2}\right)$$

$$M = \left(\frac{1}{2}, \frac{15}{2}\right)$$

**3 The gradient of is**

The gradient of $BC$ is

$$\frac{10-5}{2-(-1)} = \frac{5}{3}$$

**4 Find the gradient**

So the gradient of the perpendicular bisector is

$$-\frac{3}{5}$$

**5** **Find line equation**

Use the line through $M$:

$$y - \frac{15}{2} = -\frac{3}{5}\left(x - \frac{1}{2}\right)$$

**6** **Multiply by 10**

Multiply by 10:

$$10y - 75 = -6x + 3$$

$$10y + 6x = 78$$

**7** **Divide by 2**

Divide by 2:

$$5y + 3x = 39$$

✓ **FINAL ANSWER**

$5y + 3x = 39$

4MA1/1H NOVEMBER 2021

# Question 20

7 marks | Linear Graphs

**20** The straight line **L** passes through point $A$ $(-6, 2)$ and point $B$ $(5, 3)$
The straight line **M** is perpendicular to **L** and passes through the midpoint of $A$ and $B$.
The line **M** intersects the line $x = -1$ at point $C$.

Calculate the area of triangle $ABC$.

**(Total for Question 20 is 7 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H November 2021

**1** **Calculate value**

$$A = (-6, 2), \qquad B = (5, 3)$$

**2** **The midpoint of is**

The midpoint of $AB$ is

$$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

**3** **The gradient of is**

The gradient of $AB$ is

$$\frac{3-2}{5-(-6)} = \frac{1}{11}$$

**4** **Find the gradient**

So the perpendicular gradient is

$$-11$$

**5** **Evaluate fraction**

Line $M$ passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$, so

$$y - \frac{5}{2} = -11\left(x + \frac{1}{2}\right)$$

**6** **Evaluate fraction**

At point $C$, $x = -1$:

$$y - \frac{5}{2} = -11\left(-\frac{1}{2}\right)$$

$$y - \frac{5}{2} = \frac{11}{2}$$

$$y = 8$$

---

**7** **Calculate value**

So

$$C = (-1, 8)$$

---

**8** **Use vectors**

Now use vectors from $A$:

$$\vec{AB} = (11, 1), \qquad \vec{AC} = (5, 6)$$

---

**9** **Area of triangle is**

Area of triangle $ABC$ is

$$\frac{1}{2}|11(6) - 1(5)|$$

$$= \frac{1}{2}(61)$$

$$= 30.5$$

---

✓ **FINAL ANSWER**

$30.5$

4MA1/2HR JANUARY 2022

# Question 22

5 marks | Linear Graphs

**22** $ABC$ is a triangle in which angle $ABC = 90°$

$p$ and $q$ are integers such that

the coordinates of $A$ are $(p, 10)$
the coordinates of $B$ are $(-1, -5)$
the coordinates of $C$ are $(8, q)$

Given that the gradient of $AC$ is $-\frac{6}{7}$

work out the value of $p$ and the value of $q$

$p =$ .....

$q =$ .....

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2HR January 2022

**1 The gradient of is**

The gradient of $AC$ is $-\frac{6}{7}$.

$$\frac{q-10}{8-p} = -\frac{6}{7}$$

$$7(q-10) = -6(8-p)$$

$$7q - 70 = -48 + 6p$$

$$6p - 7q = -22$$

**2** **Find the gradient**

Since $\angle ABC = 90°$, lines $AB$ and $BC$ are perpendicular.

$$\text{gradient of } AB = \frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$$

$$\text{gradient of } BC = \frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$$

---

**3** **The product of perpendicular gradients**

The product of perpendicular gradients is $-1$:

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$5(q+5) = -3(p+1)$$

$$3p + 5q = -28$$

---

**4** **Solve**

Now solve

$$6p - 7q = -22$$

$$3p + 5q = -28$$

---

**5** **Double the second equation**

Double the second equation:

$$6p + 10q = -56$$

---

**6** **Subtract**

Subtract:

$$17q = -34$$

$$q = -2$$

---

**7** **Calculate value**

Then

$$3p + 5(-2) = -28$$

$$3p = -18$$

$$p = -6$$

---

**FINAL ANSWER**

$p = -6,\ q = -2$

4MA1/2HR JUNE 2022

# Question 20

4 marks | Linear Graphs

**20** The centre $O$ of a circle has coordinates $(4, 7)$

The point $A$, on the circle, has coordinates $(6, 11)$ and $AOP$ is a diameter of the circle.

Find an equation of the tangent to the circle at the point $P$

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR June 2022

**1 The centre of the circle**

The centre of the circle is

$$O = (4, 7)$$

**2 The point is**

The point $A$ is

$$A = (6, 11)$$

**3 Find the midpoint**

Since $AOP$ is a diameter, $O$ is the midpoint of $AP$.

**4 Let**

Let $P = (x, y)$.

$$\left(\frac{6+x}{2}, \frac{11+y}{2}\right) = (4, 7)$$

$$6 + x = 8, \qquad 11 + y = 14$$

$$P = (2, 3)$$

**5 The gradient of is**

The gradient of $OP$ is

$$\frac{3-7}{2-4} = 2$$

**6** **Use circle theorem**

The tangent is perpendicular to the radius, so its gradient is

$$-\frac{1}{2}$$

---

**7** **Evaluate fraction**

Using point $P = (2, 3)$,

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 4$$

---

✓ **FINAL ANSWER**

$$y = -\frac{1}{2}x + 4$$

4MA1/2H JUNE 2022

# Question 22

7 marks | Linear Graphs

**22** $ABCD$ is a kite, with diagonals $AC$ and $BD$, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

$A$ is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point $M$ with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the $x$ coordinate of $B$ is positive,

find the coordinates of the points $B$ and $D$.

(..... , .....)

(..... , .....)

**(Total for Question 22 is 7 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H June 2022

**1 The midpoint of is**

The midpoint of $BD$ is $M = (0, 2)$.

**2 Find the gradient**

Since $A = (-3, 4)$, the gradient of $AM$ is

$$\frac{2-4}{0-(-3)} = -\frac{2}{3}$$

**3 Find the gradient**

For the kite, diagonal $AC$ is perpendicular to $BD$, so the gradient of $BD$ is

$$\frac{3}{2}$$

**4 Use direction vector along**

Use direction vector $(2, 3)$ along $BD$.

**5** **Let**

Let

$$B = (2t, 2 + 3t)$$

**6** **Calculate value**

and

$$D = (-2t, 2 - 3t)$$

**7** **Calculate value**

Since $AB = 6.5$,

$$(2t - (-3))^2 + (2 + 3t - 4)^2 = 6.5^2$$

$$(2t + 3)^2 + (3t - 2)^2 = 42.25$$

$$4t^2 + 12t + 9 + 9t^2 - 12t + 4 = 42.25$$

$$13t^2 + 13 = 42.25$$

$$13t^2 = 29.25$$

$$t^2 = 2.25$$

$$t = 1.5$$

**8** **Use**

The $x$-coordinate of $B$ is positive, so use $t = 1.5$.

$$B = (3, 6.5)$$

$$D = (-3, -2.5)$$

✓ **FINAL ANSWER**

$B = (3, 6.5),\ D = (-3, -2.5)$

4MA1/2HR JUNE 2023

# Question 21

5 marks | Linear Graphs

**21** Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

Show clear algebraic working.

(..... , .....)

(..... , .....)

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR June 2023

**1** **The line is**

The line is

$$y - 2x = 1$$

**2** **Calculate value**

So

$$y = 2x + 1$$

**3** **Calculate value**

Substitute into

$$y^2 + xy = 7$$

$$(2x + 1)^2 + x(2x + 1) = 7$$

$$4x^2 + 4x + 1 + 2x^2 + x = 7$$

$$6x^2 + 5x - 6 = 0$$

**4** **Factorise**

Factorise:

$$(3x - 2)(2x + 3) = 0$$

**5** **Evaluate fraction**

So

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

---

**6** **Evaluate fraction**

Using $y = 2x + 1$:

$$x = \frac{2}{3} \Rightarrow y = \frac{7}{3}$$

$$x = -\frac{3}{2} \Rightarrow y = -2$$

---

✓ **FINAL ANSWER**

$\left(\frac{2}{3}, \frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -2\right)$

4MA1/2H JUNE 2023

# Question 24

6 marks | Linear Graphs

**24** $ABCD$ is a kite with $AB = AD$ and $CB = CD$

$A$ is the point with coordinates $(-2, 10)$

$B$ is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

$C$ is the point with coordinates $(4, -5)$

Work out the coordinates of $D$

(..... , .....)

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H June 2023

**1 Calculate value**

In the kite, $AB = AD$ and $CB = CD$, so $AC$ is the line of symmetry.

**2 Apply transformation**

Therefore $D$ is the reflection of $B$ in the line $AC$.

$$A = (-2, 10), \qquad B = \left(-\frac{27}{5}, 4\right), \qquad C = (4, -5)$$

**3 A direction vector for is**

A direction vector for $AC$ is

$$\begin{pmatrix} 4 - (-2) \\ -5 - 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$$

**4 Use the simpler direction vector**

Use the simpler direction vector

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

**5** **Find the gradient**

Let $H$ be the foot of the perpendicular from $B$ to $AC$.

$$\vec{AB} = \begin{pmatrix} -\frac{17}{5} \\ -6 \end{pmatrix}$$

**6** **The fraction along to is**

The fraction along $AC$ to $H$ is

$$\frac{\vec{AB} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}}{2^2 + (-5)^2} = \frac{-\frac{34}{5} + 30}{29}$$

$$= \frac{\frac{116}{5}}{29} = \frac{4}{5}$$

**7** **Use matrices**

So

$$H = A + \frac{4}{5}\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$H = \left(-2 + \frac{8}{5}, 10 - 4\right)$$

$$H = \left(-\frac{2}{5}, 6\right)$$

**8** **Find the midpoint**

Since $H$ is the midpoint of $BD$,

$$D = 2H - B$$

$$D = \left(-\frac{4}{5}, 12\right) - \left(-\frac{27}{5}, 4\right)$$

$$D = \left(\frac{23}{5}, 8\right)$$

✓ **FINAL ANSWER**

$$D = \left(\frac{23}{5}, 8\right)$$

4MA1/2HR JUNE 2023

# Question 25

6 marks | Linear Graphs

**25** The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line $AB$ where $A$ is the point with coordinates $(j, 7)$ and $B$ is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line $AB$
Show clear algebraic working.

(..... , .....)

**(Total for Question 25 is 6 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2HR June 2023

### 1 The perpendicular bisector is

The perpendicular bisector is

$$y - 2x = 7$$

### 2 Calculate value

so

$$y = 2x + 7$$

### 3 The gradient of the perpendicular

The gradient of the perpendicular bisector is $2$, so the gradient of $AB$ is

$$-\frac{1}{2}$$

**4** **Evaluate fraction**

Since $A = (j, 7)$ and $B = (6, k)$,

$$\frac{k-7}{6-j} = -\frac{1}{2}$$

$$2(k-7) = -(6-j)$$

$$j = 2k - 8$$

---

**5** **The midpoint of is**

The midpoint of $AB$ is

$$\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$$

---

**6** **Substitute**

Substitute $j = 2k - 8$:

$$\left(k-1, \frac{k+7}{2}\right)$$

---

**7** **Find the midpoint**

The midpoint lies on $y = 2x + 7$, so

$$\frac{k+7}{2} = 2(k-1) + 7$$

$$k + 7 = 4k + 10$$

$$k = -1$$

---

**8** **Calculate value**

Then

$$j = 2(-1) - 8 = -10$$

---

**9** **Find the midpoint**

Therefore the midpoint is

$$\left(\frac{-10+6}{2}, \frac{7+(-1)}{2}\right) = (-2, 3)$$

---

**FINAL ANSWER**

$$\boxed{(-2, 3)}$$

4MA1/2H NOVEMBER 2023

# Question 21

5 marks | Linear Graphs

**21** The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points $A$ and $B$

Find the coordinates of $A$ and the coordinates of $B$
Show clear algebraic working.

(..... , .....)

(..... , .....)

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H November 2023

**1 The line is**

The line is

$$x + 2y = 5$$

**2 Calculate value**

So

$$x = 5 - 2y$$

**3 Calculate value**

Substitute into

$$x^2 + 3y^2 = 13$$

$$(5 - 2y)^2 + 3y^2 = 13$$

$$25 - 20y + 4y^2 + 3y^2 = 13$$

$$7y^2 - 20y + 12 = 0$$

**4** **Factorise**

Factorise:

$$(7y - 6)(y - 2) = 0$$

**5** **Evaluate fraction**

So

$$y = \frac{6}{7} \quad \text{or} \quad y = 2$$

**6** **Evaluate fraction**

Using $x = 5 - 2y$:

$$y = \frac{6}{7} \Rightarrow x = \frac{23}{7}$$

$$y = 2 \Rightarrow x = 1$$

✓ **FINAL ANSWER**

$\left(\frac{23}{7}, \frac{6}{7}\right)$ and $(1, 2)$

4MA1/1H NOVEMBER 2023

# Question 22

6 marks | Linear Graphs

22 [In this question 1 cm = 1 unit on the $x$-axis and 1 cm = 1 unit on the $y$-axis]

$P$ is a point on a circle with centre $(0, 0)$
The coordinates of $P$ are $(8, -10)$

The line **L** is the tangent to the circle at the point $P$
**L** crosses the $x$-axis at the point $Q$ and crosses the $y$-axis at the point $R$

Work out the length of $RQ$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H November 2023

**1 The centre of the circle**

The centre of the circle is $(0, 0)$, and

$$P = (8, -10)$$

**2 The gradient of is**

The gradient of $OP$ is

$$\frac{-10}{8} = -\frac{5}{4}$$

**3 Find the gradient**

So the gradient of the tangent is

$$\frac{4}{5}$$

**4 The tangent through is**

The tangent through $P$ is

$$y + 10 = \frac{4}{5}(x - 8)$$

$$y = \frac{4}{5}x - \frac{82}{5}$$

**5** **Evaluate fraction**

At the $x$-axis, $y = 0$:

$$0 = \frac{4}{5}x - \frac{82}{5}$$

$$x = \frac{41}{2}$$

---

**6** **Evaluate fraction**

So

$$Q = \left(\frac{41}{2}, 0\right)$$

---

**7** **Evaluate fraction**

At the $y$-axis, $x = 0$:

$$R = \left(0, -\frac{82}{5}\right)$$

---

**8** **Simplify surd**

Therefore

$$RQ = \sqrt{\left(\frac{41}{2}\right)^2 + \left(\frac{82}{5}\right)^2}$$

$$RQ = 26.252...$$

---

✓ **FINAL ANSWER**

$26.3\ \text{cm}$

4MA1/2HR JUNE 2024

# Question 21

4 marks | Linear Graphs

**21** $ABCD$ is a square.

The point $A$ has coordinates $(-5, 2)$
The point $B$ has coordinates $(3, 5)$

Find an equation of the line that passes through $B$ and $C$
Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR June 2024

**1** **Calculate value**

$$A = (-5, 2), \qquad B = (3, 5)$$

**2** **The gradient of is**

The gradient of $AB$ is

$$\frac{5-2}{3-(-5)} = \frac{3}{8}$$

**3** **Find the gradient**

Since $ABCD$ is a square, $BC$ is perpendicular to $AB$.

**4** **Find the gradient**

So the gradient of $BC$ is

$$-\frac{8}{3}$$

**5** **The line through is**

The line through $B = (3, 5)$ is

$$y - 5 = -\frac{8}{3}(x - 3)$$

### 6 Multiply by 3

Multiply by 3:

$$3y - 15 = -8x + 24$$

$$8x + 3y - 39 = 0$$

---

**FINAL ANSWER**

$$8x + 3y - 39 = 0$$

4MA1/2H JUNE 2024

# Question 22

5 marks | Linear Graphs

22 The straight line **L** has equation $x + y = 5$

The curve **C** has equation $2x^2 + 3y^2 = 210$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , .....) (..... , .....)

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H June 2024

### 1 The line is

The line is

$$x + y = 5$$

### 2 Calculate value

So

$$y = 5 - x$$

### 3 Substitute into the curve

Substitute into the curve:

$$2x^2 + 3y^2 = 210$$

$$2x^2 + 3(5 - x)^2 = 210$$

$$2x^2 + 3(x^2 - 10x + 25) = 210$$

$$5x^2 - 30x + 75 = 210$$

$$5x^2 - 30x - 135 = 0$$

**4** **Divide by 5**

Divide by 5:

$$x^2 - 6x - 27 = 0$$

**5** **Factorise**

Factorise:

$$(x - 9)(x + 3) = 0$$

**6** **Calculate value**

So

$$x = 9 \quad \text{or} \quad x = -3$$

**7** **Calculate value**

Using $y = 5 - x$:

$$x = 9 \Rightarrow y = -4$$

$$x = -3 \Rightarrow y = 8$$

✓ **FINAL ANSWER**

$(9, -4)$ and $(-3, 8)$

4WM1/01R MAY 2025

# Question 23

5 marks | Linear Graphs

**23** A circle $C$ has centre $(3q, 4q)$ where $q$ is an integer.

The straight line with equation $y = -\frac{5}{4}x + \frac{21}{4}$ is the tangent to $C$ at the point $(p, p+3)$ where $p$ is an integer.

Work out the radius of $C$
Give your answer in the form $\sqrt{a}$ where $a$ is an integer.
Show your working clearly.

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4WM1/01R May 2025

**1 The tangent point lies on**

The tangent point lies on the tangent line, so

$$p + 3 = -\frac{5}{4}p + \frac{21}{4}$$

$$p = 1$$

**2 Find the gradient**

The tangent gradient is $-\frac{5}{4}$, so the radius gradient is $\frac{4}{5}$.

**3 Use trigonometry**

Using centre $(3q, 4q)$ and tangent point $(1, 4)$,

$$\frac{4 - 4q}{1 - 3q} = \frac{4}{5}$$

$$q = 2$$

**4** **Simplify surd**

So the centre is $(6, 8)$. Therefore

$$r = \sqrt{(6-1)^2 + (8-4)^2} = \sqrt{41}$$

---

✓ **FINAL ANSWER**

$\sqrt{41}$.

4MA1/2HR JUNE 2025

# Question 24

6 marks | Linear Graphs

**24** $PQ$ is a straight line drawn on a square grid, with a scale of 1 cm for 1 unit on each axis.

$P$ has coordinates $(-5,\ a)$ and $Q$ has coordinates $(7,\ 3a)$ where $a > 0$

The length of $PQ$ is $4\sqrt{10}$ cm

Find an equation of the perpendicular bisector of $PQ$
Give your answer in the form $y = mx + c$ where $m$ and $c$ are integers.

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2HR June 2025

### 1 Calculate value

$$P = (-5, a), \qquad Q = (7, 3a)$$

### 2 Simplify surd

The length of $PQ$ is $4\sqrt{10}$, so

$$\sqrt{(7-(-5))^2 + (3a-a)^2} = 4\sqrt{10}$$

$$\sqrt{12^2 + (2a)^2} = 4\sqrt{10}$$

### 3 Square both sides

Square both sides:

$$144 + 4a^2 = 160$$

$$4a^2 = 16$$

$$a^2 = 4$$

### 4 Solve inequality

Since $a > 0$,

$$a = 2$$

**5** **Calculate value**

So

$$P = (-5, 2), \qquad Q = (7, 6)$$

**6** **The midpoint of is**

The midpoint of $PQ$ is

$$\left(\frac{-5+7}{2}, \frac{2+6}{2}\right) = (1, 4)$$

**7** **The gradient of is**

The gradient of $PQ$ is

$$\frac{6-2}{7-(-5)} = \frac{4}{12} = \frac{1}{3}$$

**8** **Find the gradient**

So the gradient of the perpendicular bisector is

$$-3$$

**9** **Calculate value**

Using point $(1, 4)$:

$$y - 4 = -3(x - 1)$$

$$y = -3x + 7$$

✓ **FINAL ANSWER**

$$\boxed{y = -3x + 7}$$

4WM1/01 MAY 2025

# Question 24

5 marks | Linear Graphs

**24** $PQRS$ is a square.

$PR$ is a diagonal of the square.

$P$ is the point with coordinates $(4, 7)$
$R$ is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points $Q$ and $S$
Give your answer in the form $ay = bx + c$ where $a$, $b$ and $c$ are integers.

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4WM1/01 May 2025

**1 The diagonals of a square**

The diagonals of a square bisect each other and are perpendicular.

**2 Find the midpoint**

Midpoint of $P(4, 7)$ and $R(8, -5)$:

$$\left(\frac{4+8}{2}, \frac{7-5}{2}\right) = (6, 1)$$

**3 Find the gradient**

Gradient of $PR$:

$$\frac{-5-7}{8-4} = -3$$

**4 Find the gradient**

So the perpendicular gradient is $\frac{1}{3}$.

**5** **The line through and passes**

The line through $Q$ and $S$ passes through $(6, 1)$, so

$$y - 1 = \frac{1}{3}(x - 6)$$

$$3y - 3 = x - 6$$

$$3y = x - 3$$

---

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H JUNE 2025

# Question 25

5 marks | Linear Graphs

**25** $PQRS$ is a square.

$PR$ is a diagonal of the square.

$P$ is the point with coordinates $(4, 7)$
$R$ is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points $Q$ and $S$
Give your answer in the form $ay = bx + c$ where $a$, $b$ and $c$ are integers.

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1H June 2025

**1 Calculate value**

$$P = (4, 7), \qquad R = (8, -5)$$

**2 The gradient of diagonal is**

The gradient of diagonal $PR$ is

$$\frac{-5 - 7}{8 - 4} = -3$$

**3 The other diagonal is perpendicular**

The other diagonal $QS$ is perpendicular to $PR$, so its gradient is

$$\frac{1}{3}$$

**4 The diagonals of a square**

The diagonals of a square bisect each other.

**5 The midpoint of is**

The midpoint of $PR$ is

$$\left(\frac{4 + 8}{2}, \frac{7 + (-5)}{2}\right) = (6, 1)$$

**6** **Find the gradient**

So $QS$ passes through $(6, 1)$ with gradient $\frac{1}{3}$.

$$y - 1 = \frac{1}{3}(x - 6)$$

$$y = \frac{1}{3}x - 1$$

---

**7** **Multiply by 3**

Multiply by 3:

$$3y = x - 3$$

---

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H NOVEMBER 2025

# Question 20

3 marks | Linear Graphs

**20** The straight line **L** has equation $y = 4x + 7$

The straight line **M** is perpendicular to **L** and passes through the point with coordinates (8, 1)

Find an equation for **M**
Give your answer in the form $y = mx + c$

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H November 2025

**1 Line has equation**

Line $L$ has equation

$$y = 4x + 7$$

**2 Find the gradient**

So its gradient is

$$4$$

**3 Find the gradient**

Line $M$ is perpendicular to $L$, so its gradient is

$$-\frac{1}{4}$$

**4 Evaluate fraction**

Line $M$ passes through $(8, 1)$.

$$y - 1 = -\frac{1}{4}(x - 8)$$

$$y - 1 = -\frac{1}{4}x + 2$$

$$y = -\frac{1}{4}x + 3$$

**FINAL ANSWER**

$$y = -\frac{1}{4}x + 3$$

4WM1/01 NOVEMBER 2025

# Question 27

5 marks | Linear Graphs

27 $ABCD$ is a rhombus with $AB = BC = CD = DA$

$A$ has coordinates $(16, -9)$

$C$ has coordinates $(24, 15)$

Find an equation of the line $BD$
Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.
Show your working clearly.

**(Total for Question 27 is 5 marks)**

## WORKED SOLUTION

Question 27 · 4WM1/01 November 2025

**1 Diagonals of a rhombus bisect**

Diagonals of a rhombus bisect each other at right angles.

**2 Find the midpoint**

Midpoint of $AC$:

$$\left(\frac{16+24}{2}, \frac{-9+15}{2}\right) = (20, 3)$$

**3 Find the gradient**

Gradient of $AC$:

$$\frac{15-(-9)}{24-16} = 3$$

**4 Find the gradient**

So gradient of $BD$ is $-\frac{1}{3}$.

$$y - 3 = -\frac{1}{3}(x - 20)$$

$$x + 3y - 29 = 0$$

**FINAL ANSWER**

$x + 3y - 29 = 0$.

4MA1/1H NOVEMBER 2025

# Question 28

6 marks | Linear Graphs

**28** The curve **C** has equation $x^2 + y^2 = d - 11x$ where $d$ is an integer.
The line **L** has equation $y = 3x + e$ where $e$ is an integer.

**C** and **L** intersect at the point $A$ and at the point $B$

The coordinates of $A$ are $(0.2, 2.6)$

Work out the coordinates of $B$
Show your working clearly.

(..... , .....)

**(Total for Question 28 is 6 marks)**

## WORKED SOLUTION

Question 28 · 4MA1/1H November 2025

**1 The line is**

The line is

$$y = 3x + e$$

**2 Point lies on the line**

Point $A = (0.2, 2.6)$ lies on the line.

$$2.6 = 3(0.2) + e$$

$$2.6 = 0.6 + e$$

$$e = 2$$

**3 Calculate value**

So

$$y = 3x + 2$$

**4** **The curve is**

The curve is

$$x^2 + y^2 = d - 11x$$

**5** **Use point**

Use point $A = (0.2, 2.6)$:

$$0.2^2 + 2.6^2 = d - 11(0.2)$$

$$0.04 + 6.76 = d - 2.2$$

$$d = 9$$

**6** **Substitute into the curve**

Now substitute $y = 3x + 2$ into the curve:

$$x^2 + (3x + 2)^2 = 9 - 11x$$

$$x^2 + 9x^2 + 12x + 4 = 9 - 11x$$

$$10x^2 + 23x - 5 = 0$$

**7** **One root is**

One root is $0.2 = \frac{1}{5}$.

**8** **The product of the roots**

The product of the roots is

$$\frac{-5}{10} = -\frac{1}{2}$$

**9** **Evaluate fraction**

So the other root is

$$\frac{-\frac{1}{2}}{\frac{1}{5}} = -\frac{5}{2}$$

**10** **Evaluate fraction**

Then

$$y = 3\left(-\frac{5}{2}\right) + 2$$

$$y = -\frac{11}{2}$$

---

✓ **FINAL ANSWER**

$$B = \left(-\frac{5}{2}, -\frac{11}{2}\right)$$



TOPIC

# Graphs of Functions

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

# Question 20

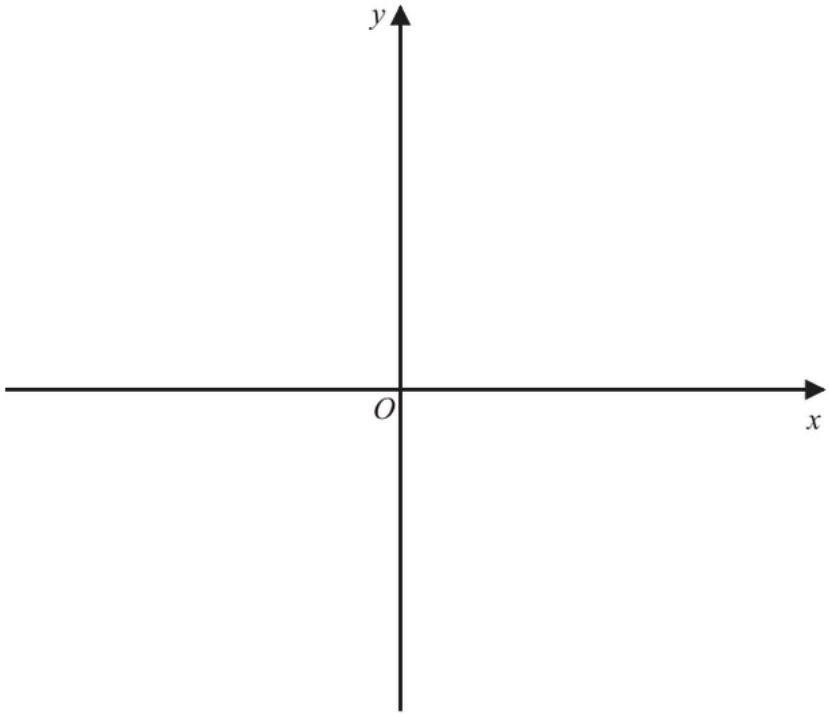
4 marks | Graphs of Functions

**20** The curve **C** has equation $y = 4(x - 1)^2 - a$ where $a > 4$

Using the axes below, sketch the curve **C**.
On your sketch show clearly, in terms of $a$,

(i) the coordinates of any points of intersection of **C** with the coordinate axes,

(ii) the coordinates of the turning point.




**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H January 2020

**1** **Calculate value**

$$y = 4(x - 1)^2 - a$$

**2** **The turning point is read**

The turning point is read directly from the completed square form:

$$(1, -a)$$

**3** **Find intercepts**

For the $y$-intercept, put $x = 0$:

$$y = 4(0-1)^2 - a$$

$$y = 4 - a$$

**4** **Find intercepts**

So the $y$-intercept is

$$(0, 4-a)$$

**5** **Find intercepts**

For the $x$-intercepts, put $y = 0$:

$$4(x-1)^2 - a = 0$$

$$4(x-1)^2 = a$$

$$(x-1)^2 = \frac{a}{4}$$

$$x - 1 = \pm\frac{\sqrt{a}}{2}$$

$$x = 1 \pm \frac{\sqrt{a}}{2}$$

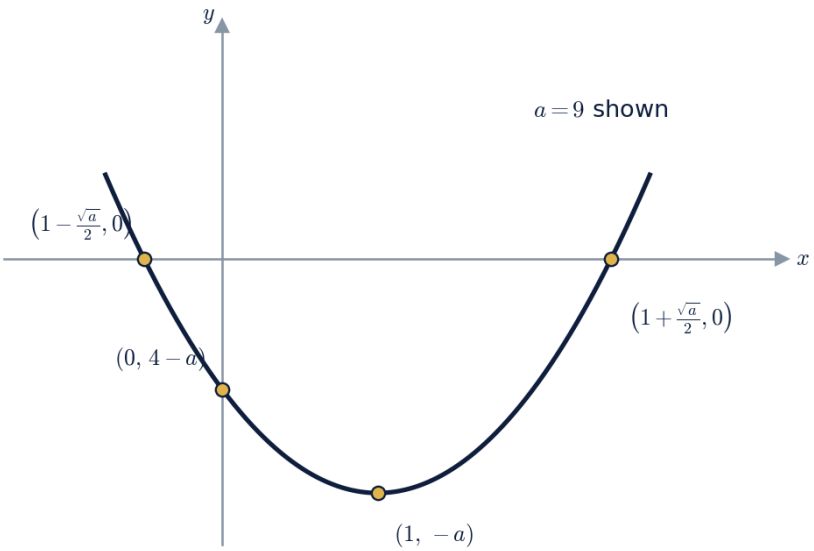
**6** **Find intercepts**

So the $x$-intercepts are

$$\left(1 - \frac{\sqrt{a}}{2}, 0\right) \quad \text{and} \quad \left(1 + \frac{\sqrt{a}}{2}, 0\right)$$

## 7 Solve inequality

Since $a > 4$, the turning point is below the $x$-axis and the curve crosses the $x$-axis twice.




**FINAL ANSWER**

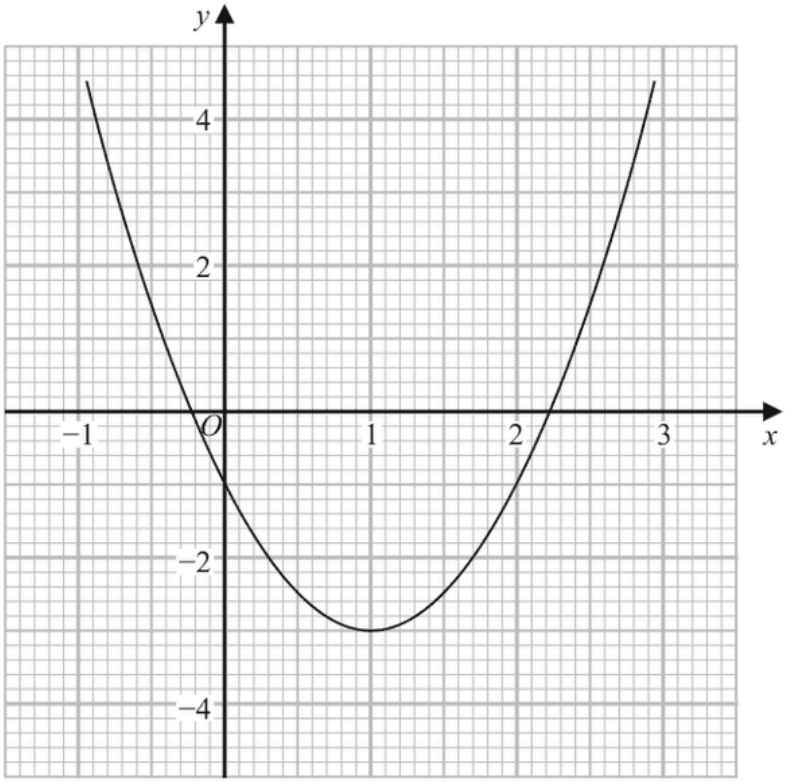
turning point $(1, -a)$, $y$-intercept $(0, 4-a)$, $x$-intercepts $\left(1-\frac{\sqrt{a}}{2}, 0\right)$ and $\left(1+\frac{\sqrt{a}}{2}, 0\right)$

4MA1/2H JANUARY 2022

# Question 21

5 marks | Graphs of Functions

**21** Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.




(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

.....

**(2)**

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

.....

**(3)**

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H January 2022

**1** **Read the graph**

For part (a), solve

$$2x^2 - 4x - 1 = 0$$

**2** **Read the graph**

Using the graph gives approximately

$$x = -0.2, \quad x = 2.2$$

**3** **Read the graph**

For part (b), we need to solve

$$x^2 - x - 1 = 0$$

**4** **Multiply by 2**

Multiply by 2:

$$2x^2 - 2x - 2 = 0$$

**5** **The given graph is**

The given graph is

$$y = 2x^2 - 4x - 1$$

**6** **Read the graph**

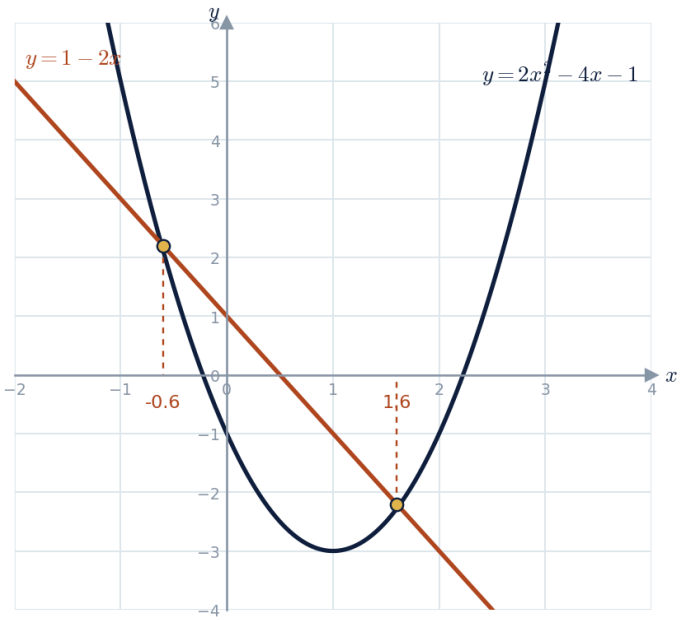
Now

$$2x^2 - 2x - 2 = (2x^2 - 4x - 1) + 2x - 1$$

**7** **Read the graph**

So draw the straight line

$$y = 1 - 2x$$



**8** **The intersections of this line**

The intersections of this line with the curve give approximately

$$x = -0.6, \quad x = 1.6$$

---

✓ **FINAL ANSWER**

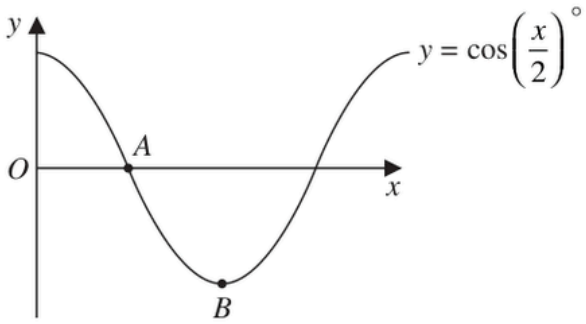
(a) $-0.2,\ 2.2$; (b) draw $y = 1 - 2x$, giving $-0.6,\ 1.6$

4MA1/2H JUNE 2022

# Question 23

2 marks | Graphs of Functions

**23** The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$




(i) Find the coordinates of the point $A$

(.....,.....)

(1)

(ii) Find the coordinates of the point $B$

(.....,.....)

(1)

**(Total for Question 23 is 2 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H June 2022

**1** **The graph is**

The graph is

$$y = \cos\left(\frac{x}{2}\right)^\circ$$

**2** **Point is the first positive**

Point $A$ is the first positive $x$-intercept, so $y = 0$.

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180$$

**3** **Read the graph**

So

$$A = (180, 0)$$

---

**4** **Point is the minimum point**

Point $B$ is the minimum point.

---

**5** **The minimum value of cosine**

The minimum value of cosine is $-1$, which occurs when

$$\frac{x}{2} = 180°$$

$$x = 360$$

---

**6** **Read the graph**

So

$$B = (360, -1)$$

---

✓ **FINAL ANSWER**

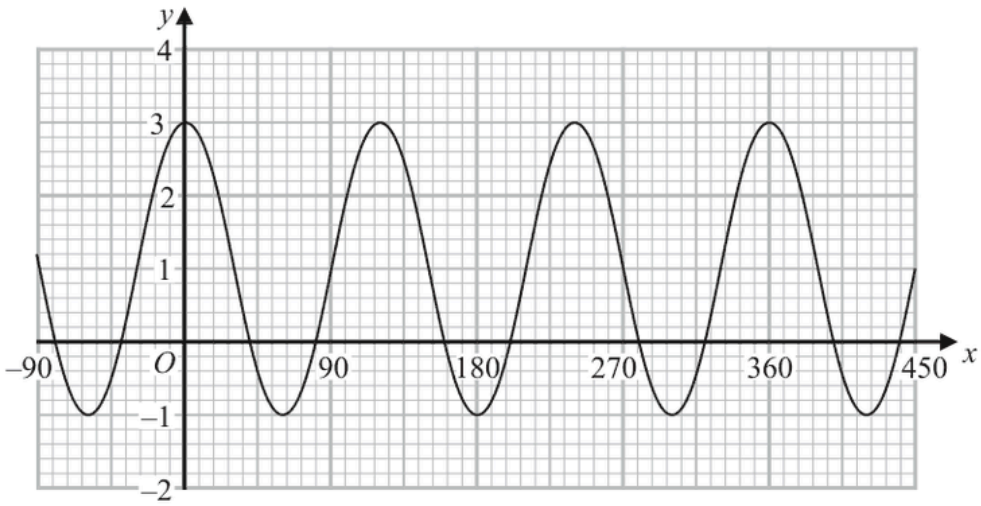
$A = (180, 0),\ B = (360, -1)$

4MA1/2HR JUNE 2023

# Question 26

3 marks | Graphs of Functions

**26** Here is a sketch of the curve with equation $y = a\cos bx° + c$ where $-90 \leqslant x \leqslant 450$




Find the value of $a$, the value of $b$ and the value of $c$

$a =$ .....

$b =$ .....

$c =$ .....

**(Total for Question 26 is 3 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2HR June 2023

**1 The curve is**

The curve is

$$y = a\cos bx° + c$$

**2 Use trigonometry**

From the graph, the maximum value is

$$3$$

**3 Use trigonometry**

and the minimum value is

$$-1$$

**4** **The amplitude is half the**

The amplitude is half the difference:

$$a = \frac{3 - (-1)}{2} = 2$$

**5** **The midline is the average**

The midline is the average:

$$c = \frac{3 + (-1)}{2} = 1$$

**6** **The distance between consecutive maxima**

The distance between consecutive maxima is $120°$, so the period is

$$120°$$

**7** **Use trigonometry**

For $y = a\cos bx° + c$, the period is

$$\frac{360°}{b}$$

**8** **Use trigonometry**

So

$$\frac{360}{b} = 120$$

$$b = 3$$

✓ **FINAL ANSWER**

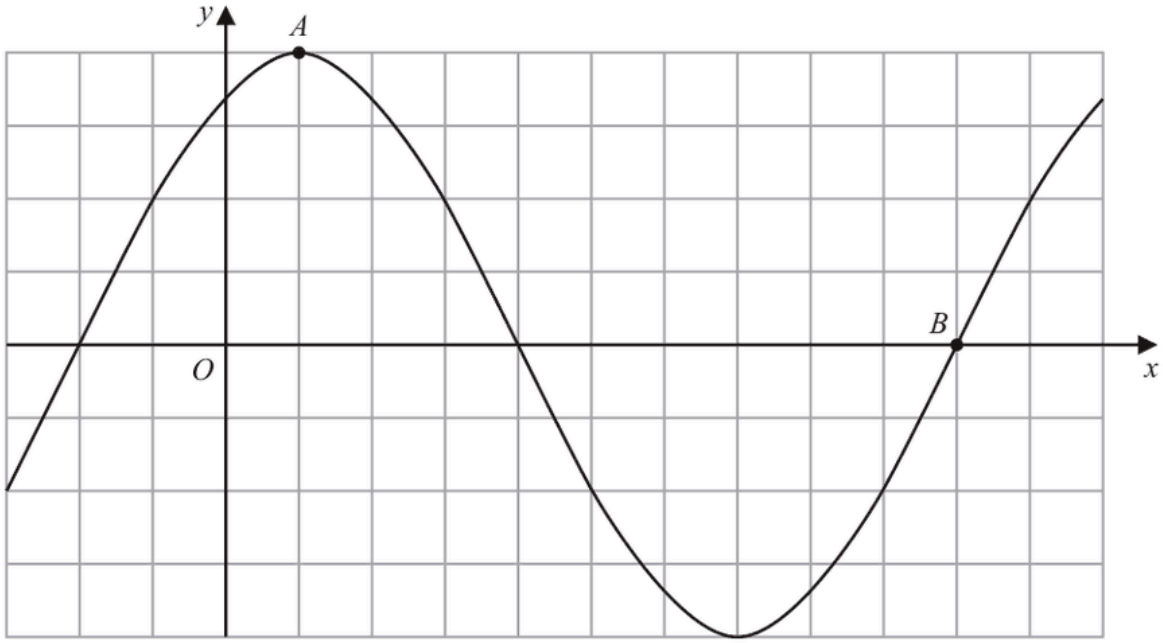
$$\boxed{a = 2,\ b = 3,\ c = 1}$$

4MA1/2H JUNE 2024

# Question 25

2 marks | Graphs of Functions

**25** The diagram shows a sketch of the graph of $y = 2\sin(x + 60)°$




(i) Find the coordinates of the point $A$

(.....  , .....)

(1)

(ii) Find the coordinates of the point $B$

(.....  , .....)

(1)

**(Total for Question 25 is 2 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H June 2024

**1** **Use trigonometry**

$$y = 2\sin(x + 60)°$$

**2** **Point is the maximum point**

Point $A$ is the maximum point.

**3** **The maximum value is**

The maximum value is

$$y = 2$$

**4** **This happens when**

This happens when

$$x + 60 = 90$$

$$x = 30$$

**5** **Read the graph**

So

$$A = (30, 2)$$

**6** **Point is the positive intercept**

Point $B$ is the positive $x$-intercept after the minimum.

$$2\sin(x + 60)° = 0$$

$$\sin(x + 60)° = 0$$

**7** **Read the graph**

For this point,

$$x + 60 = 360$$

$$x = 300$$

**8** **Read the graph**

So

$$B = (300, 0)$$

✓ **FINAL ANSWER**

$A = (30, 2),\ B = (300, 0)$

4MA1/1H NOVEMBER 2024

# Question 23

5 marks | Graphs of Functions

**23** The curve **C** has equation $y = x^2 - 8x - 9$
The straight line **L** has equation $y = k$ where $k$ is an integer.

**C** and **L** intersect at the points $A$ and $B$

The coordinates of point $A$ are $(p, k)$
The coordinates of point $B$ are $(q, k)$

Given that $p - q = 14$

find the value of $k$
Show clear algebraic working.

$k =$ .....

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H November 2024

**1 The curve is**

The curve is

$$y = x^2 - 8x - 9$$

**2 The line is**

The line is

$$y = k$$

**3 Read the graph**

At the intersection points,

$$x^2 - 8x - 9 = k$$

$$x^2 - 8x - (9 + k) = 0$$

**4** **Read the graph**

The two roots are $p$ and $q$.

**5** **Solve quadratic equation**

For this quadratic,

$$p + q = 8$$

**6** **We are also given**

We are also given

$$p - q = 14$$

**7** **Solve the two equations**

Solve the two equations:

$$2p = 22$$

$$p = 11$$

$$q = -3$$

**8** **Read the graph**

Use one root in $y = x^2 - 8x - 9$:

$$k = 11^2 - 8(11) - 9$$

$$k = 121 - 88 - 9$$

$$k = 24$$

✓ **FINAL ANSWER**

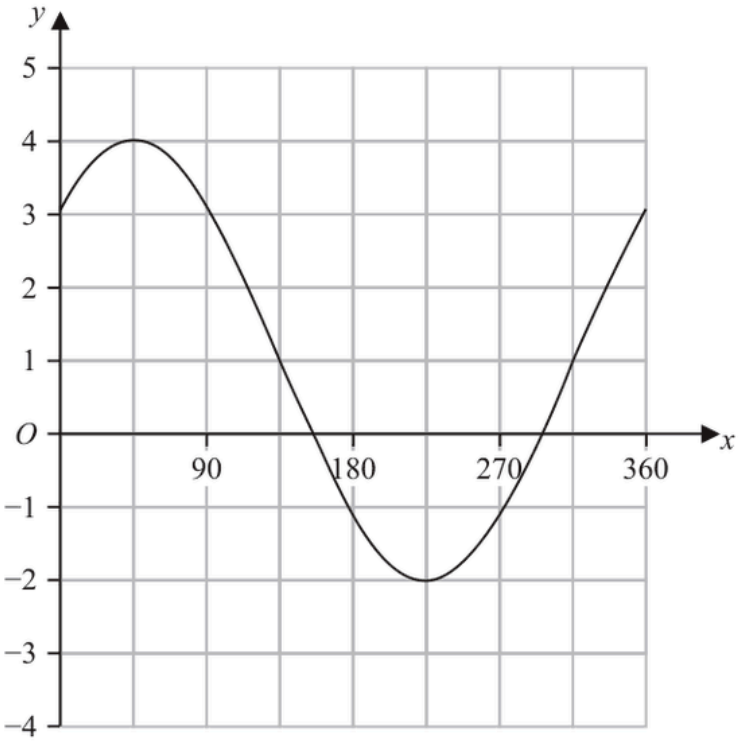
$k = 24$

4WM2/01R MAY 2025

# Question 23

3 marks | Graphs of Functions

**23** The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.




Find a suitable value for $a$, for $b$ and for $c$

$a =$ .....

$b =$ .....

$c =$ .....

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4WM2/01R May 2025

**1** **Required**

The graph of *y* = *a*sin(*x*+*b*)<sup>∘</sup> + *c* for 0 ≤ *x* ≤ 360 is shown. Find suitable values for *a*, *b*, and *c*.

**2** **Read the maximum and minimum**

Read the maximum and minimum values from the graph to find the **amplitude** *a* and **vertical shift** *c*:

$$a = \frac{\text{max} - \text{min}}{2} = \frac{4 - (-2)}{2} = 3 \quad c = \frac{\text{max} + \text{min}}{2} = \frac{4 + (-2)}{2} = 1$$

**3** **Find the phase shift b**

Find the **phase shift** *b*. The standard sin  has its peak at *x* = 90°. From the graph, the maximum occurs at *x* ≈ 45°:

*x* + *b* = 90° ⟹ 45° + *b* = 90° ⟹ *b* = 45°

---

**4** **Verify at x 0 consistent**

Verify at *x* = 0: $y = 4sin(0 + 45)^\circ = 4times fracsqrt22approx2.83$ — consistent with graph

---

**5** **Note**

The period of the graph is still 360° (unchanged), so no horizontal stretch is applied.

---

✓ **FINAL ANSWER**

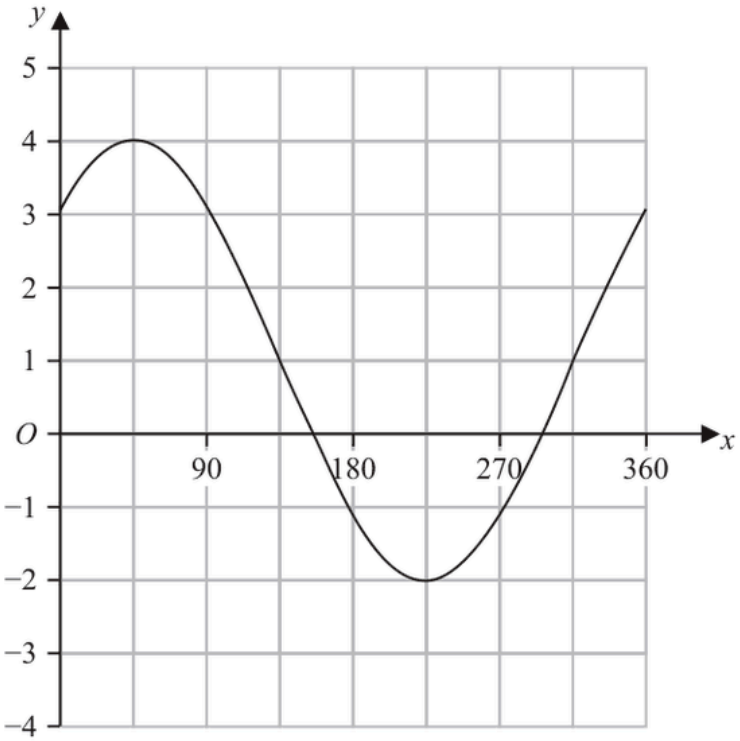
*a* = 3,*b* = 45,*c* = 1

4MA1/2HR JUNE 2025

# Question 25

3 marks | Graphs of Functions

**25** The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.




Find a suitable value for $a$, for $b$ and for $c$

$a =$ .....

$b =$ .....

$c =$ .....

**(Total for Question 25 is 3 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2HR June 2025

**1** **Read the graph**

From the graph,

$$\text{maximum} = 4$$

$$\text{minimum} = -2$$

**2** **The amplitude is**

The amplitude is

$$a = \frac{4 - (-2)}{2} = 3$$

**3** **The centre line is**

The centre line is

$$c = \frac{4 + (-2)}{2} = 1$$

**4** **Use trigonometry**

The maximum is about $45°$. For a sine graph, a maximum happens when the angle inside the sine is $90°$, so

$$45 + b = 90$$

$$b = 45$$

✓ **FINAL ANSWER**

$a = 3,\ b = 45,\ c = 1$

# Question 25

3 marks | Graphs of Functions

**25** A curve has equation $y = f(x)$

There is one minimum point on this curve.

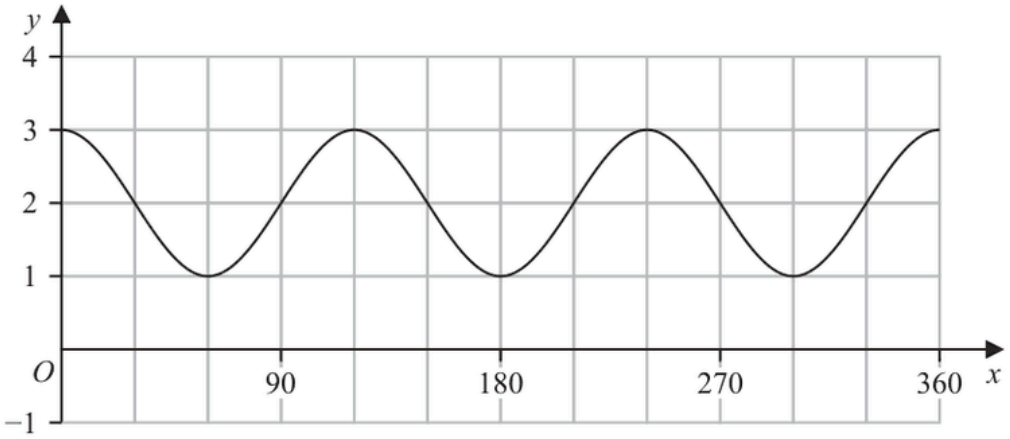
The coordinates of this minimum point are $(8, 1)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(4x)$

(.....  , .....)

**(1)**

The diagram shows the graph of $y = \cos(ax) + b$ for $0 \leqslant x \leqslant 360$




(b) (i) Find the value of $a$

$a =$ .....

**(1)**

(ii) Find the value of $b$

$b =$ .....

**(1)**

**(Total for Question 25 is 3 marks)**

## 1 The transformation y f (4

The transformation *y* = *f*(4*x*) represents a horizontal stretch by a scale factor of $frac14$. The x-coordinate is multiplied by $frac14$ while the y-coordinate stays the same.

$$textNewminimum = leftbigl(8timesfrac14, 1rightbigr) = (2, 1)$$

## 2 Answer

(2,1)

**Step 1: Find the vertical shift *b*.** The standard *y* = cos (*x*) graph fluctuates between − 1 and 1, centered at *y* = 0. The given graph fluctuates between a minimum of *y* = 1 and a maximum of *y* = 3. The center line is $frac3 + 12 = 2$. Therefore, *b* = 2.

**Step 1: Find the frequency multiplier *a*.** The period of the given graph completes one full cycle between *x* = 0 and *x* = 90<sup>∘</sup> (it starts at 3, dips to 1, and returns to 3 at 90<sup>∘</sup>). The standard cosine period is 360<sup>∘</sup>.

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**FINAL ANSWER**

*a* = 4, *b* = 2

TOPIC

# Functions

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

# Question 21

5 marks | Functions

**21** The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H January 2020

**1** **Calculate value**

$$f(x) = x^2 - 2x$$

$$g(x) = x + 3$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = (x+3)^2 - 2(x+3)$$

$$h(x) = x^2 + 4x + 3$$

$$h(x) = (x+2)^2 - 1$$

**2** **Let**

Let

$$y = (x+2)^2 - 1$$

### 3 Find inverse function

Since the domain is $x \geq -2$, take the positive square root:

$$x + 2 = \sqrt{y + 1}$$

$$x = \sqrt{y + 1} - 2$$

---

**FINAL ANSWER**

$$h^{-1}(x) = \sqrt{x + 1} - 2$$

4MA1/1H JANUARY 2021

# Question 22

3 marks | Functions

**22** The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function $f^{-1}$ in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

**(Total for Question 22 is 3 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H January 2021

**1 Calculate value**

$$f(x) = x^2 - 8x + 5$$

$$f(x) = (x-4)^2 - 11$$

**2 Let**

Let

$$y = (x-4)^2 - 11$$

$$y + 11 = (x-4)^2$$

**3 Find inverse function**

Since the domain is $x < 4$, take the negative square root:

$$x - 4 = -\sqrt{y+11}$$

$$x = 4 - \sqrt{y+11}$$

**FINAL ANSWER**

$$f^{-1}(x) = 4 - \sqrt{x+11}$$

4MA1/1H MAY 2021

# Question 24

6 marks | Functions

**24** The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

.....

(2)

(b) Express the inverse function $f^{-1}$ in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(4)

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H May 2021

**1** **(a)**

(a)

$$g(2) = 7(2) - 6 = 8$$

$$fg(2) = f(8)$$

$$f(8) = 5(8)^2 - 10(8) + 7$$

$$f(8) = 247$$

**2** **(b)**

(b)

$$f(x) = 5x^2 - 10x + 7$$

$$f(x) = 5(x - 1)^2 + 2$$

**3** **Let**

Let

$$y = 5(x-1)^2 + 2$$

$$(x-1)^2 = \frac{y-2}{5}$$

---

**4** **Find inverse function**

Since the domain is $x \geq 1$, take the positive square root:

$$x = 1 + \sqrt{\frac{y-2}{5}}$$

---

✓ **FINAL ANSWER**

(a) $247$, (b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x-2}{5}}$

4MA1/2HR JANUARY 2022

# Question 23

4 marks | Functions

**23** The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function $h^{-1}$ in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR January 2022

**1** **Calculate value**

$$f(x) = x + 25$$

$$g(x) = x^2 - 12x$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = x^2 - 12x + 25$$

$$h(x) = (x - 6)^2 - 11$$

**2** **Let**

Let

$$y = (x - 6)^2 - 11$$

$$y + 11 = (x - 6)^2$$

### 3 Find inverse function

Since the domain of $h$ is $x < 6$, take the negative square root:

$$x - 6 = -\sqrt{y + 11}$$

$$x = 6 - \sqrt{y + 11}$$

---

**FINAL ANSWER**

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$

4MA1/2H JANUARY 2022

# Question 25

5 marks | Functions

**25** The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain} \quad \{x : x \geqslant 3\}$$

(a) Express the inverse function $g^{-1}$ in the form $g^{-1}: x \mapsto \ldots$

$g^{-1}: x \mapsto$ .....

**(4)**

(b) State the domain of $g^{-1}$

.....

**(1)**

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H January 2022

**1 Calculate value**

$$g(x) = 5 + 6x - x^2$$

**2 Complete the square**

Complete the square:

$$g(x) = 14 - (x - 3)^2$$

**3 Let**

Let

$$y = 14 - (x - 3)^2$$

$$(x - 3)^2 = 14 - y$$

**4 Find inverse function**

The domain of $g$ is $x \geq 3$, so take the positive square root:

$$x - 3 = \sqrt{14 - y}$$

$$x = 3 + \sqrt{14 - y}$$

**5** **Simplify surd**

Therefore

$$g^{-1} : x \mapsto 3 + \sqrt{14 - x}$$

**6** **Calculate statistic**

The range of $g$ is $x \leq 14$, so this is the domain of $g^{-1}$.

**FINAL ANSWER**

(a) $g^{-1} : x \mapsto 3 + \sqrt{14 - x}$, (b) domain $x \leq 14$

4MA1/1H NOVEMBER 2024

# Question 25

4 marks | Functions

**25** The function f is such that $f(x) = 2x^2 - 24x + 7$ where $x \geqslant 6$

Find the inverse function $f^{-1}(x)$

$f^{-1}(x) =$ .....

**(Total for Question 25 is 4 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1H November 2024

**1 Calculate value**

$$f(x) = 2x^2 - 24x + 7$$

**2 Complete the square**

Complete the square:

$$f(x) = 2(x^2 - 12x) + 7$$

$$f(x) = 2[(x - 6)^2 - 36] + 7$$

$$f(x) = 2(x - 6)^2 - 65$$

**3 Let**

Let $y = f(x)$.

$$y = 2(x - 6)^2 - 65$$

$$y + 65 = 2(x - 6)^2$$

$$\frac{y + 65}{2} = (x - 6)^2$$

**4** **Simplify surd**

Since $x \geq 6$, take the positive square root:

$$x - 6 = \sqrt{\frac{y+65}{2}}$$

$$x = 6 + \sqrt{\frac{y+65}{2}}$$

---

**5** **Replace by**

Replace $y$ by $x$.

---

✓ **FINAL ANSWER**

$$f^{-1}(x) = 6 + \sqrt{\frac{x+65}{2}}$$

4WM2/01 MAY 2025

# Question 21

7 marks | Functions

**21** The function g is defined as

$$g(x) = \frac{7x + 20}{2x} \qquad x \neq 0$$

(a) Solve $gg(x) = 6$
Show clear algebraic working.

$x =$ .....

(3)

P81641A
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The function h is defined as

$$h(x) = 5x^2 + 30x - 7 \qquad x \geqslant -3$$

(b) Express the inverse function $h^{-1}$ in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(4)

**(Total for Question 21 is 7 marks)**

## WORKED SOLUTION

Question 21 · 4WM2/01 May 2025

**1** **Evaluate fraction**

(a) If $g(g(x)) = 6$, then

$$\frac{7g(x) + 20}{2g(x)} = 6$$

$$7g(x) + 20 = 12g(x)$$

$$g(x) = 4$$

**2** **Evaluate fraction**

So

$$\frac{7x+20}{2x}=4$$

$$7x+20=8x$$

$$x=20$$

---

**3** **(b)**

(b)

$$h(x)=5x^2+30x-7=5(x+3)^2-52$$

---

**4** **Simplify surd**

Let $y=5(x+3)^2-52$. Since $x\geq -3$, use the positive square root:

$$x+3=\sqrt{\frac{y+52}{5}}$$

$$h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$$

---

✓ **FINAL ANSWER**

$x=20$; $h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$.

4WM2/01 NOVEMBER 2025

# Question 21

7 marks | Functions

**21** The functions f and g are defined as

$$f(x) = 5x - 3$$

$$g(x) = 4x^2 + 16x - 9 \qquad \text{where} \quad x \geqslant -2$$

(a) Find g( –1)

.....

(1)

(b) Find fg($x$)

Give your answer in its simplest form.

fg($x$) = .....

(2)

$g(x) = 4x^2 + 16x - 9$ where $x \geqslant -2$

(c) Express the inverse function $g^{-1}$ in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = .....

(4)

**(Total for Question 21 is 7 marks)**

## WORKED SOLUTION

Question 21 · 4WM2/01 November 2025

**1** **(a) Find g ( 1)**

**(a) Find *g*(−1):**

*g*(−1) = 4(−1)<sup>2</sup> + 16(−1) − 9 = 4 − 16 − 9 = − 21

**2** **Answer**

$a$ − 21

**(b) Find *fg*(*x*):**

*f**g*(*x*) = *f*(*g*(*x*)) = 5(4*x*<sup>2</sup>+16*x*−9) − 3

= 20*x*<sup>2</sup> + 80*x* − 45 − 3 = 20*x*<sup>2</sup> + 80*x* − 48

---

**3** **Answer**

$b$ 20*x*<sup>2</sup> + 80*x* − 48

Let *y* = 4*x*<sup>2</sup> + 16*x* − 9. We must rearrange to make *x* the subject.

**Step 1: Complete the square for the *x* terms.**

*y* = 4(*x*<sup>2</sup>+4*x*) − 9

*y* = 4((*x*+2)<sup>2</sup>−4) − 9

*y* = 4(*x*+2)<sup>2</sup> − 16 − 9

*y* = 4(*x*+2)<sup>2</sup> − 25

**Step 2: Rearrange for *x*.**

*y* + 25 = 4(*x*+2)<sup>2</sup>

$$fracy + 254 = (x + 2)^2$$

$$x + 2 = pmsqrtfracy + 254$$

Since *x* ≥ − 2, *x* + 2 is positive, so we take the positive root.

$$x + 2 = fracsqrty + 252$$

$$x = fracsqrty + 252 - 2$$

**Step 3: Swap *y* for *x* to write the inverse function.**

---

✓ **FINAL ANSWER**

$g^{-1}(x) = fracsqrtx + 252 - 2$ When finding the inverse of a quadratic, always complete the square. The restricted domain (*x* ≥ − 2) tells you whether to take the positive or negative square root!

TOPIC

# Differentiation & Turning Points

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

# Question 23

6 marks | Differentiation & Turning Points

**23** A particle moves along a straight line.
The fixed point $O$ lies on this line.
The displacement of the particle from $O$ at time $t$ seconds, $t \geqslant 0$, is $s$ metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time $T$ seconds the velocity of $P$ is $V$ m/s where $V \geqslant -5$

Find an expression for $T$ in terms of $V$.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where $k$ and $m$ are integers to be found.

$T =$ .....

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR January 2020

**1 Calculate value**

$$s = t^3 + 4t^2 - 5t + 7$$

**2 Evaluate fraction**

Velocity is the derivative of displacement:

$$v = \frac{ds}{dt} = 3t^2 + 8t - 5$$

**3 Calculate value**

At time $T$, the velocity is $V$, so

$$V = 3T^2 + 8T - 5$$

$$3T^2 + 8T - (V + 5) = 0$$

**4** **Use the quadratic formula**

Use the quadratic formula:

$$T = \frac{-8 \pm \sqrt{8^2 - 4(3)(-(V+5))}}{2(3)}$$

$$T = \frac{-8 \pm \sqrt{64 + 12V + 60}}{6}$$

$$T = \frac{-8 \pm \sqrt{124 + 12V}}{6}$$

$$T = \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

---

**5** **Simplify surd**

Since $T \geq 0$, take the positive root:

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

---

✓ **FINAL ANSWER**

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

4MA1/1H MAY/NOVEMBER 2020

# Question 23

4 marks | Differentiation & Turning Points

**23** Curve **C** has equation $y = px^3 - mx$ where $p$ and $m$ are positive integers.

Find the range of values of $x$, in terms of $p$ and $m$, for which the gradient of **C** is negative.

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H May/November 2020

### 1 Differentiate

$$y = px^3 - mx$$

### 2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3px^2 - m$$

### 3 The gradient is negative when

The gradient is negative when

$$3px^2 - m < 0$$

$$3px^2 < m$$

$$x^2 < \frac{m}{3p}$$

### 4 Simplify surd

So

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

**FINAL ANSWER**

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

4MA1/1HR JANUARY 2021

# Question 20

5 marks | Differentiation & Turning Points

**20** A particle $P$ is moving along a straight line.
The fixed point $O$ lies on the line.

At time $t$ seconds ($t \geqslant 0$), the displacement of $P$ from $O$ is $s$ metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of $P$.

..... m/s

**(Total for Question 20 is 5 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR January 2021

**1** **Read the graph**

$$s = t^3 - 9t^2 + 33t - 6$$

**2** **Read the graph**

Velocity is the rate of change of displacement:

$$v = \frac{ds}{dt}$$

$$v = 3t^2 - 18t + 33$$

**3** **Complete the square**

Complete the square:

$$v = 3(t^2 - 6t + 11)$$

$$v = 3[(t-3)^2 + 2]$$

$$v = 3(t-3)^2 + 6$$

**4** **Read the graph**

Since $(t-3)^2 \geq 0$, the minimum value of $v$ is

$$6$$

**5** **Read the graph**

The velocity is always positive, so the minimum speed is also $6$.

---

✓ **FINAL ANSWER**

$6\ \text{m/s}$

4MA1/2H MAY 2021

# Question 21

5 marks | Differentiation & Turning Points

**21** The point $A$ is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where $k$ is a constant.

Given that the coordinates of $A$ are $\left(\frac{2}{3}, a\right)$

find the value of $a$.
Show your working clearly.

$a =$ .....

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H May 2021

### 1 Evaluate fraction

$$y = kx^2 + \frac{16}{x}$$

### 2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 2kx - \frac{16}{x^2}$$

### 3 Find the gradient

At the stationary point, $x = \frac{2}{3}$, so the gradient is 0:

$$2k\left(\frac{2}{3}\right) - \frac{16}{\left(\frac{2}{3}\right)^2} = 0$$

$$\frac{4k}{3} - \frac{16}{\frac{4}{9}} = 0$$

$$\frac{4k}{3} - 36 = 0$$

$$\frac{4k}{3} = 36$$

$$k = 27$$

### 4 Find the -coordinate

Now find $a$, the $y$-coordinate when $x = \frac{2}{3}$:

$$a = 27\left(\frac{2}{3}\right)^2 + \frac{16}{\frac{2}{3}}$$

$$a = 27 \cdot \frac{4}{9} + 24$$

$$a = 12 + 24 = 36$$

**FINAL ANSWER**

$a = 36$

4MA1/1HR JANUARY 2022

# Question 23

6 marks | Differentiation & Turning Points

**23** Two particles, $P$ and $Q$, move along a straight line.
The fixed point $O$ lies on this line.

The displacement of $P$ from $O$ at time $t$ seconds is $s$ metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of $Q$ from $O$ at time $t$ seconds is $x$ metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of $t$ where $t > 1$ for which both particles are moving in the same direction along the straight line.

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR January 2022

**1** **Particle**

Particle $P$:

$$s = t^3 - 4t^2 + 5t$$

$$\frac{ds}{dt} = 3t^2 - 8t + 5$$

$$\frac{ds}{dt} = (3t - 5)(t - 1)$$

**2** **Particle**

Particle $Q$:

$$x = t^2 - 4t + 4$$

$$\frac{dx}{dt} = 2t - 4 = 2(t - 2)$$

**3** **Differentiate**

For $t > 1$:

---

**4** **moves in the negative direction**

- $P$ moves in the negative direction when $1 < t < \frac{5}{3}$.
- $P$ moves in the positive direction when $t > \frac{5}{3}$.
- $Q$ moves in the negative direction when $1 < t < 2$.
- $Q$ moves in the positive direction when $t > 2$.

---

**5** **Differentiate**

So both particles move in the same direction when both velocities have the same sign.

---

✓ **FINAL ANSWER**

$1 < t < \frac{5}{3}$ or $t > 2$

4MA1/1H JANUARY 2022

# Question 24

6 marks | Differentiation & Turning Points

**24** The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where $a$ and $b$ are constants.

The point $A$ with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at $A$ is 16

Find the $y$ coordinate of the point on the curve whose $x$ coordinate is 3
Show clear algebraic working.

$y =$ .....

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H January 2022

### 1 Calculate value

$$y = ax^3 + bx^2 - 12x + 6$$

### 2 Point lies on the curve

Point $A = (2, -6)$ lies on the curve:

$$-6 = 8a + 4b - 24 + 6$$

$$8a + 4b = 12$$

$$2a + b = 3$$

### 3 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3ax^2 + 2bx - 12$$

**4** **The gradient at is 16**

The gradient at $x = 2$ is 16:

$$16 = 12a + 4b - 12$$

$$12a + 4b = 28$$

$$3a + b = 7$$

---

**5** **Solve**

Now solve

$$2a + b = 3$$

$$3a + b = 7$$

---

**6** **Subtracting gives**

Subtracting gives

$$a = 4$$

---

**7** **Calculate value**

Then

$$2(4) + b = 3$$

$$b = -5$$

---

**8** **Calculate value**

For $x = 3$,

$$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$$

$$y = 108 - 45 - 36 + 6$$

$$y = 33$$

---

✓ **FINAL ANSWER**

$y = 33$

4MA1/2H JANUARY 2022

# Question 24

5 marks | Differentiation & Turning Points

**24** A particle $P$ moves along a straight line that passes through the fixed point $O$

The displacement, $x$ metres, of $P$ from $O$ at time $t$ seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of $P$ reverses when $P$ is at the point $A$ on the line.

The acceleration of $P$ at the instant when $P$ is at $A$ is $a\ \text{m/s}^2$

Find the value of $a$

$a =$ .....

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H January 2022

**1 Split the ratio**

$$x = 4t^3 - 27t + 8$$

**2 Velocity is**

Velocity is

$$\frac{dx}{dt} = 12t^2 - 27$$

**3 The direction of motion reverses**

The direction of motion reverses when velocity is zero:

$$12t^2 - 27 = 0$$

$$12t^2 = 27$$

$$t^2 = \frac{9}{4}$$

**4 Split the ratio**

Since $t \geq 0$, $t = \frac{3}{2}$.

**5** **Acceleration is**

Acceleration is

$$\frac{d^2x}{dt^2} = 24t$$

---

**6** **Split the ratio**

At $t = \frac{3}{2}$,

$$a = 24\left(\frac{3}{2}\right) = 36$$

---

✓ **FINAL ANSWER**

$a = 36$

# Question 22

7 marks | Differentiation & Turning Points

**22** The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where $p$ is a positive constant.

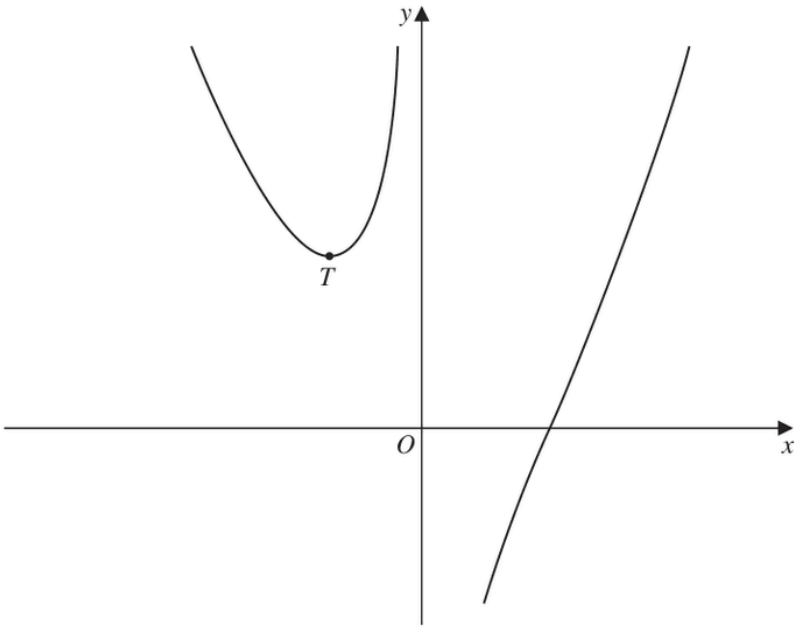



Diagram **NOT** accurately drawn

For all values of $p$, the curve has exactly one turning point and this turning point is a minimum shown as the point $T$ in the sketch.

For the curve where the $x$ coordinate of $T$ is $-3$

(a) find the value of $p$

$p =$ .....

**(4)**

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of $k$

$k =$ .....

**(3)**

**(Total for Question 22 is 7 marks)**

**1** **Find the gradient**

For part (a),

$$y = x^2 - \frac{p}{x}$$

$$\frac{dy}{dx} = 2x + \frac{p}{x^2}$$

**2** **Find the gradient**

At the minimum point, $x = -3$ and the gradient is 0.

$$2(-3) + \frac{p}{(-3)^2} = 0$$

$$-6 + \frac{p}{9} = 0$$

$$p = 54$$

**3** **Find the gradient**

For part (b),

$$y = x^2 - \frac{16}{x}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2}$$

**4** **Find the gradient**

At a horizontal tangent,

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$x^3 = -8$$

$$x = -2$$

**5** **Find the gradient**

Now substitute $x = -2$ into the curve:

$$k = (-2)^2 - \frac{16}{-2}$$

$$k = 4 + 8 = 12$$

---

✓ **FINAL ANSWER**

$p = 54,\ k = 12$

4MA1/1HR MAY 2023

# Question 21

6 marks | Differentiation & Turning Points

**21** The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

**(2)**

(b) Find the range of values of $x$ for which **T** has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

**(4)**

**(Total for Question 21 is 6 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR May 2023

**1** **(a)**

(a)

$$y = x^3 - 2x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 4x - 9$$

**2** **(b) The curve has a**

(b) The curve has a positive gradient when

$$3x^2 - 4x - 9 > 0$$

**3** **Solve**

Solve

$$3x^2 - 4x - 9 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 108}}{6}$$

$$x = \frac{2 \pm \sqrt{31}}{3}$$

---

**4** **Differentiate**

To 3 significant figures, the roots are

$$x = -1.19, \quad x = 2.52$$

---

**5** **Solve quadratic equation**

Since the quadratic opens upwards, it is positive outside the roots.

---

✓ **FINAL ANSWER**

(a) $\frac{dy}{dx} = 3x^2 - 4x - 9$, (b) $x < -1.19$ or $x > 2.52$

4MA1/1H NOVEMBER 2024

# Question 20

4 marks | Differentiation & Turning Points

**20** The curve with equation $y = 2x^4 - 64x$ has a minimum point.

Find an equation of the tangent to the curve at the minimum point.
Show clear algebraic working.

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H November 2024

**1 Find the gradient**

$$y = 2x^4 - 64x$$

**2 Differentiate**

Differentiate:

$$\frac{dy}{dx} = 8x^3 - 64$$

**3 Find the gradient**

At the minimum point, the gradient is 0.

$$8x^3 - 64 = 0$$

$$8x^3 = 64$$

$$x^3 = 8$$

$$x = 2$$

**4 Find the coordinate**

Find the $y$-coordinate:

$$y = 2(2)^4 - 64(2)$$

$$y = 32 - 128 = -96$$

### 5 The tangent at a minimum

The tangent at a minimum point is horizontal, so its equation is

$$y = -96$$

---

**FINAL ANSWER**

$y = -96$

4MA1/2HR JUNE 2025

# Question 22

5 marks | Differentiation & Turning Points

**22** The diagram shows a solid cuboid.

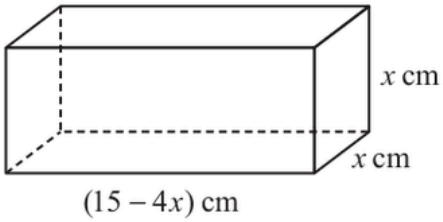



Diagram **NOT** accurately drawn

The volume of the cuboid is $V$ cm$^3$

Find the maximum value of $V$

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2HR June 2025

### 1 The cuboid has dimensions

The cuboid has dimensions

$$x, \quad x, \quad 15 - 4x$$

### 2 Calculate value

So

$$V = x^2(15 - 4x)$$

$$V = 15x^2 - 4x^3$$

### 3 Differentiate

Differentiate:

$$\frac{dV}{dx} = 30x - 12x^2$$

### 4 Calculate value

At a maximum,

$$30x - 12x^2 = 0$$

$$6x(5 - 2x) = 0$$

**5** **The useful value is**

The useful value is

$$x = 2.5$$

---

**6** **Calculate volume**

Now substitute into the volume:

$$V = (2.5)^2(15 - 4(2.5))$$

$$V = 6.25 \times 5$$

$$V = 31.25$$

---

✓ **FINAL ANSWER**

$31.25\ \text{cm}^3$

4WM2/01 MAY 2025

# Question 22

4 marks | Differentiation & Turning Points

22 Work out the range of values of $x$ for which the graph of $y = x^3 + \frac{2}{2}x^2 - 10x + 5$ has a positive gradient.
Show clear algebraic working.

.....

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4WM2/01 May 2025

**1 Evaluate fraction**

$$\frac{dy}{dx} = 3x^2 + 2x - 10$$

**2 Find the gradient**

For a positive gradient,

$$3x^2 + 2x - 10 > 0$$

**3 The roots are**

The roots are

$$x = \frac{-1 \pm \sqrt{31}}{3}$$

**4 Solve quadratic equation**

Since the quadratic opens upwards,

$$x < \frac{-1 - \sqrt{31}}{3} \quad \text{or} \quad x > \frac{-1 + \sqrt{31}}{3}$$

**FINAL ANSWER**

$x < \frac{-1 - \sqrt{31}}{3}$ or $x > \frac{-1 + \sqrt{31}}{3}$.

4WM2/01 NOVEMBER 2025

# Question 22

6 marks | Differentiation & Turning Points

**22** Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of $b$
Show clear algebraic working.

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4WM2/01 November 2025

**1** **Find the gradient**

$$fracdydx = 3x^2 - 16$$

**2** **Find the gradient**

3*x*<sup>2</sup> − 16 = 11

3*x*<sup>2</sup> = 27

*x*<sup>2</sup> = 9 ⟹ *x* = 3 or *x* = − 3

**3** **Solve inequality**

For *x* = 3: *y* = (3)<sup>3</sup> − 16(3) + 7 = 27 − 48 + 7 = − 14

**4** **Solve inequality**

For *x* = − 3: *y* = (−3)<sup>3</sup> − 16(−3) + 7 = − 27 + 48 + 7 = 28

**5** **Use trigonometry**

**Step 4: Find the y-intercept (*b*) for each tangent (*y* = 11*x* + *b*).**

Using (3,−14):

− 14 = 11(3) + *b* ⟹ − 14 = 33 + *b* ⟹ *b* = − 47

Using (−3,28):

28 = 11(−3) + *b* ⟹ 28 = − 33 + *b* ⟹ *b* = 61

**FINAL ANSWER**

*b* = − 47 and 61

4MA1/2H NOVEMBER 2025

# Question 22

6 marks | Differentiation & Turning Points

**22** Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of $b$
Show clear algebraic working.

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H November 2025

**1** **Differentiate**

$$y = x^3 - 16x + 7$$

**2** **Differentiate**

Differentiate:

$$\frac{dy}{dx} = 3x^2 - 16$$

**3** **The gradient of the tangent**

The gradient of the tangent is $11$, so

$$3x^2 - 16 = 11$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = 3 \quad \text{or} \quad x = -3$$

**4** **Differentiate**

When $x = 3$:

$$y = 3^3 - 16(3) + 7 = 27 - 48 + 7 = -14$$

**5** **Use trigonometry**

The tangent has equation $y = 11x + b$, so

$$-14 = 11(3) + b$$

$$b = -47$$

---

**6** **Differentiate**

When $x = -3$:

$$y = (-3)^3 - 16(-3) + 7 = -27 + 48 + 7 = 28$$

$$28 = 11(-3) + b$$

$$b = 61$$

---

✓ **FINAL ANSWER**

$\boxed{b = -47 \text{ or } b = 61}$

# Question 23

5 marks | Differentiation & Turning Points

**23** A particle is moving along a straight line that passes through the fixed point $O$

The displacement, $s$ metres, of the particle from $O$ at time $t$ seconds is given by

$$s = 15t^2 - t^3 + 12$$

Find the maximum velocity of the particle where $0 < t < 11$
Show clear algebraic working.

..... m/s

**(Total for Question 23 is 5 marks)**

**1** **Velocity**

$$v = \frac{ds}{dt} = 30t - 3t^2$$

**2** **Maximum when dv/dt = 0**

$$\frac{dv}{dt} = 30 - 6t = 0 \Rightarrow t = 5 \quad \left(\frac{d^2v}{dt^2} = -6 < 0, \text{ a maximum}\right)$$

**3** **Maximum velocity**

$$v = 30(5) - 3(5)^2 = 75$$

✓ **FINAL ANSWER**

$75$ m/s



TOPIC

# Transformations of Graphs

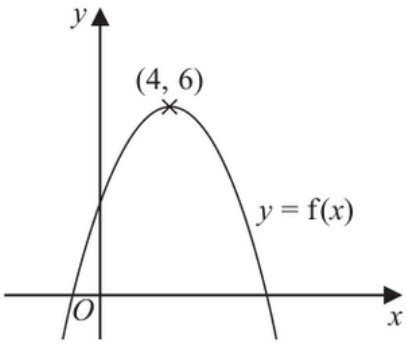
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

# Question 21

4 marks | Transformations of Graphs

**21** The diagram shows a sketch of part of the curve with equation $y = f(x)$




There is one maximum point on this curve.
The coordinates of this maximum point are (4, 6)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 4)$

(.....  , .....)

(ii) $y = f(2x)$

(.....  , .....)

**(2)**

The equation of a curve **C** is $y = x^2 + 3x + 4$

The curve **C** is transformed to curve **S** under the translation $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) Find an equation of curve **S**.

*You do not need to simplify the equation.*

.....

**(2)**

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H January 2020

**1** **The maximum point of is**

The maximum point of $y = f(x)$ is $(4, 6)$.

**2** **Calculate value**

For

$$y = f(x + 4)$$

**3** **the graph moves 4 units**

the graph moves 4 units left:

$$(4, 6) \to (0, 6)$$

**4** **Calculate value**

For

$$y = f(2x)$$

**5** **the coordinate is divided by**

the $x$-coordinate is divided by 2:

$$(4, 6) \to (2, 6)$$

**6** **Calculate value**

For part (b),

$$y = x^2 + 3x + 4$$

**7** **The translation vector is**

The translation vector is

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

**8** **Read the graph**

So the graph moves 4 units right and 6 units up.

**9** **Add 6**

Replace $x$ by $x - 4$, then add 6:

$$y = (x - 4)^2 + 3(x - 4) + 4 + 6$$

$$y = (x - 4)^2 + 3(x - 4) + 10$$

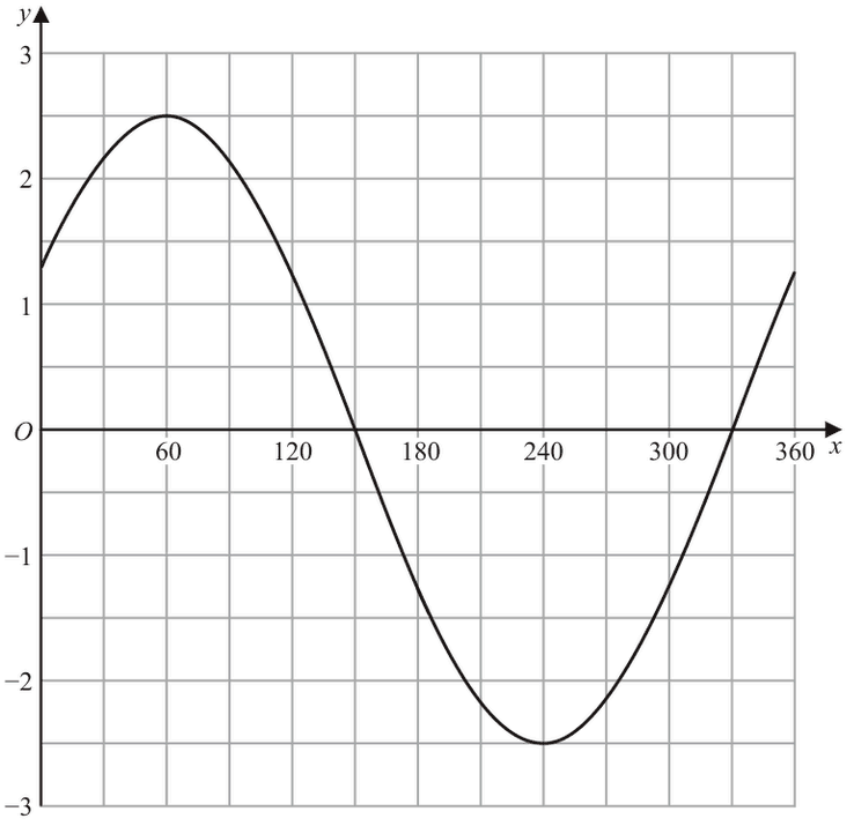
✓ **FINAL ANSWER**

$(0, 6)$, $(2, 6)$, and $y = (x - 4)^2 + 3(x - 4) + 10$

# Question 22

4 marks | Transformations of Graphs

**22** The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.




(a) Find the value of $a$ and the value of $b$.

$a =$ .....

$b =$ .....

**(2)**

Another curve $C$ has equation $y = f(x)$
The coordinates of the minimum point of $C$ are (4, 5)

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

(..... , .....)

(ii) $y = f(x) - 7$

(..... , .....)

**(2)**

**(Total for Question 22 is 4 marks)**

**1 Read the graph**

For part (a), the graph has maximum $2.5$ and minimum $-2.5$, so

$$a = 2.5$$

**2 Use trigonometry**

The maximum occurs at $x = 60°$. For $y = a\cos(x + b)°$, a maximum occurs when

$$x + b = 360$$

**3 Calculate value**

So

$$60 + b = 360$$

$$b = 300$$

**4 Calculate value**

For part (b), the minimum point of $y = f(x)$ is $(4, 5)$.

**5 Calculate value**

For

$$y = f(2x)$$

**6 the coordinate is divided by**

the $x$-coordinate is divided by 2:

$$(4, 5) \to (2, 5)$$

**7 Calculate value**

For

$$y = f(x) - 7$$

**8 the coordinate decreases by 7**

the $y$-coordinate decreases by 7:

$$(4, 5) \to (4, -2)$$

✓ **FINAL ANSWER**

$a = 2.5,\ b = 300;\ (2, 5)$ and $(4, -2)$

4MA1/1HR MAY/NOVEMBER 2020

# Question 20

2 marks | Transformations of Graphs

**20** A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are $(-3, 4)$

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

(.....  , .....)

(ii) $y = f(2x)$

(.....  , .....)

**(Total for Question 20 is 2 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR May/November 2020

**1 The maximum point of is**

The maximum point of $y = f(x)$ is $(-3, 4)$.

**2 Calculate value**

For

$$y = f(x) - 6$$

**3 the graph moves 6 units**

the graph moves 6 units down:

$$(-3, 4) \rightarrow (-3, -2)$$

**4 Calculate value**

For

$$y = f(2x)$$

**5 the coordinate is divided by**

the $x$-coordinate is divided by 2:

$$(-3, 4) \rightarrow \left(-\frac{3}{2}, 4\right)$$

**FINAL ANSWER**

$(-3, -2)$ and $\left(-\frac{3}{2}, 4\right)$

# Question 25

3 marks | Transformations of Graphs

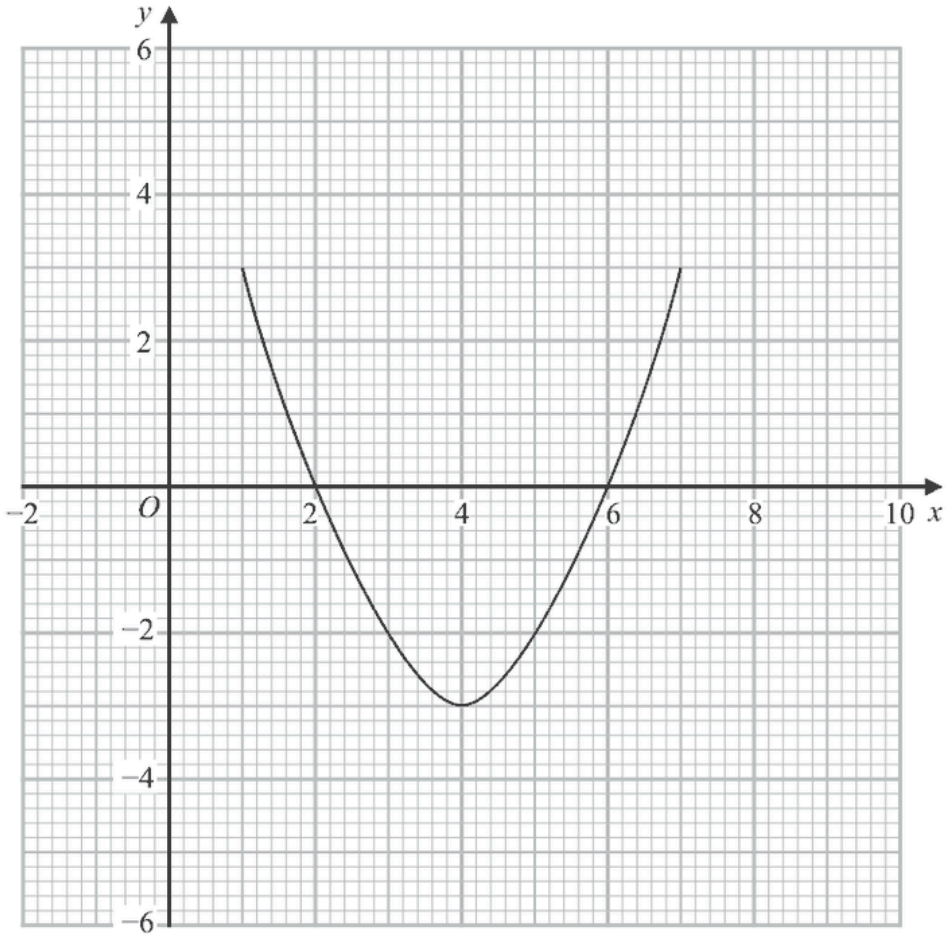
**25** The curve with equation $y = g(x)$ is transformed to the curve with equation $y = -g(x)$ by the single transformation **T**.

(a) Describe fully the transformation **T**.

.....

**(1)**

The diagram shows the graph of $y = f(x)$




(b) On the grid, draw the graph of $y = 2f(x - 1)$

**(2)**

**(Total for Question 25 is 3 marks)**

**1** **Apply transformation**

For part (a),

$$y = g(x) \rightarrow y = -g(x)$$

**2** **The coordinates stay the same**

The $x$-coordinates stay the same and the $y$-coordinates change sign.

**3** **Apply transformation**

So the transformation is a reflection in the $x$-axis.

**4** **Calculate value**

For part (b),

$$y = 2f(x - 1)$$

**5** **Read the graph**

First, $x - 1$ moves the graph 1 unit right. Then the factor 2 stretches the graph vertically by scale factor 2.

### 6 Use points from the original

Use points from the original graph:

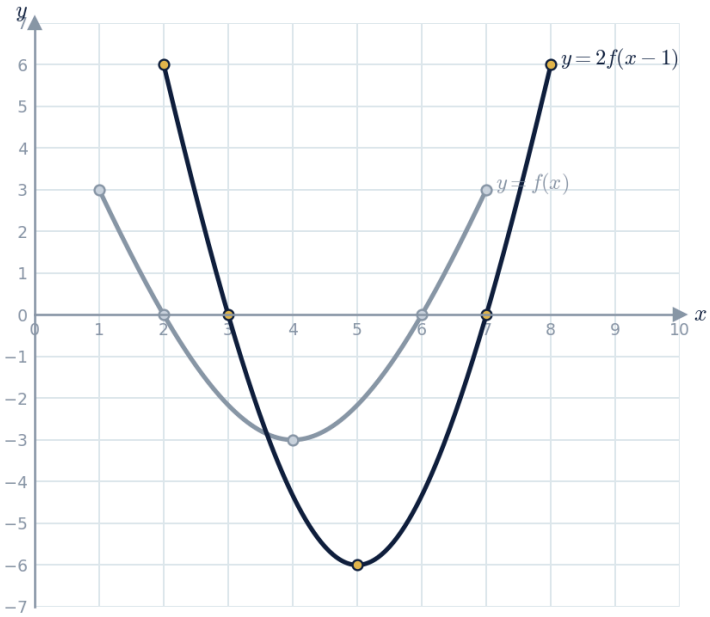
$$(1,3) \to (2,6)$$

$$(2,0) \to (3,0)$$

$$(4,-3) \to (5,-6)$$

$$(6,0) \to (7,0)$$

$$(7,3) \to (8,6)$$




### 7 Apply transformation

Draw the transformed parabola through these points.

**FINAL ANSWER**

reflection in the $x$-axis; transformed graph through $(2,6)$, $(3,0)$, $(5,-6)$, $(7,0)$, $(8,6)$

4MA1/1H JANUARY 2021

# Question 21

4 marks | Transformations of Graphs

**21** A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

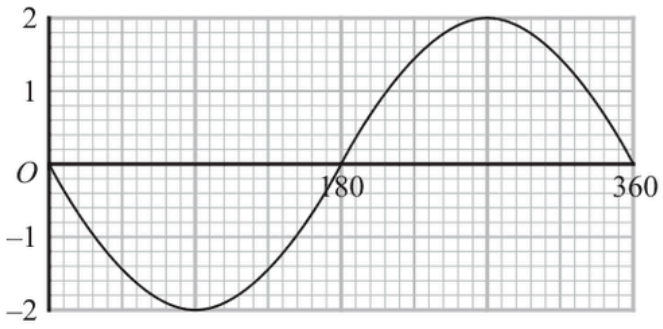
(.....  , .....)

(ii) $y = \frac{1}{3} f(x)$

(.....  , .....)

**(2)**

The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.




Given that $a > 0$ and that $0 < b < 360$

(b) find the value of $a$ and the value of $b$.

$a =$ .....

$b =$ .....

**(2)**

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H January 2021

**1** **The minimum point of is**

The minimum point of $y = f(x)$ is $(-9, 15)$.

**2** **Calculate value**

For

$$y = f(x + 3)$$

**3** **the graph moves 3 units**

the graph moves 3 units left:

$$(-9, 15) \to (-12, 15)$$

**4** **Evaluate fraction**

For

$$y = \frac{1}{3} f(x)$$

**5** **the coordinate is divided by**

the $y$-coordinate is divided by 3:

$$(-9, 15) \to (-9, 5)$$

**6** **Use trigonometry**

For part (b), the cosine graph has maximum $2$ and minimum $-2$, so

$$a = 2$$

**7** **The graph has a minimum**

The graph has a minimum at $x = 90°$. For $a\cos(x + b)°$, a minimum occurs when

$$x + b = 180$$

**8** **Calculate value**

So

$$90 + b = 180$$

$$b = 90$$

✓ **FINAL ANSWER**

$(-12, 15)$, $(-9, 5)$, $a = 2,\ b = 90$

# Question 21

6 marks | Transformations of Graphs

**21** The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point $A$ is the maximum point on **C**.

(a) Write down the coordinates of $A$.

(.....  , .....)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

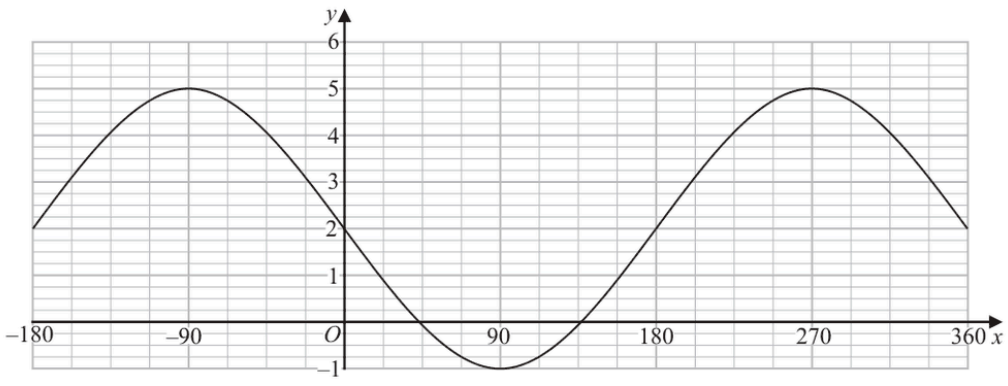
(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)

---

The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.




(d) Find the value of $a$, the value of $b$ and the value of $c$.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

**(Total for Question 21 is 6 marks)**

**1** **Calculate value**

$$f(x) = 9 - 3(x+2)^2$$

**2** **This is in completed square**

This is in completed square form, so the maximum point is

$$(-2, 9)$$

**3** **Use matrices**

For the translation

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

**4** **the graph moves 4 units**

the graph moves 4 units right. Replace $x$ by $x - 4$:

$$y = 9 - 3((x-4)+2)^2$$

$$y = 9 - 3(x-2)^2$$

**5** **The curve has equation**

The curve $T$ has equation

$$y = 3(x+2)^2 - 9$$

**6** **This is**

This is

$$y = -\left(9 - 3(x+2)^2\right)$$

**7** **Apply transformation**

So $C$ is reflected in the $x$-axis.

**8** **Use trigonometry**

For the cosine graph, the maximum is 5 and the minimum is $-1$.

$$a = \frac{5 - (-1)}{2} = 3$$

$$c = \frac{5 + (-1)}{2} = 2$$

**9** **Use trigonometry**

For

$$y = a\cos(x - b)^\circ + c$$

**10** **Read the graph**

a maximum occurs when $x - b = 0$. The graph has a maximum at $x = 270°$, so

$$b = 270$$

✓ **FINAL ANSWER**

$A = (-2, 9)$, $y = 9 - 3(x - 2)^2$, reflection in the $x$-axis, $a = 3,\ b = 270,\ c = 2$

# Question 20

6 marks | Transformations of Graphs

**20** The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

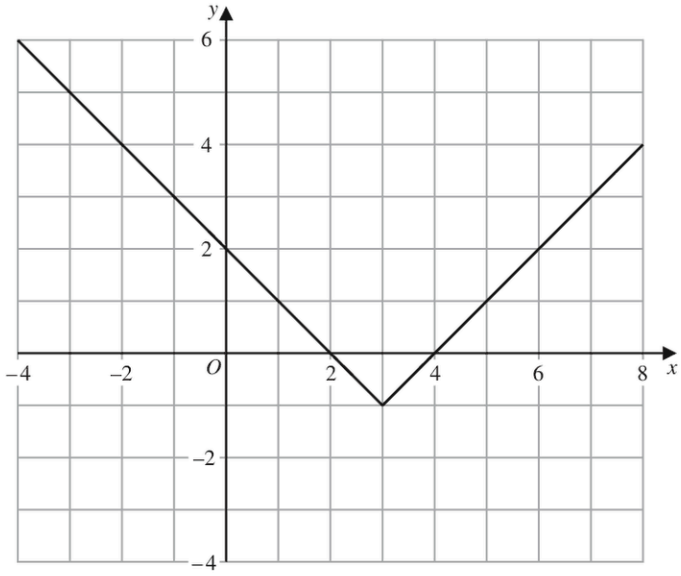
(.....,.....)

(ii) $y = f(3x)$

(.....,.....)

**(2)**

The graph of $y = g(x)$ is shown on the grid below.




(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

**(2)**

The graph of $y = h(x)$ intersects the $x$-axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where $a$ is a constant.

(c) Find the two possible values of $a$

....., .....

**(2)**

**(Total for Question 20 is 6 marks)**

**1** **The turning point of is**

The turning point of $y = f(x)$ is $(-6, -4)$.

**2** **Calculate value**

For

$$y = f(x) + 5$$

**3** **the graph moves 5 units**

the graph moves 5 units up:

$$(-6, -4) \rightarrow (-6, 1)$$

**4** **Calculate value**

For

$$y = f(3x)$$

**5** **the coordinate is divided by**

the $x$-coordinate is divided by 3:

$$(-6, -4) \rightarrow (-2, -4)$$

**6** **Read the graph**

For the sketch of $y = 2g(x)$, keep every $x$-coordinate the same and double every $y$-coordinate.

**7** **Useful points are**

Useful points are

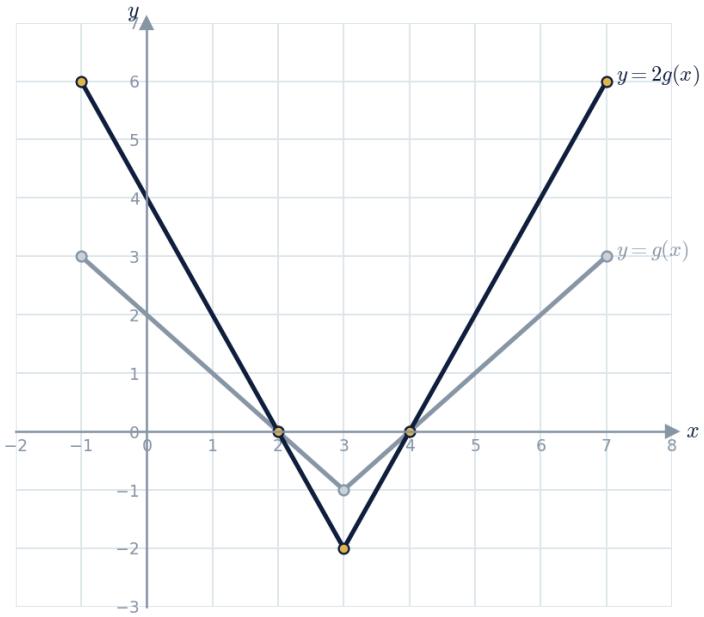
$$(-1,3) \to (-1,6)$$

$$(2,0) \to (2,0)$$

$$(3,-1) \to (3,-2)$$

$$(4,0) \to (4,0)$$

$$(7,3) \to (7,6)$$




**8** **Solve inequality**

Draw straight line segments through these points for $-1 \leq x \leq 7$.

**9** **Calculate value**

For part (c), $h(x)$ has roots at $x = -1$ and $x = 6$.

**10** **Calculate value**

For $h(x + a)$ to pass through $(2,0)$,

$$2 + a = -1 \quad \text{or} \quad 2 + a = 6$$

$$a = -3 \quad \text{or} \quad a = 4$$

**FINAL ANSWER**

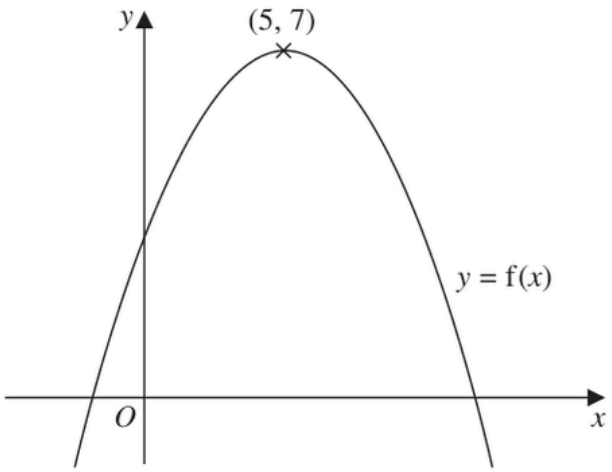
$(-6, 1)$, $(-2, -4)$, sketch through the doubled points, and $a = -3$ or $a = 4$

4MA1/1H JANUARY 2022

# Question 23

2 marks | Transformations of Graphs

**23** The diagram shows a sketch of the curve with equation $y = f(x)$




There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 9)$

(.....  , .....)

(ii) $y = f(x) + 3$

(.....  , .....)

**(Total for Question 23 is 2 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H January 2022

**1** **The maximum point of is**

The maximum point of $y = f(x)$ is $(5, 7)$.

**2** **Calculate value**

For

$$y = f(x + 9)$$

**3** **the graph moves 9 units**

the graph moves 9 units left:

$$(5, 7) \rightarrow (-4, 7)$$

**4** **Calculate value**

For

$$y = f(x) + 3$$

**5** **the graph moves 3 units**

the graph moves 3 units up:

$$(5, 7) \to (5, 10)$$

✓ **FINAL ANSWER**

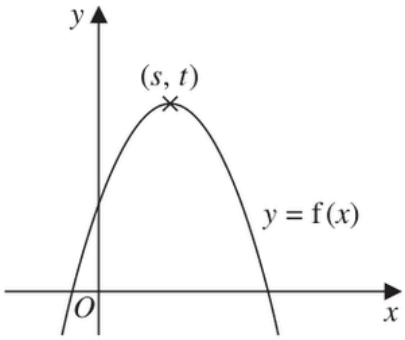
$(-4, 7)$ and $(5, 10)$

4MA1/1H JUNE 2022

# Question 20

2 marks | Transformations of Graphs

**20** The diagram shows a sketch of part of the curve with equation $y = f(x)$




There is one maximum point on this curve.
The coordinates of this maximum point are $(s, t)$

Find, in terms of $s$ and $t$, the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

(.....  , .....)
(1)

(ii) $y = 3f(x)$

(.....  , .....)
(1)

**(Total for Question 20 is 2 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H June 2022

**1 The maximum point of is**

The maximum point of $y = f(x)$ is $(s, t)$.

**2 Calculate value**

For

$$y = f(x - 2)$$

**3 the graph moves 2 units**

the graph moves 2 units right:

$$(s, t) \to (s + 2, t)$$

**4** **Calculate value**

For

$$y = 3f(x)$$

**5** **the coordinate is multiplied by**

the $y$-coordinate is multiplied by 3:

$$(s, t) \to (s, 3t)$$

✓ **FINAL ANSWER**

$(s + 2, t)$ and $(s, 3t)$

# Question 22

4 marks | Transformations of Graphs

**22** The point $A$ with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point $A$ on the straight line with equation

(i) $y = f(x) - 3$

(.....  , .....)

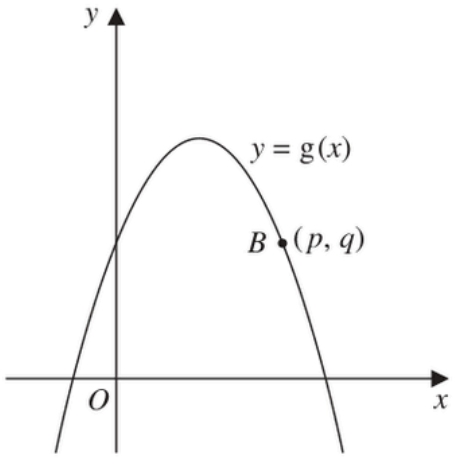
**(1)**

(ii) $y = f\left(\frac{x}{2}\right)$

(.....  , .....)

**(1)**

Here is a sketch of part of the curve with equation $y = g(x)$




The point $B$ with coordinates $(p, q)$ lies on the curve.

(b) Find the coordinates of the image of the point $B$ on the curve with equation

$$y = -g(x - c)$$

where $c$ is a constant.

(.....  , .....)

**(2)**

**(Total for Question 22 is 4 marks)**

**1** **Point lies on**

Point $A = (-3, 2)$ lies on $y = f(x)$.

**2** **Calculate value**

For

$$y = f(x) - 3$$

**3** **the graph moves 3 units**

the graph moves 3 units down:

$$(-3, 2) \to (-3, -1)$$

**4** **Evaluate fraction**

For

$$y = f\left(\frac{x}{2}\right)$$

**5** **the coordinate is multiplied by**

the $x$-coordinate is multiplied by 2:

$$(-3, 2) \to (-6, 2)$$

**6** **Calculate value**

For part (b), point $B = (p, q)$ lies on $y = g(x)$.

**7** **Calculate value**

For

$$y = -g(x - c)$$

**8** **Read the graph**

$x - c$ moves the graph $c$ units right, and the minus sign reflects the graph in the $x$-axis.

**9** **Apply transformation**

So

$$(p, q) \to (p + c, -q)$$

✓ **FINAL ANSWER**

$(-3, -1)$, $(-6, 2)$, and $(p + c, -q)$

4MA1/1HR MAY 2023

# Question 20

7 marks | Transformations of Graphs

**20** Curve **C** has equation $y = f(x)$
The graph of curve **C** has one maximum point.
The coordinates of this maximum point are (3, 5)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = 2f(x)$

(.....  , .....)
(1)

(ii) $y = f(x) - 7$

(.....  , .....)
(1)

(iii) $y = f(-x)$

(.....  , .....)
(1)

Curve **L** has equation $y = x^2 + 7x + 20$

Curve **L** is transformed to curve **S** under the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(b) Find an equation for **S**
Give your answer in the form $y = ax^2 + bx + c$

$y =$ .....
(4)

**(Total for Question 20 is 7 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR May 2023

**1** **The maximum point of is**

The maximum point of $y = f(x)$ is $(3, 5)$.

**2** **Calculate value**

For

$$y = 2f(x)$$

3 **the coordinate is multiplied by**

the $y$-coordinate is multiplied by 2:

$$(3, 5) \to (3, 10)$$

4 **Calculate value**

For

$$y = f(x) - 7$$

5 **the graph moves 7 units**

the graph moves 7 units down:

$$(3, 5) \to (3, -2)$$

6 **Calculate value**

For

$$y = f(-x)$$

7 **the graph reflects in the**

the graph reflects in the $y$-axis:

$$(3, 5) \to (-3, 5)$$

8 **Calculate value**

For part (b),

$$y = x^2 + 7x + 20$$

9 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

10 **Read the graph**

So the graph moves 2 units right.

11 **Replace by**

Replace $x$ by $x - 2$:

$$y = (x - 2)^2 + 7(x - 2) + 20$$

$$y = x^2 + 3x + 10$$

**FINAL ANSWER**

$(3, 10)$, $(3, -2)$, $(-3, 5)$, and $y = x^2 + 3x + 10$

4MA1/1H MAY 2023

# Question 21

2 marks | Transformations of Graphs

**21** A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(.....  , .....)

**(1)**

(b) $y = f(3x)$

(.....  , .....)

**(1)**

**(Total for Question 21 is 2 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H May 2023

**1 The turning point of is**

The turning point of $y = f(x)$ is $(6, 5)$.

**2 Calculate value**

For

$$y = f(x - 4)$$

**3 the graph moves 4 units**

the graph moves 4 units right:

$$(6, 5) \to (10, 5)$$

**4 Calculate value**

For

$$y = f(3x)$$

**5** **the coordinate is divided by**

the $x$-coordinate is divided by 3:

$$(6, 5) \rightarrow (2, 5)$$

---

✓ **FINAL ANSWER**

$(10, 5)$ and $(2, 5)$

# Question 24

4 marks | Transformations of Graphs

**24** The curve with equation $f(x) = 5x^2 + 9x + 2$ is transformed to the curve with equation

$g(x) = 5(x+4)^2 + 9(x+4) + 8$ by the translation $\begin{pmatrix} a \\ b \end{pmatrix}$

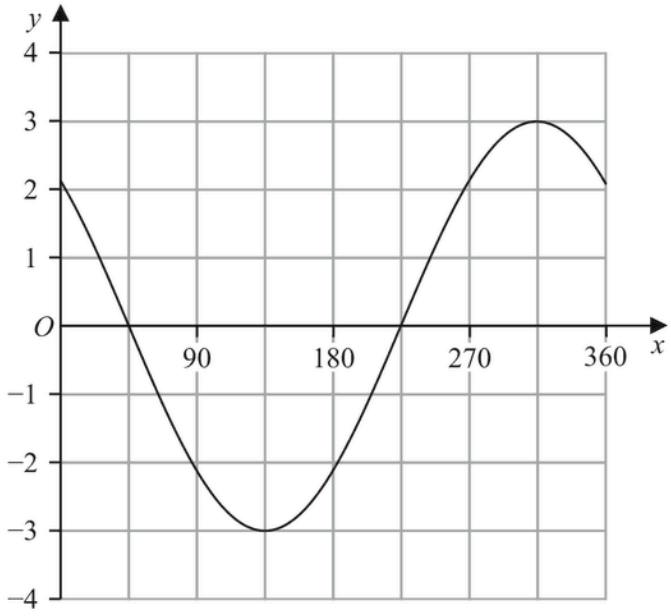
(a) Write down the value of $a$ and the value of $b$

$a =$ .....

$b =$ .....

**(2)**

The graph of $y = p\cos(x+q)^\circ$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.




Given that $p > 0$ and $0 < q < 360$

(b) find the value of $p$ and the value of $q$

$p =$ .....

$q =$ .....

**(2)**

**(Total for Question 24 is 4 marks)**

**1** **Read the graph**

For part (a),

$$f(x) = 5x^2 + 9x + 2$$

$$g(x) = 5(x+4)^2 + 9(x+4) + 8$$

**2** **Read the graph**

Since

$$f(x+4) = 5(x+4)^2 + 9(x+4) + 2$$

$$g(x) = f(x+4) + 6$$

**3** **Read the graph**

So the graph has moved 4 units left and 6 units up.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

**4** **Read the graph**

For part (b), the graph has maximum value $3$ and minimum value $-3$, so

$$p = 3$$

**5** **The minimum occurs when**

The minimum occurs when

$$x + q = 180$$

**6** **Read the graph**

From the graph, the minimum is at $x = 135$, so

$$135 + q = 180$$

$$q = 45$$

✓ **FINAL ANSWER**

$a = -4,\ b = 6,\ p = 3,\ q = 45$

4MA1/1H MAY 2024

# Question 21

2 marks | Transformations of Graphs

**21** A curve has equation $y = f(x)$

There is one minimum point on this curve.
The coordinates of this minimum point are $(5, -4)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 7)$

(.....,.....)
(1)

(ii) $y = f(x) - 6$

(.....,.....)
(1)

**(Total for Question 21 is 2 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H May 2024

**1 The minimum point of is**

The minimum point of $y = f(x)$ is $(5, -4)$.

**2 Calculate value**

For

$$y = f(x + 7)$$

**3 the graph moves 7 units**

the graph moves 7 units left:

$$(5, -4) \rightarrow (-2, -4)$$

**4 Calculate value**

For

$$y = f(x) - 6$$

**5** **the graph moves 6 units**

the graph moves 6 units down:

$$(5, -4) \rightarrow (5, -10)$$

---

✓ **FINAL ANSWER**

$(-2, -4)$ and $(5, -10)$

4MA1/2HR JUNE 2024

# Question 23

2 marks | Transformations of Graphs

**23** A curve has equation $y = f(x)$
The coordinates of the minimum point on this curve are $(6, -3)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 10$

(.....,.....)
**(1)**

(ii) $y = f(3x)$

(.....,.....)
**(1)**

**(Total for Question 23 is 2 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR June 2024

**1 The minimum point of is**

The minimum point of $y = f(x)$ is $(6, -3)$.

**2 Calculate value**

For

$$y = f(x) + 10$$

**3 the graph moves 10 units**

the graph moves 10 units up:

$$(6, -3) \to (6, 7)$$

**4 Calculate value**

For

$$y = f(3x)$$

**5 the coordinate is divided by**

the $x$-coordinate is divided by 3:

$$(6, -3) \to (2, -3)$$

**FINAL ANSWER**

$(6, 7)$ and $(2, -3)$

4MA1/1HR JUNE 2025

# Question 21

3 marks | Transformations of Graphs

**21** The curve with equation $y = f(x)$ has one turning point.
The coordinates of this turning point are $(-6, 9)$

(a) Write down the coordinates of the turning point on the curve with equation $y = f(3x)$

(.....  , .....)

(1)

The curve **C** with equation $y = g(x)$ is transformed to give the curve **S** with equation $y = g(x + a) + b$

The point $(4, -5)$ on **C** is mapped to the point $(1, -16)$ on **S**

(b) Write down the value of $a$ and the value of $b$

$a =$ .....

$b =$ .....

(2)

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR June 2025

**1 The turning point of is**

The turning point of $y = f(x)$ is $(-6, 9)$.

**2 Calculate value**

For

$$y = f(3x)$$

**3 the coordinate is divided by**

the $x$-coordinate is divided by 3:

$$(-6, 9) \to (-2, 9)$$

**4 Calculate value**

For part (b), a point $(4, -5)$ on $y = g(x)$ maps to $(1, -16)$ on

$$y = g(x + a) + b$$

**5** **Calculate value**

For the $x$-coordinate:

$$1 + a = 4$$

$$a = 3$$

---

**6** **Calculate value**

For the $y$-coordinate:

$$-5 + b = -16$$

$$b = -11$$

---

✓ **FINAL ANSWER**

$(-2, 9)$, $a = 3$, $b = -11$

# Question 27

4 marks | Transformations of Graphs

**27** A curve has equation $y = f(x)$

There is one maximum point on this curve.
The coordinates of this maximum point are $(6, -5)$

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 4)$

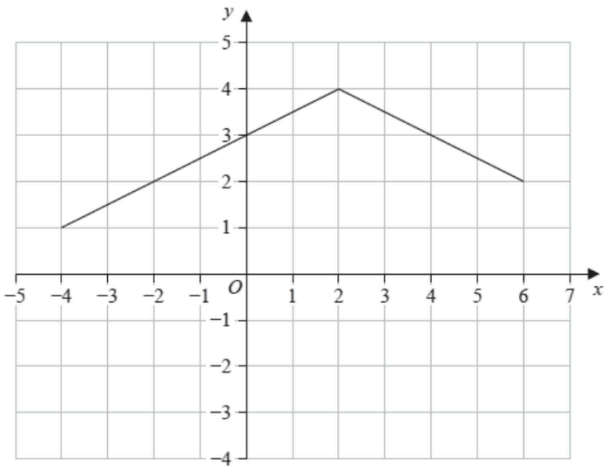
(.....)
**(1)**

(ii) $y = f(3x)$

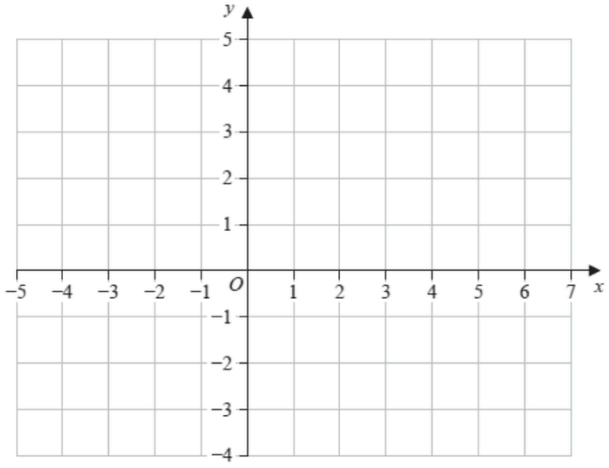
(.....)
**(1)**


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Here is the graph of $y = g(x)$




(b) On the grid below, draw the graph of $y = g(2x) - 3$




**(2)**

**(Total for Question 27 is 4 marks)**

**1** **(a)**

(a)

**2** **Read the graph**

For $y = f(x - 4)$, move the graph $4$ units right:

$$(6, -5) \to (10, -5)$$

**3** **Divide the -coordinate**

For $y = f(3x)$, divide the $x$-coordinate by $3$:

$$(6, -5) \to (2, -5)$$

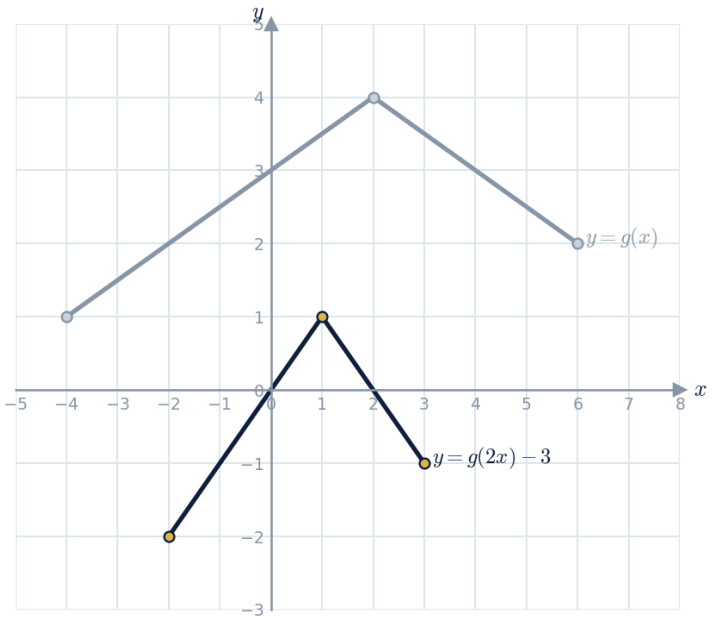
**4** **Subtract from each -coordinate**

(b) For $y = g(2x) - 3$, halve each $x$-coordinate and subtract $3$ from each $y$-coordinate.

**5** **Useful transformed points are**

Useful transformed points are

$$(-4, 1) \to (-2, -2), \quad (2, 4) \to (1, 1), \quad (6, 2) \to (3, -1)$$




**6** **Join these points with straight**

Join these points with straight line segments.

✓ **FINAL ANSWER**

$(10, -5)$, $(2, -5)$, graph through $(-2, -2), (1, 1), (3, -1)$.

4MA1/1H NOVEMBER 2025

# Question 24

3 marks | Transformations of Graphs

**24** A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (4, 3)

Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x + 5)$

(.....  , .....)
(1)

(ii) $y = f(x) + 7$

(.....  , .....)
(1)

(iii) $y = f(2x)$

(.....  , .....)
(1)

**(Total for Question 24 is 3 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1H November 2025

**1 The turning point of is**

The turning point of $y = f(x)$ is $(4, 3)$.

**2 Calculate value**

For

$$y = f(x + 5)$$

**3 the graph moves 5 units**

the graph moves 5 units left:

$$(4, 3) \rightarrow (-1, 3)$$

**4** **Calculate value**

For

$$y = f(x) + 7$$

**5** **the graph moves 7 units**

the graph moves 7 units up:

$$(4, 3) \to (4, 10)$$

**6** **Calculate value**

For

$$y = f(2x)$$

**7** **the coordinate is divided by**

the $x$-coordinate is divided by 2:

$$(4, 3) \to (2, 3)$$

✓ **FINAL ANSWER**

$(-1, 3)$, $(4, 10)$, and $(2, 3)$



TOPIC

# Angles in Polygons & Parallel Lines

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JUNE 2022

# Question 23

6 marks | Angles in Polygons & Parallel Lines

**23** A polygon has $n$ sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of $n$
Show clear algebraic working.

**Question 23 continued**

$n =$ .....

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H June 2022

**1 The interior angles form an**

The interior angles form an arithmetic sequence:

$$177,\ 175,\ 173,\ 171,\ 169, \ldots$$

**2 Calculate value**

So

$$a = 177, \quad d = -2$$

**3 Use trigonometry**

For an $n$-sided polygon, the sum of the interior angles is

$$180(n-2)$$

**4** **The sum of the arithmetic**

The sum of the arithmetic sequence is

$$\frac{n}{2}(2(177)+(n-1)(-2))$$

$$=\frac{n}{2}(354-2n+2)$$

$$=\frac{n}{2}(356-2n)$$

$$=n(178-n)$$

---

**5** **Set this equal to the**

Set this equal to the polygon angle sum:

$$n(178-n)=180(n-2)$$

$$178n-n^2=180n-360$$

$$n^2+2n-360=0$$

---

**6** **Factorise**

Factorise:

$$(n+20)(n-18)=0$$

---

**7** **Solve inequality**

Since $n>5$,

$$n=18$$

---

✓ **FINAL ANSWER**

$n=18$

TOPIC

# Perimeter & Area

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

# Question 22

6 marks | Perimeter & Area

**22** The diagram shows a regular octagon *ABCDEFGH*.

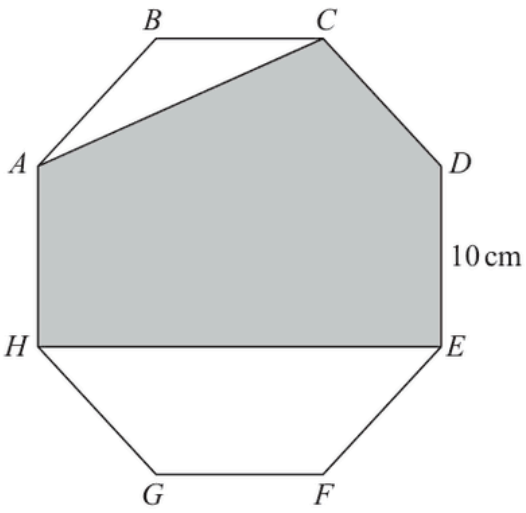



Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest $cm^2$

..... $cm^2$

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1HR May/November 2020

**1** **Each side of the regular**

Each side of the regular octagon is $10$ cm.

**2** **Calculate area**

For a regular octagon with side length $s$,

$$\text{area} = 2(1+\sqrt{2})s^2$$

**3** **Calculate area**

So the area of the whole octagon is

$$2(1+\sqrt{2})(10)^2 = 200 + 200\sqrt{2}$$

---

**4** **The unshaded triangle has**

The unshaded triangle $ABC$ has

$$AB = 10, \qquad BC = 10, \qquad \angle ABC = 135^\circ$$

---

**5** **Use trigonometry**

So

$$\text{area of triangle } ABC = \frac{1}{2}(10)(10)\sin 135^\circ = 25\sqrt{2}$$

---

**6** **Find the gradient**

At the bottom, the unshaded trapezium $HEFG$ has parallel sides

$$HE = 10 + 10\sqrt{2}, \qquad GF = 10$$

---

**7** **Simplify surd**

and height

$$5\sqrt{2}$$

---

**8** **Calculate area**

So

$$\text{area of trapezium } HEFG = \frac{1}{2}(10 + 10 + 10\sqrt{2})(5\sqrt{2})$$

$$= 50 + 50\sqrt{2}$$

---

**9** **The shaded area is**

The shaded area is

$$(200 + 200\sqrt{2}) - 25\sqrt{2} - (50 + 50\sqrt{2})$$

$$= 150 + 125\sqrt{2}$$

$$= 326.7766\ldots$$

---

**FINAL ANSWER**

$\boxed{327\ \text{cm}^2}$

4MA1/2H JANUARY 2021

# Question 20

4 marks | Perimeter & Area

**20**

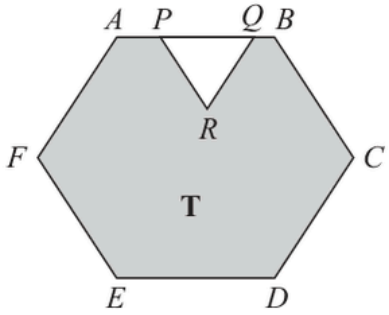



Diagram **NOT** accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle $PQR$ from a regular hexagon $ABCDEF$.

The points $P$ and $Q$ lie on $AB$ such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3}$ cm$^2$

work out the length of $PQ$.

..... cm

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H January 2021

**1 Let**

Let

$$PQ = p$$

**2 Given**

Given

$$AB = 1.5PQ$$

**3 Find the gradient**

so the side length of the regular hexagon is

$$1.5p = \frac{3}{2}p$$

**4** **Area of a regular hexagon**

Area of a regular hexagon with side length $s$ is

$$\frac{3\sqrt{3}}{2}s^2$$

**5** **Find the gradient**

So the area of the hexagon is

$$\frac{3\sqrt{3}}{2}\left(\frac{3}{2}p\right)^2$$

$$=\frac{27\sqrt{3}}{8}p^2$$

**6** **Find the gradient**

The removed triangle $PQR$ is equilateral, so its area is

$$\frac{\sqrt{3}}{4}p^2$$

**7** **The shaded area is**

The shaded area is

$$\frac{27\sqrt{3}}{8}p^2-\frac{\sqrt{3}}{4}p^2$$

$$=\frac{25\sqrt{3}}{8}p^2$$

**8** **Find the gradient**

This is $72\sqrt{3}$, so

$$\frac{25\sqrt{3}}{8}p^2=72\sqrt{3}$$

$$\frac{25}{8}p^2=72$$

$$p^2=\frac{576}{25}$$

$$p=4.8$$

**FINAL ANSWER**

$PQ = 4.8\ \text{cm}$

4WM1/01 NOVEMBER 2025

# Question 21

4 marks | Perimeter & Area

**21** The diagram shows an equilateral triangle and a circle.

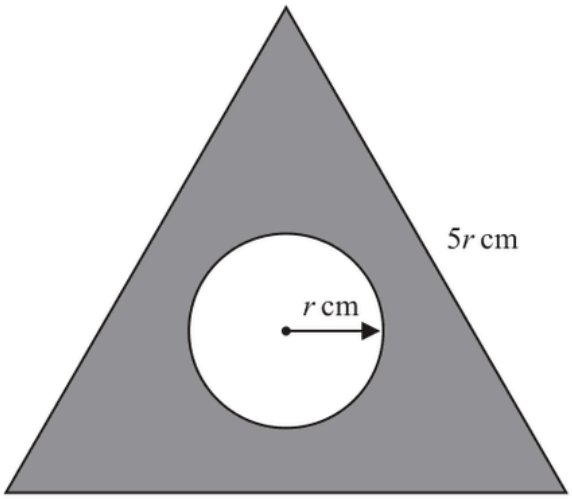



Diagram **NOT** accurately drawn

The equilateral triangle has sides of length $5r$ cm
The circle has radius $r$ cm

The area of the shaded region is $610\pi$ cm$^2$
Work out the value of $r$

Give your answer correct to 3 significant figures.

$r =$ .....

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4WM1/01 November 2025

**1** **Triangle area**

Triangle area:

$$\frac{\sqrt{3}}{4}(5r)^2 = \frac{25\sqrt{3}}{4}r^2$$

**2** **Circle area**

Circle area:

$$\pi r^2$$

**3** **Simplify surd**

So

$$\left(\frac{25\sqrt{3}}{4} - \pi\right) r^2 = 610\pi$$

$$r \approx 15.7926$$

---

✓ **FINAL ANSWER**

$r = 15.8$ to 3 significant figures.



TOPIC

# Circles, Arcs & Sectors

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2021

# Question 20

5 marks | Circles, Arcs & Sectors

**20** $A$, $B$ and $C$ are points on a circle with centre $O$.

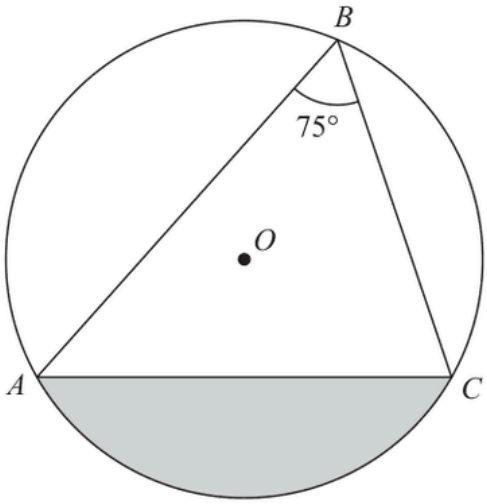



Diagram **NOT** accurately drawn

Angle $ABC = 75°$

The area of the shaded segment is $200\,\text{cm}^2$

Calculate the radius of the circle.
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 20 is 5 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR January 2021

**1** **Use trigonometry**

$$\angle ABC = 75°$$

**2** **The angle at the centre**

The angle at the centre is twice the angle at the circumference, so

$$\angle AOC = 2(75°) = 150°$$

**3** **The shaded segment is**

The shaded segment is:

$$\text{sector } AOC - \text{triangle } AOC$$

**4** **Let the radius be**

Let the radius be $r$.

**5** **Sector area**

Sector area:

$$\frac{150}{360}\pi r^2$$

**6** **Triangle area**

Triangle area:

$$\frac{1}{2}r^2 \sin 150°$$

**7** **Use trigonometry**

So

$$\frac{150}{360}\pi r^2 - \frac{1}{2}r^2 \sin 150° = 200$$

$$r^2\left(\frac{5\pi}{12} - \frac{1}{4}\right) = 200$$

$$r^2 = \frac{200}{\frac{5\pi}{12} - \frac{1}{4}}$$

$$r = 13.7426\ldots$$

✓ **FINAL ANSWER**

$\boxed{13.7 \text{ cm}}$

4MA1/2H JANUARY 2021

# Question 22

6 marks | Circles, Arcs & Sectors

**22** The diagram shows a sector $OBC$ of a circle with centre $O$ and radius $(6 + x)$ cm.

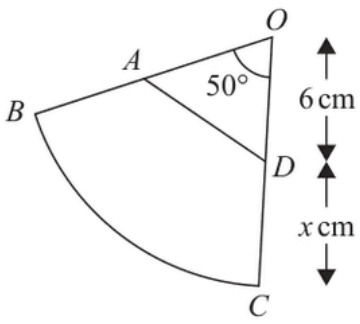



Diagram **NOT** accurately drawn

$A$ is the point on $OB$ and $D$ is the point on $OC$ such that $OA = OD = 6$ cm

Angle $BOC = 50°$

Given that

the perimeter of sector $OBC = 2 \times$ the perimeter of triangle $OAD$

find the value of $x$.
Give your answer correct to 3 significant figures.

$x =$ .....

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H January 2021

**1 The radius of sector is**

The radius of sector $OBC$ is

$$6 + x$$

**2 Use trigonometry**

The angle is $50°$, so the arc length of sector $OBC$ is

$$\frac{50}{360} \times 2\pi(6 + x)$$

**3** **Evaluate fraction**

Therefore the perimeter of sector $OBC$ is

$$2(6+x)+\frac{50}{360}\times 2\pi(6+x)$$

$$=(6+x)\left(2+\frac{5\pi}{18}\right)$$

**4** **Use trigonometry**

In triangle $OAD$,

$$OA=OD=6,\qquad \angle AOD=50^\circ$$

**5** **Use trigonometry**

So

$$AD=2(6)\sin 25^\circ=12\sin 25^\circ$$

**6** **The perimeter of triangle is**

The perimeter of triangle $OAD$ is

$$12+12\sin 25^\circ$$

**7** **Given that**

Given that

$$\text{perimeter of sector } OBC=2\times\text{perimeter of triangle } OAD$$

$$(6+x)\left(2+\frac{5\pi}{18}\right)=2(12+12\sin 25^\circ)$$

$$x=\frac{2(12+12\sin 25^\circ)}{2+\frac{5\pi}{18}}-6$$

$$x=5.8854\ldots$$

✓ **FINAL ANSWER**

$\boxed{5.89}$

4MA1/1H MAY 2021

# Question 25

6 marks | Circles, Arcs & Sectors

**25** The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

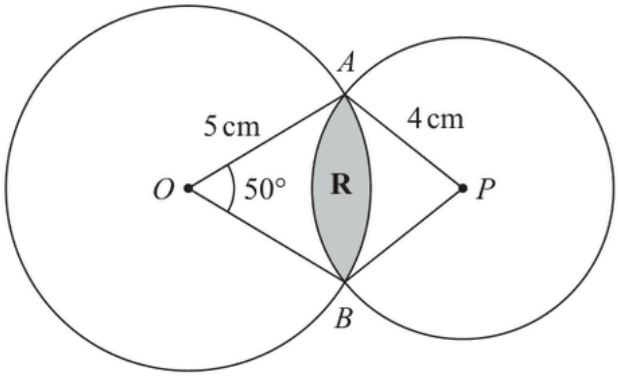



Diagram **NOT** accurately drawn

One of the circles has centre $O$ and radius 5 cm.
The other circle has centre $P$ and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

..... cm$^2$

**(Total for Question 25 is 6 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1H May 2021

**1** **Use trigonometry**

For the circle with centre $O$:

$$\angle AOB = 50°, \qquad OA = OB = 5$$

**2** **The half chord length is**

The half chord length is

$$\frac{AB}{2} = 5\sin 25°$$

**3** **Use trigonometry**

For the circle with centre $P$, let

$$\angle APB = \theta$$

---

**4** **Use trigonometry**

The radius is $4$, so

$$4\sin\left(\frac{\theta}{2}\right) = 5\sin 25°$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{5\sin 25°}{4}$$

$$\theta = 63.7777\ldots°$$

---

**5** **The common region is made**

The common region is made from two minor segments.

---

**6** **Use trigonometry**

For the circle with centre $O$:

$$\text{segment area} = \frac{50}{360}\pi(5)^2 - \frac{1}{2}(5)^2\sin 50° = 1.3328\ldots$$

---

**7** **Use trigonometry**

For the circle with centre $P$:

$$\text{segment area} = \frac{\theta}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin\theta$$

$$= 1.7284\ldots$$

---

**8** **Total area of region**

Total area of region $R$:

$$1.3328\ldots + 1.7284\ldots = 3.0611\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{3.06\ \text{cm}^2}$

4MA1/2H JUNE 2022

# Question 20

4 marks | Circles, Arcs & Sectors

**20** The diagram shows four identical circles drawn inside a square.

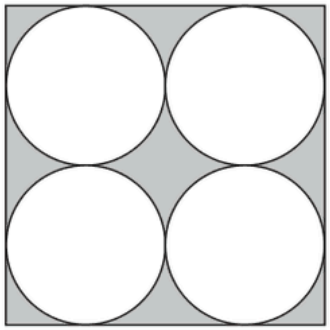


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of $40\,\text{cm}^2$

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H June 2022

**1 Let the radius of each**

Let the radius of each circle be $r$.

**2 There are two circles across**

There are two circles across the width of the square, so the side length of the square is

$$4r$$

**3 Area of the square**

Area of the square:

$$(4r)^2 = 16r^2$$

**4 Area of the four circles**

Area of the four circles:

$$4\pi r^2$$

**5** **Calculate area**

The shaded area is $40\ \text{cm}^2$, so

$$16r^2 - 4\pi r^2 = 40$$

$$r^2(16 - 4\pi) = 40$$

$$r = \sqrt{\frac{40}{16 - 4\pi}} = 3.4131\ldots$$

---

**6** **The perimeter of the square**

The perimeter of the square is

$$4(4r) = 16r$$

$$16r = 54.6101\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{54.6\ \text{cm}}$

4MA1/1H MAY 2023

# Question 22

6 marks | Circles, Arcs & Sectors

**22** The diagram shows two circles with centre $O$ and a regular pentagon $ABCDE$

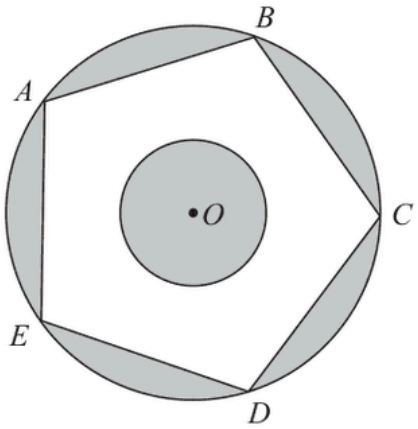



Diagram **NOT** accurately drawn

$A$, $B$, $C$, $D$ and $E$ are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H May 2023

**1** **The regular pentagon has side**

The regular pentagon has side length $8$ cm.

**2** **Use trigonometry**

For a regular pentagon,

$$R = \frac{s}{2\sin 36°}$$

**3** **Use trigonometry**

where $R$ is the radius of the larger circle.

$$R = \frac{8}{2\sin 36^\circ} = 6.8052\ldots$$

**4** **Area of the larger circle**

Area of the larger circle:

$$\pi R^2 = 145.4898\ldots$$

**5** **Area of the regular pentagon**

Area of the regular pentagon:

$$\frac{5s^2}{4\tan 36^\circ} = \frac{5(8)^2}{4\tan 36^\circ} = 110.1106\ldots$$

**6** **Let the radius of the**

Let the radius of the smaller circle be $r$.

**7** **The shaded area is**

The shaded area is:

$$\text{larger circle area} - \text{pentagon area} + \pi r^2$$

**8** **The unshaded area is**

The unshaded area is:

$$\text{pentagon area} - \pi r^2$$

**9** **Simplify surd**

These are equal, so

$$145.4898 - 110.1106 + \pi r^2 = 110.1106 - \pi r^2$$

$$2\pi r^2 = 2(110.1106) - 145.4898$$

$$\pi r^2 = 37.3657\ldots$$

$$r = \sqrt{\frac{37.3657\ldots}{\pi}} = 3.4487\ldots$$

✓ **FINAL ANSWER**

$$\boxed{3.45\text{ cm}}$$

4MA1/2H NOVEMBER 2023

# Question 20

4 marks | Circles, Arcs & Sectors

**20** The diagram shows equilateral triangle $ABC$ with sides of length 10 cm.
A circle is drawn inside the triangle.

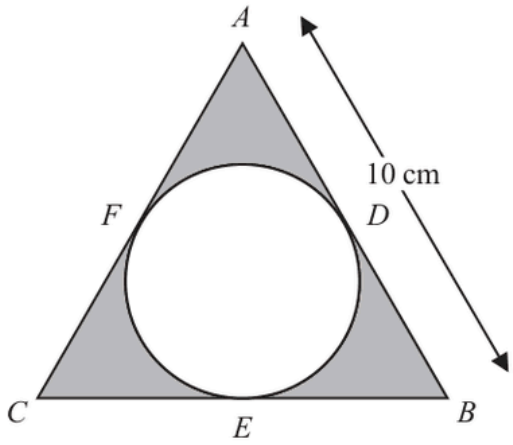



Diagram **NOT** accurately drawn

$D$, $E$ and $F$ are points on the circle.

$ADB$, $BEC$ and $CFA$ are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... $cm^2$

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H November 2023

**1** **Use trigonometry**

The triangle is equilateral with side length $10$ cm.

**2** **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$$

**3** **Use trigonometry**

For an equilateral triangle, the inradius is

$$r = \frac{s\sqrt{3}}{6}$$

**4** **Simplify surd**

So

$$r = \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

**5** **Area of the circle**

Area of the circle:

$$\pi r^2 = \pi\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{25\pi}{3}$$

**6** **The shaded area is**

The shaded area is

$$25\sqrt{3} - \frac{25\pi}{3} = 17.1213\ldots$$

✓ **FINAL ANSWER**

$\boxed{17.1\ \text{cm}^2}$

4MA1/1H MAY 2024

# Question 20

4 marks | Circles, Arcs & Sectors

**20** The diagram shows a sector $OABC$ of a circle centre $O$

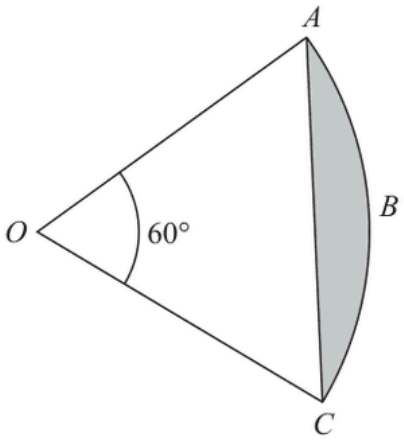



Diagram **NOT** accurately drawn

Angle $AOC = 60°$

The area of the shaded segment $ABC$ is $38\,\text{cm}^2$

Work out the perimeter of the shaded segment $ABC$
Give your answer correct to one decimal place.

..... cm

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H May 2024

**1** **Let the radius of the**

Let the radius of the sector be $r$.

**2** **The shaded segment is**

The shaded segment is

$$\text{sector area} - \text{triangle area}$$

**3** **Use trigonometry**

The angle is $60°$, so

$$38 = \frac{60}{360}\pi r^2 - \frac{1}{2}r^2 \sin 60°$$

$$38 = r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$r = 20.4815\ldots$$

**4** **The perimeter of the shaded**

The perimeter of the shaded segment is the arc $AC$ plus the chord $AC$.

**5** **Arc length**

Arc length:

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

**6** **Chord length**

Chord length:

$$2r \sin 30° = r$$

**7** **Evaluate fraction**

So the perimeter is

$$\frac{\pi r}{3} + r$$

$$= \frac{\pi(20.4815\ldots)}{3} + 20.4815\ldots$$

$$= 41.9296\ldots$$

✓ **FINAL ANSWER**

$\boxed{41.9\text{ cm}}$

4MA1/2H JUNE 2024

# Question 21

5 marks | Circles, Arcs & Sectors

**21** The diagram shows a square $ABCD$ and a circle.

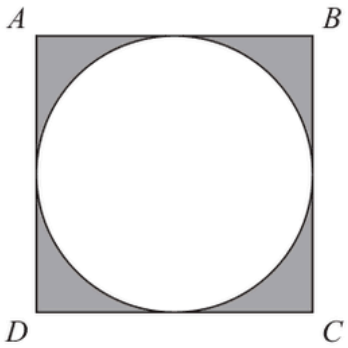



Diagram **NOT** accurately drawn

The sides of the square are tangents to the circle.

The total area of the shaded regions is $80\,\text{cm}^2$

Work out the length of $AC$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H June 2024

**1** **Let the side length**

Let the side length of the square be $s$.

**2** **Square**

The circle is inscribed in the square, so its radius is

$$\frac{s}{2}$$

**3** **The shaded area is the**

The shaded area is the area of the square minus the area of the circle:

$$s^2 - \pi\left(\frac{s}{2}\right)^2 = 80$$

$$s^2 - \frac{\pi s^2}{4} = 80$$

$$s^2\left(1 - \frac{\pi}{4}\right) = 80$$

$$s = \sqrt{\frac{80}{1 - \frac{\pi}{4}}} = 19.3076\ldots$$

---

**4** **The diagonal of the square**

The diagonal of the square is

$$AC = s\sqrt{2}$$

$$AC = 19.3076\ldots\sqrt{2} = 27.3051\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{27.3\text{ cm}}$

4MA1/2H NOVEMBER 2024

# Question 25

5 marks | Circles, Arcs & Sectors

**25** The diagram shows an equilateral triangle $ABC$ and a circle with centre $O$

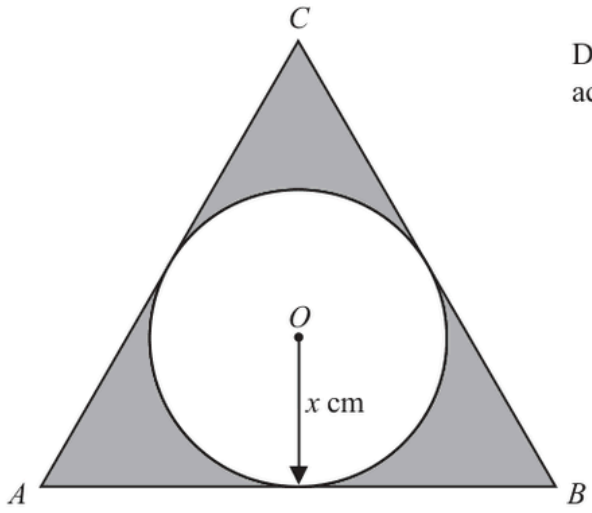



Diagram **NOT** accurately drawn

$AB$, $BC$ and $CA$ are tangents to the circle.

The radius of the circle is $x$ cm

The total area, in cm$^2$, of the regions shown shaded in the diagram is $nx^2$

Find the value of $n$
Give your answer correct to 3 significant figures.

$n =$ .....

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2H November 2024

**1** **Use trigonometry**

The circle has radius $x$, so the inradius of the equilateral triangle is $x$.

**2** **Use trigonometry**

For an equilateral triangle with side length $a$,

$$\text{inradius} = \frac{a\sqrt{3}}{6}$$

**3** **Simplify surd**

So

$$x = \frac{a\sqrt{3}}{6}$$

$$a = \frac{6x}{\sqrt{3}} = 2\sqrt{3}x$$

**4** **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(2\sqrt{3}x)^2$$

$$= \frac{\sqrt{3}}{4}(12x^2) = 3\sqrt{3}x^2$$

**5** **Area of the circle**

Area of the circle:

$$\pi x^2$$

**6** **The shaded area is**

The shaded area is

$$3\sqrt{3}x^2 - \pi x^2$$

$$= (3\sqrt{3} - \pi)x^2$$

**7** **Simplify surd**

Therefore

$$n = 3\sqrt{3} - \pi$$

✓ **FINAL ANSWER**

$$\boxed{3\sqrt{3} - \pi}$$

4WM1/01R MAY 2025

# Question 22

6 marks | Circles, Arcs & Sectors

**22**

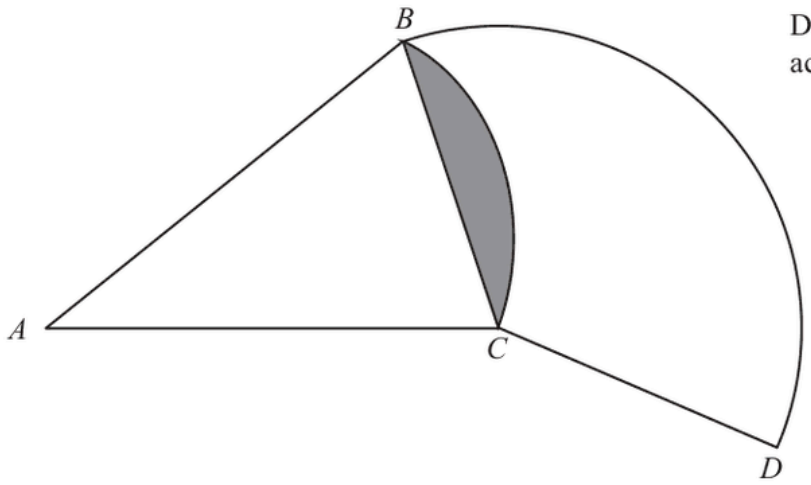



Diagram **NOT** accurately drawn

$BAC$ is a sector of a circle, centre $A$

$BCD$ is a sector of a circle, centre $C$

Angle $BAC = 40°$
Angle $BCD = 130°$
Area of shaded segment $= 28\,\text{cm}^2$

Find the length of the arc $BD$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4WM1/01R May 2025

**1 Let the radius of sector**

Let the radius of sector $BAC$ be $R$.

**2 The shaded segment is the**

The shaded segment is the segment cut off by chord $BC$ in the sector with centre $A$.

**3** **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

**4** **Use trigonometry**

Now find the chord $BC$:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

**5** **Calculate value**

In sector $BCD$, the centre is $C$, so the radius is

$$CB = CD = 21.7591\ldots$$

---

**6** **Use trigonometry**

The arc $BD$ has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/1HR JUNE 2025

# Question 25

6 marks | Circles, Arcs & Sectors

**25**

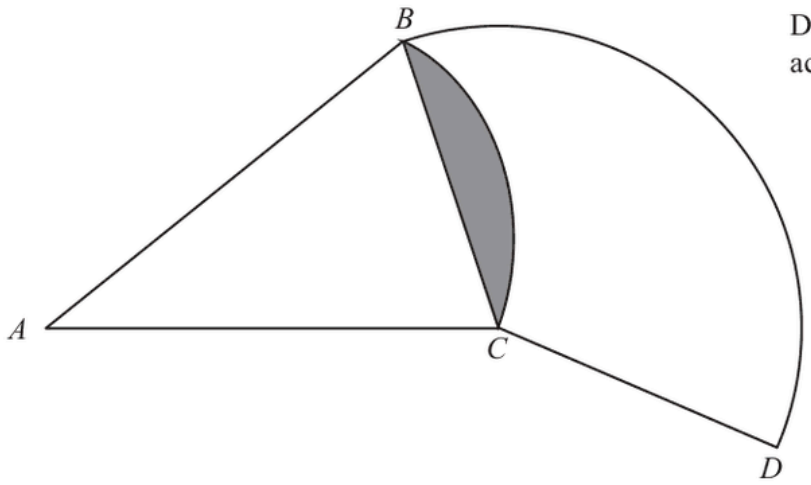



Diagram **NOT** accurately drawn

*BAC* is a sector of a circle, centre *A*

*BCD* is a sector of a circle, centre *C*

Angle *BAC* = 40°
Angle *BCD* = 130°
Area of shaded segment = 28 cm$^2$

Find the length of the arc *BD*
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 25 is 6 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1HR June 2025

**1 Let the radius of sector**

Let the radius of sector $BAC$ be $R$.

**2 The shaded segment is the**

The shaded segment is the segment cut off by chord $BC$ in the sector with centre $A$.

**3** **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

**4** **Use trigonometry**

Now find the chord $BC$:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

**5** **Calculate value**

In sector $BCD$, the centre is $C$, so the radius is

$$CB = CD = 21.7591\ldots$$

---

**6** **Use trigonometry**

The arc $BD$ has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

# Question 23

6 marks | Circles, Arcs & Sectors

**23** $OPQ$ and $OAB$ are sectors of a circle, centre $O$

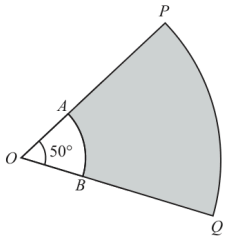



Diagram **NOT** accurately drawn

Angle $POQ = 50°$

$A$ is the point on $OP$ such that $OA = \frac{1}{4}OP$

The area of the shaded region is $\frac{243}{16}\pi\ \text{cm}^2$

Work out the perimeter of the shaded region.
Give your answer in the form $a\pi + b$ where $a$ and $b$ are constants.
Show your working clearly.

.....cm

**(Total for Question 23 is 6 marks)**

**1** **Let OP = r**

Then $OA = \frac{r}{4}$. Shaded area:

$$\frac{50}{360}\pi\left(r^2 - \frac{r^2}{16}\right) = \frac{243}{16}\pi$$

**2** **Solve for r**

$$\frac{75}{576}r^2 = \frac{243}{16} \Rightarrow r^2 = 116.64 \Rightarrow r = 10.8,\ OA = 2.7$$

**3** **Arcs**

$$\text{arc } PQ = \frac{50}{360} \times 2\pi(10.8) = 3\pi, \qquad \text{arc } AB = \frac{50}{360} \times 2\pi(2.7) = \tfrac{3}{4}\pi$$

**4** **Straight edges and total**

$AP = BQ = 10.8 - 2.7 = 8.1$:

$$3\pi + \tfrac{3}{4}\pi + 2(8.1) = \tfrac{15}{4}\pi + 16.2$$

**FINAL ANSWER**

$\frac{15}{4}\pi + 16.2$ cm



TOPIC

# Volume & Surface Area

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

# Question 22

5 marks | Volume & Surface Area

**22** A solid is made from a cone and a hemisphere.

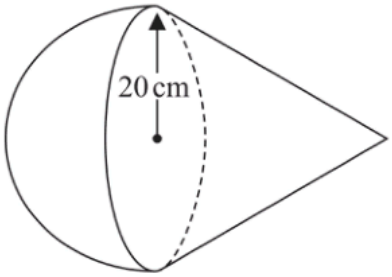



Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of $k$

$k =$ .....

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4WM2/01 Specimen

**1** **Calculate surface area**

The curved surface area of a cone is

$$\pi rl$$

**2** **Calculate area**

Here $r = 20$ and $\pi rl = 580\pi$, so

$$20l = 580$$

$$l = 29$$

**3** **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 = 29^2 - 20^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

**4** **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (20)^2 (21) = 2800\pi$$

---

**5** **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi (20)^3 = \frac{16000}{3}\pi$$

---

**6** **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

**7** **Calculate area**

Since the volume is $k\pi$,

$$k = \frac{24400}{3}$$

---

✓ **FINAL ANSWER**

$k = \frac{24400}{3}$.

# Question 26

6 marks | Volume & Surface Area

**26** Here is a sector, $AOB$, of a circle with centre $O$ and angle $AOB = x°$

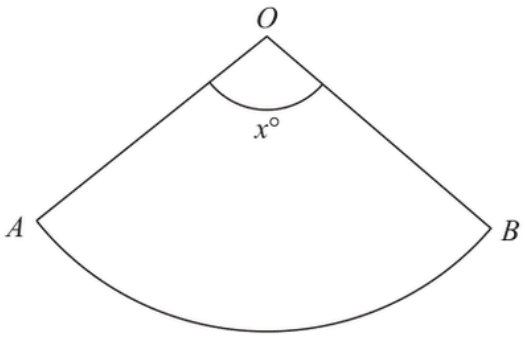



Diagram **NOT** accurately drawn

The sector can form the curved surface of a cone by joining $OA$ to $OB$.

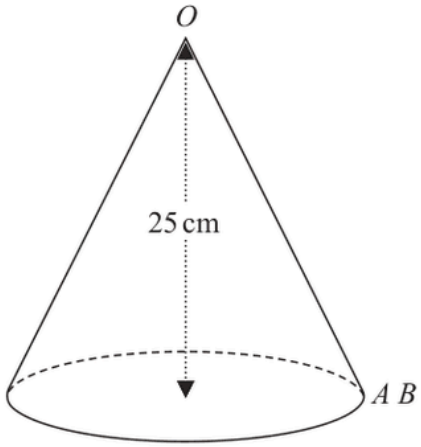



Diagram **NOT** accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 $cm^3$

Work out the value of $x$.
Give your answer correct to the nearest whole number.

$x =$ .....

**(Total for Question 26 is 6 marks)**

**1** **Let the radius of the**

Let the radius of the base of the cone be $r$, and let the slant height be $l$.

**2** **The volume of the cone**

The volume of the cone is

$$\frac{1}{3}\pi r^2 h = 1600$$

**3** **Calculate area**

The height is $25$ cm, so

$$\frac{1}{3}\pi r^2 (25) = 1600$$

$$r^2 = \frac{4800}{25\pi}$$

$$r = 7.8176\ldots$$

**4** **The slant height is found**

The slant height is found using Pythagoras:

$$l^2 = r^2 + 25^2$$

$$l = 26.1938\ldots$$

**5** **Calculate area**

When the sector is joined to form the cone, the arc length of the sector becomes the circumference of the cone base.

$$\frac{x}{360} \times 2\pi l = 2\pi r$$

**6** **Cancel**

Cancel $2\pi$:

$$\frac{x}{360}l = r$$

$$x = \frac{360r}{l}$$

$$x = \frac{360(7.8176\ldots)}{26.1938\ldots} = 107.4433\ldots$$

---

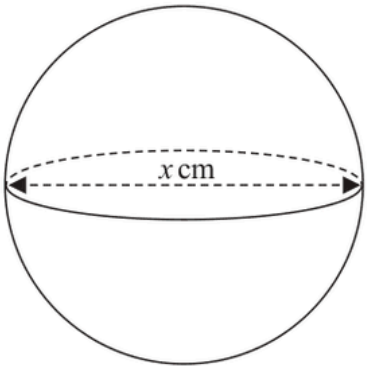
✓ **FINAL ANSWER**

$\boxed{107°}$

# Question 22

6 marks | Volume & Surface Area

**22** The diagram shows a sphere of diameter $x$ cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.



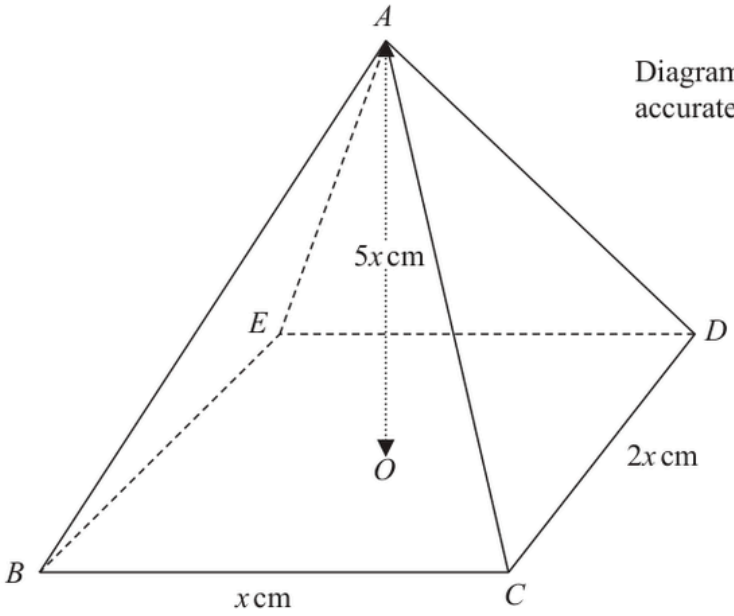




Diagram **NOT** accurately drawn

The vertex $A$ of the pyramid is vertically above the centre $O$ of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is $288\pi$ cm$^3$

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm$^2$

..... cm$^2$

**(Total for Question 22 is 6 marks)**

**1** **Use trigonometry**

The sphere has diameter $x$, so its radius is

$$\frac{x}{2}$$

**2** **The volume of the sphere**

The volume of the sphere is $288\pi\ \text{cm}^3$:

$$\frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 288\pi$$

$$\frac{\pi x^3}{6} = 288\pi$$

$$x^3 = 1728$$

$$x = 12$$

**3** **The rectangular base of the**

The rectangular base of the pyramid has dimensions $x$ and $2x$, so its area is

$$x(2x) = 2x^2$$

**4** **Use trigonometry**

For the two faces with base $x$, the slant height is

$$\sqrt{(5x)^2 + x^2} = x\sqrt{26}$$

**5** **Their total area is**

Their total area is

$$2\left(\frac{1}{2}x \cdot x\sqrt{26}\right) = x^2\sqrt{26}$$

**6** **Use trigonometry**

For the two faces with base $2x$, the slant height is

$$\sqrt{(5x)^2 + \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{101}}{2}$$

**7** **Their total area is**

Their total area is

$$2\left(\frac{1}{2}(2x)\cdot\frac{x\sqrt{101}}{2}\right)=x^2\sqrt{101}$$

**8** **Total surface area**

Total surface area:

$$2x^2+x^2\sqrt{26}+x^2\sqrt{101}$$

**9** **Substitute**

Substitute $x=12$:

$$144(2+\sqrt{26}+\sqrt{101})=2469.440\ldots$$

**10** **Correct to the nearest**

Correct to the nearest $\mathrm{cm}^2$:

$$2469$$

✓ **FINAL ANSWER**

$$\boxed{2469\ \mathrm{cm}^2}$$

4MA1/1H JUNE 2022

# Question 22

5 marks | Volume & Surface Area

**22** A solid is made from a cone and a hemisphere.

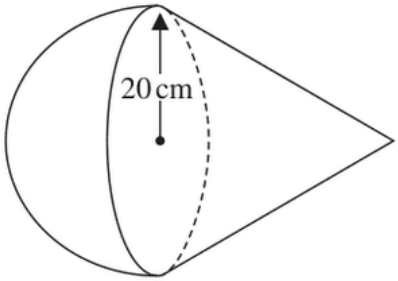



Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi \text{ cm}^2$

The volume of the solid is $k\pi \text{ cm}^3$

Work out the exact value of $k$

$k =$ .....

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H June 2022

**1 The cone has radius cm**

The cone has radius $20$ cm.

**2 Calculate surface area**

The curved surface area of the cone is

$$\pi rl$$

**3 Calculate value**

So

$$\pi(20)l = 580\pi$$

$$l = 29$$

**4** **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 + 20^2 = 29^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

**5** **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(20)^2(21) = 2800\pi$$

---

**6** **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(20)^3 = \frac{16000}{3}\pi$$

---

**7** **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

**8** **Calculate volume**

The volume is $k\pi$, so

$$k = \frac{24400}{3}$$

---

✓ **FINAL ANSWER**

$$\boxed{\frac{24400}{3}}$$

4MA1/2HR JUNE 2023

# Question 24

5 marks | Volume & Surface Area

**24** The surface area of sphere **A** is nine times the surface area of sphere **B**
The difference between the volume of sphere **A** and the volume of sphere **B** is $117\pi$ cm$^3$

Find the radius of the smaller sphere.
Show your working clearly.

..... cm

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2HR June 2023

**1** **Let the radius of the**

Let the radius of the smaller sphere be $r$.

**2** **Let the radius of the**

Let the radius of the larger sphere be $R$.

**3** **Surface area of a sphere**

Surface area of a sphere is

$$4\pi r^2$$

**4** **The surface area of sphere**

The surface area of sphere $A$ is nine times the surface area of sphere $B$, so

$$4\pi R^2 = 9(4\pi r^2)$$

$$R^2 = 9r^2$$

$$R = 3r$$

### 5 The difference between the volumes

The difference between the volumes is $117\pi$:

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

$$\frac{4}{3}\pi(27r^3 - r^3) = 117\pi$$

$$\frac{104}{3}\pi r^3 = 117\pi$$

---

### 6 Cancel

Cancel $\pi$:

$$\frac{104}{3}r^3 = 117$$

$$r^3 = \frac{117 \times 3}{104}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

---

✓ **FINAL ANSWER**

$\boxed{1.5\text{ cm}}$

4MA1/1H NOVEMBER 2023

# Question 21

5 marks | Volume & Surface Area

**21** Here is a triangular prism $ABCDEF$

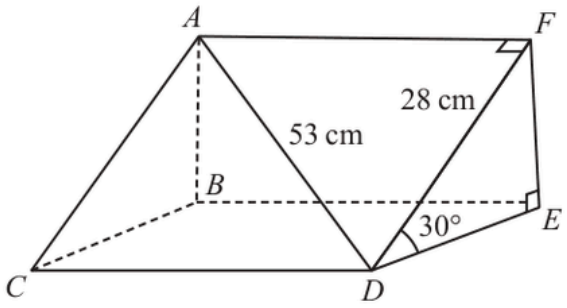



Diagram **NOT** accurately drawn

$AD = 53$ cm
$DF = 28$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

..... $cm^3$

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H November 2023

**1 The triangular cross section is**

The triangular cross-section is triangle $DEF$.

**2 It is right angled at**

It is right-angled at $E$, with

$$DF = 28$$

**3 Use trigonometry**

and

$$\angle FDE = 30°$$

**4 Use trigonometry**

So

$$FE = 28 \sin 30° = 14$$

**5** **Use trigonometry**

and

$$DE = 28\cos 30^\circ = 14\sqrt{3}$$

---

**6** **Area of triangle**

Area of triangle $DEF$:

$$\frac{1}{2}(14)(14\sqrt{3}) = 98\sqrt{3}$$

---

**7** **The length of the prism**

The length of the prism is

$$AD = 53$$

---

**8** **Volume of the prism**

Volume of the prism:

$$98\sqrt{3} \times 53$$

$$= 5194\sqrt{3}$$

$$= 8996.2719\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{8996\ \text{cm}^3}$

4MA1/1H MAY 2024

# Question 23

5 marks | Volume & Surface Area

**23** A solid shape is made by removing a hemisphere, shown shaded, from a cone as shown in the diagram.

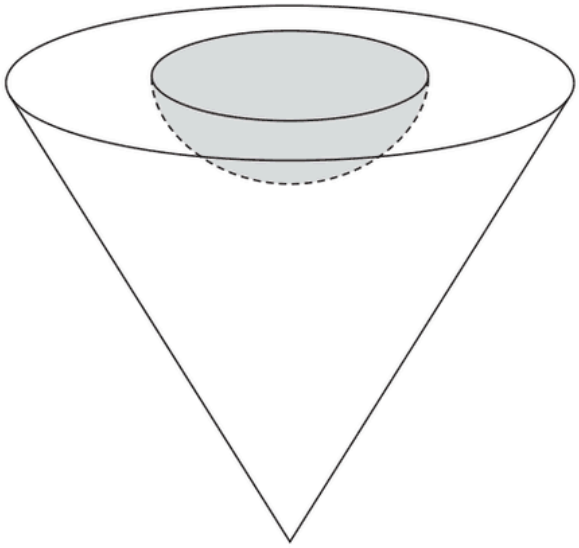


Diagram **NOT** accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the solid shape is $6948\pi$ cm$^3$

Work out the **total** surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer.

..... cm$^2$

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H May 2024

**1** **Define cone dimensions**

The cone has radius $5x$ and height $6x$.

**2** **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi(5x)^2(6x)$$

$$= 50\pi x^3$$

**3** **The removed hemisphere has radius**

The removed hemisphere has radius $2x$.

**4** **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi(2x)^3$$

$$= \frac{16}{3}\pi x^3$$

**5** **The remaining solid has volume**

The remaining solid has volume $6948\pi$, so

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\frac{134}{3}\pi x^3 = 6948\pi$$

**6** **Cancel**

Cancel $\pi$:

$$\frac{134}{3}x^3 = 6948$$

$$x^3 = \frac{10422}{67}$$

$$x = 5.3781\ldots$$

**7** **The solid hemisphere removed has**

The solid hemisphere removed has radius $2x$.

**8** **Total surface area of a**

Total surface area of a solid hemisphere is

$$3\pi r^2$$

**9** **Calculate value**

So

$$3\pi(2x)^2 = 12\pi x^2$$

$$12\pi(5.3781\ldots)^2 = 1090.3903\ldots$$

✓ **FINAL ANSWER**

$\boxed{1090\ \text{cm}^2}$

4MA1/1H NOVEMBER 2024

# Question 22

3 marks | Volume & Surface Area

**22** The diagram shows a bowl in the shape of a hemisphere.
The bowl is made from metal.

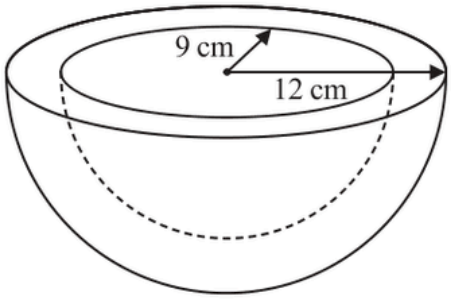



Diagram **NOT** accurately drawn

The outer radius of the bowl is 12 cm
The inner radius of the bowl is 9 cm

The thickness of the bowl is uniform.

Work out the volume of the metal.
Give your answer correct to the nearest whole number.

..... $cm^3$

**(Total for Question 22 is 3 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H November 2024

**1** **The bowl is a hemispherical**

The bowl is a hemispherical shell.

**2** **The volume of the metal**

The volume of the metal is:

$$\text{outer hemisphere volume} - \text{inner hemisphere volume}$$

**3** **Volume of a hemisphere**

Volume of a hemisphere:

$$\frac{2}{3}\pi r^3$$

**4** **Calculate volume**

So

$$\text{metal volume} = \frac{2}{3}\pi(12)^3 - \frac{2}{3}\pi(9)^3$$

$$= \frac{2}{3}\pi(12^3 - 9^3)$$

$$= \frac{2}{3}\pi(1728 - 729)$$

$$= 666\pi$$

$$= 2092.3007\ldots$$

---

✓ **FINAL ANSWER**

$\boxed{2092\ \text{cm}^3}$

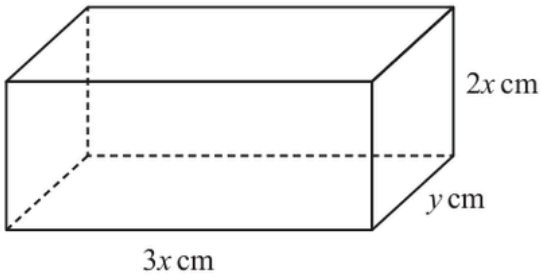
4MA1/2H JUNE 2025

# Question 21

3 marks | Volume & Surface Area

**21** The diagram shows a cuboid.

Diagram **NOT** accurately drawn




The cuboid measures $3x$ cm by $2x$ cm by $y$ cm

The volume of the cuboid is $1014\,\text{cm}^3$
The total surface area of the cuboid is $A\,\text{cm}^2$

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H June 2025

**1** **The cuboid has dimensions**

The cuboid has dimensions

$$3x, \quad 2x, \quad y$$

**2** **Calculate volume**

Its volume is $1014\ \text{cm}^3$, so

$$(3x)(2x)(y) = 1014$$

$$6x^2y = 1014$$

$$y = \frac{1014}{6x^2}$$

$$y = \frac{169}{x^2}$$

**3** **The total surface area is**

The total surface area is

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$A = 2(6x^2 + 5xy)$$

$$A = 12x^2 + 10xy$$

**4** **Substitute**

Substitute $y = \frac{169}{x^2}$:

$$A = 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$A = 12x^2 + \frac{1690}{x}$$

✓ **FINAL ANSWER**

$$\boxed{A = 12x^2 + \frac{1690}{x}}$$

4WM2/01R MAY 2025

# Question 21

5 marks | Volume & Surface Area

**21** The diagram shows a solid cuboid.

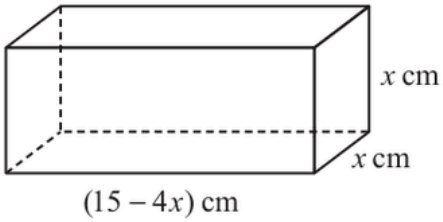



Diagram **NOT** accurately drawn

The volume of the cuboid is $V$ cm$^3$

Find the maximum value of $V$

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4WM2/01R May 2025

**1 Required**

Cuboid has a square cross-section of side *x* cm and length (15−4*x*) cm. Find the maximum value of the volume *V*.

**2 Write V as a function**

Write *V* as a function of *x*:

*V* = *x* · *x* · (15−4*x*) = *x*<sup>2</sup>(15−4*x*) = 15*x*<sup>2</sup> − 4*x*<sup>3</sup>

**3 Differentiate**

Differentiate:

$$fracdVdx = 30x - 12x^2$$

**4 Set ( x 0 gives**

Set $dfracdVdx = 0$:

$$30x - 12x^2 = 0implies6x(5 - 2x) = 0implies x = 0; textor; x = tfrac52 = 2.5$$

(*x* = 0 gives zero volume, so the meaningful solution is *x* = 2.5.)

**5** **Second derivative test to confirm**

Second derivative test to confirm maximum:

$$fracd^2Vdx^2 = 30 - 24x; Bigg|_x = 2.5 = 30 - 60 = -30\<0quadcheckmarktext(maximum)$$

**6** **Calculate V max V max**

Calculate *V*<sub>max</sub>:

*V*<sub>max</sub> = (2.5)<sup>2</sup> × (15−4×2.5) = 6.25 × 5 = 31.25 cm<sup>3</sup>

**7** **Note**

Calculus optimisation: always verify the nature of the stationary point using $dfracd^2Vdx^2$. Also check *x* \> 0 and 15 − 4*x* \> 0 (i.e. *x* \< 3.75) to ensure a valid cuboid.

✓ **FINAL ANSWER**

Maximum volume *V* = 31.25 cm<sup>3</sup>

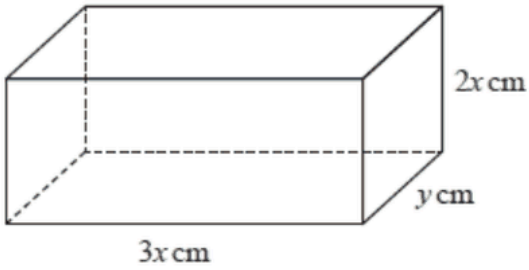
4WM2/01 MAY 2025

# Question 23

3 marks | Volume & Surface Area

**23** The diagram shows a cuboid.

Diagram **NOT** accurately drawn




The cuboid measures $3x$ cm by $2x$ cm by $y$ cm

The volume of the cuboid is 1014 cm$^3$
The total surface area of the cuboid is $A$ cm$^2$

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4WM2/01 May 2025

**1** **Evaluate fraction**

$$3x \times 2x \times y = 1014$$

$$6x^2y = 1014$$

$$y = \frac{169}{x^2}$$

## 2 Surface area

Surface area:

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$= 12x^2 + 10xy$$

$$= 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$= 12x^2 + \frac{1690}{x}$$

---

**FINAL ANSWER**

$A = 12x^2 + \dfrac{1690}{x}$.

4WM2/01R MAY 2025

# Question 25

5 marks | Volume & Surface Area

**25** The diagram shows a solid hemisphere, $H$

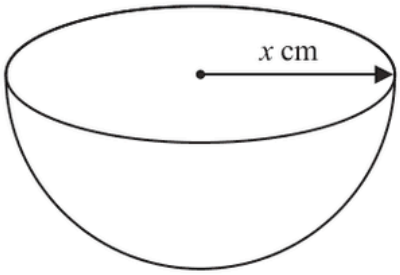



Diagram **NOT** accurately drawn

The radius of $H$ is $x$ cm
The volume of $H$ is $6174\pi$ cm$^3$

A bowl is made by removing a solid hemisphere from $H$ such that the uniform thickness of the bowl is 2 cm

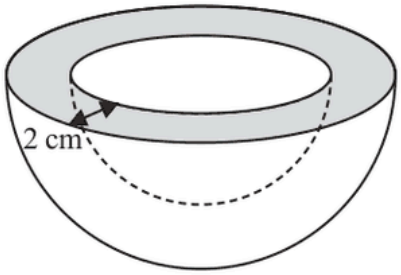



Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of $\pi$

..... cm$^2$

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4WM2/01R May 2025

**1** **Required**

Hemisphere H has radius *x* cm and volume 6174*π* cm<sup>3</sup>. A bowl is made by removing an inner hemisphere, leaving 2 cm thickness. Find total surface area in terms of *π*.

**2** **Calculate volume**

Find outer radius *x*. Volume of hemisphere $= \frac{2}{3}\pi r^3$:

$$\frac{2}{3}\pi x^3 = 6174\pi \implies x^3 = \frac{6174}{3} \times 2 = 9261 \implies x = \sqrt[3]{9261} = 21\,\text{cm}$$

**3** **Find inner radius r inner**

Find inner radius:

*r*<sub>inner</sub> = 21 − 2 = 19 cm

**4** **Add 1**

Three surfaces to add:

**1. Outer curved surface** (half-sphere, radius 21):

*A*<sub>1</sub> = 2*π*(21)<sup>2</sup> = 2*π* × 441 = 882*π*

**2. Inner curved surface** (half-sphere, radius 19):

*A*<sub>2</sub> = 2*π*(19)<sup>2</sup> = 2*π* × 361 = 722*π*

**3. Flat annular ring** at the top rim:

*A*<sub>3</sub> = *π*(21)<sup>2</sup> − *π*(19)<sup>2</sup> = *π*(441−361) = 80*π*

**5** **Total surface area A total**

Total surface area:

*A*<sub>total</sub> = 882*π* + 722*π* + 80*π* = 1684*π*

**6** **Note**

Three separate surfaces! Missing the flat annular ring is the most common error on this type of question.

✓ **FINAL ANSWER**

Total surface area = 1684*π* cm<sup>2</sup>

4MA1/2HR JUNE 2025

# Question 26

5 marks | Volume & Surface Area

**26** The diagram shows a solid hemisphere, $H$

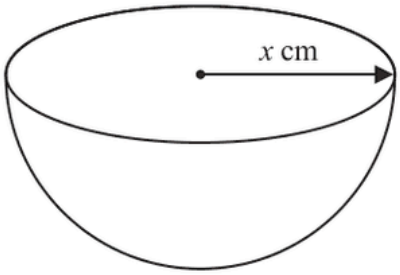



Diagram **NOT** accurately drawn

The radius of $H$ is $x$ cm
The volume of $H$ is $6174\pi$ cm$^3$

A bowl is made by removing a solid hemisphere from $H$ such that the uniform thickness of the bowl is 2 cm

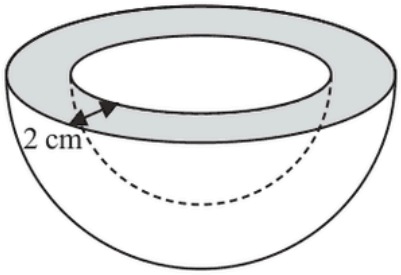



Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of $\pi$

..... cm$^2$

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2HR June 2025

**1** **The volume of a hemisphere**

The volume of a hemisphere is

$$\frac{2}{3}\pi r^3$$

**2** **Evaluate fraction**

For the original hemisphere $H$,

$$\frac{2}{3}\pi x^3 = 6174\pi$$

**3** **Cancel**

Cancel $\pi$:

$$\frac{2}{3}x^3 = 6174$$

$$x^3 = 9261$$

$$x = 21$$

**4** **The outer radius of the**

The outer radius of the bowl is $21$ cm.

**5** **Calculate value**

The thickness is $2$ cm, so the inner radius is

$$21 - 2 = 19$$

**6** **Total surface area of the**

Total surface area of the bowl is:

**7** **outer curved hemisphere inner curved**

- outer curved hemisphere
- inner curved hemisphere
- flat annular rim

**8** **Outer curved surface area**

Outer curved surface area:

$$2\pi(21)^2 = 882\pi$$

**9** **Inner curved surface area**

Inner curved surface area:

$$2\pi(19)^2 = 722\pi$$

**10** **Flat rim area**

Flat rim area:

$$\pi(21^2 - 19^2) = 80\pi$$

---

**11** **Total**

Total:

$$882\pi + 722\pi + 80\pi = 1684\pi$$

---

✓ **FINAL ANSWER**

$\boxed{1684\pi \text{ cm}^2}$

# Question 20

5 marks | Volume & Surface Area

**20** The diagram shows a solid shape.

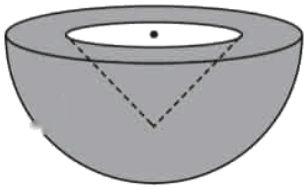


Diagram **NOT** accurately drawn

The solid shape is made by removing a cone from a hemisphere.

The radius of the cone is $x$ cm
The height of the cone is $x$ cm

The radius of the hemisphere is $2x$ cm

The volume of the solid shape is $\frac{3645}{8}\pi$ cm$^3$

Find the outer curved surface area of the solid shape.
Give your answer correct to the nearest whole number.

..... cm$^2$

**(Total for Question 20 is 5 marks)**

**1** **Volume of the solid**

$$\tfrac{2}{3}\pi(2x)^3 - \tfrac{1}{3}\pi x^2(x) = \tfrac{16}{3}\pi x^3 - \tfrac{1}{3}\pi x^3 = 5\pi x^3$$

**2** **Solve for x**

$$5\pi x^3 = \frac{3645}{8}\pi \Rightarrow x^3 = \frac{729}{8} \Rightarrow x = 4.5$$

**3** **Outer curved surface = curved hemisphere**

Radius $2x = 9$:

$$2\pi(9)^2 = 162\pi = 508.9\ldots$$

✓ **FINAL ANSWER**

$509$ cm$^2$



TOPIC

# 3D Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2021

# Question 22

4 marks | 3D Pythagoras & Trigonometry

**22** $ABC$ is an isosceles triangle in a horizontal plane.
The point $T$ is vertically above $B$.

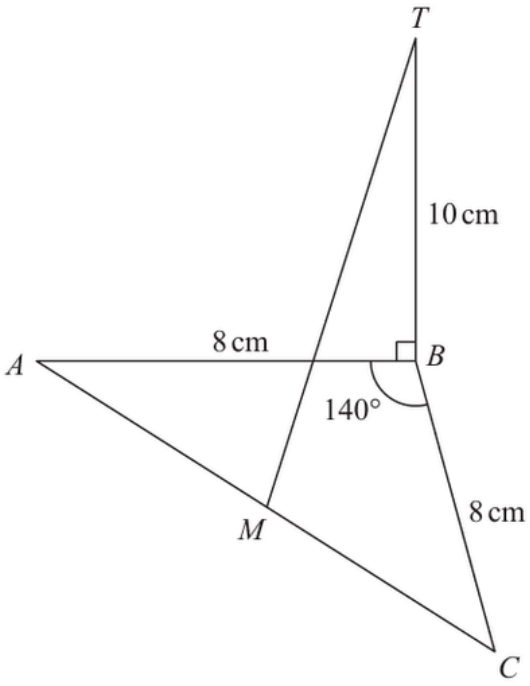



Diagram **NOT** accurately drawn

Angle $ABC = 140°$
$AB = BC = 8$ cm
$TB = 10$ cm
$M$ is the midpoint of $AC$.

Calculate the size of the angle between $MT$ and the horizontal plane $ABC$.
Give your answer correct to one decimal place.

.....°

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2HR January 2021

**1** **Use trigonometry**

Since $AB = BC = 8$, triangle $ABC$ is isosceles.

**2** **The line from to the**

The line from $B$ to the midpoint $M$ bisects angle $ABC$.

$$\angle ABM = \frac{140^\circ}{2} = 70^\circ$$

**3** **Use trigonometry**

In right triangle $ABM$,

$$\cos 70^\circ = \frac{BM}{8}$$

$$BM = 8\cos 70^\circ$$

**4** **Use trigonometry**

Since $T$ is vertically above $B$, the projection of $MT$ onto the horizontal plane is $MB$.

**5** **Let the required angle be**

Let the required angle be $\theta$.

$$\tan\theta = \frac{TB}{BM}$$

$$\tan\theta = \frac{10}{8\cos 70^\circ}$$

$$\theta = 74.7^\circ$$

**FINAL ANSWER**

$74.7^\circ$

4MA1/1H MAY 2021

# Question 22

4 marks | 3D Pythagoras & Trigonometry

**22** The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

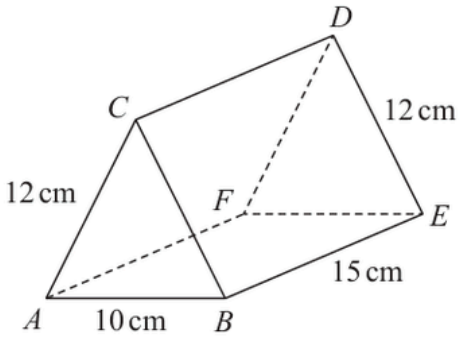



Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between $AD$ and the base $ABEF$.
Give your answer correct to 3 significant figures.

.....°

**(Total for Question 22 is 4 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H May 2021

### 1 Calculate value

The triangular end is isosceles, with

$$AC = BC = 12, \qquad AB = 10$$

### 2 The perpendicular height of the

The perpendicular height of the triangle is found by halving $AB$:

$$AM = 5$$

$$CM^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$CM = \sqrt{119}$$

### 3 The projection of on the

The projection of $AD$ on the horizontal base goes from $A$ to the midpoint of $FE$. Its length is

$$\sqrt{15^2 + 5^2} = \sqrt{250}$$

---

### 4 Use trigonometry

So, for the angle $\theta$ between $AD$ and the base,

$$\tan\theta = \frac{\sqrt{119}}{\sqrt{250}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.602\ldots$$

---

**FINAL ANSWER**

$34.6°$

4MA1/1H JANUARY 2022

# Question 21

5 marks | 3D Pythagoras & Trigonometry

**21** The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

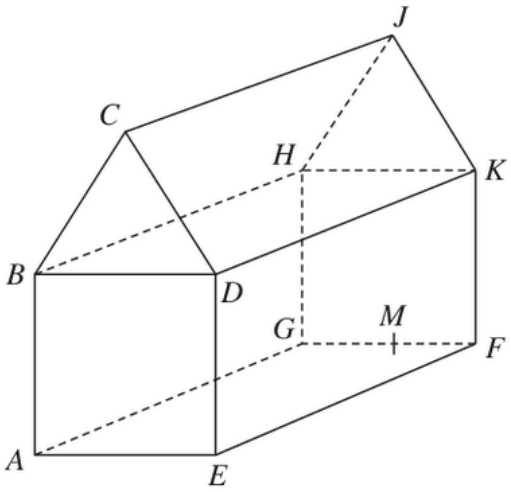



Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
$BCD$ is an equilateral triangle

$EF = 2 \times AE$
$M$ is the midpoint of $GF$ so that $JM$ is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of $T$, giving your answer in the form $\dfrac{\sqrt{p} + \sqrt{q}}{17}$ where $p$ and $q$ are integers.

$T =$ .....

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H January 2022

**1** **Let**

Let

$$AE = x$$

**2** **Use trigonometry**

Since $ABDE$ is a square,

$$AB = BD = DE = AE = x$$

**3** **Use trigonometry**

Since $BCD$ is an equilateral triangle, the height from $C$ to $BD$ is

$$\frac{\sqrt{3}}{2}x$$

**4** **Use trigonometry**

Therefore the vertical height from the base to $J$ is

$$JM = x + \frac{\sqrt{3}}{2}x$$

$$JM = \frac{x(2+\sqrt{3})}{2}$$

**5** **Use trigonometry**

Also,

$$EF = 2AE = 2x$$

**6** **Find the midpoint**

Since $M$ is the midpoint of $GF$, the horizontal distance from $A$ to $M$ is

$$AM = \sqrt{\left(\frac{x}{2}\right)^2 + (2x)^2}$$

$$AM = \sqrt{\frac{x^2}{4} + 4x^2}$$

$$AM = \frac{x\sqrt{17}}{2}$$

**7** **Use trigonometry**

In right triangle $AMJ$,

$$\tan y^\circ = \frac{JM}{AM}$$

$$T = \frac{\frac{x(2+\sqrt{3})}{2}}{\frac{x\sqrt{17}}{2}}$$

$$T = \frac{2+\sqrt{3}}{\sqrt{17}}$$

---

**8** **Rationalise the denominator**

Rationalise the denominator:

$$T = \frac{(2+\sqrt{3})\sqrt{17}}{17}$$

$$T = \frac{2\sqrt{17}+\sqrt{51}}{17}$$

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

---

✓ **FINAL ANSWER**

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

# Question 22

5 marks | 3D Pythagoras & Trigonometry

**22** Here is a cuboid *ABCDEFGH*

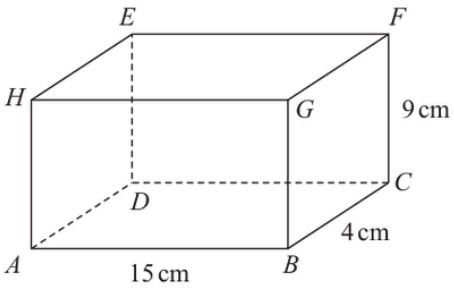



Diagram **NOT** accurately drawn

$AB = 15\,\text{cm}$ $BC = 4\,\text{cm}$ $CF = 9\,\text{cm}$

(a) Work out the length of *BE*
Give your answer correct to 3 significant figures.

..... cm

**(2)**

Here is a cuboid *PQRSTUVW*

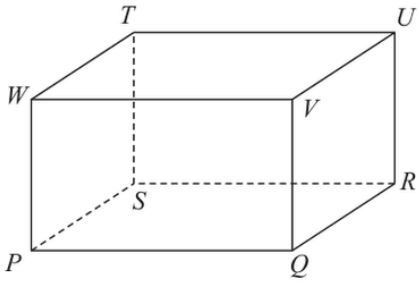



Diagram **NOT** accurately drawn

$PR = 42\,\text{cm}$

The size of the angle between *PU* and the plane *PQRS* is 30°

*M* is the midpoint of *PR*

(b) Work out the size of angle *UMR*
Give your answer correct to 3 significant figures.

.....°

**(3)**

**(Total for Question 22 is 5 marks)**

**1** **Find the gradient**

For part (a), $BE$ is the space diagonal using the three perpendicular lengths:

$$AB = 15, \qquad BC = 4, \qquad CF = 9$$

$$BE^2 = 15^2 + 4^2 + 9^2$$

$$BE^2 = 225 + 16 + 81 = 322$$

$$BE = \sqrt{322} = 17.9 \text{ cm}$$

**2** **Use trigonometry**

For part (b), the projection of $PU$ onto the plane $PQRS$ is $PR$.

$$PR = 42$$

**3** **The angle between and the**

The angle between $PU$ and the plane is $30°$, so

$$\tan 30° = \frac{RU}{PR}$$

$$RU = 42 \tan 30°$$

**4** **Find the midpoint**

Since $M$ is the midpoint of $PR$,

$$MR = 21$$

**5** **Use trigonometry**

In right triangle $MRU$,

$$\tan \angle UMR = \frac{RU}{MR}$$

$$\tan \angle UMR = \frac{42 \tan 30°}{21}$$

$$\tan \angle UMR = 2 \tan 30°$$

$$\angle UMR = 49.1°$$

**FINAL ANSWER**

$BE = 17.9\ \text{cm},\ \angle UMR = 49.1°$

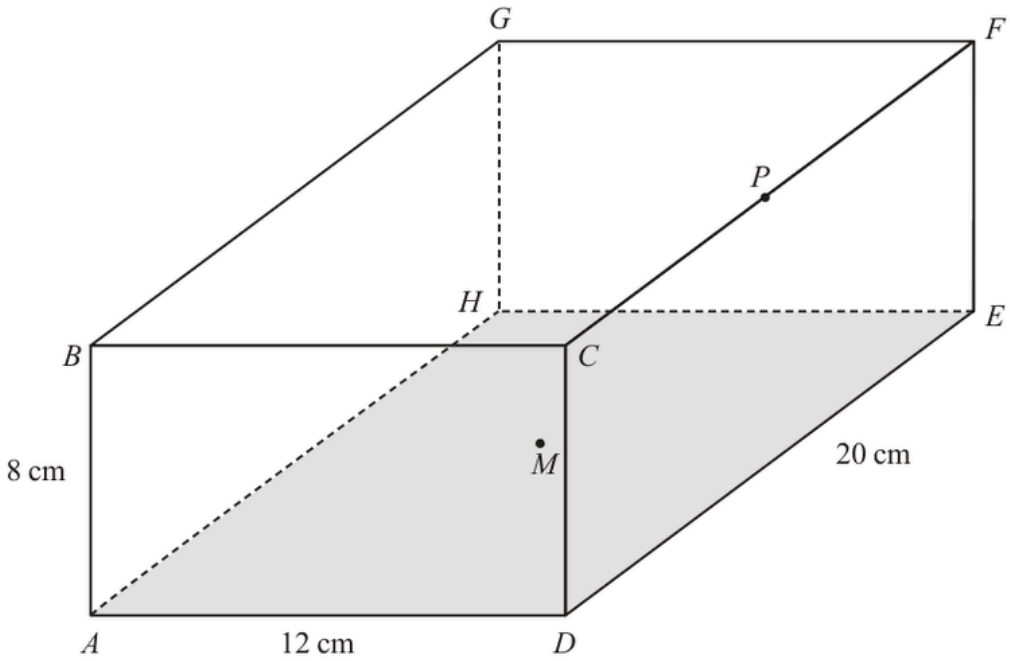
4MA1/1HR MAY 2024

# Question 22

6 marks | 3D Pythagoras & Trigonometry

**22** The diagram shows a cuboid $ABCDEFGH$ with horizontal base $ADEH$

Diagram **NOT** accurately drawn




$AB = 8$ cm $\quad AD = 12$ cm $\quad DE = 20$ cm

$M$ is the midpoint of the base $ADEH$ and $P$ is the midpoint of the edge $CF$

Work out the size of angle $BMP$
Give your answer correct to one decimal place.

.....°

**(Total for Question 22 is 6 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1HR May 2024

**1** **Use trigonometry**

Use coordinates with

$$A = (0, 0, 0)$$

**2** **Use trigonometry**

Since

$$AD = 12, \qquad DE = 20, \qquad AB = 8$$

**3** **take**

take

$$B = (0, 0, 8)$$

**4** **The centre of the rectangular**

The centre of the rectangular base $ADEH$ is

$$M = (6, 10, 0)$$

**5** **Find the midpoint**

Point $P$ is the midpoint of $CF$.

$$C = (12, 0, 8), \qquad F = (12, 20, 8)$$

$$P = (12, 10, 8)$$

**6** **Use trigonometry**

Now

$$\vec{MB} = B - M = (-6, -10, 8)$$

$$\vec{MP} = P - M = (6, 0, 8)$$

**7** **Use the scalar product formula**

Use the scalar product formula:

$$\vec{MB} \cdot \vec{MP} = (-6)(6) + (-10)(0) + (8)(8)$$

$$\vec{MB} \cdot \vec{MP} = 28$$

$$|\vec{MB}| = \sqrt{(-6)^2 + (-10)^2 + 8^2} = \sqrt{200}$$

$$|\vec{MP}| = \sqrt{6^2 + 0^2 + 8^2} = 10$$

$$\cos \angle BMP = \frac{28}{10\sqrt{200}}$$

$$\angle BMP = 78.6^\circ$$

---

**FINAL ANSWER**

$78.6^\circ$

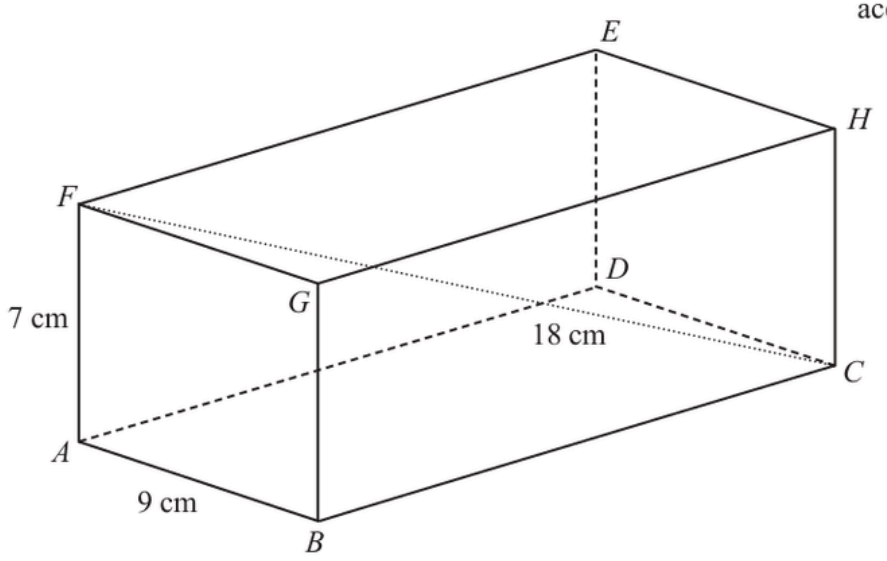
4MA1/1HR JUNE 2025

# Question 20

3 marks | 3D Pythagoras & Trigonometry

**20** The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn




$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of $BC$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR June 2025

**1** **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal $FC$:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

**2** **Use trigonometry**

Now use Pythagoras in triangle $ABC$:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

---

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

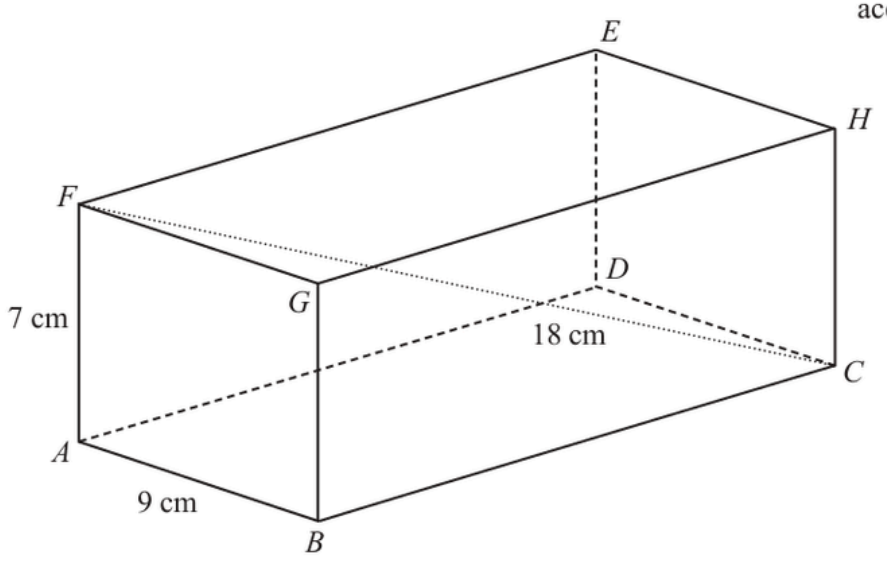
4WM1/01R MAY 2025

# Question 20

3 marks | 3D Pythagoras & Trigonometry

**20** The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn




$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of $BC$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4WM1/01R May 2025

**1** **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal $FC$:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

**2** **Use trigonometry**

Now use Pythagoras in triangle $ABC$:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

---

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

4MA1/2HR JUNE 2025

# Question 23

3 marks | 3D Pythagoras & Trigonometry

**23** The diagram shows a triangular prism, $ABCDEF$, with a horizontal rectangular base $ABCD$

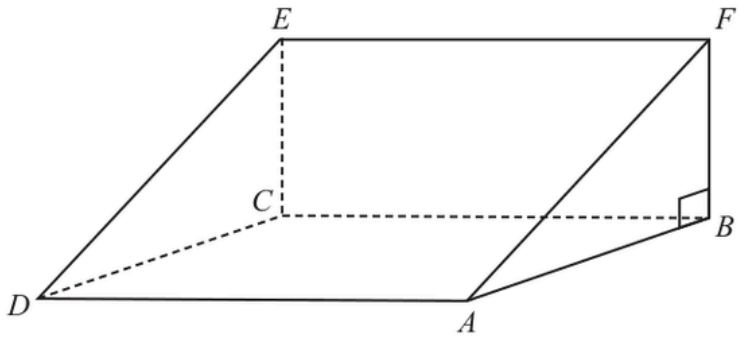



Diagram **NOT** accurately drawn

$M$ is the midpoint of the line $AC$

$AC = 40\,\text{cm}$ angle $CAE = 35°$ angle $ABF = 90°$

Work out the size of angle $CME$
Give your answer correct to 3 significant figures.

.....°

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2HR June 2025

### 1 Use trigonometry

$$AC = 40$$

### 2 The point is vertically above

The point $E$ is vertically above $C$, so triangle $ACE$ is right-angled at $C$.

$$\angle CAE = 35°$$

$$\tan 35° = \frac{CE}{AC}$$

$$CE = 40 \tan 35°$$

**3** **Find the midpoint**

Since $M$ is the midpoint of $AC$,

$$CM = 20$$

---

**4** **Use trigonometry**

In right triangle $CME$,

$$\tan \angle CME = \frac{CE}{CM}$$

$$\tan \angle CME = \frac{40 \tan 35°}{20}$$

$$\tan \angle CME = 2 \tan 35°$$

$$\angle CME = 54.5°$$

---

✓ **FINAL ANSWER**

$54.5°$

4WM1/01 MAY 2025

# Question 23

6 marks | 3D Pythagoras & Trigonometry

**23** The diagram shows cuboid $ABCDEFGH$

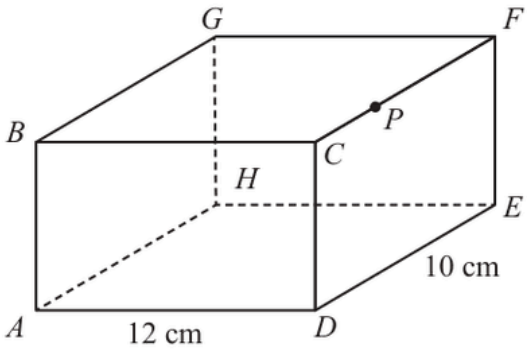



Diagram **NOT** accurately drawn

The point $P$ lies on $CF$

$AD = 12$ cm $\quad DE = 10$ cm $\quad CP = 3$ cm

The angle of elevation of $P$ from $A$ is $24°$

Calculate the size of angle $APG$
Give your answer correct to the nearest degree.
Show your working clearly.

.....°

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4WM1/01 May 2025

**1** **Point is cm from along**

Point $P$ is $3$ cm from $C$ along $CF$. The horizontal distance from $A$ to the point directly below $P$ is

$$\sqrt{12^2 + 3^2} = \sqrt{153}$$

**2** **Use trigonometry**

Using the angle of elevation,

$$h = \sqrt{153}\tan 24° = 5.507\ldots$$

**3** **Use trigonometry**

So

$$AP = \frac{\sqrt{153}}{\cos 24^\circ} = 13.5399\ldots$$

**4** **Use trigonometry**

On the top face,

$$PG = \sqrt{12^2 + 7^2} = \sqrt{193} = 13.8924\ldots$$

**5** **Use trigonometry**

Also,

$$AG = \sqrt{12^2 + 10^2 + h^2} = 16.561\ldots$$

**6** **Use cosine rule**

Use the cosine rule in triangle $APG$.

$$\cos \angle APG = \frac{AP^2 + PG^2 - AG^2}{2(AP)(PG)}$$

$$\angle APG = 49.163\ldots^\circ$$

**7** **Use trigonometry**

To the nearest degree,

$$49^\circ$$

✓ **FINAL ANSWER**

$49^\circ$

4MA1/2H JUNE 2025

# Question 24

4 marks | 3D Pythagoras & Trigonometry

**24** The diagram shows a square-based pyramid $ABCDE$

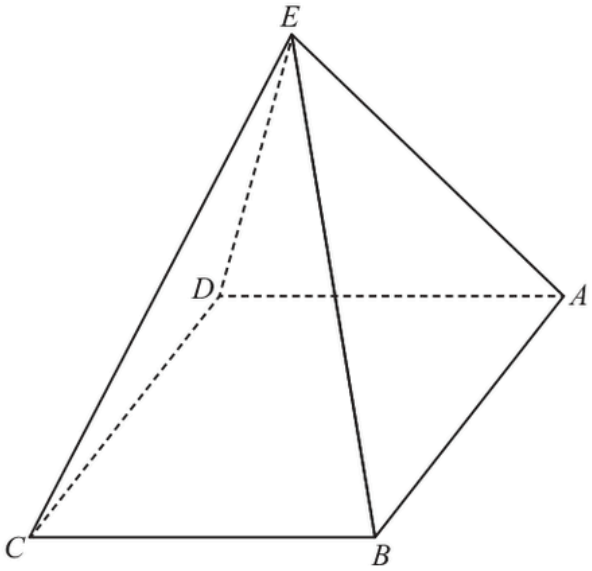



Diagram **NOT** accurately drawn

$EA = EB = EC = ED$

$M$ is the centre of the horizontal square base $ABCD$
$Q$ is the midpoint of $AB$
Angle $EQM = 80°$

$EA : AB = n : 1$

Find the value of $n$
Give your answer correct to 3 significant figures.

$n =$ .....

**(Total for Question 24 is 4 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H June 2025

**1 Let**

Let

$$AB = s$$

**2** **Find the midpoint**

Since $M$ is the centre of the square base and $Q$ is the midpoint of $AB$,

$$QM = \frac{s}{2}$$

**3** **Use trigonometry**

Since $EA = EB = EC = ED$, the point $E$ is vertically above $M$.

**4** **Use trigonometry**

In right triangle $EQM$,

$$\tan 80° = \frac{EM}{QM}$$

$$EM = \frac{s}{2}\tan 80°$$

**5** **The distance from the centre**

The distance from the centre of a square to a vertex is half the diagonal:

$$AM = \frac{s\sqrt{2}}{2}$$

**6** **Use trigonometry**

In right triangle $EMA$,

$$EA^2 = EM^2 + AM^2$$

$$EA^2 = \left(\frac{s}{2}\tan 80°\right)^2 + \left(\frac{s\sqrt{2}}{2}\right)^2$$

$$EA = s\sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

**7** **Use trigonometry**

So

$$\frac{EA}{AB} = \sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

$$n = 2.92$$

**FINAL ANSWER**

$n = 2.92$

# Question 26

5 marks | 3D Pythagoras & Trigonometry

**26** The diagram shows prism $ABCDEFGH$

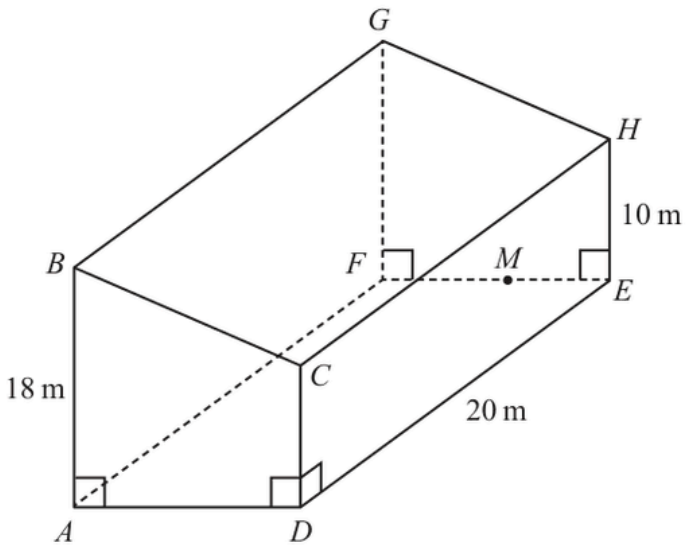



Diagram **NOT** accurately drawn

$$AB = FG = 18\,\text{m} \qquad DC = EH = 10\,\text{m} \qquad AD = FE$$
$$DE = CH = AF = BG = 20\,\text{m}$$
$$\text{angle } BAD = \text{angle } ADC = \text{angle } CDE = 90°$$

$M$ is the point on $FE$ such that $FM = \frac{3}{5}FE$

Given that angle $GMF = 60°$

calculate the angle of elevation of $G$ from $D$
Give your answer correct to one decimal place.
Show your working clearly.

.....°

**(Total for Question 26 is 5 marks)**

**1** **Use trigonometry**

From triangle $GMF$,

$$\tan 60^\circ = \frac{FG}{FM} = \frac{18}{FM}$$

$$FM = \frac{18}{\tan 60^\circ} = 6\sqrt{3}$$

**2** **Simplify surd**

Since $FM = \frac{3}{5}FE$,

$$FE = 10\sqrt{3}$$

**3** **Simplify surd**

The horizontal distance from $D$ to the point below $G$ is

$$\sqrt{20^2 + (10\sqrt{3})^2} = 10\sqrt{7}$$

**4** **Use trigonometry**

If $\theta$ is the angle of elevation,

$$\tan\theta = \frac{18}{10\sqrt{7}}$$

$$\theta \approx 34.2289^\circ$$

✓ **FINAL ANSWER**

$34.2^\circ$.

4MA1/1H NOVEMBER 2025

# Question 26

5 marks | 3D Pythagoras & Trigonometry

**26** Here is a prism $ABCDEFGH$ with a horizontal, rectangular base $ABGF$

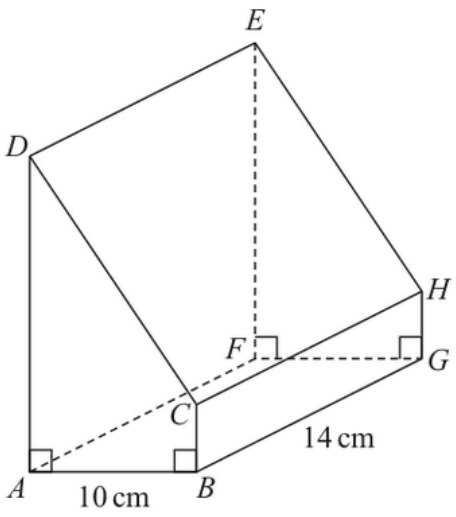



Diagram **NOT** accurately drawn

$AB = 10\,\text{cm}$ $BG = 14\,\text{cm}$

angle $DAB$ = angle $ABC$ = angle $EFG$ = angle $FGH = 90°$

$BC = \frac{1}{5}AD$

The angle of elevation of $E$ from $B$ is $50°$

Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... $\text{cm}^3$

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/1H November 2025

**1** **The horizontal base is a**

The horizontal base $ABGF$ is a rectangle.

$$AB = 10, \qquad BG = 14$$

**2** **Use trigonometry**

So the horizontal distance from $B$ to $F$ is

$$BF = \sqrt{10^2 + 14^2} = \sqrt{296}$$

---

**3** **Use trigonometry**

The angle of elevation of $E$ from $B$ is $50°$.

---

**4** **Use trigonometry**

Since $E$ is vertically above $F$,

$$\tan 50° = \frac{FE}{BF}$$

$$FE = \sqrt{296} \tan 50°$$

---

**5** **The prism has the same**

The prism has the same cross section throughout, so

$$AD = FE = \sqrt{296} \tan 50°$$

---

**6** **Use trigonometry**

Also

$$BC = \frac{1}{5} AD$$

---

**7** **Area of the trapezium cross**

Area of the trapezium cross section $ABCD$:

$$\frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}\left(AD + \frac{1}{5}AD\right) \times 10$$

$$= 6AD$$

---

**8** **Volume of the prism**

Volume of the prism:

$$6AD \times 14 = 84AD$$

$$= 84\sqrt{296}\tan 50°$$

$$= 1722.311\ldots$$

**9** **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1720$$

✓ **FINAL ANSWER**

$$\boxed{1720\ \text{cm}^3}$$



TOPIC

# Sine & Cosine Rules

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

# Question 21

6 marks | Sine & Cosine Rules

**21** The diagram shows the prism $ABCDEF$ with cross section triangle $ABC$.

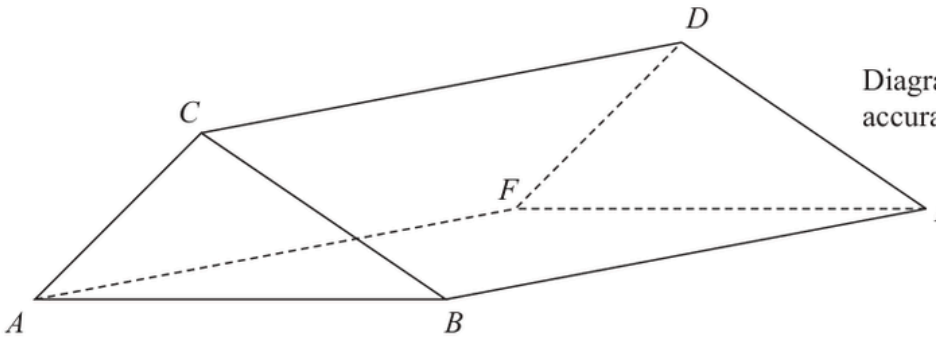



Diagram **NOT** accurately drawn

Angle $BEC = 40°$ and angle $ACB$ is obtuse.
$AC = 6\,\text{cm}$ and $CE = 13\,\text{cm}$

The area of triangle $ABC$ is $22\,\text{cm}^2$

Calculate the length of $AB$.
Give your answer correct to one decimal place.

..... cm

**(Total for Question 21 is 6 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR January 2020

**1 Use trigonometry**

In triangle $BEC$, $CE = 13$ and $\angle BEC = 40°$.

**2 Find the gradient**

Since this is a prism, $BE$ is perpendicular to the cross section, so triangle $BEC$ is right-angled at $B$.

$$BC = 13 \sin 40°$$

$$BC = 8.356238\ldots$$

**3 The area of triangle is**

The area of triangle $ABC$ is $22\ \text{cm}^2$.

**4** **Use trigonometry**

Using

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$22 = \frac{1}{2}(AC)(BC)\sin(\angle ACB)$$

$$22 = \frac{1}{2}(6)(8.356238\ldots)\sin(\angle ACB)$$

$$\sin(\angle ACB) = 0.877587\ldots$$

---

**5** **Use trigonometry**

The angle $ACB$ is obtuse, so

$$\angle ACB = 180° - \sin^{-1}(0.877587\ldots)$$

$$\angle ACB = 118.647\ldots°$$

---

**6** **Use the cosine rule**

Use the cosine rule:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos(\angle ACB)$$

$$AB^2 = 6^2 + (8.356238\ldots)^2 - 2(6)(8.356238\ldots)\cos(118.647\ldots°)$$

$$AB = 12.4056\ldots$$

---

**7** **Correct to one decimal place**

Correct to one decimal place:

$$12.4$$

---

✓ **FINAL ANSWER**

$\boxed{12.4\text{ cm}}$

4MA1/2H JANUARY 2022

# Question 23

6 marks | Sine & Cosine Rules

**23** The diagram shows triangle $PQR$

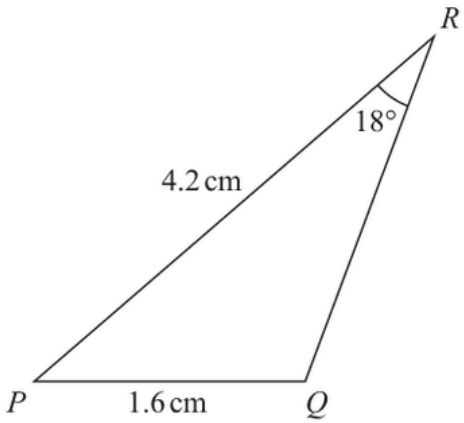



Diagram **NOT** accurately drawn

$PQ = 1.6\,\text{cm}$ $PR = 4.2\,\text{cm}$ Angle $PRQ = 18°$

Given that angle $PQR$ is obtuse,

work out the area of triangle $PQR$
Give your answer correct to 3 significant figures.

..... $\text{cm}^2$

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H January 2022

**1** **Use trigonometry**

In triangle $PQR$,

$$PQ = 1.6, \quad PR = 4.2, \quad \angle PRQ = 18°$$

**2** **Use the sine rule**

Use the sine rule:

$$\frac{\sin Q}{PR} = \frac{\sin R}{PQ}$$

$$\frac{\sin Q}{4.2} = \frac{\sin 18^\circ}{1.6}$$

$$\sin Q = \frac{4.2 \sin 18^\circ}{1.6}$$

$$\sin Q = 0.811169\ldots$$

---

**3** **Use trigonometry**

Since angle $PQR$ is obtuse,

$$Q = 180^\circ - \sin^{-1}(0.811169\ldots)$$

$$Q = 125.7896\ldots^\circ$$

---

**4** **Use trigonometry**

Now find angle $P$:

$$P = 180^\circ - 18^\circ - 125.7896\ldots^\circ$$

$$P = 36.2103\ldots^\circ$$

---

**5** **Area of triangle**

Area of triangle $PQR$:

$$\frac{1}{2}(PQ)(PR)\sin P$$

$$= \frac{1}{2}(1.6)(4.2)\sin(36.2103\ldots^\circ)$$

$$= 1.984925\ldots$$

---

**6** **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1.98$$

---

**FINAL ANSWER**

$\boxed{1.98\ \text{cm}^2}$

4MA1/2H JUNE 2023

# Question 22

5 marks | Sine & Cosine Rules

**22** The diagram shows a triangle $ABC$ and a flagpole $BF$

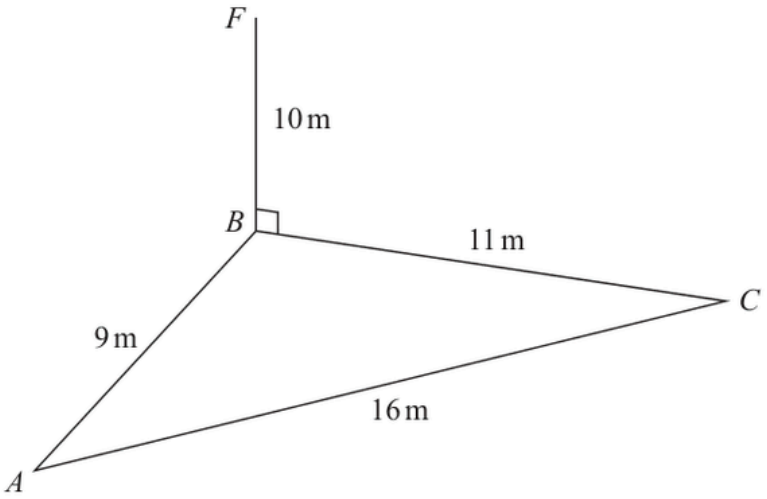



Diagram **NOT** accurately drawn

$A$, $B$ and $C$ are points on horizontal ground.

$BF$ is vertical.

$AB = 9\,\text{m}$ $BC = 11\,\text{m}$ $AC = 16\,\text{m}$ $BF = 10\,\text{m}$

$D$ is the point on $AC$ such that angle $BDC = 90°$

Work out the size of the angle of elevation of the point $F$ from the point $D$
Give your answer correct to one decimal place.

.....°

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/2H June 2023

**1** **Use trigonometry**

In triangle $ABC$,

$$AB = 9, \qquad BC = 11, \qquad AC = 16$$

**2** **Use Heron s formula**

Use Heron's formula.

$$s = \frac{9 + 11 + 16}{2} = 18$$

$$\text{area} = \sqrt{18(18 - 9)(18 - 11)(18 - 16)}$$

$$= \sqrt{18 \cdot 9 \cdot 7 \cdot 2} = 47.6235\ldots$$

---

**3** **is the foot of the**

$D$ is the foot of the perpendicular from $B$ to $AC$, so $BD$ is the height of triangle $ABC$.

$$\text{area} = \frac{1}{2} \times AC \times BD$$

$$47.6235\ldots = \frac{1}{2} \times 16 \times BD$$

$$BD = 5.9529\ldots$$

---

**4** **Use trigonometry**

In right-angled triangle $BDF$,

$$\tan\theta = \frac{BF}{BD}$$

$$\tan\theta = \frac{10}{5.9529\ldots}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.9529\ldots}\right) = 59.2349\ldots^\circ$$

---

✓ **FINAL ANSWER**

$\boxed{59.2^\circ}$

4MA1/1HR MAY 2023

# Question 25

6 marks | Sine & Cosine Rules

**25** Here is a triangle $ABC$

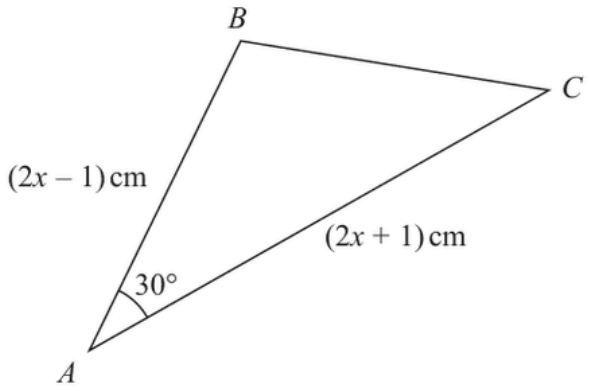



Diagram **NOT** accurately drawn

The area of the triangle is $(x^2 + x - 3.75)\,\text{cm}^2$

Find the size of the largest angle in triangle $ABC$
Give your answer correct to the nearest degree.

.....°

**(Total for Question 25 is 6 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/1HR May 2023

**1** **The two sides around the**

The two sides around the $30°$ angle are

$$2x - 1 \quad \text{and} \quad 2x + 1$$

**2** **Use trigonometry**

Using the area formula,

$$\text{area} = \frac{1}{2}ab\sin C$$

$$x^2 + x - 3.75 = \frac{1}{2}(2x - 1)(2x + 1)\sin 30°$$

**3** **Use trigonometry**

Since $\sin 30° = \frac{1}{2}$,

$$x^2 + x - 3.75 = \frac{1}{4}(2x - 1)(2x + 1)$$

$$x^2 + x - 3.75 = \frac{1}{4}(4x^2 - 1)$$

$$x^2 + x - 3.75 = x^2 - 0.25$$

$$x = 3.5$$

---

**4** **Use trigonometry**

So

$$AB = 2x - 1 = 6, \qquad AC = 2x + 1 = 8$$

---

**5** **Use cosine rule**

Use the cosine rule to find $BC$:

$$BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 30°$$

$$BC = 4.1063\ldots$$

---

**6** **Use trigonometry**

The largest side is $AC = 8$, so the largest angle is $\angle B$.

---

**7** **Use trigonometry**

Using the cosine rule again:

$$\cos B = \frac{6^2 + BC^2 - 8^2}{2(6)(BC)}$$

$$B = 103.0643\ldots°$$

---

✓ **FINAL ANSWER**

$\boxed{103°}$

# Question 20

5 marks | Sine & Cosine Rules

**20**

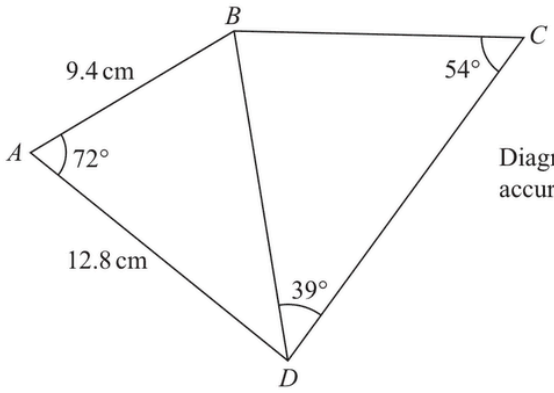



Diagram **NOT** accurately drawn

Work out the length of $BC$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 20 is 5 marks)**

**1** **Use trigonometry**

First use triangle $ABD$.

$$AB = 9.4, \quad AD = 12.8, \quad \angle BAD = 72°$$

**2** **Use cosine rule**

Use the cosine rule to find $BD$:

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

**3** **Use trigonometry**

In triangle $BCD$,

$$\angle BDC = 39°, \quad \angle BCD = 54°$$

**4** **Use the sine rule**

Use the sine rule:

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = \frac{13.3356\ldots \sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

**5** **Use trigonometry**

Correct to 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

$10.4$ cm

4WM1/01 MAY 2025

# Question 21

5 marks | Sine & Cosine Rules

**21**

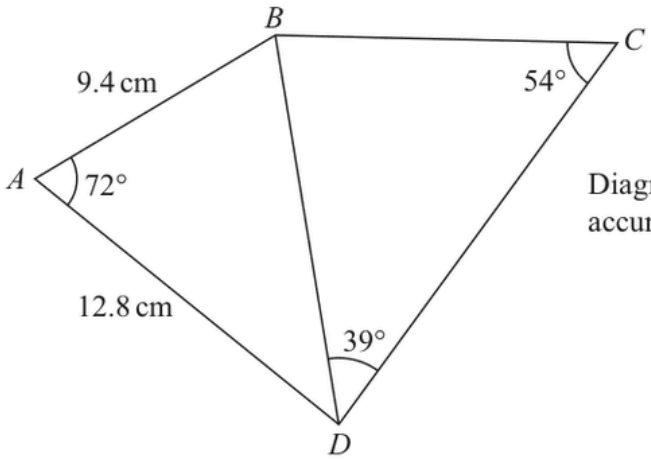



Diagram **NOT** accurately drawn

Work out the length of $BC$
Give your answer correct to 3 significant figures.

..... cm

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4WM1/01 May 2025

**1** **Use trigonometry**

First find $BD$ in triangle $ABD$ using the cosine rule.

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

**2** **Use trigonometry**

In triangle $BCD$, use the sine rule.

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = 13.3356\ldots \times \frac{\sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

**3** **Use trigonometry**

To 3 significant figures,

$$BC = 10.4$$

**FINAL ANSWER**

$10.4\ \text{cm}$



TOPIC

# Area & Volume of Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

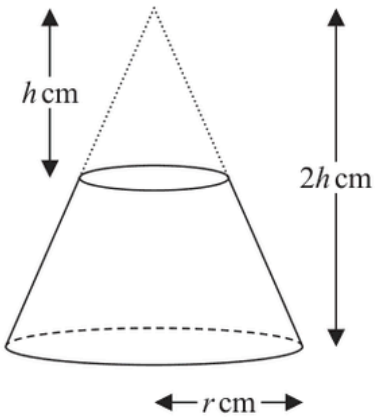
4MA1/1HR JANUARY 2020

# Question 20

5 marks | Area & Volume of Similar Shapes

**20** The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.



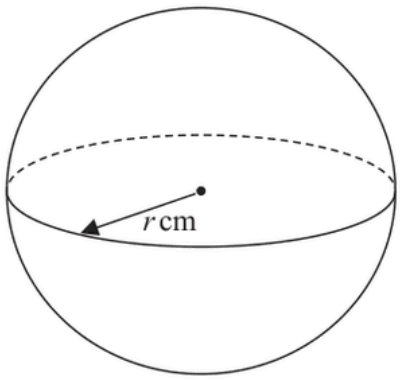




Diagram **NOT** accurately drawn

The height of the small cone is $h$ cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is $r$ cm.

The radius of the sphere is $r$ cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for $r$ in terms of $h$.
Give your expression in its simplest form.

$r =$ .....

**(Total for Question 20 is 5 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR January 2020

**1** **The two cones are similar**

The two cones are similar.

**2** **The large cone has height**

The large cone has height $2h$ and base radius $r$.

**3** **The small cone has height**

The small cone has height $h$, so its linear scale factor compared with the large cone is

$$\frac{h}{2h} = \frac{1}{2}$$

**4** **Define cone dimensions**

So the small cone has radius

$$\frac{r}{2}$$

**5** **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$

**6** **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h = \frac{1}{12}\pi r^2 h$$

**7** **Calculate volume**

So the volume of the frustum is

$$\frac{2}{3}\pi r^2 h - \frac{1}{12}\pi r^2 h = \frac{7}{12}\pi r^2 h$$

**8** **Calculate volume**

The sphere has radius $r$, so its volume is

$$\frac{4}{3}\pi r^3$$

**9** **The two volumes are equal**

The two volumes are equal:

$$\frac{7}{12}\pi r^2 h = \frac{4}{3}\pi r^3$$

**10** **Divide by**

Divide by $\pi r^2$:

$$\frac{7}{12}h = \frac{4}{3}r$$

$$7h = 16r$$

$$r = \frac{7h}{16}$$

---

✓ **FINAL ANSWER**

$$r = \frac{7h}{16}$$

4MA1/2HR JANUARY 2020

# Question 20

4 marks | Area & Volume of Similar Shapes

**20** The diagram shows two similar vases, **A** and **B**.

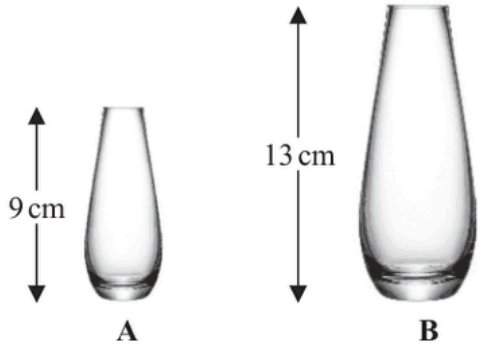



Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

surface area of vase **A** + surface area of vase **B** = 1800 cm$^2$

calculate the surface area of vase **A**.

..... cm$^2$

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR January 2020

**1** **The two vases are similar**

The two vases are similar.

**2** **Their heights are in the**

Their heights are in the ratio

$$9 : 13$$

**3** **Split the ratio**

So their surface areas are in the ratio

$$9^2 : 13^2 = 81 : 169$$

**4** **The total surface area is**

The total surface area is $1800\ \text{cm}^2$, so there are

$$81 + 169 = 250$$

**5** **parts altogether**

parts altogether.

**6** **Surface area of vase A**

Surface area of vase A:

$$1800 \times \frac{81}{250} = 583.2$$

✓ **FINAL ANSWER**

$\boxed{583.2\ \text{cm}^2}$

4MA1/1H MAY/NOVEMBER 2020

# Question 20

3 marks | Area & Volume of Similar Shapes

**20** **R** and **S** are two similar solid shapes.

Shape **R** has surface area 108 cm$^2$ and volume 135 cm$^3$
Shape **S** has surface area 300 cm$^2$

Work out the volume of shape **S**.

..... cm$^3$

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H May/November 2020

**1** **Shape R has surface area**

Shape R has surface area $108\ \text{cm}^2$ and volume $135\ \text{cm}^3$.

**2** **Shape S has surface area**

Shape S has surface area $300\ \text{cm}^2$.

**3** **Surface area ratio S R**

Surface area ratio S:R:

$$300 : 108 = 25 : 9$$

**4** **Split the ratio**

So the linear ratio S:R is

$$5 : 3$$

**5** **Split the ratio**

Therefore the volume ratio S:R is

$$5^3 : 3^3 = 125 : 27$$

**6** **Volume of S**

Volume of S:

$$135 \times \frac{125}{27} = 625$$

**FINAL ANSWER**

$\boxed{625\ \text{cm}^3}$

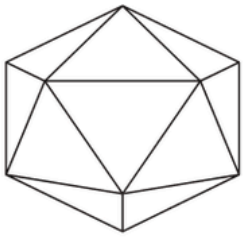
4MA1/1H JANUARY 2021

# Question 20

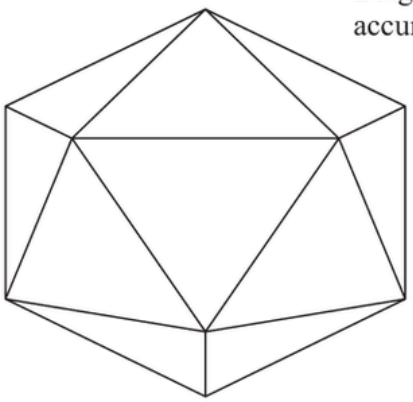
3 marks | Area & Volume of Similar Shapes

20 **A** and **B** are two similar solids.

Diagram **NOT** accurately drawn



**A**



**B**

**A** has a volume of 1836 cm$^3$
**B** has a volume of 4352 cm$^3$

**B** has a total surface area of 1120 cm$^2$

Work out the total surface area of **A**.

..... cm$^2$

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H January 2021

### 1 The volume ratio of solid

The volume ratio of solid A to solid B is

$$1836 : 4352$$

### 2 Simplify

Simplify:

$$\frac{1836}{4352} = \frac{27}{64}$$

### 3 Split the ratio

So the linear ratio is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

**4** **Surface area ratio**

Surface area ratio:

$$3^2 : 4^2 = 9 : 16$$

---

**5** **Solid B has surface area**

Solid B has surface area $1120\ \text{cm}^2$, so solid A has surface area

$$1120 \times \frac{9}{16} = 630$$

---

✓ **FINAL ANSWER**

$\boxed{630\ \text{cm}^2}$

4MA1/2H MAY 2021

# Question 20

3 marks | Area & Volume of Similar Shapes

**20** Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is $300\,\text{cm}^3$

Given that

the surface area of each small block : the surface area of each large block = 25 : 36

work out the volume of each large block.

..... $\text{cm}^3$

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H May 2021

**1 Split the ratio**

The surface area ratio of the small block to the large block is

$$25 : 36$$

**2 Split the ratio**

So the linear ratio is

$$\sqrt{25} : \sqrt{36} = 5 : 6$$

**3 Split the ratio**

Therefore the volume ratio is

$$5^3 : 6^3 = 125 : 216$$

**4 The small block has volume**

The small block has volume $300\ \text{cm}^3$.

**5 Large block volume**

Large block volume:

$$300 \times \frac{216}{125} = 518.4$$

**FINAL ANSWER**

$\boxed{518.4\ \text{cm}^3}$

4MA1/1HR JUNE 2022

# Question 20

4 marks | Area & Volume of Similar Shapes

**20** The diagram shows two similar metal statues.



A



B

Diagram **NOT** accurately drawn

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of $k$
Give your answer correct to 3 significant figures.

$k =$ .....

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1HR June 2022

**1 The volume of statue is**

The volume of statue $B$ is $20\%$ less than statue $A$, so the volume scale factor is $0.8$.

**2 Calculate area**

For similar solids, the linear scale factor is the cube root of the volume scale factor:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

**3 Surface area scale factor is**

Surface area scale factor is the square of the linear scale factor:

$$\text{surface area scale factor} = (\sqrt[3]{0.8})^2 = 0.8^{2/3} = 0.861773\ldots$$

**4** **Calculate area**

So statue $B$ has $86.1773...\%$ of the surface area of statue $A$.

$k = 100 - 86.1773... = 13.8226...$

---

✓ **FINAL ANSWER**

$k = 13.8$ to 3 significant figures.

# Question 24

6 marks | Area & Volume of Similar Shapes

**24** The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

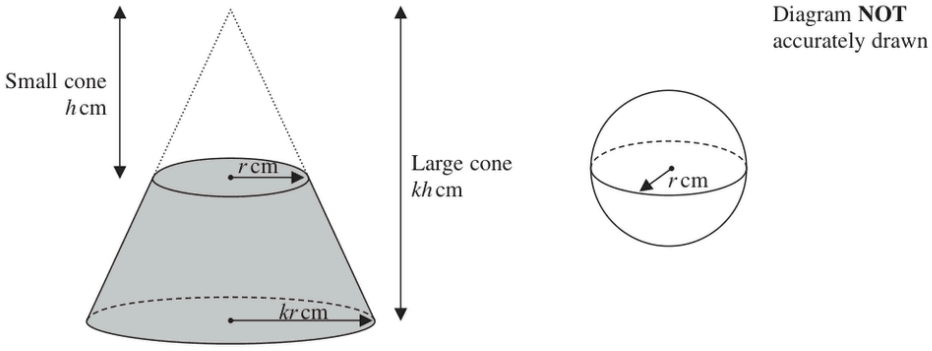



Diagram **NOT** accurately drawn

The height of the small cone is $h$ cm and the radius of the base of the small cone is $r$ cm.
The height of the large cone is $kh$ cm and the radius of the base of the large cone is $kr$ cm.
The radius of the sphere is $r$ cm.

The sphere is divided into two hemispheres, each of radius $r$ cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

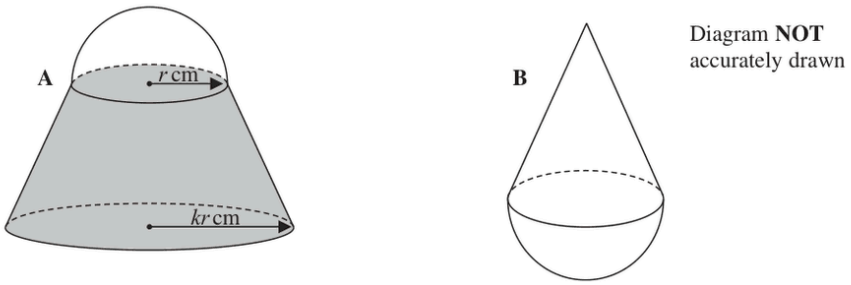



Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for $h$ in terms of $k$ and $r$

$h =$ .....

**(Total for Question 24 is 6 marks)**

### 1 Small cone volume

Small cone volume:

$$V_s = \frac{1}{3}\pi r^2 h$$

### 2 The larger similar cone has

The larger similar cone has radius $kr$ and height $kh$, so its volume is

$$V_L = \frac{1}{3}\pi (kr)^2 (kh) = \frac{1}{3}\pi k^3 r^2 h$$

### 3 Calculate volume

So the frustum volume is

$$V_L - V_s = \frac{1}{3}\pi r^2 h(k^3 - 1)$$

### 4 The volume of one hemisphere

The volume of one hemisphere of radius $r$ is

$$\frac{2}{3}\pi r^3$$

### 5 Solid A is the frustum

Solid A is the frustum plus one hemisphere:

$$V_A = \frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3$$

### 6 Solid B is the small

Solid B is the small cone plus one hemisphere:

$$V_B = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

### 7 Given

Given $V_A = 6V_B$:

$$\frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

**8** **Divide by**

Divide by $\frac{1}{3}\pi r^2$:

$$h(k^3-1)+2r=6h+12r$$

$$h(k^3-7)=10r$$

$$h=\frac{10r}{k^3-7}$$

---

✓ **FINAL ANSWER**

$$h=\frac{10r}{k^3-7}$$

4MA1/1H MAY 2023

# Question 23

5 marks | Area & Volume of Similar Shapes

**23** A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

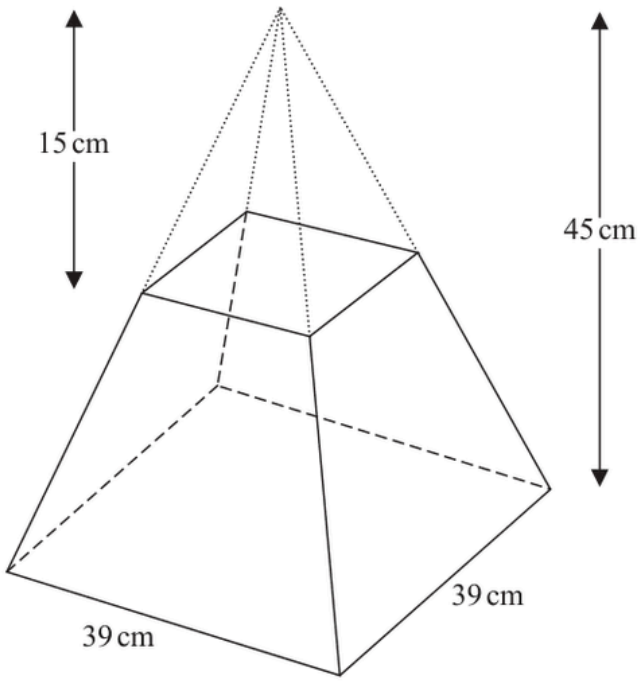



Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

..... $cm^2$

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H May 2023

**1** **The small and large square**

The small and large square-based pyramids are similar.

**2** **Their heights are**

Their heights are

$$15 : 45 = 1 : 3$$

---

**3** **Square of the frustum**

So the side length of the top square of the frustum is

$$\frac{39}{3} = 13 \text{ cm}$$

---

**4** **The vertical height of the**

The vertical height of the frustum is

$$45 - 15 = 30 \text{ cm}$$

---

**5** **Evaluate fraction**

For one trapezium face, the difference between the half-side lengths is

$$\frac{39 - 13}{2} = 13 \text{ cm}$$

---

**6** **Simplify surd**

So the slant height of the trapezium face is

$$\sqrt{30^2 + 13^2} = \sqrt{1069}$$

---

**7** **Area of one trapezium face**

Area of one trapezium face:

$$\frac{1}{2}(39 + 13)\sqrt{1069} = 26\sqrt{1069}$$

---

**8** **Area of four trapezium faces**

Area of four trapezium faces:

$$4(26\sqrt{1069}) = 104\sqrt{1069}$$

---

**9** **Add the top and bottom**

Add the top and bottom square faces:

$$13^2 + 39^2 = 169 + 1521 = 1690$$

---

**10** **Total surface area**

Total surface area:

$$1690 + 104\sqrt{1069} = 5090.338\ldots$$

---

## 11 Correct to the nearest whole

Correct to the nearest whole number:

$$5090$$

---

**FINAL ANSWER**

$$\boxed{5090\ \text{cm}^2}$$

4MA1/1H NOVEMBER 2023

# Question 23

5 marks | Area & Volume of Similar Shapes

**23** Solid **A** is similar to solid **B**

Here is some information about solid **A** and solid **B**

| | solid **A** | solid **B** |
|---|---|---|
| Height (cm) | $3^x$ | |
| Area ($cm^2$) | 7776 | 486 |
| Volume ($cm^3$) | $8^x$ | $2^{x+4}$ |

Work out the height of solid **B**
Give your answer as a decimal.

..... cm

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H November 2023

**1** **The area ratio of solid**

The area ratio of solid A to solid B is

$$7776 : 486 = 16 : 1$$

**2** **Split the ratio**

So the linear ratio A:B is

$$\sqrt{16} : 1 = 4 : 1$$

**3** **Split the ratio**

Therefore the volume ratio A:B is

$$4^3 : 1^3 = 64 : 1$$

**4** **Calculate volume**

The volumes are $8^x$ and $2^{x+4}$, so

$$\frac{8^x}{2^{x+4}} = 64$$

**5** **Use index laws**

Write $8^x$ as a power of 2:

$$8^x = (2^3)^x = 2^{3x}$$

**6** **Evaluate fraction**

So

$$\frac{2^{3x}}{2^{x+4}} = 2^6$$

$$2^{3x-(x+4)} = 2^6$$

$$2x - 4 = 6$$

$$x = 5$$

**7** **The height of solid A**

The height of solid A is

$$3^x = 3^5 = 243$$

**8** **Split the ratio**

Since the linear ratio A:B is $4 : 1$, the height of solid B is

$$\frac{243}{4} = 60.75$$

✓ **FINAL ANSWER**

$\boxed{60.75 \text{ cm}}$

4MA1/2H NOVEMBER 2023

# Question 23

5 marks | Area & Volume of Similar Shapes

**23** Here is a frustum of a cone.
The frustum is made by removing a small cone from a similar large cone.

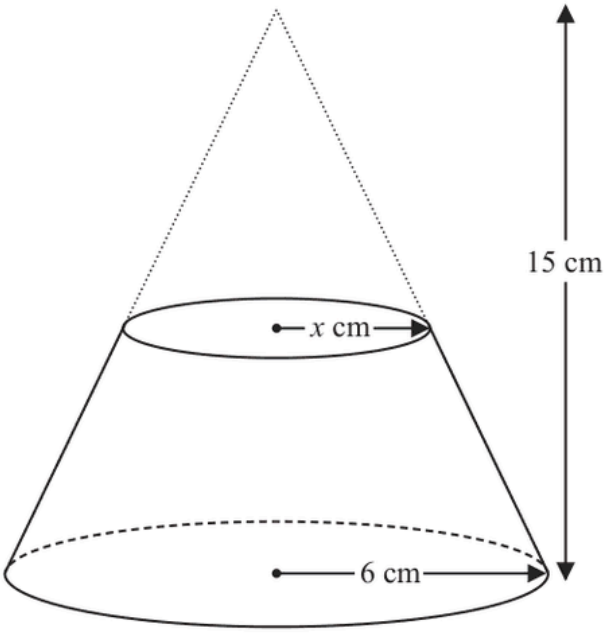



Diagram **NOT** accurately drawn

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is $x$ cm.

Given that the volume of the frustum is $\frac{4212}{25}\pi$ cm$^3$

work out the value of $x$
Show clear algebraic working.

$x =$ .....

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H November 2023

**1** **The large cone has height**

The large cone has height $\mathbf{15}$ cm and radius $\mathbf{6}$ cm.

**2** **The small cone has radius**

The small cone has radius $x$. Since the cones are similar,

$$\frac{\text{radius}}{\text{height}} = \frac{6}{15} = \frac{2}{5}$$

**3** **Evaluate fraction**

So for the small cone,

$$\frac{x}{h} = \frac{2}{5}$$

$$h = \frac{5}{2}x$$

**4** **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi(6)^2(15) = 180\pi$$

**5** **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$$

**6** **The frustum volume is**

The frustum volume is

$$\frac{4212}{25}\pi$$

**7** **Evaluate fraction**

So

$$180\pi - \frac{5}{6}\pi x^3 = \frac{4212}{25}\pi$$

**8** **Cancel**

Cancel $\pi$:

$$180 - \frac{5}{6}x^3 = \frac{4212}{25}$$

$$\frac{5}{6}x^3 = 180 - \frac{4212}{25}$$

$$\frac{5}{6}x^3 = \frac{288}{25}$$

$$x^3 = \frac{1728}{125}$$

$$x = \frac{12}{5} = 2.4$$

---

✓ **FINAL ANSWER**

$\boxed{2.4 \text{ cm}}$

# Question 24

5 marks | Area & Volume of Similar Shapes

**24** The diagram shows a solid, **S**, made from a cone and a hemisphere.

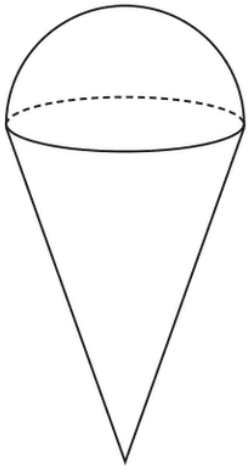


Diagram **NOT** accurately drawn

The centre of the circular face of the cone coincides with the centre of the flat surface of the hemisphere.

The radius of the circular face of the cone, $x$ cm, is equal to the radius of the hemisphere.

The total height of **S** is 4 × the radius of the hemisphere.

A separate sphere has radius $kx$ cm.
The volume of this sphere is 12.5 × the volume of **S**

(a) Work out the value of $k$

$k =$ .....

**(4)**

A solid, **T**, is similar to solid **S**

The volume of **T** is 512 × the volume of **S**

The total surface area of **T** is $d$ × the total surface area of **S**

(b) Find the value of $d$

$d =$ .....

**(1)**

**(Total for Question 24 is 5 marks)**

**1** **The hemisphere has radius**

The hemisphere has radius $x$, so its height is $x$.

**2** **The total height of solid**

The total height of solid S is $4x$, so the cone height is

$$4x - x = 3x$$

**3** **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi x^2(3x) = \pi x^3$$

**4** **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

**5** **Calculate volume**

So the volume of solid S is

$$\pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

**6** **Calculate volume**

The sphere has radius $kx$, so its volume is

$$\frac{4}{3}\pi(kx)^3 = \frac{4}{3}\pi k^3 x^3$$

**7** **This is times the volume**

This is $12.5$ times the volume of S:

$$\frac{4}{3}\pi k^3 x^3 = 12.5\left(\frac{5}{3}\pi x^3\right)$$

**8** **Cancel**

Cancel $\frac{1}{3}\pi x^3$:

$$4k^3 = 62.5$$

$$k^3 = 15.625 = \frac{125}{8}$$

$$k = 2.5$$

---

**9** **Calculate volume**

For solid T, the volume scale factor is $512$.

$$512 = 8^3$$

---

**10** **Calculate area**

So the linear scale factor is $8$, and the surface area scale factor is

$$8^2 = 64$$

---

✓ **FINAL ANSWER**

$$\boxed{k = 2.5,\ d = 64}$$

4WM2/01 MAY 2025

# Question 25

4 marks | Area & Volume of Similar Shapes

25 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** − volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

**(Total for Question 25 is 4 marks)**

## WORKED SOLUTION

Question 25 · 4WM2/01 May 2025

### 1 Linear scale factor from solid

Linear scale factor from solid $A$ to solid $B$:

$$\frac{18.6}{31} = 0.6$$

### 2 Calculate volume

So the volume scale factor is

$$0.6^3 = 0.216$$

### 3 Let the volume of be

Let the volume of $A$ be $V$. Then

$$V - 0.216V = 735$$

$$0.784V = 735$$

$$V = 937.5$$

**FINAL ANSWER**

$937.5\ \text{cm}^3$.

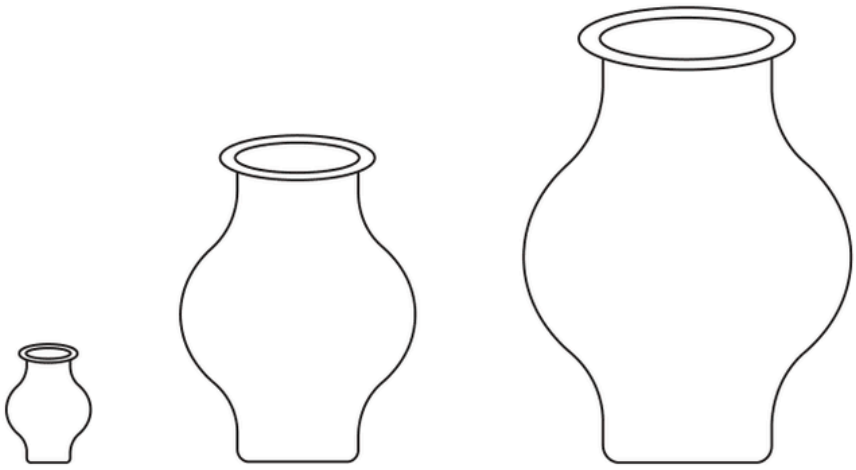
4MA1/1HR JUNE 2025

# Question 26

4 marks | Area & Volume of Similar Shapes

**26** **R**, **S** and **T** are three similar vases.

Diagram **NOT** accurately drawn




The volume of vase **S** is 72.8% more than the volume of vase **R**

The height of vase **R** is $h$ cm
The height of vase **T** is $6h$ cm

The surface area of vase **S** is $A$ cm$^2$
The surface area of vase **T** is $kA$ cm$^2$

Work out the value of $k$

$k =$ .....

**(Total for Question 26 is 4 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/1HR June 2025

**1** **The volume of vase is**

The volume of vase $S$ is $72.8\%$ more than vase $R$, so

$$\frac{V_S}{V_R} = 1.728$$

**2** **Calculate area**

For similar solids, volume scale factor is the cube of the linear scale factor:

$$\text{linear scale factor from } R \text{ to } S = \sqrt[3]{1.728} = 1.2$$

**3** **The height of vase is**

The height of vase $T$ is $6h$, so the linear scale factor from $R$ to $T$ is $6$.

**4** **Calculate area**

Therefore the linear scale factor from $S$ to $T$ is

$$\frac{6}{1.2} = 5$$

**5** **Surface area scale factor is**

Surface area scale factor is the square of the linear scale factor:

$k = 5^2 = 25$

✓ **FINAL ANSWER**

$k = 25$

4MA1/2H JUNE 2025

# Question 26

4 marks | Area & Volume of Similar Shapes

**26** **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** – volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

**(Total for Question 26 is 4 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2H June 2025

**1** **The heights of solids A**

The heights of solids A and B are $31$ cm and $18.6$ cm.

$$31 : 18.6 = 5 : 3$$

**2** **Split the ratio**

So the volumes are in the ratio

$$5^3 : 3^3 = 125 : 27$$

**3** **The difference in volume is**

The difference in volume is

$$125 - 27 = 98$$

**4** **parts**

parts.

**5** **Evaluate fraction**

This difference is $735\ \text{cm}^3$, so one part is

$$\frac{735}{98} = 7.5$$

### 6 Volume of solid A

Volume of solid A:

$$125 \times 7.5 = 937.5$$

---

**FINAL ANSWER**

$\boxed{937.5\ \text{cm}^3}$

4WM2/01 NOVEMBER 2025

# Question 23

3 marks | Area & Volume of Similar Shapes

**23** Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

| | Surface area ($cm^2$) | Volume ($cm^3$) |
|---|---|---|
| Shape **P** | 200 | 672 |
| Shape **Q** | 450 | |

Work out the volume of shape **Q**

..... $cm^3$

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4WM2/01 November 2025

**1 Calculate area**

$$textASF = fractextAreaofQtextAreaofP = frac450200 = frac94 = 2.25$$

**2 Simplify surd**

$$textLSF = sqrttextASF = sqrtfrac94 = frac32 = 1.5$$

**3 Calculate volume**

$$textVSF = (textLSF)^3 = left(frac32right)^3 = frac278 = 3.375$$

**4 Calculate volume**

$$textVolumeofQ = textVolumeofPtimestextVSF = 672timesfrac278$$

= 84 × 27 = 2268

**FINAL ANSWER**

2268 cm<sup>3</sup>

4MA1/2H NOVEMBER 2025

# Question 23

3 marks | Area & Volume of Similar Shapes

**23** Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

| | Surface area ($cm^2$) | Volume ($cm^3$) |
|---|---|---|
| Shape **P** | 200 | 672 |
| Shape **Q** | 450 | |

Work out the volume of shape **Q**

..... $cm^3$

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H November 2025

**1** **Surface area ratio Q P**

Surface area ratio Q:P:

$$450 : 200 = 9 : 4$$

**2** **Split the ratio**

So the linear ratio Q:P is

$$3 : 2$$

**3** **Split the ratio**

Therefore the volume ratio Q:P is

$$3^3 : 2^3 = 27 : 8$$

**4** **Shape P has volume**

Shape P has volume $672\ \text{cm}^3$.

**5** **Volume of Q**

Volume of Q:

$$672 \times \frac{27}{8} = 2268$$

**FINAL ANSWER**

$\boxed{2268\ \text{cm}^3}$

Dr Eslam Ahmed · eliteigcse.com

TOPIC

# Circle Theorems

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JUNE 2023

# Question 20

2 marks | Circle Theorems

**20** $A$, $B$ and $C$ are points on a circle.

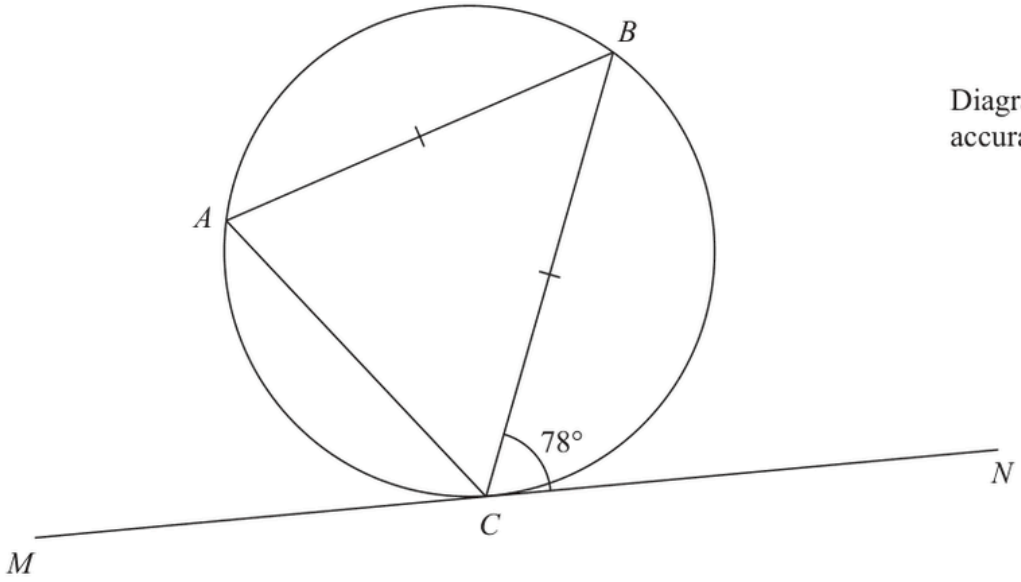



Diagram **NOT** accurately drawn

$MN$ is the tangent to the circle at $C$

$AB = CB$

Angle $BCN = 78°$

Find the size of angle $ABC$

.....°

**(Total for Question 20 is 2 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR June 2023

**1** **The angle between the tangent**

The angle between the tangent $CN$ and the chord $CB$ is $78°$.

**2** **Use trigonometry**

By the alternate segment theorem,

$$\angle BAC = 78°$$

**3** **Calculate value**

Also,

$$AB = CB$$

---

**4** **Use trigonometry**

so triangle $ABC$ is isosceles and the base angles are equal:

$$\angle BAC = \angle ACB = 78°$$

---

**5** **Use trigonometry**

Using the angle sum of a triangle:

$$\angle ABC = 180° - 78° - 78°$$

$$\angle ABC = 24°$$

---

✓ **FINAL ANSWER**

$\boxed{24°}$

4MA1/1HR JUNE 2025

# Question 23

5 marks | Circle Theorems

**23**

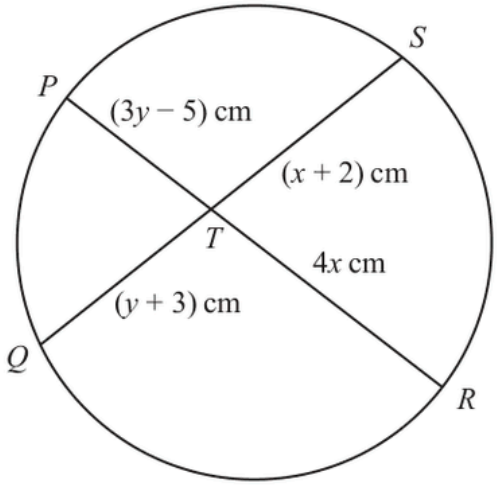



Diagram **NOT** accurately drawn

$PTR$ and $QTS$ are chords of a circle.

$PT = (3y - 5)\,\text{cm}$ $QT = (y + 3)\,\text{cm}$ $RT = 4x\,\text{cm}$ $ST = (x + 2)\,\text{cm}$

Find an expression for $y$ in terms of $x$

$y =$ .....

**Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR June 2025

**1 Use chord theorem**

For two intersecting chords in a circle,

$$PT \times TR = QT \times TS$$

**2 Substitute the given lengths**

Substitute the given lengths:

$$(3y - 5)(4x) = (y + 3)(x + 2)$$

**3 Expand**

Expand:

$$12xy - 20x = xy + 2y + 3x + 6$$

**4** **Collect the terms on one**

Collect the $y$ terms on one side:

$$12xy - xy - 2y = 20x + 3x + 6$$

$$11xy - 2y = 23x + 6$$

---

**5** **Factorise**

Factorise:

$$y(11x - 2) = 23x + 6$$

$$y = \frac{23x + 6}{11x - 2}$$

---

✓ **FINAL ANSWER**

$$\boxed{y = \frac{23x + 6}{11x - 2}}$$



TOPIC

# Bearings

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2024

# Question 24

6 marks | Bearings

**24** $A$, $B$ and $C$ are three points on horizontal ground.

$AB = 8.4$ metres $BC = 9.2$ metres

$B$ is on a bearing of 067° from $A$
$C$ is on a bearing of 129° from $B$

Calculate the bearing of $A$ from $C$
Give your answer correct to the nearest degree.

.....°

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H November 2024

**1 Resolve each journey into east**

Resolve each journey into east and north components.

**2 Use trigonometry**

From $A$ to $B$:

$$\text{east} = 8.4 \sin 67° = 7.7322\ldots$$

$$\text{north} = 8.4 \cos 67° = 3.2821\ldots$$

**3 Use trigonometry**

From $B$ to $C$:

$$\text{east} = 9.2 \sin 129° = 7.1497\ldots$$

$$\text{north} = 9.2 \cos 129° = -5.7897\ldots$$

**4** **Calculate value**

So from $A$ to $C$:

$$\text{east} = 14.8820\ldots$$

$$\text{north} = -2.5076\ldots$$

---

**5** **Calculate value**

Therefore from $C$ to $A$:

$$\text{east} = -14.8820\ldots, \qquad \text{north} = 2.5076\ldots$$

---

**6** **Use trigonometry**

This direction is west of north.

$$\theta = \tan^{-1}\left(\frac{14.8820\ldots}{2.5076\ldots}\right) = 80.4355\ldots^\circ$$

---

**7** **Use bearings**

Bearing of $A$ from $C$:

$$360^\circ - 80.4355\ldots^\circ = 279.5644\ldots^\circ$$

---

✓ **FINAL ANSWER**

$\boxed{280^\circ}$

TOPIC

# Vectors

Every past-paper question on this topic, 2020 to 2026.

# Question 23

5 marks | Vectors

**23**

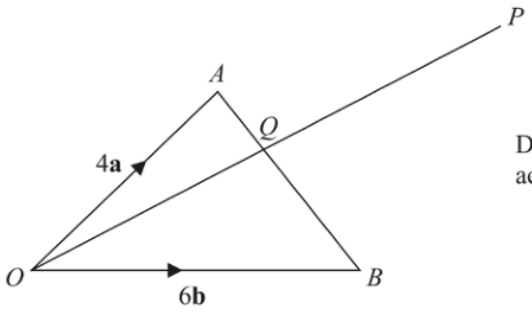



Diagram **NOT** accurately drawn

$OAB$ is a triangle.
$Q$ is the point on $AB$ such that $OQP$ is a straight line.

$\overrightarrow{OA} = \mathbf{4a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AP} = \mathbf{2a} + \mathbf{8b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

**(Total for Question 23 is 5 marks)**

**1** **Use vectors**

$$\overrightarrow{OA} = 4a, \qquad \overrightarrow{OB} = 6b, \qquad \overrightarrow{AP} = 2a + 8b$$

**2** **Use vectors**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4a + (2a + 8b) = 6a + 8b$$

**3** **Use vectors**

Since $O, Q, P$ are collinear, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

**4** **Use vectors**

Also $Q$ lies on $AB$. Let

$$\overrightarrow{AQ} = \mu\overrightarrow{AB}$$

**5** **Use vectors**

Then

$$\overrightarrow{AB} = 6b - 4a$$

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a)$$

$$\overrightarrow{OQ} = (4 - 4\mu)a + 6\mu b$$

**6** **Use vectors**

Compare coefficients with

$$\lambda(6a + 8b) = 6\lambda a + 8\lambda b$$

**7** **Use vectors**

So

$$4 - 4\mu = 6\lambda, \qquad 6\mu = 8\lambda$$

**8** **Use vectors**

From $6\mu = 8\lambda$,

$$\lambda = \frac{3}{4}\mu$$

---

**9** **Substitute**

Substitute:

$$4 - 4\mu = 6\left(\frac{3}{4}\mu\right)$$

$$4 - 4\mu = \frac{9}{2}\mu$$

$$8 = 17\mu$$

$$\mu = \frac{8}{17}$$

---

**10** **Use vectors**

Therefore

$$AQ : QB = \frac{8}{17} : \frac{9}{17} = 8 : 9$$

---

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$.

4MA1/2H JANUARY 2020

# Question 23

3 marks | Vectors

**23**

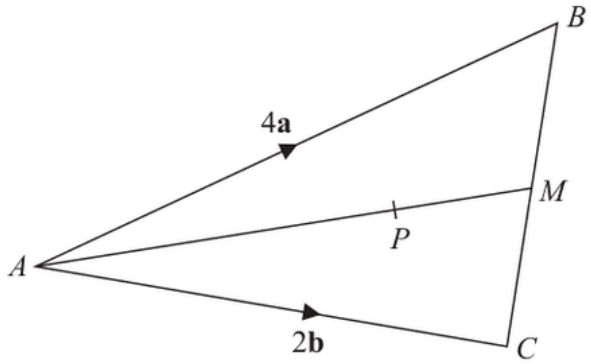



Diagram **NOT** accurately drawn

$ABC$ is a triangle.
The midpoint of $BC$ is $M$.
$P$ is a point on $AM$.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H January 2020

**1** **Split the ratio**

Since $M$ is the midpoint of $BC$,

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = \frac{4a + 2b}{2} = 2a + b$$

**2** **Given**

Given

$$\overrightarrow{AP} = \frac{3}{2}a + \frac{3}{4}b$$

**3** **Factorise**

Factorise:

$$\overrightarrow{AP} = \frac{3}{4}(2a + b) = \frac{3}{4}\overrightarrow{AM}$$

**4** **Split the ratio**

So $AP$ is $\frac{3}{4}$ of $AM$, leaving $PM$ as $\frac{1}{4}$ of $AM$.

✓ **FINAL ANSWER**

$AP : PM = 3 : 1$

4MA1/2HR JANUARY 2020

# Question 26

5 marks | Vectors

**26** The diagram shows trapezium $OACB$.

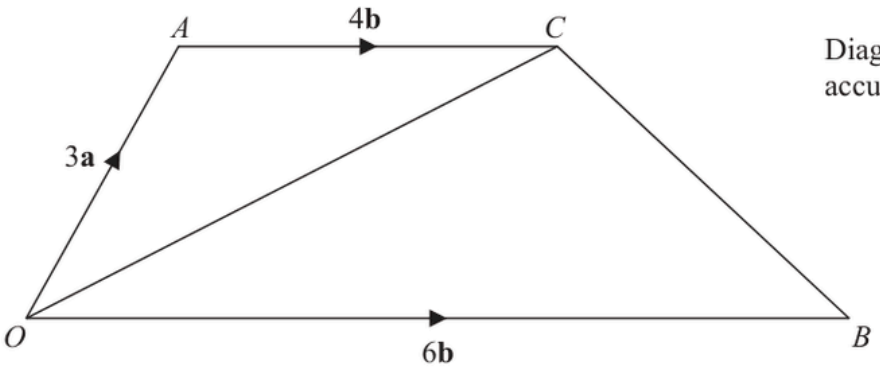



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AC} = 4\mathbf{b}$

$N$ is the point on $OC$ such that $ANB$ is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/2HR January 2020

**1** **Read the graph**

$$\vec{OA} = 3\mathbf{a}, \quad \vec{OB} = 6\mathbf{b}, \quad \vec{AC} = 4\mathbf{b}$$

**2** **Read the graph**

So

$$\vec{OC} = 3\mathbf{a} + 4\mathbf{b}$$

**3** **Read the graph**

Point $N$ lies on $OC$, so

$$\vec{ON} = t(3\mathbf{a} + 4\mathbf{b})$$

**4** **Read the graph**

Point $N$ also lies on $AB$, so

$$\vec{ON} = 3\mathbf{a} + s(6\mathbf{b} - 3\mathbf{a})$$

$$\vec{ON} = (3 - 3s)\mathbf{a} + 6s\mathbf{b}$$

---

**5** **Equate coefficients**

Equate coefficients:

$$3t = 3 - 3s$$

$$4t = 6s$$

---

**6** **Read the graph**

So

$$t = 1 - s$$

---

**7** **Read the graph**

and

$$t = \frac{3}{2}s$$

$$\frac{3}{2}s = 1 - s$$

$$5s = 2$$

$$s = \frac{2}{5}$$

---

**8** **Read the graph**

Then

$$t = \frac{3}{5}$$

---

**9** **Read the graph**

Therefore

$$\vec{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$$

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

---

✓ **FINAL ANSWER**

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

4MA1/1H JANUARY 2021

# Question 23

6 marks | Vectors

**23** $OAB$ is a triangle.

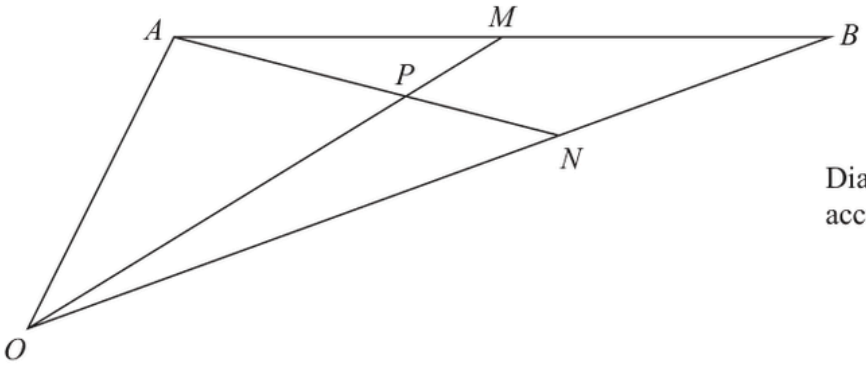



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

$M$ is the midpoint of $AB$.
$N$ is the point on $OB$ such that $ON : NB = 2 : 1$

$P$ is the point on $AN$ such that $OPM$ is a straight line.

Use a vector method to find $OP : PM$
Show your working clearly.

**(Total for Question 23 is 6 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1H January 2021

**1** **Use vectors**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{OB} = 2\mathbf{b}$$

**2** **Find the midpoint**

Since $M$ is the midpoint of $AB$,

$$\vec{OM} = \frac{2\mathbf{a} + 2\mathbf{b}}{2}$$

$$\vec{OM} = \mathbf{a} + \mathbf{b}$$

**3** **Use vectors**

Since $ON : NB = 2 : 1$,

$$\vec{ON} = \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \frac{2}{3}(2\mathbf{b}) = \frac{4}{3}\mathbf{b}$$

---

**4** **Use vectors**

Point $P$ lies on $OM$, so

$$\vec{OP} = t(\mathbf{a} + \mathbf{b})$$

---

**5** **Use vectors**

Point $P$ also lies on $AN$, so

$$\vec{OP} = 2\mathbf{a} + s\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$$

$$\vec{OP} = (2 - 2s)\mathbf{a} + \frac{4s}{3}\mathbf{b}$$

---

**6** **Use vectors**

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$:

$$t = 2 - 2s$$

$$t = \frac{4s}{3}$$

---

**7** **Use vectors**

So

$$\frac{4s}{3} = 2 - 2s$$

$$4s = 6 - 6s$$

$$10s = 6$$

$$s = \frac{3}{5}$$

---

**8** **Use vectors**

Then

$$t = \frac{4}{5}$$

**9** **Use vectors**

So $P$ is $\frac{4}{5}$ of the way from $O$ to $M$.

✓ **FINAL ANSWER**

$OP : PM = 4 : 1$

4MA1/1H MAY 2021

# Question 26

5 marks | Vectors

**26** $OACB$ is a trapezium.

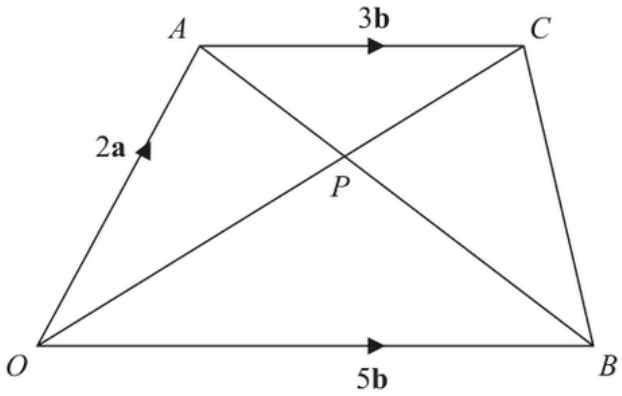



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = 5\mathbf{b}$ $\overrightarrow{AC} = 3\mathbf{b}$

The diagonals, $OC$ and $AB$, of the trapezium intersect at the point $P$.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{OP} =$ .....

**(Total for Question 26 is 5 marks)**

## WORKED SOLUTION

Question 26 · 4MA1/1H May 2021

**1** **Use vectors**

$$\overrightarrow{OA} = 2\mathbf{a}, \quad \overrightarrow{OB} = 5\mathbf{b}, \quad \overrightarrow{AC} = 3\mathbf{b}$$

**2** **Use vectors**

So

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 2\mathbf{a} + 3\mathbf{b}$$

**3** **Use vectors**

Since $P$ lies on $OC$, let

$$\overrightarrow{OP} = t(2\mathbf{a} + 3\mathbf{b})$$

---

**4** **Use vectors**

Since $P$ also lies on $AB$, let

$$\overrightarrow{OP} = \overrightarrow{OA} + s\overrightarrow{AB}$$

---

**5** **Use vectors**

Now

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{b} - 2\mathbf{a}$$

---

**6** **Use vectors**

So

$$\overrightarrow{OP} = 2\mathbf{a} + s(5\mathbf{b} - 2\mathbf{a})$$

$$\overrightarrow{OP} = (2 - 2s)\mathbf{a} + 5s\mathbf{b}$$

---

**7** **Use vectors**

Compare this with

$$t(2\mathbf{a} + 3\mathbf{b}) = 2t\mathbf{a} + 3t\mathbf{b}$$

---

**8** **Equating coefficients**

Equating coefficients:

$$2t = 2 - 2s$$

$$3t = 5s$$

---

**9** **Use vectors**

From $t = 1 - s$,

$$3(1 - s) = 5s$$

$$3 = 8s$$

$$s = \frac{3}{8}$$

---

**10** **Use vectors**

So

$$t = 1 - \frac{3}{8} = \frac{5}{8}$$

---

**11** **Use vectors**

Therefore

$$\overrightarrow{OP} = \frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$$

$$\overrightarrow{OP} = \frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$$

---

✓ **FINAL ANSWER**

$$\boxed{\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}}$$

4MA1/1H JANUARY 2022

# Question 22

5 marks | Vectors

**22** The diagram shows triangle $OAB$

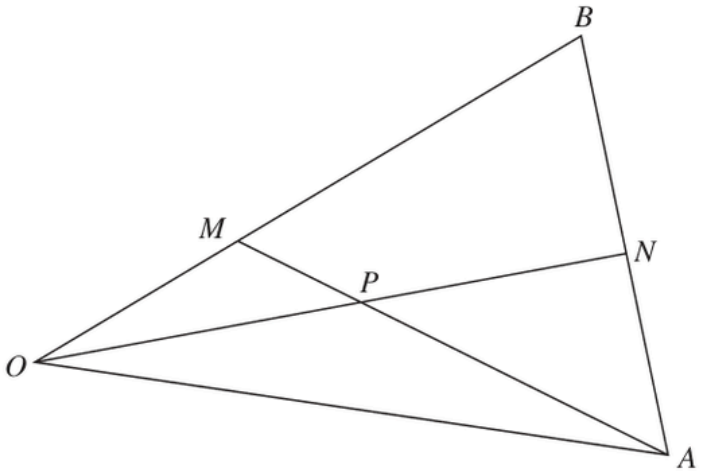



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

$M$ is the point on $OB$ such that $OM : MB = 1 : 2$
$N$ is the midpoint of $AB$
$P$ is the point of intersection of $ON$ and $AM$

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$\overrightarrow{OP} =$ .....

**(Total for Question 22 is 5 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H January 2022

**1** **Use vectors**

$$\vec{OA} = 8\mathbf{a}, \qquad \vec{OB} = 6\mathbf{b}$$

**2** **Use vectors**

Since $OM : MB = 1 : 2$,

$$\vec{OM} = \frac{1}{3}\vec{OB}$$

$$\vec{OM} = 2\mathbf{b}$$

---

**3** **Find the midpoint**

Since $N$ is the midpoint of $AB$,

$$\vec{ON} = \frac{8\mathbf{a} + 6\mathbf{b}}{2}$$

$$\vec{ON} = 4\mathbf{a} + 3\mathbf{b}$$

---

**4** **Use vectors**

Point $P$ lies on $ON$, so

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b})$$

---

**5** **Use vectors**

Point $P$ also lies on $AM$, so

$$\vec{OP} = 8\mathbf{a} + \mu(2\mathbf{b} - 8\mathbf{a})$$

$$\vec{OP} = (8 - 8\mu)\mathbf{a} + 2\mu\mathbf{b}$$

---

**6** **Equate coefficients**

Equate coefficients:

$$4\lambda = 8 - 8\mu$$

$$3\lambda = 2\mu$$

---

**7** **Use vectors**

From $3\lambda = 2\mu$,

$$\mu = \frac{3}{2}\lambda$$

---

**8** **Substitute**

Substitute:

$$4\lambda = 8 - 8\left(\frac{3}{2}\lambda\right)$$

$$4\lambda = 8 - 12\lambda$$

$$16\lambda = 8$$

$$\lambda = \frac{1}{2}$$

---

**9** **Use vectors**

Therefore

$$\vec{OP} = \frac{1}{2}(4\mathbf{a} + 3\mathbf{b})$$

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

---

✓ **FINAL ANSWER**

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

4MA1/1HR JANUARY 2022

# Question 24

5 marks | Vectors

**24** $OXYZ$ is a trapezium.

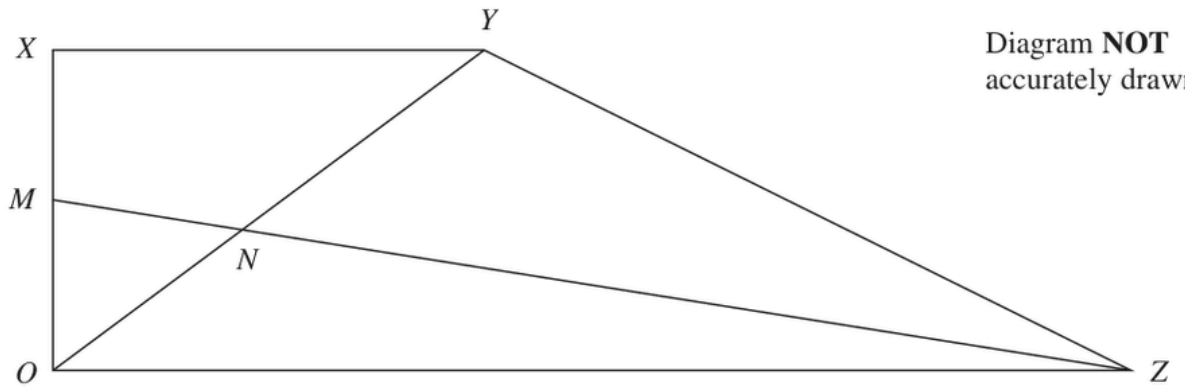



Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

$M$ is the midpoint of $OX$
$N$ is the point such that $MNZ$ and $ONY$ are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of $\lambda$

$\lambda =$ .....

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1HR January 2022

**1** **Take as the origin**

Take $O$ as the origin.

$$\vec{OX} = \mathbf{a}, \qquad \vec{XY} = \mathbf{b}, \qquad \vec{OZ} = 3\mathbf{b}$$

**2** **Use vectors**

So

$$\vec{OY} = \mathbf{a} + \mathbf{b}$$

---

**3** **Find the midpoint**

Since $M$ is the midpoint of $OX$,

$$\vec{OM} = \frac{1}{2}\mathbf{a}$$

---

**4** **Use vectors**

Given $ON : OY = \lambda : 1$,

$$\vec{ON} = \lambda(\mathbf{a} + \mathbf{b})$$

---

**5** **Use vectors**

Point $N$ also lies on $MZ$, so

$$\vec{ON} = \frac{1}{2}\mathbf{a} + t\left(3\mathbf{b} - \frac{1}{2}\mathbf{a}\right)$$

$$\vec{ON} = \left(\frac{1}{2} - \frac{t}{2}\right)\mathbf{a} + 3t\mathbf{b}$$

---

**6** **Use vectors**

Equate coefficients with $\lambda\mathbf{a} + \lambda\mathbf{b}$:

$$\lambda = \frac{1}{2} - \frac{t}{2}$$

$$\lambda = 3t$$

---

**7** **Use vectors**

So

$$t = \frac{\lambda}{3}$$

---

**8** **Substitute**

Substitute:

$$\lambda = \frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{3}\right)$$

$$\lambda = \frac{1}{2} - \frac{\lambda}{6}$$

$$\frac{7}{6}\lambda = \frac{1}{2}$$

$$\lambda = \frac{3}{7}$$

---

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}$$

4MA1/2H JUNE 2022

# Question 21

5 marks | Vectors

**21**

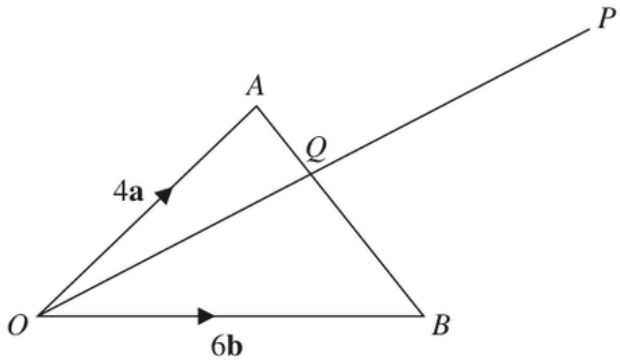



Diagram **NOT** accurately drawn

$OAB$ is a triangle.
$Q$ is the point on $AB$ such that $OQP$ is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H June 2022

**1 Split the ratio**

$$\overrightarrow{OA} = 4a, \quad \overrightarrow{OB} = 6b, \quad \overrightarrow{AP} = 2a + 8b$$

**2 Split the ratio**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4a + (2a + 8b) = 6a + 8b$$

**3 Split the ratio**

Since $Q$ lies on $OP$, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

**4** **Split the ratio**

Since $Q$ lies on $AB$, let

$$\overrightarrow{OQ} = \overrightarrow{OA} + \mu\overrightarrow{AB}$$

$$\overrightarrow{AB} = 6b - 4a$$

---

**5** **Split the ratio**

So

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a) = (4 - 4\mu)a + 6\mu b$$

---

**6** **Equating coefficients**

Equating coefficients:

$$6\lambda = 4 - 4\mu, \quad 8\lambda = 6\mu$$

---

**7** **Split the ratio**

From $8\lambda = 6\mu$, $\mu = \frac{4}{3}\lambda$.

$$6\lambda = 4 - \frac{16}{3}\lambda$$

$$34\lambda = 12, \quad \lambda = \frac{6}{17}$$

$$\mu = \frac{8}{17}$$

---

**8** **Split the ratio**

So $AQ : QB = \frac{8}{17} : \frac{9}{17}$.

---

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$

4MA1/2HR JUNE 2022

# Question 25

5 marks | Vectors

**25** $ABCD$ is a parallelogram and $ADM$ is a straight line.

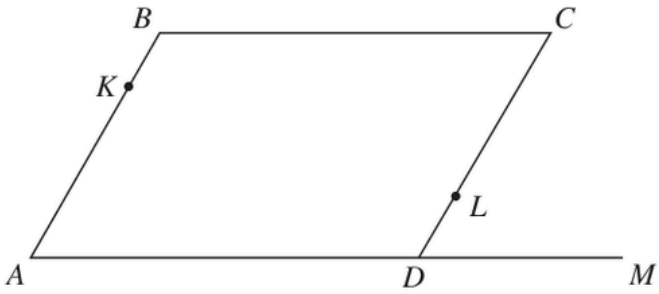



Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$ $\overrightarrow{DM} = \frac{1}{2}\mathbf{b}$

$K$ is the point on $AB$ such that $AK : AB = \lambda : 1$
$L$ is the point on $CD$ such that $CL : CD = \mu : 1$
$KLM$ is a straight line.

Given that $\lambda : \mu = 1 : 2$

use a vector method to find the value of $\lambda$ and the value of $\mu$

$\lambda =$ .....

$\mu =$ .....

**(Total for Question 25 is 5 marks)**

## WORKED SOLUTION

Question 25 · 4MA1/2HR June 2022

**1 Let be the origin**

Let $A$ be the origin.

$$\vec{AB} = \mathbf{a}, \qquad \vec{BC} = \mathbf{b}$$

**2 Read the graph**

So

$$B = \mathbf{a}, \qquad D = \mathbf{b}, \qquad C = \mathbf{a} + \mathbf{b}$$

**3** **Read the graph**

Also

$$\vec{DM} = \frac{1}{2}\mathbf{b}$$

---

**4** **Read the graph**

So

$$M = \frac{3}{2}\mathbf{b}$$

---

**5** **Point is on**

Point $K$ is on $AB$:

$$K = \lambda\mathbf{a}$$

---

**6** **Read the graph**

Point $L$ is on $CD$, and $\vec{CD} = -\mathbf{a}$:

$$L = C + \mu(D - C)$$

$$L = \mathbf{a} + \mathbf{b} + \mu(-\mathbf{a})$$

$$L = (1 - \mu)\mathbf{a} + \mathbf{b}$$

---

**7** **Given**

Given

$$\lambda : \mu = 1 : 2$$

$$\mu = 2\lambda$$

---

**8** **Read the graph**

Since $K, L, M$ are collinear, $\vec{KM}$ is a multiple of $\vec{KL}$.

$$\vec{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$$

$$\vec{KL} = (1 - \mu - \lambda)\mathbf{a} + \mathbf{b}$$

$$\vec{KL} = (1 - 3\lambda)\mathbf{a} + \mathbf{b}$$

---

**9** **Compare coefficients using**

Compare coefficients using

$$\vec{KM} = \frac{3}{2}\vec{KL}$$

$$-\lambda = \frac{3}{2}(1 - 3\lambda)$$

$$-2\lambda = 3 - 9\lambda$$

$$7\lambda = 3$$

$$\lambda = \frac{3}{7}$$

---

**10** **Read the graph**

Then

$$\mu = 2\lambda = \frac{6}{7}$$

---

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}, \quad \mu = \frac{6}{7}$$

# Question 22

5 marks | Vectors

**22**

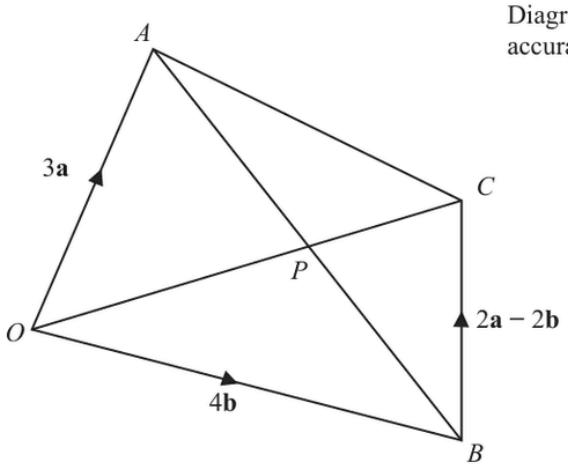



Diagram **NOT** accurately drawn

$OACB$ is a quadrilateral.

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{4b}$ $\overrightarrow{BC} = \mathbf{2a} - \mathbf{2b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$\overrightarrow{OC} =$ .....

**(1)**

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .....

**(1)**

The point $P$ lies on $AB$ and on $OC$

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

.....

**(3)**

**(Total for Question 22 is 5 marks)**

**1** **Split the ratio**

$$\overrightarrow{OA} = 3a, \quad \overrightarrow{OB} = 4b, \quad \overrightarrow{BC} = 2a - 2b$$

**2** **Part (a)**

Part (a):

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 4b + (2a - 2b) = 2a + 2b$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4b - 3a$$

**3** **Split the ratio**

For part (b), let $P$ divide $AB$ so that

$$\overrightarrow{AP} = \mu\overrightarrow{AB}$$

**4** **Split the ratio**

Then

$$\overrightarrow{OP} = 3a + \mu(4b - 3a) = (3 - 3\mu)a + 4\mu b$$

**5** **Split the ratio**

Since $P$ lies on $OC$, also let

$$\overrightarrow{OP} = \lambda(2a + 2b)$$

**6** **Equating coefficients**

Equating coefficients:

$$3 - 3\mu = 2\lambda, \quad 4\mu = 2\lambda$$

**7** **Split the ratio**

From $4\mu = 2\lambda$, $\lambda = 2\mu$.

$$3 - 3\mu = 4\mu$$

$$\mu = \frac{3}{7}$$

**8** **Split the ratio**

So $AP : PB = \frac{3}{7} : \frac{4}{7}$.

**FINAL ANSWER**

$\overrightarrow{OC} = 2\boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{AB} = 4\boldsymbol{b} - 3\boldsymbol{a}$, and $AP : PB = 3 : 4$

4MA1/2H JUNE 2024

# Question 24

6 marks | Vectors

**24**

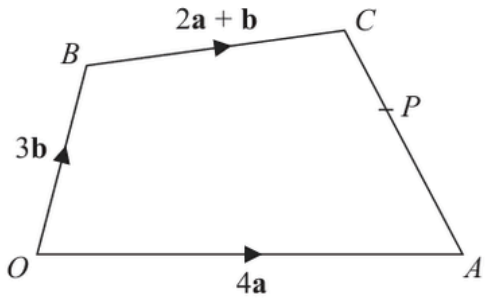



Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OACB$ in which

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 3\mathbf{b}$ $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{AC} =$ .....

(2)

The point $P$ lies on $AC$ such that $AP : PC = 3 : 2$

The point $Q$ is such that $OPQ$ and $BCQ$ are straight lines.

(b) Using a vector method, find $\overrightarrow{OQ}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OQ} =$ .....

(4)

**(Total for Question 24 is 6 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/2H June 2024

**1** **Read the graph**

$$\vec{OA} = 4\mathbf{a}, \quad \vec{OB} = 3\mathbf{b}, \quad \vec{BC} = 2\mathbf{a} + \mathbf{b}$$

**2** **Read the graph**

So

$$\vec{OC} = \vec{OB} + \vec{BC}$$

$$\vec{OC} = 2\mathbf{a} + 4\mathbf{b}$$

---

**3** **Read the graph**

For part (a),

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\vec{AC} = (2\mathbf{a} + 4\mathbf{b}) - 4\mathbf{a}$$

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$$

---

**4** **Read the graph**

For part (b), $AP : PC = 3 : 2$, so

$$\vec{OP} = \vec{OA} + \frac{3}{5}\vec{AC}$$

$$\vec{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a})$$

$$\vec{OP} = \frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

---

**5** **Read the graph**

Since $O, P, Q$ are collinear,

$$\vec{OQ} = t\vec{OP}$$

$$\vec{OQ} = t\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

---

**6** **Read the graph**

Since $B, C, Q$ are collinear,

$$\vec{OQ} = \vec{OB} + s\vec{BC}$$

$$\vec{OQ} = 3\mathbf{b} + s(2\mathbf{a} + \mathbf{b})$$

$$\vec{OQ} = 2s\mathbf{a} + (3 + s)\mathbf{b}$$

---

**7** **Equate coefficients**

Equate coefficients:

$$\frac{14}{5}t = 2s$$

$$\frac{12}{5}t = 3 + s$$

---

**8** **Solve equation**

From the first equation,

$$s = \frac{7}{5}t$$

---

**9** **Substitute into the second**

Substitute into the second:

$$\frac{12}{5}t = 3 + \frac{7}{5}t$$

$$t = 3$$

---

**10** **Read the graph**

Therefore

$$\vec{OQ} = 3\vec{OP}$$

$$\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

---

✓ **FINAL ANSWER**

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a},\ \vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

# Question 25

6 marks | Vectors

**25** $OPQR$ is a parallelogram.

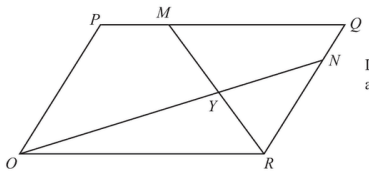



Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point $M$ lies on $PQ$ such that $PM = \frac{1}{4}PQ$

The point $N$ lies on $RQ$ such that $RN = \frac{4}{5}RQ$

(a) Find, in terms of **a** and **b**, giving your answers in simplest form

(i) $\overrightarrow{ON}$

.....

(1)

(ii) $\overrightarrow{MR}$

.....

(1)

$MR$ and $ON$ intersect at the point $Y$

Given that

$$OY = k \times ON$$

(b) use a vector method to find the value of $k$

$k =$ .....

(4)

**(Total for Question 25 is 6 marks)**

**1** **Calculate value**

$$\overrightarrow{OP} = 2a, \qquad \overrightarrow{OR} = 3b$$

**2** **Calculate value**

So

$$\overrightarrow{OQ} = 2a + 3b$$

**3** **Evaluate fraction**

Since $PM : PQ = 1 : 4$,

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{4}(3b) = \frac{3}{4}b$$

$$\overrightarrow{OM} = 2a + \frac{3}{4}b$$

**4** **Evaluate fraction**

Since $RN : RQ = 4 : 5$,

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ}$$

$$\overrightarrow{RN} = \frac{4}{5}(2a) = \frac{8}{5}a$$

$$\overrightarrow{ON} = 3b + \frac{8}{5}a$$

**5** **Evaluate fraction**

So

$$\overrightarrow{MR} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{MR} = 3b - \left(2a + \frac{3}{4}b\right)$$

$$\overrightarrow{MR} = -2a + \frac{9}{4}b$$

---

**6** **Let be on**

Let $Y$ be on $ON$, so

$$\overrightarrow{OY} = k\overrightarrow{ON} = k\left(\frac{8}{5}a + 3b\right)$$

---

**7** **Evaluate fraction**

Also $Y$ is on $MR$, so

$$\overrightarrow{OY} = \overrightarrow{OM} + t\overrightarrow{MR}$$

$$\overrightarrow{OY} = 2a + \frac{3}{4}b + t\left(-2a + \frac{9}{4}b\right)$$

---

**8** **Evaluate fraction**

Compare coefficients of $a$ and $b$:

$$\frac{8}{5}k = 2 - 2t$$

$$3k = \frac{3}{4} + \frac{9}{4}t$$

---

**9** **Solve equation**

From the first equation,

$$t = 1 - \frac{4}{5}k$$

---

**10** **Substitute into the second equation**

Substitute into the second equation:

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = 3 - \frac{9}{5}k$$

$$\frac{24}{5}k = 3$$

$$k = \frac{5}{8}$$

---

✓ **FINAL ANSWER**

$\overrightarrow{ON} = \frac{8}{5}a + 3b$, $\overrightarrow{MR} = -2a + \frac{9}{4}b$, and $k = \frac{5}{8}$

# Question 24

6 marks | Vectors

**24** $OAB$ is a triangle.

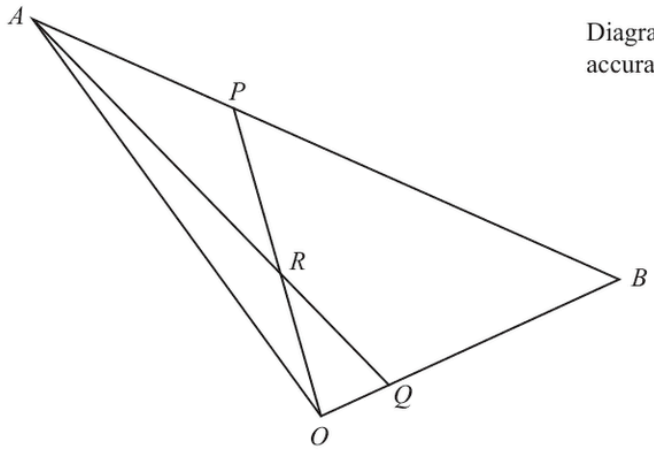



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 10\mathbf{a}$ $\overrightarrow{OB} = 10\mathbf{b}$

$ARQ$ and $ORP$ are straight lines.

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

Write the following vectors in terms of **a** and **b**
Simplify your answers.

(i) $\overrightarrow{AQ}$

.....

(1)

(ii) $\overrightarrow{OP}$

.....

(1)

(iii) $\overrightarrow{OR}$

.....

(4)

**(Total for Question 24 is 6 marks)**

**1** **Read the graph**

$$\vec{OA} = 10\mathbf{a}, \qquad \vec{OB} = 10\mathbf{b}$$

**2** **Read the graph**

Also,

$$\vec{AP} = \frac{1}{4}\vec{AB}$$

**3** **Read the graph**

and

$$\vec{OQ} = \frac{1}{5}\vec{OB}$$

**4** **Read the graph**

For part (i),

$$\vec{OQ} = 2\mathbf{b}$$

$$\vec{AQ} = \vec{OQ} - \vec{OA}$$

$$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$$

**5** **Read the graph**

For part (ii),

$$\vec{AB} = 10\mathbf{b} - 10\mathbf{a}$$

$$\vec{AP} = \frac{1}{4}(10\mathbf{b} - 10\mathbf{a})$$

$$\vec{AP} = \frac{5}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

**6** **Read the graph**

For part (iii), $R$ lies on $OP$, so

$$\vec{OR} = t\vec{OP}$$

**7** **Read the graph**

It also lies on $AQ$, so

$$\vec{OR} = \vec{OA} + s\vec{AQ}$$

$$\vec{OR} = 10\mathbf{a} + s(2\mathbf{b} - 10\mathbf{a})$$

$$\vec{OR} = (10 - 10s)\mathbf{a} + 2s\mathbf{b}$$

**8** **Read the graph**

Equate with

$$t\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$$

$$\frac{15}{2}t = 10 - 10s$$

$$\frac{5}{2}t = 2s$$

**9** **Solve equation**

From the second equation,

$$s = \frac{5}{4}t$$

**10** **Substitute**

Substitute:

$$\frac{15}{2}t = 10 - \frac{25}{2}t$$

$$20t = 10$$

$$t = \frac{1}{2}$$

**11** **Read the graph**

So

$$\vec{OR} = \frac{1}{2}\vec{OP}$$

$$\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$$

---

✓ **FINAL ANSWER**

$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$, $\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$, $\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$

# Question 23

5 marks | Vectors

**23**

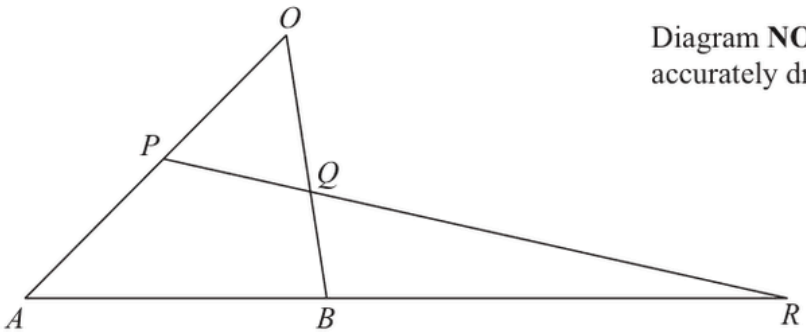



Diagram **NOT** accurately drawn

$OAB$ is a triangle.
$P$ is the midpoint of $OA$
$Q$ is a point on $OB$

$ABR$ and $PQR$ are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\overrightarrow{OB} = 8\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**

.....

**(1)**

$AB : BR = 1 : 2$ $\overrightarrow{OQ} = n\mathbf{b}$

(b) Use a vector method to find the value of $n$

$n =$ .....

**(4)**

**(Total for Question 23 is 5 marks)**

**1** **Use vectors**

For part (a),

$$\vec{OA} = 12\mathbf{a}, \qquad \vec{OB} = 8\mathbf{b}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}$$

**2** **Find the midpoint**

For part (b), $P$ is the midpoint of $OA$, so

$$\vec{OP} = 6\mathbf{a}$$

**3** **Given**

Given

$$AB : BR = 1 : 2$$

$$\vec{BR} = 2\vec{AB}$$

$$\vec{BR} = 16\mathbf{b} - 24\mathbf{a}$$

**4** **Use vectors**

Therefore

$$\vec{OR} = \vec{OB} + \vec{BR}$$

$$\vec{OR} = 8\mathbf{b} + 16\mathbf{b} - 24\mathbf{a}$$

$$\vec{OR} = -24\mathbf{a} + 24\mathbf{b}$$

**5** **Use vectors**

Since $P, Q, R$ are collinear, write $Q$ on $PR$:

$$\vec{OQ} = \vec{OP} + t(\vec{OR} - \vec{OP})$$

$$\vec{OQ} = 6\mathbf{a} + t(-30\mathbf{a} + 24\mathbf{b})$$

$$\vec{OQ} = (6 - 30t)\mathbf{a} + 24t\mathbf{b}$$

---

**6** **Use vectors**

But $Q$ lies on $OB$, so there is no $\mathbf{a}$ component:

$$6 - 30t = 0$$

$$t = \frac{1}{5}$$

---

**7** **Use vectors**

Then

$$\vec{OQ} = 24\left(\frac{1}{5}\right)\mathbf{b}$$

$$\vec{OQ} = \frac{24}{5}\mathbf{b}$$

---

**8** **Use vectors**

Since $\vec{OQ} = n\mathbf{b}$,

$$n = \frac{24}{5}$$

---

✓ **FINAL ANSWER**

$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}, \quad n = \frac{24}{5}$

# Question 26

5 marks | Vectors

**26**

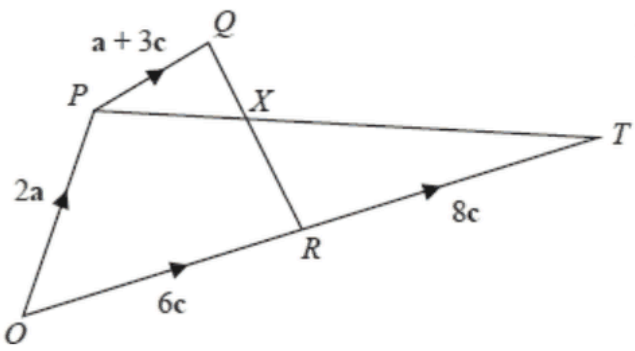



Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OPQR$ in which

$\overrightarrow{OP} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{c}$ $\overrightarrow{PQ} = \mathbf{a} + 3\mathbf{c}$

(a) Find in terms of **a** and **c**

$\overrightarrow{QR}$ = .....

**(1)**

The point $T$ is such that $\overrightarrow{RT} = 8\mathbf{c}$
The point $X$ lies on $QR$ such that $PXT$ is a straight line.

(b) Using a vector method, find the ratio $QX : XR$
Give your answer in the form $m : n$ where $m$ and $n$ are integers.
Show your working clearly.


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$QX : XR$ = .....

**(4)**

**(Total for Question 26 is 5 marks)**

## 1 (a)

(a)

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$\overrightarrow{OQ} = 2a + (a + 3c) = 3a + 3c$$

$$\overrightarrow{QR} = 6c - (3a + 3c) = 3c - 3a$$

## 2 (b) Let divide so that

(b) Let $X$ divide $QR$ so that

$$\overrightarrow{QX} = \lambda\overrightarrow{QR}$$

## 3 Calculate value

Then

$$\overrightarrow{OX} = 3a + 3c + \lambda(3c - 3a)$$

## 4 Calculate value

Also $X$ lies on $PT$, where $\overrightarrow{OT} = 14c$, so

$$\overrightarrow{OX} = 2a + \mu(14c - 2a)$$

## 5 Equating coefficients gives . Therefore

Equating coefficients gives $\lambda = \frac{5}{9}$. Therefore

$$QX : XR = \frac{5}{9} : \frac{4}{9} = 5 : 4$$

**FINAL ANSWER**

$\overrightarrow{QR} = 3c - 3a$; $QX : XR = 5 : 4$.

# Question 24

6 marks | Vectors

**24** The diagram shows a quadrilateral $OABC$

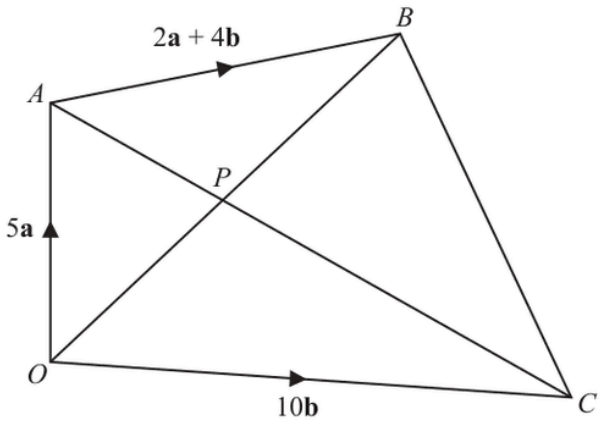



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 5\mathbf{a}$ $\overrightarrow{OC} = 10\mathbf{b}$ $\overrightarrow{AB} = 2\mathbf{a} + 4\mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**

$\overrightarrow{AC} =$ .....

**(1)**

(b) Find $\overrightarrow{OB}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{OB} =$ .....

**(1)**

The point $P$ is such that $APC$ and $OPB$ are straight lines.

(c) Using a vector method, find $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OP} =$ .....

**(4)**

**(Total for Question 24 is 6 marks)**

1 **(a) Find** $overrightarrow{AC}$

$overrightarrow{AC} = overrightarrow{AO} + overrightarrow{OC} = -overrightarrow{OA} + overrightarrow{OC} = -5mathbf{a} + 10mathbf{b}$

2 **Answer**

$a$ − 5**a** + 10**b**

3 **(b) Find** $overrightarrow{OB}$

$overrightarrow{OB} = overrightarrow{OA} + overrightarrow{AB} = 5mathbf{a} + (2mathbf{a} + 4mathbf{b}) = 7mathbf{a} + 4mathbf{b}$

4 **Answer**

$b$ 7**a** + 4**b**

**Step 1: Set up vector equations for** $overrightarrow{OP}$**.** Since *O**P**B* is a straight line, $overrightarrow{OP}$ is a scalar multiple of $overrightarrow{OB}$. Let this be *λ*.

$$overrightarrow{OP} = lambda(7mathbf{a} + 4mathbf{b}) = 7lambdamathbf{a} + 4lambdamathbf{b}$$

Since *P* lies on the straight line *A**C*, $overrightarrow{AP} = muoverrightarrow{AC}$ for some scalar *μ*.

$$overrightarrow{OP} = overrightarrow{OA} + overrightarrow{AP} = 5mathbf{a} + mu(-5mathbf{a} + 10mathbf{b})$$

$$overrightarrow{OP} = (5 - 5mu)mathbf{a} + 10mumathbf{b}$$

**Step 2: Equate the coefficients of** a **and** b**.**

7*λ* = 5 − 5*μ* — (1)

$$4lambda = 10mu implies mu = frac{4}{10}lambda = 0.4lambda quad text{---(2)}$$

**Step 3: Solve for *λ*.** Substitute (2) into (1):

7*λ* = 5 − 5(0.4*λ*)

7*λ* = 5 − 2*λ*

$$9lambda = 5 implies lambda = frac{5}{9}$$

5 **Evaluate fraction**

$$overrightarrow{OP} = frac{5}{9}(7mathbf{a} + 4mathbf{b}) = frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

✓ **FINAL ANSWER**

$$frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

4MA1/1H NOVEMBER 2025

# Question 27

6 marks | Vectors

27

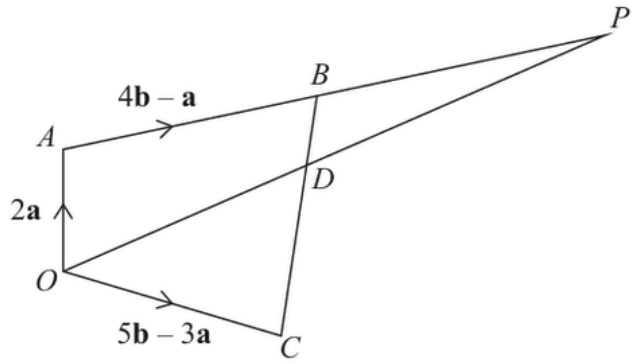



Diagram **NOT** accurately drawn

$OABC$ is a quadrilateral.
$ABP$ and $ODP$ are straight lines.

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{AB} = 4\mathbf{b} - \mathbf{a}$ $\overrightarrow{OC} = 5\mathbf{b} - 3\mathbf{a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

.....

(2)

The point $D$ lies on $BC$ such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of $n$

$n =$ .....

(4)

**(Total for Question 27 is 6 marks)**

## WORKED SOLUTION

Question 27 · 4MA1/1H November 2025

**1** **Read the graph**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{AB} = 4\mathbf{b} - \mathbf{a}, \qquad \vec{OC} = 5\mathbf{b} - 3\mathbf{a}$$

**2** **Read the graph**

First find $\vec{OB}$:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = 2\mathbf{a} + 4\mathbf{b} - \mathbf{a}$$

$$\vec{OB} = \mathbf{a} + 4\mathbf{b}$$

---

**3** **Read the graph**

For part (a),

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$\vec{BC} = (5\mathbf{b} - 3\mathbf{a}) - (\mathbf{a} + 4\mathbf{b})$$

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}$$

---

**4** **Read the graph**

For part (b), $BD : DC = 1 : 3$, so

$$\vec{OD} = \vec{OB} + \frac{1}{4}\vec{BC}$$

$$\vec{OD} = \mathbf{a} + 4\mathbf{b} + \frac{1}{4}(\mathbf{b} - 4\mathbf{a})$$

$$\vec{OD} = \frac{17}{4}\mathbf{b}$$

---

**5** **Read the graph**

Since $A, B, P$ are collinear, let

$$\vec{OP} = \vec{OA} + s\vec{AB}$$

$$\vec{OP} = 2\mathbf{a} + s(4\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = (2 - s)\mathbf{a} + 4s\mathbf{b}$$

---

**6** **Read the graph**

But $O, D, P$ are collinear, and $\vec{OD}$ has only a $\mathbf{b}$ component.

---

**7** **Read the graph**

So

$$2 - s = 0$$

$$s = 2$$

---

**8** **Read the graph**

Then

$$\vec{OP} = 8\mathbf{b}$$

---

**9** **Given**

Given

$$\vec{OP} = n\vec{OD}$$

$$8\mathbf{b} = n\left(\frac{17}{4}\mathbf{b}\right)$$

$$n = \frac{32}{17}$$

---

✓ **FINAL ANSWER**

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}, \quad n = \frac{32}{17}$$

TOPIC

# Histograms

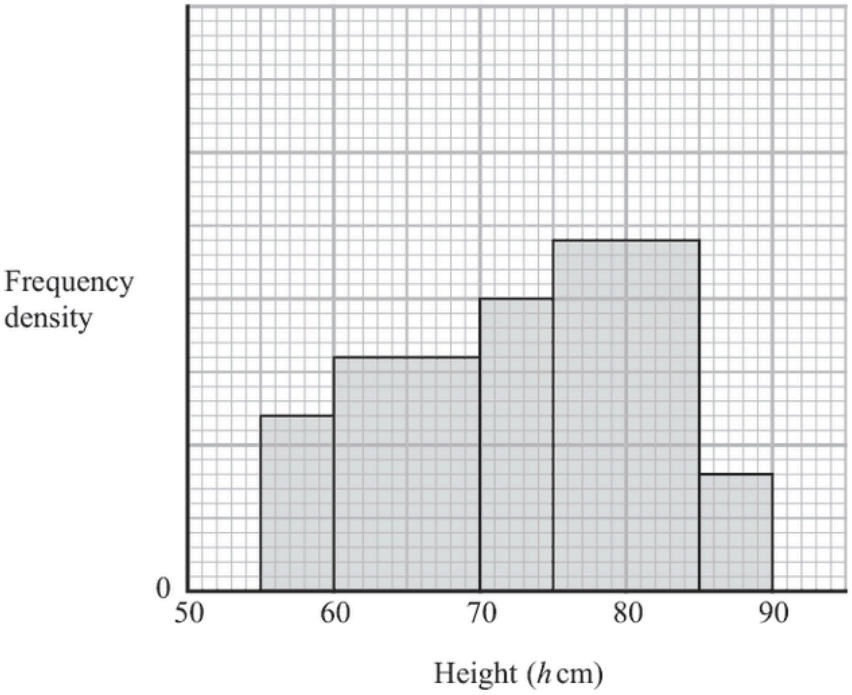
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2022

# Question 20

4 marks | Histograms

**20** The histogram gives information about the heights, $h$ cm, of some tomato plants.




There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H January 2022

### 1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

### 2 Use frequency table

For $75 < h \leq 85$, the frequency is 12 and the class width is 10, so

$$\text{frequency density} = \frac{12}{10} = 1.2$$

### 3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$0.6,\ 0.8,\ 1.0,\ 1.2,\ 0.4$$

### 4 The total number of tomato

The total number of tomato plants is

$$(5)(0.6) + (10)(0.8) + (5)(1.0) + (10)(1.2) + (5)(0.4)$$

$$= 3 + 8 + 5 + 12 + 2 = 30$$

### 5 Solve inequality

For $h > 82.5$:

$$(2.5)(1.2) + (5)(0.4) = 3 + 2 = 5$$

### 6 Calculate probability

So the probability is

$$\frac{5}{30} = \frac{1}{6}$$

**FINAL ANSWER**

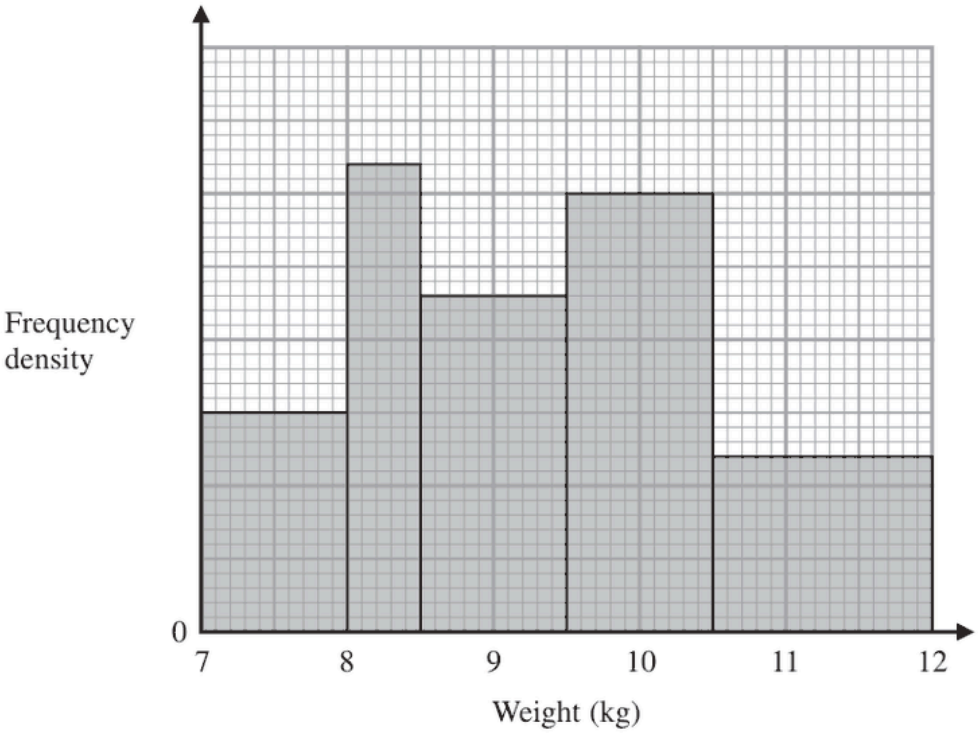
$\frac{1}{6}$

4MA1/1HR JANUARY 2022

# Question 21

3 marks | Histograms

**21**




The histogram gives information about the weights, in kg, of all the watermelons in a field.

There are 16 watermelons with a weight between 8 kg and 8.5 kg

Work out the total number of watermelons in the field.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR January 2022

**1** **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

**2** **Use frequency table**

For $8 \leq w < 8.5$, the frequency is 16 and the class width is $0.5$, so

$$\text{frequency density} = \frac{16}{0.5} = 32$$

### 3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$15,\ 32,\ 23,\ 30,\ 12$$

### 4 The total frequency is

The total frequency is

$$(1)(15) + (0.5)(32) + (1)(23) + (1)(30) + (1.5)(12)$$

$$= 15 + 16 + 23 + 30 + 18$$

$$= 102$$

**FINAL ANSWER**

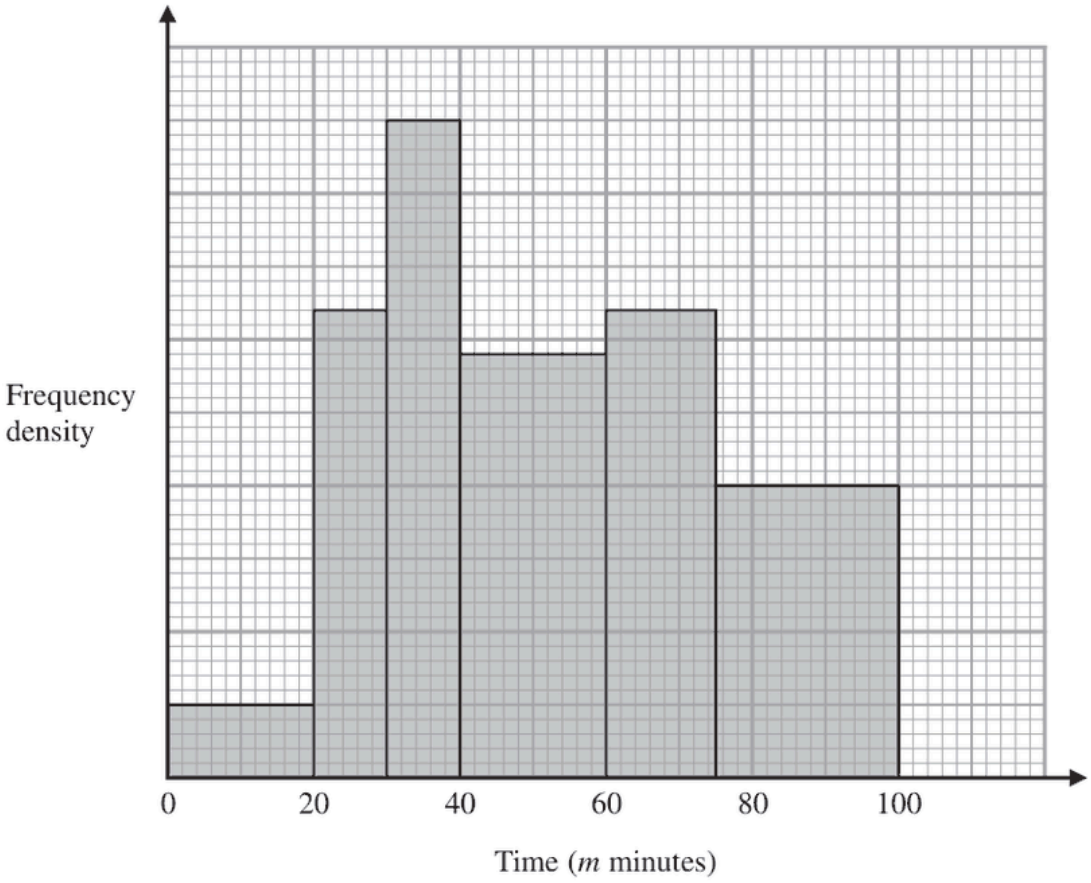
$102$ watermelons

4MA1/1H JUNE 2022

# Question 21

3 marks | Histograms

**21** The histogram shows information about the total time, $m$ minutes, taken by each child in a school to walk to school every day for one week.




There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$

Work out an estimate for the number of children for whom $50 < m \leqslant 80$

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H June 2022

**1** **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

**2** **Use frequency table**

For $m \leq 20$, the frequency is 10 and the class width is 20, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

---

**3** **Use histogram**

Using this scale from the histogram:

$$40 < m \leq 60 \quad \text{has frequency density } 2.9$$

$$60 < m \leq 75 \quad \text{has frequency density } 3.2$$

$$75 < m \leq 100 \quad \text{has frequency density } 2.0$$

---

**4** **Use part of each relevant**

For $50 < m \leq 80$, use part of each relevant bar:

$$(10)(2.9) + (15)(3.2) + (5)(2.0)$$

$$= 29 + 48 + 10$$

$$= 87$$

---

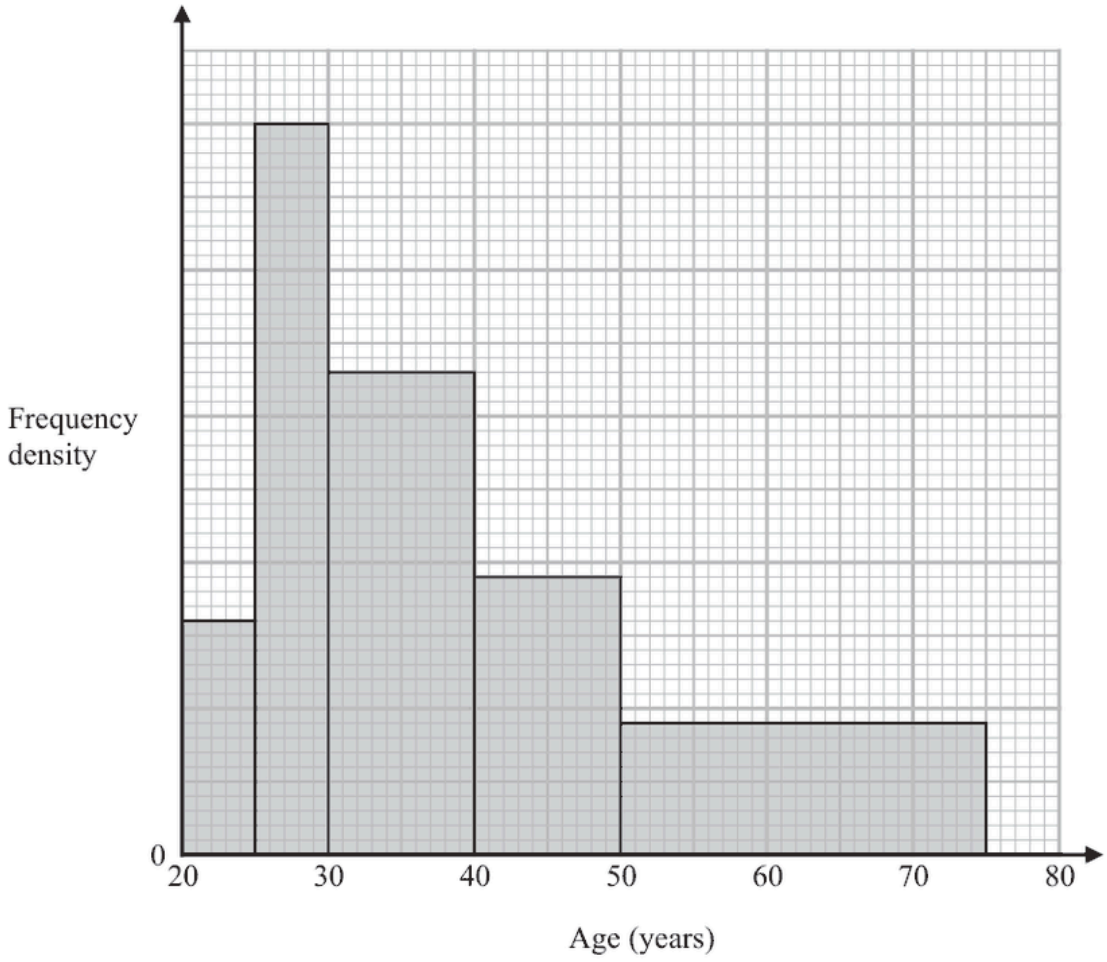
✓ **FINAL ANSWER**

87 children

# Question 22

4 marks | Histograms

**22** Some people attend a concert.
The histogram shows information about the ages of these people.




Work out an estimate for the percentage of these people who are aged more than 55 years.
Give your answer correct to one decimal place.

.....%

**(Total for Question 22 is 4 marks)**

## 1 The scale of the frequency

The scale of the frequency density axis is not needed because the percentage is found from areas.

## 2 Calculate area

Using the bar heights from the histogram, the relative areas are:

$$(5)(277) + (5)(866) + (10)(571) + (10)(329) + (25)(155)$$

$$= 1385 + 4330 + 5710 + 3290 + 3875$$

$$= 18590$$

## 3 Calculate value

For ages more than 55, use the part of the last bar from 55 to 75:

$$(20)(155) = 3100$$

## 4 Calculate percentage

So the estimated percentage is

$$\frac{3100}{18590} \times 100 = 16.675\ldots$$

**FINAL ANSWER**

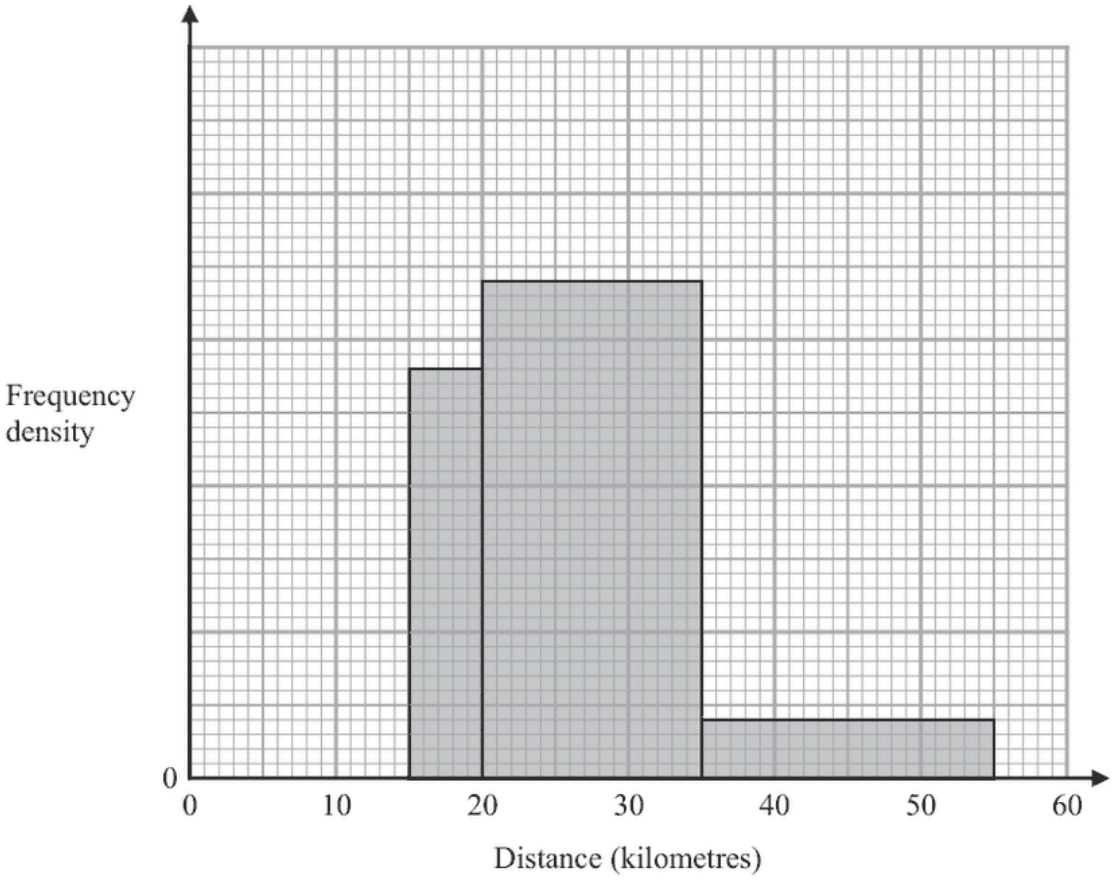
$16.7\%$

4MA1/1H MAY 2024

# Question 22

3 marks | Histograms

22 The incomplete histogram shows some information about the distances, in kilometres, that 100 adults ran last week.




All of the adults ran at least 5 kilometres.
None of the adults ran more than 55 kilometres.
14 adults ran between 15 kilometres and 20 kilometres.

Complete the histogram.

**(Total for Question 22 is 3 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H May 2024

**1 Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

**2** **Use frequency table**

For $15 \leq d < 20$, the frequency is 14 and the class width is 5, so

$$\text{frequency density} = \frac{14}{5} = 2.8$$

**3** **Use histogram**

Using this scale from the histogram:

$$20 \leq d < 35 \quad \text{has frequency density } 3.4$$

$$35 \leq d \leq 55 \quad \text{has frequency density } 0.4$$

**4** **Calculate value**

So the known frequencies are

$$14 + (15)(3.4) + (20)(0.4)$$

$$= 14 + 51 + 8 = 73$$

**5** **Use frequency table**

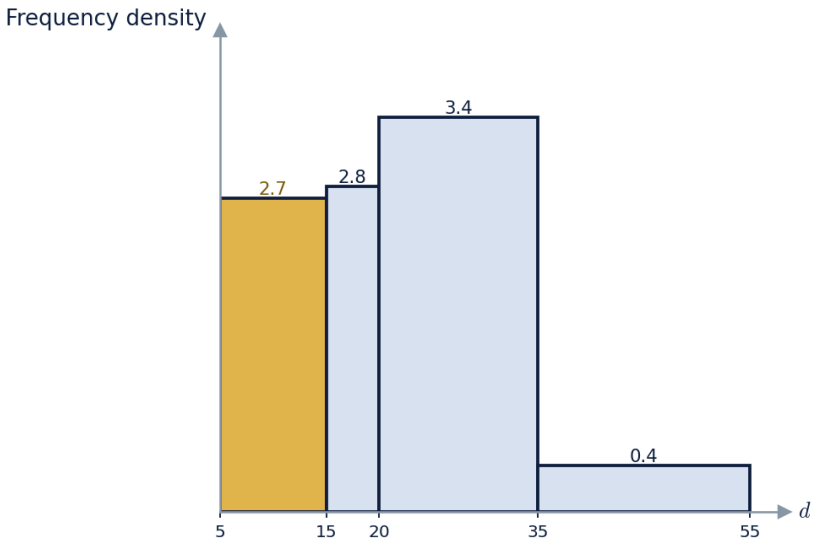
There are 100 adults in total, so the missing frequency from 5 km to 15 km is

$$100 - 73 = 27$$

**6** **The missing class width is**

The missing class width is 10, so its frequency density is

$$\frac{27}{10} = 2.7$$



**FINAL ANSWER**

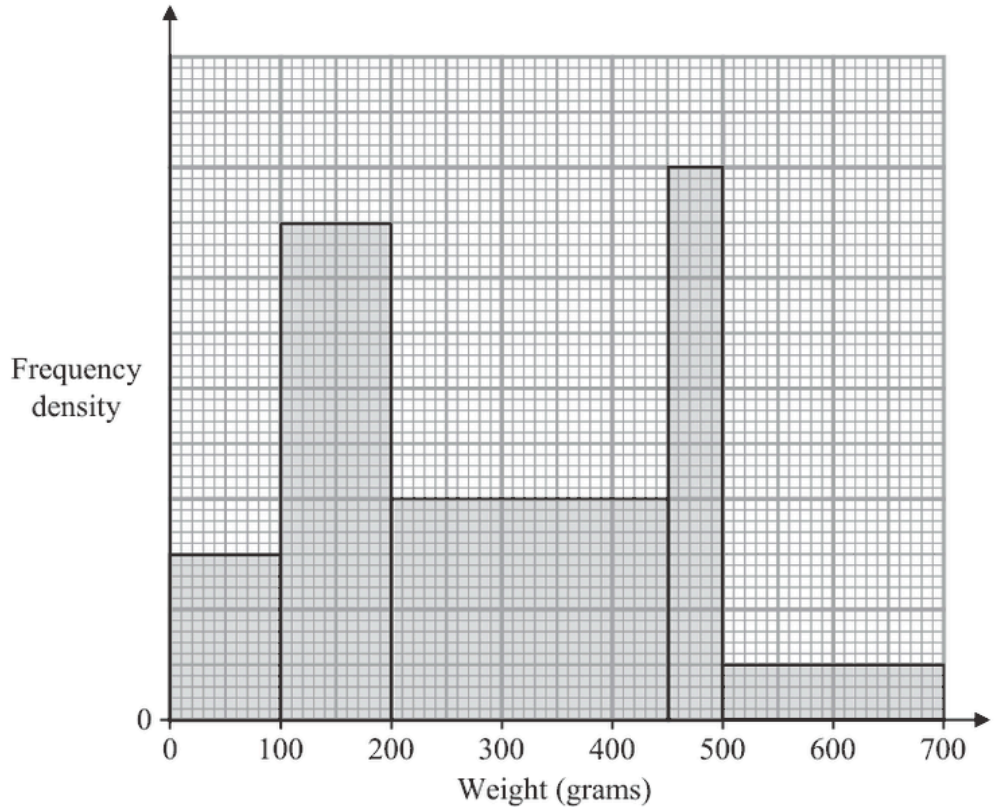
draw the missing bar from $5$ to $15$ with frequency density $2.7$

4MA1/2HR JUNE 2025

# Question 20

4 marks | Histograms

**20** The histogram gives some information about the weights, in grams, of some books.




75 books weigh less than 100 grams.

A book is chosen at random.

Find an estimate for the probability that this book weighs between 400 grams and 600 grams.

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2HR June 2025

**1** **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

**2** **Use frequency table**

For books weighing less than 100 g, the frequency is 75 and the class width is 100, so

$$\text{frequency density} = \frac{75}{100} = 0.75$$

**3** **Use histogram**

Using this scale from the histogram, the frequency densities are:

$$0.75,\ 2.25,\ 1.0,\ 2.5,\ 0.25$$

**4** **The total number of books**

The total number of books is

$$(100)(0.75) + (100)(2.25) + (250)(1.0) + (50)(2.5) + (200)(0.25)$$

$$= 75 + 225 + 250 + 125 + 50 = 725$$

**5** **Calculate value**

For weights between 400 g and 600 g:

$$(50)(1.0) + (50)(2.5) + (100)(0.25)$$

$$= 50 + 125 + 25 = 200$$

**6** **Calculate probability**

So the probability is

$$\frac{200}{725} = \frac{8}{29}$$

✓ **FINAL ANSWER**

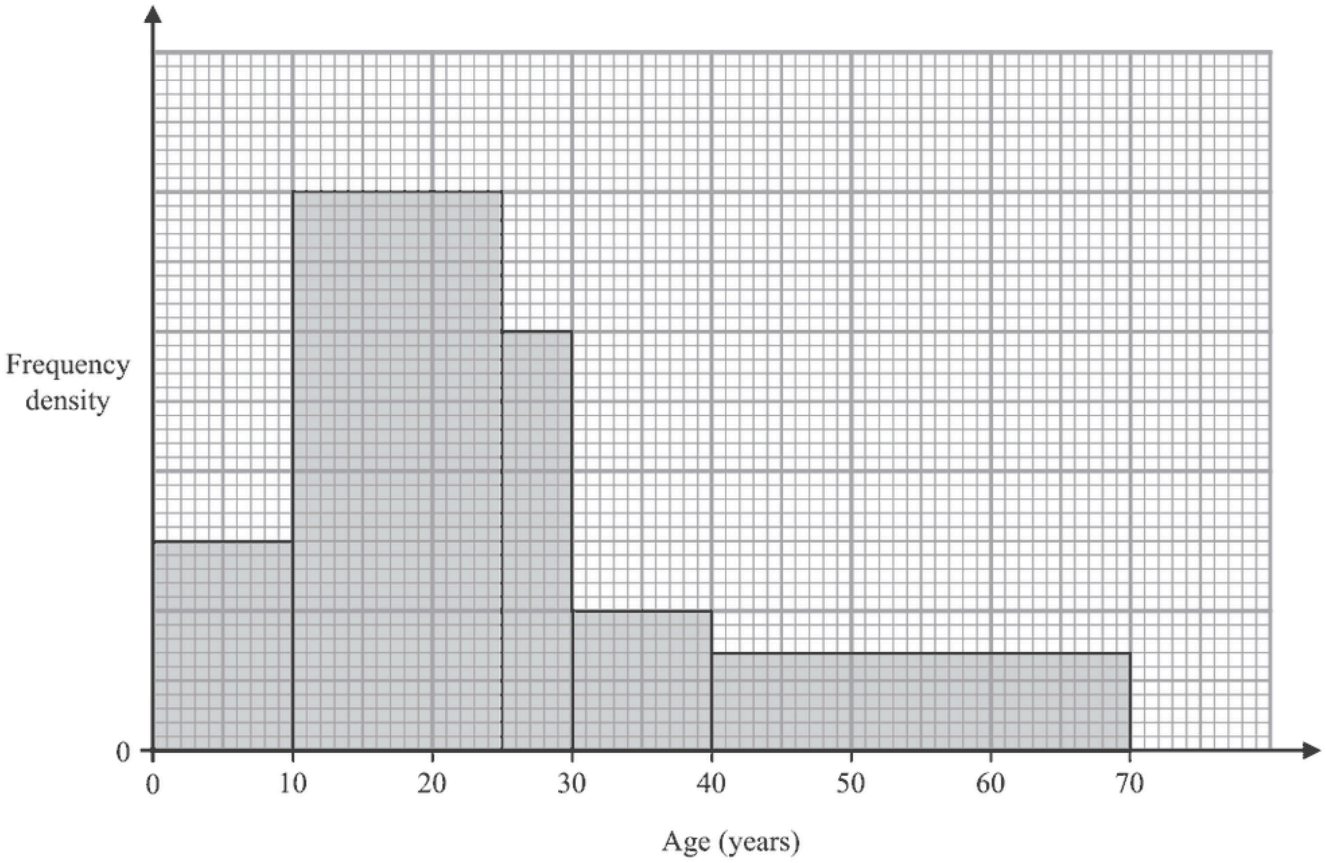
$\frac{8}{29}$ or $0.276$ to 3 significant figures

4WM1/01R MAY 2025

# Question 21

3 marks | Histograms

**21** The histogram shows information about the ages of the members of a chess club.




There are 60 members aged between 10 and 25
There are no members older than 70

Work out an estimate for the number of members who are over 35 years of age.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4WM1/01R May 2025

**1** **The interval to has width**

The interval $10$ to $25$ has width $15$ and frequency $60$, so its frequency density is

$$\frac{60}{15} = 4$$

**2** **Use histogram**

Using that scale from the histogram:

$$35 \text{ to } 40 : 5 \times 1 = 5$$

$$40 \text{ to } 70 : 30 \times 0.75 = 22.5$$

$$5 + 22.5 = 27.5$$

---

✓ **FINAL ANSWER**

about $28$ members.

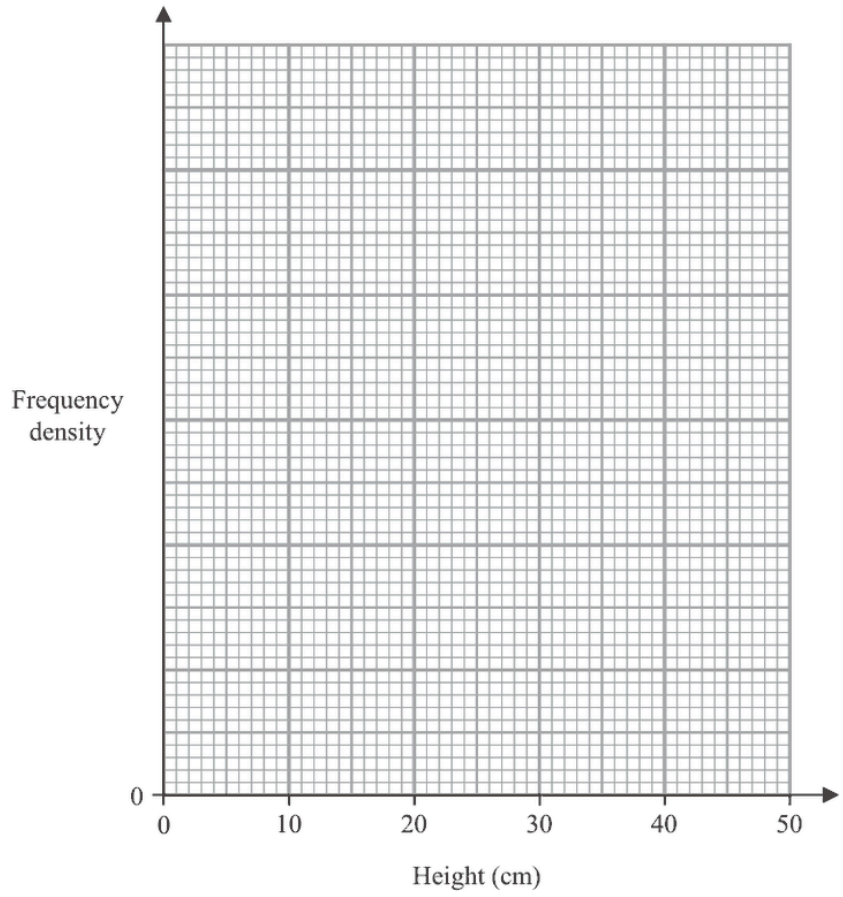
# Question 20

3 marks | Histograms

**20** The table gives information about the heights, in centimetres, of some plants.

| Height ($h$ cm) | Frequency |
|---|---|
| $0 < h \leqslant 5$ | 12 |
| $5 < h \leqslant 15$ | 34 |
| $15 < h \leqslant 30$ | 45 |
| $30 < h \leqslant 50$ | 28 |

On the grid, draw a histogram for this information.
You must include a scale on the vertical axis.




**(Total for Question 20 is 3 marks)**

## 1 Frequency density

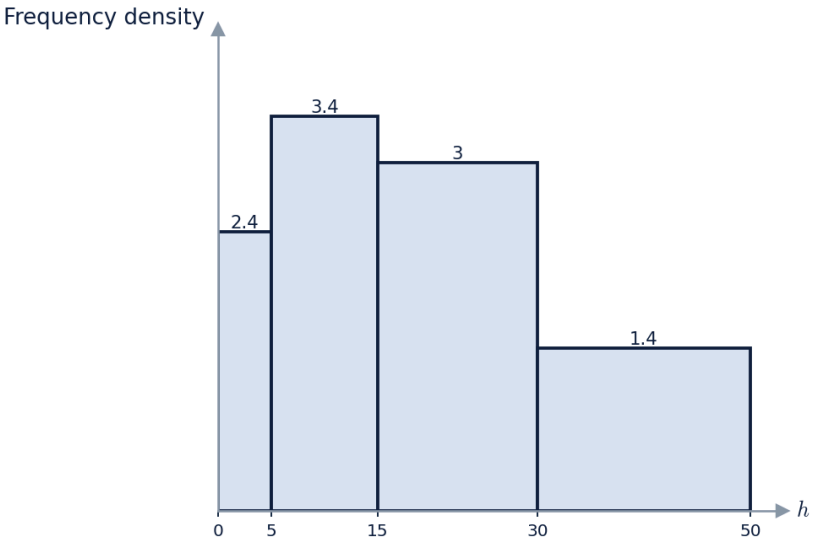
Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$.

$$0 < h \leq 5 : \frac{12}{5} = 2.4$$

$$5 < h \leq 15 : \frac{34}{10} = 3.4$$

$$15 < h \leq 30 : \frac{45}{15} = 3$$

$$30 < h \leq 50 : \frac{28}{20} = 1.4$$




**FINAL ANSWER**

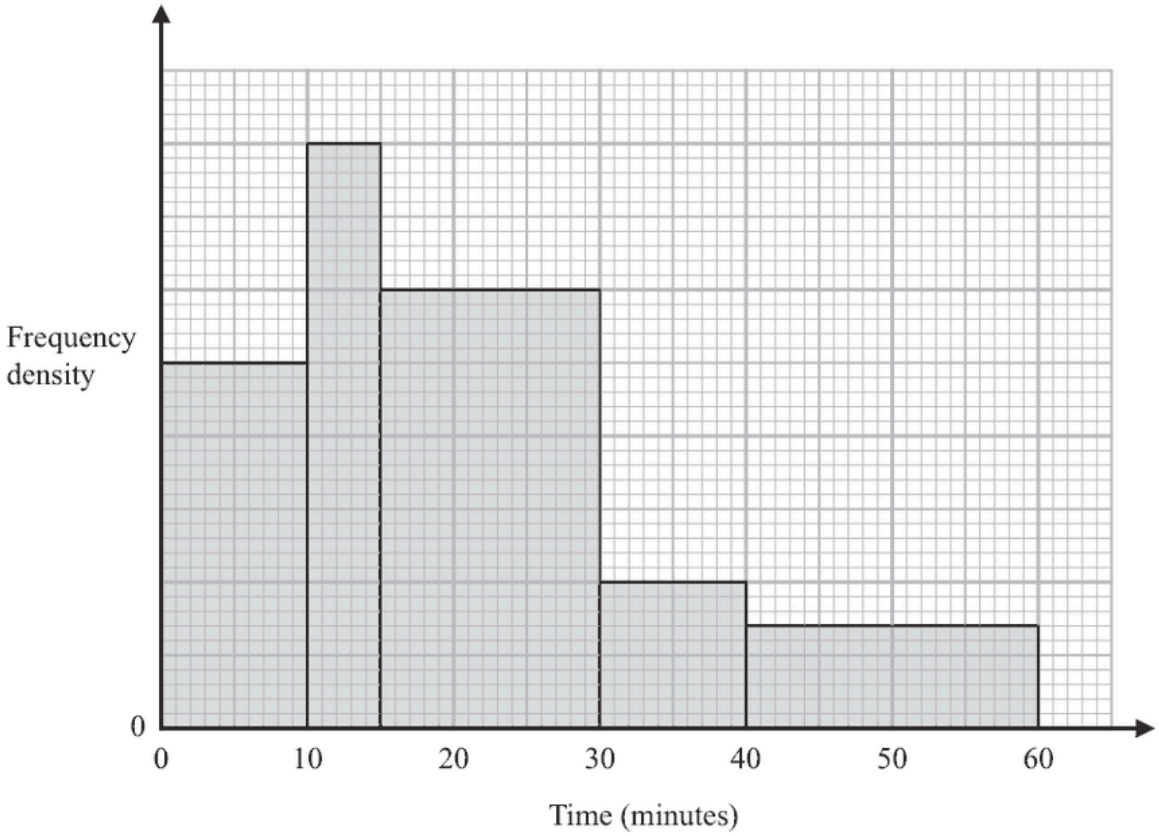
draw bars with densities $2.4, 3.4, 3, 1.4$.

4MA1/2H NOVEMBER 2025

# Question 21

3 marks | Histograms

**21** The histogram shows information about the times, in minutes, that some trains arrived late at a station one day.




20 of these trains arrived between 10 minutes late and 15 minutes late.

No trains arrived more than 60 minutes late.

Work out an estimate for the number of these trains that arrived at least 25 minutes late.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H November 2025

**1** **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

**2** **Use frequency table**

For $10 < t \leq 15$, the frequency is 20 and the class width is 5, so

$$\text{frequency density} = \frac{20}{5} = 4$$

---

**3** **Use histogram**

Using this scale from the histogram:

$$15 < t \leq 30 \quad \text{has frequency density } 3$$

$$30 < t \leq 40 \quad \text{has frequency density } 1$$

$$40 < t \leq 60 \quad \text{has frequency density } 0.7$$

---

**4** **Calculate value**

For trains at least 25 minutes late:

$$(5)(3) + (10)(1) + (20)(0.7)$$

$$= 15 + 10 + 14$$

$$= 39$$

---

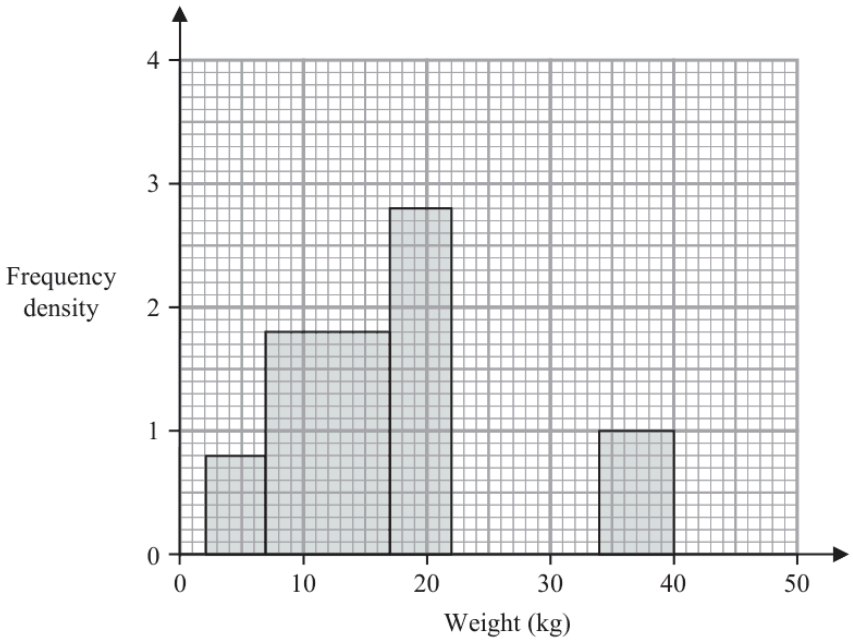
✓ **FINAL ANSWER**

$39$ trains

# Question 21

3 marks | Histograms

**21** The incomplete histogram shows information about the weights, in kg, of some parcels.




No parcels weigh less than 2 kg
No parcels weigh more than 40 kg
Half of the parcels weigh between 22 kg and 34 kg

Complete the histogram by drawing a bar to represent the parcels that weigh between 22 kg and 34 kg

**(Total for Question 21 is 3 marks)**

**1** **Frequency = width × height**

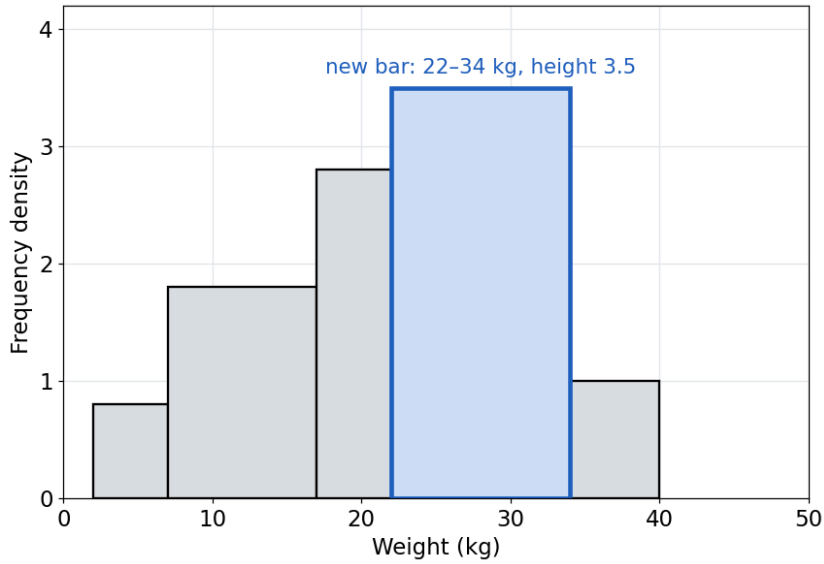
$$5(0.8) + 10(1.8) + 5(2.8) + 6(1) = 4 + 18 + 14 + 6 = 42$$

**2** **Half are in 22–34**

So the other half is also $42$ parcels.

**3** **Height of the new bar**

$$\text{f.d.} = \frac{42}{34 - 22} = 3.5$$




**FINAL ANSWER**

Bar from $22$ to $34$ kg, height $3.5$

TOPIC

# Probability Toolkit

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

# Question 23

4 marks | Probability Toolkit

**23** In a bag, there are only

3 blue beads
4 white beads
and $x$ orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

**(Total for Question 23 is 4 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/1HR May/November 2020

### 1 The total number of beads

The total number of beads is

$$x + 7$$

### 2 The probability of choosing two

The probability of choosing two beads of the same colour is

$$\frac{\binom{3}{2} + \binom{4}{2} + \binom{x}{2}}{\binom{x+7}{2}}$$

**3** **Evaluate fraction**

This is equal to $\frac{3}{8}$, so

$$\frac{3+6+\frac{x(x-1)}{2}}{\frac{(x+7)(x+6)}{2}}=\frac{3}{8}$$

$$\frac{x^2-x+18}{(x+7)(x+6)}=\frac{3}{8}$$

$$8(x^2-x+18)=3(x+7)(x+6)$$

$$8x^2-8x+144=3x^2+39x+126$$

$$5x^2-47x+18=0$$

$$(x-9)(5x-2)=0$$

**4** **Calculate value**

Since $x$ must be a whole number,

$$x=9$$

**5** **Calculate value**

So the total number of beads is

$$9+7=16$$

**FINAL ANSWER**

$16$ beads

4MA1/2HR JANUARY 2021

# Question 21

6 marks | Probability Toolkit

**21** A bag contains $n$ beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of $n$.

$n =$ .....

**(Total for Question 21 is 6 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2HR January 2021

**1 There are beads altogether**

There are $n$ beads altogether.

**2 Red beads**

Red beads:

$$6$$

**3 Blue beads**

Blue beads:

$$n - 6$$

**4 The probability of taking two**

The probability of taking two beads of the same colour is

$$P(RR) + P(BB) = \frac{9}{17}$$

**5** **Use trigonometry**

So

$$\frac{6}{n} \cdot \frac{5}{n-1} + \frac{n-6}{n} \cdot \frac{n-7}{n-1} = \frac{9}{17}$$

$$\frac{30 + (n-6)(n-7)}{n(n-1)} = \frac{9}{17}$$

**6** **Expand**

Expand:

$$(n-6)(n-7) = n^2 - 13n + 42$$

**7** **Use trigonometry**

So

$$\frac{n^2 - 13n + 72}{n(n-1)} = \frac{9}{17}$$

**8** **Cross multiply**

Cross multiply:

$$17(n^2 - 13n + 72) = 9n(n-1)$$

$$17n^2 - 221n + 1224 = 9n^2 - 9n$$

$$8n^2 - 212n + 1224 = 0$$

**9** **Divide by 4**

Divide by 4:

$$2n^2 - 53n + 306 = 0$$

**10** **Factor**

Factor:

$$(2n - 17)(n - 18) = 0$$

**11** **Use trigonometry**

So

$$n = \frac{17}{2} \quad \text{or} \quad n = 18$$

12 **The number of beads must**

The number of beads must be a whole number, so

$$n = 18$$

---

✓ **FINAL ANSWER**

$\boxed{18}$

4MA1/2H MAY 2021

# Question 23

5 marks | Probability Toolkit

**23** Pippa has a box containing $N$ pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of $N$.
Show clear algebraic working.

**(Total for Question 23 is 5 marks)**

## WORKED SOLUTION

Question 23 · 4MA1/2H May 2021

**1 Let the number of red**

Let the number of red pens be $r$.

**2 List probability cases**

Then the number of black pens is

$$r + 3$$

**3 Calculate value**

and

$$N = 2r + 3$$

**4 Evaluate fraction**

So

$$r = \frac{N - 3}{2}$$

**5** **Evaluate fraction**

and

$$r+3=\frac{N+3}{2}$$

**6** **The probability of black followed**

The probability of black followed by red is

$$\frac{(N+3)/2}{N}\cdot\frac{(N-3)/2}{N-1}=\frac{9}{35}$$

$$\frac{N^2-9}{4N(N-1)}=\frac{9}{35}$$

$$35(N^2-9)=36N(N-1)$$

$$35N^2-315=36N^2-36N$$

$$N^2-36N+315=0$$

$$(N-15)(N-21)=0$$

✓ **FINAL ANSWER**

$N=15$ or $N=21$

4MA1/1HR JUNE 2022

# Question 24

5 marks | Probability Toolkit

**24** Elliot has $x$ counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of $x$
Show clear algebraic working.

$x =$ .....

**(Total for Question 24 is 5 marks)**

## WORKED SOLUTION

Question 24 · 4MA1/1HR June 2022

**1 There are counters. Initially 5**

There are $x$ counters. Initially 5 show red, so $x - 5$ show green.

**2 List probability cases**

After two turns, there are still 5 red faces if:

**3 the same counter is picked**

- the same counter is picked twice, or
- one red counter and one green counter are picked.

**4 The probability of picking the**

The probability of picking the same counter twice is

$$\frac{1}{x}$$

**5** **The probability of picking one**

The probability of picking one red and one green in either order is

$$\frac{5}{x} \cdot \frac{x-5}{x} + \frac{x-5}{x} \cdot \frac{5}{x}$$

$$= \frac{10(x-5)}{x^2}$$

---

**6** **Evaluate fraction**

So

$$\frac{1}{x} + \frac{10(x-5)}{x^2} = \frac{19}{32}$$

$$\frac{11x-50}{x^2} = \frac{19}{32}$$

$$32(11x-50) = 19x^2$$

$$19x^2 - 352x + 1600 = 0$$

$$(x-8)(19x-200) = 0$$

---

**7** **Calculate value**

Since $x$ must be a whole number,

$$x = 8$$

---

✓ **FINAL ANSWER**

$x = 8$

4MA1/1H NOVEMBER 2023

# Question 20

4 marks | Probability Toolkit

**20** A bag contains only 10 cent coins and 20 cent coins.

Josip takes at random a coin from the bag, records its value and replaces it in the bag. He then takes at random a second coin from the bag, records its value and replaces it in the bag.

Josip finds the mean value of the two coins.

The probability that the two coins have a mean value of 10 cents is $\frac{49}{121}$

Work out the probability that the two coins have a mean value of 15 cents.

**(Total for Question 20 is 4 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/1H November 2023

**1 Let the probability of taking**

Let the probability of taking a 10 cent coin be $p$.

**2 The mean is 10 cents**

The mean is 10 cents only when both coins are 10 cent coins.

$$p^2 = \frac{49}{121}$$

$$p = \frac{7}{11}$$

**3 Calculate probability**

So the probability of taking a 20 cent coin is

$$1 - \frac{7}{11} = \frac{4}{11}$$

**4 The mean is 15 cents**

The mean is 15 cents when one coin is 10 cents and the other is 20 cents.

**5** **There are two possible orders**

There are two possible orders:

$$P(10, 20) + P(20, 10)$$

$$= \frac{7}{11} \cdot \frac{4}{11} + \frac{4}{11} \cdot \frac{7}{11}$$

$$= \frac{56}{121}$$

---

✓ **FINAL ANSWER**

$\frac{56}{121}$

4MA1/1HR MAY 2024

# Question 21

5 marks | Probability Toolkit

**21** There are 25 counters in a bag such that

6 counters are blue
$x$ counters are orange, where $x > 9$
the rest of the counters are pink

Maalam takes at random two of the counters from the bag.

The probability that Maalam takes one orange counter and one pink counter is $\frac{22}{75}$

Calculate the probability that Maalam takes 2 pink counters from the bag.
Show clear algebraic working.

**(Total for Question 21 is 5 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1HR May 2024

**1** **There are 25 counters**

There are 25 counters.

**2** **Blue counters**

Blue counters:

$$6$$

**3** **Orange counters**

Orange counters:

$$x$$

**4** **Pink counters**

Pink counters:

$$25 - 6 - x = 19 - x$$

**5** **The probability of taking one**

The probability of taking one orange and one pink in either order is

$$\frac{x}{25} \cdot \frac{19-x}{24} + \frac{19-x}{25} \cdot \frac{x}{24}$$

$$= \frac{2x(19-x)}{25 \cdot 24}$$

**6** **Evaluate fraction**

This is equal to $\frac{22}{75}$, so

$$\frac{2x(19-x)}{600} = \frac{22}{75}$$

$$\frac{x(19-x)}{300} = \frac{22}{75}$$

$$x(19-x) = 88$$

$$19x - x^2 = 88$$

$$x^2 - 19x + 88 = 0$$

$$(x-8)(x-11) = 0$$

**7** **Solve inequality**

Since $x > 9$,

$$x = 11$$

**8** **Calculate value**

So the number of pink counters is

$$19 - 11 = 8$$

**9** **The probability of taking 2**

The probability of taking 2 pink counters is

$$\frac{8}{25} \cdot \frac{7}{24} = \frac{56}{600}$$

$$= \frac{7}{75}$$

**FINAL ANSWER**

$$\frac{7}{75}$$

4MA1/1H JUNE 2025

# Question 21

3 marks | Probability Toolkit

**21** A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

**(Total for Question 21 is 3 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/1H June 2025

**1 There are**

There are

$$\binom{20}{3} = 1140$$

**2 ways to choose 3 counters**

ways to choose 3 counters.

**3 Exactly two counters are the**

Exactly two counters are the same colour:

$$\binom{9}{2}(11) + \binom{7}{2}(13) + \binom{4}{2}(16)$$

$$= 36(11) + 21(13) + 6(16)$$

$$= 396 + 273 + 96 = 765$$

**4 Calculate probability**

So the probability is

$$\frac{765}{1140} = \frac{51}{76}$$

**FINAL ANSWER**

$$\frac{51}{76}$$

4WM1/01 MAY 2025

# Question 22

3 marks | Probability Toolkit

**22** A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

**(Total for Question 22 is 3 marks)**

## WORKED SOLUTION

Question 22 · 4WM1/01 May 2025

### 1 Total ways to choose counters

Total ways to choose $3$ counters:

$$\binom{20}{3} = 1140$$

### 2 Exactly two red

Exactly two red:

$$\binom{9}{2}\binom{11}{1} = 396$$

### 3 Exactly two yellow

Exactly two yellow:

$$\binom{7}{2}\binom{13}{1} = 273$$

### 4 Exactly two green

Exactly two green:

$$\binom{4}{2}\binom{16}{1} = 96$$

**5** **Favourable outcomes**

Favourable outcomes:

$$396 + 273 + 96 = 765$$

$$P = \frac{765}{1140} = \frac{51}{76}$$

**FINAL ANSWER**

$\frac{51}{76}$



TOPIC

# Tree Diagrams & Conditional Probability

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JUNE 2023

# Question 20

3 marks | Tree Diagrams & Conditional Probability

**20** There are 12 counters in a bag.

3 of the counters are red
9 of the counters are green

Ameya, Jack and Ella each take at random one counter from the bag.

Work out the probability that at least one red counter is still in the bag.

**(Total for Question 20 is 3 marks)**

## WORKED SOLUTION

Question 20 · 4MA1/2H June 2023

**1 There are 3 red counters**

There are 3 red counters.

**2 At least one red counter**

At least one red counter is still in the bag unless all 3 red counters are taken.

**3 Use the complement**

So use the complement:

$$P(\text{all 3 red are taken}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$$

$$= \frac{6}{1320}$$

$$= \frac{1}{220}$$

**4 Evaluate fraction**

Therefore,

$$P(\text{at least one red is still in the bag}) = 1 - \frac{1}{220}$$

$$= \frac{219}{220}$$

**FINAL ANSWER**

$$\frac{219}{220}$$

4MA1/1H NOVEMBER 2025

# Question 22

3 marks | Tree Diagrams & Conditional Probability

**22** There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

**(Total for Question 22 is 3 marks)**

## WORKED SOLUTION

Question 22 · 4MA1/1H November 2025

**1 There are 2 pink buttons**

There are 2 pink buttons.

**2 At least one pink button**

At least one pink button is still in the box unless both pink buttons are taken.

**3 Use the complement**

So use the complement.

**4 The probability that both pink**

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

**5** **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

---

✓ **FINAL ANSWER**

$\frac{34}{35}$

4WM1/01 NOVEMBER 2025

# Question 23

3 marks | Tree Diagrams & Conditional Probability

**23** There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

**(Total for Question 23 is 3 marks)**

## WORKED SOLUTION

Question 23 · 4WM1/01 November 2025

**1 There are 2 pink buttons**

There are 2 pink buttons.

**2 At least one pink button**

At least one pink button is still in the box unless both pink buttons are taken.

**3 Use the complement**

So use the complement.

**4 The probability that both pink**

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

**5** **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

---

✓ **FINAL ANSWER**

$\frac{34}{35}$

# Question 22

5 marks | Tree Diagrams & Conditional Probability

**22** A bag contains $x$ marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of $x$
Show clear algebraic working.

$x =$ .....

**(Total for Question 22 is 5 marks)**

**1** **Pink then yellow, or yellow then pink**

$$2 \times \frac{12}{x} \times \frac{x-12}{x-1} = \frac{18}{47}$$

**2** **Cross-multiply**

$$1128(x-12) = 18x(x-1) \Rightarrow 18x^2 - 1146x + 13536 = 0$$

**3** **Simplify (÷6) and solve**

$$3x^2 - 191x + 2256 = 0 \Rightarrow (x-48)(3x-47) = 0$$

**4** **x must be a whole number**

$$x = 48$$

✓ **FINAL ANSWER**

$x = 48$



TOPIC

# Set Notation & Venn Diagrams

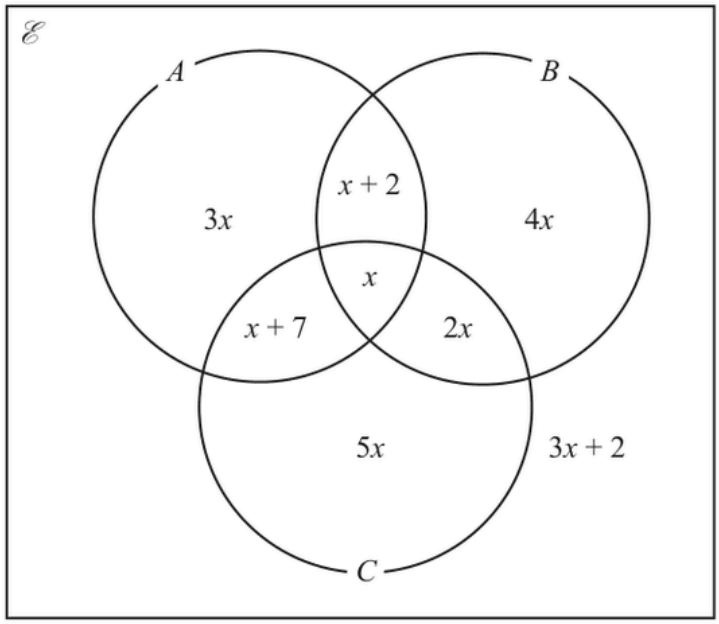
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2024

# Question 21

4 marks | Set Notation & Venn Diagrams

**21** The Venn diagram shows information about the numbers of items in set $A$, set $B$ and set $C$, where $x$ is an integer.




Given that $\text{n}(A \cup B)' = 26$

find $\text{n}(A' \cap C)$

$\text{n}(A' \cap C) =$ .....

**(Total for Question 21 is 4 marks)**

## WORKED SOLUTION

Question 21 · 4MA1/2H November 2024

**1 The complement of contains the**

The complement of $A \cup B$ contains the regions outside both $A$ and $B$. These are $5x$ and $3x + 2$.

**2 Calculate value**

So $5x + (3x + 2) = 26$, hence $8x + 2 = 26$ and $x = 3$.

**3 Calculate statistic**

Now $A' \cap C$ means inside $C$, but not inside $A$. These are the regions $5x$ and $2x$.

**4 Calculate value**

So $n(A' \cap C) = 5x + 2x = 7x = 7(3) = 21$.

**FINAL ANSWER**

21

# Question 20

6 marks | Set Notation & Venn Diagrams

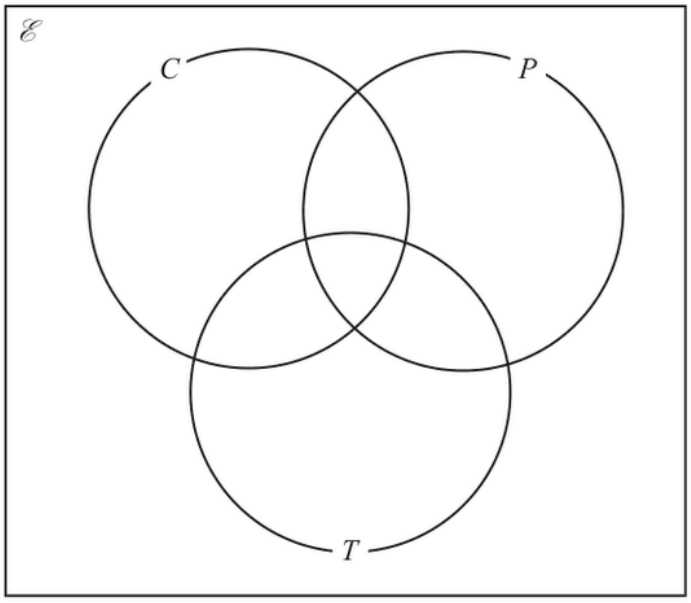
**20** 120 gardeners were asked if they grow carrots ($C$) or potatoes ($P$) or tomatoes ($T$)

Of these gardeners

43 grow carrots
12 grow carrots and potatoes and tomatoes
18 grow carrots and potatoes
27 grow carrots and tomatoes
32 grow potatoes and tomatoes
29 do not grow carrots or potatoes or tomatoes

The number of these gardeners who grow only potatoes is equal to the number of these gardeners who grow only tomatoes.

(a) Complete the Venn diagram to show this information.




**(3)**

(b) Find $n(T' \cap C)$

.....

**(1)**

One of the gardeners who grows carrots is chosen at random.

(c) Calculate the probability that this gardener also grows potatoes.

.....

**(2)**

**(Total for Question 20 is 6 marks)**

**1** **Calculate value**

There are 120 gardeners in total and 29 are outside all three sets, so

$$n(C \cup P \cup T) = 120 - 29 = 91$$

**2** **The triple intersection is**

The triple intersection is

$$n(C \cap P \cap T) = 12$$

**3** **Calculate value**

So the pair-only regions are:

$$C \cap P \text{ only} = 18 - 12 = 6$$

$$C \cap T \text{ only} = 27 - 12 = 15$$

$$P \cap T \text{ only} = 32 - 12 = 20$$

**4** **Calculate value**

Since 43 grow carrots,

$$C \text{ only} = 43 - 6 - 15 - 12 = 10$$

**5** **Let the number who grow**

Let the number who grow only potatoes be $y$. The number who grow only tomatoes is also $y$.

**6** **Add the regions union**

Now add the regions in the union:

$$10 + y + y + 6 + 15 + 20 + 12 = 91$$

$$63 + 2y = 91$$

$$2y = 28$$

$$y = 14$$

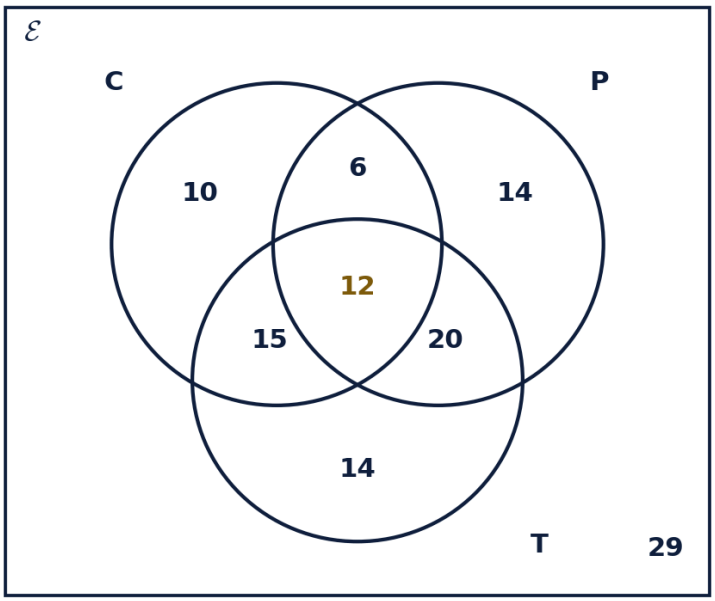
**7** **Calculate value**

So the Venn regions are:

$$C \text{ only} = 10, \quad P \text{ only} = 14, \quad T \text{ only} = 14$$

$$C \cap P \text{ only} = 6, \quad C \cap T \text{ only} = 15, \quad P \cap T \text{ only} = 20$$

$$C \cap P \cap T = 12, \quad \text{outside} = 29$$




**8** **Calculate value**

For part (b),

$$n(T' \cap C) = C \text{ only} + C \cap P \text{ only}$$

$$= 10 + 6 = 16$$

**9** **Evaluate fraction**

For part (c), one gardener who grows carrots is chosen.

$$P(\text{also grows potatoes}) = \frac{n(C \cap P)}{n(C)}$$

$$= \frac{18}{43}$$

✓ **FINAL ANSWER**

(b) $16$, (c) $\frac{18}{43}$