Elite Mathematics
Pearson Edexcel International GCSE

IGCSE MODULAR · UNIT 2 · 4WM2

Modular Unit 2 Expertise With Answers

Q20+ questions with Dr Eslam Ahmed's worked solutions.

137	2020–2026	Questions with worked solutions
topic placements	49 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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TOPIC

Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

Question 21

3 marks | Algebraic Fractions

21 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 21 is 3 marks)

1 **Simplify fraction**

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 **Simplify fraction**

Substitute for x:

$$a = \frac{14}{3\left(\frac{7}{4y-3}\right) - 7}$$

3 **Simplify the denominator**

Simplify the denominator:

$$3\left(\frac{7}{4y - 3}\right) - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

4 **Simplify fraction**

So

$$a = \frac{14}{\frac{42-28y}{4y-3}} = \frac{14(4y - 3)}{42 - 28y}$$

$$a = \frac{4y - 3}{3 - 2y}$$

✓ **FINAL ANSWER**

$a = \dfrac{4y - 3}{3 - 2y}$.

TOPIC

Graphs of Functions

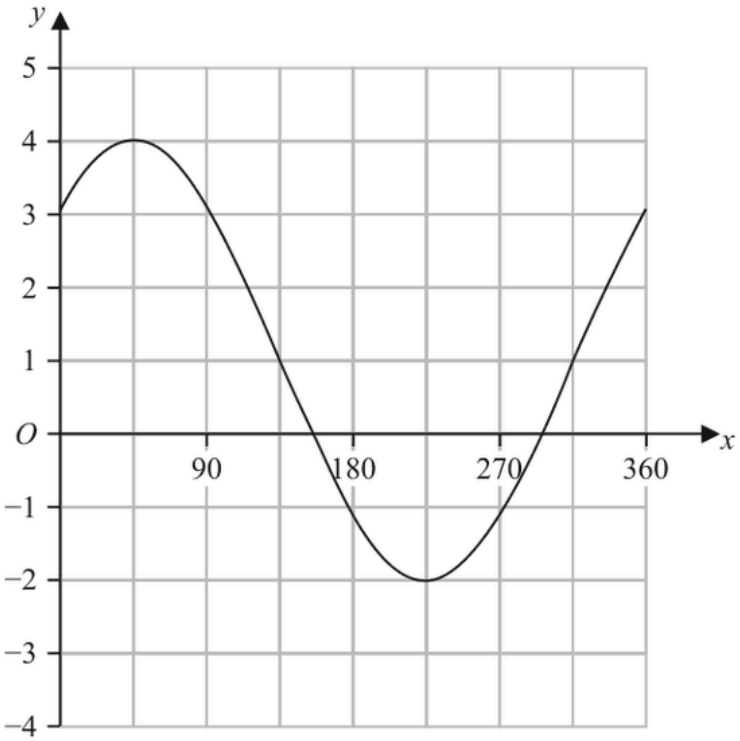
Every past-paper question on this topic, 2020 to 2026.

4WM2/01R MAY 2025

Question 23

3 marks | Graphs of Functions

23 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01R May 2025

1 Required

The graph of *y* = *a*sin(*x*+*b*)[∘] + *c* for 0 ≤ *x* ≤ 360 is shown. Find suitable values for *a*, *b*, and *c*.

2 Read the maximum and minimum

Read the maximum and minimum values from the graph to find the **amplitude** *a* and **vertical shift** *c*:

$$a = \frac{\text{max} - \text{min}}{2} = \frac{4 - (-2)}{2} = 3 \quad c = \frac{\text{max} + \text{min}}{2} = \frac{4 + (-2)}{2} = 1$$

3 Find the phase shift b

Find the **phase shift** *b*. The standard sin has its peak at *x* = 90°. From the graph, the maximum occurs at *x* ≈ 45°:

x + *b* = 90° ⟹ 45° + *b* = 90° ⟹ *b* = 45°

---

4 Verify at x 0 consistent

Verify at *x* = 0: $y = 4sin(0 + 45)^\circ = 4times fracsqrt22approx2.83$ — consistent with graph

---

5 Note

The period of the graph is still 360° (unchanged), so no horizontal stretch is applied.

✓ **FINAL ANSWER**

a = 3,*b* = 45,*c* = 1

Question 25

3 marks | Graphs of Functions

25 A curve has equation $y = f(x)$

There is one minimum point on this curve.

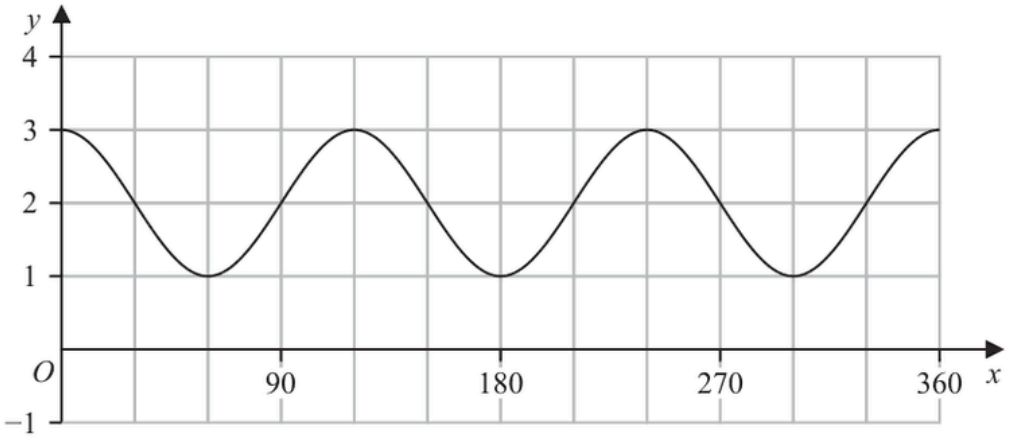
The coordinates of this minimum point are $(8, 1)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(4x)$

(..... , ...)

(1)

The diagram shows the graph of $y = \cos(ax) + b$ for $0 \leqslant x \leqslant 360$



(b) (i) Find the value of a

$a =$ .....

(1)

(ii) Find the value of b

$b =$ .....

(1)

(Total for Question 25 is 3 marks)

1 The transformation y f (4

The transformation *y* = *f*(4*x*) represents a horizontal stretch by a scale factor of $frac14$. The x-coordinate is multiplied by $frac14$ while the y-coordinate stays the same.

$$textNewminimum = leftbegin{8times}frac14, 1rightend = (2, 1)$$

2 Answer

(2,1)

Step 1: Find the vertical shift *b*. The standard *y* = cos (*x*) graph fluctuates between − 1 and 1, centered at *y* = 0. The given graph fluctuates between a minimum of *y* = 1 and a maximum of *y* = 3. The center line is $frac3 + 12 = 2$. Therefore, *b* = 2.

Step 1: Find the frequency multiplier *a*. The period of the given graph completes one full cycle between *x* = 0 and *x* = 90[∘] (it starts at 3, dips to 1, and returns to 3 at 90[∘]). The standard cosine period is 360[∘].

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Missing open brace for superscript

FINAL ANSWER

a = 4, *b* = 2

TOPIC

Forming & Solving Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2025

Question 20

3 marks | Forming & Solving Equations

20 Here is a quadratic equation.

$$ax^2 + 4x + c = 0$$

The solutions of this equation are given by $x = \dfrac{-4 \pm 2\sqrt{39}}{10}$

Find the value of a and the value of c
Show your working clearly.

$a =$ .....

$c =$ .....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2025

1 Calculate value

For

$$ax^2 + 4x + c = 0$$

2 the quadratic formula gives

the quadratic formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4ac}}{2a}$$

3 The given solutions are

The given solutions are

$$x = \frac{-4 \pm 2\sqrt{39}}{10}$$

4 Compare denominators

Compare denominators:

$$2a = 10$$

$$a = 5$$

5 **Compare the square root parts**

Compare the square root parts:

$$\sqrt{16 - 4ac} = 2\sqrt{39}$$

6 **Square both sides**

Square both sides:

$$16 - 4ac = 156$$

7 **Use**

Use $a = 5$:

$$16 - 20c = 156$$

$$-20c = 140$$

$$c = -7$$

✓ **FINAL ANSWER**

$a = 5,\ c = -7$

TOPIC

Simultaneous Equations

Every past-paper question on this topic, 2020 to 2026.

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

1 **Solve simultaneous equations**

From

$$x - 2y = 3$$

$$x = 2y + 3$$

2 **Solve simultaneous equations**

Substitute into $x^2 - y^2 + 2x = 10$:

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

3 **Solve simultaneous equations**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

4 **Solve simultaneous equations**

If $y = -\frac{1}{3}$,

$$x = 2\left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

5 **Solve simultaneous equations**

If $y = -5$,

$$x = 2(-5) + 3 = -7$$

✓ **FINAL ANSWER**

$\left(\frac{7}{3}, -\frac{1}{3}\right)$ and $(-7, -5)$.

4MA1/2HR JUNE 2022

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2022

1 **Calculate value**

$$x - 2y = 3$$

2 **Calculate value**

So

$$x = 2y + 3$$

3 **Calculate value**

Substitute into

$$x^2 - y^2 + 2x = 10$$

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

4 **Evaluate fraction**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

5 **Find**

Find x:

$$y = -\frac{1}{3} \Rightarrow x = \frac{7}{3}$$

$$y = -5 \Rightarrow x = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{7}{3}, -\frac{1}{3}\right)$ or $(-7, -5)$

4MA1/2H JUNE 2023

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2023

1 Calculate value

$$2x^2 + 3y^2 = 11$$

$$x = 3y - 1$$

2 Substitute

Substitute:

$$2(3y - 1)^2 + 3y^2 = 11$$

$$18y^2 - 12y + 2 + 3y^2 = 11$$

$$21y^2 - 12y - 9 = 0$$

$$7y^2 - 4y - 3 = 0$$

$$(7y + 3)(y - 1) = 0$$

3 Evaluate fraction

So

$$y = -\frac{3}{7} \quad \text{or} \quad y = 1$$

4 **Find**

Find x:

$$y = -\frac{3}{7} \Rightarrow x = -\frac{16}{7}$$

$$y = 1 \Rightarrow x = 2$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{16}{7}, -\frac{3}{7}\right)$ or $(2, 1)$

4MA1/2HR JUNE 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = y + 11$$
$$y = 3x - 1$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2024

1 Calculate value

$$x^2 + y^2 = y + 11$$

$$y = 3x - 1$$

2 Substitute

Substitute:

$$x^2 + (3x - 1)^2 = 3x - 1 + 11$$

$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

$$10x^2 - 9x - 9 = 0$$

$$(5x + 3)(2x - 3) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{3}{2}$$

4 **Find**

Find y:

$$x = -\frac{3}{5} \Rightarrow y = -\frac{14}{5}$$

$$x = \frac{3}{2} \Rightarrow y = \frac{7}{2}$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, -\frac{14}{5}\right)$ or $\left(\frac{3}{2}, \frac{7}{2}\right)$

4MA1/2H NOVEMBER 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 + y = 3$$

$$x + 2 = y$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2024

1 Calculate value

$$x^2 + y^2 + y = 3$$

$$y = x + 2$$

2 Substitute

Substitute:

$$x^2 + (x + 2)^2 + (x + 2) = 3$$

$$x^2 + x^2 + 4x + 4 + x + 2 = 3$$

$$2x^2 + 5x + 3 = 0$$

$$(2x + 3)(x + 1) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

4 **Find**

Find y:

$$x = -\frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$x = -1 \Rightarrow y = 1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{2}, \frac{1}{2}\right)$ or $(-1, 1)$

4MA1/1H JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + 3y + y^2 = 7$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2025

1 Calculate value

$$x^2 + y^2 + 3y = 7$$

$$y = x + 2$$

2 Substitute

Substitute:

$$x^2 + (x+2)^2 + 3(x+2) = 7$$

$$x^2 + x^2 + 4x + 4 + 3x + 6 = 7$$

$$2x^2 + 7x + 3 = 0$$

$$(2x+1)(x+3) = 0$$

3 Evaluate fraction

So

$$x = -\frac{1}{2} \quad \text{or} \quad x = -3$$

4 **Find**

Find y:

$$x = -\frac{1}{2} \Rightarrow y = \frac{3}{2}$$

$$x = -3 \Rightarrow y = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{1}{2}, \frac{3}{2}\right)$ or $(-3, -1)$

4MA1/1HR JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = 41$$
$$2x + y = 3$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR June 2025

1 **Calculate value**

$$x^2 + y^2 = 41$$

$$2x + y = 3$$

2 **Calculate value**

So

$$y = 3 - 2x$$

3 **Substitute**

Substitute:

$$x^2 + (3 - 2x)^2 = 41$$

$$x^2 + 9 - 12x + 4x^2 = 41$$

$$5x^2 - 12x - 32 = 0$$

$$(5x + 8)(x - 4) = 0$$

4 **Evaluate fraction**

So

$$x = -\frac{8}{5} \quad \text{or} \quad x = 4$$

5 **Find**

Find y:

$$x = -\frac{8}{5} \Rightarrow y = \frac{31}{5}$$

$$x = 4 \Rightarrow y = -5$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{8}{5}, \frac{31}{5}\right)$ or $(4, -5)$

4WM2/01R MAY 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 + 5y + x^2 = 12$$
$$x = y + 5$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01R May 2025

1 Required

Solve *y*² + 5*y* + *x*² = 12 and *x* = *y* + 5.

2 Substitute x y + 5

Substitute *x* = *y* + 5 into the first equation:

*y*² + 5*y* + (*y*+5)² = 12

3 Expand (y +5) 2

Expand (*y*+5)² = *y*² + 10*y* + 25:

*y*² + 5*y* + *y*² + 10*y* + 25 = 12 ⟹ 2*y*² + 15*y* + 13 = 0

4 Factorise

Factorise:

$$(2y + 13)(y + 1) = 0 implies y = -tfrac132; textor; y = -1$$

5 Find corresponding x values using

Find corresponding *x* values using *x* = *y* + 5:

FINAL ANSWER

(*x*,*y*) = (4, −1)and$left(-dfrac32, , -dfrac132right)$

4WM2/01 NOVEMBER 2025

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x^2 + y^2 = 3xy - 11$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 November 2025

1 Substitute into equation

Step 1: Substitute *y* = (*x*+2) into the quadratic equation.

*x*² + (*x*+2)² = 3*x*(*x*+2) − 11

2 Expand brackets

*x*² + (*x*²+4*x*+4) = 3*x*² + 6*x* − 11

2*x*² + 4*x* + 4 = 3*x*² + 6*x* − 11

3 Solve equation

0 = 3*x*² − 2*x*² + 6*x* − 4*x* − 11 − 4

*x*² + 2*x* − 15 = 0

4 Solve quadratic equation

(*x*+5)(*x*−3) = 0

So, *x* = − 5 or *x* = 3.

Step 5: Find corresponding *y* values using *y* = *x* + 2.

If *x* = − 5 ⟹ *y* = − 5 + 2 = − 3

If *x* = 3 ⟹ *y* = 3 + 2 = 5

✓ FINAL ANSWER

(−5,−3) and (3,5)

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

1 **Substitute y = 3x − 5**

$$(3x-5)^2 = x^2 + 7 \Rightarrow 9x^2 - 30x + 25 = x^2 + 7$$

2 **Simplify and factorise**

$$8x^2 - 30x + 18 = 0 \Rightarrow 4x^2 - 15x + 9 = 0 \Rightarrow (x-3)(4x-3) = 0$$

3 **Find y for each x**

$x = 3 \Rightarrow y = 4$;\quad $x = \frac{3}{4} \Rightarrow y = \frac{9}{4} - 5 = -\frac{11}{4}$

✓ **FINAL ANSWER**

$x = 3,\ y = 4$ and $x = \frac{3}{4},\ y = -\frac{11}{4}$

TOPIC

Inequalities (Solving & Graphing)

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2022

Question 22

5 marks | Inequalities (Solving & Graphing)

22 Here is a rectangle.

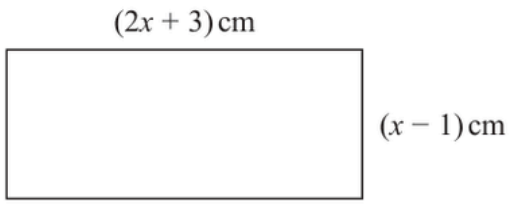


Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than $75\,\text{cm}^2$

find the range of possible values of x

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2022

1 **The rectangle has side lengths**

The rectangle has side lengths

$$2x + 3$$

2 **Use Functions Solving Inequalities**

and

$$x - 1$$

3 **Solve inequality**

For the side lengths to be possible,

$$x > 1$$

4 **The area is less than**

The area is less than 75:

$$(2x + 3)(x - 1) < 75$$

$$2x^2 + x - 3 < 75$$

$$2x^2 + x - 78 < 0$$

5 **Factorise**

Factorise:

$$2x^2 + x - 78 = (2x + 13)(x - 6)$$

---

6 **Evaluate fraction**

So

$$-\frac{13}{2} < x < 6$$

---

7 **Solve inequality**

Combining with $x > 1$:

$$1 < x < 6$$

✓ **FINAL ANSWER**

$1 < x < 6$

4MA1/2HR JUNE 2024

Question 20

4 marks | Inequalities (Solving & Graphing)

20 Solve $6x^2 - 7x - 20 > 0$
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2024

1 Solve quadratic equation

$$6x^2 - 7x - 20 > 0$$

2 Factorise

Factorise:

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{4}{3} \quad \text{and} \quad x = \frac{5}{2}$$

4 Solve quadratic equation

Since the coefficient of x^2 is positive, the quadratic is greater than 0 outside the roots.

✓ **FINAL ANSWER**

$x < -\frac{4}{3}$ or $x > \frac{5}{2}$

4MA1/2H NOVEMBER 2024

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $10x^2 + 11x - 21 < 0$
Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2024

1 Solve inequality

$$10x^2 + 11x - 21 < 0$$

2 Factorise

Factorise:

$$10x^2 + 11x - 21 = (10x + 21)(x - 1)$$

3 The critical values are

The critical values are

$$x = -\frac{21}{10} \quad \text{and} \quad x = 1$$

4 The quadratic is less than

The quadratic is less than 0 between the roots.

✓ FINAL ANSWER

$$-\frac{21}{10} < x < 1$$

4WM2/01R MAY 2025

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01R May 2025

1 Required

Solve 2*x*² − 7*x* − 15 \> 0. Show clear algebraic working.

2 Factorise the quadratic 2 x

Factorise the quadratic:

2*x*² − 7*x* − 15 = (2*x*+3)(*x*−5)

Check: (2*x*+3)(*x*−5) = 2*x*² − 10*x* + 3*x* − 15 = 2*x*² − 7*x* − 15

3 Set each factor zero

Roots: set each factor to zero:

$$2x + 3 = 0 implies x = -tfrac32; qquad x - 5 = 0 implies x = 5$$

4 Solve inequality

Since the coefficient of *x*² is + 2 \> 0, the parabola opens **upward**. The expression is \> 0 *outside* the roots:

FINAL ANSWER

$x \< - dfrac32$ or *x* \> 5

4MA1/2HR JUNE 2025

Question 21

3 marks | Inequalities (Solving & Graphing)

21 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2025

1 Solve inequality

$$2x^2 - 7x - 15 > 0$$

2 Factorise

Factorise:

$$2x^2 - 7x - 15 = (2x + 3)(x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{3}{2} \quad \text{and} \quad x = 5$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

$x < -\frac{3}{2}$ or $x > 5$

4MA1/1H NOVEMBER 2025

Question 25

3 marks | Inequalities (Solving & Graphing)

25 Solve the inequality $2x^2 + x - 28 > 0$
Show clear algebraic working.

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2025

1 Solve inequality

$$2x^2 + x - 28 > 0$$

2 Factorise

Factorise:

$$2x^2 + x - 28 = (2x - 7)(x + 4)$$

3 The critical values are

The critical values are

$$x = \frac{7}{2} \quad \text{and} \quad x = -4$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

$x < -4$ or $x > \frac{7}{2}$

TOPIC

Algebraic Proof

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

Question 25

3 marks | Algebraic Proof

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR January 2020

1 Use trigonometry

Since N is a multiple of 5, let

$$N = 5k$$

2 where is an integer

where k is an integer.

3 Given

Given

$$A = N + 1, \qquad B = N - 1$$

4 Use trigonometry

So

$$A^2 - B^2 = (N+1)^2 - (N-1)^2$$

5 Expand

Expand:

$$(N+1)^2 = N^2 + 2N + 1$$

$$(N-1)^2 = N^2 - 2N + 1$$

6 **Use trigonometry**

Therefore

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$

$$= 4N$$

---

7 **Substitute**

Substitute $N = 5k$:

$$4N = 4(5k) = 20k$$

---

8 **Use trigonometry**

Since k is an integer, $20k$ is a multiple of 20.

✓ **FINAL ANSWER**

$A^2 - B^2$ is always a multiple of 20.

4MA1/2H NOVEMBER 2024

Question 23

4 marks | Algebraic Proof

23 Show that $\dfrac{16x^2 - 36}{x - 7} \div \dfrac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x) = n$

where n is an integer to be found.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2024

1 Evaluate fraction

$$\frac{16x^2 - 36}{x - 7} \div \frac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x)$$

2 Factorise

Factorise:

$$16x^2 - 36 = 4(2x - 3)(2x + 3)$$

$$2x^2 + 7x + 6 = (2x + 3)(x + 2)$$

$$x^2 - 5x - 14 = (x - 7)(x + 2)$$

3 Evaluate fraction

So

$$\frac{4(2x-3)(2x+3)}{x-7} \div \frac{(2x+3)(x+2)}{(x-7)(x+2)} - (7+8x)$$

$$= \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)} - (7+8x)$$

$$= 4(2x-3) - (7+8x)$$

$$= 8x - 12 - 7 - 8x$$

$$= -19$$

4 Calculate value

So $n = -19$, which is an integer.

FINAL ANSWER

$n = -19$

4WM2/01 MAY 2025

Question 20

3 marks | Algebraic Proof

20 a, b and c are three consecutive even numbers where $a < b < c$

Prove algebraically that $b^2 = ac + 4$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 May 2025

1 Let

Let

$$a = 2n, \qquad b = 2n + 2, \qquad c = 2n + 4$$

2 Calculate value

Then

$$ac + 4 = (2n)(2n + 4) + 4$$

$$= 4n^2 + 8n + 4$$

$$= (2n + 2)^2 = b^2$$

3 Calculate value

So $b^2 = ac + 4$.

FINAL ANSWER

So $b^2 = ac + 4$.

TOPIC

Sequences

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 24

5 marks | Sequences

24 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.
The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

.....

(Total for Question 24 is 5 marks)

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

2 Find the nth term

So

$$S_{10} = 5(2a + 9d)$$

3 Find the nth term

and

$$S_5 = \frac{5}{2}(2a + 4d)$$

4 Given

Given $S_{10} = 4S_5$:

$$5(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$10a + 45d = 20a + 40d$$

$$5d = 10a$$

$$d = 2a$$

5 The 8th term is

The 8th term is 45:

$$a + 7d = 45$$

6 Substitute

Substitute $d = 2a$:

$$a + 14a = 45$$

$$15a = 45$$

$$a = 3$$

FINAL ANSWER

the first term is $\mathbf{3}$.

4MA1/2HR JANUARY 2020

Question 22

4 marks | Sequences

22 The first term of an arithmetic series S is -6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2020

1 Find the nth term

First term:

$$a = -6$$

$$S_{30} = 2865$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$2865 = \frac{30}{2}(2(-6) + 29d)$$

$$2865 = 15(-12 + 29d)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7$$

3 **The 9th term is**

The 9th term is

$$u_9 = a + 8d$$

$$u_9 = -6 + 8(7) = 50$$

✓ **FINAL ANSWER**

50

4MA1/2H JANUARY 2020

Question 25

3 marks | Sequences

25 Mario is going to save \$50 in the year 2021

He is going to continue to save, up to and including the year 2070, by increasing the amount he saves each year by \$$k$

Mario will save a total of \$33 125 from 2021 to 2070

Work out the value of k.

$k =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2020

1 Mario saves from 2021 up

Mario saves from 2021 up to and including 2070.

2 Number of years

Number of years:

$$2070 - 2021 + 1 = 50$$

3 This is an arithmetic sequence

This is an arithmetic sequence.

4 Use trigonometry

First term:

$$50$$

5 Common difference

Common difference:

$$k$$

6 Last term

Last term:

$$50 + 49k$$

7 Use the arithmetic series formula

Use the arithmetic series formula:

$$S_n = \frac{n}{2}(a + l)$$

$$33125 = \frac{50}{2}(50 + (50 + 49k))$$

$$33125 = 25(100 + 49k)$$

$$1325 = 100 + 49k$$

$$49k = 1225$$

$$k = 25$$

FINAL ANSWER

$$\boxed{k = 25}$$

4MA1/1H MAY/NOVEMBER 2020

Question 24

4 marks | Sequences

24 Here are the first five terms of an arithmetic sequence.

8 15 22 29 36

Work out the sum of all the terms from the 50th term to the 100th term inclusive.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May/November 2020

1 The arithmetic sequence is

The arithmetic sequence is

$$8,\ 15,\ 22,\ 29,\ 36,\ldots$$

2 Calculate value

So

$$a = 8, \quad d = 7$$

3 The th term is

The nth term is

$$u_n = 8 + 7(n-1) = 7n + 1$$

$$u_{50} = 7(50) + 1 = 351$$

$$u_{100} = 7(100) + 1 = 701$$

4 There are

There are

$$100 - 50 + 1 = 51$$

5 **Find the nth term**

terms from the 50th to the 100th inclusive.

$$\text{sum} = \frac{51}{2}(351 + 701)$$

$$= 26826$$

✓ **FINAL ANSWER**

26826

4MA1/1HR JANUARY 2021

Question 21

6 marks | Sequences

21 The nth term of an arithmetic series is u_n where $u_n > 0$ for all n
The sum to n terms of the series is S_n

Given that $u_4 = 6$ and that $S_{11} = (u_6)^2 + 18$

find the value of u_{20}

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2021

1 Let

Let $u_6 = v$.

2 Use series formula

For an arithmetic series,

$$S_{11} = \frac{11}{2}(u_1 + u_{11})$$

3 The average of and is

The average of u_1 and u_{11} is u_6, so

$$S_{11} = 11u_6 = 11v$$

4 **Given**

Given

$$S_{11} = (u_6)^2 + 18$$

$$11v = v^2 + 18$$

$$v^2 - 11v + 18 = 0$$

$$(v - 2)(v - 9) = 0$$

5 **Calculate value**

So

$$u_6 = 2 \quad \text{or} \quad u_6 = 9$$

6 **Find the nth term**

But $u_4 = 6$. If $u_6 = 2$, then the common difference is negative and later terms would not all be positive. Therefore,

$$u_6 = 9$$

$$d = \frac{u_6 - u_4}{2} = \frac{9 - 6}{2} = \frac{3}{2}$$

$$u_{20} = u_6 + 14d$$

$$u_{20} = 9 + 14\left(\frac{3}{2}\right) = 30$$

✓ **FINAL ANSWER**

$u_{20} = 30$

4MA1/1H JANUARY 2021

Question 24

4 marks | Sequences

24 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$a =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2021

1 Evaluate fraction

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{2n} = n(2a + (2n-1)d)$$

2 Given

Given

$$S_{2n} = 4S_n$$

$$n(2a + (2n-1)d) = 4 \cdot \frac{n}{2}(2a + (n-1)d)$$

3 Divide by

Divide by n:

$$2a + (2n-1)d = 2(2a + (n-1)d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$d = 2a$$

$$a = \frac{d}{2}$$

FINAL ANSWER

$$a = \frac{d}{2}$$

4MA1/1H MAY 2021

Question 23

5 marks | Sequences

23 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$N =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2021

1 **The 2nd term is**

The 2nd term is

$$a + d = 8.5$$

2 **The 5th term is**

The 5th term is

$$a + 4d = 13$$

3 **Subtract**

Subtract:

$$3d = 4.5$$

$$d = 1.5$$

$$a = 8.5 - 1.5 = 7$$

4 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$292 = \frac{N}{2}(14 + 1.5(N-1))$$

$$292 = \frac{N}{2}(1.5N + 12.5)$$

$$3N^2 + 25N - 1168 = 0$$

$$N = \frac{-25 + 121}{6} = 16$$

✓ **FINAL ANSWER**

$N = 16$

4MA1/2H NOVEMBER 2021

Question 20

5 marks | Sequences

20 Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2021

1 The first four terms are

The first four terms are

$$k, \quad \frac{3k}{4}, \quad \frac{k}{2}, \quad \frac{k}{4}$$

2 The common difference is

The common difference is

$$d = -\frac{k}{4}$$

3 The 15th term is

The 15th term is

$$u_{15} = k + 14\left(-\frac{k}{4}\right)$$

$$u_{15} = k - \frac{7k}{2} = -\frac{5k}{2}$$

4 **Given**

Given

$$-\frac{5k}{2} = 90 + 2k$$

$$-5k = 180 + 4k$$

$$k = -20$$

---

5 **Evaluate fraction**

So

$$a = -20, \quad d = 5$$

$$S_{30} = \frac{30}{2}(2(-20) + 29(5))$$

$$S_{30} = 15(105) = 1575$$

✓ **FINAL ANSWER**

1575

4MA1/2H JANUARY 2022

Question 26

5 marks | Sequences

26 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$d =$ .....

$m =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H January 2022

1 Calculate value

$$S_m = 39$$

$$S_{2m} = 320$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

3 Calculate value

Then

$$2S_m = m(2a + (m-1)d) = 78$$

4 Calculate value

and

$$S_{2m} = m(2a + (2m-1)d) = 320$$

5 **Subtract**

Subtract:

$$S_{2m} - 2S_m = m[(2m-1)d - (m-1)d]$$

$$320 - 78 = m(md)$$

$$242 = m^2 d$$

$$242 = 2 \times 11^2$$

---

6 **Square factor**

Since d is prime and m^2 is a square factor,

$$d = 2, \quad m^2 = 121$$

$$m = 11$$

✓ **FINAL ANSWER**

$d = 2,\ m = 11$

4MA1/1HR JUNE 2022

Question 25

5 marks | Sequences

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2022

1 **Calculate value**

$$S_{10} = 4S_5$$

2 **Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

3 **Calculate value**

So

$$5(2a + 9d) = 4 \cdot 5(a + 2d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

4 **The 8th term is 45**

The 8th term is 45:

$$u_8 = a + 7d = 45$$

$$a + 7(2a) = 45$$

$$15a = 45$$

$$a = 3$$

✓ **FINAL ANSWER**

The first term is $\mathbf{3}$

4MA1/1H MAY 2023

Question 20

6 marks | Sequences

20 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

$X =$ .....

(Total for Question 20 is 6 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2023

1 Evaluate fraction

$$S_{80} = 470$$

$$40(2a + 79d) = 470$$

$$2a + 79d = \frac{47}{4}$$

2 The 75th term is

The 75th term is

$$a + 74d = 14.5 = \frac{29}{2}$$

3 Double it

Double it:

$$2a + 148d = 29$$

4 **Subtract**

Subtract:

$$69d = 29 - \frac{47}{4} = \frac{69}{4}$$

$$d = \frac{1}{4}$$

$$a + 74\left(\frac{1}{4}\right) = \frac{29}{2}$$

$$a = -4$$

---

5 **Evaluate fraction**

Now

$$S_X = 171$$

$$171 = \frac{X}{2}\left(2(-4) + (X-1)\frac{1}{4}\right)$$

$$171 = \frac{X(X-33)}{8}$$

$$X^2 - 33X - 1368 = 0$$

$$X = \frac{33 + 81}{2} = 57$$

✓ **FINAL ANSWER**

$X = 57$

4MA1/2HR JUNE 2023

Question 23

5 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$8p \qquad 7p - 3 \qquad 4p + 2$$

The sum of the first n terms of the sequence is -1914

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2023

1 The first three terms are

The first three terms are

$$8p, \quad 7p - 3, \quad 4p + 2$$

2 Find the nth term

For an arithmetic sequence, consecutive differences are equal:

$$(7p - 3) - 8p = (4p + 2) - (7p - 3)$$

$$-p - 3 = -3p + 5$$

$$2p = 8$$

$$p = 4$$

3 Find the nth term

So the first term is

$$8p = 32$$

4 **Find common difference**

and the common difference is

$$(7p-3)-8p=25-32=-7$$

5 **The sum of the first**

The sum of the first n terms is

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$-1914=\frac{n}{2}(64-7(n-1))$$

$$-1914=\frac{n}{2}(71-7n)$$

$$-3828=n(71-7n)$$

$$7n^2-71n-3828=0$$

$$n=\frac{71\pm 335}{14}$$

6 **Only the positive value is**

Only the positive value is possible:

$$n=29$$

✓ **FINAL ANSWER**

$n=29$

4MA1/1HR MAY 2024

Question 23

4 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$(4x-14) \quad , \quad (x+2) \quad , \quad (7x-9)$$

Find, as an integer, the sum of the first 40 terms of the sequence.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2024

1 **The first three terms are**

The first three terms are

$$4x - 14, \quad x + 2, \quad 7x - 9$$

2 **Equal differences give**

Equal differences give

$$(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$$

$$-3x + 16 = 6x - 11$$

$$9x = 27$$

$$x = 3$$

3 **Find the nth term**

So the sequence is

$$-2,\ 5,\ 12, \ldots$$

4 **with common difference**

with common difference 7.

$$S_{40} = \frac{40}{2}(2(-2) + 39(7))$$

$$S_{40} = 20(-4 + 273) = 5380$$

✓ **FINAL ANSWER**

5380

4MA1/1H MAY 2024

Question 24

6 marks | Sequences

24 A polygon has n sides, where $n > 5$

The interior angles of the polygon form an arithmetic sequence.

The smallest angle of the polygon is 84°
The common difference of the sequence is 4°

Work out the sum of the interior angles of the polygon.
Show clear algebraic working.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2024

1 The interior angles form an

The interior angles form an arithmetic sequence with first term 84 and common difference 4.

2 Use trigonometry

So the sum of the angles is

$$\frac{n}{2}(2(84) + (n-1)4)$$

$$= \frac{n}{2}(168 + 4n - 4)$$

$$= 2n^2 + 82n$$

3 The sum of the interior

The sum of the interior angles of an n-sided polygon is

$$180(n-2)$$

4 **Calculate value**

So

$$2n^2 + 82n = 180(n - 2)$$

$$2n^2 - 98n + 360 = 0$$

$$n^2 - 49n + 180 = 0$$

$$(n - 45)(n - 4) = 0$$

---

5 **Solve inequality**

Since $n > 5$,

$$n = 45$$

---

6 **Use trigonometry**

Therefore the sum of the interior angles is

$$180(45 - 2) = 7740$$

✓ **FINAL ANSWER**

7740 degrees

4MA1/1H JUNE 2025

Question 24

5 marks | Sequences

24 An arithmetic series has 30 terms.

The first term is a
The common difference is d

The 20th term is 123
The sum of the 30 terms is 2880

Work out the value of a and the value of d
Show clear algebraic working.

$a =$ .....

$d =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2025

1 Evaluate fraction

$$u_{20} = a + 19d = 123$$

$$S_{30} = \frac{30}{2}(2a + 29d) = 2880$$

$$2a + 29d = 192$$

2 Double the first equation

Double the first equation:

$$2a + 38d = 246$$

3 **Subtract**

Subtract:

$$9d = 54$$

$$d = 6$$

$$a + 19(6) = 123$$

$$a = 9$$

✓ **FINAL ANSWER**

$a = 9,\ d = 6$

4MA1/1HR JUNE 2025

Question 24

6 marks | Sequences

24 The first 3 terms of an arithmetic series are

$$(2x + 5) \qquad (3y - 4) \qquad (4x - 2)$$

where x and y are constants.

The sum of the first 9 terms of the series is 216

Find the value of x and the value of y
Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2025

1 The first 3 terms are

The first 3 terms are

$$2x + 5, \quad 3y - 4, \quad 4x - 2$$

2 Equal differences give

Equal differences give

$$(3y - 4) - (2x + 5) = (4x - 2) - (3y - 4)$$

$$6y - 6x = 11$$

3 Use series formula

For an arithmetic series,

$$S_9 = 9(a + 4d)$$

4 **Here**

Here

$$a = 2x + 5$$

$$d = (3y - 4) - (2x + 5) = 3y - 2x - 9$$

5 **Given**

Given $S_9 = 216$:

$$9[(2x + 5) + 4(3y - 2x - 9)] = 216$$

$$12y - 6x = 55$$

6 **Solve**

Now solve

$$6y - 6x = 11$$

$$12y - 6x = 55$$

7 **Subtracting gives**

Subtracting gives

$$6y = 44$$

$$y = \frac{22}{3}$$

8 **Evaluate fraction**

Then

$$6\left(\frac{22}{3}\right) - 6x = 11$$

$$x = \frac{11}{2}$$

✓ **FINAL ANSWER**

$$x = \frac{11}{2},\ y = \frac{22}{3}$$

4WM2/01 MAY 2025

Question 24

4 marks | Sequences

24 The 21st term of an arithmetic series is 109
The sum of the first 52 terms of the series is 4381

Work out the 5th term of the series.
Show clear algebraic working.

.....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM2/01 May 2025

1 Use series formula

For an arithmetic series,

$$a + 20d = 109$$

2 Evaluate fraction

and

$$\frac{52}{2}(2a + 51d) = 4381$$

$$26(2a + 51d) = 4381$$

3 Solving gives

Solving gives

$$d = -4.5, \qquad a = 199$$

4 The fifth term is

The fifth term is

$$a + 4d = 199 + 4(-4.5) = 181$$

FINAL ANSWER

181.

4WM2/01R MAY 2025

Question 24

6 marks | Sequences

24 An arithmetic series P has first term a and common difference d

The sum of the first 40 terms of P is 2070

An arithmetic series Q has first term $2a$ and common difference $3d$

The 25th term of Q is 186

Work out the sum of the first 40 terms of Q
Show clear algebraic working.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4WM2/01R May 2025

1 Required

Series P: first term *a*, c.d. *d*; *S*₄₀(*P*) = 2070. Series Q: first term 2*a*, c.d. 3*d*; 25th term of Q = 186. Find *S*₄₀(*Q*).

2 Form equation from S 40

Form equation from *S*₄₀(*P*) = 2070:

$$S_40 = frac402\[2a + 39d\] = 20(2a + 39d) = 2070implies2a + 39d = 103.5tag1$$

3 Form equation from 25th term

Form equation from 25th term of Q = 186:

*T*₂₅(*Q*) = 2*a* + 24(3*d*) = 2*a* + 72*d* = 186

4 Solve simultaneous equations

Solve simultaneously — subtract (1) from (2):

33*d* = 82.5 ⟹ *d* = 2.5

5 Substitute d 2 5 (1)

Substitute *d* = 2.5 into (1):

2*a* + 39(2.5) = 103.5 ⟹ 2*a* = 6 ⟹ *a* = 3

6 Compute S 40 (Q

Compute *S*₄₀(*Q*), with first term = 2*a* = 6 and c.d. = 3*d* = 7.5:

$$S_40(Q) = frac402bigl\[2(6) + 39(7.5)bigr\] = 20bigl\[12 + 292.5bigr\] = 20times304.5 = 6090$$

FINAL ANSWER

*S*₄₀(*Q*) = 6090

4MA1/2H NOVEMBER 2025

Question 25

5 marks | Sequences

25 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2025

1 Calculate value

$$a = 10$$

$$u_{20} = 86$$

$$10 + 19d = 86$$

$$d = 4$$

2 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$5194 = \frac{N}{2}(20 + 4(N-1))$$

$$5194 = \frac{N}{2}(4N + 16)$$

$$5194 = 2N(N+4)$$

$$N^2 + 4N - 2597 = 0$$

$$N = \frac{-4 + 102}{2} = 49$$

FINAL ANSWER

$N = 49$

4WM2/01 NOVEMBER 2025

Question 26

5 marks | Sequences

26 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4WM2/01 November 2025

1 Find common difference

Step 1: Find the common difference *d*. The *n*th term is given by *u*_{*n*} = *a* + (*n*−1)*d*.

*u*₂₀ = 10 + 19*d* = 86

19*d* = 76

$$d = frac7619 = 4$$

2 Use sum formula

Step 2: Use the sum formula to form an equation in terms of *N*.

$$S_N = fracN2left\[2a + (N-1)dright\]$$

Substitute the known values *S*_{*N*} = 5194, *a* = 10, and *d* = 4:

$$5194 = fracN2left\[2(10) + (N-1)(4)right\]$$

3 Solve quadratic equation

Step 3: Simplify and solve the quadratic equation. Multiply by 2 to clear the fraction:

10388 = *N*

$$20 + 4 * N * -4$$

10388 = *N*(4*N*+16)

4*N*² + 16*N* − 10388 = 0

4 Divide the entire equation by

Divide the entire equation by 4 to make it simpler:

*N*² + 4*N* − 2597 = 0

5 Use quadratic formula

Step 4: Use the quadratic formula to find *N*.

$$N = frac-bpmsqrtb^2 - 4ac2a$$

$$N = frac-4pmsqrt16 - 4(1)(-2597)2$$

$$N = frac-4pmsqrt16 + 103882$$

$$N = frac-4pmsqrt104042$$

$$N = frac-4pm1022$$

6 Evaluate fraction

Since *N* must be a positive integer, we discard the negative root:

$$N = frac-4 + 1022 = frac982 = 49$$

✓ FINAL ANSWER

N = 49 Always double-check that your *N* (number of terms) evaluates to a positive whole integer. If it ends up as a decimal or fraction, recheck your arithmetic progression formulas.

Question 25

6 marks | Sequences

25 Here are the first four terms of an arithmetic series.

$$(k + 2) \qquad (k + 5) \qquad (k + 8) \qquad (k + 11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first n terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

(Total for Question 25 is 6 marks)

1 **First term and difference**

$a = k + 2$, $d = 3$.

2 **Use the sum formula**

$$\frac{2n}{2}\left[2a + (2n-1)3\right] = 4 \times \frac{n}{2}\left[2a + (n-1)3\right]$$

3 **Simplify (divide by n)**

$$2a + 6n - 3 = 4a + 6n - 6 \Rightarrow 2a = 3 \Rightarrow a = 1.5,\ k = -\tfrac{1}{2}$$

4 **100th term**

$$a + 99d = 1.5 + 99(3) = 298.5$$

✓ **FINAL ANSWER**

298.5

TOPIC

Direct & Inverse Proportion

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2022

Question 20

4 marks | Direct & Inverse Proportion

20 y is inversely proportional to $\sqrt{x}$
x is directly proportional to T^3

Given that $y = 8$ when $T = 25$

find the exact value of T when $y = 27$

$T =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2022

1 **Find inverse function**

$$y \propto \frac{1}{\sqrt{x}}$$

2 **Use Direct Inverse Proportion correct**

and

$$x \propto T^3$$

3 **Find inverse function**

So

$$\sqrt{x} \propto T^{3/2}$$

4 **Find inverse function**

Therefore

$$y \propto \frac{1}{T^{3/2}}$$

5 **Use Direct Inverse Proportion correct**

So $yT^{3/2}$ is constant.

6 **Find inverse function**

When $y = 8$ and $T = 25$,

$$8(25)^{3/2} = 8(125) = 1000$$

7 **Find inverse function**

When $y = 27$,

$$27T^{3/2} = 1000$$

$$T^{3/2} = \frac{1000}{27} = \left(\frac{10}{3}\right)^3$$

$$T = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

✓ **FINAL ANSWER**

$$\frac{100}{9}$$

TOPIC

Functions

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 21

5 marks | Functions

21 The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2020

1 **Calculate value**

$$f(x) = x^2 - 2x$$

$$g(x) = x + 3$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = (x+3)^2 - 2(x+3)$$

$$h(x) = x^2 + 4x + 3$$

$$h(x) = (x+2)^2 - 1$$

2 **Let**

Let

$$y = (x+2)^2 - 1$$

3 Find inverse function

Since the domain is $x \geq -2$, take the positive square root:

$$x + 2 = \sqrt{y + 1}$$

$$x = \sqrt{y + 1} - 2$$

FINAL ANSWER

$$h^{-1}(x) = \sqrt{x + 1} - 2$$

4MA1/1H JANUARY 2021

Question 22

3 marks | Functions

22 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2021

1 Calculate value

$$f(x) = x^2 - 8x + 5$$

$$f(x) = (x-4)^2 - 11$$

2 Let

Let

$$y = (x-4)^2 - 11$$

$$y + 11 = (x-4)^2$$

3 Find inverse function

Since the domain is $x < 4$, take the negative square root:

$$x - 4 = -\sqrt{y+11}$$

$$x = 4 - \sqrt{y+11}$$

FINAL ANSWER

$$f^{-1}(x) = 4 - \sqrt{x+11}$$

4MA1/1H MAY 2021

Question 24

6 marks | Functions

24 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2021

1 **(a)**

(a)

$$g(2) = 7(2) - 6 = 8$$

$$fg(2) = f(8)$$

$$f(8) = 5(8)^2 - 10(8) + 7$$

$$f(8) = 247$$

2 **(b)**

(b)

$$f(x) = 5x^2 - 10x + 7$$

$$f(x) = 5(x - 1)^2 + 2$$

3 **Let**

Let

$$y = 5(x-1)^2 + 2$$

$$(x-1)^2 = \frac{y-2}{5}$$

---

4 **Find inverse function**

Since the domain is $x \geq 1$, take the positive square root:

$$x = 1 + \sqrt{\frac{y-2}{5}}$$

✓ **FINAL ANSWER**

(a) 247, (b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x-2}{5}}$

4MA1/2HR JANUARY 2022

Question 23

4 marks | Functions

23 The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function h^{-1} in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2022

1 **Calculate value**

$$f(x) = x + 25$$

$$g(x) = x^2 - 12x$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = x^2 - 12x + 25$$

$$h(x) = (x - 6)^2 - 11$$

2 **Let**

Let

$$y = (x - 6)^2 - 11$$

$$y + 11 = (x - 6)^2$$

3 Find inverse function

Since the domain of h is $x < 6$, take the negative square root:

$$x - 6 = -\sqrt{y + 11}$$

$$x = 6 - \sqrt{y + 11}$$

FINAL ANSWER

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$

4MA1/2H JANUARY 2022

Question 25

5 marks | Functions

25 The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain} \quad \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

$g^{-1}: x \mapsto$ .....

(4)

(b) State the domain of g^{-1}

.....

(1)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2022

1 **Calculate value**

$$g(x) = 5 + 6x - x^2$$

2 **Complete the square**

Complete the square:

$$g(x) = 14 - (x - 3)^2$$

3 **Let**

Let

$$y = 14 - (x - 3)^2$$

$$(x - 3)^2 = 14 - y$$

4 **Find inverse function**

The domain of g is $x \geq 3$, so take the positive square root:

$$x - 3 = \sqrt{14 - y}$$

$$x = 3 + \sqrt{14 - y}$$

5 **Simplify surd**

Therefore

$$g^{-1} : x \mapsto 3 + \sqrt{14 - x}$$

6 **Calculate statistic**

The range of g is $x \leq 14$, so this is the domain of g^{-1}.

✓ **FINAL ANSWER**

(a) $g^{-1} : x \mapsto 3 + \sqrt{14 - x}$, (b) domain $x \leq 14$

4MA1/1H NOVEMBER 2024

Question 25

4 marks | Functions

25 The function f is such that $f(x) = 2x^2 - 24x + 7$ where $x \geqslant 6$

Find the inverse function $f^{-1}(x)$

$f^{-1}(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2024

1 Calculate value

$$f(x) = 2x^2 - 24x + 7$$

2 Complete the square

Complete the square:

$$f(x) = 2(x^2 - 12x) + 7$$

$$f(x) = 2[(x - 6)^2 - 36] + 7$$

$$f(x) = 2(x - 6)^2 - 65$$

3 Let

Let $y = f(x)$.

$$y = 2(x - 6)^2 - 65$$

$$y + 65 = 2(x - 6)^2$$

$$\frac{y + 65}{2} = (x - 6)^2$$

4 **Simplify surd**

Since $x \geq 6$, take the positive square root:

$$x - 6 = \sqrt{\frac{y+65}{2}}$$

$$x = 6 + \sqrt{\frac{y+65}{2}}$$

---

5 **Replace by**

Replace y by x.

✓ **FINAL ANSWER**

$$f^{-1}(x) = 6 + \sqrt{\frac{x+65}{2}}$$

4WM2/01 MAY 2025

Question 21

7 marks | Functions

21 The function g is defined as

$$g(x) = \frac{7x + 20}{2x} \qquad x \neq 0$$

(a) Solve $gg(x) = 6$
Show clear algebraic working.

$x =$ .....

(3)

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The function h is defined as

$$h(x) = 5x^2 + 30x - 7 \qquad x \geqslant -3$$

(b) Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 May 2025

1 **Evaluate fraction**

(a) If $g(g(x)) = 6$, then

$$\frac{7g(x) + 20}{2g(x)} = 6$$

$$7g(x) + 20 = 12g(x)$$

$$g(x) = 4$$

2 **Evaluate fraction**

So

$$\frac{7x+20}{2x}=4$$

$$7x+20=8x$$

$$x=20$$

---

3 **(b)**

(b)

$$h(x)=5x^2+30x-7=5(x+3)^2-52$$

---

4 **Simplify surd**

Let $y=5(x+3)^2-52$. Since $x\geq -3$, use the positive square root:

$$x+3=\sqrt{\frac{y+52}{5}}$$

$$h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$$

✓ **FINAL ANSWER**

$x=20$; $h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$.

4WM2/01 NOVEMBER 2025

Question 21

7 marks | Functions

21 The functions f and g are defined as

$$f(x) = 5x - 3$$

$$g(x) = 4x^2 + 16x - 9 \qquad \text{where} \quad x \geqslant -2$$

(a) Find g(–1)

.....

(1)

(b) Find fg(x)

Give your answer in its simplest form.

fg(x) = ...

(2)

$g(x) = 4x^2 + 16x - 9$ where $x \geqslant -2$

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 November 2025

1 **(a) Find g (1)**

(a) Find *g*(−1):

g(−1) = 4(−1)² + 16(−1) − 9 = 4 − 16 − 9 = − 21

2 **Answer**

a − 21

(b) Find *fg*(*x*):**

*f**g*(*x*) = *f*(*g*(*x*)) = 5(4*x*²+16*x*−9) − 3

= 20*x*² + 80*x* − 45 − 3 = 20*x*² + 80*x* − 48

---

3 **Answer**

b 20*x*² + 80*x* − 48

Let *y* = 4*x*² + 16*x* − 9. We must rearrange to make *x* the subject.

Step 1: Complete the square for the *x* terms.

y = 4(*x*²+4*x*) − 9

y = 4((*x*+2)²−4) − 9

y = 4(*x*+2)² − 16 − 9

y = 4(*x*+2)² − 25

Step 2: Rearrange for *x*.

y + 25 = 4(*x*+2)²

$$fracy + 254 = (x + 2)^2$$

$$x + 2 = pmsqrtfracy + 254$$

Since *x* ≥ − 2, *x* + 2 is positive, so we take the positive root.

$$x + 2 = fracsqrty + 252$$

$$x = fracsqrty + 252 - 2$$

Step 3: Swap *y* for *x* to write the inverse function.

✓ **FINAL ANSWER**

$g^{-1}(x) = fracsqrtx + 252 - 2$ When finding the inverse of a quadratic, always complete the square. The restricted domain (*x* ≥ − 2) tells you whether to take the positive or negative square root!

TOPIC

Differentiation & Turning Points

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 23

6 marks | Differentiation & Turning Points

23 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$T =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2020

1 Calculate value

$$s = t^3 + 4t^2 - 5t + 7$$

2 Evaluate fraction

Velocity is the derivative of displacement:

$$v = \frac{ds}{dt} = 3t^2 + 8t - 5$$

3 Calculate value

At time T, the velocity is V, so

$$V = 3T^2 + 8T - 5$$

$$3T^2 + 8T - (V + 5) = 0$$

4 **Use the quadratic formula**

Use the quadratic formula:

$$T = \frac{-8 \pm \sqrt{8^2 - 4(3)(-(V+5))}}{2(3)}$$

$$T = \frac{-8 \pm \sqrt{64 + 12V + 60}}{6}$$

$$T = \frac{-8 \pm \sqrt{124 + 12V}}{6}$$

$$T = \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

---

5 **Simplify surd**

Since $T \geq 0$, take the positive root:

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

✓ **FINAL ANSWER**

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

4MA1/1H MAY/NOVEMBER 2020

Question 23

4 marks | Differentiation & Turning Points

23 Curve **C** has equation $y = px^3 - mx$ where p and m are positive integers.

Find the range of values of x, in terms of p and m, for which the gradient of **C** is negative.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May/November 2020

1 Differentiate

$$y = px^3 - mx$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3px^2 - m$$

3 The gradient is negative when

The gradient is negative when

$$3px^2 - m < 0$$

$$3px^2 < m$$

$$x^2 < \frac{m}{3p}$$

4 Simplify surd

So

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

FINAL ANSWER

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

4MA1/1HR JANUARY 2021

Question 20

5 marks | Differentiation & Turning Points

20 A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

..... m/s

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2021

1 **Read the graph**

$$s = t^3 - 9t^2 + 33t - 6$$

2 **Read the graph**

Velocity is the rate of change of displacement:

$$v = \frac{ds}{dt}$$

$$v = 3t^2 - 18t + 33$$

3 **Complete the square**

Complete the square:

$$v = 3(t^2 - 6t + 11)$$

$$v = 3[(t-3)^2 + 2]$$

$$v = 3(t-3)^2 + 6$$

4 **Read the graph**

Since $(t-3)^2 \geq 0$, the minimum value of v is

$$6$$

5 **Read the graph**

The velocity is always positive, so the minimum speed is also 6.

✓ **FINAL ANSWER**

$6\ \text{m/s}$

4MA1/2H MAY 2021

Question 21

5 marks | Differentiation & Turning Points

21 The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a.
Show your working clearly.

$a =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H May 2021

1 Evaluate fraction

$$y = kx^2 + \frac{16}{x}$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 2kx - \frac{16}{x^2}$$

3 Find the gradient

At the stationary point, $x = \frac{2}{3}$, so the gradient is 0:

$$2k\left(\frac{2}{3}\right) - \frac{16}{\left(\frac{2}{3}\right)^2} = 0$$

$$\frac{4k}{3} - \frac{16}{\frac{4}{9}} = 0$$

$$\frac{4k}{3} - 36 = 0$$

$$\frac{4k}{3} = 36$$

$$k = 27$$

4 Find the -coordinate

Now find a, the y-coordinate when $x = \frac{2}{3}$:

$$a = 27\left(\frac{2}{3}\right)^2 + \frac{16}{\frac{2}{3}}$$

$$a = 27 \cdot \frac{4}{9} + 24$$

$$a = 12 + 24 = 36$$

FINAL ANSWER

$a = 36$

4MA1/1HR JANUARY 2022

Question 23

6 marks | Differentiation & Turning Points

23 Two particles, P and Q, move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2022

1 **Particle**

Particle P:

$$s = t^3 - 4t^2 + 5t$$

$$\frac{ds}{dt} = 3t^2 - 8t + 5$$

$$\frac{ds}{dt} = (3t - 5)(t - 1)$$

2 **Particle**

Particle Q:

$$x = t^2 - 4t + 4$$

$$\frac{dx}{dt} = 2t - 4 = 2(t - 2)$$

3 **Differentiate**

For $t > 1$:

---

4 **moves in the negative direction**

- P moves in the negative direction when $1 < t < \frac{5}{3}$.
- P moves in the positive direction when $t > \frac{5}{3}$.
- Q moves in the negative direction when $1 < t < 2$.
- Q moves in the positive direction when $t > 2$.

---

5 **Differentiate**

So both particles move in the same direction when both velocities have the same sign.

✓ **FINAL ANSWER**

$1 < t < \frac{5}{3}$ or $t > 2$

4MA1/1H JANUARY 2022

Question 24

6 marks | Differentiation & Turning Points

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2022

1 Calculate value

$$y = ax^3 + bx^2 - 12x + 6$$

2 Point lies on the curve

Point $A = (2, -6)$ lies on the curve:

$$-6 = 8a + 4b - 24 + 6$$

$$8a + 4b = 12$$

$$2a + b = 3$$

3 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3ax^2 + 2bx - 12$$

4 **The gradient at is 16**

The gradient at $x = 2$ is 16:

$$16 = 12a + 4b - 12$$

$$12a + 4b = 28$$

$$3a + b = 7$$

---

5 **Solve**

Now solve

$$2a + b = 3$$

$$3a + b = 7$$

---

6 **Subtracting gives**

Subtracting gives

$$a = 4$$

---

7 **Calculate value**

Then

$$2(4) + b = 3$$

$$b = -5$$

---

8 **Calculate value**

For $x = 3$,

$$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$$

$$y = 108 - 45 - 36 + 6$$

$$y = 33$$

✓ **FINAL ANSWER**

$y = 33$

4MA1/2H JANUARY 2022

Question 24

5 marks | Differentiation & Turning Points

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a\ \text{m/s}^2$

Find the value of a

$a =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2022

1 Split the ratio

$$x = 4t^3 - 27t + 8$$

2 Velocity is

Velocity is

$$\frac{dx}{dt} = 12t^2 - 27$$

3 The direction of motion reverses

The direction of motion reverses when velocity is zero:

$$12t^2 - 27 = 0$$

$$12t^2 = 27$$

$$t^2 = \frac{9}{4}$$

4 Split the ratio

Since $t \geq 0$, $t = \frac{3}{2}$.

5 **Acceleration is**

Acceleration is

$$\frac{d^2x}{dt^2} = 24t$$

---

6 **Split the ratio**

At $t = \frac{3}{2}$,

$$a = 24\left(\frac{3}{2}\right) = 36$$

✓ **FINAL ANSWER**

$a = 36$

Question 22

7 marks | Differentiation & Turning Points

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

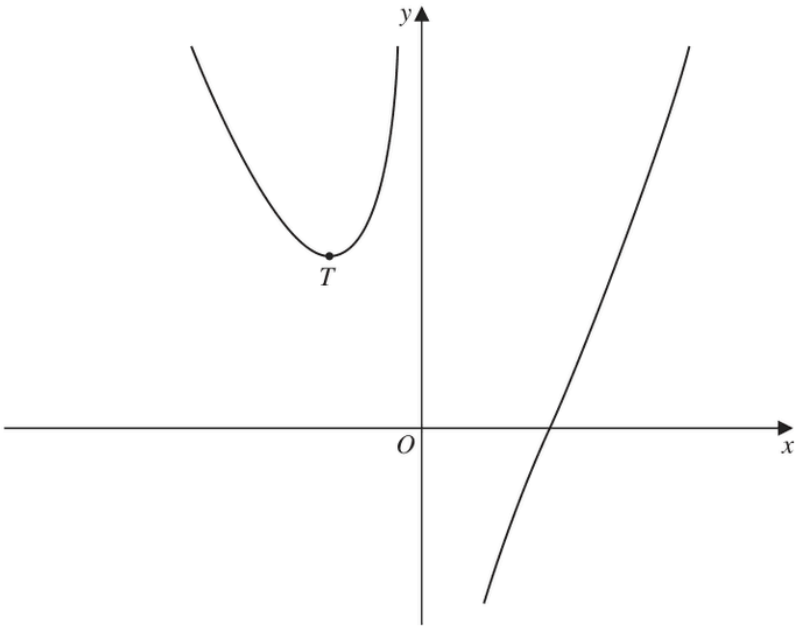


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

$p =$ .....

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$k =$ .....

(3)

(Total for Question 22 is 7 marks)

1 **Find the gradient**

For part (a),

$$y = x^2 - \frac{p}{x}$$

$$\frac{dy}{dx} = 2x + \frac{p}{x^2}$$

2 **Find the gradient**

At the minimum point, $x = -3$ and the gradient is 0.

$$2(-3) + \frac{p}{(-3)^2} = 0$$

$$-6 + \frac{p}{9} = 0$$

$$p = 54$$

3 **Find the gradient**

For part (b),

$$y = x^2 - \frac{16}{x}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2}$$

4 **Find the gradient**

At a horizontal tangent,

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$x^3 = -8$$

$$x = -2$$

5 **Find the gradient**

Now substitute $x = -2$ into the curve:

$$k = (-2)^2 - \frac{16}{-2}$$

$$k = 4 + 8 = 12$$

✓ **FINAL ANSWER**

$p = 54,\ k = 12$

4MA1/1HR MAY 2023

Question 21

6 marks | Differentiation & Turning Points

21 The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the range of values of x for which **T** has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

(4)

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2023

1 **(a)**

(a)

$$y = x^3 - 2x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 4x - 9$$

2 **(b) The curve has a**

(b) The curve has a positive gradient when

$$3x^2 - 4x - 9 > 0$$

3 **Solve**

Solve

$$3x^2 - 4x - 9 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 108}}{6}$$

$$x = \frac{2 \pm \sqrt{31}}{3}$$

---

4 **Differentiate**

To 3 significant figures, the roots are

$$x = -1.19, \quad x = 2.52$$

---

5 **Solve quadratic equation**

Since the quadratic opens upwards, it is positive outside the roots.

✓ **FINAL ANSWER**

(a) $\frac{dy}{dx} = 3x^2 - 4x - 9$, (b) $x < -1.19$ or $x > 2.52$

4MA1/1H NOVEMBER 2024

Question 20

4 marks | Differentiation & Turning Points

20 The curve with equation $y = 2x^4 - 64x$ has a minimum point.

Find an equation of the tangent to the curve at the minimum point.
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2024

1 Find the gradient

$$y = 2x^4 - 64x$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 8x^3 - 64$$

3 Find the gradient

At the minimum point, the gradient is 0.

$$8x^3 - 64 = 0$$

$$8x^3 = 64$$

$$x^3 = 8$$

$$x = 2$$

4 Find the coordinate

Find the y-coordinate:

$$y = 2(2)^4 - 64(2)$$

$$y = 32 - 128 = -96$$

5 The tangent at a minimum

The tangent at a minimum point is horizontal, so its equation is

$$y = -96$$

FINAL ANSWER

$y = -96$

4MA1/2HR JUNE 2025

Question 22

5 marks | Differentiation & Turning Points

22 The diagram shows a solid cuboid.

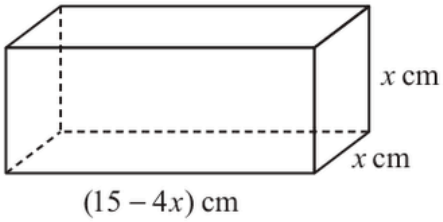


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2025

1 The cuboid has dimensions

The cuboid has dimensions

$$x, \quad x, \quad 15 - 4x$$

2 Calculate value

So

$$V = x^2(15 - 4x)$$

$$V = 15x^2 - 4x^3$$

3 Differentiate

Differentiate:

$$\frac{dV}{dx} = 30x - 12x^2$$

4 Calculate value

At a maximum,

$$30x - 12x^2 = 0$$

$$6x(5 - 2x) = 0$$

5 **The useful value is**

The useful value is

$$x = 2.5$$

---

6 **Calculate volume**

Now substitute into the volume:

$$V = (2.5)^2(15 - 4(2.5))$$

$$V = 6.25 \times 5$$

$$V = 31.25$$

✓ **FINAL ANSWER**

$31.25\ \text{cm}^3$

4WM2/01 MAY 2025

Question 22

4 marks | Differentiation & Turning Points

22 Work out the range of values of x for which the graph of $y = x^3 + \frac{1}{2}x^2 - 10x + 5$ has a positive gradient.
Show clear algebraic working.

.....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 May 2025

1 Evaluate fraction

$$\frac{dy}{dx} = 3x^2 + 2x - 10$$

2 Find the gradient

For a positive gradient,

$$3x^2 + 2x - 10 > 0$$

3 The roots are

The roots are

$$x = \frac{-1 \pm \sqrt{31}}{3}$$

4 Solve quadratic equation

Since the quadratic opens upwards,

$$x < \frac{-1 - \sqrt{31}}{3} \quad \text{or} \quad x > \frac{-1 + \sqrt{31}}{3}$$

FINAL ANSWER

$x < \frac{-1 - \sqrt{31}}{3}$ or $x > \frac{-1 + \sqrt{31}}{3}$.

4WM2/01 NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 November 2025

1 **Find the gradient**

$$fracdydx = 3x^2 - 16$$

2 **Find the gradient**

3*x*² − 16 = 11

3*x*² = 27

*x*² = 9 ⟹ *x* = 3 or *x* = − 3

3 **Solve inequality**

For *x* = 3: *y* = (3)³ − 16(3) + 7 = 27 − 48 + 7 = − 14

4 **Solve inequality**

For *x* = − 3: *y* = (−3)³ − 16(−3) + 7 = − 27 + 48 + 7 = 28

5 **Use trigonometry**

Step 4: Find the y-intercept (*b*) for each tangent (*y* = 11*x* + *b*).

Using (3,−14):

− 14 = 11(3) + *b* ⟹ − 14 = 33 + *b* ⟹ *b* = − 47

Using (−3,28):

28 = 11(−3) + *b* ⟹ 28 = − 33 + *b* ⟹ *b* = 61

✓ **FINAL ANSWER**

b = − 47 and 61

4MA1/2H NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2025

1 **Differentiate**

$$y = x^3 - 16x + 7$$

2 **Differentiate**

Differentiate:

$$\frac{dy}{dx} = 3x^2 - 16$$

3 **The gradient of the tangent**

The gradient of the tangent is 11, so

$$3x^2 - 16 = 11$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = 3 \quad \text{or} \quad x = -3$$

4 **Differentiate**

When $x = 3$:

$$y = 3^3 - 16(3) + 7 = 27 - 48 + 7 = -14$$

5 **Use trigonometry**

The tangent has equation $y = 11x + b$, so

$$-14 = 11(3) + b$$

$$b = -47$$

---

6 **Differentiate**

When $x = -3$:

$$y = (-3)^3 - 16(-3) + 7 = -27 + 48 + 7 = 28$$

$$28 = 11(-3) + b$$

$$b = 61$$

✓ **FINAL ANSWER**

$\boxed{b = -47 \text{ or } b = 61}$

Question 23

5 marks | Differentiation & Turning Points

23 A particle is moving along a straight line that passes through the fixed point O

The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 15t^2 - t^3 + 12$$

Find the maximum velocity of the particle where $0 < t < 11$
Show clear algebraic working.

..... m/s

(Total for Question 23 is 5 marks)

1 **Velocity**

$$v = \frac{ds}{dt} = 30t - 3t^2$$

2 **Maximum when dv/dt = 0**

$$\frac{dv}{dt} = 30 - 6t = 0 \Rightarrow t = 5 \quad \left(\frac{d^2v}{dt^2} = -6 < 0, \text{ a maximum}\right)$$

3 **Maximum velocity**

$$v = 30(5) - 3(5)^2 = 75$$

✓ **FINAL ANSWER**

75 m/s

TOPIC

Transformations of Graphs

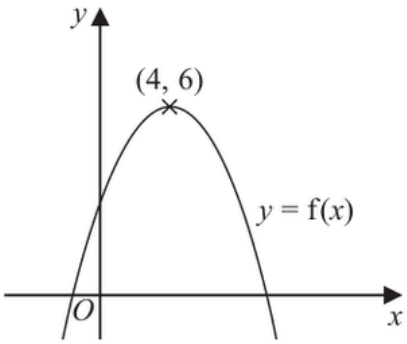
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 21

4 marks | Transformations of Graphs

21 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (4, 6)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 4)$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(2)

The equation of a curve **C** is $y = x^2 + 3x + 4$

The curve **C** is transformed to curve **S** under the translation $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) Find an equation of curve **S**.

You do not need to simplify the equation.

.....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2020

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(4, 6)$.

2 **Calculate value**

For

$$y = f(x + 4)$$

3 **the graph moves 4 units**

the graph moves 4 units left:

$$(4, 6) \to (0, 6)$$

4 **Calculate value**

For

$$y = f(2x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 6) \to (2, 6)$$

6 **Calculate value**

For part (b),

$$y = x^2 + 3x + 4$$

7 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

8 **Read the graph**

So the graph moves 4 units right and 6 units up.

9 **Add 6**

Replace x by $x - 4$, then add 6:

$$y = (x - 4)^2 + 3(x - 4) + 4 + 6$$

$$y = (x - 4)^2 + 3(x - 4) + 10$$

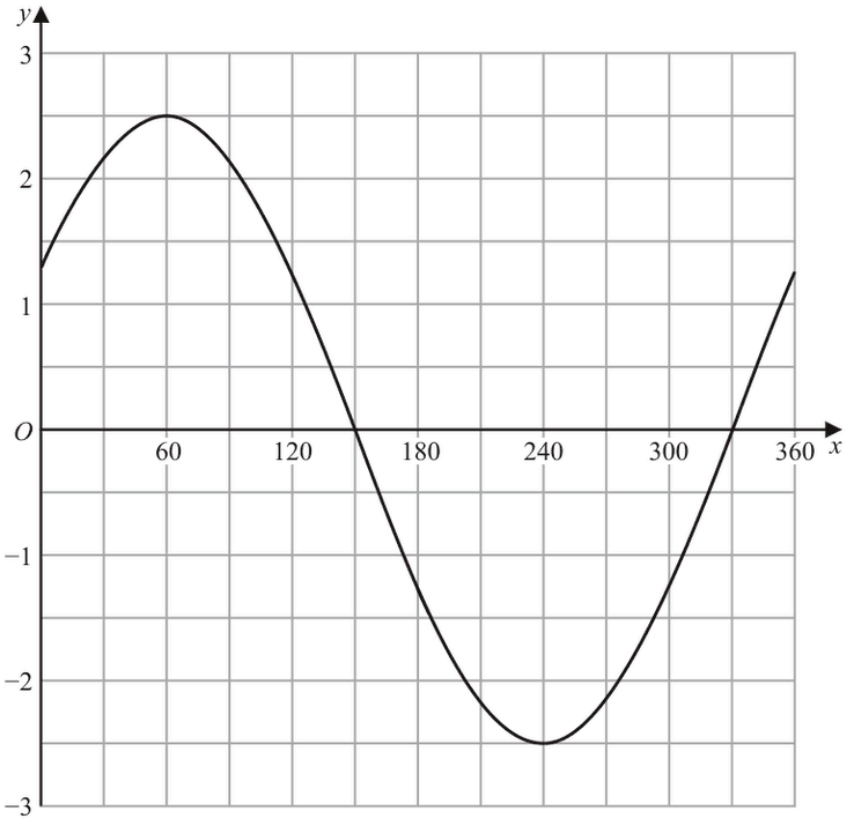
✓ **FINAL ANSWER**

$(0, 6)$, $(2, 6)$, and $y = (x - 4)^2 + 3(x - 4) + 10$

Question 22

4 marks | Transformations of Graphs

22 The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.



(a) Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

Another curve C has equation $y = f(x)$
The coordinates of the minimum point of C are (4, 5)

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

(..... , ...)

(ii) $y = f(x) - 7$

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Read the graph**

For part (a), the graph has maximum 2.5 and minimum -2.5, so

$$a = 2.5$$

2 **Use trigonometry**

The maximum occurs at $x = 60°$. For $y = a\cos(x + b)°$, a maximum occurs when

$$x + b = 360$$

3 **Calculate value**

So

$$60 + b = 360$$

$$b = 300$$

4 **Calculate value**

For part (b), the minimum point of $y = f(x)$ is $(4, 5)$.

5 **Calculate value**

For

$$y = f(2x)$$

6 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 5) \to (2, 5)$$

7 **Calculate value**

For

$$y = f(x) - 7$$

8 **the coordinate decreases by 7**

the y-coordinate decreases by 7:

$$(4, 5) \to (4, -2)$$

✓ **FINAL ANSWER**

$a = 2.5,\ b = 300;\ (2, 5)$ and $(4, -2)$

4MA1/1HR MAY/NOVEMBER 2020

Question 20

2 marks | Transformations of Graphs

20 A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are $(-3, 4)$

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May/November 2020

1 The maximum point of is

The maximum point of $y = f(x)$ is $(-3, 4)$.

2 Calculate value

For

$$y = f(x) - 6$$

3 the graph moves 6 units

the graph moves 6 units down:

$$(-3, 4) \rightarrow (-3, -2)$$

4 Calculate value

For

$$y = f(2x)$$

5 the coordinate is divided by

the x-coordinate is divided by 2:

$$(-3, 4) \rightarrow \left(-\frac{3}{2}, 4\right)$$

FINAL ANSWER

$(-3, -2)$ and $\left(-\frac{3}{2}, 4\right)$

Question 25

3 marks | Transformations of Graphs

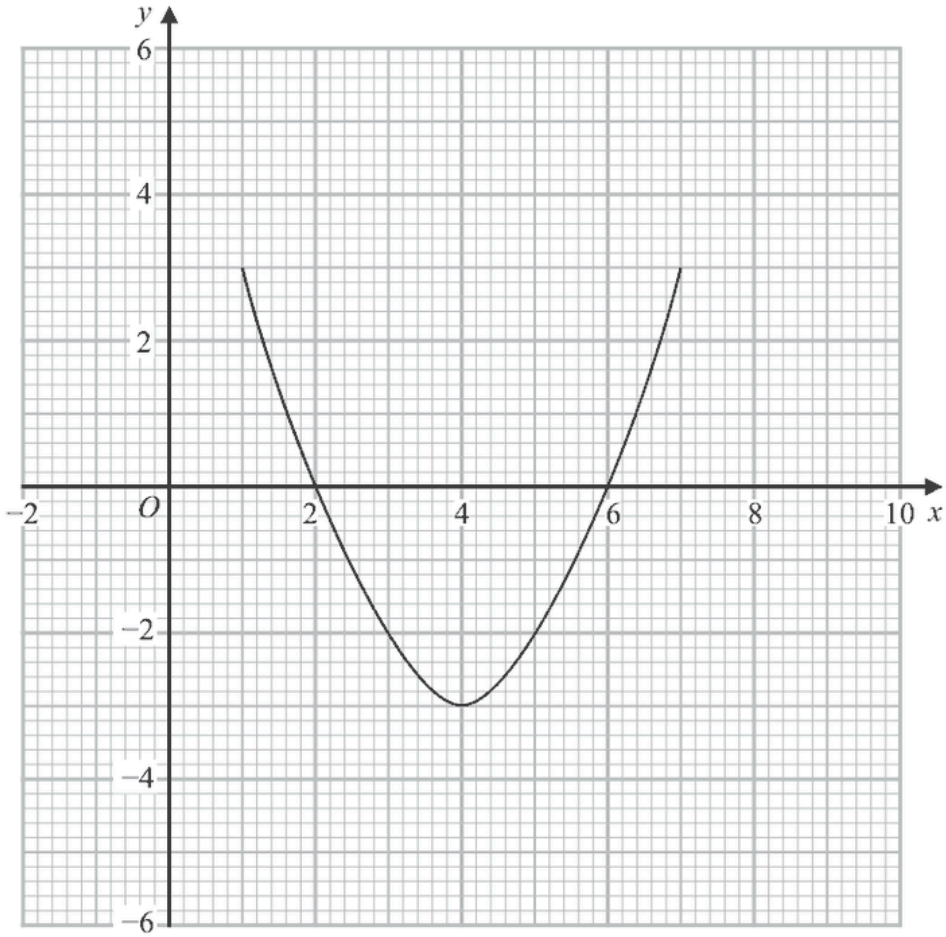
25 The curve with equation $y = g(x)$ is transformed to the curve with equation $y = -g(x)$ by the single transformation **T**.

(a) Describe fully the transformation **T**.

.....

(1)

The diagram shows the graph of $y = f(x)$



(b) On the grid, draw the graph of $y = 2f(x - 1)$

(2)

(Total for Question 25 is 3 marks)

1 **Apply transformation**

For part (a),

$$y = g(x) \rightarrow y = -g(x)$$

2 **The coordinates stay the same**

The x-coordinates stay the same and the y-coordinates change sign.

3 **Apply transformation**

So the transformation is a reflection in the x-axis.

4 **Calculate value**

For part (b),

$$y = 2f(x - 1)$$

5 **Read the graph**

First, $x - 1$ moves the graph 1 unit right. Then the factor 2 stretches the graph vertically by scale factor 2.

6 Use points from the original

Use points from the original graph:

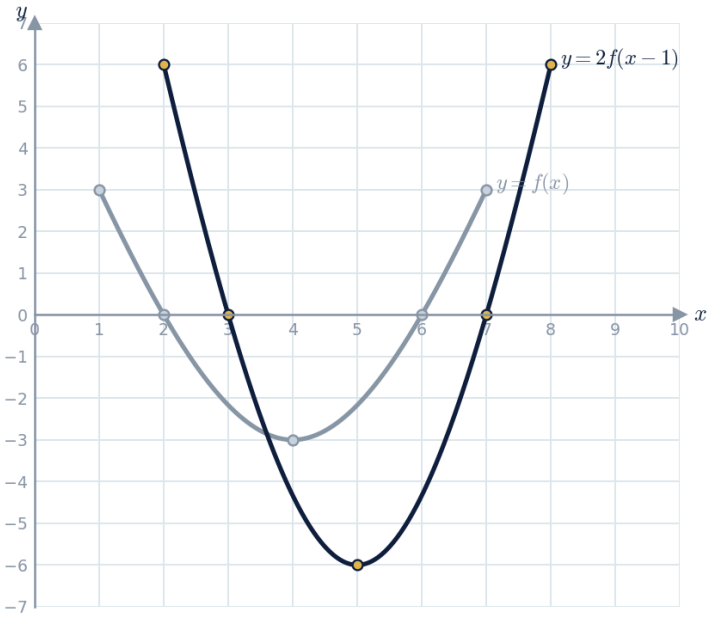
$$(1,3) \to (2,6)$$

$$(2,0) \to (3,0)$$

$$(4,-3) \to (5,-6)$$

$$(6,0) \to (7,0)$$

$$(7,3) \to (8,6)$$



7 Apply transformation

Draw the transformed parabola through these points.

FINAL ANSWER

reflection in the x-axis; transformed graph through $(2,6)$, $(3,0)$, $(5,-6)$, $(7,0)$, $(8,6)$

4MA1/1H JANUARY 2021

Question 21

4 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

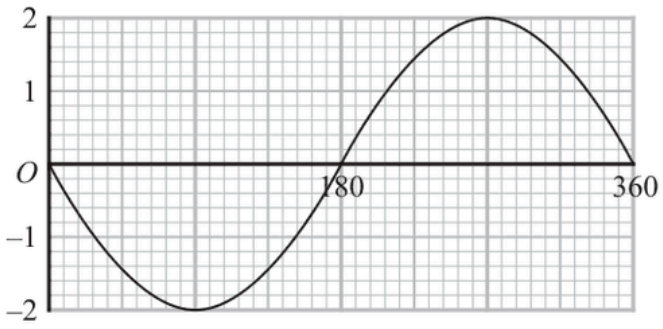
(..... , ...)

(ii) $y = \frac{1}{3} f(x)$

(..... , ...)

(2)

The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2021

1 **The minimum point of is**

The minimum point of $y = f(x)$ is $(-9, 15)$.

2 **Calculate value**

For

$$y = f(x + 3)$$

3 **the graph moves 3 units**

the graph moves 3 units left:

$$(-9, 15) \to (-12, 15)$$

4 **Evaluate fraction**

For

$$y = \frac{1}{3} f(x)$$

5 **the coordinate is divided by**

the y-coordinate is divided by 3:

$$(-9, 15) \to (-9, 5)$$

6 **Use trigonometry**

For part (b), the cosine graph has maximum 2 and minimum -2, so

$$a = 2$$

7 **The graph has a minimum**

The graph has a minimum at $x = 90°$. For $a\cos(x + b)°$, a minimum occurs when

$$x + b = 180$$

8 **Calculate value**

So

$$90 + b = 180$$

$$b = 90$$

✓ **FINAL ANSWER**

$(-12, 15)$, $(-9, 5)$, $a = 2,\ b = 90$

Question 21

6 marks | Transformations of Graphs

21 The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point A is the maximum point on **C**.

(a) Write down the coordinates of A.

(..... , ...)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

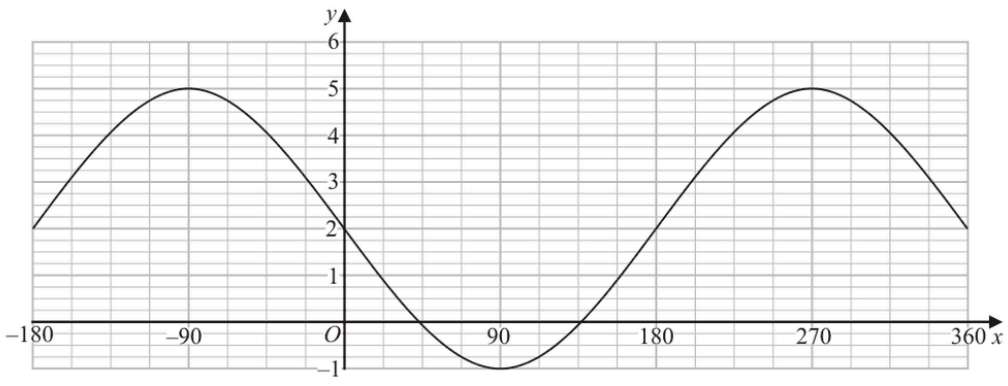
The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)

The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

(Total for Question 21 is 6 marks)

1 **Calculate value**

$$f(x) = 9 - 3(x+2)^2$$

2 **This is in completed square**

This is in completed square form, so the maximum point is

$$(-2, 9)$$

3 **Use matrices**

For the translation

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

4 **the graph moves 4 units**

the graph moves 4 units right. Replace x by $x - 4$:

$$y = 9 - 3((x-4)+2)^2$$

$$y = 9 - 3(x-2)^2$$

5 **The curve has equation**

The curve T has equation

$$y = 3(x+2)^2 - 9$$

6 **This is**

This is

$$y = -\left(9 - 3(x+2)^2\right)$$

7 **Apply transformation**

So C is reflected in the x-axis.

8 **Use trigonometry**

For the cosine graph, the maximum is 5 and the minimum is -1.

$$a = \frac{5 - (-1)}{2} = 3$$

$$c = \frac{5 + (-1)}{2} = 2$$

9 **Use trigonometry**

For

$$y = a\cos(x - b)^\circ + c$$

10 **Read the graph**

a maximum occurs when $x - b = 0$. The graph has a maximum at $x = 270°$, so

$$b = 270$$

✓ **FINAL ANSWER**

$A = (-2, 9)$, $y = 9 - 3(x - 2)^2$, reflection in the x-axis, $a = 3,\ b = 270,\ c = 2$

Question 20

6 marks | Transformations of Graphs

20 The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

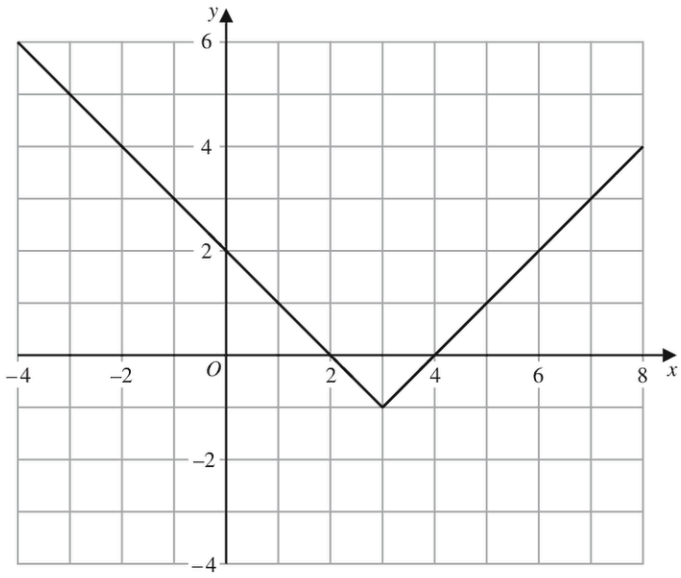
(.....,.....)

(ii) $y = f(3x)$

(.....,.....)

(2)

The graph of $y = g(x)$ is shown on the grid below.



(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

(2)

The graph of $y = h(x)$ intersects the x-axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where a is a constant.

(c) Find the two possible values of a

....., ...

(2)

(Total for Question 20 is 6 marks)

1 **The turning point of is**

The turning point of $y = f(x)$ is $(-6, -4)$.

2 **Calculate value**

For

$$y = f(x) + 5$$

3 **the graph moves 5 units**

the graph moves 5 units up:

$$(-6, -4) \rightarrow (-6, 1)$$

4 **Calculate value**

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(-6, -4) \rightarrow (-2, -4)$$

6 **Read the graph**

For the sketch of $y = 2g(x)$, keep every x-coordinate the same and double every y-coordinate.

7 **Useful points are**

Useful points are

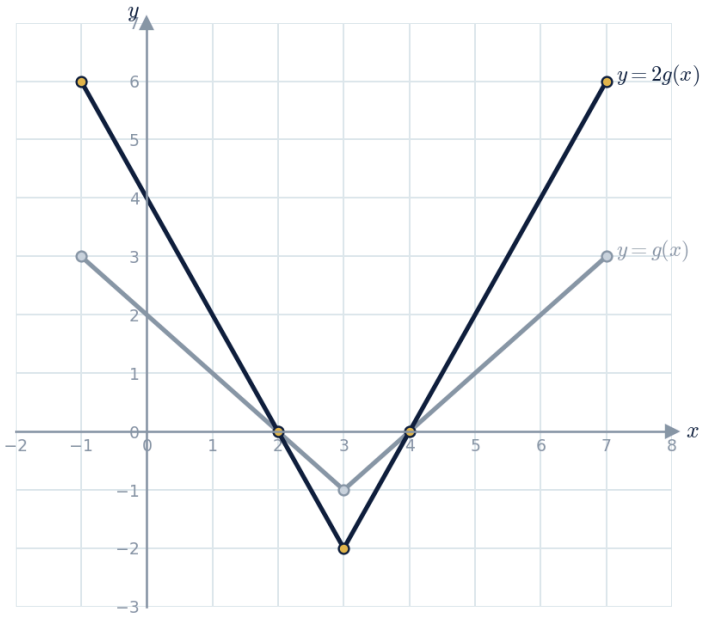
$$(-1,3) \to (-1,6)$$

$$(2,0) \to (2,0)$$

$$(3,-1) \to (3,-2)$$

$$(4,0) \to (4,0)$$

$$(7,3) \to (7,6)$$



---

8 **Solve inequality**

Draw straight line segments through these points for $-1 \leq x \leq 7$.

---

9 **Calculate value**

For part (c), $h(x)$ has roots at $x = -1$ and $x = 6$.

---

10 **Calculate value**

For $h(x + a)$ to pass through $(2,0)$,

$$2 + a = -1 \quad \text{or} \quad 2 + a = 6$$

$$a = -3 \quad \text{or} \quad a = 4$$

FINAL ANSWER

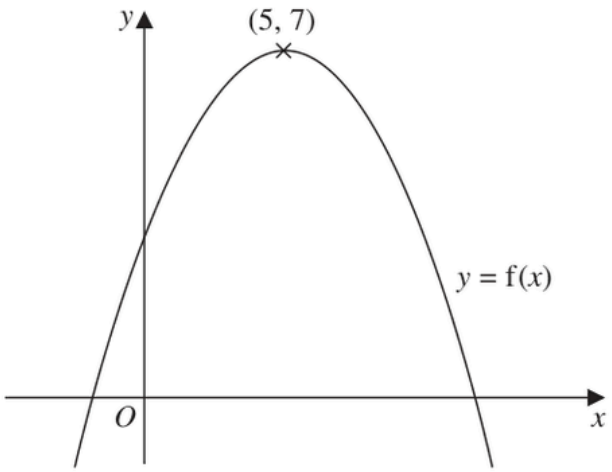
$(-6, 1)$, $(-2, -4)$, sketch through the doubled points, and $a = -3$ or $a = 4$

4MA1/1H JANUARY 2022

Question 23

2 marks | Transformations of Graphs

23 The diagram shows a sketch of the curve with equation $y = f(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 9)$

(..... , ...)

(ii) $y = f(x) + 3$

(..... , ...)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2022

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(5, 7)$.

2 **Calculate value**

For

$$y = f(x + 9)$$

3 **the graph moves 9 units**

the graph moves 9 units left:

$$(5, 7) \rightarrow (-4, 7)$$

4 **Calculate value**

For

$$y = f(x) + 3$$

5 **the graph moves 3 units**

the graph moves 3 units up:

$$(5, 7) \to (5, 10)$$

✓ **FINAL ANSWER**

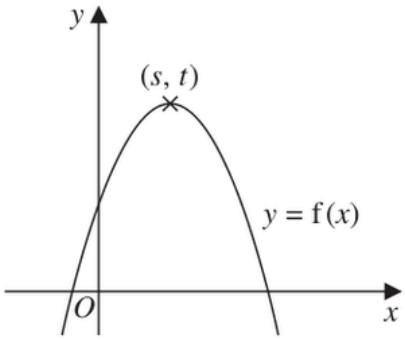
$(-4, 7)$ and $(5, 10)$

4MA1/1H JUNE 2022

Question 20

2 marks | Transformations of Graphs

20 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (s, t)

Find, in terms of s and t, the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

(.....,.....)
(1)

(ii) $y = 3f(x)$

(.....,.....)
(1)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H June 2022

1 The maximum point of is

The maximum point of $y = f(x)$ is (s, t).

2 Calculate value

For

$$y = f(x - 2)$$

3 the graph moves 2 units

the graph moves 2 units right:

$$(s, t) \to (s + 2, t)$$

4 **Calculate value**

For

$$y = 3f(x)$$

5 **the coordinate is multiplied by**

the y-coordinate is multiplied by 3:

$$(s, t) \to (s, 3t)$$

✓ **FINAL ANSWER**

$(s + 2, t)$ and $(s, 3t)$

Question 22

4 marks | Transformations of Graphs

22 The point A with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point A on the straight line with equation

(i) $y = f(x) - 3$

(..... , ...)

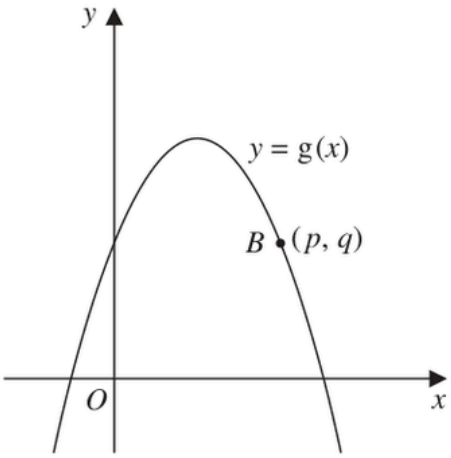
(1)

(ii) $y = f\left(\frac{x}{2}\right)$

(..... , ...)

(1)

Here is a sketch of part of the curve with equation $y = g(x)$



The point B with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of the point B on the curve with equation

$$y = -g(x - c)$$

where c is a constant.

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Point lies on**

Point $A = (-3, 2)$ lies on $y = f(x)$.

2 **Calculate value**

For

$$y = f(x) - 3$$

3 **the graph moves 3 units**

the graph moves 3 units down:

$$(-3, 2) \to (-3, -1)$$

4 **Evaluate fraction**

For

$$y = f\left(\frac{x}{2}\right)$$

5 **the coordinate is multiplied by**

the x-coordinate is multiplied by 2:

$$(-3, 2) \to (-6, 2)$$

6 **Calculate value**

For part (b), point $B = (p, q)$ lies on $y = g(x)$.

7 **Calculate value**

For

$$y = -g(x - c)$$

8 **Read the graph**

$x - c$ moves the graph c units right, and the minus sign reflects the graph in the x-axis.

9 **Apply transformation**

So

$$(p, q) \to (p + c, -q)$$

✓ **FINAL ANSWER**

$(-3, -1)$, $(-6, 2)$, and $(p + c, -q)$

4MA1/1HR MAY 2023

Question 20

7 marks | Transformations of Graphs

20 Curve **C** has equation $y = f(x)$
The graph of curve **C** has one maximum point.
The coordinates of this maximum point are (3, 5)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = 2f(x)$

(..... , ...)
(1)

(ii) $y = f(x) - 7$

(..... , ...)
(1)

(iii) $y = f(-x)$

(..... , ...)
(1)

Curve **L** has equation $y = x^2 + 7x + 20$

Curve **L** is transformed to curve **S** under the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(b) Find an equation for **S**
Give your answer in the form $y = ax^2 + bx + c$

$y =$ .....
(4)

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May 2023

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(3, 5)$.

2 **Calculate value**

For

$$y = 2f(x)$$

3 **the coordinate is multiplied by**

the y-coordinate is multiplied by 2:

$$(3, 5) \to (3, 10)$$

4 **Calculate value**

For

$$y = f(x) - 7$$

5 **the graph moves 7 units**

the graph moves 7 units down:

$$(3, 5) \to (3, -2)$$

6 **Calculate value**

For

$$y = f(-x)$$

7 **the graph reflects in the**

the graph reflects in the y-axis:

$$(3, 5) \to (-3, 5)$$

8 **Calculate value**

For part (b),

$$y = x^2 + 7x + 20$$

9 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

10 **Read the graph**

So the graph moves 2 units right.

11 **Replace by**

Replace x by $x - 2$:

$$y = (x - 2)^2 + 7(x - 2) + 20$$

$$y = x^2 + 3x + 10$$

FINAL ANSWER

$(3, 10)$, $(3, -2)$, $(-3, 5)$, and $y = x^2 + 3x + 10$

4MA1/1H MAY 2023

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(..... , ...)

(1)

(b) $y = f(3x)$

(..... , ...)

(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2023

1 The turning point of is

The turning point of $y = f(x)$ is $(6, 5)$.

2 Calculate value

For

$$y = f(x - 4)$$

3 the graph moves 4 units

the graph moves 4 units right:

$$(6, 5) \to (10, 5)$$

4 Calculate value

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(6, 5) \rightarrow (2, 5)$$

✓ **FINAL ANSWER**

$(10, 5)$ and $(2, 5)$

Question 24

4 marks | Transformations of Graphs

24 The curve with equation $f(x) = 5x^2 + 9x + 2$ is transformed to the curve with equation

$g(x) = 5(x+4)^2 + 9(x+4) + 8$ by the translation $\begin{pmatrix} a \\ b \end{pmatrix}$

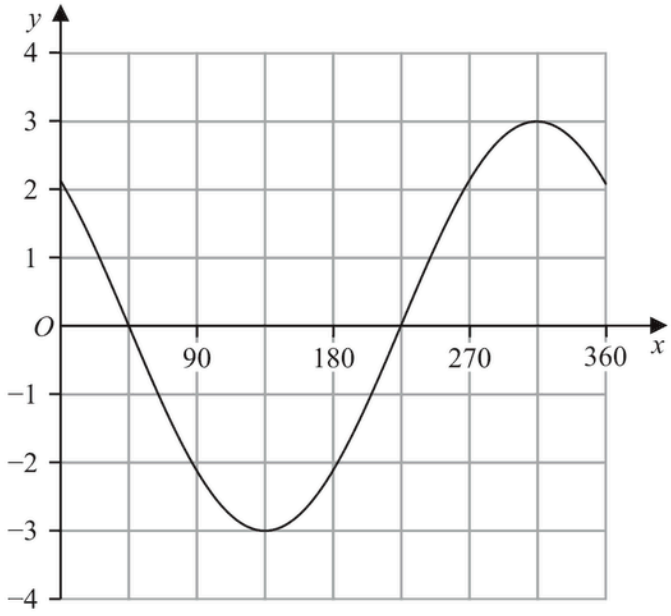
(a) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

The graph of $y = p\cos(x+q)^\circ$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $p > 0$ and $0 < q < 360$

(b) find the value of p and the value of q

$p =$ .....

$q =$ .....

(2)

(Total for Question 24 is 4 marks)

1 **Read the graph**

For part (a),

$$f(x) = 5x^2 + 9x + 2$$

$$g(x) = 5(x+4)^2 + 9(x+4) + 8$$

2 **Read the graph**

Since

$$f(x+4) = 5(x+4)^2 + 9(x+4) + 2$$

$$g(x) = f(x+4) + 6$$

3 **Read the graph**

So the graph has moved 4 units left and 6 units up.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

4 **Read the graph**

For part (b), the graph has maximum value 3 and minimum value -3, so

$$p = 3$$

5 **The minimum occurs when**

The minimum occurs when

$$x + q = 180$$

6 **Read the graph**

From the graph, the minimum is at $x = 135$, so

$$135 + q = 180$$

$$q = 45$$

✓ **FINAL ANSWER**

$a = -4,\ b = 6,\ p = 3,\ q = 45$

4MA1/1H MAY 2024

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is one minimum point on this curve.
The coordinates of this minimum point are $(5, -4)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 7)$

(.....,.....)
(1)

(ii) $y = f(x) - 6$

(.....,.....)
(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(5, -4)$.

2 Calculate value

For

$$y = f(x + 7)$$

3 the graph moves 7 units

the graph moves 7 units left:

$$(5, -4) \rightarrow (-2, -4)$$

4 Calculate value

For

$$y = f(x) - 6$$

5 **the graph moves 6 units**

the graph moves 6 units down:

$$(5, -4) \rightarrow (5, -10)$$

✓ **FINAL ANSWER**

$(-2, -4)$ and $(5, -10)$

4MA1/2HR JUNE 2024

Question 23

2 marks | Transformations of Graphs

23 A curve has equation $y = f(x)$
The coordinates of the minimum point on this curve are $(6, -3)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 10$

(.....,.....)
(1)

(ii) $y = f(3x)$

(.....,.....)
(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(6, -3)$.

2 Calculate value

For

$$y = f(x) + 10$$

3 the graph moves 10 units

the graph moves 10 units up:

$$(6, -3) \to (6, 7)$$

4 Calculate value

For

$$y = f(3x)$$

5 the coordinate is divided by

the x-coordinate is divided by 3:

$$(6, -3) \to (2, -3)$$

FINAL ANSWER

$(6, 7)$ and $(2, -3)$

4MA1/1HR JUNE 2025

Question 21

3 marks | Transformations of Graphs

21 The curve with equation $y = f(x)$ has one turning point.
The coordinates of this turning point are $(-6, 9)$

(a) Write down the coordinates of the turning point on the curve with equation $y = f(3x)$

(..... , ...)

(1)

The curve **C** with equation $y = g(x)$ is transformed to give the curve **S** with equation $y = g(x + a) + b$

The point $(4, -5)$ on **C** is mapped to the point $(1, -16)$ on **S**

(b) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(-6, 9)$.

2 Calculate value

For

$$y = f(3x)$$

3 the coordinate is divided by

the x-coordinate is divided by 3:

$$(-6, 9) \rightarrow (-2, 9)$$

4 Calculate value

For part (b), a point $(4, -5)$ on $y = g(x)$ maps to $(1, -16)$ on

$$y = g(x + a) + b$$

5 **Calculate value**

For the x-coordinate:

$$1 + a = 4$$

$$a = 3$$

---

6 **Calculate value**

For the y-coordinate:

$$-5 + b = -16$$

$$b = -11$$

✓ **FINAL ANSWER**

$(-2, 9)$, $a = 3,\ b = -11$

Question 27

4 marks | Transformations of Graphs

27 A curve has equation $y = f(x)$

There is one maximum point on this curve.
The coordinates of this maximum point are $(6, -5)$

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 4)$

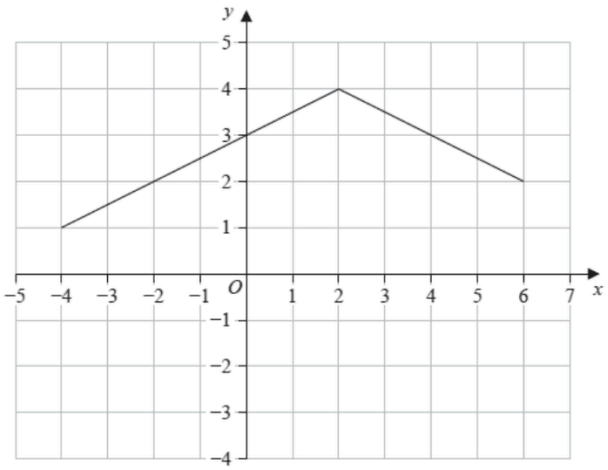
(.....)
(1)

(ii) $y = f(3x)$

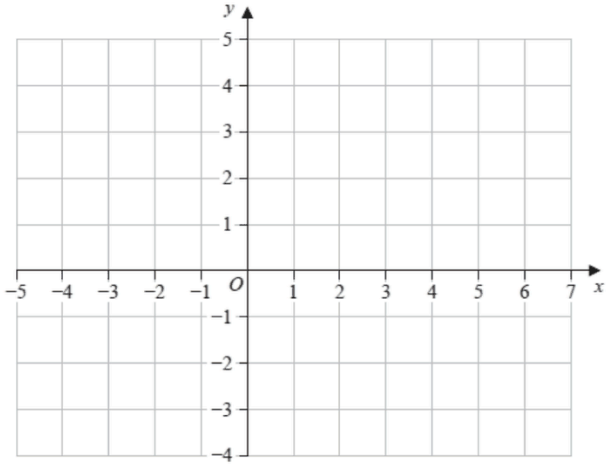
(.....)
(1)

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Here is the graph of $y = g(x)$



(b) On the grid below, draw the graph of $y = g(2x) - 3$



(2)

(Total for Question 27 is 4 marks)

1 **(a)**

(a)

2 **Read the graph**

For $y = f(x - 4)$, move the graph 4 units right:

$$(6, -5) \to (10, -5)$$

3 **Divide the -coordinate**

For $y = f(3x)$, divide the x-coordinate by 3:

$$(6, -5) \to (2, -5)$$

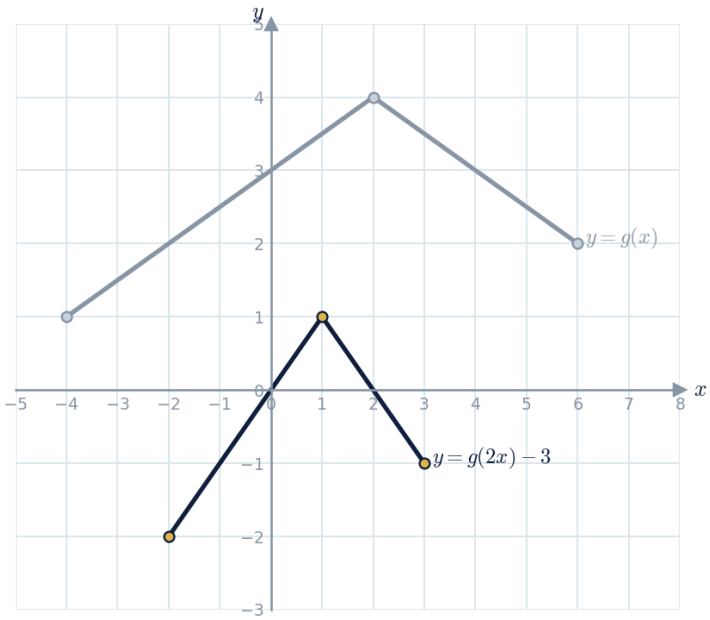
4 **Subtract from each -coordinate**

(b) For $y = g(2x) - 3$, halve each x-coordinate and subtract 3 from each y-coordinate.

5 **Useful transformed points are**

Useful transformed points are

$$(-4, 1) \to (-2, -2), \quad (2, 4) \to (1, 1), \quad (6, 2) \to (3, -1)$$



6 **Join these points with straight**

Join these points with straight line segments.

✓ **FINAL ANSWER**

$(10, -5)$, $(2, -5)$, graph through $(-2, -2), (1, 1), (3, -1)$.

4MA1/1H NOVEMBER 2025

Question 24

3 marks | Transformations of Graphs

24 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (4, 3)

Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x + 5)$

(..... , ...)
(1)

(ii) $y = f(x) + 7$

(..... , ...)
(1)

(iii) $y = f(2x)$

(..... , ...)
(1)

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H November 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(4, 3)$.

2 Calculate value

For

$$y = f(x + 5)$$

3 the graph moves 5 units

the graph moves 5 units left:

$$(4, 3) \rightarrow (-1, 3)$$

4 **Calculate value**

For

$$y = f(x) + 7$$

5 **the graph moves 7 units**

the graph moves 7 units up:

$$(4, 3) \to (4, 10)$$

6 **Calculate value**

For

$$y = f(2x)$$

7 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 3) \to (2, 3)$$

✓ **FINAL ANSWER**

$(-1, 3)$, $(4, 10)$, and $(2, 3)$

TOPIC

Angles in Polygons & Parallel Lines

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JUNE 2022

Question 23

6 marks | Angles in Polygons & Parallel Lines

23 A polygon has n sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of n
Show clear algebraic working.

Question 23 continued

$n =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H June 2022

1 The interior angles form an

The interior angles form an arithmetic sequence:

$$177,\ 175,\ 173,\ 171,\ 169, \ldots$$

2 Calculate value

So

$$a = 177, \quad d = -2$$

3 Use trigonometry

For an n-sided polygon, the sum of the interior angles is

$$180(n-2)$$

4 **The sum of the arithmetic**

The sum of the arithmetic sequence is

$$\frac{n}{2}(2(177)+(n-1)(-2))$$

$$=\frac{n}{2}(354-2n+2)$$

$$=\frac{n}{2}(356-2n)$$

$$=n(178-n)$$

---

5 **Set this equal to the**

Set this equal to the polygon angle sum:

$$n(178-n)=180(n-2)$$

$$178n-n^2=180n-360$$

$$n^2+2n-360=0$$

---

6 **Factorise**

Factorise:

$$(n+20)(n-18)=0$$

---

7 **Solve inequality**

Since $n>5$,

$$n=18$$

✓ **FINAL ANSWER**

$n=18$

TOPIC

Volume & Surface Area

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

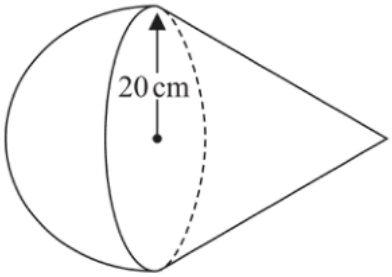


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 Specimen

1 **Calculate surface area**

The curved surface area of a cone is

$$\pi rl$$

2 **Calculate area**

Here $r = 20$ and $\pi rl = 580\pi$, so

$$20l = 580$$

$$l = 29$$

3 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 = 29^2 - 20^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

4 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (20)^2 (21) = 2800\pi$$

---

5 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi (20)^3 = \frac{16000}{3}\pi$$

---

6 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

7 **Calculate area**

Since the volume is $k\pi$,

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$k = \frac{24400}{3}$.

Question 26

6 marks | Volume & Surface Area

26 Here is a sector, AOB, of a circle with centre O and angle $AOB = x°$

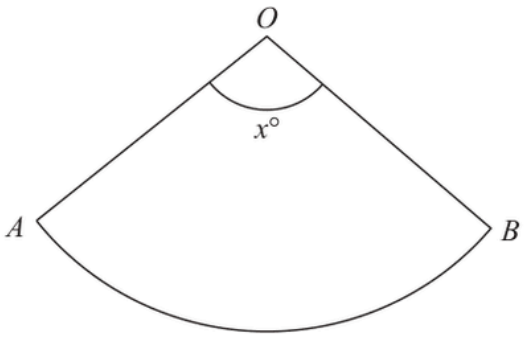


Diagram **NOT** accurately drawn

The sector can form the curved surface of a cone by joining OA to OB.

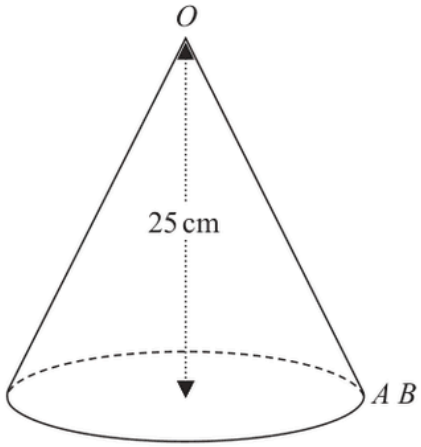


Diagram **NOT** accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 cm^3

Work out the value of x.
Give your answer correct to the nearest whole number.

$x =$ .....

(Total for Question 26 is 6 marks)

1 **Let the radius of the**

Let the radius of the base of the cone be r, and let the slant height be l.

2 **The volume of the cone**

The volume of the cone is

$$\frac{1}{3}\pi r^2 h = 1600$$

3 **Calculate area**

The height is 25 cm, so

$$\frac{1}{3}\pi r^2 (25) = 1600$$

$$r^2 = \frac{4800}{25\pi}$$

$$r = 7.8176\ldots$$

4 **The slant height is found**

The slant height is found using Pythagoras:

$$l^2 = r^2 + 25^2$$

$$l = 26.1938\ldots$$

5 **Calculate area**

When the sector is joined to form the cone, the arc length of the sector becomes the circumference of the cone base.

$$\frac{x}{360} \times 2\pi l = 2\pi r$$

6 **Cancel**

Cancel 2π:

$$\frac{x}{360}l = r$$

$$x = \frac{360r}{l}$$

$$x = \frac{360(7.8176\ldots)}{26.1938\ldots} = 107.4433\ldots$$

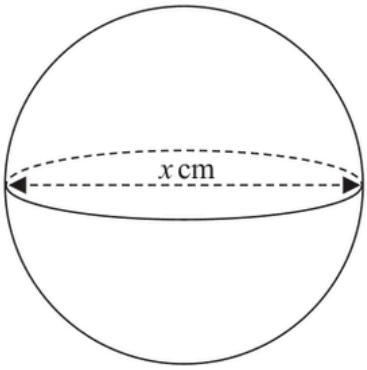
✓ **FINAL ANSWER**

$\boxed{107°}$

Question 22

6 marks | Volume & Surface Area

22 The diagram shows a sphere of diameter x cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.



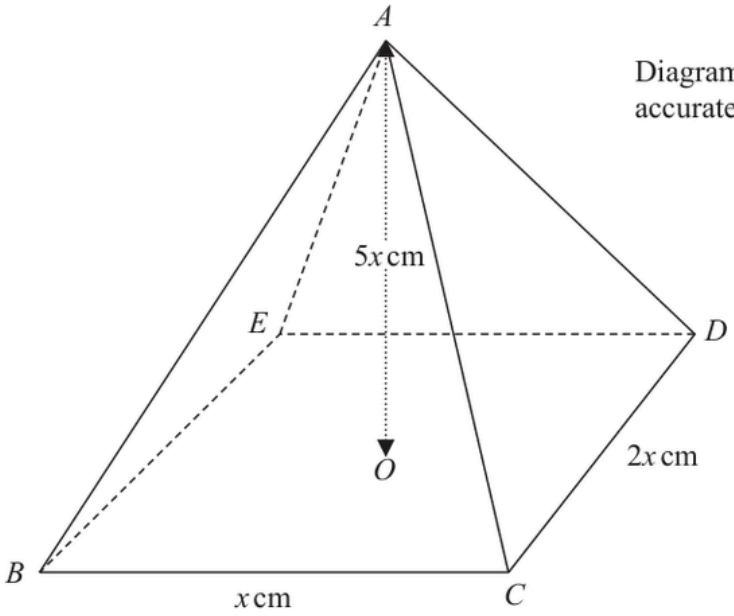


Diagram **NOT** accurately drawn

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is 288π cm^3

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

1 **Use trigonometry**

The sphere has diameter x, so its radius is

$$\frac{x}{2}$$

2 **The volume of the sphere**

The volume of the sphere is $288\pi\ \text{cm}^3$:

$$\frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 288\pi$$

$$\frac{\pi x^3}{6} = 288\pi$$

$$x^3 = 1728$$

$$x = 12$$

3 **The rectangular base of the**

The rectangular base of the pyramid has dimensions x and $2x$, so its area is

$$x(2x) = 2x^2$$

4 **Use trigonometry**

For the two faces with base x, the slant height is

$$\sqrt{(5x)^2 + x^2} = x\sqrt{26}$$

5 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}x \cdot x\sqrt{26}\right) = x^2\sqrt{26}$$

6 **Use trigonometry**

For the two faces with base $2x$, the slant height is

$$\sqrt{(5x)^2 + \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{101}}{2}$$

7 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}(2x)\cdot\frac{x\sqrt{101}}{2}\right)=x^2\sqrt{101}$$

8 **Total surface area**

Total surface area:

$$2x^2+x^2\sqrt{26}+x^2\sqrt{101}$$

9 **Substitute**

Substitute $x=12$:

$$144(2+\sqrt{26}+\sqrt{101})=2469.440\ldots$$

10 **Correct to the nearest**

Correct to the nearest cm^2:

$$2469$$

✓ **FINAL ANSWER**

$$\boxed{2469\ \mathrm{cm}^2}$$

4MA1/1H JUNE 2022

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

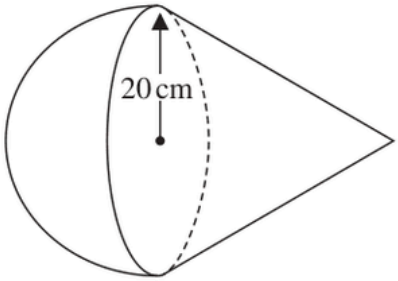


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi \text{ cm}^2$

The volume of the solid is $k\pi \text{ cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2022

1 The cone has radius cm

The cone has radius 20 cm.

2 Calculate surface area

The curved surface area of the cone is

$$\pi rl$$

3 Calculate value

So

$$\pi(20)l = 580\pi$$

$$l = 29$$

4 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 + 20^2 = 29^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

5 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(20)^2(21) = 2800\pi$$

---

6 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(20)^3 = \frac{16000}{3}\pi$$

---

7 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

8 **Calculate volume**

The volume is $k\pi$, so

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{24400}{3}}$$

4MA1/2HR JUNE 2023

Question 24

5 marks | Volume & Surface Area

24 The surface area of sphere **A** is nine times the surface area of sphere **B**
The difference between the volume of sphere **A** and the volume of sphere **B** is 117π cm^3

Find the radius of the smaller sphere.
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2023

1 **Let the radius of the**

Let the radius of the smaller sphere be r.

2 **Let the radius of the**

Let the radius of the larger sphere be R.

3 **Surface area of a sphere**

Surface area of a sphere is

$$4\pi r^2$$

4 **The surface area of sphere**

The surface area of sphere A is nine times the surface area of sphere B, so

$$4\pi R^2 = 9(4\pi r^2)$$

$$R^2 = 9r^2$$

$$R = 3r$$

5 **The difference between the volumes**

The difference between the volumes is 117π:

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

$$\frac{4}{3}\pi(27r^3 - r^3) = 117\pi$$

$$\frac{104}{3}\pi r^3 = 117\pi$$

---

6 **Cancel**

Cancel π:

$$\frac{104}{3}r^3 = 117$$

$$r^3 = \frac{117 \times 3}{104}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

✓ **FINAL ANSWER**

$\boxed{1.5\text{ cm}}$

4MA1/1H NOVEMBER 2023

Question 21

5 marks | Volume & Surface Area

21 Here is a triangular prism $ABCDEF$

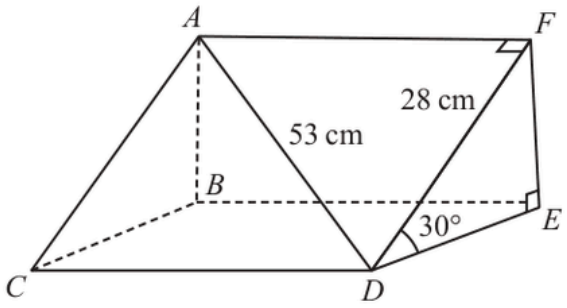


Diagram **NOT** accurately drawn

$AD = 53$ cm
$DF = 28$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2023

1 The triangular cross section is

The triangular cross-section is triangle DEF.

2 It is right angled at

It is right-angled at E, with

$$DF = 28$$

3 Use trigonometry

and

$$\angle FDE = 30°$$

4 Use trigonometry

So

$$FE = 28 \sin 30° = 14$$

5 **Use trigonometry**

and

$$DE = 28\cos 30^\circ = 14\sqrt{3}$$

6 **Area of triangle**

Area of triangle DEF:

$$\frac{1}{2}(14)(14\sqrt{3}) = 98\sqrt{3}$$

7 **The length of the prism**

The length of the prism is

$$AD = 53$$

8 **Volume of the prism**

Volume of the prism:

$$98\sqrt{3} \times 53$$

$$= 5194\sqrt{3}$$

$$= 8996.2719\ldots$$

✓ **FINAL ANSWER**

$\boxed{8996\ \text{cm}^3}$

4MA1/1H MAY 2024

Question 23

5 marks | Volume & Surface Area

23 A solid shape is made by removing a hemisphere, shown shaded, from a cone as shown in the diagram.

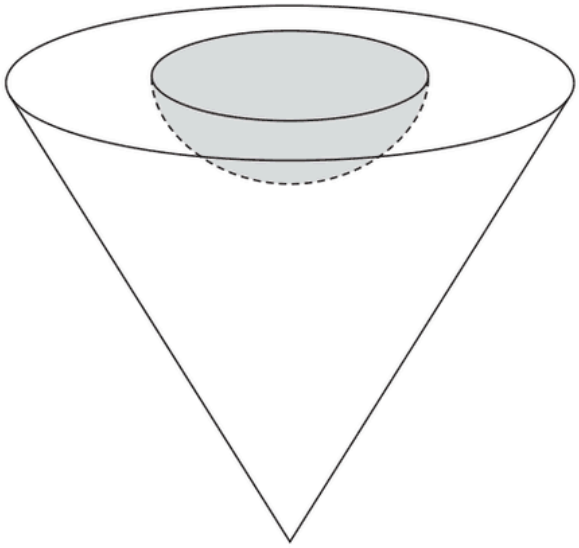


Diagram **NOT** accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the solid shape is 6948π cm^3

Work out the **total** surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2024

1 **Define cone dimensions**

The cone has radius $5x$ and height $6x$.

2 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi(5x)^2(6x)$$

$$= 50\pi x^3$$

3 **The removed hemisphere has radius**

The removed hemisphere has radius $2x$.

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi(2x)^3$$

$$= \frac{16}{3}\pi x^3$$

5 **The remaining solid has volume**

The remaining solid has volume 6948π, so

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\frac{134}{3}\pi x^3 = 6948\pi$$

6 **Cancel**

Cancel π:

$$\frac{134}{3}x^3 = 6948$$

$$x^3 = \frac{10422}{67}$$

$$x = 5.3781\ldots$$

7 **The solid hemisphere removed has**

The solid hemisphere removed has radius $2x$.

8 **Total surface area of a**

Total surface area of a solid hemisphere is

$$3\pi r^2$$

---

9 **Calculate value**

So

$$3\pi(2x)^2 = 12\pi x^2$$

$$12\pi(5.3781\ldots)^2 = 1090.3903\ldots$$

✓ **FINAL ANSWER**

$\boxed{1090\ \text{cm}^2}$

4MA1/1H NOVEMBER 2024

Question 22

3 marks | Volume & Surface Area

22 The diagram shows a bowl in the shape of a hemisphere.
The bowl is made from metal.

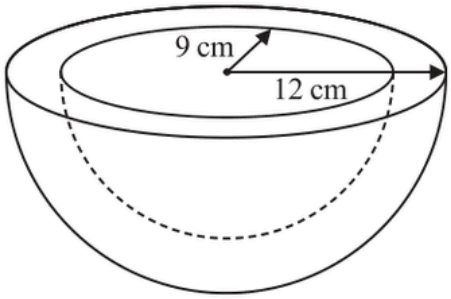


Diagram **NOT** accurately drawn

The outer radius of the bowl is 12 cm
The inner radius of the bowl is 9 cm

The thickness of the bowl is uniform.

Work out the volume of the metal.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2024

1 **The bowl is a hemispherical**

The bowl is a hemispherical shell.

2 **The volume of the metal**

The volume of the metal is:

$$\text{outer hemisphere volume} - \text{inner hemisphere volume}$$

3 **Volume of a hemisphere**

Volume of a hemisphere:

$$\frac{2}{3}\pi r^3$$

4 **Calculate volume**

So

$$\text{metal volume} = \frac{2}{3}\pi(12)^3 - \frac{2}{3}\pi(9)^3$$

$$= \frac{2}{3}\pi(12^3 - 9^3)$$

$$= \frac{2}{3}\pi(1728 - 729)$$

$$= 666\pi$$

$$= 2092.3007\ldots$$

✓ **FINAL ANSWER**

$\boxed{2092\ \text{cm}^3}$

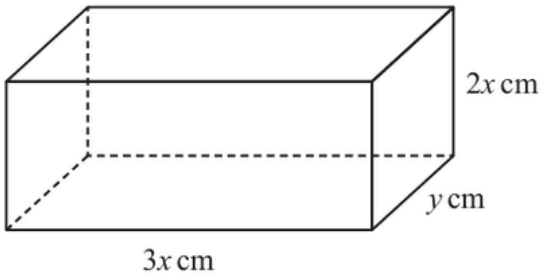
4MA1/2H JUNE 2025

Question 21

3 marks | Volume & Surface Area

21 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is $1014\,\text{cm}^3$
The total surface area of the cuboid is $A\,\text{cm}^2$

Show that $A = 12x^2 + \frac{1690}{x}$

You must show all the stages of your working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2025

1 **The cuboid has dimensions**

The cuboid has dimensions

$$3x, \quad 2x, \quad y$$

2 **Calculate volume**

Its volume is $1014\ \text{cm}^3$, so

$$(3x)(2x)(y) = 1014$$

$$6x^2y = 1014$$

$$y = \frac{1014}{6x^2}$$

$$y = \frac{169}{x^2}$$

3 **The total surface area is**

The total surface area is

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$A = 2(6x^2 + 5xy)$$

$$A = 12x^2 + 10xy$$

4 **Substitute**

Substitute $y = \frac{169}{x^2}$:

$$A = 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$A = 12x^2 + \frac{1690}{x}$$

✓ **FINAL ANSWER**

$$\boxed{A = 12x^2 + \frac{1690}{x}}$$

4WM2/01R MAY 2025

Question 21

5 marks | Volume & Surface Area

21 The diagram shows a solid cuboid.

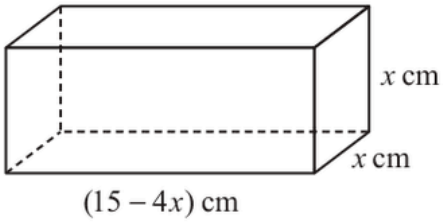


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM2/01R May 2025

1 Required

Cuboid has a square cross-section of side *x* cm and length (15−4*x*) cm. Find the maximum value of the volume *V*.

2 Write V as a function

Write *V* as a function of *x*:

V = *x* · *x* · (15−4*x*) = *x*²(15−4*x*) = 15*x*² − 4*x*³

3 Differentiate

Differentiate:

$$fracdVdx = 30x - 12x^2$$

4 Set (x 0 gives

Set $dfracdVdx = 0$:

$$30x - 12x^2 = 0implies6x(5 - 2x) = 0impliesx = 0; textor; x = tfrac52 = 2.5$$

(*x* = 0 gives zero volume, so the meaningful solution is *x* = 2.5.)

5 **Second derivative test to confirm**

Second derivative test to confirm maximum:

$$fracd^2Vdx^2 = 30 - 24x; Bigg|_x = 2.5 = 30 - 60 = -30\<0quadcheckmarktext(maximum)$$

6 **Calculate V max V max**

Calculate *V*_{max}:

*V*_{max} = (2.5)² × (15−4×2.5) = 6.25 × 5 = 31.25 cm³

7 **Note**

Calculus optimisation: always verify the nature of the stationary point using $dfracd^2Vdx^2$. Also check *x* \> 0 and 15 − 4*x* \> 0 (i.e. *x* \< 3.75) to ensure a valid cuboid.

✓ **FINAL ANSWER**

Maximum volume *V* = 31.25 cm³

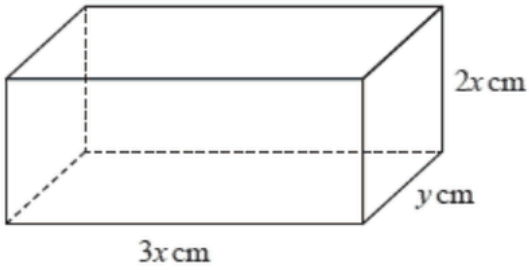
4WM2/01 MAY 2025

Question 23

3 marks | Volume & Surface Area

23 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is 1014 cm^3
The total surface area of the cuboid is A cm^2

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 May 2025

1 **Evaluate fraction**

$$3x \times 2x \times y = 1014$$

$$6x^2y = 1014$$

$$y = \frac{169}{x^2}$$

2 Surface area

Surface area:

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$= 12x^2 + 10xy$$

$$= 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$= 12x^2 + \frac{1690}{x}$$

FINAL ANSWER

$A = 12x^2 + \dfrac{1690}{x}$.

4WM2/01R MAY 2025

Question 25

5 marks | Volume & Surface Area

25 The diagram shows a solid hemisphere, H

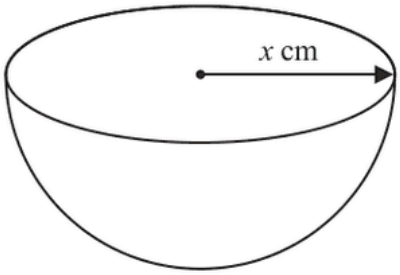


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

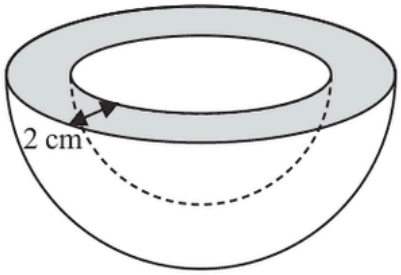


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM2/01R May 2025

1 **Required**

Hemisphere H has radius *x* cm and volume 6174*π* cm³. A bowl is made by removing an inner hemisphere, leaving 2 cm thickness. Find total surface area in terms of *π*.

2 **Calculate volume**

Find outer radius *x*. Volume of hemisphere $= \frac{2}{3}\pi r^3$:

$$\frac{2}{3}\pi x^3 = 6174 \pi i \implies x^3 = \frac{6174}{\pi} \times \frac{3}{2} = 9261 \implies x = \sqrt[3]{9261} = 21 \text{ cm}$$

3 **Find inner radius r inner**

Find inner radius:

*r*_{inner} = 21 − 2 = 19 cm

4 **Add 1**

Three surfaces to add:

1. Outer curved surface (half-sphere, radius 21):

*A*₁ = 2*π*(21)² = 2*π* × 441 = 882*π*

2. Inner curved surface (half-sphere, radius 19):

*A*₂ = 2*π*(19)² = 2*π* × 361 = 722*π*

3. Flat annular ring at the top rim:

*A*₃ = *π*(21)² − *π*(19)² = *π*(441−361) = 80*π*

5 **Total surface area A total**

Total surface area:

*A*_{total} = 882*π* + 722*π* + 80*π* = 1684*π*

6 **Note**

Three separate surfaces! Missing the flat annular ring is the most common error on this type of question.

✓ **FINAL ANSWER**

Total surface area = 1684*π* cm²

4MA1/2HR JUNE 2025

Question 26

5 marks | Volume & Surface Area

26 The diagram shows a solid hemisphere, H

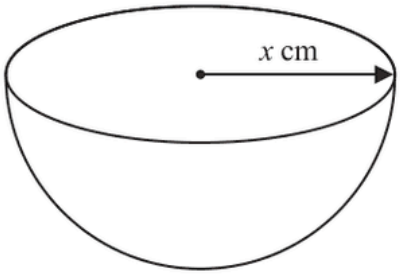


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

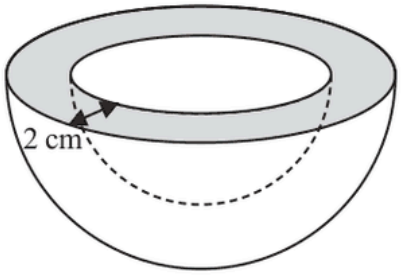


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2025

1 **The volume of a hemisphere**

The volume of a hemisphere is

$$\frac{2}{3}\pi r^3$$

2 **Evaluate fraction**

For the original hemisphere H,

$$\frac{2}{3}\pi x^3 = 6174\pi$$

3 **Cancel**

Cancel π:

$$\frac{2}{3}x^3 = 6174$$

$$x^3 = 9261$$

$$x = 21$$

4 **The outer radius of the**

The outer radius of the bowl is 21 cm.

5 **Calculate value**

The thickness is 2 cm, so the inner radius is

$$21 - 2 = 19$$

6 **Total surface area of the**

Total surface area of the bowl is:

7 **outer curved hemisphere inner curved**

- outer curved hemisphere
- inner curved hemisphere
- flat annular rim

8 **Outer curved surface area**

Outer curved surface area:

$$2\pi(21)^2 = 882\pi$$

9 **Inner curved surface area**

Inner curved surface area:

$$2\pi(19)^2 = 722\pi$$

10 **Flat rim area**

Flat rim area:

$$\pi(21^2 - 19^2) = 80\pi$$

11 **Total**

Total:

$$882\pi + 722\pi + 80\pi = 1684\pi$$

✓ **FINAL ANSWER**

$\boxed{1684\pi \text{ cm}^2}$

Question 20

5 marks | Volume & Surface Area

20 The diagram shows a solid shape.

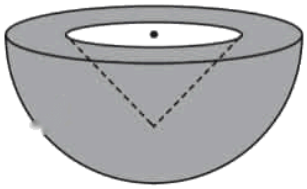


Diagram **NOT** accurately drawn

The solid shape is made by removing a cone from a hemisphere.

The radius of the cone is x cm
The height of the cone is x cm

The radius of the hemisphere is $2x$ cm

The volume of the solid shape is $\frac{3645}{8}\pi$ cm^3

Find the outer curved surface area of the solid shape.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 20 is 5 marks)

1 **Volume of the solid**

$$\tfrac{2}{3}\pi(2x)^3 - \tfrac{1}{3}\pi x^2(x) = \tfrac{16}{3}\pi x^3 - \tfrac{1}{3}\pi x^3 = 5\pi x^3$$

2 **Solve for x**

$$5\pi x^3 = \frac{3645}{8}\pi \Rightarrow x^3 = \frac{729}{8} \Rightarrow x = 4.5$$

3 **Outer curved surface = curved hemisphere**

Radius $2x = 9$:

$$2\pi(9)^2 = 162\pi = 508.9\ldots$$

✓ **FINAL ANSWER**

509 cm^2

TOPIC

Area & Volume of Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

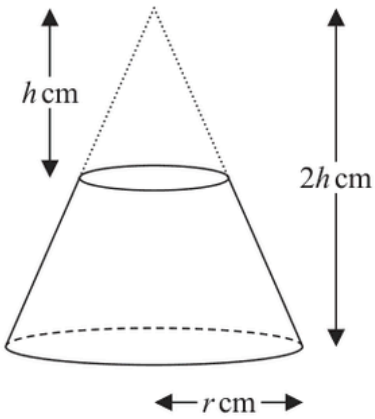
4MA1/1HR JANUARY 2020

Question 20

5 marks | Area & Volume of Similar Shapes

20 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.



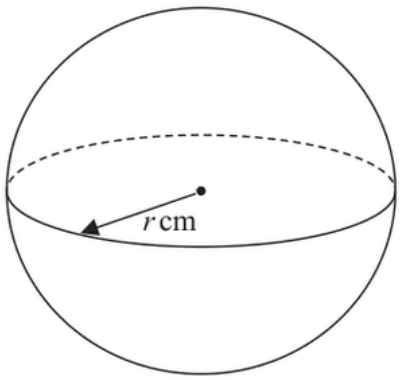


Diagram **NOT** accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h.
Give your expression in its simplest form.

$r =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2020

1 **The two cones are similar**

The two cones are similar.

2 **The large cone has height**

The large cone has height $2h$ and base radius r.

3 **The small cone has height**

The small cone has height h, so its linear scale factor compared with the large cone is

$$\frac{h}{2h} = \frac{1}{2}$$

4 **Define cone dimensions**

So the small cone has radius

$$\frac{r}{2}$$

5 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$

6 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h = \frac{1}{12}\pi r^2 h$$

7 **Calculate volume**

So the volume of the frustum is

$$\frac{2}{3}\pi r^2 h - \frac{1}{12}\pi r^2 h = \frac{7}{12}\pi r^2 h$$

8 **Calculate volume**

The sphere has radius r, so its volume is

$$\frac{4}{3}\pi r^3$$

9 **The two volumes are equal**

The two volumes are equal:

$$\frac{7}{12}\pi r^2 h = \frac{4}{3}\pi r^3$$

10 **Divide by**

Divide by πr^2:

$$\frac{7}{12}h = \frac{4}{3}r$$

$$7h = 16r$$

$$r = \frac{7h}{16}$$

✓ **FINAL ANSWER**

$$r = \frac{7h}{16}$$

4MA1/2HR JANUARY 2020

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar vases, **A** and **B**.

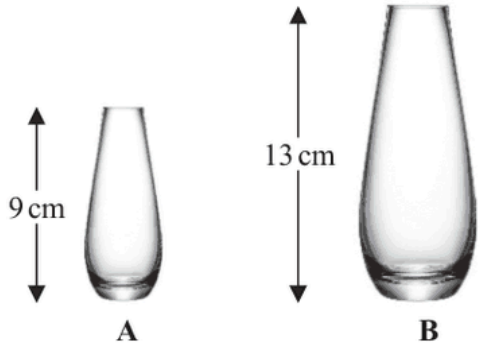


Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

surface area of vase **A** + surface area of vase **B** = 1800 cm^2

calculate the surface area of vase **A**.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2020

1 The two vases are similar

The two vases are similar.

2 Their heights are in the

Their heights are in the ratio

$$9 : 13$$

3 Split the ratio

So their surface areas are in the ratio

$$9^2 : 13^2 = 81 : 169$$

4 **The total surface area is**

The total surface area is $1800\ \text{cm}^2$, so there are

$$81 + 169 = 250$$

5 **parts altogether**

parts altogether.

6 **Surface area of vase A**

Surface area of vase A:

$$1800 \times \frac{81}{250} = 583.2$$

✓ **FINAL ANSWER**

$\boxed{583.2\ \text{cm}^2}$

4MA1/1H MAY/NOVEMBER 2020

Question 20

3 marks | Area & Volume of Similar Shapes

20 **R** and **S** are two similar solid shapes.

Shape **R** has surface area 108 cm^2 and volume 135 cm^3
Shape **S** has surface area 300 cm^2

Work out the volume of shape **S**.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May/November 2020

1 **Shape R has surface area**

Shape R has surface area $108\ \text{cm}^2$ and volume $135\ \text{cm}^3$.

2 **Shape S has surface area**

Shape S has surface area $300\ \text{cm}^2$.

3 **Surface area ratio S R**

Surface area ratio S:R:

$$300 : 108 = 25 : 9$$

4 **Split the ratio**

So the linear ratio S:R is

$$5 : 3$$

5 **Split the ratio**

Therefore the volume ratio S:R is

$$5^3 : 3^3 = 125 : 27$$

6 **Volume of S**

Volume of S:

$$135 \times \frac{125}{27} = 625$$

FINAL ANSWER

$\boxed{625\ \text{cm}^3}$

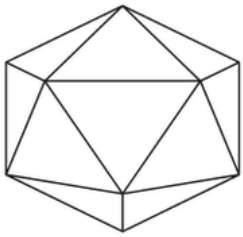
4MA1/1H JANUARY 2021

Question 20

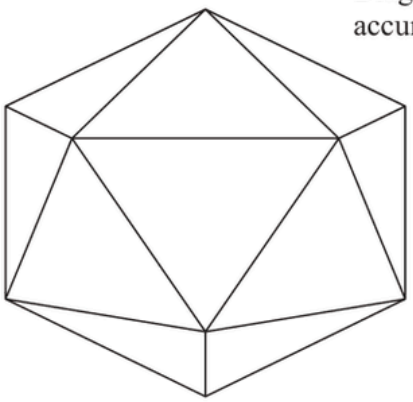
3 marks | Area & Volume of Similar Shapes

20 **A** and **B** are two similar solids.

Diagram **NOT** accurately drawn



A



B

A has a volume of 1836 cm^3
B has a volume of 4352 cm^3

B has a total surface area of 1120 cm^2

Work out the total surface area of **A**.

..... cm^2

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2021

1 The volume ratio of solid

The volume ratio of solid A to solid B is

$$1836 : 4352$$

2 Simplify

Simplify:

$$\frac{1836}{4352} = \frac{27}{64}$$

3 Split the ratio

So the linear ratio is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

4 **Surface area ratio**

Surface area ratio:

$$3^2 : 4^2 = 9 : 16$$

---

5 **Solid B has surface area**

Solid B has surface area $1120\ \text{cm}^2$, so solid A has surface area

$$1120 \times \frac{9}{16} = 630$$

✓ **FINAL ANSWER**

$\boxed{630\ \text{cm}^2}$

4MA1/2H MAY 2021

Question 20

3 marks | Area & Volume of Similar Shapes

20 Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is $300\,\text{cm}^3$

Given that

the surface area of each small block : the surface area of each large block = 25 : 36

work out the volume of each large block.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H May 2021

1 Split the ratio

The surface area ratio of the small block to the large block is

$$25 : 36$$

2 Split the ratio

So the linear ratio is

$$\sqrt{25} : \sqrt{36} = 5 : 6$$

3 Split the ratio

Therefore the volume ratio is

$$5^3 : 6^3 = 125 : 216$$

4 The small block has volume

The small block has volume $300\ \text{cm}^3$.

5 Large block volume

Large block volume:

$$300 \times \frac{216}{125} = 518.4$$

FINAL ANSWER

$\boxed{518.4\ \text{cm}^3}$

4MA1/1HR JUNE 2022

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar metal statues.



A



B

Diagram **NOT** accurately drawn

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2022

1 The volume of statue is

The volume of statue B is 20% less than statue A, so the volume scale factor is 0.8.

2 Calculate area

For similar solids, the linear scale factor is the cube root of the volume scale factor:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 Surface area scale factor is

Surface area scale factor is the square of the linear scale factor:

$$\text{surface area scale factor} = (\sqrt[3]{0.8})^2 = 0.8^{2/3} = 0.861773\ldots$$

4 **Calculate area**

So statue B has $86.1773...\%$ of the surface area of statue A.

$k = 100 - 86.1773... = 13.8226...$

✓ **FINAL ANSWER**

$k = 13.8$ to 3 significant figures.

Question 24

6 marks | Area & Volume of Similar Shapes

24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

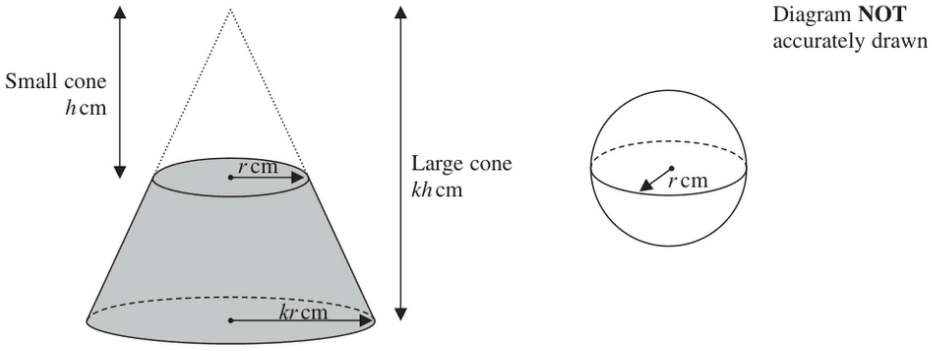


Diagram **NOT** accurately drawn

The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

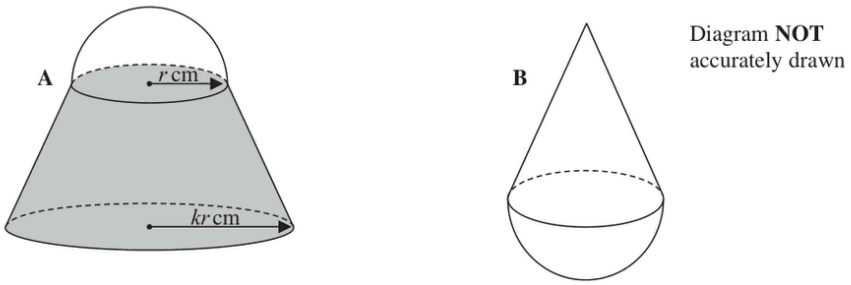


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

$h =$ .....

(Total for Question 24 is 6 marks)

1 Small cone volume

Small cone volume:

$$V_s = \frac{1}{3}\pi r^2 h$$

2 The larger similar cone has

The larger similar cone has radius kr and height kh, so its volume is

$$V_L = \frac{1}{3}\pi (kr)^2 (kh) = \frac{1}{3}\pi k^3 r^2 h$$

3 Calculate volume

So the frustum volume is

$$V_L - V_s = \frac{1}{3}\pi r^2 h(k^3 - 1)$$

4 The volume of one hemisphere

The volume of one hemisphere of radius r is

$$\frac{2}{3}\pi r^3$$

5 Solid A is the frustum

Solid A is the frustum plus one hemisphere:

$$V_A = \frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3$$

6 Solid B is the small

Solid B is the small cone plus one hemisphere:

$$V_B = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

7 Given

Given $V_A = 6V_B$:

$$\frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

8 **Divide by**

Divide by $\frac{1}{3}\pi r^2$:

$$h(k^3-1)+2r=6h+12r$$

$$h(k^3-7)=10r$$

$$h=\frac{10r}{k^3-7}$$

✓ **FINAL ANSWER**

$$h=\frac{10r}{k^3-7}$$

4MA1/1H MAY 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

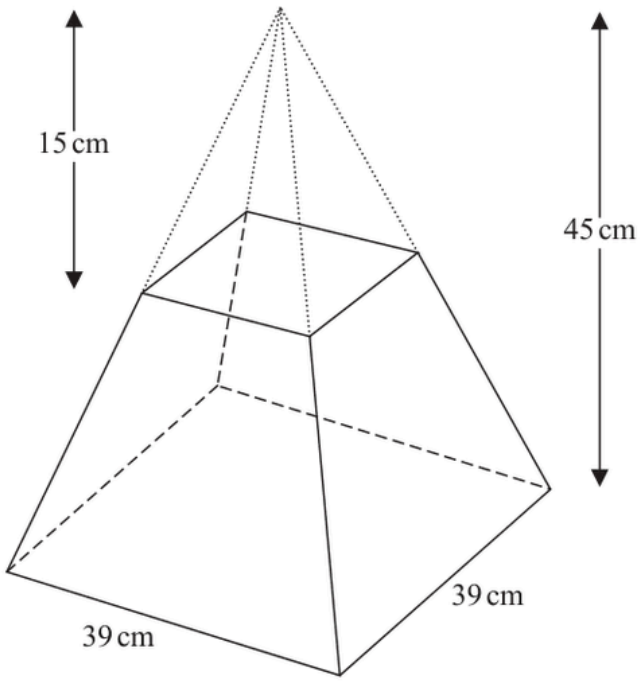


Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2023

1 **The small and large square**

The small and large square-based pyramids are similar.

2 **Their heights are**

Their heights are

$$15 : 45 = 1 : 3$$

---

3 **Square of the frustum**

So the side length of the top square of the frustum is

$$\frac{39}{3} = 13 \text{ cm}$$

---

4 **The vertical height of the**

The vertical height of the frustum is

$$45 - 15 = 30 \text{ cm}$$

---

5 **Evaluate fraction**

For one trapezium face, the difference between the half-side lengths is

$$\frac{39 - 13}{2} = 13 \text{ cm}$$

---

6 **Simplify surd**

So the slant height of the trapezium face is

$$\sqrt{30^2 + 13^2} = \sqrt{1069}$$

---

7 **Area of one trapezium face**

Area of one trapezium face:

$$\frac{1}{2}(39 + 13)\sqrt{1069} = 26\sqrt{1069}$$

---

8 **Area of four trapezium faces**

Area of four trapezium faces:

$$4(26\sqrt{1069}) = 104\sqrt{1069}$$

---

9 **Add the top and bottom**

Add the top and bottom square faces:

$$13^2 + 39^2 = 169 + 1521 = 1690$$

---

10 **Total surface area**

Total surface area:

$$1690 + 104\sqrt{1069} = 5090.338\ldots$$

---

11 Correct to the nearest whole

Correct to the nearest whole number:

$$5090$$

FINAL ANSWER

$$\boxed{5090\ \text{cm}^2}$$

4MA1/1H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Solid **A** is similar to solid **B**

Here is some information about solid **A** and solid **B**

	solid **A**	solid **B**
Height (cm)	3^x	
Area (cm^2)	7776	486
Volume (cm^3)	8^x	2^{x+4}

Work out the height of solid **B**
Give your answer as a decimal.

..... cm

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2023

1 **The area ratio of solid**

The area ratio of solid A to solid B is

$$7776 : 486 = 16 : 1$$

2 **Split the ratio**

So the linear ratio A:B is

$$\sqrt{16} : 1 = 4 : 1$$

3 **Split the ratio**

Therefore the volume ratio A:B is

$$4^3 : 1^3 = 64 : 1$$

4 **Calculate volume**

The volumes are 8^x and 2^{x+4}, so

$$\frac{8^x}{2^{x+4}} = 64$$

5 **Use index laws**

Write 8^x as a power of 2:

$$8^x = (2^3)^x = 2^{3x}$$

6 **Evaluate fraction**

So

$$\frac{2^{3x}}{2^{x+4}} = 2^6$$

$$2^{3x-(x+4)} = 2^6$$

$$2x - 4 = 6$$

$$x = 5$$

7 **The height of solid A**

The height of solid A is

$$3^x = 3^5 = 243$$

8 **Split the ratio**

Since the linear ratio A:B is $4 : 1$, the height of solid B is

$$\frac{243}{4} = 60.75$$

✓ **FINAL ANSWER**

$\boxed{60.75 \text{ cm}}$

4MA1/2H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Here is a frustum of a cone.
The frustum is made by removing a small cone from a similar large cone.

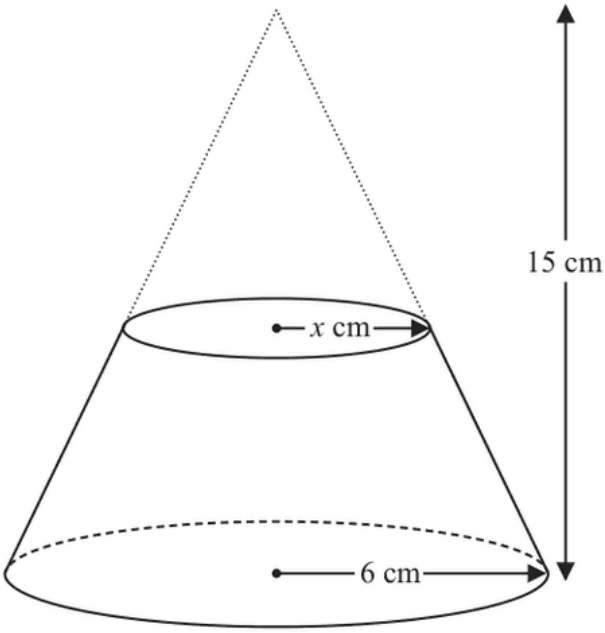


Diagram **NOT** accurately drawn

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is x cm.

Given that the volume of the frustum is $\frac{4212}{25}\pi$ cm^3

work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2023

1 **The large cone has height**

The large cone has height $\mathbf{15}$ cm and radius $\mathbf{6}$ cm.

2 **The small cone has radius**

The small cone has radius x. Since the cones are similar,

$$\frac{\text{radius}}{\text{height}} = \frac{6}{15} = \frac{2}{5}$$

3 **Evaluate fraction**

So for the small cone,

$$\frac{x}{h} = \frac{2}{5}$$

$$h = \frac{5}{2}x$$

4 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi(6)^2(15) = 180\pi$$

5 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$$

6 **The frustum volume is**

The frustum volume is

$$\frac{4212}{25}\pi$$

7 **Evaluate fraction**

So

$$180\pi - \frac{5}{6}\pi x^3 = \frac{4212}{25}\pi$$

8 **Cancel**

Cancel π:

$$180 - \frac{5}{6}x^3 = \frac{4212}{25}$$

$$\frac{5}{6}x^3 = 180 - \frac{4212}{25}$$

$$\frac{5}{6}x^3 = \frac{288}{25}$$

$$x^3 = \frac{1728}{125}$$

$$x = \frac{12}{5} = 2.4$$

✓ **FINAL ANSWER**

$\boxed{2.4 \text{ cm}}$

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows a solid, **S**, made from a cone and a hemisphere.

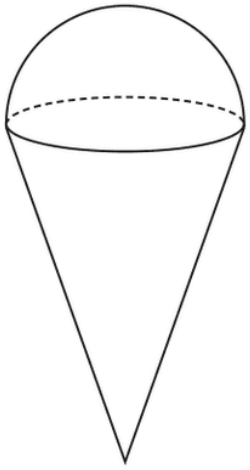


Diagram **NOT** accurately drawn

The centre of the circular face of the cone coincides with the centre of the flat surface of the hemisphere.

The radius of the circular face of the cone, x cm, is equal to the radius of the hemisphere.

The total height of **S** is 4 × the radius of the hemisphere.

A separate sphere has radius kx cm.
The volume of this sphere is 12.5 × the volume of **S**

(a) Work out the value of k

$k =$ .....

(4)

A solid, **T**, is similar to solid **S**

The volume of **T** is 512 × the volume of **S**

The total surface area of **T** is d × the total surface area of **S**

(b) Find the value of d

$d =$ .....

(1)

(Total for Question 24 is 5 marks)

1 **The hemisphere has radius**

The hemisphere has radius x, so its height is x.

2 **The total height of solid**

The total height of solid S is $4x$, so the cone height is

$$4x - x = 3x$$

3 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi x^2(3x) = \pi x^3$$

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

5 **Calculate volume**

So the volume of solid S is

$$\pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

6 **Calculate volume**

The sphere has radius kx, so its volume is

$$\frac{4}{3}\pi(kx)^3 = \frac{4}{3}\pi k^3 x^3$$

7 **This is times the volume**

This is 12.5 times the volume of S:

$$\frac{4}{3}\pi k^3 x^3 = 12.5\left(\frac{5}{3}\pi x^3\right)$$

8 **Cancel**

Cancel $\frac{1}{3}\pi x^3$:

$$4k^3 = 62.5$$

$$k^3 = 15.625 = \frac{125}{8}$$

$$k = 2.5$$

---

9 **Calculate volume**

For solid T, the volume scale factor is 512.

$$512 = 8^3$$

---

10 **Calculate area**

So the linear scale factor is 8, and the surface area scale factor is

$$8^2 = 64$$

✓ **FINAL ANSWER**

$$\boxed{k = 2.5,\ d = 64}$$

4WM2/01 MAY 2025

Question 25

4 marks | Area & Volume of Similar Shapes

25 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** − volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4WM2/01 May 2025

1 Linear scale factor from solid

Linear scale factor from solid A to solid B:

$$\frac{18.6}{31} = 0.6$$

2 Calculate volume

So the volume scale factor is

$$0.6^3 = 0.216$$

3 Let the volume of be

Let the volume of A be V. Then

$$V - 0.216V = 735$$

$$0.784V = 735$$

$$V = 937.5$$

FINAL ANSWER

$937.5\ \text{cm}^3$.

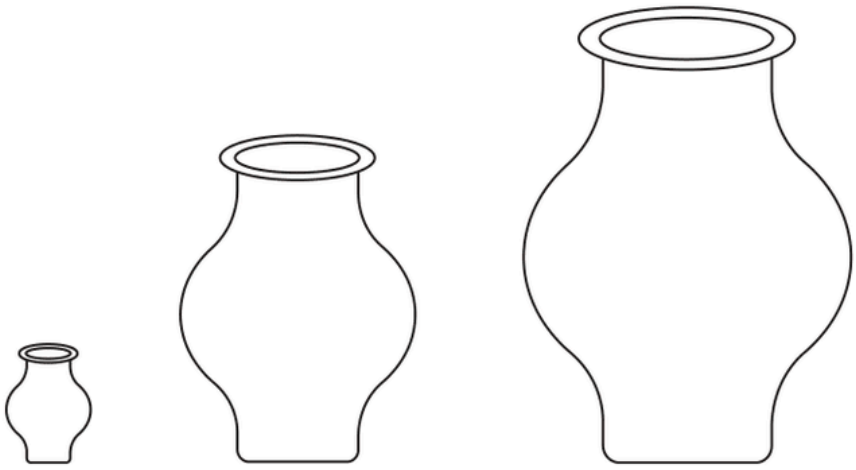
4MA1/1HR JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **R**, **S** and **T** are three similar vases.

Diagram **NOT** accurately drawn



The volume of vase **S** is 72.8% more than the volume of vase **R**

The height of vase **R** is h cm
The height of vase **T** is $6h$ cm

The surface area of vase **S** is A cm^2
The surface area of vase **T** is kA cm^2

Work out the value of k

$k =$ .....

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/1HR June 2025

1 **The volume of vase is**

The volume of vase S is 72.8% more than vase R, so

$$\frac{V_S}{V_R} = 1.728$$

2 **Calculate area**

For similar solids, volume scale factor is the cube of the linear scale factor:

$$\text{linear scale factor from } R \text{ to } S = \sqrt[3]{1.728} = 1.2$$

3 **The height of vase is**

The height of vase T is $6h$, so the linear scale factor from R to T is 6.

4 **Calculate area**

Therefore the linear scale factor from S to T is

$$\frac{6}{1.2} = 5$$

5 **Surface area scale factor is**

Surface area scale factor is the square of the linear scale factor:

$k = 5^2 = 25$

✓ **FINAL ANSWER**

$k = 25$

4MA1/2H JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** – volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H June 2025

1 **The heights of solids A**

The heights of solids A and B are 31 cm and 18.6 cm.

$$31 : 18.6 = 5 : 3$$

2 **Split the ratio**

So the volumes are in the ratio

$$5^3 : 3^3 = 125 : 27$$

3 **The difference in volume is**

The difference in volume is

$$125 - 27 = 98$$

4 **parts**

parts.

5 **Evaluate fraction**

This difference is $735\ \text{cm}^3$, so one part is

$$\frac{735}{98} = 7.5$$

6 Volume of solid A

Volume of solid A:

$$125 \times 7.5 = 937.5$$

FINAL ANSWER

$\boxed{937.5\ \text{cm}^3}$

4WM2/01 NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 November 2025

1 Calculate area

$$textASF = fractextAreaofQtextAreaofP = frac450200 = frac94 = 2.25$$

2 Simplify surd

$$textLSF = sqrttextASF = sqrtfrac94 = frac32 = 1.5$$

3 Calculate volume

$$textVSF = (textLSF)^3 = left(frac32right)^3 = frac278 = 3.375$$

4 Calculate volume

$$textVolumeofQ = textVolumeofPtimestextVSF = 672timesfrac278$$

= 84 × 27 = 2268

FINAL ANSWER

2268 cm³

4MA1/2H NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2025

1 **Surface area ratio Q P**

Surface area ratio Q:P:

$$450 : 200 = 9 : 4$$

2 **Split the ratio**

So the linear ratio Q:P is

$$3 : 2$$

3 **Split the ratio**

Therefore the volume ratio Q:P is

$$3^3 : 2^3 = 27 : 8$$

4 **Shape P has volume**

Shape P has volume $672\ \text{cm}^3$.

5 **Volume of Q**

Volume of Q:

$$672 \times \frac{27}{8} = 2268$$

FINAL ANSWER

$\boxed{2268\ \text{cm}^3}$

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TOPIC

Circle Theorems

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JUNE 2023

Question 20

2 marks | Circle Theorems

20 A, B and C are points on a circle.

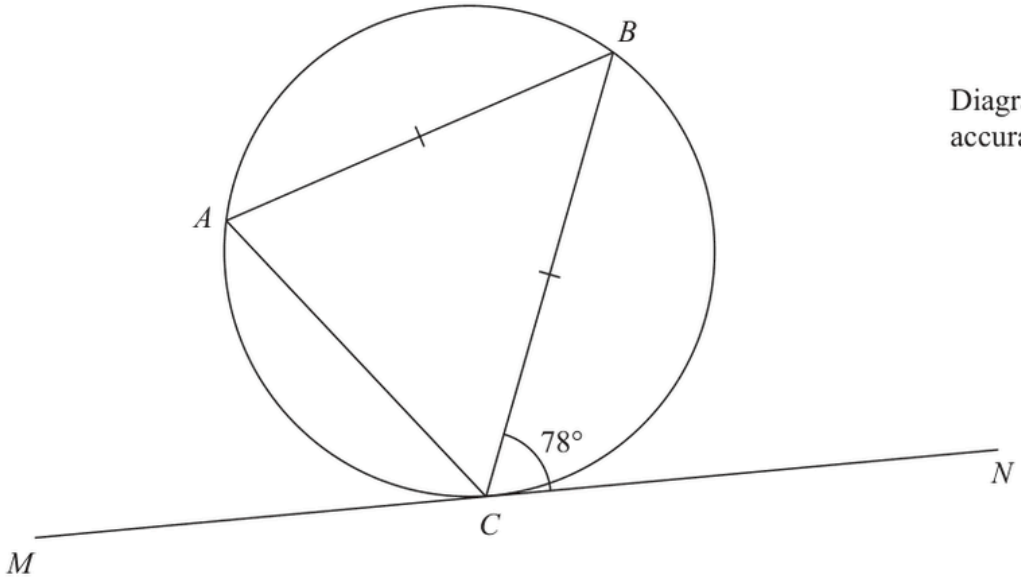


Diagram **NOT** accurately drawn

MN is the tangent to the circle at C

$AB = CB$

Angle $BCN = 78°$

Find the size of angle ABC

.....°

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2023

1 **The angle between the tangent**

The angle between the tangent CN and the chord CB is $78°$.

2 **Use trigonometry**

By the alternate segment theorem,

$$\angle BAC = 78°$$

3 **Calculate value**

Also,

$$AB = CB$$

---

4 **Use trigonometry**

so triangle ABC is isosceles and the base angles are equal:

$$\angle BAC = \angle ACB = 78^\circ$$

---

5 **Use trigonometry**

Using the angle sum of a triangle:

$$\angle ABC = 180^\circ - 78^\circ - 78^\circ$$

$$\angle ABC = 24^\circ$$

✓ **FINAL ANSWER**

$\boxed{24^\circ}$

4MA1/1HR JUNE 2025

Question 23

5 marks | Circle Theorems

23

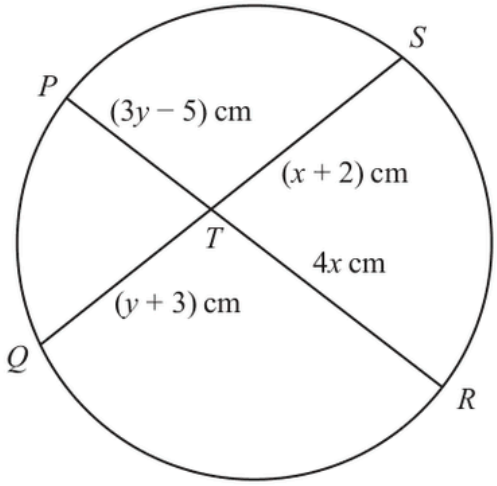


Diagram **NOT** accurately drawn

PTR and QTS are chords of a circle.

$PT = (3y - 5)\,\text{cm}$ $QT = (y + 3)\,\text{cm}$ $RT = 4x\,\text{cm}$ $ST = (x + 2)\,\text{cm}$

Find an expression for y in terms of x

$y =$ .....

Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2025

1 Use chord theorem

For two intersecting chords in a circle,

$$PT \times TR = QT \times TS$$

2 Substitute the given lengths

Substitute the given lengths:

$$(3y - 5)(4x) = (y + 3)(x + 2)$$

3 Expand

Expand:

$$12xy - 20x = xy + 2y + 3x + 6$$

4 **Collect the terms on one**

Collect the y terms on one side:

$$12xy - xy - 2y = 20x + 3x + 6$$

$$11xy - 2y = 23x + 6$$

---

5 **Factorise**

Factorise:

$$y(11x - 2) = 23x + 6$$

$$y = \frac{23x + 6}{11x - 2}$$

✓ **FINAL ANSWER**

$$\boxed{y = \frac{23x + 6}{11x - 2}}$$

TOPIC

Bearings

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2024

Question 24

6 marks | Bearings

24 A, B and C are three points on horizontal ground.

$AB = 8.4$ metres $BC = 9.2$ metres

B is on a bearing of 067° from A
C is on a bearing of 129° from B

Calculate the bearing of A from C
Give your answer correct to the nearest degree.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2024

1 Resolve each journey into east

Resolve each journey into east and north components.

2 Use trigonometry

From A to B:

$$\text{east} = 8.4 \sin 67° = 7.7322\ldots$$

$$\text{north} = 8.4 \cos 67° = 3.2821\ldots$$

3 Use trigonometry

From B to C:

$$\text{east} = 9.2 \sin 129° = 7.1497\ldots$$

$$\text{north} = 9.2 \cos 129° = -5.7897\ldots$$

4 **Calculate value**

So from A to C:

$$\text{east} = 14.8820\ldots$$

$$\text{north} = -2.5076\ldots$$

---

5 **Calculate value**

Therefore from C to A:

$$\text{east} = -14.8820\ldots, \qquad \text{north} = 2.5076\ldots$$

---

6 **Use trigonometry**

This direction is west of north.

$$\theta = \tan^{-1}\left(\frac{14.8820\ldots}{2.5076\ldots}\right) = 80.4355\ldots^\circ$$

---

7 **Use bearings**

Bearing of A from C:

$$360^\circ - 80.4355\ldots^\circ = 279.5644\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{280^\circ}$

TOPIC

Vectors

Every past-paper question on this topic, 2020 to 2026.

Question 23

5 marks | Vectors

23

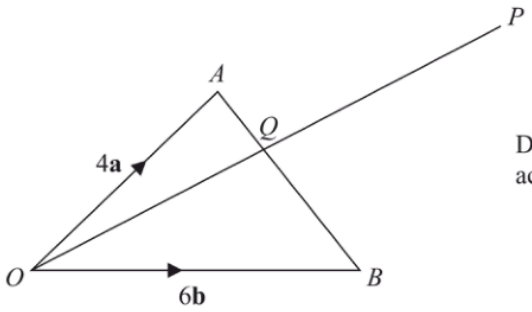


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = \mathbf{4a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AP} = \mathbf{2a} + \mathbf{8b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 23 is 5 marks)

1 **Use vectors**

$$\overrightarrow{OA} = 4a, \qquad \overrightarrow{OB} = 6b, \qquad \overrightarrow{AP} = 2a + 8b$$

2 **Use vectors**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4a + (2a + 8b) = 6a + 8b$$

3 **Use vectors**

Since O, Q, P are collinear, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Use vectors**

Also Q lies on AB. Let

$$\overrightarrow{AQ} = \mu\overrightarrow{AB}$$

5 **Use vectors**

Then

$$\overrightarrow{AB} = 6b - 4a$$

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a)$$

$$\overrightarrow{OQ} = (4 - 4\mu)a + 6\mu b$$

6 **Use vectors**

Compare coefficients with

$$\lambda(6a + 8b) = 6\lambda a + 8\lambda b$$

7 **Use vectors**

So

$$4 - 4\mu = 6\lambda, \qquad 6\mu = 8\lambda$$

8 **Use vectors**

From $6\mu = 8\lambda$,

$$\lambda = \frac{3}{4}\mu$$

---

9 **Substitute**

Substitute:

$$4 - 4\mu = 6\left(\frac{3}{4}\mu\right)$$

$$4 - 4\mu = \frac{9}{2}\mu$$

$$8 = 17\mu$$

$$\mu = \frac{8}{17}$$

---

10 **Use vectors**

Therefore

$$AQ : QB = \frac{8}{17} : \frac{9}{17} = 8 : 9$$

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$.

4MA1/2H JANUARY 2020

Question 23

3 marks | Vectors

23

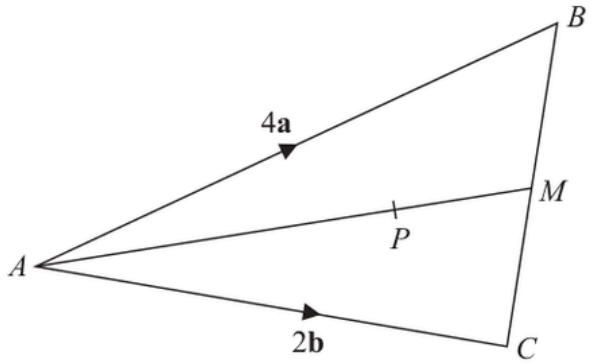


Diagram **NOT** accurately drawn

ABC is a triangle.
The midpoint of BC is M.
P is a point on AM.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2020

1 Split the ratio

Since M is the midpoint of BC,

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = \frac{4a + 2b}{2} = 2a + b$$

2 Given

Given

$$\overrightarrow{AP} = \frac{3}{2}a + \frac{3}{4}b$$

3 **Factorise**

Factorise:

$$\overrightarrow{AP} = \frac{3}{4}(2a + b) = \frac{3}{4}\overrightarrow{AM}$$

4 **Split the ratio**

So AP is $\frac{3}{4}$ of AM, leaving PM as $\frac{1}{4}$ of AM.

✓ **FINAL ANSWER**

$AP : PM = 3 : 1$

4MA1/2HR JANUARY 2020

Question 26

5 marks | Vectors

26 The diagram shows trapezium $OACB$.

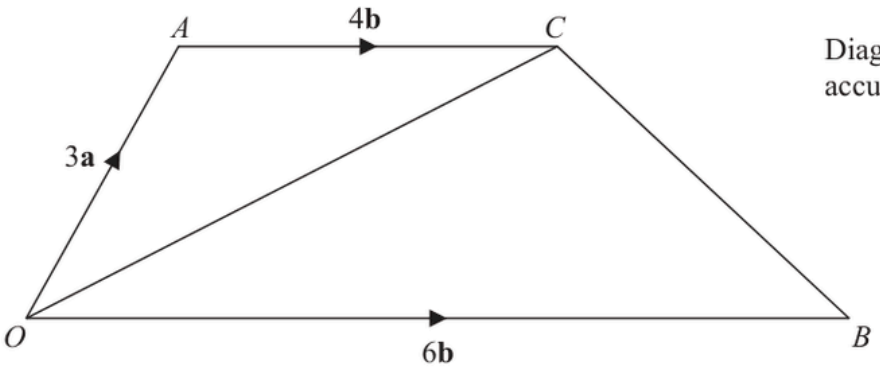


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AC} = \mathbf{4b}$

N is the point on OC such that ANB is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR January 2020

1 **Read the graph**

$$\vec{OA} = \mathbf{3a}, \quad \vec{OB} = \mathbf{6b}, \quad \vec{AC} = \mathbf{4b}$$

2 **Read the graph**

So

$$\vec{OC} = \mathbf{3a} + \mathbf{4b}$$

3 **Read the graph**

Point N lies on OC, so

$$\vec{ON} = t(\mathbf{3a} + \mathbf{4b})$$

4 **Read the graph**

Point N also lies on AB, so

$$\vec{ON} = 3\mathbf{a} + s(6\mathbf{b} - 3\mathbf{a})$$

$$\vec{ON} = (3 - 3s)\mathbf{a} + 6s\mathbf{b}$$

---

5 **Equate coefficients**

Equate coefficients:

$$3t = 3 - 3s$$

$$4t = 6s$$

---

6 **Read the graph**

So

$$t = 1 - s$$

---

7 **Read the graph**

and

$$t = \frac{3}{2}s$$

$$\frac{3}{2}s = 1 - s$$

$$5s = 2$$

$$s = \frac{2}{5}$$

---

8 **Read the graph**

Then

$$t = \frac{3}{5}$$

---

9 **Read the graph**

Therefore

$$\vec{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$$

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

4MA1/1H JANUARY 2021

Question 23

6 marks | Vectors

23 OAB is a triangle.

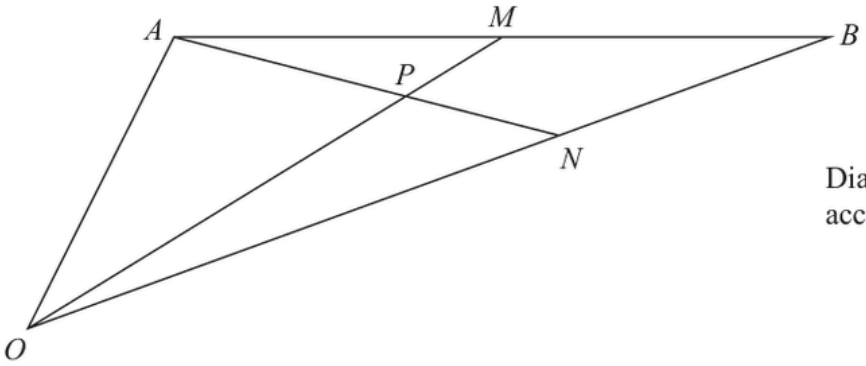


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

M is the midpoint of AB.
N is the point on OB such that $ON : NB = 2 : 1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP : PM$
Show your working clearly.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2021

1 **Use vectors**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{OB} = 2\mathbf{b}$$

2 **Find the midpoint**

Since M is the midpoint of AB,

$$\vec{OM} = \frac{2\mathbf{a} + 2\mathbf{b}}{2}$$

$$\vec{OM} = \mathbf{a} + \mathbf{b}$$

3 **Use vectors**

Since $ON : NB = 2 : 1$,

$$\vec{ON} = \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \frac{2}{3}(2\mathbf{b}) = \frac{4}{3}\mathbf{b}$$

---

4 **Use vectors**

Point P lies on OM, so

$$\vec{OP} = t(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AN, so

$$\vec{OP} = 2\mathbf{a} + s\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$$

$$\vec{OP} = (2 - 2s)\mathbf{a} + \frac{4s}{3}\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$:

$$t = 2 - 2s$$

$$t = \frac{4s}{3}$$

---

7 **Use vectors**

So

$$\frac{4s}{3} = 2 - 2s$$

$$4s = 6 - 6s$$

$$10s = 6$$

$$s = \frac{3}{5}$$

---

8 **Use vectors**

Then

$$t = \frac{4}{5}$$

9 **Use vectors**

So P is $\frac{4}{5}$ of the way from O to M.

✓ **FINAL ANSWER**

$OP : PM = 4 : 1$

4MA1/1H MAY 2021

Question 26

5 marks | Vectors

26 $OACB$ is a trapezium.

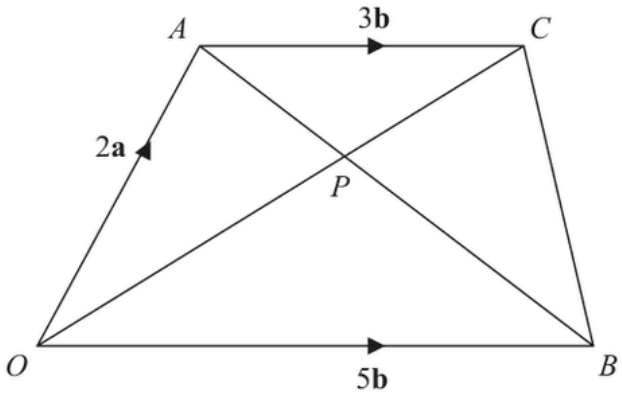


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = 5\mathbf{b}$ $\overrightarrow{AC} = 3\mathbf{b}$

The diagonals, OC and AB, of the trapezium intersect at the point P.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H May 2021

1 **Use vectors**

$$\overrightarrow{OA} = 2\mathbf{a}, \quad \overrightarrow{OB} = 5\mathbf{b}, \quad \overrightarrow{AC} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 2\mathbf{a} + 3\mathbf{b}$$

3 **Use vectors**

Since P lies on OC, let

$$\overrightarrow{OP} = t(2\mathbf{a} + 3\mathbf{b})$$

4 **Use vectors**

Since P also lies on AB, let

$$\overrightarrow{OP} = \overrightarrow{OA} + s\overrightarrow{AB}$$

5 **Use vectors**

Now

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{b} - 2\mathbf{a}$$

6 **Use vectors**

So

$$\overrightarrow{OP} = 2\mathbf{a} + s(5\mathbf{b} - 2\mathbf{a})$$

$$\overrightarrow{OP} = (2 - 2s)\mathbf{a} + 5s\mathbf{b}$$

7 **Use vectors**

Compare this with

$$t(2\mathbf{a} + 3\mathbf{b}) = 2t\mathbf{a} + 3t\mathbf{b}$$

8 **Equating coefficients**

Equating coefficients:

$$2t = 2 - 2s$$

$$3t = 5s$$

9 **Use vectors**

From $t = 1 - s$,

$$3(1 - s) = 5s$$

$$3 = 8s$$

$$s = \frac{3}{8}$$

---

10 **Use vectors**

So

$$t = 1 - \frac{3}{8} = \frac{5}{8}$$

---

11 **Use vectors**

Therefore

$$\overrightarrow{OP} = \frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$$

$$\overrightarrow{OP} = \frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}}$$

4MA1/1H JANUARY 2022

Question 22

5 marks | Vectors

22 The diagram shows triangle OAB

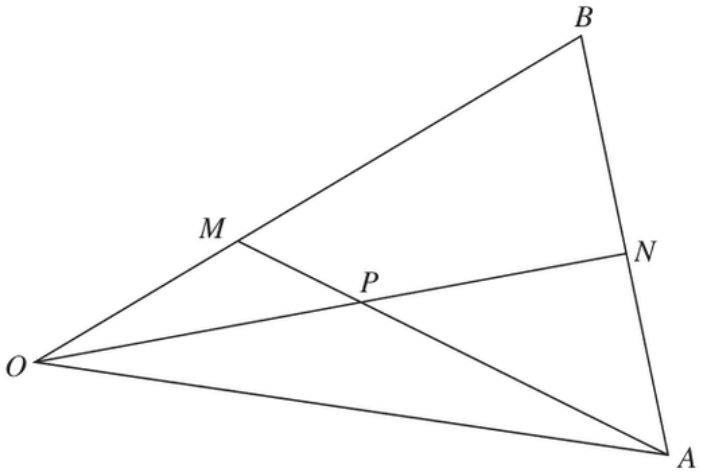


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

M is the point on OB such that $OM : MB = 1 : 2$
N is the midpoint of AB
P is the point of intersection of ON and AM

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2022

1 **Use vectors**

$$\vec{OA} = 8\mathbf{a}, \qquad \vec{OB} = 6\mathbf{b}$$

2 **Use vectors**

Since $OM : MB = 1 : 2$,

$$\vec{OM} = \frac{1}{3}\vec{OB}$$

$$\vec{OM} = 2\mathbf{b}$$

---

3 **Find the midpoint**

Since N is the midpoint of AB,

$$\vec{ON} = \frac{8\mathbf{a} + 6\mathbf{b}}{2}$$

$$\vec{ON} = 4\mathbf{a} + 3\mathbf{b}$$

---

4 **Use vectors**

Point P lies on ON, so

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AM, so

$$\vec{OP} = 8\mathbf{a} + \mu(2\mathbf{b} - 8\mathbf{a})$$

$$\vec{OP} = (8 - 8\mu)\mathbf{a} + 2\mu\mathbf{b}$$

---

6 **Equate coefficients**

Equate coefficients:

$$4\lambda = 8 - 8\mu$$

$$3\lambda = 2\mu$$

---

7 **Use vectors**

From $3\lambda = 2\mu$,

$$\mu = \frac{3}{2}\lambda$$

---

8 **Substitute**

Substitute:

$$4\lambda = 8 - 8\left(\frac{3}{2}\lambda\right)$$

$$4\lambda = 8 - 12\lambda$$

$$16\lambda = 8$$

$$\lambda = \frac{1}{2}$$

---

9 **Use vectors**

Therefore

$$\vec{OP} = \frac{1}{2}(4\mathbf{a} + 3\mathbf{b})$$

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

4MA1/1HR JANUARY 2022

Question 24

5 marks | Vectors

24 $OXYZ$ is a trapezium.

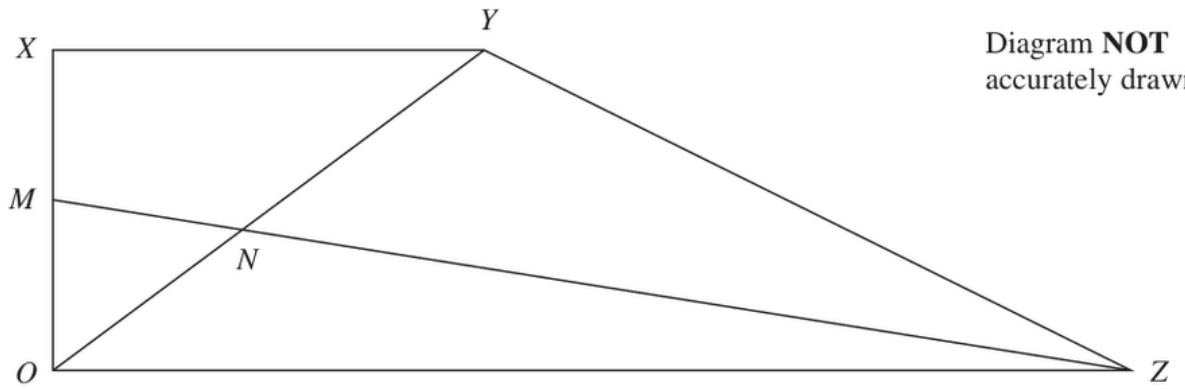


Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

M is the midpoint of OX
N is the point such that MNZ and ONY are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of λ

$\lambda =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR January 2022

1 **Take as the origin**

Take O as the origin.

$$\vec{OX} = \mathbf{a}, \qquad \vec{XY} = \mathbf{b}, \qquad \vec{OZ} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\vec{OY} = \mathbf{a} + \mathbf{b}$$

---

3 **Find the midpoint**

Since M is the midpoint of OX,

$$\vec{OM} = \frac{1}{2}\mathbf{a}$$

---

4 **Use vectors**

Given $ON : OY = \lambda : 1$,

$$\vec{ON} = \lambda(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point N also lies on MZ, so

$$\vec{ON} = \frac{1}{2}\mathbf{a} + t\left(3\mathbf{b} - \frac{1}{2}\mathbf{a}\right)$$

$$\vec{ON} = \left(\frac{1}{2} - \frac{t}{2}\right)\mathbf{a} + 3t\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients with $\lambda\mathbf{a} + \lambda\mathbf{b}$:

$$\lambda = \frac{1}{2} - \frac{t}{2}$$

$$\lambda = 3t$$

---

7 **Use vectors**

So

$$t = \frac{\lambda}{3}$$

---

8 **Substitute**

Substitute:

$$\lambda = \frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{3}\right)$$

$$\lambda = \frac{1}{2} - \frac{\lambda}{6}$$

$$\frac{7}{6}\lambda = \frac{1}{2}$$

$$\lambda = \frac{3}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}$$

4MA1/2H JUNE 2022

Question 21

5 marks | Vectors

21

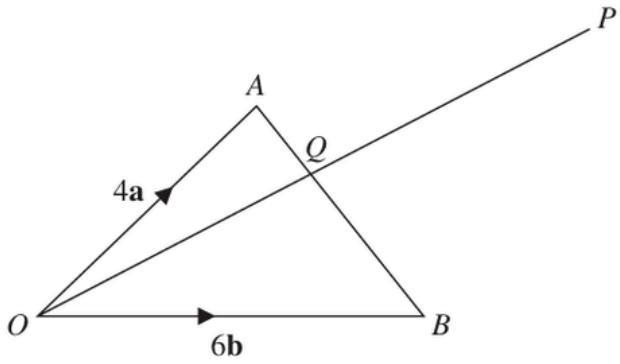


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2022

1 Split the ratio

$$\overrightarrow{OA} = 4a, \quad \overrightarrow{OB} = 6b, \quad \overrightarrow{AP} = 2a + 8b$$

2 Split the ratio

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4a + (2a + 8b) = 6a + 8b$$

3 Split the ratio

Since Q lies on OP, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Split the ratio**

Since Q lies on AB, let

$$\overrightarrow{OQ} = \overrightarrow{OA} + \mu\overrightarrow{AB}$$

$$\overrightarrow{AB} = 6b - 4a$$

---

5 **Split the ratio**

So

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a) = (4 - 4\mu)a + 6\mu b$$

---

6 **Equating coefficients**

Equating coefficients:

$$6\lambda = 4 - 4\mu, \quad 8\lambda = 6\mu$$

---

7 **Split the ratio**

From $8\lambda = 6\mu$, $\mu = \frac{4}{3}\lambda$.

$$6\lambda = 4 - \frac{16}{3}\lambda$$

$$34\lambda = 12, \quad \lambda = \frac{6}{17}$$

$$\mu = \frac{8}{17}$$

---

8 **Split the ratio**

So $AQ : QB = \frac{8}{17} : \frac{9}{17}$.

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$

4MA1/2HR JUNE 2022

Question 25

5 marks | Vectors

25 $ABCD$ is a parallelogram and ADM is a straight line.

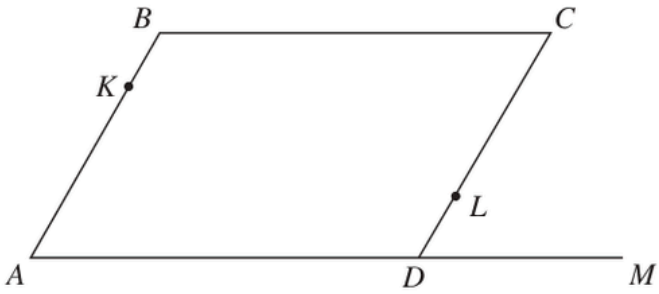


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$ $\overrightarrow{DM} = \frac{1}{2}\mathbf{b}$

K is the point on AB such that $AK : AB = \lambda : 1$
L is the point on CD such that $CL : CD = \mu : 1$
KLM is a straight line.

Given that $\lambda : \mu = 1 : 2$

use a vector method to find the value of λ and the value of μ

$\lambda =$ .....

$\mu =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2022

1 Let be the origin

Let A be the origin.

$$\vec{AB} = \mathbf{a}, \qquad \vec{BC} = \mathbf{b}$$

2 Read the graph

So

$$B = \mathbf{a}, \qquad D = \mathbf{b}, \qquad C = \mathbf{a} + \mathbf{b}$$

3 **Read the graph**

Also

$$\vec{DM} = \frac{1}{2}\mathbf{b}$$

---

4 **Read the graph**

So

$$M = \frac{3}{2}\mathbf{b}$$

---

5 **Point is on**

Point K is on AB:

$$K = \lambda\mathbf{a}$$

---

6 **Read the graph**

Point L is on CD, and $\vec{CD} = -\mathbf{a}$:

$$L = C + \mu(D - C)$$

$$L = \mathbf{a} + \mathbf{b} + \mu(-\mathbf{a})$$

$$L = (1 - \mu)\mathbf{a} + \mathbf{b}$$

---

7 **Given**

Given

$$\lambda : \mu = 1 : 2$$

$$\mu = 2\lambda$$

---

8 **Read the graph**

Since K, L, M are collinear, $\vec{KM}$ is a multiple of $\vec{KL}$.

$$\vec{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$$

$$\vec{KL} = (1 - \mu - \lambda)\mathbf{a} + \mathbf{b}$$

$$\vec{KL} = (1 - 3\lambda)\mathbf{a} + \mathbf{b}$$

---

9 **Compare coefficients using**

Compare coefficients using

$$\vec{KM} = \frac{3}{2}\vec{KL}$$

$$-\lambda = \frac{3}{2}(1 - 3\lambda)$$

$$-2\lambda = 3 - 9\lambda$$

$$7\lambda = 3$$

$$\lambda = \frac{3}{7}$$

---

10 **Read the graph**

Then

$$\mu = 2\lambda = \frac{6}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}, \quad \mu = \frac{6}{7}$$

Question 22

5 marks | Vectors

22

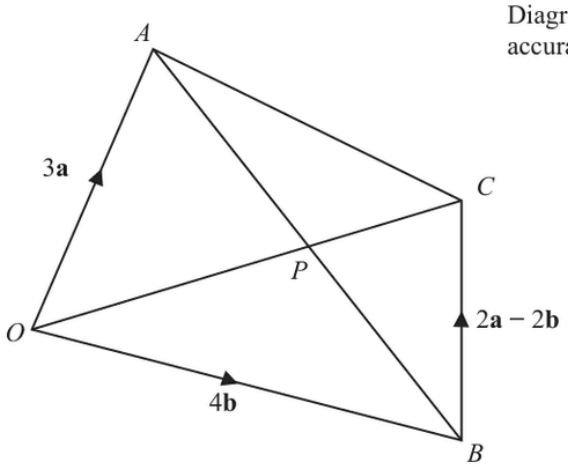


Diagram **NOT** accurately drawn

$OACB$ is a quadrilateral.

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{4b}$ $\overrightarrow{BC} = \mathbf{2a} - \mathbf{2b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$\overrightarrow{OC} =$ .....

(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .....

(1)

The point P lies on AB and on OC

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

.....

(3)

(Total for Question 22 is 5 marks)

1 **Split the ratio**

$$\overrightarrow{OA} = 3a, \quad \overrightarrow{OB} = 4b, \quad \overrightarrow{BC} = 2a - 2b$$

2 **Part (a)**

Part (a):

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 4b + (2a - 2b) = 2a + 2b$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4b - 3a$$

3 **Split the ratio**

For part (b), let P divide AB so that

$$\overrightarrow{AP} = \mu\overrightarrow{AB}$$

4 **Split the ratio**

Then

$$\overrightarrow{OP} = 3a + \mu(4b - 3a) = (3 - 3\mu)a + 4\mu b$$

5 **Split the ratio**

Since P lies on OC, also let

$$\overrightarrow{OP} = \lambda(2a + 2b)$$

6 **Equating coefficients**

Equating coefficients:

$$3 - 3\mu = 2\lambda, \quad 4\mu = 2\lambda$$

7 **Split the ratio**

From $4\mu = 2\lambda$, $\lambda = 2\mu$.

$$3 - 3\mu = 4\mu$$

$$\mu = \frac{3}{7}$$

8 **Split the ratio**

So $AP : PB = \frac{3}{7} : \frac{4}{7}$.

FINAL ANSWER

$\overrightarrow{OC} = 2\boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{AB} = 4\boldsymbol{b} - 3\boldsymbol{a}$, and $AP : PB = 3 : 4$

4MA1/2H JUNE 2024

Question 24

6 marks | Vectors

24

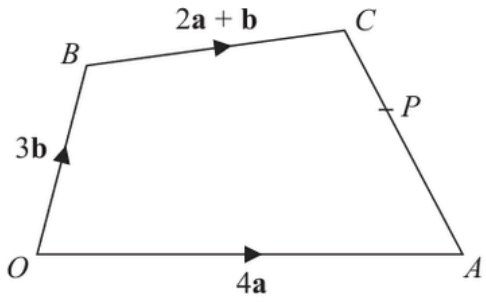


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OACB$ in which

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 3\mathbf{b}$ $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{AC} =$ .....

(2)

The point P lies on AC such that $AP : PC = 3 : 2$

The point Q is such that OPQ and BCQ are straight lines.

(b) Using a vector method, find $\overrightarrow{OQ}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OQ} =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2024

1 **Read the graph**

$$\vec{OA} = 4\mathbf{a}, \quad \vec{OB} = 3\mathbf{b}, \quad \vec{BC} = 2\mathbf{a} + \mathbf{b}$$

2 **Read the graph**

So

$$\vec{OC} = \vec{OB} + \vec{BC}$$

$$\vec{OC} = 2\mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\vec{AC} = (2\mathbf{a} + 4\mathbf{b}) - 4\mathbf{a}$$

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$$

---

4 **Read the graph**

For part (b), $AP : PC = 3 : 2$, so

$$\vec{OP} = \vec{OA} + \frac{3}{5}\vec{AC}$$

$$\vec{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a})$$

$$\vec{OP} = \frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

---

5 **Read the graph**

Since O, P, Q are collinear,

$$\vec{OQ} = t\vec{OP}$$

$$\vec{OQ} = t\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

---

6 **Read the graph**

Since B, C, Q are collinear,

$$\vec{OQ} = \vec{OB} + s\vec{BC}$$

$$\vec{OQ} = 3\mathbf{b} + s(2\mathbf{a} + \mathbf{b})$$

$$\vec{OQ} = 2s\mathbf{a} + (3 + s)\mathbf{b}$$

---

7 **Equate coefficients**

Equate coefficients:

$$\frac{14}{5}t = 2s$$

$$\frac{12}{5}t = 3 + s$$

---

8 **Solve equation**

From the first equation,

$$s = \frac{7}{5}t$$

---

9 **Substitute into the second**

Substitute into the second:

$$\frac{12}{5}t = 3 + \frac{7}{5}t$$

$$t = 3$$

---

10 **Read the graph**

Therefore

$$\vec{OQ} = 3\vec{OP}$$

$$\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$, $\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$

Question 25

6 marks | Vectors

25 $OPQR$ is a parallelogram.

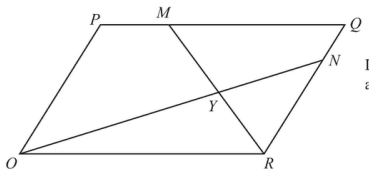


Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point M lies on PQ such that $PM = \frac{1}{4}PQ$

The point N lies on RQ such that $RN = \frac{4}{5}RQ$

(a) Find, in terms of **a** and **b**, giving your answers in simplest form

(i) $\overrightarrow{ON}$

.....

(1)

(ii) $\overrightarrow{MR}$

.....

(1)

MR and ON intersect at the point Y

Given that

$$OY = k \times ON$$

(b) use a vector method to find the value of k

$k =$ .....

(4)

(Total for Question 25 is 6 marks)

1 **Calculate value**

$$\overrightarrow{OP} = 2a, \qquad \overrightarrow{OR} = 3b$$

2 **Calculate value**

So

$$\overrightarrow{OQ} = 2a + 3b$$

3 **Evaluate fraction**

Since $PM : PQ = 1 : 4$,

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{4}(3b) = \frac{3}{4}b$$

$$\overrightarrow{OM} = 2a + \frac{3}{4}b$$

4 **Evaluate fraction**

Since $RN : RQ = 4 : 5$,

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ}$$

$$\overrightarrow{RN} = \frac{4}{5}(2a) = \frac{8}{5}a$$

$$\overrightarrow{ON} = 3b + \frac{8}{5}a$$

5 **Evaluate fraction**

So

$$\overrightarrow{MR} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{MR} = 3b - \left(2a + \frac{3}{4}b\right)$$

$$\overrightarrow{MR} = -2a + \frac{9}{4}b$$

---

6 **Let be on**

Let Y be on ON, so

$$\overrightarrow{OY} = k\overrightarrow{ON} = k\left(\frac{8}{5}a + 3b\right)$$

---

7 **Evaluate fraction**

Also Y is on MR, so

$$\overrightarrow{OY} = \overrightarrow{OM} + t\overrightarrow{MR}$$

$$\overrightarrow{OY} = 2a + \frac{3}{4}b + t\left(-2a + \frac{9}{4}b\right)$$

---

8 **Evaluate fraction**

Compare coefficients of a and b:

$$\frac{8}{5}k = 2 - 2t$$

$$3k = \frac{3}{4} + \frac{9}{4}t$$

---

9 **Solve equation**

From the first equation,

$$t = 1 - \frac{4}{5}k$$

---

10 **Substitute into the second equation**

Substitute into the second equation:

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = 3 - \frac{9}{5}k$$

$$\frac{24}{5}k = 3$$

$$k = \frac{5}{8}$$

✓ **FINAL ANSWER**

$\overrightarrow{ON} = \frac{8}{5}a + 3b$, $\overrightarrow{MR} = -2a + \frac{9}{4}b$, and $k = \frac{5}{8}$

Question 24

6 marks | Vectors

24 OAB is a triangle.

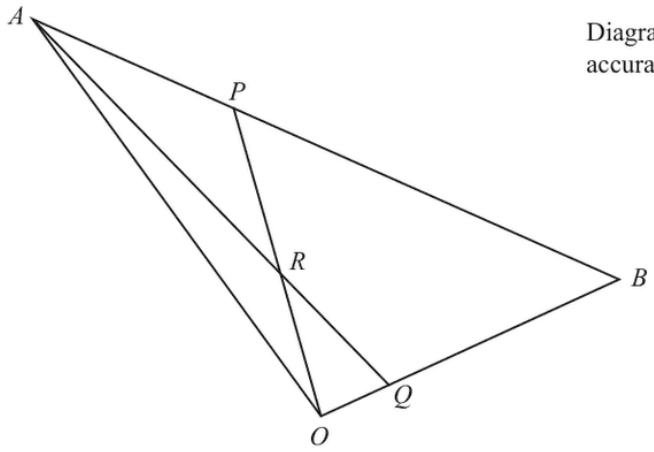


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 10\mathbf{a}$ $\overrightarrow{OB} = 10\mathbf{b}$

ARQ and ORP are straight lines.

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

Write the following vectors in terms of **a** and **b**
Simplify your answers.

(i) $\overrightarrow{AQ}$

.....

(1)

(ii) $\overrightarrow{OP}$

.....

(1)

(iii) $\overrightarrow{OR}$

.....

(4)

(Total for Question 24 is 6 marks)

1 **Read the graph**

$$\vec{OA} = 10\mathbf{a}, \qquad \vec{OB} = 10\mathbf{b}$$

2 **Read the graph**

Also,

$$\vec{AP} = \frac{1}{4}\vec{AB}$$

3 **Read the graph**

and

$$\vec{OQ} = \frac{1}{5}\vec{OB}$$

4 **Read the graph**

For part (i),

$$\vec{OQ} = 2\mathbf{b}$$

$$\vec{AQ} = \vec{OQ} - \vec{OA}$$

$$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$$

5 **Read the graph**

For part (ii),

$$\vec{AB} = 10\mathbf{b} - 10\mathbf{a}$$

$$\vec{AP} = \frac{1}{4}(10\mathbf{b} - 10\mathbf{a})$$

$$\vec{AP} = \frac{5}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

6 **Read the graph**

For part (iii), R lies on OP, so

$$\vec{OR} = t\vec{OP}$$

7 **Read the graph**

It also lies on AQ, so

$$\vec{OR} = \vec{OA} + s\vec{AQ}$$

$$\vec{OR} = 10\mathbf{a} + s(2\mathbf{b} - 10\mathbf{a})$$

$$\vec{OR} = (10 - 10s)\mathbf{a} + 2s\mathbf{b}$$

8 **Read the graph**

Equate with

$$t\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$$

$$\frac{15}{2}t = 10 - 10s$$

$$\frac{5}{2}t = 2s$$

9 **Solve equation**

From the second equation,

$$s = \frac{5}{4}t$$

10 **Substitute**

Substitute:

$$\frac{15}{2}t = 10 - \frac{25}{2}t$$

$$20t = 10$$

$$t = \frac{1}{2}$$

11 **Read the graph**

So

$$\vec{OR} = \frac{1}{2}\vec{OP}$$

$$\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$$

✓ **FINAL ANSWER**

$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$, $\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$, $\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$

Question 23

5 marks | Vectors

23

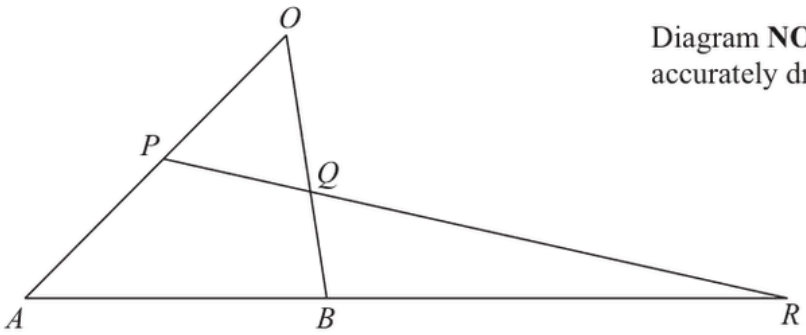


Diagram **NOT** accurately drawn

OAB is a triangle.
P is the midpoint of OA
Q is a point on OB

ABR and PQR are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\overrightarrow{OB} = 8\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

$AB : BR = 1 : 2$ $\overrightarrow{OQ} = n\mathbf{b}$

(b) Use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 23 is 5 marks)

1 **Use vectors**

For part (a),

$$\vec{OA} = 12\mathbf{a}, \qquad \vec{OB} = 8\mathbf{b}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}$$

2 **Find the midpoint**

For part (b), P is the midpoint of OA, so

$$\vec{OP} = 6\mathbf{a}$$

3 **Given**

Given

$$AB : BR = 1 : 2$$

$$\vec{BR} = 2\vec{AB}$$

$$\vec{BR} = 16\mathbf{b} - 24\mathbf{a}$$

4 **Use vectors**

Therefore

$$\vec{OR} = \vec{OB} + \vec{BR}$$

$$\vec{OR} = 8\mathbf{b} + 16\mathbf{b} - 24\mathbf{a}$$

$$\vec{OR} = -24\mathbf{a} + 24\mathbf{b}$$

5 **Use vectors**

Since P, Q, R are collinear, write Q on PR:

$$\vec{OQ} = \vec{OP} + t(\vec{OR} - \vec{OP})$$

$$\vec{OQ} = 6\mathbf{a} + t(-30\mathbf{a} + 24\mathbf{b})$$

$$\vec{OQ} = (6 - 30t)\mathbf{a} + 24t\mathbf{b}$$

---

6 **Use vectors**

But Q lies on OB, so there is no $\mathbf{a}$ component:

$$6 - 30t = 0$$

$$t = \frac{1}{5}$$

---

7 **Use vectors**

Then

$$\vec{OQ} = 24\left(\frac{1}{5}\right)\mathbf{b}$$

$$\vec{OQ} = \frac{24}{5}\mathbf{b}$$

---

8 **Use vectors**

Since $\vec{OQ} = n\mathbf{b}$,

$$n = \frac{24}{5}$$

✓ **FINAL ANSWER**

$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}, \quad n = \frac{24}{5}$

Question 26

5 marks | Vectors

26

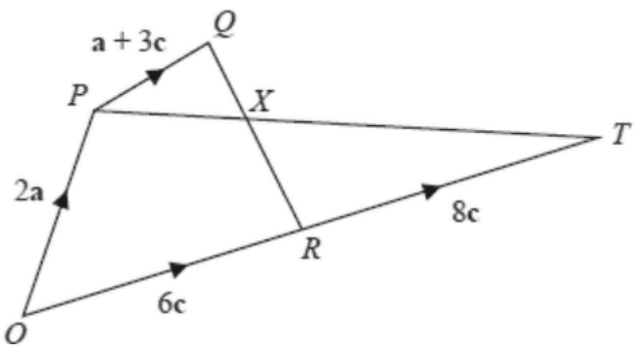


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OPQR$ in which

$\overrightarrow{OP} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{c}$ $\overrightarrow{PQ} = \mathbf{a} + 3\mathbf{c}$

(a) Find in terms of **a** and **c**

$\overrightarrow{QR}$ = ...

(1)

The point T is such that $\overrightarrow{RT} = 8\mathbf{c}$
The point X lies on QR such that PXT is a straight line.

(b) Using a vector method, find the ratio $QX : XR$
Give your answer in the form $m : n$ where m and n are integers.
Show your working clearly.

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$QX : XR$ = ...

(4)

(Total for Question 26 is 5 marks)

1 **(a)**

(a)

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$\overrightarrow{OQ} = 2a + (a + 3c) = 3a + 3c$$

$$\overrightarrow{QR} = 6c - (3a + 3c) = 3c - 3a$$

2 **(b) Let divide so that**

(b) Let X divide QR so that

$$\overrightarrow{QX} = \lambda\overrightarrow{QR}$$

3 **Calculate value**

Then

$$\overrightarrow{OX} = 3a + 3c + \lambda(3c - 3a)$$

4 **Calculate value**

Also X lies on PT, where $\overrightarrow{OT} = 14c$, so

$$\overrightarrow{OX} = 2a + \mu(14c - 2a)$$

5 **Equating coefficients gives . Therefore**

Equating coefficients gives $\lambda = \frac{5}{9}$. Therefore

$$QX : XR = \frac{5}{9} : \frac{4}{9} = 5 : 4$$

✓ **FINAL ANSWER**

$\overrightarrow{QR} = 3c - 3a$; $QX : XR = 5 : 4$.

Question 24

6 marks | Vectors

24 The diagram shows a quadrilateral $OABC$

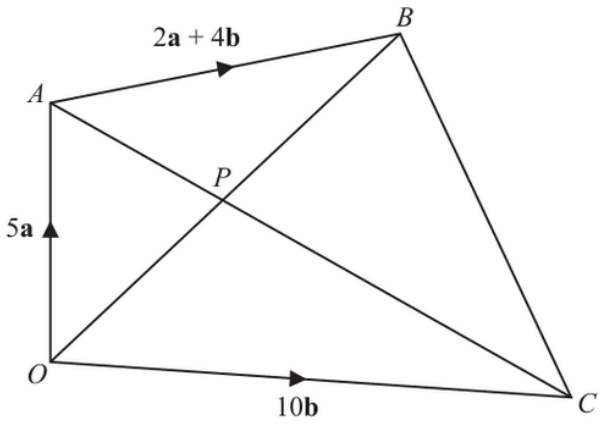


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 5\mathbf{a}$ $\overrightarrow{OC} = 10\mathbf{b}$ $\overrightarrow{AB} = 2\mathbf{a} + 4\mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**

$\overrightarrow{AC} =$ .....

(1)

(b) Find $\overrightarrow{OB}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{OB} =$ .....

(1)

The point P is such that APC and OPB are straight lines.

(c) Using a vector method, find $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OP} =$ .....

(4)

(Total for Question 24 is 6 marks)

1

(a) Find $overrightarrow{AC}$

$$overrightarrow{AC} = overrightarrow{AO} + overrightarrow{OC} = -overrightarrow{OA} + overrightarrow{OC} = -5mathbf{a} + 10mathbf{b}$$

2

Answer

a − 5**a** + 10**b**

3

(b) Find $overrightarrow{OB}$

$$overrightarrow{OB} = overrightarrow{OA} + overrightarrow{AB} = 5mathbf{a} + (2mathbf{a} + 4mathbf{b}) = 7mathbf{a} + 4mathbf{b}$$

4

Answer

b 7**a** + 4**b**

Step 1: Set up vector equations for $overrightarrow{OP}$. Since *O**P**B* is a straight line, $overrightarrow{OP}$ is a scalar multiple of $overrightarrow{OB}$. Let this be *λ*.

$$overrightarrow{OP} = lambda(7mathbf{a} + 4mathbf{b}) = 7lambdamathbf{a} + 4lambdamathbf{b}$$

Since *P* lies on the straight line *A**C*, $overrightarrow{AP} = muoverrightarrow{AC}$ for some scalar *μ*.

$$overrightarrow{OP} = overrightarrow{OA} + overrightarrow{AP} = 5mathbf{a} + mu(-5mathbf{a} + 10mathbf{b})$$

$$overrightarrow{OP} = (5 - 5mu)mathbf{a} + 10mumathbf{b}$$

Step 2: Equate the coefficients of a **and** b**.**

7*λ* = 5 − 5*μ* — (1)

$$4lambda = 10mu implies mu = frac{4}{10}lambda = 0.4lambda quad text{— — —} (2)$$

Step 3: Solve for *λ*. Substitute (2) into (1):

7*λ* = 5 − 5(0.4*λ*)

7*λ* = 5 − 2*λ*

$$9lambda = 5 implies lambda = frac{5}{9}$$

5

Evaluate fraction

$$overrightarrow{OP} = frac{5}{9}(7mathbf{a} + 4mathbf{b}) = frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

✓ **FINAL ANSWER**

$$frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

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Question 27

6 marks | Vectors

27

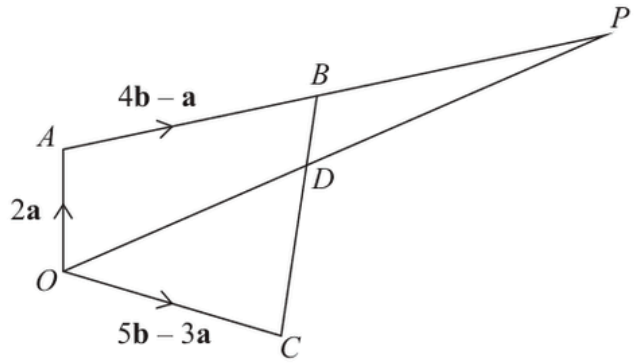


Diagram **NOT** accurately drawn

$OABC$ is a quadrilateral.
ABP and ODP are straight lines.

$\overrightarrow{OA} = \mathbf{2a}$ $\overrightarrow{AB} = \mathbf{4b - a}$ $\overrightarrow{OC} = \mathbf{5b - 3a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

.....

(2)

The point D lies on BC such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 27 is 6 marks)

WORKED SOLUTION

Question 27 · 4MA1/1H November 2025

1 Read the graph

$$\vec{OA} = \mathbf{2a}, \qquad \vec{AB} = \mathbf{4b - a}, \qquad \vec{OC} = \mathbf{5b - 3a}$$

2 **Read the graph**

First find $\vec{OB}$:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = 2\mathbf{a} + 4\mathbf{b} - \mathbf{a}$$

$$\vec{OB} = \mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$\vec{BC} = (5\mathbf{b} - 3\mathbf{a}) - (\mathbf{a} + 4\mathbf{b})$$

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}$$

---

4 **Read the graph**

For part (b), $BD : DC = 1 : 3$, so

$$\vec{OD} = \vec{OB} + \frac{1}{4}\vec{BC}$$

$$\vec{OD} = \mathbf{a} + 4\mathbf{b} + \frac{1}{4}(\mathbf{b} - 4\mathbf{a})$$

$$\vec{OD} = \frac{17}{4}\mathbf{b}$$

---

5 **Read the graph**

Since A, B, P are collinear, let

$$\vec{OP} = \vec{OA} + s\vec{AB}$$

$$\vec{OP} = 2\mathbf{a} + s(4\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = (2 - s)\mathbf{a} + 4s\mathbf{b}$$

---

6 **Read the graph**

But O, D, P are collinear, and $\vec{OD}$ has only a $\mathbf{b}$ component.

---

7 **Read the graph**

So

$$2 - s = 0$$

$$s = 2$$

---

8 **Read the graph**

Then

$$\vec{OP} = 8\mathbf{b}$$

---

9 **Given**

Given

$$\vec{OP} = n\vec{OD}$$

$$8\mathbf{b} = n\left(\frac{17}{4}\mathbf{b}\right)$$

$$n = \frac{32}{17}$$

✓ **FINAL ANSWER**

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}, \quad n = \frac{32}{17}$$