Elite Mathematics
Pearson Edexcel International GCSE

IGCSE MODULAR · UNIT 2 · 4WM2

Modular Unit 2 Classified With Answers

Every question followed by Dr Eslam Ahmed's worked solution and final answer.

697	2020–2026	Questions with worked solutions
topic placements	49 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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TOPIC

Number Toolkit

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JUNE 2026

Question 1

3 marks | Number Toolkit

1 Write 4500 as a product of powers of its prime factors.
Show your working clearly.

.....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2026

1 Solution | Step 1

$$4500 = 45 \times 100 = (9 \times 5)(4 \times 25).$$

Split into factors that are easy to prime-factorise.

2 Solution | Step 2

$$4500 = (3^2 \times 5)(2^2 \times 5^2) = 2^2 \times 3^2 \times 5^3.$$

Express each factor as prime powers and combine powers of 5.

FINAL ANSWER

$2^2 \times 3^2 \times 5^3$

4MA1/1HR JUNE 2026

Question 2

3 marks | Number Toolkit

2 Write 504 as a product of powers of its prime factors.
Show your working clearly.

.....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2026

1 Solution | Step 1

$$504 = 2 \times 252 = 2^2 \times 126 = 2^3 \times 63.$$

Divide repeatedly by the prime 2.

2 Solution | Step 2

$$63 = 3 \times 21 = 3^2 \times 7, \qquad 504 = 2^3 \times 3^2 \times 7.$$

All remaining factors are prime; group repeated factors as powers.

✓ FINAL ANSWER

$504 = 2^3 \times 3^2 \times 7$

TOPIC

Standard & Compound Units

Every past-paper question on this topic, 2020 to 2026.

Question 7

4 marks | Standard & Compound Units

7 A cylinder is placed on the ground.

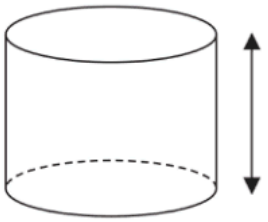


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 7 is 4 marks)

1 **Calculate area**

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

2 **Calculate area**

So the area of the base is

$$\text{area} = \frac{72}{1.4} = 51.428571\ldots$$

3 **The volume is**

The volume is

$$51.428571\ldots \times 18 = 925.714285\ldots$$

4 **Calculate volume**

Correct to 3 significant figures,

$$925.714\ldots \approx 926$$

✓ **FINAL ANSWER**

$926\ \text{cm}^3$.

TOPIC

Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

Question 21

3 marks | Algebraic Fractions

21 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 21 is 3 marks)

1 **Simplify fraction**

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 **Simplify fraction**

Substitute for x:

$$a = \frac{14}{3\left(\frac{7}{4y-3}\right) - 7}$$

3 **Simplify the denominator**

Simplify the denominator:

$$3\left(\frac{7}{4y - 3}\right) - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

4 **Simplify fraction**

So

$$a = \frac{14}{\frac{42-28y}{4y-3}} = \frac{14(4y - 3)}{42 - 28y}$$

$$a = \frac{4y - 3}{3 - 2y}$$

✓ **FINAL ANSWER**

$a = \dfrac{4y - 3}{3 - 2y}$.

TOPIC

Algebraic Roots & Indices

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 3

2 marks | Algebraic Roots & Indices

3 $\frac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 Specimen

1 **Use index laws**

$$4^n = (2^2)^n = 2^{2n}$$

2 **Use index laws**

So

$$\frac{2^k}{4^n} = \frac{2^k}{2^{2n}} = 2^{k-2n}$$

3 **Use index laws**

Since this equals 2^x,

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

TOPIC

Completing the Square

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JUNE 2026

Question 24

5 marks | Completing the Square

24 (a) Express $10 - 24x - 3x^2$ in the form $a - b(x + c)^2$ where a, b and c are integers.

.....

(3)

T is the curve with equation $y = 10 - 24(x - 1) - 3(x - 1)^2$
The point P is the maximum point on **T**

(b) Use your answer to part (a) to find the coordinates of P

(..... , ...)

(1)

(c) Write down the equation for the line of symmetry of **T**

.....

(1)

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$10 - 24x - 3x^2 = 10 - 3(x^2 + 8x).$$

Factor -3 from the terms involving x.

2 **Part (a) | Step 2**

$$x^2 + 8x = (x + 4)^2 - 16.$$

Half the coefficient of x is 4.

3 **Part (a) | Step 3**

$$10 - 3\big((x + 4)^2 - 16\big) = 58 - 3(x + 4)^2.$$

Thus a=58, b=3 and c=4.

4 **Part (b) | Step 1**

$$y = 58 - 3\big((x - 1) + 4\big)^2 = 58 - 3(x + 3)^2.$$

Use (a) with x replaced throughout by x-1.

5 **Part (b) | Step 2**

Since $(x+3)^2 \geq 0$, the greatest y is 58, attained at $x = -3$. Thus $P = (-3, 58)$.

The negative square term is largest when its square is zero.

6 **Part (c) | Step 1**

The parabola is symmetric about its maximum, so its line of symmetry is $x = -3$.

In 58-3(x+3)^2, equal positive and negative offsets from -3 give the same y.

✓ **FINAL ANSWER**

(a) $58 - 3(x+4)^2$ (b) $P = (-3, 58)$ (c) $x = -3$

TOPIC

Graphs of Functions

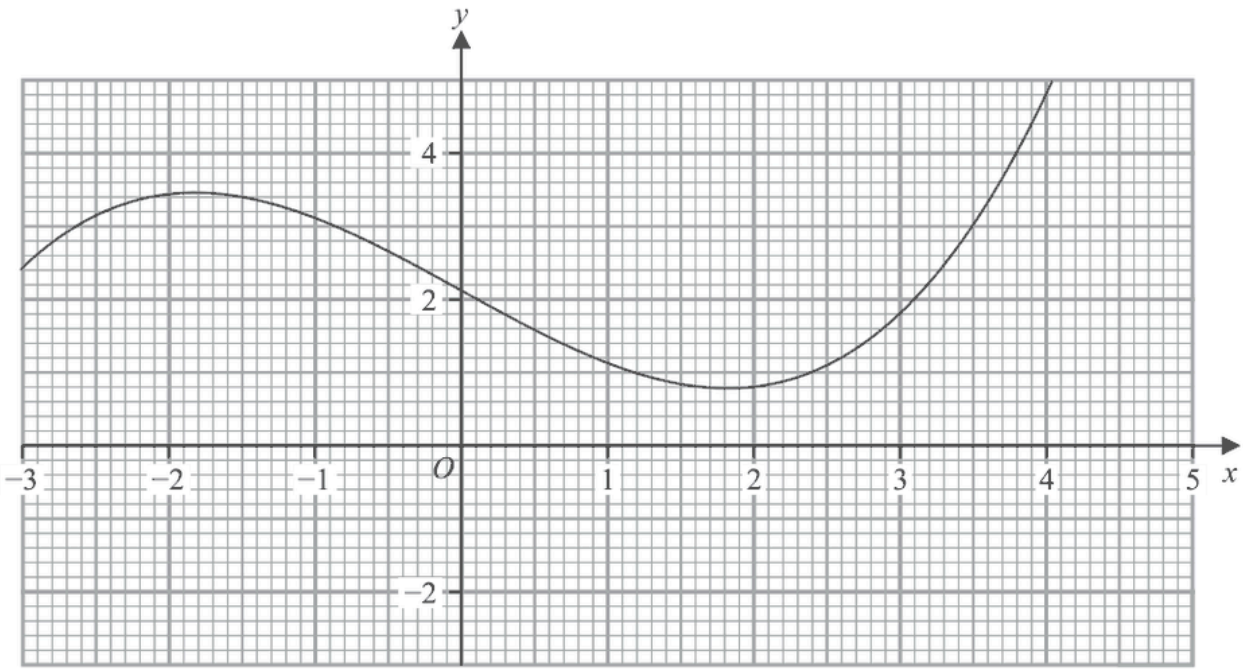
Every past-paper question on this topic, 2020 to 2026.

4WM2/01R MAY 2025

Question 19

3 marks | Graphs of Functions

19 The diagram shows the graph of $y = f(x)$



Find an estimate for the gradient of the graph at the point where $x = 3$
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM2/01R May 2025

1 **Required**

Estimate the gradient of *y* = *f*(*x*) at the point where *x* = 3.

2 **Draw a tangent to the**

Draw a tangent to the curve at *x* = 3. From the graph, a suitable tangent passes through approximately (1, −2) and (5, 6):

3 **Calculate gradient of the tangent**

Calculate gradient of the tangent:

$$textGradient = fracDeltayDeltax = frac6 - (-2)5 - 1 = frac84 = 2$$

4 **Note**

Answers in the range 1 to 3 are typically accepted. Draw the tangent triangle as large as possible for the most accurate estimate.

✓ **FINAL ANSWER**

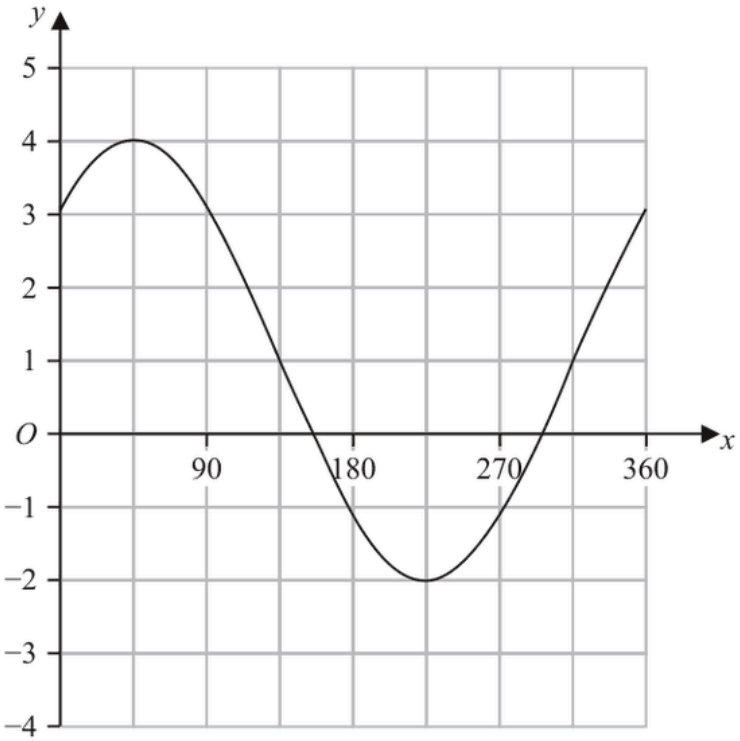
Gradient ≈ 2 at *x* = 3

4WM2/01R MAY 2025

Question 23

3 marks | Graphs of Functions

23 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01R May 2025

1 **Required**

The graph of *y* = *a*sin(*x*+*b*)[∘] + *c* for 0 ≤ *x* ≤ 360 is shown. Find suitable values for *a*, *b*, and *c*.

2 **Read the maximum and minimum**

Read the maximum and minimum values from the graph to find the **amplitude** *a* and **vertical shift** *c*:

$$a = \frac{\text{max} - \text{min}}{2} = \frac{4 - (-2)}{2} = 3 \quad c = \frac{\text{max} + \text{min}}{2} = \frac{4 + (-2)}{2} = 1$$

3 Find the phase shift b

Find the **phase shift** *b*. The standard sin has its peak at *x* = 90°. From the graph, the maximum occurs at *x* ≈ 45°:

x + *b* = 90° ⟹ 45° + *b* = 90° ⟹ *b* = 45°

---

4 Verify at x 0 consistent

Verify at *x* = 0: $y = 4sin(0 + 45)^\circ = 4times fracsqrt22approx2.83$ — consistent with graph

---

5 Note

The period of the graph is still 360° (unchanged), so no horizontal stretch is applied.

✓ **FINAL ANSWER**

a = 3,*b* = 45,*c* = 1

Question 25

3 marks | Graphs of Functions

25 A curve has equation $y = f(x)$

There is one minimum point on this curve.

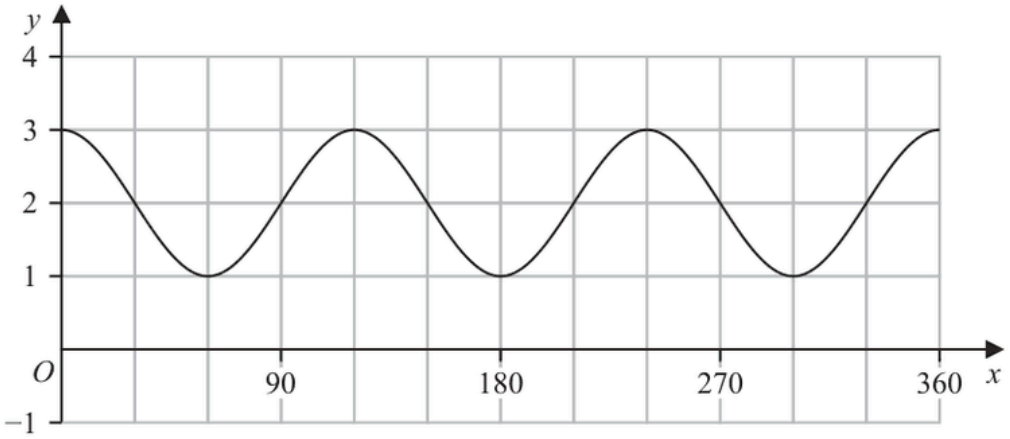
The coordinates of this minimum point are $(8, 1)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = f(4x)$

(..... , ...)

(1)

The diagram shows the graph of $y = \cos(ax) + b$ for $0 \leqslant x \leqslant 360$



(b) (i) Find the value of a

$a =$ .....

(1)

(ii) Find the value of b

$b =$ .....

(1)

(Total for Question 25 is 3 marks)

1 The transformation y f (4

The transformation *y* = *f*(4*x*) represents a horizontal stretch by a scale factor of $frac14$. The x-coordinate is multiplied by $frac14$ while the y-coordinate stays the same.

$$textNewminimum = leftbigl(8timesfrac14, 1rightbigr) = (2, 1)$$

2 Answer

(2,1)

Step 1: Find the vertical shift *b*. The standard *y* = cos (*x*) graph fluctuates between − 1 and 1, centered at *y* = 0. The given graph fluctuates between a minimum of *y* = 1 and a maximum of *y* = 3. The center line is $frac3 + 12 = 2$. Therefore, *b* = 2.

Step 1: Find the frequency multiplier *a*. The period of the given graph completes one full cycle between *x* = 0 and *x* = 90[∘] (it starts at 3, dips to 1, and returns to 3 at 90[∘]). The standard cosine period is 360[∘].

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Missing open brace for superscript

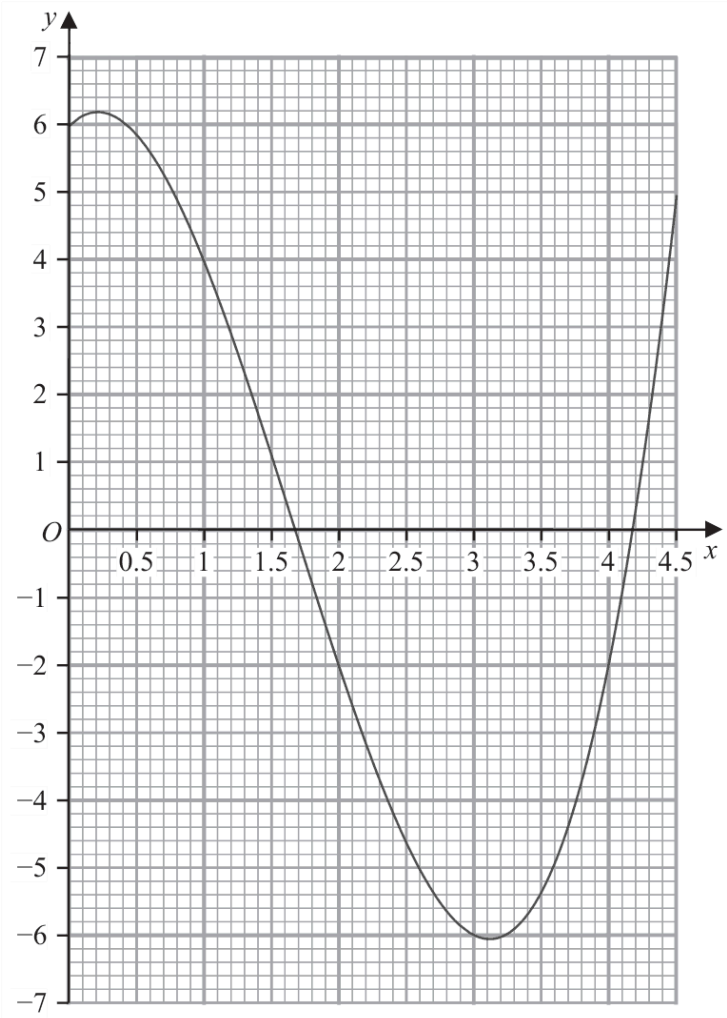
FINAL ANSWER

a = 4, *b* = 2

Question 21

5 marks | Graphs of Functions

21 The curve with equation $y = f(x)$ is drawn on a grid.



(a) By drawing a suitable straight line (at least 3 cm long) on the grid, work out an estimate for the gradient of the curve at the point where $x = 1$
Show your working clearly.

.....

(2)

(b) By drawing a suitable straight line on the grid, find the solutions of the equation $x = 3 + \frac{1}{2}f(x)$
Show your working clearly.

.....

(3)

(Total for Question 21 is 5 marks)

1 **Part (a) | Step 1**

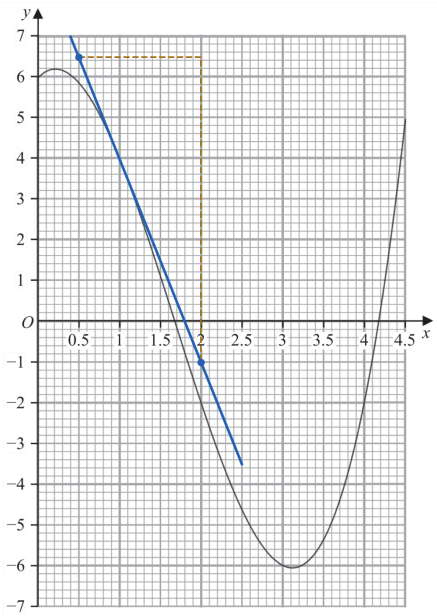
At $x = 1$, the point is approximately $(1, 4)$. Draw a tangent there, extending at least $3\ \text{cm}$. A suitable approximate tangent is $y = -5x + 9$, through $(0.5, 6.5)$ and $(2, -1)$.

The gradient at a point is estimated by the gradient of its tangent; use two well-separated points on the line.

2 **Part (a) | Step 2**

$$\text{Gradient} \approx \frac{-1 - 6.5}{2 - 0.5} = \frac{-7.5}{1.5} = -5.$$

Calculate change in y divided by change in x using the axis values.



3 **Part (b) | Step 1**

$$x = 3 + \tfrac{1}{2} f(x) \quad \Rightarrow \quad f(x) = 2x - 6.$$

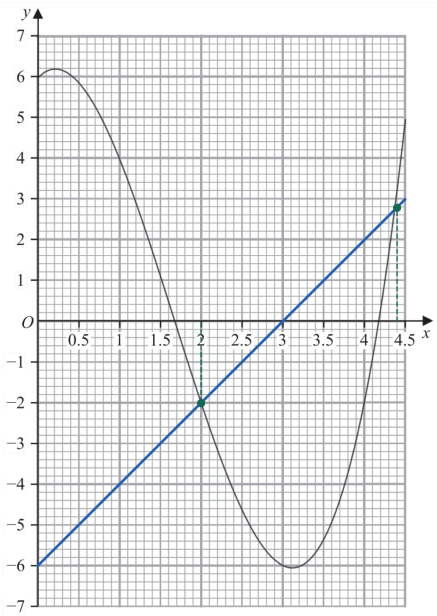
Rearrange so the given curve y=f(x) can be intersected by a straight line.

4

Part (b) | Step 2

Draw $y = 2x - 6$ through $(0, -6), (3, 0), (4.5, 3)$ on the same grid. Read the x-coordinates of its two intersections with the curve: approximately 2.0 and 4.4.

Each intersection has equal y-values on the line and curve and therefore solves the original equation.



FINAL ANSWER

(a) Gradient ≈ -5. (b) $x \approx 2.0$ or $x \approx 4.4$.

TOPIC

Prime Factors, HCF & LCM

Every past-paper question on this topic, 2020 to 2026.

Question 2

4 marks | Prime Factors, HCF & LCM

2 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

.....

(2)

(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.

.....

(2)

(Total for Question 2 is 4 marks)

1 **Find the LCM**

$$56 = 2^3 \times 7, \qquad 84 = 2^2 \times 3 \times 7$$

2 **The common prime factors are**

The common prime factors are 2^2 and 7, so

$$\text{HCF} = 2^2 \times 7 = 28$$

3 **Find the LCM**

Also

$$60 = 2^2 \times 3 \times 5, \qquad 72 = 2^3 \times 3^2$$

4 **Find the LCM**

Use the highest powers of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

✓ **FINAL ANSWER**

HCF $= 28$, LCM $= 360$.

4MA1/2HR JANUARY 2020

Question 1

3 marks | Prime Factors, HCF & LCM

1 (a) Write $5^{17} \times 5^2$ as a single power of 5

.....

(1)

(b) Write 800 as a product of its prime factors.
Show your working clearly.

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2020

1 Use the index law

Use the index law $a^m \times a^n = a^{m+n}$:

$$5^{17} \times 5^2 = 5^{19}$$

2 Find the LCM

Now write 800 as a product of prime factors:

$$800 = 8 \times 100$$

$$800 = 2^3 \times 2^2 \times 5^2$$

$$800 = 2^5 \times 5^2$$

FINAL ANSWER

(a) 5^{19}, (b) $2^5 \times 5^2$

4MA1/1H JANUARY 2020

Question 2

2 marks | Prime Factors, HCF & LCM

2 Find the lowest common multiple (LCM) of 28 and 105

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2020

1 Write both numbers as products

Write both numbers as products of prime factors:

$$28 = 2^2 \times 7$$

$$105 = 3 \times 5 \times 7$$

2 Find the LCM

For the lowest common multiple, take the highest power of each prime:

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

FINAL ANSWER

420

4MA1/1H MAY/NOVEMBER 2020

Question 10

3 marks | Prime Factors, HCF & LCM

10 $A = 2 \times 3^{43}$
$B = 16 \times 3^{37}$

(a) Find the highest common factor (HCF) of A and B.

.....

(1)

(b) Express the number $A \times B$ as a product of powers of its prime factors.
Give your answer in its simplest form.

.....

(2)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May/November 2020

1 Find the LCM

$$A = 2 \times 3^{43}$$

$$B = 16 \times 3^{37} = 2^4 \times 3^{37}$$

2 Find the LCM

For the HCF, take the smaller powers:

$$\text{HCF} = 2 \times 3^{37}$$

3 Find the LCM

For $A \times B$,

$$A \times B = (2 \times 3^{43})(2^4 \times 3^{37})$$

$$A \times B = 2^5 \times 3^{80}$$

FINAL ANSWER

(a) 2×3^{37}, (b) $2^5 \times 3^{80}$

4MA1/1HR JANUARY 2021

Question 9

4 marks | Prime Factors, HCF & LCM

9 $A = 2^8 \times 3^5 \times 11^4$ $B = 2^6 \times 3 \times 11^8$

(a) Find the highest common factor (HCF) of A and B.

.....

(2)

(b) Find the lowest common multiple (LCM) of $2A$ and $3B$.
Give the LCM as a product of powers of its prime factors.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2021

1 Find the LCM

$$A = 2^8 \times 3^5 \times 11^4$$

$$B = 2^6 \times 3 \times 11^8$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 2^6 \times 3 \times 11^4$$

3 Find the LCM

Now

$$2A = 2^9 \times 3^5 \times 11^4$$

4 Find the LCM

and

$$3B = 2^6 \times 3^2 \times 11^8$$

5 Find the LCM

For the LCM of $2A$ and $3B$, take the larger power of each prime:

$$\text{LCM} = 2^9 \times 3^5 \times 11^8$$

FINAL ANSWER

(a) $2^6 \times 3 \times 11^4$, (b) $2^9 \times 3^5 \times 11^8$

4MA1/2H JANUARY 2021

Question 9

2 marks | Prime Factors, HCF & LCM

9 $A = 2^3 \times 3^2 \times 5^2 \times 11$
$B = 2^4 \times 3 \times 5^4 \times 13$

Find the lowest common multiple (LCM) of A and B.
Give your answer as a product of powers of prime numbers.

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2021

1 Find the LCM

$$A = 2^3 \times 3^2 \times 5^2 \times 11$$

$$B = 2^4 \times 3 \times 5^4 \times 13$$

2 Find the LCM

For the LCM, take the highest power of every prime that appears:

$$\text{LCM} = 2^4 \times 3^2 \times 5^4 \times 11 \times 13$$

FINAL ANSWER

$2^4 \times 3^2 \times 5^4 \times 11 \times 13$

4MA1/2H MAY 2021

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 600 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H May 2021

1 Find the LCM

$$600 = 6 \times 100$$

$$600 = (2 \times 3)(2^2 \times 5^2)$$

$$600 = 2^3 \times 3 \times 5^2$$

FINAL ANSWER

$2^3 \times 3 \times 5^2$

4MA1/1H NOVEMBER 2021

Question 7

3 marks | Prime Factors, HCF & LCM

7 Write 3.6×10^3 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2021

1 Find the LCM

$$3.6 \times 10^3 = 3600$$

$$3600 = 36 \times 100$$

$$36 = 2^2 \times 3^2, \qquad 100 = 2^2 \times 5^2$$

2 Find the LCM

So

$$3600 = 2^4 \times 3^2 \times 5^2$$

✓ **FINAL ANSWER**

$2^4 \times 3^2 \times 5^2$

4MA1/1HR JANUARY 2022

Question 6

4 marks | Prime Factors, HCF & LCM

6 (a) Work out the lowest common multiple (LCM) of 36 and 120

.....

(2)

$A = 5^2 \times 7^4 \times 11^p$
$B = 5^m \times 7^{n-5} \times 11$

m, n and p are integers such that
$m > 2$
$n > 10$
$p > 1$

(b) Find the highest common factor (HCF) of A and B
Give your answer as a product of powers of its prime factors.

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2022

1 **Find the LCM**

$$36 = 2^2 \times 3^2$$

$$120 = 2^3 \times 3 \times 5$$

2 **Find the LCM**

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

3 **Find the LCM**

For part (b),

$$A = 5^2 \times 7^4 \times 11^p$$

$$B = 5^m \times 7^{n-5} \times 11$$

4 **Find the LCM**

Since $m > 2$, the smaller power of 5 is 5^2.

5 **Find the LCM**

Since $n > 10$, $n - 5 > 5$, so the smaller power of 7 is 7^4.

6 **Find the LCM**

Since $p > 1$, the smaller power of 11 is 11^1.

$$\text{HCF} = 5^2 \times 7^4 \times 11$$

✓ **FINAL ANSWER**

(a) 360, (b) $5^2 \times 7^4 \times 11$

4MA1/2H JANUARY 2022

Question 7

3 marks | Prime Factors, HCF & LCM

7 Find the lowest common multiple (LCM) of 28, 42 and 63
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2022

1 Find the LCM

$$28 = 2^2 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3^2 \times 7$$

2 Find the LCM

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^2 \times 3^2 \times 7 = 252$$

FINAL ANSWER

252

4MA1/2HR JUNE 2022

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1200 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2022

1 Find the LCM

$$1200 = 12 \times 100$$

$$1200 = (2^2 \times 3)(2^2 \times 5^2)$$

$$1200 = 2^4 \times 3 \times 5^2$$

FINAL ANSWER

$2^4 \times 3 \times 5^2$

4MA1/1H JUNE 2022

Question 3

4 marks | Prime Factors, HCF & LCM

3 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

.....

(2)

(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2022

1 Find the LCM

For the HCF of 56 and 84:

$$56 = 2^3 \times 7$$

$$84 = 2^2 \times 3 \times 7$$

2 Take the smaller powers

Take the smaller powers:

$$\text{HCF} = 2^2 \times 7 = 28$$

3 Find the LCM

For the LCM of 60 and 72:

$$60 = 2^2 \times 3 \times 5$$

$$72 = 2^3 \times 3^2$$

4 Take the larger powers

Take the larger powers:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

FINAL ANSWER

(a) 28, (b) 360

4MA1/1HR JUNE 2022

Question 12

3 marks | Prime Factors, HCF & LCM

12 $P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(a) Write down the highest common factor (HCF) of P and Q

.....

(1)

$P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(b) Work out the value of $P^3 \times Q$
Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

.....

(2)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR June 2022

1 Find the LCM

$$P = 3^3 \times 5^2 \times 7$$

$$Q = 3^2 \times 5 \times 7^2$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 3^2 \times 5 \times 7$$

3 Find the LCM

For $P^3 \times Q$:

$$P^3 = (3^3 \times 5^2 \times 7)^3 = 3^9 \times 5^6 \times 7^3$$

4 **Find the LCM**

Now multiply by Q:

$$P^3 \times Q = (3^9 \times 5^6 \times 7^3)(3^2 \times 5 \times 7^2)$$

$$P^3 \times Q = 3^{11} \times 5^7 \times 7^5$$

✓ **FINAL ANSWER**

(a) $3^2 \times 5 \times 7$, (b) $3^{11} \times 5^7 \times 7^5$

4MA1/1HR MAY 2023

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 2250 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May 2023

1 Find the LCM

$$2250 = 225 \times 10$$

$$225 = 3^2 \times 5^2, \qquad 10 = 2 \times 5$$

2 Find the LCM

So

$$2250 = 3^2 \times 5^2 \times 2 \times 5$$

$$2250 = 2 \times 3^2 \times 5^3$$

✓ **FINAL ANSWER**

$2 \times 3^2 \times 5^3$

4MA1/2HR JUNE 2023

Question 2

4 marks | Prime Factors, HCF & LCM

2 Sandeep wants to buy some packets of pens and some boxes of pencils for his stationery shop.

Each packet of pens contains 9 pens.
Each box of pencils contains 12 pencils.

Each packet of pens costs £7.60
Each box of pencils costs £4.80

Sandeep can only buy full packets of pens and full boxes of pencils.
He wants to buy exactly the same number of pens as pencils.

Work out the minimum amount Sandeep needs to pay.

£.....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2023

1 Find the LCM

Pens come in packs of 9 and pencils come in boxes of 12.

2 Find the LCM

The smallest equal number of pens and pencils is

$$\text{LCM}(9, 12) = 36$$

3 Find the LCM

So Sandeep buys

$$\text{4 packs of pens}$$

4 Find the LCM

and

$$\text{3 boxes of pencils}$$

5 Cost

Cost:

$$4(7.60) + 3(4.80) = 30.40 + 14.40 = 44.80$$

FINAL ANSWER

\pounds44.80.

4MA1/1H MAY 2023

Question 8

4 marks | Prime Factors, HCF & LCM

8 (a) Write 300 as a product of its prime factors.
Show your working clearly.

.....

(2)

$A = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$B = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

(b) Find the lowest common multiple (LCM) of $5A$ and $7B$
Show your working clearly.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2023

1 **Find the LCM**

First write 300 as a product of prime factors:

$$300 = 3 \times 100 = 3 \times 2^2 \times 5^2$$

$$300 = 2^2 \times 3 \times 5^2$$

2 **Find the LCM**

Now

$$A = 2^3 \times 3^2 \times 5$$

3 **Find the LCM**

and

$$B = 2^2 \times 3^3 \times 5$$

4 **Find the LCM**

So

$$5A = 2^3 \times 3^2 \times 5^2$$

5 **Find the LCM**

and

$$7B = 2^2 \times 3^3 \times 5 \times 7$$

---

6 **Find the LCM**

For the LCM of $5A$ and $7B$, take the highest power of every prime:

$$\text{LCM} = 2^3 \times 3^3 \times 5^2 \times 7$$

✓ **FINAL ANSWER**

(a) $2^2 \times 3 \times 5^2$, (b) $2^3 \times 3^3 \times 5^2 \times 7$

4MA1/1H NOVEMBER 2023

Question 7

2 marks | Prime Factors, HCF & LCM

7 $A = 5^3 \times 7^3 \times 11^6$ and $B = 5^6 \times 7^2 \times 11^4$

Find the highest common factor (HCF) of A and B
Give your answer as a product of powers of its prime factors.

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2023

1 Find the LCM

$$A = 5^3 \times 7^3 \times 11^6$$

$$B = 5^6 \times 7^2 \times 11^4$$

2 The HCF takes the smaller

The HCF takes the smaller power of each common prime:

$$\text{HCF} = 5^3 \times 7^2 \times 11^4$$

FINAL ANSWER

$5^3 \times 7^2 \times 11^4$

4MA1/2HR JUNE 2024

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2024

1 Find the LCM

$$1400 = 14 \times 100$$

$$1400 = (2 \times 7)(2^2 \times 5^2)$$

$$1400 = 2^3 \times 5^2 \times 7$$

FINAL ANSWER

$2^3 \times 5^2 \times 7$

4MA1/1H MAY 2024

Question 3

2 marks | Prime Factors, HCF & LCM

3 Find the highest common factor (HCF) of 72 and 108
Show your working clearly.

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2024

1 Find the LCM

$$72 = 2^3 \times 3^2$$

$$108 = 2^2 \times 3^3$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 2^2 \times 3^2$$

$$2^2 \times 3^2 = 4 \times 9 = 36$$

✓ **FINAL ANSWER**

36

4MA1/1HR MAY 2024

Question 11

4 marks | Prime Factors, HCF & LCM

11 $A = 2^5 \times 5 \times 7^2$

$B = 2^3 \times 5^3 \times 7^4$

(a) Write down the highest common factor (HCF) of $5A$ and $2B$
Give your answer as a product of prime factors.

.....

(2)

$A = 2^5 \times 5 \times 7^2$

$B = 2^3 \times 5^3 \times 7^4$

(b) Work out the value of $(AB)^2$
Give your answer as a product of prime factors.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR May 2024

1 Find the LCM

$$A = 2^5 \times 5 \times 7^2$$

$$B = 2^3 \times 5^3 \times 7^4$$

2 Find the LCM

For part (a),

$$5A = 2^5 \times 5^2 \times 7^2$$

$$2B = 2^4 \times 5^3 \times 7^4$$

3 The HCF takes the smaller

The HCF takes the smaller powers:

$$\text{HCF} = 2^4 \times 5^2 \times 7^2$$

4 **Find the LCM**

For part (b),

$$AB = 2^{5+3} \times 5^{1+3} \times 7^{2+4}$$

$$AB = 2^{8} \times 5^{4} \times 7^{6}$$

---

5 **Find the LCM**

Therefore

$$(AB)^2 = 2^{16} \times 5^{8} \times 7^{12}$$

✓ **FINAL ANSWER**

(a) $2^4 \times 5^2 \times 7^2$, (b) $2^{16} \times 5^8 \times 7^{12}$

4MA1/1H NOVEMBER 2024

Question 6

2 marks | Prime Factors, HCF & LCM

6 $A = 2^3 \times 5^4 \times 7 \times 11$

$B = 2^2 \times 5^2 \times 7^2$

$C = 2^2 \times 5^3 \times 7^4$

Find the highest common factor (HCF) of A, B and C
Write your answer as a product of prime factors.

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2024

1 Find the LCM

$$A = 2^3 \times 5^4 \times 7 \times 11$$

$$B = 2^2 \times 5^2 \times 7^2$$

$$C = 2^2 \times 5^3 \times 7^4$$

2 Find the LCM

The HCF of A, B, and C uses the smallest power of each prime common to all three.

$$2^2, \quad 5^2, \quad 7$$

3 Find the LCM

The prime 11 is not in all three numbers, so it is not included.

$$\text{HCF} = 2^2 \times 5^2 \times 7$$

✓ **FINAL ANSWER**

$2^2 \times 5^2 \times 7$

4MA1/1HR JUNE 2025

Question 1

2 marks | Prime Factors, HCF & LCM

1 Find the lowest common multiple (LCM) of 45 and 70

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2025

1 **Find the LCM**

$$45 = 3^2 \times 5$$

$$70 = 2 \times 5 \times 7$$

2 **Find the LCM**

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2 \times 3^2 \times 5 \times 7 = 630$$

✓ **FINAL ANSWER**

630

4MA1/2HR JUNE 2025

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2025

1 Find the LCM

$$1400 = 14 \times 100$$

$$1400 = (2 \times 7)(2^2 \times 5^2)$$

$$1400 = 2^3 \times 5^2 \times 7$$

FINAL ANSWER

$2^3 \times 5^2 \times 7$

4WM2/01R MAY 2025

Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM2/01R May 2025

1 Required

Write 1400 as a product of powers of its prime factors.

2 Divide repeatedly by the smallest

Divide repeatedly by the smallest prime factor each time:

$$1400;\ xrightarrowdiv2;\ 700;\ xrightarrowdiv2;\ 350;\ xrightarrowdiv2;\ 175;\ xrightarrowdiv5;\ 35;\ xrightarrowdiv5;\ 7;\ xrightarrowdiv7;\ 1$$

3 Collect the prime factors

Collect the prime factors:

$$1400 = underbrace2times2times2_2^3timesunderbrace5times5_5^2times7$$

FINAL ANSWER

1400 = 2³ × 5² × 7

4WM2/01 MAY 2025

Question 17

4 marks | Prime Factors, HCF & LCM

17 $A = 3^2 \times 5^4$ $B = 2^5 \times 3^2$ $C = 2^3 \times 5^6$

(a) Work out the value of $(ABC)^3$

Give your answer as a product of powers of its prime factors.

.....

(2)

(b) Work out $(5 \times 10^{150}) \times (3 \times 10^{140})$

Give your answer as a product of powers of its prime factors.

.....

(2)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM2/01 May 2025

1 **(a)**

(a)

$$ABC = (3^2 \times 5^4)(2^5 \times 3^2)(2^3 \times 5^6)$$

$$= 2^8 \times 3^4 \times 5^{10}$$

$$(ABC)^3 = 2^{24} \times 3^{12} \times 5^{30}$$

2 **(b)**

(b)

$$(5 \times 10^{150})(3 \times 10^{140}) = 15 \times 10^{290}$$

$$= 3 \times 5 \times (2 \times 5)^{290}$$

$$= 2^{290} \times 3 \times 5^{291}$$

FINAL ANSWER

$2^{24} \times 3^{12} \times 5^{30}$; $2^{290} \times 3 \times 5^{291}$.

4MA1/2H JUNE 2026

Question 11

2 marks | Prime Factors, HCF & LCM

11 Find the highest prime number that is a factor of 3.6×10^{24}

.....

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2026

1 **Solution | Step 1**

$$3.6 \times 10^{24} = 36 \times 10^{23} = (2^2 \times 3^2)(2 \times 5)^{23} = 2^{25}3^2 5^{23}.$$

Rewrite with an integer coefficient, then decompose 36 and 10 into primes.

2 **Solution | Step 2**

The only prime factors are $2, 3, 5$, so the highest is 5.

A larger composite factor is not a prime factor.

✓ **FINAL ANSWER**

5

TOPIC

Percentages

Every past-paper question on this topic, 2020 to 2026.

Question 12

5 marks | Percentages

12 Zimo is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Zimo pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{2}{5}$ of the total cost
Payment 2	45% of the total cost
Payment 3	£405

Work out how much Zimo has to pay for Payment 2

£.....

(Total for Question 12 is 5 marks)

1 **Calculate percentage**

$$\frac{2}{5} = 40\%$$

2 **Calculate percentage**

So Payments 1 and 2 together are

$$40\% + 45\% = 85\%$$

3 **This means Payment 3 is**

This means Payment 3 is

$$100\% - 85\% = 15\%$$

4 **Calculate percentage**

So

$$15\% = 405$$

$$1\% = 27$$

5 **Payment 2 is**

Payment 2 is 45%:

$$45 \times 27 = 1215$$

FINAL ANSWER

$\$1215$.

4MA1/2H JANUARY 2021

Question 10

3 marks | Percentages

10 The people working for a company work in Team A or in Team B.

number of people in Team A : number of people in Team B = 3 : 4

$\frac{4}{5}$ of Team A work full time.

24% of Team B work full time.

Work out what fraction of the people working for the company work full time.
Give your fraction in its simplest form.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2021

1 The ratio of Team A

The ratio of Team A to Team B is $3 : 4$.

2 Use parts for Team A

Use 3 parts for Team A and 4 parts for Team B.

3 Full time part from Team

Full-time part from Team A:

$$\frac{4}{5} \times 3 = \frac{12}{5}$$

4 Full time part from Team

Full-time part from Team B:

$$24\% \times 4 = \frac{24}{100} \times 4 = \frac{24}{25}$$

5 Total full time parts

Total full-time parts:

$$\frac{12}{5} + \frac{24}{25} = \frac{60}{25} + \frac{24}{25} = \frac{84}{25}$$

6 **Total parts**

Total parts:

$$3+4=7$$

7 **Simplify fraction**

So the fraction who work full time is

$$\frac{\frac{84}{25}}{7}=\frac{84}{175}=\frac{12}{25}$$

✓ **FINAL ANSWER**

$$\frac{12}{25}$$

4MA1/1H NOVEMBER 2021

Question 8

3 marks | Percentages

8 In 2018, the population of Sydney was 5.48 million.
This was 22% of the total population of Australia.

Work out the total population of Australia in 2018
Give your answer correct to 3 significant figures.

..... million

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2021

1 Calculate percentage

Sydney's population was 22% of Australia's total population.

$$0.22 \times \text{total} = 5.48$$

$$\text{total} = \frac{5.48}{0.22} = 24.909\ldots$$

2 Calculate percentage

Correct to 3 significant figures,

$$24.909\ldots \approx 24.9$$

FINAL ANSWER

24.9 million

4MA1/1HR JUNE 2022

Question 4

4 marks | Percentages

4 The language department of a college has 180 students.
Each student studies exactly one of French, German, Italian or Spanish.

15 students study French.
45% of the students study German.

Express the percentage of students studying Italian or Spanish as a percentage of those studying French or German.

.....%

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2022

1 Total students

Total students:

$$180$$

2 French

French:

$$15$$

3 German

German:

$$45\% \times 180 = 81$$

4 French or German

French or German:

$$15 + 81 = 96$$

5 Italian or Spanish

Italian or Spanish:

$$180 - 96 = 84$$

6 **Convert standard form**

Write 84 as a percentage of 96:

$$\frac{84}{96} \times 100 = 87.5$$

✓ **FINAL ANSWER**

87.5%

4MA1/2H JUNE 2022

Question 8

5 marks | Percentages

8 Behnaz makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7:5:3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.
36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2022

1 The ratio has parts

The ratio $7 : 5 : 3$ has 15 parts.

2 Birthday cards

Birthday cards:

$$\frac{7}{15} \times 300 = 140$$

3 Anniversary cards

Anniversary cards:

$$\frac{5}{15} \times 300 = 100$$

4 Congratulations cards

Congratulations cards:

$$\frac{3}{15} \times 300 = 60$$

5 Birthday cards with numbers

Birthday cards with numbers:

$$\frac{2}{5} \times 140 = 56$$

6 **Anniversary cards with numbers**

Anniversary cards with numbers:

$$36\% \times 100 = 36$$

7 **Congratulations cards with numbers**

Congratulations cards with numbers:

$$0$$

8 **Total with numbers**

Total with numbers:

$$56 + 36 = 92$$

9 **Fraction of all cards**

Fraction of all cards:

$$\frac{92}{300} = \frac{23}{75}$$

✓ **FINAL ANSWER**

$$\frac{23}{75}$$

4MA1/1H JUNE 2022

Question 13

5 marks | Percentages

13 Antoine is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Antoine pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{3}{8}$ of the total cost
Payment 2	45% of the total cost
Payment 3	\$406

Work out how much Antoine has to pay for Payment 2

\$ .....

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2022

1 **Payment 1 is**

Payment 1 is

$$\frac{3}{8}$$

2 **Payment 2 is**

Payment 2 is

$$45\% = \frac{45}{100} = \frac{9}{20}$$

3 **Together**

Together:

$$\frac{3}{8} + \frac{9}{20} = \frac{15}{40} + \frac{18}{40} = \frac{33}{40}$$

4 **Calculate percentage**

So Payment 3 is

$$1 - \frac{33}{40} = \frac{7}{40}$$

5 **Calculate percentage**

Payment 3 is $406, so

$$\frac{7}{40} \times \text{total} = 406$$

$$\text{total} = 406 \times \frac{40}{7} = 2320$$

6 **Payment 2 is**

Payment 2 is

$$45\% \times 2320 = 1044$$

✓ **FINAL ANSWER**

$1044

4MA1/2H JUNE 2024

Question 8

5 marks | Percentages

8 Ishir plants 600 bulbs in a garden.
He plants tulip bulbs, crocus bulbs and daffodil bulbs so that

number of tulip bulbs : number of crocus bulbs : number of daffodil bulbs = 9 : 4 : 2

45% of the tulip bulbs are for yellow flowers.

$\frac{5}{8}$ of the crocus bulbs are for yellow flowers.

All of the daffodil bulbs are for yellow flowers.

Work out the number of bulbs that are for yellow flowers.

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2024

1 The ratio parts are

The ratio parts are

$$9 + 4 + 2 = 15$$

$$\text{tulips} = 600 \times \frac{9}{15} = 360, \quad \text{crocuses} = 600 \times \frac{4}{15} = 160, \quad \text{daffodils} = 600 \times \frac{2}{15} = 80$$

2 Yellow tulips

Yellow tulips:

$$45\% \times 360 = 0.45 \times 360 = 162$$

3 Yellow crocuses

Yellow crocuses:

$$\frac{5}{8} \times 160 = 100$$

4 All the daffodil bulbs are

All the daffodil bulbs are yellow, so

$$162 + 100 + 80 = 342$$

FINAL ANSWER

342 bulbs

4MA1/1H NOVEMBER 2024

Question 7

4 marks | Percentages

7 Shop **A** and Shop **B** have offers for buying the same type of laptop.

The normal price of the laptop in Shop **A** is different to the normal price of the laptop in Shop **B**

Shop A	**Shop B**
Our normal price £475 Get 16% off our normal price	Get 15% off our normal price Only pay £408

Which shop gives more money off their normal price?
Show your working clearly.

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2024

1 Shop A discount

Shop A discount:

$$16\% \text{ of } 475 = 0.16 \times 475 = 76$$

2 Calculate percentage

Shop B: 408 euros is 85% of the normal price.

$$\text{normal price} = 408 \div 0.85 = 480$$

3 Shop B discount

Shop B discount:

$$480 - 408 = 72$$

4 Calculate percentage

Since

$$76 > 72$$

5

Shop A gives the greater

Shop A gives the greater discount.

✓ **FINAL ANSWER**

Shop A gives the greater discount.

4WM2/01 NOVEMBER 2025

Question 4

4 marks | Percentages

4 Pete plants a total of 960 trees in a park.
He plants only apple trees, cherry trees and pear trees such that

number of apple trees : number of cherry trees : number of pear trees = 6 : 7 : 3

65% of the cherry trees will grow morello cherries.

Work out the number of trees that will grow morello cherries.

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4WM2/01 November 2025

1 Split the ratio

Total parts = 6 + 7 + 3 = 16 parts

2 Evaluate fraction

$$1textpart = fracfrac96016 = 60texttrees$$

3 Calculate value

Cherry trees = 7 parts × 60 = 420 trees

4 Calculate percentage

Morello trees = 0.65 × 420 = 273

FINAL ANSWER

273 Always read carefully what the final question asks. A common mistake is stopping at the total number of cherry trees (420) instead of calculating the 65% morello portion.

4MA1/1H NOVEMBER 2025

Question 9

3 marks | Percentages

9 Hari scored 140 points playing a game on Tuesday.
This was 12% more points than he scored playing the game on Monday.

Work out the number of points that Hari scored playing the game on Monday.

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2025

1 Tuesday s score was more

Tuesday's score was 12% more than Monday's score, so Tuesday was 112% of Monday.

$$1.12 \times \text{Monday} = 140$$

$$\text{Monday} = \frac{140}{1.12} = 125$$

FINAL ANSWER

125 points

4MA1/2HR JUNE 2026

Question 27

4 marks | Percentages

27 Bryan owns a house.

During 2024, the value of Bryan's house increased by $x\%$
During 2025, the value of Bryan's house increased by $2x\%$

Overall, during 2024 and 2025, the value of Bryan's house increased by 38.88%

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 27 is 4 marks)

WORKED SOLUTION

Question 27 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\left(1+\frac{x}{100}\right)\left(1+\frac{2x}{100}\right)=1.3888.$$

The second increase acts on the already-increased house value.

2 Solution | Step 2

$$1+\frac{3x}{100}+\frac{2x^2}{10000}=1.3888 \quad \Rightarrow \quad x^2+150x-1944=0.$$

Expand, subtract 1, and multiply by 5000.

3 Solution | Step 3

$$(x-12)(x+162)=0 \quad \Rightarrow \quad x=12 \text{ or } x=-162.$$

Factorize the quadratic.

4 Solution | Step 4

Reject $x=-162$, since both changes are increases, so $x>0$.
The positive root gives multipliers 1.12 and 1.24, whose product is 1.3888.

FINAL ANSWER

$x=12$.

TOPIC

Compound Interest & Depreciation

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 8

6 marks | Compound Interest & Depreciation

8 In 2021, the value of Asha's apartment was 634 400 euros.
The value of Asha's apartment had increased by 4% from its value in 2020

(a) Work out the value of Asha's apartment in 2020

..... euros
(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

..... %
(3)

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4WM2/01 Specimen

1 **Use compound interest**

For part (a), the 2021 value is 104% of the 2020 value.

$$\text{2020 value} = \frac{634400}{1.04} = 610000$$

2 **Use compound interest**

For part (b), after one year the multiplier is 0.85. After two years:

$$0.85^2 = 0.7225$$

3 **Use compound interest**

So 72.25% of the value remains, and the total depreciation is

$$100\% - 72.25\% = 27.75\%$$

✓ **FINAL ANSWER**

610000 euros; 27.75%.

4MA1/2HR JANUARY 2020

Question 8

3 marks | Compound Interest & Depreciation

8 Hamish buys a new car for $20 000
The car depreciates in value by 19% each year.

Work out the value of the car at the end of 3 years.
Give your answer to the nearest $.

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2020

1 A depreciation of gives a

A depreciation of 19% gives a multiplier of

$$1 - 0.19 = 0.81$$

2 Use compound interest

After 3 years,

$$20000(0.81)^3 = 10628.82$$

3 Use compound interest

Correct to the nearest dollar,

$$10628.82 \approx 10629$$

✓ **FINAL ANSWER**

$10629

4MA1/1H JANUARY 2020

Question 11

3 marks | Compound Interest & Depreciation

11 Max invests \$6000 in a savings account for 3 years.
The account pays compound interest at a rate of 1.5% per year for the first 2 years.

The compound interest rate changes for the third year.
At the end of 3 years, there is a total of \$6311.16 in the account.

Work out the compound interest rate for the third year.
Give your answer correct to 1 decimal place.

..... %

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H January 2020

1 **Use compound interest**

After the first 2 years,

$$6000(1.015)^2 = 6181.35$$

2 **Let the third year interest**

Let the third-year interest rate be $r\%$.

$$6181.35\left(1+\frac{r}{100}\right)=6311.16$$

$$1+\frac{r}{100}=\frac{6311.16}{6181.35}=1.021000\ldots$$

$$r=2.1000\ldots$$

✓ **FINAL ANSWER**

2.1%

4MA1/1HR MAY/NOVEMBER 2020

Question 8

3 marks | Compound Interest & Depreciation

8 Harold bought an antique clock for £1200
The clock increased in value by 8% per year.

Find the value of the clock exactly 3 years after Harold bought the clock.
Give your answer correct to the nearest £.

£.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May/November 2020

1 The multiplier for an increase

The multiplier for an 8% increase is

$$1.08$$

2 Value after 3 years

Value after 3 years:

$$1200(1.08)^3 = 1511.6544$$

3 Use compound interest

Correct to the nearest pound,

$$1511.6544 \approx 1512$$

FINAL ANSWER

1512 pounds

4MA1/1H MAY/NOVEMBER 2020

Question 13

3 marks | Compound Interest & Depreciation

13 Jan invests \$8000 in a savings account.
The account pays compound interest at a rate of $x\%$ per year.

At the end of 6 years, there is a total of \$8877.62 in the account.

Work out the value of x.
Give your answer correct to 2 decimal places.

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May/November 2020

1 Let the compound interest rate

Let the compound interest rate be $x\%$.

$$8000\left(1+\frac{x}{100}\right)^6 = 8877.62$$

$$1+\frac{x}{100} = \sqrt[6]{\frac{8877.62}{8000}} = 1.017500\ldots$$

$$x = 1.7500\ldots$$

2 Use compound interest

Correct to 2 decimal places,

$$x = 1.75$$

FINAL ANSWER

$x = 1.75$

4MA1/1H JANUARY 2021

Question 7

6 marks | Compound Interest & Depreciation

7 Chen invests 40 000 yuan in a fixed-term bond for 3 years.

The fixed-term bond pays compound interest at a rate of 3.5% each year.

(a) Work out the value of Chen's investment at the end of 3 years.
Give your answer to the nearest yuan.

..... yuan

(3)

Wang invested P yuan.
The value of his investment decreased by 6.5% each year.

At the end of the first year, the value of Wang's investment was 30 481 yuan.

(b) Work out the value of P.

$P =$ .....

(3)

(Total for Question 7 is 6 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2021

1 Use compound interest

For part (a),

$$40000(1.035)^3 = 44348.715$$

2 Use compound interest

Correct to the nearest yuan,

$$44348.715 \approx 44349$$

3 Use compound interest

For part (b), after one year the value is 93.5% of P.

$$0.935P = 30481$$

$$P = \frac{30481}{0.935} = 32600$$

FINAL ANSWER

(a) $\mathbf{44349}$ yuan, (b) $\boldsymbol{P = 32600}$

4MA1/1HR JANUARY 2021

Question 7

3 marks | Compound Interest & Depreciation

7 Kuro invests 50 000 yen for 3 years in a savings account.
She gets 2.4% per year compound interest.

Work out how much money Kuro will have in her savings account at the end of the 3 years.
Give your answer correct to the nearest yen.

..... yen

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2021

1 The multiplier for a increase

The multiplier for a 2.4% increase is

$$1.024$$

2 Use compound interest

After 3 years,

$$50000(1.024)^3 = 53687.0912$$

3 Use compound interest

Correct to the nearest yen,

$$53687.0912 \approx 53687$$

✓ **FINAL ANSWER**

53687 yen

4MA1/2HR JANUARY 2021

Question 14

4 marks | Compound Interest & Depreciation

14 Simon bought a house at the beginning of 2018
The value of Simon's house had decreased by 15% by the end of 2018

The house increased in value during both 2019 and 2020
The percentage increases in the value of the house during 2019 and 2020 were the same.

The value of Simon's house at the end of 2020 was 2.85% greater than the amount he paid for his house at the beginning of 2018

Calculate the percentage increase in the value of the house during 2019

.....%

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR January 2021

1 **Let the original house value**

Let the original house value be V.

2 **Use compound interest**

After the 15% decrease in 2018:

$$0.85V$$

3 **Use compound interest**

Let the percentage increase in each of 2019 and 2020 be r.

4 **The final value is greater**

The final value is 2.85% greater than the original value:

$$1.0285V$$

5 **Use compound interest**

So

$$0.85V(1+r)^2 = 1.0285V$$

$$(1+r)^2 = \frac{1.0285}{0.85} = 1.21$$

$$1+r = \sqrt{1.21} = 1.1$$

$$r = 0.1 = 10\%$$

✓ **FINAL ANSWER**

10%

4MA1/2H MAY 2021

Question 11

6 marks | Compound Interest & Depreciation

11 Chengbo sold a house for 180 000 yuan.

The amount for which he sold the house is 24% more than the amount he paid for the house.

(a) Work out how much Chengbo paid for the house.
Give your answer correct to 3 significant figures.

..... yuan
(3)

Zhi bought a house on 1st January 2017
When she bought the house, its value was 120 000 yuan.

The value of the house increased by 1.8% per year.

(b) Work out the value of Zhi's house on 1st January 2020
Give your answer correct to 3 significant figures.

..... yuan
(3)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H May 2021

1 **Use compound interest**

For Chengbo's house, 180000 yuan is 124% of the price paid.

$$\text{price paid} = 180000 \div 1.24 = 145161.29\ldots$$

2 **To 3 significant figures**

To 3 significant figures:

$$145000$$

3 **Use compound interest**

For Zhi's house, the value increases by 1.8% each year for 3 years.

$$120000 \times 1.018^3 = 126598.2\ldots$$

4 **To 3 significant figures**

To 3 significant figures:

$$127000$$

✓ **FINAL ANSWER**

Chengbo paid 145000 yuan. Zhi's house was worth 127000 yuan.

4MA1/2H NOVEMBER 2021

Question 7

4 marks | Compound Interest & Depreciation

7 Ali and Badia each have 25 000 dollars to invest.

Cyclone Bank	Tornado Bank
Invest 25 000 dollars 4.5% compound interest per year for 3 years	Invest 25 000 dollars Receive 1150 dollars interest each year for 3 years

Ali invests in the Cyclone Bank for 3 years.
Badia invests in the Tornado Bank for 3 years.

By the end of the 3 years, Ali will have received more interest than Badia.

How much more?
Show your working clearly.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2021

1 Ali s interest from Cyclone

Ali's interest from Cyclone Bank is

$$25000(1.045)^3 - 25000 = 3529.153\ldots$$

2 Badia s interest from Tornado

Badia's interest from Tornado Bank is

$$1150 \times 3 = 3450$$

3 Difference

Difference:

$$3529.153\ldots - 3450 = 79.153\ldots$$

4 Use compound interest

Correct to the nearest dollar,

$$79.153\ldots \approx 79$$

FINAL ANSWER

$79

4MA1/2HR JANUARY 2022

Question 5

3 marks | Compound Interest & Depreciation

5 Shane invests 7200 dollars for 3 years in a savings account.
He gets 2.5% per year compound interest.

How much money will Shane have in his savings account at the end of 3 years?
Give your answer to the nearest dollar.

..... dollars

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2022

1 The multiplier for compound interest

The multiplier for 2.5% compound interest is

$$1.025$$

2 Use compound interest

After 3 years,

$$7200(1.025)^3 = 7753.6125$$

3 Use compound interest

Correct to the nearest dollar,

$$7753.6125 \approx 7754$$

✓ **FINAL ANSWER**

7754 dollars

4MA1/1H JANUARY 2022

Question 11

3 marks | Compound Interest & Depreciation

11 Himari invests 200000 yen for 3 years in a savings account paying compound interest.

The rate of interest is 1.8% for the first year and x% for each of the second year and the third year.

The value of the investment at the end of the third year is 209754 yen.

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H January 2022

1 The first year multiplier is

The first-year multiplier is

$$1.018$$

2 The second and third years

The second and third years both have multiplier $1 + \frac{x}{100}$, so

$$200000(1.018)\left(1 + \frac{x}{100}\right)^2 = 209754$$

$$\left(1 + \frac{x}{100}\right)^2 = \frac{209754}{200000(1.018)}$$

$$1 + \frac{x}{100} = 1.015000\ldots$$

$$x = 1.5000\ldots$$

FINAL ANSWER

$x = 1.5$

4MA1/1H JUNE 2022

Question 6

6 marks | Compound Interest & Depreciation

6 Asha bought an apartment.

The table gives information about the value of apartments, in euros, and the annual service charge band.

Value (x euros)	Service charge band
$x \geqslant 700\,000$	A
$600\,000 \leqslant x < 700\,000$	B
$500\,000 \leqslant x < 600\,000$	C
$400\,000 \leqslant x < 500\,000$	D
$0 < x < 400\,000$	E

In 2021, the value of Asha's apartment was 634 400 euros.

The value of Asha's apartment had increased by 4% from its value in 2020

(a) Has the annual service charge band changed for Asha's apartment?
Show your working clearly.

(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

..... %

(3)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2022

1 Use compound interest

For part (a), the value in 2021 was 4% more than in 2020.

$$\text{2020 value} = \frac{634400}{1.04} = 610000$$

2 **Use compound interest**

In 2020, $600000 < x < 700000$, so the service charge band was B.

3 **Use compound interest**

In 2021, 634400 is also between 600000 and 700000, so the band was still B.

4 **Use compound interest**

For part (b), a depreciation of 15% has multiplier 0.85.

5 **Use compound interest**

After 2 years, the multiplier is

$$0.85^2 = 0.7225$$

6 **Use compound interest**

So the value left is 72.25%, meaning the total depreciation is

$$100\% - 72.25\% = 27.75\%$$

✓ **FINAL ANSWER**

(a) No, it stayed in band B. (b) 27.75%

4MA1/1HR JUNE 2022

Question 8

3 marks | Compound Interest & Depreciation

8 Jane bought a new car for $18 000
The car depreciates in value by 15% each year.

Work out the value of the car at the end of 4 years.
Give your answer correct to the nearest $

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2022

1 A depreciation of gives multiplier

A depreciation of 15% gives multiplier

$$0.85$$

2 Use compound interest

After 4 years,

$$18000(0.85)^4 = 9396.1125$$

3 Use compound interest

Correct to the nearest dollar,

$$9396.1125 \approx 9396$$

✓ **FINAL ANSWER**

$9396

4MA1/2H JUNE 2022

Question 9

3 marks | Compound Interest & Depreciation

9 Pasha invests 50 000 dollars in a savings account for 4 years.
He gets 1.3% per year compound interest.

Work out how much money Pasha will have in his savings account at the end of 4 years.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2022

1 The multiplier for compound interest

The multiplier for 1.3% compound interest is

$$1.013$$

2 Use compound interest

After 4 years,

$$50000(1.013)^4 = 52651.1408\ldots$$

3 Use compound interest

Correct to the nearest dollar,

$$52651.1408\ldots \approx 52651$$

✓ **FINAL ANSWER**

52651 dollars

4MA1/2H JUNE 2023

Question 8

3 marks | Compound Interest & Depreciation

8 Charlotte buys a painting for $680
The value of the painting increases by 4% each year.

Work out the value of the painting at the end of 3 years.
Give your answer correct to the nearest $

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2023

1 A increase each year gives

A 4% increase each year gives a multiplier of

$$1.04$$

2 Use compound interest

After 3 years, the value is

$$680(1.04)^3$$

$$= 680(1.124864)$$

$$= 764.90752$$

3 Correct to the nearest dollar

Correct to the nearest dollar:

$$765$$

✓ **FINAL ANSWER**

$765

4MA1/1HR MAY 2023

Question 9

3 marks | Compound Interest & Depreciation

9 Kazi buys a car for 700 000 taka.
The value of the car depreciates by 12% each year.

Work out the value of the car at the end of 3 years.
Give your answer correct to the nearest taka.

..... taka

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May 2023

1 A depreciation of gives multiplier

A depreciation of 12% gives multiplier

$$0.88$$

2 Use compound interest

After 3 years,

$$700000(0.88)^3 = 477030.4$$

3 Use compound interest

Correct to the nearest taka,

$$477030.4 \approx 477030$$

FINAL ANSWER

477030 taka

4MA1/2HR JUNE 2023

Question 13

3 marks | Compound Interest & Depreciation

13 Feruzi invests 80 000 Kenyan shillings (KES)
He invests the money for 3 years at $x\%$ compound interest each year.

At the end of 3 years, the total interest he receives is 6151.25 KES

Work out the value of x

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2023

1 The final value is

The final value is

$$80000 + 6151.25 = 86151.25$$

2 Let the annual compound interest

Let the annual compound interest rate be $x\%$.

$$80000\left(1 + \frac{x}{100}\right)^3 = 86151.25$$

$$1 + \frac{x}{100} = \sqrt[3]{\frac{86151.25}{80000}} = 1.025$$

$$x = 2.5$$

FINAL ANSWER

$x = 2.5$

4MA1/2H NOVEMBER 2023

Question 7

3 marks | Compound Interest & Depreciation

7 Hermione buys a boat for $26 800
The value of the boat depreciates by 8% each year.

Work out the value of the boat at the end of 3 years.
Give your answer correct to the nearest dollar.

$.....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2023

1 A depreciation of gives multiplier

A depreciation of 8% gives multiplier

$$0.92$$

2 Use compound interest

After 3 years,

$$26800(0.92)^3 = 20868.8384$$

3 Use compound interest

Correct to the nearest dollar,

$$20868.8384 \approx 20869$$

✓ **FINAL ANSWER**

$20869

4MA1/1HR MAY 2024

Question 5

3 marks | Compound Interest & Depreciation

5 Slavomir invests 5200 euros in a savings account for 4 years.
He gets 2.5% per year compound interest.

Work out how much money Slavomir will have in the savings account at the end of 4 years.
Give your answer correct to the nearest euro.

..... euros

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May 2024

1 The multiplier for compound interest

The multiplier for 2.5% compound interest is

$$1.025$$

2 Use compound interest

After 4 years,

$$5200(1.025)^4 = 5739.827\ldots$$

3 Use compound interest

Correct to the nearest euro,

$$5739.827\ldots \approx 5740$$

FINAL ANSWER

5740 euros

4MA1/2H JUNE 2024

Question 9

3 marks | Compound Interest & Depreciation

9 Giovanni invests 4500 koruna in a savings account for 4 years.
He gets 2.4% per year compound interest.

Work out how much money Giovanni will have in the savings account at the end of 4 years.
Give your answer correct to the nearest koruna.

..... koruna

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2024

1 The multiplier for compound interest

The multiplier for 2.4% compound interest is

$$1.024$$

2 Use compound interest

After 4 years,

$$4500(1.024)^4 = 4947.8023\ldots$$

3 Use compound interest

Correct to the nearest koruna,

$$4947.8023\ldots \approx 4948$$

FINAL ANSWER

4948 koruna

4MA1/2H NOVEMBER 2024

Question 9

3 marks | Compound Interest & Depreciation

9 Nisha invests 20 000 euros for 3 years in a savings account.
She gets 3.5% per year compound interest.

Work out how much money Nisha will have in her savings account at the end of the 3 years.
Give your answer correct to the nearest euro.

..... euros

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2024

1 The multiplier for compound interest

The multiplier for 3.5% compound interest is

$$1.035$$

2 Use compound interest

After 3 years,

$$20000(1.035)^3 = 22174.3575$$

3 Use compound interest

Correct to the nearest euro,

$$22174.3575 \approx 22174$$

FINAL ANSWER

22174 euros

4MA1/2H JUNE 2025

Question 7

3 marks | Compound Interest & Depreciation

7 Tim buys a bracelet for 4000 Swiss francs.
The value of the bracelet increases by 7% each year.

Work out the value of the bracelet at the end of 3 years.
Give your answer correct to the nearest Swiss franc.

..... Swiss francs

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2025

1 The multiplier for a increase

The multiplier for a 7% increase is

$$1.07$$

2 Use compound interest

After 3 years,

$$4000(1.07)^3 = 4900.172$$

3 Correct to the nearest Swiss

Correct to the nearest Swiss franc,

$$4900.172 \approx 4900$$

✓ FINAL ANSWER

4900 Swiss francs

4WM2/01 MAY 2025

Question 9

3 marks | Compound Interest & Depreciation

9 Tim buys a bracelet for 4000 Swiss francs.
The value of the bracelet increases by 7% each year.

Work out the value of the bracelet at the end of 3 years.
Give your answer correct to the nearest Swiss franc.

..... Swiss francs

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 May 2025

1 The multiplier for a increase

The multiplier for a 7% increase is

$$1.07$$

2 Use compound interest

After 3 years,

$$4000(1.07)^3 = 4900.172$$

3 Correct to the nearest Swiss

Correct to the nearest Swiss franc,

$$4900.172 \approx 4900$$

FINAL ANSWER

4900 Swiss francs

4MA1/1HR JUNE 2025

Question 11

5 marks | Compound Interest & Depreciation

11 Zhou invests some money for 2 years.
He invests \$2500 with Bank A and \$3000 with Bank B.

Bank A
Invests \$2500
amount of money invested : total amount of interest after 2 years = 20 : 3

Bank B
Invests \$3000
4% per year compound interest for 2 years

Zhou receives more **interest** from Bank A than from Bank B.

How much more?

\$ .....

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR June 2025

1 **Use compound interest**

For Bank A,

$$\text{amount invested} : \text{interest} = 20 : 3$$

2 **Use compound interest**

So the interest from Bank A is

$$2500 \times \frac{3}{20} = 375$$

3 Use compound interest

For Bank B, the compound interest after 2 years is

$$3000(1.04)^2 - 3000$$

$$= 3000(1.0816) - 3000 = 244.80$$

4 Difference

Difference:

$$375 - 244.80 = 130.20$$

FINAL ANSWER

$130.20

4MA1/2HR JUNE 2025

Question 12

3 marks | Compound Interest & Depreciation

12 Osman buys a car for \$16 000

The car depreciates at a rate of 12% each year for the first 2 years.
In the third year, the car depreciates at a rate of $x\%$

At the end of 3 years, the value of the car is \$11 461.12

Work out the value of x

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2025

1 Use compound interest

After the first 2 years,

$$16000(0.88)^2 = 12390.40$$

2 Use compound interest

In the third year, the value becomes 11461.12, so

$$12390.40\left(1 - \frac{x}{100}\right) = 11461.12$$

$$1 - \frac{x}{100} = \frac{11461.12}{12390.40} = 0.925$$

$$x = 7.5$$

FINAL ANSWER

$x = 7.5$

4WM2/01R MAY 2025

Question 12

3 marks | Compound Interest & Depreciation

12 Osman buys a car for $16 000

The car depreciates at a rate of 12% each year for the first 2 years.
In the third year, the car depreciates at a rate of $x\%$

At the end of 3 years, the value of the car is $11 461.12

Work out the value of x

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01R May 2025

1 Required

Car bought for $16\,000; depreciates 12 11\,461.12$. Find *x*.

2 Value after 2 years at

Value after 2 years at 12% depreciation:

*V*₂ = 16000 × (0.88)² = 16000 × 0.7744 = $12 390.40

3 Use compound interest

Apply year-3 depreciation and set equal to final value:

$$12390.40 times ! left(1 - frac x 100 right) = 11461.12$$

4 Solve

Solve:

$$1 - frac x 100 = frac 11461.12 12390.40 = 0.925 implies frac x 100 = 0.075 implies x = 7.5$$

✓ FINAL ANSWER

x = 7.5

4MA1/1H JUNE 2025

Question 13

3 marks | Compound Interest & Depreciation

13 Charlotte bought an apartment for \$750 000

In the first year, the value of the apartment decreased by 4%
In the second year, the value of the apartment decreased by 6.5%

In the third year, the value of the apartment increased by $x\%$

At the end of the third year, the value of Charlotte's apartment was \$698 445

Work out the value of x

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2025

1 **Use compound interest**

After the first two years, the value is

$$750000(0.96)(0.935) = 673200$$

2 **Use compound interest**

Then the value increases by $x\%$ to 698445, so

$$673200\left(1+\frac{x}{100}\right) = 698445$$

$$1+\frac{x}{100} = \frac{698445}{673200} = 1.0375$$

$$x = 3.75$$

FINAL ANSWER

$x = 3.75$

4WM2/01 MAY 2025

Question 15

3 marks | Compound Interest & Depreciation

15 Fred buys a painting.

The value of the painting increases by 12% during the first year.
The value of the painting then decreases by 15% during the second year.

Work out the overall percentage decrease in the value of the painting during this two-year period.

.....%

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM2/01 May 2025

1 Use multipliers

Use multipliers:

$$1.12 \times 0.85 = 0.952$$

2 Calculate percentage

So the final value is 95.2% of the original.

$$100 - 95.2 = 4.8$$

FINAL ANSWER

4.8% decrease.

4MA1/2H NOVEMBER 2025

Question 5

6 marks | Compound Interest & Depreciation

5 Otis sells ice creams.

On Friday, Otis sells 75 ice creams.
On Saturday, Otis sells 87 ice creams.

(a) Work out the percentage increase in the number of ice creams Otis sells from Friday to Saturday.

.....%
(3)

Claudia buys an ice cream machine for 960 Swiss francs.

The value of the ice cream machine depreciates by 20% each year.

(b) Work out the value of the ice cream machine at the end of 3 years.

..... Swiss francs
(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2025

1 Use compound interest

For part (a), the increase is

$$87 - 75 = 12$$

2 Percentage increase

Percentage increase:

$$\frac{12}{75} \times 100 = 16$$

3 Use compound interest

For part (b), a depreciation of 20% gives multiplier

$$0.80$$

4 **Use compound interest**

After 3 years,

$$960(0.80)^3 = 491.52$$

✓ **FINAL ANSWER**

(a) 16%, (b) 491.52 Swiss francs

4WM2/01 NOVEMBER 2025

Question 6

6 marks | Compound Interest & Depreciation

6 Otis sells ice creams.

On Friday, Otis sells 75 ice creams.
On Saturday, Otis sells 87 ice creams.

(a) Work out the percentage increase in the number of ice creams Otis sells from Friday to Saturday.

..... %
(3)

Claudia buys an ice cream machine for 960 Swiss francs.

The value of the ice cream machine depreciates by 20% each year.

(b) Work out the value of the ice cream machine at the end of 3 years.

..... Swiss francs
(3)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 November 2025

1 Calculate value

Increase = 87 − 75 = 12

2 Convert standard form

Extra open brace or missing close brace

3 Answer

16%

Step 1: Determine the depreciation multiplier. A 20% decrease means the multiplier is 1 − 0.20 = 0.80.

4 Solve inequality

Final Value = 960 × (0.80)³

Final Value = 960 × 0.512 = 491.52

FINAL ANSWER

491.52 Swiss francs

4WM2/01 NOVEMBER 2025

Question 12

3 marks | Compound Interest & Depreciation

12 The value of a house

- decreased by 5% from the end of 2021 to the end of 2022
- increased by 20% from the end of 2022 to the end of 2023
- increased by 6.5% from the end of 2023 to the end of 2024

Calculate the percentage by which the value of the house increased from the end of 2021 to the end of 2024

..... %

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01 November 2025

1 Calculate percentage

Decrease by 5% $\Rightarrow$ Multiplier = 1 − 0.05 = 0.95

Increase by 20% $\Rightarrow$ Multiplier = 1 + 0.20 = 1.20

Increase by 6.5% $\Rightarrow$ Multiplier = 1 + 0.065 = 1.065

2 Calculate value

Overall Multiplier = 0.95 × 1.20 × 1.065 = 1.2141

3 Calculate percentage

1.2141 − 1 = 0.2141 = 21.41%

✓ **FINAL ANSWER**

21.41%

4MA1/1HR JUNE 2026

Question 10

3 marks | Compound Interest & Depreciation

10 Arwa invests 200 000 yen in a savings account for 3 years.
She gets 1.4% per year compound interest.

Work out how much money Arwa will have in her savings account at the end of 3 years.
Give your answer correct to the nearest yen.

..... yen

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2026

1 Solution | Step 1

$$\text{annual multiplier} = 1 + \frac{1.4}{100} = 1.014.$$

An increase of 1.4% retains the original amount and adds 1.4%.

2 Solution | Step 2

$$A = 200000(1.014)^3 = 208518.1488 \text{ yen}.$$

Apply the multiplier once for each of the 3 years.

3 Solution | Step 3

$$A \approx 208518 \text{ yen}.$$

Round to the nearest whole yen.

FINAL ANSWER

208518 yen

4MA1/2H JUNE 2026

Question 10

4 marks | Compound Interest & Depreciation

10 Johan owns a car.

The current value of the car is 32 000 euros.
The value of the car will now depreciate by 15% per year.

Lena is going to save up and buy Johan's car in 3 years' time.
She has already saved 2516 euros.

She will save an equal amount of money each month for the next 3 years.

Work out how much money Lena needs to save each month.

..... euros

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2026

1 Solution | Step 1

The annual multiplier is $1 - 0.15 = 0.85$. After 3 years the value is

$$32000(0.85)^3 = 19652 \text{ euros}.$$

Apply the decrease to each successive year, giving a power of 3.

2 Solution | Step 2

Lena still needs $19652 - 2516 = 17136$ euros. There are $3 \times 12 = 36$ months.

Subtract her existing savings and convert years to months.

3 Solution | Step 3

$$\text{Monthly saving} = \frac{17136}{36} = 476 \text{ euros}.$$

Divide the remaining target equally over the months.

FINAL ANSWER

476 euros per month

4MA1/1H JUNE 2026

Question 14

3 marks | Compound Interest & Depreciation

14 In September, the sales in a shop increased by 20%
In October, the sales in the same shop decreased by x%

Over these two months, the sales in the shop decreased by 10%

Work out the value of x

$x =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2026

1 **Solution | Step 1**

Let the original sales be 100 units. After September they are $100 \times 1.20 = 120$; after both months they are $100 \times 0.90 = 90$.
Both overall percentages refer to the original sales.

2 **Solution | Step 2**

$$x = \frac{120 - 90}{120} \times 100 = 25.$$

The October decrease is measured relative to the September sales of 120.

✓ **FINAL ANSWER**

$x = 25$

TOPIC

Reverse Percentages

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 7

3 marks | Reverse Percentages

7 In a sale, the normal price of a hat is reduced by 15%
The sale price of the hat is 20.40 euros.

Work out the normal price of the hat.

..... euros

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2020

1 The hat is reduced by

The hat is reduced by 15%, so the sale price is

$$100\% - 15\% = 85\%$$

2 of the normal price

of the normal price.

$$0.85 \times \text{normal price} = 20.40$$

$$\text{normal price} = 20.40 \div 0.85 = 24$$

FINAL ANSWER

24 euros.

4MA1/2H JANUARY 2020

Question 8

7 marks | Reverse Percentages

8 The table shows the cost, in euros, of Brigitte's car insurance in each of the years 2016, 2017 and 2018

Year	2016	2017	2018
Cost of insurance (euros)	500	545	592

Brigitte says,

"The percentage increase in the cost of my car insurance from 2017 to 2018 is more than the percentage increase in the cost of my car insurance from 2016 to 2017"

(a) Is Brigitte correct?
You must show how you get your answer.

(4)

Henri wants to insure his car.

He gets a discount of 15% off the normal price.
Henri pays 952 euros for his car insurance after the discount.

(b) Work out the discount that Henri gets.

..... euros

(3)

(Total for Question 8 is 7 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2020

1 **Convert standard form**

From 2016 to 2017, the increase is

$$545 - 500 = 45$$

$$\frac{45}{500} \times 100 = 9\%$$

2 **Convert standard form**

From 2017 to 2018, the increase is

$$592 - 545 = 47$$

$$\frac{47}{545} \times 100 \approx 8.62\%$$

3 **Calculate percentage**

So Brigitte is not correct because the second percentage increase is smaller.

4 **Calculate percentage**

For Henri, 952 is 85% of the original price because there is a 15% discount.

$$\text{original price} = 952 \div 0.85 = 1120$$

5 **The discount is**

The discount is

$$1120 - 952 = 168$$

✓ **FINAL ANSWER**

Brigitte is not correct; the discount is 168 euros.

4MA1/1HR MAY/NOVEMBER 2020

Question 5

6 marks | Reverse Percentages

5 In 2017, the population of a village was 7500
In 2019, the population of the village was 8265

(a) Work out the percentage increase in the population of the village from 2017 to 2019

..... %

(3)

In a sale, normal prices are reduced by 30%
The sale price of a T-shirt was 31.50 euros.

(b) Work out the normal price of the T-shirt.

..... euros

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May/November 2020

1 The population increase is

The population increase is

$$8265 - 7500 = 765$$

2 Percentage increase

Percentage increase:

$$\frac{765}{7500} \times 100 = 10.2\%$$

3 The T shirt sale price

The T-shirt sale price is after a 30% reduction, so it is 70% of the normal price.

$$0.70 \times \text{normal price} = 31.50$$

$$\text{normal price} = 31.50 \div 0.70 = 45$$

FINAL ANSWER

(a) 10.2%, (b) 45 euros.

4MA1/1H MAY/NOVEMBER 2020

Question 9

3 marks | Reverse Percentages

9 In a sale, normal prices are reduced by 20%

A bag costs 1080 rupees in the sale.

Work out the normal price of the bag.

..... rupees

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May/November 2020

1 A reduction of means the

A reduction of 20% means the sale price is 80% of the normal price.

$$0.80 \times \text{normal price} = 1080$$

$$\text{normal price} = \frac{1080}{0.80} = 1350$$

FINAL ANSWER

1350 rupees

4MA1/1HR MAY/NOVEMBER 2020

Question 10

5 marks | Reverse Percentages

10 Alex makes 80 cakes to sell.
He makes chocolate cakes, lemon cakes and fruit cakes where

$$\text{number of chocolate cakes} : \text{number of lemon cakes} : \text{number of fruit cakes} = 3 : 2 : 5$$

Alex sells

all of the chocolate cakes

$\frac{3}{4}$ of the lemon cakes

$\frac{7}{8}$ of the fruit cakes

The profit he makes on each cake he sells is shown in the table.

Type of cake	Profit per cake he sells
chocolate	£2.00
lemon	£1.70
fruit	£2.40

Work out the total profit that Alex makes from the cakes he sells.

£.....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR May/November 2020

1 **The ratio parts are**

The ratio parts are

$$3 + 2 + 5 = 10$$

2 Numbers of cakes made

Numbers of cakes made:

$$\text{chocolate} = 80 \times \frac{3}{10} = 24$$

$$\text{lemon} = 80 \times \frac{2}{10} = 16$$

$$\text{fruit} = 80 \times \frac{5}{10} = 40$$

3 Cakes sold

Cakes sold:

$$\text{chocolate} = 24, \quad \text{lemon} = \frac{3}{4} \times 16 = 12, \quad \text{fruit} = \frac{7}{8} \times 40 = 35$$

4 Total profit

Total profit:

$$24(2.00) + 12(1.70) + 35(2.40) = 48 + 20.40 + 84 = 152.40$$

FINAL ANSWER

152.40 pounds

4MA1/2H JANUARY 2021

Question 3

6 marks | Reverse Percentages

3 Gladys buys a table for \$465 to sell in her shop.

She sells the table for \$520

(a) Work out the percentage profit that Gladys makes from the sale of the table.
Give your answer correct to 3 significant figures.

.....%

(3)

Gladys has a sale in her shop.

She decreases all the normal prices by 12%
The normal price of an armchair was \$550

(b) Work out the sale price of the armchair.

\$.....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2021

1 Profit

Profit:

$$520 - 465 = 55$$

2 Percentage profit

Percentage profit:

$$\frac{55}{465} \times 100 = 11.827\ldots$$

3 Correct to 3 significant figures

Correct to 3 significant figures:

$$11.8\%$$

4 **Calculate statistic**

For the sale price, a 12% decrease means 88% of the original price:

$$550 \times 0.88 = 484$$

✓ **FINAL ANSWER**

(a) 11.8%, (b) $\$484$

4MA1/2HR JANUARY 2021

Question 6

3 marks | Reverse Percentages

6 On Wednesday, the price of 1 litre of petrol was £1.26
The price of petrol on Wednesday was 5% more than the price of petrol on the previous Monday.

Calculate the price of 30 litres of petrol on the previous Monday.

£.....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR January 2021

1 Wednesday s price was more

Wednesday's price was 5% more than Monday's price.

2 Calculate percentage

So

$$1.26 = 1.05 \times \text{Monday price}$$

$$\text{Monday price} = \frac{1.26}{1.05} = 1.20$$

3 The cost of litres on

The cost of 30 litres on Monday was

$$30 \times 1.20 = 36$$

FINAL ANSWER

$\pounds 36$

4MA1/2H MAY 2021

Question 6

4 marks | Reverse Percentages

6 Mariana sells bags of bird food.

The bags that Mariana sold last week each contained 12 kg of seeds.

The bags that she is going to sell next week will each contain a mixture of nuts and seeds where for each bag

$$\text{weight of nuts} : \text{weight of seeds} = 4 : 5$$

The total weight of the nuts and the seeds in each bag will be 19.35 kg

The weight of seeds in each bag that Mariana sells next week will be less than the weight of seeds in each bag that Mariana sold last week.

Work out this decrease as a percentage of the weight of seeds in each bag that Mariana sold last week.
Give your answer correct to one decimal place.

..... %

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H May 2021

1 Split the ratio

In each new bag, the ratio

$$\text{nuts} : \text{seeds} = 4 : 5$$

2 has total parts

has 9 total parts.

3 Weight of seeds in each

Weight of seeds in each new bag:

$$\frac{5}{9} \times 19.35 = 10.75 \text{ kg}$$

4 Decrease in seed weight

Decrease in seed weight:

$$12 - 10.75 = 1.25 \text{ kg}$$

5 **Percentage decrease**

Percentage decrease:

$$\frac{1.25}{12} \times 100 = 10.416\ldots\%$$

✓ **FINAL ANSWER**

10.4%

4MA1/2H NOVEMBER 2021

Question 6

3 marks | Reverse Percentages

6 Victor buys 12 bottles of apple juice for a total cost of $21
Victor sells all 12 bottles at $2.45 each bottle.

Work out Victor's percentage profit.

.....%

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2021

1 Total selling price

Total selling price:

$$12 \times 2.45 = 29.40$$

2 Profit

Profit:

$$29.40 - 21 = 8.40$$

3 Percentage profit

Percentage profit:

$$\frac{8.40}{21} \times 100 = 40$$

FINAL ANSWER

40%

Question 2

7 marks | Reverse Percentages

2 Mary saves for a holiday each year.

In 2020 she saved a total of \$720
In 2021, each month she saved \$78

The total amount Mary saved in 2021 was P% more than the total she saved in 2020

(a) Work out the value of P

.....

(4)

Roberto is going to go on holiday.

He has two coupons that will save him money on his holiday.

Coupon A	**Coupon B**
18% off the cost of the accommodation	12.5% off the total cost of the accommodation **and** the flights

For Roberto's holiday
the cost of the accommodation is \$1600
the cost of the flights is \$800

Roberto can only use one of the coupons.
He wants to save as much money as he can.

(b) Which of the two coupons, **A** or **B**, should he use?
Show your working clearly.

(3)

(Total for Question 2 is 7 marks)

1 Mary saved in 2021

Mary saved in 2021:

$$78 \times 12 = 936$$

2 Increase from 2020

Increase from 2020:

$$936 - 720 = 216$$

3 Percentage increase

Percentage increase:

$$\frac{216}{720} \times 100 = 30$$

4 Calculate percentage

So $P = 30$.

5 Coupon A saves

Coupon A saves

$$18\% \times 1600 = 0.18 \times 1600 = 288$$

6 Coupon B saves of the

Coupon B saves 12.5% of the accommodation and flights together:

$$1600 + 800 = 2400$$

$$12.5\% \times 2400 = 0.125 \times 2400 = 300$$

7 Coupon B saves more money

Coupon B saves more money.

FINAL ANSWER

(a) $P = 30$, (b) Coupon B

4MA1/1H JANUARY 2022

Question 4

5 marks | Reverse Percentages

4 Sarah makes and sells mugs.

One day she makes 150 mugs.
Her total cost for making these mugs is £1140

Of these mugs

$\frac{2}{5}$ are small mugs

32% are medium mugs

and the rest are large mugs

Here is Sarah's price list for selling each mug.

MUGS	
Small	**£8.50**
Medium	**£11.20**
Large	**£14.20**

Sarah sells all 150 mugs.

Work out her percentage profit.
Give your answer correct to the nearest whole number.

..... %

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2022

1 **Small mugs**

Small mugs:

$$\frac{2}{5} \times 150 = 60$$

2 **Medium mugs**

Medium mugs:

$$32\% \times 150 = 48$$

3 **Large mugs**

Large mugs:

$$150 - 60 - 48 = 42$$

4 **Revenue**

Revenue:

$$60(8.50) + 48(11.20) + 42(14.20)$$

$$= 510 + 537.60 + 596.40 = 1644$$

5 **Profit**

Profit:

$$1644 - 1140 = 504$$

6 **Percentage profit**

Percentage profit:

$$\frac{504}{1140} \times 100 = 44.21\ldots$$

7 **Correct to the nearest whole**

Correct to the nearest whole number:

$$44\%$$

✓ **FINAL ANSWER**

44%

4MA1/2H JANUARY 2022

Question 8

5 marks | Reverse Percentages

8 The table gives information about the average house price in England in 2018 and in 2019

Year	2017	2018	2019
Average house price (£)		228 314	231 776

(a) Work out the percentage increase in the average house price from 2018 to 2019
Give your answer correct to one decimal place.

..... %

(2)

The average house price in 2019 was 7.7% greater than the average house price in 2017

(b) Work out the average house price in 2017
Give your answer correct to 3 significant figures.

£ .....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2022

1 Increase from 2018 to 2019

Increase from 2018 to 2019:

$$231776 - 228314 = 3462$$

2 Percentage increase

Percentage increase:

$$\frac{3462}{228314} \times 100 = 1.516\ldots$$

3 Correct to one decimal place

Correct to one decimal place:

$$1.5\%$$

4 **Calculate percentage**

For part (b), the 2019 price was 7.7% greater than the 2017 price.

$$231776 = 1.077 \times 2017\ \text{price}$$

$$2017\ \text{price} = \frac{231776}{1.077} = 215205.19\ldots$$

5 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$215000$$

FINAL ANSWER

(a) 1.5%, (b) $\pounds 215000$

4MA1/2HR JUNE 2022

Question 8

3 marks | Reverse Percentages

8 A cinema increased the cost of an adult ticket by 12%

After the increase, the cost of an adult ticket was £18.20

Work out the cost of an adult ticket before the increase.

£.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2022

1 **Calculate percentage**

After a 12% increase, the ticket price is 112% of the original price.

$$18.20 = 1.12 \times \text{original price}$$

$$\text{original price} = \frac{18.20}{1.12} = 16.25$$

FINAL ANSWER

$\pounds 16.25$

4MA1/1H MAY 2023

Question 3

4 marks | Reverse Percentages

3 Nanette buys 60 notebooks for a total cost of 780 dirhams.

Nanette sells 70% of the notebooks for 22 dirhams each.
She sells the remaining notebooks for 19 dirhams each.

Work out Nanette's percentage profit.
Give your answer correct to 3 significant figures.

.....%

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2023

1 Number sold for 22 dirhams

Number sold for 22 dirhams:

$$70\% \times 60 = 42$$

2 Number remaining

Number remaining:

$$60 - 42 = 18$$

3 Total selling price

Total selling price:

$$42 \times 22 + 18 \times 19 = 924 + 342 = 1266$$

4 Profit

Profit:

$$1266 - 780 = 486$$

5 Percentage profit

Percentage profit:

$$\frac{486}{780} \times 100 = 62.307\ldots\%$$

FINAL ANSWER

62.3%

4MA1/2HR JUNE 2023

Question 6

7 marks | Reverse Percentages

6 In his previous job, Pierre was paid 400 euros in total for working a 5-day week.

In his new job, Pierre is paid 14 euros per hour.
In his new job, Pierre works for 7 hours each day for a 5-day week.

(a) Work out the percentage increase in the amount that Pierre is paid for a 5-day week.

..... %
(4)

Marie changes her job.

Her salary decreases by 6%
Her new salary is 23 030 euros.

(b) Work out Marie's salary before she changes her job.

..... euros
(3)

(Total for Question 6 is 7 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2023

1 **Pierre s old weekly pay**

Pierre's old weekly pay is 400 euros.

2 **His new weekly pay is**

His new weekly pay is

$$14 \times 7 \times 5 = 490$$

3 **The increase is**

The increase is

$$490 - 400 = 90$$

4 **Percentage increase**

Percentage increase:

$$\frac{90}{400} \times 100 = 22.5\%$$

5 **Marie s new salary is**

Marie's new salary is after a 6% decrease, so it is 94% of her old salary.

$$0.94 \times \text{old salary} = 23030$$

$$\text{old salary} = 23030 \div 0.94 = 24500$$

✓ **FINAL ANSWER**

Pierre's pay increased by 22.5%. Marie's old salary was 24500 euros.

4MA1/1H NOVEMBER 2023

Question 3

4 marks | Reverse Percentages

3 Joshua buys a car for $12 500

He sells the car to Nina.

Nina pays

- a deposit of $1500
- followed by 36 monthly payments of $450

Work out Joshua's percentage profit.

.....%

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2023

1 Nina pays

Nina pays

$$1500 + 36(450) = 1500 + 16200 = 17700$$

2 Joshua s profit is

Joshua's profit is

$$17700 - 12500 = 5200$$

3 Percentage profit

Percentage profit:

$$\frac{5200}{12500} \times 100 = 41.6$$

FINAL ANSWER

41.6%

4MA1/1H NOVEMBER 2023

Question 6

4 marks | Reverse Percentages

6 Juan wants to buy a ticket to fly from Madrid to Berlin.

He finds two different types of ticket he can buy in a sale, ticket **A** and ticket **B**

ticket **A**	ticket **B**
$\frac{1}{6}$ off normal price	20% off normal price

The sale price of ticket **A** is 140 euros.
The sale price of ticket **B** is 136 euros.

Work out the difference between the normal price of ticket **A** and the normal price of ticket **B**

..... euros

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2023

1 Calculate percentage

Ticket A is sold for $\frac{5}{6}$ of its normal price.

$$\text{normal price of A} = 140 \times \frac{6}{5} = 168$$

2 Calculate percentage

Ticket B has a 20% discount, so its sale price is 80% of the normal price.

$$\text{normal price of B} = 136 \div 0.80 = 170$$

3 The difference is

The difference is

$$170 - 168 = 2$$

FINAL ANSWER

The difference in normal prices is 2 euros.

4MA1/1HR MAY 2024

Question 3

5 marks | Reverse Percentages

3 Norberto sells white loaves of bread and brown loaves of bread.

He sells a total of 200 loaves such that

the number of white loaves sold : the number of brown loaves sold = 3 : 2

Norberto sells the white loaves for £1.50 each.
He sells the brown loaves for £1.75 each.

40% of the price of a white loaf is profit.
60% of the price of a brown loaf is profit.

Work out Norberto's total profit when he sells all 200 loaves.

£.....

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May 2024

1 The ratio parts are

The ratio parts are

$$3 + 2 = 5$$

2 White loaves

White loaves:

$$200 \times \frac{3}{5} = 120$$

3 Brown loaves

Brown loaves:

$$200 \times \frac{2}{5} = 80$$

4 Profit on each white loaf

Profit on each white loaf:

$$40\% \times 1.50 = 0.60$$

5 **Profit on each brown loaf**

Profit on each brown loaf:

$$60\% \times 1.75 = 1.05$$

6 **Total profit**

Total profit:

$$120 \times 0.60 + 80 \times 1.05 = 72 + 84 = 156$$

✓ **FINAL ANSWER**

156 pounds

4MA1/1H MAY 2024

Question 4

3 marks | Reverse Percentages

4 Ava records the number of kilometres she drives each month.

In April, Ava drove 943 kilometres.
This is 15% more than the number of kilometres she drove in March.

Work out the number of kilometres Ava drove in March.

..... kilometres

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May 2024

1 Calculate percentage

April is 15% more than March, so April is 115% of March.

$$1.15 \times \text{March} = 943$$

$$\text{March} = \frac{943}{1.15} = 820$$

FINAL ANSWER

820 kilometres

4MA1/2HR JUNE 2024

Question 8

3 marks | Reverse Percentages

8 In a sale, all normal prices are reduced by 17%

The sale price of a fridge is 6225 rupees.

Work out the normal price of the fridge.

..... rupees

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2024

1 **A reduction of means the**

A reduction of 17% means the sale price is 83% of the normal price.

$$0.83 \times \text{normal price} = 6225$$

$$\text{normal price} = \frac{6225}{0.83} = 7500$$

FINAL ANSWER

7500 rupees

4MA1/1H MAY 2024

Question 14

5 marks | Reverse Percentages

14 The combined savings of Abel and Bahira are 15 435 dinars.

The savings of Bahira are 45% more than the savings of Abel.

The savings of Bahira are $\frac{3}{2}$ times the savings of Chanda.

Work out the savings of Chanda.

..... dinars

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2024

1 Let Abel s savings be

Let Abel's savings be A.

2 Calculate percentage

Bahira has 45% more than Abel, so

$$B = 1.45A$$

3 Their combined savings are

Their combined savings are

$$A + 1.45A = 15435$$

$$2.45A = 15435$$

$$A = 6300$$

4 Calculate percentage

So

$$B = 1.45 \times 6300 = 9135$$

5 **Bahira s savings are times**

Bahira's savings are $\frac{3}{2}$ times Chanda's savings, so

$$9135 = \frac{3}{2}C$$

$$C = 9135 \times \frac{2}{3} = 6090$$

✓ **FINAL ANSWER**

6090 dinars

4MA1/2H NOVEMBER 2024

Question 4

5 marks | Reverse Percentages

4 N is a number.
17% of N is 357

(a) Work out the value of N

$N =$ .....

(2)

In 2019, the population of a village was 650
In 2020, the population of the village was 806

(b) Work out the percentage increase in the population.

..... %

(3)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2024

1 **Calculate percentage**

For part (a),

$$17\% \text{ of } N = 357$$

$$0.17N = 357$$

$$N = \frac{357}{0.17} = 2100$$

2 **Calculate percentage**

For part (b), the increase in population is

$$806 - 650 = 156$$

3 **Percentage increase**

Percentage increase:

$$\frac{156}{650} \times 100 = 24$$

FINAL ANSWER

(a) $N = 2100$, (b) 24%

4MA1/1H JUNE 2025

Question 2

6 marks | Reverse Percentages

2 Anna makes cups.
Each cup costs 6 Swiss francs to make.

Anna puts the cups into boxes to sell.
Each box contains 4 cups.

Anna sells 80 boxes of cups for a total of 2160 Swiss francs.

(a) Work out the percentage profit Anna makes.
Show your working clearly.

.....%
(4)

The height of each cup is 9 cm, correct to the nearest cm

(b) Write down the lower bound of the height.

.....cm
(1)

The weight of each cup is 120 g, correct to the nearest 10 g

(c) Write down the upper bound of the weight.

.....g
(1)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2025

1 Number of cups

Number of cups:

$$80 \times 4 = 320$$

2 Cost to make the cups

Cost to make the cups:

$$320 \times 6 = 1920$$

3 **Profit**

Profit:

$$2160 - 1920 = 240$$

4 **Percentage profit**

Percentage profit:

$$\frac{240}{1920} \times 100 = 12.5$$

5 **Find lower bound**

The height 9 cm correct to the nearest cm has lower bound

$$8.5 \text{ cm}$$

6 **Find upper bound**

The weight 120 g correct to the nearest 10 g has upper bound

$$125 \text{ g}$$

✓ **FINAL ANSWER**

(a) 12.5%, (b) 8.5 cm, (c) 125 g

4WM2/01 MAY 2025

Question 3

3 marks | Reverse Percentages

3 Bernice bought a car for $8000
She sold the car for $5600

Work out Bernice's percentage loss when she sold the car.

.....%

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 May 2025

1 Convert standard form

$$\text{loss} = 8000 - 5600 = 2400$$

$$\text{percentage loss} = \frac{2400}{8000} \times 100 = 30\%$$

FINAL ANSWER

30%.

4MA1/2HR JUNE 2025

Question 4

5 marks | Reverse Percentages

4 Joshua is going to cover a floor with tiles for a customer.
The area of the floor is 45 m^2

Joshua buys one box of tiles for each 1.5 m^2 of floor area.
Each box of tiles costs £64

Joshua also buys 5 bags of tile adhesive.
Each bag of tile adhesive costs £12

Joshua charges the customer £3000

Work out his percentage profit.
Give your answer correct to one decimal place.

..... %

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2025

1 Number of boxes of tiles

Number of boxes of tiles:

$$\frac{45}{1.5} = 30$$

2 Cost of tiles

Cost of tiles:

$$30 \times 64 = 1920$$

3 Cost of adhesive

Cost of adhesive:

$$5 \times 12 = 60$$

4 Total cost

Total cost:

$$1920 + 60 = 1980$$

5 **Profit**

Profit:

$$3000 - 1980 = 1020$$

6 **Percentage profit**

Percentage profit:

$$\frac{1020}{1980} \times 100 = 51.515\ldots\% $$

✓ **FINAL ANSWER**

51.5%

4MA1/1H JUNE 2025

Question 5

3 marks | Reverse Percentages

5 In a sale, normal prices are reduced by 28%
The sale price of a watch is 198 euros.

Work out the normal price of the watch.

..... euros

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2025

1 The watch is reduced by

The watch is reduced by 28%, so the sale price is

$$100\% - 28\% = 72\%$$

2 of the normal price

of the normal price.

$$0.72 \times \text{normal price} = 198$$

$$\text{normal price} = 198 \div 0.72 = 275$$

FINAL ANSWER

275 euros.

4WM2/01R MAY 2025

Question 5

5 marks | Reverse Percentages

5 Joshua is going to cover a floor with tiles for a customer.
The area of the floor is 45 m^2

Joshua buys one box of tiles for each 1.5 m^2 of floor area.
Each box of tiles costs £64

Joshua also buys 5 bags of tile adhesive.
Each bag of tile adhesive costs £12

Joshua charges the customer £3000

Work out his percentage profit.
Give your answer correct to one decimal place.

.....%

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4WM2/01R May 2025

1 Required

Calculate Joshua's percentage profit on the flooring job. Give answer to 1 d.p.

2 Number of boxes of tiles

Number of boxes of tiles needed:

$$frac45, textm^2 1.5, textm^2 / textbox = 30 textboxes$$

3 Cost of tiles 30 64

Cost of tiles:

30 × £64 = £1920

4 Cost of adhesive 5 12

Cost of adhesive:

5 × £12 = £60

5 Total cost (cost price) 1920

Total cost (cost price):

£1920 + £60 = £1980

6 **Profit and percentage profit Profit**

Profit and percentage profit:

Profit = £3000 − £1980 = £1020

7 **Note**

Always divide profit by the **cost price** (£1980), not the selling price (£3000). Dividing by £3000 gives 34.0% — a common but wrong answer.

✓ **FINAL ANSWER**

Percentage profit = 51.5%

4WM2/01R MAY 2025

Question 8

3 marks | Reverse Percentages

8 In a sale, normal prices are reduced by 15%

The sale price of a dishwasher is 612 Swiss francs.

Work out the normal price of the dishwasher.

..... Swiss francs

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM2/01R May 2025

1 Required

Sale price (after 15% reduction) = 612 Swiss francs. Find the normal price.

2 A 15% reduction means the

A 15% reduction means the sale price is 85% of the normal price:

612 = 0.85 × Normal price

3 Solve for the normal price

Solve for the normal price:

$$textNormalprice = frac6120.85 = 720$$

4 Note

Never calculate 15% of 612 and add it on — that gives 703.80 (wrong!). Always divide by the decimal multiplier.

✓ **FINAL ANSWER**

Normal price = 720 Swiss francs

4MA1/2HR JUNE 2025

Question 9

3 marks | Reverse Percentages

9 In a sale, normal prices are reduced by 15%

The sale price of a dishwasher is 612 Swiss francs.

Work out the normal price of the dishwasher.

..... Swiss francs

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2025

1 **A reduction of means the**

A reduction of 15% means the sale price is 85% of the normal price.

$$0.85 \times \text{normal price} = 612$$

$$\text{normal price} = \frac{612}{0.85} = 720$$

FINAL ANSWER

720 Swiss francs

4WM2/01 NOVEMBER 2025

Question 8

4 marks | Reverse Percentages

8 Shop A and shop B have offers for buying the same type of chair.

Shop A	Shop B
25% off the normal price	$\frac{2}{9}$ off the normal price
Sale price 240 euros	Sale price 245 euros

The normal price of the chair in shop A is more than the normal price of the chair in shop B

How much more?
Show your working clearly.

..... euros

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4WM2/01 November 2025

1 Calculate normal price

Step 1: Calculate Normal Price for Shop A. A 25% discount means the sale price is 75% of the normal price.

0.75 × Normal_{*A*} = 240

$$\text{Normal}_A = \frac{240}{0.75} = 320\text{ euros}$$

2 Calculate normal price

Step 2: Calculate Normal Price for Shop B. A discount of $\frac{2}{9}$ means the sale price is $1 - \frac{2}{9} = \frac{7}{9}$ of the normal price.

$$\frac{7}{9} \times \text{Normal}_B = 245$$

$$\text{Normal}_B = 245 \times \frac{9}{7} = 35 \times 9 = 315\text{ euros}$$

3 Calculate value

Difference = 320 − 315 = 5 euros

FINAL ANSWER

5 euros

4MA1/2H JUNE 2026

Question 4

3 marks | Reverse Percentages

4 Sara sells water bottles.

In July, she sold 450 water bottles.
In August, she sold 612 water bottles.

Work out the percentage increase in the number of water bottles she sold from July to August.

..... %

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2026

1 Solution | Step 1

$$\text{Increase} = 612 - 450 = 162.$$

Find the change from the original number.

2 Solution | Step 2

$$\text{Percentage increase} = \frac{162}{450} \times 100 = 36\%.$$

Express the increase as a percentage of the July value, not the August value.

FINAL ANSWER

36%

4MA1/2H JUNE 2026

Question 8

3 marks | Reverse Percentages

8 A golf club increases the cost of membership by 17%
After the increase, the cost of membership is 3276 zloty.

Work out the cost of membership before the increase.

..... zloty

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2026

1 Solution | Step 1

Let the original membership cost be C zloty. After a 17% increase, $1.17C = 3276$.

The new value is 117% of the original.

2 Solution | Step 2

$$C = \frac{3276}{1.17} = 2800 \text{ zloty.}$$

Divide by the increase multiplier.

FINAL ANSWER

2800 zloty

4MA1/2HR JUNE 2026

Question 8

3 marks | Reverse Percentages

8 Lauren's salary increased by 4%
After the increase, her salary was 36 400 euros.

Work out Lauren's salary before the increase.

..... euros

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2026

1 Solution | Step 1

Let the old salary be s euros. Then

$$1.04s = 36400.$$

The increased salary is 104% of the original.

2 Solution | Step 2

$$s = \frac{36400}{1.04} = 35000.$$

Divide by the percentage multiplier.

FINAL ANSWER

35000 euros.

TOPIC

Standard Form

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 9

2 marks | Standard Form

9 (a) Write 0.000 089 in standard form.

.....

(1)

(b) Write 8.34×10^4 as an ordinary number.

.....

(1)

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 Specimen

1 Convert standard form

$$0.000089 = 8.9 \times 10^{-5}$$

2 Convert standard form

Also

$$8.34 \times 10^4 = 83400$$

FINAL ANSWER

8.9×10^{-5}, 83400.

4MA1/1H JANUARY 2020

Question 6

3 marks | Standard Form

6 (a) Write 7.8×10^{-4} as an ordinary number.

.....

(1)

(b) Work out $\dfrac{5.6 \times 10^4 + 7 \times 10^3}{2.8 \times 10^{-3}}$

Give your answer in standard form.

.....

(2)

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2020

1 Convert standard form

$$7.8 \times 10^{-4} = 0.00078$$

2 Convert standard form

For part (b),

$$5.6 \times 10^4 + 7 \times 10^3 = 56000 + 7000 = 63000$$

3 Convert standard form

Then

$$\frac{63000}{2.8 \times 10^{-3}} = \frac{63000}{0.0028}$$

$$= 22500000 = 2.25 \times 10^7$$

FINAL ANSWER

(a) 0.00078, (b) 2.25×10^7

4MA1/2HR JANUARY 2020

Question 10

3 marks | Standard Form

10 The table shows information about the surface area of each of the world's oceans.

Ocean	Surface area in square kilometres
Pacific	1.56×10^8
Indian	6.86×10^7
Southern	2.03×10^7
Arctic	1.41×10^7
Atlantic	1.06×10^8

(a) Work out the difference, in square kilometres, between the surface area of the Atlantic Ocean and the surface area of the Indian Ocean.
Give your answer in standard form.

..... square kilometres
(2)

The surface area of the Pacific Ocean is k times the surface area of the Arctic Ocean.

(b) Work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....
(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2020

1 Atlantic surface area

Atlantic surface area:

$$1.06 \times 10^8 = 106000000$$

2 Indian surface area

Indian surface area:

$$6.86 \times 10^7 = 68600000$$

3 **Difference**

Difference:

$$106000000 - 68600000 = 37400000$$

$$37400000 = 3.74 \times 10^{7}$$

---

4 **Convert standard form**

For part (b),

$$k = \frac{1.56 \times 10^{8}}{1.41 \times 10^{7}}$$

$$k = 11.063\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 3.74×10^{7}, (b) 11

4MA1/1H MAY/NOVEMBER 2020

Question 5

4 marks | Standard Form

5 (a) Write 5.7×10^{-3} as an ordinary number.

.....

(1)

(b) Write 800 000 in standard form.

.....

(1)

(c) Work out $\dfrac{3 \times 10^5 - 2.7 \times 10^4}{6 \times 10^{-2}}$

.....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May/November 2020

1 **Convert standard form**

$$5.7 \times 10^{-3} = 0.0057$$

$$800000 = 8 \times 10^5$$

2 **Convert standard form**

For part (c), first simplify the numerator:

$$3 \times 10^5 - 2.7 \times 10^4 = 300000 - 27000 = 273000$$

3 **Convert standard form**

Then

$$\frac{273000}{6 \times 10^{-2}} = \frac{273000}{0.06} = 4550000$$

$$4550000 = 4.55 \times 10^6$$

FINAL ANSWER

(a) 0.0057, (b) 8×10^5, (c) 4.55×10^6

4MA1/1H JANUARY 2021

Question 6

2 marks | Standard Form

6 (a) Write 2 840 000 000 in standard form.

.....

(1)

(b) Write 2.5×10^{-4} as an ordinary number.

.....

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2021

1 Convert standard form

$$2840000000 = 2.84 \times 10^{9}$$

2 Convert standard form

Also,

$$2.5 \times 10^{-4} = 0.00025$$

FINAL ANSWER

(a) 2.84×10^{9}, (b) 0.00025

4MA1/2H JANUARY 2021

Question 6

4 marks | Standard Form

6 (a) Write down the value of y^0

.....

(1)

(b) Work out $\dfrac{9.6 \times 10^{141} + 6.4 \times 10^{140}}{3.2 \times 10^{16}}$

Give your answer in standard form.

.....

(3)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2021

1 Convert standard form

Any non-zero value to the power 0 is 1, so

$$y^0 = 1$$

2 Convert standard form

For part (b),

$$9.6 \times 10^{141} + 6.4 \times 10^{140} = 9.6 \times 10^{141} + 0.64 \times 10^{141}$$

$$= 10.24 \times 10^{141} = 1.024 \times 10^{142}$$

3 Convert standard form

Now divide:

$$\frac{1.024 \times 10^{142}}{3.2 \times 10^{16}} = 0.32 \times 10^{126}$$

$$= 3.2 \times 10^{125}$$

✓ **FINAL ANSWER**

(a) 1, (b) 3.2×10^{125}

4MA1/2HR JANUARY 2021

Question 8

3 marks | Standard Form

8 The table gives the length of the coastline, in kilometres, of each of five oceans.

Ocean	Length of coastline (km)
Arctic	4.539×10^4
Atlantic	1.119×10^5
Pacific	1.357×10^5
Indian	6.653×10^4
Southern	1.797×10^4

(a) Which ocean has the greatest length of coastline?

.....

(1)

(b) Calculate the difference between the length of the Atlantic Ocean's coastline and the length of the Southern Ocean's coastline.
Give your answer in standard form.

..... km

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2021

1 **The greatest coastline is the**

The greatest coastline is the largest number in standard form.

$$1.357 \times 10^5$$

2 **Convert standard form**

is the largest, so the answer is the Pacific Ocean.

3 Convert standard form

For part (b),

$$1.119 \times 10^5 = 111900$$

$$1.797 \times 10^4 = 17970$$

4 Difference

Difference:

$$111900 - 17970 = 93930$$

$$93930 = 9.393 \times 10^4$$

FINAL ANSWER

(a) Pacific Ocean, (b) 9.393×10^4 km

4MA1/1H MAY 2021

Question 8

4 marks | Standard Form

8 The table shows the populations of five countries.

Country	Population
China	1.4×10^9
Germany	8.2×10^7
Sweden	9.9×10^6
Fiji	9.1×10^5
Malta	4.3×10^5

(a) Work out the difference between the population of China and the population of Germany.
Give your answer in standard form.

.....

(2)

Given that

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

(b) work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2021

1 **Convert standard form**

For part (a),

$$1.4 \times 10^9 = 1400000000$$

$$8.2 \times 10^7 = 82000000$$

2 **Difference**

Difference:

$$1400000000 - 82000000 = 1318000000$$

$$1318000000 = 1.318 \times 10^{9}$$

---

3 **Convert standard form**

For part (b),

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

---

4 **Convert standard form**

So

$$k = \frac{\text{population of Sweden}}{\text{population of Fiji}}$$

$$k = \frac{9.9 \times 10^{6}}{9.1 \times 10^{5}} = 10.879\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 1.318×10^{9}, (b) 11

4MA1/2H NOVEMBER 2021

Question 9

5 marks | Standard Form

9 A rainwater tank contains 2.4×10^7 raindrops.
The rainwater tank also contains 1.75×10^6 bacteria.

(a) Work out the number of bacteria per raindrop in the tank.
Give your answer in standard form correct to 2 significant figures.

.....

(3)

A drop of rainwater contains 5.01×10^{21} atoms.

In a drop of rainwater the number of atoms is 3 times the number of molecules.

(b) Work out the number of molecules in the rainwater tank.
Give your answer in standard form correct to one significant figure.

..... molecules

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2021

1 Convert standard form

For part (a), divide the number of bacteria by the number of raindrops:

$$\frac{1.75 \times 10^6}{2.4 \times 10^7} = \frac{1.75}{2.4} \times 10^{-1}$$

$$= 0.072916\ldots$$

2 Correct to 2 significant figures

Correct to 2 significant figures:

$$7.3 \times 10^{-2}$$

3 Convert standard form

For part (b), one drop contains 5.01×10^{21} atoms.

4 **Convert standard form**

The number of molecules in one drop is

$$\frac{5.01 \times 10^{21}}{3} = 1.67 \times 10^{21}$$

5 **Convert standard form**

There are 2.4×10^{7} drops, so the number of molecules in the tank is

$$(1.67 \times 10^{21})(2.4 \times 10^{7}) = 4.008 \times 10^{28}$$

6 **Correct to one significant figure**

Correct to one significant figure:

$$4 \times 10^{28}$$

✓ **FINAL ANSWER**

(a) 7.3×10^{-2}, (b) 4×10^{28} molecules

4MA1/1H JANUARY 2022

Question 8

2 marks | Standard Form

8 $a = 4.2 \times 10^{-24}$ $b = 3 \times 10^{145}$

Work out the value of $a \times b$
Give your answer in standard form.

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2022

1 Convert standard form

$$a = 4.2 \times 10^{-24}$$

$$b = 3 \times 10^{145}$$

2 Convert standard form

So

$$ab = (4.2 \times 3) \times 10^{-24+145}$$

$$ab = 12.6 \times 10^{121}$$

3 Write this in standard form

Write this in standard form:

$$12.6 \times 10^{121} = 1.26 \times 10^{122}$$

FINAL ANSWER

1.26×10^{122}

4MA1/1HR JANUARY 2022

Question 8

4 marks | Standard Form

8 (a) Write 5×10^4 as an ordinary number.

.....

(1)

(b) Write 0.000 06 in standard form.

.....

(1)

(c) Work out $(4 \times 10^{512}) \div (1.6 \times 10^{700})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2022

1 Convert standard form

$$5 \times 10^4 = 50000$$

$$0.00006 = 6 \times 10^{-5}$$

2 Convert standard form

For part (c),

$$\frac{4 \times 10^{512}}{1.6 \times 10^{700}} = \frac{4}{1.6} \times 10^{512-700}$$

$$= 2.5 \times 10^{-188}$$

FINAL ANSWER

(a) 50000, (b) 6×10^{-5}, (c) 2.5×10^{-188}

4MA1/2H JANUARY 2022

Question 12

3 marks | Standard Form

12 $a = 6 \times 10^{40}$

Work out the value of a^3
Give your answer in standard form.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2022

1 Convert standard form

$$a = 6 \times 10^{40}$$

$$a^3 = (6 \times 10^{40})^3$$

$$a^3 = 6^3 \times 10^{120}$$

$$a^3 = 216 \times 10^{120}$$

2 Write this in standard form

Write this in standard form:

$$216 \times 10^{120} = 2.16 \times 10^{122}$$

FINAL ANSWER

2.16×10^{122}

4MA1/1H JUNE 2022

Question 8

2 marks | Standard Form

8 (a) Write $0.000\,089$ in standard form.

.....

(1)

(b) Write 8.34×10^4 as an ordinary number.

.....

(1)

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2022

1 Convert standard form

$$0.000089 = 8.9 \times 10^{-5}$$

$$8.34 \times 10^4 = 83400$$

FINAL ANSWER

(a) 8.9×10^{-5}, (b) 83400

4MA1/2HR JUNE 2022

Question 9

4 marks | Standard Form

9 The table gives information about the population, correct to 2 significant figures, of each of five cities in 2018

City	Population (2018)
Ahmedabad	7.7×10^6
Barcelona	5.5×10^6
Chicago	8.8×10^6
Lagos	1.3×10^7
Tokyo	3.7×10^7

(a) Write 8.8×10^6 as an ordinary number.

.....

(1)

(b) Which of these cities had the least population in 2018?

.....

(1)

(c) Work out the difference between the population of Tokyo and the population of Ahmedabad in 2018
Give your answer in standard form correct to 2 significant figures.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2022

1 Convert standard form

$$8.8 \times 10^6 = 8800000$$

2 The least population in the

The least population in the table is Barcelona, with

$$5.5 \times 10^6$$

3 Convert standard form

For the difference between Tokyo and Ahmedabad:

$$3.7 \times 10^7 - 7.7 \times 10^6$$

$$37000000 - 7700000 = 29300000$$

$$29300000 = 2.93 \times 10^7$$

4 Correct to 2 significant figures

Correct to 2 significant figures:

$$2.9 \times 10^7$$

FINAL ANSWER

(a) 8800000, (b) Barcelona, (c) 2.9×10^7

4MA1/2H JUNE 2023

Question 6

2 marks | Standard Form

6 (a) Write $76\,000\,000$ in standard form.

.....

(1)

(b) Write 5.4×10^{-4} as an ordinary number.

.....

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2023

1 Convert standard form

$$76000000 = 7.6 \times 10^{7}$$

$$5.4 \times 10^{-4} = 0.00054$$

FINAL ANSWER

(a) 7.6×10^{7}, (b) 0.00054

4MA1/1HR MAY 2023

Question 8

3 marks | Standard Form

8 (a) Write 5.6×10^{-3} as an ordinary number.

.....

(1)

(b) Work out $\dfrac{6 \times 10^{3}}{2.1 \times 10^{-4} + 9 \times 10^{-5}}$

Give your answer in standard form.

.....

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May 2023

1 Convert standard form

$$5.6 \times 10^{-3} = 0.0056$$

2 Convert standard form

For part (b), simplify the denominator:

$$2.1 \times 10^{-4} + 9 \times 10^{-5} = 0.00021 + 0.00009$$

$$= 0.00030 = 3 \times 10^{-4}$$

3 Convert standard form

Then

$$\frac{6 \times 10^{3}}{3 \times 10^{-4}} = 2 \times 10^{3-(-4)}$$

$$= 2 \times 10^{7}$$

FINAL ANSWER

(a) 0.0056, (b) 2×10^{7}

4MA1/1H MAY 2023

Question 15

6 marks | Standard Form

15 (a) $\sqrt{2} \div \dfrac{8^3}{16^{\frac{3}{2}}} = 2^n$

Work out the value of n
Show your working clearly.

$n =$ .....

(3)

(b) Find 4% of 4.5×10^{157}
Give your answer in standard form.

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2023

1 **Convert standard form**

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$8^3 = (2^3)^3 = 2^9$$

$$16^{\frac{3}{2}} = (2^4)^{\frac{3}{2}} = 2^6$$

2 **Convert standard form**

So

$$\frac{8^3}{16^{\frac{3}{2}}} = \frac{2^9}{2^6} = 2^3$$

3 **Convert standard form**

Therefore

$$\sqrt{2} \div \frac{8^3}{16^{\frac{3}{2}}} = 2^{\frac{1}{2}} \div 2^3$$

$$= 2^{\frac{1}{2}-3} = 2^{-\frac{5}{2}}$$

4 **Convert standard form**

So

$$n = -\frac{5}{2}$$

---

5 **Convert standard form**

For part (b),

$$4\% = 0.04 = 4 \times 10^{-2}$$

$$0.04(4.5 \times 10^{157}) = 18 \times 10^{155}$$

$$= 1.8 \times 10^{156}$$

✓ **FINAL ANSWER**

(a) $n = -\frac{5}{2}$, (b) 1.8×10^{156}

4MA1/1H NOVEMBER 2023

Question 9

4 marks | Standard Form

9 (a) Write 5.87×10^{-4} as an ordinary number.

.....

(1)

(b) Write 84 000 000 in standard form.

.....

(1)

The number of neurons in a human brain is 8.5×10^{10}
The number of neurons in a monkey brain is 1.47×10^{9}

The number of neurons in a human brain is K × the number of neurons in a monkey brain.

(c) Work out the value of K
Give your answer correct to one decimal place.

$K =$ .....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2023

1 Convert standard form

$$5.87 \times 10^{-4} = 0.000587$$

$$84000000 = 8.4 \times 10^{7}$$

2 Convert standard form

For part (c),

$$K = \frac{8.5 \times 10^{10}}{1.47 \times 10^{9}}$$

$$K = \frac{8.5}{1.47} \times 10$$

$$K = 57.823\ldots$$

3 **Convert standard form**

Correct to one decimal place,

$$K = 57.8$$

✓ **FINAL ANSWER**

(a) 0.000587, (b) 8.4×10^{7}, (c) 57.8

4MA1/2HR JUNE 2024

Question 9

4 marks | Standard Form

9 (a) Write 6.04×10^5 as an ordinary number.

.....

(1)

(b) Write 0.000 07 in standard form.

.....

(1)

(c) Work out $\dfrac{7.6 \times 10^{10}}{4 \times 10^5 - 2 \times 10^4}$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2024

1 **Convert standard form**

$$6.04 \times 10^5 = 604000$$

$$0.00007 = 7 \times 10^{-5}$$

2 **Convert standard form**

For part (c), first simplify the denominator:

$$4 \times 10^5 - 2 \times 10^4 = 400000 - 20000 = 380000$$

$$380000 = 3.8 \times 10^5$$

3 **Convert standard form**

Then

$$\frac{7.6 \times 10^{10}}{3.8 \times 10^5} = 2 \times 10^5$$

FINAL ANSWER

(a) 604000, (b) 7×10^{-5}, (c) 2×10^{5}

4MA1/1H MAY 2024

Question 9

3 marks | Standard Form

9 The table gives the amount of rice produced by each of two countries in 2020

Country	Amount of rice (tonnes)
Indonesia	3.5×10^7
Argentina	8.2×10^5

(a) Write 3.5×10^7 as an ordinary number.

.....

(1)

In 2020, Japan produced 6 780 000 more tonnes of rice than Argentina.

(b) Work out the amount of rice Japan produced in 2020
Give your answer in standard form.

..... tonnes

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2024

1 Convert standard form

$$3.5 \times 10^7 = 35000000$$

2 Argentina produced

Argentina produced

$$8.2 \times 10^5 = 820000$$

3 tonnes of rice

tonnes of rice.

4 Japan produced more tonnes than

Japan produced 6780000 more tonnes than Argentina, so

$$820000 + 6780000 = 7600000$$

5 **Convert standard form**

In standard form,

$$7600000 = 7.6 \times 10^6$$

✓ **FINAL ANSWER**

(a) 35000000, (b) 7.6×10^6 tonnes

4MA1/1H NOVEMBER 2024

Question 9

3 marks | Standard Form

9 (a) Write 8.4×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(6.5 \times 10^{-40}) \times (8 \times 10^{185})$
Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2024

1 Convert standard form

$$8.4 \times 10^{-5} = 0.000084$$

2 Convert standard form

For part (b),

$$(6.5 \times 10^{-40})(8 \times 10^{185}) = 52 \times 10^{145}$$

3 Write this in standard form

Write this in standard form:

$$52 \times 10^{145} = 5.2 \times 10^{146}$$

✓ **FINAL ANSWER**

(a) 0.000084, (b) 5.2×10^{146}

4WM2/01R MAY 2025

Question 6

3 marks | Standard Form

6 (a) Write $82\,000\,000$ in standard form.

.....

(1)

(b) Write 5.8×10^{-6} as an ordinary number.

.....

(1)

(c) Write 160×10^{300} in standard form.

.....

(1)

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM2/01R May 2025

1 **Required**

a Write 82 000 000 in standard form. (b) Write 5.8 × 10^{−6} as an ordinary number. (c) Write 160 × 10³⁰⁰ in standard form.

2 **Part (a)**

82 000 000 = 8.2 × 10⁷

3 **Answer**

8.2 × 10⁷

4 **Part (b)**

Move the decimal 6 places **left**:

5.8 × 10^{−6} = 0.0000058

5 **Answer**

0.0000058

6 **Part (c)**

160 × 10³⁰⁰ = 1.6 × 10¹ × 10³⁰⁰ = 1.6 × 10³⁰¹

FINAL ANSWER

1.6 × 10³⁰¹

4MA1/1H JUNE 2025

Question 8

4 marks | Standard Form

8 (a) Write 520 million in standard form.

.....

(1)

(b) Write 8.79×10^{-5} as an ordinary number.

.....

(1)

(c) Work out $(5 \times 10^{42}) \times (7 \times 10^{-180})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2025

1 Convert standard form

$$520 \text{ million} = 520000000 = 5.2 \times 10^8$$

$$8.79 \times 10^{-5} = 0.0000879$$

2 Convert standard form

For part (c),

$$(5 \times 10^{42})(7 \times 10^{-180}) = 35 \times 10^{-138}$$

3 Write this in standard form

Write this in standard form:

$$35 \times 10^{-138} = 3.5 \times 10^{-137}$$

FINAL ANSWER

(a) 5.2×10^8, (b) 0.0000879, (c) 3.5×10^{-137}

4WM2/01 MAY 2025

Question 12

3 marks | Standard Form

12 (a) Write 8.6×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(3 \times 10^{143}) \div (6 \times 10^{137})$

Give your answer in standard form.

.....

(2)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4WM2/01 May 2025

1 **(a)**

(a)

$$8.6 \times 10^{-5} = 0.000086$$

2 **(b)**

(b)

$$(3 \times 10^{43}) \div (6 \times 10^{37})$$

$$= 0.5 \times 10^{6} = 5 \times 10^{5}$$

✓ **FINAL ANSWER**

0.000086, 5×10^{5}.

4WM2/01R MAY 2025

Question 13

3 marks | Standard Form

13 The distance from Earth to Andromeda Galaxy is 1.6×10^{11} astronomical units.

- 1 astronomical unit $= 1.5 \times 10^{8}$ km
- 1 light year $= 9.5 \times 10^{12}$ km

Work out the distance, in light years, from Earth to Andromeda Galaxy.
Give your answer in standard form, correct to 2 significant figures.

..... light years

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4WM2/01R May 2025

1 Required

Distance to Andromeda: 1.6 × 10¹¹ AU. Convert to light years (2 s.f., standard form).

2 Convert AU km 1 6

Convert AU → km:
1.6 × 10¹¹ × 1.5 × 10⁸ = (1.6×1.5) × 10¹⁹ = 2.4 × 10¹⁹ km

3 Convert km light years

Convert km → light years:

$$frac2.4times10^{19}9.5times10^{12} = frac2.49.5times10^{7} = 0.2526ldotstimes10^{7}$$

4 Convert standard form

Write in standard form to 2 s.f.:
= 2.526… × 10⁶ ≈ 2.5 × 10⁶

✓ **FINAL ANSWER**

Distance = 2.5 × 10⁶ light years

4MA1/1HR JUNE 2025

Question 14

2 marks | Standard Form

14 Work out $6.7 \times 10^{135} + 3 \times 10^{134}$

Give your answer in standard form.

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR June 2025

1 Convert standard form

$$6.7 \times 10^{135} + 3 \times 10^{134}$$

2 Write both terms using

Write both terms using 10^{135}:

$$3 \times 10^{134} = 0.3 \times 10^{135}$$

3 Convert standard form

So

$$6.7 \times 10^{135} + 0.3 \times 10^{135} = 7.0 \times 10^{135}$$

$$= 7 \times 10^{135}$$

FINAL ANSWER

7×10^{135}

4MA1/2H NOVEMBER 2025

Question 8

3 marks | Standard Form

8 (a) Write 5.76×10^4 as an ordinary number.

.....

(1)

(b) Work out $\dfrac{3 \times 10^5 + 8 \times 10^3}{4 \times 10^{-2}}$

Give your answer in standard form.

.....

(2)

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2025

1 Convert standard form

$$5.76 \times 10^4 = 57600$$

2 Convert standard form

For part (b), simplify the numerator:

$$3 \times 10^5 + 8 \times 10^3 = 300000 + 8000 = 308000$$

3 Convert standard form

Then

$$\frac{308000}{4 \times 10^{-2}} = \frac{308000}{0.04} = 7700000$$

4 In standard form

In standard form:

$$7700000 = 7.7 \times 10^6$$

FINAL ANSWER

(a) 57600, (b) 7.7×10^6

4WM2/01 NOVEMBER 2025

Question 9

3 marks | Standard Form

9 (a) Write 5.76×10^4 as an ordinary number.

.....

(1)

(b) Work out $\dfrac{3 \times 10^5 + 8 \times 10^3}{4 \times 10^{-2}}$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 November 2025

1 Move the decimal point 4

Move the decimal point 4 places to the right:

5.76 × 10⁴ = 57600

2 Answer

57600

3 Expand brackets

3 × 10⁵ = 300,000

8 × 10³ = 8,000

Numerator = 300,000 + 8,000 = 308,000 = 3.08 × 10⁵

4 Convert standard form

$$frac3.08times10^54times10^{-2} = left(frac3.084right)times10^{5-(-2)}$$

= 0.77 × 10⁷

Step 3: Convert to strict standard form (1 ≤ *A* \< 10).

0.77 × 10⁷ = 7.7 × 10⁶

FINAL ANSWER

7.7 × 10⁶

4WM2/01 NOVEMBER 2025

Question 13

2 marks | Standard Form

13 Work out $\left(9 \times 10^{80}\right)^3$

Give your answer in standard form.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4WM2/01 November 2025

1 Use index laws

(9×10⁸⁰)³ = 9³ × (10⁸⁰)³

2 Solve inequality

9³ = 729

(10⁸⁰)³ = 10^{80 × 3} = 10²⁴⁰

3 Convert standard form

729 × 10²⁴⁰ = 7.29 × 10² × 10²⁴⁰ = 7.29 × 10²⁴²

✓ **FINAL ANSWER**

7.29 × 10²⁴²

4MA1/2HR JUNE 2026

Question 7

3 marks | Standard Form

7 (a) Write 4×10^{-5} as an ordinary number.

.....

(1)

(b) Work out $(6 \times 10^{158}) \times (2 \times 10^{72})$
Give your answer in standard form.

.....

(2)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2026

1 Part (a) | Step 1

$$4 \times 10^{-5} = \frac{4}{100000} = 0.00004.$$

Move the decimal point five places left.

2 Part (b) | Step 1

$$(6 \times 10^{158})(2 \times 10^{72}) = 12 \times 10^{230} = 1.2 \times 10^{231}.$$

Multiply the coefficients and add the exponents; normalize the coefficient into [1,10).

FINAL ANSWER

(a) 0.00004. (b) 1.2×10^{231}.

4MA1/1H JUNE 2026

Question 9

3 marks | Standard Form

9 (a) Write 0.0651 in standard form.

.....

(1)

(b) Work out $\left(4 \times 10^{56}\right) \times \left(8 \times 10^{74}\right)$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2026

1 Part (a) | Step 1

$$0.0651 = 6.51 \times 10^{-2}.$$

The coefficient must be at least 1 and less than 10.

2 Part (b) | Step 1

$$(4 \times 10^{56})(8 \times 10^{74}) = 32 \times 10^{130} = 3.2 \times 10^{131}.$$

Multiply the coefficients, add the exponents, and adjust the coefficient to standard form.

FINAL ANSWER

(a) 6.51×10^{-2} (b) 3.2×10^{131}

TOPIC

Ratio Toolkit

Every past-paper question on this topic, 2020 to 2026.

Question 10

5 marks | Ratio Toolkit

10 Payel makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7 : 5 : 3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.

36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

.....

(Total for Question 10 is 5 marks)

1 **The total number of parts**

The total number of parts is

$$7 + 5 + 3 = 15$$

2 **Each part is**

Each part is

$$300 \div 15 = 20$$

3 **Split the ratio**

So the numbers of cards are:

$$140, \quad 100, \quad 60$$

4 **The birthday cards with numbers**

The birthday cards with numbers are

$$\frac{2}{5} \times 140 = 56$$

5 **The anniversary cards with numbers**

The anniversary cards with numbers are

$$36\% \times 100 = 36$$

6 **Split the ratio**

So the total with numbers is

$$56 + 36 = 92$$

7 **The fraction of all cards**

The fraction of all cards is

$$\frac{92}{300} = \frac{23}{75}$$

✓ **FINAL ANSWER**

$\frac{23}{75}$.

4MA1/1HR JANUARY 2020

Question 1

3 marks | Ratio Toolkit

1 Brendon, Asha and Julie share some money in the ratios 3 : 2 : 6
The **total** amount of money that Asha and Julie receive is $36

Work out the amount of money that Brendon receives.

$.....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2020

1 The ratio is

The ratio is

$$\text{Brendon} : \text{Asha} : \text{Julie} = 3 : 2 : 6$$

2 Asha and Julie together have

Asha and Julie together have

$$2 + 6 = 8 \text{ parts}$$

3 These 8 parts are worth

These 8 parts are worth $\$36$, so one part is

$$36 \div 8 = 4.50$$

4 Brendon has 3 parts

Brendon has 3 parts:

$$3 \times 4.50 = 13.50$$

FINAL ANSWER

Brendon receives $\$13.50$.

4MA1/1H JANUARY 2020

Question 5

6 marks | Ratio Toolkit

5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3 : 5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2020

1 Split the ratio

Girls : boys $= 3 : 5$, so there are 8 parts.

$$\text{girls} = 120 \times \frac{3}{8} = 45$$

$$\text{boys} = 120 \times \frac{5}{8} = 75$$

2 Split the ratio

Boys climbing:

$$\frac{16}{25} \times 75 = 48$$

3 Split the ratio

Boys sailing:

$$75 - 48 = 27$$

4 Split the ratio

For the girls, twice as many go sailing as climbing, so the girls are split in the ratio

$$\text{sailing} : \text{climbing} = 2 : 1$$

5 **Split the ratio**

Girls sailing:

$$\frac{2}{3} \times 45 = 30$$

6 **Total sailing**

Total sailing:

$$27 + 30 = 57$$

✓ **FINAL ANSWER**

57

4MA1/2H JANUARY 2020

Question 12

4 marks | Ratio Toolkit

12 Astrid wants to buy some oil.
She can buy the oil from either Dane Oil or Arctic Oil.

Here is information about the price that each company will charge Astrid.

Dane Oil	Arctic Oil
(4.2×10^5) litres for 2 500 000 Krone	(8.6×10^5) litres for 770 000 Dollars

Astrid wants to get the better value for money for the oil.

1 Dollar = 6.57 Krone

From which company should she buy her oil, Dane Oil or Arctic Oil?
You must show your working.

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2020

1 Convert standard form

Dane Oil costs 2500000 Krone for

$$4.2 \times 10^5 = 420000$$

2 litres

litres.

$$\text{Dane cost per litre} = \frac{2500000}{420000} = 5.952\ldots$$

3 Convert standard form

So Dane Oil costs about 5.95 Krone per litre.

4 Arctic Oil costs dollars. Convert

Arctic Oil costs 770000 dollars. Convert this to Krone:

$$770000 \times 6.57 = 5058900$$

5 **Arctic Oil gives**

Arctic Oil gives

$$8.6 \times 10^5 = 860000$$

6 **litres**

litres.

$$\text{Arctic cost per litre} = \frac{5058900}{860000} = 5.882\ldots$$

7 **Arctic Oil has the lower**

Arctic Oil has the lower cost per litre.

✓ **FINAL ANSWER**

Arctic Oil

4MA1/1H JANUARY 2021

Question 1

4 marks | Ratio Toolkit

1 Pieter owns a currency conversion shop.

Last Monday, Pieter changed a total of 20 160 rand into a number of different currencies.

He changed $\frac{3}{10}$ of the 20 160 rand into euros.

He changed the rest of the rands into dollars, rupees and francs in the ratios $9:5:2$

Pieter changed more rands into dollars than he changed into francs.

Work out how many more.

..... rand

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2021

1 **of the money is changed**

$\frac{3}{10}$ of the money is changed to euros, so the rest is

$$\frac{7}{10} \times 20160 = 14112$$

2 **Split the ratio**

This rest is divided in the ratio

$$\text{dollars} : \text{rupees} : \text{francs} = 9 : 5 : 2$$

3 **There are**

There are

$$9 + 5 + 2 = 16 \text{ parts}$$

4 **Split the ratio**

The difference between dollars and francs is

$$9 - 2 = 7 \text{ parts}$$

5 **Split the ratio**

So the difference is

$$\frac{7}{16} \times 14112 = 6174$$

✓ **FINAL ANSWER**

6174 rand more is changed into dollars than francs.

4MA1/1H MAY 2021

Question 4

3 marks | Ratio Toolkit

4 On a farm there are chickens, ducks and pigs.

The ratio of the number of chickens to the number of ducks is 7 : 2
The ratio of the number of ducks to the number of pigs is 5 : 9
There are 36 pigs on the farm.

Work out the number of chickens on the farm.

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May 2021

1 Ducks pigs

Ducks : pigs $= \mathbf{5 : 9}$.

2 Split the ratio

There are $\mathbf{36}$ pigs, so

$$9 \text{ parts} = 36$$

$$1 \text{ part} = 4$$

$$\text{ducks} = 5 \times 4 = 20$$

3 Chickens ducks

Chickens : ducks $= \mathbf{7 : 2}$.

$$2 \text{ parts} = 20$$

$$1 \text{ part} = 10$$

$$\text{chickens} = 7 \times 10 = 70$$

FINAL ANSWER

70

4MA1/1H JANUARY 2022

Question 2

3 marks | Ratio Toolkit

2 Danil, Gabriel and Hadley share some money in the ratios $3:5:9$
The difference between the amount of money that Gabriel receives and the amount of money that Hadley receives is 196 euros.

Work out the amount of money that Danil receives.

..... euros

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2022

1 The ratio is

The ratio is

$$\text{Danil} : \text{Gabriel} : \text{Hadley} = 3 : 5 : 9$$

2 Split the ratio

The difference between Gabriel and Hadley is

$$9 - 5 = 4 \text{ parts}$$

3 Split the ratio

These 4 parts are 196 euros, so one part is

$$196 \div 4 = 49$$

4 Danil has 3 parts

Danil has 3 parts:

$$3 \times 49 = 147$$

FINAL ANSWER

Danil receives 147 euros.

4MA1/1H MAY 2023

Question 1

3 marks | Ratio Toolkit

1 Last season, the number of goals scored by Arjun, by Simon and by Kath for their football team were in the ratios $2:5:8$

Simon scored 12 more goals than Arjun.

Work out the number of goals scored by Kath.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2023

1 The ratio is

The ratio is

$$\text{Arjun} : \text{Simon} : \text{Kath} = 2 : 5 : 8$$

2 Simon scored more than Arjun

Simon scored 12 more than Arjun.

$$5 - 2 = 3 \text{ parts}$$

$$3 \text{ parts} = 12$$

$$1 \text{ part} = 4$$

3 Kath scored

Kath scored

$$8 \times 4 = 32$$

FINAL ANSWER

32

4MA1/2H JUNE 2023

Question 5

4 marks | Ratio Toolkit

5 Nancy has some coins with a total value of 85 pence.
She has only 2 pence coins and 5 pence coins.
The ratio

$$\text{number of 2 pence coins} : \text{number of 5 pence coins} = 1 : 3$$

Nancy has more 5 pence coins than 2 pence coins.

How many more?

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2023

1 Let the number of pence

Let the number of 2 pence coins be x.

2 Split the ratio

Then the number of 5 pence coins is $3x$.

3 Total value

Total value:

$$2x + 5(3x) = 85$$

$$17x = 85$$

$$x = 5$$

4 Split the ratio

So there are 5 two-pence coins and 15 five-pence coins.

$$15 - 5 = 10$$

FINAL ANSWER

10 more 5 pence coins

4MA1/2H NOVEMBER 2023

Question 5

3 marks | Ratio Toolkit

5 Avril bakes a cake.

She uses flour, butter and sugar such that

weight of flour : weight of butter = 6 : 5
weight of butter : weight of sugar = 3 : 2

Avril uses 120 grams of sugar.

Work out the weight of flour Avril uses.

..... grams

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2023

1 Butter sugar

Butter : sugar $= \mathbf{3 : 2}$.

2 Split the ratio

Sugar is 120 g, so

$$2 \text{ parts} = 120$$

$$1 \text{ part} = 60$$

$$\text{butter} = 3 \times 60 = 180$$

3 Flour butter

Flour : butter $= \mathbf{6 : 5}$.

$$5 \text{ parts} = 180$$

$$1 \text{ part} = 36$$

$$\text{flour} = 6 \times 36 = 216$$

FINAL ANSWER

216 grams

4MA1/2H JUNE 2025

Question 6

4 marks | Ratio Toolkit

6 Pau, Sam and Tia share £240 in the ratios $3:4:5$

Sam and Tia each give £10 of their share to Pau.

Work out the ratios of the amounts of money that Pau, Sam and Tia now have.
Give your answer in its simplest form.

..... : ..... : .....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2025

1 The original ratio is

The original ratio is

$$\text{Pau} : \text{Sam} : \text{Tia} = 3 : 4 : 5$$

2 There are

There are

$$3 + 4 + 5 = 12 \text{ parts}$$

3 Each part is

Each part is

$$240 \div 12 = 20$$

4 Split the ratio

So the original amounts are

$$60,\ 80,\ 100$$

5 Sam gives 10 to Pau

Sam gives 10 to Pau and Tia gives 10 to Pau, so the new amounts are

$$80,\ 70,\ 90$$

6 **Divide by 10**

Divide by 10:

$$80 : 70 : 90 = 8 : 7 : 9$$

✓ **FINAL ANSWER**

The new ratio is $8 : 7 : 9$.

4WM2/01 MAY 2025

Question 7

4 marks | Ratio Toolkit

7 Pau, Sam and Tia share £240 in the ratios $3 : 4 : 5$

Sam and Tia each give £10 of their share to Pau.

Work out the ratios of the amounts of money that Pau, Sam and Tia now have.
Give your answer in its simplest form.

..... : ..... : .....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4WM2/01 May 2025

1 The original ratio is

The original ratio is

$$\text{Pau} : \text{Sam} : \text{Tia} = 3 : 4 : 5$$

2 There are

There are

$$3 + 4 + 5 = 12 \text{ parts}$$

3 Each part is

Each part is

$$240 \div 12 = 20$$

4 Split the ratio

So the original amounts are

$$60,\ 80,\ 100$$

5 Sam gives 10 to Pau

Sam gives 10 to Pau and Tia gives 10 to Pau, so the new amounts are

$$80,\ 70,\ 90$$

6 Divide by 10

Divide by 10:

$$80 : 70 : 90 = 8 : 7 : 9$$

FINAL ANSWER

The new ratio is $8 : 7 : 9$.

4MA1/1H NOVEMBER 2025

Question 6

4 marks | Ratio Toolkit

6 Eli and Peta share \$275 in the ratio 2 : 3

Eli gives $\frac{3}{11}$ of his share to charity.

Peta gives 0.32 of her share to charity.

Work out the total amount that Eli and Peta give to charity.

\$.....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2025

1 The ratio is

The ratio is

$$\text{Eli} : \text{Peta} = 2 : 3$$

2 Split the ratio

There are 5 parts, so one part is

$$275 \div 5 = 55$$

3 Eli gets

Eli gets

$$2 \times 55 = 110$$

4 Peta gets

Peta gets

$$3 \times 55 = 165$$

5 Eli gives

Eli gives

$$\frac{3}{11} \times 110 = 30$$

6 **Peta gives**

Peta gives

$$0.32 \times 165 = 52.8$$

7 **Total given to charity**

Total given to charity:

$$30 + 52.8 = 82.8$$

✓ **FINAL ANSWER**

82.8.

4MA1/2HR JUNE 2026

Question 5

5 marks | Ratio Toolkit

5 Sarah and Dej each give some money to charity **A** and to charity **B**

Sarah shares \$400 between charity **A** and charity **B** in the ratio 2 : 3

Dej shares \$375 between the two charities.
He gives twice as much money to charity **B** as he gives to charity **A**

Dej gives more money to charity **B** than Sarah gives to charity **B**

How much more?

\$.....

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\text{Sarah to B} = 400 \times \frac{3}{2+3} = 240 \text{ dollars.}$$

Charity B receives 3 of the 5 equal shares.

2 Solution | Step 2

Dej gives to A and B in the ratio $1 : 2$, so

$$\text{Dej to B} = 375 \times \frac{2}{3} = 250 \text{ dollars.}$$

Twice as much to B means that B receives 2 of the 3 shares.

3 Solution | Step 3

$$250 - 240 = 10 \text{ dollars.}$$

Subtract the two donations to charity B.

FINAL ANSWER

\$10 more.

TOPIC

Forming & Solving Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

Question 3

4 marks | Forming & Solving Equations

3 Three tins, A, B and C, each contain buttons.

Tin A contains x buttons.
Tin B contains 4 times the number of buttons that tin A contains.
Tin C contains 7 fewer buttons than tin A.

The total number of buttons in the three tins is 137

Work out the number of buttons in tin C.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2020

1 Write each tin in terms of x

Tin A has x buttons, tin B has $4x$, tin C has $x - 7$.

2 Form and solve the equation

$$x + 4x + (x - 7) = 137 \Rightarrow 6x - 7 = 137 \Rightarrow 6x = 144 \Rightarrow x = 24$$

3 Buttons in tin C

$$24 - 7 = 17$$

✓ **FINAL ANSWER**

17 buttons

4MA1/2H NOVEMBER 2021

Question 2

4 marks | Forming & Solving Equations

2 Here are two rectangles, rectangle A and rectangle B.

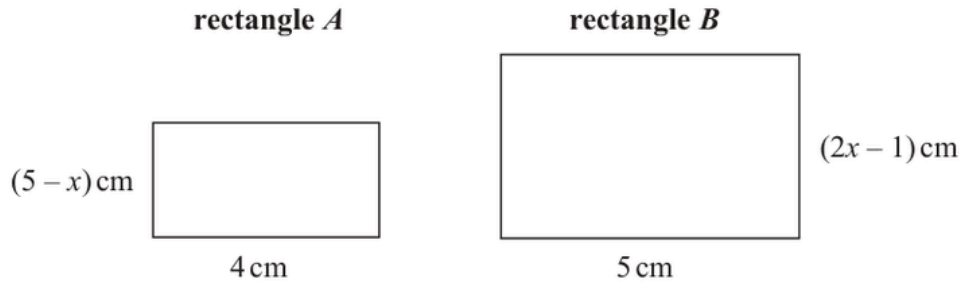


Diagram **NOT** accurately drawn

The area of rectangle B is twice the area of rectangle A.

Work out the value of x.
Show your working clearly.

$x =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2021

1 Area of rectangle A

Area of rectangle A:

$$4(5 - x)$$

2 Area of rectangle B

Area of rectangle B:

$$5(2x - 1)$$

3 **Rectangle B has twice the**

Rectangle B has twice the area of rectangle A:

$$5(2x - 1) = 2 \cdot 4(5 - x)$$

$$10x - 5 = 40 - 8x$$

$$18x = 45$$

$$x = 2.5$$

✓ **FINAL ANSWER**

$x = 2.5$.

4MA1/1H NOVEMBER 2021

Question 5

5 marks | Forming & Solving Equations

5 Yasmin has some identical rectangular tiles.
Each tile is L cm by W cm.

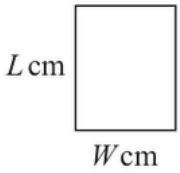


Diagram **NOT** accurately drawn

Using 9 of her tiles, Yasmin makes rectangle $ABCD$, shown in the diagram below.

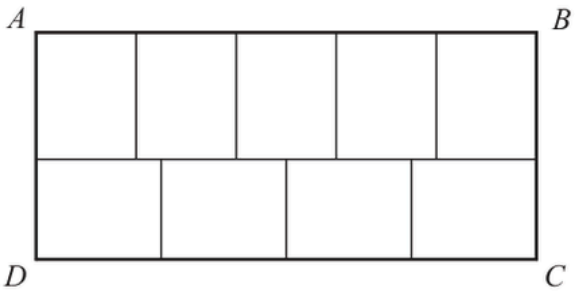


Diagram **NOT** accurately drawn

The area of $ABCD$ is $1620\,\text{cm}^2$

Work out the value of L and the value of W.

$L =$ ..... $W =$ .....

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2021

1 The top row has 5

The top row has 5 tile widths and the bottom row has 4 tile lengths, so

$$5W = 4L$$

$$L = \frac{5}{4}W$$

2 Use trigonometry

The rectangle has width $5W$ and height $L + W$:

$$5W(L + W) = 1620$$

3 **Substitute**

Substitute $L = \frac{5}{4}W$:

$$5W\left(\frac{5}{4}W + W\right) = 1620$$

$$5W\left(\frac{9}{4}W\right) = 1620$$

$$\frac{45}{4}W^2 = 1620$$

$$W^2 = 144$$

$$W = 12, \qquad L = \frac{5}{4}(12) = 15$$

✓ **FINAL ANSWER**

$L = 15\ \text{cm}$, $W = 12\ \text{cm}$.

4MA1/1H NOVEMBER 2023

Question 2

6 marks | Forming & Solving Equations

2 (a) Factorise fully $12pq - 18p$

.....

(2)

There are 56 metal bars in a box.
Each metal bar is gold or silver or zinc.

y metal bars are gold.
$(3y + 7)$ metal bars are silver.
$(2y - 5)$ metal bars are zinc.

(b) Work out the number of metal bars that are zinc.
Show clear algebraic working.

.....

(4)

(Total for Question 2 is 6 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2023

1 **(a)**

(a)

$$12pq - 18p = 6p(2q - 3)$$

2 **Solve equation**

(b) The total number of bars is 56, so

$$y + (3y + 7) + (2y - 5) = 56$$

$$6y + 2 = 56$$

$$y = 9$$

3 **The number of zinc bars**

The number of zinc bars is

$$2y - 5 = 2(9) - 5 = 13$$

FINAL ANSWER

(a) $6p(2q - 3)$, (b) 13 zinc bars

4MA1/2H NOVEMBER 2025

Question 20

3 marks | Forming & Solving Equations

20 Here is a quadratic equation.

$$ax^2 + 4x + c = 0$$

The solutions of this equation are given by $x = \dfrac{-4 \pm 2\sqrt{39}}{10}$

Find the value of a and the value of c
Show your working clearly.

$a =$ .....

$c =$ .....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2025

1 Calculate value

For

$$ax^2 + 4x + c = 0$$

2 the quadratic formula gives

the quadratic formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4ac}}{2a}$$

3 The given solutions are

The given solutions are

$$x = \frac{-4 \pm 2\sqrt{39}}{10}$$

4 Compare denominators

Compare denominators:

$$2a = 10$$

$$a = 5$$

5 **Compare the square root parts**

Compare the square root parts:

$$\sqrt{16 - 4ac} = 2\sqrt{39}$$

6 **Square both sides**

Square both sides:

$$16 - 4ac = 156$$

7 **Use**

Use $a = 5$:

$$16 - 20c = 156$$

$$-20c = 140$$

$$c = -7$$

✓ **FINAL ANSWER**

$a = 5,\ c = -7$

TOPIC

Rearranging Formulae

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 3

8 marks | Rearranging Formulae

3 (a) Make a the subject of $d = g + 2ac$

.....

(2)

(b) Factorise fully $9ef - 12f$

.....

(2)

(c) Expand and simplify $(x + 2)(x - 5)$

.....

(2)

(d) Simplify fully $\dfrac{n^4 \times n^7}{n^5}$

.....

(2)

(Total for Question 3 is 8 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2020

1 **(a)**

(a)

$$d = g + 2ac$$

$$d - g = 2ac$$

$$a = \frac{d - g}{2c}$$

2 **(b)**

(b)

$$9ef - 12f = 3f(3e - 4)$$

3 **(c)**

(c)

$$(x+2)(x-5) = x^2 - 3x - 10$$

---

4 **(d)**

(d)

$$\frac{n^4 \times n^7}{n^5} = n^{4+7-5} = n^6$$

✓ **FINAL ANSWER**

$a = \frac{d-g}{2c}$, $3f(3e-4)$, $x^2 - 3x - 10$, n^6

4MA1/2H JANUARY 2020

Question 16

4 marks | Rearranging Formulae

16 Make x the subject of $y = \sqrt{\dfrac{x+1}{x-4}}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H January 2020

1 Rearrange formula

$$y = \sqrt{\frac{x+1}{x-4}}$$

2 Square both sides

Square both sides:

$$y^2 = \frac{x+1}{x-4}$$

$$y^2(x-4) = x+1$$

$$y^2x - 4y^2 = x+1$$

$$x(y^2-1) = 4y^2+1$$

FINAL ANSWER

$$x = \frac{4y^2+1}{y^2-1}$$

4MA1/1HR JANUARY 2021

Question 6

5 marks | Rearranging Formulae

6 (a) Make c the subject of $A = \frac{c}{y} - 5z$

.....

(2)

(b) Write down the value of g^0

.....

(1)

(c) Factorise $x^2 - 11x + 24$

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$A = \frac{c}{y} - 5z$$

$$A + 5z = \frac{c}{y}$$

$$c = y(A + 5z)$$

2 **(b)**

(b) $g^0 = 1$.

3 **(c)**

(c)

$$x^2 - 11x + 24 = (x - 3)(x - 8)$$

✓ **FINAL ANSWER**

$c = y(A + 5z)$, 1, $(x - 3)(x - 8)$

4MA1/2H JANUARY 2021

Question 14

7 marks | Rearranging Formulae

14 (a) Solve $\frac{9a-7}{5} - \frac{3a-7}{4} = 4.55$

Show clear algebraic working.

$a =$ .....

(3)

(b) Make c the subject of the formula $p = \sqrt{\frac{ac+8}{3+c}}$

.....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2021

1 **(a)**

(a)

$$\frac{9a-7}{5} - \frac{3a-7}{4} = 4.55$$

2 **Multiply by 20**

Multiply by 20:

$$4(9a-7) - 5(3a-7) = 91$$

$$36a - 28 - 15a + 35 = 91$$

$$21a + 7 = 91$$

$$a = 4$$

3 **(b)**

(b)

$$p = \sqrt{\frac{ac+8}{3+c}}$$

$$p^2 = \frac{ac+8}{3+c}$$

$$p^2(3+c) = ac+8$$

$$c(p^2 - a) = 8 - 3p^2$$

FINAL ANSWER

(a) $a = 4$, (b) $c = \frac{8-3p^2}{p^2-a}$

4MA1/1H MAY 2021

Question 5

6 marks | Rearranging Formulae

5 (a) Expand and simplify $3x(2x + 3) - x(3x + 5)$

.....

(2)

(b) Make t the subject of the formula $p = at - d$

.....

(2)

Given that $\frac{w^5 \times w^n}{w^3} = w^{10}$

(c) work out the value of n.

$n =$ .....

(2)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2021

1 **(a)**

(a)

$$3x(2x+3) - x(3x+5) = 6x^2 + 9x - 3x^2 - 5x$$

$$= 3x^2 + 4x$$

2 **(b)**

(b)

$$p = at - d$$

$$p + d = at$$

$$t = \frac{p+d}{a}$$

3 **(c)**

(c)

$$\frac{w^5 \times w^n}{w^3} = w^{n+2} = w^{10}$$

4 **Rearrange formula**

So $n + 2 = 10$, hence $n = 8$.

FINAL ANSWER

(a) $3x^2 + 4x$, (b) $t = \frac{p+d}{a}$, (c) $n = 8$

4MA1/1H NOVEMBER 2021

Question 17

8 marks | Rearranging Formulae

17 (a) Factorise $6y^2 - y - 5$

.....

(2)

(b) Make f the subject of $w = \dfrac{2f + 3}{8 - f}$

.....

(3)

(c) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

(Total for Question 17 is 8 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2021

1 **(a)**

(a)

$$6y^2 - y - 5 = (6y + 5)(y - 1)$$

2 **(b)**

(b)

$$w = \frac{2f + 3}{8 - f}$$

$$w(8 - f) = 2f + 3$$

$$8w - 3 = f(w + 2)$$

$$f = \frac{8w - 3}{w + 2}$$

3 **(c)**

(c)

$$4x^2 - 8x + 7 = 4(x^2 - 2x) + 7$$

$$= 4\left((x-1)^2 - 1\right) + 7$$

$$= 4(x-1)^2 + 3$$

FINAL ANSWER

$(6y+5)(y-1)$, $f = \frac{8w-3}{w+2}$, $4(x-1)^2 + 3$

4MA1/2HR JANUARY 2022

Question 15

4 marks | Rearranging Formulae

15 Make t the subject of $n^2 = \dfrac{4d + t^3}{t^3}$

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR January 2022

1 Rearrange formula

$$n^2 = \frac{4d + t^3}{t^3}$$

$$n^2 t^3 = 4d + t^3$$

$$t^3(n^2 - 1) = 4d$$

$$t^3 = \frac{4d}{n^2 - 1}$$

FINAL ANSWER

$$t = \sqrt[3]{\frac{4d}{n^2 - 1}}$$

4MA1/2H JUNE 2022

Question 14

7 marks | Rearranging Formulae

14 (a) Expand and simplify $(5 - x)(2x + 3)(x + 4)$
Show your working clearly.

.....

(3)

(b) Make c the subject of $g = \dfrac{c + 3}{4 + c} - 7$

.....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2022

1 **(a)**

(a)

$$(5 - x)(2x + 3)(x + 4)$$

$$(2x + 3)(x + 4) = 2x^2 + 11x + 12$$

$$(5 - x)(2x^2 + 11x + 12) = -2x^3 - x^2 + 43x + 60$$

2 **(b)**

(b)

$$g = \frac{c + 3}{4 + c} - 7$$

$$g + 7 = \frac{c + 3}{c + 4}$$

$$(g + 7)(c + 4) = c + 3$$

$$c(g + 6) = -4g - 25$$

FINAL ANSWER

(a) $-2x^3 - x^2 + 43x + 60$, (b) $c = \frac{-4g-25}{g+6}$

4MA1/2H JUNE 2023

Question 17

7 marks | Rearranging Formulae

17 (a) Expand and simplify $(x + 6)(3x - 2)(x + 6)$

.....

(3)

(b) Make e the subject of $w = \sqrt{\dfrac{e+g}{ef-d}}$

.....

(4)

(Total for Question 17 is 7 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2023

1 **(a)**

(a)

$$(x+6)(3x-2)(x+6) = (x+6)^2(3x-2)$$

$$= (x^2+12x+36)(3x-2)$$

$$= 3x^3+34x^2+84x-72$$

2 **(b)**

(b)

$$w = \sqrt{\frac{e+g}{ef-d}}$$

$$w^2 = \frac{e+g}{ef-d}$$

$$w^2(ef-d) = e+g$$

$$e(fw^2-1) = g+dw^2$$

FINAL ANSWER

(a) $3x^3 + 34x^2 + 84x - 72$, (b) $e = \frac{g+dw^2}{fw^2-1}$

4MA1/2HR JUNE 2023

Question 17

4 marks | Rearranging Formulae

17 Make x the subject of $y = \sqrt[3]{\dfrac{6+5x}{x+4}}$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2023

1 Rearrange formula

$$y = \sqrt[3]{\frac{6+5x}{x+4}}$$

$$y^3 = \frac{6+5x}{x+4}$$

$$y^3(x+4) = 6+5x$$

$$x(y^3-5) = 6-4y^3$$

FINAL ANSWER

$$x = \frac{6-4y^3}{y^3-5}$$

4MA1/1H NOVEMBER 2023

Question 15

3 marks | Rearranging Formulae

15 Make n the subject of the formula $x = \dfrac{3p+n}{3n-4}$

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2023

1 Rearrange formula

$$x = \frac{3p+n}{3n-4}$$

$$x(3n-4) = 3p+n$$

$$3xn - 4x = 3p+n$$

$$n(3x-1) = 3p+4x$$

FINAL ANSWER

$n = \frac{3p+4x}{3x-1}$

4MA1/2H JUNE 2024

Question 15

7 marks | Rearranging Formulae

15 (a) Make g the subject of $e = \sqrt{\dfrac{7g+5}{11+2g}}$

.....

(4)

(b) Solve the inequality $3y^2 + 4y - 32 > 0$
Show your working clearly.

.....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2024

1 **(a)**

(a)

$$e = \sqrt{\frac{7g+5}{11+2g}}$$

$$e^2 = \frac{7g+5}{11+2g}$$

$$e^2(11+2g) = 7g+5$$

$$g(2e^2-7) = 5-11e^2$$

$$g = \frac{5-11e^2}{2e^2-7}$$

2 **(b)**

(b)

$$3y^2+4y-32 = (3y-8)(y+4)$$

3 Solve quadratic equation

The critical values are $y = -4$ and $y = \frac{8}{3}$. Since the quadratic is positive outside the roots,

$$y < -4 \quad \text{or} \quad y > \frac{8}{3}$$

FINAL ANSWER

(a) $g = \frac{5-11e^2}{2e^2-7}$, (b) $y < -4$ or $y > \frac{8}{3}$

4MA1/2H NOVEMBER 2024

Question 12

7 marks | Rearranging Formulae

12 (a) Solve $\frac{3x+2}{5} - \frac{2x+1}{3} = x$

Show clear algebraic working.

$x =$ .....

(3)

(b) Make c the subject of the formula $f = \sqrt{\frac{a+bc}{c-d}}$

.....

(4)

(Total for Question 12 is 7 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2024

1 **(a)**

(a)

$$\frac{3x+2}{5} - \frac{2x+1}{3} = x$$

2 **Multiply by 15**

Multiply by 15:

$$3(3x+2) - 5(2x+1) = 15x$$

$$9x + 6 - 10x - 5 = 15x$$

$$1 = 16x$$

$$x = \frac{1}{16}$$

3 **(b)**

(b)

$$f = \sqrt{\frac{a+bc}{c-d}}$$

$$f^2 = \frac{a+bc}{c-d}$$

$$f^2(c-d) = a+bc$$

$$c(f^2-b) = a+df^2$$

FINAL ANSWER

(a) $x = \frac{1}{16}$, (b) $c = \frac{a+df^2}{f^2-b}$

4WM2/01R MAY 2025

Question 14

4 marks | Rearranging Formulae

14 Make k the subject of $p = \dfrac{8k^2 + 5}{7 - 3k^2}$

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4WM2/01R May 2025

1 Required

Make *k* the subject of $displaystylep = dfrac8k^2 + 57 - 3k^2$.

2 Multiply both sides by (7

Multiply both sides by (7−3*k*²):

p(7−3*k*²) = 8*k*² + 5

3 Expand 7 p 3 p

Expand:

7*p* − 3*p***k*² = 8*k*² + 5

4 Collect all k 2 terms

Collect all *k*² terms on one side:

7*p* − 5 = 8*k*² + 3*p***k*²

5 Factorise and solve for k

Factorise and solve for *k*:

$$7p - 5 = k^2(3p + 8) impliesk^2 = frac7p - 53p + 8$$

$$k = pmsqrtdfrac7p - 53p + 8$$

6 Note

Include both ± roots unless the context specifies the sign of *k*.

FINAL ANSWER

$k = pmsqrtdfrac7p - 53p + 8$

4MA1/2H JUNE 2025

Question 16

4 marks | Rearranging Formulae

16 Make t the subject of the formula $c = \dfrac{t^2 + 3}{7 - 8t^2}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2025

1 Rearrange formula

$$c = \frac{t^2 + 3}{7 - 8t^2}$$

$$c(7 - 8t^2) = t^2 + 3$$

$$7c - 3 = t^2(8c + 1)$$

$$t^2 = \frac{7c - 3}{8c + 1}$$

FINAL ANSWER

$$t = \pm\sqrt{\frac{7c-3}{8c+1}}$$

4MA1/2HR JUNE 2025

Question 17

4 marks | Rearranging Formulae

17 Make k the subject of $p = \dfrac{8k^2 + 5}{7 - 3k^2}$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2025

1 Rearrange formula

$$p = \frac{8k^2 + 5}{7 - 3k^2}$$

$$p(7 - 3k^2) = 8k^2 + 5$$

$$7p - 5 = k^2(3p + 8)$$

$$k^2 = \frac{7p - 5}{3p + 8}$$

FINAL ANSWER

$$k = \pm\sqrt{\frac{7p-5}{3p+8}}$$

4WM2/01 MAY 2025

Question 19

4 marks | Rearranging Formulae

19 Make t the subject of the formula $c = \frac{t^2 + 3}{7 - 8t^2}$

.....

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4WM2/01 May 2025

1 Evaluate fraction

$$c = \frac{t^2 + 3}{7 - 8t^2}$$

$$c(7 - 8t^2) = t^2 + 3$$

$$7c - 3 = t^2(8c + 1)$$

$$t^2 = \frac{7c - 3}{8c + 1}$$

FINAL ANSWER

$$t = \pm\sqrt{\frac{7c - 3}{8c + 1}}.$$

4WM2/01 NOVEMBER 2025

Question 18

4 marks | Rearranging Formulae

18 Make w the subject of the formula $p = \sqrt{\dfrac{7w + y}{cw + k}}$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 November 2025

1 Multiply both sides denominator

$$p^2 = frac7w + ycw + k$$

Step 2: Multiply both sides by the denominator (*cw*+*k*).**

*p*²(*c**w*+*k*) = 7*w* + *y*

2 Expand brackets

*p*²*c**w* + *p*²*k* = 7*w* + *y*

Step 4: Collect all terms involving *w* on one side.

*p*²*c**w* − 7*w* = *y* − *p*²*k*

Step 5: Factor out *w*.

w(*p*²*c*−7) = *y* − *p*²*k*

Step 6: Divide to isolate *w*.

$$w = fracy - p^2kp^2c - 7$$

FINAL ANSWER

$$w = fracy - p^2kp^2c - 7$$

4MA1/1H NOVEMBER 2025

Question 18

4 marks | Rearranging Formulae

18 Make y the subject of $p = \sqrt{\dfrac{y+w}{3y-t}}$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2025

1 Rearrange formula

$$p = \sqrt{\frac{y+w}{3y-t}}$$

$$p^2 = \frac{y+w}{3y-t}$$

$$p^2(3y-t) = y+w$$

$$3p^2y - p^2t = y+w$$

$$y(3p^2-1) = w+p^2t$$

FINAL ANSWER

$y = \frac{w+p^2t}{3p^2-1}$

4MA1/2HR JUNE 2026

Question 4

2 marks | Rearranging Formulae

$P = c^2 - 5d$

(b) Work out the value of P when $c = -7$ and $d = 3$

$P =$ .....

(2)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2026

1 Part (b) | Step 1

$$P = (-7)^2 - 5(3) = 49 - 15 = 34.$$

The square applies to the whole negative number, so (-7) squared is positive.

FINAL ANSWER

(b) $P = 34$.

4MA1/1H JUNE 2026

Question 5

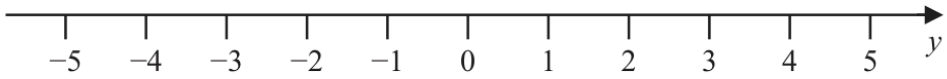
4 marks | Rearranging Formulae

(c) Make x the subject of $t = ax - 2n$

.....

(2)

(d) On the number line, represent the inequality $-1 < y \leqslant 3$



(2)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2026

1 Part (c) | Step 1

From $t = ax - 2n$, add $2n$ to give $t + 2n = ax$.

Use the same operation on both sides.

2 Part (c) | Step 2

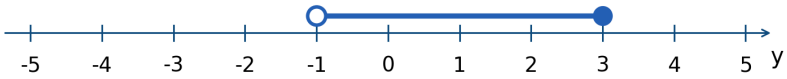
$$x = \frac{t + 2n}{a} \quad (a \neq 0).$$

Divide by the coefficient of x.

3 Part (d) | Step 1

Draw an open circle at $y = -1$, a filled circle at $y = 3$, and a straight segment joining them. The represented interval is $(-1, 3]$.

The strict lower inequality excludes -1; the non-strict upper inequality includes 3.



FINAL ANSWER

(c) $x = (t + 2n)/a$ (d) $(-1, 3]$

4MA1/2H JUNE 2026

Question 18

4 marks | Rearranging Formulae

18 Make x the subject of the formula $a = \dfrac{3w - x^2}{x^2 + 6}$

.....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2026

1 Solution | Step 1

$$a = \frac{3w - x^2}{x^2 + 6} \quad \Rightarrow \quad a(x^2 + 6) = 3w - x^2.$$

Multiply by x^2+6, which is positive for real x.

2 Solution | Step 2

$$ax^2 + 6a = 3w - x^2 \quad \Rightarrow \quad ax^2 + x^2 = 3w - 6a.$$

Collect both terms involving x^2 on one side.

3 Solution | Step 3

$$(a + 1)x^2 = 3w - 6a \quad \Rightarrow \quad x^2 = \frac{3w - 6a}{a + 1}.$$

Factor out x^2 and divide by a+1, assuming a is not -1.

4 Solution | Step 4

$$x = \pm\sqrt{\frac{3w - 6a}{a + 1}} \quad (a \neq -1).$$

For real solutions the radicand must be nonnegative. If $a = -1$, the original equation instead requires $w = -2$; then every real x satisfies it.

Taking square roots gives both signs; handle separately the value excluded by division.

FINAL ANSWER

$x = \pm\sqrt{\dfrac{3w - 6a}{a + 1}}$, for $a \neq -1$ and nonnegative radicand.

4WM2/01R JUNE 2026

Question 19

3 marks | Rearranging Formulae

19 Make x the subject of $y = \frac{3x+5}{4-2x}$

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM2/01R June 2026

1 Multiply out the fraction

$$y(4-2x) = 3x+5 \Rightarrow 4y - 2xy = 3x + 5$$

2 Collect x terms on one side

$$4y - 5 = 3x + 2xy = x(3+2y)$$

3 Divide

$$x = \frac{4y-5}{2y+3}$$

✓ **FINAL ANSWER**

$$x = \frac{4y-5}{2y+3}$$

4MA1/2HR JUNE 2026

Question 20

3 marks | Rearranging Formulae

20 Make x the subject of $y = \dfrac{3x+5}{4-2x}$

.....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2026

1 Solution | Step 1

$$y(4-2x) = 3x+5 \quad \Rightarrow \quad 4y-2xy = 3x+5.$$

Multiply by the denominator, which is nonzero in the original expression.

2 Solution | Step 2

$$4y-5 = 3x+2xy = x(3+2y).$$

Collect both x-terms together and factor out x.

3 Solution | Step 3

$$x = \frac{4y-5}{2y+3}.$$

Divide by 2y+3. The inverse formula requires y not equal to -3/2; the original requires x not equal to 2.

FINAL ANSWER

$x = \frac{4y-5}{2y+3}$.

TOPIC

Simultaneous Equations

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 11

3 marks | Simultaneous Equations

11 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4WM2/01 Specimen

1 **Solve simultaneous equations**

From

$$3x - y = 7$$

$$y = 3x - 7$$

2 **Solve simultaneous equations**

Substitute into $7x + 3y = 3$:

$$7x + 3(3x - 7) = 3$$

$$16x - 21 = 3$$

$$16x = 24$$

$$x = 1.5$$

---

3 **Solve simultaneous equations**

Then

$$y = 3(1.5) - 7 = -2.5$$

✓ **FINAL ANSWER**

$x = 1.5,\ y = -2.5$.

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

1 **Solve simultaneous equations**

From

$$x - 2y = 3$$

$$x = 2y + 3$$

2 **Solve simultaneous equations**

Substitute into $x^2 - y^2 + 2x = 10$:

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

3 **Solve simultaneous equations**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

4 **Solve simultaneous equations**

If $y = -\frac{1}{3}$,

$$x = 2\left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

5 **Solve simultaneous equations**

If $y = -5$,

$$x = 2(-5) + 3 = -7$$

✓ **FINAL ANSWER**

$\left(\frac{7}{3}, -\frac{1}{3}\right)$ and $(-7, -5)$.

4MA1/2HR JANUARY 2020

Question 7

3 marks | Simultaneous Equations

7 Solve the simultaneous equations

$$3x + 5y = 6$$
$$7x - 5y = -11$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2020

1 Solve simultaneous equations

$$3x + 5y = 6$$

$$7x - 5y = -11$$

2 Add the equations

Add the equations:

$$10x = -5$$

$$x = -\frac{1}{2}$$

3 **Solve simultaneous equations**

Substitute into $3x + 5y = 6$:

$$3\left(-\frac{1}{2}\right) + 5y = 6$$

$$-\frac{3}{2} + 5y = 6$$

$$5y = \frac{15}{2}$$

$$y = \frac{3}{2}$$

✓ **FINAL ANSWER**

$x = -\frac{1}{2}$, $y = \frac{3}{2}$.

4MA1/1H MAY/NOVEMBER 2020

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$7x - 2y = 34$$
$$3x + 5y = -3$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May/November 2020

1 **Solve simultaneous equations**

$$7x - 2y = 34$$

$$3x + 5y = -3$$

2 **Multiply the first equation by**

Multiply the first equation by 5:

$$35x - 10y = 170$$

3 **Multiply the second equation by**

Multiply the second equation by 2:

$$6x + 10y = -6$$

4 **Add**

Add:

$$41x = 164$$

$$x = 4$$

5 **Solve simultaneous equations**

Substitute into $3x + 5y = -3$:

$$12 + 5y = -3$$

$$y = -3$$

FINAL ANSWER

$x = 4$, $y = -3$.

4MA1/1HR MAY/NOVEMBER 2020

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$7x + 2y = 5.5$$
$$3x - 5y = 17$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR May/November 2020

1 Solve simultaneous equations

$$7x + 2y = 5.5$$

$$3x - 5y = 17$$

2 Multiply the first equation by

Multiply the first equation by 5:

$$35x + 10y = 27.5$$

3 Multiply the second equation by

Multiply the second equation by 2:

$$6x - 10y = 34$$

4 Add

Add:

$$41x = 61.5$$

$$x = 1.5$$

5 **Solve simultaneous equations**

Substitute into $7x + 2y = 5.5$:

$$10.5 + 2y = 5.5$$

$$y = -2.5$$

FINAL ANSWER

$x = 1.5$, $y = -2.5$.

4MA1/2HR JANUARY 2021

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations $2x + 7y = 17$
$5x + 3y = -1$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2021

1 **Solve simultaneous equations**

$$2x + 7y = 17$$

$$5x + 3y = -1$$

2 **Multiply the first equation by**

Multiply the first equation by 5:

$$10x + 35y = 85$$

3 **Multiply the second equation by**

Multiply the second equation by 2:

$$10x + 6y = -2$$

4 **Subtract**

Subtract:

$$29y = 87$$

$$y = 3$$

5 **Solve simultaneous equations**

Substitute into $2x + 7y = 17$:

$$2x + 21 = 17$$

$$2x = -4$$

$$x = -2$$

✓ **FINAL ANSWER**

$x = -2$, $y = 3$.

4MA1/1HR JANUARY 2021

Question 17

5 marks | Simultaneous Equations

17 Solve the simultaneous equations

$$x - 6y = 5$$
$$xy - 2y^2 = 6$$

Show clear algebraic working.

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR January 2021

1 Solve simultaneous equations

$$x - 6y = 5$$

$$xy - 2y^2 = 6$$

2 From the first equation

From the first equation:

$$x = 6y + 5$$

3 Substitute into the second equation

Substitute into the second equation:

$$y(6y + 5) - 2y^2 = 6$$

$$4y^2 + 5y - 6 = 0$$

4 Factorise

Factorise:

$$(4y - 3)(y + 2) = 0$$

5 Solve simultaneous equations

So

$$y = \frac{3}{4} \quad \text{or} \quad y = -2$$

6 **Solve simultaneous equations**

If $y = \frac{3}{4}$:

$$x = 6\left(\frac{3}{4}\right) + 5 = \frac{19}{2}$$

---

7 **Solve simultaneous equations**

If $y = -2$:

$$x = 6(-2) + 5 = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{19}{2}, \frac{3}{4}\right)$ or $(-7, -2)$.

4MA1/1H JANUARY 2021

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$
$$x + 2y - 1 = 0$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2021

1 Solve simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$

$$x + 2y - 1 = 0$$

2 From the second equation

From the second equation:

$$x = 1 - 2y$$

3 Substitute into the first equation

Substitute into the first equation:

$$(1 - 2y)^2 - 9y - (1 - 2y) = 2y^2 - 12$$

$$4y^2 - 11y = 2y^2 - 12$$

$$2y^2 - 11y + 12 = 0$$

4 Factorise

Factorise:

$$(2y - 3)(y - 4) = 0$$

5 **Solve simultaneous equations**

So

$$y = \frac{3}{2} \quad \text{or} \quad y = 4$$

---

6 **Solve simultaneous equations**

If $y = \frac{3}{2}$:

$$x = 1 - 2\left(\frac{3}{2}\right) = -2$$

---

7 **Solve simultaneous equations**

If $y = 4$:

$$x = 1 - 8 = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(-2, \frac{3}{2}\right)$ or $(-7, 4)$.

4MA1/2H MAY 2021

Question 8

3 marks | Simultaneous Equations

8 Solve the simultaneous equations

$$5a + 2c = 10$$
$$2a - 4c = 7$$

Show clear algebraic working.

$a =$ .....

$c =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H May 2021

1 **Solve simultaneous equations**

$$5a + 2c = 10$$

$$2a - 4c = 7$$

2 **Double the first equation**

Double the first equation:

$$10a + 4c = 20$$

3 **Add this to the second**

Add this to the second equation:

$$12a = 27$$

$$a = \frac{9}{4}$$

4 **Solve simultaneous equations**

Substitute into $5a + 2c = 10$:

$$5\left(\frac{9}{4}\right) + 2c = 10$$

$$\frac{45}{4} + 2c = \frac{40}{4}$$

$$2c = -\frac{5}{4}$$

$$c = -\frac{5}{8}$$

✓ **FINAL ANSWER**

$a = \frac{9}{4}$, $c = -\frac{5}{8}$.

4MA1/2H MAY 2021

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$y = 3 - 2x$$
$$x^2 + y^2 = 18$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H May 2021

1 **Solve simultaneous equations**

$$y = 3 - 2x$$

$$x^2 + y^2 = 18$$

2 **Substitute**

Substitute $y = 3 - 2x$:

$$x^2 + (3 - 2x)^2 = 18$$

$$x^2 + 9 - 12x + 4x^2 = 18$$

$$5x^2 - 12x - 9 = 0$$

3 **Factorise**

Factorise:

$$(5x + 3)(x - 3) = 0$$

4 **Solve simultaneous equations**

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = 3$$

5 **Solve simultaneous equations**

If $x = -\frac{3}{5}$, then

$$y = 3 - 2\left(-\frac{3}{5}\right) = \frac{21}{5}$$

---

6 **Solve simultaneous equations**

If $x = 3$, then

$$y = 3 - 6 = -3$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, \frac{21}{5}\right)$ or $(3, -3)$.

4MA1/1H NOVEMBER 2021

Question 6

5 marks | Simultaneous Equations

6 Alison buys 5 apples and 3 pears for a total cost of $1.96
Greg buys 3 apples and 2 pears for a total cost of $1.22

Michael buys 10 apples and 10 pears.

Work out how much Michael pays for his 10 apples and 10 pears.
Show your working clearly.

$.....

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2021

1 Solve simultaneous equations

Let a be the cost of one apple and p be the cost of one pear.

$$5a + 3p = 1.96$$

$$3a + 2p = 1.22$$

2 Multiply the first equation by

Multiply the first equation by 2:

$$10a + 6p = 3.92$$

3 Multiply the second equation by

Multiply the second equation by 3:

$$9a + 6p = 3.66$$

4 Subtract

Subtract:

$$a = 0.26$$

5 **Solve simultaneous equations**

Then

$$3(0.26) + 2p = 1.22$$

$$2p = 0.44$$

$$p = 0.22$$

---

6 **Solve simultaneous equations**

Michael buys 10 apples and 10 pears:

$$10(0.26) + 10(0.22) = 4.80$$

✓ **FINAL ANSWER**

$\$4.80$.

4MA1/2H JANUARY 2022

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$3x + 5y = 3.1$$
$$6x + 3y = 3.75$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2022

1 Solve simultaneous equations

$$3x + 5y = 3.1$$

$$6x + 3y = 3.75$$

2 Double the first equation

Double the first equation:

$$6x + 10y = 6.2$$

3 Subtract the second equation

Subtract the second equation:

$$7y = 2.45$$

$$y = 0.35$$

4 **Solve simultaneous equations**

Substitute into $3x + 5y = 3.1$:

$$3x + 5(0.35) = 3.1$$

$$3x + 1.75 = 3.1$$

$$3x = 1.35$$

$$x = 0.45$$

FINAL ANSWER

$x = 0.45$, $y = 0.35$.

4MA1/1H JANUARY 2022

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$3x^2 + y^2 - xy = 5$$
$$y = 2x - 3$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2022

1 Solve simultaneous equations

$$3x^2 + y^2 - xy = 5$$

$$y = 2x - 3$$

2 Substitute

Substitute $y = 2x - 3$:

$$3x^2 + (2x - 3)^2 - x(2x - 3) = 5$$

$$3x^2 + 4x^2 - 12x + 9 - 2x^2 + 3x = 5$$

$$5x^2 - 9x + 9 = 5$$

$$5x^2 - 9x + 4 = 0$$

3 Factorise

Factorise:

$$(5x - 4)(x - 1) = 0$$

4 Solve simultaneous equations

So

$$x = \frac{4}{5} \quad \text{or} \quad x = 1$$

5 **Solve simultaneous equations**

If $x = \frac{4}{5}$:

$$y = 2\left(\frac{4}{5}\right) - 3 = -\frac{7}{5}$$

---

6 **Solve simultaneous equations**

If $x = 1$:

$$y = 2(1) - 3 = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{4}{5}, -\frac{7}{5}\right)$ or $(1, -1)$.

4MA1/2H JUNE 2022

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2022

1 Solve simultaneous equations

$$7x + 3y = 3$$

$$3x - y = 7$$

2 From the second equation

From the second equation:

$$y = 3x - 7$$

3 Substitute into the first equation

Substitute into the first equation:

$$7x + 3(3x - 7) = 3$$

$$7x + 9x - 21 = 3$$

$$16x = 24$$

$$x = \frac{3}{2}$$

4 **Solve simultaneous equations**

Then

$$y = 3\left(\frac{3}{2}\right) - 7 = \frac{9}{2} - 7 = -\frac{5}{2}$$

✓ **FINAL ANSWER**

$x = \frac{3}{2}, y = -\frac{5}{2}$.

4MA1/1HR JUNE 2022

Question 14

4 marks | Simultaneous Equations

14 Solve the simultaneous equations

$$3x - 5y = 25$$
$$4x + 3y = 14$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR June 2022

1 Solve simultaneous equations

$$3x - 5y = 25$$

$$4x + 3y = 14$$

2 Multiply the first equation by

Multiply the first equation by 3:

$$9x - 15y = 75$$

3 Multiply the second equation by

Multiply the second equation by 5:

$$20x + 15y = 70$$

4 Add

Add:

$$29x = 145$$

$$x = 5$$

5 **Solve simultaneous equations**

Substitute into $3x - 5y = 25$:

$$15 - 5y = 25$$

$$-5y = 10$$

$$y = -2$$

✓ **FINAL ANSWER**

$x = 5$, $y = -2$.

4MA1/2HR JUNE 2022

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$\begin{aligned} x - 2y &= 3 \\ x^2 - y^2 + 2x &= 10 \end{aligned}$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2022

1 **Calculate value**

$$x - 2y = 3$$

2 **Calculate value**

So

$$x = 2y + 3$$

3 **Calculate value**

Substitute into

$$x^2 - y^2 + 2x = 10$$

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

4 **Evaluate fraction**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

5 **Find**

Find x:

$$y = -\frac{1}{3} \Rightarrow x = \frac{7}{3}$$

$$y = -5 \Rightarrow x = -7$$

FINAL ANSWER

$(x, y) = \left(\frac{7}{3}, -\frac{1}{3}\right)$ or $(-7, -5)$

4MA1/1H MAY 2023

Question 9

3 marks | Simultaneous Equations

9 Solve the simultaneous equations

$$2x + 9y = 14.5$$
$$7x + 3y = 8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2023

1 Solve simultaneous equations

$$2x + 9y = 14.5$$

$$7x + 3y = 8$$

2 Multiply the second equation by

Multiply the second equation by 3:

$$21x + 9y = 24$$

3 Subtract the first equation

Subtract the first equation:

$$19x = 9.5$$

$$x = 0.5$$

4 **Solve simultaneous equations**

Substitute into $7x + 3y = 8$:

$$7(0.5) + 3y = 8$$

$$3.5 + 3y = 8$$

$$y = 1.5$$

✓ **FINAL ANSWER**

$x = 0.5$, $y = 1.5$.

4MA1/2H JUNE 2023

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2023

1 Calculate value

$$2x^2 + 3y^2 = 11$$

$$x = 3y - 1$$

2 Substitute

Substitute:

$$2(3y - 1)^2 + 3y^2 = 11$$

$$18y^2 - 12y + 2 + 3y^2 = 11$$

$$21y^2 - 12y - 9 = 0$$

$$7y^2 - 4y - 3 = 0$$

$$(7y + 3)(y - 1) = 0$$

3 Evaluate fraction

So

$$y = -\frac{3}{7} \quad \text{or} \quad y = 1$$

4 **Find**

Find x:

$$y = -\frac{3}{7} \Rightarrow x = -\frac{16}{7}$$

$$y = 1 \Rightarrow x = 2$$

✓ **FINAL ANSWER**

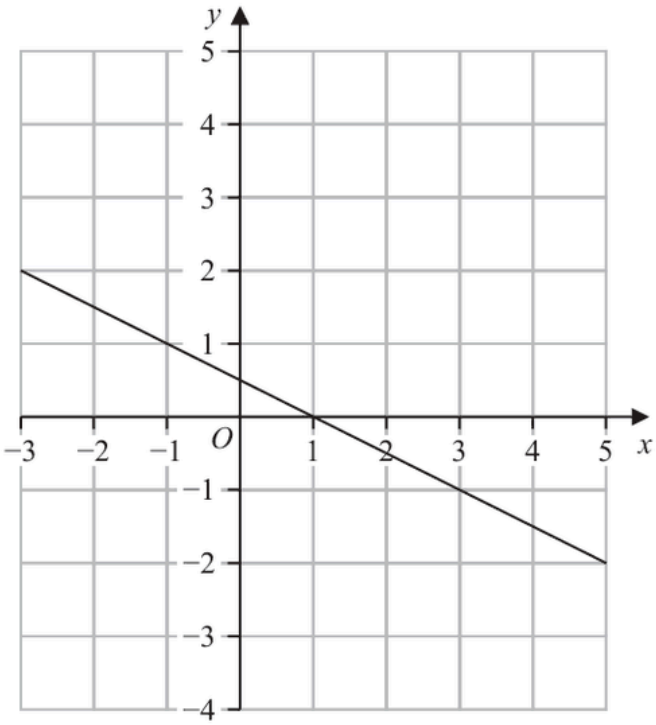
$(x, y) = \left(-\frac{16}{7}, -\frac{3}{7}\right)$ or $(2, 1)$

4MA1/2H NOVEMBER 2023

Question 14

3 marks | Simultaneous Equations

14 Here is the graph of the equation $2y + x = 1$ drawn on a grid.



By drawing another straight line on the grid, solve the simultaneous equations

$$y - x - 2 = 0$$
$$2y + x = 1$$

$x =$ .....

$y =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H November 2023

1 **The equations are**

The equations are

$$y - x - 2 = 0$$

$$2y + x = 1$$

2 **Solve simultaneous equations**

From the first equation,

$$y = x + 2$$

3 **Substitute into the second equation**

Substitute into the second equation:

$$2(x + 2) + x = 1$$

$$3x + 4 = 1$$

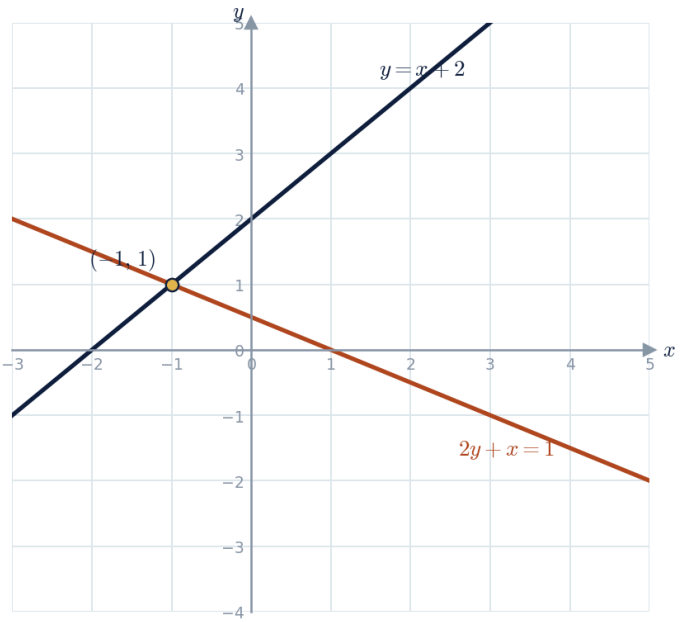
$$3x = -3$$

$$x = -1$$

4 **Solve simultaneous equations**

Then

$$y = -1 + 2 = 1$$



FINAL ANSWER

$x = -1, y = 1.$

4MA1/2H JUNE 2024

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$6x + 4y = 1$$
$$3x + 5y = 8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2024

1 **Solve simultaneous equations**

$$6x + 4y = 1$$

$$3x + 5y = 8$$

2 **Double the second equation**

Double the second equation:

$$6x + 10y = 16$$

3 **Subtract the first equation**

Subtract the first equation:

$$6y = 15$$

$$y = \frac{5}{2}$$

4 **Solve simultaneous equations**

Substitute into $3x + 5y = 8$:

$$3x + 5\left(\frac{5}{2}\right) = 8$$

$$3x + \frac{25}{2} = 8$$

$$3x = -\frac{9}{2}$$

$$x = -\frac{3}{2}$$

✓ **FINAL ANSWER**

$x = -\frac{3}{2}$, $y = \frac{5}{2}$.

4MA1/1HR MAY 2024

Question 12

4 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$4x + 3y = 9.6$$
$$6x + 5y = 16.8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR May 2024

1 Solve simultaneous equations

$$4x + 3y = 9.6$$

$$6x + 5y = 16.8$$

2 Multiply the first equation by

Multiply the first equation by 5:

$$20x + 15y = 48$$

3 Multiply the second equation by

Multiply the second equation by 3:

$$18x + 15y = 50.4$$

4 Subtract

Subtract:

$$2x = -2.4$$

$$x = -1.2$$

5 **Solve simultaneous equations**

Substitute into $4x + 3y = 9.6$:

$$4(-1.2) + 3y = 9.6$$

$$3y = 14.4$$

$$y = 4.8$$

✓ **FINAL ANSWER**

$x = -1.2$, $y = 4.8$.

4MA1/2HR JUNE 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = y + 11$$
$$y = 3x - 1$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2024

1 **Calculate value**

$$x^2 + y^2 = y + 11$$

$$y = 3x - 1$$

2 **Substitute**

Substitute:

$$x^2 + (3x - 1)^2 = 3x - 1 + 11$$

$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

$$10x^2 - 9x - 9 = 0$$

$$(5x + 3)(2x - 3) = 0$$

3 **Evaluate fraction**

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{3}{2}$$

4 **Find**

Find y:

$$x = -\frac{3}{5} \Rightarrow y = -\frac{14}{5}$$

$$x = \frac{3}{2} \Rightarrow y = \frac{7}{2}$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, -\frac{14}{5}\right)$ or $\left(\frac{3}{2}, \frac{7}{2}\right)$

4MA1/2H NOVEMBER 2024

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 + y = 3$$

$$x + 2 = y$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2024

1 Calculate value

$$x^2 + y^2 + y = 3$$

$$y = x + 2$$

2 Substitute

Substitute:

$$x^2 + (x + 2)^2 + (x + 2) = 3$$

$$x^2 + x^2 + 4x + 4 + x + 2 = 3$$

$$2x^2 + 5x + 3 = 0$$

$$(2x + 3)(x + 1) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{2} \quad \text{or} \quad x = -1$$

4 **Find**

Find y:

$$x = -\frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$x = -1 \Rightarrow y = 1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{2}, \frac{1}{2}\right)$ or $(-1, 1)$

4MA1/2HR JUNE 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$3x + 2y = 10$$
$$3x - 4y = 16$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2025

1 Solve simultaneous equations

$$3x + 2y = 10$$

$$3x - 4y = 16$$

2 Solve simultaneous equations

Subtract the second equation from the first:

$$6y = -6$$

$$y = -1$$

3 Solve simultaneous equations

Substitute into $3x + 2y = 10$:

$$3x + 2(-1) = 10$$

$$3x = 12$$

$$x = 4$$

FINAL ANSWER

$x = 4,\ y = -1.$

4WM2/01R MAY 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$3x + 2y = 10$$
$$3x - 4y = 16$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01R May 2025

1 Required

Solve: 3*x* + 2*y* = 10 and 3*x* − 4*y* = 16 algebraically.

2 Label the equations

Label the equations:

Misplaced &

3 Subtract (2) from (1) eliminate

Subtract (2) from (1) to eliminate *x*:

$$(1) - (2) : quad6y = -6impliesboxedy = -1$$

4 Substitute y 1 into (1)

Substitute *y* = − 1 into (1):

$$3x + 2(-1) = 10implies3x = 12impliesboxedx = 4$$

5 Check in equation (2) 3(4)

Check in equation (2): 3(4) − 4(−1) = 12 + 4 = 16

FINAL ANSWER

x = 4, *y* = − 1

4MA1/2H JUNE 2025

Question 8

3 marks | Simultaneous Equations

8 Solve the simultaneous equations

$$3x + 5y = 8$$
$$4x + y = -3.5$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2025

1 Solve simultaneous equations

$$3x + 5y = 8$$

$$4x + y = -3.5$$

2 Solve simultaneous equations

From the second equation,

$$y = -3.5 - 4x$$

3 Substitute into the first equation

Substitute into the first equation:

$$3x + 5(-3.5 - 4x) = 8$$

$$3x - 17.5 - 20x = 8$$

$$-17x = 25.5$$

$$x = -1.5$$

4 **Solve simultaneous equations**

Then

$$y = -3.5 - 4(-1.5) = 2.5$$

✓ **FINAL ANSWER**

$x = -1.5$, $y = 2.5$.

4WM2/01 MAY 2025

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$3x + 5y = 8$$
$$4x + y = -3.5$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM2/01 May 2025

1 Calculate value

From

$$4x + y = -3.5$$

$$y = -3.5 - 4x$$

2 Substitute into equation

Substitute into $3x + 5y = 8$:

$$3x + 5(-3.5 - 4x) = 8$$

$$-17x = 25.5$$

$$x = -1.5$$

3 Calculate value

Then

$$y = -3.5 - 4(-1.5) = 2.5$$

✓ **FINAL ANSWER**

$x = -1.5$, $y = 2.5$.

4MA1/1H JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + 3y + y^2 = 7$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2025

1 **Calculate value**

$$x^2 + y^2 + 3y = 7$$

$$y = x + 2$$

2 **Substitute**

Substitute:

$$x^2 + (x+2)^2 + 3(x+2) = 7$$

$$x^2 + x^2 + 4x + 4 + 3x + 6 = 7$$

$$2x^2 + 7x + 3 = 0$$

$$(2x+1)(x+3) = 0$$

3 **Evaluate fraction**

So

$$x = -\frac{1}{2} \quad \text{or} \quad x = -3$$

4 **Find**

Find y:

$$x = -\frac{1}{2} \Rightarrow y = \frac{3}{2}$$

$$x = -3 \Rightarrow y = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{1}{2}, \frac{3}{2}\right)$ or $(-3, -1)$

4MA1/1HR JUNE 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = 41$$
$$2x + y = 3$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR June 2025

1 **Calculate value**

$$x^2 + y^2 = 41$$

$$2x + y = 3$$

2 **Calculate value**

So

$$y = 3 - 2x$$

3 **Substitute**

Substitute:

$$x^2 + (3 - 2x)^2 = 41$$

$$x^2 + 9 - 12x + 4x^2 = 41$$

$$5x^2 - 12x - 32 = 0$$

$$(5x + 8)(x - 4) = 0$$

4 **Evaluate fraction**

So

$$x = -\frac{8}{5} \quad \text{or} \quad x = 4$$

5 **Find**

Find y:

$$x = -\frac{8}{5} \Rightarrow y = \frac{31}{5}$$

$$x = 4 \Rightarrow y = -5$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{8}{5}, \frac{31}{5}\right)$ or $(4, -5)$

4WM2/01R MAY 2025

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 + 5y + x^2 = 12$$
$$x = y + 5$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01R May 2025

1 Required

Solve *y*² + 5*y* + *x*² = 12 and *x* = *y* + 5.

2 Substitute x y + 5

Substitute *x* = *y* + 5 into the first equation:

*y*² + 5*y* + (*y*+5)² = 12

3 Expand (y +5) 2

Expand (*y*+5)² = *y*² + 10*y* + 25:

*y*² + 5*y* + *y*² + 10*y* + 25 = 12 ⟹ 2*y*² + 15*y* + 13 = 0

4 Factorise

Factorise:

$$(2y + 13)(y + 1) = 0 implies y = -tfrac132; textor; y = -1$$

5 Find corresponding x values using

Find corresponding *x* values using *x* = *y* + 5:

FINAL ANSWER

(*x*,*y*) = (4, −1)and$left(-dfrac32, , -dfrac132right)$

4MA1/2H NOVEMBER 2025

Question 2

3 marks | Simultaneous Equations

2 Solve the simultaneous equations

$$5x + y = 11$$
$$3x - y = 9$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2025

1 Solve simultaneous equations

$$5x + y = 11$$

$$3x - y = 9$$

2 Add the equations

Add the equations:

$$8x = 20$$

$$x = 2.5$$

3 Solve simultaneous equations

Substitute into $5x + y = 11$:

$$5(2.5) + y = 11$$

$$y = -1.5$$

FINAL ANSWER

$x = 2.5$, $y = -1.5$.

4WM2/01 NOVEMBER 2025

Question 3

3 marks | Simultaneous Equations

3 Solve the simultaneous equations

$$5x + y = 11$$
$$3x - y = 9$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 November 2025

1 Label the equations

Label the equations:

Misplaced &

2 Add (1) and (2) eliminate

Step 1: Add (1) and (2) to eliminate *y*.

Misplaced &

3 Solve equation

Step 2: Substitute *x* = 2.5 into equation (1) to find *y*.

Misplaced &

FINAL ANSWER

x = 2.5, *y* = − 1.5

4WM2/01 NOVEMBER 2025

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x^2 + y^2 = 3xy - 11$$
$$y = x + 2$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 November 2025

1 Substitute into equation

Step 1: Substitute *y* = (*x*+2) into the quadratic equation.

*x*² + (*x*+2)² = 3*x*(*x*+2) − 11

2 Expand brackets

*x*² + (*x*²+4*x*+4) = 3*x*² + 6*x* − 11

2*x*² + 4*x* + 4 = 3*x*² + 6*x* − 11

3 Solve equation

0 = 3*x*² − 2*x*² + 6*x* − 4*x* − 11 − 4

*x*² + 2*x* − 15 = 0

4 Solve quadratic equation

(*x*+5)(*x*−3) = 0

So, *x* = − 5 or *x* = 3.

Step 5: Find corresponding *y* values using *y* = *x* + 2.

If *x* = − 5 ⟹ *y* = − 5 + 2 = − 3

If *x* = 3 ⟹ *y* = 3 + 2 = 5

FINAL ANSWER

(−5,−3) and (3,5)

4MA1/1H JUNE 2026

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$4x + 3y = 9$$
$$2x - 2y = -13$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H June 2026

1 Solution | Step 1

$$4x + 3y = 9, \qquad 2x - 2y = -13.$$

Use both equations.

2 Solution | Step 2

Double the second equation: $4x - 4y = -26$. Subtract this from the first: $7y = 35$, so $y = 5$.

Equal x coefficients allow elimination.

3 Solution | Step 3

$$2x - 2(5) = -13 \implies 2x = -3 \implies x = -\frac{3}{2}.$$

Substitute back to find the other unknown.

FINAL ANSWER

$x = -\frac{3}{2}$, $y = 5$

4MA1/1HR JUNE 2026

Question 12

3 marks | Simultaneous Equations

12 Solve the simultaneous equations

$$4x + 4y = 5$$
$$x + 2y = 1$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR June 2026

1 Solution | Step 1

$$4x + 4y = 5, \qquad x + 2y = 1.$$

Use the two given equations.

2 Solution | Step 2

$$4(x + 2y) = 4 \implies 4x + 8y = 4.$$

Multiply the second equation by 4 to match the coefficients of x.

3 Solution | Step 3

$$(4x + 8y) - (4x + 4y) = 4 - 5 \implies 4y = -1, \quad y = -\frac{1}{4}.$$

Subtract the first equation to eliminate x.

4 Solution | Step 4

$$x + 2\left(-\frac{1}{4}\right) = 1 \implies x = \frac{3}{2}.$$

Substitute into the second original equation.

FINAL ANSWER

$x = \frac{3}{2},\ y = -\frac{1}{4}$

Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

1 **Substitute y = 3x − 5**

$$(3x-5)^2 = x^2 + 7 \Rightarrow 9x^2 - 30x + 25 = x^2 + 7$$

2 **Simplify and factorise**

$$8x^2 - 30x + 18 = 0 \Rightarrow 4x^2 - 15x + 9 = 0 \Rightarrow (x-3)(4x-3) = 0$$

3 **Find y for each x**

$x = 3 \Rightarrow y = 4$;\quad $x = \frac{3}{4} \Rightarrow y = \frac{9}{4} - 5 = -\frac{11}{4}$

✓ **FINAL ANSWER**

$x = 3,\ y = 4$ and $x = \frac{3}{4},\ y = -\frac{11}{4}$

4MA1/2HR JUNE 2026

Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$y^2 = x^2 + 7$$
$$y = 3x - 5$$

Show clear algebraic working.

.....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2026

1 Solution | Step 1

$$(3x - 5)^2 = x^2 + 7 \quad \Rightarrow \quad 9x^2 - 30x + 25 = x^2 + 7.$$

Substitute the linear expression for y into the nonlinear equation.

2 Solution | Step 2

$$8x^2 - 30x + 18 = 0 \quad \Rightarrow \quad 4x^2 - 15x + 9 = 0.$$

Collect all terms on one side and divide by 2.

3 Solution | Step 3

$$(4x - 3)(x - 3) = 0 \quad \Rightarrow \quad x = \frac{3}{4} \text{ or } x = 3.$$

Factorize the quadratic and set each factor to zero.

4 Solution | Step 4

For $x = \frac{3}{4}$, $y = 3(\frac{3}{4}) - 5 = -\frac{11}{4}$. For $x = 3$, $y = 9 - 5 = 4$.

Find the corresponding y-value for each x, preserving the pairs.

FINAL ANSWER

$(x, y) = (\frac{3}{4}, -\frac{11}{4})$ or $(3, 4)$.

4MA1/2H JUNE 2026

Question 23

5 marks | Simultaneous Equations

23 The straight line **L** has equation $y = 3x - 2$

The curve **C** has equation $x^2 + y^2 - 2y = 12$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , ...) and (..... , ...)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2026

1 Solution | Step 1

Substitute $y = 3x - 2$ into $x^2 + y^2 - 2y = 12$:

$$x^2 + (3x - 2)^2 - 2(3x - 2) = 12.$$

Both equations must hold at an intersection.

2 Solution | Step 2

$$x^2 + 9x^2 - 12x + 4 - 6x + 4 = 12$$

$$10x^2 - 18x - 4 = 0 \quad \Rightarrow \quad 5x^2 - 9x - 2 = 0.$$

Expand carefully, collect like terms, and divide by 2.

3 Solution | Step 3

$$(5x + 1)(x - 2) = 0 \quad \Rightarrow \quad x = -\frac{1}{5} \text{ or } x = 2.$$

Use factorisation and the zero-product property.

4 Solution | Step 4

When $x = -\frac{1}{5}$, $y = 3(-\frac{1}{5}) - 2 = -\frac{13}{5}$. When $x = 2$, $y = 3(2) - 2 = 4$.
Substitute each root into the line, preserving the coordinate pairings.

FINAL ANSWER

$\left(-\frac{1}{5}, -\frac{13}{5}\right)$ and $(2, 4)$.

Dr Eslam Ahmed · eliteigcse.com

TOPIC

Inequalities (Solving & Graphing)

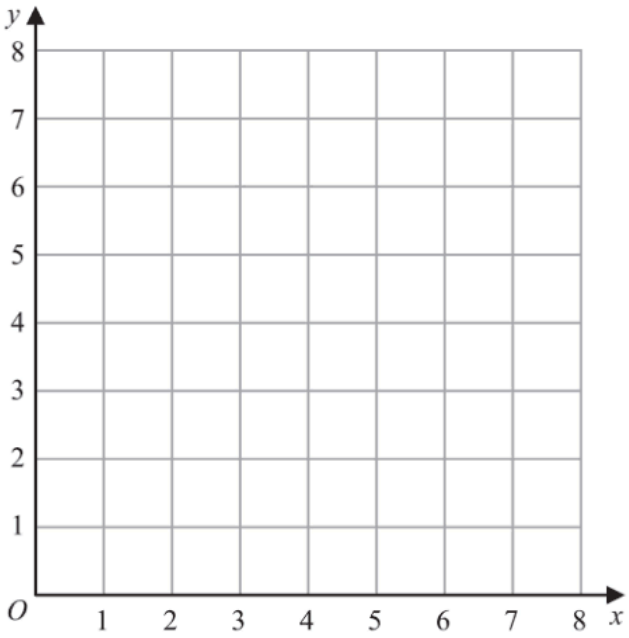
Every past-paper question on this topic, 2020 to 2026.

Question 5

4 marks | Inequalities (Solving & Graphing)

5 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**

(1)

(Total for Question 5 is 4 marks)

1 Draw the three boundary lines

Draw the three boundary lines:

$$y = 1, \qquad x = 2, \qquad x + y = 7$$

2 Read the graph

For $x + y = 7$, use points such as $(0, 7)$ and $(7, 0)$.

3 The required region is

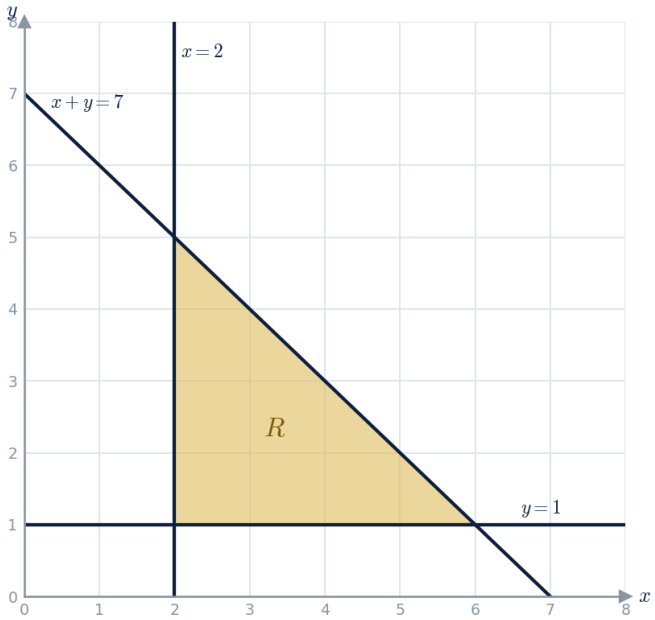
The required region is:

$$y \geq 1, \qquad x \geq 2, \qquad x + y \leq 7$$

4 Read the graph

So shade the triangular region above $y = 1$, to the right of $x = 2$, and below the line $x + y = 7$. Its vertices are

$$(2, 1), \quad (2, 5), \quad (6, 1)$$



FINAL ANSWER

the shaded region R is the triangle with vertices $(2, 1)$, $(2, 5)$, and $(6, 1)$.

4WM2/01 SPECIMEN

Question 18

3 marks | Inequalities (Solving & Graphing)

18 Solve the inequality $2y^2 - 7y - 30 \leqslant 0$
Show your working clearly.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 Specimen

1 **Factorise**

Factorise:

$$2y^2 - 7y - 30 = (2y + 5)(y - 6)$$

2 **Solve inequality**

So

$$(2y + 5)(y - 6) \leq 0$$

3 **The roots are**

The roots are

$$y = -\frac{5}{2}, \qquad y = 6$$

---

4 **Solve quadratic equation**

The quadratic opens upwards, so it is less than or equal to zero between the roots.

✓ **FINAL ANSWER**

$-\frac{5}{2} \leq y \leq 6$.

4MA1/2H JANUARY 2020

Question 7

5 marks | Inequalities (Solving & Graphing)

7 (a) Solve the inequality $2x + 7 > 4$

.....

(2)

(b) Solve $x^2 - 3x - 40 = 0$
Show clear algebraic working.

.....

(3)

(Total for Question 7 is 5 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2020

1 **(a)**

(a)

$$2x + 7 > 4$$

$$2x > -3$$

$$x > -\frac{3}{2}$$

2 **(b)**

(b)

$$x^2 - 3x - 40 = 0$$

3 **Factorise**

Factorise:

$$(x - 8)(x + 5) = 0$$

4 **Solve quadratic equation**

So

$$x = 8 \quad \text{or} \quad x = -5$$

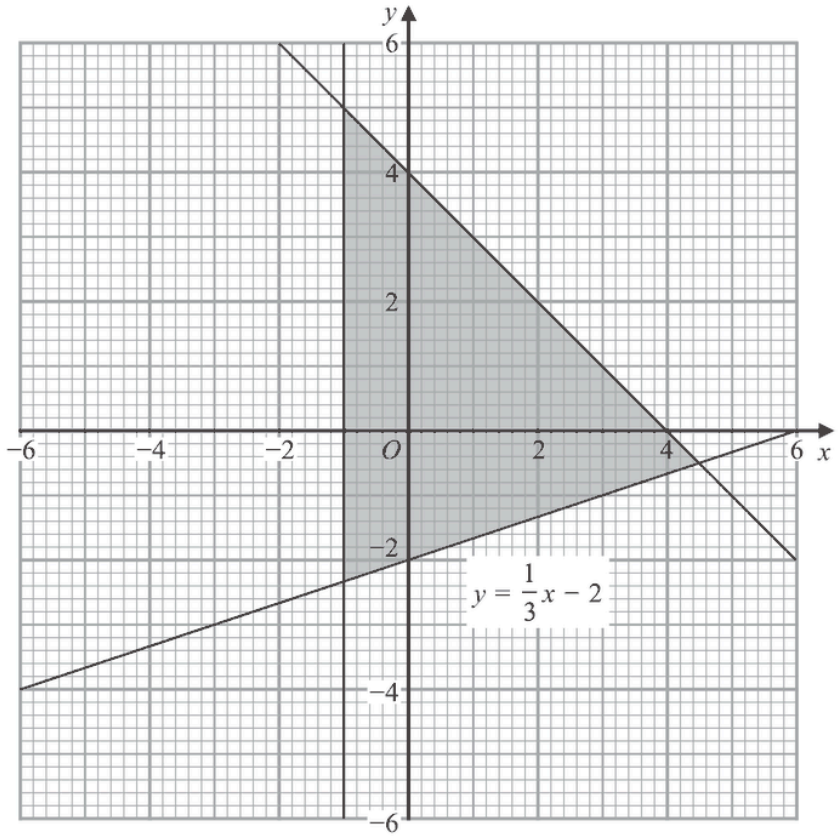
FINAL ANSWER

$x > -\frac{3}{2}$, and $x = 8$ or $x = -5$.

Question 10

3 marks | Inequalities (Solving & Graphing)

10 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down the three inequalities that define the shaded region.

.....

.....

.....

(Total for Question 10 is 3 marks)

1 **The shaded region is left**

The shaded region is left of the y-axis:

$$x \leq 0$$

2 **It is below the line**

It is below the line through $(0, 4)$ and $(4, 0)$:

$$y \leq -x + 4$$

3 **It is above the line**

It is above the line

$$y = \frac{1}{3}x - 2$$

4 **Evaluate fraction**

So

$$y \geq \frac{1}{3}x - 2$$

✓ **FINAL ANSWER**

$x \leq 0$, $y \leq -x + 4$, $y \geq \frac{1}{3}x - 2$.

4MA1/2HR JANUARY 2020

Question 11

4 marks | Inequalities (Solving & Graphing)

11 (a) Write down the integer values of x that satisfy the inequality $-2 < x \leqslant 4$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.

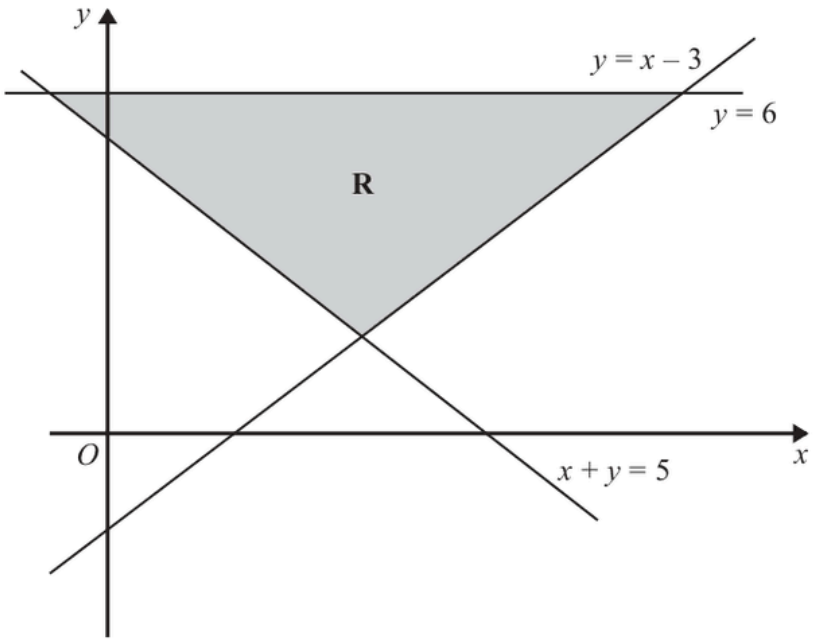


Diagram **NOT** accurately drawn

(b) Write down the three inequalities that define the region **R**.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR January 2020

1 **Solve inequality**

For part (a),

$$2 < x \leq 4$$

2 **The integer values are**

The integer values are

$$3, 4$$

3 **Calculate value**

For the region R, the boundaries are

$$y = x - 3, \qquad x + y = 5, \qquad y = 6$$

4 **The shaded region is above**

The shaded region is above the two sloping lines and below the horizontal line:

$$y \geq x - 3$$

$$x + y \geq 5$$

$$y \leq 6$$

✓ **FINAL ANSWER**

(a) $3, 4$. (b) $y \geq x - 3$, $x + y \geq 5$, $y \leq 6$.

4MA1/2HR JANUARY 2020

Question 19

6 marks | Inequalities (Solving & Graphing)

19 (a) Solve $\frac{4-3x}{5} - \frac{3x-5}{2} = -3$

Show clear algebraic working.

$x =$ .....

(3)

(b) Solve the inequality $5y^2 - 17y \leqslant 40$

.....

(3)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2020

1 **(a)**

(a)

$$\frac{4-3x}{5} - \frac{3x-5}{2} = -3$$

2 **Multiply by 10**

Multiply by 10:

$$2(4-3x) - 5(3x-5) = -30$$

$$8 - 6x - 15x + 25 = -30$$

$$33 - 21x = -30$$

$$-21x = -63$$

$$x = 3$$

3 **(b)**

(b)

$$5y^2 - 17y \leq 40$$

$$5y^2 - 17y - 40 \leq 0$$

---

4 **Factorise**

Factorise:

$$(5y + 8)(y - 5) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{8}{5}$ and 5. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

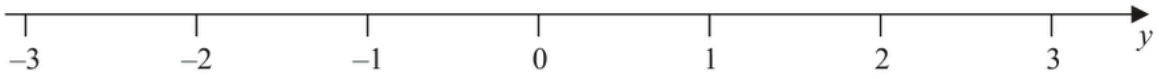
$x = 3$, and $-\frac{8}{5} \leq y \leq 5$.

4MA1/1HR JANUARY 2021

Question 3

4 marks | Inequalities (Solving & Graphing)

3 (a) On the number line, show the inequality $-2 \leqslant y < 1$



(2)

n is an integer.

(b) Write down all the values of n that satisfy $-3.4 < n \leqslant 2$

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2021

1 Solve inequality

(a) For

$$-2 \leq y < 1$$

2 use a closed circle at

use a closed circle at -2, an open circle at 1, and shade the line between them.

3 (b)

(b)

$$-3.4 < n \leq 2$$

4 The integer values are

The integer values are

$$-3,\ -2,\ -1,\ 0,\ 1,\ 2$$

FINAL ANSWER

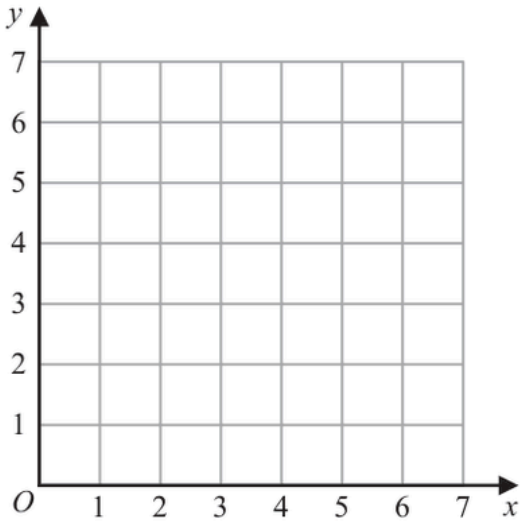
Closed at -2, open at 1; integers $-3, -2, -1, 0, 1, 2$.

4MA1/2H JANUARY 2021

Question 4

4 marks | Inequalities (Solving & Graphing)

4



(a) On the grid, draw and **label** the straight line with equation

(i) $x = 1.5$

(ii) $y = x$

(iii) $x + y = 6$

(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$$x \geqslant 1.5 \qquad y \geqslant x \qquad x + y \leqslant 6$$

Label the region **R**.

(1)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2021

1 **Draw the three boundary lines**

Draw the three boundary lines:

$$x = 1.5, \qquad y = x, \qquad x + y = 6$$

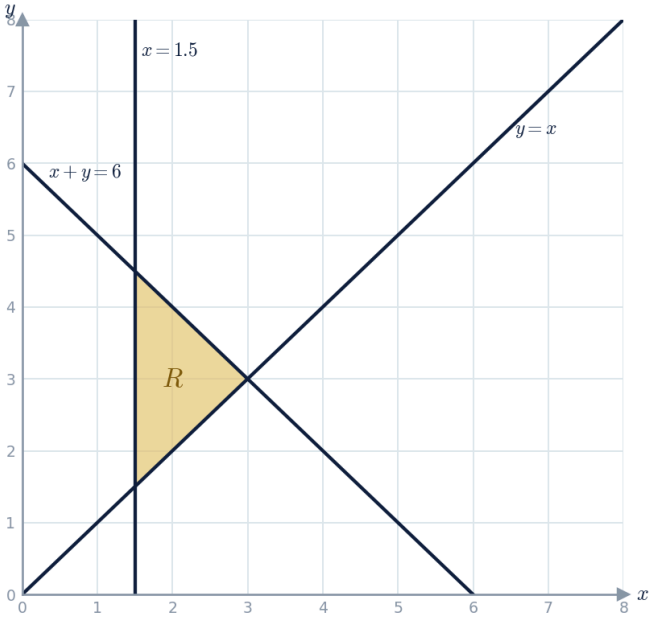
2 **The required region satisfies**

The required region satisfies

$$x > 1.5$$

$$y \geq x$$

$$x + y \leq 6$$



FINAL ANSWER

Shade the region $x > 1.5$, $y \geq x$, $x + y \leq 6$.

4MA1/2H JANUARY 2021

Question 5

7 marks | Inequalities (Solving & Graphing)

5 (a) Expand and simplify $4x(2x + 5) - 3x(2x - 3)$

.....

(2)

Given that $\dfrac{y^5 \times y^n}{y^6} = y^{13}$

(b) work out the value of n.

$n =$ .....

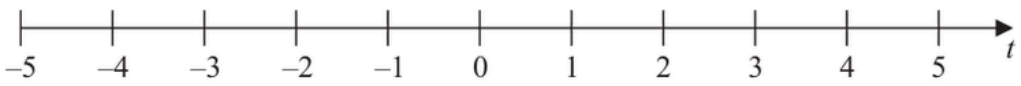
(2)

(c) (i) Solve the inequality $7t - 8 < 2t + 7$

.....

(2)

(ii) On the number line below, represent the solution set of the inequality solved in part (c)(i)



(1)

(Total for Question 5 is 7 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H January 2021

1 **(a)**

(a)

$$4x(2x+5) - 3x(2x-3) = 8x^2 + 20x - 6x^2 + 9x = 2x^2 + 29x$$

2 **Expand brackets**

(b)

$$\frac{y^5 \times y^n}{y^6} = y^{n-1} = y^{13}$$

so $n - 1 = 13$, hence $n = 14$.

3 **(c)**

(c)

$$7t - 8 < 2t + 7$$

$$5t < 15$$

$$t < 3$$

---

4 **Expand brackets**

On the number line, use an open circle at 3 and shade to the left.

✓ **FINAL ANSWER**

(a) $2x^2 + 29x$, (b) $n = 14$, (c) $t < 3$

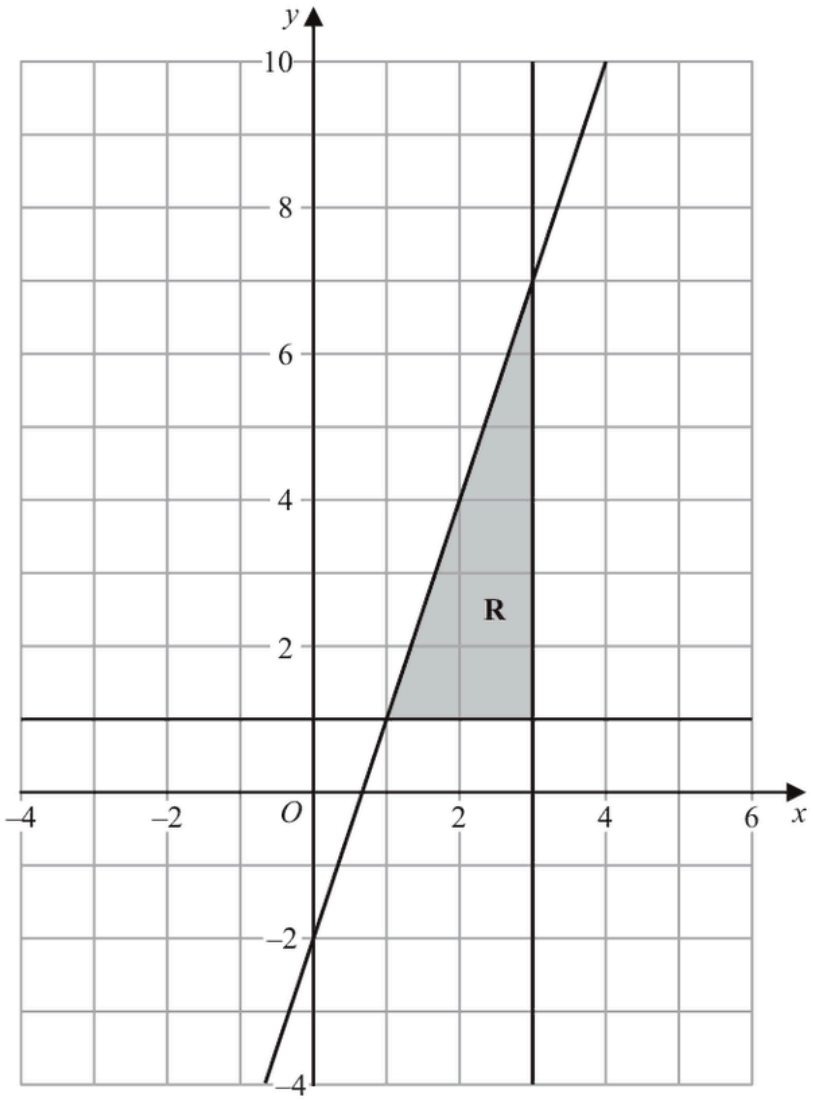
4MA1/2HR JANUARY 2021

Question 7

3 marks | Inequalities (Solving & Graphing)

7 The shaded region **R**, shown in the diagram below, is bounded by the straight line with equation $y = 3x - 2$ and by two other straight lines.

Write down the three inequalities that define region **R**.



(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2021

1 The sloping boundary is

The sloping boundary is

$$y = 3x - 2$$

2 **The shaded region is below**

The shaded region is below this line:

$$y \leq 3x - 2$$

3 **The vertical boundary is so**

The vertical boundary is

$$x = 3$$

so the region has

$$x \leq 3$$

4 **The lower boundary is the**

The lower boundary is the x-axis, so

$$y \geq 0$$

✓ **FINAL ANSWER**

$y \leq 3x - 2$, $x \leq 3$, $y \geq 0$.

4MA1/1H MAY 2021

Question 7

4 marks | Inequalities (Solving & Graphing)

7 $-4 \leqslant 2y < 6$

y is an integer.

(a) Write down all the possible values of y.

.....

(2)

(b) Solve the inequality $7t - 3 \leqslant 2t + 31$

Show your working clearly.

.....

(2)

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2021

1 **(a)**

(a)

$$-4 \leq 2y < 6$$

2 **Divide by 2**

Divide by 2:

$$-2 \leq y < 3$$

3 **Calculate value**

Since y is an integer,

$$y = -2, -1, 0, 1, 2$$

4 **(b)**

(b)

$$7t - 3 \leq 2t + 31$$

$$5t \leq 34$$

$$t \leq \frac{34}{5}$$

FINAL ANSWER

$y = -2, -1, 0, 1, 2$, and $t \leq \frac{34}{5}$.

4MA1/1H NOVEMBER 2021

Question 9

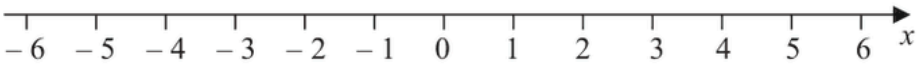
5 marks | Inequalities (Solving & Graphing)

9 (i) Solve the inequalities $-7 \leqslant 2x - 3 < 5$

.....

(3)

(ii) On the number line, represent the solution set to part (i)



(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2021

1 Solve inequality

$$-7 \leq 2x - 3 < 5$$

2 Add 3 to each part

Add 3 to each part:

$$-4 \leq 2x < 8$$

3 Divide by 2

Divide by 2:

$$-2 \leq x < 4$$

4 Use a closed circle

On the number line, use a closed circle at -2, an open circle at 4, and shade between them.

✓ FINAL ANSWER

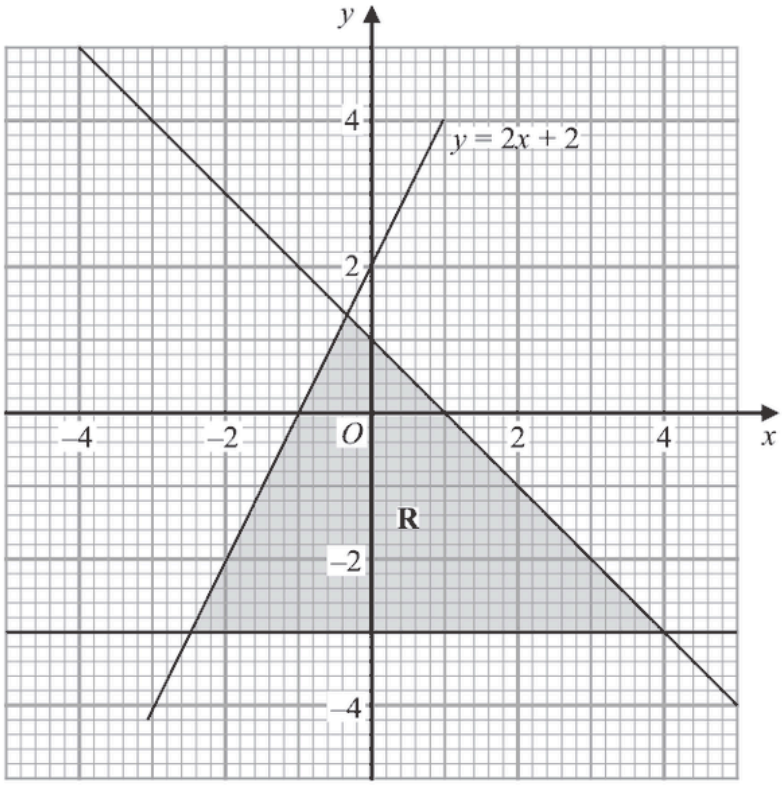
$-2 \leq x < 4$.

4MA1/2H NOVEMBER 2021

Question 13

3 marks | Inequalities (Solving & Graphing)

13



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Write down the three inequalities that define **R**.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2021

1 The bottom boundary is so

The bottom boundary is

$$y = -3$$

so

$$y \geq -3$$

2 **The rising boundary is so**

The rising boundary is

$$y = 2x + 2$$

so the region has

$$y \leq 2x + 2$$

---

3 **Calculate value**

The falling boundary goes through $(0, 1)$ and $(1, 0)$, so

$$x + y = 1$$

---

4 **The region is below this**

The region is below this line:

$$x + y \leq 1$$

✓ **FINAL ANSWER**

$y \geq -3$, $y \leq 2x + 2$, $x + y \leq 1$.

4MA1/1HR JANUARY 2022

Question 1

3 marks | Inequalities (Solving & Graphing)

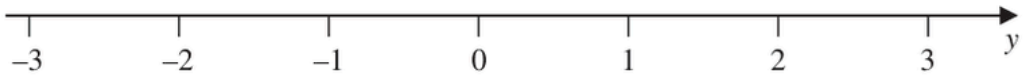
1 n is an integer.

(a) Write down all the values of n such that $-2 \leqslant n < 3$

.....

(2)

(b) On the number line, represent the inequality $y \leqslant 1$



(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$-2 \leq n < 3$$

2 **Calculate value**

Since n is an integer,

$$n = -2, -1, 0, 1, 2$$

3 **Solve inequality**

(b) For

$$y \leq 1$$

4 **use a closed circle at**

use a closed circle at 1 and shade to the left.

✓ **FINAL ANSWER**

$n = -2, -1, 0, 1, 2$; closed at 1, shaded left.

4MA1/1H JANUARY 2022

Question 6

5 marks | Inequalities (Solving & Graphing)

6 (a) Solve the inequality $5x - 7 \leqslant 2$

.....

(2)

(b) (i) Factorise $y^2 - 2y - 35$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 35 = 0$

.....

(1)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x - 7 < 2$$

$$5x < 9$$

$$x < \frac{9}{5}$$

2 **(b)(i)**

(b)(i)

$$y^2 - 2y - 35 = (y - 7)(y + 5)$$

3 **Factorise expression**

(b)(ii) $y = 7$ or $y = -5$.

FINAL ANSWER

(a) $x < \frac{9}{5}$, (b)(i) $(y - 7)(y + 5)$, (b)(ii) $y = 7$ or $y = -5$

4MA1/2HR JANUARY 2022

Question 7

5 marks | Inequalities (Solving & Graphing)

7 (a) Write down an equation of the straight line with gradient -3 and which passes through the point with coordinates $(0, 5)$

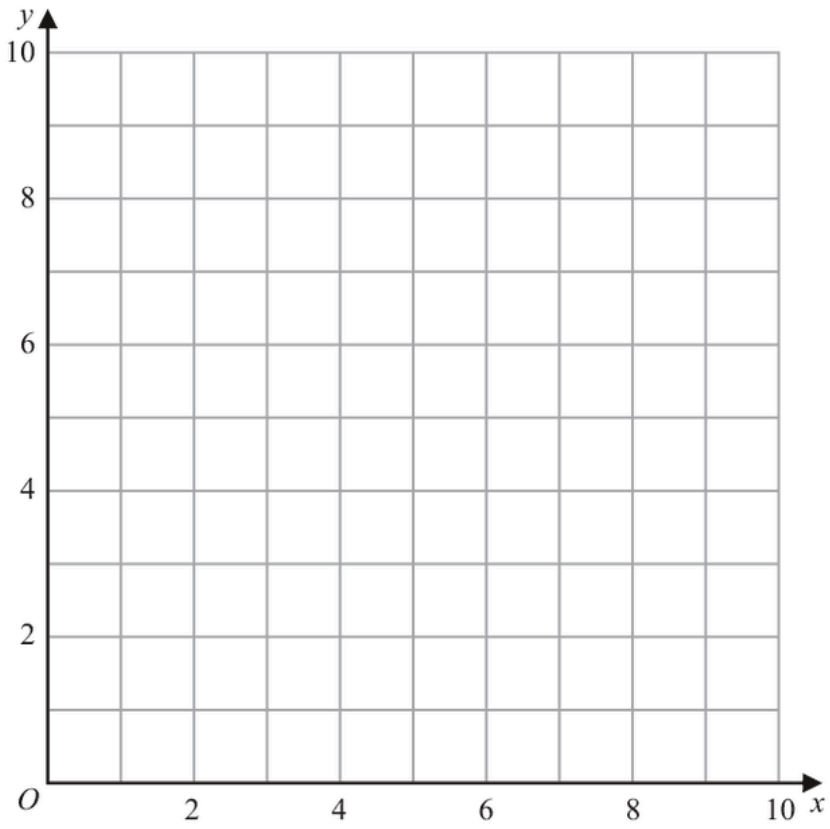
.....

(2)

(b) Show, by shading on the grid, the region defined by **all three** of the inequalities

$$x \leqslant 6 \qquad y \geqslant 2 \qquad y \leqslant x + 1$$

Label the region **R**



(3)

(Total for Question 7 is 5 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2022

1 **A straight line with gradient**

A straight line with gradient -3 passing through $(0, 5)$ has equation

$$y = -3x + 5$$

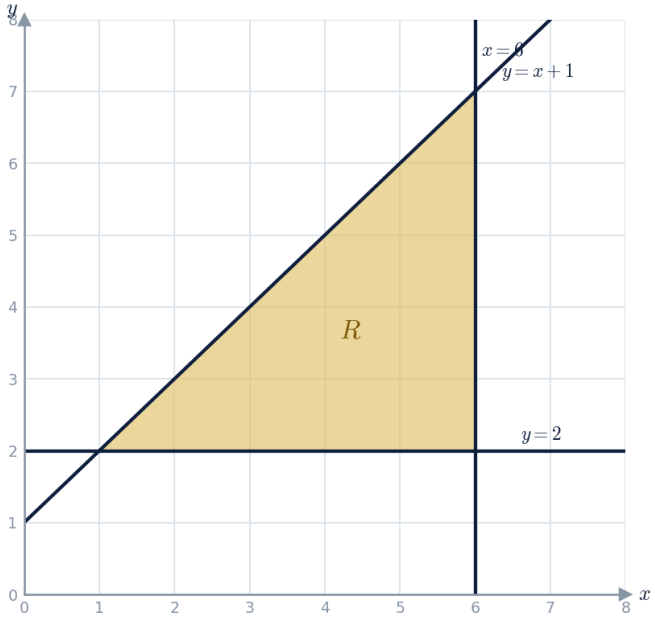
2 **Use the inequalities**

For the region, use the inequalities:

$$x < 6$$

$$y > 2$$

$$y \leq x + 1$$



FINAL ANSWER

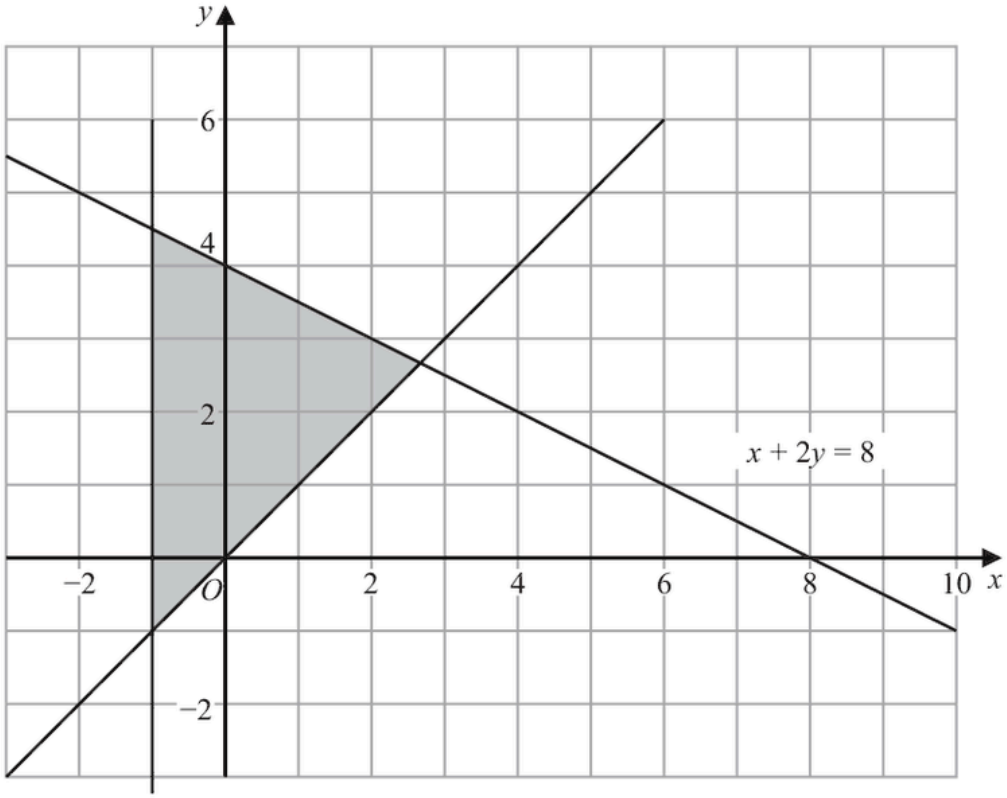
(a) $y = -3x + 5$. (b) Shade $x < 6$, $y > 2$, $y \leq x + 1$.

4MA1/2H JANUARY 2022

Question 13

3 marks | Inequalities (Solving & Graphing)

13 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down three inequalities that define the shaded region.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2022

1 **The region is to the**

The region is to the right of the y-axis:

$$x \geq 0$$

2 **It is above the line**

It is above the line $y = x$:

$$y \geq x$$

3 It is below the line

It is below the line

$$x + 2y = 8$$

so

$$x + 2y \leq 8$$

FINAL ANSWER

$x \geq 0$, $y \geq x$, $x + 2y \leq 8$.

4MA1/2H JANUARY 2022

Question 22

5 marks | Inequalities (Solving & Graphing)

22 Here is a rectangle.

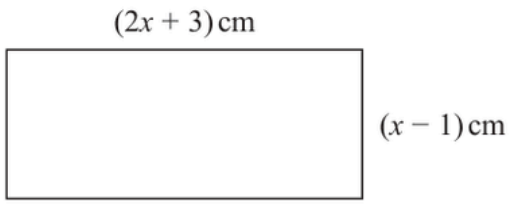


Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than $75\,\text{cm}^2$

find the range of possible values of x

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2022

1 The rectangle has side lengths

The rectangle has side lengths

$$2x + 3$$

2 Use Functions Solving Inequalities

and

$$x - 1$$

3 Solve inequality

For the side lengths to be possible,

$$x > 1$$

4 The area is less than

The area is less than 75:

$$(2x + 3)(x - 1) < 75$$

$$2x^2 + x - 3 < 75$$

$$2x^2 + x - 78 < 0$$

5 **Factorise**

Factorise:

$$2x^2 + x - 78 = (2x + 13)(x - 6)$$

---

6 **Evaluate fraction**

So

$$-\frac{13}{2} < x < 6$$

---

7 **Solve inequality**

Combining with $x > 1$:

$$1 < x < 6$$

✓ **FINAL ANSWER**

$1 < x < 6$

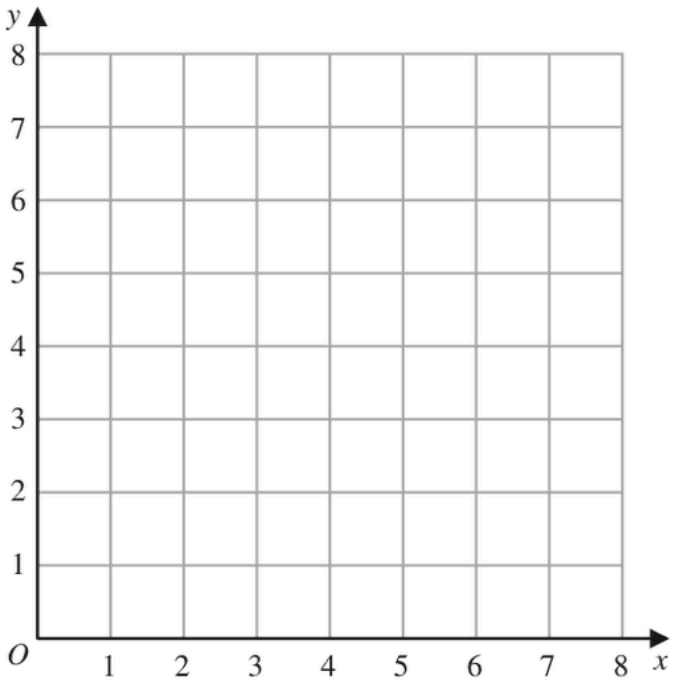
4MA1/2H JUNE 2022

Question 2

4 marks | Inequalities (Solving & Graphing)

2 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**.

(1)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2022

1 **Draw the boundary lines**

Draw the boundary lines:

$$y = 1, \quad x = 2, \quad x + y = 7$$

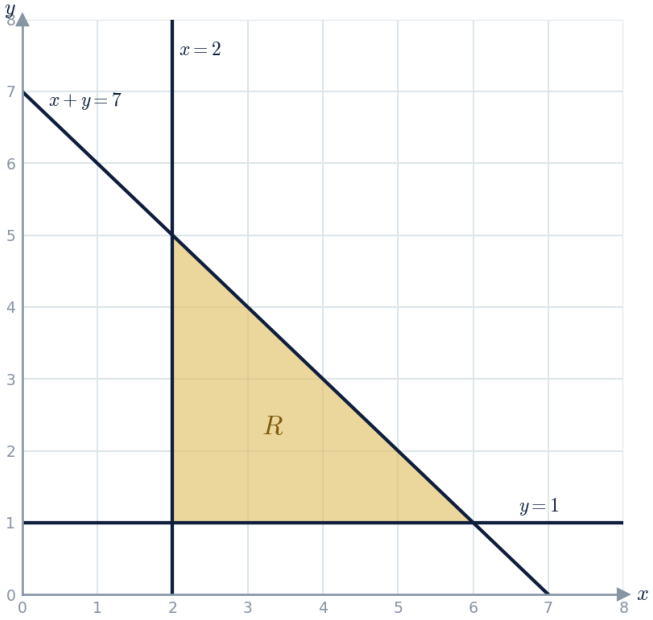
2 **The required region is**

The required region is

$$y \geq 1$$

$$x \geq 2$$

$$x + y \leq 7$$



FINAL ANSWER

Shade $y \geq 1$, $x \geq 2$, $x + y \leq 7$.

4MA1/1HR JUNE 2022

Question 9

2 marks | Inequalities (Solving & Graphing)

9 Solve the inequality $3 - 4x \leqslant 11$

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2022

1 Solve inequality

$$3 - 4x \leq 11$$

$$-4x \leq 8$$

2 Solve inequality

Divide by -4, reversing the inequality:

$$x \geq -2$$

FINAL ANSWER

$x \geq -2$.

4MA1/2H JUNE 2022

Question 15

7 marks | Inequalities (Solving & Graphing)

15 (a) Solve $\frac{4x+5}{3} - \frac{3-2x}{2} = 13$

Show clear algebraic working.

$x =$ .....

(4)

(b) Solve the inequality $2y^2 - 7y - 30 \leqslant 0$

Show your working clearly.

.....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2022

1 (a)

(a)

$$\frac{4x+5}{3} - \frac{3-2x}{2} = 13$$

2 Multiply by 6

Multiply by 6:

$$2(4x+5) - 3(3-2x) = 78$$

$$8x + 10 - 9 + 6x = 78$$

$$14x + 1 = 78$$

$$14x = 77$$

$$x = \frac{77}{14} = \frac{11}{2}$$

3 **(b)**

(b)

$$2y^2 - 7y - 30 \leq 0$$

---

4 **Factorise**

Factorise:

$$(2y + 5)(y - 6) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{5}{2}$ and 6. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

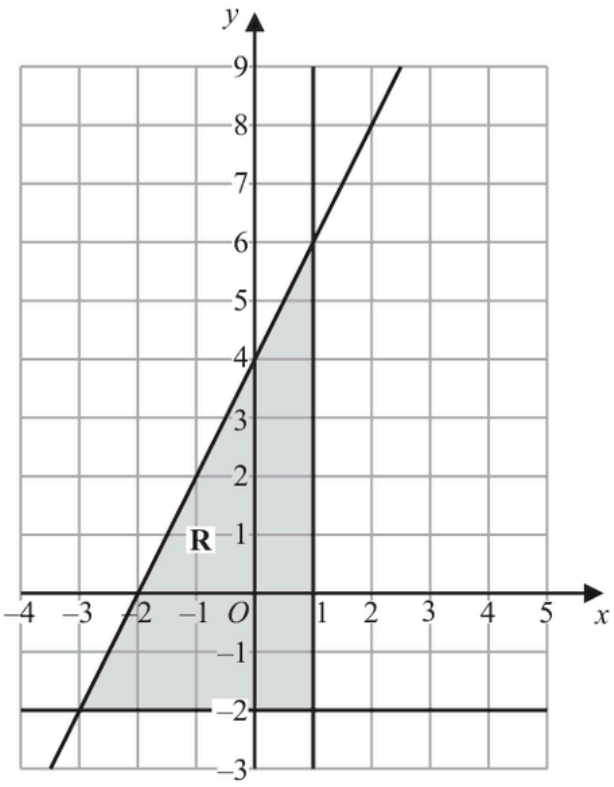
$x = \frac{11}{2}$, and $-\frac{5}{2} \leq y \leq 6$.

4MA1/1H MAY 2023

Question 7

4 marks | Inequalities (Solving & Graphing)

7



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Find the inequalities that define **R**

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2023

1 **The vertical boundary is so**

The vertical boundary is

$$x = 1$$

so the region has

$$x \leq 1$$

2 **The horizontal boundary is so**

The horizontal boundary is

$$y = -2$$

so

$$y \geq -2$$

---

3 **Find the gradient**

The sloping line passes through $(0, 4)$ and has gradient 2, so

$$y = 2x + 4$$

---

4 **The shaded region is below**

The shaded region is below this line:

$$y \leq 2x + 4$$

✓ **FINAL ANSWER**

$x \leq 1$, $y \geq -2$, $y \leq 2x + 4$.

4MA1/2HR JUNE 2023

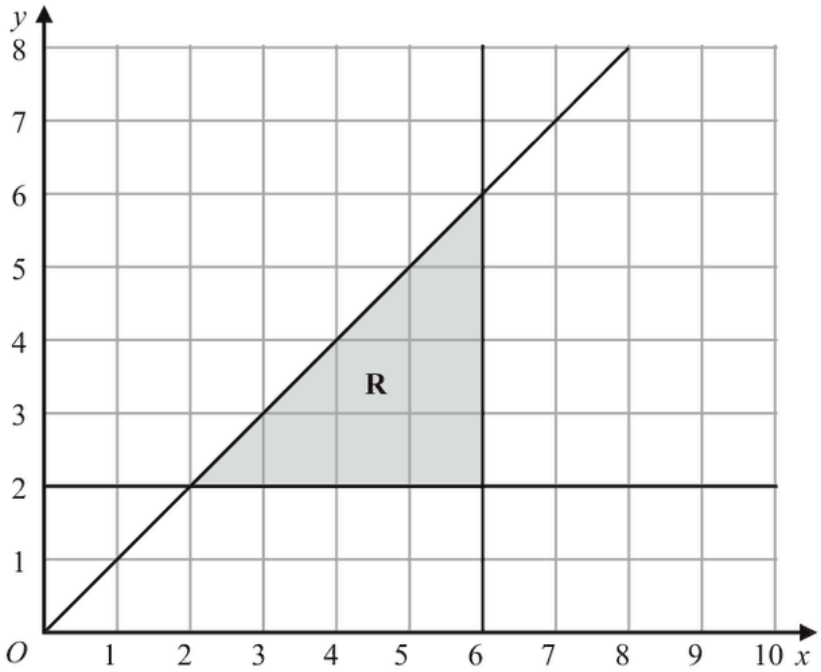
Question 8

5 marks | Inequalities (Solving & Graphing)

8 (a) Solve $9 - 4x > 17$

.....

(2)



(b) Write down the three inequalities that represent the shaded region **R**

.....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2023

1 **Read the graph**

For part (a),

$$9 - 4x > 17$$

$$-4x > 8$$

$$x < -2$$

2 **Read the graph**

For part (b), the region is above $y = 2$, left of $x = 6$, and below $y = x$:

$$y \geq 2$$

$$x \leq 6$$

$$y \leq x$$

✓ **FINAL ANSWER**

(a) $x < -2$. (b) $y \geq 2$, $x \leq 6$, $y \leq x$.

Question 8

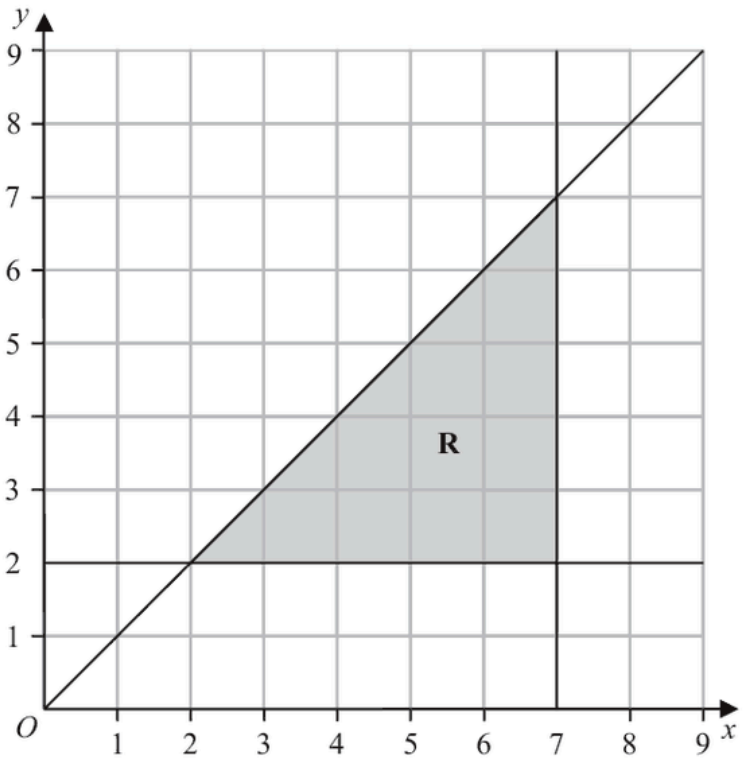
5 marks | Inequalities (Solving & Graphing)

8 (a) Solve the inequality $8x - 4 \geqslant 3x - 10$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 8 is 5 marks)

1 Evaluate fraction

For part (a),

$$8x - 4 \geq 3x - 10$$

$$5x \geq -6$$

$$x \geq -\frac{6}{5}$$

2 Solve inequality

For part (b), the region is above $y = 2$, left of $x = 7$, and below $y = x$:

$$y \geq 2$$

$$x \leq 7$$

$$y \leq x$$

FINAL ANSWER

(a) $x \geq -\frac{6}{5}$. (b) $y \geq 2$, $x \leq 7$, $y \leq x$.

4MA1/2H JUNE 2024

Question 2

4 marks | Inequalities (Solving & Graphing)

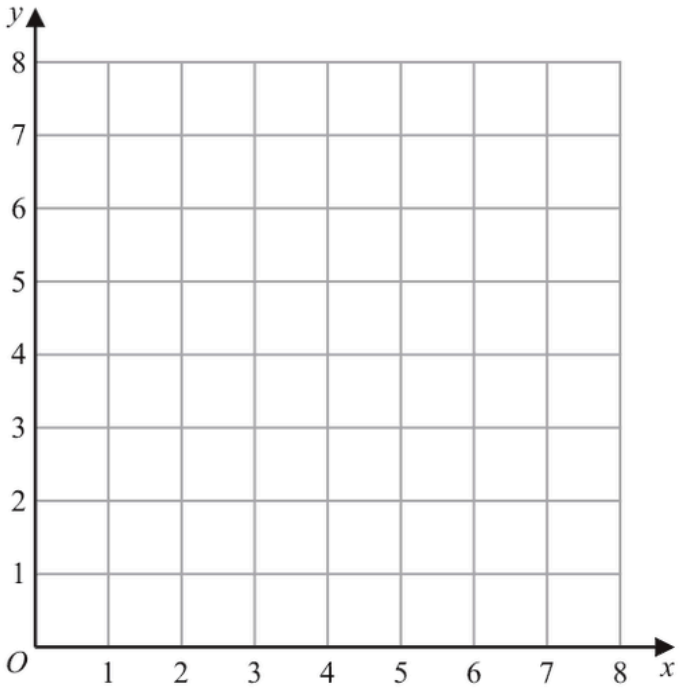
2 (a) On the grid, draw the straight line with equation

(i) $y = 2$

(ii) $x = 6$

(iii) $y = x + 1$

Label each line with its equation.



(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$y \geqslant 2$ $x \leqslant 6$ $y \leqslant x + 1$

Label the region **R**

(1)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2024

1 **Draw the boundary lines**

Draw the boundary lines:

$$y = 2, \quad x = 6, \quad y = x + 1$$

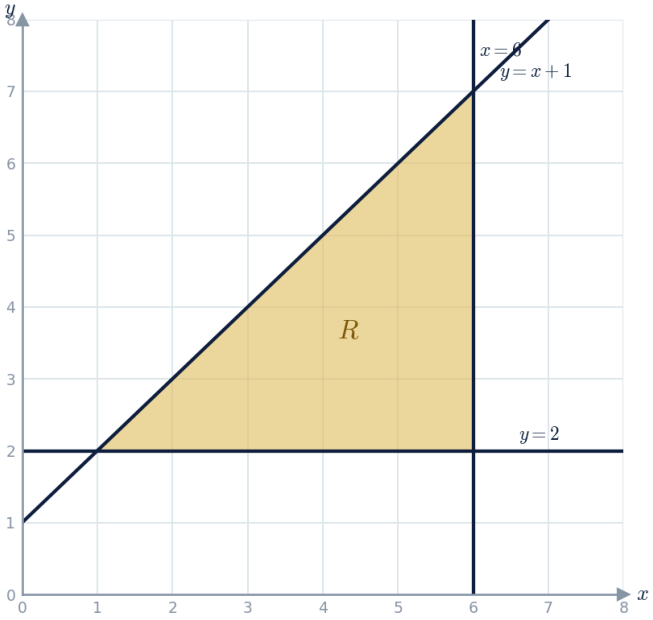
2 **The required region is**

The required region is

$$y \geq 2$$

$$x \leq 6$$

$$y \leq x + 1$$



FINAL ANSWER

Shade $y \geq 2$, $x \leq 6$, $y \leq x + 1$.

4MA1/1HR MAY 2024

Question 7

9 marks | Inequalities (Solving & Graphing)

7 (a) Simplify $g^9 \div g^2$

.....

(1)

(b) Expand $5k^2(k^3 + 4)$

.....

(2)

(c) (i) Factorise $x^2 - 2x - 63$

.....

(2)

(ii) Hence, solve $x^2 - 2x - 63 = 0$

.....

(1)

(d) Solve the inequality $7 - 2y < 3y - 12$

.....

(3)

(Total for Question 7 is 9 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May 2024

1 **(a)**

(a) $g^9 \div g^2 = g^7$.

2 **(b)**

(b) $5k^2(k^3 + 4) = 5k^5 + 20k^2$.

3 **(c)(i)**

(c)(i)

$$x^2 - 2x - 63 = (x - 9)(x + 7)$$

4 **Solve quadratic equation**

(c)(ii) $x = 9$ or $x = -7$.

5 **(d)**

(d)

$$7 - 2y < 3y - 12$$

$$19 < 5y$$

FINAL ANSWER

(a) g^7, (b) $5k^5 + 20k^2$, (c) $(x - 9)(x + 7)$, $x = 9$ or $x = -7$, (d) $y > \frac{19}{5}$

4MA1/2HR JUNE 2024

Question 20

4 marks | Inequalities (Solving & Graphing)

20 Solve $6x^2 - 7x - 20 > 0$
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2024

1 Solve quadratic equation

$$6x^2 - 7x - 20 > 0$$

2 Factorise

Factorise:

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{4}{3} \quad \text{and} \quad x = \frac{5}{2}$$

4 Solve quadratic equation

Since the coefficient of x^2 is positive, the quadratic is greater than 0 outside the roots.

✓ **FINAL ANSWER**

$$x < -\frac{4}{3} \text{ or } x > \frac{5}{2}$$

4MA1/2H NOVEMBER 2024

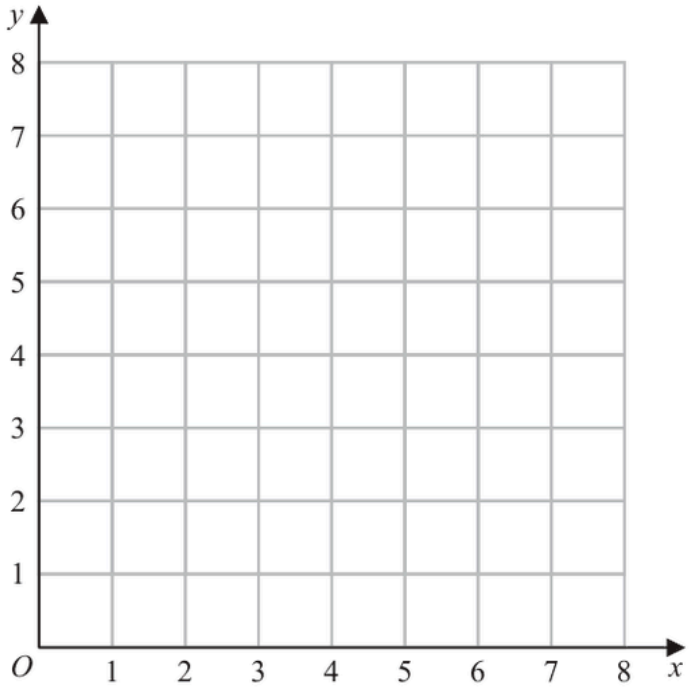
Question 7

4 marks | Inequalities (Solving & Graphing)

7 (a) On the grid, draw the straight line with equation

(i) $x = 3$ (ii) $y = 1$ (iii) $x + y = 7$

Label each line with its equation.



(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$x \geqslant 3$ $y \geqslant 1$ $x + y \leqslant 7$

Label the region **R**

(1)

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2024

1 Draw the boundary lines

Draw the boundary lines:

$$x = 3, \quad y = 1, \quad x + y = 7$$

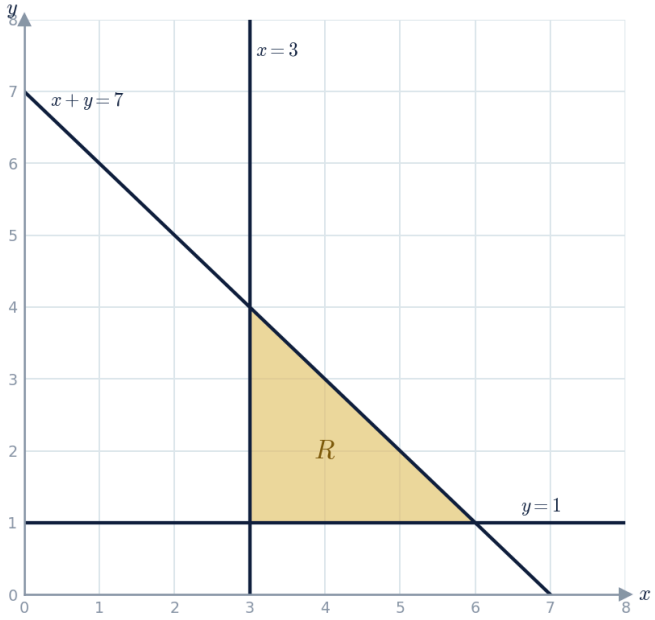
2 **The required region is**

The required region is

$$x \geq 3$$

$$y \geq 1$$

$$x + y \leq 7$$



FINAL ANSWER

Shade $x \geq 3$, $y \geq 1$, $x + y \leq 7$.

4MA1/1H NOVEMBER 2024

Question 8

6 marks | Inequalities (Solving & Graphing)

8 (a) (i) Factorise $x^2 + 5x - 24$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

.....

(1)

(b) Solve the inequality $3y + 5 > 7y - 10$

Show clear algebraic working.

.....

(3)

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2024

1 **(a)(i)**

(a)(i)

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

2 **Solve quadratic equation**

(a)(ii) $x = -8$ or $x = 3$.

3 **(b)**

(b)

$$3y + 5 > 7y - 10$$

$$15 > 4y$$

$$y < \frac{15}{4}$$

✓ **FINAL ANSWER**

$(x + 8)(x - 3)$, $x = -8$ or $x = 3$, $y < \frac{15}{4}$

4MA1/2H NOVEMBER 2024

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $10x^2 + 11x - 21 < 0$
Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2024

1 Solve inequality

$$10x^2 + 11x - 21 < 0$$

2 Factorise

Factorise:

$$10x^2 + 11x - 21 = (10x + 21)(x - 1)$$

3 The critical values are

The critical values are

$$x = -\frac{21}{10} \quad \text{and} \quad x = 1$$

4 The quadratic is less than

The quadratic is less than 0 between the roots.

✓ FINAL ANSWER

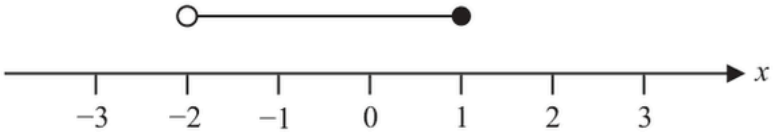
$$-\frac{21}{10} < x < 1$$

4MA1/1HR JUNE 2025

Question 6

4 marks | Inequalities (Solving & Graphing)

6



(a) Write down the inequality shown on the number line.

.....

(2)

(b) Solve the inequality $7a - 5 \leqslant 3a + 28$
Show clear algebraic working.

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2025

1 **(a) The number line has**

(a) The number line has an open circle at -2, a closed circle at 1, and shading between them.

$$-2 < x \leq 1$$

2 **(b)**

(b)

$$7a - 5 \leq 3a + 28$$

$$4a \leq 33$$

$$a \leq \frac{33}{4}$$

FINAL ANSWER

$-2 < x \leq 1$, and $a \leq \frac{33}{4}$.

Question 9

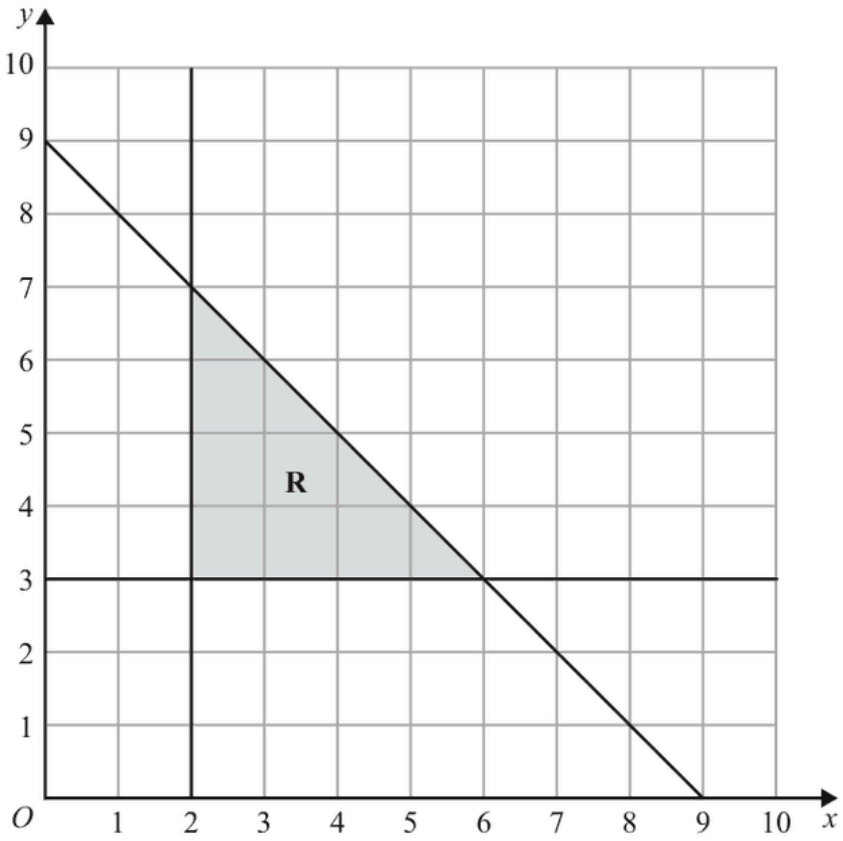
5 marks | Inequalities (Solving & Graphing)

9 (a) Solve the inequality $7 - 3t < 2t + 15$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down three inequalities that define the region **R**

.....

(3)

(Total for Question 9 is 5 marks)

1 Evaluate fraction

For part (a),

$$7 - 3x < 2x + 15$$

$$-8 < 5x$$

$$x > -\frac{8}{5}$$

2 Calculate value

For part (b), the boundaries are $x = 2$, $y = 3$, and $x + y = 9$.

3 The shaded region is

The shaded region is

$$x \geq 2$$

$$y \geq 3$$

$$x + y \leq 9$$

FINAL ANSWER

(a) $x > -\frac{8}{5}$. (b) $x \geq 2$, $y \geq 3$, $x + y \leq 9$.

4WM2/01R MAY 2025

Question 9

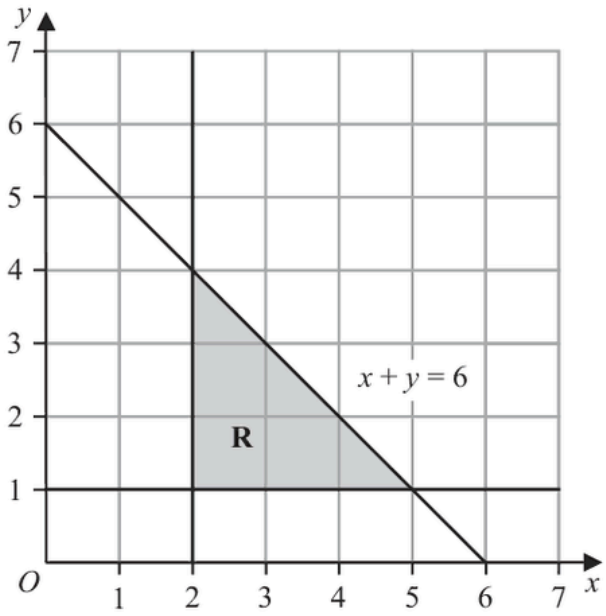
6 marks | Inequalities (Solving & Graphing)

9 (a) Solve the inequality $3x - 22 > 8x - 40$
Show clear algebraic working.

.....

(3)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 9 is 6 marks)

WORKED SOLUTION

Question 9 · 4WM2/01R May 2025

1 **Required**

a Solve 3*x* − 22 \> 8*x* − 40 algebraically. (b) Write the three inequalities defining region R.

2 **Find the nth term**

Move all *x* terms to the right and constants to the left:

3*x* − 22 \> 8*x* − 40 ⟹ − 22 + 40 \> 8*x* − 3*x* ⟹ 18 \> 5*x*

3 Divide both sides by 5

Divide both sides by 5:

$$x\backslash<frac185 = 3.6$$

4 Answer

x \< 3.6

5 Identify the three boundary lines

Identify the three boundary lines and their regions using the graph. The shaded region R is bounded by *x* = 2, *y* = 1, and *x* + *y* = 6.

FINAL ANSWER

x ≥ 2,*y* ≥ 1,*x* + *y* ≤ 6

Question 11

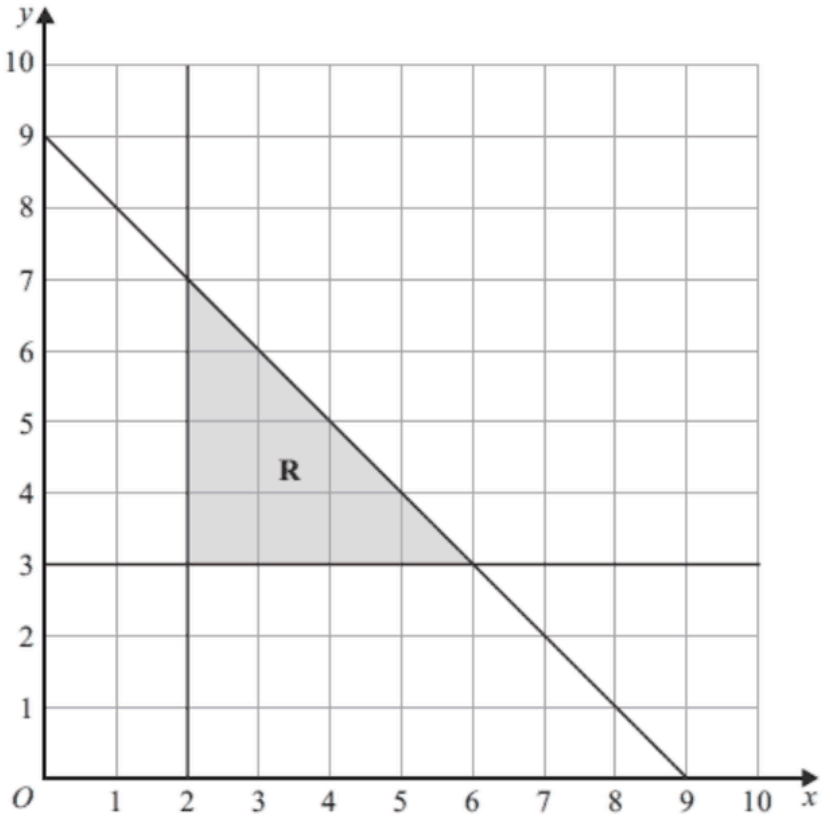
5 marks | Inequalities (Solving & Graphing)

11 (a) Solve the inequality $7 - 3t < 2t + 15$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down three inequalities that define the region **R**

.....

.....

.....

(3)

(Total for Question 11 is 5 marks)

1 **(a)**

(a)

$$7 - 3t < 2t + 15$$

$$-8 < 5t$$

$$t > -1.6$$

2 **(b) The three boundary lines**

(b) The three boundary lines are $x = 2$, $y = 3$, and $x + y = 9$. The shaded side gives

$$x \geq 2, \qquad y \geq 3, \qquad x + y \leq 9$$

✓ **FINAL ANSWER**

$t > -1.6$; $x \geq 2$, $y \geq 3$, $x + y \leq 9$.

4MA1/2H JUNE 2025

Question 19

3 marks | Inequalities (Solving & Graphing)

19 Solve the inequality $4x^2 + 4x - 15 < 0$
Show clear algebraic working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2025

1 Solve inequality

$$4x^2 + 4x - 15 < 0$$

2 Factorise

Factorise:

$$(2x + 5)(2x - 3) < 0$$

3 The roots are

The roots are

$$x = -\frac{5}{2}, \qquad x = \frac{3}{2}$$

4 The quadratic is negative between

The quadratic is negative between the roots.

FINAL ANSWER

$-\frac{5}{2} < x < \frac{3}{2}$.

4WM2/01R MAY 2025

Question 20

3 marks | Inequalities (Solving & Graphing)

20 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01R May 2025

1 Required

Solve 2*x*² − 7*x* − 15 \> 0. Show clear algebraic working.

2 Factorise the quadratic 2 x

Factorise the quadratic:

2*x*² − 7*x* − 15 = (2*x*+3)(*x*−5)

Check: (2*x*+3)(*x*−5) = 2*x*² − 10*x* + 3*x* − 15 = 2*x*² − 7*x* − 15

3 Set each factor zero

Roots: set each factor to zero:

$$2x + 3 = 0 implies x = -tfrac32; qquad x - 5 = 0 implies x = 5$$

4 Solve inequality

Since the coefficient of *x*² is + 2 \> 0, the parabola opens **upward**. The expression is \> 0 *outside* the roots:

FINAL ANSWER

$x \< - dfrac32$ or *x* \> 5

4MA1/2HR JUNE 2025

Question 21

3 marks | Inequalities (Solving & Graphing)

21 Solve the inequality $2x^2 - 7x - 15 > 0$

Show clear algebraic working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2025

1 Solve inequality

$$2x^2 - 7x - 15 > 0$$

2 Factorise

Factorise:

$$2x^2 - 7x - 15 = (2x + 3)(x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{3}{2} \quad \text{and} \quad x = 5$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

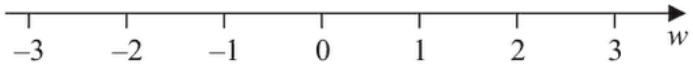
$x < -\frac{3}{2}$ or $x > 5$

4MA1/2H NOVEMBER 2025

Question 10

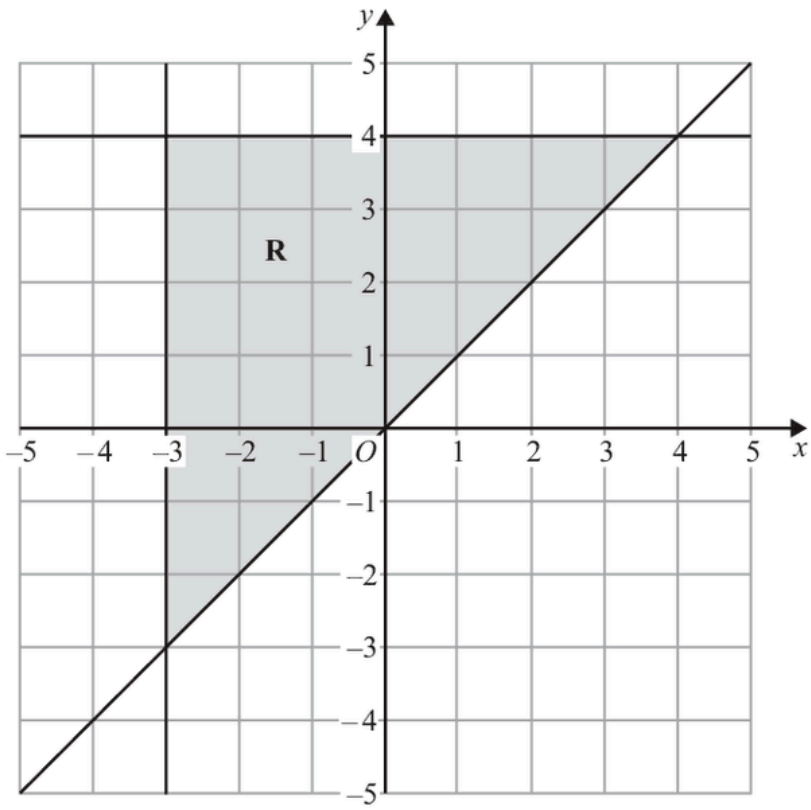
4 marks | Inequalities (Solving & Graphing)

10 (a) On the number line, represent the inequality $w < 1$



(1)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2025

1 **Read the graph**

For part (a), $w < 1$ is shown with an open circle at 1 and an arrow to the left.

2 **Read the graph**

For part (b), the region is right of $x = -3$, below $y = 4$, and above $y = x$:

$$x \geq -3$$

$$y \leq 4$$

$$y \geq x$$

FINAL ANSWER

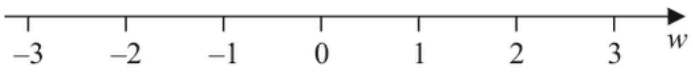
(a) Open circle at 1, shade left. (b) $x \geq -3$, $y \leq 4$, $y \geq x$.

4WM2/01 NOVEMBER 2025

Question 11

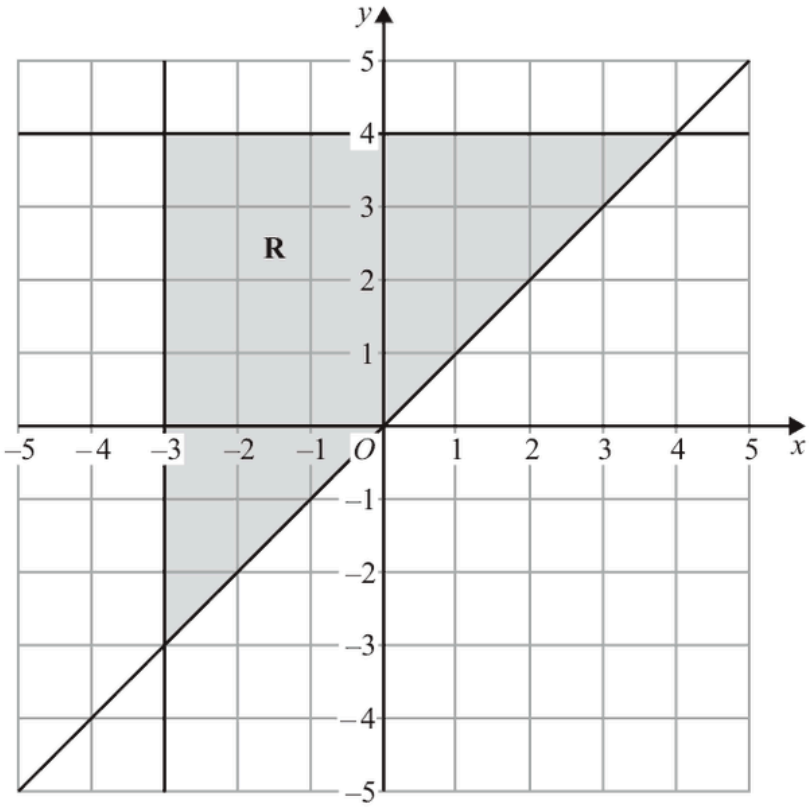
4 marks | Inequalities (Solving & Graphing)

11 (a) On the number line, represent the inequality $w < 1$



(1)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.



(b) Write down the three inequalities that define the region **R**

.....

(3)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4WM2/01 November 2025

1 **Draw an open circle at**

Draw an open circle at 1 and an arrow pointing to the left.

2 **Identify vertical boundary**

Step 1: Identify the vertical boundary. The region is to the right of the vertical line at *x* = − 3. Inequality 1: *x* ≥ − 3

3 **Solve inequality**

Step 2: Identify the horizontal boundary. The region is below the horizontal line at *y* = 4. Inequality 2: *y* ≤ 4

4 **Calculate statistic**

Step 3: Identify the diagonal boundary. The line passes through (0,0), (1,1), (−1,−1), etc., meaning its equation is *y* = *x*. The shaded region is above this line. Inequality 3: *y* ≥ *x*

✓ **FINAL ANSWER**

x ≥ − 3, *y* ≤ 4, *y* ≥ *x* Always pick a test point inside the shaded region, like (0,2), to verify your inequalities. For *y* ≥ *x*, 2 ≥ 0 is true!

4MA1/1H NOVEMBER 2025

Question 25

3 marks | Inequalities (Solving & Graphing)

25 Solve the inequality $2x^2 + x - 28 > 0$
Show clear algebraic working.

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2025

1 Solve inequality

$$2x^2 + x - 28 > 0$$

2 Factorise

Factorise:

$$2x^2 + x - 28 = (2x - 7)(x + 4)$$

3 The critical values are

The critical values are

$$x = \frac{7}{2} \quad \text{and} \quad x = -4$$

4 The quadratic is greater than

The quadratic is greater than 0 outside the roots.

FINAL ANSWER

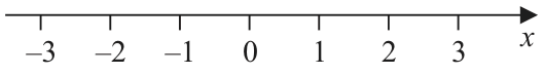
$x < -4$ or $x > \frac{7}{2}$

4MA1/1HR JUNE 2026

Question 7

5 marks | Inequalities (Solving & Graphing)

7



(a) Represent the inequality $x < 1$ on the number line above.

(1)

$2 \leqslant y + 3 < 7$
y is an integer.

(b) Write down all the possible values of y

.....

(2)

(c) Solve the inequality $8 - 5w > -10$

.....

(2)

(Total for Question 7 is 5 marks)

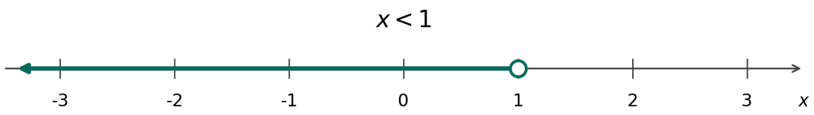
WORKED SOLUTION

Question 7 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

Place an open circle at 1 and draw a horizontal ray from it to the left, with a left-pointing arrow. Do not fill the circle.

The strict inequality excludes 1 and includes every real number less than 1.



2 **Part (b) | Step 1**

$$2 \leq y + 3 < 7 \implies -1 \leq y < 4.$$

Subtract 3 from all three expressions.

3 **Part (b) | Step 2**

The integer values in this interval are $-1, 0, 1, 2, 3$.

Include the lower endpoint but exclude 4.

---

4 **Part (c) | Step 1**

$$8 - 5w > -10 \quad \Longrightarrow \quad -5w > -18.$$

Subtract 8 from both sides.

---

5 **Part (c) | Step 2**

$$w < \frac{18}{5} = 3.6.$$

Division by a negative number reverses the inequality sign.

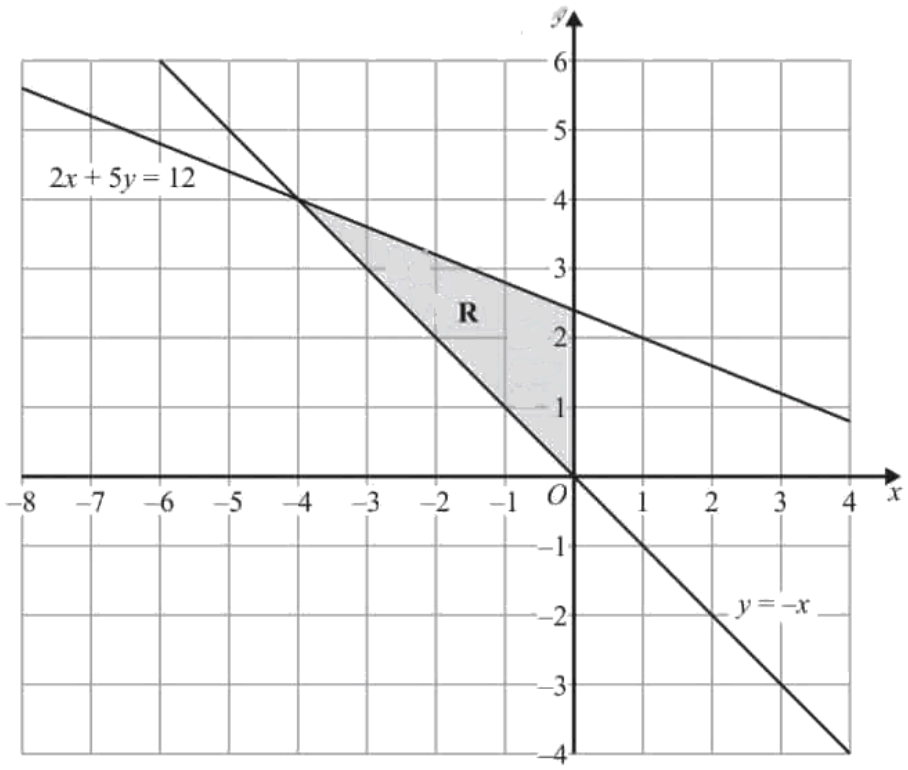
✓ **FINAL ANSWER**

(a) Open circle at 1, ray to the left: $(-\infty, 1)$. (b) $y = -1, 0, 1, 2, 3$ (c) $w < 3.6$

Question 12

3 marks | Inequalities (Solving & Graphing)

12



The region **R**, shown shaded in the diagram, is bounded by the y-axis and the straight lines with equations

$$y = -x$$

$$2x + 5y = 12$$

Write down the three inequalities that define **R**

(Total for Question 12 is 3 marks)

1 **Boundary lines**

R lies to the left of the y-axis, above $y = -x$ and below $2x + 5y = 12$.

2 **Inequalities**

$$x \leq 0, \qquad y \geq -x, \qquad 2x + 5y \leq 12$$

3 **Check a point inside R**

$(-1, 2)$: $-1 \leq 0$, $2 \geq 1$, $-2 + 10 = 8 \leq 12$ ✓

✓ **FINAL ANSWER**

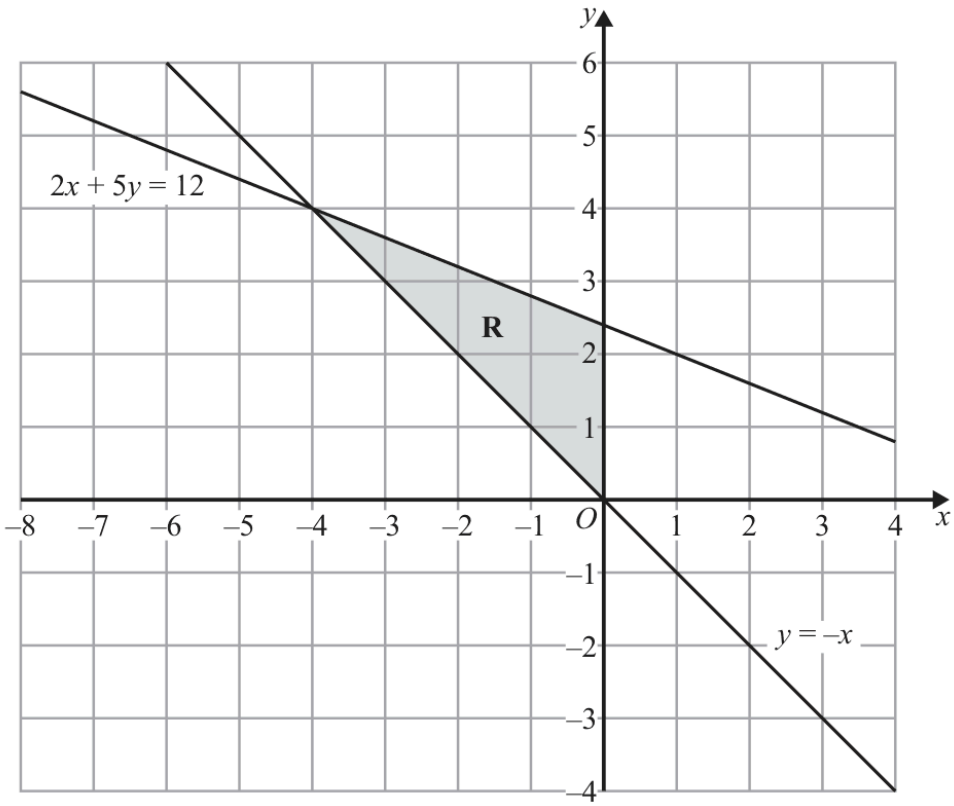
$x \leq 0$, $y \geq -x$, $2x + 5y \leq 12$

4MA1/2HR JUNE 2026

Question 12

3 marks | Inequalities (Solving & Graphing)

12



The region **R**, shown shaded in the diagram, is bounded by the y-axis and the straight lines with equations

$$y = -x$$

$$2x + 5y = 12$$

Write down the three inequalities that define **R**

.....

.....

.....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$x \leq 0$.

R lies to the left of the y-axis.

2 **Solution | Step 2**

$y \geq -x$.

R lies above the line y=-x.

3 **Solution | Step 3**

$2x + 5y \leq 12$.

R lies below the other line. All three boundaries are solid, so equality is included.

✓ **FINAL ANSWER**

$x \leq 0, \quad y \geq -x, \quad 2x + 5y \leq 12$.

TOPIC

Algebraic Proof

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 13

4 marks | Algebraic Proof

13 (a) Use algebra to show that $0.5\dot{7}\dot{2} = \frac{63}{110}$

(2)

Given that y is a prime number,

(b) express $\frac{3}{2-\sqrt{y}}$ in the form $\frac{a+b\sqrt{y}}{c-y}$ where a, b and c are integers.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR January 2020

1 **(a) Let**

(a) Let

$$x = 0.5727272\ldots$$

2 **Calculate value**

Then

$$1000x = 572.727272\ldots$$

3 **Calculate value**

and

$$10x = 5.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 567$$

$$x = \frac{567}{990} = \frac{63}{110}$$

5 **Evaluate fraction**

So

$$0.5\dot{7}\dot{2} = \frac{63}{110}$$

---

6 **(b)**

(b)

$$\frac{3}{2-\sqrt{y}} \times \frac{2+\sqrt{y}}{2+\sqrt{y}} = \frac{3(2+\sqrt{y})}{4-y}$$

$$= \frac{6+3\sqrt{y}}{4-y}$$

✓ **FINAL ANSWER**

(a) shown, (b) $\frac{6+3\sqrt{y}}{4-y}$

4MA1/2H JANUARY 2020

Question 17

3 marks | Algebraic Proof

17 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H January 2020

1 Let the two consecutive square

Let the two consecutive square numbers be n^2 and $(n+1)^2$.

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$$

2 is one more than an

$2n + 1$ is one more than an even number, so it is always odd.

FINAL ANSWER

The difference is always odd.

4MA1/2HR JANUARY 2020

Question 25

3 marks | Algebraic Proof

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR January 2020

1 Use trigonometry

Since N is a multiple of 5, let

$$N = 5k$$

2 where is an integer

where k is an integer.

3 Given

Given

$$A = N + 1, \qquad B = N - 1$$

4 Use trigonometry

So

$$A^2 - B^2 = (N+1)^2 - (N-1)^2$$

5 Expand

Expand:

$$(N+1)^2 = N^2 + 2N + 1$$

$$(N-1)^2 = N^2 - 2N + 1$$

6 **Use trigonometry**

Therefore

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$

$$= 4N$$

---

7 **Substitute**

Substitute $N = 5k$:

$$4N = 4(5k) = 20k$$

---

8 **Use trigonometry**

Since k is an integer, $20k$ is a multiple of 20.

✓ **FINAL ANSWER**

$A^2 - B^2$ is always a multiple of 20.

4MA1/1HR JANUARY 2021

Question 15

5 marks | Algebraic Proof

15 (a) Use algebra to show that $4.\dot{5}\dot{7} = 4\frac{19}{33}$

(2)

(b) Show that $\frac{2}{6-3\sqrt{2}}$ can be written in the form $\frac{a+\sqrt{a}}{b}$

where a and b are integers.

Show your working clearly.

(3)

(Total for Question 15 is 5 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2021

1 (a) Let

(a) Let

$$x = 4.\dot{5}\dot{7}$$

2 Calculate value

Then

$$100x = 457.\dot{5}\dot{7}$$

3 Subtract

Subtract:

$$100x - x = 457.\dot{5}\dot{7} - 4.\dot{5}\dot{7}$$

$$99x = 453$$

$$x = \frac{453}{99} = \frac{151}{33} = 4\frac{19}{33}$$

4 **(b)**

(b)

$$\frac{2}{6-3\sqrt{2}} \times \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = \frac{12+6\sqrt{2}}{36-18}$$

$$= \frac{12+6\sqrt{2}}{18} = \frac{2+\sqrt{2}}{3}$$

---

5 **Simplify surd**

So this is in the form $\frac{a+\sqrt{a}}{b}$, with $a = 2$ and $b = 3$.

FINAL ANSWER

$4.\dot{5}\dot{7} = 4\frac{19}{33}$ and $\frac{2}{6-3\sqrt{2}} = \frac{2+\sqrt{2}}{3}$.

4MA1/2HR JANUARY 2021

Question 15

4 marks | Algebraic Proof

15 Prove algebraically that the product of any two odd numbers is always an odd number.

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR January 2021

1 Let the two odd numbers

Let the two odd numbers be

$$2m+1 \quad \text{and} \quad 2n+1$$

2 Their product is

Their product is

$$(2m+1)(2n+1)$$

$$= 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1$$

3 This is of the form

This is of the form

$$2k+1$$

where k is an integer.

4 Use algebraic proof asks general

So the product is odd.

FINAL ANSWER

The product of any two odd numbers is always odd.

4MA1/1H MAY 2021

Question 17

3 marks | Algebraic Proof

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2021

1 Let the three consecutive even

Let the three consecutive even numbers be

$$2n-2, \quad 2n, \quad 2n+2$$

2 The middle number is

The middle number is $2n$.

3 The difference between the square

The difference between the square of the largest and the square of the smallest is

$$(2n+2)^2-(2n-2)^2$$

$$=(4n^2+8n+4)-(4n^2-8n+4)$$

$$=16n$$

4 Calculate value

Since $8 \times \text{middle number} = 8(2n) = 16n$, the result is 8 times the middle number.

FINAL ANSWER

The difference is always 8 times the middle number.

Question 19

6 marks | Algebraic Proof

19 *ABCED* is a five-sided shape.

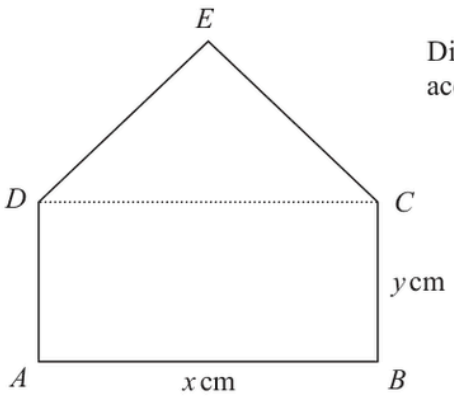


Diagram **NOT** accurately drawn

ABCD is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $BC = y$ cm

The perimeter of *ABCED* is 100 cm.
The area of *ABCED* is R cm^2

(a) Show that $R = \frac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

(3)

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\frac{p}{q - \sqrt{3}}$ where p and q are integers.

$x =$ .....

(2)

(ii) Explain why the maximum value of R is given by this value of x.

.....

(1)

(Total for Question 19 is 6 marks)

1 **Rearrange formula**

The perimeter is made from three sides of length x and two sides of length y:

$$3x + 2y = 100$$

2 **Rearrange formula**

So

$$y = \frac{100 - 3x}{2}$$

3 **The area is the area**

The area is the area of the rectangle plus the area of the equilateral triangle:

$$R = xy + \frac{\sqrt{3}}{4}x^2$$

4 **Substitute**

Substitute $y = \frac{100 - 3x}{2}$:

$$R = x\left(\frac{100 - 3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$

$$R = 50x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$

$$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$$

5 **Rearrange formula**

For the maximum, write

$$R = -\frac{6 - \sqrt{3}}{4}x^2 + 50x$$

6 **The maximum occurs at**

The maximum occurs at

$$x = \frac{-b}{2a} = \frac{-50}{2\left(-\frac{6-\sqrt{3}}{4}\right)}$$

$$x = \frac{100}{6 - \sqrt{3}}$$

7 **Solve quadratic equation**

The coefficient of x^2 is negative, so the quadratic graph opens downwards and this vertex gives a maximum.

✓ **FINAL ANSWER**

$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$, and $x = \frac{100}{6 - \sqrt{3}}$.

4MA1/2HR JANUARY 2022

Question 13

3 marks | Algebraic Proof

13 $a = \sqrt{8} + 4$

$b = \sqrt{8} - 4$

$(a - b)(a + b)$ can be written in the form $y\sqrt{4y}$

Find the value of y
Show your working clearly.

$y =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2022

1 Simplify surd

$$a = \sqrt{8} + 4, \qquad b = \sqrt{8} - 4$$

$$a - b = (\sqrt{8} + 4) - (\sqrt{8} - 4) = 8$$

$$a + b = (\sqrt{8} + 4) + (\sqrt{8} - 4) = 2\sqrt{8} = 4\sqrt{2}$$

2 Simplify surd

So

$$(a - b)(a + b) = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

3 The question says this equals

The question says this equals $y\sqrt{4y}$. Try $y = 8$:

$$8\sqrt{32} = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

FINAL ANSWER

$y = 8$.

4MA1/1HR JANUARY 2022

Question 15

3 marks | Algebraic Proof

15 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2022

1 Let the three consecutive whole

Let the three consecutive whole numbers be

$$n, \quad n+1, \quad n+2$$

2 The sum of the squares

The sum of the squares of the smallest and largest is

$$n^2 + (n+2)^2 = n^2 + n^2 + 4n + 4 = 2n^2 + 4n + 4$$

3 Twice the square of the

Twice the square of the middle number is

$$2(n+1)^2 = 2(n^2 + 2n + 1) = 2n^2 + 4n + 2$$

4 Calculate value

So

$$2n^2 + 4n + 4 = (2n^2 + 4n + 2) + 2$$

FINAL ANSWER

The sum is always 2 more than twice the square of the middle number.

4MA1/2H JANUARY 2022

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a+\sqrt{b}$ where a and b are integers.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H January 2022

1 Simplify surd

$$\frac{\sqrt{12}}{\sqrt{3}+2}=\frac{2\sqrt{3}}{\sqrt{3}+2}$$

2 Multiply by the conjugate

Multiply by the conjugate:

$$\frac{2\sqrt{3}}{\sqrt{3}+2}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2\sqrt{3}(2-\sqrt{3})}{4-3}$$

$$=4\sqrt{3}-6$$

3 Simplify surd

Since $4\sqrt{3}=\sqrt{48}$,

$$4\sqrt{3}-6=-6+\sqrt{48}$$

FINAL ANSWER

$-6+\sqrt{48}$, so $a=-6$ and $b=48$.

4MA1/2H JANUARY 2022

Question 18

3 marks | Algebraic Proof

18 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H January 2022

1 Let the two consecutive odd

Let the two consecutive odd numbers be

$$2n+1 \quad \text{and} \quad 2n+3$$

2 The sum of their squares

The sum of their squares is

$$(2n+1)^2+(2n+3)^2$$

$$=4n^2+4n+1+4n^2+12n+9$$

$$=8n^2+16n+10$$

$$=8(n^2+2n+1)+2=8(n+1)^2+2$$

3 Use Algebraic proof correct

This is a multiple of 8 plus 2, so the remainder is always 2.

FINAL ANSWER

The remainder is always 2.

4MA1/2HR JUNE 2022

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.3\dot{4}\dot{5} = \frac{19}{55}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2022

1 Let

Let

$$x = 0.3\dot{4}\dot{5}$$

2 Calculate value

Then

$$10x = 3.\dot{4}\dot{5}$$

3 Calculate value

and

$$1000x = 345.\dot{4}\dot{5}$$

4 Subtract

Subtract:

$$1000x - 10x = 345.\dot{4}\dot{5} - 3.\dot{4}\dot{5}$$

$$990x = 342$$

$$x = \frac{342}{990} = \frac{19}{55}$$

FINAL ANSWER

$0.3\dot{4}\dot{5} = \frac{19}{55}$.

4MA1/2H JUNE 2023

Question 1

3 marks | Algebraic Proof

1 Show that $4\frac{2}{3} \div 1\frac{1}{5} = 3\frac{8}{9}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2023

1 **Evaluate fraction**

$$4\frac{2}{3} = \frac{14}{3}, \qquad 1\frac{1}{5} = \frac{6}{5}$$

$$4\frac{2}{3} \div 1\frac{1}{5} = \frac{14}{3} \div \frac{6}{5}$$

$$= \frac{14}{3} \times \frac{5}{6} = \frac{70}{18} = \frac{35}{9}$$

$$\frac{35}{9} = 3\frac{8}{9}$$

FINAL ANSWER

$3\frac{8}{9}$.

4MA1/2HR JUNE 2023

Question 16

3 marks | Algebraic Proof

16 Show that $\dfrac{2\sqrt{3}}{\sqrt{3}-1}$ can be written in the form $a+\sqrt{a}$ where a is an integer.

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2023

1 Simplify surd

$$\frac{2\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{2\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$=\frac{6+2\sqrt{3}}{2}=3+\sqrt{3}$$

2 This is in the form

This is in the form $a+\sqrt{a}$, where $a=3$.

FINAL ANSWER

$3+\sqrt{3}$, so $a=3$.

4MA1/1HR MAY 2023

Question 16

2 marks | Algebraic Proof

16 Use algebra to show that $0.1\dot{7}\dot{6} = \frac{35}{198}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May 2023

1 Let

Let

$$x = 0.1767676\ldots$$

2 Calculate value

Then

$$10x = 1.767676\ldots$$

3 Calculate value

and

$$1000x = 176.767676\ldots$$

4 Subtract

Subtract:

$$990x = 175$$

$$x = \frac{175}{990} = \frac{35}{198}$$

FINAL ANSWER

$0.1767676\ldots = \frac{35}{198}$.

4MA1/2H NOVEMBER 2023

Question 6

3 marks | Algebraic Proof

6 Show that $3\frac{3}{7} \div 2\frac{2}{3} = 1\frac{2}{7}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2023

1 Evaluate fraction

$$3\frac{3}{7} = \frac{24}{7}, \qquad 2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{7} \div 2\frac{2}{3} = \frac{24}{7} \div \frac{8}{3}$$

$$= \frac{24}{7} \times \frac{3}{8} = \frac{9}{7} = 1\frac{2}{7}$$

FINAL ANSWER

$1\frac{2}{7}$.

4MA1/2H NOVEMBER 2023

Question 15

5 marks | Algebraic Proof

15 (a) Use algebra to show that $0.3\dot{7}\dot{2} = \frac{41}{110}$

(2)

(b) Express $\frac{\sqrt{125} + \sqrt{80}}{\sqrt{3}}$ in the form $\sqrt{n}$ where n is an integer.

Show your working clearly.

.....

(3)

(Total for Question 15 is 5 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2023

1 **(a) Let**

(a) Let

$$x = 0.3727272\ldots$$

2 **Calculate value**

Then

$$10x = 3.727272\ldots$$

3 **Calculate value**

and

$$1000x = 372.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 369$$

$$x = \frac{369}{990} = \frac{41}{110}$$

5 (b)

(b)

$$\frac{\sqrt{125}+\sqrt{80}}{\sqrt{3}} = \frac{5\sqrt{5}+4\sqrt{5}}{\sqrt{3}}$$

$$= \frac{9\sqrt{5}}{\sqrt{3}}$$

---

6 Rationalise

Rationalise:

$$\frac{9\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{15}}{3} = 3\sqrt{15}$$

$$3\sqrt{15} = \sqrt{135}$$

FINAL ANSWER

$0.3727272\ldots = \frac{41}{110}$, and $n = 135$.

4MA1/1HR MAY 2024

Question 4

3 marks | Algebraic Proof

4 Show that $2\frac{1}{3} \div 5\frac{1}{4} = \frac{4}{9}$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May 2024

1 Evaluate fraction

$$2\frac{1}{3} = \frac{7}{3}, \qquad 5\frac{1}{4} = \frac{21}{4}$$

$$2\frac{1}{3} \div 5\frac{1}{4} = \frac{7}{3} \div \frac{21}{4}$$

$$= \frac{7}{3} \times \frac{4}{21} = \frac{28}{63} = \frac{4}{9}$$

✓ **FINAL ANSWER**

$\frac{4}{9}$.

4MA1/2H JUNE 2024

Question 14

7 marks | Algebraic Proof

14 (a) Show that $3y(2y + 5)(y + 7)$ can be written in the form $ay^3 + by^2 + cy$ where a, b and c are integers.

(3)

(b) Solve $\frac{2x + 3}{5} + \frac{6x - 5}{4} = \frac{163}{100}$

Show clear algebraic working.

$x =$ .....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2024

1 **(a)**

(a)

$$3y(2y+5)(y+7)$$

2 **Expand brackets**

First expand the two brackets:

$$(2y+5)(y+7) = 2y^2 + 14y + 5y + 35 = 2y^2 + 19y + 35$$

3 **Multiply by**

Now multiply by $3y$:

$$3y(2y^2 + 19y + 35) = 6y^3 + 57y^2 + 105y$$

4 **Calculate value**

So $a = 6$, $b = 57$, and $c = 105$.

5 **(b)**

(b)

$$\frac{2x+3}{5} + \frac{6x-5}{4} = \frac{163}{100}$$

6 **Multiply by 100**

Multiply by 100:

$$20(2x+3)+25(6x-5)=163$$

$$40x+60+150x-125=163$$

$$190x-65=163$$

$$190x=228$$

$$x=\frac{228}{190}=\frac{6}{5}$$

FINAL ANSWER

$6y^3+57y^2+105y$, and $x=\frac{6}{5}$.

4MA1/1H MAY 2024

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{1+\sqrt{5}}{3-\sqrt{5}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers.

Show each stage of your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2024

1 Simplify surd

$$\frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(1+\sqrt{5})(3+\sqrt{5})}{9-5}$$

$$= \frac{3+\sqrt{5}+3\sqrt{5}+5}{4}$$

$$= \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$$

FINAL ANSWER

$2+\sqrt{5}$, so $a=2$ and $b=5$.

4MA1/2HR JUNE 2024

Question 18

2 marks | Algebraic Proof

18 Use algebra to show that $0.\dot{3}0\dot{6} = \frac{34}{111}$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2024

1 Let

Let

$$x = 0.\dot{3}0\dot{6}$$

2 Calculate value

Then

$$1000x = 306.\dot{3}0\dot{6}$$

3 Subtract

Subtract:

$$1000x - x = 306$$

$$999x = 306$$

$$x = \frac{306}{999} = \frac{34}{111}$$

FINAL ANSWER

$0.\dot{3}0\dot{6} = \frac{34}{111}$.

4MA1/2H NOVEMBER 2024

Question 1

3 marks | Algebraic Proof

1 Show that $1\frac{5}{7} \times 2\frac{3}{16} = 3\frac{3}{4}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2024

1 **Evaluate fraction**

$$1\frac{5}{7} = \frac{12}{7}, \qquad 2\frac{3}{16} = \frac{35}{16}$$

$$1\frac{5}{7} \times 2\frac{3}{16} = \frac{12}{7} \times \frac{35}{16}$$

$$= \frac{420}{112} = \frac{15}{4} = 3\frac{3}{4}$$

✓ **FINAL ANSWER**

$3\frac{3}{4}$.

4MA1/2H NOVEMBER 2024

Question 15

2 marks | Algebraic Proof

15 Use algebra to show that $0.7\dot{6}\dot{3} = \frac{42}{55}$

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2024

1 Let

Let

$$x = 0.7636363\ldots$$

2 Calculate value

Then

$$10x = 7.636363\ldots$$

3 Calculate value

and

$$1000x = 763.636363\ldots$$

4 Subtract

Subtract:

$$990x = 756$$

$$x = \frac{756}{990} = \frac{42}{55}$$

FINAL ANSWER

$0.7636363\ldots = \frac{42}{55}$.

4MA1/1H NOVEMBER 2024

Question 17

4 marks | Algebraic Proof

17 (a) Express $\sqrt{675}$ in the form $n\sqrt{27}$ where n is a positive integer.

.....

(1)

(b) Show that $\dfrac{5-\sqrt{2}}{\sqrt{2}-1}$ can be written in the form $a+b\sqrt{2}$ where a and b are integers.

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2024

1 **(a)**

(a)

$$\sqrt{675} = \sqrt{25 \times 27} = 5\sqrt{27}$$

2 **Calculate value**

So $n = 5$.

3 **(b)**

(b)

$$\frac{5-\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(5-\sqrt{2})(\sqrt{2}+1)}{2-1}$$

$$= 5\sqrt{2} + 5 - 2 - \sqrt{2}$$

$$= 3 + 4\sqrt{2}$$

✓ **FINAL ANSWER**

$n = 5$, and $3 + 4\sqrt{2}$.

4MA1/2H NOVEMBER 2024

Question 23

4 marks | Algebraic Proof

23 Show that $\dfrac{16x^2 - 36}{x - 7} \div \dfrac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x) = n$

where n is an integer to be found.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2024

1 Evaluate fraction

$$\frac{16x^2 - 36}{x - 7} \div \frac{2x^2 + 7x + 6}{x^2 - 5x - 14} - (7 + 8x)$$

2 Factorise

Factorise:

$$16x^2 - 36 = 4(2x - 3)(2x + 3)$$

$$2x^2 + 7x + 6 = (2x + 3)(x + 2)$$

$$x^2 - 5x - 14 = (x - 7)(x + 2)$$

3 Evaluate fraction

So

$$\frac{4(2x-3)(2x+3)}{x-7} \div \frac{(2x+3)(x+2)}{(x-7)(x+2)} - (7+8x)$$

$$= \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)} - (7+8x)$$

$$= 4(2x-3) - (7+8x)$$

$$= 8x - 12 - 7 - 8x$$

$$= -19$$

4 Calculate value

So $n = -19$, which is an integer.

FINAL ANSWER

$n = -19$

4MA1/1HR JUNE 2025

Question 3

3 marks | Algebraic Proof

3 Show that $2\frac{1}{4} \times 1\frac{5}{7} = 3\frac{6}{7}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2025

1 **Evaluate fraction**

$$2\frac{1}{4} = \frac{9}{4}, \qquad 1\frac{5}{7} = \frac{12}{7}$$

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{9}{4} \times \frac{12}{7}$$

$$= \frac{108}{28} = \frac{27}{7} = 3\frac{6}{7}$$

✓ **FINAL ANSWER**

$3\frac{6}{7}$.

4MA1/2H JUNE 2025

Question 3

3 marks | Algebraic Proof

3 Show that $7\frac{1}{3} - 3\frac{4}{7} = 3\frac{16}{21}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2025

1 **Evaluate fraction**

$$7\frac{1}{3} = \frac{22}{3}, \qquad 5\frac{4}{7} = \frac{39}{7}$$

$$7\frac{1}{3} - 5\frac{4}{7} = \frac{22}{3} - \frac{39}{7}$$

$$= \frac{154}{21} - \frac{117}{21} = \frac{37}{21}$$

$$\frac{37}{21} = 1\frac{16}{21}$$

FINAL ANSWER

$1\frac{16}{21}$.

Question 15

4 marks | Algebraic Proof

15 $(\sqrt{3})^5 = k\sqrt{3}$ where k is an integer.

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{21}{3-\sqrt{2}}$ can be written in the form $c + \sqrt{d}$

where c and d are integers.
Show each stage of your working clearly.

(3)

(Total for Question 15 is 4 marks)

1 **(a)**

(a)

$$(\sqrt{3})^5 = (\sqrt{3})^4\sqrt{3} = 9\sqrt{3}$$

2 **Calculate value**

So $k = 9$.

3 **(b)**

(b)

$$\frac{21}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{21(3+\sqrt{2})}{9-2}$$

$$= 3(3+\sqrt{2}) = 9+3\sqrt{2}$$

4 **Simplify surd**

Since $3\sqrt{2} = \sqrt{18}$,

$$9+3\sqrt{2} = 9+\sqrt{18}$$

FINAL ANSWER

$k = 9$, and $\frac{21}{3-\sqrt{2}} = 9+\sqrt{18}$.

4MA1/2HR JUNE 2025

Question 16

3 marks | Algebraic Proof

16 Show that $\frac{4}{3\sqrt{5}+7}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show each stage of your working.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2025

1 **Simplify surd**

$$\frac{4}{3\sqrt{5}+7} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}} = \frac{4(7-3\sqrt{5})}{49-45}$$

$$= 7-3\sqrt{5}$$

2 **Simplify surd**

Since $3\sqrt{5} = \sqrt{45}$,

$$7-3\sqrt{5} = 7-\sqrt{45}$$

✓ **FINAL ANSWER**

$7-\sqrt{45}$, so $a = 7$ and $b = 45$.

4MA1/1HR JUNE 2025

Question 17

3 marks | Algebraic Proof

17 Prove that, for any three numbers which are consecutive multiples of 4, the difference between the square of the largest number and the square of the smallest number is always a multiple of 64

Show clear algebraic working.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2025

1 Let the three consecutive multiples

Let the three consecutive multiples of 4 be

$$4n, \quad 4n+4, \quad 4n+8$$

2 The difference between the square

The difference between the square of the largest and the square of the smallest is

$$(4n+8)^2 - (4n)^2$$

$$= 16n^2 + 64n + 64 - 16n^2$$

$$= 64n + 64 = 64(n+1)$$

3 This is always a multiple

This is always a multiple of 64.

FINAL ANSWER

The difference is always a multiple of 64.

4MA1/2H JUNE 2025

Question 18

2 marks | Algebraic Proof

18 Use algebra to show that $0.\dot{7}0\dot{2} = \frac{26}{37}$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2025

1 Let

Let

$$x = 0.\dot{7}0\dot{2}$$

2 Calculate value

Then

$$1000x = 702.\dot{7}0\dot{2}$$

3 Subtract

Subtract:

$$999x = 702$$

$$x = \frac{702}{999} = \frac{26}{37}$$

FINAL ANSWER

$0.\dot{7}0\dot{2} = \frac{26}{37}$.

4WM2/01 MAY 2025

Question 20

3 marks | Algebraic Proof

20 a, b and c are three consecutive even numbers where $a < b < c$

Prove algebraically that $b^2 = ac + 4$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM2/01 May 2025

1 Let

Let

$$a = 2n, \qquad b = 2n + 2, \qquad c = 2n + 4$$

2 Calculate value

Then

$$ac + 4 = (2n)(2n + 4) + 4$$

$$= 4n^2 + 8n + 4$$

$$= (2n + 2)^2 = b^2$$

3 Calculate value

So $b^2 = ac + 4$.

FINAL ANSWER

So $b^2 = ac + 4$.

4MA1/1H NOVEMBER 2025

Question 5

3 marks | Algebraic Proof

5 Show that $3\frac{5}{7} + 1\frac{2}{3} = 5\frac{8}{21}$

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2025

1 Evaluate fraction

$$\frac{3}{7} + \frac{1}{3}$$

2 Use denominator 21

Use denominator 21:

$$\frac{3}{7} = \frac{9}{21}, \qquad \frac{1}{3} = \frac{7}{21}$$

$$\frac{9}{21} + \frac{7}{21} = \frac{16}{21}$$

FINAL ANSWER

$\frac{16}{21}$.

4MA1/2H NOVEMBER 2025

Question 16

2 marks | Algebraic Proof

16 Use algebra to show that $0.3\dot{7}\dot{8} = \frac{25}{66}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2025

1 Let

Let

$$x = 0.3787878\ldots$$

2 Calculate value

Then

$$10x = 3.787878\ldots$$

3 Calculate value

and

$$1000x = 378.787878\ldots$$

4 Subtract

Subtract:

$$990x = 375$$

$$x = \frac{375}{990} = \frac{25}{66}$$

FINAL ANSWER

$0.3787878\ldots = \frac{25}{66}$.

4MA1/1H NOVEMBER 2025

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{2\sqrt{7}+2}{\sqrt{7}-3}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2025

1 **Simplify surd**

$$\frac{2\sqrt{7}+2}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{(2\sqrt{7}+2)(\sqrt{7}+3)}{7-9}$$

$$=\frac{14+6\sqrt{7}+2\sqrt{7}+6}{-2}$$

$$=\frac{20+8\sqrt{7}}{-2}=-10-4\sqrt{7}$$

2 **Simplify surd**

Since $4\sqrt{7}=\sqrt{112}$,

$$-10-4\sqrt{7}=-10-\sqrt{112}$$

FINAL ANSWER

$-10-\sqrt{112}$, so $a=-10$ and $b=112$.

TOPIC

Sequences

Every past-paper question on this topic, 2020 to 2026.

Question 24

5 marks | Sequences

24 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.
The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

.....

(Total for Question 24 is 5 marks)

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

2 Find the nth term

So

$$S_{10} = 5(2a + 9d)$$

3 Find the nth term

and

$$S_5 = \frac{5}{2}(2a + 4d)$$

4 Given

Given $S_{10} = 4S_5$:

$$5(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$10a + 45d = 20a + 40d$$

$$5d = 10a$$

$$d = 2a$$

5 The 8th term is

The 8th term is 45:

$$a + 7d = 45$$

6 Substitute

Substitute $d = 2a$:

$$a + 14a = 45$$

$$15a = 45$$

$$a = 3$$

FINAL ANSWER

the first term is $\mathbf{3}$.

4MA1/2HR JANUARY 2020

Question 22

4 marks | Sequences

22 The first term of an arithmetic series S is -6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2020

1 Find the nth term

First term:

$$a = -6$$

$$S_{30} = 2865$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$2865 = \frac{30}{2}(2(-6) + 29d)$$

$$2865 = 15(-12 + 29d)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7$$

3 **The 9th term is**

The 9th term is

$$u_9 = a + 8d$$

$$u_9 = -6 + 8(7) = 50$$

✓ **FINAL ANSWER**

50

4MA1/2H JANUARY 2020

Question 25

3 marks | Sequences

25 Mario is going to save \$50 in the year 2021

He is going to continue to save, up to and including the year 2070, by increasing the amount he saves each year by \$$k$

Mario will save a total of \$33 125 from 2021 to 2070

Work out the value of k.

$k =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2020

1 Mario saves from 2021 up

Mario saves from 2021 up to and including 2070.

2 Number of years

Number of years:

$$2070 - 2021 + 1 = 50$$

3 This is an arithmetic sequence

This is an arithmetic sequence.

4 Use trigonometry

First term:

$$50$$

5 Common difference

Common difference:

$$k$$

6 Last term

Last term:

$$50 + 49k$$

7 **Use the arithmetic series formula**

Use the arithmetic series formula:

$$S_n = \frac{n}{2}(a + l)$$

$$33125 = \frac{50}{2}(50 + (50 + 49k))$$

$$33125 = 25(100 + 49k)$$

$$1325 = 100 + 49k$$

$$49k = 1225$$

$$k = 25$$

✓ **FINAL ANSWER**

$\boxed{k = 25}$

4MA1/1HR MAY/NOVEMBER 2020

Question 18

4 marks | Sequences

18 In an arithmetic series, the 6th term is 39
In the same arithmetic series, the 19th term is 7.8

Work out the sum of the first 25 terms of the arithmetic series.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May/November 2020

1 Let the first term be

Let the first term be a and the common difference be d.

2 The 6th term is 39

The 6th term is 39:

$$a + 5d = 39$$

3 Find the nth term

The 19th term is 7.8:

$$a + 18d = 7.8$$

4 Subtract

Subtract:

$$13d = -31.2$$

$$d = -2.4$$

5 Find the nth term

Then

$$a + 5(-2.4) = 39$$

$$a = 51$$

6 **The sum of the first**

The sum of the first 25 terms is

$$S_{25} = \frac{25}{2}(2a + 24d)$$

$$S_{25} = \frac{25}{2}(102 - 57.6) = 555$$

✓ **FINAL ANSWER**

555.

4MA1/1H MAY/NOVEMBER 2020

Question 24

4 marks | Sequences

24 Here are the first five terms of an arithmetic sequence.

8 15 22 29 36

Work out the sum of all the terms from the 50th term to the 100th term inclusive.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May/November 2020

1 The arithmetic sequence is

The arithmetic sequence is

$$8,\ 15,\ 22,\ 29,\ 36,\ldots$$

2 Calculate value

So

$$a = 8, \quad d = 7$$

3 The th term is

The nth term is

$$u_n = 8 + 7(n-1) = 7n + 1$$

$$u_{50} = 7(50) + 1 = 351$$

$$u_{100} = 7(100) + 1 = 701$$

4 There are

There are

$$100 - 50 + 1 = 51$$

5 **Find the nth term**

terms from the 50th to the 100th inclusive.

$$\text{sum} = \frac{51}{2}(351 + 701)$$

$$= 26826$$

✓ **FINAL ANSWER**

26826

4MA1/1HR JANUARY 2021

Question 21

6 marks | Sequences

21 The nth term of an arithmetic series is u_n where $u_n > 0$ for all n
The sum to n terms of the series is S_n

Given that $u_4 = 6$ and that $S_{11} = (u_6)^2 + 18$

find the value of u_{20}

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2021

1 Let

Let $u_6 = v$.

2 Use series formula

For an arithmetic series,

$$S_{11} = \frac{11}{2}(u_1 + u_{11})$$

3 The average of and is

The average of u_1 and u_{11} is u_6, so

$$S_{11} = 11u_6 = 11v$$

4 **Given**

Given

$$S_{11} = (u_6)^2 + 18$$

$$11v = v^2 + 18$$

$$v^2 - 11v + 18 = 0$$

$$(v - 2)(v - 9) = 0$$

---

5 **Calculate value**

So

$$u_6 = 2 \quad \text{or} \quad u_6 = 9$$

---

6 **Find the nth term**

But $u_4 = 6$. If $u_6 = 2$, then the common difference is negative and later terms would not all be positive. Therefore,

$$u_6 = 9$$

$$d = \frac{u_6 - u_4}{2} = \frac{9 - 6}{2} = \frac{3}{2}$$

$$u_{20} = u_6 + 14d$$

$$u_{20} = 9 + 14\left(\frac{3}{2}\right) = 30$$

✓ **FINAL ANSWER**

$u_{20} = 30$

4MA1/1H JANUARY 2021

Question 24

4 marks | Sequences

24 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$a =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2021

1 Evaluate fraction

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{2n} = n(2a + (2n-1)d)$$

2 Given

Given

$$S_{2n} = 4S_n$$

$$n(2a + (2n-1)d) = 4 \cdot \frac{n}{2}(2a + (n-1)d)$$

3 Divide by

Divide by n:

$$2a + (2n-1)d = 2(2a + (n-1)d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$d = 2a$$

$$a = \frac{d}{2}$$

FINAL ANSWER

$$a = \frac{d}{2}$$

4MA1/1H MAY 2021

Question 2

2 marks | Sequences

2 Here are the first 4 terms of an arithmetic sequence.

85 79 73 67

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2021

1 The sequence is

The sequence is

$$85,\ 79,\ 73,\ 67$$

2 The common difference is

The common difference is -6.

$$\text{nth term} = 85 + (n-1)(-6)$$

$$= 85 - 6n + 6 = 91 - 6n$$

FINAL ANSWER

$91 - 6n$.

4MA1/1H MAY 2021

Question 23

5 marks | Sequences

23 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$N =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2021

1 **The 2nd term is**

The 2nd term is

$$a + d = 8.5$$

2 **The 5th term is**

The 5th term is

$$a + 4d = 13$$

3 **Subtract**

Subtract:

$$3d = 4.5$$

$$d = 1.5$$

$$a = 8.5 - 1.5 = 7$$

4 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$292 = \frac{N}{2}(14 + 1.5(N-1))$$

$$292 = \frac{N}{2}(1.5N + 12.5)$$

$$3N^2 + 25N - 1168 = 0$$

$$N = \frac{-25 + 121}{6} = 16$$

✓ **FINAL ANSWER**

$N = 16$

4MA1/2H NOVEMBER 2021

Question 20

5 marks | Sequences

20 Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2021

1 The first four terms are

The first four terms are

$$k, \quad \frac{3k}{4}, \quad \frac{k}{2}, \quad \frac{k}{4}$$

2 The common difference is

The common difference is

$$d = -\frac{k}{4}$$

3 The 15th term is

The 15th term is

$$u_{15} = k + 14\left(-\frac{k}{4}\right)$$

$$u_{15} = k - \frac{7k}{2} = -\frac{5k}{2}$$

4 **Given**

Given

$$-\frac{5k}{2} = 90 + 2k$$

$$-5k = 180 + 4k$$

$$k = -20$$

---

5 **Evaluate fraction**

So

$$a = -20, \quad d = 5$$

$$S_{30} = \frac{30}{2}(2(-20) + 29(5))$$

$$S_{30} = 15(105) = 1575$$

✓ **FINAL ANSWER**

1575

4MA1/1HR JANUARY 2022

Question 16

4 marks | Sequences

16 An arithmetic series has first term 1 and common difference 4

Find the sum of all terms of the series from the 41st term to the 100th term inclusive.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2022

1 Find the nth term

The first term is 1 and the common difference is 4.

2 The 41st term is

The 41st term is

$$1 + 40(4) = 161$$

3 The 100th term is

The 100th term is

$$1 + 99(4) = 397$$

4 There are

There are

$$100 - 41 + 1 = 60$$

5 Find the nth term

terms from the 41st to the 100th inclusive.

$$S = \frac{60}{2}(161 + 397)$$

$$S = 30 \times 558 = 16740$$

FINAL ANSWER

16740.

4MA1/2H JANUARY 2022

Question 26

5 marks | Sequences

26 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$d =$ .....

$m =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H January 2022

1 Calculate value

$$S_m = 39$$

$$S_{2m} = 320$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

3 Calculate value

Then

$$2S_m = m(2a + (m-1)d) = 78$$

4 Calculate value

and

$$S_{2m} = m(2a + (2m-1)d) = 320$$

5 **Subtract**

Subtract:

$$S_{2m} - 2S_m = m[(2m-1)d - (m-1)d]$$

$$320 - 78 = m(md)$$

$$242 = m^2 d$$

$$242 = 2 \times 11^2$$

---

6 **Square factor**

Since d is prime and m^2 is a square factor,

$$d = 2, \quad m^2 = 121$$

$$m = 11$$

✓ **FINAL ANSWER**

$d = 2,\ m = 11$

4MA1/1H JUNE 2022

Question 1

3 marks | Sequences

1 Here are the first five terms of an arithmetic sequence.

1 5 9 13 17

(a) Find an expression, in terms of n, for the nth term of this sequence.

.....

(2)

The nth term of another arithmetic sequence is $3n + 5$

(b) Find an expression, in terms of m, for the $(2m)$th term of this sequence.

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2022

1 The sequence is

The sequence is

$$1,\ 5,\ 9,\ 13,\ 17$$

2 Find the nth term

The common difference is 4, so the nth term is

$$4n - 3$$

3 Find the nth term

For the second sequence, the nth term is $3n + 5$.

4 The th term is

The $(2m)$th term is

$$3(2m) + 5 = 6m + 5$$

FINAL ANSWER

$4n - 3$, and $6m + 5$.

4MA1/1HR JUNE 2022

Question 2

4 marks | Sequences

2 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

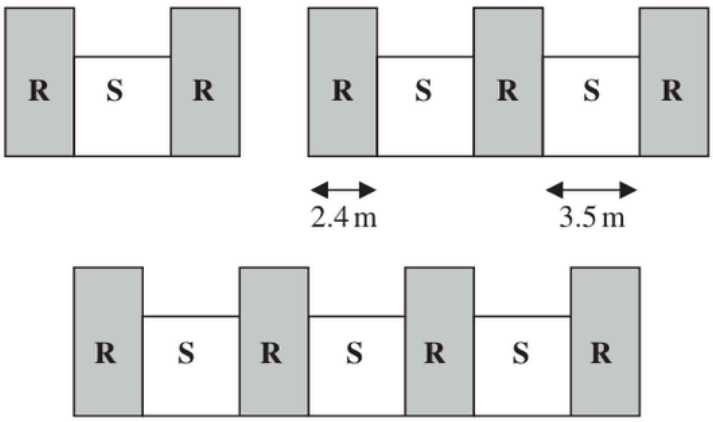


Diagram **NOT** accurately drawn

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

..... m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
Give your answer in its simplest form.

$W =$ .....

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2022

1 **Find the nth term**

For a shelving unit with 6 type R shelves, there are 5 type S shelves.

$$6(2.4) + 5(3.5) = 14.4 + 17.5 = 31.9$$

2 **Find the nth term**

For n type R shelves, there are $n-1$ type S shelves.

$$W = 2.4n + 3.5(n-1)$$

$$W = 2.4n + 3.5n - 3.5 = 5.9n - 3.5$$

✓ **FINAL ANSWER**

$31.9\ \text{m}$, and $W = 5.9n - 3.5$.

4MA1/1HR JUNE 2022

Question 25

5 marks | Sequences

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2022

1 **Calculate value**

$$S_{10} = 4S_5$$

2 **Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

3 **Calculate value**

So

$$5(2a + 9d) = 4 \cdot 5(a + 2d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

4 **The 8th term is 45**

The 8th term is 45:

$$u_8 = a + 7d = 45$$

$$a + 7(2a) = 45$$

$$15a = 45$$

$$a = 3$$

✓ **FINAL ANSWER**

The first term is $\mathbf{3}$

4MA1/2HR JUNE 2023

Question 4

2 marks | Sequences

4 Here are the first four terms of an arithmetic sequence.

38 31 24 17

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2023

1 The sequence is

The sequence is

$$38,\ 31,\ 24,\ 17$$

2 The common difference is

The common difference is -7.

$$\text{nth term} = 38 + (n-1)(-7)$$

$$= 38 - 7n + 7 = 45 - 7n$$

FINAL ANSWER

$45 - 7n$.

4MA1/1H MAY 2023

Question 20

6 marks | Sequences

20 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

$X =$ .....

(Total for Question 20 is 6 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2023

1 Evaluate fraction

$$S_{80} = 470$$

$$40(2a + 79d) = 470$$

$$2a + 79d = \frac{47}{4}$$

2 The 75th term is

The 75th term is

$$a + 74d = 14.5 = \frac{29}{2}$$

3 Double it

Double it:

$$2a + 148d = 29$$

4 **Subtract**

Subtract:

$$69d = 29 - \frac{47}{4} = \frac{69}{4}$$

$$d = \frac{1}{4}$$

$$a + 74\left(\frac{1}{4}\right) = \frac{29}{2}$$

$$a = -4$$

---

5 **Evaluate fraction**

Now

$$S_X = 171$$

$$171 = \frac{X}{2}\left(2(-4) + (X-1)\frac{1}{4}\right)$$

$$171 = \frac{X(X-33)}{8}$$

$$X^2 - 33X - 1368 = 0$$

$$X = \frac{33 + 81}{2} = 57$$

✓ **FINAL ANSWER**

$X = 57$

4MA1/2HR JUNE 2023

Question 23

5 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$8p \qquad 7p - 3 \qquad 4p + 2$$

The sum of the first n terms of the sequence is -1914

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2023

1 The first three terms are

The first three terms are

$$8p, \quad 7p - 3, \quad 4p + 2$$

2 Find the nth term

For an arithmetic sequence, consecutive differences are equal:

$$(7p - 3) - 8p = (4p + 2) - (7p - 3)$$

$$-p - 3 = -3p + 5$$

$$2p = 8$$

$$p = 4$$

3 Find the nth term

So the first term is

$$8p = 32$$

4 **Find common difference**

and the common difference is

$$(7p-3)-8p=25-32=-7$$

5 **The sum of the first**

The sum of the first n terms is

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$-1914=\frac{n}{2}(64-7(n-1))$$

$$-1914=\frac{n}{2}(71-7n)$$

$$-3828=n(71-7n)$$

$$7n^2-71n-3828=0$$

$$n=\frac{71\pm 335}{14}$$

6 **Only the positive value is**

Only the positive value is possible:

$$n=29$$

✓ **FINAL ANSWER**

$n=29$

4MA1/1H NOVEMBER 2023

Question 19

6 marks | Sequences

19 Here are the first 4 terms in an arithmetic sequence.

3 7 11 15

The last term of the sequence is x
The sum of the terms of the sequence is 7260

Find the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2023

1 The sequence is

The sequence is

$$3,\ 7,\ 11,\ 15$$

2 Find the nth term

The common difference is 4, so the nth term is

$$4n - 1$$

3 Find the nth term

The last term is x, so

$$x = 4n - 1$$

$$n = \frac{x + 1}{4}$$

4 The sum is 7260

The sum is 7260:

$$\frac{n}{2}(3 + x) = 7260$$

5 **Substitute**

Substitute $n = \dfrac{x+1}{4}$:

$$\frac{x+1}{8}(x+3) = 7260$$

$$(x+1)(x+3) = 58080$$

$$x^2 + 4x - 58077 = 0$$

---

6 **Factorise**

Factorise:

$$(x-239)(x+243) = 0$$

---

7 **Find the nth term**

The last term is positive, so

$$x = 239$$

✓ **FINAL ANSWER**

$x = 239$.

4MA1/1H MAY 2024

Question 1

3 marks | Sequences

1 Here are the first four terms of an arithmetic sequence.

1 4 7 10

(a) Find an expression, in terms of n, for the nth term of this sequence.

.....

(2)

The nth term of a different arithmetic sequence is $5n + 17$

(b) Find the 12th term of this sequence.

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2024

1 The sequence is

The sequence is

$$1,\ 4,\ 7,\ 10$$

2 The common difference is

The common difference is 3.

$$\text{nth term} = 1 + 3(n-1) = 3n - 2$$

3 Find the nth term

For the other sequence,

$$5n + 17$$

4 The 12th term is

The 12th term is

$$5(12) + 17 = 77$$

FINAL ANSWER

$3n - 2$, and 77.

4MA1/1HR MAY 2024

Question 23

4 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$(4x-14) \quad , \quad (x+2) \quad , \quad (7x-9)$$

Find, as an integer, the sum of the first 40 terms of the sequence.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2024

1 **The first three terms are**

The first three terms are

$$4x - 14, \quad x + 2, \quad 7x - 9$$

2 **Equal differences give**

Equal differences give

$$(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$$

$$-3x + 16 = 6x - 11$$

$$9x = 27$$

$$x = 3$$

3 **Find the nth term**

So the sequence is

$$-2,\ 5,\ 12, \ldots$$

4 **with common difference**

with common difference 7.

$$S_{40} = \frac{40}{2}(2(-2) + 39(7))$$

$$S_{40} = 20(-4 + 273) = 5380$$

✓ **FINAL ANSWER**

5380

4MA1/1H MAY 2024

Question 24

6 marks | Sequences

24 A polygon has n sides, where $n > 5$

The interior angles of the polygon form an arithmetic sequence.

The smallest angle of the polygon is 84°
The common difference of the sequence is 4°

Work out the sum of the interior angles of the polygon.
Show clear algebraic working.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2024

1 The interior angles form an

The interior angles form an arithmetic sequence with first term 84 and common difference 4.

2 Use trigonometry

So the sum of the angles is

$$\frac{n}{2}(2(84) + (n-1)4)$$

$$= \frac{n}{2}(168 + 4n - 4)$$

$$= 2n^2 + 82n$$

3 The sum of the interior

The sum of the interior angles of an n-sided polygon is

$$180(n-2)$$

4 **Calculate value**

So

$$2n^2 + 82n = 180(n - 2)$$

$$2n^2 - 98n + 360 = 0$$

$$n^2 - 49n + 180 = 0$$

$$(n - 45)(n - 4) = 0$$

---

5 **Solve inequality**

Since $n > 5$,

$$n = 45$$

---

6 **Use trigonometry**

Therefore the sum of the interior angles is

$$180(45 - 2) = 7740$$

✓ **FINAL ANSWER**

7740 degrees

4MA1/1H NOVEMBER 2024

Question 19

5 marks | Sequences

19 An arithmetic series has first term a and common difference d

The sum of the first 30 terms of the arithmetic series is 4395

The sum of the 10th term and the 20th term is 284

Work out the sum of the first 45 terms of the arithmetic series.
Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2024

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_{30} = 4395$$

$$\frac{30}{2}(2a + 29d) = 4395$$

$$2a + 29d = 293$$

2 The 8th term plus the

The 8th term plus the 20th term is 284:

$$(a + 7d) + (a + 19d) = 284$$

$$2a + 26d = 284$$

3 Subtract the equations

Subtract the equations:

$$3d = 9$$

$$d = 3$$

4 **Find the nth term**

Then

$$2a + 26(3) = 284$$

$$2a = 206$$

$$a = 103$$

---

5 **Find the nth term**

Now find S_{45}:

$$S_{45} = \frac{45}{2}(2a + 44d)$$

$$S_{45} = \frac{45}{2}(206 + 132) = 7605$$

✓ **FINAL ANSWER**

7605.

4WM2/01 MAY 2025

Question 5

3 marks | Sequences

5 Here are the first five terms of a number sequence.

11 15 19 23 27

(a) Find an expression in terms of n for the nth term of this sequence.

.....

(2)

The nth term of another number sequence is $5n + 4$

Salim says no term in this sequence is a prime number.

(b) Find a term in this sequence that shows Salim is wrong.

.....

(1)

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM2/01 May 2025

1 Find the nth term

The sequence increases by 4, so

$$u_n = 4n + 7$$

2 Find the nth term

For the sequence $5n + 4$, take $n = 3$:

$$5(3) + 4 = 19$$

3 Check prime condition

and 19 is prime.

FINAL ANSWER

$4n + 7$; a suitable counterexample is 19.

4MA1/1H JUNE 2025

Question 24

5 marks | Sequences

24 An arithmetic series has 30 terms.

The first term is a
The common difference is d

The 20th term is 123
The sum of the 30 terms is 2880

Work out the value of a and the value of d
Show clear algebraic working.

$a =$ .....

$d =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2025

1 Evaluate fraction

$$u_{20} = a + 19d = 123$$

$$S_{30} = \frac{30}{2}(2a + 29d) = 2880$$

$$2a + 29d = 192$$

2 Double the first equation

Double the first equation:

$$2a + 38d = 246$$

3 **Subtract**

Subtract:

$$9d = 54$$

$$d = 6$$

$$a + 19(6) = 123$$

$$a = 9$$

✓ **FINAL ANSWER**

$a = 9,\ d = 6$

4MA1/1HR JUNE 2025

Question 24

6 marks | Sequences

24 The first 3 terms of an arithmetic series are

$$(2x + 5) \qquad (3y - 4) \qquad (4x - 2)$$

where x and y are constants.

The sum of the first 9 terms of the series is 216

Find the value of x and the value of y
Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2025

1 **The first 3 terms are**

The first 3 terms are

$$2x + 5, \quad 3y - 4, \quad 4x - 2$$

2 **Equal differences give**

Equal differences give

$$(3y - 4) - (2x + 5) = (4x - 2) - (3y - 4)$$

$$6y - 6x = 11$$

3 **Use series formula**

For an arithmetic series,

$$S_9 = 9(a + 4d)$$

4 **Here**

Here

$$a = 2x + 5$$

$$d = (3y - 4) - (2x + 5) = 3y - 2x - 9$$

5 **Given**

Given $S_9 = 216$:

$$9[(2x + 5) + 4(3y - 2x - 9)] = 216$$

$$12y - 6x = 55$$

6 **Solve**

Now solve

$$6y - 6x = 11$$

$$12y - 6x = 55$$

7 **Subtracting gives**

Subtracting gives

$$6y = 44$$

$$y = \frac{22}{3}$$

8 **Evaluate fraction**

Then

$$6\left(\frac{22}{3}\right) - 6x = 11$$

$$x = \frac{11}{2}$$

✓ **FINAL ANSWER**

$$x = \frac{11}{2},\ y = \frac{22}{3}$$

4WM2/01 MAY 2025

Question 24

4 marks | Sequences

24 The 21st term of an arithmetic series is 109
The sum of the first 52 terms of the series is 4381

Work out the 5th term of the series.
Show clear algebraic working.

.....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM2/01 May 2025

1 Use series formula

For an arithmetic series,

$$a + 20d = 109$$

2 Evaluate fraction

and

$$\frac{52}{2}(2a + 51d) = 4381$$

$$26(2a + 51d) = 4381$$

3 Solving gives

Solving gives

$$d = -4.5, \qquad a = 199$$

4 The fifth term is

The fifth term is

$$a + 4d = 199 + 4(-4.5) = 181$$

FINAL ANSWER

181.

4WM2/01R MAY 2025

Question 24

6 marks | Sequences

24 An arithmetic series P has first term a and common difference d

The sum of the first 40 terms of P is 2070

An arithmetic series Q has first term $2a$ and common difference $3d$

The 25th term of Q is 186

Work out the sum of the first 40 terms of Q
Show clear algebraic working.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4WM2/01R May 2025

1 Required

Series P: first term *a*, c.d. *d*; *S*₄₀(*P*) = 2070. Series Q: first term 2*a*, c.d. 3*d*; 25th term of Q = 186. Find *S*₄₀(*Q*).

2 Form equation from S 40

Form equation from *S*₄₀(*P*) = 2070:

$$S_40 = frac402\[2a + 39d\] = 20(2a + 39d) = 2070implies2a + 39d = 103.5tag1$$

3 Form equation from 25th term

Form equation from 25th term of Q = 186:

*T*₂₅(*Q*) = 2*a* + 24(3*d*) = 2*a* + 72*d* = 186

4 Solve simultaneous equations

Solve simultaneously — subtract (1) from (2):

33*d* = 82.5 ⟹ *d* = 2.5

5 Substitute d 2 5 (1)

Substitute *d* = 2.5 into (1):

2*a* + 39(2.5) = 103.5 ⟹ 2*a* = 6 ⟹ *a* = 3

6 Compute S 40 (Q

Compute *S*₄₀(*Q*), with first term = 2*a* = 6 and c.d. = 3*d* = 7.5:

$$S_40(Q) = frac402bigl\[2(6) + 39(7.5)bigr\] = 20bigl\[12 + 292.5bigr\] = 20times304.5 = 6090$$

FINAL ANSWER

*S*₄₀(*Q*) = 6090

4MA1/2H NOVEMBER 2025

Question 25

5 marks | Sequences

25 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2025

1 Calculate value

$$a = 10$$

$$u_{20} = 86$$

$$10 + 19d = 86$$

$$d = 4$$

2 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$5194 = \frac{N}{2}(20 + 4(N-1))$$

$$5194 = \frac{N}{2}(4N + 16)$$

$$5194 = 2N(N+4)$$

$$N^2 + 4N - 2597 = 0$$

$$N = \frac{-4 + 102}{2} = 49$$

FINAL ANSWER

$N = 49$

4WM2/01 NOVEMBER 2025

Question 26

5 marks | Sequences

26 The first term of an arithmetic series is 10
The 20th term of the series is 86

The sum of the first N terms of the series is 5194

Work out the value of N
Show your working clearly.

$N =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4WM2/01 November 2025

1 Find common difference

Step 1: Find the common difference *d*. The *n*th term is given by *u*_{*n*} = *a* + (*n*−1)*d*.

*u*₂₀ = 10 + 19*d* = 86

19*d* = 76

$$d = frac7619 = 4$$

2 Use sum formula

Step 2: Use the sum formula to form an equation in terms of *N*.

$$S_N = fracN2left\[2a + (N-1)dright\]$$

Substitute the known values *S*_{*N*} = 5194, *a* = 10, and *d* = 4:

$$5194 = fracN2left\[2(10) + (N-1)(4)right\]$$

3 Solve quadratic equation

Step 3: Simplify and solve the quadratic equation. Multiply by 2 to clear the fraction:

10388 = *N*

$$20 + 4 * N * -4$$

10388 = *N*(4*N*+16)

4*N*² + 16*N* − 10388 = 0

4 Divide the entire equation by

Divide the entire equation by 4 to make it simpler:

*N*² + 4*N* − 2597 = 0

5 Use quadratic formula

Step 4: Use the quadratic formula to find *N*.

$$N = frac-bpmsqrtb^2 - 4ac2a$$

$$N = frac-4pmsqrt16 - 4(1)(-2597)2$$

$$N = frac-4pmsqrt16 + 103882$$

$$N = frac-4pmsqrt104042$$

$$N = frac-4pm1022$$

6 Evaluate fraction

Since *N* must be a positive integer, we discard the negative root:

$$N = frac-4 + 1022 = frac982 = 49$$

✓ FINAL ANSWER

N = 49 Always double-check that your *N* (number of terms) evaluates to a positive whole integer. If it ends up as a decimal or fraction, recheck your arithmetic progression formulas.

4MA1/2H JUNE 2026

Question 2

5 marks | Sequences

2 The nth term of an arithmetic sequence is given by $8n - 5$

(a) Work out the ratio of the 40th term of the sequence to the 60th term of the sequence.
Give your ratio in its simplest form.

.....

(3)

Here are the first five terms of a different arithmetic sequence.

3 8 13 18 23

(b) Find an expression for the nth term of this sequence.

.....

(2)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

$$u_{40} = 8(40) - 5 = 315, \qquad u_{60} = 8(60) - 5 = 475.$$

Substitute the two term numbers into the given rule.

2 **Part (a) | Step 2**

$$315 : 475 = (315 \div 5) : (475 \div 5) = 63 : 95.$$

Divide both entries by their highest common factor, 5.

3 **Part (b) | Step 1**

The common difference is $8 - 3 = 5$, so the rule has the form $5n + c$.

A constant difference gives a linear nth term.

4 **Part (b) | Step 2**

At $n = 1$, $5 + c = 3$, giving $c = -2$. Thus $u_n = 5n - 2$.

Use the first term to find the constant.

✓ **FINAL ANSWER**

(a) $63 : 95$ (b) $5n - 2$

4MA1/2HR JUNE 2026

Question 2

2 marks | Sequences

2 Here are the first five terms of an arithmetic sequence.

14 8 2 –4 –10

Find an expression, in terms of n, for the nth term of this sequence.

.....

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2026

1 Solution | Step 1

The common difference is $8 - 14 = -6$.

Each successive term decreases by 6.

2 Solution | Step 2

$$u_n = 14 + (n-1)(-6) = 20 - 6n.$$

Use first term plus (n-1) common differences.

FINAL ANSWER

$u_n = 20 - 6n.$

Question 25

6 marks | Sequences

25 Here are the first four terms of an arithmetic series.

$$(k + 2) \qquad (k + 5) \qquad (k + 8) \qquad (k + 11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first n terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

(Total for Question 25 is 6 marks)

1 First term and difference

$a = k + 2$, $d = 3$.

2 Use the sum formula

$$\frac{2n}{2}\left[2a + (2n-1)3\right] = 4 \times \frac{n}{2}\left[2a + (n-1)3\right]$$

3 Simplify (divide by n)

$$2a + 6n - 3 = 4a + 6n - 6 \Rightarrow 2a = 3 \Rightarrow a = 1.5,\ k = -\tfrac{1}{2}$$

4 100th term

$$a + 99d = 1.5 + 99(3) = 298.5$$

FINAL ANSWER

298.5

4MA1/2H JUNE 2026

Question 25

5 marks | Sequences

25 An arithmetic series has first term 3 and common difference 8

The nth term of the series is U_n
The sum of the first n terms of the series is S_n

Given that $S_{5k} - S_{3k} = 25\,560$

find the value of U_{4k}
Show clear algebraic working.

.....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2026

1 Solution | Step 1

With first term 3 and difference 8,

$$S_n = \frac{n}{2}[2(3) + (n-1)8] = \frac{n}{2}(8n - 2) = 4n^2 - n.$$

Use the arithmetic-series sum formula with the given first term and common difference.

2 Solution | Step 2

$$S_{5k} = 100k^2 - 5k, \qquad S_{3k} = 36k^2 - 3k.$$

$$(100k^2 - 5k) - (36k^2 - 3k) = 25560 \quad \Rightarrow \quad 64k^2 - 2k - 25560 = 0.$$

Substitute 5k and 3k as the term counts, then subtract the complete expressions.

3 Solution | Step 3

$$32k^2 - k - 12780 = 0 \quad \Rightarrow \quad (k - 20)(32k + 639) = 0.$$

The roots are $k = 20$ and $k = -639/32$. Reject the negative root because the numbers of terms must be positive.
Factor the quadratic and apply the contextual restriction.

4 Solution | Step 4

$$U_n = 3 + (n-1)8 = 8n - 5, \qquad U_{4k} = U_{80} = 8(80) - 5 = 635.$$

Use the nth-term formula with 4k=80, not with k=20.

FINAL ANSWER

$U_{4k} = 635$

4MA1/2HR JUNE 2026

Question 26

6 marks | Sequences

26 Here are the first four terms of an arithmetic series.

$$(k+2) \qquad (k+5) \qquad (k+8) \qquad (k+11)$$

The sum of the first $2n$ terms of the series is 4 times the sum of the first n terms of the series.

Work out the numerical value of the 100th term of the series.
Show your working clearly.

.....

(Total for Question 26 is 6 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2026

Solution | Step 1

The first term is $a = k + 2$ and the common difference is $d = 3$.

Successive listed terms differ by 3.

Solution | Step 2

$$S_n = \frac{n}{2}\{2(k+2) + 3(n-1)\} = \frac{n}{2}(2k + 1 + 3n).$$

Apply the arithmetic-series sum formula.

Solution | Step 3

$$S_{2n} = n\{2(k+2) + 3(2n-1)\} = n(2k + 1 + 6n).$$

Use 2n in place of n throughout the sum formula.

Solution | Step 4

Using $S_{2n} = 4S_n$ and $n > 0$,

$$2k + 1 + 6n = 2(2k + 1 + 3n) = 4k + 2 + 6n.$$

Divide the equation by n, then expand the right-hand side.

5 Solution | Step 5

$$2k + 1 = 4k + 2 \quad \Rightarrow \quad k = -\frac{1}{2}.$$

The n-terms cancel, leaving a linear equation for k.

6 Solution | Step 6

$$u_{100} = k + 2 + 99(3) = -\frac{1}{2} + 2 + 297 = 298.5.$$

The 100th term is the first term plus 99 common differences.

FINAL ANSWER

The 100th term is $\mathbf{298.5}$.

TOPIC

Direct & Inverse Proportion

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 16

4 marks | Direct & Inverse Proportion

16 M is directly proportional to h^3
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

$h =$ .....

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 Specimen

1 **Find inverse function**

Since M is directly proportional to h^3,

$$M = kh^3$$

2 **Use when**

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$4 = 0.125k$$

$$k = 32$$

---

3 **Use**

Now use $M = 500$:

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

✓ **FINAL ANSWER**

$h = 2.5$.

4MA1/1H MAY/NOVEMBER 2020

Question 14

3 marks | Direct & Inverse Proportion

14 F is inversely proportional to the square of v.

Given that $F = 6.5$ when $v = 4$

find a formula for F in terms of v.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May/November 2020

1 Find inverse function

F is inversely proportional to v^2, so

$$F = \frac{k}{v^2}$$

2 Use when

Use $F = 6.5$ when $v = 4$:

$$6.5 = \frac{k}{4^2}$$

$$k = 6.5 \times 16 = 104$$

3 Find inverse function

Therefore

$$F = \frac{104}{v^2}$$

FINAL ANSWER

$F = \frac{104}{v^2}$.

Question 16

6 marks | Direct & Inverse Proportion

16 A is inversely proportional to the square of r

$A = 5$ when $r = 0.3$

(a) Find a formula for A in terms of r

.....

(3)

(b) Find the value of A when $r = 7.5A$

$A =$ .....

(3)

(Total for Question 16 is 6 marks)

1 **Find inverse function**

A is inversely proportional to r^2, so

$$A = \frac{k}{r^2}$$

2 **Use when**

Use $A = 5$ when $r = 0.3$:

$$5 = \frac{k}{0.3^2}$$

$$k = 5 \times 0.09 = 0.45$$

3 **Find inverse function**

So

$$A = \frac{0.45}{r^2}$$

4 **Rearrange formula**

For part (b), $r = 7.5A$. Substitute into the formula:

$$A = \frac{0.45}{(7.5A)^2}$$

$$A = \frac{0.45}{56.25A^2}$$

$$56.25A^3 = 0.45$$

$$A^3 = 0.008$$

$$A = 0.2$$

✓ **FINAL ANSWER**

(a) $A = \frac{0.45}{r^2}$, (b) $A = 0.2$.

4MA1/1H JANUARY 2022

Question 15

3 marks | Direct & Inverse Proportion

15 A is inversely proportional to C^2

$A = 40$ when $C = 1.5$

Calculate the value of C when $A = 1000$

$C =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2022

1 Find inverse function

A is inversely proportional to C^2, so

$$A = \frac{k}{C^2}$$

2 Use when

Use $A = 40$ when $C = 1.5$:

$$40 = \frac{k}{1.5^2}$$

$$k = 40 \times 1.5^2 = 90$$

3 Find inverse function

So

$$A = \frac{90}{C^2}$$

4 Find inverse function

When $A = 1000$,

$$1000 = \frac{90}{C^2}$$

$$C^2 = 0.09$$

$$C = 0.3$$

FINAL ANSWER

$C = 0.3$.

4MA1/2HR JANUARY 2022

Question 20

4 marks | Direct & Inverse Proportion

20 y is inversely proportional to $\sqrt{x}$
x is directly proportional to T^3

Given that $y = 8$ when $T = 25$

find the exact value of T when $y = 27$

$T =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2022

1 Find inverse function

y is inversely proportional to $\sqrt{x}$, so

$$y \propto \frac{1}{\sqrt{x}}$$

2 Find inverse function

x is directly proportional to T^3, so $\sqrt{x} \propto T^{3/2}$.

3 Find inverse function

Therefore

$$y \propto \frac{1}{T^{3/2}}$$

4 Find inverse function

So

$$yT^{3/2} = k$$

5 Use when

Use $y = 8$ when $T = 25$:

$$k = 8 \times 25^{3/2} = 8 \times 125 = 1000$$

6 **Find inverse function**

When $y = 27$,

$$27T^{3/2} = 1000$$

$$T^{3/2} = \frac{1000}{27} = \left(\frac{10}{3}\right)^3$$

---

7 **Raise both sides to the**

Raise both sides to the power $\frac{2}{3}$:

$$T = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

✓ **FINAL ANSWER**

$T = \frac{100}{9}$.

4MA1/2H JUNE 2022

Question 17

4 marks | Direct & Inverse Proportion

17 M varies directly as the cube of h
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2022

1 Find inverse function

M varies directly as h^3, so

$$M = kh^3$$

2 Use when

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$k = \frac{4}{0.125} = 32$$

3 Find inverse function

So

$$M = 32h^3$$

4 Find inverse function

When $M = 500$,

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

FINAL ANSWER

$h = 2.5$.

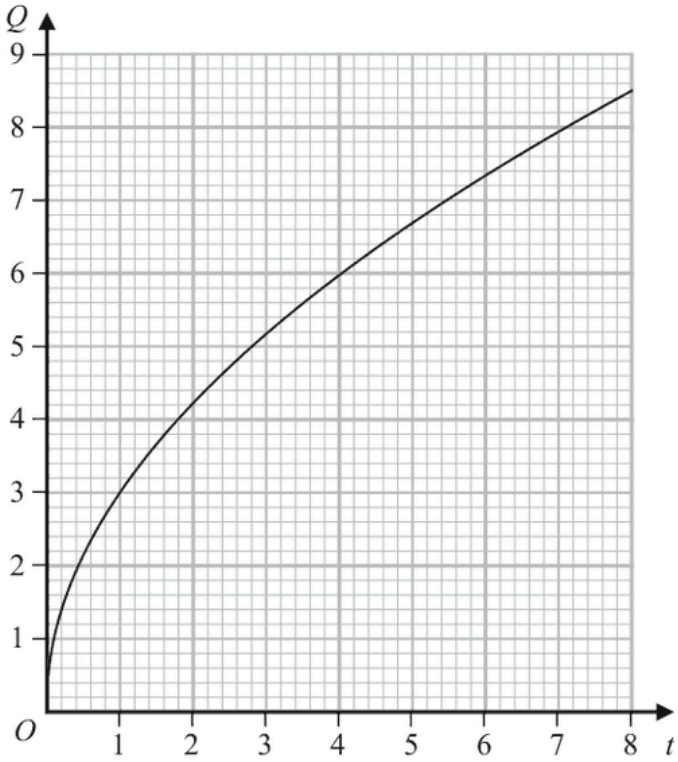
4MA1/2H JUNE 2023

Question 16

5 marks | Direct & Inverse Proportion

16 Q is directly proportional to $\sqrt{t}$

The graph shows the relationship between Q and t for $0 < t < 8$



(a) Find a formula for Q in terms of t

.....

(3)

Q is increased by 20%

(b) Find the percentage increase in t

.....%

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2023

1 **Find inverse function**

$$Q \propto \sqrt{t}$$

2 **Find inverse function**

So

$$Q = k\sqrt{t}$$

3 **Read the graph**

From the graph, when $t = 4$, $Q = 6$.

$$6 = k\sqrt{4}$$

$$6 = 2k$$

$$k = 3$$

4 **Rearrange formula**

So the formula is

$$Q = 3\sqrt{t}$$

5 **Find inverse function**

For part (b), Q is increased by 20%, so Q is multiplied by 1.2.

6 **Find inverse function**

Since $Q \propto \sqrt{t}$,

$$t \propto Q^2$$

7 **Find inverse function**

So t is multiplied by

$$1.2^2 = 1.44$$

8 **This is a increase**

This is a 44% increase.

✓ **FINAL ANSWER**

(a) $Q = 3\sqrt{t}$, (b) 44%

4MA1/1HR MAY 2023

Question 17

4 marks | Direct & Inverse Proportion

17 F is inversely proportional to the square of r

$F = 36$ when $r = 4$

(a) Find a formula for F in terms of r

.....

(3)

(b) Work out the value of F when $r = 48$

.....

(1)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May 2023

1 Find inverse function

F is inversely proportional to r^2, so

$$F = \frac{k}{r^2}$$

2 Use when

Use $F = 36$ when $r = 4$:

$$36 = \frac{k}{4^2}$$

$$k = 36 \times 16 = 576$$

3 Find inverse function

So

$$F = \frac{576}{r^2}$$

4 Find inverse function

When $r = 48$,

$$F = \frac{576}{48^2} = \frac{576}{2304} = 0.25$$

FINAL ANSWER

$F = \frac{576}{r^2}$, and $F = 0.25$ when $r = 48$.

4MA1/2H JUNE 2024

Question 17

3 marks | Direct & Inverse Proportion

17 Q is directly proportional to the square root of d

$Q = 4.5$ when $d = 324$

Find a formula for Q in terms of d

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2024

1 Find inverse function

Q is directly proportional to $\sqrt{d}$, so

$$Q = k\sqrt{d}$$

2 Use when

Use $Q = 4.5$ when $d = 324$:

$$4.5 = k\sqrt{324}$$

$$4.5 = 18k$$

$$k = 0.25$$

3 Find inverse function

Therefore

$$Q = \frac{1}{4}\sqrt{d}$$

FINAL ANSWER

$Q = \frac{1}{4}\sqrt{d}$.

4MA1/1H NOVEMBER 2024

Question 14

3 marks | Direct & Inverse Proportion

14 B is inversely proportional to the square of d

$B = 0.25$ when $d = 12$

Find a formula for B in terms of d

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2024

1 Find inverse function

B is inversely proportional to d^2, so

$$B = \frac{k}{d^2}$$

2 Use when

Use $B = 0.25$ when $d = 12$:

$$0.25 = \frac{k}{12^2}$$

$$k = 0.25 \times 144 = 36$$

3 Find inverse function

Therefore

$$B = \frac{36}{d^2}$$

FINAL ANSWER

$B = \frac{36}{d^2}$.

4MA1/1H JUNE 2025

Question 18

4 marks | Direct & Inverse Proportion

18 y is inversely proportional to x^3

$y = 4$ when $x = 3$

Work out the value of x when $y = 864$

$x =$ .....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2025

1 Find inverse function

y is inversely proportional to x^3, so

$$y = \frac{k}{x^3}$$

2 Use when

Use $y = 4$ when $x = 3$:

$$4 = \frac{k}{3^3}$$

$$k = 4 \times 27 = 108$$

3 Find inverse function

So

$$y = \frac{108}{x^3}$$

4 **Find inverse function**

When $y = 864$,

$$864 = \frac{108}{x^3}$$

$$x^3 = \frac{108}{864} = \frac{1}{8}$$

$$x = \frac{1}{2}$$

FINAL ANSWER

$x = \frac{1}{2}$.

4MA1/1HR JUNE 2025

Question 18

5 marks | Direct & Inverse Proportion

18 F is inversely proportional to the cube of r

$F = 6$ when $r = 2$

(a) Find a formula for F in terms of r

.....

(3)

(b) Find the value of r when $F = 3072$

$r =$ .....

(2)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2025

1 Find inverse function

F is inversely proportional to r^3, so

$$F = \frac{k}{r^3}$$

2 Use when

Use $F = 6$ when $r = 2$:

$$6 = \frac{k}{2^3}$$

$$k = 48$$

3 Find inverse function

So

$$F = \frac{48}{r^3}$$

4 **Find inverse function**

When $F = 3072$,

$$3072 = \frac{48}{r^3}$$

$$r^3 = \frac{48}{3072} = \frac{1}{64}$$

$$r = \frac{1}{4}$$

✓ **FINAL ANSWER**

$F = \frac{48}{r^3}$, and $r = \frac{1}{4}$.

4WM2/01 MAY 2025

Question 18

3 marks | Direct & Inverse Proportion

18 W is inversely proportional to y^2

$W = 50$ when $y = 4$

Find a formula for W in terms of y

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 May 2025

1 Find inverse function

Since W is inversely proportional to y^2,

$$W = \frac{k}{y^2}$$

2 Evaluate fraction

Using $W = 50$ when $y = 4$:

$$50 = \frac{k}{16}$$

$$k = 800$$

FINAL ANSWER

$$W = \frac{800}{y^2}.$$

4WM2/01 NOVEMBER 2025

Question 17

3 marks | Direct & Inverse Proportion

17 T is inversely proportional to $\sqrt{m}$

$T = 15$ when $m = 36$

Find a formula for T in terms of m

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4WM2/01 November 2025

1 Solve equation

$$T = fracksqrtm$$

Step 2: Substitute the given values to find constant *k*.

$$15 = fracksqrt36$$

$$15 = frack6$$

k = 15 × 6 = 90

2 Rearrange formula

$$T = frac90sqrtm$$

FINAL ANSWER

$$T = frac90sqrtm$$

4MA1/2H NOVEMBER 2025

Question 17

3 marks | Direct & Inverse Proportion

17 T is inversely proportional to $\sqrt{m}$

$T = 15$ when $m = 36$

Find a formula for T in terms of m

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2025

1 Find inverse function

T is inversely proportional to m, so

$$T = \frac{k}{m}$$

2 Use when

Use $T = 15$ when $m = 36$:

$$15 = \frac{k}{36}$$

$$k = 540$$

3 Find inverse function

Therefore

$$T = \frac{540}{m}$$

FINAL ANSWER

$T = \frac{540}{m}$.

4WM2/01R JUNE 2026

Question 16

3 marks | Direct & Inverse Proportion

16 m is directly proportional to r^2

$m = 180$ when $r = 6$

Find a formula for m in terms of r

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM2/01R June 2026

1 Write the proportion

$$m = kr^2$$

2 Find k

$$180 = k(6)^2 \Rightarrow k = 5$$

3 Formula

$$m = 5r^2$$

FINAL ANSWER

$m = 5r^2$

4MA1/1H JUNE 2026

Question 19

4 marks | Direct & Inverse Proportion

19 M is directly proportional to p^2

$M = 2$ when $p = 5$

(a) Find a formula for M in terms of p

.....

(3)

(b) Work out the value of M when $p = 15$

$M =$ .....

(1)

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$M = kp^2, \qquad 2 = k(5^2) \implies k = \frac{2}{25}.$$

Introduce a constant of proportionality and use the given pair.

2 **Part (a) | Step 2**

$$M = \frac{2p^2}{25}.$$

Substitute the constant into the general relationship.

3 **Part (b) | Step 1**

$$M = \frac{2(15)^2}{25} = 18.$$

Use the formula from (a).

FINAL ANSWER

(a) $M = 2p^2/25$ (b) $M = 18$

4MA1/1HR JUNE 2026

Question 20

5 marks | Direct & Inverse Proportion

20 y is directly proportional to t^2
$y = 40$ when $t = 4$

t is directly proportional to the cube root of x
$t = 12$ when $x = 8$

Work out the value of y when $x = 27$

$y =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2026

1 Solution | Step 1

$$y = kt^2, \qquad 40 = k(4^2) \implies k = \frac{5}{2}.$$

Direct proportionality to t squared means a constant times t squared.

2 Solution | Step 2

$$t = c\sqrt[3]{x}, \qquad 12 = c\sqrt[3]{8} = 2c \implies c = 6.$$

Find the second proportionality constant using the given pair of values.

3 Solution | Step 3

$$x = 27 \implies t = 6\sqrt[3]{27} = 18.$$

Use the cube-root relationship first.

4 Solution | Step 4

$$y = \frac{5}{2}(18)^2 = 810.$$

Substitute the new value of t into the square relationship.

FINAL ANSWER

$y = 810$

TOPIC

Functions

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 13

4 marks | Functions

13 The function f is defined as

$$f : x \mapsto \frac{2x}{x-6}$$

(a) Find f(10)

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1} : x \mapsto \ldots$

$f^{-1} : x \mapsto$ .....

(3)

WORKED SOLUTION

Question 13 · 4WM2/01 Specimen

1 **Evaluate fraction**

$$f(10) = \frac{2(10)}{10-6} = \frac{20}{4} = 5$$

2 **Let**

For the inverse, let

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$xy - 6y = 2x$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

---

3 **Evaluate fraction**

So

$$f^{-1} : x \mapsto \frac{6x}{x-2}$$

✓ **FINAL ANSWER**

5, and $f^{-1} : x \mapsto \frac{6x}{x-2}$.

4MA1/1HR JANUARY 2020

Question 17

4 marks | Functions

17 The function f is such that $f(x) = (x - 4)^2$ for all values of x.

(a) Find f(1)

.....

(1)

(b) State the range of the function f.

.....

(1)

The function g is such that $g(x) = \dfrac{4}{x+3}$ $\quad x \neq -3$

(c) Work out fg(2)

.....

(2)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR January 2020

1 **Calculate value**

$$f(x) = (x - 4)^2$$

2 **(a)**

(a)

$$f(1) = (1 - 4)^2 = 9$$

3 **(b) A square is always**

(b) A square is always greater than or equal to zero, so the range is

$$f(x) \geq 0$$

4 **(c)**

(c)

$$g(x) = \frac{4}{x+3}$$

$$g(2) = \frac{4}{2+3} = \frac{4}{5}$$

FINAL ANSWER

$f(1) = 9$, range $f(x) \geq 0$, and $g(2) = \frac{4}{5}$.

4MA1/1H JANUARY 2020

Question 21

5 marks | Functions

21 The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2020

1 **Calculate value**

$$f(x) = x^2 - 2x$$

$$g(x) = x + 3$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = (x+3)^2 - 2(x+3)$$

$$h(x) = x^2 + 4x + 3$$

$$h(x) = (x+2)^2 - 1$$

2 **Let**

Let

$$y = (x+2)^2 - 1$$

3 Find inverse function

Since the domain is $x \geq -2$, take the positive square root:

$$x + 2 = \sqrt{y + 1}$$

$$x = \sqrt{y + 1} - 2$$

FINAL ANSWER

$$h^{-1}(x) = \sqrt{x + 1} - 2$$

4MA1/1HR MAY/NOVEMBER 2020

Question 16

9 marks | Functions

16 The functions f and g are defined as

$$f: x \mapsto 5x - 7$$

$$g: x \mapsto \frac{5x}{x+4}$$

(a) Write down the value of x that must be excluded from any domain of g

.....

(1)

(b) Find gf(2.6)

.....

(2)

(c) Solve $fg(x) = 2$

$x =$ .....

(3)

(d) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto ...$

$g^{-1}: x \mapsto$ .....

(3)

(Total for Question 16 is 9 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May/November 2020

1 **Evaluate fraction**

$$f(x) = 5x - 7, \qquad g(x) = \frac{5x}{x+4}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero, so

$$x = -4$$

3 **(b)**

(b)

$$f(2.6) = 5(2.6) - 7 = 6$$

$$gf(2.6) = g(6) = \frac{5(6)}{6+4} = 3$$

---

4 **(c)**

(c)

$$fg(x) = 2$$

$$f(g(x)) = 2$$

$$5g(x) - 7 = 2$$

$$g(x) = \frac{9}{5}$$

$$\frac{5x}{x+4} = \frac{9}{5}$$

$$25x = 9x + 36$$

$$16x = 36$$

$$x = \frac{9}{4}$$

---

5 **(d) Let**

(d) Let

$$y = \frac{5x}{x+4}$$

$$y(x+4) = 5x$$

$$yx + 4y = 5x$$

$$x(y-5) = -4y$$

$$x = \frac{4y}{5-y}$$

6 **Evaluate fraction**

So

$$g^{-1}(x) = \frac{4x}{5-x}$$

FINAL ANSWER

-4, 3, $x = \frac{9}{4}$, and $g^{-1}(x) = \frac{4x}{5-x}$.

4MA1/2HR JANUARY 2021

Question 17

9 marks | Functions

17 The functions f and g are defined as

$f(x) = x^2 + 6$
$g(x) = x - 10$

(a) Find fg(3)

.....

(2)

(b) Solve the equation $fg(x) = f(x)$
Show clear algebraic working.

.....

(3)

The function h is defined as

$$h(x) = \frac{2x - 4}{x}$$

(c) State the value of x that cannot be included in the domain of h

.....

(1)

(d) Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(3)

(Total for Question 17 is 9 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR January 2021

1 **Solve quadratic equation**

$$f(x) = x^2 + 6, \qquad g(x) = x - 10$$

2 **(a)**

(a)

$$g(3) = 3 - 10 = -7$$

$$fg(3) = f(-7) = (-7)^2 + 6 = 55$$

3 **(b)**

(b)

$$fg(x) = f(g(x)) = (x-10)^2 + 6$$

4 **Solve**

Solve $fg(x) = f(x)$:

$$(x-10)^2 + 6 = x^2 + 6$$

$$x^2 - 20x + 100 = x^2$$

$$-20x + 100 = 0$$

$$x = 5$$

5 **(c)**

(c)

$$h(x) = \frac{2x-4}{x}$$

6 **Solve quadratic equation**

The denominator cannot be zero, so $x = 0$ cannot be included.

7 **(d) Let**

(d) Let

$$y = \frac{2x-4}{x}$$

$$y = 2 - \frac{4}{x}$$

$$y - 2 = -\frac{4}{x}$$

$$x = \frac{4}{2-y}$$

8 **Solve quadratic equation**

Therefore

$$h^{-1}(x) = \frac{4}{2-x}$$

FINAL ANSWER

$fg(3) = 55$, $x = 5$, excluded value 0, and $h^{-1}(x) = \dfrac{4}{2 - x}$.

4MA1/1H JANUARY 2021

Question 22

3 marks | Functions

22 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2021

1 Calculate value

$$f(x) = x^2 - 8x + 5$$

$$f(x) = (x-4)^2 - 11$$

2 Let

Let

$$y = (x-4)^2 - 11$$

$$y + 11 = (x-4)^2$$

3 Find inverse function

Since the domain is $x < 4$, take the negative square root:

$$x - 4 = -\sqrt{y+11}$$

$$x = 4 - \sqrt{y+11}$$

FINAL ANSWER

$$f^{-1}(x) = 4 - \sqrt{x+11}$$

4MA1/1H MAY 2021

Question 24

6 marks | Functions

24 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2021

1 **(a)**

(a)

$$g(2) = 7(2) - 6 = 8$$

$$fg(2) = f(8)$$

$$f(8) = 5(8)^2 - 10(8) + 7$$

$$f(8) = 247$$

2 **(b)**

(b)

$$f(x) = 5x^2 - 10x + 7$$

$$f(x) = 5(x - 1)^2 + 2$$

3 **Let**

Let

$$y = 5(x-1)^2 + 2$$

$$(x-1)^2 = \frac{y-2}{5}$$

---

4 **Find inverse function**

Since the domain is $x \geq 1$, take the positive square root:

$$x = 1 + \sqrt{\frac{y-2}{5}}$$

✓ **FINAL ANSWER**

(a) 247, (b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x-2}{5}}$

4MA1/2H NOVEMBER 2021

Question 15

6 marks | Functions

15 The functions f and g are such that

$$f(x) = 2x - 3$$

$$g(x) = \frac{x}{3x + 1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find gf(x)
Simplify your answer.

gf(x) = ...

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2021

1 **Evaluate fraction**

$$f(x) = 2x - 3, \qquad g(x) = \frac{x}{3x + 1}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{2x-3}{3(2x-3)+1}$$

$$gf(x) = \frac{2x-3}{6x-8}$$

---

4 **(c) Let**

(c) Let

$$y = \frac{x}{3x+1}$$

$$y(3x+1) = x$$

$$3xy + y = x$$

$$x(3y-1) = -y$$

$$x = \frac{y}{1-3y}$$

---

5 **Evaluate fraction**

So

$$g^{-1}(x) = \frac{x}{1-3x}$$

✓ **FINAL ANSWER**

Excluded value $-\frac{1}{3}$, $gf(x) = \frac{2x-3}{6x-8}$, and $g^{-1}(x) = \frac{x}{1-3x}$.

4MA1/2HR JANUARY 2022

Question 23

4 marks | Functions

23 The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function h^{-1} in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2022

1 **Calculate value**

$$f(x) = x + 25$$

$$g(x) = x^2 - 12x$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = x^2 - 12x + 25$$

$$h(x) = (x - 6)^2 - 11$$

2 **Let**

Let

$$y = (x - 6)^2 - 11$$

$$y + 11 = (x - 6)^2$$

3 Find inverse function

Since the domain of h is $x < 6$, take the negative square root:

$$x - 6 = -\sqrt{y + 11}$$

$$x = 6 - \sqrt{y + 11}$$

FINAL ANSWER

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$

4MA1/2H JANUARY 2022

Question 25

5 marks | Functions

25 The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain} \quad \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

$g^{-1}: x \mapsto$ .....

(4)

(b) State the domain of g^{-1}

.....

(1)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H January 2022

1 Calculate value

$$g(x) = 5 + 6x - x^2$$

2 Complete the square

Complete the square:

$$g(x) = 14 - (x - 3)^2$$

3 Let

Let

$$y = 14 - (x - 3)^2$$

$$(x - 3)^2 = 14 - y$$

4 Find inverse function

The domain of g is $x \geq 3$, so take the positive square root:

$$x - 3 = \sqrt{14 - y}$$

$$x = 3 + \sqrt{14 - y}$$

5 **Simplify surd**

Therefore

$$g^{-1} : x \mapsto 3 + \sqrt{14 - x}$$

6 **Calculate statistic**

The range of g is $x \leq 14$, so this is the domain of g^{-1}.

FINAL ANSWER

(a) $g^{-1} : x \mapsto 3 + \sqrt{14 - x}$, (b) domain $x \leq 14$

4MA1/1H JUNE 2022

Question 14

4 marks | Functions

14 The function f is defined as

$$f: x \mapsto \frac{2x}{x-6} \qquad x \neq 6$$

(a) Find $f(10)$

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}: x \mapsto \ldots$

$f^{-1}: x \mapsto$ .....

(3)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2022

1 **Evaluate fraction**

$$f(x) = \frac{2x}{x-6}$$

2 **(a)**

(a)

$$f(10) = \frac{2(10)}{10-6} = \frac{20}{4} = 5$$

3 **(b) Let**

(b) Let

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$yx - 6y = 2x$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

4 **Evaluate fraction**

So

$$f^{-1}(x) = \frac{6x}{x-2}$$

✓ **FINAL ANSWER**

$f(10) = 5$, and $f^{-1}(x) = \frac{6x}{x-2}$.

4MA1/2HR JUNE 2022

Question 19

4 marks | Functions

19 $f(x) = x^2 - 4$

$g(x) = 2x + 1$

Solve $fg(x) > 0$

Show clear algebraic working.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2022

1 Solve quadratic equation

$$f(x) = x^2 - 4, \qquad g(x) = 2x + 1$$

$$fg(x) = f(g(x)) = (2x + 1)^2 - 4$$

$$= 4x^2 + 4x + 1 - 4 = 4x^2 + 4x - 3$$

2 Solve

Solve

$$4x^2 + 4x - 3 > 0$$

3 Factorise

Factorise:

$$(2x + 3)(2x - 1) > 0$$

4 The critical values are

The critical values are

$$x = -\frac{3}{2}, \qquad x = \frac{1}{2}$$

5 The product is positive outside

The product is positive outside the roots.

FINAL ANSWER

$x < -\frac{3}{2}$ or $x > \frac{1}{2}$.

4MA1/1H MAY 2023

Question 17

4 marks | Functions

17 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(a) What value of x must be excluded from any domain of g?

.....

(1)

(b) Solve $gh(x) = 1$

.....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2023

1 Find inverse function

$$g(x) = \frac{11}{2x - 5}, \qquad h(x) = x^2 + 4, \quad x \geq 0$$

2 (a) The denominator of cannot

(a) The denominator of $g(x)$ cannot be zero.

$$2x - 5 = 0$$

$$x = \frac{5}{2}$$

3 Find inverse function

So $x = \frac{5}{2}$ must be excluded.

4 **(b)**

(b)

$$gh(x) = g(h(x)) = \frac{11}{2(x^2+4)-5}$$

$$= \frac{11}{2x^2+3}$$

---

5 **Solve**

Solve

$$\frac{11}{2x^2+3} = 1$$

$$2x^2 + 3 = 11$$

$$2x^2 = 8$$

$$x^2 = 4$$

---

6 **Solve inequality**

Since $x \geq 0$,

$$x = 2$$

✓ **FINAL ANSWER**

Exclude $x = \frac{5}{2}$, and $x = 2$.

4MA1/2H NOVEMBER 2023

Question 19

7 marks | Functions

19 The functions f and g are such that

$$f: x \mapsto 5x + 7$$

$$g: x \mapsto \frac{5}{2x - 9}$$

(a) State which value of x cannot be included in any domain of g

.....

(1)

(b) Find fg(4)

.....

(2)

The function h is such that

$$h: x \mapsto 3x^2 - 12x + 8 \quad \text{where } x > 2$$

(c) Express the inverse function h^{-1} in the form $h^{-1}: x \mapsto ...$

$h^{-1}: x \mapsto$ .....

(4)

(Total for Question 19 is 7 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2023

1 **Evaluate fraction**

$$f(x) = 5x + 7, \qquad g(x) = \frac{5}{2x - 9}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$2x - 9 = 0$$

$$x = \frac{9}{2}$$

3 **(b)**

(b)

$$g(4) = \frac{5}{8-9} = -5$$

$$fg(4) = f(-5) = 5(-5) + 7 = -18$$

---

4 **(c)**

(c)

$$h(x) = 3x^2 - 12x + 8, \qquad x > 2$$

---

5 **Complete the square**

Complete the square:

$$h(x) = 3(x-2)^2 - 4$$

---

6 **Let**

Let

$$y = 3(x-2)^2 - 4$$

$$y + 4 = 3(x-2)^2$$

$$x - 2 = \sqrt{\frac{y+4}{3}}$$

---

7 **Simplify surd**

Since $x > 2$, take the positive root:

$$x = 2 + \sqrt{\frac{y+4}{3}}$$

---

8 **Simplify surd**

So

$$h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$$

✓ **FINAL ANSWER**

Excluded value $\frac{9}{2}$, $fg(4) = -18$, and $h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$.

4MA1/1H MAY 2024

Question 15

4 marks | Functions

15 The function f is defined as

$$f: x \mapsto \frac{3x+1}{x-2}$$

(a) State the value of x that cannot be included in any domain of the function f

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(3)

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2024

1 **Evaluate fraction**

$$f(x) = \frac{3x+1}{x-2}$$

2 **(a) The denominator cannot be**

(a) The denominator cannot be zero, so

$$x - 2 = 0$$

$$x = 2$$

3 **(b) Let**

(b) Let

$$y = \frac{3x+1}{x-2}$$

$$y(x-2) = 3x+1$$

$$yx - 2y = 3x + 1$$

$$x(y-3) = 2y+1$$

$$x = \frac{2y+1}{y-3}$$

---

4 **Evaluate fraction**

So

$$f^{-1}(x) = \frac{2x+1}{x-3}$$

✓ **FINAL ANSWER**

Excluded value 2, and $f^{-1}(x) = \frac{2x+1}{x-3}$.

4MA1/2HR JUNE 2024

Question 17

3 marks | Functions

17 $f(x) = \dfrac{x}{2x-4}$ $\quad g(x) = 3x + 1$

Given that $fg(k) = 2$

work out the value of k

$k =$ .....

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2024

1 Evaluate fraction

$$f(x) = \frac{k}{2x-4}, \qquad g(x) = 3x + 1$$

2 Given

Given

$$fg(k) = 2$$

3 This means

This means

$$f(g(k)) = 2$$

4 Calculate value

Now

$$g(k) = 3k + 1$$

5 **Evaluate fraction**

So

$$f(g(k)) = \frac{k}{2(3k+1)-4} = \frac{k}{6k-2}$$

$$\frac{k}{6k-2} = 2$$

$$k = 12k - 4$$

$$11k = 4$$

$$k = \frac{4}{11}$$

✓ **FINAL ANSWER**

$k = \frac{4}{11}$.

4MA1/1H NOVEMBER 2024

Question 25

4 marks | Functions

25 The function f is such that $f(x) = 2x^2 - 24x + 7$ where $x \geqslant 6$

Find the inverse function $f^{-1}(x)$

$f^{-1}(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H November 2024

1 Calculate value

$$f(x) = 2x^2 - 24x + 7$$

2 Complete the square

Complete the square:

$$f(x) = 2(x^2 - 12x) + 7$$

$$f(x) = 2[(x - 6)^2 - 36] + 7$$

$$f(x) = 2(x - 6)^2 - 65$$

3 Let

Let $y = f(x)$.

$$y = 2(x - 6)^2 - 65$$

$$y + 65 = 2(x - 6)^2$$

$$\frac{y + 65}{2} = (x - 6)^2$$

4 Simplify surd

Since $x \geq 6$, take the positive square root:

$$x - 6 = \sqrt{\frac{y+65}{2}}$$

$$x = 6 + \sqrt{\frac{y+65}{2}}$$

5 Replace by

Replace y by x.

FINAL ANSWER

$$f^{-1}(x) = 6 + \sqrt{\frac{x+65}{2}}$$

4MA1/1H JUNE 2025

Question 16

4 marks | Functions

16 $f(x) = 9 - \sqrt{x}$ where $x \geqslant 0$

$g(x) = 4x^2$

(a) Find $f(9)$

.....

(1)

(b) Solve $fg(x) < 0$

Show clear algebraic working.

.....

(3)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2025

1 **Solve quadratic equation**

$$f(x) = 9 - \sqrt{x}, \qquad g(x) = 4x^2$$

2 **(a)**

(a)

$$f(9) = 9 - \sqrt{9} = 6$$

3 **(b)**

(b)

$$fg(x) = f(g(x)) = 9 - \sqrt{4x^2}$$

$$= 9 - 2|x|$$

4 **Solve**

Solve

$$9 - 2|x| < 0$$

$$2|x| > 9$$

$$|x| > \frac{9}{2}$$

---

5 **Solve quadratic equation**

So

$$x < -\frac{9}{2} \quad \text{or} \quad x > \frac{9}{2}$$

---

6 **Solve quadratic equation**

If the input is restricted to $x \geq 0$, this becomes $x > \frac{9}{2}$.

✓ **FINAL ANSWER**

$f(9) = 6$, and $x < -\frac{9}{2}$ or $x > \frac{9}{2}$.

4WM2/01R MAY 2025

Question 17

4 marks | Functions

17 The function g is defined as

$$g(x) = \frac{3x}{x-2}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots$

$g^{-1}(x) =$ .....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM2/01R May 2025

1 Required

$g(x) = dfrac3xx - 2$. (a) Value excluded from domain. (b) Find *g*^{−1}(*x*).

2 Part (a)

The denominator must not equal zero: *x* − 2 ≠ 0.

3 Answer

x = 2 cannot be in any domain of *g*

4 Set y g (x

Set *y* = *g*(*x*) and swap *x* ↔ *y*:

$$x = frac3yy - 2$$

5 Multiply both sides by (

Multiply both sides by (*y*−2):

x(*y*−2) = 3*y* ⟹ *x**y* − 2*x* = 3*y*

6 Collect y terms and factorise

Collect *y* terms and factorise:

*x**y* − 3*y* = 2*x* ⟹ *y*(*x*−3) = 2*x*

7 **Solve for y**

Solve for *y*:

$$y = frac2xx - 3$$

8 **Note**

g^{−1}(*x*) is undefined at *x* = 3, which is the range restriction of *g*(*x*) (horizontal asymptote).

✓ **FINAL ANSWER**

$$g^{-1}(x) = dfrac2xx - 3$$

4MA1/2HR JUNE 2025

Question 19

3 marks | Functions

19 The functions f and g are such that

$$f(x) = 3x - 4 \text{ where } x > 2$$

$$g(x) = \frac{x}{2x+1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find $gf(x)$
Give your answer in its simplest form.

$gf(x) =$ .....

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2025

1 **Evaluate fraction**

$$f(x) = 3x - 4, \qquad g(x) = \frac{x}{2x+1}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$2x + 1 = 0$$

$$x = -\frac{1}{2}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{3x-4}{2(3x-4)+1}$$

$$gf(x) = \frac{3x-4}{6x-7}$$

FINAL ANSWER

Excluded value $-\frac{1}{2}$, and $gf(x) = \dfrac{3x-4}{6x-7}$.

4WM2/01 MAY 2025

Question 21

7 marks | Functions

21 The function g is defined as

$$g(x) = \frac{7x + 20}{2x} \qquad x \neq 0$$

(a) Solve $gg(x) = 6$
Show clear algebraic working.

$x =$ .....

(3)

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The function h is defined as

$$h(x) = 5x^2 + 30x - 7 \qquad x \geqslant -3$$

(b) Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h^{-1}(x) =$ .....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 May 2025

1 **Evaluate fraction**

(a) If $g(g(x)) = 6$, then

$$\frac{7g(x) + 20}{2g(x)} = 6$$

$$7g(x) + 20 = 12g(x)$$

$$g(x) = 4$$

2 **Evaluate fraction**

So

$$\frac{7x+20}{2x}=4$$

$$7x+20=8x$$

$$x=20$$

---

3 **(b)**

(b)

$$h(x)=5x^2+30x-7=5(x+3)^2-52$$

---

4 **Simplify surd**

Let $y=5(x+3)^2-52$. Since $x\geq -3$, use the positive square root:

$$x+3=\sqrt{\frac{y+52}{5}}$$

$$h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$$

✓ **FINAL ANSWER**

$x=20$; $h^{-1}(x)=\sqrt{\frac{x+52}{5}}-3$.

4MA1/2H NOVEMBER 2025

Question 19

3 marks | Functions

19 The functions f and g are such that

$$f(x) = 3x - 2 \qquad g(x) = \frac{x}{2x - 1}$$

(a) Find g(3)

.....

(1)

(b) Find gf(x)
Give your answer in its simplest form.

gf(x) = ...

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2025

1 **Evaluate fraction**

$$f(x) = 3x - 2, \qquad g(x) = \frac{x}{x - 1}$$

2 **(a)**

(a)

$$g(3) = \frac{3}{3 - 1} = \frac{3}{2}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{3x - 2}{(3x - 2) - 1}$$

$$gf(x) = \frac{3x - 2}{3x - 3}$$

FINAL ANSWER

$g(3) = \frac{3}{2}$, and $gf(x) = \dfrac{3x-2}{3x-3}$.

4WM2/01 NOVEMBER 2025

Question 21

7 marks | Functions

21 The functions f and g are defined as

$$f(x) = 5x - 3$$

$$g(x) = 4x^2 + 16x - 9 \qquad \text{where} \quad x \geqslant -2$$

(a) Find g(–1)

.....

(1)

(b) Find fg(x)

Give your answer in its simplest form.

fg(x) = ...

(2)

$g(x) = 4x^2 + 16x - 9$ where $x \geqslant -2$

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4WM2/01 November 2025

1 **(a) Find g (1)**

(a) Find *g*(−1):

g(−1) = 4(−1)² + 16(−1) − 9 = 4 − 16 − 9 = − 21

2 **Answer**

a − 21

(b) Find *fg*(*x*):**

*f**g*(*x*) = *f*(*g*(*x*)) = 5(4*x*²+16*x*−9) − 3

= 20*x*² + 80*x* − 45 − 3 = 20*x*² + 80*x* − 48

---

3 **Answer**

b 20*x*² + 80*x* − 48

Let *y* = 4*x*² + 16*x* − 9. We must rearrange to make *x* the subject.

Step 1: Complete the square for the *x* terms.

y = 4(*x*²+4*x*) − 9

y = 4((*x*+2)²−4) − 9

y = 4(*x*+2)² − 16 − 9

y = 4(*x*+2)² − 25

Step 2: Rearrange for *x*.

y + 25 = 4(*x*+2)²

$$fracy + 254 = (x + 2)^2$$

$$x + 2 = pmsqrtfracy + 254$$

Since *x* ≥ − 2, *x* + 2 is positive, so we take the positive root.

$$x + 2 = fracsqrty + 252$$

$$x = fracsqrty + 252 - 2$$

Step 3: Swap *y* for *x* to write the inverse function.

✓ **FINAL ANSWER**

$g^{-1}(x) = fracsqrtx + 252 - 2$ When finding the inverse of a quadratic, always complete the square. The restricted domain (*x* ≥ − 2) tells you whether to take the positive or negative square root!

4MA1/2H JUNE 2026

Question 16

5 marks | Functions

16 The functions f and g are such that

$$f(x) = 4 + x \qquad g(x) = 3x^2 + 2$$

(a) Find $g(4)$

.....

(1)

(b) Find $f^{-1}(7)$

.....

(2)

(c) Express $gf(x)$ in the form $ax^2 + bx + c$ where a, b and c are integers.

$gf(x) =$ .....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

$$g(4) = 3(4)^2 + 2 = 48 + 2 = 50.$$

Substitute 4 for the function input.

2 **Part (b) | Step 1**

If $y = 4 + x$, then $x = y - 4$, so $f^{-1}(x) = x - 4$.

Reverse the input-output relationship.

3 **Part (b) | Step 2**

$$f^{-1}(7) = 7 - 4 = 3.$$

Apply the inverse function to 7.

4 **Part (c) | Step 1**

$$gf(x) = g(f(x)) = g(x + 4) = 3(x + 4)^2 + 2.$$

Apply f first and use its whole output as the input of g.

5 **Part (c) | Step 2**

$$3(x^2 + 8x + 16) + 2 = 3x^2 + 24x + 50.$$

Expand and collect the constant terms.

✓ **FINAL ANSWER**

(a) 50 (b) 3 (c) $gf(x) = 3x^2 + 24x + 50$

Question 17

4 marks | Functions

17 The functions f and g are such that

$$f(x) = x^2 - 5$$
$$g(x) = 2x + 1$$

Solve $fg(x) = 2gf(x)$
Show clear algebraic working.

$x =$ .....

(Total for Question 17 is 4 marks)

1 **fg(x): put g into f**

$$\mathrm{fg}(x) = (2x+1)^2 - 5 = 4x^2 + 4x - 4$$

2 **gf(x): put f into g**

$$\mathrm{gf}(x) = 2(x^2-5) + 1 = 2x^2 - 9, \qquad 2\,\mathrm{gf}(x) = 4x^2 - 18$$

3 **Solve**

$$4x^2 + 4x - 4 = 4x^2 - 18 \Rightarrow 4x = -14 \Rightarrow x = -3.5$$

✓ **FINAL ANSWER**

$x = -3.5$

4MA1/1HR JUNE 2026

Question 23

7 marks | Functions

23 The function f is such that $f(x) = \frac{1}{x}$

(a) Write down the value of x that cannot be included in any domain of f

.....

(1)

The function g is such that $g(x) = 4x^2 - 8x + 15$
The domain of g is $x \geqslant 1$

(b) Work out $gf(0.5)$

.....

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x) =$ .....

(4)

(Total for Question 23 is 7 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

For $f(x) = \frac{1}{x}$, the denominator is zero when $x = 0$.
Division by zero is undefined, so this input must be excluded.

2 **Part (b) | Step 1**

$$f(0.5) = \frac{1}{0.5} = 2.$$

In gf, apply f first.

3 **Part (b) | Step 2**

$$gf(0.5) = g(2) = 4(2)^2 - 8(2) + 15 = 16 - 16 + 15 = 15.$$

The input 2 lies in the stated domain x>=1 of g.

4 **Part (c) | Step 1**

$$y = 4x^2 - 8x + 15 = 4(x-1)^2 + 11.$$

Complete the square to isolate the expression containing x.

5 **Part (c) | Step 2**

$$(x-1)^2 = \frac{y-11}{4}, \qquad x-1 = \frac{\sqrt{y-11}}{2}.$$

Choose the nonnegative root because the domain of g is x>=1.

6 **Part (c) | Step 3**

$$x = 1 + \frac{\sqrt{y-11}}{2}.$$

Rearrange to make the original input the subject.

7 **Part (c) | Step 4**

$$g^{-1}(x) = 1 + \frac{\sqrt{x-11}}{2}, \qquad x \geq 11.$$

Exchange the input and output labels; the range of g becomes the domain of the inverse.

✓ **FINAL ANSWER**

(a) $x = 0$ must be excluded. (b) $gf(0.5) = 15$ (c) $g^{-1}(x) = 1 + \frac{\sqrt{x-11}}{2}, \quad x \geq 11$

TOPIC

Differentiation & Turning Points

Every past-paper question on this topic, 2020 to 2026.

Question 17

5 marks | Differentiation & Turning Points

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

1 **Differentiate**

$$s = 4t^2 + \frac{125}{t}$$

2 **Differentiate to find velocity**

Differentiate to find velocity:

$$v = \frac{ds}{dt} = 8t - \frac{125}{t^2}$$

3 **Set**

Set $v = 0$:

$$8t - \frac{125}{t^2} = 0$$

$$8t^3 = 125$$

$$t^3 = \frac{125}{8}$$

$$t = 2.5$$

4 **Differentiate**

Now find s:

$$s = 4(2.5)^2 + \frac{125}{2.5}$$

$$s = 25 + 50 = 75$$

✓ **FINAL ANSWER**

75 m.

4MA1/1H JANUARY 2020

Question 15

8 marks | Differentiation & Turning Points

15

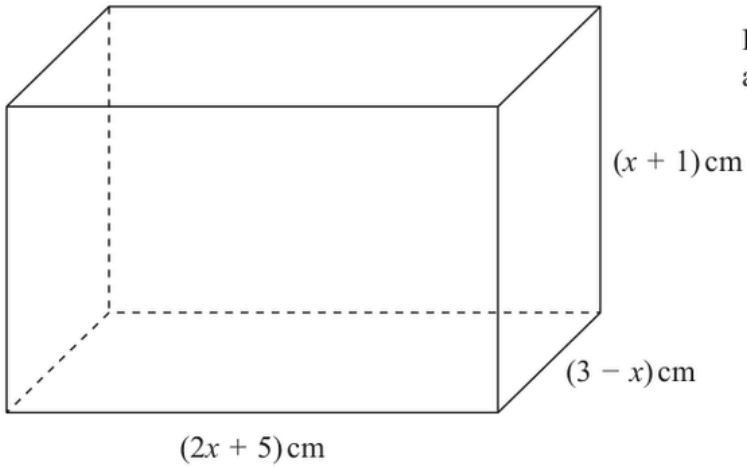


Diagram **NOT** accurately drawn

The diagram shows a cuboid of volume $V\,\text{cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

(3)

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x.
Show your working clearly.
Give your answer correct to 3 significant figures.

$x =$ .....

(5)

(Total for Question 15 is 8 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2020

1 The volume is

The volume is

$$V = (2x + 5)(x + 1)(3 - x)$$

2 **Expand brackets**

First expand:

$$(2x+5)(x+1) = 2x^2 + 7x + 5$$

$$V = (2x^2 + 7x + 5)(3 - x)$$

$$V = 15 + 16x - x^2 - 2x^3$$

---

3 **Calculate volume**

For the maximum volume,

$$\frac{dV}{dx} = 0$$

$$\frac{dV}{dx} = 16 - 2x - 6x^2$$

$$16 - 2x - 6x^2 = 0$$

$$3x^2 + x - 8 = 0$$

---

4 **Solve quadratic equation**

Using the quadratic formula,

$$x = \frac{-1 \pm \sqrt{97}}{6}$$

---

5 **The valid positive value is**

The valid positive value is

$$x = 1.4748\ldots$$

✓ **FINAL ANSWER**

$x = 1.47$ to 3 significant figures.

4MA1/1HR JANUARY 2020

Question 23

6 marks | Differentiation & Turning Points

23 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$T =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2020

1 Calculate value

$$s = t^3 + 4t^2 - 5t + 7$$

2 Evaluate fraction

Velocity is the derivative of displacement:

$$v = \frac{ds}{dt} = 3t^2 + 8t - 5$$

3 Calculate value

At time T, the velocity is V, so

$$V = 3T^2 + 8T - 5$$

$$3T^2 + 8T - (V + 5) = 0$$

4 **Use the quadratic formula**

Use the quadratic formula:

$$T = \frac{-8 \pm \sqrt{8^2 - 4(3)(-(V+5))}}{2(3)}$$

$$T = \frac{-8 \pm \sqrt{64 + 12V + 60}}{6}$$

$$T = \frac{-8 \pm \sqrt{124 + 12V}}{6}$$

$$T = \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

---

5 **Simplify surd**

Since $T \geq 0$, take the positive root:

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

✓ **FINAL ANSWER**

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

4MA1/1HR MAY/NOVEMBER 2020

Question 13

6 marks | Differentiation & Turning Points

13 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

.....

(4)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR May/November 2020

1 **Find the gradient**

$$y = 5x^3 - x^2 - 6x + 4$$

$$\frac{dy}{dx} = 15x^2 - 2x - 6$$

2 **Find the gradient**

The gradient is 2, so

$$15x^2 - 2x - 6 = 2$$

$$15x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(15)(-8)}}{30}$$

$$x = \frac{2 \pm 22}{30}$$

$$x = \frac{4}{5} \quad \text{or} \quad x = -\frac{2}{3}$$

FINAL ANSWER

$x = -\frac{2}{3}$ and $x = \frac{4}{5}$.

4MA1/1H MAY/NOVEMBER 2020

Question 23

4 marks | Differentiation & Turning Points

23 Curve **C** has equation $y = px^3 - mx$ where p and m are positive integers.

Find the range of values of x, in terms of p and m, for which the gradient of **C** is negative.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May/November 2020

1 Differentiate

$$y = px^3 - mx$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3px^2 - m$$

3 The gradient is negative when

The gradient is negative when

$$3px^2 - m < 0$$

$$3px^2 < m$$

$$x^2 < \frac{m}{3p}$$

4 Simplify surd

So

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

FINAL ANSWER

$$-\sqrt{\frac{m}{3p}} < x < \sqrt{\frac{m}{3p}}$$

Question 17

5 marks | Differentiation & Turning Points

17 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

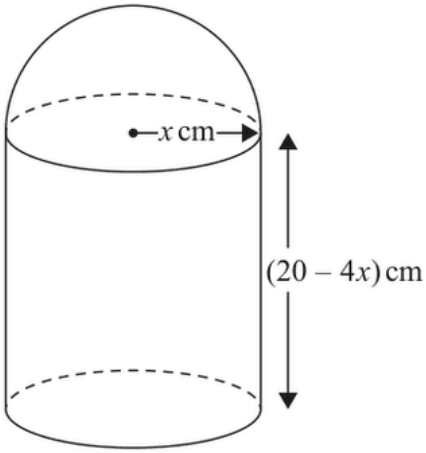


Diagram **NOT** accurately drawn

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is $V\,\text{cm}^3$ where $V = \frac{1}{3}\pi y$

Find the maximum value of y.
Show clear algebraic working.

(Total for Question 17 is 5 marks)

1 **Volume of the solid is**

Volume of the solid is cylinder plus hemisphere:

$$V = \pi x^2(20 - 4x) + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - 4\pi x^3 + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - \frac{10}{3}\pi x^3$$

2 **The question gives**

The question gives

$$V = \frac{1}{3}\pi y$$

3 **Differentiate**

So

$$y = 60x^2 - 10x^3$$

4 **Differentiate**

Differentiate:

$$\frac{dy}{dx} = 120x - 30x^2$$

5 **Differentiate**

For a maximum,

$$120x - 30x^2 = 0$$

$$30x(4 - x) = 0$$

6 **Differentiate**

So the valid value is

$$x = 4$$

$$y = 60(4^2) - 10(4^3) = 960 - 640 = 320$$

FINAL ANSWER

The maximum value of y is 320.

4MA1/2H JANUARY 2021

Question 19

5 marks | Differentiation & Turning Points

19 A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds where $t \geqslant 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 5t^2 - 8t + 10$$

Find the displacement of P from O when P is instantaneously at rest.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

..... metres

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2021

1 Calculate statistic

$$s = t^3 + 5t^2 - 8t + 10$$

2 Velocity is

Velocity is

$$\frac{ds}{dt} = 3t^2 + 10t - 8$$

3 Calculate statistic

At rest,

$$3t^2 + 10t - 8 = 0$$

4 Factorise

Factorise:

$$(3t - 2)(t + 4) = 0$$

5 Calculate statistic

So

$$t = \frac{2}{3} \quad \text{or} \quad t = -4$$

6 **Calculate statistic**

Since $t \geq 0$,

$$t = \frac{2}{3}$$

---

7 **Calculate statistic**

Now find s:

$$s = \left(\frac{2}{3}\right)^3 + 5\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) + 10$$

$$s = \frac{8}{27} + \frac{20}{9} - \frac{16}{3} + 10$$

$$s = \frac{194}{27}$$

✓ **FINAL ANSWER**

$\frac{194}{27}$ metres.

4MA1/1HR JANUARY 2021

Question 20

5 marks | Differentiation & Turning Points

20 A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

..... m/s

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2021

1 **Read the graph**

$$s = t^3 - 9t^2 + 33t - 6$$

2 **Read the graph**

Velocity is the rate of change of displacement:

$$v = \frac{ds}{dt}$$

$$v = 3t^2 - 18t + 33$$

3 **Complete the square**

Complete the square:

$$v = 3(t^2 - 6t + 11)$$

$$v = 3[(t-3)^2 + 2]$$

$$v = 3(t-3)^2 + 6$$

4 **Read the graph**

Since $(t-3)^2 \geq 0$, the minimum value of v is

$$6$$

5 **Read the graph**

The velocity is always positive, so the minimum speed is also 6.

✓ **FINAL ANSWER**

$6\ \text{m/s}$

4MA1/2H MAY 2021

Question 21

5 marks | Differentiation & Turning Points

21 The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a.
Show your working clearly.

$a =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H May 2021

1 Evaluate fraction

$$y = kx^2 + \frac{16}{x}$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 2kx - \frac{16}{x^2}$$

3 Find the gradient

At the stationary point, $x = \frac{2}{3}$, so the gradient is 0:

$$2k\left(\frac{2}{3}\right) - \frac{16}{\left(\frac{2}{3}\right)^2} = 0$$

$$\frac{4k}{3} - \frac{16}{\frac{4}{9}} = 0$$

$$\frac{4k}{3} - 36 = 0$$

$$\frac{4k}{3} = 36$$

$$k = 27$$

4 Find the -coordinate

Now find a, the y-coordinate when $x = \frac{2}{3}$:

$$a = 27\left(\frac{2}{3}\right)^2 + \frac{16}{\frac{2}{3}}$$

$$a = 27 \cdot \frac{4}{9} + 24$$

$$a = 12 + 24 = 36$$

FINAL ANSWER

$a = 36$

4MA1/1HR JANUARY 2022

Question 23

6 marks | Differentiation & Turning Points

23 Two particles, P and Q, move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2022

1 **Particle**

Particle P:

$$s = t^3 - 4t^2 + 5t$$

$$\frac{ds}{dt} = 3t^2 - 8t + 5$$

$$\frac{ds}{dt} = (3t - 5)(t - 1)$$

2 **Particle**

Particle Q:

$$x = t^2 - 4t + 4$$

$$\frac{dx}{dt} = 2t - 4 = 2(t - 2)$$

3 **Differentiate**

For $t > 1$:

---

4 **moves in the negative direction**

- P moves in the negative direction when $1 < t < \frac{5}{3}$.
- P moves in the positive direction when $t > \frac{5}{3}$.
- Q moves in the negative direction when $1 < t < 2$.
- Q moves in the positive direction when $t > 2$.

---

5 **Differentiate**

So both particles move in the same direction when both velocities have the same sign.

✓ **FINAL ANSWER**

$1 < t < \frac{5}{3}$ or $t > 2$

4MA1/1H JANUARY 2022

Question 24

6 marks | Differentiation & Turning Points

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2022

1 Calculate value

$$y = ax^3 + bx^2 - 12x + 6$$

2 Point lies on the curve

Point $A = (2, -6)$ lies on the curve:

$$-6 = 8a + 4b - 24 + 6$$

$$8a + 4b = 12$$

$$2a + b = 3$$

3 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3ax^2 + 2bx - 12$$

4 **The gradient at is 16**

The gradient at $x = 2$ is 16:

$$16 = 12a + 4b - 12$$

$$12a + 4b = 28$$

$$3a + b = 7$$

---

5 **Solve**

Now solve

$$2a + b = 3$$

$$3a + b = 7$$

---

6 **Subtracting gives**

Subtracting gives

$$a = 4$$

---

7 **Calculate value**

Then

$$2(4) + b = 3$$

$$b = -5$$

---

8 **Calculate value**

For $x = 3$,

$$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$$

$$y = 108 - 45 - 36 + 6$$

$$y = 33$$

✓ **FINAL ANSWER**

$y = 33$

4MA1/2H JANUARY 2022

Question 24

5 marks | Differentiation & Turning Points

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a\ \text{m/s}^2$

Find the value of a

$a =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2022

1 Split the ratio

$$x = 4t^3 - 27t + 8$$

2 Velocity is

Velocity is

$$\frac{dx}{dt} = 12t^2 - 27$$

3 The direction of motion reverses

The direction of motion reverses when velocity is zero:

$$12t^2 - 27 = 0$$

$$12t^2 = 27$$

$$t^2 = \frac{9}{4}$$

4 Split the ratio

Since $t \geq 0$, $t = \frac{3}{2}$.

5 **Acceleration is**

Acceleration is

$$\frac{d^2x}{dt^2} = 24t$$

---

6 **Split the ratio**

At $t = \frac{3}{2}$,

$$a = 24\left(\frac{3}{2}\right) = 36$$

✓ **FINAL ANSWER**

$a = 36$

4MA1/1H JUNE 2022

Question 17

5 marks | Differentiation & Turning Points

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2022

1 Differentiate

$$s = 4t^2 + \frac{125}{t}$$

2 Velocity is

Velocity is

$$v = \frac{ds}{dt} = 8t - \frac{125}{t^2}$$

3 Differentiate

When $v = 0$,

$$8t - \frac{125}{t^2} = 0$$

$$8t^3 = 125$$

$$t^3 = 15.625$$

$$t = 2.5$$

4 **Differentiate**

Now find s:

$$s = 4(2.5)^2 + \frac{125}{2.5}$$

$$s = 25 + 50 = 75$$

✓ **FINAL ANSWER**

$75\ \text{m}$.

Question 22

7 marks | Differentiation & Turning Points

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

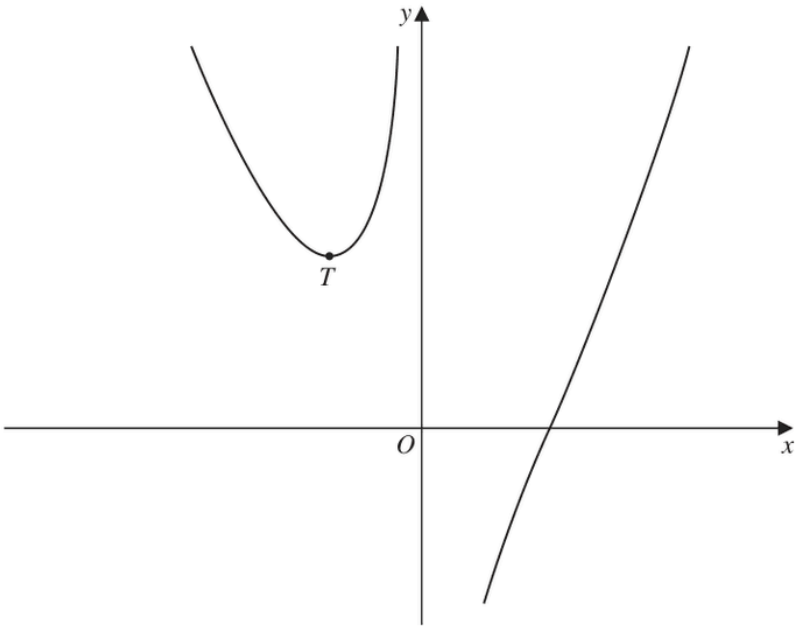


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

$p =$ .....

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$k =$ .....

(3)

(Total for Question 22 is 7 marks)

1 **Find the gradient**

For part (a),

$$y = x^2 - \frac{p}{x}$$

$$\frac{dy}{dx} = 2x + \frac{p}{x^2}$$

2 **Find the gradient**

At the minimum point, $x = -3$ and the gradient is 0.

$$2(-3) + \frac{p}{(-3)^2} = 0$$

$$-6 + \frac{p}{9} = 0$$

$$p = 54$$

3 **Find the gradient**

For part (b),

$$y = x^2 - \frac{16}{x}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2}$$

4 **Find the gradient**

At a horizontal tangent,

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$x^3 = -8$$

$$x = -2$$

5 **Find the gradient**

Now substitute $x = -2$ into the curve:

$$k = (-2)^2 - \frac{16}{-2}$$

$$k = 4 + 8 = 12$$

✓ **FINAL ANSWER**

$p = 54,\ k = 12$

4MA1/1H MAY 2023

Question 11

6 marks | Differentiation & Turning Points

11 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the x coordinates of the points on **C** where the gradient is 4
Show clear algebraic working.

.....

(4)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2023

1 **Find the gradient**

$$y = 4x^3 + x^2 - 20x$$

2 **(a)**

(a)

$$\frac{dy}{dx} = 12x^2 + 2x - 20$$

3 **Find the gradient**

(b) The gradient is 4, so

$$12x^2 + 2x - 20 = 4$$

$$12x^2 + 2x - 24 = 0$$

$$6x^2 + x - 12 = 0$$

4 **Factorise**

Factorise:

$$(3x - 4)(2x + 3) = 0$$

---

5 **Find the gradient**

So

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

✓ **FINAL ANSWER**

$\frac{dy}{dx} = 12x^2 + 2x - 20$, and $x = \frac{4}{3}$ or $x = -\frac{3}{2}$.

4MA1/1HR MAY 2023

Question 21

6 marks | Differentiation & Turning Points

21 The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the range of values of x for which **T** has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

(4)

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2023

1 **(a)**

(a)

$$y = x^3 - 2x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 4x - 9$$

2 **(b) The curve has a**

(b) The curve has a positive gradient when

$$3x^2 - 4x - 9 > 0$$

3 **Solve**

Solve

$$3x^2 - 4x - 9 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 108}}{6}$$

$$x = \frac{2 \pm \sqrt{31}}{3}$$

---

4 **Differentiate**

To 3 significant figures, the roots are

$$x = -1.19, \quad x = 2.52$$

---

5 **Solve quadratic equation**

Since the quadratic opens upwards, it is positive outside the roots.

✓ **FINAL ANSWER**

(a) $\frac{dy}{dx} = 3x^2 - 4x - 9$, (b) $x < -1.19$ or $x > 2.52$

4MA1/1H NOVEMBER 2023

Question 16

4 marks | Differentiation & Turning Points

16 A curve has equation $y = 4x^3 - 8x + 5$

Find the x coordinates of the two points on the curve where the gradient is $\frac{1}{3}$

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2023

1 Find the gradient

$$y = 4x^3 - 8x + 5$$

$$\frac{dy}{dx} = 12x^2 - 8$$

2 Find the gradient

The gradient is $\frac{1}{3}$, so

$$12x^2 - 8 = \frac{1}{3}$$

$$12x^2 = \frac{25}{3}$$

$$x^2 = \frac{25}{36}$$

$$x = \frac{5}{6} \quad \text{or} \quad x = -\frac{5}{6}$$

FINAL ANSWER

$x = \frac{5}{6}$ or $x = -\frac{5}{6}$.

4MA1/2H NOVEMBER 2023

Question 18

4 marks | Differentiation & Turning Points

18 A particle is moving along a straight line that passes through the fixed point O
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 2t^3 - 5t^2 + 6t - 5$$

Find the value of t when the acceleration of the particle is 5 m/s²

$t =$ .....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2023

1 Split the ratio

$$s = 2t^3 - 5t^2 + 6t - 5$$

2 Velocity is

Velocity is

$$\frac{ds}{dt} = 6t^2 - 10t + 6$$

3 Acceleration is

Acceleration is

$$\frac{d^2s}{dt^2} = 12t - 10$$

4 Split the ratio

The acceleration is 5, so

$$12t - 10 = 5$$

$$12t = 15$$

$$t = \frac{15}{12} = \frac{5}{4}$$

FINAL ANSWER

$t = \frac{5}{4}$ seconds

4MA1/1H MAY 2024

Question 18

5 marks | Differentiation & Turning Points

18 A curve C has equation $y = x^3 - 40x + 1$

Find the coordinates of both the points on C at which the gradient is 8

(.....,.....)

(.....,.....)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H May 2024

1 Differentiate

$$y = x^3 - 40x + 1$$

$$\frac{dy}{dx} = 3x^2 - 40$$

2 Find the gradient

The gradient is 8, so

$$3x^2 - 40 = 8$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = 4 \quad \text{or} \quad x = -4$$

3 Differentiate

When $x = 4$:

$$y = 4^3 - 40(4) + 1 = -95$$

4 Differentiate

When $x = -4$:

$$y = (-4)^3 - 40(-4) + 1 = 97$$

FINAL ANSWER

$(-4, 97)$ and $(4, -95)$.

4MA1/1HR MAY 2024

Question 19

5 marks | Differentiation & Turning Points

19 A curve C has equation $y = x^3 - 8x^2 - 12x + 5$

Curve C has exactly two stationary points, one at point A and one at point B such that

x coordinate of point $A > x$ coordinate of point B

Find the coordinates of point A
Show clear algebraic working.

(..... , ...)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May 2024

1 Differentiate

$$y = x^3 - 8x^2 - 12x + 5$$

$$\frac{dy}{dx} = 3x^2 - 16x - 12$$

2 Stationary points have

Stationary points have

$$3x^2 - 16x - 12 = 0$$

$$x = \frac{16 \pm \sqrt{(-16)^2 - 4(3)(-12)}}{6}$$

$$x = \frac{16 \pm 20}{6}$$

$$x = 6 \quad \text{or} \quad x = -\frac{2}{3}$$

3 **Differentiate**

Point A has the greater x-coordinate, so $x = 6$.

$$y = 6^3 - 8(6^2) - 12(6) + 5$$

$$y = 216 - 288 - 72 + 5 = -139$$

✓ **FINAL ANSWER**

$A = (6, -139)$.

4MA1/1H NOVEMBER 2024

Question 20

4 marks | Differentiation & Turning Points

20 The curve with equation $y = 2x^4 - 64x$ has a minimum point.

Find an equation of the tangent to the curve at the minimum point.
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2024

1 Find the gradient

$$y = 2x^4 - 64x$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 8x^3 - 64$$

3 Find the gradient

At the minimum point, the gradient is 0.

$$8x^3 - 64 = 0$$

$$8x^3 = 64$$

$$x^3 = 8$$

$$x = 2$$

4 Find the coordinate

Find the y-coordinate:

$$y = 2(2)^4 - 64(2)$$

$$y = 32 - 128 = -96$$

5 The tangent at a minimum

The tangent at a minimum point is horizontal, so its equation is

$$y = -96$$

FINAL ANSWER

$y = -96$

Question 17

6 marks | Differentiation & Turning Points

17 $y = 4x^3 + 5x^2 + 2x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the coordinates of the turning points on the graph with equation $y = 4x^3 + 5x^2 + 2x$
Show clear algebraic working.

.....

(4)

(Total for Question 17 is 6 marks)

1 **Differentiate**

$$y = 4x^3 + 5x^2 + 2x$$

$$\frac{dy}{dx} = 12x^2 + 10x + 2$$

2 **Differentiate**

At a turning point,

$$12x^2 + 10x + 2 = 0$$

$$2(6x^2 + 5x + 1) = 0$$

$$(3x + 1)(2x + 1) = 0$$

$$x = -\frac{1}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

3 **Differentiate**

When $x = -\frac{1}{3}$,

$$y = -\frac{7}{27}$$

4 **Differentiate**

When $x = -\frac{1}{2}$,

$$y = -\frac{1}{4}$$

FINAL ANSWER

$\left(-\frac{1}{2}, -\frac{1}{4}\right)$ and $\left(-\frac{1}{3}, -\frac{7}{27}\right)$.

4MA1/2HR JUNE 2025

Question 22

5 marks | Differentiation & Turning Points

22 The diagram shows a solid cuboid.

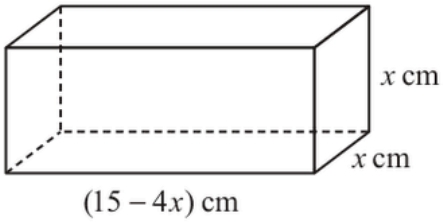


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2025

1 **The cuboid has dimensions**

The cuboid has dimensions

$$x, \quad x, \quad 15 - 4x$$

2 **Calculate value**

So

$$V = x^2(15 - 4x)$$

$$V = 15x^2 - 4x^3$$

3 **Differentiate**

Differentiate:

$$\frac{dV}{dx} = 30x - 12x^2$$

4 **Calculate value**

At a maximum,

$$30x - 12x^2 = 0$$

$$6x(5 - 2x) = 0$$

5 **The useful value is**

The useful value is

$$x = 2.5$$

6 **Calculate volume**

Now substitute into the volume:

$$V = (2.5)^2(15 - 4(2.5))$$

$$V = 6.25 \times 5$$

$$V = 31.25$$

✓ **FINAL ANSWER**

$31.25\ \text{cm}^3$

4WM2/01 MAY 2025

Question 22

4 marks | Differentiation & Turning Points

22 Work out the range of values of x for which the graph of $y = x^3 + \frac{2}{2}x^2 - 10x + 5$ has a positive gradient.
Show clear algebraic working.

.....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 May 2025

1 Evaluate fraction

$$\frac{dy}{dx} = 3x^2 + 2x - 10$$

2 Find the gradient

For a positive gradient,

$$3x^2 + 2x - 10 > 0$$

3 The roots are

The roots are

$$x = \frac{-1 \pm \sqrt{31}}{3}$$

4 Solve quadratic equation

Since the quadratic opens upwards,

$$x < \frac{-1 - \sqrt{31}}{3} \quad \text{or} \quad x > \frac{-1 + \sqrt{31}}{3}$$

FINAL ANSWER

$x < \frac{-1 - \sqrt{31}}{3}$ or $x > \frac{-1 + \sqrt{31}}{3}$.

4WM2/01 NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 November 2025

1 **Find the gradient**

$$fracdydx = 3x^2 - 16$$

2 **Find the gradient**

3*x*² − 16 = 11

3*x*² = 27

*x*² = 9 ⟹ *x* = 3 or *x* = − 3

3 **Solve inequality**

For *x* = 3: *y* = (3)³ − 16(3) + 7 = 27 − 48 + 7 = − 14

4 **Solve inequality**

For *x* = − 3: *y* = (−3)³ − 16(−3) + 7 = − 27 + 48 + 7 = 28

5 **Use trigonometry**

Step 4: Find the y-intercept (*b*) for each tangent (*y* = 11*x* + *b*).

Using (3,−14):

− 14 = 11(3) + *b* ⟹ − 14 = 33 + *b* ⟹ *b* = − 47

Using (−3,28):

28 = 11(−3) + *b* ⟹ 28 = − 33 + *b* ⟹ *b* = 61

✓ **FINAL ANSWER**

b = − 47 and 61

4MA1/2H NOVEMBER 2025

Question 22

6 marks | Differentiation & Turning Points

22 Curve **C** has equation $y = x^3 - 16x + 7$

At two points on **C**, the gradient is 11
The tangents to **C** at these two points have equations of the form $y = ax + b$

Work out the two possible values of b
Show clear algebraic working.

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H November 2025

1 Differentiate

$$y = x^3 - 16x + 7$$

2 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3x^2 - 16$$

3 The gradient of the tangent

The gradient of the tangent is 11, so

$$3x^2 - 16 = 11$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = 3 \quad \text{or} \quad x = -3$$

4 Differentiate

When $x = 3$:

$$y = 3^3 - 16(3) + 7 = 27 - 48 + 7 = -14$$

5 **Use trigonometry**

The tangent has equation $y = 11x + b$, so

$$-14 = 11(3) + b$$

$$b = -47$$

---

6 **Differentiate**

When $x = -3$:

$$y = (-3)^3 - 16(-3) + 7 = -27 + 48 + 7 = 28$$

$$28 = 11(-3) + b$$

$$b = 61$$

✓ **FINAL ANSWER**

$\boxed{b = -47 \text{ or } b = 61}$

4MA1/1HR JUNE 2026

Question 17

5 marks | Differentiation & Turning Points

17 $y = x^3 + 5x^2 - 8x$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} =$ .....

(2)

The curve with equation $y = x^3 + 5x^2 - 8x$ has two stationary points, P and Q
The x-coordinate of P is less than the x-coordinate of Q

(b) Work out the coordinates of P
Show clear algebraic working.

(..... , ...)

(3)

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\frac{dy}{dx} = 3x^2 + 10x - 8.$$

Apply the power rule to each term of y=x^3+5x^2-8x.

2 **Part (b) | Step 1**

$$3x^2 + 10x - 8 = 0 \quad \Longrightarrow \quad (3x - 2)(x + 4) = 0.$$

At a stationary point the derivative is zero.

3 **Part (b) | Step 2**

$$x = \frac{2}{3} \quad \text{or} \quad x = -4.$$

Set each factor equal to zero.

4

Part (b) | Step 3

P has the smaller x-coordinate, so $x_P = -4$. Then

$$y_P = (-4)^3 + 5(-4)^2 - 8(-4) = -64 + 80 + 32 = 48.$$

Choose the left-hand stationary point and substitute into the original curve equation.

FINAL ANSWER

(a) $\frac{dy}{dx} = 3x^2 + 10x - 8$ (b) $P = (-4, 48)$

4MA1/1H JUNE 2026

Question 21

5 marks | Differentiation & Turning Points

21 The diagram shows a solid cuboid.

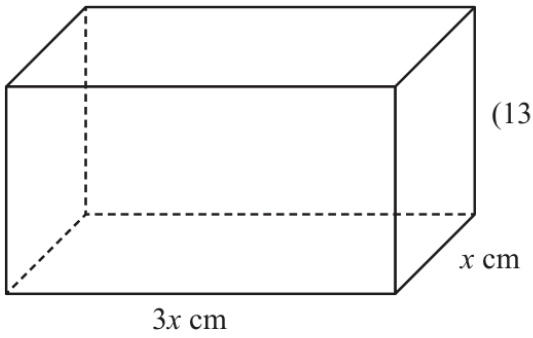


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V
Show clear algebraic working.

$V =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2026

1 **Solution | Step 1**

$$V = (3x)(x)(13.5 - 2x) = 40.5x^2 - 6x^3, \qquad 0 < x < 6.75.$$

Multiply the three dimensions and require them all to be positive.

2 **Solution | Step 2**

$$\frac{dV}{dx} = 81x - 18x^2 = 9x(9 - 2x).$$

Differentiate each power of x.

3 **Solution | Step 3**

At a stationary point, $9x(9 - 2x) = 0$, giving $x = 0$ or $x = 4.5$. Only $x = 4.5$ is inside the domain.
A stationary volume has zero derivative.

4 **Solution | Step 4**

The derivative is positive for $0 < x < 4.5$ and negative for $4.5 < x < 6.75$. Hence $x = 4.5$ gives the maximum.
The volume increases then decreases; its limiting endpoint volumes are zero.

5 Solution | Step 5

$$V_{\max} = 3(4.5)^2(13.5 - 9) = 273.375 \text{ cm}^3.$$

Substitute the maximising value into the original volume.

FINAL ANSWER

273.375 cm^3

Question 23

5 marks | Differentiation & Turning Points

23 A particle is moving along a straight line that passes through the fixed point O

The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 15t^2 - t^3 + 12$$

Find the maximum velocity of the particle where $0 < t < 11$
Show clear algebraic working.

..... m/s

(Total for Question 23 is 5 marks)

1 **Velocity**

$$v = \frac{ds}{dt} = 30t - 3t^2$$

2 **Maximum when dv/dt = 0**

$$\frac{dv}{dt} = 30 - 6t = 0 \Rightarrow t = 5 \quad \left(\frac{d^2v}{dt^2} = -6 < 0, \text{ a maximum}\right)$$

3 **Maximum velocity**

$$v = 30(5) - 3(5)^2 = 75$$

✓ **FINAL ANSWER**

75 m/s

TOPIC

Transformations of Graphs

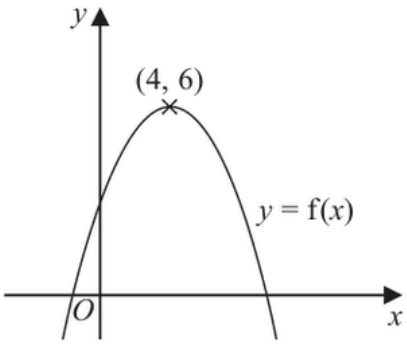
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 21

4 marks | Transformations of Graphs

21 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (4, 6)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 4)$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(2)

The equation of a curve **C** is $y = x^2 + 3x + 4$

The curve **C** is transformed to curve **S** under the translation $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(b) Find an equation of curve **S**.

You do not need to simplify the equation.

.....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2020

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(4, 6)$.

2 **Calculate value**

For

$$y = f(x + 4)$$

3 **the graph moves 4 units**

the graph moves 4 units left:

$$(4, 6) \to (0, 6)$$

4 **Calculate value**

For

$$y = f(2x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 6) \to (2, 6)$$

6 **Calculate value**

For part (b),

$$y = x^2 + 3x + 4$$

7 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

8 **Read the graph**

So the graph moves 4 units right and 6 units up.

9 **Add 6**

Replace x by $x - 4$, then add 6:

$$y = (x - 4)^2 + 3(x - 4) + 4 + 6$$

$$y = (x - 4)^2 + 3(x - 4) + 10$$

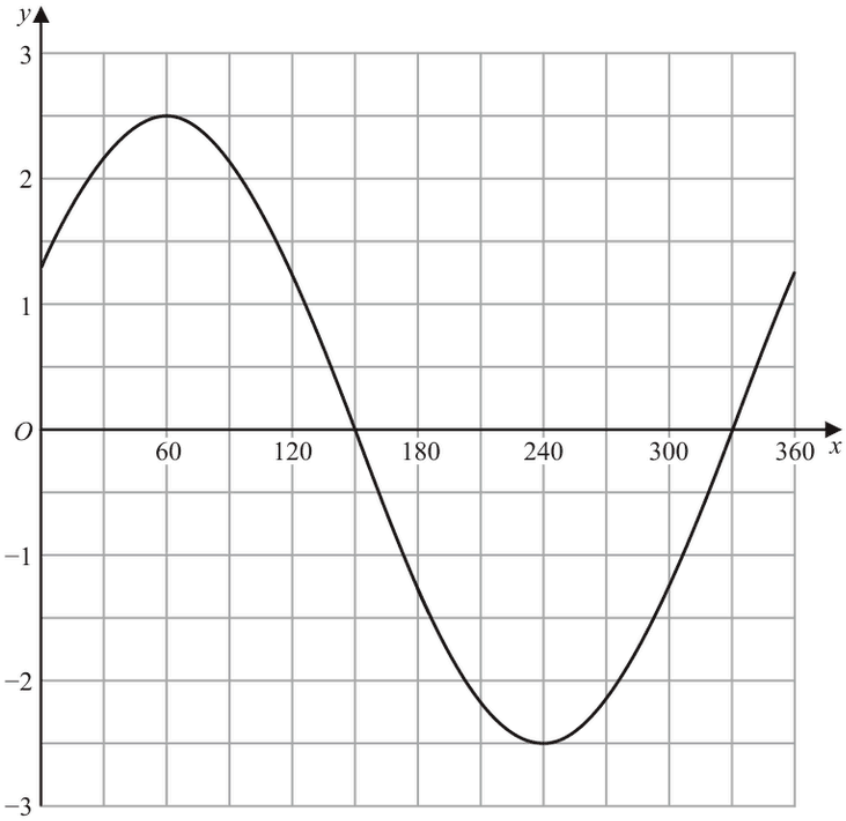
✓ **FINAL ANSWER**

$(0, 6)$, $(2, 6)$, and $y = (x - 4)^2 + 3(x - 4) + 10$

Question 22

4 marks | Transformations of Graphs

22 The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.



(a) Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

Another curve C has equation $y = f(x)$
The coordinates of the minimum point of C are (4, 5)

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

(..... , ...)

(ii) $y = f(x) - 7$

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Read the graph**

For part (a), the graph has maximum 2.5 and minimum -2.5, so

$$a = 2.5$$

2 **Use trigonometry**

The maximum occurs at $x = 60°$. For $y = a\cos(x + b)°$, a maximum occurs when

$$x + b = 360$$

3 **Calculate value**

So

$$60 + b = 360$$

$$b = 300$$

4 **Calculate value**

For part (b), the minimum point of $y = f(x)$ is $(4, 5)$.

5 **Calculate value**

For

$$y = f(2x)$$

6 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 5) \to (2, 5)$$

7 **Calculate value**

For

$$y = f(x) - 7$$

8 **the coordinate decreases by 7**

the y-coordinate decreases by 7:

$$(4, 5) \to (4, -2)$$

✓ **FINAL ANSWER**

$a = 2.5,\ b = 300;\ (2, 5)$ and $(4, -2)$

4MA1/1HR MAY/NOVEMBER 2020

Question 20

2 marks | Transformations of Graphs

20 A curve has equation $y = f(x)$
There is only one maximum point on the curve.
The coordinates of this maximum point are $(-3, 4)$

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x) - 6$

(..... , ...)

(ii) $y = f(2x)$

(..... , ...)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May/November 2020

1 The maximum point of is

The maximum point of $y = f(x)$ is $(-3, 4)$.

2 Calculate value

For

$$y = f(x) - 6$$

3 the graph moves 6 units

the graph moves 6 units down:

$$(-3, 4) \rightarrow (-3, -2)$$

4 Calculate value

For

$$y = f(2x)$$

5 the coordinate is divided by

the x-coordinate is divided by 2:

$$(-3, 4) \rightarrow \left(-\frac{3}{2}, 4\right)$$

FINAL ANSWER

$(-3, -2)$ and $\left(-\frac{3}{2}, 4\right)$

Question 25

3 marks | Transformations of Graphs

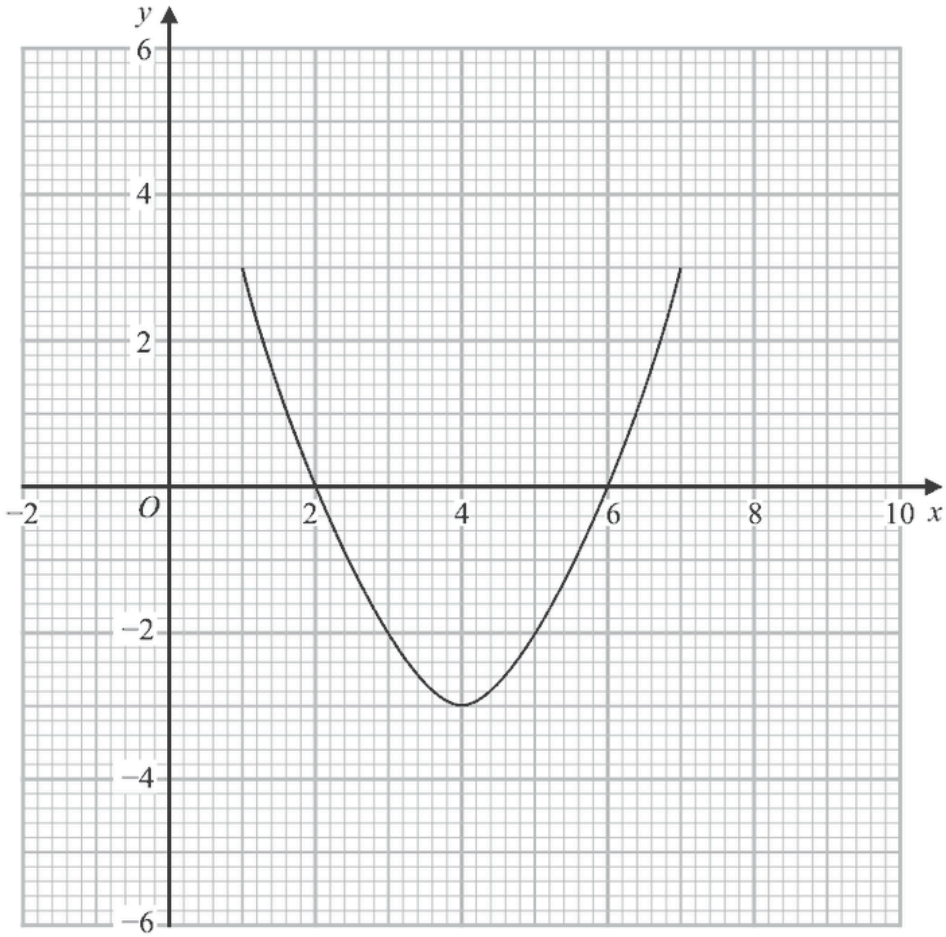
25 The curve with equation $y = g(x)$ is transformed to the curve with equation $y = -g(x)$ by the single transformation **T**.

(a) Describe fully the transformation **T**.

.....

(1)

The diagram shows the graph of $y = f(x)$



(b) On the grid, draw the graph of $y = 2f(x - 1)$

(2)

(Total for Question 25 is 3 marks)

1 **Apply transformation**

For part (a),

$$y = g(x) \rightarrow y = -g(x)$$

2 **The coordinates stay the same**

The x-coordinates stay the same and the y-coordinates change sign.

3 **Apply transformation**

So the transformation is a reflection in the x-axis.

4 **Calculate value**

For part (b),

$$y = 2f(x - 1)$$

5 **Read the graph**

First, $x - 1$ moves the graph 1 unit right. Then the factor 2 stretches the graph vertically by scale factor 2.

6 Use points from the original

Use points from the original graph:

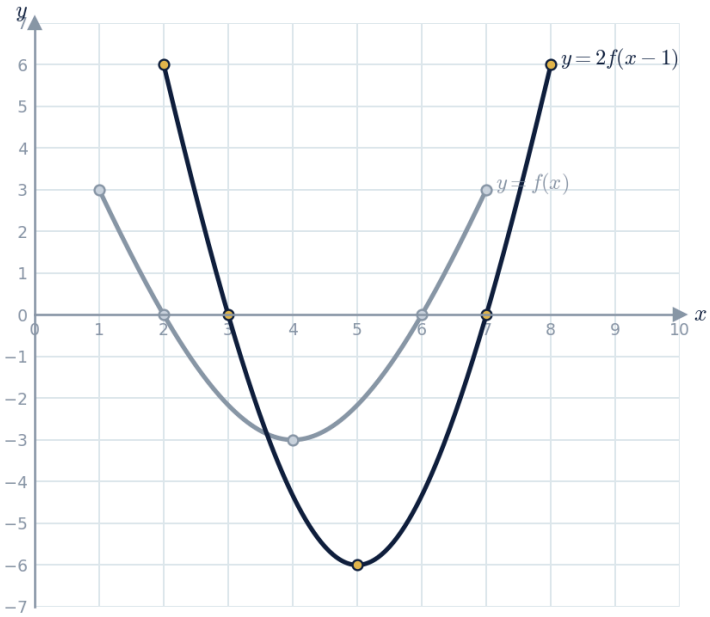
$$(1,3) \to (2,6)$$

$$(2,0) \to (3,0)$$

$$(4,-3) \to (5,-6)$$

$$(6,0) \to (7,0)$$

$$(7,3) \to (8,6)$$



7 Apply transformation

Draw the transformed parabola through these points.

FINAL ANSWER

reflection in the x-axis; transformed graph through $(2,6)$, $(3,0)$, $(5,-6)$, $(7,0)$, $(8,6)$

4MA1/1H JANUARY 2021

Question 21

4 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

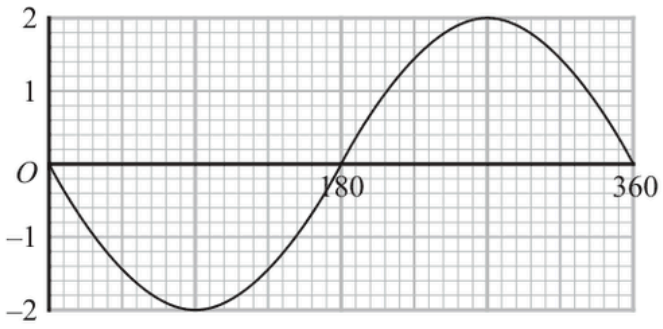
(..... , ...)

(ii) $y = \frac{1}{3} f(x)$

(..... , ...)

(2)

The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2021

1 **The minimum point of is**

The minimum point of $y = f(x)$ is $(-9, 15)$.

2 **Calculate value**

For

$$y = f(x + 3)$$

3 **the graph moves 3 units**

the graph moves 3 units left:

$$(-9, 15) \to (-12, 15)$$

4 **Evaluate fraction**

For

$$y = \frac{1}{3} f(x)$$

5 **the coordinate is divided by**

the y-coordinate is divided by 3:

$$(-9, 15) \to (-9, 5)$$

6 **Use trigonometry**

For part (b), the cosine graph has maximum 2 and minimum -2, so

$$a = 2$$

7 **The graph has a minimum**

The graph has a minimum at $x = 90°$. For $a\cos(x + b)°$, a minimum occurs when

$$x + b = 180$$

8 **Calculate value**

So

$$90 + b = 180$$

$$b = 90$$

✓ **FINAL ANSWER**

$(-12, 15)$, $(-9, 5)$, $a = 2,\ b = 90$

Question 21

6 marks | Transformations of Graphs

21 The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point A is the maximum point on **C**.

(a) Write down the coordinates of A.

(..... , ...)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

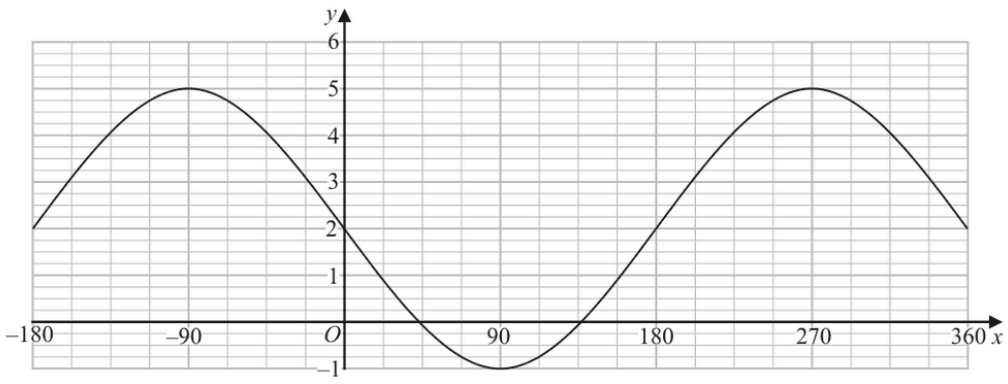
The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)

The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

(Total for Question 21 is 6 marks)

1 **Calculate value**

$$f(x) = 9 - 3(x + 2)^2$$

2 **This is in completed square**

This is in completed square form, so the maximum point is

$$(-2, 9)$$

3 **Use matrices**

For the translation

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

4 **the graph moves 4 units**

the graph moves 4 units right. Replace x by $x - 4$:

$$y = 9 - 3((x - 4) + 2)^2$$

$$y = 9 - 3(x - 2)^2$$

5 **The curve has equation**

The curve T has equation

$$y = 3(x + 2)^2 - 9$$

6 **This is**

This is

$$y = -\left(9 - 3(x + 2)^2\right)$$

7 **Apply transformation**

So C is reflected in the x-axis.

8 **Use trigonometry**

For the cosine graph, the maximum is 5 and the minimum is -1.

$$a = \frac{5 - (-1)}{2} = 3$$

$$c = \frac{5 + (-1)}{2} = 2$$

9 **Use trigonometry**

For

$$y = a\cos(x - b)^\circ + c$$

10 **Read the graph**

a maximum occurs when $x - b = 0$. The graph has a maximum at $x = 270°$, so

$$b = 270$$

✓ **FINAL ANSWER**

$A = (-2, 9)$, $y = 9 - 3(x - 2)^2$, reflection in the x-axis, $a = 3,\ b = 270,\ c = 2$

Question 20

6 marks | Transformations of Graphs

20 The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

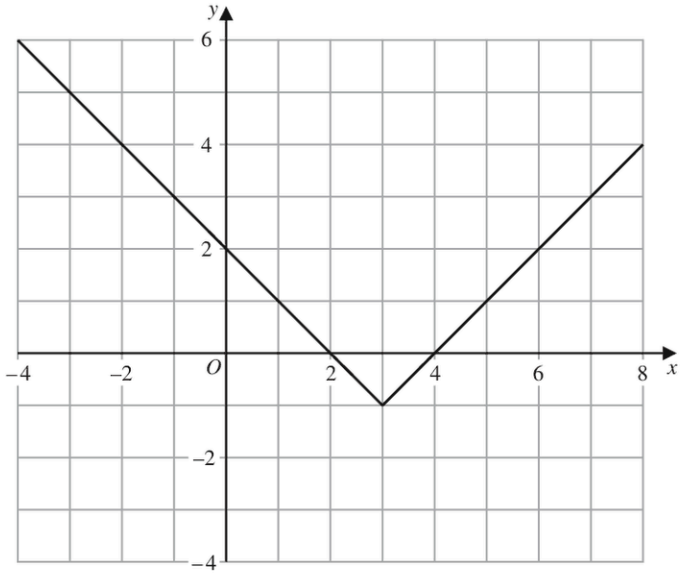
(.....,.....)

(ii) $y = f(3x)$

(.....,.....)

(2)

The graph of $y = g(x)$ is shown on the grid below.



(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

(2)

The graph of $y = h(x)$ intersects the x-axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where a is a constant.

(c) Find the two possible values of a

....., ...

(2)

(Total for Question 20 is 6 marks)

1 **The turning point of is**

The turning point of $y = f(x)$ is $(-6, -4)$.

2 **Calculate value**

For

$$y = f(x) + 5$$

3 **the graph moves 5 units**

the graph moves 5 units up:

$$(-6, -4) \rightarrow (-6, 1)$$

4 **Calculate value**

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(-6, -4) \rightarrow (-2, -4)$$

6 **Read the graph**

For the sketch of $y = 2g(x)$, keep every x-coordinate the same and double every y-coordinate.

7 **Useful points are**

Useful points are

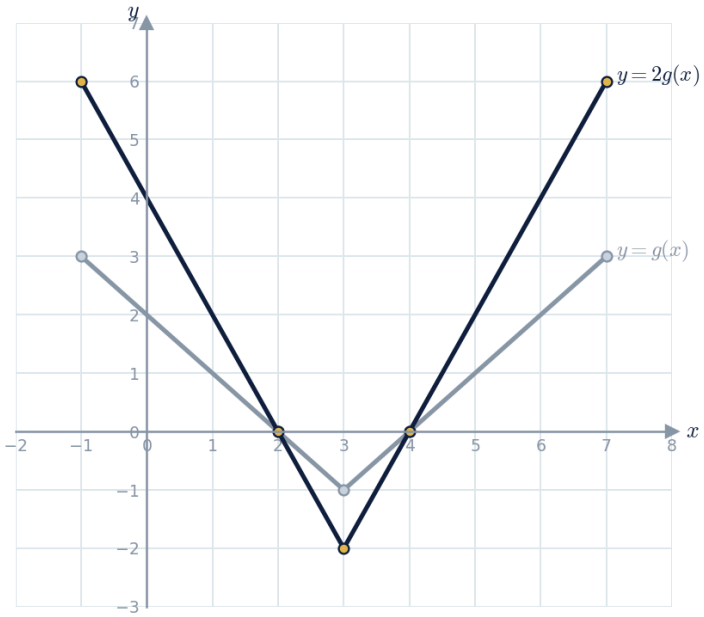
$$(-1,3) \to (-1,6)$$

$$(2,0) \to (2,0)$$

$$(3,-1) \to (3,-2)$$

$$(4,0) \to (4,0)$$

$$(7,3) \to (7,6)$$



---

8 **Solve inequality**

Draw straight line segments through these points for $-1 \leq x \leq 7$.

---

9 **Calculate value**

For part (c), $h(x)$ has roots at $x = -1$ and $x = 6$.

---

10 **Calculate value**

For $h(x+a)$ to pass through $(2,0)$,

$$2 + a = -1 \quad \text{or} \quad 2 + a = 6$$

$$a = -3 \quad \text{or} \quad a = 4$$

FINAL ANSWER

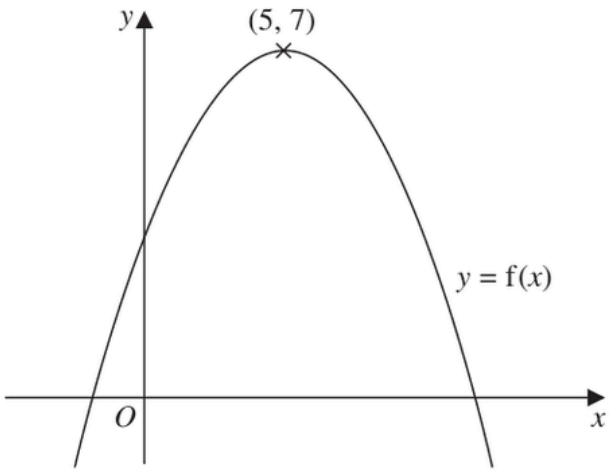
$(-6, 1)$, $(-2, -4)$, sketch through the doubled points, and $a = -3$ or $a = 4$

4MA1/1H JANUARY 2022

Question 23

2 marks | Transformations of Graphs

23 The diagram shows a sketch of the curve with equation $y = f(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 9)$

(..... , ...)

(ii) $y = f(x) + 3$

(..... , ...)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2022

1 The maximum point of is

The maximum point of $y = f(x)$ is $(5, 7)$.

2 Calculate value

For

$$y = f(x + 9)$$

3 the graph moves 9 units

the graph moves 9 units left:

$$(5, 7) \rightarrow (-4, 7)$$

4 **Calculate value**

For

$$y = f(x) + 3$$

5 **the graph moves 3 units**

the graph moves 3 units up:

$$(5, 7) \to (5, 10)$$

✓ **FINAL ANSWER**

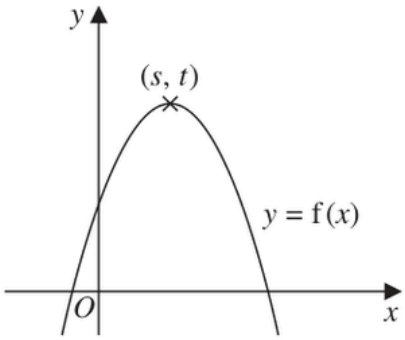
$(-4, 7)$ and $(5, 10)$

4MA1/1H JUNE 2022

Question 20

2 marks | Transformations of Graphs

20 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (s, t)

Find, in terms of s and t, the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

(..... , ...)
(1)

(ii) $y = 3f(x)$

(..... , ...)
(1)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H June 2022

1 The maximum point of is

The maximum point of $y = f(x)$ is (s, t).

2 Calculate value

For

$$y = f(x - 2)$$

3 the graph moves 2 units

the graph moves 2 units right:

$$(s, t) \to (s + 2, t)$$

4 **Calculate value**

For

$$y = 3f(x)$$

5 **the coordinate is multiplied by**

the y-coordinate is multiplied by 3:

$$(s, t) \to (s, 3t)$$

✓ **FINAL ANSWER**

$(s + 2, t)$ and $(s, 3t)$

Question 22

4 marks | Transformations of Graphs

22 The point A with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point A on the straight line with equation

(i) $y = f(x) - 3$

(..... , ...)

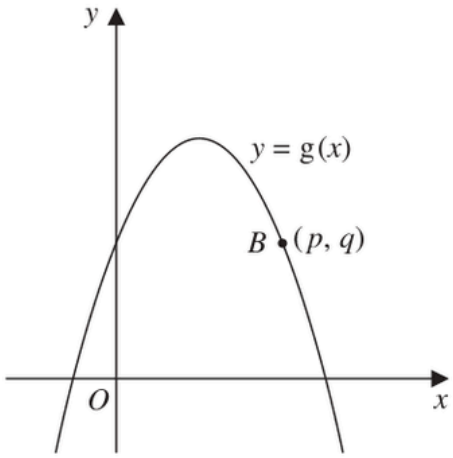
(1)

(ii) $y = f\left(\frac{x}{2}\right)$

(..... , ...)

(1)

Here is a sketch of part of the curve with equation $y = g(x)$



The point B with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of the point B on the curve with equation

$$y = -g(x - c)$$

where c is a constant.

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Point lies on**

Point $A = (-3, 2)$ lies on $y = f(x)$.

2 **Calculate value**

For

$$y = f(x) - 3$$

3 **the graph moves 3 units**

the graph moves 3 units down:

$$(-3, 2) \to (-3, -1)$$

4 **Evaluate fraction**

For

$$y = f\left(\frac{x}{2}\right)$$

5 **the coordinate is multiplied by**

the x-coordinate is multiplied by 2:

$$(-3, 2) \to (-6, 2)$$

6 **Calculate value**

For part (b), point $B = (p, q)$ lies on $y = g(x)$.

7 **Calculate value**

For

$$y = -g(x - c)$$

8 **Read the graph**

$x - c$ moves the graph c units right, and the minus sign reflects the graph in the x-axis.

9 **Apply transformation**

So

$$(p, q) \to (p + c, -q)$$

✓ **FINAL ANSWER**

$(-3, -1)$, $(-6, 2)$, and $(p + c, -q)$

4MA1/1HR MAY 2023

Question 20

7 marks | Transformations of Graphs

20 Curve **C** has equation $y = f(x)$
The graph of curve **C** has one maximum point.
The coordinates of this maximum point are (3, 5)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = 2f(x)$

(..... , ...)
(1)

(ii) $y = f(x) - 7$

(..... , ...)
(1)

(iii) $y = f(-x)$

(..... , ...)
(1)

Curve **L** has equation $y = x^2 + 7x + 20$

Curve **L** is transformed to curve **S** under the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(b) Find an equation for **S**
Give your answer in the form $y = ax^2 + bx + c$

$y =$ .....
(4)

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR May 2023

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(3, 5)$.

2 **Calculate value**

For

$$y = 2f(x)$$

3 **the coordinate is multiplied by**

the y-coordinate is multiplied by 2:

$$(3, 5) \to (3, 10)$$

4 **Calculate value**

For

$$y = f(x) - 7$$

5 **the graph moves 7 units**

the graph moves 7 units down:

$$(3, 5) \to (3, -2)$$

6 **Calculate value**

For

$$y = f(-x)$$

7 **the graph reflects in the**

the graph reflects in the y-axis:

$$(3, 5) \to (-3, 5)$$

8 **Calculate value**

For part (b),

$$y = x^2 + 7x + 20$$

9 **The translation vector is**

The translation vector is

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

10 **Read the graph**

So the graph moves 2 units right.

11 **Replace by**

Replace x by $x - 2$:

$$y = (x - 2)^2 + 7(x - 2) + 20$$

$$y = x^2 + 3x + 10$$

FINAL ANSWER

$(3, 10)$, $(3, -2)$, $(-3, 5)$, and $y = x^2 + 3x + 10$

4MA1/1H MAY 2023

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(..... , ...)

(1)

(b) $y = f(3x)$

(..... , ...)

(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2023

1 The turning point of is

The turning point of $y = f(x)$ is $(6, 5)$.

2 Calculate value

For

$$y = f(x - 4)$$

3 the graph moves 4 units

the graph moves 4 units right:

$$(6, 5) \to (10, 5)$$

4 Calculate value

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(6, 5) \rightarrow (2, 5)$$

✓ **FINAL ANSWER**

$(10, 5)$ and $(2, 5)$

Question 24

4 marks | Transformations of Graphs

24 The curve with equation $f(x) = 5x^2 + 9x + 2$ is transformed to the curve with equation

$g(x) = 5(x+4)^2 + 9(x+4) + 8$ by the translation $\begin{pmatrix} a \\ b \end{pmatrix}$

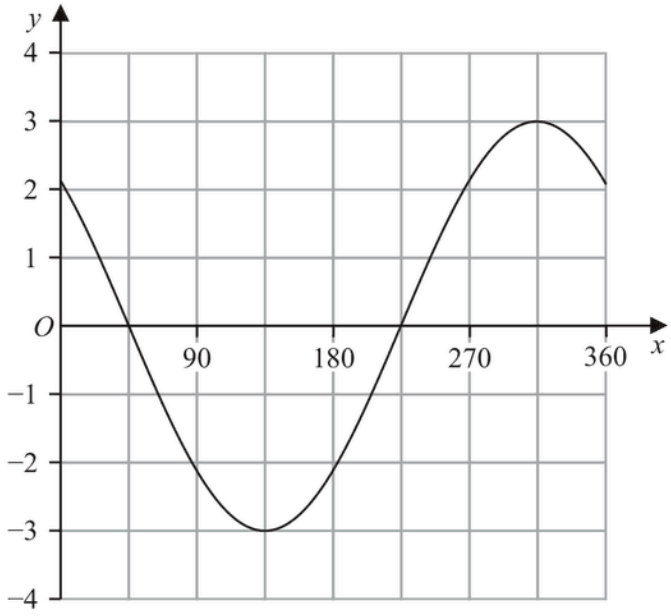
(a) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

The graph of $y = p\cos(x+q)^\circ$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $p > 0$ and $0 < q < 360$

(b) find the value of p and the value of q

$p =$ .....

$q =$ .....

(2)

(Total for Question 24 is 4 marks)

1 **Read the graph**

For part (a),

$$f(x) = 5x^2 + 9x + 2$$

$$g(x) = 5(x+4)^2 + 9(x+4) + 8$$

2 **Read the graph**

Since

$$f(x+4) = 5(x+4)^2 + 9(x+4) + 2$$

$$g(x) = f(x+4) + 6$$

3 **Read the graph**

So the graph has moved 4 units left and 6 units up.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

4 **Read the graph**

For part (b), the graph has maximum value 3 and minimum value -3, so

$$p = 3$$

5 **The minimum occurs when**

The minimum occurs when

$$x + q = 180$$

6 **Read the graph**

From the graph, the minimum is at $x = 135$, so

$$135 + q = 180$$

$$q = 45$$

✓ **FINAL ANSWER**

$a = -4,\ b = 6,\ p = 3,\ q = 45$

4MA1/1H MAY 2024

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is one minimum point on this curve.
The coordinates of this minimum point are $(5, -4)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 7)$

(.....,.....)
(1)

(ii) $y = f(x) - 6$

(.....,.....)
(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(5, -4)$.

2 Calculate value

For

$$y = f(x + 7)$$

3 the graph moves 7 units

the graph moves 7 units left:

$$(5, -4) \rightarrow (-2, -4)$$

4 Calculate value

For

$$y = f(x) - 6$$

5 **the graph moves 6 units**

the graph moves 6 units down:

$$(5, -4) \rightarrow (5, -10)$$

✓ **FINAL ANSWER**

$(-2, -4)$ and $(5, -10)$

4MA1/2HR JUNE 2024

Question 23

2 marks | Transformations of Graphs

23 A curve has equation $y = f(x)$
The coordinates of the minimum point on this curve are $(6, -3)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 10$

(.....,.....)
(1)

(ii) $y = f(3x)$

(.....,.....)
(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is $(6, -3)$.

2 Calculate value

For

$$y = f(x) + 10$$

3 the graph moves 10 units

the graph moves 10 units up:

$$(6, -3) \to (6, 7)$$

4 Calculate value

For

$$y = f(3x)$$

5 the coordinate is divided by

the x-coordinate is divided by 3:

$$(6, -3) \to (2, -3)$$

FINAL ANSWER

$(6, 7)$ and $(2, -3)$

4MA1/2H NOVEMBER 2024

Question 19

3 marks | Transformations of Graphs

19 A curve has equation $y = f(x)$

There is only one minimum point on the curve.
The coordinates of this minimum point are (5, 4)

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 5)$

(..... , ...)
(1)

(ii) $y = 3f(x)$

(..... , ...)
(1)

(iii) $y = f(x) - 7$

(..... , ...)
(1)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2024

1 The minimum point of is

The minimum point of $y = f(x)$ is

$$(5, 4)$$

2 Read the graph

(i) For

$$y = f(x + 5)$$

3 the graph moves 5 units

the graph moves 5 units left, so the minimum point becomes

$$(0, 4)$$

4 **Read the graph**

(ii) For

$$y = 3f(x)$$

5 **the y coordinate is multiplied**

the y-coordinate is multiplied by 3, so the minimum point becomes

$$(5, 12)$$

6 **Read the graph**

(iii) For

$$y = f(x) - 7$$

7 **the graph moves 7 units**

the graph moves 7 units down, so the minimum point becomes

$$(5, -3)$$

✓ **FINAL ANSWER**

$(0, 4)$, $(5, 12)$, and $(5, -3)$.

4MA1/1H JUNE 2025

Question 19

2 marks | Transformations of Graphs

19 The equation of a curve is $y = f(x)$

There is only one minimum point on the curve.
The coordinates of this minimum point are $(8, -12)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 3$

(..... , ...)

(1)

(ii) $y = f(2x)$

(..... , ...)

(1)

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2025

1 **The minimum point of is**

The minimum point of $y = f(x)$ is

$$(8, 12)$$

2 **Read the graph**

For

$$y = f(x) + 5$$

the graph moves up by 5:

$$(8, 12) \rightarrow (8, 17)$$

3 **Read the graph**

For

$$y = f(2x)$$

the graph is horizontally compressed by scale factor $\frac{1}{2}$, so the x-coordinate is halved:

$$(8, 12) \to (4, 12)$$

✓ **FINAL ANSWER**

(i) $(8, 17)$ and (ii) $(4, 12)$.

4MA1/1HR JUNE 2025

Question 21

3 marks | Transformations of Graphs

21 The curve with equation $y = f(x)$ has one turning point.
The coordinates of this turning point are $(-6, 9)$

(a) Write down the coordinates of the turning point on the curve with equation $y = f(3x)$

(..... , ...)

(1)

The curve **C** with equation $y = g(x)$ is transformed to give the curve **S** with equation $y = g(x + a) + b$

The point $(4, -5)$ on **C** is mapped to the point $(1, -16)$ on **S**

(b) Write down the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(-6, 9)$.

2 Calculate value

For

$$y = f(3x)$$

3 the coordinate is divided by

the x-coordinate is divided by 3:

$$(-6, 9) \to (-2, 9)$$

4 Calculate value

For part (b), a point $(4, -5)$ on $y = g(x)$ maps to $(1, -16)$ on

$$y = g(x + a) + b$$

5 **Calculate value**

For the x-coordinate:

$$1 + a = 4$$

$$a = 3$$

---

6 **Calculate value**

For the y-coordinate:

$$-5 + b = -16$$

$$b = -11$$

✓ **FINAL ANSWER**

$(-2, 9)$, $a = 3$, $b = -11$

Question 27

4 marks | Transformations of Graphs

27 A curve has equation $y = f(x)$

There is one maximum point on this curve.
The coordinates of this maximum point are $(6, -5)$

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 4)$

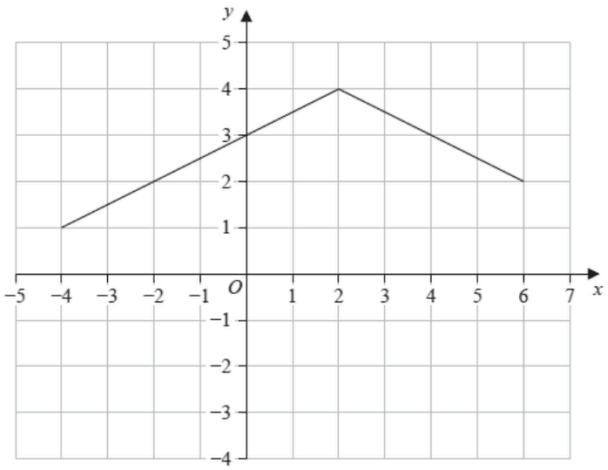
(.....)
(1)

(ii) $y = f(3x)$

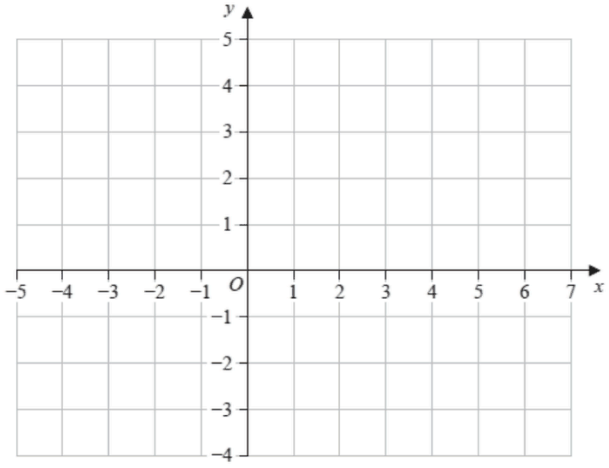
(.....)
(1)

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Here is the graph of $y = g(x)$



(b) On the grid below, draw the graph of $y = g(2x) - 3$



(2)

(Total for Question 27 is 4 marks)

1 **(a)**

(a)

2 **Read the graph**

For $y = f(x - 4)$, move the graph 4 units right:

$$(6, -5) \to (10, -5)$$

3 **Divide the -coordinate**

For $y = f(3x)$, divide the x-coordinate by 3:

$$(6, -5) \to (2, -5)$$

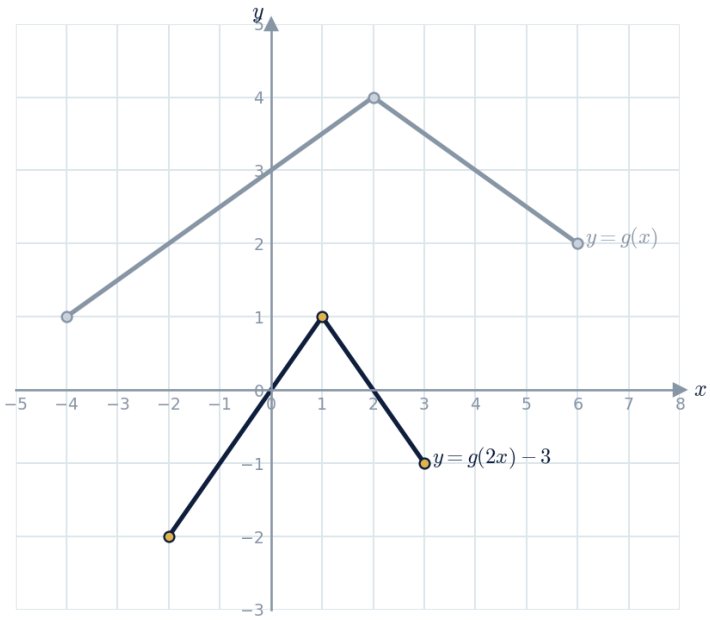
4 **Subtract from each -coordinate**

(b) For $y = g(2x) - 3$, halve each x-coordinate and subtract 3 from each y-coordinate.

5 **Useful transformed points are**

Useful transformed points are

$$(-4, 1) \to (-2, -2), \quad (2, 4) \to (1, 1), \quad (6, 2) \to (3, -1)$$



6 **Join these points with straight**

Join these points with straight line segments.

✓ **FINAL ANSWER**

$(10, -5)$, $(2, -5)$, graph through $(-2, -2), (1, 1), (3, -1)$.

4MA1/1H NOVEMBER 2025

Question 24

3 marks | Transformations of Graphs

24 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are (4, 3)

Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x + 5)$

(..... , ...)
(1)

(ii) $y = f(x) + 7$

(..... , ...)
(1)

(iii) $y = f(2x)$

(..... , ...)
(1)

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H November 2025

1 The turning point of is

The turning point of $y = f(x)$ is $(4, 3)$.

2 Calculate value

For

$$y = f(x + 5)$$

3 the graph moves 5 units

the graph moves 5 units left:

$$(4, 3) \rightarrow (-1, 3)$$

4 **Calculate value**

For

$$y = f(x) + 7$$

5 **the graph moves 7 units**

the graph moves 7 units up:

$$(4, 3) \to (4, 10)$$

6 **Calculate value**

For

$$y = f(2x)$$

7 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 3) \to (2, 3)$$

✓ **FINAL ANSWER**

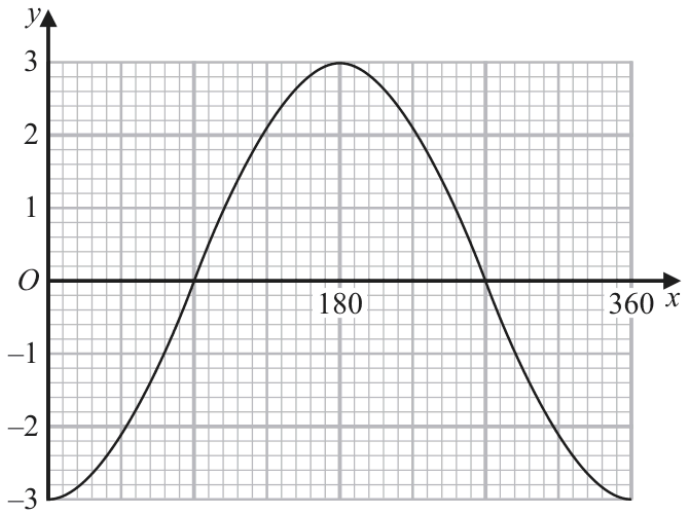
$(-1, 3)$, $(4, 10)$, and $(2, 3)$

4MA1/2HR JUNE 2026

Question 25

4 marks | Transformations of Graphs

25 The graph of $y = a\sin(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a < 0$ and that $0 < b < 360$

(a) find the value of a and the value of b

$a =$ .....

$b =$ .....

(2)

A curve has equation $y = f(x)$
The point T with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of T on the curve with equation

$$y = f(2x) + 5$$

Give your coordinates in terms of p and q

(..... , ...)

(2)

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2026

1 **Part (a) | Step 1**

The graph has maximum 3 and minimum -3, so $|a| = 3$. Since $a < 0$, $a = -3$.

The amplitude is half the vertical range.

2 **Part (a) | Step 2**

At $x = 0$, $y = -3$, so $-3 \sin b° = -3$ and $\sin b° = 1$. Hence $b = 90$ for $0 < b < 360$.

The minimum at x=0 fixes the phase when the amplitude is negative.

---

3 **Part (b) | Step 1**

To use the original input p, set $2x = p$, giving $x = p/2$.

The replacement of x by 2x halves horizontal coordinates.

---

4 **Part (b) | Step 2**

The corresponding output is $f(p) + 5 = q + 5$.

Adding 5 translates every output upwards by 5.

✓ **FINAL ANSWER**

(a) $a = -3, \qquad b = 90$. (b) $\left(\frac{p}{2}, q + 5\right)$.

Question 26

5 marks | Transformations of Graphs

26 The curve **P** has equation $y = f(x)$
The coordinates of the minimum point on **P** are $(-5, 4)$

(a) Write down the coordinates of the minimum point on the curve with equation $y = 3f(x)$

(..... , ...)

(1)

The curve ***Q*** has equation $y = 2 - (x + 3)^2$

The curve ***R*** has equation $y = 2 - (3 - x)^2$

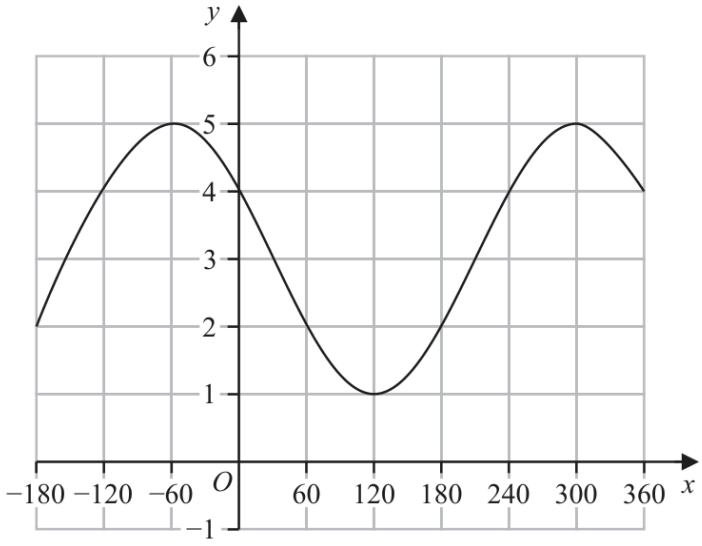
(b) Describe fully the single transformation that maps ***Q*** to ***R***

.....

.....

(1)

The graph of $y = a\cos(x + b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid.



Given that $a > 0$ and $0 < b < 360$

(c) find the value of a, the value of b and the value of c

a = ...

b = ...

c = ...

(3)

(Total for Question 26 is 5 marks)

1 **Part (a) | Step 1**

For $y = 3f(x)$, x-coordinates are unchanged and y-coordinates are multiplied by 3. Thus $(-5, 4) \mapsto (-5, 12)$.

The vertical scale factor is positive, so a minimum remains a minimum.

2 **Part (b) | Step 1**

For $Q(x) = 2 - (x + 3)^2$,

$$Q(-x) = 2 - (-x + 3)^2 = 2 - (3 - x)^2 = R(x).$$

Replacing x by $-x$ reflects the graph in the y-axis.

The point mapping is (x,y) to (-x,y).

3 **Part (c) | Step 1**

The maximum is 5 and the minimum is 1, so

$$a = \frac{5 - 1}{2} = 2, \qquad c = \frac{5 + 1}{2} = 3.$$

The positive amplitude is half the range; the vertical shift is the midline.

4 **Part (c) | Step 2**

A maximum occurs at $x = -60°$. For positive a, cosine is maximal when $x + b = 0°$ modulo $360°$, giving $b = 60$ in the stated interval. Hence $y = 2\cos(x + 60)° + 3$.

Read the phase from a maximum and apply 0<b<360.

✓ **FINAL ANSWER**

(a) $(-5, 12)$ (b) Reflection in the y-axis. (c) $a = 2, \quad b = 60, \quad c = 3$.

TOPIC

Angles in Polygons & Parallel Lines

Every past-paper question on this topic, 2020 to 2026.

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

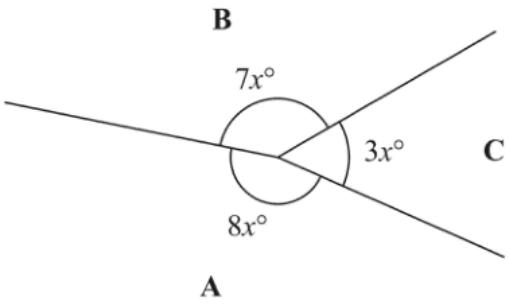


Diagram **NOT** accurately drawn

Polygon **B** has n sides.
Work out the value of n

$n =$ .....

(Total for Question 4 is 4 marks)

1 **Angles around a point add**

Angles around a point add to $360°$:

$$8x + 7x + 3x = 360$$

$$18x = 360$$

$$x = 20$$

2 **The interior angle of polygon**

The interior angle of polygon B is

$$7x = 7(20) = 140°$$

3 **Find the gradient**

For a regular n-sided polygon,

$$\frac{180(n-2)}{n} = 140$$

$$180n - 360 = 140n$$

$$40n = 360$$

$$n = 9$$

FINAL ANSWER

$n = 9$.

4MA1/2H JANUARY 2020

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows a triangle.

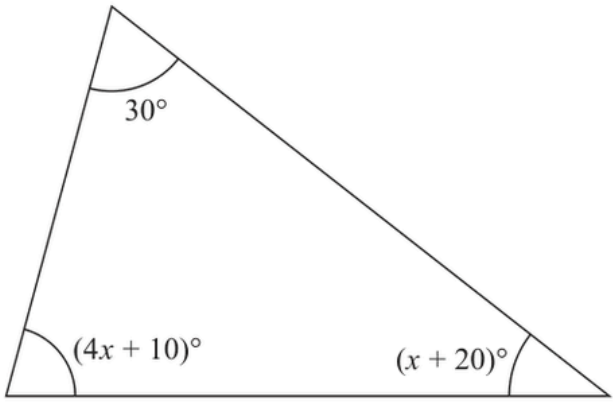


Diagram **NOT** accurately drawn

Work out the value of x.

$x =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2020

1 Angles in a triangle add

Angles in a triangle add to $180°$:

$$30 + (4x + 10) + (x + 20) = 180$$

$$5x + 60 = 180$$

$$5x = 120$$

$$x = 24$$

FINAL ANSWER

$x = 24$.

4MA1/1H MAY/NOVEMBER 2020

Question 8

4 marks | Angles in Polygons & Parallel Lines

8 Here is a 10-sided polygon.

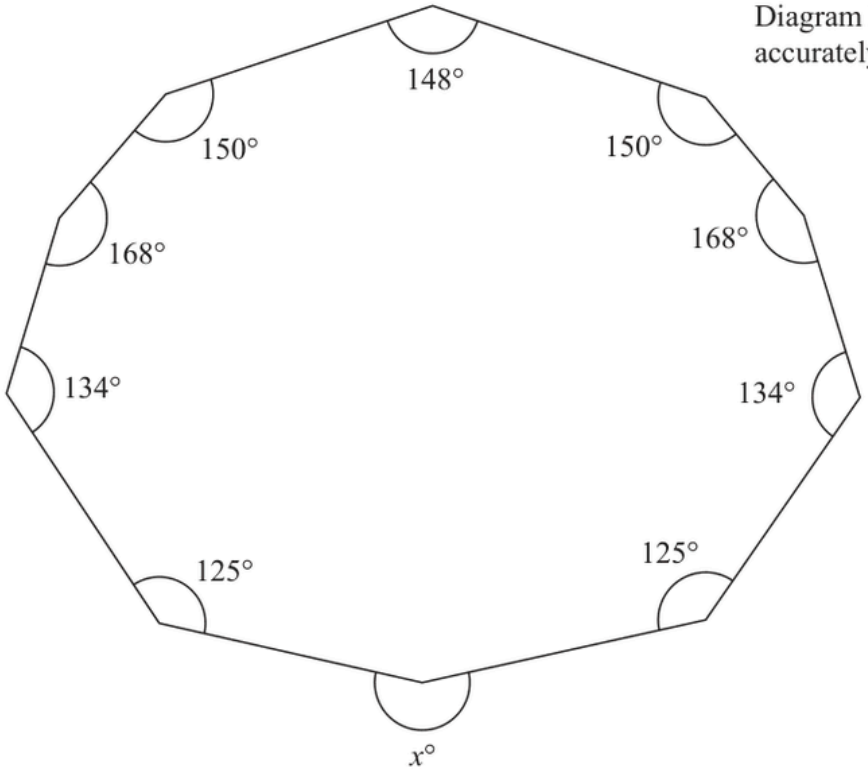


Diagram **NOT** accurately drawn

Work out the value of x.

$x =$ .....

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May/November 2020

1 **The sum of the interior**

The sum of the interior angles of a 10-sided polygon is

$$(10 - 2) \times 180 = 1440°$$

2 **The sum of the 9**

The sum of the 9 given interior angles is

$$150 + 168 + 134 + 125 + 125 + 134 + 168 + 150 + 148 = 1302°$$

3 Find the gradient

So the missing interior angle is

$$1440^\circ - 1302^\circ = 138^\circ$$

4 The angle is the reflex

The angle x is the reflex angle outside the polygon at this vertex:

$$x = 360^\circ - 138^\circ = 222^\circ$$

FINAL ANSWER

222°.

4MA1/1HR JANUARY 2021

Question 8

5 marks | Angles in Polygons & Parallel Lines

8 The diagram shows a regular pentagon, $ABCDE$, a regular hexagon, $CFGHID$, and a quadrilateral, $EDIJ$.

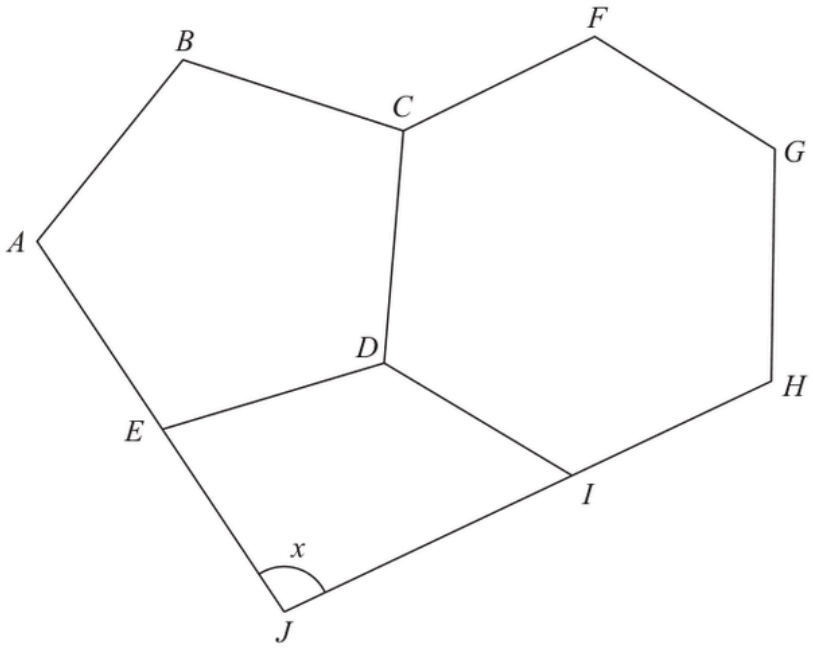


Diagram **NOT** accurately drawn

AEJ and HIJ are straight lines.

Work out the size of the angle marked x.
Show your working clearly.

.....°

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2021

1 **A regular pentagon has interior**

A regular pentagon has interior angle

$$\frac{(5-2)180}{5} = 108°$$

2 **A regular hexagon has interior**

A regular hexagon has interior angle

$$\frac{(6-2)180}{6} = 120°$$

3 **Find the gradient**

At E, since AEJ is a straight line,

$$\angle DEJ = 180 - 108 = 72^\circ$$

---

4 **Find the gradient**

At D, the angles around the point include the pentagon angle 108° and the hexagon angle 120°, so

$$\angle EDI = 360 - 108 - 120 = 132^\circ$$

---

5 **Find the gradient**

At I, since HIJ is a straight line,

$$\angle DIJ = 180 - 120 = 60^\circ$$

---

6 **Find the gradient**

In quadrilateral $EDIJ$, angles sum to 360°:

$$x = 360 - 72 - 132 - 60 = 96$$

✓ **FINAL ANSWER**

96°.

4MA1/2H MAY 2021

Question 5

5 marks | Angles in Polygons & Parallel Lines

5

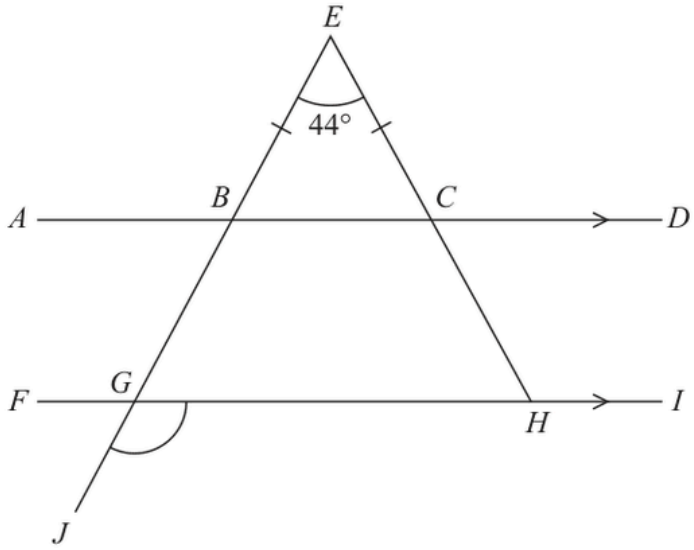


Diagram **NOT** accurately drawn

ABCD and *FGHI* are parallel straight lines.
EBGJ and *ECH* are straight lines.

$BE = CE$
Angle $BEC = 44°$

Work out the size of angle *JGH*.
Give a reason for each stage of your working.

.....°

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H May 2021

1 Triangle is isosceles because

Triangle EBC is isosceles because

$$BE = CE$$

2 Find the gradient

So the base angles are equal.

$$\angle EBC = \angle ECB = \frac{180° - 44°}{2} = 68°$$

3 **The two horizontal lines are**

The two horizontal lines are parallel, so the corresponding acute angle at G is $68°$.

4 **The required angle is supplementary**

The required angle is supplementary to this angle:

$$180° - 68° = 112°$$

✓ **FINAL ANSWER**

$112°$.

4MA1/2H NOVEMBER 2021

Question 5

5 marks | Angles in Polygons & Parallel Lines

5 The diagram shows a regular octagon $ABCDHIJK$ and a pentagon $DEFGH$.

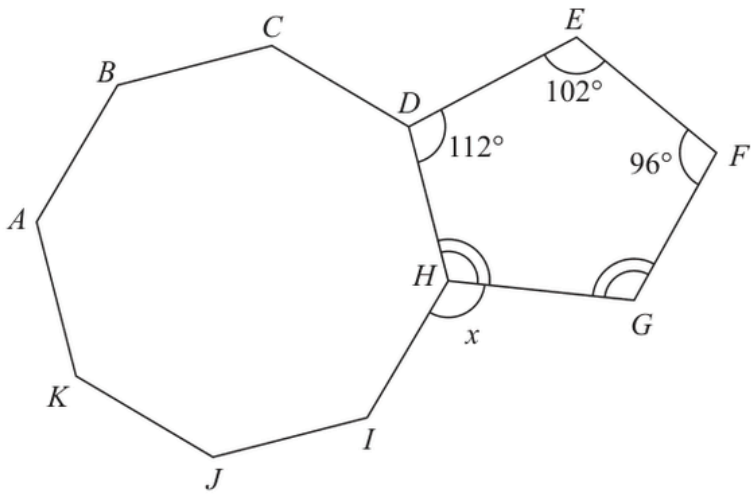


Diagram **NOT** accurately drawn

Angle GHD = angle FGH.

Work out the size of the angle marked x.
Show your working clearly.

.....°

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2021

1 Find the gradient

For pentagon $DEFGH$, the interior angle sum is

$$(5-2)\times 180 = 540°$$

2 Let

Let

$$\angle GHD = \angle FGH = a$$

3 **Find the gradient**

Then

$$112 + 102 + 96 + a + a = 540$$

$$2a = 230$$

$$a = 115°$$

---

4 **A regular octagon has interior**

A regular octagon has interior angle

$$\frac{(8-2) \times 180}{8} = 135°$$

---

5 **Find the gradient**

At H, angles around a point add to $360°$:

$$x = 360° - 135° - 115° = 110°$$

✓ **FINAL ANSWER**

$110°$.

4MA1/2HR JANUARY 2022

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows a regular octagon $ABCDEFGH$ and a regular pentagon $ABIJK$

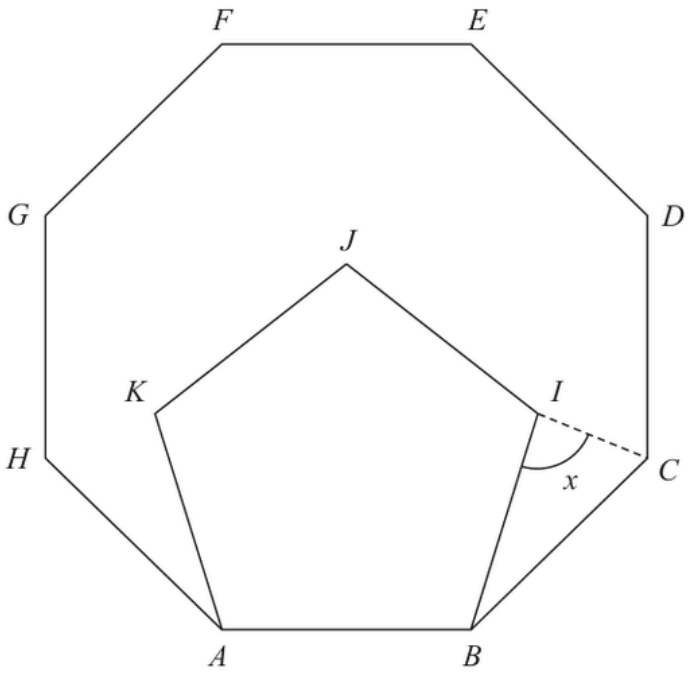


Diagram **NOT** accurately drawn

Work out the size of the angle x

.....°

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2022

1 **The interior angle of a**

The interior angle of a regular octagon is

$$\frac{(8-2)\times 180}{8} = 135°$$

2 **The interior angle of a**

The interior angle of a regular pentagon is

$$\frac{(5-2)\times 180}{5} = 108°$$

3 **Find the gradient**

At B,

$$\angle IBC = 135° - 108° = 27°$$

4 **Find the gradient**

Since both polygons share side AB,

$$BI = BC$$

5 **Find the gradient**

So triangle BIC is isosceles.

$$x = \frac{180° - 27°}{2} = 76.5°$$

✓ **FINAL ANSWER**

$76.5°$.

4MA1/2H JANUARY 2022

Question 11

4 marks | Angles in Polygons & Parallel Lines

11 The diagram shows a regular 10-sided polygon, *ABCDEFGHIJ*

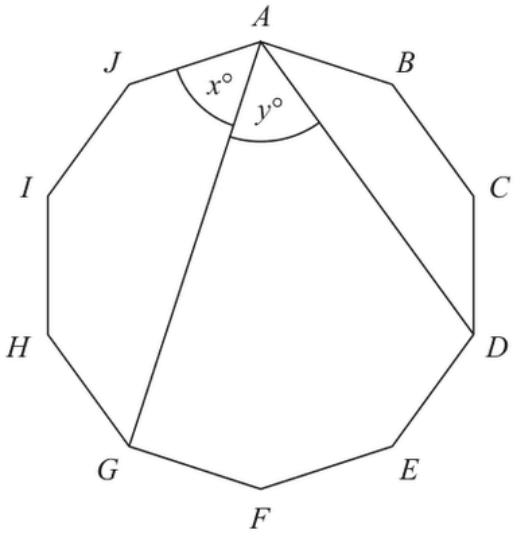


Diagram **NOT** accurately drawn

Show that $x = y$

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2022

1 **A regular 10 sided polygon**

A regular 10-sided polygon can be divided into 10 equal arcs around its circumcircle.

2 **The angle subtends the arc**

The angle x subtends the arc from J to G, which covers 3 equal sides of the decagon.

3 **The angle subtends the arc**

The angle y subtends the arc from G to D, which also covers 3 equal sides of the decagon.

4 **Equal arcs subtend equal angles**

Equal arcs subtend equal angles at the circumference, so

$$x = y$$

✓ **FINAL ANSWER**

$x = y$.

4MA1/1H JUNE 2022

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

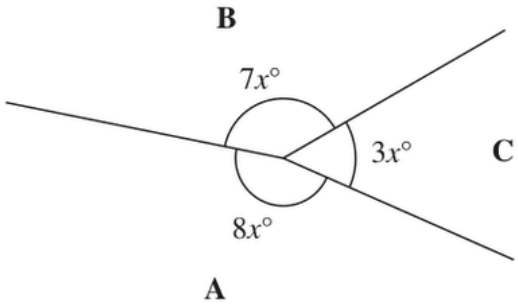


Diagram **NOT** accurately drawn

Polygon **B** has n sides.

Work out the value of n.

$n =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2022

1 **The angles around the point**

The angles around the point add to $360°$:

$$7x + 3x + 8x = 360$$

$$18x = 360$$

$$x = 20$$

2 **The interior angle of polygon**

The interior angle of polygon B is

$$7x = 7 \times 20 = 140°$$

3 **Find the gradient**

So the exterior angle is

$$180° - 140° = 40°$$

4 **Find the gradient**

For a regular polygon,

$$n = \frac{360}{40} = 9$$

✓ **FINAL ANSWER**

$n = 9$.

4MA1/1H JUNE 2022

Question 23

6 marks | Angles in Polygons & Parallel Lines

23 A polygon has n sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of n
Show clear algebraic working.

Question 23 continued

$n =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H June 2022

1 The interior angles form an

The interior angles form an arithmetic sequence:

$$177,\ 175,\ 173,\ 171,\ 169, \ldots$$

2 Calculate value

So

$$a = 177, \quad d = -2$$

3 Use trigonometry

For an n-sided polygon, the sum of the interior angles is

$$180(n-2)$$

4 **The sum of the arithmetic**

The sum of the arithmetic sequence is

$$\frac{n}{2}(2(177)+(n-1)(-2))$$

$$=\frac{n}{2}(354-2n+2)$$

$$=\frac{n}{2}(356-2n)$$

$$=n(178-n)$$

---

5 **Set this equal to the**

Set this equal to the polygon angle sum:

$$n(178-n)=180(n-2)$$

$$178n-n^2=180n-360$$

$$n^2+2n-360=0$$

---

6 **Factorise**

Factorise:

$$(n+20)(n-18)=0$$

---

7 **Solve inequality**

Since $n>5$,

$$n=18$$

✓ **FINAL ANSWER**

$n=18$

4MA1/2H JUNE 2023

Question 3

4 marks | Angles in Polygons & Parallel Lines

3 $ABCD$ is a trapezium.

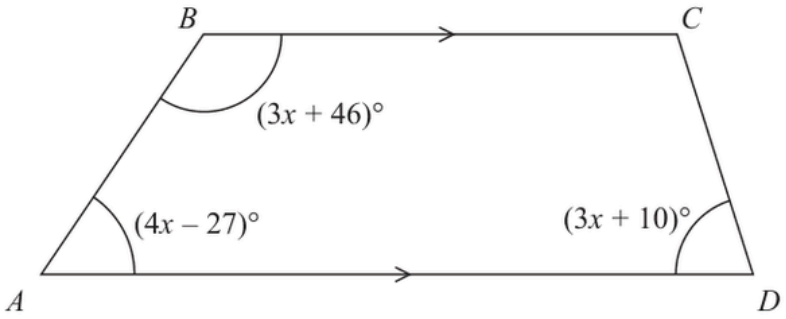


Diagram **NOT** accurately drawn

BC is parallel to AD

Find the size of the largest angle inside the trapezium.

.....°

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2023

1 **Find the gradient**

Since $BC \parallel AD$, the co-interior angles on side AB add to $180°$:

$$(4x - 27) + (3x + 46) = 180$$

$$7x + 19 = 180$$

$$7x = 161$$

$$x = 23$$

2 **Find the gradient**

Now find the angles:

$$\angle A = 4(23) - 27 = 65°$$

$$\angle B = 3(23) + 46 = 115°$$

$$\angle D = 3(23) + 10 = 79°$$

---

3 **Find the gradient**

Also, co-interior angles on side CD add to $180°$:

$$\angle C = 180° - 79° = 101°$$

---

4 **The largest angle is**

The largest angle is $115°$.

✓ **FINAL ANSWER**

$115°$.

4MA1/2H JUNE 2023

Question 11

3 marks | Angles in Polygons & Parallel Lines

11 Here is a 9-sided regular polygon $ABCDEFGHJ$, with centre O

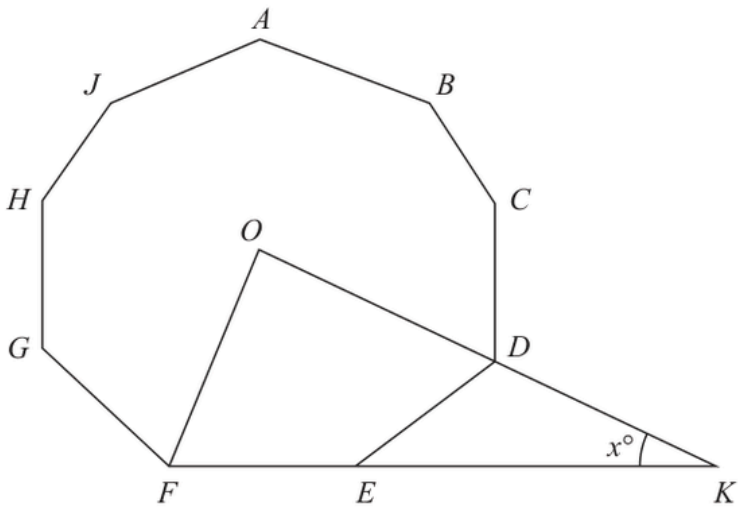


Diagram **NOT** accurately drawn

ODK and FEK are straight lines.

Work out the value of x

$x =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2023

1 **Find the gradient**

For a regular 9-sided polygon, the angle at the centre between adjacent vertices is

$$\frac{360}{9} = 40°$$

2 **Find the gradient**

The side FE is perpendicular to the radius drawn to the midpoint of side FE.

3 **From to this midpoint radius**

From OD to this midpoint radius is

$$40° + 20° = 60°$$

4 **Find the gradient**

So the angle between OD and the side FE is

$$90° - 60° = 30°$$

5 **Find the gradient**

Since ODK and FEK are straight lines, this is the angle x.

✓ **FINAL ANSWER**

$x = 30°$.

4MA1/1HR MAY 2023

Question 15

4 marks | Angles in Polygons & Parallel Lines

15

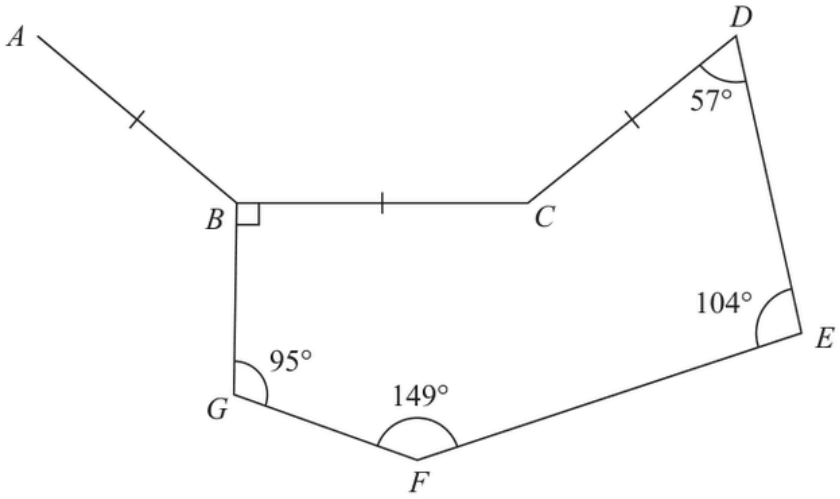


Diagram **NOT** accurately drawn

BCDEFG is a hexagon.
AB, *BC* and *CD* are three sides of a regular *n*-sided polygon.

Calculate the value of *n*
Show your working clearly.

$n =$ .....

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR May 2023

1 The sum of the interior

The sum of the interior angles of a hexagon is

$$(6-2)\times 180 = 720^\circ$$

2 Find the gradient

The reflex angle at C in the hexagon is

$$720^\circ - (90^\circ + 95^\circ + 149^\circ + 104^\circ + 57^\circ) = 225^\circ$$

3 Find the gradient

So the smaller angle BCD is

$$360^\circ - 225^\circ = 135^\circ$$

4 **This is the interior angle**

This is the interior angle of the regular n-sided polygon.

5 **Find the gradient**

So the exterior angle is

$$180^\circ - 135^\circ = 45^\circ$$

$$n = \frac{360}{45} = 8$$

✓ **FINAL ANSWER**

$n = 8$.

4MA1/1H MAY 2024

Question 5

4 marks | Angles in Polygons & Parallel Lines

5 In the diagram, $ABCDE$ is a regular pentagon.

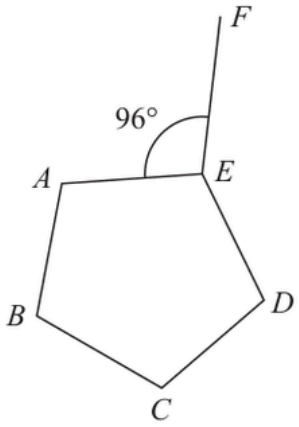


Diagram **NOT** accurately drawn

Angle $AEF = 96°$

Work out the size of the obtuse angle FED
Show your working clearly.

.....°

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2024

1 **The interior angle of a**

The interior angle of a regular pentagon is

$$\frac{(5-2)\times 180}{5} = 108°$$

2 **Find the gradient**

At E, angles around a point add to $360°$.

$$\angle FED = 360° - 96° - 108° = 156°$$

✓ **FINAL ANSWER**

$156°$.

4MA1/2H NOVEMBER 2024

Question 11

5 marks | Angles in Polygons & Parallel Lines

11

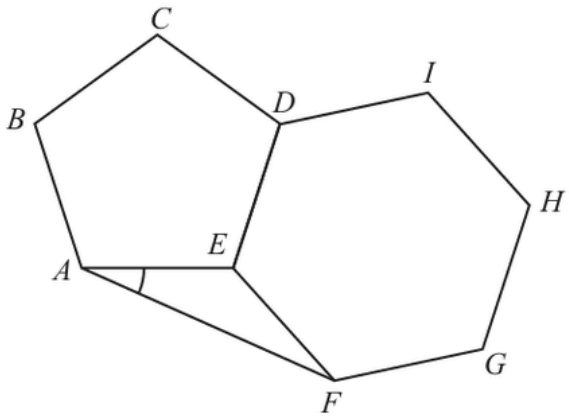


Diagram **NOT** accurately drawn

ABCDE is a regular pentagon.
DEFGHI is a regular hexagon.

AF is a straight line.

Work out the size of angle *EAF*

.....°

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2024

1 A regular pentagon has interior

A regular pentagon has interior angle

$$108°$$

2 A regular hexagon has interior

A regular hexagon has interior angle

$$120°$$

3 Use trigonometry

At E, the angle inside triangle AEF is

$$360° - 108° - 120° = 132°$$

4 **Use trigonometry**

Also $AE = ED$ and $ED = EF$, so

$$AE = EF$$

---

5 **Use trigonometry**

Triangle AEF is isosceles, so the base angles are equal:

$$\angle EAF = \frac{180^\circ - 132^\circ}{2} = 24^\circ$$

✓ **FINAL ANSWER**

24°.

4MA1/1H JUNE 2025

Question 3

4 marks | Angles in Polygons & Parallel Lines

3

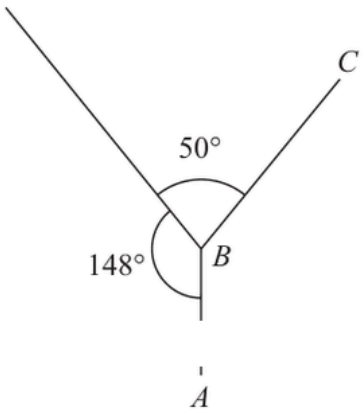


Diagram **NOT** accurately drawn

AB and BC are two sides of a regular polygon with n sides.

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2025

1 Find the gradient

Angles around point B add to $360°$.

2 The interior angle of the

The interior angle of the regular polygon is

$$360° - 140° - 50° = 170°$$

3 Find the gradient

So the exterior angle is

$$180° - 170° = 10°$$

4 Find the gradient

For a regular polygon,

$$n = \frac{360}{10} = 36$$

FINAL ANSWER

$n = 36$.

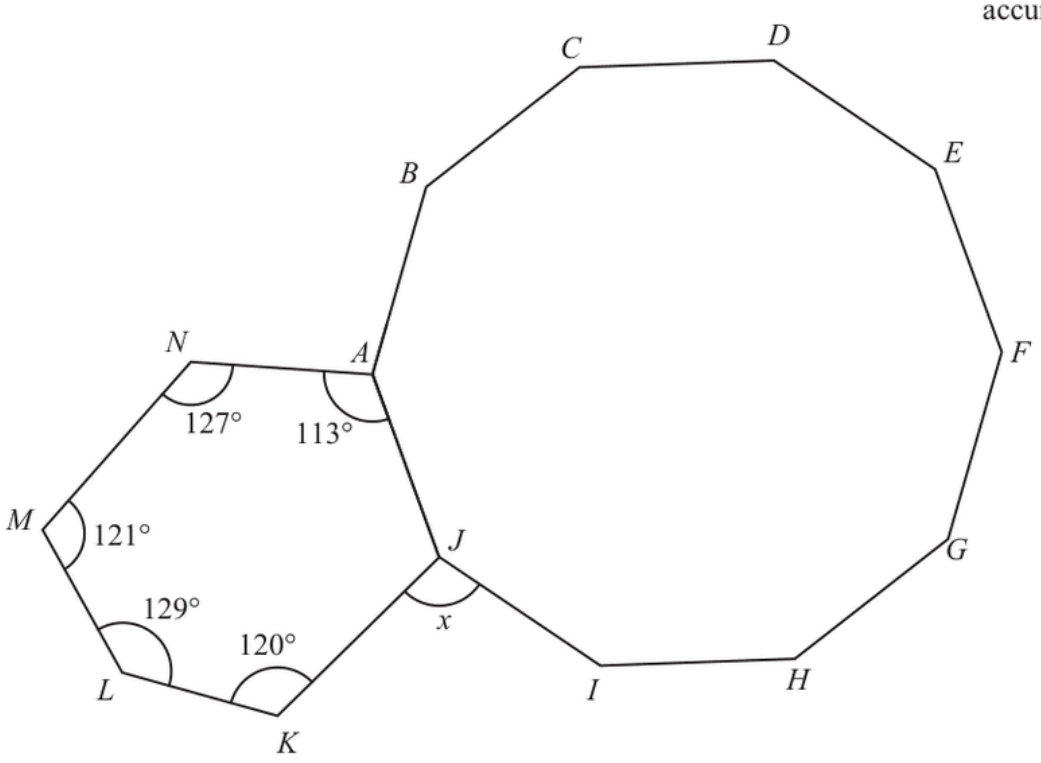
4WM2/01R MAY 2025

Question 10

5 marks | Angles in Polygons & Parallel Lines

10 The diagram shows a regular 10-sided polygon *ABCDEFGHIJ* and a hexagon *AJKLMN*

Diagram **NOT** accurately drawn



Work out the size of the angle marked x
Show your working clearly.

.....°

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM2/01R May 2025

1 **Required**

Find angle *x* in the figure combining a regular 10-gon (ABCDEFGHIJ) and hexagon AJKLMN.

2 **Interior angle of a regular**

Interior angle of a regular 10-sided polygon:

$$textInteriorangle = frac(10 - 2)times180^\circ 10 = frac1440^\circ 10 = 144^\circ$$

3 Use trigonometry

Find the interior angle of the hexagon at vertex *A*. At point *A*: decagon angle + 113° + hexagon angle at *A* = 360°:

∠*x* = 360° − 144° − 110° = 106°

✓ **FINAL ANSWER**

x = 106°

4WM2/01 NOVEMBER 2025

Question 7

4 marks | Angles in Polygons & Parallel Lines

7 *ABCDE* is a regular pentagon and *APQE* is a rectangle.

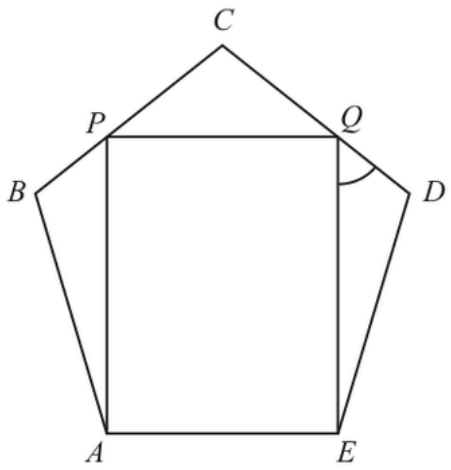


Diagram **NOT** accurately drawn

BPC and *DQC* are straight lines.

Work out the size of angle *EQD*
Show your working clearly.

.....°

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4WM2/01 November 2025

1 Use trigonometry

Missing open brace for superscript

So, ∠*AED* = 108[∘].

Step 2: Find angles within the corner triangle △*DEQ*. Since *APQE* is a rectangle, the angle ∠*AEQ* = 90[∘]. Therefore, the angle ∠*DE*Q* is:

∠*DEQ* = ∠*AED* − ∠*AEQ* = 108[∘] − 90[∘] = 18[∘]

Step 3: Analyze △*DEQ*. The problem states *DQC* is a straight line, meaning *Q* lies on the side *DC*. Therefore, △*DEQ* is formed by *D*, *E*, *Q*. In △*DEQ*, the angle at vertex *D* is the pentagon's interior angle ∠*CD*E* = 108[∘].

Using angles in a triangle summing to 180[∘]:

∠*EQD* = 180[∘] − (∠*DEQ*+∠*EDQ*) = 180[∘] − (18[∘]+108[∘]) = 180[∘] − 126[∘] = 54[∘]

FINAL ANSWER

54[∘]

4MA1/1H NOVEMBER 2025

Question 12

4 marks | Angles in Polygons & Parallel Lines

12

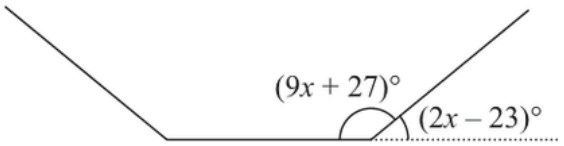


Diagram **NOT** accurately drawn

The diagram shows part of a regular n-sided polygon with

an interior angle of $(9x + 27)°$
an exterior angle of $(2x – 23)°$

Work out the value of n

$n =$ .....

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H November 2025

1 The interior angle and exterior

The interior angle and exterior angle of a regular polygon add to $180°$:

$$(9x + 27) + (2x - 23) = 180$$

$$11x + 4 = 180$$

$$x = 16$$

2 The exterior angle is

The exterior angle is

$$2x - 23 = 2(16) - 23 = 9°$$

3 Find the gradient

For a regular polygon,

$$n = \frac{360}{9} = 40$$

FINAL ANSWER

$n = 40$.

4MA1/2H JUNE 2026

Question 6

3 marks | Angles in Polygons & Parallel Lines

6 $ABCDE$ is a pentagon.

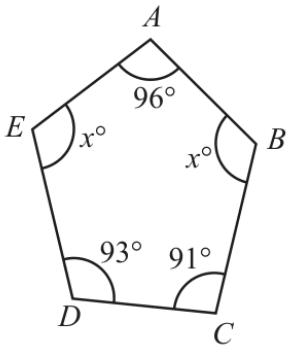


Diagram **NOT** accurately drawn

angle $EAB = 96°$ angle $BCD = 91°$ angle $CDE = 93°$ angle DEA = angle $ABC = x°$

Work out the value of x

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2026

1 **Solution | Step 1**

The sum of the interior angles of a pentagon is $(5-2)180° = 540°$.

Use the interior-angle sum formula.

2 **Solution | Step 2**

$$96 + 91 + 93 + x + x = 540 \quad \Rightarrow \quad 2x = 260 \quad \Rightarrow \quad x = 130.$$

Subtract the three known angles and divide the remainder equally.

✓ **FINAL ANSWER**

$x = 130$

4MA1/1HR JUNE 2026

Question 15

3 marks | Angles in Polygons & Parallel Lines

15

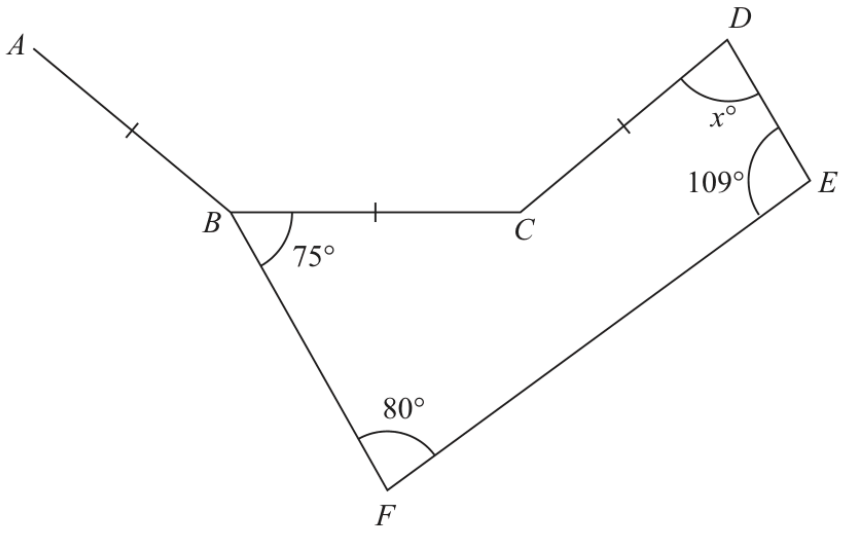


Diagram **NOT** accurately drawn

BCDEF is a pentagon.
AB, *BC* and *CD* are three sides of a regular 10-sided polygon.

Find the value of x

$x =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR June 2026

1 Solution | Step 1

$$\text{interior angle of a regular decagon} = \frac{(10-2)180^\circ}{10} = 144^\circ.$$

BC and CD are consecutive sides of the regular decagon.

2 Solution | Step 2

$$\text{reflex interior angle at C} = 360^\circ - 144^\circ = 216^\circ.$$

The pentagon BCDEF is concave at C, and its interior lies on the other side from the decagon interior angle.

3 Solution | Step 3

$$75^\circ + 216^\circ + x + 109^\circ + 80^\circ = (5-2)180^\circ = 540^\circ,$$

The interior-angle sum formula also applies to a simple concave pentagon.

4 Solution | Step 4

$$x = 540^\circ - 480^\circ = 60^\circ.$$

Subtract the four known interior angles.

FINAL ANSWER

$x = 60^\circ$

4MA1/2HR JUNE 2026

Question 24

4 marks | Angles in Polygons & Parallel Lines

24

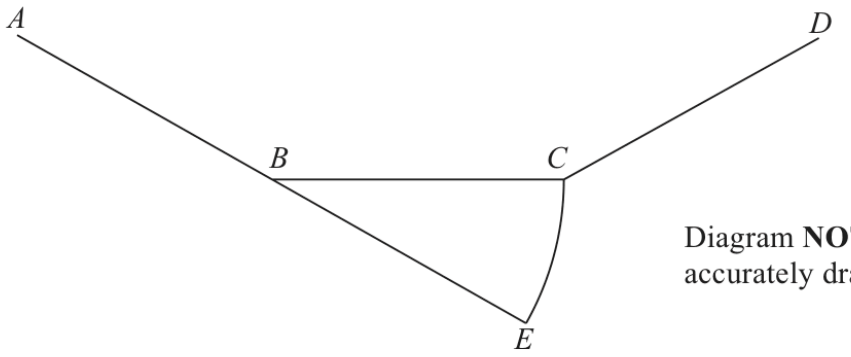


Diagram **NOT** accurately drawn

AB, BC and CD are three sides of a regular polygon.
ABE is a straight line.

BCE is a sector of a circle with centre B and radius 4.5 cm

The length of arc CE is $\frac{3}{10}\pi$ cm

Work out the sum of the interior angles of the regular polygon.

.....°

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2026

1 **Solution | Step 1**

Let $\theta = \angle CBE$. Then

$$\frac{\theta}{360}\,2\pi(4.5) = \frac{3}{10}\pi \quad \Rightarrow \quad \theta = 12^\circ.$$

The arc is theta/360 of a full circumference.

2 **Solution | Step 2**

$\angle ABC = 180^\circ - 12^\circ = 168^\circ$, so the regular polygon has exterior angle 12°.
A, B and E are collinear, so the sector angle is supplementary to the interior angle.

3 **Solution | Step 3**

$$n = \frac{360}{12} = 30, \qquad \text{sum} = (30-2)180^\circ = 5040^\circ.$$

Exterior angles sum to 360 degrees, and an n-gon has interior-angle sum (n-2) times 180 degrees.

✓ **FINAL ANSWER**

5040°.

TOPIC

Constructions & Loci

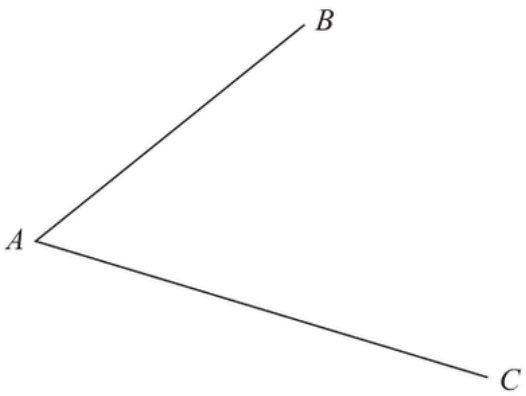
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 5

2 marks | Constructions & Loci

5 Use ruler and compasses to construct the bisector of angle BAC.
You must show all your construction lines.



(Total for Question 5 is 2 marks)

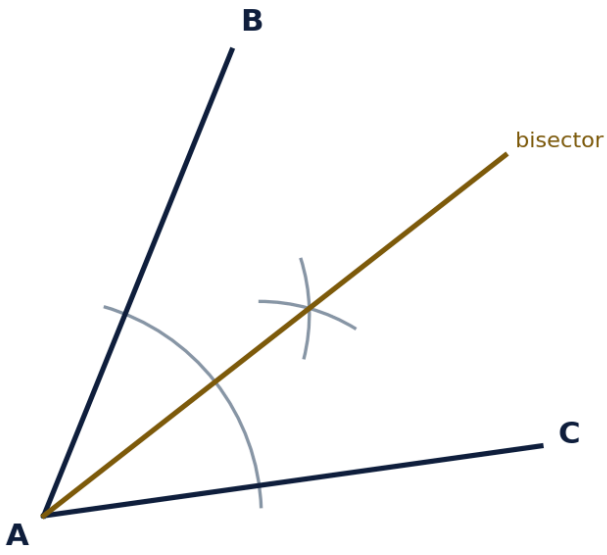
WORKED SOLUTION

Question 5 · 4MA1/2H January 2020

1 **Construction answer**

To construct the bisector of angle BAC:

1. Put the compass point at A and draw an arc that cuts AB and AC.
2. From the two points where the arc cuts the arms, draw equal arcs inside the angle.
3. Draw a straight line from A through the intersection of those two arcs.



FINAL ANSWER

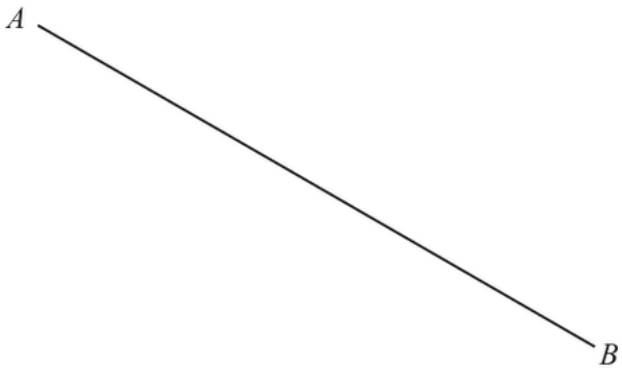
The ray from A through the arc intersection is the angle bisector of BAC.

4MA1/2HR JANUARY 2020

Question 6

2 marks | Constructions & Loci

6 Use ruler and compasses only to construct the perpendicular bisector of the line AB.
You must show all your construction lines.



(Total for Question 6 is 2 marks)

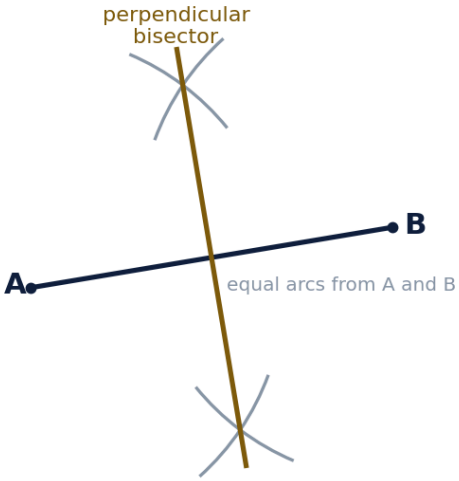
WORKED SOLUTION

Question 6 · 4MA1/2HR January 2020

1 Construction answer

To construct the perpendicular bisector of AB:

1. Set the compass radius to more than half of AB.
2. With centre A, draw arcs above and below the line.
3. With the same radius and centre B, draw arcs crossing the first pair.
4. Draw the straight line through the two arc intersections.



FINAL ANSWER

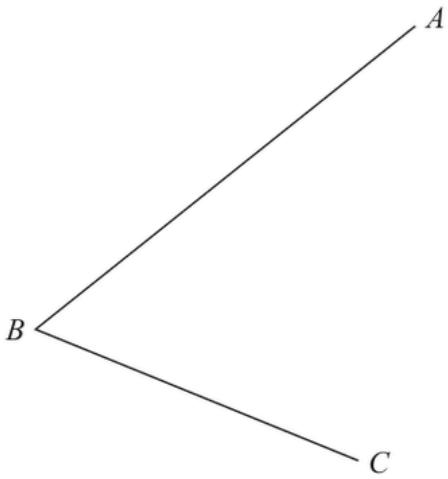
This line is the perpendicular bisector of AB.

4MA1/1HR MAY/NOVEMBER 2020

Question 2

2 marks | Constructions & Loci

2 Using ruler and compasses only, construct the bisector of angle ABC.
You must show all your construction lines.

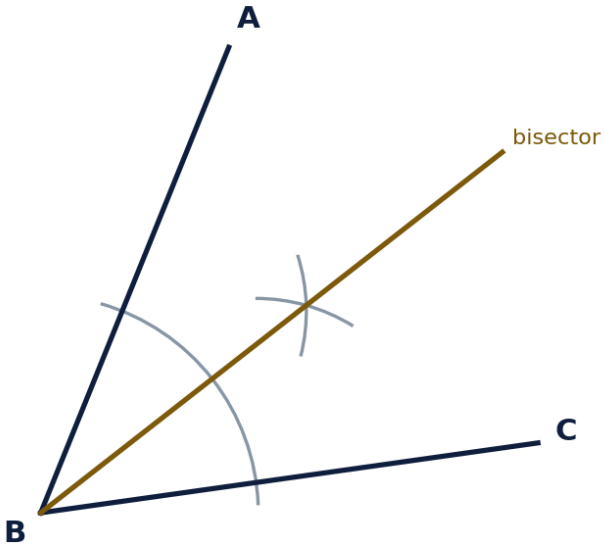


(Total for Question 2 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Put the compass point at B and draw an arc cutting BA and BC.
2. From those two cut points, draw equal arcs inside the angle.
3. Draw a straight line from B through the point where the two arcs cross.



FINAL ANSWER

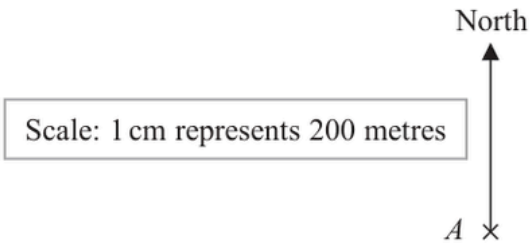
This line bisects angle ABC.

4MA1/2H JANUARY 2022

Question 5

4 marks | Constructions & Loci

5 The scale diagram shows the position on a map of a house, A



House C is on a bearing of 110° from A
The distance from A to C is 700 m

(a) Mark the position of C on the diagram with a cross (×)
Label your cross C

(3)

(b) Write the scale of the map in the form $1 : n$

1 : .....

(1)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H January 2022

1 The distance from to is

The distance from A to C is $700\ \text{m}$.

2 The scale is

The scale is

$$1\ \text{cm represents } 200\ \text{m}$$

3 Use bearings

So the map distance is

$$700 \div 200 = 3.5\ \text{cm}$$

4 **Use bearings**

Part (a): from A, draw a bearing of $110°$ and mark C at $3.5\ \text{cm}$ from A.

5 **Use bearings**

For part (b),

$$200\ \text{m} = 20000\ \text{cm}$$

6 **Use bearings**

So the scale is

$$1 : 20000$$

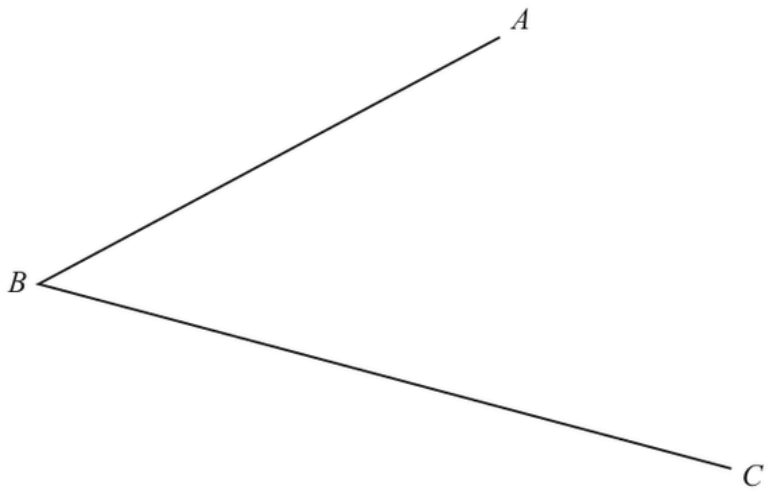
✓ **FINAL ANSWER**

Draw C 3.5 cm from A on a bearing of $110°$, and the scale is $1 : 20000$.

Question 2

2 marks | Constructions & Loci

2 Use ruler and compasses only to construct the bisector of angle ABC
You must show all your construction lines.

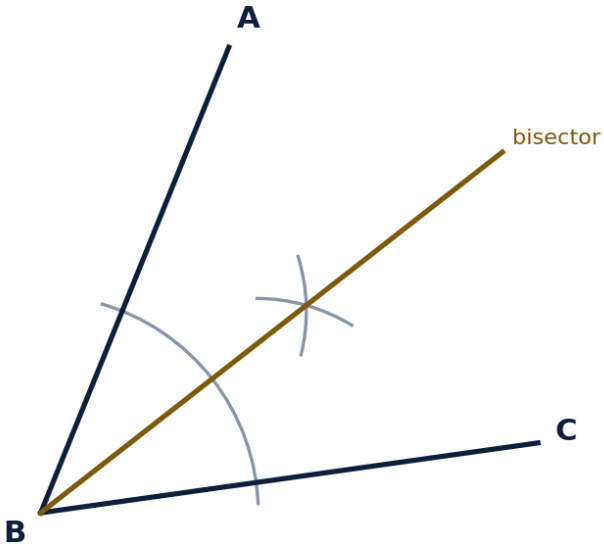


(Total for Question 2 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Draw an arc centred at B cutting both arms of the angle.
2. From the two cut points, draw equal arcs inside the angle.
3. Draw the line from B through the intersection of these arcs.



FINAL ANSWER

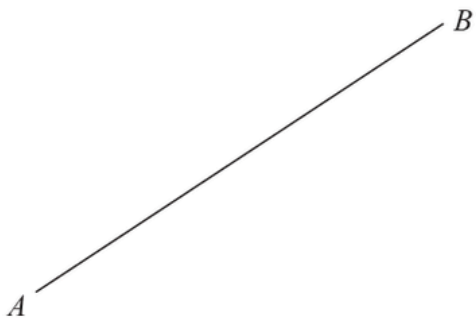
The constructed ray is the angle bisector of ABC.

4MA1/2H JUNE 2025

Question 4

2 marks | Constructions & Loci

4 Using ruler and compasses only, construct the perpendicular bisector of the line AB
Show all your construction lines.



(Total for Question 4 is 2 marks)

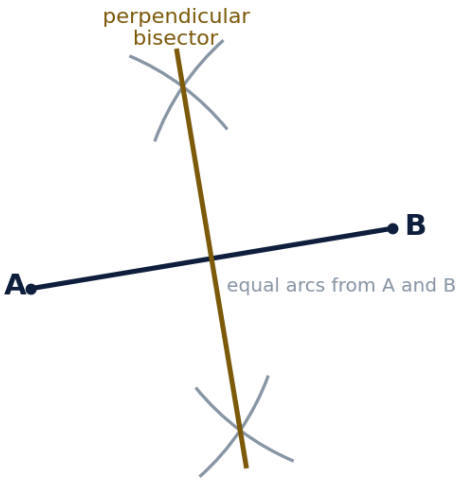
WORKED SOLUTION

Question 4 · 4MA1/2H June 2025

1 **Construction answer**

To construct the perpendicular bisector of AB:

1. Open the compass to more than half the length of AB.
2. Draw equal arcs from A above and below AB.
3. Without changing the compass width, draw equal arcs from B.
4. Join the two arc intersections with a ruler.



FINAL ANSWER

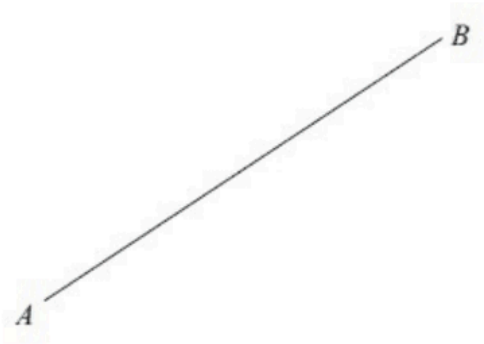
The joined line is the perpendicular bisector of AB.

4WM2/01 MAY 2025

Question 4

2 marks | Constructions & Loci

4 Using ruler and compasses only, construct the perpendicular bisector of the line AB
Show all your construction lines.



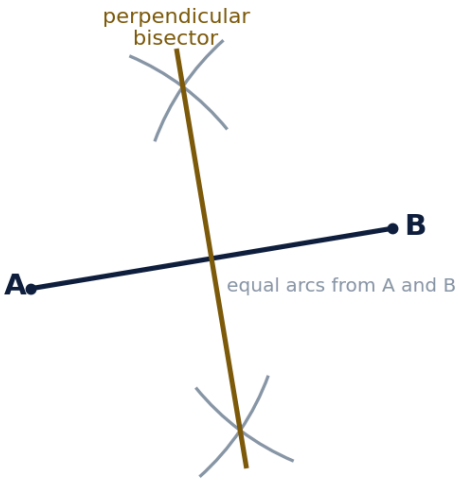
(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4WM2/01 May 2025

1 **Use a compass radius greater**

Use a compass radius greater than half of AB. Draw matching arcs from A above and below the line, then draw matching arcs from B with the same radius. Join the two arc-intersection points.



✓ **FINAL ANSWER**

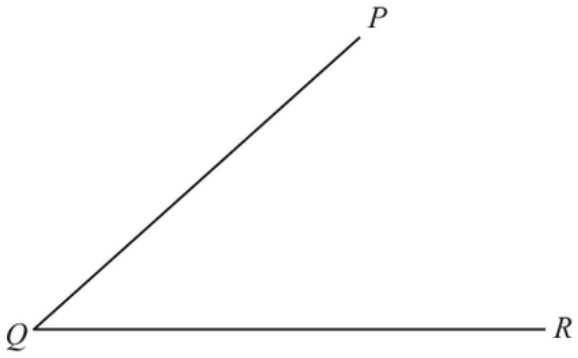
the joined line is the perpendicular bisector of AB.

4WM2/01 NOVEMBER 2025

Question 1

2 marks | Constructions & Loci

1 Using ruler and compasses only, construct the bisector of angle PQR
You must show all your construction lines.



(Total for Question 1 is 2 marks)

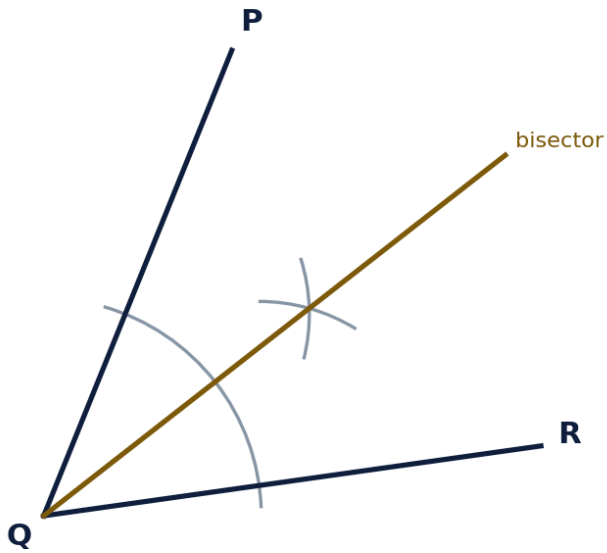
1 Construct angle bisector

Step 1: Place the point of the compass at the vertex *Q*. Draw an arc that intersects both arms *Q**P*** **and *Q**R*. Let's call these intersection points *A* and *B*.

Step 2: Without changing the compass width, place the compass point on *A* and draw an arc inside the angle.

Step 3: Place the compass point on *B* and draw another arc to intersect the previous arc. Let this intersection be point *C*.

Step 4: Draw a straight line from *Q* through *C*. This is the angle bisector.



FINAL ANSWER

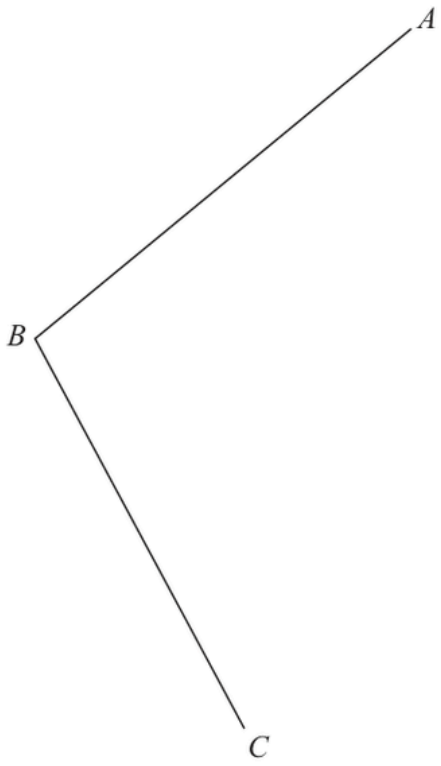
This is the angle bisector.

4MA1/1H NOVEMBER 2025

Question 3

2 marks | Constructions & Loci

3 Using ruler and compasses only, construct the bisector of angle ABC
Show all your construction lines.

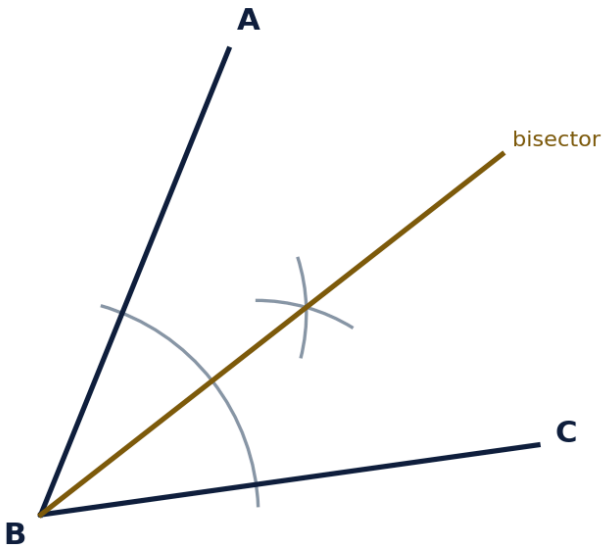


(Total for Question 3 is 2 marks)

1 Construction answer

To construct the bisector of angle ABC:

1. Use centre B to draw an arc crossing BA and BC.
2. Use the same compass width from the two crossing points to make two arcs inside the angle.
3. Join B to the point where these two arcs meet.



FINAL ANSWER

That line is the required angle bisector.

TOPIC

Volume & Surface Area

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

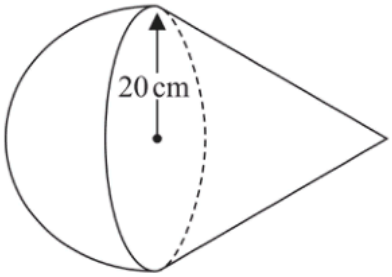


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 Specimen

1 **Calculate surface area**

The curved surface area of a cone is

$$\pi rl$$

2 **Calculate area**

Here $r = 20$ and $\pi rl = 580\pi$, so

$$20l = 580$$

$$l = 29$$

3 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 = 29^2 - 20^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

4 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (20)^2 (21) = 2800\pi$$

---

5 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi (20)^3 = \frac{16000}{3}\pi$$

---

6 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

7 **Calculate area**

Since the volume is $k\pi$,

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$k = \frac{24400}{3}$.

4MA1/2HR JANUARY 2020

Question 13

3 marks | Volume & Surface Area

13

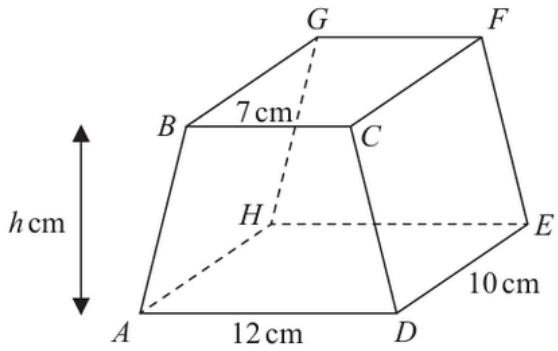


Diagram **NOT** accurately drawn

The diagram shows a prism $ABCDEFGH$ in which $ABCD$ is a trapezium with BC parallel to AD and $CDEF$ is a rectangle.

$BC = 7\,\text{cm}$ $AD = 12\,\text{cm}$ $DE = 10\,\text{cm}$

The height of trapezium $ABCD$ is $h\,\text{cm}$
The volume of the prism is $608\,\text{cm}^3$

Work out the value of h.

$h =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2020

1 Area of the trapezium cross

Area of the trapezium cross-section:

$$\frac{1}{2}(7+12)h = 9.5h$$

2 Volume of the prism

Volume of the prism:

$$9.5h \times 10 = 608$$

$$95h = 608$$

$$h = 6.4$$

FINAL ANSWER

$h = 6.4\ \text{cm}$.

Question 26

6 marks | Volume & Surface Area

26 Here is a sector, AOB, of a circle with centre O and angle $AOB = x°$

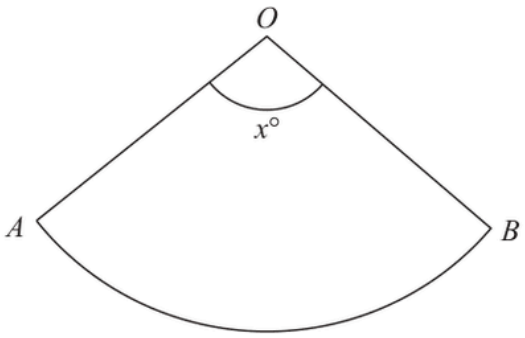


Diagram **NOT** accurately drawn

The sector can form the curved surface of a cone by joining OA to OB.

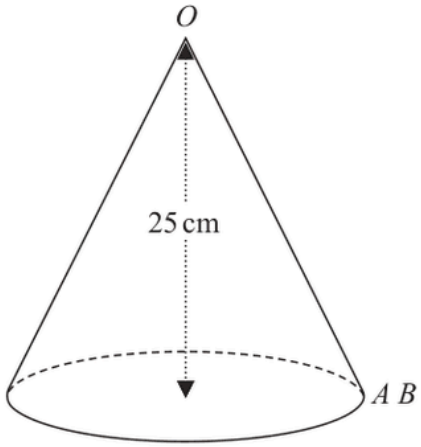


Diagram **NOT** accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 cm^3

Work out the value of x.
Give your answer correct to the nearest whole number.

$x =$ .....

(Total for Question 26 is 6 marks)

1 **Let the radius of the**

Let the radius of the base of the cone be r, and let the slant height be l.

2 **The volume of the cone**

The volume of the cone is

$$\frac{1}{3}\pi r^2 h = 1600$$

3 **Calculate area**

The height is 25 cm, so

$$\frac{1}{3}\pi r^2 (25) = 1600$$

$$r^2 = \frac{4800}{25\pi}$$

$$r = 7.8176\ldots$$

4 **The slant height is found**

The slant height is found using Pythagoras:

$$l^2 = r^2 + 25^2$$

$$l = 26.1938\ldots$$

5 **Calculate area**

When the sector is joined to form the cone, the arc length of the sector becomes the circumference of the cone base.

$$\frac{x}{360} \times 2\pi l = 2\pi r$$

6 **Cancel**

Cancel 2π:

$$\frac{x}{360}l = r$$

$$x = \frac{360r}{l}$$

$$x = \frac{360(7.8176\ldots)}{26.1938\ldots} = 107.4433\ldots$$

✓ **FINAL ANSWER**

$\boxed{107°}$

4MA1/1H JANUARY 2021

Question 3

4 marks | Volume & Surface Area

3 The diagram shows a container for water in the shape of a prism.

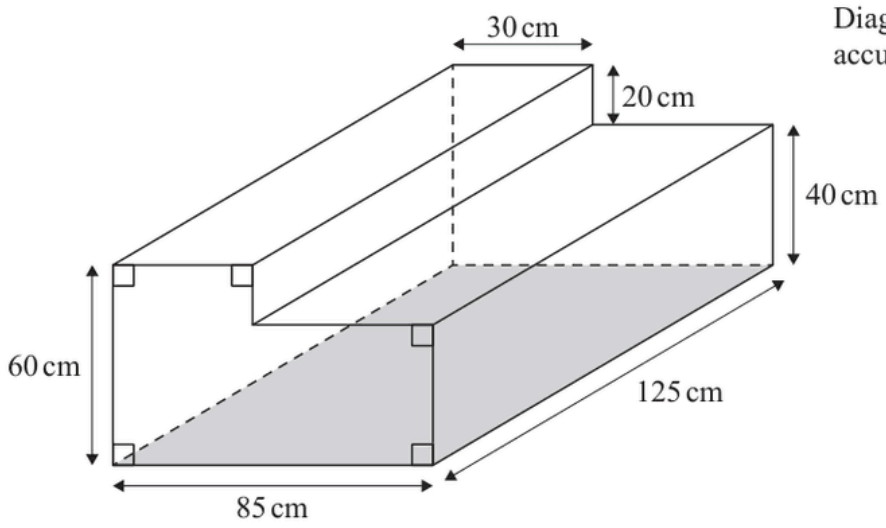


Diagram **NOT** accurately drawn

The rectangular base of the prism, shown shaded in the diagram, is horizontal.
The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 10 30 and at 12 00 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2021

1 The cross section is an

The cross-section is an L-shape.

2 Area of the lower rectangle

Area of the lower rectangle:

$$85 \times 40 = 3400$$

3 **Area of the upper rectangle**

Area of the upper rectangle:

$$30 \times 20 = 600$$

4 **Calculate area**

So the cross-sectional area is

$$3400 + 600 = 4000 \text{ cm}^2$$

5 **The full volume is**

The full volume is

$$4000 \times 125 = 500000 \text{ cm}^3$$

6 **Calculate area**

In the first 1.5 hours, the water level drops by 20 cm. This is the top rectangular part:

$$30 \times 20 \times 125 = 75000 \text{ cm}^3$$

7 **Calculate area**

So the pumping rate is

$$\frac{75000}{1.5} = 50000 \text{ cm}^3/\text{hour}$$

8 **Time to empty the whole**

Time to empty the whole container:

$$\frac{500000}{50000} = 10 \text{ hours}$$

9 **Calculate area**

Starting at $10{:}30$, ten hours later is

$$20{:}30$$

✓ **FINAL ANSWER**

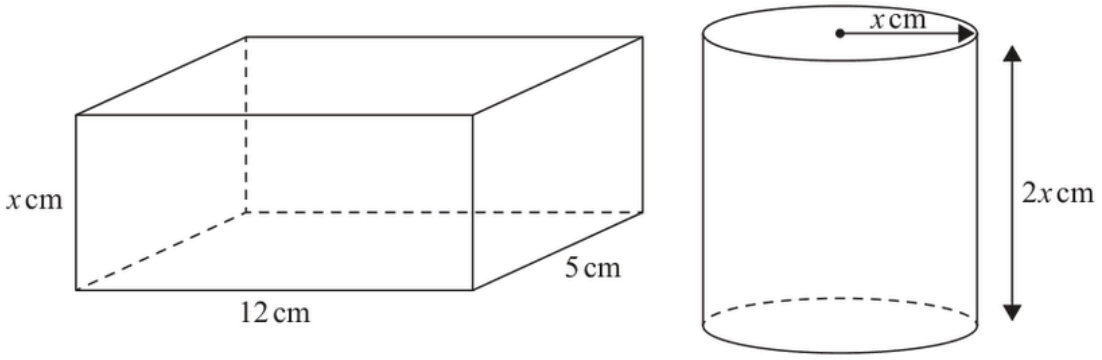
$20{:}30$

Question 5

4 marks | Volume & Surface Area

5 The diagram shows a cuboid and a cylinder.

Diagram **NOT** accurately drawn



The dimensions of the cuboid are x cm by 12 cm by 5 cm.
The volume of the cuboid is 270 cm^3

The radius of the cylinder is x cm.
The height of the cylinder is $2x$ cm.

(a) Work out the volume of the cylinder.
Give your answer correct to the nearest whole number.

..... cm^3

(3)

(b) Change 1 m^3 to cm^3

..... cm^3

(1)

(Total for Question 5 is 4 marks)

1 **Calculate area**

For the cuboid,

$$12 \times 5 \times x = 270$$

$$60x = 270$$

$$x = 4.5$$

2 **Calculate area**

The cylinder has radius $4.5\ \text{cm}$ and height $9\ \text{cm}$.

$$V = \pi r^2 h = \pi(4.5)^2(9) = 572.555\ldots$$

3 **Calculate area**

Also,

$$1\ \text{m}^3 = 100^3 = 1000000\ \text{cm}^3$$

FINAL ANSWER

$573\ \text{cm}^3$, and $1\ \text{m}^3 = 1000000\ \text{cm}^3$.

Question 17

7 marks | Volume & Surface Area

17 The diagram shows a solid prism *ABCDEFGH*.

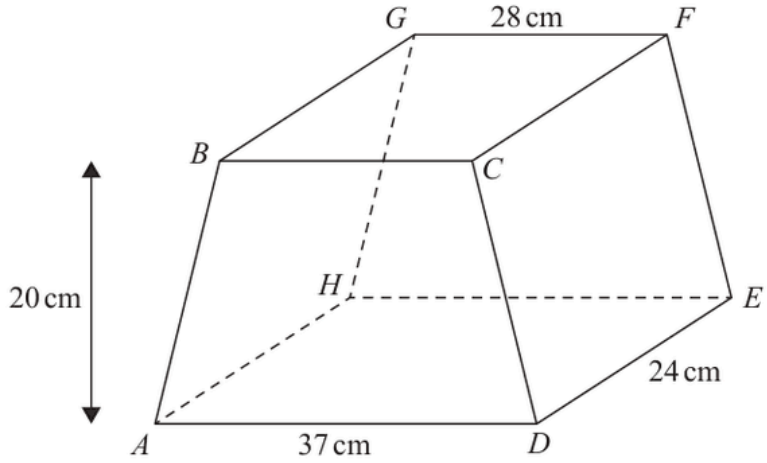


Diagram **NOT** accurately drawn

The trapezium *ABCD*, in which *AD* is parallel to *BC*, is a cross section of the prism.
The base *ADEH* of the prism is a horizontal plane.
ADEH and *BCFG* are rectangles.
The midpoint of *BC* is vertically above the midpoint of *AD* so that $BA = CD$.

$AD = 37$ cm $GF = 28$ cm $DE = 24$ cm

The perpendicular distance between edges *AD* and *BC* is 20 cm.

(a) Work out the total surface area of the prism.

..... cm²

(4)

(b) Calculate the size of the angle between *AF* and the plane *ADEH*.
Give your answer correct to one decimal place.

.....°

(3)

(Total for Question 17 is 7 marks)

1 **The cross section is a**

The cross-section is a trapezium with parallel sides 37 cm and 28 cm, and perpendicular height 20 cm.

$$\text{Area of cross-section} = \frac{1}{2}(37+28)(20) = 650$$

2 **Calculate area**

Since the trapezium is isosceles, the horizontal difference on each side is

$$\frac{37-28}{2} = 4.5$$

3 **Calculate area**

So

$$AB = CD = \sqrt{20^2 + 4.5^2} = 20.5$$

4 **The perimeter of the cross**

The perimeter of the cross-section is

$$37 + 28 + 20.5 + 20.5 = 106$$

5 **The length of the prism**

The length of the prism is 24 cm, so the total surface area is

$$2(650) + 106(24) = 3844$$

6 **Use trigonometry**

For the angle between AF and the horizontal plane, the horizontal projection has length

$$\sqrt{24^2 + 4.5^2}$$

7 **Use trigonometry**

The vertical height is 20, so

$$\tan\theta = \frac{20}{\sqrt{24^2 + 4.5^2}}$$

$$\theta = 39.3°$$

FINAL ANSWER

$3844\ \text{cm}^2$, and $39.3°$.

4MA1/2H MAY 2021

Question 10

5 marks | Volume & Surface Area

10 Here is a triangular prism.

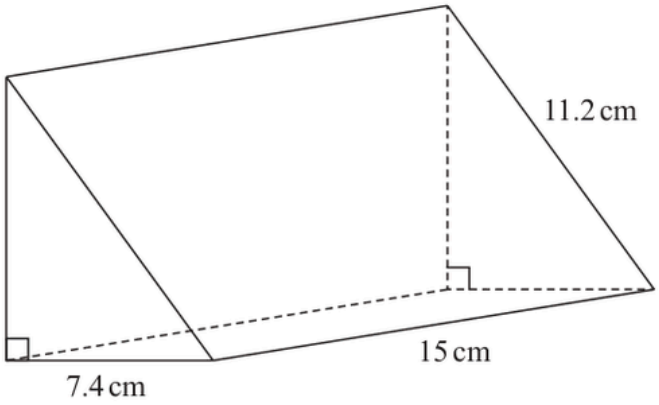


Diagram **NOT** accurately drawn

Work out the volume of the prism.
Give your answer correct to 3 significant figures.

..... cm³

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H May 2021

1 Find the height of the

Find the height of the triangular cross-section:

$$h^2 = 11.2^2 - 7.4^2$$

$$h = 8.407\ldots$$

2 Area of the triangular cross

Area of the triangular cross-section:

$$\frac{1}{2}(7.4)(8.407\ldots) = 31.106\ldots$$

3 Volume of the prism

Volume of the prism:

$$31.106\ldots \times 15 = 466.596\ldots$$

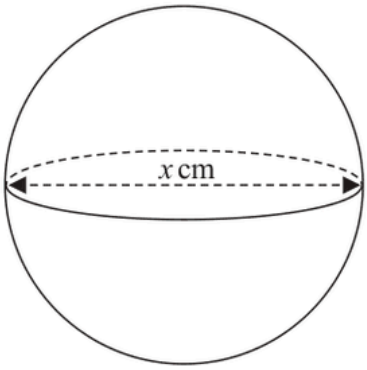
FINAL ANSWER

$467\ \text{cm}^3$.

Question 22

6 marks | Volume & Surface Area

22 The diagram shows a sphere of diameter x cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.



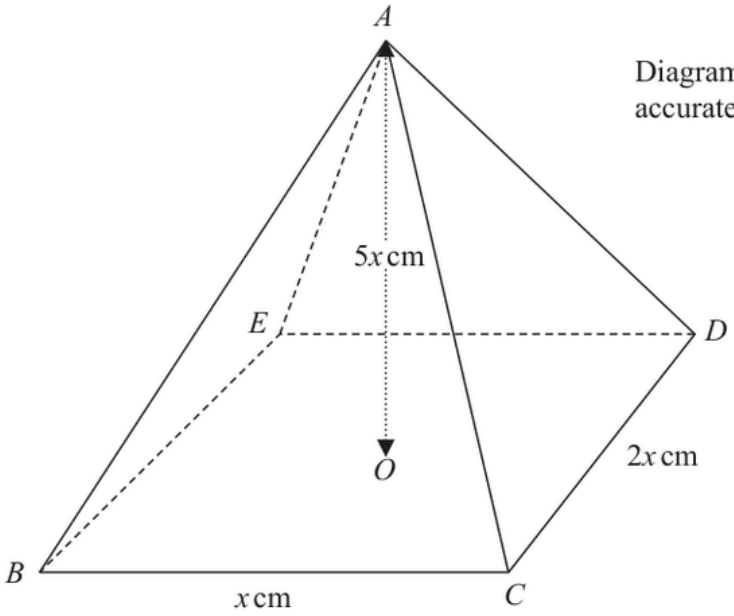


Diagram **NOT** accurately drawn

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is 288π cm^3

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

1 **Use trigonometry**

The sphere has diameter x, so its radius is

$$\frac{x}{2}$$

2 **The volume of the sphere**

The volume of the sphere is $288\pi\ \text{cm}^3$:

$$\frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 288\pi$$

$$\frac{\pi x^3}{6} = 288\pi$$

$$x^3 = 1728$$

$$x = 12$$

3 **The rectangular base of the**

The rectangular base of the pyramid has dimensions x and $2x$, so its area is

$$x(2x) = 2x^2$$

4 **Use trigonometry**

For the two faces with base x, the slant height is

$$\sqrt{(5x)^2 + x^2} = x\sqrt{26}$$

5 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}x \cdot x\sqrt{26}\right) = x^2\sqrt{26}$$

6 **Use trigonometry**

For the two faces with base $2x$, the slant height is

$$\sqrt{(5x)^2 + \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{101}}{2}$$

7 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}(2x)\cdot\frac{x\sqrt{101}}{2}\right)=x^2\sqrt{101}$$

8 **Total surface area**

Total surface area:

$$2x^2+x^2\sqrt{26}+x^2\sqrt{101}$$

9 **Substitute**

Substitute $x=12$:

$$144(2+\sqrt{26}+\sqrt{101})=2469.440\ldots$$

10 **Correct to the nearest**

Correct to the nearest cm^2:

$$2469$$

✓ **FINAL ANSWER**

$$\boxed{2469\ \mathrm{cm}^2}$$

4MA1/1HR JANUARY 2022

Question 10

6 marks | Volume & Surface Area

10 Jonty has a storage container in the shape of a cuboid, as shown in the diagram.

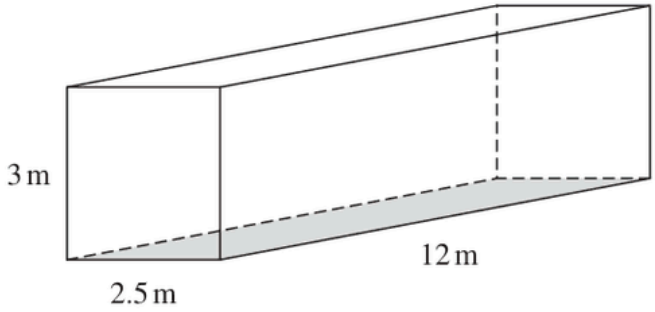


Diagram **NOT** accurately drawn

Jonty is going to paint the outside of his storage container, apart from the base which is shown shaded in the diagram.
He needs enough paint to cover the four sides and the top.

Each tin of paint covers an area of $15\,m^2$

The cost of each tin of paint recently increased by 10%
After the increase, the cost of each tin of paint is £26.95

Jonty says

"**Before** the increase, I could have bought enough paint for less than £200"

Show that Jonty is correct.
Show your working clearly.

(Total for Question 10 is 6 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2022

1 **Jonty paints the four sides**

Jonty paints the four sides and the top, but not the base.

2 **Top**

Top:

$$12 \times 2.5 = 30$$

3 **Two long sides**

Two long sides:

$$2(12 \times 3) = 72$$

4 **Two short sides**

Two short sides:

$$2(2.5 \times 3) = 15$$

5 **Total area**

Total area:

$$30 + 72 + 15 = 117\ \text{m}^2$$

6 **Calculate area**

Each tin covers $15\ \text{m}^2$, so the number of tins needed is

$$\frac{117}{15} = 7.8$$

7 **He must buy tins**

He must buy 8 tins.

8 **Calculate area**

After the 10% increase, each tin costs $\pounds 26.95$. Before the increase:

$$\frac{26.95}{1.10} = 24.50$$

9 **Calculate area**

Before the increase, 8 tins would cost

$$8 \times 24.50 = 196$$

10 **Calculate area**

Since $\pounds 196 < \pounds 200$, Jonty is correct.

✓ **FINAL ANSWER**

Jonty is correct; before the increase the paint would have cost $\pounds 196$.

4MA1/1H JUNE 2022

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

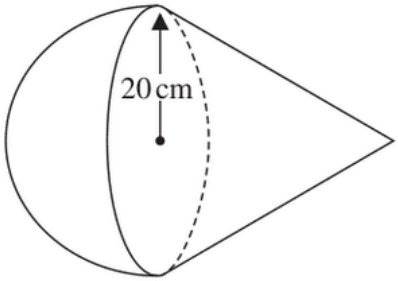


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi \text{ cm}^2$

The volume of the solid is $k\pi \text{ cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H June 2022

1 The cone has radius cm

The cone has radius 20 cm.

2 Calculate surface area

The curved surface area of the cone is

$$\pi rl$$

3 Calculate value

So

$$\pi(20)l = 580\pi$$

$$l = 29$$

4 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 + 20^2 = 29^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

5 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(20)^2(21) = 2800\pi$$

---

6 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(20)^3 = \frac{16000}{3}\pi$$

---

7 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

8 **Calculate volume**

The volume is $k\pi$, so

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{24400}{3}}$$

4MA1/2HR JUNE 2023

Question 9

6 marks | Volume & Surface Area

9 The diagram shows a rectangular sheet of metal $ABCD$

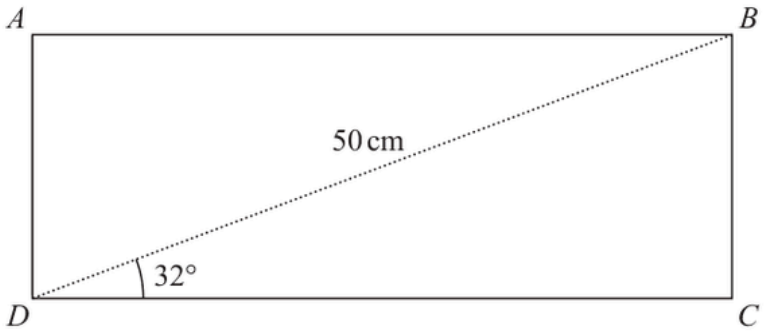


Diagram **NOT** accurately drawn

$BD = 50\,\text{cm}$ and angle $BDC = 32°$

Nasser joins side AD to side BC to form a cylinder.

BC is the height of the cylinder.
DC is the circumference of the cross section of the cylinder.

Work out the volume, in cm^3, of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 9 is 6 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2023

1 **Use trigonometry**

In triangle DBC,

$$DC = 50\cos 32° = 42.402\ldots, \qquad BC = 50\sin 32° = 26.496\ldots$$

2 **The circumference of the cylinder**

The circumference of the cylinder is DC, so

$$2\pi r = DC$$

$$r = \frac{42.402\ldots}{2\pi}$$

---

3 **Volume**

Volume:

$$V = \pi r^2 h = \pi\left(\frac{42.402\ldots}{2\pi}\right)^2 (26.496\ldots) = 3790.974\ldots$$

✓ **FINAL ANSWER**

$3.79 \times 10^3\ \text{cm}^3$ to 3 significant figures.

4MA1/2HR JUNE 2023

Question 24

5 marks | Volume & Surface Area

24 The surface area of sphere **A** is nine times the surface area of sphere **B**
The difference between the volume of sphere **A** and the volume of sphere **B** is 117π cm^3

Find the radius of the smaller sphere.
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2023

1 Let the radius of the

Let the radius of the smaller sphere be r.

2 Let the radius of the

Let the radius of the larger sphere be R.

3 Surface area of a sphere

Surface area of a sphere is

$$4\pi r^2$$

4 The surface area of sphere

The surface area of sphere A is nine times the surface area of sphere B, so

$$4\pi R^2 = 9(4\pi r^2)$$

$$R^2 = 9r^2$$

$$R = 3r$$

5 **The difference between the volumes**

The difference between the volumes is 117π:

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

$$\frac{4}{3}\pi(27r^3 - r^3) = 117\pi$$

$$\frac{104}{3}\pi r^3 = 117\pi$$

---

6 **Cancel**

Cancel π:

$$\frac{104}{3}r^3 = 117$$

$$r^3 = \frac{117 \times 3}{104}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

✓ **FINAL ANSWER**

$\boxed{1.5\text{ cm}}$

4MA1/1H NOVEMBER 2023

Question 21

5 marks | Volume & Surface Area

21 Here is a triangular prism $ABCDEF$

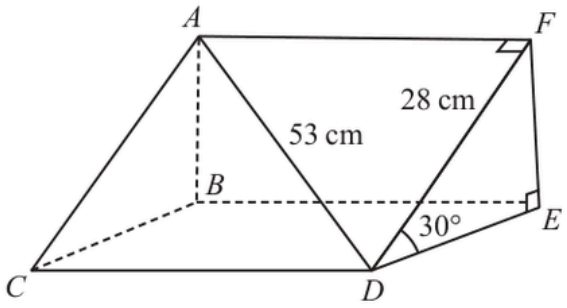


Diagram **NOT** accurately drawn

$AD = 53$ cm
$DF = 28$ cm
Angle $FDE = 30°$

Work out the volume of the triangular prism.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2023

1 The triangular cross section is

The triangular cross-section is triangle DEF.

2 It is right angled at

It is right-angled at E, with

$$DF = 28$$

3 Use trigonometry

and

$$\angle FDE = 30°$$

4 Use trigonometry

So

$$FE = 28 \sin 30° = 14$$

5 **Use trigonometry**

and

$$DE = 28\cos 30^\circ = 14\sqrt{3}$$

6 **Area of triangle**

Area of triangle DEF:

$$\frac{1}{2}(14)(14\sqrt{3}) = 98\sqrt{3}$$

7 **The length of the prism**

The length of the prism is

$$AD = 53$$

8 **Volume of the prism**

Volume of the prism:

$$98\sqrt{3} \times 53$$

$$= 5194\sqrt{3}$$

$$= 8996.2719\ldots$$

✓ **FINAL ANSWER**

$\boxed{8996\ \text{cm}^3}$

4MA1/1H MAY 2024

Question 23

5 marks | Volume & Surface Area

23 A solid shape is made by removing a hemisphere, shown shaded, from a cone as shown in the diagram.

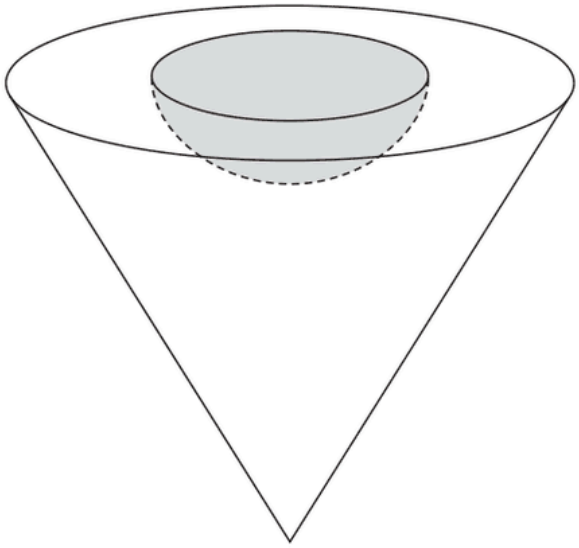


Diagram **NOT** accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the solid shape is 6948π cm^3

Work out the **total** surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2024

1 **Define cone dimensions**

The cone has radius $5x$ and height $6x$.

2 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi(5x)^2(6x)$$

$$= 50\pi x^3$$

3 **The removed hemisphere has radius**

The removed hemisphere has radius $2x$.

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi(2x)^3$$

$$= \frac{16}{3}\pi x^3$$

5 **The remaining solid has volume**

The remaining solid has volume 6948π, so

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\frac{134}{3}\pi x^3 = 6948\pi$$

6 **Cancel**

Cancel π:

$$\frac{134}{3}x^3 = 6948$$

$$x^3 = \frac{10422}{67}$$

$$x = 5.3781\ldots$$

7 **The solid hemisphere removed has**

The solid hemisphere removed has radius $2x$.

8 **Total surface area of a**

Total surface area of a solid hemisphere is

$$3\pi r^2$$

---

9 **Calculate value**

So

$$3\pi(2x)^2 = 12\pi x^2$$

$$12\pi(5.3781\ldots)^2 = 1090.3903\ldots$$

✓ **FINAL ANSWER**

$\boxed{1090\ \text{cm}^2}$

4MA1/1H NOVEMBER 2024

Question 5

4 marks | Volume & Surface Area

5 The diagram shows a cylinder.

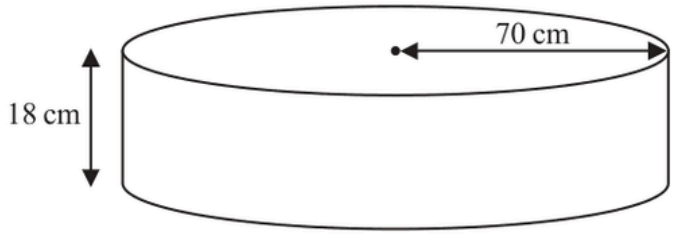


Diagram **NOT** accurately drawn

The radius of the cylinder is 70 cm
The height of the cylinder is 18 cm

Work out the volume of the cylinder.
Give your answer in litres correct to the nearest litre.

..... litres

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2024

1 Calculate area

$$V = \pi r^2 h = \pi(70)^2(18)$$

$$V = 277088.472\ldots \text{ cm}^3$$

2 Calculate area

Since $1000 \text{ cm}^3 = 1$ litre,

$$277088.472\ldots \text{ cm}^3 = 277.088\ldots \text{ litres}$$

FINAL ANSWER

277 litres.

4MA1/2H NOVEMBER 2024

Question 6

3 marks | Volume & Surface Area

6 The diagram shows a solid triangular prism.

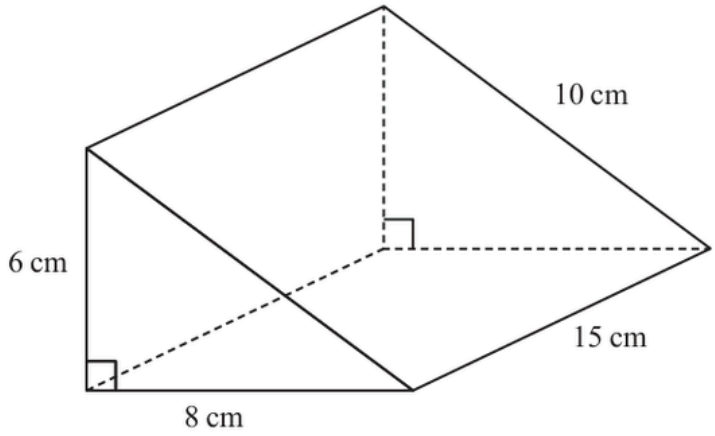


Diagram **NOT** accurately drawn

Work out the **total** surface area of the triangular prism.

..... cm^2

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2024

1 **The triangular end is a**

The triangular end is a right triangle with sides 6, 8, and 10.

2 **Area of the two triangular**

Area of the two triangular ends:

$$2\left(\frac{1}{2}\times 6\times 8\right)=48$$

3 **Area of the three rectangles**

Area of the three rectangles:

$$15(6+8+10)=360$$

4 **Total surface area**

Total surface area:

$$48+360=408$$

FINAL ANSWER

$408\ \text{cm}^2$.

4MA1/1H NOVEMBER 2024

Question 22

3 marks | Volume & Surface Area

22 The diagram shows a bowl in the shape of a hemisphere.
The bowl is made from metal.

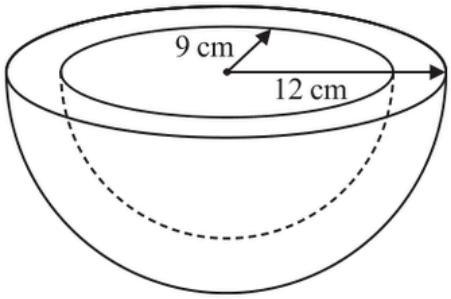


Diagram **NOT** accurately drawn

The outer radius of the bowl is 12 cm
The inner radius of the bowl is 9 cm

The thickness of the bowl is uniform.

Work out the volume of the metal.
Give your answer correct to the nearest whole number.

..... cm^3

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2024

1 **The bowl is a hemispherical**

The bowl is a hemispherical shell.

2 **The volume of the metal**

The volume of the metal is:

$$\text{outer hemisphere volume} - \text{inner hemisphere volume}$$

3 **Volume of a hemisphere**

Volume of a hemisphere:

$$\frac{2}{3}\pi r^3$$

4 **Calculate volume**

So

$$\text{metal volume} = \frac{2}{3}\pi(12)^3 - \frac{2}{3}\pi(9)^3$$

$$= \frac{2}{3}\pi(12^3 - 9^3)$$

$$= \frac{2}{3}\pi(1728 - 729)$$

$$= 666\pi$$

$$= 2092.3007\ldots$$

✓ **FINAL ANSWER**

$\boxed{2092\ \text{cm}^3}$

4WM2/01 MAY 2025

Question 2

3 marks | Volume & Surface Area

2 The diagram shows a solid prism with a cross section in the shape of a trapezium.

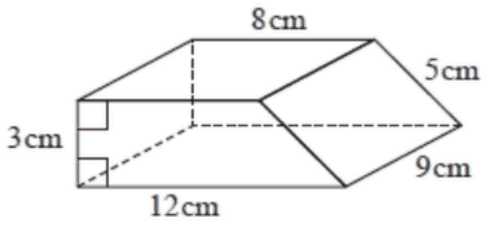


Diagram **NOT** accurately drawn

Work out the total surface area of the prism.

..... cm^2

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM2/01 May 2025

1 Area of one trapezium

Area of one trapezium:

$$\frac{1}{2}(8+12)\times 3 = 30$$

2 Perimeter of the trapezium

Perimeter of the trapezium:

$$8+12+3+5 = 28$$

3 Rectangular faces

Rectangular faces:

$$28 \times 9 = 252$$

4 Total surface area

Total surface area:

$$252 + 2(30) = 312$$

FINAL ANSWER

$312\ \text{cm}^2$.

4MA1/1H JUNE 2025

Question 7

3 marks | Volume & Surface Area

7 Here is a solid cylinder.

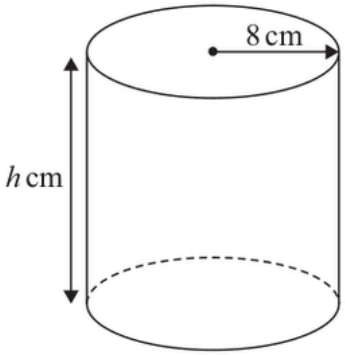


Diagram **NOT** accurately drawn

The radius of the cylinder is 8 cm
The height of the cylinder is h cm

The volume of the cylinder is $3892\,\text{cm}^3$

Work out the value of h
Give your answer correct to one decimal place.

$h =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2025

1 Calculate area

For a cylinder,

$$V = \pi r^2 h$$

$$3892 = \pi(8)^2 h$$

$$h = \frac{3892}{64\pi} = 19.357\ldots$$

FINAL ANSWER

$19.4\ \text{cm}$.

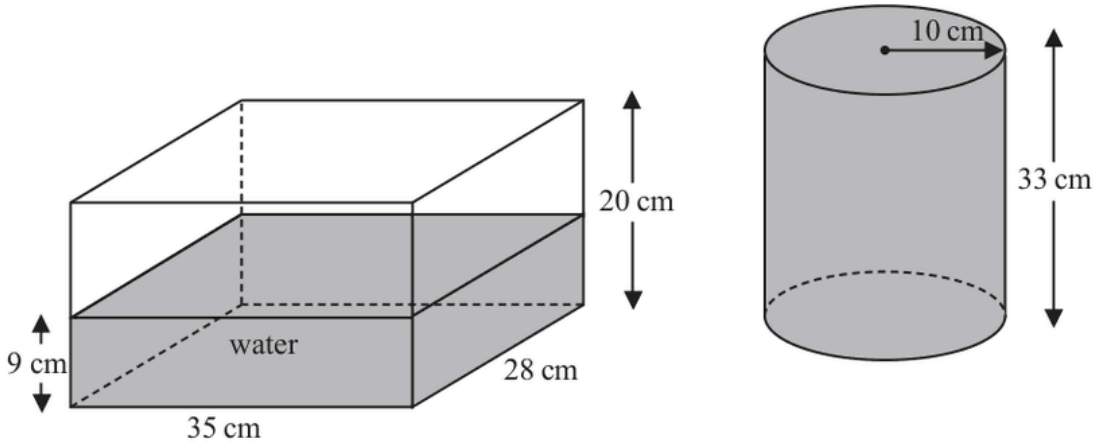
4MA1/1HR JUNE 2025

Question 10

3 marks | Volume & Surface Area

10 The diagram shows two water containers.
One is a cuboid and one is a cylinder.

Diagram **NOT** accurately drawn



The cuboid measures 35 cm by 28 cm by 20 cm
The surface of the water in the cuboid is 9 cm above the base of the cuboid.

The cylinder has a radius of 10 cm and a height of 33 cm
The cylinder is completely full of water.

Izzy is going to pour all the water from the cylinder into the cuboid.

Show that the cuboid will not be completely full of water.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2025

1 Capacity of the cuboid

Capacity of the cuboid:

$$35 \times 28 \times 20 = 19600 \text{ cm}^3$$

2 Water already in the cuboid

Water already in the cuboid:

$$35 \times 28 \times 9 = 8820 \text{ cm}^3$$

3 **Volume of the full cylinder**

Volume of the full cylinder:

$$\pi(10)^2(33) = 10367.255\ldots \text{ cm}^3$$

4 **Total water after pouring**

Total water after pouring:

$$8820 + 10367.255\ldots = 19187.255\ldots$$

5 **Calculate area**

Since

$$19187.255\ldots < 19600$$

6 **the cuboid will not be**

the cuboid will not be completely full.

✓ **FINAL ANSWER**

The cuboid will not be completely full.

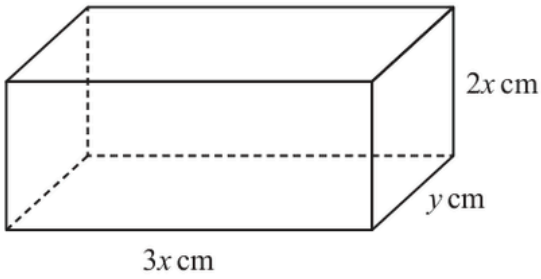
4MA1/2H JUNE 2025

Question 21

3 marks | Volume & Surface Area

21 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is $1014\,\text{cm}^3$
The total surface area of the cuboid is $A\,\text{cm}^2$

Show that $A = 12x^2 + \frac{1690}{x}$

You must show all the stages of your working.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2025

1 **The cuboid has dimensions**

The cuboid has dimensions

$$3x, \quad 2x, \quad y$$

2 **Calculate volume**

Its volume is $1014\ \text{cm}^3$, so

$$(3x)(2x)(y) = 1014$$

$$6x^2y = 1014$$

$$y = \frac{1014}{6x^2}$$

$$y = \frac{169}{x^2}$$

3 **The total surface area is**

The total surface area is

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$A = 2(6x^2 + 5xy)$$

$$A = 12x^2 + 10xy$$

4 **Substitute**

Substitute $y = \frac{169}{x^2}$:

$$A = 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$A = 12x^2 + \frac{1690}{x}$$

✓ **FINAL ANSWER**

$$\boxed{A = 12x^2 + \frac{1690}{x}}$$

4WM2/01R MAY 2025

Question 21

5 marks | Volume & Surface Area

21 The diagram shows a solid cuboid.

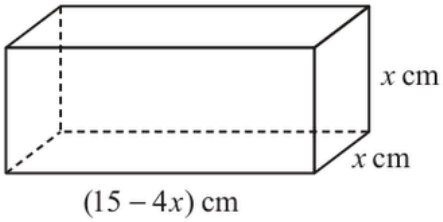


Diagram **NOT** accurately drawn

The volume of the cuboid is V cm^3

Find the maximum value of V

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM2/01R May 2025

1 Required

Cuboid has a square cross-section of side *x* cm and length (15−4*x*) cm. Find the maximum value of the volume *V*.

2 Write V as a function

Write *V* as a function of *x*:

V = *x* · *x* · (15−4*x*) = *x*²(15−4*x*) = 15*x*² − 4*x*³

3 Differentiate

Differentiate:

$$fracdVdx = 30x - 12x^2$$

4 Set (x 0 gives

Set $dfracdVdx = 0$:

$$30x - 12x^2 = 0implies6x(5 - 2x) = 0impliesx = 0; textor; x = tfrac52 = 2.5$$

(*x* = 0 gives zero volume, so the meaningful solution is *x* = 2.5.)

5 **Second derivative test to confirm**

Second derivative test to confirm maximum:

$$fracd^2Vdx^2 = 30 - 24x; Bigg|_x = 2.5 = 30 - 60 = -30\<0quadcheckmarktext(maximum)$$

6 **Calculate V max V max**

Calculate *V*_{max}:

*V*_{max} = (2.5)² × (15−4×2.5) = 6.25 × 5 = 31.25 cm³

7 **Note**

Calculus optimisation: always verify the nature of the stationary point using $dfracd^2Vdx^2$. Also check *x* \> 0 and 15 − 4*x* \> 0 (i.e. *x* \< 3.75) to ensure a valid cuboid.

✓ **FINAL ANSWER**

Maximum volume *V* = 31.25 cm³

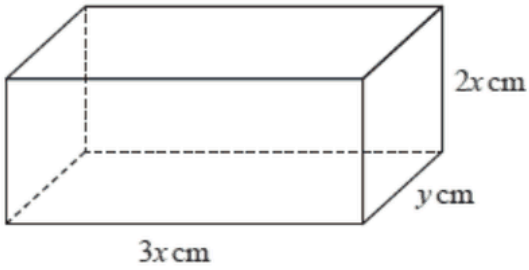
4WM2/01 MAY 2025

Question 23

3 marks | Volume & Surface Area

23 The diagram shows a cuboid.

Diagram **NOT** accurately drawn



The cuboid measures $3x$ cm by $2x$ cm by y cm

The volume of the cuboid is 1014 cm^3
The total surface area of the cuboid is A cm^2

Show that $A = 12x^2 + \dfrac{1690}{x}$

You must show all the stages of your working.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 May 2025

1 Evaluate fraction

$$3x \times 2x \times y = 1014$$

$$6x^2y = 1014$$

$$y = \frac{169}{x^2}$$

2 Surface area

Surface area:

$$A = 2(3x \cdot 2x + 3x \cdot y + 2x \cdot y)$$

$$= 12x^2 + 10xy$$

$$= 12x^2 + 10x\left(\frac{169}{x^2}\right)$$

$$= 12x^2 + \frac{1690}{x}$$

FINAL ANSWER

$A = 12x^2 + \dfrac{1690}{x}$.

4WM2/01R MAY 2025

Question 25

5 marks | Volume & Surface Area

25 The diagram shows a solid hemisphere, H

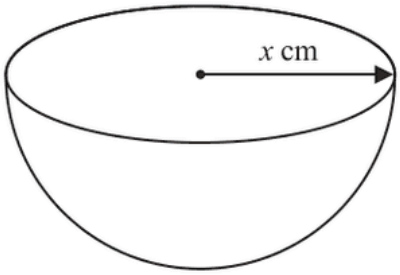


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

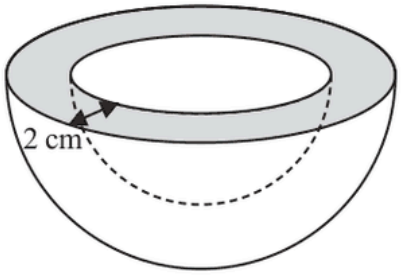


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM2/01R May 2025

1 **Required**

Hemisphere H has radius *x* cm and volume 6174*π* cm³. A bowl is made by removing an inner hemisphere, leaving 2 cm thickness. Find total surface area in terms of *π*.

2 **Calculate volume**

Find outer radius *x*. Volume of hemisphere $= \frac{2}{3}\pi r^3$:

$$\frac{2}{3}\pi x^3 = 6174\pi \implies x^3 = \frac{6174}{3} \times 2 = 9261 \implies x = \sqrt[3]{9261} = 21\,\text{cm}$$

3 **Find inner radius r inner**

Find inner radius:

*r*_{inner} = 21 − 2 = 19 cm

4 **Add 1**

Three surfaces to add:

1. Outer curved surface (half-sphere, radius 21):

*A*₁ = 2*π*(21)² = 2*π* × 441 = 882*π*

2. Inner curved surface (half-sphere, radius 19):

*A*₂ = 2*π*(19)² = 2*π* × 361 = 722*π*

3. Flat annular ring at the top rim:

*A*₃ = *π*(21)² − *π*(19)² = *π*(441−361) = 80*π*

5 **Total surface area A total**

Total surface area:

*A*_{total} = 882*π* + 722*π* + 80*π* = 1684*π*

6 **Note**

Three separate surfaces! Missing the flat annular ring is the most common error on this type of question.

✓ **FINAL ANSWER**

Total surface area = 1684*π* cm²

4MA1/2HR JUNE 2025

Question 26

5 marks | Volume & Surface Area

26 The diagram shows a solid hemisphere, H

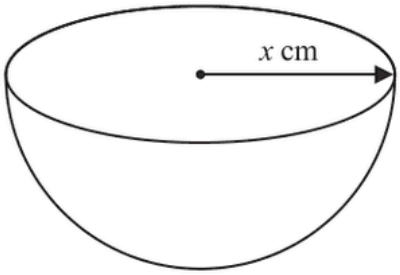


Diagram **NOT** accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A bowl is made by removing a solid hemisphere from H such that the uniform thickness of the bowl is 2 cm

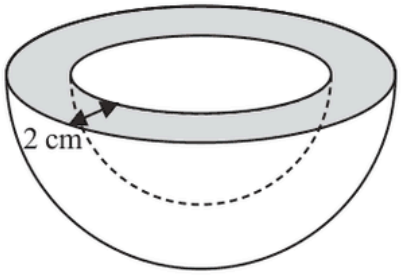


Diagram **NOT** accurately drawn

Work out the **total** surface area of the bowl.
Give your answer in terms of π

..... cm^2

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2025

1 **The volume of a hemisphere**

The volume of a hemisphere is

$$\frac{2}{3}\pi r^3$$

2 **Evaluate fraction**

For the original hemisphere H,

$$\frac{2}{3}\pi x^3 = 6174\pi$$

3 **Cancel**

Cancel π:

$$\frac{2}{3}x^3 = 6174$$

$$x^3 = 9261$$

$$x = 21$$

4 **The outer radius of the**

The outer radius of the bowl is 21 cm.

5 **Calculate value**

The thickness is 2 cm, so the inner radius is

$$21 - 2 = 19$$

6 **Total surface area of the**

Total surface area of the bowl is:

7 **outer curved hemisphere inner curved**

- outer curved hemisphere
- inner curved hemisphere
- flat annular rim

8 **Outer curved surface area**

Outer curved surface area:

$$2\pi(21)^2 = 882\pi$$

9 **Inner curved surface area**

Inner curved surface area:

$$2\pi(19)^2 = 722\pi$$

10 **Flat rim area**

Flat rim area:

$$\pi(21^2 - 19^2) = 80\pi$$

---

11 **Total**

Total:

$$882\pi + 722\pi + 80\pi = 1684\pi$$

✓ **FINAL ANSWER**

$\boxed{1684\pi \text{ cm}^2}$

4WM2/01 NOVEMBER 2025

Question 19

2 marks | Volume & Surface Area

19 A sphere has a diameter of 24 cm

Find the surface area of the sphere.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4WM2/01 November 2025

1 Evaluate fraction

$$r = frac242 = 12textcm$$

2 Calculate area

Surface Area = 4*π**r*²

= 4 × *π* × 12²

= 4 × *π* × 144

= 576*π* ≈ 1809.557... cm²

Step 3: Round to 3 significant figures. 1809.557 → 1810 (the 1, 8, and 1 are the first three sig figs).

FINAL ANSWER

1810 cm²

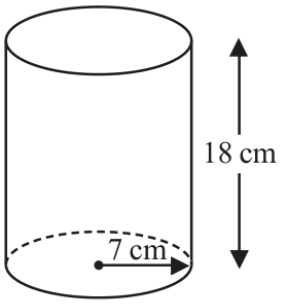
4MA1/2H JUNE 2026

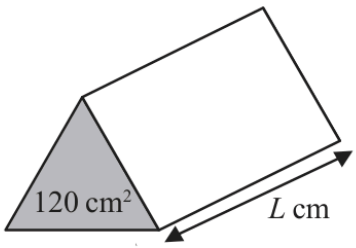
Question 7

3 marks | Volume & Surface Area

7 The diagram shows a cylinder and a prism.

Diagram **NOT** accurately drawn





The radius of the cylinder is 7 cm
The height of the cylinder is 18 cm

The area of the cross section of the prism, shown shaded in the diagram, is $120\ \text{cm}^2$
The length of the prism is L cm

The volume of the cylinder is equal to the volume of the prism.

Work out the value of L
Give your answer correct to 3 significant figures.

$L =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2026

1 Solution | Step 1

$$V_{\text{cylinder}} = \pi r^2 h = \pi(7)^2(18) = 882\pi\ \text{cm}^3.$$

Use the stated radius, not a diameter.

2 Solution | Step 2

$$V_{\text{prism}} = 120L, \qquad 120L = 882\pi \quad \Rightarrow \quad L = \frac{882\pi}{120} = 23.090706\ldots.$$

Prism volume equals cross-sectional area times length; equate the two volumes.

FINAL ANSWER

$L = 23.1\ \text{cm}$ (3 significant figures).

Question 11

4 marks | Volume & Surface Area

11 Here is a solid cylinder.

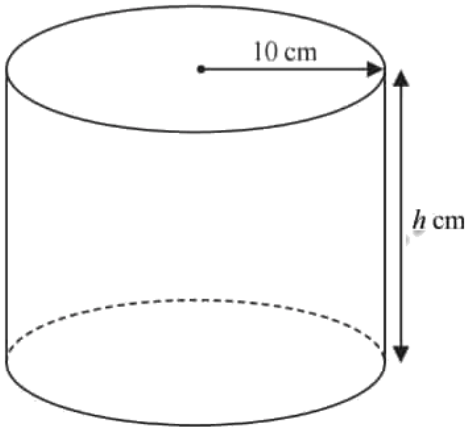


Diagram **NOT** accurately drawn

The radius of the cylinder is 10 cm
The height of the cylinder is h cm

The volume of the cylinder is 5341 cm^3

Work out the **total** surface area of the cylinder.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 11 is 4 marks)

1 **Find h from the volume**

$$\pi(10)^2 h = 5341 \Rightarrow h = \frac{5341}{100\pi} = 17.00\ldots \text{ cm}$$

2 **Curved surface**

$$2\pi r h = 2\pi(10)(17.00\ldots) = 1068.2$$

3 **Two circular ends**

$$2\pi r^2 = 2\pi(10)^2 = 200\pi = 628.31\ldots$$

4 **Total**

$$1068.2 + 628.31\ldots = 1696.5\ldots$$

Keep h unrounded; using $h = 17$ gives 1696.

✓ **FINAL ANSWER**

1697 cm^2

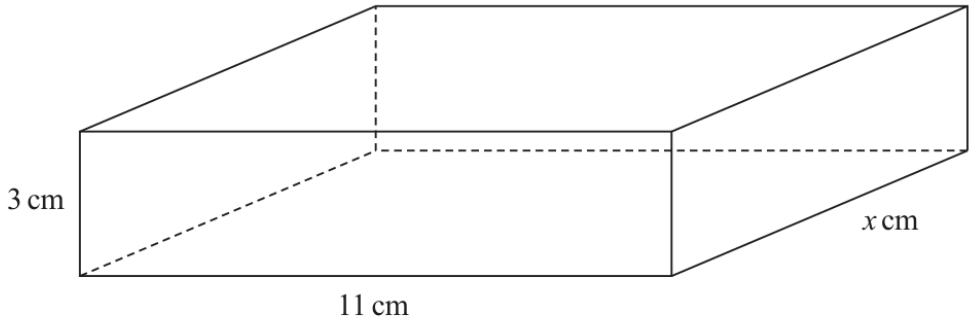
4MA1/1HR JUNE 2026

Question 11

4 marks | Volume & Surface Area

11 The diagram shows a solid cuboid.

Diagram **NOT** accurately drawn



The cuboid measures 3 cm by 11 cm by x cm

The volume of the cuboid is 264 cm^3

Work out the total surface area of the cuboid.

..... cm^2

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$3 \times 11 \times x = 264, \qquad x = \frac{264}{33} = 8 \text{ cm}.$$

The volume is the product of the three perpendicular edge lengths.

2 **Solution | Step 2**

$$S = 2(3 \times 11) + 2(3 \times 8) + 2(11 \times 8).$$

There are two faces of each of the three rectangular sizes.

3 **Solution | Step 3**

$$S = 66 + 48 + 176 = 290 \text{ cm}^2.$$

Add all six face areas.

FINAL ANSWER

$290\ \text{cm}^2$

Question 20

5 marks | Volume & Surface Area

20 The diagram shows a solid shape.

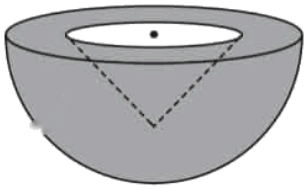


Diagram **NOT** accurately drawn

The solid shape is made by removing a cone from a hemisphere.

The radius of the cone is x cm
The height of the cone is x cm

The radius of the hemisphere is $2x$ cm

The volume of the solid shape is $\frac{3645}{8}\pi$ cm^3

Find the outer curved surface area of the solid shape.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 20 is 5 marks)

1 **Volume of the solid**

$$\tfrac{2}{3}\pi(2x)^3 - \tfrac{1}{3}\pi x^2(x) = \tfrac{16}{3}\pi x^3 - \tfrac{1}{3}\pi x^3 = 5\pi x^3$$

2 **Solve for x**

$$5\pi x^3 = \frac{3645}{8}\pi \Rightarrow x^3 = \frac{729}{8} \Rightarrow x = 4.5$$

3 **Outer curved surface = curved hemisphere**

Radius $2x = 9$:

$$2\pi(9)^2 = 162\pi = 508.9\ldots$$

FINAL ANSWER

509 cm^2

TOPIC

Congruent Shapes

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2020

Question 12

3 marks | Congruent Shapes

12 The diagram shows two congruent isosceles triangles and parts of two congruent regular polygons, **X** and **Y**.

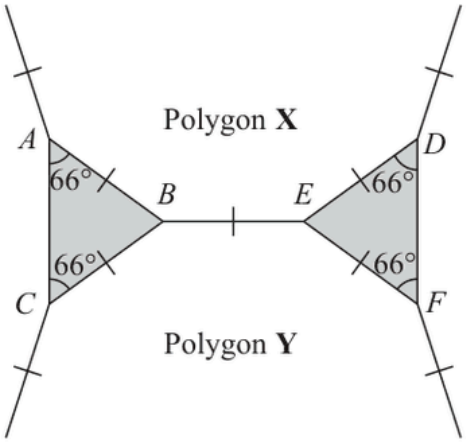


Diagram **NOT** accurately drawn

The two regular polygons each have n sides.

Work out the value of n.

$n =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2020

1 **Each congruent isosceles triangle has**

Each congruent isosceles triangle has two angles of $66°$.

2 **Use trigonometry**

So the angle at the point between the two polygons is

$$180° - 66° - 66° = 48°$$

3 **The two regular polygons have**

The two regular polygons have equal interior angles. Around the point:

$$48^\circ + I + I = 360^\circ$$

$$2I = 312^\circ$$

$$I = 156^\circ$$

4 **The exterior angle is**

The exterior angle is

$$180^\circ - 156^\circ = 24^\circ$$

$$n = \frac{360}{24} = 15$$

✓ **FINAL ANSWER**

$n = 15$.

TOPIC

Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2021

Question 4

4 marks | Similar Shapes

4 *ABC* and *DEF* are similar triangles.

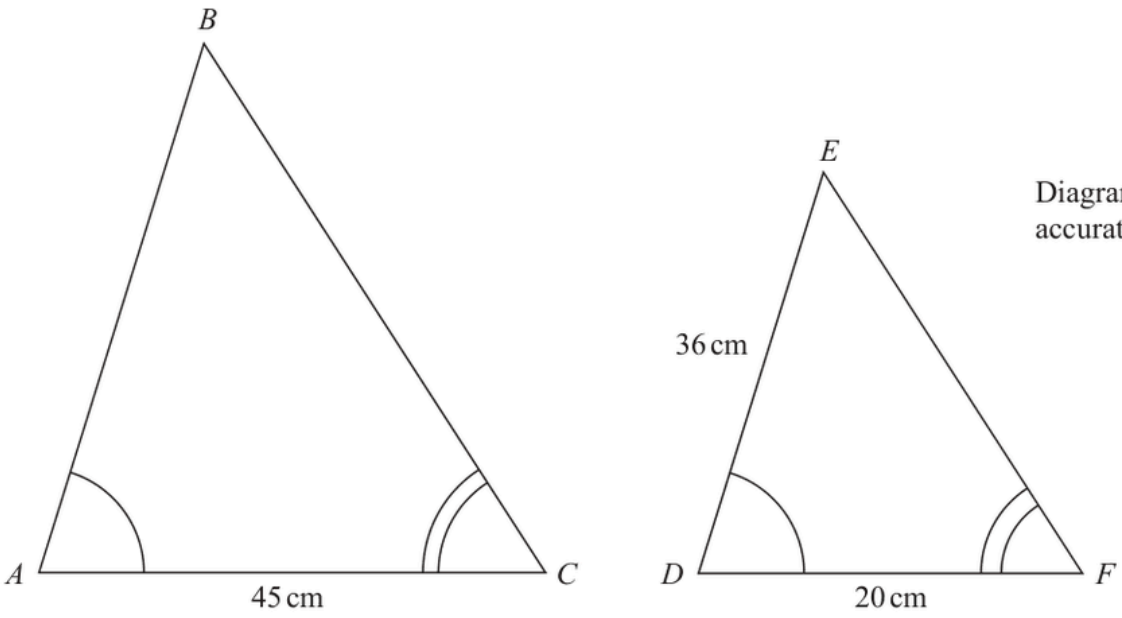


Diagram **NOT** accurately drawn

(a) Work out the length of *AB*.

..... cm

(2)

Given that $BC = 54$ cm,

(b) work out the length of *EF*.

..... cm

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2021

1 **The scale factor from triangle**

The scale factor from triangle DEF to triangle ABC is

$$\frac{AC}{DF} = \frac{45}{20} = 2.25$$

$$AB = DE \times 2.25 = 36 \times 2.25 = 81$$

2 **Evaluate fraction**

Given $BC = 54$,

$$EF = \frac{54}{2.25} = 24$$

FINAL ANSWER

$AB = 81\ \text{cm}$, and $EF = 24\ \text{cm}$.

4MA1/2H JANUARY 2022

Question 3

3 marks | Similar Shapes

3

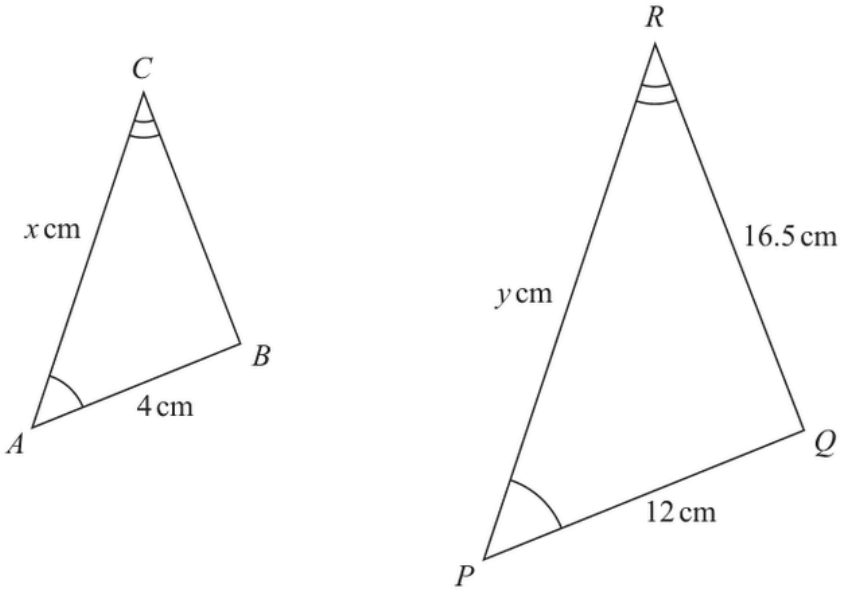


Diagram **NOT** accurately drawn

Triangle ABC is similar to triangle PQR

$AB = 4\,\text{cm}$ $PQ = 12\,\text{cm}$ $RQ = 16.5\,\text{cm}$ $AC = x\,\text{cm}$ $PR = y\,\text{cm}$

(a) Calculate the length of BC

..... cm

(2)

(b) Write down an expression for y in terms of x

$y =$ .....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2022

1 **The scale factor from triangle**

The scale factor from triangle ABC to triangle PQR is

$$\frac{PQ}{AB} = \frac{12}{4} = 3$$

2 **Evaluate fraction**

So

$$BC = \frac{RQ}{3} = \frac{16.5}{3} = 5.5$$

3 **Calculate value**

Also PR corresponds to AC, so

$$y = 3x$$

✓ **FINAL ANSWER**

$BC = 5.5\ \text{cm}$, and $y = 3x$.

4MA1/2HR JUNE 2022

Question 12

3 marks | Similar Shapes

12

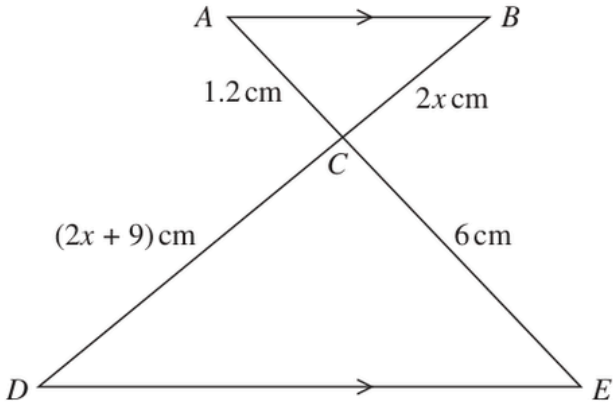


Diagram **NOT** accurately drawn

ACE and *BCD* are straight lines.
AB is parallel to *DE*

Work out the value of x

$x =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2022

1 Find the gradient

Since AB is parallel to DE, triangles ACB and ECD are similar.

2 Split the ratio

So corresponding sides are in the same ratio:

$$\frac{AC}{CE} = \frac{BC}{CD}$$

3 Substitute the lengths

Substitute the lengths:

$$\frac{1.2}{6} = \frac{2x}{2x+9}$$

$$\frac{1}{5} = \frac{2x}{2x+9}$$

$$2x + 9 = 10x$$

$$8x = 9$$

$$x = \frac{9}{8}$$

FINAL ANSWER

$x = \frac{9}{8}$.

4MA1/1HR MAY 2023

Question 3

4 marks | Similar Shapes

3 $ABCD$ and $EFGH$ are similar quadrilaterals.

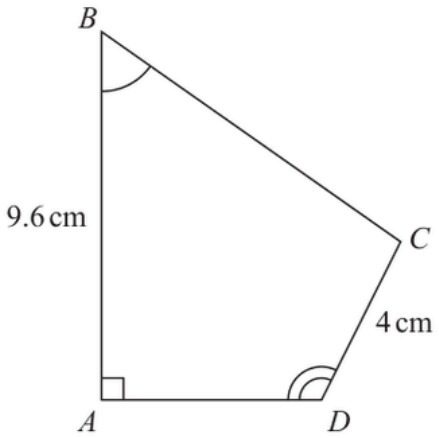
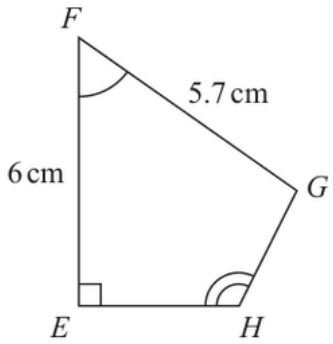


Diagram **NOT** accurately drawn



(a) Work out the length of GH

..... cm

(2)

(b) Work out the length of BC

..... cm

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May 2023

1 **Evaluate fraction**

The scale factor from $EFGH$ to $ABCD$ is

$$\frac{9.6}{6} = 1.6$$

2 **Evaluate fraction**

Since GH corresponds to CD,

$$GH = \frac{4}{1.6} = 2.5$$

3 **Calculate value**

Since FG corresponds to BC,

$$BC = 5.7 \times 1.6 = 9.12$$

✓ **FINAL ANSWER**

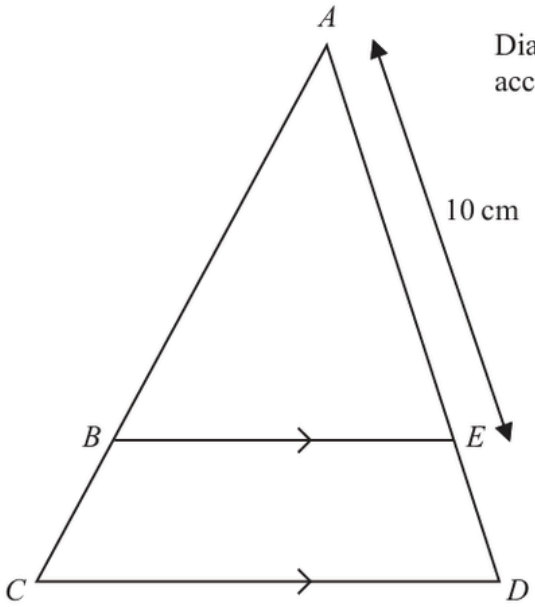
$GH = 2.5\ \text{cm}$, and $BC = 9.12\ \text{cm}$.

4MA1/2HR JUNE 2024

Question 12

4 marks | Similar Shapes

12



In the diagram, ABC and AED are straight lines.
BE is parallel to CD

$$AE = 10\,\text{cm} \qquad \text{and} \qquad CD = 1.5 \times BE$$

(a) Work out the length of ED

..... cm

(2)

$AB = (2x + 5)\,\text{cm}$ and $BC = (3x - 5)\,\text{cm}$

(b) Work out the value of x

$x =$ .....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2024

1 **Find the gradient**

Since $BE \parallel CD$, triangles ABE and ACD are similar.

2 **Find the gradient**

Also

$$CD = 1.5BE$$

so

$$\frac{BE}{CD} = \frac{1}{1.5} = \frac{2}{3}$$

3 **Find the gradient**

Therefore

$$\frac{AE}{AD} = \frac{2}{3}$$

4 **Find the gradient**

Given $AE = 10$,

$$\frac{10}{AD} = \frac{2}{3}$$

$$AD = 15$$

5 **Find the gradient**

So

$$ED = AD - AE = 15 - 10 = 5$$

6 **Find the gradient**

For part (b),

$$AB = 2x + 5, \qquad BC = 3x - 5$$

$$AC = AB + BC = 5x$$

7 **Find the gradient**

Using similarity,

$$\frac{AB}{AC} = \frac{2}{3}$$

$$\frac{2x+5}{5x} = \frac{2}{3}$$

$$3(2x+5) = 10x$$

$$6x + 15 = 10x$$

$$x = 3.75$$

✓ **FINAL ANSWER**

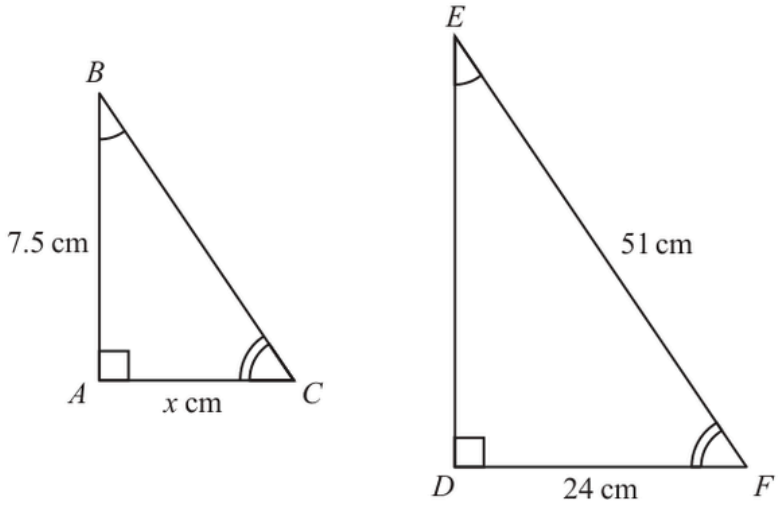
(a) $ED = 5\ \text{cm}$, (b) $x = 3.75$.

4MA1/1H NOVEMBER 2024

Question 10

5 marks | Similar Shapes

10 Here are two similar triangles.



Diagrams **NOT** accurately drawn

Work out the value of x
Show your working clearly.

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2024

1 Use trigonometry

In the larger triangle,

$$DE^2 = 51^2 - 24^2$$

$$DE = 45$$

2 The scale factor from the

The scale factor from the large triangle to the small triangle is

$$\frac{7.5}{45} = \frac{1}{6}$$

3 Evaluate fraction

So

$$x = 24 \times \frac{1}{6} = 4$$

FINAL ANSWER

$x = 4$.

4MA1/2H JUNE 2025

Question 5

4 marks | Similar Shapes

5 The diagram shows two similar quadrilaterals, $ABCD$ and $EFGH$

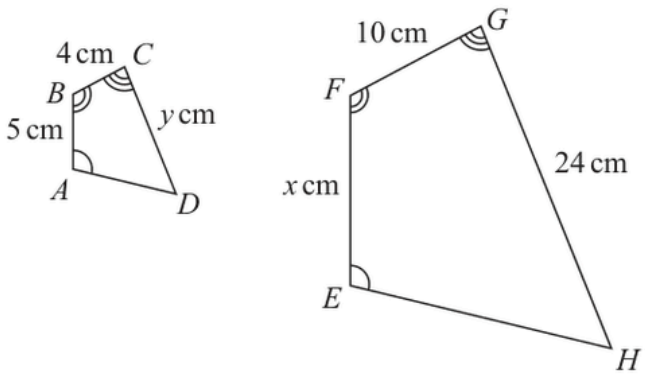


Diagram **NOT** accurately drawn

$AB = 5\,\text{cm}$ $BC = 4\,\text{cm}$ $CD = y\,\text{cm}$

$EF = x\,\text{cm}$ $FG = 10\,\text{cm}$ $GH = 24\,\text{cm}$

(a) Work out the value of x

$x =$ .....

(2)

(b) Work out the value of y

$y =$ .....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2025

1 **Evaluate fraction**

The scale factor from $ABCD$ to $EFGH$ is

$$\frac{FG}{BC} = \frac{10}{4} = 2.5$$

2 **Calculate value**

Since AB corresponds to EF,

$$x = EF = 5 \times 2.5 = 12.5$$

3 **Calculate value**

Since CD corresponds to GH,

$$2.5y = 24$$

$$y = 9.6$$

✓ **FINAL ANSWER**

$x = 12.5$, $y = 9.6$.

4WM2/01 MAY 2025

Question 6

4 marks | Similar Shapes

6 The diagram shows two similar quadrilaterals, *ABCD* and *EFGH*

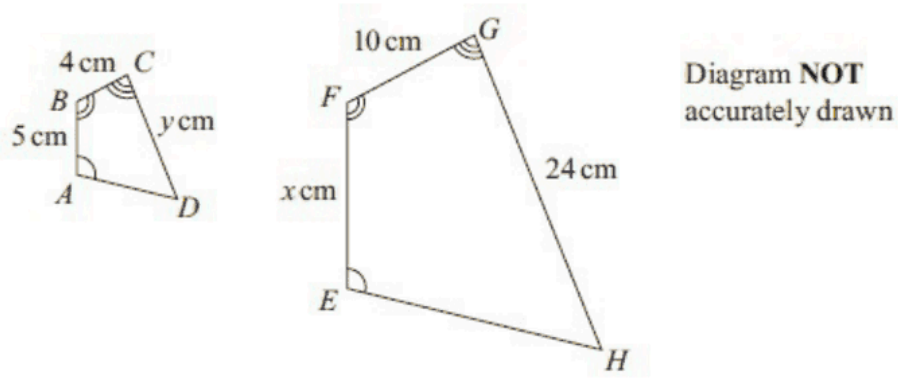


Diagram **NOT** accurately drawn

$AB = 5$ cm $BC = 4$ cm $CD = y$ cm

$EF = x$ cm $FG = 10$ cm $GH = 24$ cm

(a) Work out the value of x

$x =$ .....

(2)

(b) Work out the value of y

$y =$ .....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 May 2025

1 Evaluate fraction

The scale factor from $ABCD$ to $EFGH$ is

$$\frac{FG}{BC} = \frac{10}{4} = 2.5$$

2 Calculate value

So

$$x = EF = 5 \times 2.5 = 12.5$$

3 **Evaluate fraction**

and

$$y = CD = \frac{24}{2.5} = 9.6$$

✓ **FINAL ANSWER**

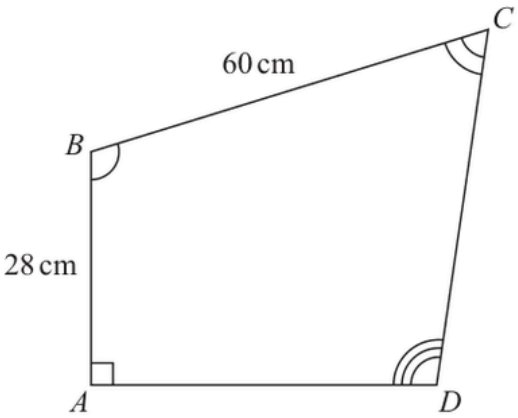
$x = 12.5$, $y = 9.6$.

4MA1/2H NOVEMBER 2025

Question 4

3 marks | Similar Shapes

4 Here are three similar quadrilaterals.



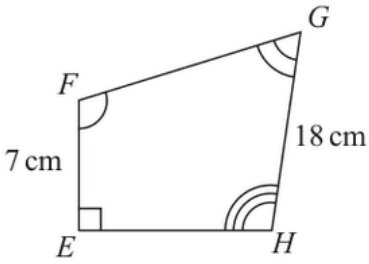
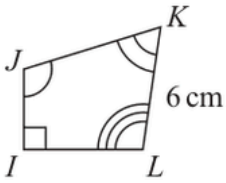


Diagram **NOT** accurately drawn



Work out the length of JK

..... cm

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2025

1 The scale factor from the

The scale factor from the largest quadrilateral to the middle quadrilateral is

$$\frac{7}{28} = \frac{1}{4}$$

2 Evaluate fraction

So the side corresponding to $60\ \text{cm}$ in the middle shape is

$$60 \times \frac{1}{4} = 15$$

3 The scale factor from the

The scale factor from the middle quadrilateral to the smallest quadrilateral is

$$\frac{6}{18} = \frac{1}{3}$$

$$JK = 15 \times \frac{1}{3} = 5$$

FINAL ANSWER

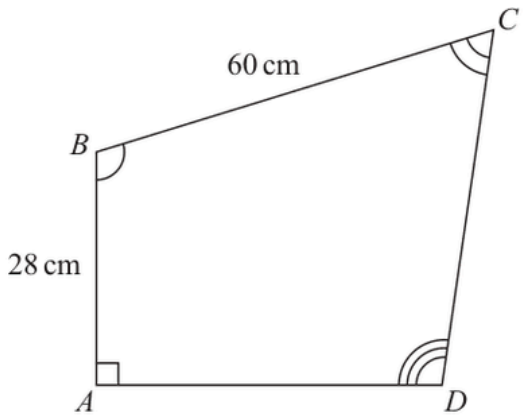
5 cm.

4WM2/01 NOVEMBER 2025

Question 5

3 marks | Similar Shapes

5 Here are three similar quadrilaterals.



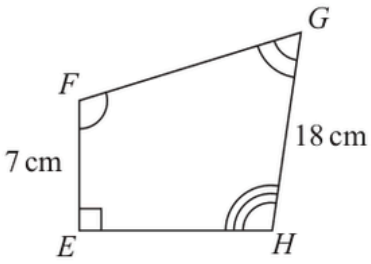
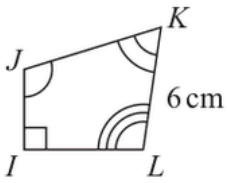


Diagram **NOT** accurately drawn



Work out the length of JK

..... cm

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM2/01 November 2025

1 Identify matching sides

Step 1: Identify corresponding sides from the similar shapes. Looking at the given angles/orientations:

2 Left side A B corresponds

Left side *A**B* corresponds to *E**F* and *I**J*.

3 Top side B C corresponds

Top side *B**C* corresponds to *F**G* and *J**K*.

4 Right side C D corresponds

Right side *C**D* corresponds to *G**H* and *K**L*.

5 **We are given C D**

We are given *C**D* = 60 (wait, diagram check: *B**C* is the top side labeled 60, *A**B* = 28. On middle shape *E**F* = 7, *G**H* = 18. On right shape *K**L* = 6, *J**K* is unknown). Let's use the explicit lengths:

Misplaced &

---

6 **Find scale factors**

Step 2: Find scale factors. Scale factor from *A**B**C**D* to *E**F**G**H* using the left sides:

$$textSF_1 = fracEFAB = frac728 = frac14$$

So, $FG = BCtimesfrac14 = 60timesfrac14 = 15textcm$.

---

7 **Scale factor from E F**

Scale factor from *E**F**G**H* to *I**J**K**L* using the right sides (*G**H* and *K**L*):

$$textSF_2 = fracKLGH = frac618 = frac13$$

---

8 **Calculate J K**

Step 3: Calculate *JK*. Side *J**K* corresponds to *F**G*.

$$JK = FGtimestextSF_2 = 15timesfrac13 = 5textcm$$

✓ **FINAL ANSWER**

5 cm

TOPIC

Area & Volume of Similar Shapes

Every past-paper question on this topic, 2020 to 2026.

Question 19

4 marks | Area & Volume of Similar Shapes

19 The diagram shows two similar metal statues.

Diagram **NOT** accurately drawn



A **B**

The volume of statue **B** is 20% less than the volume of statue **A**
The surface area of statue **B** is k% less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 19 is 4 marks)

1 **The volume of statue is**

The volume of statue B is 80% of the volume of statue A, so the volume scale factor is

$$0.8$$

2 **Calculate area**

For similar solids:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 **The surface area scale factor**

The surface area scale factor is therefore

$$\left(\sqrt[3]{0.8}\right)^2 = 0.8^{2/3}$$

$$0.8^{2/3} = 0.861773\ldots$$

4 **Convert standard form**

So the percentage decrease in surface area is

$$(1 - 0.861773\ldots) \times 100 = 13.8226\ldots$$

5 **Calculate area**

Correct to 3 significant figures,

$$k = 13.8$$

✓ **FINAL ANSWER**

$k = 13.8$.

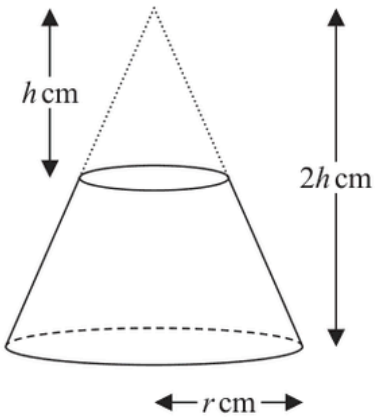
4MA1/1HR JANUARY 2020

Question 20

5 marks | Area & Volume of Similar Shapes

20 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.



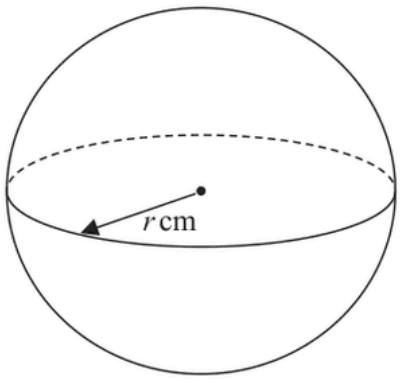


Diagram **NOT** accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h.
Give your expression in its simplest form.

$r =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2020

1 **The two cones are similar**

The two cones are similar.

2 **The large cone has height**

The large cone has height $2h$ and base radius r.

3 **The small cone has height**

The small cone has height h, so its linear scale factor compared with the large cone is

$$\frac{h}{2h} = \frac{1}{2}$$

4 **Define cone dimensions**

So the small cone has radius

$$\frac{r}{2}$$

5 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$

6 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h = \frac{1}{12}\pi r^2 h$$

7 **Calculate volume**

So the volume of the frustum is

$$\frac{2}{3}\pi r^2 h - \frac{1}{12}\pi r^2 h = \frac{7}{12}\pi r^2 h$$

8 **Calculate volume**

The sphere has radius r, so its volume is

$$\frac{4}{3}\pi r^3$$

9 **The two volumes are equal**

The two volumes are equal:

$$\frac{7}{12}\pi r^2 h = \frac{4}{3}\pi r^3$$

10 **Divide by**

Divide by πr^2:

$$\frac{7}{12}h = \frac{4}{3}r$$

$$7h = 16r$$

$$r = \frac{7h}{16}$$

✓ **FINAL ANSWER**

$$r = \frac{7h}{16}$$

4MA1/2HR JANUARY 2020

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar vases, **A** and **B**.

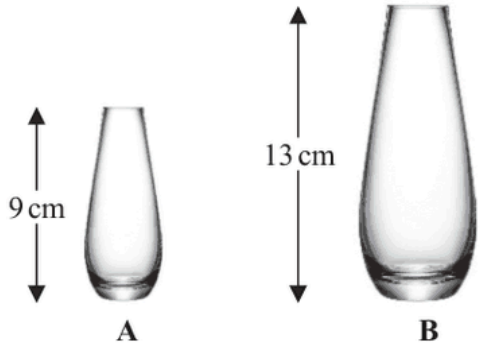


Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

surface area of vase **A** + surface area of vase **B** = 1800 cm^2

calculate the surface area of vase **A**.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2020

1 **The two vases are similar**

The two vases are similar.

2 **Their heights are in the**

Their heights are in the ratio

$$9 : 13$$

3 **Split the ratio**

So their surface areas are in the ratio

$$9^2 : 13^2 = 81 : 169$$

4 **The total surface area is**

The total surface area is $1800\ \text{cm}^2$, so there are

$$81 + 169 = 250$$

5 **parts altogether**

parts altogether.

6 **Surface area of vase A**

Surface area of vase A:

$$1800 \times \frac{81}{250} = 583.2$$

✓ **FINAL ANSWER**

$\boxed{583.2\ \text{cm}^2}$

4MA1/1H MAY/NOVEMBER 2020

Question 20

3 marks | Area & Volume of Similar Shapes

20 **R** and **S** are two similar solid shapes.

Shape **R** has surface area 108 cm^2 and volume 135 cm^3
Shape **S** has surface area 300 cm^2

Work out the volume of shape **S**.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May/November 2020

1 **Shape R has surface area**

Shape R has surface area $108\ \text{cm}^2$ and volume $135\ \text{cm}^3$.

2 **Shape S has surface area**

Shape S has surface area $300\ \text{cm}^2$.

3 **Surface area ratio S R**

Surface area ratio S:R:

$$300 : 108 = 25 : 9$$

4 **Split the ratio**

So the linear ratio S:R is

$$5 : 3$$

5 **Split the ratio**

Therefore the volume ratio S:R is

$$5^3 : 3^3 = 125 : 27$$

6 **Volume of S**

Volume of S:

$$135 \times \frac{125}{27} = 625$$

FINAL ANSWER

$\boxed{625\ \text{cm}^3}$

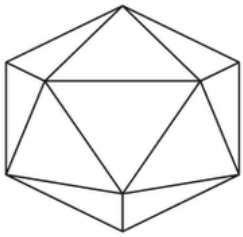
4MA1/1H JANUARY 2021

Question 20

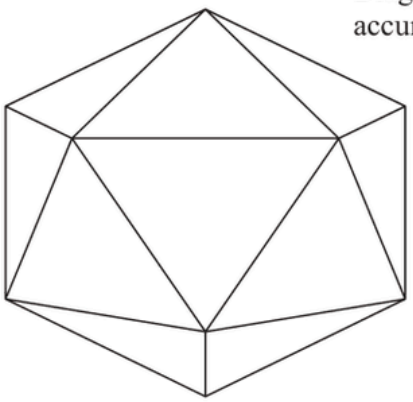
3 marks | Area & Volume of Similar Shapes

20 **A** and **B** are two similar solids.

Diagram **NOT** accurately drawn



A



B

A has a volume of $1836\,\text{cm}^3$
B has a volume of $4352\,\text{cm}^3$

B has a total surface area of $1120\,\text{cm}^2$

Work out the total surface area of **A**.

..... cm^2

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2021

1 The volume ratio of solid

The volume ratio of solid A to solid B is

$$1836 : 4352$$

2 Simplify

Simplify:

$$\frac{1836}{4352} = \frac{27}{64}$$

3 Split the ratio

So the linear ratio is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

4 **Surface area ratio**

Surface area ratio:

$$3^2 : 4^2 = 9 : 16$$

---

5 **Solid B has surface area**

Solid B has surface area $1120\ \text{cm}^2$, so solid A has surface area

$$1120 \times \frac{9}{16} = 630$$

✓ **FINAL ANSWER**

$\boxed{630\ \text{cm}^2}$

4MA1/2H MAY 2021

Question 20

3 marks | Area & Volume of Similar Shapes

20 Mathematically similar wooden blocks are made in a workshop.

There are small blocks and there are large blocks.

The volume of each small block is $300\,\text{cm}^3$

Given that

the surface area of each small block : the surface area of each large block = 25 : 36

work out the volume of each large block.

..... cm^3

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H May 2021

1 Split the ratio

The surface area ratio of the small block to the large block is

$$25 : 36$$

2 Split the ratio

So the linear ratio is

$$\sqrt{25} : \sqrt{36} = 5 : 6$$

3 Split the ratio

Therefore the volume ratio is

$$5^3 : 6^3 = 125 : 216$$

4 The small block has volume

The small block has volume $300\ \text{cm}^3$.

5 Large block volume

Large block volume:

$$300 \times \frac{216}{125} = 518.4$$

FINAL ANSWER

$\boxed{518.4\ \text{cm}^3}$

4MA1/1H JANUARY 2022

Question 17

4 marks | Area & Volume of Similar Shapes

17 **A** and **B** are two similar vases.

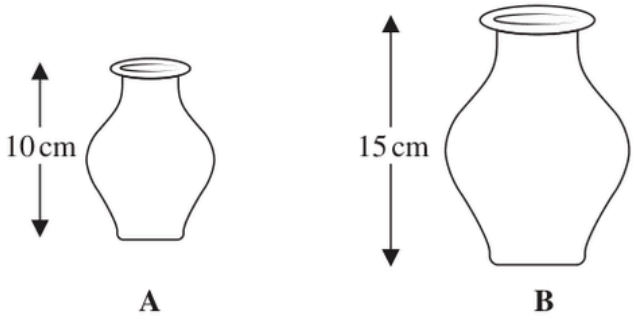


Diagram **NOT** accurately drawn

Vase **A** has height 10 cm.
Vase **B** has height 15 cm.

The difference between the volume of vase **A** and the volume of vase **B** is 1197 cm^3

Calculate the volume of vase **A**

..... cm^3

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H January 2022

1 **Calculate area**

The linear scale factor from vase A to vase B is

$$\frac{15}{10} = 1.5$$

2 **Calculate area**

So the volume scale factor is

$$1.5^3 = 3.375$$

3 **Let the volume of vase**

Let the volume of vase A be V.

$$3.375V - V = 1197$$

$$2.375V = 1197$$

$$V = 504$$

✓ **FINAL ANSWER**

$504\ \text{cm}^3$.

4MA1/1HR JANUARY 2022

Question 18

4 marks | Area & Volume of Similar Shapes

18 The three solids **A**, **B** and **C** are similar such that

the surface area of **A** : the surface area of **B** = 4 : 9

and

the volume of **B** : the volume of **C** = 125 : 343

Work out the ratio

the height of **A** : the height of **C**

Give your ratio in its simplest form.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2022

1 Split the ratio

Surface area ratio $A : B = 4 : 9$, so the length ratio is

$$\sqrt{4} : \sqrt{9} = 2 : 3$$

2 Split the ratio

Volume ratio $B : C = 125 : 343$, so the length ratio is

$$\sqrt[3]{125} : \sqrt[3]{343} = 5 : 7$$

3 Split the ratio

Therefore

$$\frac{A}{C} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

FINAL ANSWER

$10 : 21$

4MA1/1HR JUNE 2022

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar metal statues.



A



B

Diagram **NOT** accurately drawn

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2022

1 The volume of statue is

The volume of statue B is 20% less than statue A, so the volume scale factor is 0.8.

2 Calculate area

For similar solids, the linear scale factor is the cube root of the volume scale factor:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 Surface area scale factor is

Surface area scale factor is the square of the linear scale factor:

$$\text{surface area scale factor} = (\sqrt[3]{0.8})^2 = 0.8^{2/3} = 0.861773\ldots$$

4 **Calculate area**

So statue B has $86.1773...\%$ of the surface area of statue A.

$k = 100 - 86.1773... = 13.8226...$

✓ **FINAL ANSWER**

$k = 13.8$ to 3 significant figures.

Question 24

6 marks | Area & Volume of Similar Shapes

24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

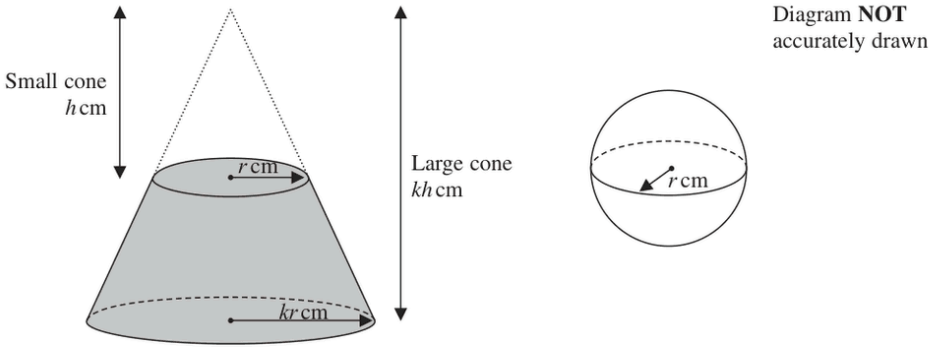


Diagram **NOT** accurately drawn

The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

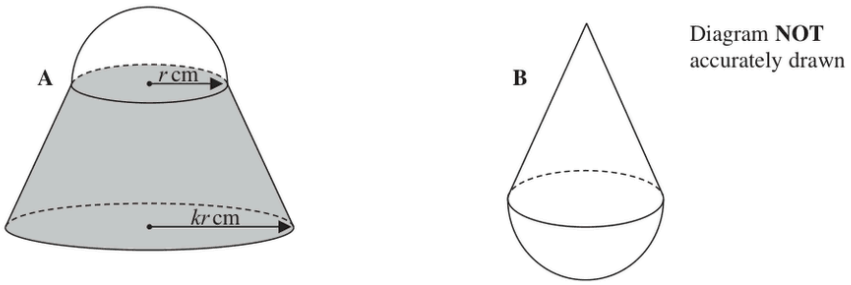


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

$h =$ .....

(Total for Question 24 is 6 marks)

1 Small cone volume

Small cone volume:

$$V_s = \frac{1}{3}\pi r^2 h$$

2 The larger similar cone has

The larger similar cone has radius kr and height kh, so its volume is

$$V_L = \frac{1}{3}\pi (kr)^2 (kh) = \frac{1}{3}\pi k^3 r^2 h$$

3 Calculate volume

So the frustum volume is

$$V_L - V_s = \frac{1}{3}\pi r^2 h(k^3 - 1)$$

4 The volume of one hemisphere

The volume of one hemisphere of radius r is

$$\frac{2}{3}\pi r^3$$

5 Solid A is the frustum

Solid A is the frustum plus one hemisphere:

$$V_A = \frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3$$

6 Solid B is the small

Solid B is the small cone plus one hemisphere:

$$V_B = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

7 Given

Given $V_A = 6V_B$:

$$\frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

8 **Divide by**

Divide by $\frac{1}{3}\pi r^2$:

$$h(k^3 - 1) + 2r = 6h + 12r$$

$$h(k^3 - 7) = 10r$$

$$h = \frac{10r}{k^3 - 7}$$

✓ **FINAL ANSWER**

$$h = \frac{10r}{k^3 - 7}$$

4MA1/1H MAY 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

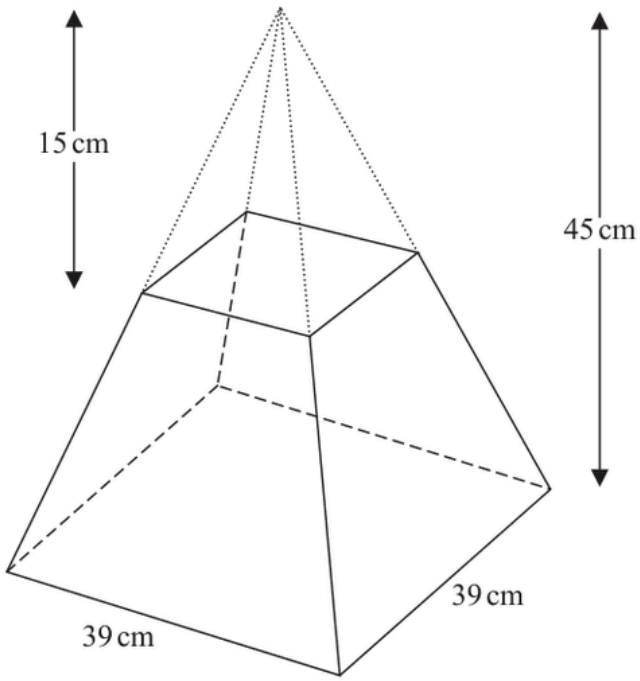


Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2023

1 The small and large square

The small and large square-based pyramids are similar.

2 **Their heights are**

Their heights are

$$15 : 45 = 1 : 3$$

---

3 **Square of the frustum**

So the side length of the top square of the frustum is

$$\frac{39}{3} = 13 \text{ cm}$$

---

4 **The vertical height of the**

The vertical height of the frustum is

$$45 - 15 = 30 \text{ cm}$$

---

5 **Evaluate fraction**

For one trapezium face, the difference between the half-side lengths is

$$\frac{39 - 13}{2} = 13 \text{ cm}$$

---

6 **Simplify surd**

So the slant height of the trapezium face is

$$\sqrt{30^2 + 13^2} = \sqrt{1069}$$

---

7 **Area of one trapezium face**

Area of one trapezium face:

$$\frac{1}{2}(39 + 13)\sqrt{1069} = 26\sqrt{1069}$$

---

8 **Area of four trapezium faces**

Area of four trapezium faces:

$$4(26\sqrt{1069}) = 104\sqrt{1069}$$

---

9 **Add the top and bottom**

Add the top and bottom square faces:

$$13^2 + 39^2 = 169 + 1521 = 1690$$

---

10 **Total surface area**

Total surface area:

$$1690 + 104\sqrt{1069} = 5090.338\ldots$$

---

11 Correct to the nearest whole

Correct to the nearest whole number:

$$5090$$

FINAL ANSWER

$$\boxed{5090\ \text{cm}^2}$$

4MA1/1H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Solid **A** is similar to solid **B**

Here is some information about solid **A** and solid **B**

	solid **A**	solid **B**
Height (cm)	3^x	
Area (cm^2)	7776	486
Volume (cm^3)	8^x	2^{x+4}

Work out the height of solid **B**
Give your answer as a decimal.

..... cm

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2023

1 **The area ratio of solid**

The area ratio of solid A to solid B is

$$7776 : 486 = 16 : 1$$

2 **Split the ratio**

So the linear ratio A:B is

$$\sqrt{16} : 1 = 4 : 1$$

3 **Split the ratio**

Therefore the volume ratio A:B is

$$4^3 : 1^3 = 64 : 1$$

4 **Calculate volume**

The volumes are 8^x and 2^{x+4}, so

$$\frac{8^x}{2^{x+4}} = 64$$

5 **Use index laws**

Write 8^x as a power of 2:

$$8^x = (2^3)^x = 2^{3x}$$

6 **Evaluate fraction**

So

$$\frac{2^{3x}}{2^{x+4}} = 2^6$$

$$2^{3x-(x+4)} = 2^6$$

$$2x - 4 = 6$$

$$x = 5$$

7 **The height of solid A**

The height of solid A is

$$3^x = 3^5 = 243$$

8 **Split the ratio**

Since the linear ratio A:B is $4 : 1$, the height of solid B is

$$\frac{243}{4} = 60.75$$

✓ **FINAL ANSWER**

$\boxed{60.75 \text{ cm}}$

4MA1/2H NOVEMBER 2023

Question 23

5 marks | Area & Volume of Similar Shapes

23 Here is a frustum of a cone.
The frustum is made by removing a small cone from a similar large cone.

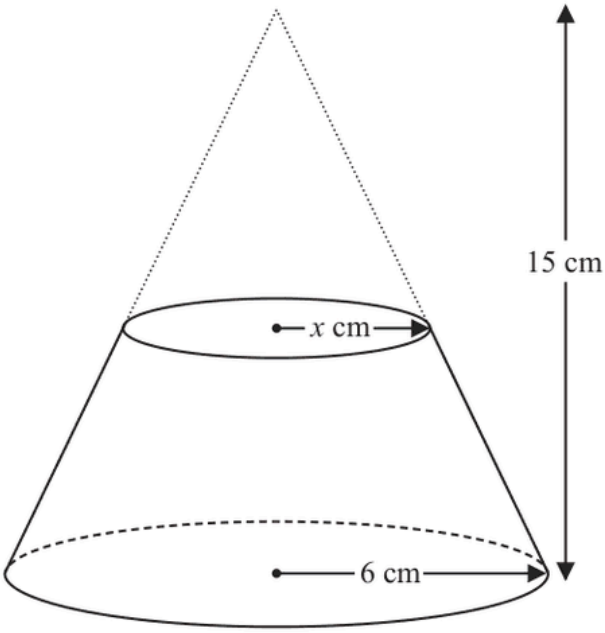


Diagram **NOT** accurately drawn

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is x cm.

Given that the volume of the frustum is $\frac{4212}{25}\pi$ cm^3

work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2023

1 **The large cone has height**

The large cone has height $\mathbf{15}$ cm and radius $\mathbf{6}$ cm.

2 **The small cone has radius**

The small cone has radius x. Since the cones are similar,

$$\frac{\text{radius}}{\text{height}} = \frac{6}{15} = \frac{2}{5}$$

3 **Evaluate fraction**

So for the small cone,

$$\frac{x}{h} = \frac{2}{5}$$

$$h = \frac{5}{2}x$$

4 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi(6)^2(15) = 180\pi$$

5 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$$

6 **The frustum volume is**

The frustum volume is

$$\frac{4212}{25}\pi$$

7 **Evaluate fraction**

So

$$180\pi - \frac{5}{6}\pi x^3 = \frac{4212}{25}\pi$$

8 **Cancel**

Cancel π:

$$180 - \frac{5}{6}x^3 = \frac{4212}{25}$$

$$\frac{5}{6}x^3 = 180 - \frac{4212}{25}$$

$$\frac{5}{6}x^3 = \frac{288}{25}$$

$$x^3 = \frac{1728}{125}$$

$$x = \frac{12}{5} = 2.4$$

✓ **FINAL ANSWER**

$\boxed{2.4 \text{ cm}}$

4MA1/1HR MAY 2024

Question 18

4 marks | Area & Volume of Similar Shapes

18 The diagram shows two similar vases, **A** and **B**

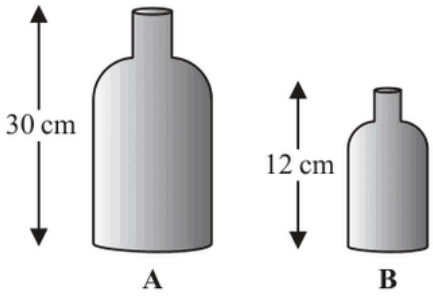


Diagram **NOT** accurately drawn

The height of vase **A** is 30 cm
The height of vase **B** is 12 cm

Given that

surface area of vase **A** – surface area of vase **B** = 178.5 cm²

find the surface area of vase **A**

..... cm²

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May 2024

1 **Calculate area**

The linear scale factor from vase B to vase A is

$$\frac{30}{12} = 2.5$$

2 **Calculate area**

So the surface area scale factor is

$$2.5^2 = 6.25$$

3 Calculate surface area

Let the surface area of vase B be S.

$$6.25S - S = 178.5$$

$$5.25S = 178.5$$

$$S = 34$$

4 Calculate area

Therefore the surface area of vase A is

$$6.25 \times 34 = 212.5$$

FINAL ANSWER

$212.5\ \text{cm}^2$.

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows a solid, **S**, made from a cone and a hemisphere.

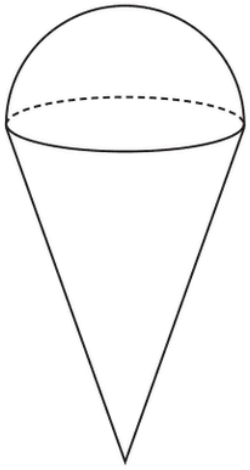


Diagram **NOT** accurately drawn

The centre of the circular face of the cone coincides with the centre of the flat surface of the hemisphere.

The radius of the circular face of the cone, x cm, is equal to the radius of the hemisphere.

The total height of **S** is 4 × the radius of the hemisphere.

A separate sphere has radius kx cm.
The volume of this sphere is 12.5 × the volume of **S**

(a) Work out the value of k

$k =$ .....

(4)

A solid, **T**, is similar to solid **S**

The volume of **T** is 512 × the volume of **S**

The total surface area of **T** is d × the total surface area of **S**

(b) Find the value of d

$d =$ .....

(1)

(Total for Question 24 is 5 marks)

1 **The hemisphere has radius**

The hemisphere has radius x, so its height is x.

2 **The total height of solid**

The total height of solid S is $4x$, so the cone height is

$$4x - x = 3x$$

3 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi x^2(3x) = \pi x^3$$

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

5 **Calculate volume**

So the volume of solid S is

$$\pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

6 **Calculate volume**

The sphere has radius kx, so its volume is

$$\frac{4}{3}\pi(kx)^3 = \frac{4}{3}\pi k^3 x^3$$

7 **This is times the volume**

This is 12.5 times the volume of S:

$$\frac{4}{3}\pi k^3 x^3 = 12.5\left(\frac{5}{3}\pi x^3\right)$$

8 **Cancel**

Cancel $\frac{1}{3}\pi x^3$:

$$4k^3 = 62.5$$

$$k^3 = 15.625 = \frac{125}{8}$$

$$k = 2.5$$

---

9 **Calculate volume**

For solid T, the volume scale factor is 512.

$$512 = 8^3$$

---

10 **Calculate area**

So the linear scale factor is 8, and the surface area scale factor is

$$8^2 = 64$$

✓ **FINAL ANSWER**

$$\boxed{k = 2.5,\ d = 64}$$

4WM2/01 MAY 2025

Question 25

4 marks | Area & Volume of Similar Shapes

25 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** − volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4WM2/01 May 2025

1 Linear scale factor from solid

Linear scale factor from solid A to solid B:

$$\frac{18.6}{31} = 0.6$$

2 Calculate volume

So the volume scale factor is

$$0.6^3 = 0.216$$

3 Let the volume of be

Let the volume of A be V. Then

$$V - 0.216V = 735$$

$$0.784V = 735$$

$$V = 937.5$$

FINAL ANSWER

$937.5\ \text{cm}^3$.

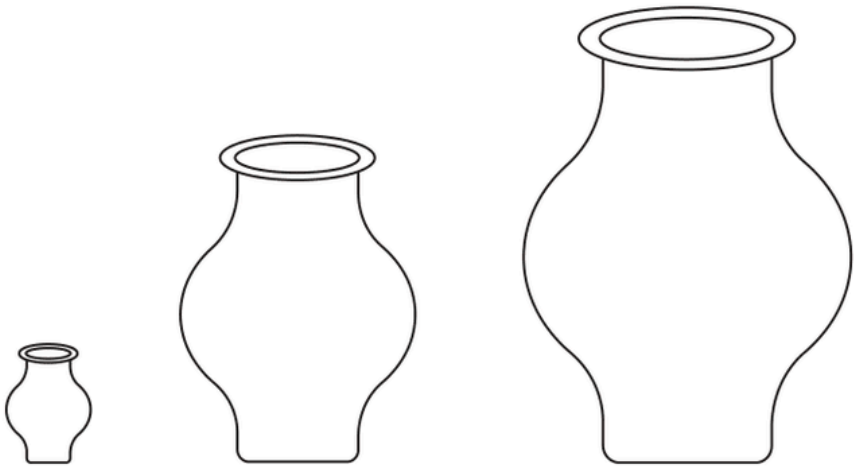
4MA1/1HR JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **R**, **S** and **T** are three similar vases.

Diagram **NOT** accurately drawn



The volume of vase **S** is 72.8% more than the volume of vase **R**

The height of vase **R** is h cm
The height of vase **T** is $6h$ cm

The surface area of vase **S** is A cm^2
The surface area of vase **T** is kA cm^2

Work out the value of k

$k =$ .....

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/1HR June 2025

1 **The volume of vase is**

The volume of vase S is 72.8% more than vase R, so

$$\frac{V_S}{V_R} = 1.728$$

2 **Calculate area**

For similar solids, volume scale factor is the cube of the linear scale factor:

$$\text{linear scale factor from } R \text{ to } S = \sqrt[3]{1.728} = 1.2$$

3 **The height of vase is**

The height of vase T is $6h$, so the linear scale factor from R to T is 6.

4 **Calculate area**

Therefore the linear scale factor from S to T is

$$\frac{6}{1.2} = 5$$

5 **Surface area scale factor is**

Surface area scale factor is the square of the linear scale factor:

$k = 5^2 = 25$

✓ **FINAL ANSWER**

$k = 25$

4MA1/2H JUNE 2025

Question 26

4 marks | Area & Volume of Similar Shapes

26 **A** and **B** are two similar solids.

The height of solid **A** is 31 cm
The height of solid **B** is 18.6 cm

Given that

volume of solid **A** – volume of solid **B** = 735 cm³

find the volume of solid **A**

..... cm³

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H June 2025

1 **The heights of solids A**

The heights of solids A and B are 31 cm and 18.6 cm.

$$31 : 18.6 = 5 : 3$$

2 **Split the ratio**

So the volumes are in the ratio

$$5^3 : 3^3 = 125 : 27$$

3 **The difference in volume is**

The difference in volume is

$$125 - 27 = 98$$

4 **parts**

parts.

5 **Evaluate fraction**

This difference is $735\ \text{cm}^3$, so one part is

$$\frac{735}{98} = 7.5$$

6 Volume of solid A

Volume of solid A:

$$125 \times 7.5 = 937.5$$

FINAL ANSWER

$\boxed{937.5\ \text{cm}^3}$

4WM2/01 NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM2/01 November 2025

1 Calculate area

$$textASF = fractextAreaofQtextAreaofP = frac450200 = frac94 = 2.25$$

2 Simplify surd

$$textLSF = sqrttextASF = sqrtfrac94 = frac32 = 1.5$$

3 Calculate volume

$$textVSF = (textLSF)^3 = left(frac32right)^3 = frac278 = 3.375$$

4 Calculate volume

$$textVolumeofQ = textVolumeofPtimestextVSF = 672timesfrac278$$

= 84 × 27 = 2268

FINAL ANSWER

2268 cm³

4MA1/2H NOVEMBER 2025

Question 23

3 marks | Area & Volume of Similar Shapes

23 Shape **P** is similar to shape **Q**

The table shows some information about shape **P** and shape **Q**

	Surface area (cm^2)	Volume (cm^3)
Shape **P**	200	672
Shape **Q**	450	

Work out the volume of shape **Q**

..... cm^3

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2025

1 **Surface area ratio Q P**

Surface area ratio Q:P:

$$450 : 200 = 9 : 4$$

2 **Split the ratio**

So the linear ratio Q:P is

$$3 : 2$$

3 **Split the ratio**

Therefore the volume ratio Q:P is

$$3^3 : 2^3 = 27 : 8$$

4 **Shape P has volume**

Shape P has volume $672\ \text{cm}^3$.

5 **Volume of Q**

Volume of Q:

$$672 \times \frac{27}{8} = 2268$$

FINAL ANSWER

$\boxed{2268\ \text{cm}^3}$

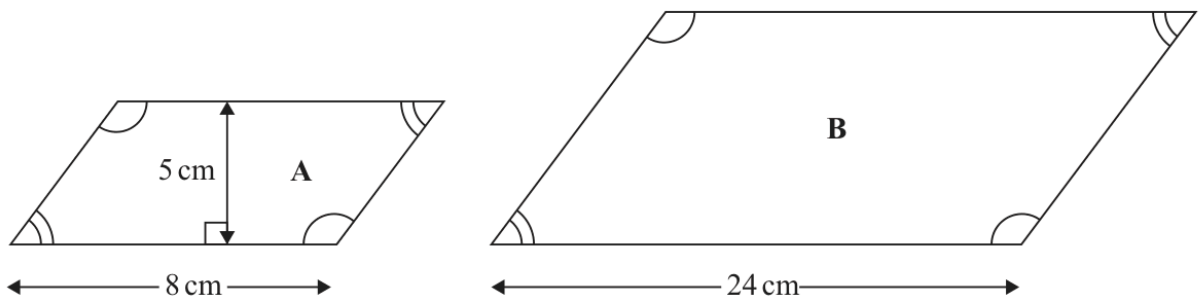
4MA1/1HR JUNE 2026

Question 6

3 marks | Area & Volume of Similar Shapes

6

Diagram **NOT** accurately drawn



A and **B** are similar parallelograms.

Work out the area of **B**

..... cm^2

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2026

1 Solution | Step 1

$$k = \frac{24}{8} = 3.$$

The marked corresponding bases give the linear scale factor from A to B.

2 Solution | Step 2

$$\text{area}(A) = 8 \times 5 = 40 \text{ cm}^2.$$

The 5 cm measurement is the perpendicular height.

3 Solution | Step 3

$$\text{area}(B) = k^2 \, \text{area}(A) = 3^2 \times 40 = 360 \text{ cm}^2.$$

Areas scale by the square of the linear scale factor.

FINAL ANSWER

$360\ \text{cm}^2$

4MA1/2HR JUNE 2026

Question 23

5 marks | Area & Volume of Similar Shapes

23 **K**, **L** and **M** are three similar solids.

volume of **K** : volume of **L** = 125 : 216

surface area of **K** : surface area of **M** = 4 : 9

The height of **M** is y% greater than the height of **L**

Work out the value of y
Show your working clearly.

$y =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2026

1 Solution | Step 1

$$h_K : h_L = \sqrt[3]{125} : \sqrt[3]{216} = 5 : 6.$$

Corresponding lengths have the cube-root ratio of volumes.

2 Solution | Step 2

$$h_K : h_M = \sqrt{4} : \sqrt{9} = 2 : 3.$$

Corresponding lengths have the square-root ratio of surface areas.

3 Solution | Step 3

Take $h_K = 10$ units. Then $h_L = 12$ and $h_M = 15$, so

$$y = \frac{15 - 12}{12} \times 100 = 25.$$

Align the shared K-length and compare the increase relative to L.

FINAL ANSWER

$y = 25$, so the height is 25% greater.

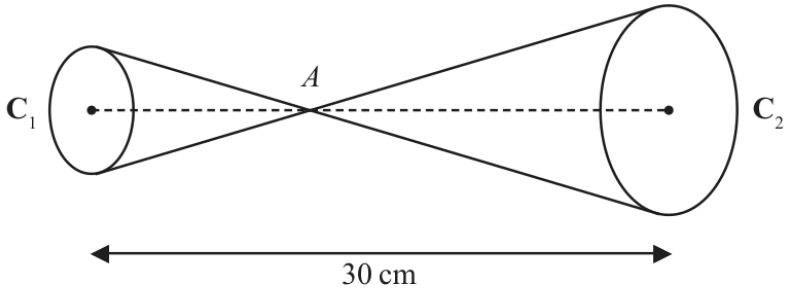
4MA1/2H JUNE 2026

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows two cones $\mathbf{C}_1$ and $\mathbf{C}_2$

Diagram **NOT** accurately drawn



The vertex of $\mathbf{C}_1$ coincides with the vertex of $\mathbf{C}_2$ at the point A
$\mathbf{C}_1$ and $\mathbf{C}_2$ have the same axis of symmetry.

$$\text{height of } \mathbf{C}_1 + \text{height of } \mathbf{C}_2 = 30 \text{ cm}$$

$$\text{total surface area of } \mathbf{C}_1 : \text{total surface area of } \mathbf{C}_2 = 4 : 9$$

$$\text{volume of } \mathbf{C}_1 = 23.04\pi \text{ cm}^3$$

Work out the radius of the base of $\mathbf{C}_2$
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2026

1 **Solution | Step 1**

The straight generators shown through A give equal vertex angles, so the cones are similar. The area ratio $4 : 9$ gives length ratio $\sqrt{4} : \sqrt{9} = 2 : 3$.
For similar solids, surface areas scale as the square of corresponding lengths.

2 **Solution | Step 2**

The heights total 30 cm, so

$$h_1 = \frac{2}{5}(30) = 12 \text{ cm}, \qquad h_2 = \frac{3}{5}(30) = 18 \text{ cm}.$$

Split the total in the common length ratio 2:3.

3 **Solution | Step 3**

$$\frac{1}{3}\pi r_1^2(12) = 23.04\pi \quad \Rightarrow \quad 4r_1^2 = 23.04 \quad \Rightarrow \quad r_1 = \sqrt{5.76} = 2.4 \text{ cm}.$$

Use the volume of the first cone and choose a positive radius.

4 **Solution | Step 4**

$$r_2 = \frac{3}{2}r_1 = \frac{3}{2}(2.4) = 3.6 \text{ cm}.$$

Radii scale by the same factor as the heights.

✓ **FINAL ANSWER**

3.6 cm

TOPIC

Circle Theorems

Every past-paper question on this topic, 2020 to 2026.

Question 14

5 marks | Circle Theorems

14 A, B, C and D are points on a circle, centre O

EBF is the tangent to the circle at B

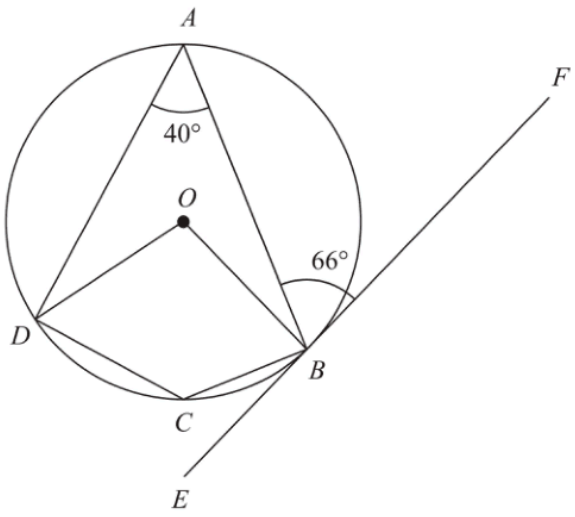


Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle DCB

.....

(1)

(ii) Give a reason for your answer to (a)(i)

.....

.....

(1)

(b) Work out the size of angle ADO

..... °

(3)

(Total for Question 14 is 5 marks)

1 **Use trigonometry**

In cyclic quadrilateral $ADCB$, opposite angles add to $180°$.

$$\angle DCB = 180° - 40° = 140°$$

2 **Reason opposite angles in a**

Reason: opposite angles in a cyclic quadrilateral add to $180°$.

3 **Use trigonometry**

By the alternate segment theorem,

$$\angle ADB = 66°$$

4 **Use trigonometry**

In triangle ADB,

$$\angle ABD = 180° - 40° - 66° = 74°$$

5 **The angle at the centre**

The angle at the centre is twice the angle at the circumference standing on the same chord AD:

$$\angle AOD = 2(74°) = 148°$$

6 **Use trigonometry**

Since $OA = OD$, triangle AOD is isosceles:

$$\angle ADO = \frac{180° - 148°}{2} = 16°$$

FINAL ANSWER

$\angle DCB = 140°$; reason: opposite angles in a cyclic quadrilateral; $\angle ADO = 16°$.

4MA1/2H JANUARY 2020

Question 13

4 marks | Circle Theorems

13

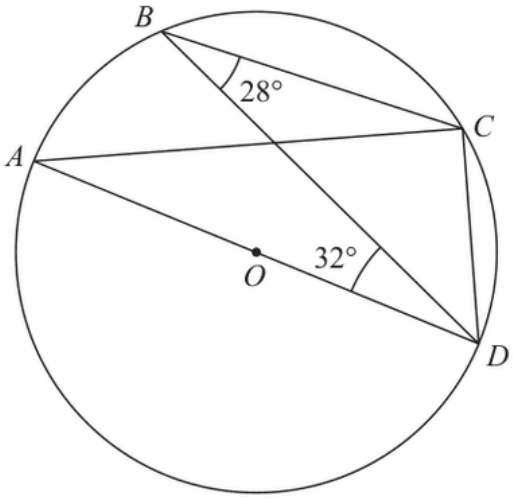


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O.
AOD is a diameter of the circle.

Angle $CBD = 28°$
Angle $BDA = 32°$

Find the size of angle BDC.
Give a reason for each stage of your working.

.....°

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2020

1 Use trigonometry

Since AD is a diameter,

$$\angle ABD = 90°$$

2 Given

Given

$$\angle CBD = 28°$$

3 **Use trigonometry**

So

$$\angle ABC = 90° - 28° = 62°$$

---

4 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$:

$$\angle ADC = 180° - 62° = 118°$$

---

5 **Use trigonometry**

At D,

$$\angle ADC = \angle BDA + \angle BDC$$

$$118° = 32° + \angle BDC$$

$$\angle BDC = 86°$$

✓ **FINAL ANSWER**

$86°$.

4MA1/1HR JANUARY 2020

Question 14

4 marks | Circle Theorems

14

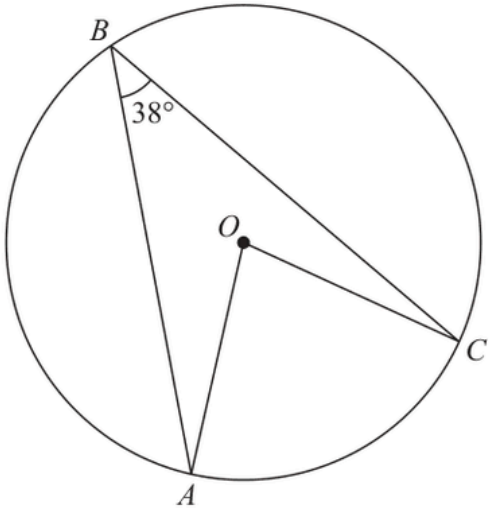


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O.
Angle $ABC = 38°$

Work out the size of angle OAC.
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2020

1 Angle and central angle stand

Angle ABC and central angle AOC stand on the same arc AC.

2 The angle at the centre

The angle at the centre is twice the angle at the circumference:

$$\angle AOC = 2 \times 38° = 76°$$

3 Use trigonometry

Since $OA = OC$, triangle AOC is isosceles.

$$\angle OAC = \angle OCA = \frac{180° - 76°}{2} = 52°$$

FINAL ANSWER

$52°$.

4MA1/2HR JANUARY 2020

Question 18

2 marks | Circle Theorems

18

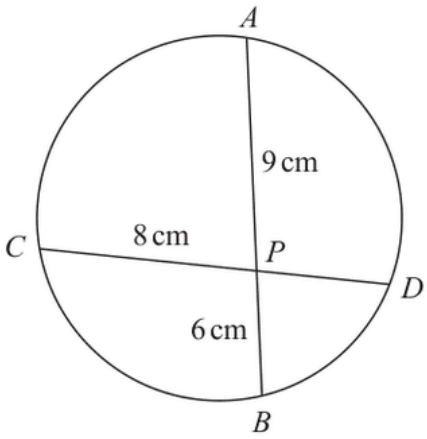


Diagram **NOT** accurately drawn

APB and *CPD* are chords of a circle.

$AP = 9\,\text{cm}$ $PB = 6\,\text{cm}$ $CP = 8\,\text{cm}$

Calculate the length of *PD*.

..... cm

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2020

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$AP \times PB = CP \times PD$$

2 Substitute the values

Substitute the values:

$$9 \times 6 = 8 \times PD$$

$$54 = 8PD$$

$$PD = 6.75$$

FINAL ANSWER

6.75 cm.

Question 15

4 marks | Circle Theorems

15

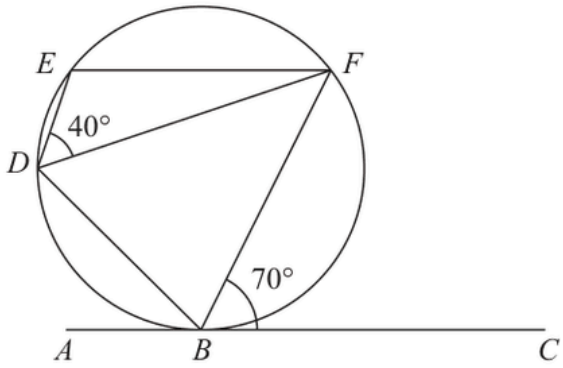


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle.
ABC is the tangent to the circle at B.

Angle $EDF = 40°$
Angle $FBC = 70°$

Prove that the tangent ABC is parallel to EF.
Give a reason for each stage of your working.

(Total for Question 15 is 4 marks)

1 Proof

By the alternate segment theorem, the angle between tangent BC and chord BF equals the angle in the opposite segment:

$$\angle BEF = 70°$$

Given

$$\angle EDF = 40°$$

Angles EDF and EBF stand on the same chord EF, so

$$\angle EBF = 40°$$

In triangle BEF,

$$\angle EFB = 180° - 70° - 40° = 70°$$

So

$$\angle EFB = \angle FBC = 70°$$

These are alternate interior angles, so

$$EF \parallel ABC$$

FINAL ANSWER

The tangent ABC is parallel to EF.

4MA1/1H MAY/NOVEMBER 2020

Question 16

4 marks | Circle Theorems

16

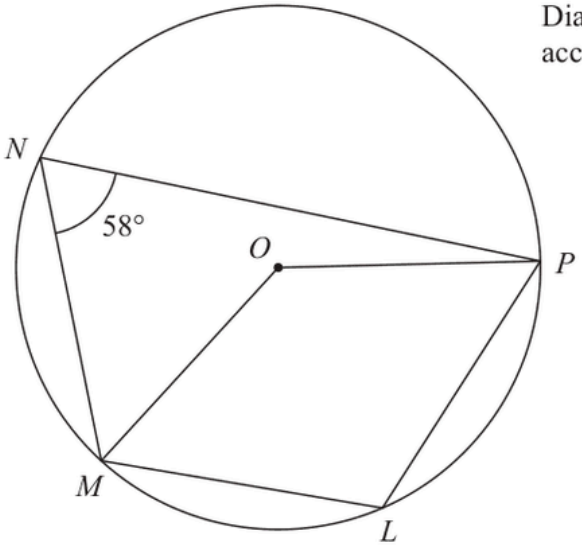


Diagram **NOT** accurately drawn

L, M, N and P are points on a circle, centre O

Angle $MNP = 58°$

(a) (i) Find the size of angle MLP

.....°

(ii) Give a reason for your answer.

.....

.....

(2)

(b) Find the size of the reflex angle MOP

.....°

.....

(2)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May/November 2020

1 (a) Opposite angles in cyclic

(a) Opposite angles in cyclic quadrilateral $MNPL$ add to $180°$:

$$\angle MLP = 180° - 58° = 122°$$

2 **(b) The angle at the**

(b) The angle at the centre is twice the angle at the circumference standing on the same arc MP:

$$\angle MOP = 2 \times 58^\circ = 116^\circ$$

---

3 **The reflex angle is**

The reflex angle is

$$360^\circ - 116^\circ = 244^\circ$$

✓ **FINAL ANSWER**

$\angle MLP = 122^\circ$, and reflex $\angle MOP = 244^\circ$.

4MA1/2H JANUARY 2021

Question 13

3 marks | Circle Theorems

13 P, Q, R, S and T are points on a circle with centre O.

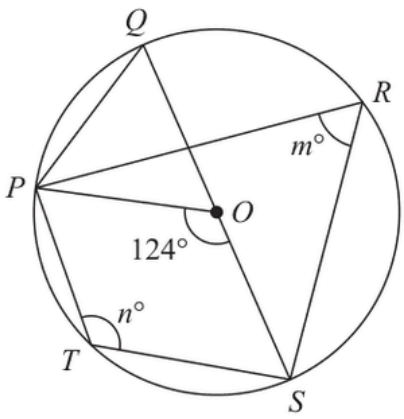


Diagram **NOT** accurately drawn

QOS is a diameter of the circle.

angle $POS = 124°$ angle $PRS = m°$ angle $PTS = n°$

(a) Find the value of

(i) m

.....

(ii) n

.....

(2)

(b) Find the size of angle QPO.

..... °

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2021

1 **Use trigonometry**

The central angle $\angle POS$ is $124°$. The angle at the circumference standing on the same chord PS is half this angle.

$$m = \frac{124}{2} = 62$$

2 **Use circle theorem**

Angles opposite each other in a cyclic quadrilateral add to $180°$, so

$$n = 180 - 62 = 118$$

---

3 **Use trigonometry**

Since QOS is a diameter,

$$\angle QOP = 180° - 124° = 56°$$

---

4 **Use trigonometry**

Triangle QOP is isosceles because $OQ = OP$. Therefore

$$\angle QPO = \frac{180 - 56}{2} = 62°$$

✓ **FINAL ANSWER**

$m = 62$, $n = 118$, and $\angle QPO = 62°$.

4MA1/1HR JANUARY 2021

Question 14

4 marks | Circle Theorems

14

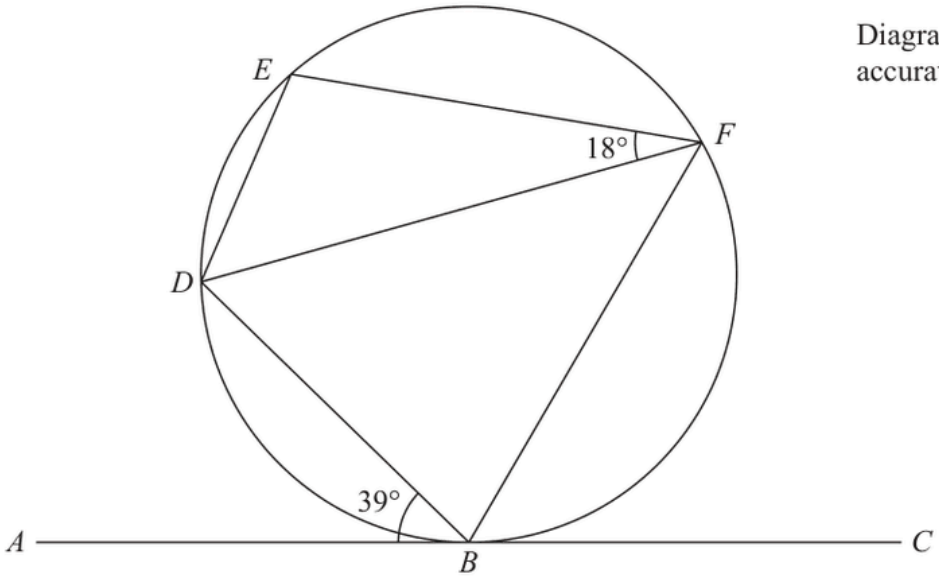


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle.

ABC is the tangent at B to the circle.
Angle $ABD = 39°$
Angle $EFD = 18°$

Work out the size of angle BDE.
Give reasons for your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2021

1 Use trigonometry

By the alternate segment theorem, the angle between the tangent and chord BD is equal to the angle in the opposite segment:

$$\angle BFD = 39°$$

2 Given

Given

$$\angle EFD = 18°$$

3 Use trigonometry

So

$$\angle BFE = 39^\circ + 18^\circ = 57^\circ$$

4 Opposite angles in cyclic quadrilateral

Opposite angles in cyclic quadrilateral $BDEF$ add to 180°:

$$\angle BDE = 180^\circ - 57^\circ = 123^\circ$$

FINAL ANSWER

123°.

4MA1/1H MAY 2021

Question 15

4 marks | Circle Theorems

15 P, Q and R are points on a circle, centre O.
TRV is the tangent to the circle at R.

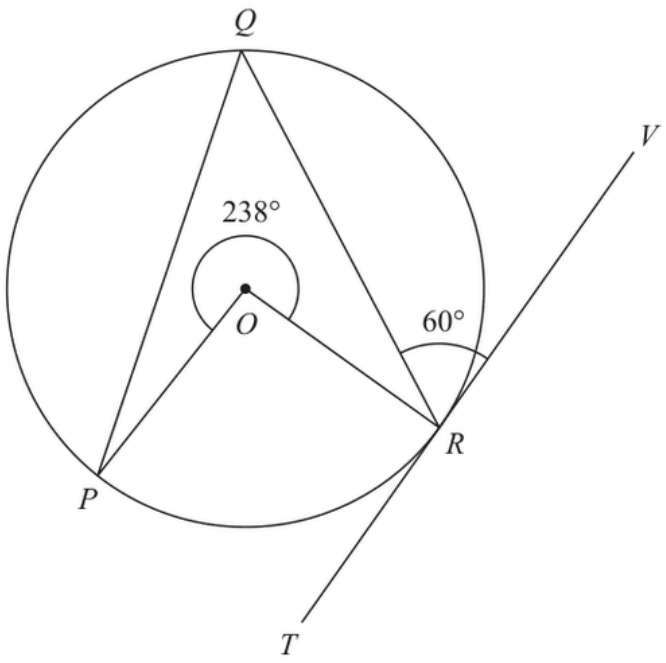


Diagram **NOT** accurately drawn

Reflex angle $POR = 238°$
Angle $QRV = 60°$

Calculate the size of angle OPQ.
Give a reason for each stage of your working.

.....°

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2021

1 **Use trigonometry**

The reflex angle POR is $238°$, so the smaller angle is

$$360° - 238° = 122°$$

2 **Use trigonometry**

Since $OP = OR$, triangle OPR is isosceles:

$$\angle OPR = \frac{180° - 122°}{2} = 29°$$

3 Use trigonometry

By the alternate segment theorem,

$$\angle QPR = 60°$$

4 Use trigonometry

Therefore

$$\angle QPO = 60° - 29° = 31°$$

FINAL ANSWER

$31°$.

4MA1/2H MAY 2021

Question 18

6 marks | Circle Theorems

18

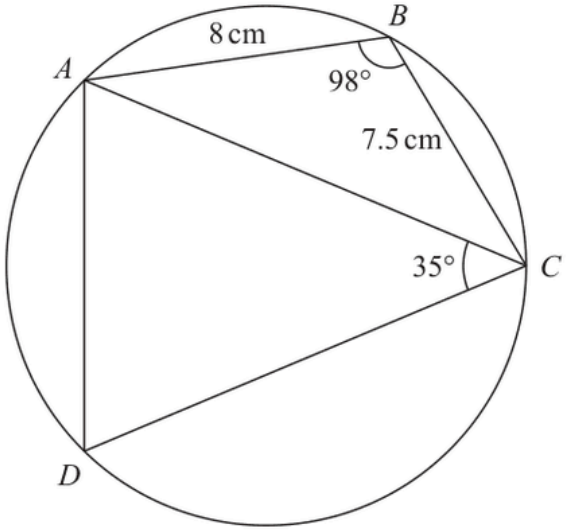


Diagram **NOT** accurately drawn

$ABCD$ is a quadrilateral where A, B, C and D are points on a circle.

$AB = 8$ cm
$BC = 7.5$ cm
Angle $ABC = 98°$
Angle $ACD = 35°$

Work out the perimeter of quadrilateral $ABCD$.
Give your answer correct to one decimal place.

..... cm

(Total for Question 18 is 6 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H May 2021

1 **Use trigonometry**

In triangle ABC, use the cosine rule:

$$AC^2 = 8^2 + 7.5^2 - 2(8)(7.5)\cos 98°$$

$$AC = 11.702\ldots$$

2 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$, so

$$\angle ADC = 180° - 98° = 82°$$

3 **Use trigonometry**

In triangle ACD,

$$\angle CAD = 180° - 82° - 35° = 63°$$

4 **Use the sine rule**

Use the sine rule:

$$AD = \frac{AC \sin 35°}{\sin 82°} = 6.778\ldots$$

$$CD = \frac{AC \sin 63°}{\sin 82°} = 10.529\ldots$$

5 **Perimeter**

Perimeter:

$$8 + 7.5 + 6.778\ldots + 10.529\ldots = 32.807\ldots$$

✓ **FINAL ANSWER**

$32.8\ \text{cm}$.

4MA1/2H NOVEMBER 2021

Question 11

3 marks | Circle Theorems

11

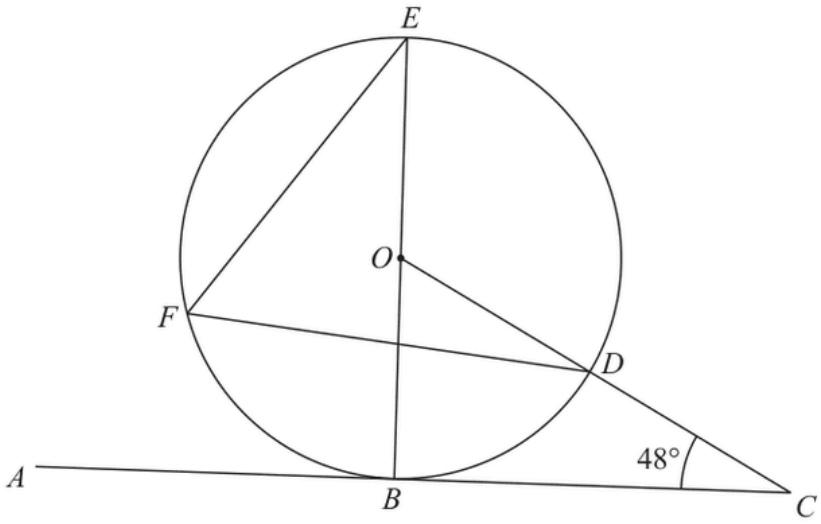


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle, centre O.
ABC is a tangent to the circle.
ODC is a straight line.

BOE is a diameter of the circle.

Angle $BCD = 48°$

Find the size of angle DFE.

.....°

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2021

1 The tangent at is perpendicular

The tangent at B is perpendicular to the radius OB, so

$$\angle OBC = 90°$$

2 Use trigonometry

In triangle OBC,

$$\angle BOC = 180° - 90° - 48° = 42°$$

3 **Use trigonometry**

Since O, D, C are on a straight line,

$$\angle BOD = 42°$$

---

4 **Use trigonometry**

BOK is a diameter, so BOE is a straight line and

$$\angle DOE = 180° - 42° = 138°$$

---

5 **The angle at the circumference**

The angle at the circumference is half the angle at the centre:

$$\angle DFE = \frac{138°}{2} = 69°$$

✓ **FINAL ANSWER**

$69°$.

4MA1/1HR JANUARY 2022

Question 14

4 marks | Circle Theorems

14

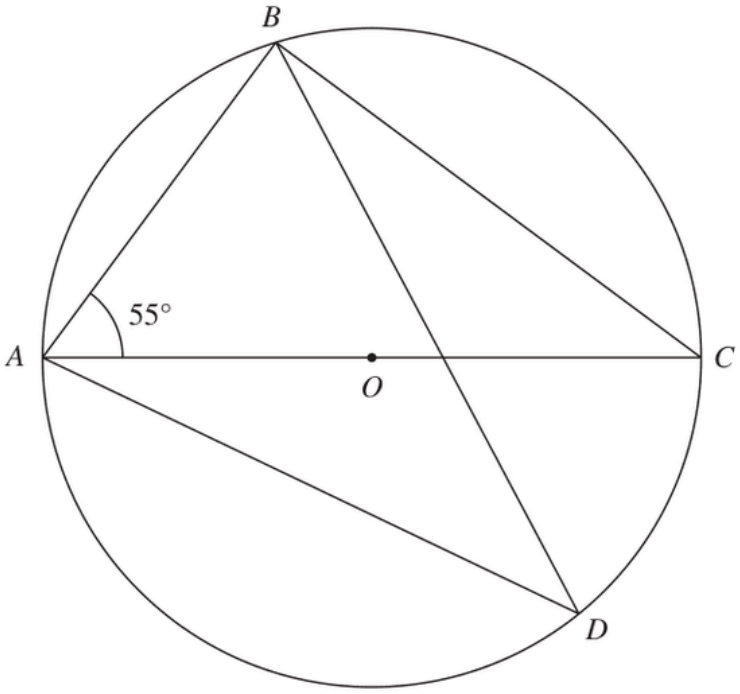


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O
AOC is a diameter of the circle.

Angle $BAC = 55°$

Work out the size of angle ADB
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2022

1 **Use trigonometry**

Since AC is a diameter,

$$\angle ABC = 90°$$

2 **Use trigonometry**

In triangle ABC,

$$\angle ACB = 180° - 90° - 55° = 35°$$

3

Angles in the same segment

Angles in the same segment are equal, so

$$\angle ADB = \angle ACB = 35^\circ$$

✓ **FINAL ANSWER**

35°.

4MA1/2H JANUARY 2022

Question 16

4 marks | Circle Theorems

16 D, E, F and G are points on a circle, centre O

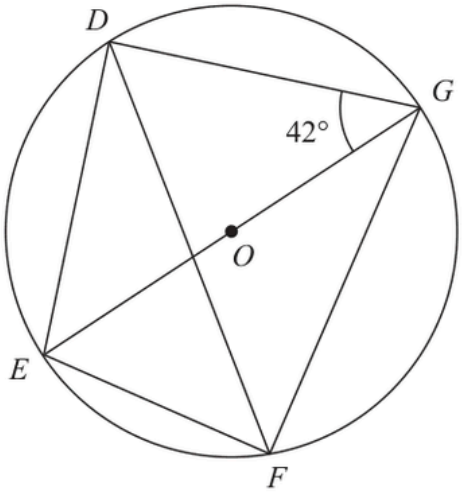


Diagram **NOT** accurately drawn

EOG is a diameter of the circle.

Angle $EGD = 42°$

Calculate the size of angle DFG
Give a reason for each stage of your working.

Angle DFG = ... °

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H January 2022

1 Use trigonometry

Since EOG is a diameter,

$$\angle EDG = 90°$$

2 Use trigonometry

In triangle EDG,

$$\angle DEG = 180° - 90° - 42° = 48°$$

3 Angles and stand on the

Angles DEG and DFG stand on the same chord DG, so they are equal.

$$\angle DFG = 48°$$

FINAL ANSWER

$48°$.

4MA1/2H JANUARY 2022

Question 19

3 marks | Circle Theorems

19

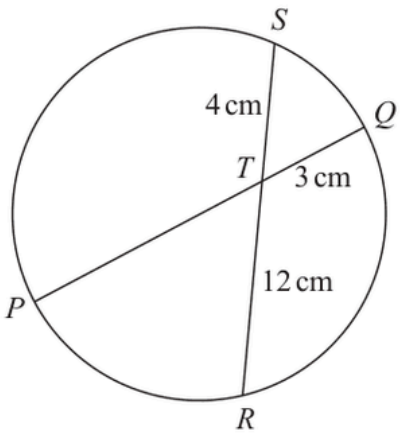


Diagram **NOT** accurately drawn

PTQ is a diameter of a circle.
RTS is a chord of the circle.

$TQ = 3\,\text{cm}$ $ST = 4\,\text{cm}$ $TR = 12\,\text{cm}$

Calculate the radius of the circle.

..... cm

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2022

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$PT \times TQ = ST \times TR$$

$$PT \times 3 = 4 \times 12$$

$$PT = 16$$

2 Use circle theorem

Since PTQ is a diameter,

$$PQ = PT + TQ = 16 + 3 = 19$$

3 Use circle theorem

So the radius is

$$19 \div 2 = 9.5$$

FINAL ANSWER

$9.5\ \text{cm}$.

Question 15

3 marks | Circle Theorems

15

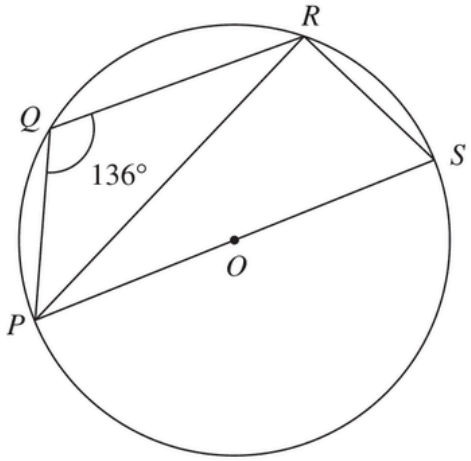


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136°$

Work out the size of angle RPS

.....°

(Total for Question 15 is 3 marks)

1 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$:

$$\angle PSR = 180° - 136° = 44°$$

2 **Use trigonometry**

Since PS is a diameter,

$$\angle PRS = 90°$$

3 **Use trigonometry**

In triangle PRS,

$$\angle RPS = 180° - 90° - 44° = 46°$$

✓ **FINAL ANSWER**

$46°$.

4MA1/2H JUNE 2023

Question 7

3 marks | Circle Theorems

7

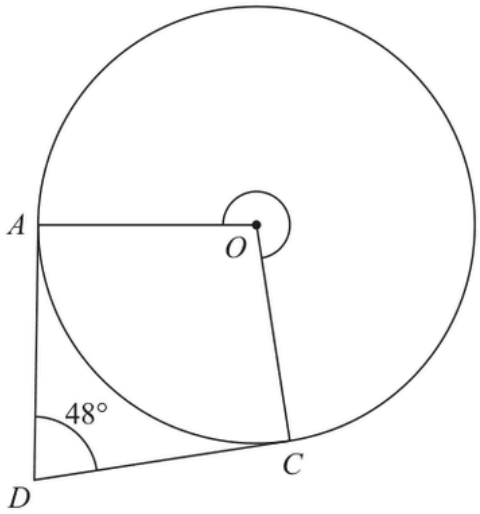


Diagram **NOT** accurately drawn

A and C are points on a circle, centre O
DA is the tangent to the circle at A and DC is the tangent to the circle at C

Angle $ADC = 48°$

Work out the size of reflex angle AOC

.....°

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2023

1 Use circle theorem

A radius is perpendicular to a tangent, so

$$\angle OAD = 90°$$

2 Use trigonometry

and

$$\angle OCD = 90°$$

3 Use trigonometry

In quadrilateral $AOCD$,

$$\angle AOC = 360° - 90° - 90° - 48° = 132°$$

4 **The question asks for the**

The question asks for the reflex angle:

$$360° - 132° = 228°$$

✓ **FINAL ANSWER**

$228°$.

Question 14

4 marks | Circle Theorems

14

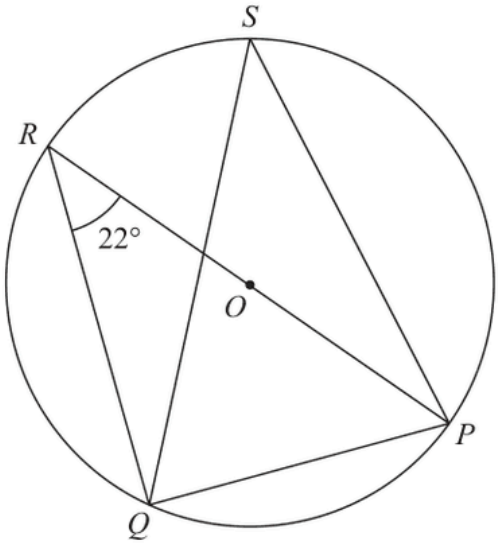


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle, centre O
ROP is a diameter of the circle.
Angle $PRQ = 22°$

(a) (i) Find the size of angle RQP

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Find the size of angle PSQ

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 14 is 4 marks)

1 **Use trigonometry**

(a) Since RP is a diameter,

$$\angle RQP = 90°$$

2 **Reason the angle in a**

Reason: the angle in a semicircle is $90°$.

3 **(b) Angles and stand on**

(b) Angles PRQ and PSQ stand on the same chord PQ, so they are equal.

$$\angle PSQ = 22°$$

✓ **FINAL ANSWER**

$\angle RQP = 90°$, and $\angle PSQ = 22°$.

4MA1/2HR JUNE 2023

Question 18

3 marks | Circle Theorems

18

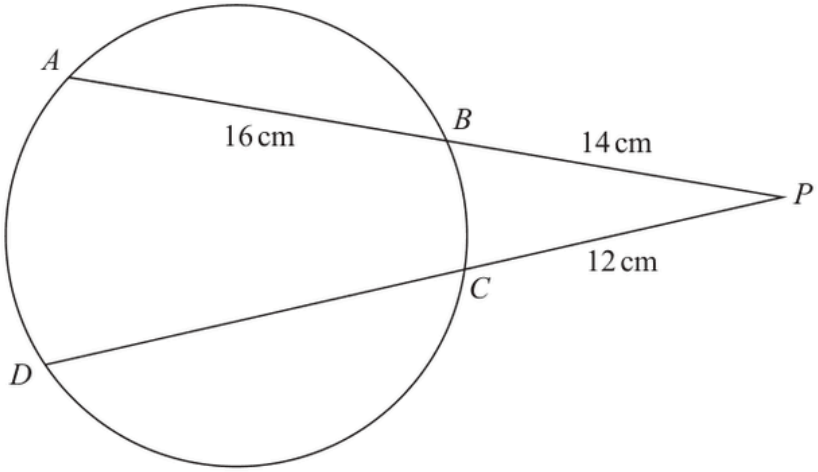


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle.

ABP and DCP are straight lines.

$AB = 16\,\text{cm}$ $BP = 14\,\text{cm}$ $CP = 12\,\text{cm}$

Work out the length of DC

..... cm

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2023

1 Use circle theorem

From the external point P, two secants meet the circle.

2 Use circle theorem

For the upper secant,

$$PB = 14, \qquad PA = 14 + 16 = 30$$

3 Use circle theorem

For the lower secant,

$$PC = 12, \qquad PD = 12 + DC$$

4 **Use circle theorem**

Using the secant theorem:

$$PB \times PA = PC \times PD$$

$$14 \times 30 = 12(12 + DC)$$

$$420 = 144 + 12DC$$

$$276 = 12DC$$

$$DC = 23$$

✓ **FINAL ANSWER**

$DC = 23$ cm.

4MA1/2HR JUNE 2023

Question 20

2 marks | Circle Theorems

20 A, B and C are points on a circle.

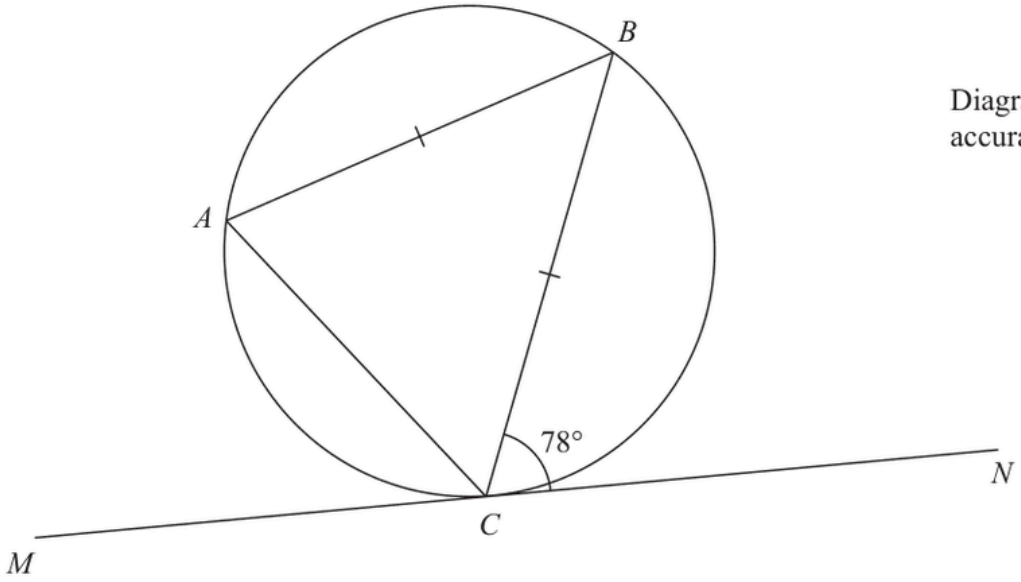


Diagram **NOT** accurately drawn

MN is the tangent to the circle at C

$AB = CB$

Angle $BCN = 78°$

Find the size of angle ABC

.....°

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2023

1 **The angle between the tangent**

The angle between the tangent CN and the chord CB is $78°$.

2 **Use trigonometry**

By the alternate segment theorem,

$$\angle BAC = 78°$$

3 **Calculate value**

Also,

$$AB = CB$$

---

4 **Use trigonometry**

so triangle ABC is isosceles and the base angles are equal:

$$\angle BAC = \angle ACB = 78^\circ$$

---

5 **Use trigonometry**

Using the angle sum of a triangle:

$$\angle ABC = 180^\circ - 78^\circ - 78^\circ$$

$$\angle ABC = 24^\circ$$

✓ **FINAL ANSWER**

$\boxed{24^\circ}$

4MA1/1H NOVEMBER 2023

Question 14

3 marks | Circle Theorems

14

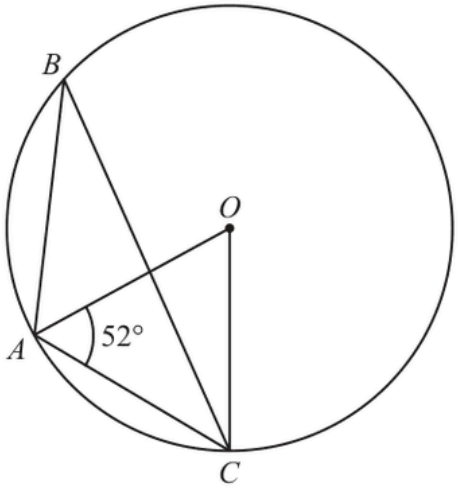


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O

Angle $OAC = 52°$

Find the size of angle ABC
Give reasons for your working.

.....°

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2023

1 Use trigonometry

Since $OA = OC$, triangle AOC is isosceles.

$$\angle ACO = \angle OAC = 52°$$

2 Use trigonometry

So

$$\angle AOC = 180° - 52° - 52° = 76°$$

3 The angle at the circumference

The angle at the circumference is half the angle at the centre standing on the same arc:

$$\angle ABC = \frac{76°}{2} = 38°$$

FINAL ANSWER

$38°$.

4MA1/1HR MAY 2024

Question 13

5 marks | Circle Theorems

13

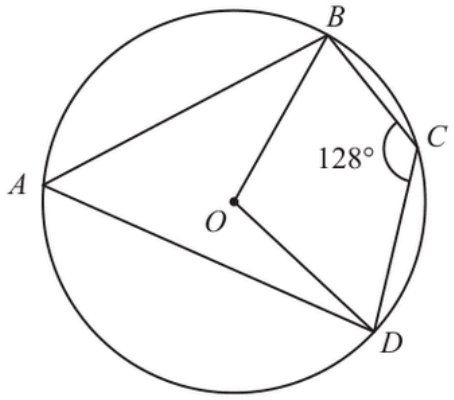


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

Angle $BCD = 128°$

Work out the size of angle OBD
Give a reason for each stage of your working.

angle OBD = ... °

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR May 2024

1 Opposite angles in cyclic quadrilateral

Opposite angles in cyclic quadrilateral $ABCD$ add to $180°$:

$$\angle BAD = 180° - 128° = 52°$$

2 The angle at the centre

The angle at the centre is twice the angle at the circumference standing on the same arc BD:

$$\angle BOD = 2 \times 52° = 104°$$

3 Use trigonometry

Since $OB = OD$, triangle OBD is isosceles:

$$\angle OBD = \frac{180° - 104°}{2} = 38°$$

FINAL ANSWER

$38°$.

Question 16

5 marks | Circle Theorems

16 A, B, C and D are points on a circle, centre O

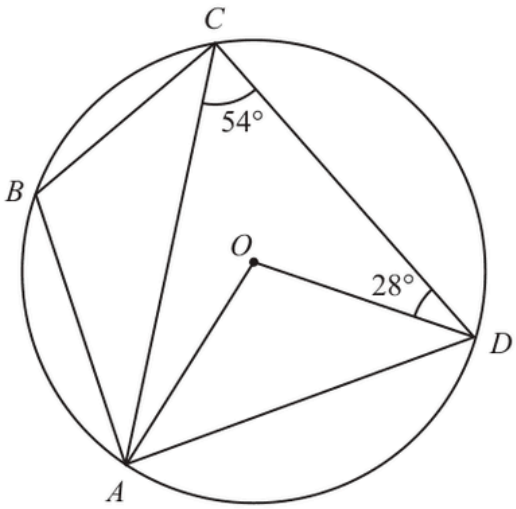


Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle AOD

.....°

(1)

(ii) Give a reason for your answer to part (a)(i)

.....

(1)

(b) Work out the size of angle CAO

.....°

(1)

(c) Work out the size of angle ABC

.....°

(2)

(Total for Question 16 is 5 marks)

1 **(a) The angle at the**

(a) The angle at the centre is twice the angle at the circumference standing on the same arc AD:

$$\angle AOD = 2 \times 54° = 108°$$

2 **Use trigonometry**

(b) Since $OC = OD$, triangle COD is isosceles:

$$\angle OCD = \angle CDO = 28°$$

3 **Use trigonometry**

So

$$\angle COD = 180° - 28° - 28° = 124°$$

4 **Use circle theorem**

Angles around O add to $360°$:

$$\angle AOC = 360° - 108° - 124° = 128°$$

5 **Use trigonometry**

Since $OA = OC$,

$$\angle CAO = \frac{180° - 128°}{2} = 26°$$

6 **(c) Angle stands on the**

(c) Angle ABC stands on the major arc AC, so

$$\angle ABC = \frac{360° - 128°}{2} = 116°$$

✓ **FINAL ANSWER**

$\angle AOD = 108°$, $\angle CAO = 26°$, and $\angle ABC = 116°$.

Question 13

4 marks | Circle Theorems

13 The diagram shows a circle, centre O

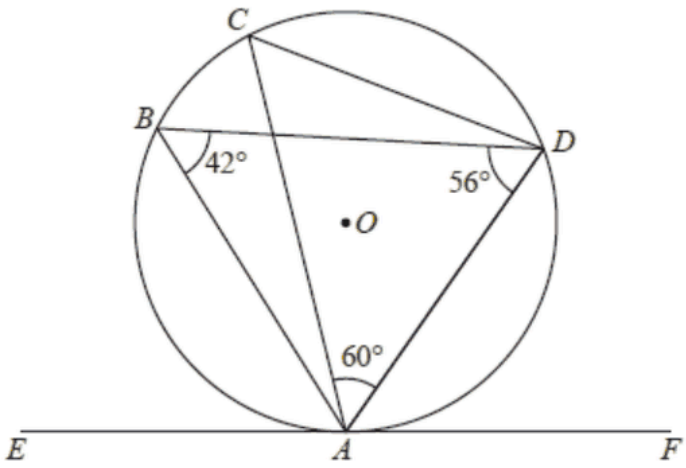


Diagram **NOT** accurately drawn

A, B, C and D are points on the circle.
EAF is the tangent to the circle at A

angle ABD = 42° **angle BDA = 56°** **angle CAD = 60°**

(a) (i) Find the size of angle EAB

..... °

(1)

(ii) Give a reason for your answer to (a) (i)

.....

.....

(1)

(b) Work out the size of angle CDB

..... °

(2)

(Total for Question 13 is 4 marks)

1 **Use trigonometry**

By the alternate segment theorem,

$$\angle EAB = \angle ADB = 56^\circ$$

2 **Use trigonometry**

In triangle ABD,

$$\angle BAD = 180^\circ - 42^\circ - 56^\circ = 82^\circ$$

3 **Use trigonometry**

So

$$\angle BAC = 82^\circ - 60^\circ = 22^\circ$$

4 **Angles subtended by the same**

Angles subtended by the same chord CB are equal, so

$$\angle CDB = 22^\circ$$

FINAL ANSWER

56°, alternate segment theorem, 22°.

4MA1/2HR JUNE 2025

Question 15

3 marks | Circle Theorems

15 A, B and C are points on a circle, centre O

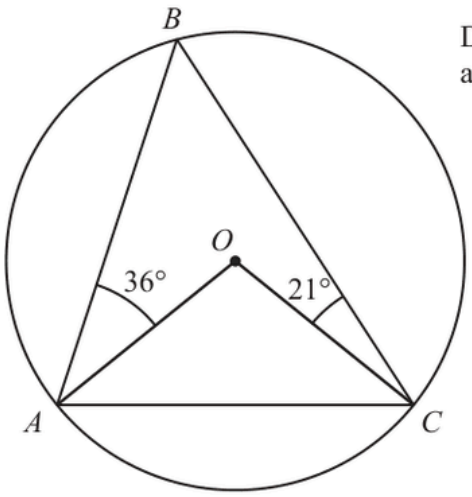


Diagram **NOT** accurately drawn

Angle $BAO = 36°$
Angle $BCO = 21°$

Work out the size of angle ACO

.....°

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2025

1 Use trigonometry

Since $OA = OB$, triangle AOB is isosceles:

$$\angle ABO = 36°$$

2 Use trigonometry

So

$$\angle AOB = 180° - 36° - 36° = 108°$$

3 Use trigonometry

Since $OB = OC$, triangle BOC is isosceles:

$$\angle OBC = 21°$$

4 **Use trigonometry**

So

$$\angle BOC = 180° - 21° - 21° = 138°$$

5 **Use circle theorem**

Angles around O add to $360°$:

$$\angle AOC = 360° - 108° - 138° = 114°$$

6 **Use trigonometry**

Since $OA = OC$,

$$\angle ACO = \frac{180° - 114°}{2} = 33°$$

✓ **FINAL ANSWER**

$33°$.

4WM2/01R MAY 2025

Question 15

3 marks | Circle Theorems

15 A, B and C are points on a circle, centre O

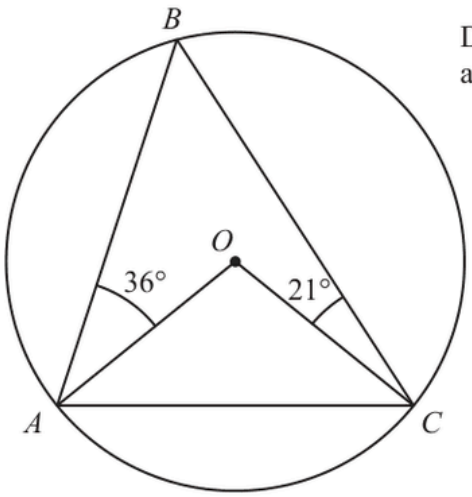


Diagram **NOT** accurately drawn

Angle $BAO = 36°$
Angle $BCO = 21°$

Work out the size of angle ACO

.....°

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM2/01R May 2025

1 Required

A, B, C on circle centre O. ∠*B**A**O* = 36°, ∠*B**C**O* = 21°. Find ∠*A**C**O*.

2 O A B is isosceles

△*O**A**B* is isosceles (*O**A*** = ***O**B*= radius):

∠*O**A**B* = ∠*O**B**A* = 36° ⟹ ∠*A**O**B* = 180° − 36° − 36° = 108°

3 O B C is isosceles

△*O**B**C* is isosceles (*O**B*** = ***O**C*):

∠*O**B**C* = ∠*O**C**B* = 21° ⟹ ∠*B**O**C* = 180° − 21° − 21° = 138°

4 Angles at centre O (on

Angles at centre O (on the same side):

∠*A**O**C* = 360° − 108° − 138° = 114°

5 O A C is isosceles

△*O**A**C* is isosceles (*O**A*** = ***O**C*):

$$angle OAC + angle OCA = 180^\circ - 114^\circ = 66^\circ implies angle ACO = frac66^\circ 2 = 33^\circ$$

FINAL ANSWER

∠*A**C**O* = 33°

4WM2/01R MAY 2025

Question 18

3 marks | Circle Theorems

18

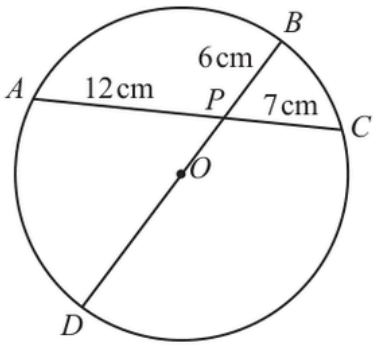


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

DB is a diameter of the circle.
AC and DB intersect at the point P

$AP = 12\,\text{cm}$ $PC = 7\,\text{cm}$ $PB = 6\,\text{cm}$

Work out the area of the circle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01R May 2025

1 **Required**

DB is a diameter; chords AC and DB intersect at P. AP = 12 cm, PC = 7 cm, PB = 6 cm. Find area of circle (3 s.f.).

2 **Apply the Intersecting Chords Theorem**

Apply the **Intersecting Chords Theorem**: *A**P** × *PC* = *B**P** × *PD*

$$12 times 7 = 6 times PD implies PD = frac{84}{6} = 14 textcm$$

3 **Diameter D B D B**

Diameter *D**B*:

*D**B** = *PD* + *P**B* = 14 + 6 = 20 cm ⟹ *r* = 10 cm

4 **Area of circle Area r**

Area of circle:

Area = *π**r*² = 100*π* ≈ 314.159… cm²

5 **Note**

Intersecting Chords Theorem: if two chords cross inside a circle at P, then *A**P* × **P**C* = *B**P* × **P**D*.

✓ **FINAL ANSWER**

Area of circle = 314 cm² (3 s.f.)

4MA1/1HR JUNE 2025

Question 23

5 marks | Circle Theorems

23

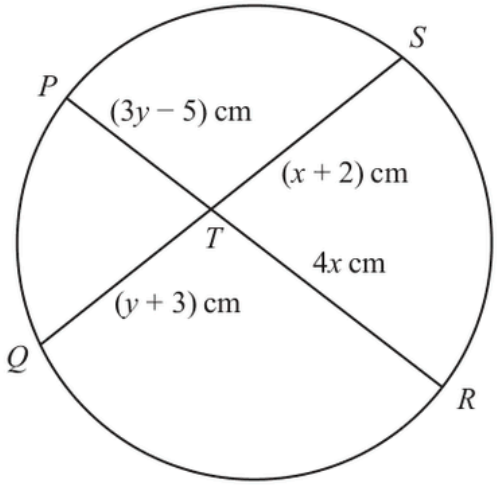


Diagram **NOT** accurately drawn

PTR and QTS are chords of a circle.

$PT = (3y - 5)\,\text{cm}$ $QT = (y + 3)\,\text{cm}$ $RT = 4x\,\text{cm}$ $ST = (x + 2)\,\text{cm}$

Find an expression for y in terms of x

$y =$ .....

Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2025

1 Use chord theorem

For two intersecting chords in a circle,

$$PT \times TR = QT \times TS$$

2 Substitute the given lengths

Substitute the given lengths:

$$(3y - 5)(4x) = (y + 3)(x + 2)$$

3 Expand

Expand:

$$12xy - 20x = xy + 2y + 3x + 6$$

4 **Collect the terms on one**

Collect the y terms on one side:

$$12xy - xy - 2y = 20x + 3x + 6$$

$$11xy - 2y = 23x + 6$$

---

5 **Factorise**

Factorise:

$$y(11x - 2) = 23x + 6$$

$$y = \frac{23x + 6}{11x - 2}$$

✓ **FINAL ANSWER**

$$\boxed{y = \frac{23x + 6}{11x - 2}}$$

Question 15

4 marks | Circle Theorems

15 A, B, C and D are points on a circle with centre O

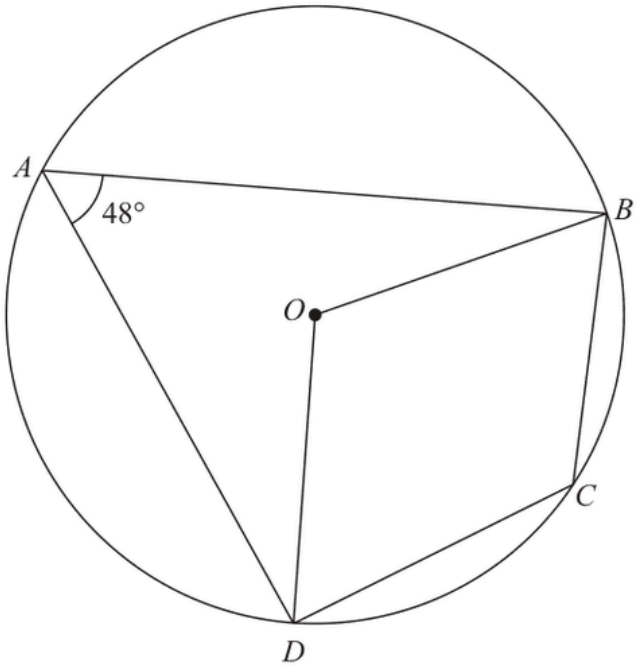


Diagram **NOT** accurately drawn

Angle $DAB = 48°$

(a) (i) Work out the size of the obtuse angle DOB

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Work out the size of angle BCD

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 15 is 4 marks)

1 **Part (a)(i) Angle D O**

Part (a)(i) Angle *DOB*:

∠*D**O**B* = 2 × 48[∘] = 96[∘]

2 **Answer**

(a)(i) 96[∘]

Part (a)(ii) Reason: The angle subtended by an arc at the centre is twice the angle subtended at the circumference.

Part (b)(i) Angle *BCD*:

∠*B**C**D* = 180[∘] − 48[∘] = 132[∘]

✓ **FINAL ANSWER**

(b)(i) 132[∘] **Part (b)(ii) Reason:** Opposite angles in a cyclic quadrilateral sum to 180[∘].

Question 15

4 marks | Circle Theorems

15 A, B, C and D are points on a circle with centre O

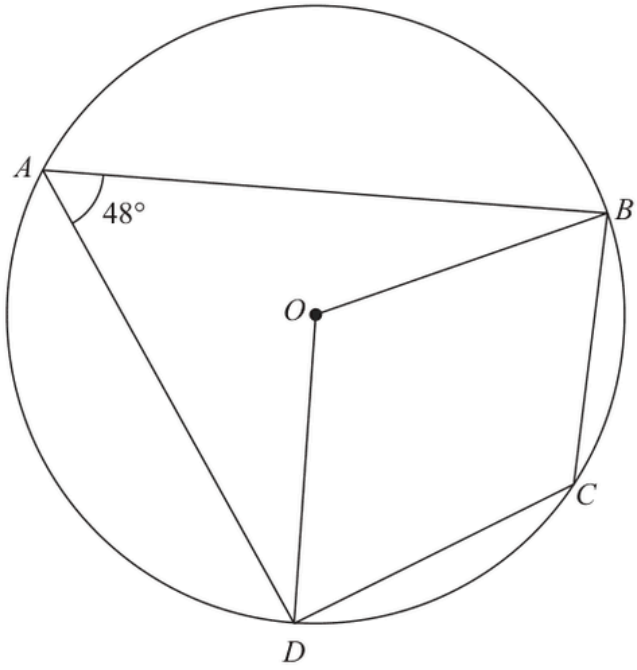


Diagram **NOT** accurately drawn

Angle $DAB = 48°$

(a) (i) Work out the size of the obtuse angle DOB

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(b) (i) Work out the size of angle BCD

.....°

(1)

(ii) Give a reason for your answer.

.....

(1)

(Total for Question 15 is 4 marks)

1 **(a) The angle at the**

(a) The angle at the centre is twice the angle at the circumference standing on chord DB:

$$\angle DOB = 2 \times 48° = 96°$$

2 **(b) Opposite angles in cyclic**

(b) Opposite angles in cyclic quadrilateral $ABCD$ add to $180°$:

$$\angle BCD = 180° - 48° = 132°$$

✓ **FINAL ANSWER**

$\angle DOB = 96°$, and $\angle BCD = 132°$.

Question 14

4 marks | Circle Theorems

14

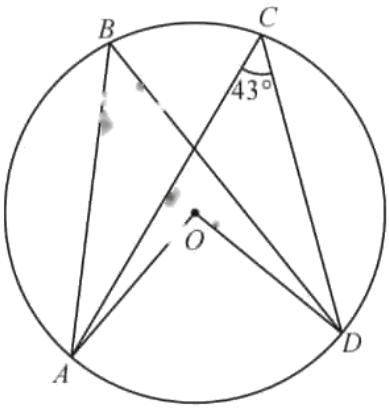


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O

angle $ACD = 43°$

(a) (i) Find the size of angle ABD

..... °

(1)

(ii) Give a reason for your answer.

.....

.....

(1)

(b) Find the size of the reflex angle AOD

..... °

(2)

(Total for Question 14 is 4 marks)

1 **(a)(i) Same segment**

$$\angle ABD = \angle ACD = 43^\circ$$

2 **(a)(ii) Reason**

Angles in the same segment are equal (both stand on the arc AD).

3 **(b) Angle at the centre**

$$\angle AOD = 2 \times 43^\circ = 86^\circ \quad \text{(angle at centre is twice the angle at the circumference)}$$

4 **Reflex angle**

$$360^\circ - 86^\circ = 274^\circ$$

✓ **FINAL ANSWER**

(a) 43°, angles in the same segment are equal (b) 274°

4MA1/1HR JUNE 2026

Question 19

5 marks | Circle Theorems

19 P, Q, R and S are points on a circle, centre O

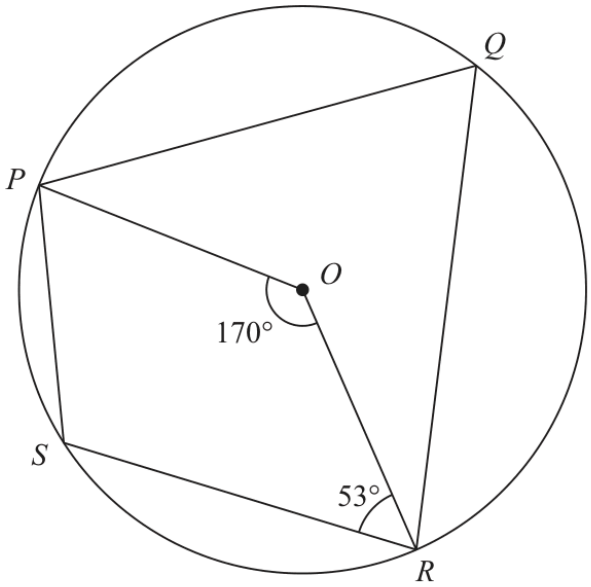


Diagram **NOT** accurately drawn

Angle $POR = 170°$ Angle $ORS = 53°$

Find the size of angle SPO
Give a reason for each stage of your working.

.....°

.....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$\angle PQR = \frac{170°}{2} = 85°.$$

The angle at the centre is twice the angle at the circumference subtended by the same arc PR containing S.

2 **Solution | Step 2**

$$\angle PSR = 180° - 85° = 95°.$$

Opposite angles of cyclic quadrilateral PQRS sum to 180 degrees.

3 Solution | Step 3

$$\angle SPO = 360^\circ - (170^\circ + 53^\circ + 95^\circ) = 42^\circ.$$

The interior angles of quadrilateral PSRO sum to 360 degrees; its other angles are POR, ORS and PSR.

FINAL ANSWER

$\angle SPO = 42^\circ$

4MA1/2H JUNE 2026

Question 19

2 marks | Circle Theorems

19 B, D and E are points on a circle.

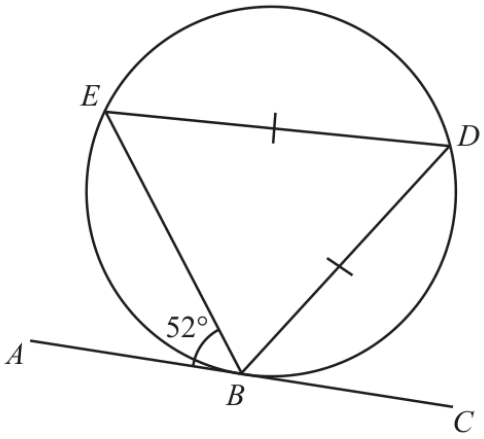


Diagram **NOT** accurately drawn

ABC is the tangent to the circle at B

angle $ABE = 52°$ $ED = BD$

Work out the size of angle BED

.....°

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2026

1 **Solution | Step 1**

$\angle BDE = \angle ABE = 52°$, by the alternate segment theorem for tangent AB and chord BE.
The angle between a tangent and chord equals the angle in the opposite segment.

2 **Solution | Step 2**

Since $ED = BD$, the base angles $\angle BED$ and $\angle EBD$ are equal. Thus

$$\angle BED = \frac{180° - 52°}{2} = 64°.$$

Equal sides in triangle EBD subtend equal opposite angles, and its angles sum to 180 degrees.

✓ **FINAL ANSWER**

$64°$

4MA1/1HR JUNE 2026

Question 24

4 marks | Circle Theorems

24 The centre of a circle is the point with coordinates (3, 1)

The point P with coordinates (7, 6) lies on the circle.

The line **L** is the tangent to the circle at P

Find an equation for **L**
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

.....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2026

1 Solution | Step 1

$$m_{CP} = \frac{6-1}{7-3} = \frac{5}{4}.$$

Find the gradient of the radius from the centre C=(3,1) to P=(7,6).

2 Solution | Step 2

$$m_L = -\frac{4}{5}.$$

The tangent is perpendicular to the radius at P, so its gradient is the negative reciprocal.

3 Solution | Step 3

$$y - 6 = -\frac{4}{5}(x - 7).$$

Use the tangent gradient and the given point of contact.

4 Solution | Step 4

$$5y - 30 = -4x + 28 \quad \Longrightarrow \quad 4x + 5y - 58 = 0.$$

Multiply by 5 and collect all terms on one side to obtain integer coefficients.

✓ **FINAL ANSWER**

$$4x + 5y - 58 = 0$$

TOPIC

Bearings

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2021

Question 16

5 marks | Bearings

16 The diagram shows the positions of three ships, A, B and C.

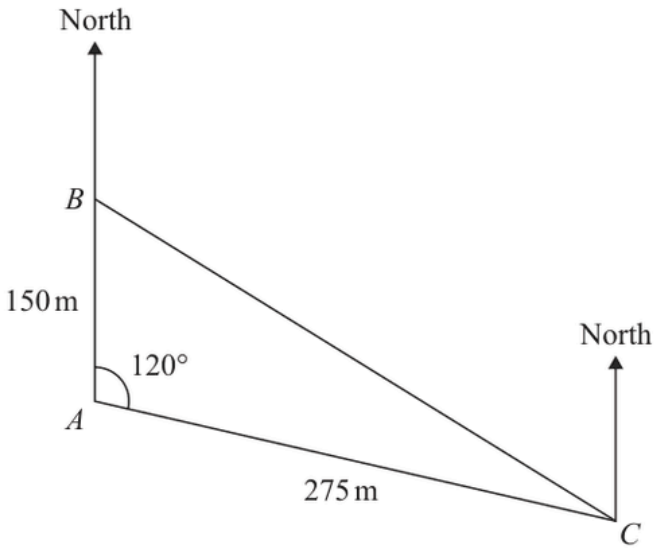


Diagram **NOT** accurately drawn

Ship B is due north of ship A.

The bearing of ship C from ship A is 120°

Calculate the bearing of ship C from ship B.
Give your answer correct to the nearest degree.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2021

1 **Use bearings**

From A, ship C is on a bearing of $120°$, and $AC = 275\ \text{m}$.

2 **Resolve**

Resolve AC:

$$\text{east component} = 275\sin 120^\circ = 238.16\ldots$$

$$\text{north component} = 275\cos 120^\circ = -137.5$$

---

3 **Use bearings**

Ship B is $150\ \text{m}$ north of A, so from B to C:

$$\text{east} = 238.16\ldots, \qquad \text{north} = -137.5 - 150 = -287.5$$

---

4 **Use bearings**

The bearing is therefore in the south-east quadrant:

$$180^\circ - \tan^{-1}\left(\frac{238.16\ldots}{287.5}\right) = 140.36\ldots^\circ$$

✓ **FINAL ANSWER**

140°.

4MA1/2H MAY 2021

Question 3

2 marks | Bearings

3 The bearing of Paris from London is 149°

Work out the bearing of London from Paris.

.....°

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H May 2021

1 Use bearings

The bearing of Paris from London is the reverse bearing of London from Paris.

2 Reverse bearings differ by

Reverse bearings differ by $180°$:

$$149° + 180° = 329°$$

FINAL ANSWER

$329°$.

4MA1/1H NOVEMBER 2023

Question 18

5 marks | Bearings

18 The diagram shows the positions of three villages, A, B and C

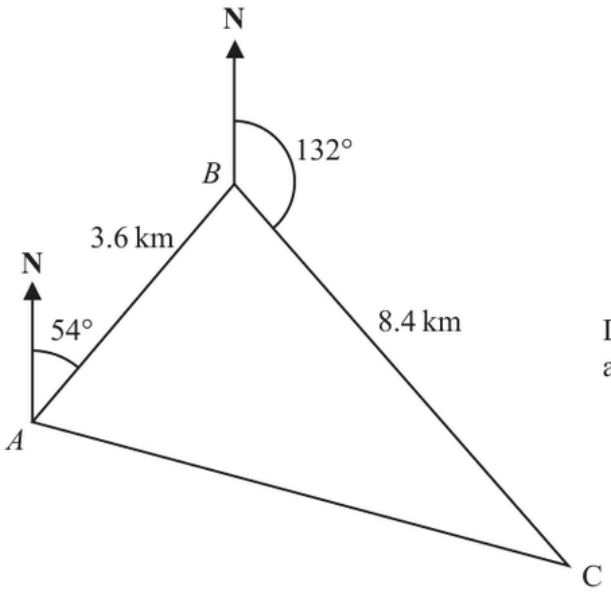


Diagram **NOT** accurately drawn

The bearing of B from A is 054°
The bearing of C from B is 132°

Melur walks from A to B
She then walks from B to C and from C to A

Melur walks at an average speed of 6 km/h

Work out the total time Melur takes.
Give your answer in hours and minutes.

..... hours ..... minutes

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2023

1 The bearing of from is

The bearing of B from A is $054°$, so the bearing of A from B is

$$054° + 180° = 234°$$

2 The bearing of from is

The bearing of C from B is $132°$.

3 **Use bearings**

So

$$\angle ABC = 234° - 132° = 102°$$

---

4 **Use cosine rule**

Use the cosine rule to find AC:

$$AC^2 = 3.6^2 + 8.4^2 - 2(3.6)(8.4)\cos 102°$$

$$AC = 9.8028\ldots \text{ km}$$

---

5 **Total distance walked**

Total distance walked:

$$3.6 + 8.4 + 9.8028\ldots = 21.8028\ldots \text{ km}$$

---

6 **Use bearings**

At 6 km/h, time is

$$21.8028\ldots \div 6 = 3.6338\ldots \text{ hours}$$

$$0.6338\ldots \times 60 = 38.0\ldots \text{ minutes}$$

✓ **FINAL ANSWER**

3 hours 38 minutes.

4MA1/2H NOVEMBER 2024

Question 24

6 marks | Bearings

24 A, B and C are three points on horizontal ground.

$AB = 8.4$ metres $BC = 9.2$ metres

B is on a bearing of 067° from A
C is on a bearing of 129° from B

Calculate the bearing of A from C
Give your answer correct to the nearest degree.

.....°

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2024

1 Resolve each journey into east

Resolve each journey into east and north components.

2 Use trigonometry

From A to B:

$$\text{east} = 8.4 \sin 67° = 7.7322\ldots$$

$$\text{north} = 8.4 \cos 67° = 3.2821\ldots$$

3 Use trigonometry

From B to C:

$$\text{east} = 9.2 \sin 129° = 7.1497\ldots$$

$$\text{north} = 9.2 \cos 129° = -5.7897\ldots$$

4 **Calculate value**

So from A to C:

$$\text{east} = 14.8820\ldots$$

$$\text{north} = -2.5076\ldots$$

---

5 **Calculate value**

Therefore from C to A:

$$\text{east} = -14.8820\ldots, \qquad \text{north} = 2.5076\ldots$$

---

6 **Use trigonometry**

This direction is west of north.

$$\theta = \tan^{-1}\left(\frac{14.8820\ldots}{2.5076\ldots}\right) = 80.4355\ldots^\circ$$

---

7 **Use bearings**

Bearing of A from C:

$$360^\circ - 80.4355\ldots^\circ = 279.5644\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{280^\circ}$

TOPIC

Transformations

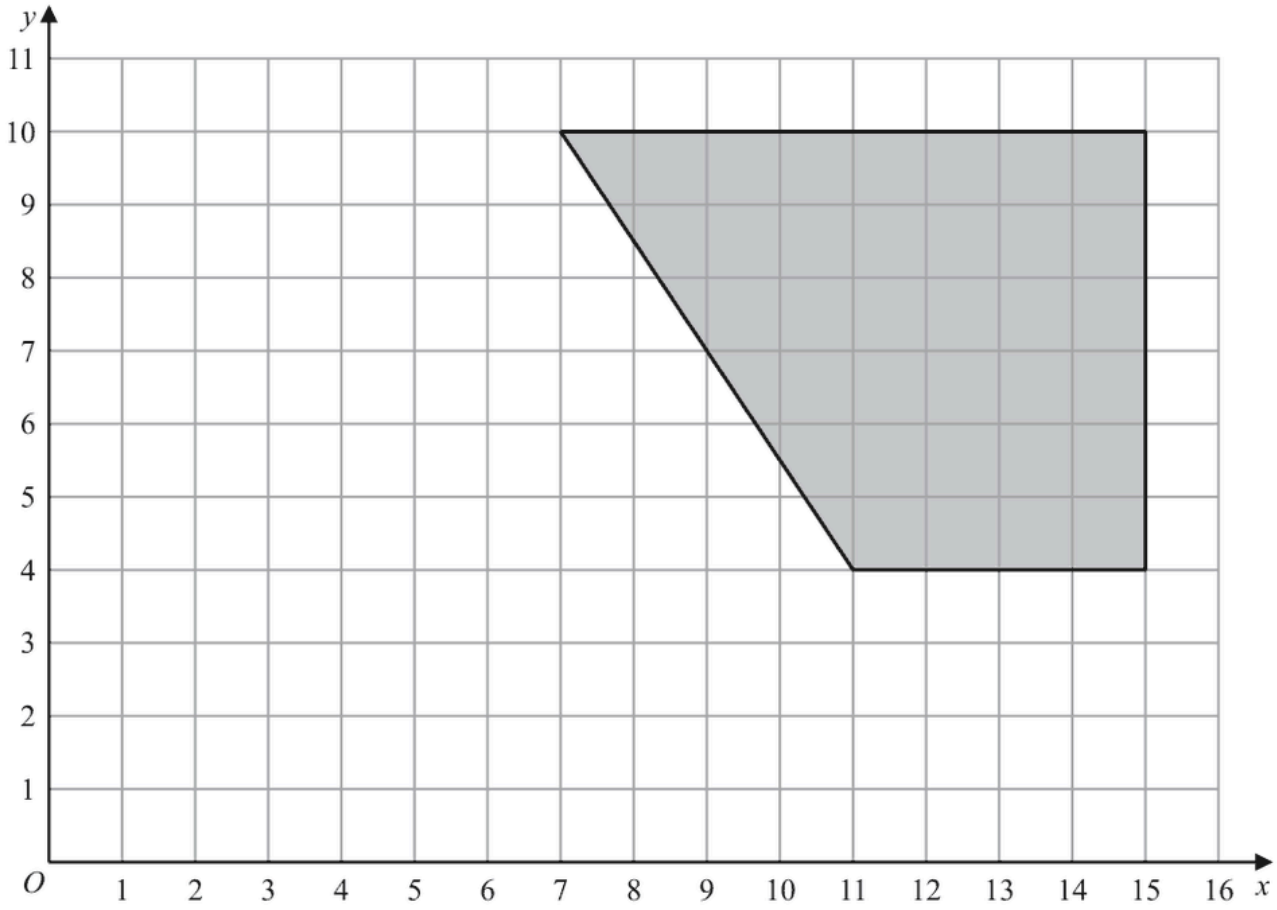
Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 8

2 marks | Transformations

8



On the grid, enlarge the shaded shape with scale factor $\frac{1}{2}$ and centre (1, 2)

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2020

1 **Use**

Use

$$\text{new point} = (1, 2) + \frac{1}{2}(\text{old point} - (1, 2))$$

2 **The vertices**

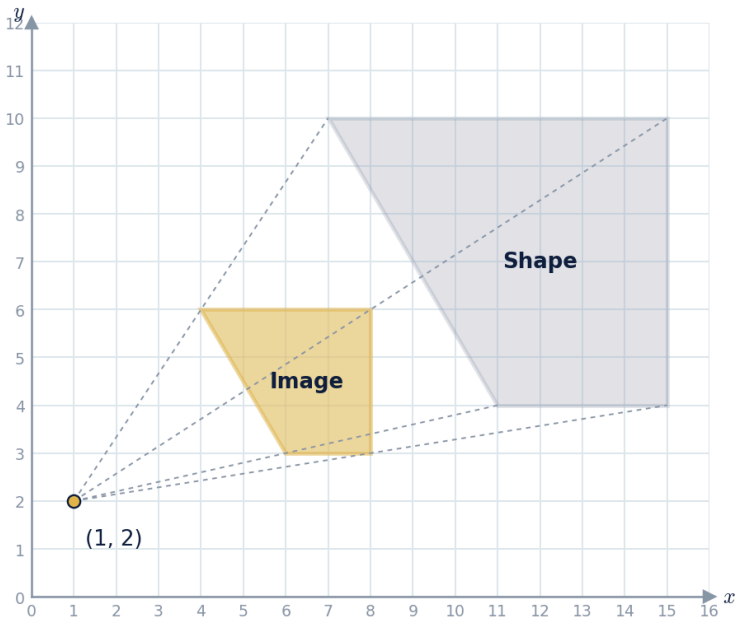
The vertices

$$(7, 10),\ (15, 10),\ (15, 4),\ (11, 4)$$

3 **become**

become

$$(4, 6),\ (8, 6),\ (8, 3),\ (6, 3)$$



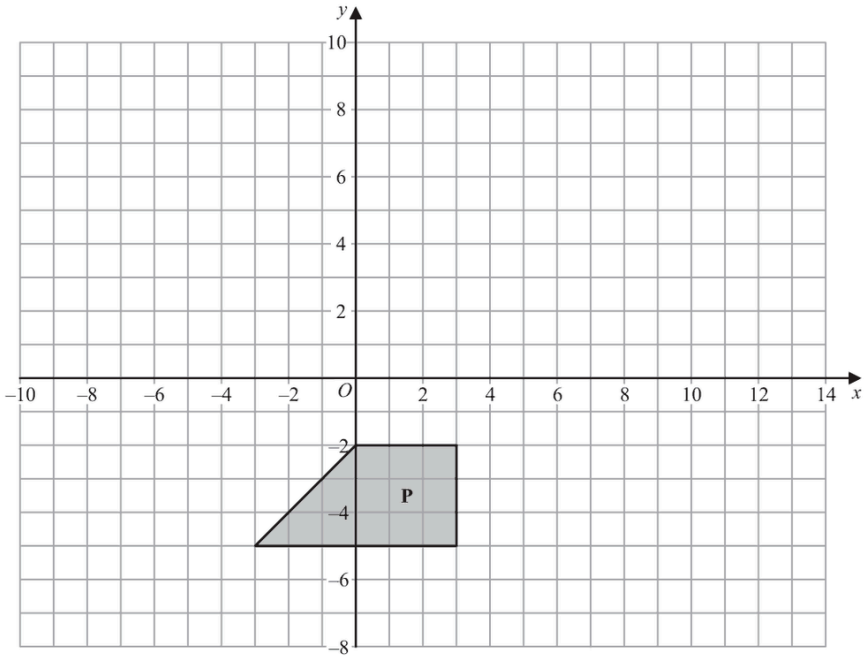
FINAL ANSWER

draw the enlarged shape with vertices $(4, 6), (8, 6), (8, 3), (6, 3)$.

Question 5

6 marks | Transformations

5 Here is a shape **P** drawn on a grid of squares.



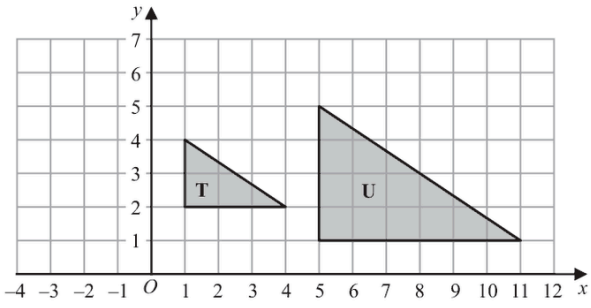
(a) On the grid, rotate shape **P** 180° about the point (–3, 2)
Label the new shape **Q**.

(2)

(b) On the grid, translate shape **P** by the vector $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$
Label the new shape **R**.

(1)

Here are triangle **T** and triangle **U** drawn on a grid of squares.



(c) Describe fully the single transformation that maps triangle **T** onto triangle **U**.

.....

(3)

(Total for Question 5 is 6 marks)

1 **(a) For a rotation about**

(a) For a $180°$ rotation about $(-3, 2)$,

$$(x, y) \mapsto (-6 - x,\ 4 - y)$$

2 **The image has vertices**

The image has vertices

$$(-3, 9),\ (-9, 9),\ (-9, 6),\ (-6, 6)$$

3 **(b) Translate by**

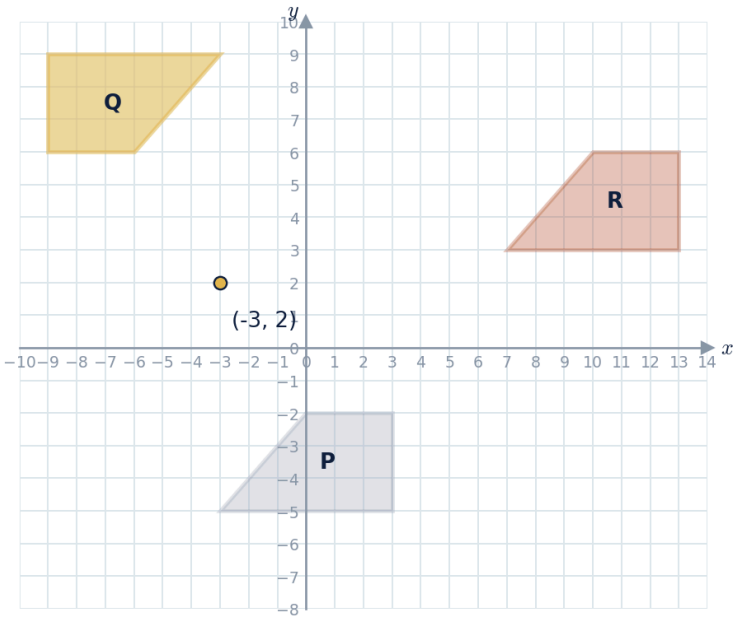
(b) Translate by

$$\begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

4 **The image has vertices**

The image has vertices

$$(7, 3),\ (13, 3),\ (13, 6),\ (10, 6)$$



5 **Use trigonometry**

(c) Triangle T is mapped to triangle U by an enlargement with scale factor 2 and centre $(-3, 3)$.

✓ **FINAL ANSWER**

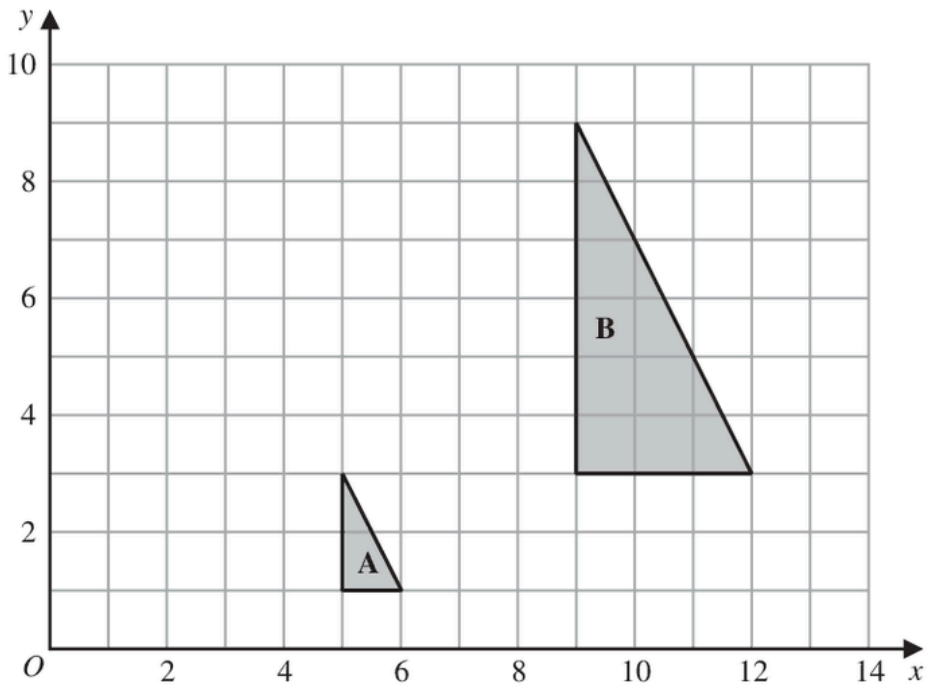
(a) vertices $(-3, 9), (-9, 9), (-9, 6), (-6, 6)$; (b) vertices $(7, 3), (13, 3), (13, 6), (10, 6)$; (c) enlargement, scale factor 2, centre $(-3, 3)$.

4MA1/2HR JUNE 2022

Question 1

4 marks | Transformations

1



(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(b) On the grid above, translate triangle **A** by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Label your triangle **C**

(1)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2022

1 **Use trigonometry**

(a) Triangle A is enlarged to triangle B.

2 **The scale factor is**

The scale factor is

3 **Use Transformations tag correct**

Using corresponding vertices gives centre $(3, 0)$.

4 **Use Transformations tag correct**

So the transformation is an enlargement, scale factor 3, centre $(3, 0)$.

5 **(b) Translating triangle by**

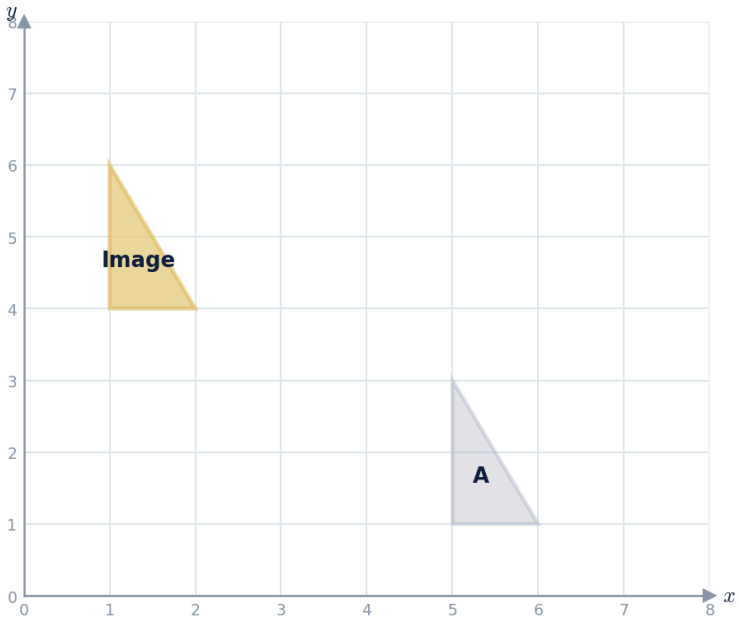
(b) Translating triangle A by

$$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

6 **gives vertices**

gives vertices

$$(1, 4),\ (2, 4),\ (1, 6)$$



FINAL ANSWER

(a) enlargement, scale factor 3, centre $(3, 0)$; (b) vertices $(1, 4), (2, 4), (1, 6)$.

Question 4

8 marks | Transformations

4 The diagram shows a sketch of triangle **A** and triangle **B** on a coordinate grid.

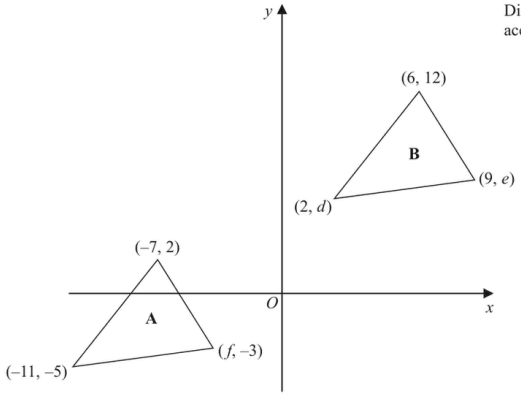


Diagram **NOT** accurately drawn

(a) Given that triangle **A** has been translated to give triangle **B**, work out the value of d, the value of e and the value of f

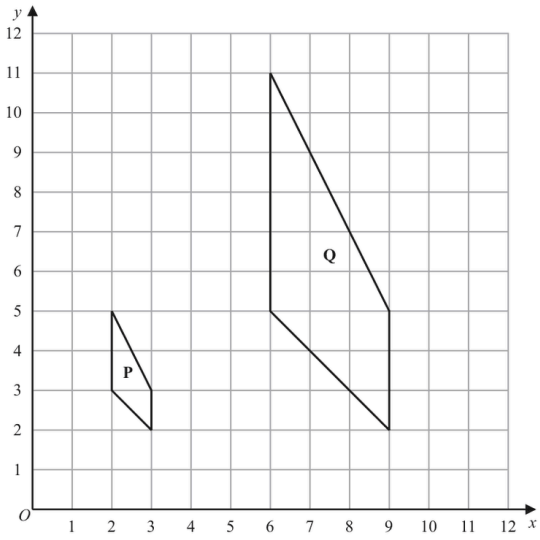
$d =$ .....

$e =$ .....

$f =$ .....

(3)

The diagram shows shape **P** and shape **Q** drawn on a grid.



(b) Describe fully the single transformation that maps shape **P** onto shape **Q**

.....

(3)

(c) On the grid above, rotate shape **P** 90° clockwise about (3, 5)
Label your shape **R**

(2)

(Total for Question 4 is 8 marks)

1 **Use matrices**

(a) The translation from $(-7, 2)$ to $(6, 12)$ is

$$\begin{pmatrix} 13 \\ 10 \end{pmatrix}$$

2 **Use Transformations tag correct**

So

$$(-11, -5) \mapsto (2, 5)$$

3 **Use Transformations tag correct**

and

$$(f, -3) \mapsto (9, 7)$$

4 **Apply transformation**

Therefore

$$d = 5, \qquad e = 7, \qquad f = -4$$

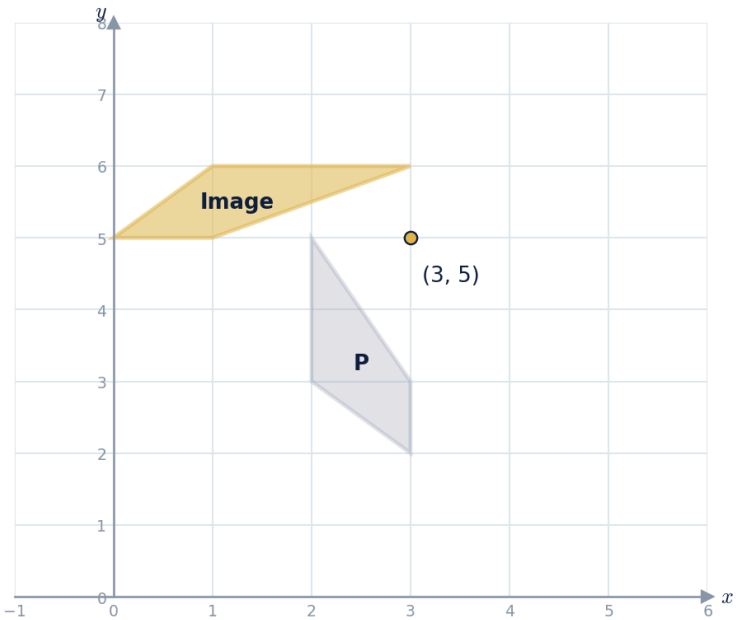
5 **(b) Shape maps to by**

(b) Shape P maps to Q by an enlargement with scale factor 3 and centre $(0, 2)$.

6 **(c) Rotate clockwise about**

(c) Rotate P $90°$ clockwise about $(3, 5)$. The image has vertices

$$(3, 6),\ (1, 6),\ (0, 5),\ (1, 5)$$



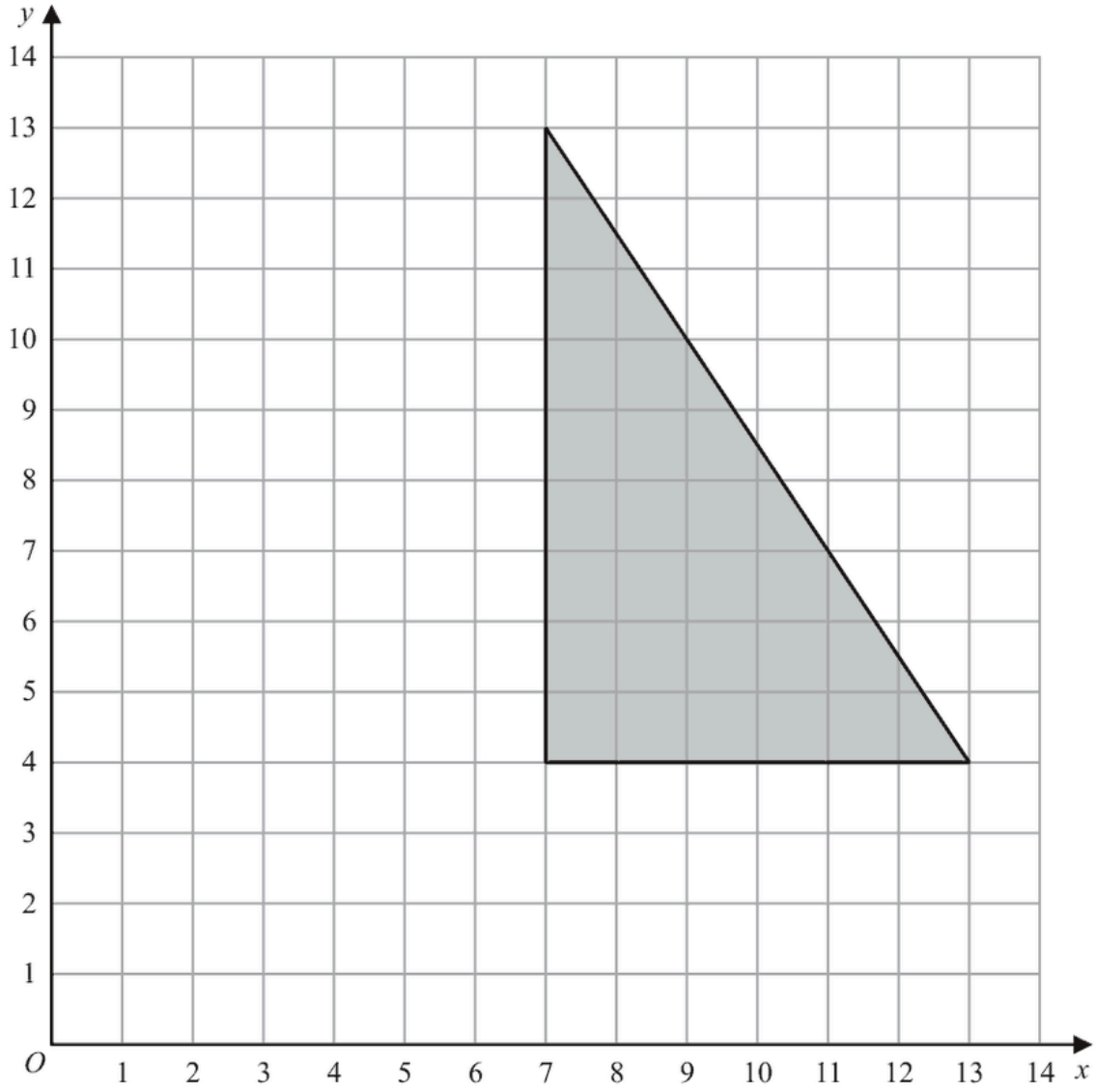
FINAL ANSWER

(a) $d = 5,\ e = 7,\ f = -4$; (b) enlargement, scale factor 3, centre $(0, 2)$; (c) vertices $(3, 6), (1, 6), (0, 5), (1, 5)$.

Question 10

2 marks | Transformations

10



On the grid, enlarge the shaded shape with scale factor $\frac{1}{3}$ and centre (1, 7)

(Total for Question 10 is 2 marks)

1 **Use**

Use

$$\text{new point} = (1,7) + \frac{1}{3}(\text{old point} - (1,7))$$

2 **The original vertices are**

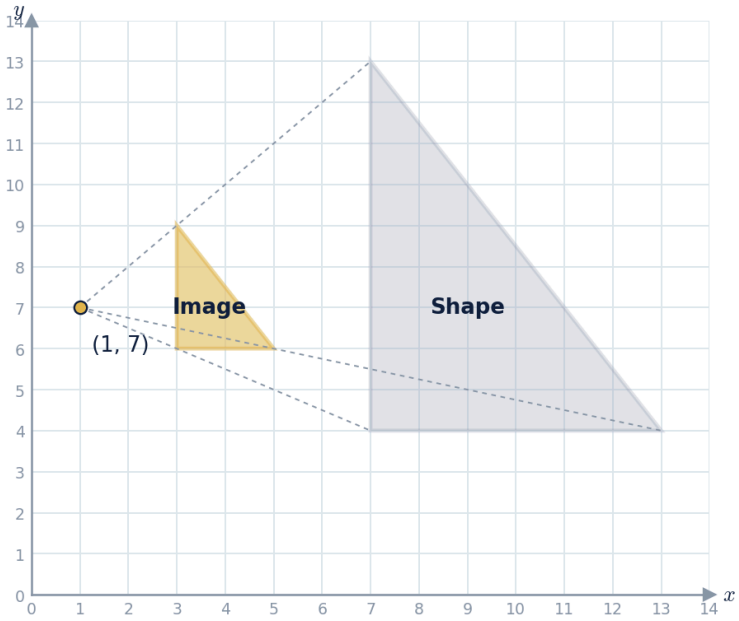
The original vertices are

$$(7,4),\ (13,4),\ (7,13)$$

3 **They become**

They become

$$(3,6),\ (5,6),\ (3,9)$$



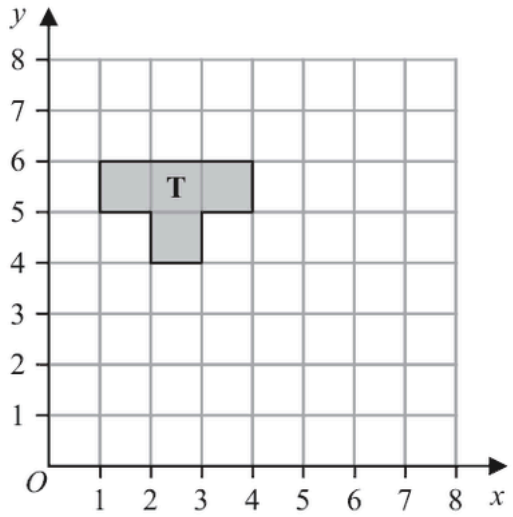
FINAL ANSWER

draw the triangle with vertices $(3,6), (5,6), (3,9)$.

Question 3

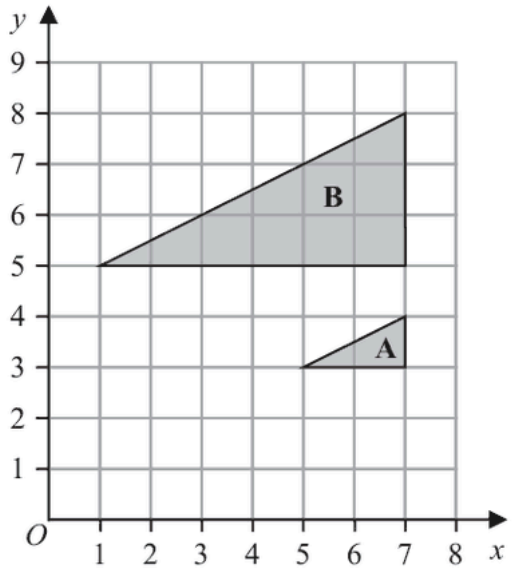
5 marks | Transformations

3



(a) Reflect shape **T** in the line $y = x$

(2)



(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(Total for Question 3 is 5 marks)

1 (a) Reflection in swaps the

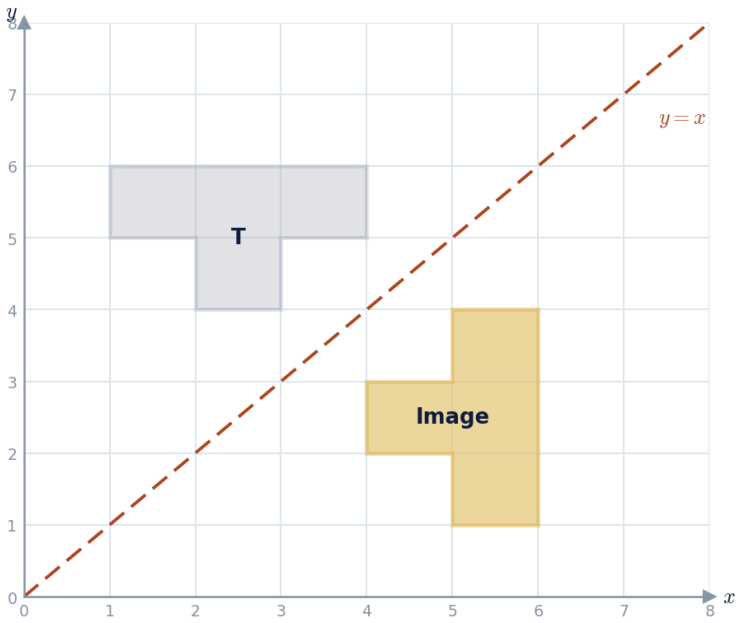
(a) Reflection in $y = x$ swaps the coordinates:

$$(x, y) \mapsto (y, x)$$

2 The reflected T shape has

The reflected T-shape has vertices

$$(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$$



3 Use trigonometry

(b) Triangle A is mapped to B by an enlargement with scale factor 3 and centre $(7, 2)$.

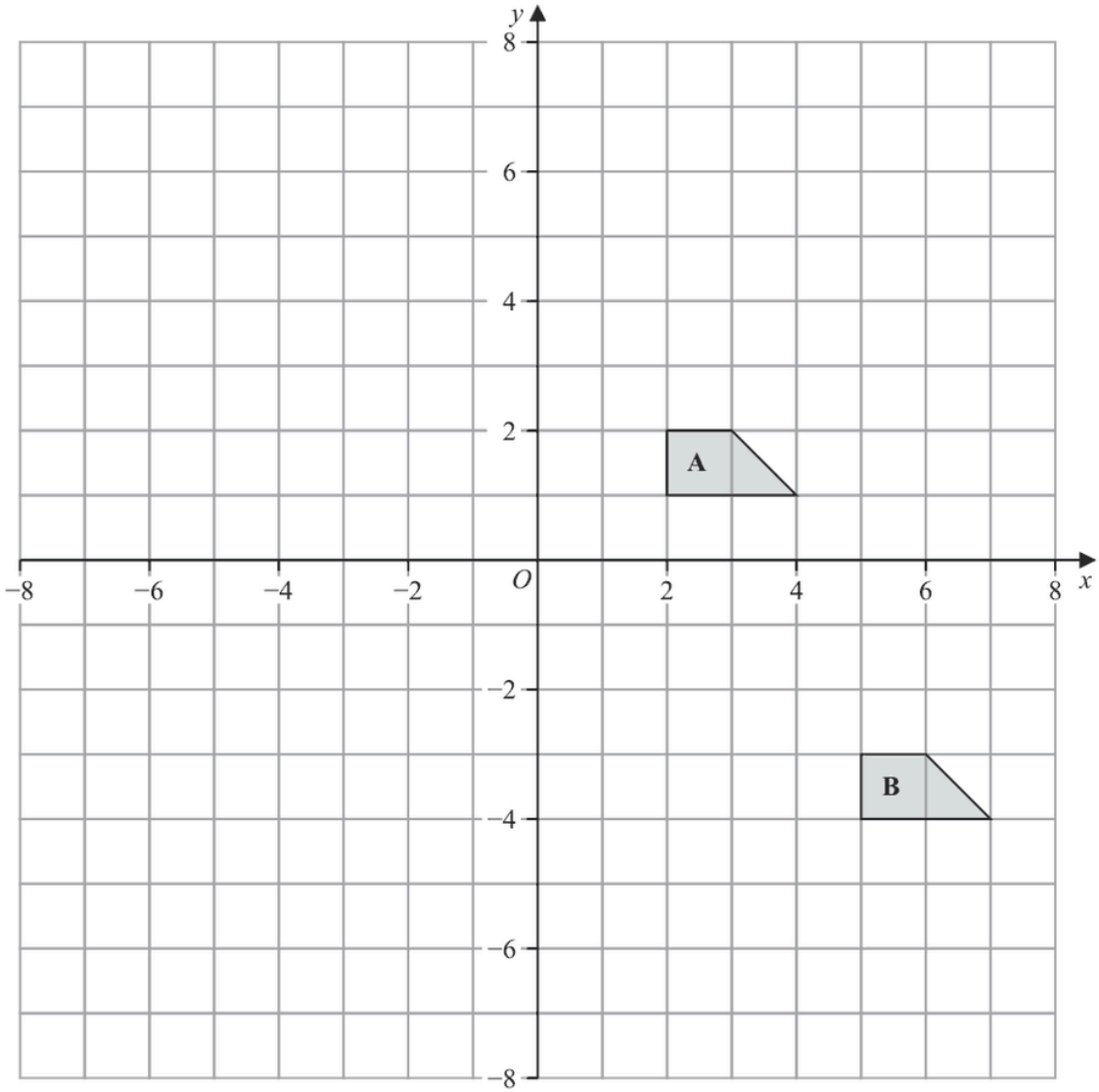
FINAL ANSWER

(a) reflected vertices $(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$; (b) enlargement, scale factor 3, centre $(7, 2)$.

Question 2

4 marks | Transformations

2



(a) Describe fully the single transformation that maps shape **A** onto shape **B**

.....

.....

(2)

(b) On the grid above, rotate shape **A** 180° about (−1, 0)
Label your shape **C**

(2)

(Total for Question 2 is 4 marks)

1 **(a) Shape moves to shape**

(a) Shape A moves to shape B by the vector

$$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

2 **Use matrices**

So the transformation is a translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

3 **(b) For a rotation about**

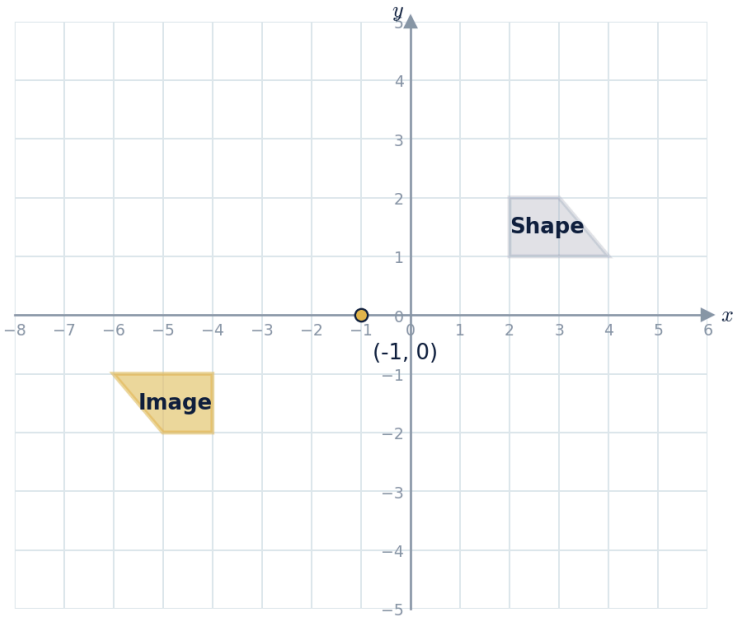
(b) For a $180°$ rotation about $(-1, 0)$,

$$(x, y) \mapsto (-2 - x, \ -y)$$

4 **The image has vertices**

The image has vertices

$$(-4, -1),\ (-6, -1),\ (-5, -2),\ (-4, -2)$$



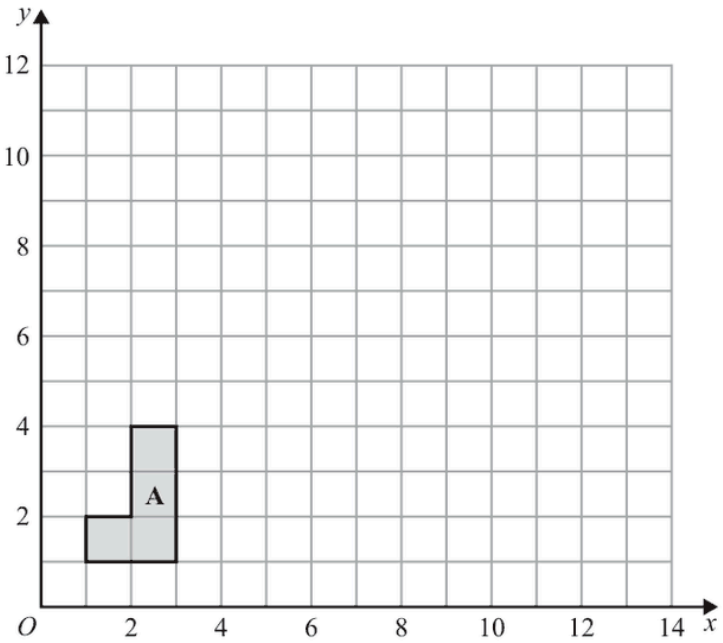
FINAL ANSWER

(a) translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$; (b) vertices $(-4, -1), (-6, -1), (-5, -2), (-4, -2)$.

Question 2

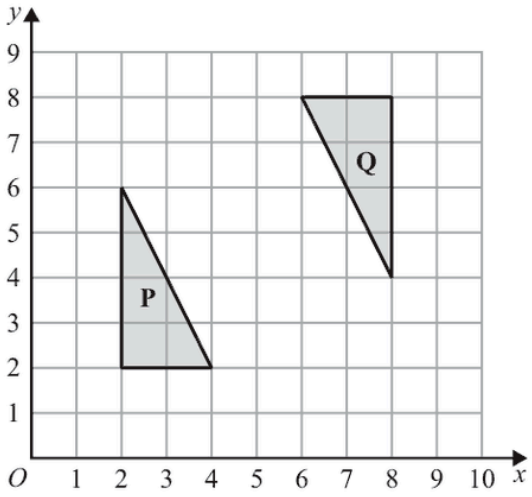
5 marks | Transformations

2



(a) On the grid, enlarge shape **A** with scale factor 3 and centre (0, 1)

(2)



(b) Describe fully the single transformation that maps triangle **P** onto triangle **Q**

.....

.....

(3)

(Total for Question 2 is 5 marks)

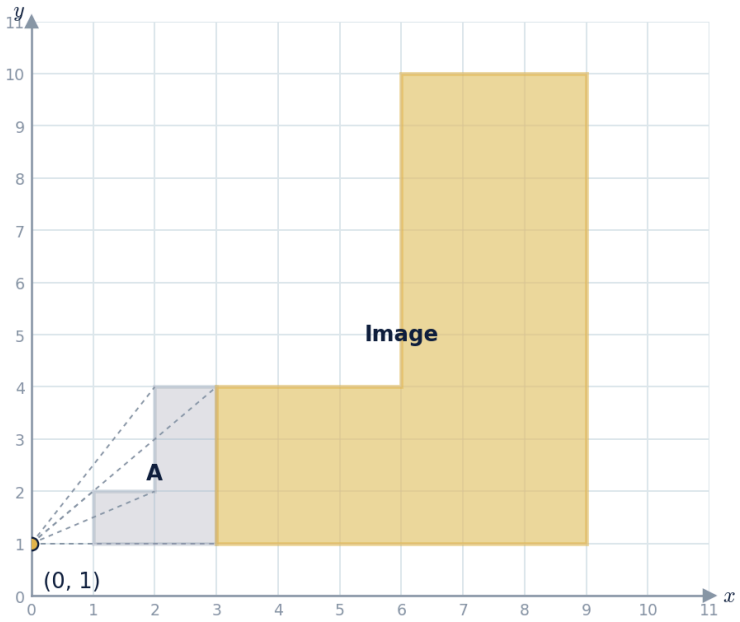
1 **Use Transformations correct**

(a) Enlarge from centre $(0, 1)$ with scale factor 3.

2 **The vertices become**

The vertices become

$$(3, 1),\ (9, 1),\ (9, 10),\ (6, 10),\ (6, 4),\ (3, 4)$$



3 **Use trigonometry**

(b) Triangle P is mapped to triangle Q by a $180°$ rotation about $(5, 5)$.

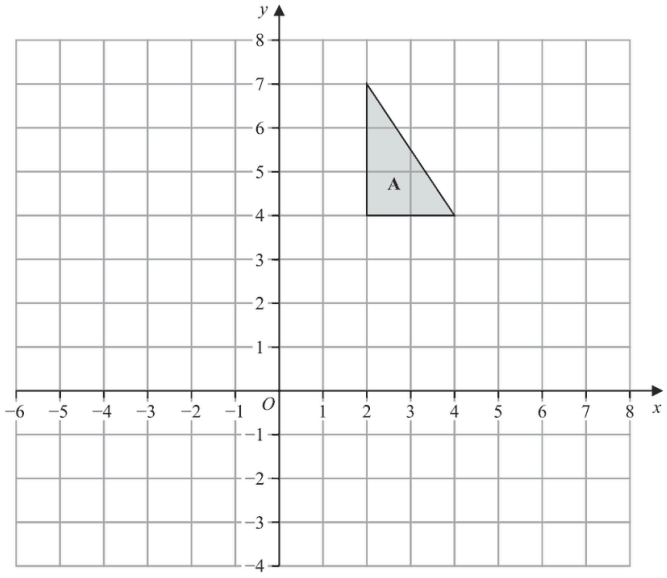
✓ **FINAL ANSWER**

(a) vertices $(3, 1), (9, 1), (9, 10), (6, 10), (6, 4), (3, 4)$; (b) rotation $180°$ about $(5, 5)$.

Question 4

4 marks | Transformations

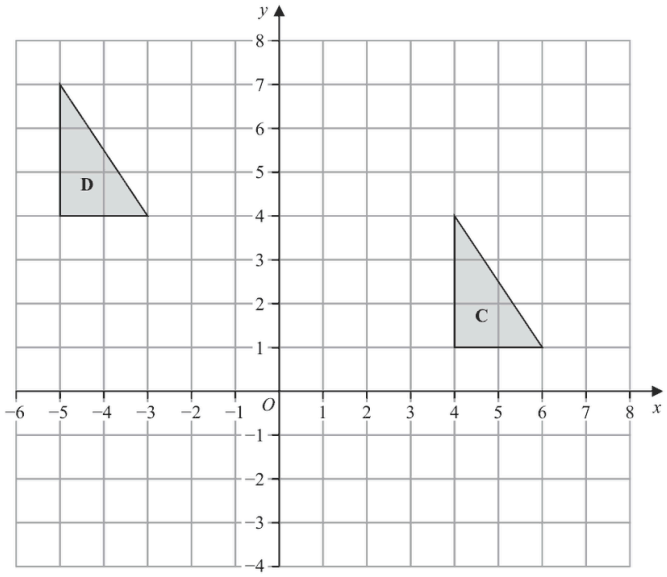
4



(a) On the grid above, rotate triangle **A** 90° anticlockwise about $(1, 2)$

Label your triangle **B**

(2)



(b) Describe fully the single transformation that maps triangle **C** onto triangle **D**

(2)

(Total for Question 4 is 4 marks)

1 Required

a Rotate triangle A 90° anticlockwise about (1,2); label triangle B. (b) Describe fully the transformation mapping C onto D.

2 The rule for 90 anticlockwise

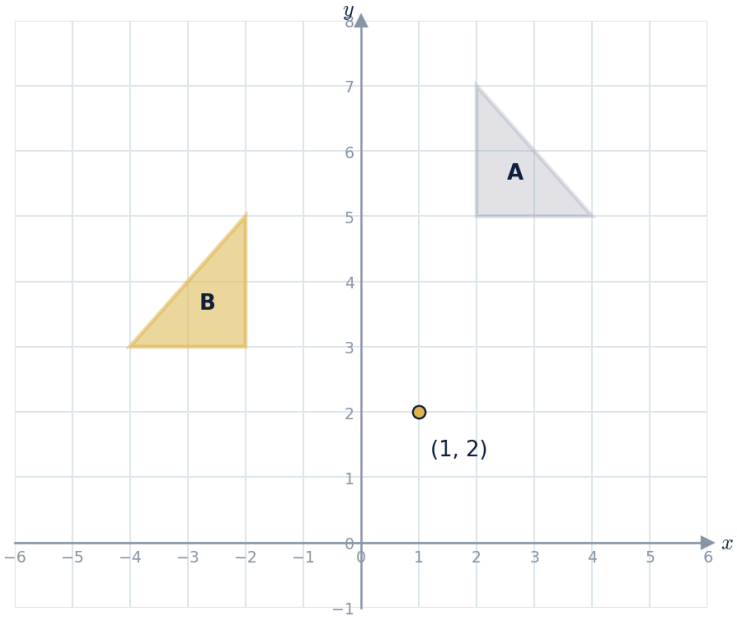
The rule for 90° anticlockwise rotation about (*a*,*b*):
(*x*, *y*) → (*a* − (*y*−*b*), *b* + (*x*−*a*))
About (1,2): (*x*,*y*) → (3−*y*, *x*+1)

3 Triangle A has vertices A

Triangle A has vertices *A*₁(2,7), *A*₂(4,5), *A*₃(2,5).
Apply the rule:
(2,7) → (−4,3), (4,5) → (−2,5), (2,5) → (−2,3)

4 Answer

Triangle B has vertices (−4,3), (−2,5), (−2,3)



5 Identify corresponding vertices of C

Identify corresponding vertices of C and D from the diagram and compute the vector:

$$(4, , 4) longrightarrow (-5, , 7) : quad vec{v} = begin{pmatrix} - 9 3 end{pmatrix}$$

Check: (6,1) → (−3,4) gives the same vector $begin{pmatrix} - 9 3 end{pmatrix}$

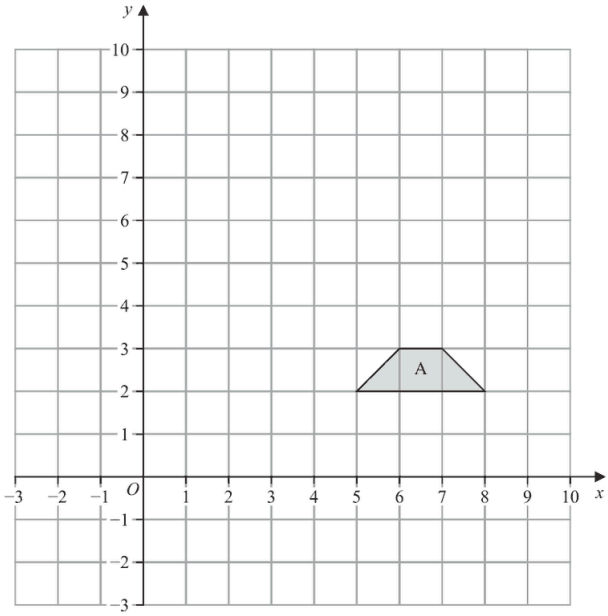
FINAL ANSWER

Translation by vector $begin{pmatrix} - 9 3 end{pmatrix}$

Question 5

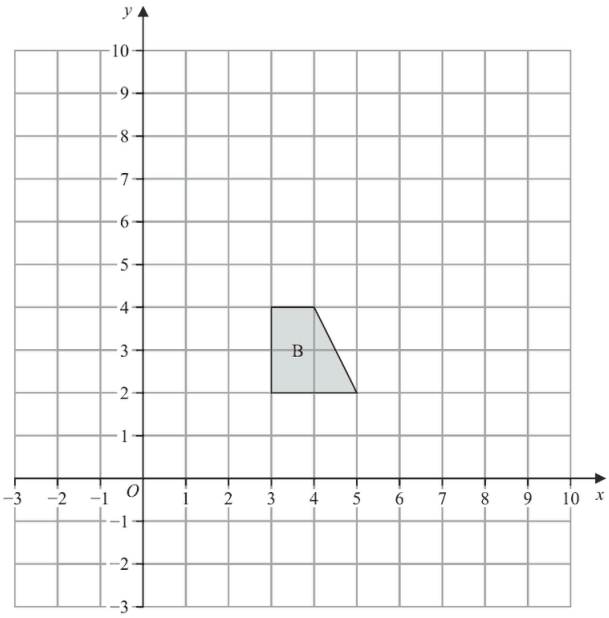
4 marks | Transformations

5



(a) On the grid above, reflect shape **A** in the line $y = x$

(2)



(b) On the grid above, enlarge shape **B** by scale factor 2 with centre (1, 1)

(2)

(Total for Question 5 is 4 marks)

1 (a) Reflection in swaps each

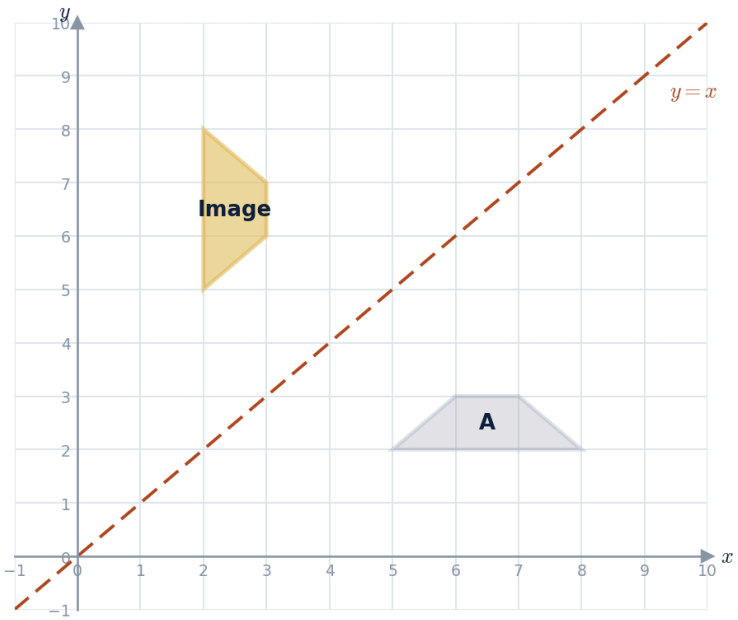
(a) Reflection in $y = x$ swaps each coordinate:

$$(x, y) \mapsto (y, x)$$

2 The image of has vertices

The image of A has vertices

$$(2, 5),\ (2, 8),\ (3, 7),\ (3, 6)$$



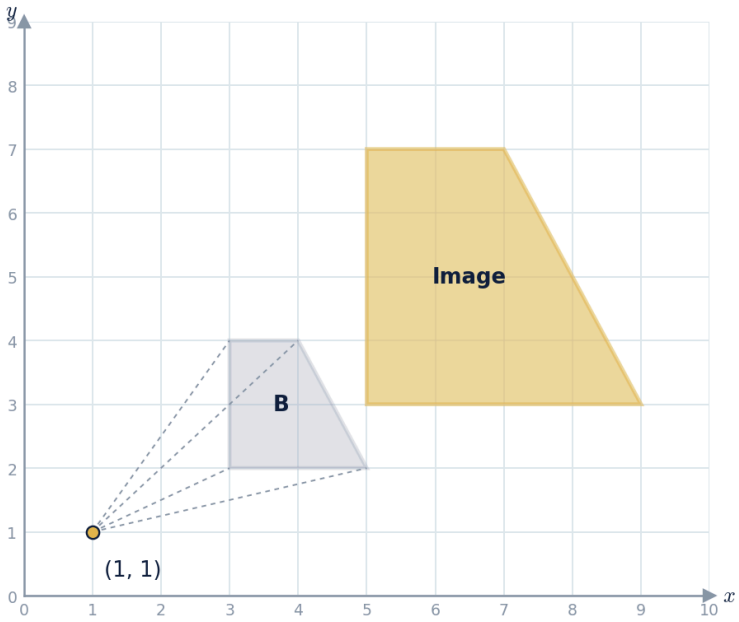
3 (b) Enlarge by scale factor

(b) Enlarge B by scale factor 2 from centre $(1, 1)$.

4 **The vertices become**

The vertices become

$$(5,3),\ (9,3),\ (7,7),\ (5,7)$$



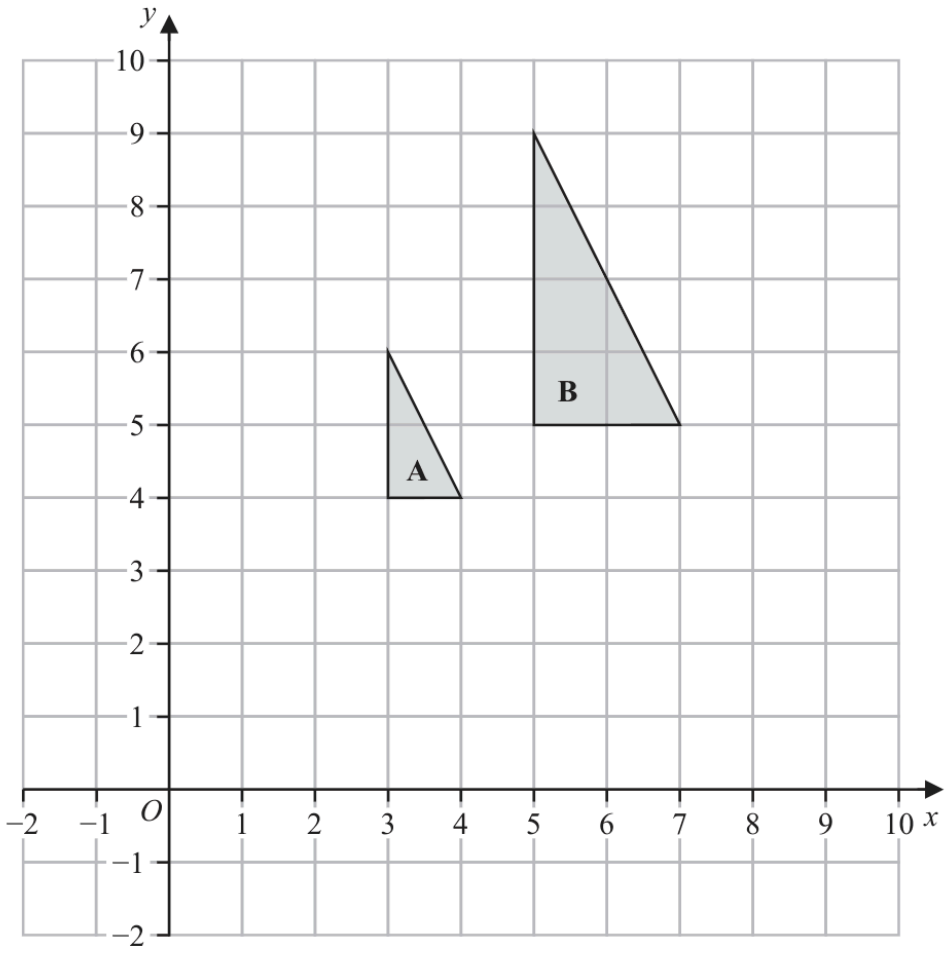
FINAL ANSWER

(a) vertices $(2,5),(2,8),(3,7),(3,6)$; (b) vertices $(5,3),(9,3),(7,7),(5,7)$.

Question 5

4 marks | Transformations

5



(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(b) On the grid, translate triangle **A** by the vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Label your triangle **C**

(1)

(Total for Question 5 is 4 marks)

1

Part (a) | Step 1

Triangle A has vertices $(3,4),(4,4),(3,6)$; the corresponding vertices of B are $(5,5),(7,5),(5,9)$.

The lengths have doubled.

Read corresponding vertices and compare side lengths.

2

Part (a) | Step 2

For enlargement factor 2, the centre O satisfies $B = O + 2(A - O)$, so $O = 2A - B$. Using $(3,4) \mapsto (5,5)$ gives $O = (6,8) - (5,5) = (1,3)$.

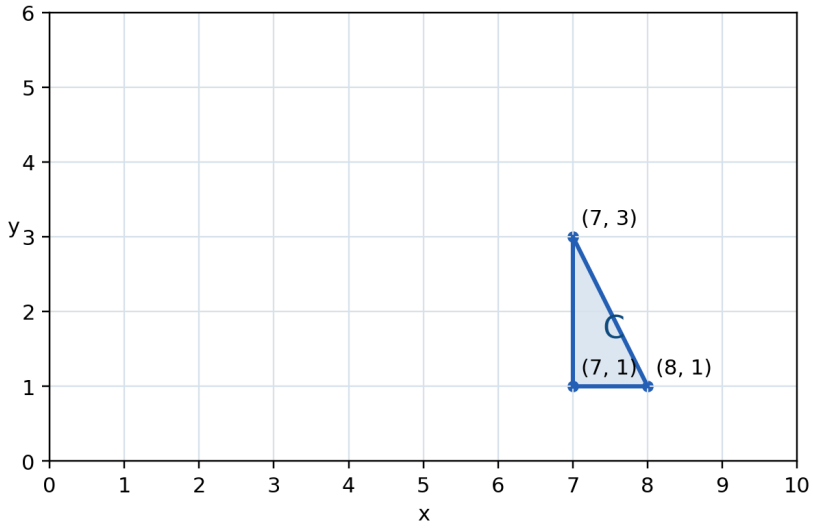
Find the fixed centre from a corresponding pair; the other pairs give the same centre.

3

Part (b) | Step 1

Add $(4,-3)$ to every vertex: $(3,4) \mapsto (7,1)$, $(4,4) \mapsto (8,1)$, and $(3,6) \mapsto (7,3)$. Plot these three points, join them with straight sides and label the triangle C.

A translation moves every point by the same vector.



FINAL ANSWER

(a) Enlargement with scale factor 2 and centre $(1,3)$. (b) $C : (7,1),(8,1),(7,3)$.

TOPIC

Vectors

Every past-paper question on this topic, 2020 to 2026.

Question 23

5 marks | Vectors

23

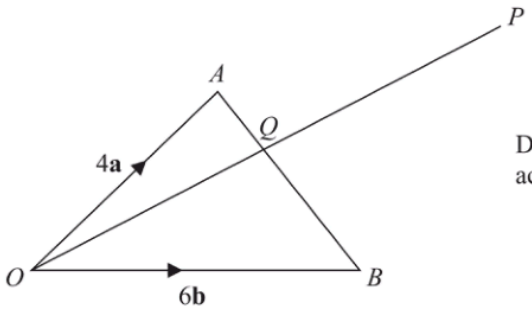


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = \mathbf{4a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AP} = \mathbf{2a} + \mathbf{8b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 23 is 5 marks)

1 **Use vectors**

$$\overrightarrow{OA} = 4a, \qquad \overrightarrow{OB} = 6b, \qquad \overrightarrow{AP} = 2a + 8b$$

2 **Use vectors**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4a + (2a + 8b) = 6a + 8b$$

3 **Use vectors**

Since O, Q, P are collinear, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Use vectors**

Also Q lies on AB. Let

$$\overrightarrow{AQ} = \mu\overrightarrow{AB}$$

5 **Use vectors**

Then

$$\overrightarrow{AB} = 6b - 4a$$

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a)$$

$$\overrightarrow{OQ} = (4 - 4\mu)a + 6\mu b$$

6 **Use vectors**

Compare coefficients with

$$\lambda(6a + 8b) = 6\lambda a + 8\lambda b$$

7 **Use vectors**

So

$$4 - 4\mu = 6\lambda, \qquad 6\mu = 8\lambda$$

8 **Use vectors**

From $6\mu = 8\lambda$,

$$\lambda = \frac{3}{4}\mu$$

---

9 **Substitute**

Substitute:

$$4 - 4\mu = 6\left(\frac{3}{4}\mu\right)$$

$$4 - 4\mu = \frac{9}{2}\mu$$

$$8 = 17\mu$$

$$\mu = \frac{8}{17}$$

---

10 **Use vectors**

Therefore

$$AQ : QB = \frac{8}{17} : \frac{9}{17} = 8 : 9$$

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$.

4MA1/2H JANUARY 2020

Question 23

3 marks | Vectors

23

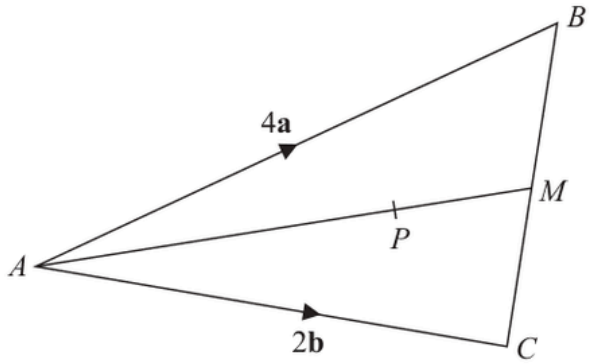


Diagram **NOT** accurately drawn

ABC is a triangle.
The midpoint of BC is M.
P is a point on AM.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2020

1 **Split the ratio**

Since M is the midpoint of BC,

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = \frac{4a + 2b}{2} = 2a + b$$

2 **Given**

Given

$$\overrightarrow{AP} = \frac{3}{2}a + \frac{3}{4}b$$

3 **Factorise**

Factorise:

$$\overrightarrow{AP} = \frac{3}{4}(2a + b) = \frac{3}{4}\overrightarrow{AM}$$

4 **Split the ratio**

So AP is $\frac{3}{4}$ of AM, leaving PM as $\frac{1}{4}$ of AM.

✓ **FINAL ANSWER**

$AP : PM = 3 : 1$

4MA1/2HR JANUARY 2020

Question 26

5 marks | Vectors

26 The diagram shows trapezium $OACB$.

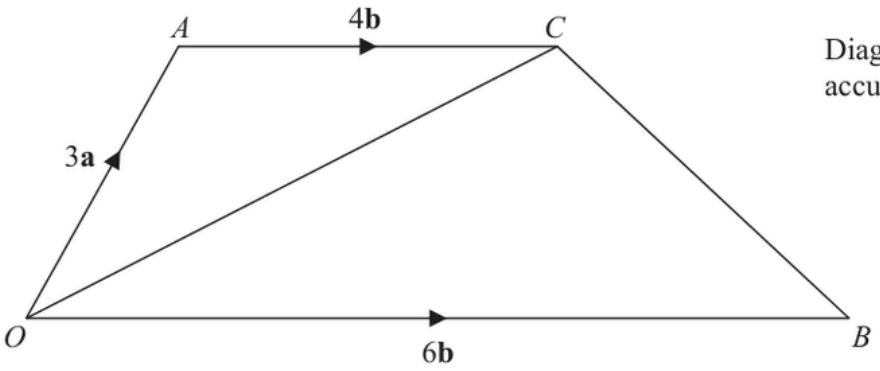


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AC} = 4\mathbf{b}$

N is the point on OC such that ANB is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR January 2020

1 **Read the graph**

$$\vec{OA} = 3\mathbf{a}, \quad \vec{OB} = 6\mathbf{b}, \quad \vec{AC} = 4\mathbf{b}$$

2 **Read the graph**

So

$$\vec{OC} = 3\mathbf{a} + 4\mathbf{b}$$

3 **Read the graph**

Point N lies on OC, so

$$\vec{ON} = t(3\mathbf{a} + 4\mathbf{b})$$

4 **Read the graph**

Point N also lies on AB, so

$$\vec{ON} = 3\mathbf{a} + s(6\mathbf{b} - 3\mathbf{a})$$

$$\vec{ON} = (3 - 3s)\mathbf{a} + 6s\mathbf{b}$$

---

5 **Equate coefficients**

Equate coefficients:

$$3t = 3 - 3s$$

$$4t = 6s$$

---

6 **Read the graph**

So

$$t = 1 - s$$

---

7 **Read the graph**

and

$$t = \frac{3}{2}s$$

$$\frac{3}{2}s = 1 - s$$

$$5s = 2$$

$$s = \frac{2}{5}$$

---

8 **Read the graph**

Then

$$t = \frac{3}{5}$$

---

9 **Read the graph**

Therefore

$$\vec{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$$

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

4MA1/1HR JANUARY 2021

Question 13

2 marks | Vectors

13 Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Find, as a column vector, $\overrightarrow{AC}$

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR January 2021

1 **Use vectors**

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

2 **Use vectors**

and

$$\overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

3 **Use vectors**

So

$$\overrightarrow{BC} = -\overrightarrow{CB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

✓ **FINAL ANSWER**

$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$.

4MA1/1H JANUARY 2021

Question 23

6 marks | Vectors

23 OAB is a triangle.

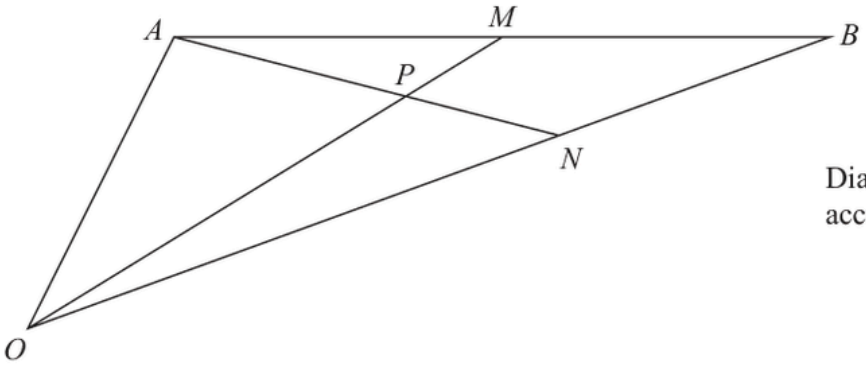


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

M is the midpoint of AB.
N is the point on OB such that $ON : NB = 2 : 1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP : PM$
Show your working clearly.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2021

1 **Use vectors**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{OB} = 2\mathbf{b}$$

2 **Find the midpoint**

Since M is the midpoint of AB,

$$\vec{OM} = \frac{2\mathbf{a} + 2\mathbf{b}}{2}$$

$$\vec{OM} = \mathbf{a} + \mathbf{b}$$

3 **Use vectors**

Since $ON : NB = 2 : 1$,

$$\vec{ON} = \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \frac{2}{3}(2\mathbf{b}) = \frac{4}{3}\mathbf{b}$$

---

4 **Use vectors**

Point P lies on OM, so

$$\vec{OP} = t(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AN, so

$$\vec{OP} = 2\mathbf{a} + s\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$$

$$\vec{OP} = (2 - 2s)\mathbf{a} + \frac{4s}{3}\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$:

$$t = 2 - 2s$$

$$t = \frac{4s}{3}$$

---

7 **Use vectors**

So

$$\frac{4s}{3} = 2 - 2s$$

$$4s = 6 - 6s$$

$$10s = 6$$

$$s = \frac{3}{5}$$

---

8 **Use vectors**

Then

$$t = \frac{4}{5}$$

9 **Use vectors**

So P is $\frac{4}{5}$ of the way from O to M.

FINAL ANSWER

$OP : PM = 4 : 1$

4MA1/1H MAY 2021

Question 26

5 marks | Vectors

26 $OACB$ is a trapezium.

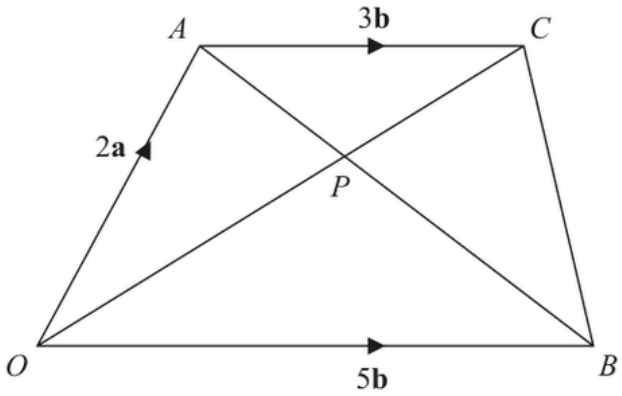


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = 5\mathbf{b}$ $\overrightarrow{AC} = 3\mathbf{b}$

The diagonals, OC and AB, of the trapezium intersect at the point P.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H May 2021

1 **Use vectors**

$$\overrightarrow{OA} = 2\mathbf{a}, \quad \overrightarrow{OB} = 5\mathbf{b}, \quad \overrightarrow{AC} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 2\mathbf{a} + 3\mathbf{b}$$

3 **Use vectors**

Since P lies on OC, let

$$\overrightarrow{OP} = t(2\mathbf{a} + 3\mathbf{b})$$

4 **Use vectors**

Since P also lies on AB, let

$$\overrightarrow{OP} = \overrightarrow{OA} + s\overrightarrow{AB}$$

5 **Use vectors**

Now

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{b} - 2\mathbf{a}$$

6 **Use vectors**

So

$$\overrightarrow{OP} = 2\mathbf{a} + s(5\mathbf{b} - 2\mathbf{a})$$

$$\overrightarrow{OP} = (2 - 2s)\mathbf{a} + 5s\mathbf{b}$$

7 **Use vectors**

Compare this with

$$t(2\mathbf{a} + 3\mathbf{b}) = 2t\mathbf{a} + 3t\mathbf{b}$$

8 **Equating coefficients**

Equating coefficients:

$$2t = 2 - 2s$$

$$3t = 5s$$

9 **Use vectors**

From $t = 1 - s$,

$$3(1 - s) = 5s$$

$$3 = 8s$$

$$s = \frac{3}{8}$$

---

10 **Use vectors**

So

$$t = 1 - \frac{3}{8} = \frac{5}{8}$$

---

11 **Use vectors**

Therefore

$$\overrightarrow{OP} = \frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$$

$$\overrightarrow{OP} = \frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}}$$

Question 14

8 marks | Vectors

14 *ABCDEF* and *GHIJKL* are regular hexagons each with centre *O*.

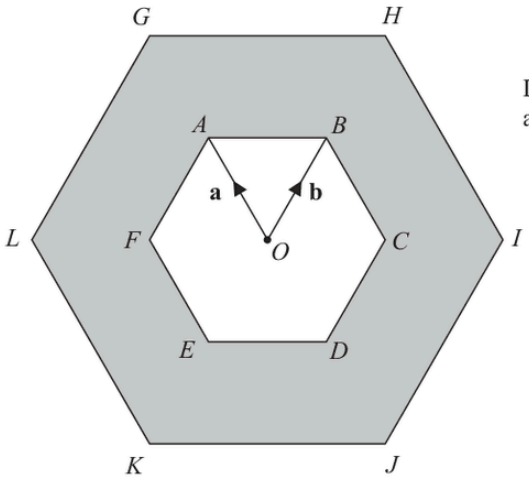


Diagram **NOT** accurately drawn

GHIJKL is an enlargement of *ABCDEF*, with centre *O* and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

(a) Write the following vectors, in terms of **a** and **b**.
Simplify your answers.

(i) $\overrightarrow{AB}$

.....

(1)

(ii) $\overrightarrow{KI}$

.....

(2)

(iii) $\overrightarrow{LD}$

.....

(2)

The triangle *OAB* has an area of 5 cm^2

(b) Calculate the area of the shaded region.

..... cm^2

(3)

(Total for Question 14 is 8 marks)

1 **Use vectors**

$$\overrightarrow{OA} = \mathbf{a}, \qquad \overrightarrow{OB} = \mathbf{b}$$

2 **(a)(i)**

(a)(i)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$$

3 **Use vectors**

For a regular hexagon,

$$\overrightarrow{OC} = \mathbf{b} - \mathbf{a}, \quad \overrightarrow{OD} = -\mathbf{a}, \quad \overrightarrow{OE} = -\mathbf{b}, \quad \overrightarrow{OF} = \mathbf{a} - \mathbf{b}$$

4 **The outer hexagon has scale**

The outer hexagon has scale factor $\mathbf{2}$, so

$$\overrightarrow{OI} = 2(\mathbf{b} - \mathbf{a}), \quad \overrightarrow{OK} = -2\mathbf{b}, \quad \overrightarrow{OL} = 2(\mathbf{a} - \mathbf{b})$$

5 **(ii)**

(ii)

$$\overrightarrow{KI} = \overrightarrow{OI} - \overrightarrow{OK} = 2(\mathbf{b} - \mathbf{a}) - (-2\mathbf{b}) = 4\mathbf{b} - 2\mathbf{a}$$

6 **(iii)**

(iii)

$$\overrightarrow{LD} = \overrightarrow{OD} - \overrightarrow{OL} = -\mathbf{a} - 2(\mathbf{a} - \mathbf{b}) = 2\mathbf{b} - 3\mathbf{a}$$

7 **(b) The inner hexagon is**

(b) The inner hexagon is made from 6 equal triangles like OAB:

$$\text{inner area} = 6 \times 5 = 30$$

8 **The outer hexagon is an**

The outer hexagon is an enlargement with scale factor $\mathbf{2}$, so area scale factor is $\mathbf{2^2 = 4}$:

$$\text{outer area} = 4 \times 30 = 120$$

$$\text{shaded area} = 120 - 30 = 90$$

FINAL ANSWER

(i) $\mathbf{b} - \mathbf{a}$, (ii) $4\mathbf{b} - 2\mathbf{a}$, (iii) $2\mathbf{b} - 3\mathbf{a}$, (b) $90\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 22

5 marks | Vectors

22 The diagram shows triangle OAB

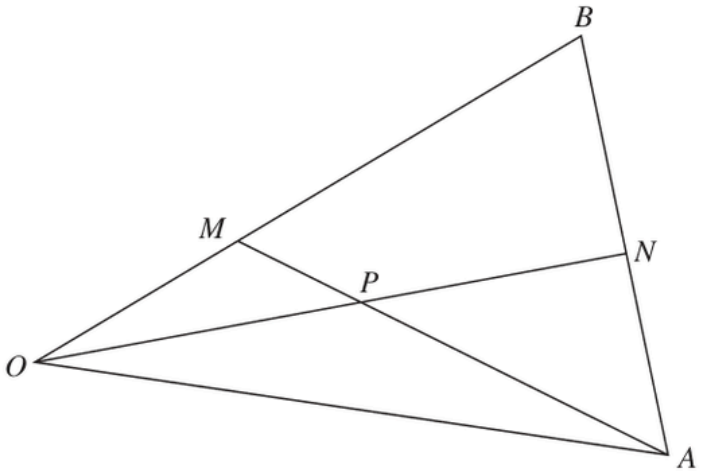


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

M is the point on OB such that $OM : MB = 1 : 2$
N is the midpoint of AB
P is the point of intersection of ON and AM

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2022

1 **Use vectors**

$$\vec{OA} = 8\mathbf{a}, \qquad \vec{OB} = 6\mathbf{b}$$

2 **Use vectors**

Since $OM : MB = 1 : 2$,

$$\vec{OM} = \frac{1}{3}\vec{OB}$$

$$\vec{OM} = 2\mathbf{b}$$

---

3 **Find the midpoint**

Since N is the midpoint of AB,

$$\vec{ON} = \frac{8\mathbf{a} + 6\mathbf{b}}{2}$$

$$\vec{ON} = 4\mathbf{a} + 3\mathbf{b}$$

---

4 **Use vectors**

Point P lies on ON, so

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AM, so

$$\vec{OP} = 8\mathbf{a} + \mu(2\mathbf{b} - 8\mathbf{a})$$

$$\vec{OP} = (8 - 8\mu)\mathbf{a} + 2\mu\mathbf{b}$$

---

6 **Equate coefficients**

Equate coefficients:

$$4\lambda = 8 - 8\mu$$

$$3\lambda = 2\mu$$

---

7 **Use vectors**

From $3\lambda = 2\mu$,

$$\mu = \frac{3}{2}\lambda$$

---

8 **Substitute**

Substitute:

$$4\lambda = 8 - 8\left(\frac{3}{2}\lambda\right)$$

$$4\lambda = 8 - 12\lambda$$

$$16\lambda = 8$$

$$\lambda = \frac{1}{2}$$

---

9 **Use vectors**

Therefore

$$\vec{OP} = \frac{1}{2}(4\mathbf{a} + 3\mathbf{b})$$

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

4MA1/1HR JANUARY 2022

Question 24

5 marks | Vectors

24 $OXYZ$ is a trapezium.

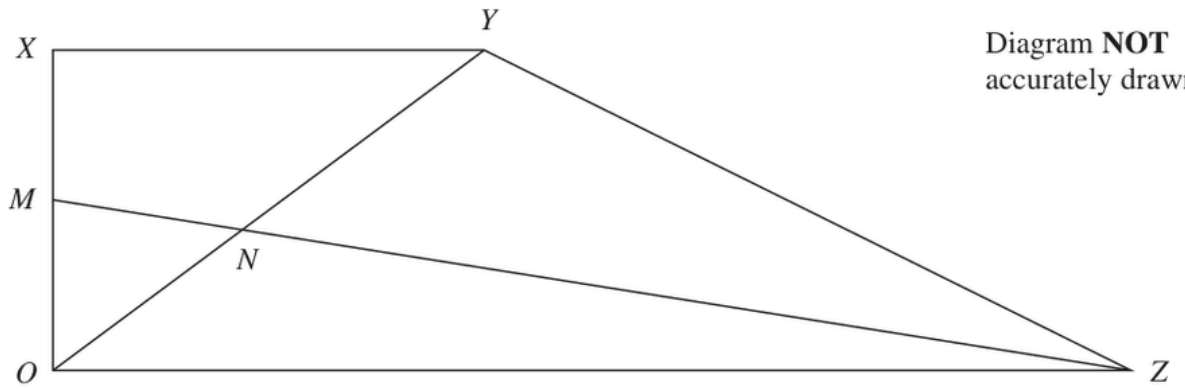


Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

M is the midpoint of OX
N is the point such that MNZ and ONY are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of λ

$\lambda =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR January 2022

1 **Take as the origin**

Take O as the origin.

$$\vec{OX} = \mathbf{a}, \qquad \vec{XY} = \mathbf{b}, \qquad \vec{OZ} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\vec{OY} = \mathbf{a} + \mathbf{b}$$

3 **Find the midpoint**

Since M is the midpoint of OX,

$$\vec{OM} = \frac{1}{2}\mathbf{a}$$

4 **Use vectors**

Given $ON : OY = \lambda : 1$,

$$\vec{ON} = \lambda(\mathbf{a} + \mathbf{b})$$

5 **Use vectors**

Point N also lies on MZ, so

$$\vec{ON} = \frac{1}{2}\mathbf{a} + t\left(3\mathbf{b} - \frac{1}{2}\mathbf{a}\right)$$

$$\vec{ON} = \left(\frac{1}{2} - \frac{t}{2}\right)\mathbf{a} + 3t\mathbf{b}$$

6 **Use vectors**

Equate coefficients with $\boldsymbol{\lambda}\mathbf{a} + \boldsymbol{\lambda}\mathbf{b}$:

$$\lambda = \frac{1}{2} - \frac{t}{2}$$

$$\lambda = 3t$$

7 **Use vectors**

So

$$t = \frac{\lambda}{3}$$

8 **Substitute**

Substitute:

$$\lambda = \frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{3}\right)$$

$$\lambda = \frac{1}{2} - \frac{\lambda}{6}$$

$$\frac{7}{6}\lambda = \frac{1}{2}$$

$$\lambda = \frac{3}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}$$

4MA1/2H JUNE 2022

Question 21

5 marks | Vectors

21

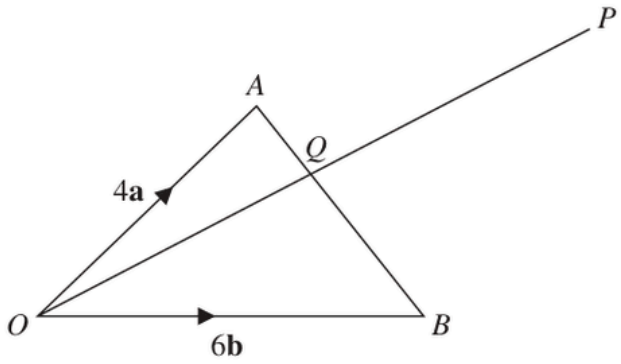


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2022

1 Split the ratio

$$\overrightarrow{OA} = 4a, \quad \overrightarrow{OB} = 6b, \quad \overrightarrow{AP} = 2a + 8b$$

2 Split the ratio

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4a + (2a + 8b) = 6a + 8b$$

3 Split the ratio

Since Q lies on OP, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Split the ratio**

Since Q lies on AB, let

$$\overrightarrow{OQ} = \overrightarrow{OA} + \mu\overrightarrow{AB}$$

$$\overrightarrow{AB} = 6b - 4a$$

---

5 **Split the ratio**

So

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a) = (4 - 4\mu)a + 6\mu b$$

---

6 **Equating coefficients**

Equating coefficients:

$$6\lambda = 4 - 4\mu, \quad 8\lambda = 6\mu$$

---

7 **Split the ratio**

From $8\lambda = 6\mu$, $\mu = \frac{4}{3}\lambda$.

$$6\lambda = 4 - \frac{16}{3}\lambda$$

$$34\lambda = 12, \quad \lambda = \frac{6}{17}$$

$$\mu = \frac{8}{17}$$

---

8 **Split the ratio**

So $AQ : QB = \frac{8}{17} : \frac{9}{17}$.

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$

4MA1/2HR JUNE 2022

Question 25

5 marks | Vectors

25 $ABCD$ is a parallelogram and ADM is a straight line.

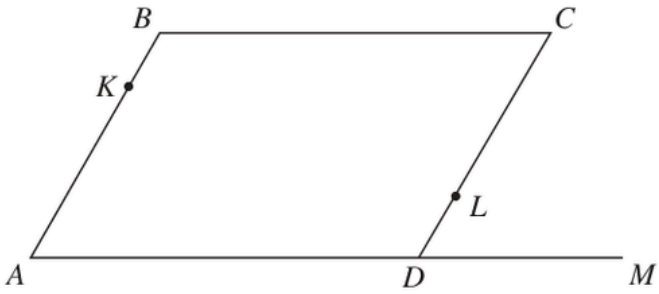


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$ $\overrightarrow{DM} = \frac{1}{2}\mathbf{b}$

K is the point on AB such that $AK : AB = \lambda : 1$
L is the point on CD such that $CL : CD = \mu : 1$
KLM is a straight line.

Given that $\lambda : \mu = 1 : 2$

use a vector method to find the value of λ and the value of μ

$\lambda =$ .....

$\mu =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2022

1 Let be the origin

Let A be the origin.

$$\vec{AB} = \mathbf{a}, \qquad \vec{BC} = \mathbf{b}$$

2 Read the graph

So

$$B = \mathbf{a}, \qquad D = \mathbf{b}, \qquad C = \mathbf{a} + \mathbf{b}$$

3 **Read the graph**

Also

$$\vec{DM} = \frac{1}{2}\mathbf{b}$$

---

4 **Read the graph**

So

$$M = \frac{3}{2}\mathbf{b}$$

---

5 **Point is on**

Point K is on AB:

$$K = \lambda\mathbf{a}$$

---

6 **Read the graph**

Point L is on CD, and $\vec{CD} = -\mathbf{a}$:

$$L = C + \mu(D - C)$$

$$L = \mathbf{a} + \mathbf{b} + \mu(-\mathbf{a})$$

$$L = (1 - \mu)\mathbf{a} + \mathbf{b}$$

---

7 **Given**

Given

$$\lambda : \mu = 1 : 2$$

$$\mu = 2\lambda$$

---

8 **Read the graph**

Since K, L, M are collinear, $\vec{KM}$ is a multiple of $\vec{KL}$.

$$\vec{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$$

$$\vec{KL} = (1 - \mu - \lambda)\mathbf{a} + \mathbf{b}$$

$$\vec{KL} = (1 - 3\lambda)\mathbf{a} + \mathbf{b}$$

---

9 **Compare coefficients using**

Compare coefficients using

$$\vec{KM} = \frac{3}{2}\vec{KL}$$

$$-\lambda = \frac{3}{2}(1 - 3\lambda)$$

$$-2\lambda = 3 - 9\lambda$$

$$7\lambda = 3$$

$$\lambda = \frac{3}{7}$$

---

10 **Read the graph**

Then

$$\mu = 2\lambda = \frac{6}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}, \quad \mu = \frac{6}{7}$$

4MA1/1H MAY 2023

Question 19

5 marks | Vectors

19

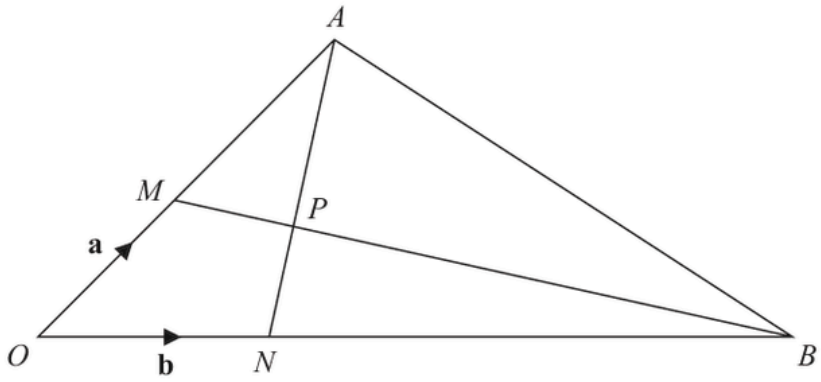


Diagram **NOT** accurately drawn

OMA, *ONB*, *MPB* and *NPA* are straight lines.

M is the midpoint of *OA*

$ON : NB = 1 : 5$

$\overrightarrow{OM} = \mathbf{a}$ $\overrightarrow{ON} = \mathbf{b}$

(a) Find in terms of **a** and **b** the vector $\overrightarrow{AN}$

.....

(1)

(b) Use a vector method to find the ratio $AP : PN$

$AP : PN =$ .....

(4)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2023

1 **Split the ratio**

Since M is the midpoint of OA,

$$\overrightarrow{OA} = 2a$$

2 **Split the ratio**

Also,

$$\overrightarrow{ON} = b$$

3 **Split the ratio**

So

$$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA} = b - 2a$$

4 **Let**

Let

$$\overrightarrow{AP} = \lambda \overrightarrow{AN}$$

5 **Split the ratio**

Then

$$\overrightarrow{OP} = 2a + \lambda(b - 2a) = (2 - 2\lambda)a + \lambda b$$

6 **Split the ratio**

Since $ON : NB = 1 : 5$,

$$\overrightarrow{OB} = 6b$$

7 **Split the ratio**

Point P is also on MB, so

$$\overrightarrow{OP} = a + \mu(6b - a) = (1 - \mu)a + 6\mu b$$

8 **Compare coefficients**

Compare coefficients:

$$2 - 2\lambda = 1 - \mu$$

$$\lambda = 6\mu$$

9 **Split the ratio**

From $\mu = 2\lambda - 1$,

$$\lambda = 6(2\lambda - 1)$$

$$11\lambda = 6$$

$$\lambda = \frac{6}{11}$$

---

10 **Split the ratio**

So

$$AP : PN = \frac{6}{11} : \frac{5}{11} = 6 : 5$$

✓ **FINAL ANSWER**

(a) $b - 2a$, (b) $6 : 5$

4MA1/1HR MAY 2023

Question 19

5 marks | Vectors

19 The diagram shows a triangle ABC where A, B and C represent the positions of three towns.

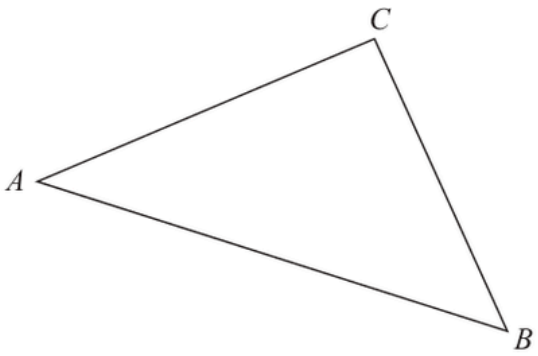


Diagram **NOT** accurately drawn

$$\overrightarrow{AB} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

Pru travels directly from A to B and then directly from B to C

Yang travels directly from A to C

Given that the values for $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are in kilometres,

work out how much further Pru travels than Yang travels.
Give your answer in km, correct to one decimal place.

..... km

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May 2023

1 **Pru travels**

Pru travels

$$|AB| + |BC| = \sqrt{7^2 + (-2)^2} + \sqrt{(-3)^2 + 5^2}$$

$$= \sqrt{53} + \sqrt{34} = 13.111\ldots$$

2 **Simplify surd**

Yang travels from A to C.

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$|AC| = \sqrt{4^2 + 3^2} = 5$$

---

3 **Difference**

Difference:

$$13.111\ldots - 5 = 8.111\ldots$$

FINAL ANSWER

Pru travels $8.1\ \text{km}$ further.

Question 22

5 marks | Vectors

22

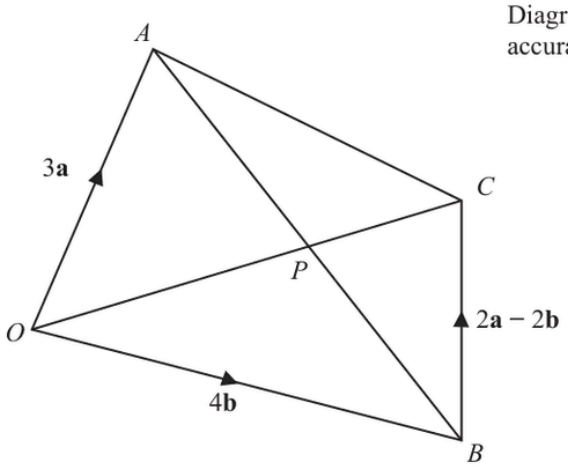


Diagram **NOT** accurately drawn

$OACB$ is a quadrilateral.

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{4b}$ $\overrightarrow{BC} = \mathbf{2a} - \mathbf{2b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$\overrightarrow{OC} =$ .....

(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .....

(1)

The point P lies on AB and on OC

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

.....

(3)

(Total for Question 22 is 5 marks)

1 **Split the ratio**

$$\overrightarrow{OA} = 3a, \quad \overrightarrow{OB} = 4b, \quad \overrightarrow{BC} = 2a - 2b$$

2 **Part (a)**

Part (a):

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 4b + (2a - 2b) = 2a + 2b$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4b - 3a$$

3 **Split the ratio**

For part (b), let P divide AB so that

$$\overrightarrow{AP} = \mu\overrightarrow{AB}$$

4 **Split the ratio**

Then

$$\overrightarrow{OP} = 3a + \mu(4b - 3a) = (3 - 3\mu)a + 4\mu b$$

5 **Split the ratio**

Since P lies on OC, also let

$$\overrightarrow{OP} = \lambda(2a + 2b)$$

6 **Equating coefficients**

Equating coefficients:

$$3 - 3\mu = 2\lambda, \quad 4\mu = 2\lambda$$

7 **Split the ratio**

From $4\mu = 2\lambda$, $\lambda = 2\mu$.

$$3 - 3\mu = 4\mu$$

$$\mu = \frac{3}{7}$$

8 **Split the ratio**

So $AP : PB = \frac{3}{7} : \frac{4}{7}$.

FINAL ANSWER

$\overrightarrow{OC} = 2\boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{AB} = 4\boldsymbol{b} - 3\boldsymbol{a}$, and $AP : PB = 3 : 4$

4MA1/2H JUNE 2024

Question 24

6 marks | Vectors

24

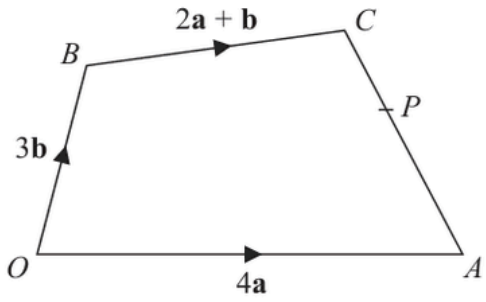


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OACB$ in which

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 3\mathbf{b}$ $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{AC} =$ .....

(2)

The point P lies on AC such that $AP : PC = 3 : 2$

The point Q is such that OPQ and BCQ are straight lines.

(b) Using a vector method, find $\overrightarrow{OQ}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OQ} =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2024

1 Read the graph

$$\vec{OA} = 4\mathbf{a}, \quad \vec{OB} = 3\mathbf{b}, \quad \vec{BC} = 2\mathbf{a} + \mathbf{b}$$

2 **Read the graph**

So

$$\vec{OC} = \vec{OB} + \vec{BC}$$

$$\vec{OC} = 2\mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\vec{AC} = (2\mathbf{a} + 4\mathbf{b}) - 4\mathbf{a}$$

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a}$$

---

4 **Read the graph**

For part (b), $AP : PC = 3 : 2$, so

$$\vec{OP} = \vec{OA} + \frac{3}{5}\vec{AC}$$

$$\vec{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a})$$

$$\vec{OP} = \frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

---

5 **Read the graph**

Since O, P, Q are collinear,

$$\vec{OQ} = t\vec{OP}$$

$$\vec{OQ} = t\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

---

6 **Read the graph**

Since B, C, Q are collinear,

$$\vec{OQ} = \vec{OB} + s\vec{BC}$$

$$\vec{OQ} = 3\mathbf{b} + s(2\mathbf{a} + \mathbf{b})$$

$$\vec{OQ} = 2s\mathbf{a} + (3 + s)\mathbf{b}$$

---

7 **Equate coefficients**

Equate coefficients:

$$\frac{14}{5}t = 2s$$

$$\frac{12}{5}t = 3 + s$$

---

8 **Solve equation**

From the first equation,

$$s = \frac{7}{5}t$$

---

9 **Substitute into the second**

Substitute into the second:

$$\frac{12}{5}t = 3 + \frac{7}{5}t$$

$$t = 3$$

---

10 **Read the graph**

Therefore

$$\vec{OQ} = 3\vec{OP}$$

$$\vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{AC} = 4\mathbf{b} - 2\mathbf{a},\ \vec{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

Question 25

6 marks | Vectors

25 $OPQR$ is a parallelogram.

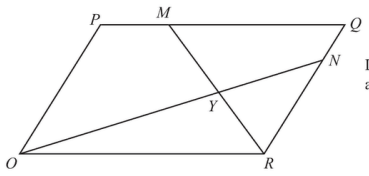


Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point M lies on PQ such that $PM = \frac{1}{4}PQ$

The point N lies on RQ such that $RN = \frac{4}{5}RQ$

(a) Find, in terms of **a** and **b**, giving your answers in simplest form

(i) $\overrightarrow{ON}$

.....

(1)

(ii) $\overrightarrow{MR}$

.....

(1)

MR and ON intersect at the point Y

Given that

$$OY = k \times ON$$

(b) use a vector method to find the value of k

$k =$ .....

(4)

(Total for Question 25 is 6 marks)

1 **Calculate value**

$$\overrightarrow{OP} = 2a, \qquad \overrightarrow{OR} = 3b$$

2 **Calculate value**

So

$$\overrightarrow{OQ} = 2a + 3b$$

3 **Evaluate fraction**

Since $PM : PQ = 1 : 4$,

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{4}(3b) = \frac{3}{4}b$$

$$\overrightarrow{OM} = 2a + \frac{3}{4}b$$

4 **Evaluate fraction**

Since $RN : RQ = 4 : 5$,

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ}$$

$$\overrightarrow{RN} = \frac{4}{5}(2a) = \frac{8}{5}a$$

$$\overrightarrow{ON} = 3b + \frac{8}{5}a$$

5 **Evaluate fraction**

So

$$\overrightarrow{MR} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{MR} = 3b - \left(2a + \frac{3}{4}b\right)$$

$$\overrightarrow{MR} = -2a + \frac{9}{4}b$$

---

6 **Let be on**

Let Y be on ON, so

$$\overrightarrow{OY} = k\overrightarrow{ON} = k\left(\frac{8}{5}a + 3b\right)$$

---

7 **Evaluate fraction**

Also Y is on MR, so

$$\overrightarrow{OY} = \overrightarrow{OM} + t\overrightarrow{MR}$$

$$\overrightarrow{OY} = 2a + \frac{3}{4}b + t\left(-2a + \frac{9}{4}b\right)$$

---

8 **Evaluate fraction**

Compare coefficients of a and b:

$$\frac{8}{5}k = 2 - 2t$$

$$3k = \frac{3}{4} + \frac{9}{4}t$$

---

9 **Solve equation**

From the first equation,

$$t = 1 - \frac{4}{5}k$$

---

10 **Substitute into the second equation**

Substitute into the second equation:

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = 3 - \frac{9}{5}k$$

$$\frac{24}{5}k = 3$$

$$k = \frac{5}{8}$$

✓ **FINAL ANSWER**

$\overrightarrow{ON} = \frac{8}{5}a + 3b$, $\overrightarrow{MR} = -2a + \frac{9}{4}b$, and $k = \frac{5}{8}$

4MA1/2H NOVEMBER 2024

Question 17

3 marks | Vectors

17 Here are two vectors.

$$\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad \overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$$

Calculate the magnitude of the vector $\overrightarrow{HF}$

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2024

1 Use vectors

$$\overrightarrow{GF} = -\overrightarrow{FG} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\overrightarrow{HF} = \overrightarrow{HG} + \overrightarrow{GF} = \begin{pmatrix} 4 \\ 14 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

2 Magnitude

Magnitude:

$$|\overrightarrow{HF}| = \sqrt{9^2 + 12^2} = 15$$

FINAL ANSWER

15.

Question 24

6 marks | Vectors

24 OAB is a triangle.

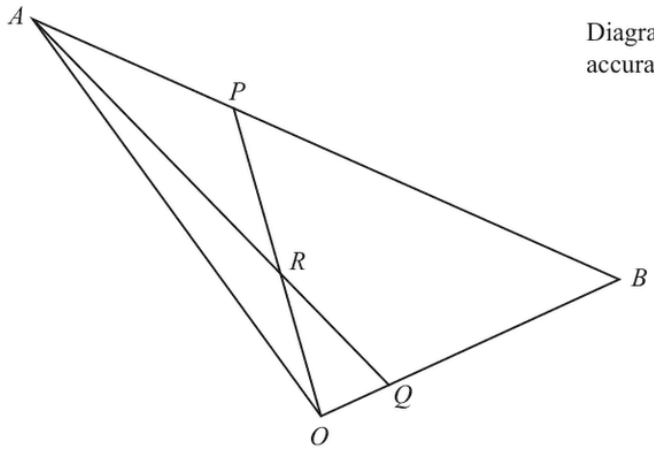


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 10\mathbf{a}$ $\overrightarrow{OB} = 10\mathbf{b}$

ARQ and ORP are straight lines.

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$ and $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

Write the following vectors in terms of **a** and **b**
Simplify your answers.

(i) $\overrightarrow{AQ}$

.....

(1)

(ii) $\overrightarrow{OP}$

.....

(1)

(iii) $\overrightarrow{OR}$

.....

(4)

(Total for Question 24 is 6 marks)

1 **Read the graph**

$$\vec{OA} = 10\mathbf{a}, \qquad \vec{OB} = 10\mathbf{b}$$

2 **Read the graph**

Also,

$$\vec{AP} = \frac{1}{4}\vec{AB}$$

3 **Read the graph**

and

$$\vec{OQ} = \frac{1}{5}\vec{OB}$$

4 **Read the graph**

For part (i),

$$\vec{OQ} = 2\mathbf{b}$$

$$\vec{AQ} = \vec{OQ} - \vec{OA}$$

$$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$$

5 **Read the graph**

For part (ii),

$$\vec{AB} = 10\mathbf{b} - 10\mathbf{a}$$

$$\vec{AP} = \frac{1}{4}(10\mathbf{b} - 10\mathbf{a})$$

$$\vec{AP} = \frac{5}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

6 **Read the graph**

For part (iii), R lies on OP, so

$$\vec{OR} = t\vec{OP}$$

---

7 **Read the graph**

It also lies on AQ, so

$$\vec{OR} = \vec{OA} + s\vec{AQ}$$

$$\vec{OR} = 10\mathbf{a} + s(2\mathbf{b} - 10\mathbf{a})$$

$$\vec{OR} = (10 - 10s)\mathbf{a} + 2s\mathbf{b}$$

---

8 **Read the graph**

Equate with

$$t\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$$

$$\frac{15}{2}t = 10 - 10s$$

$$\frac{5}{2}t = 2s$$

---

9 **Solve equation**

From the second equation,

$$s = \frac{5}{4}t$$

---

10 **Substitute**

Substitute:

$$\frac{15}{2}t = 10 - \frac{25}{2}t$$

$$20t = 10$$

$$t = \frac{1}{2}$$

---

11 **Read the graph**

So

$$\vec{OR} = \frac{1}{2}\vec{OP}$$

$$\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$$

✓ **FINAL ANSWER**

$\vec{AQ} = 2\mathbf{b} - 10\mathbf{a}$, $\vec{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$, $\vec{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$

4WM2/01R MAY 2025

Question 16

3 marks | Vectors

16 OAB is a triangle.

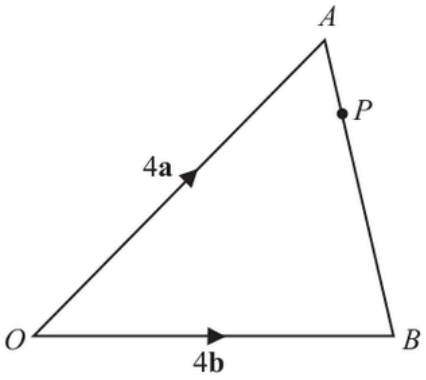


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{4a}$

$\overrightarrow{OB} = \mathbf{4b}$

P is the point on AB such that $AP : PB = 1 : 3$

(a) Write down $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

(b) Express $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM2/01R May 2025

1 Required

$overrightarrowOA = 4mathbfa$, $overrightarrowOB = 4mathbfb$, AP:PB = 1:3. (a) Find $overrightarrowAB$. (b) Express $overrightarrowOP$ in simplest form.

2 Part (a)

$$overrightarrowAB = overrightarrowAO + overrightarrowOB = -4mathbfa + 4mathbfb$$

3 Answer

$$overrightarrowAB = 4mathbfb - 4mathbfa$$

4 **AP PB 1 3 means**

AP:PB = 1:3 means $overrightarrowAP = tfrac14overrightarrowAB$:

$$overrightarrowAP = tfrac14(4mathbfb - 4mathbfa) = mathbfb - mathbfa$$

---

5 **Use**

Use $overrightarrowOP = overrightarrowOA + overrightarrowAP$:

$$overrightarrowOP = 4mathbfa + (mathbfb - mathbfa) = 3mathbfa + mathbfb$$

✓ **FINAL ANSWER**

$$overrightarrowOP = 3mathbfa + mathbfb$$

4MA1/2HR JUNE 2025

Question 18

3 marks | Vectors

18 OAB is a triangle.

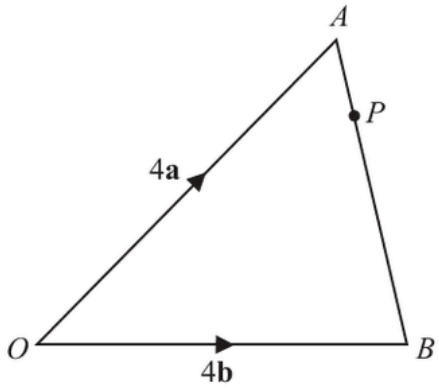


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 4\mathbf{a}$

$\overrightarrow{OB} = 4\mathbf{b}$

P is the point on AB such that $AP : PB = 1 : 3$

(a) Write down $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

(b) Express $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2025

1 **Use vectors**

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4\mathbf{b} - 4\mathbf{a}$$

2 **Use vectors**

Since $AP : PB = 1 : 3$, point P is one quarter of the way from A to B.

$$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}(4\mathbf{b} - 4\mathbf{a}) = \mathbf{b} - \mathbf{a}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4\mathbf{a} + \mathbf{b} - \mathbf{a} = 3\mathbf{a} + \mathbf{b}$$

FINAL ANSWER

$\overrightarrow{AB} = 4\mathbf{b} - 4\mathbf{a}$, and $\overrightarrow{OP} = 3\mathbf{a} + \mathbf{b}$.

Question 23

5 marks | Vectors

23

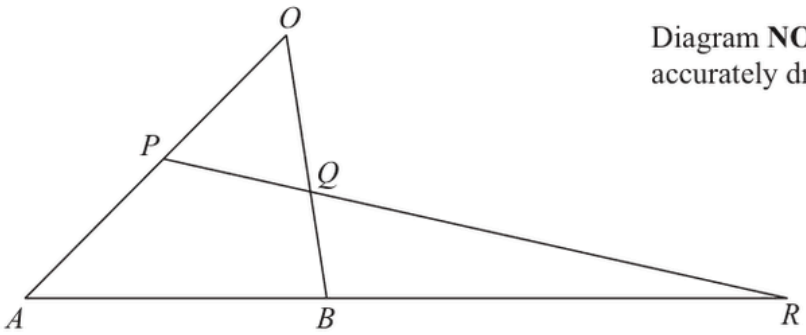


Diagram **NOT** accurately drawn

OAB is a triangle.
P is the midpoint of OA
Q is a point on OB

ABR and PQR are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\overrightarrow{OB} = 8\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**

.....

(1)

$AB : BR = 1 : 2$ $\overrightarrow{OQ} = n\mathbf{b}$

(b) Use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 23 is 5 marks)

1 **Use vectors**

For part (a),

$$\vec{OA} = 12\mathbf{a}, \qquad \vec{OB} = 8\mathbf{b}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}$$

2 **Find the midpoint**

For part (b), P is the midpoint of OA, so

$$\vec{OP} = 6\mathbf{a}$$

3 **Given**

Given

$$AB : BR = 1 : 2$$

$$\vec{BR} = 2\vec{AB}$$

$$\vec{BR} = 16\mathbf{b} - 24\mathbf{a}$$

4 **Use vectors**

Therefore

$$\vec{OR} = \vec{OB} + \vec{BR}$$

$$\vec{OR} = 8\mathbf{b} + 16\mathbf{b} - 24\mathbf{a}$$

$$\vec{OR} = -24\mathbf{a} + 24\mathbf{b}$$

5 **Use vectors**

Since P, Q, R are collinear, write Q on PR:

$$\vec{OQ} = \vec{OP} + t(\vec{OR} - \vec{OP})$$

$$\vec{OQ} = 6\mathbf{a} + t(-30\mathbf{a} + 24\mathbf{b})$$

$$\vec{OQ} = (6 - 30t)\mathbf{a} + 24t\mathbf{b}$$

---

6 **Use vectors**

But Q lies on OB, so there is no $\mathbf{a}$ component:

$$6 - 30t = 0$$

$$t = \frac{1}{5}$$

---

7 **Use vectors**

Then

$$\vec{OQ} = 24\left(\frac{1}{5}\right)\mathbf{b}$$

$$\vec{OQ} = \frac{24}{5}\mathbf{b}$$

---

8 **Use vectors**

Since $\vec{OQ} = n\mathbf{b}$,

$$n = \frac{24}{5}$$

✓ **FINAL ANSWER**

$\vec{AB} = 8\mathbf{b} - 12\mathbf{a}, \quad n = \frac{24}{5}$

Question 26

5 marks | Vectors

26

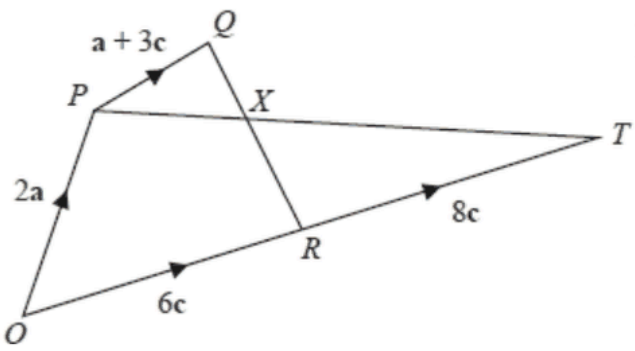


Diagram **NOT** accurately drawn

The diagram shows a quadrilateral $OPQR$ in which

$\overrightarrow{OP} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{c}$ $\overrightarrow{PQ} = \mathbf{a} + 3\mathbf{c}$

(a) Find in terms of **a** and **c**

$\overrightarrow{QR}$ = ...

(1)

The point T is such that $\overrightarrow{RT} = 8\mathbf{c}$
The point X lies on QR such that PXT is a straight line.

(b) Using a vector method, find the ratio $QX : XR$
Give your answer in the form $m : n$ where m and n are integers.
Show your working clearly.

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$QX : XR$ = ...

(4)

(Total for Question 26 is 5 marks)

1 **(a)**

(a)

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$\overrightarrow{OQ} = 2a + (a + 3c) = 3a + 3c$$

$$\overrightarrow{QR} = 6c - (3a + 3c) = 3c - 3a$$

2 **(b) Let divide so that**

(b) Let X divide QR so that

$$\overrightarrow{QX} = \lambda\overrightarrow{QR}$$

3 **Calculate value**

Then

$$\overrightarrow{OX} = 3a + 3c + \lambda(3c - 3a)$$

4 **Calculate value**

Also X lies on PT, where $\overrightarrow{OT} = 14c$, so

$$\overrightarrow{OX} = 2a + \mu(14c - 2a)$$

5 **Equating coefficients gives . Therefore**

Equating coefficients gives $\lambda = \frac{5}{9}$. Therefore

$$QX : XR = \frac{5}{9} : \frac{4}{9} = 5 : 4$$

✓ **FINAL ANSWER**

$\overrightarrow{QR} = 3c - 3a$; $QX : XR = 5 : 4$.

Question 24

6 marks | Vectors

24 The diagram shows a quadrilateral $OABC$

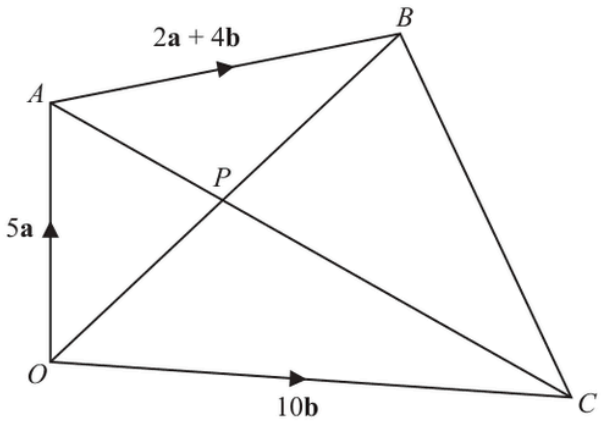


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 5\mathbf{a}$ $\overrightarrow{OC} = 10\mathbf{b}$ $\overrightarrow{AB} = 2\mathbf{a} + 4\mathbf{b}$

(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**

$\overrightarrow{AC} =$ .....

(1)

(b) Find $\overrightarrow{OB}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{OB} =$ .....

(1)

The point P is such that APC and OPB are straight lines.

(c) Using a vector method, find $\overrightarrow{OP}$ in terms of **a** and **b**
Give your answer in its simplest form.
Show your working clearly.

$\overrightarrow{OP} =$ .....

(4)

(Total for Question 24 is 6 marks)

1

(a) Find $overrightarrow{AC}$

$$overrightarrow{AC} = overrightarrow{AO} + overrightarrow{OC} = -overrightarrow{OA} + overrightarrow{OC} = -5mathbf{a} + 10mathbf{b}$$

2

Answer

a − 5**a** + 10**b**

3

(b) Find $overrightarrow{OB}$

$$overrightarrow{OB} = overrightarrow{OA} + overrightarrow{AB} = 5mathbf{a} + (2mathbf{a} + 4mathbf{b}) = 7mathbf{a} + 4mathbf{b}$$

4

Answer

b 7**a** + 4**b**

Step 1: Set up vector equations for $overrightarrow{OP}$. Since *O**P**B* is a straight line, $overrightarrow{OP}$ is a scalar multiple of $overrightarrow{OB}$. Let this be *λ*.

$$overrightarrow{OP} = lambda(7mathbf{a} + 4mathbf{b}) = 7lambdamathbf{a} + 4lambdamathbf{b}$$

Since *P* lies on the straight line *A**C*, $overrightarrow{AP} = muoverrightarrow{AC}$ for some scalar *μ*.

$$overrightarrow{OP} = overrightarrow{OA} + overrightarrow{AP} = 5mathbf{a} + mu(-5mathbf{a} + 10mathbf{b})$$

$$overrightarrow{OP} = (5 - 5mu)mathbf{a} + 10mumathbf{b}$$

Step 2: Equate the coefficients of a **and** b**.**

7*λ* = 5 − 5*μ* — (1)

$$4lambda = 10mu implies mu = frac{4}{10}lambda = 0.4lambda quad text{---(2)}$$

Step 3: Solve for *λ*. Substitute (2) into (1):

7*λ* = 5 − 5(0.4*λ*)

7*λ* = 5 − 2*λ*

$$9lambda = 5 implies lambda = frac{5}{9}$$

5

Evaluate fraction

$$overrightarrow{OP} = frac{5}{9}(7mathbf{a} + 4mathbf{b}) = frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

✓ **FINAL ANSWER**

$$frac{35}{9}mathbf{a} + frac{20}{9}mathbf{b}$$

4MA1/1H NOVEMBER 2025

Question 27

6 marks | Vectors

27

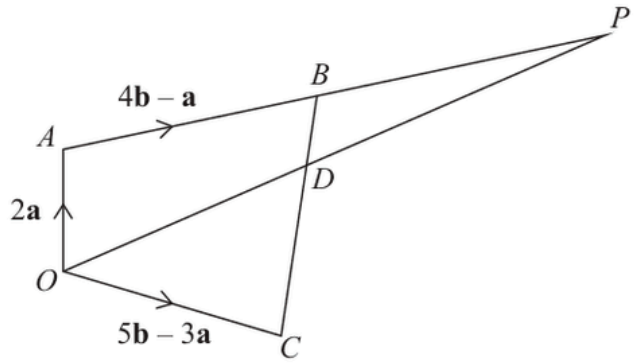


Diagram **NOT** accurately drawn

$OABC$ is a quadrilateral.
ABP and ODP are straight lines.

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{AB} = 4\mathbf{b} - \mathbf{a}$ $\overrightarrow{OC} = 5\mathbf{b} - 3\mathbf{a}$

(a) Find an expression in terms of **a** and **b** for the vector $\overrightarrow{BC}$
Simplify your answer.

.....

(2)

The point D lies on BC such that $BD : DC = 1 : 3$

Given that $\overrightarrow{OP} = n\overrightarrow{OD}$

(b) use a vector method to find the value of n

$n =$ .....

(4)

(Total for Question 27 is 6 marks)

WORKED SOLUTION

Question 27 · 4MA1/1H November 2025

1 Read the graph

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{AB} = 4\mathbf{b} - \mathbf{a}, \qquad \vec{OC} = 5\mathbf{b} - 3\mathbf{a}$$

2 **Read the graph**

First find $\vec{OB}$:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = 2\mathbf{a} + 4\mathbf{b} - \mathbf{a}$$

$$\vec{OB} = \mathbf{a} + 4\mathbf{b}$$

---

3 **Read the graph**

For part (a),

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$\vec{BC} = (5\mathbf{b} - 3\mathbf{a}) - (\mathbf{a} + 4\mathbf{b})$$

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}$$

---

4 **Read the graph**

For part (b), $BD : DC = 1 : 3$, so

$$\vec{OD} = \vec{OB} + \frac{1}{4}\vec{BC}$$

$$\vec{OD} = \mathbf{a} + 4\mathbf{b} + \frac{1}{4}(\mathbf{b} - 4\mathbf{a})$$

$$\vec{OD} = \frac{17}{4}\mathbf{b}$$

---

5 **Read the graph**

Since A, B, P are collinear, let

$$\vec{OP} = \vec{OA} + s\vec{AB}$$

$$\vec{OP} = 2\mathbf{a} + s(4\mathbf{b} - \mathbf{a})$$

$$\vec{OP} = (2 - s)\mathbf{a} + 4s\mathbf{b}$$

---

6 **Read the graph**

But O, D, P are collinear, and $\vec{OD}$ has only a $\mathbf{b}$ component.

---

7 **Read the graph**

So

$$2 - s = 0$$

$$s = 2$$

---

8 **Read the graph**

Then

$$\vec{OP} = 8\mathbf{b}$$

---

9 **Given**

Given

$$\vec{OP} = n\vec{OD}$$

$$8\mathbf{b} = n\left(\frac{17}{4}\mathbf{b}\right)$$

$$n = \frac{32}{17}$$

✓ **FINAL ANSWER**

$$\vec{BC} = \mathbf{b} - 4\mathbf{a}, \quad n = \frac{32}{17}$$

Question 18

5 marks | Vectors

18 Here are two vectors.

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \qquad \overrightarrow{BA} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$$

(a) Find $2\overrightarrow{AO}$ as a column vector.

$2\overrightarrow{AO} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$

(1)

(b) Find $\overrightarrow{OB}$ as a column vector.

$\overrightarrow{OB} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$

(2)

(c) Calculate the magnitude of the vector $\overrightarrow{BA}$

.....

(2)

(Total for Question 18 is 5 marks)

1 **(a) Reverse OA and double**

$$\overrightarrow{AO} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \qquad 2\overrightarrow{AO} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

2 **(b) Route O → A → B**

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 15 \\ -8 \end{pmatrix} = \begin{pmatrix} 19 \\ -11 \end{pmatrix}$$

3 **(c) Magnitude**

$$|\overrightarrow{BA}| = \sqrt{(-15)^2 + 8^2} = \sqrt{289} = 17$$

✓ **FINAL ANSWER**

(a) $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ (b) $\begin{pmatrix} 19 \\ -11 \end{pmatrix}$ (c) 17

4MA1/2HR JUNE 2026

Question 18

2 marks | Vectors

18 Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$$

Find $\overrightarrow{AC}$ as a column vector.

$$\overrightarrow{AC} = \begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AB} - \overrightarrow{CB}.$$

Reverse CB to travel from B to C.

2 Solution | Step 2

$$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}.$$

Subtract corresponding components.

FINAL ANSWER

$$\overrightarrow{AC} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}.$$

4MA1/2H JUNE 2026

Question 22

4 marks | Vectors

22 The diagram shows a quadrilateral $OABC$ where O, A, B and C represent the positions of four houses.

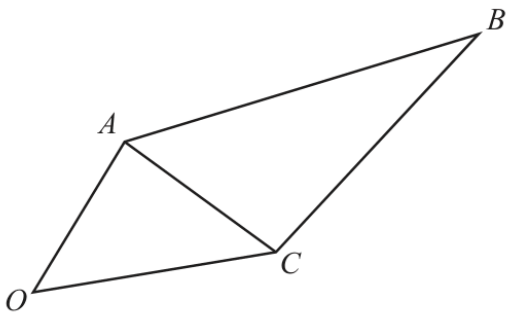


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$

Ria walks directly from A to B
Anna walks directly from A to C and then walks directly from C to B

$\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are given in metres.

Work out how much further Anna walks than Ria walks.
Give your answer correct to one decimal place.

..... metres

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2026

1 **Solution | Step 1**

$$\overrightarrow{AB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 19 \\ 6 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 12 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix},$$

$$\overrightarrow{CB} = \begin{pmatrix} 21 \\ 14 \end{pmatrix} - \begin{pmatrix} 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \end{pmatrix}.$$

Subtract the starting position vector from the finishing position vector.

2 **Solution | Step 2**

$$AB = \sqrt{19^2 + 6^2} = \sqrt{397}, \quad AC = \sqrt{10^2 + (-5)^2} = \sqrt{125}, \quad CB = \sqrt{9^2 + 11^2} = \sqrt{202}.$$

The modulus of a two-dimensional vector is the square root of the sum of squares of its components.

3 Solution | Step 3

$$\text{Extra distance} = AC + CB - AB = \sqrt{125} + \sqrt{202} - \sqrt{397} = 5.468151\ldots \text{ m}.$$

Compare Anna's two-stage route with Ria's direct route; do not round the intermediate lengths.

FINAL ANSWER

Anna walks 5.5 m further (1 decimal place).

4MA1/1HR JUNE 2026

Question 25

6 marks | Vectors

25

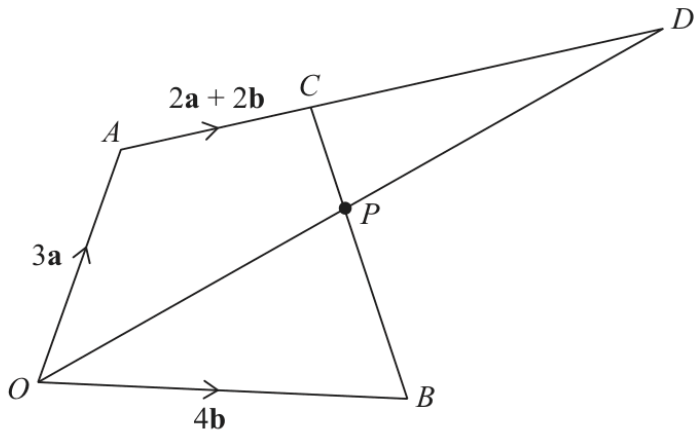


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = 4\mathbf{b}$ $\overrightarrow{AC} = 2\mathbf{a} + 2\mathbf{b}$ $\overrightarrow{CD} = \frac{3}{2} \times \overrightarrow{AC}$

P is the point on BC such that OPD is a straight line.

(a) Find $\overrightarrow{OD}$ in terms of **a** and **b**
Give your answer in its simplest form.

.....

(2)

(b) Find the ratio $OP : PD$
Show your working clearly.

.....

(4)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\overrightarrow{CD} = \frac{3}{2}(2\mathbf{a} + 2\mathbf{b}) = 3\mathbf{a} + 3\mathbf{b}.$$

Use the stated relationship between CD and AC.

2 **Part (a) | Step 2**

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD} = 3\mathbf{a} + (2\mathbf{a} + 2\mathbf{b}) + (3\mathbf{a} + 3\mathbf{b}) = 8\mathbf{a} + 5\mathbf{b}.$$

Add consecutive vectors along the route O-A-C-D.

3 **Part (b) | Step 1**

$$\overrightarrow{OC} = 3\mathbf{a} + (2\mathbf{a} + 2\mathbf{b}) = 5\mathbf{a} + 2\mathbf{b}, \qquad \overrightarrow{BC} = 5\mathbf{a} - 2\mathbf{b}.$$

Use OC=OA+AC and BC=OC-OB.

4 **Part (b) | Step 2**

Write $\overrightarrow{OP} = t\overrightarrow{OD} = 8t\mathbf{a} + 5t\mathbf{b}$. Also, because P lies on BC,

$$\overrightarrow{OP} = \overrightarrow{OB} + s\overrightarrow{BC} = 5s\mathbf{a} + (4 - 2s)\mathbf{b}.$$

Use one scalar parameter on each of the two intersecting lines.

5 **Part (b) | Step 3**

$$8t = 5s, \qquad 5t = 4 - 2s, \qquad s = \frac{8}{5}t.$$

Equate the coefficients of the two nonparallel base vectors.

6 **Part (b) | Step 4**

$$5t = 4 - \frac{16}{5}t \quad \Longrightarrow \quad 41t = 20 \quad \Longrightarrow \quad t = \frac{20}{41}.$$

Substitute the first relation into the second and solve.

7 **Part (b) | Step 5**

$$OP : PD = t : (1 - t) = \frac{20}{41} : \frac{21}{41} = 20 : 21.$$

OD is divided into the fractions t and 1-t at P.

✓ **FINAL ANSWER**

(a) $\overrightarrow{OD} = 8\mathbf{a} + 5\mathbf{b}$ (b) $OP : PD = 20 : 21$

TOPIC

Statistics Toolkit

Every past-paper question on this topic, 2020 to 2026.

4WM2/01 SPECIMEN

Question 1

3 marks | Statistics Toolkit

1 Here are five cards, where x represents a number.

15	7	−2	23	x

The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM2/01 Specimen

1 **Calculate statistic**

The mean is 12, so the total of the five numbers is

$$5 \times 12 = 60$$

2 **Calculate value**

The known numbers add to

$$15 + 7 - 2 + 23 = 43$$

3 **Calculate value**

Therefore

$$43 + x = 60$$

$$x = 17$$

FINAL ANSWER

$x = 17$.

4WM2/01 SPECIMEN

Question 6

3 marks | Statistics Toolkit

6 Here are some integers where $a < b < c < d$

$$a \quad b \quad c \quad d \quad d \quad d$$

The mode of the integers is 9
The range of the integers is 4
The median of the integers is 8

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 Specimen

1 **Calculate statistic**

The mode is 9, and d appears three times, so

$$d = 9$$

2 **Calculate statistic**

The range is 4, so

$$d - a = 4$$

$$9 - a = 4$$

$$a = 5$$

3 Calculate statistic

There are six values, so the median is the mean of the 3rd and 4th values:

$$\frac{c+d}{2}=8$$

$$\frac{c+9}{2}=8$$

$$c=7$$

4 Solve inequality

Since $a<b<c<d$, the remaining integer is $b=6$.

FINAL ANSWER

$a=5,\ b=6,\ c=7,\ d=9$.

4MA1/2HR JANUARY 2020

Question 2

3 marks | Statistics Toolkit

2 The table gives information about the amount of money, in £, that Fiona spent in a grocery store each week during 2019

Amount spent (£x)	Frequency
$0 \leqslant x < 20$	5
$20 \leqslant x < 40$	11
$40 \leqslant x < 60$	8
$60 \leqslant x < 80$	19
$80 \leqslant x < 100$	9

Work out an estimate for the total amount of money that Fiona spent in the grocery store during 2019

£.....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR January 2020

1 Use midpoints

Use midpoints $10, 30, 50, 70, 90$.

$$10(5) + 30(11) + 50(8) + 70(19) + 90(9)$$

$$= 50 + 330 + 400 + 1330 + 810 = 2920$$

FINAL ANSWER

$\pounds 2920$.

4MA1/1HR JANUARY 2020

Question 8

3 marks | Statistics Toolkit

8 5 children are playing on a trampoline.
The mean weight of the 5 children is 28 kg.

2 of the children get off the trampoline.
The mean weight of these 2 children is 26.5 kg.

Work out the mean weight of the 3 children who remain on the trampoline.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2020

1 Total weight of 5 children

Total weight of 5 children:

$$5 \times 28 = 140 \text{ kg}$$

2 Total weight of the 2

Total weight of the 2 children who get off:

$$2 \times 26.5 = 53 \text{ kg}$$

3 Total weight of the 3

Total weight of the 3 children left:

$$140 - 53 = 87 \text{ kg}$$

4 Mean weight

Mean weight:

$$\frac{87}{3} = 29$$

FINAL ANSWER

29 kg.

4MA1/2HR JANUARY 2020

Question 14

3 marks | Statistics Toolkit

14 Max kept a record of the marks he scored in each of the 11 spelling tests he took one term.

Here are his marks.

18 5 7 12 11 18 15 16 17 13 14

Find the interquartile range of the marks.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR January 2020

1 **Put the marks in order**

Put the marks in order:

$$5,\ 7,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 18$$

$$Q_1 = 11, \qquad Q_3 = 17$$

$$\text{IQR} = 17 - 11 = 6$$

FINAL ANSWER

6.

4MA1/1H MAY/NOVEMBER 2020

Question 4

4 marks | Statistics Toolkit

4 Here is a list of six numbers written in order of size.

$$4 \quad 7 \quad x \quad 10 \quad y \quad y$$

The numbers have

a median of 9
a mean of 11

Find the value of x and the value of y.

$x =$ .....

$y =$ .....

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May/November 2020

1 The median is

The median is 9:

$$\frac{x + 10}{2} = 9$$

$$x = 8$$

2 Calculate statistic

The mean is 11, so the total is

$$6 \times 11 = 66$$

$$4 + 7 + 8 + 10 + y + y = 66$$

$$29 + 2y = 66$$

$$2y = 37$$

$$y = 18.5$$

FINAL ANSWER

$x = 8,\ y = 18.5.$

4MA1/1HR JANUARY 2021

Question 2

4 marks | Statistics Toolkit

2 Given that $a < b < c$

the four whole numbers a, a, b and c have

a mode of 7
a median of 8.5
a mean of 9

Work out the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2021

1 Calculate statistic

The mode is 7, and the numbers are a, a, b, c, so

$$a = 7$$

2 The median of four numbers

The median of four numbers is

$$\frac{a+b}{2}$$

3 Evaluate fraction

So

$$\frac{7+b}{2} = 8.5$$

$$7 + b = 17$$

$$b = 10$$

4 **Calculate statistic**

The mean is 9, so the total is

$$4 \times 9 = 36$$

$$7 + 7 + 10 + c = 36$$

$$c = 12$$

✓ **FINAL ANSWER**

$a = 7,\ b = 10,\ c = 12$.

4MA1/2H JANUARY 2021

Question 2

3 marks | Statistics Toolkit

2 Ava writes down five whole numbers.

For these five numbers

the median is 7

the mode is 8

the range is 5

Find a possible value for each of the five numbers that Ava writes down.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2021

1 Calculate statistic

The median is 7, so the middle number is 7.

2 Calculate statistic

The mode is 8, so use two 8s.

3 Calculate statistic

The range is 5, so choose the smallest and largest numbers to differ by 5.

One possible set is

$$3,\ 6,\ 7,\ 8,\ 8$$

4 Range

Range:

$$8 - 3 = 5$$

5 Mode

Mode:

$$8$$

6 **Median**

Median:

$$7$$

✓ **FINAL ANSWER**

$3,\ 6,\ 7,\ 8,\ 8.$

4MA1/2H JANUARY 2021

Question 7

3 marks | Statistics Toolkit

7 There are 5 cocoa pods in a bag.
The mean weight of the 5 cocoa pods is 398 grams.

A sixth cocoa pod is put into the bag.
The mean weight of the 6 cocoa pods is 401 grams.

Work out the weight of the sixth cocoa pod that is put into the bag.

..... grams

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2021

1 Total weight of 5 cocoa

Total weight of 5 cocoa pods:

$$5 \times 398 = 1990 \text{ g}$$

2 Total weight of 6 cocoa

Total weight of 6 cocoa pods:

$$6 \times 401 = 2406 \text{ g}$$

3 Weight of the sixth cocoa

Weight of the sixth cocoa pod:

$$2406 - 1990 = 416$$

FINAL ANSWER

416 grams.

4MA1/2H JANUARY 2021

Question 12

2 marks | Statistics Toolkit

12 15 people were asked how long, in minutes, they had been waiting for a bus.

Here are the results.

2	3	3	4	5	6	6	8	9	10	11	13	14	15	18

Find the interquartile range of these times.

..... minutes

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2021

1 Use Statistics Toolkit correct

The data is already in order:

$$2, 3, 3, 4, 5, 6, 6, 8, 9, 10, 11, 13, 14, 15, 18$$

2 Calculate value

For 15 values,

$$Q_1 = 4, \qquad Q_3 = 13$$

$$\text{IQR} = 13 - 4 = 9$$

FINAL ANSWER

9 minutes.

4MA1/1H MAY 2021

Question 10

3 marks | Statistics Toolkit

10 Jethro has sat 5 tests.
Each test was marked out of 100 and Jethro's mean mark for the 5 tests is 74

Jethro has to sit one more test that is also to be marked out of 100

Jethro wants his mean mark for all 6 tests to be at least 77

Work out the least mark that Jethro needs to get for the last test.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2021

1 **The total for the first**

The total for the first 5 tests is

$$5 \times 74 = 370$$

2 **Calculate value**

For an average of at least 77 after 6 tests, the total must be at least

$$6 \times 77 = 462$$

3 **Calculate value**

So the last test score must be at least

$$462 - 370 = 92$$

✓ **FINAL ANSWER**

at least 92.

4MA1/1H MAY 2021

Question 14

2 marks | Statistics Toolkit

14 Here is the number of goals that Henri's team scored one summer in each water polo match.

5 8 9 11 13 13 14 15 16 17 20

Find the interquartile range of the numbers of goals.
Show your working clearly.

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2021

1 **The goals in order are**

The goals in order are

$$5,\ 8,\ 9,\ 11,\ 13,\ 13,\ 14,\ 15,\ 16,\ 17,\ 20$$

$$Q_1 = 9, \qquad Q_3 = 16$$

$$\text{IQR} = 16 - 9 = 7$$

FINAL ANSWER

7.

4MA1/1H JANUARY 2022

Question 5

6 marks | Statistics Toolkit

5 Jenny has six cards.

Each card has a whole number written on it so that

the smallest number is 5
the largest number is 24
the median of the six numbers is 14
the mode of the six numbers is 8

Jenny arranges her cards so that the numbers are in order of size.



(a) For the remaining four cards, write on each dotted line a number that could be on the card.

(3)

A basketball team plays 6 games.
After playing 5 games, the team has a mean score of 21 points per game.
After playing 6 games, the team has a mean score of 23 points per game.

(b) Work out the number of points the team scored in its 6th game.

.....

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2022

1 **One possible ordered set is**

One possible ordered set is

$$5,\ 8,\ 8,\ 20,\ 21,\ 24$$

2 **Check**

Check:

$$\text{median} = \frac{8 + 20}{2} = 14$$

3 **Calculate statistic**

The mode is 8, the smallest number is 5, and the largest number is 24.

4 **Calculate value**

For the basketball scores:

$$5 \times 21 = 105$$

$$6 \times 23 = 138$$

$$138 - 105 = 33$$

FINAL ANSWER

$5, 8, 8, 20, 21, 24$ and 33.

4MA1/1HR JANUARY 2022

Question 5

4 marks | Statistics Toolkit

5 Yusuf sat 8 examinations.

Here are his marks for 5 of the examinations.

68 72 75 77 80

For his results in all 8 examinations

the mode of his marks is 80
the median of his marks is 74
the range of his marks is 16

Find Yusuf's marks for each of the other 3 examinations.

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2022

1 The range is . Since

The range is 16. Since the greatest known mark is 80,

$$80 - 16 = 64$$

2 Use Statistics Toolkit tag correct

So one missing mark is 64.

3 Calculate statistic

The mode is 80, so another missing mark is 80.

4 Use Statistics Toolkit tag correct

Now the marks in order are

$$64,\ 68,\ 72,\ x,\ 75,\ 77,\ 80,\ 80$$

5 Calculate statistic

The median is 74, so

$$\frac{x + 75}{2} = 74$$

$$x = 73$$

FINAL ANSWER

64, 73, 80.

4MA1/2HR JANUARY 2022

Question 8

3 marks | Statistics Toolkit

8 A scientist is investigating the weight of 50 tigers.

Here is some information about these tigers.

	Type of tiger	
	Siberian	**Bengal**
Number of tigers	22	28
Mean weight of tigers (kg)	260	

The mean weight of all 50 tigers is 218 kg

Work out the mean weight of the Bengal tigers.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2022

1 Total mass of all 50

Total mass of all 50 tigers:

$$50 \times 218 = 10900$$

2 Total mass of the 22

Total mass of the 22 Siberian tigers:

$$22 \times 260 = 5720$$

3 Total mass of the 28

Total mass of the 28 Bengal tigers:

$$10900 - 5720 = 5180$$

4 Mean mass of the Bengal

Mean mass of the Bengal tigers:

$$\frac{5180}{28} = 185$$

FINAL ANSWER

185 kg.

4MA1/2H JANUARY 2022

Question 15

3 marks | Statistics Toolkit

15 Diyar recorded the distance, in kilometres, that he cycled each day for 11 days.
Here are his results.

8 10 12 13 5 23 21 7 5 16 14

Find the interquartile range of his results.

..... km

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H January 2022

1 Put the distances in order

Put the distances in order:

$$5,\ 5,\ 7,\ 8,\ 10,\ 12,\ 13,\ 14,\ 16,\ 21,\ 23$$

2 Calculate statistic

There are 11 values, so the median is the 6th value.

3 The lower half is

The lower half is

$$5,\ 5,\ 7,\ 8,\ 10$$

4 Calculate value

so

$$Q_1 = 7$$

5 The upper half is

The upper half is

$$13,\ 14,\ 16,\ 21,\ 23$$

6 **Calculate value**

so

$$Q_3 = 16$$

$$\text{IQR} = 16 - 7 = 9$$

✓ **FINAL ANSWER**

$9\ \text{km}$.

4MA1/2H JUNE 2022

Question 1

3 marks | Statistics Toolkit

1 Here are some integers where $a < b < c < d$

$$a \quad b \quad c \quad d \quad d \quad d$$

The mode of the integers is 9
The median of the integers is 8
The range of the integers is 4

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2022

1 **Calculate statistic**

The mode is $\mathbf{9}$, so

$$d = 9$$

2 **The range is**

The range is $\mathbf{4}$:

$$9 - a = 4$$

$$a = 5$$

3 **Calculate statistic**

The median is 8, so

$$\frac{c+d}{2}=8$$

$$\frac{c+9}{2}=8$$

$$c=7$$

4 **The integers are in order**

The integers are in order and $a<b<c$, so

$$b=6$$

FINAL ANSWER

$a=5,\ b=6,\ c=7,\ d=9$.

4MA1/1HR JUNE 2022

Question 3

3 marks | Statistics Toolkit

3 Here are five cards.

Each card has a number written on it.



The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2022

1 The mean of the five

The mean of the five cards is 12, so the total is

$$5 \times 12 = 60$$

2 The known total is

The known total is

$$15 + 7 - 2 + 23 = 43$$

$$x = 60 - 43 = 17$$

FINAL ANSWER

$x = 17$.

4MA1/2HR JUNE 2022

Question 3

3 marks | Statistics Toolkit

3 Alberto, Bill, Candela and Diana are four friends.

Here is some information about the height of each of these friends.

Alberto's height is 158 cm.
Bill's height is 175 cm.
Candela's height is greater than Diana's height.

The median height of these four friends is 160 cm.
The range of the heights of these four friends is 21 cm.

Work out Candela's height and Diana's height.

Candela ..... cm

Diana ..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2022

1 The median is . Since

The median is 160. Since the ordered heights are

$$\text{Diana},\ 158,\ \text{Candela},\ 175$$

$$\frac{158 + \text{Candela}}{2} = 160$$

$$\text{Candela} = 162$$

2 The range is

The range is 21:

$$175 - \text{Diana} = 21$$

$$\text{Diana} = 154$$

FINAL ANSWER

Candela $= 162$ cm, Diana $= 154$ cm.

4MA1/2H JUNE 2022

Question 12

3 marks | Statistics Toolkit

12 Larry is a delivery man.

He has 7 parcels to deliver.
The mean weight of the 7 parcels is 2.7 kg

Larry delivers 3 of the parcels.
Each of these 3 parcels has a weight of W kg

The mean weight of the other 4 parcels is 3.3 kg

Work out the value of W

$W =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2022

1 Total mass of the 7

Total mass of the 7 parcels:

$$7 \times 2.7 = 18.9$$

2 Total mass of the 4

Total mass of the 4 remaining parcels:

$$4 \times 3.3 = 13.2$$

3 Total mass of the 3

Total mass of the 3 delivered parcels:

$$18.9 - 13.2 = 5.7$$

4 Each delivered parcel has mass

Each delivered parcel has mass W:

$$3W = 5.7$$

$$W = 1.9$$

FINAL ANSWER

1.9 kg.

4MA1/1HR JUNE 2022

Question 13

2 marks | Statistics Toolkit

13 Here is the number of runs scored by a baseball team in each of its 15 games this season.

The number of runs have been arranged in order of size.

0 1 1 3 5 6 7 7 8 9 9 12 12 15 16

Work out the interquartile range of the number of runs.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2022

1 Use Statistics Toolkit correct

The runs are already in order:

$$0,\ 1,\ 1,\ 3,\ 5,\ 6,\ 7,\ 7,\ 8,\ 9,\ 9,\ 12,\ 12,\ 15,\ 16$$

2 Calculate statistic

There are 15 values, so ignore the median when finding the quartiles.

$$Q_1 = 3, \qquad Q_3 = 12$$

$$\text{IQR} = 12 - 3 = 9$$

FINAL ANSWER

9.

4MA1/1H MAY 2023

Question 2

3 marks | Statistics Toolkit

2 The table gives information about the number of minutes that Abby spent walking each day in September.

Number of minutes (M)	Frequency
$0 < M \leqslant 30$	5
$30 < M \leqslant 60$	6
$60 < M \leqslant 90$	8
$90 < M \leqslant 120$	9
$120 < M \leqslant 150$	2

Work out an estimate for the total number of minutes that Abby spent walking in September.

..... minutes

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2023

1 Use midpoints

Use midpoints $15, 45, 75, 105, 135$.

$$15(5) + 45(6) + 75(8) + 105(9) + 135(2) = 2160$$

✓ **FINAL ANSWER**

2160 minutes.

4MA1/2H JUNE 2023

Question 10

4 marks | Statistics Toolkit

10 Team **A** and Team **B** take part in a quiz league.

After 11 rounds, Team **A** has a mean score per round of 17
After 9 rounds, Team **B** has a mean score per round of 18

Both teams take part in a further round.
After this round, both teams have a mean score per round of 18.5

In the further round, Team **A** scored more points than Team **B**.

How many more?

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2023

1 **Team A**

Team A:

$$11 \times 17 = 187$$

2 **Estimate the value**

After the next round:

$$12 \times 18.5 = 222$$

$$\text{next score} = 222 - 187 = 35$$

3 **Team B**

Team B:

$$9 \times 18 = 162$$

4 **Estimate the value**

After the next round:

$$10 \times 18.5 = 185$$

$$\text{next score} = 185 - 162 = 23$$

5 Difference

Difference:

$$35 - 23 = 12$$

FINAL ANSWER

12.

4MA1/2HR JUNE 2023

Question 10

3 marks | Statistics Toolkit

10 Gemara works as a taxi driver.
Last week, he recorded the following information about the distances he drove.

For the 5 days from Monday to Friday, the mean number of kilometres he drove was 104
For the 7 days from Monday to Sunday, the mean number of kilometres he drove was 127

On Saturday, Gemara drove 132 kilometres.

Work out the number of kilometres he drove on Sunday.

..... kilometres

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2023

1 Total for Monday to Friday

Total for Monday to Friday:

$$5 \times 104 = 520$$

2 Total for Monday to Sunday

Total for Monday to Sunday:

$$7 \times 127 = 889$$

3 Calculate value

So the total for Saturday and Sunday is

$$889 - 520 = 369$$

4 Calculate value

Saturday was $132\ \text{km}$, so

$$369 - 132 = 237$$

FINAL ANSWER

$237\ \text{km}$.

4MA1/1H MAY 2023

Question 10

2 marks | Statistics Toolkit

10 Here are the test marks of 15 students.

7 10 14 15 16 17 18 19 20 20 23 25 30 39 40

Find the interquartile range of these marks.

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2023

1 The marks in order are

The marks in order are

$$7,\ 10,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20,\ 20,\ 23,\ 25,\ 30,\ 39,\ 40$$

$$Q_1 = 15, \qquad Q_3 = 25$$

$$\text{IQR} = 25 - 15 = 10$$

FINAL ANSWER

10.

4MA1/2HR JUNE 2023

Question 14

4 marks | Statistics Toolkit

14 Akari played a computer game eleven times.
Here are her scores.

25 20 28 27 26 22 23 29 20 29 26

(a) Find the interquartile range of her scores.

.....

(3)

Machi played the same computer game eleven times.

The interquartile range for Machi's scores was 9

(b) Who had the more consistent scores, Akari or Machi?
Give a reason for your answer.

.....

(1)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2023

1 **Akari s scores in order**

Akari's scores in order are

$$20,\ 20,\ 22,\ 23,\ 25,\ 26,\ 26,\ 27,\ 28,\ 29,\ 29$$

$$Q_1 = 22, \qquad Q_3 = 28$$

$$\text{IQR} = 28 - 22 = 6$$

2 **Machi s interquartile range is**

Machi's interquartile range is $\mathbf{9}$.

3 **Akari is more consistent because**

Akari is more consistent because her interquartile range is smaller.

FINAL ANSWER

Akari's IQR is $\mathbf{6}$, and Akari is more consistent.

4MA1/2H NOVEMBER 2023

Question 8

3 marks | Statistics Toolkit

8 The mean number of goals scored by a hockey team in 8 matches is 6
The team plays 2 more matches and scores k goals in each match.
The mean number of goals scored by the hockey team in the 10 matches is 7

Work out the value of k

$k =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2023

1 The total for the first

The total for the first 8 matches is

$$8 \times 6 = 48$$

2 The total for all 10

The total for all 10 matches is

$$10 \times 7 = 70$$

3 Compare density

So the last two matches have total

$$70 - 48 = 22$$

4 The last two scores are

The last two scores are k and $k + 10$:

$$k + (k + 10) = 22$$

$$2k + 10 = 22$$

$$k = 6$$

FINAL ANSWER

$k = 6$.

4MA1/2H NOVEMBER 2023

Question 13

2 marks | Statistics Toolkit

13 Robert asked 11 people how many meetings they attended last week.

Here are the results in numerical order.

1 2 4 6 6 8 11 12 13 14 17

Find the interquartile range of the number of meetings.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2023

1 The results in order are

The results in order are

$$1,\ 2,\ 4,\ 6,\ 6,\ 8,\ 11,\ 12,\ 13,\ 14,\ 17$$

2 There are 11 values

There are 11 values.

$$Q_1 = 4, \qquad Q_3 = 13$$

$$\text{IQR} = 13 - 4 = 9$$

FINAL ANSWER

9.

4MA1/2H JUNE 2024

Question 1

3 marks | Statistics Toolkit

1 Here are eight numbers written in order of size

$$h \quad 6 \quad 7 \quad 8 \quad j \quad 16 \quad k \quad k$$

where h, j and k are integers.

The median of the eight numbers is 10

The mode of the eight numbers is 18

The range of the eight numbers is 13

Work out the value of h, the value of j and the value of k

$h =$ .....

$j =$ .....

$k =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2024

1 **The median is**

The median is 10:

$$\frac{8+j}{2} = 10$$

$$j = 12$$

2 **Calculate statistic**

The mode is 18, so

$$k = 18$$

3 **The range is**

The range is 13:

$$18 - h = 13$$

$$h = 5$$

✓ **FINAL ANSWER**

$h = 5,\ j = 12,\ k = 18.$

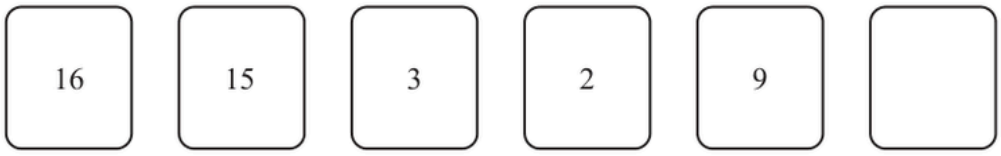
4MA1/1HR MAY 2024

Question 1

3 marks | Statistics Toolkit

1 Here are six cards.

Five of the cards have a number written on them.



Work out the number that should be written on the last card so that the mean of the six numbers will be 11

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May 2024

1 The total of the 6

The total of the 6 cards is

$$6 \times 11 = 66$$

2 The known total is

The known total is

$$16 + 15 + 3 + 2 + 9 = 45$$

$$x = 66 - 45 = 21$$

FINAL ANSWER

$x = 21$.

4MA1/2HR JUNE 2024

Question 3

2 marks | Statistics Toolkit

3 Here is a list of four numbers written in ascending order of size

$$x \quad x \quad y \quad 15$$

where x and y are integers.

The numbers have

a median of 12.5
a range of 4

Find the value of x and the value of y

$x =$ .....

$y =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2024

1 The range is

The range is 4:

$$15 - x = 4$$

$$x = 11$$

2 The median is

The median is 12.5:

$$\frac{x + y}{2} = 12.5$$

$$x + y = 25$$

$$y = 25 - 11 = 14$$

FINAL ANSWER

$x = 11,\ y = 14.$

4MA1/1HR MAY 2024

Question 10

2 marks | Statistics Toolkit

10 Here are the numbers of goals scored by a hockey team in its 11 games this season.

0 1 2 2 3 4 4 6 7 9 11

Work out the interquartile range of the numbers of goals.

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR May 2024

1 The goals in order are

The goals in order are

$$0,\ 1,\ 2,\ 2,\ 3,\ 4,\ 4,\ 6,\ 7,\ 9,\ 11$$

$$Q_1 = 2, \qquad Q_3 = 7$$

$$\text{IQR} = 7 - 2 = 5$$

FINAL ANSWER

5.

4MA1/2H JUNE 2024

Question 12

3 marks | Statistics Toolkit

12 Ali uses a fitness tracker to count the number of steps he walks each day for 7 days.

For the first 4 days, his mean number of steps is 11 800
For the next 3 days, his mean number of steps is 13 207

Work out his mean number of steps for the 7 days.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2024

1 Total for the first 4

Total for the first 4 days:

$$4 \times 11800 = 47200$$

2 Total for the next 3

Total for the next 3 days:

$$3 \times 13207 = 39621$$

3 Total for all 7 days

Total for all 7 days:

$$47200 + 39621 = 86821$$

4 Mean for the 7 days

Mean for the 7 days:

$$\frac{86821}{7} = 12403$$

FINAL ANSWER

12403.

4MA1/2H NOVEMBER 2024

Question 8

3 marks | Statistics Toolkit

8 Kim puts 4 bananas in a bag.
The mean weight of the 4 bananas in the bag is 145 grams.

Andy puts one more banana into the bag.
The mean weight of the 5 bananas in the bag is 142 grams.

Work out the weight of the banana that Andy puts into the bag.

..... grams

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2024

1 **The total weight of the**

The total weight of the first 4 bananas is

$$4 \times 145 = 580$$

2 **The total weight of the**

The total weight of the 5 bananas is

$$5 \times 142 = 710$$

3 **Weight of the banana Andy**

Weight of the banana Andy added:

$$710 - 580 = 130$$

✓ **FINAL ANSWER**

130 g.

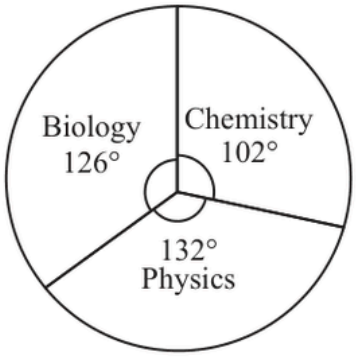
4MA1/2H NOVEMBER 2024

Question 10

5 marks | Statistics Toolkit

10 All the students in year 10 and all the students in year 11 named their favourite science subject from Biology, Chemistry and Physics.

The pie chart shows information about the results for the year 10 students.
The table shows information about the results for the year 11 students.



Pie chart for year 10

science subject	number of students
Biology	$3x + 6$
Chemistry	$5x + 8$
Physics	$7x - 9$

Table for year 11

There are 300 students in year 10
There are 320 students in year 11

More students in year 10 than in year 11 said Biology was their favourite science subject.

How many more?

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2024

1 **Evaluate fraction**

For year 10, Biology is $126°$ out of $360°$.

$$\frac{126}{360} \times 300 = 105$$

2 **Calculate value**

For year 11:

$$(3x + 6) + (5x + 8) + (7x - 9) = 320$$

$$15x + 5 = 320$$

$$x = 21$$

---

3 **Year 11 Biology**

Year 11 Biology:

$$3(21) + 6 = 69$$

---

4 **Difference**

Difference:

$$105 - 69 = 36$$

✓ **FINAL ANSWER**

36 more students.

4MA1/2H JUNE 2025

Question 1

3 marks | Statistics Toolkit

1 Rio has six cards.
He writes a number on each card so that

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

Rio arranges the cards so that the numbers are in order of size.

		6		10	14

Three of the numbers are hidden.

Complete the cards above to show the three numbers that are hidden.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2025

1 **Calculate statistic**

The range is 10, and the greatest card is 14:

$$14 - \text{smallest} = 10$$

2 **Use Statistics Toolkit tag correct**

So the smallest card is 4.

3 **Calculate statistic**

The median is 7.5. For 6 cards, the median is the mean of the 3rd and 4th cards:

$$\frac{6 + x}{2} = 7.5$$

$$x = 9$$

4 **Calculate statistic**

The mode is 6, so the other missing card must be 6.

FINAL ANSWER

$4,\ 6,\ 9$.

4WM2/01 MAY 2025

Question 1

3 marks | Statistics Toolkit

1 Rio has six cards.
He writes a number on each card so that

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

Rio arranges the cards so that the numbers are in order of size.

		6		10	14

Three of the numbers are hidden.
Complete the cards above to show the three numbers that are hidden.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM2/01 May 2025

1 **Calculate statistic**

The range is 10, so the first number is

$$14 - 10 = 4$$

2 **Calculate statistic**

For six numbers, the median is the mean of the third and fourth values.

$$\frac{6 + \text{fourth value}}{2} = 7.5$$

3 **Calculate statistic**

So the fourth value is 9. To make 6 the mode, the second value must also be 6.

✓ **FINAL ANSWER**

the hidden numbers are $4, 6, 9$.

4WM2/01R MAY 2025

Question 7

3 marks | Statistics Toolkit

7 The mean of 7 numbers is 60

The mean of 3 of the numbers is 46

Work out the mean of the other 4 numbers.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4WM2/01R May 2025

1 Required

Mean of 7 numbers = 60; mean of 3 of them = 46. Find the mean of the other 4.

2 Sum of all 7 numbers

Sum of all 7 numbers:

$7 \times 60 = 420$

3 Sum of the 3 numbers

Sum of the 3 numbers:

$3 \times 46 = 138$

4 Sum of the remaining 4

Sum of the remaining 4:

$420 - 138 = 282$

5 Mean of the 4 numbers

Mean of the 4 numbers:

$$frac2824 = 70.5$$

✓ FINAL ANSWER

Mean of the other 4 numbers = 70.5

4MA1/2HR JUNE 2025

Question 8

3 marks | Statistics Toolkit

8 The mean of 7 numbers is 60

The mean of 3 of the numbers is 46

Work out the mean of the other 4 numbers.

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2025

1 The total of the 7

The total of the 7 numbers is

$$7 \times 60 = 420$$

2 The total of the 3

The total of the 3 numbers is

$$3 \times 46 = 138$$

3 The total of the other

The total of the other 4 numbers is

$$420 - 138 = 282$$

4 Their mean is

Their mean is

$$\frac{282}{4} = 70.5$$

FINAL ANSWER

70.5.

4MA1/1HR JUNE 2025

Question 13

2 marks | Statistics Toolkit

13 Here are the numbers of eggs laid by some hens on each of 11 days in June.

9 10 12 15 17 18 19 19 20 25 26

Work out the interquartile range of the numbers of eggs.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2025

1 Put the numbers in order

Put the numbers in order:

$$9,\ 10,\ 12,\ 15,\ 17,\ 18,\ 19,\ 19,\ 20,\ 25,\ 26$$

$$Q_1 = 12, \qquad Q_3 = 20$$

$$\text{IQR} = 20 - 12 = 8$$

FINAL ANSWER

8.

4WM2/01 MAY 2025

Question 16

2 marks | Statistics Toolkit

16 Here are the numbers of goals scored by a netball team in its 11 games this season.

7 8 10 11 13 14 16 17 19 21 24

Work out the interquartile range for the number of goals.

.....

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 May 2025

1 There are values. Excluding the

There are 11 values. Excluding the median 14:

2 Calculate value

Lower half: $7, 8, 10, 11, 13$, so $Q_1 = 10$.

3 Calculate value

Upper half: $16, 17, 19, 21, 24$, so $Q_3 = 19$.

$$\text{IQR} = 19 - 10 = 9$$

✓ **FINAL ANSWER**

9.

4MA1/1H NOVEMBER 2025

Question 4

3 marks | Statistics Toolkit

4 e, f and g represent integers such that

$$e \quad f \quad 7 \quad g \quad 12 \quad 17$$

is a list of integers written in order of size.

The integers have

a range of 15
a median of 8.5
a mean of 9

Work out the value of e, the value of f and the value of g

$e =$ .....

$f =$ .....

$g =$ .....

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2025

1 The range is

The range is 15:

$$17 - e = 15$$

$$e = 2$$

2 Calculate statistic

The mean is 9, so the total is

$$6 \times 9 = 54$$

$$2 + f + 7 + g + 12 + 17 = 54$$

$$f + g = 16$$

3 **The median is**

The median is 8.5:

$$\frac{7+g}{2} = 8.5$$

$$g = 10$$

$$f = 16 - 10 = 6$$

✓ **FINAL ANSWER**

$e = 2,\ f = 6,\ g = 10$.

4WM2/01 NOVEMBER 2025

Question 10

3 marks | Statistics Toolkit

10 There are 5 potatoes in a bag.
The mean weight of the 5 potatoes is 217 grams.

One more potato, of weight 175 grams, is put into the bag.

Work out the mean weight of the 6 potatoes in the bag.

..... grams

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM2/01 November 2025

1 Calculate statistic

Total₅ = Mean × Number = 217 × 5 = 1085 grams

2 Solve inequality

Total₆ = 1085 + 175 = 1260 grams

3 Calculate statistic

$$textNewMean = frac12606 = 210textgrams$$

FINAL ANSWER

210

4WM2/01 NOVEMBER 2025

Question 16

2 marks | Statistics Toolkit

16 11 people were asked how long, in minutes, they stayed in a coffee shop last Friday.

Here are the results.

14 15 16 18 19 20 22 25 28 40 50

Work out the interquartile range of these times.

..... minutes

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 November 2025

1 Calculate value

Data (sorted): 14, 15, 16, 18, 19, 20, 22, 25, 28, 40, 50.

Number of data points *n* = 11.

2 Find median

Step 1: Find the Median (Q2). The median is the $left(frac11 + 12right) = 6^{textth}$ value.

Median = 20

3 Find median

Step 2: Find Lower Quartile (Q1) and Upper Quartile (Q3). Q1 is the median of the lower 5 values (14, 15, **16**, 18, 19).

*Q*1 = 16

Q3 is the median of the upper 5 values (22, 25, **28**, 40, 50).

*Q*3 = 28

4 Calculate statistic

IQR = *Q*3 − *Q*1 = 28 − 16 = 12

✓ **FINAL ANSWER**

12 minutes

4MA1/1H NOVEMBER 2025

Question 16

2 marks | Statistics Toolkit

16 Ivan asked 15 people how many books they read last year.

Here are his results.

1 1 3 4 6 7 8 9 10 12 15 25 30 37 50

Work out the interquartile range of the number of books read.

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2025

1 Use Statistics Toolkit correct

The data are already in order:

$$1,\ 1,\ 3,\ 4,\ 6,\ 7,\ 8,\ 9,\ 10,\ 12,\ 15,\ 25,\ 30,\ 37,\ 50$$

2 There are 15 values

There are 15 values.

$$Q_1 = 4, \qquad Q_3 = 25$$

$$\text{IQR} = 25 - 4 = 21$$

FINAL ANSWER

21.

4MA1/2HR JUNE 2026

Question 6

4 marks | Statistics Toolkit

6 Mika has six cards.
There is a number on each card.
The cards are arranged with the numbers in order of size.
Three of the numbers are hidden.



The mode of the six numbers is 8
The range of the six numbers is 7
The sum of the six numbers is 31

Work out the median of the six numbers.

.....

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2026

1 **Solution | Step 1**

The largest card is 8, so the smallest is $8 - 7 = 1$.

Range is largest minus smallest.

2 **Solution | Step 2**

The fifth card must be 8, since 8 is the mode.

Only the fifth position can supply another 8 in the ordered cards.

3 **Solution | Step 3**

If the third card is z, then

$$1 + 3 + z + 7 + 8 + 8 = 31 \quad \Rightarrow \quad z = 4.$$

Use the total of all six cards.

4 **Solution | Step 4**

The ordered cards are $1, 3, 4, 7, 8, 8$, so

$$\text{median} = \frac{4 + 7}{2} = 5.5.$$

With six values, average the third and fourth.

FINAL ANSWER

Median $= \mathbf{5.5}$.

4MA1/1HR JUNE 2026

Question 9

3 marks | Statistics Toolkit

9 Caleb owns 11 sheep.

The mean weight of all 11 sheep is 77 kg
The mean weight of 8 of the sheep is 75.2 kg

Work out the mean weight of the other 3 sheep.

..... kg

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2026

1 Solution | Step 1

$$W_{11} = 11(77) = 847 \text{ kg}, \qquad W_8 = 8(75.2) = 601.6 \text{ kg}.$$

Total weight equals number of sheep multiplied by their mean weight.

2 Solution | Step 2

$$W_3 = 847 - 601.6 = 245.4 \text{ kg}, \qquad \overline{w}_3 = \frac{245.4}{3} = 81.8 \text{ kg}.$$

Subtract the known subgroup total and divide the remaining total by 3.

FINAL ANSWER

81.8 kg

4MA1/2H JUNE 2026

Question 13

4 marks | Statistics Toolkit

13 Leo collects apples from three trees.
The incomplete table gives some information about the apples he collects from these trees.

Tree	Number of apples	Mean weight of apples (grams)	Total weight of apples (grams)
A	12	150	
B	20		2700
C		145	2610

Work out the mean weight of all the apples Leo collects from these three trees.

..... grams

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H June 2026

1 Solution | Step 1

Tree A contributes $12 \times 150 = 1800$ grams.

Total weight equals count times mean weight.

2 Solution | Step 2

Tree C has $2610 \div 145 = 18$ apples.

Number of items equals total divided by mean.

3 Solution | Step 3

The total weight is $1800 + 2700 + 2610 = 7110$ grams and the total number of apples is $12 + 20 + 18 = 50$.

Add weights and counts separately; do not average the three group means.

4 Solution | Step 4

$$\text{Combined mean} = \frac{7110}{50} = 142.2 \text{ grams.}$$

Divide the total weight by the total count.

FINAL ANSWER

142.2 grams

4WM2/01R JUNE 2026

Question 15

2 marks | Statistics Toolkit

15 Mina recorded the times taken, in minutes, by 11 people to complete a puzzle.
Here are her results.

2 2 3 6 7 8 9 12 13 17 20

Find the interquartile range of these times.

..... minutes

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4WM2/01R June 2026

1 Positions of the quartiles

$n = 11$: LQ is the $\frac{11+1}{4} = 3$rd value, UQ is the 9th value.

2 Read from the ordered list

$2, 2, \mathbf{3}, 6, 7, 8, 9, 12, \mathbf{13}, 17, 20$: LQ $= \mathbf{3}$, UQ $= \mathbf{13}$.

3 Interquartile range

$$13 - 3 = 10$$

✓ **FINAL ANSWER**

10 minutes

4MA1/2HR JUNE 2026

Question 15

2 marks | Statistics Toolkit

15 Mina recorded the times taken, in minutes, by 11 people to complete a puzzle.
Here are her results.

2 2 3 6 7 9 9 12 13 17 20

Find the interquartile range of these times.

..... minutes

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2026

1 Solution | Step 1

The overall median is the sixth value, 9. The lower five values are $2, 2, 3, 6, 7$, and the upper five are $9, 12, 13, 17, 20$.

Exclude the overall median when forming the two halves.

2 Solution | Step 2

$$Q_1 = 3, \quad Q_3 = 13, \quad \text{IQR} = 13 - 3 = 10 \text{ minutes}.$$

Take each half-median, then subtract the lower quartile from the upper.

✓ FINAL ANSWER

10 minutes.

TOPIC

Averages from Frequency Tables

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 4

7 marks | Averages from Frequency Tables

4 The table shows information about the weights, in kilograms, of 40 babies.

Weight (w kg)	Frequency
$2 < w \leqslant 3$	12
$3 < w \leqslant 4$	16
$4 < w \leqslant 5$	9
$5 < w \leqslant 6$	2
$6 < w \leqslant 7$	1

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean weight of the 40 babies.

..... kg

(4)

One of the 40 babies is going to be chosen at random.

(c) Find the probability that this baby has a weight of more than 5 kg.

.....

(2)

(Total for Question 4 is 7 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2020

1 **(a) The greatest frequency is**

(a) The greatest frequency is 16, so the modal class is

$$3 < w \leq 4$$

2 **(b) Use class midpoints**

(b) Use class midpoints:

$$\frac{2.5(12) + 3.5(16) + 4.5(9) + 5.5(2) + 6.5(1)}{40} = 3.6$$

3 (c) Weights greater than kg

(c) Weights greater than 4 kg:

$$9 + 2 + 1 = 12$$

$$P(w > 4) = \frac{12}{40} = \frac{3}{10}$$

FINAL ANSWER

(a) $3 < w \leq 4$, (b) $3.6\ \text{kg}$, (c) $\frac{3}{10}$.

4MA1/1HR MAY/NOVEMBER 2020

Question 4

5 marks | Averages from Frequency Tables

4 The table gives information about the length of time, in minutes, that each of 60 students took to travel to school on Monday.

Length of time (t minutes)	Frequency
$0 < t \leqslant 10$	4
$10 < t \leqslant 20$	10
$20 < t \leqslant 30$	15
$30 < t \leqslant 40$	25
$40 < t \leqslant 50$	6

(a) Write down the modal class interval.

.....

(1)

(b) Work out an estimate for the mean length of time taken by these 60 students to travel to school on Monday.
Give your answer correct to one decimal place.

..... minutes

(4)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May/November 2020

1 **The modal class is the**

The modal class is the class with the greatest frequency:

$$30 < t \leq 40$$

2 **Use midpoints**

Use midpoints $5, 15, 25, 35, 45$.

$$5(4) + 15(10) + 25(15) + 35(25) + 45(6) = 1690$$

$$\text{estimated mean} = \frac{1690}{60} = 28.166\ldots$$

✓ **FINAL ANSWER**

modal class $30 < t \leq 40$, estimated mean 28.2 minutes.

4MA1/1H JANUARY 2021

Question 2

5 marks | Averages from Frequency Tables

2 The table gives information about the speeds, in kilometres per hour, of 80 motorbikes as each pass under a bridge.

Speed (s kilometres per hour)	Frequency
$40 < s \leqslant 50$	10
$50 < s \leqslant 60$	16
$60 < s \leqslant 70$	19
$70 < s \leqslant 80$	23
$80 < s \leqslant 90$	12

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean speed of the motorbikes as they pass under the bridge. Give your answer correct to 3 significant figures.

..... kilometres per hour

(4)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2021

1 **(a) The greatest frequency is**

(a) The greatest frequency is $\mathbf{23}$, so the modal class is

$$70 < s \leq 80$$

2 **(b) Use midpoints**

(b) Use midpoints $45, 55, 65, 75, 85$:

$$\frac{45(10) + 55(16) + 65(19) + 75(23) + 85(12)}{80} = 66.375$$

3 **To 3 significant figures**

To 3 significant figures:

$$66.4$$

✓ **FINAL ANSWER**

(a) $70 < s \leq 80$, (b) $66.4\ \text{km/h}$.

4MA1/2HR JANUARY 2021

Question 10

4 marks | Averages from Frequency Tables

10 A mathematics teacher at a school asked a group of students how far, in kilometres, each student had travelled to get to school that day.

The table gives information about their answers.

Distance travelled (d km)	Number of students
$0 < d \leqslant 2$	x
$2 < d \leqslant 4$	11
$4 < d \leqslant 6$	8
$6 < d \leqslant 8$	6
$8 < d \leqslant 10$	5

The teacher calculated that an estimate for the mean distance travelled by the whole group of students was 4.25 km.

Work out the value of x.
Show your working clearly.

$x =$.....

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2021

1 Use class midpoints

Use class midpoints $1, 3, 5, 7, 9$.

2 The total frequency is

The total frequency is

$$x + 11 + 8 + 6 + 5 = x + 30$$

3 The total estimated distance is

The total estimated distance is

$$1x + 3(11) + 5(8) + 7(6) + 9(5) = x + 160$$

4 **The estimated mean is**

The estimated mean is 4.25:

$$\frac{x+160}{x+30}=4.25$$

$$x+160=4.25x+127.5$$

$$32.5=3.25x$$

$$x=10$$

✓ **FINAL ANSWER**

$x=10$.

4MA1/2H NOVEMBER 2021

Question 3

5 marks | Averages from Frequency Tables

3 The table gives information about the amounts of money, in euros, that 70 of Anjali's friends spent last Saturday.

Money spent (S euros)	Frequency
$0 < S \leqslant 8$	6
$8 < S \leqslant 16$	14
$16 < S \leqslant 24$	19
$24 < S \leqslant 32$	25
$32 < S \leqslant 40$	6

One of Anjali's 70 friends is going to be chosen at random.

(a) Find the probability that this friend spent more than 24 euros last Saturday.

.....

(1)

(b) Work out an estimate for the mean amount of money spent by Anjali's friends last Saturday. Give your answer correct to 2 decimal places.

..... euros

(4)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2021

1 **More than euros**

More than 24 euros:

$$25 + 6 = 31$$

2 **Total frequency**

Total frequency:

$$6 + 14 + 19 + 25 + 6 = 70$$

$$P(S > 24) = \frac{31}{70}$$

3 **Use midpoints**

Use midpoints $4, 12, 20, 28, 36$.

$$4(6) + 12(14) + 20(19) + 28(25) + 36(6) = 1488$$

$$\text{estimated mean} = \frac{1488}{70} = 21.257\ldots$$

✓ **FINAL ANSWER**

$\frac{31}{70}$, estimated mean $\mathbf{21.3}$ euros.

4MA1/2H JANUARY 2022

Question 9

4 marks | Averages from Frequency Tables

9 The frequency table gives information about the number of points scored by a player.

Number of points	Frequency
0	13
1	17
2	8
3	x
4	11

The mean number of points scored is 2

Work out the value of x

$x =$ .....

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2022

1 The total number of points

The total number of points is

$$0(13) + 1(17) + 2(8) + 3x + 4(11) = 77 + 3x$$

2 The total frequency is

The total frequency is

$$13 + 17 + 8 + x + 11 = 49 + x$$

3 Calculate statistic

The mean is 2, so

$$\frac{77+3x}{49+x}=2$$

$$77+3x=98+2x$$

$$x=21$$

FINAL ANSWER

$x=21$.

4MA1/2H NOVEMBER 2023

Question 1

4 marks | Averages from Frequency Tables

1 The table shows information about the lengths, in minutes, of 50 telephone calls.

Length of telephone call (m minutes)	Frequency
$0 < m \leqslant 5$	8
$5 < m \leqslant 10$	2
$10 < m \leqslant 15$	6
$15 < m \leqslant 20$	4
$20 < m \leqslant 25$	12
$25 < m \leqslant 30$	18

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the total length, in minutes, of these telephone calls.

..... minutes

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2023

1 The modal class is the

The modal class is the class with the greatest frequency:

$$25 < m \leq 30$$

2 Use midpoints

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5, 27.5$.

$$2.5(8) + 7.5(2) + 12.5(6) + 17.5(4) + 22.5(12) + 27.5(18) = 945$$

FINAL ANSWER

modal class $25 < m \leq 30$, estimated total 945 minutes.

4MA1/1H NOVEMBER 2024

Question 1

5 marks | Averages from Frequency Tables

1 The table shows some information about the hourly rates of pay of 60 workers.

Hourly rate of pay (p dollars)	Frequency
$10 < p \leqslant 15$	18
$15 < p \leqslant 20$	16
$20 < p \leqslant 25$	14
$25 < p \leqslant 30$	8
$30 < p \leqslant 35$	4

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean hourly rate of pay of the 60 workers.

..... dollars

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2024

1 **The modal class is the**

The modal class is the class with the greatest frequency:

$$10 < p \leq 15$$

2 **Use midpoints**

Use midpoints $12.5, 17.5, 22.5, 27.5, 32.5$.

$$12.5(18) + 17.5(16) + 22.5(14) + 27.5(8) + 32.5(4) = 1170$$

$$\text{estimated mean} = \frac{1170}{60} = 19.5$$

FINAL ANSWER

modal class $10 < p \leq 15$, estimated mean 19.5 dollars.

4MA1/1H JUNE 2025

Question 1

5 marks | Averages from Frequency Tables

1 The table gives information about the distances 100 adults travel to work.

Distance (d km)	Frequency
$0 < d \leqslant 5$	26
$5 < d \leqslant 10$	40
$10 < d \leqslant 15$	16
$15 < d \leqslant 20$	10
$20 < d \leqslant 25$	8

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean distance.

..... km

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2025

1 The modal class is the

The modal class is the class with the greatest frequency.

2 Modal class

Modal class: $5 < d \leq 10$.

3 Use midpoints

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5$.

$$2.5(26) + 7.5(40) + 12.5(16) + 17.5(10) + 22.5(8) = 920$$

$$\text{estimated mean} = \frac{920}{100} = 9.2$$

FINAL ANSWER

modal class $5 < d \leq 10$, estimated mean $9.2\ \text{km}$.

4WM2/01R MAY 2025

Question 1

5 marks | Averages from Frequency Tables

1 The table shows information about the weights, in kilograms, of 50 dogs.

Weight (w kg)	Frequency
$4 < w \leqslant 8$	5
$8 < w \leqslant 12$	17
$12 < w \leqslant 16$	15
$16 < w \leqslant 20$	9
$20 < w \leqslant 24$	4

(a) Write down the modal class.

..... kg

(1)

(b) Work out an estimate for the mean weight of the 50 dogs.

..... kg

(4)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4WM2/01R May 2025

1 Required

a Write down the modal class. (b) Estimate the mean weight of the 50 dogs.

2 Part (a)

The **modal class** is the interval with the highest frequency. From the table, the highest frequency is **17**, which belongs to:

3 Answer

Modal class: 8 \< *w* ≤ 12 kg

4 **Find the midpoint of each**

Find the midpoint of each class interval:

5 **Apply the estimated mean formula**

Apply the estimated mean formula:

$$barx = fracsumf!xsumf = frac66050 = 13.2textkg$$

✓ **FINAL ANSWER**

Estimated mean weight = 13.2 kg

Question 8

4 marks | Averages from Frequency Tables

8 The table gives information about the lengths of time, in minutes, that some people were in a coffee shop.

Length of time (m minutes)	Frequency
$0 < m \leqslant 10$	x
$10 < m \leqslant 20$	3
$20 < m \leqslant 30$	2
$30 < m \leqslant 40$	4
$40 < m \leqslant 50$	2

The information in the table is used to calculate an estimate for the mean length of time these people were in the coffee shop.

The estimate for the mean length of time is 20

Work out the value of x

$x =$ .....

(Total for Question 8 is 4 marks)

1 Use class midpoints

Use class midpoints $5, 15, 25, 35, 45$.

$$\frac{5x + 15(3) + 25(2) + 35(4) + 45(2)}{x + 3 + 2 + 4 + 2} = 20$$

$$\frac{5x + 325}{x + 11} = 20$$

$$5x + 325 = 20x + 220$$

$$105 = 15x$$

$$x = 7$$

FINAL ANSWER

$x = 7$.

4MA1/1H NOVEMBER 2025

Question 1

4 marks | Averages from Frequency Tables

1 The table gives information about the number of kilometres that Ted cycled on each of the 30 days in April.

Number of kilometres (K)	Frequency
$0 \leqslant K < 5$	8
$5 \leqslant K < 10$	7
$10 \leqslant K < 15$	3
$15 \leqslant K < 20$	10
$20 \leqslant K < 25$	2

Calculate an estimate for the mean number of kilometres that Ted cycled on each day in April.

..... kilometres

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2025

1 **Use midpoints**

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5$.

$$2.5(8) + 7.5(7) + 12.5(3) + 17.5(10) + 22.5(2) = 330$$

$$\text{estimated mean} = \frac{330}{30} = 11$$

FINAL ANSWER

11 km.

4WM2/01 NOVEMBER 2025

Question 2

4 marks | Averages from Frequency Tables

2 The table gives information about the heights, in cm, of 60 flowers in Abby's garden.

Height (h cm)	Frequency
$0 < h \leqslant 10$	17
$10 < h \leqslant 20$	9
$20 < h \leqslant 30$	16
$30 < h \leqslant 40$	14
$40 < h \leqslant 50$	4

Work out an estimate for the mean height of these flowers.

..... cm

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4WM2/01 November 2025

1 Find class midpoints

Step 1: Find the midpoints (*x*) for each class interval.

2 0 h 10 x 5

0 \< *h* ≤ 10 ⟹ *x* = 5

3 10 h 20 x 15

10 \< *h* ≤ 20 ⟹ *x* = 15

4 20 h 30 x 25

20 \< *h* ≤ 30 ⟹ *x* = 25

5 30 h 40 x 35

30 \< *h* ≤ 40 ⟹ *x* = 35

6 40 h 50 x 45

40 \< *h* ≤ 50 ⟹ *x* = 45

7 **Find the midpoint**

Step 2: Multiply frequency (*f*) by midpoint (*x*) to find *f*x*.

8 **Calculate statistic**

$$textMean = fracsumfxsumf = frac129060 = 21.5$$

✓ **FINAL ANSWER**

21.5 cm

4MA1/2H JUNE 2026

Question 3

5 marks | Averages from Frequency Tables

3 The table shows information about the number of hours each of 40 candles burned.

Number of hours (h)	Frequency
$15 < h \leqslant 19$	2
$19 < h \leqslant 23$	5
$23 < h \leqslant 27$	12
$27 < h \leqslant 31$	15
$31 < h \leqslant 35$	6

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean number of hours.

..... hours

(4)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

The greatest frequency is 15, in the interval $27 < h \leq 31$.

The modal class is the class with the highest frequency.

2 **Part (b) | Step 1**

The class midpoints are $17, 21, 25, 29, 33$ hours.

Use the midpoint of each class as a representative value.

3 **Part (b) | Step 2**

$$\sum fm = 2(17) + 5(21) + 12(25) + 15(29) + 6(33)$$

$$= 34 + 105 + 300 + 435 + 198 = 1072.$$

Multiply each midpoint by its frequency and add.

4 **Part (b) | Step 3**

$$\text{Estimated mean} = \frac{1072}{2+5+12+15+6} = \frac{1072}{40} = 26.8 \text{ hours}.$$

Divide the estimated total by the number of candles.

✓ **FINAL ANSWER**

(a) $27 < h \leq 31$ (b) 26.8 hours

TOPIC

Cumulative Frequency Diagrams

Every past-paper question on this topic, 2020 to 2026.

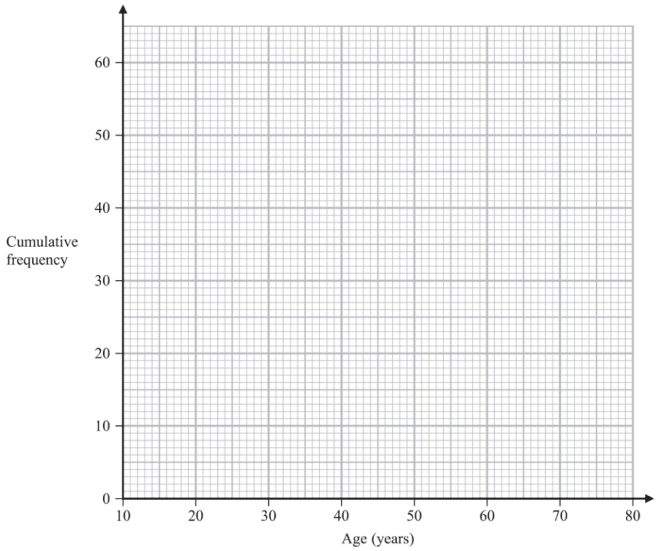
Question 15

7 marks | Cumulative Frequency Diagrams

15 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

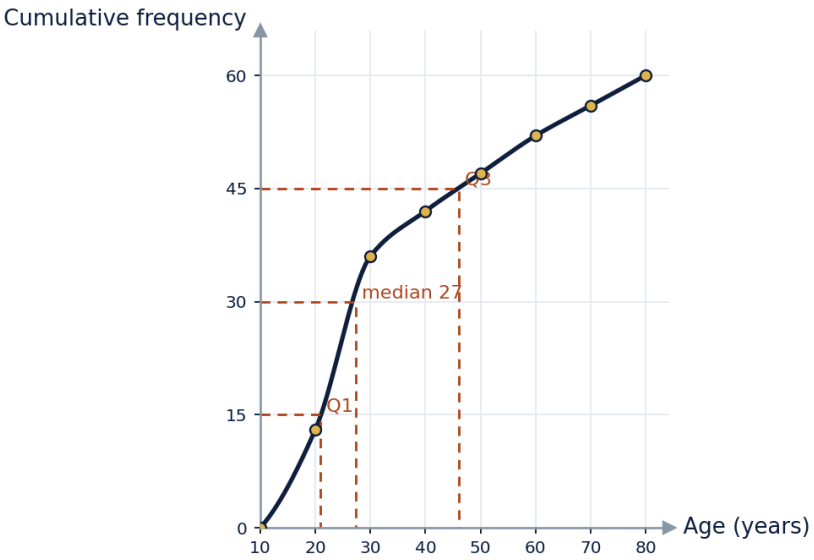
(2)

(Total for Question 15 is 7 marks)

1 Plot the cumulative frequency points

Plot the cumulative frequency points and join with a smooth increasing curve:

$$(10, 0),\ (20, 13),\ (30, 36),\ (40, 42),\ (50, 47),\ (60, 52),\ (70, 56),\ (80, 60)$$



2 Use cumulative frequency

There are 60 people, so the median is read at cumulative frequency 30.

3 Convert standard form

Using linear interpolation between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 \approx 27.4$$

4 Use cumulative frequency

So the median is about 27 years.

5 Use cumulative frequency

For the interquartile range, read Q_1 at cumulative frequency 15 and Q_3 at cumulative frequency 45:

$$Q_1 \approx 20.9, \qquad Q_3 \approx 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 Use cumulative frequency

So the IQR is about 25 years.

7 Use cumulative frequency

At age 55, the cumulative frequency is about 49.5, so the number older than 55 is

$$60 - 49.5 \approx 10.5$$

8 This gives about people

This gives about 11 people.

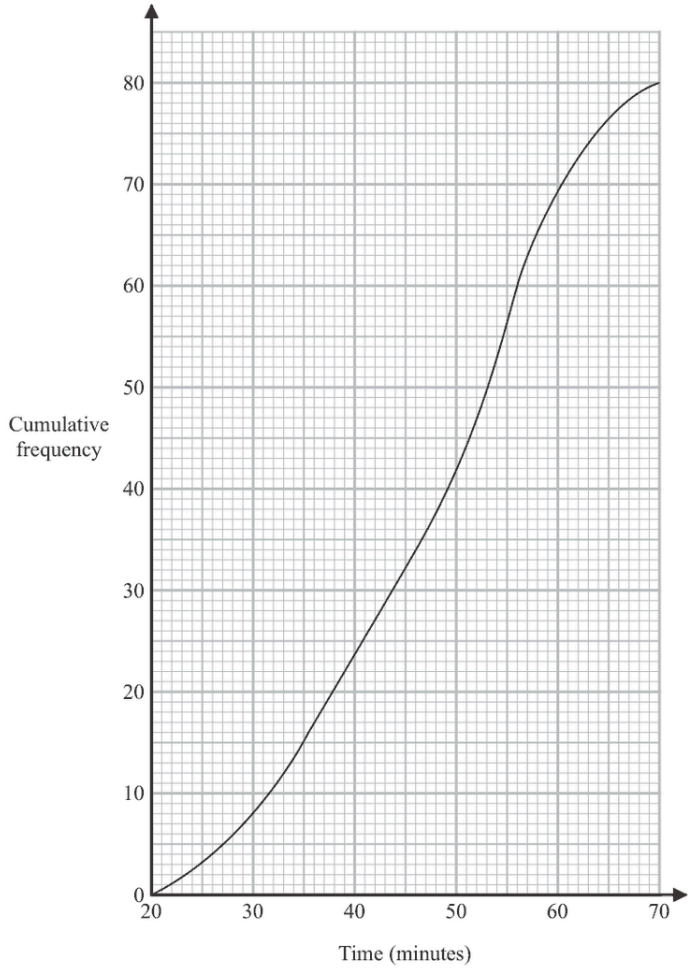
FINAL ANSWER

median about 27 years, IQR about 25 years, about 11 people older than 55.

Question 12

5 marks | Cumulative Frequency Diagrams

12 A total of 80 men and women took part in a race.
The cumulative frequency graph gives information about the times, in minutes, they took for the race.



(a) Use the graph to find an estimate for the interquartile range.

..... minutes
(2)

60% of the men took 50 minutes or less for the race.
No women took 50 minutes or less for the race.

(b) Work out an estimate for the number of men who took part in the race.

.....
(3)

(Total for Question 12 is 5 marks)

1 **There are people**

There are 80 people.

2 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at cumulative frequency } 20$$

$$Q_3 \text{ is at cumulative frequency } 60$$

3 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 38, \qquad Q_3 \approx 56$$

$$\text{IQR} \approx 56 - 38 = 18$$

4 **Use cumulative frequency**

At 50 minutes, the cumulative frequency is about 42.

5 **Use cumulative frequency**

No women took 50 minutes or less, so these 42 people are men.

$$0.6 \times \text{number of men} = 42$$

$$\text{number of men} = \frac{42}{0.6} = 70$$

FINAL ANSWER

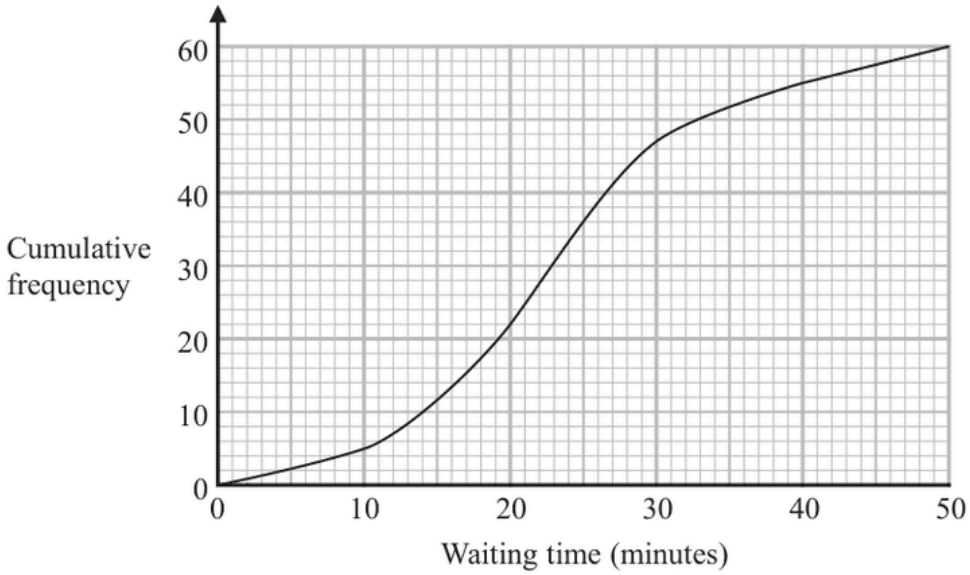
IQR about 18 minutes, and about 70 men.

4MA1/1HR JANUARY 2020

Question 12

5 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph gives information about the waiting times, in minutes, of people with appointments at Hospital A.



(a) Use the graph to find an estimate of the median waiting time at Hospital A.

..... minutes

(1)

(b) Use the graph to find an estimate of the interquartile range of the waiting times at Hospital A.

..... minutes

(2)

At a different hospital, Hospital B, the median waiting time is 28 minutes and the interquartile range of the waiting times is 19 minutes.

(c) Compare the waiting times at Hospital A with the waiting times at Hospital B.

.....

(2)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2020

1 **There are people**

There are **60** people.

2 **The median is at cumulative**

The median is at cumulative frequency 30.

3 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 22.5 \text{ minutes}$$

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 16.5, \qquad Q_3 \approx 28.5$$

$$\text{IQR} \approx 28.5 - 16.5 = 12$$

6 **Hospital B has median minutes**

Hospital B has median 28 minutes and IQR 19 minutes.

7 **Use cumulative frequency**

So Hospital A has a shorter median waiting time and a smaller spread.

✓ **FINAL ANSWER**

median about 22.5 minutes, IQR about 12 minutes. Hospital A is quicker and more consistent.

Question 11

5 marks | Cumulative Frequency Diagrams

11 The frequency table gives information about the ages of the 80 people in a train carriage.

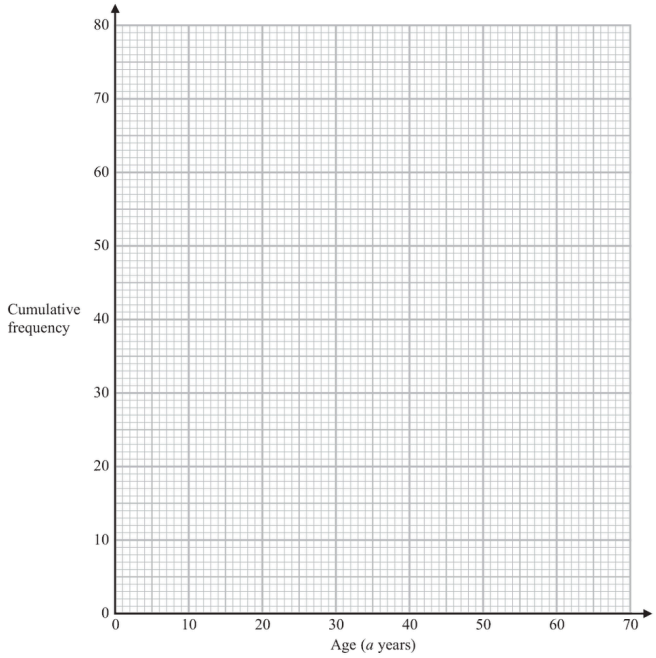
Age (a years)	Frequency
$0 < a \leqslant 20$	9
$20 < a \leqslant 30$	19
$30 < a \leqslant 40$	17
$40 < a \leqslant 50$	18
$50 < a \leqslant 60$	13
$60 < a \leqslant 70$	4

(a) Complete the cumulative frequency table.

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	
$0 < a \leqslant 30$	
$0 < a \leqslant 40$	
$0 < a \leqslant 50$	
$0 < a \leqslant 60$	
$0 < a \leqslant 70$	

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median age of the people in the train carriage.

..... years

(2)

(Total for Question 11 is 5 marks)

1 **(a) Running totals**

$$9,\ 28,\ 45,\ 63,\ 76,\ 80$$

2 **(b) Plot at the upper class boundaries**

Plot $(20, 9), (30, 28), (40, 45), (50, 63), (60, 76), (70, 80)$, start at $(0, 0)$, and join with a smooth curve.

3 **(c) Median**

Read across from $\frac{80}{2} = 40$: about 37 years.

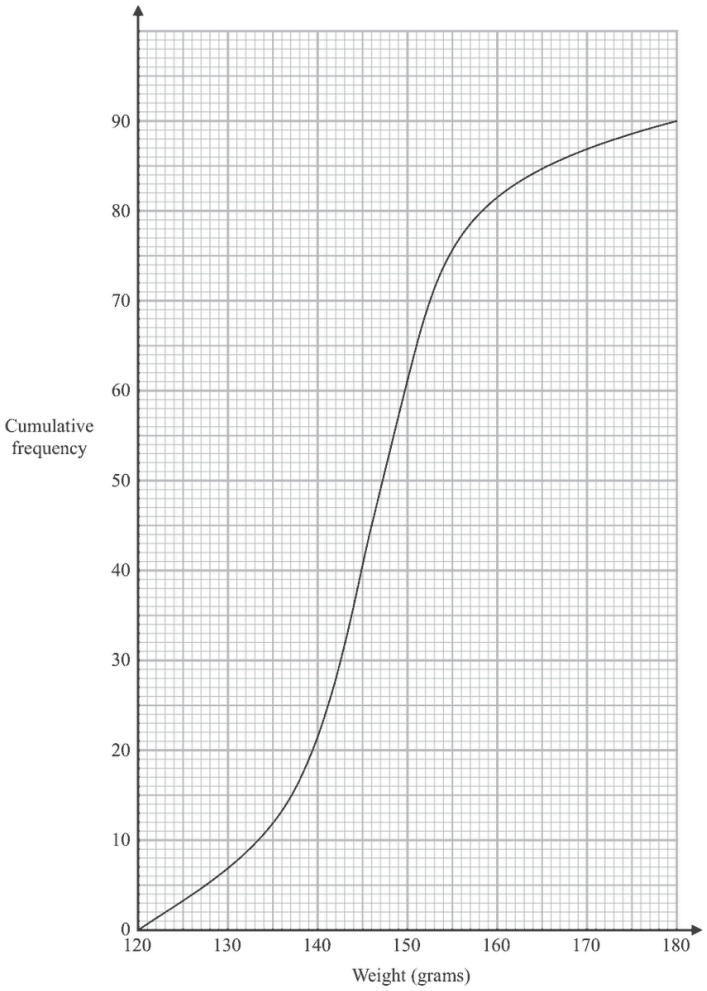
✓ **FINAL ANSWER**

(a) $9, 28, 45, 63, 76, 80$ (c) about 37 years

Question 11

5 marks | Cumulative Frequency Diagrams

11 The cumulative frequency graph gives information about the weights, in grams, of 90 bags of sweets.



(a) Find an estimate for the median of the weights of these bags of sweets.

..... grams

(2)

Roberto sells the bags of sweets to raise money for charity.
Bags with a weight greater than d grams are labelled large bags and sold for 3.75 euros each bag.

The total amount of money he receives by selling all the large bags is 93.75 euros.

(b) Find the value of d.

$d =$ .....

(3)

(Total for Question 11 is 5 marks)

1 **Use cumulative frequency**

There are 90 bags, so the median is at cumulative frequency 45.

2 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 146 \text{ g}$$

3 **Large bags sold**

Large bags sold:

$$\frac{93.75}{3.75} = 25$$

4 **Use cumulative frequency**

So 25 bags have weight greater than d.

5 **That means**

That means

$$90 - 25 = 65$$

6 **Use cumulative frequency**

bags have weight d grams or less.

7 **Use cumulative frequency**

From the graph, cumulative frequency 65 gives

$$d \approx 150$$

✓ **FINAL ANSWER**

median about 146 g, and $d \approx 150$.

Question 11

6 marks | Cumulative Frequency Diagrams

11 The manager of a call centre asked the 120 people, who rang the call centre last week, how long they each waited before their call was answered.

The table gives information about their replies.

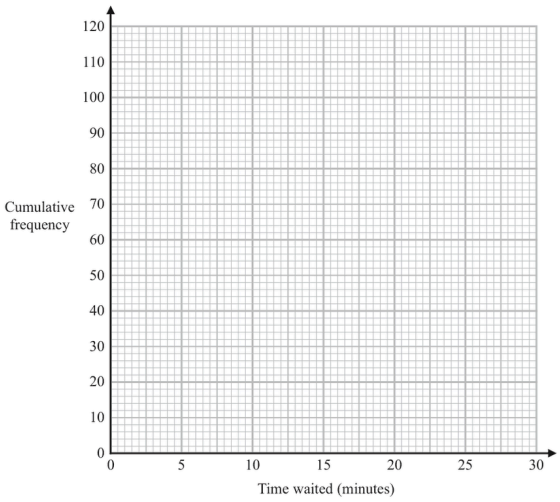
Time waited (t minutes)	Frequency
$0 < t \leqslant 5$	8
$5 < t \leqslant 10$	15
$10 < t \leqslant 15$	17
$15 < t \leqslant 20$	28
$20 < t \leqslant 25$	33
$25 < t \leqslant 30$	19

(a) Complete the cumulative frequency table.

Time waited (t minutes)	Cumulative frequency
$0 < t \leqslant 5$	
$0 < t \leqslant 10$	
$0 < t \leqslant 15$	
$0 < t \leqslant 20$	
$0 < t \leqslant 25$	
$0 < t \leqslant 30$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median of the times waited.

..... minutes

(1)

(d) Using your graph, find an estimate for the percentage of the 120 people who said that they waited longer than 23 minutes before their call was answered.
Show your working clearly.

.....%

(2)

(Total for Question 11 is 6 marks)

1 Complete the cumulative frequencies

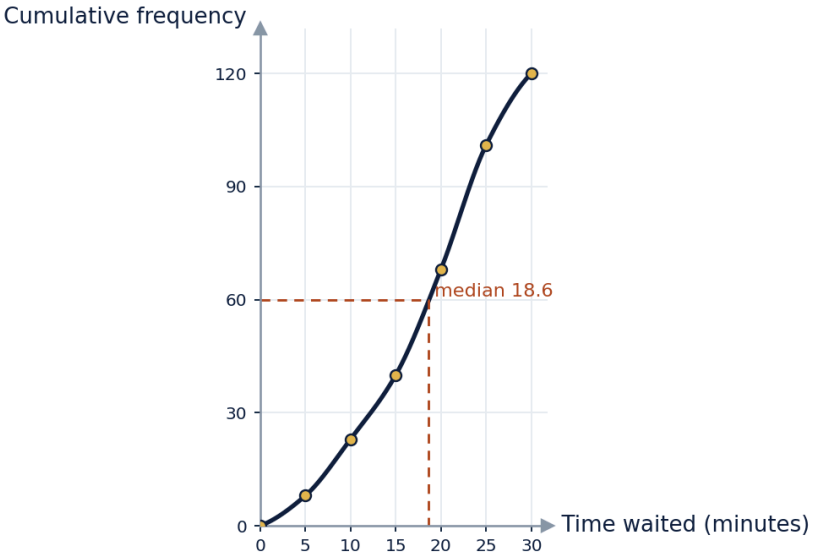
Complete the cumulative frequencies:

$$8,\ 23,\ 40,\ 68,\ 101,\ 120$$

2 Plot the points

Plot the points

$$(0, 0), (5, 8), (10, 23), (15, 40), (20, 68), (25, 101), (30, 120)$$



3 The median is at cumulative

The median is at cumulative frequency 60.

4 Use cumulative frequency

This lies between $(15, 40)$ and $(20, 68)$:

$$15 + \frac{60 - 40}{68 - 40} \times 5 = 18.57\ldots$$

5 Use cumulative frequency

So the median is about 18.6 minutes.

6 At minutes

At 23 minutes:

$$68 + \frac{3}{5}(101 - 68) = 87.8$$

7 Number who waited longer than

Number who waited longer than 23 minutes:

$$120 - 87.8 = 32.2$$

$$\frac{32.2}{120} \times 100 = 26.8\%$$

FINAL ANSWER

cumulative frequencies $8, 23, 40, 68, 101, 120$, median about 18.6 minutes, percentage about 27%.

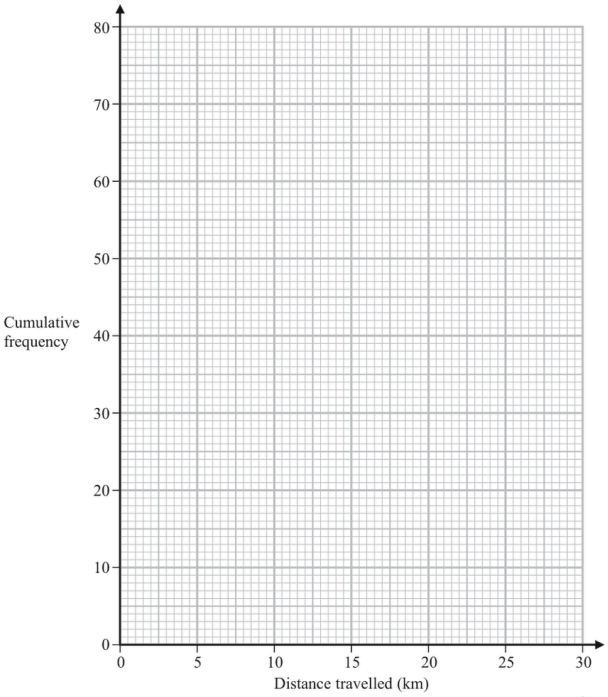
Question 12

7 marks | Cumulative Frequency Diagrams

12 The cumulative frequency table gives information about the distance, in kilometres, that each of 80 workers travel from home to work at Office *A*.

Distance travelled (d km)	Cumulative frequency
$0 < d \leqslant 5$	17
$0 < d \leqslant 10$	32
$0 < d \leqslant 15$	57
$0 < d \leqslant 20$	70
$0 < d \leqslant 25$	76
$0 < d \leqslant 30$	80

(a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the median distance travelled.

..... km

(1)

(c) Use your graph to find an estimate for the interquartile range of the distances travelled.

..... km

(2)

For Office *B*, the median distance workers travel from home to work is 15 km and the interquartile range is 5 km.

(d) Use the information above to compare the distances that workers at Office *A* and workers at Office *B* travel from home to work.
Write down **two** comparisons.

1 .....

2 .....

.....

(2)

(Total for Question 12 is 7 marks)

1 **(a) Plot the table**

Plot $(5, 17), (10, 32), (15, 57), (20, 70), (25, 76), (30, 80)$, start at $(0, 0)$, and join with a smooth curve.

2 **(b) Median**

Read across from 40: about 11.6 km.

3 **(c) Interquartile range**

Lower quartile at 20: about 6 km. Upper quartile at 60: about 16.2 km.

$$\text{IQR} \approx 16.2 - 6 = 10.2 \text{ km}$$

4 **(d) Two comparisons**

1. Office A has a lower median (about 11.6 km against 15 km), so on average workers at Office A travel a shorter distance.
2. Office A has a larger IQR (about 10 km against 5 km), so the distances at Office A are more varied (less consistent).

✓ **FINAL ANSWER**

(b) about 11.6 km (c) about 10 km (d) A shorter on average; A more spread out

Question 11

5 marks | Cumulative Frequency Diagrams

11 The table gives information about the times taken by 90 runners to complete a 10 km race.

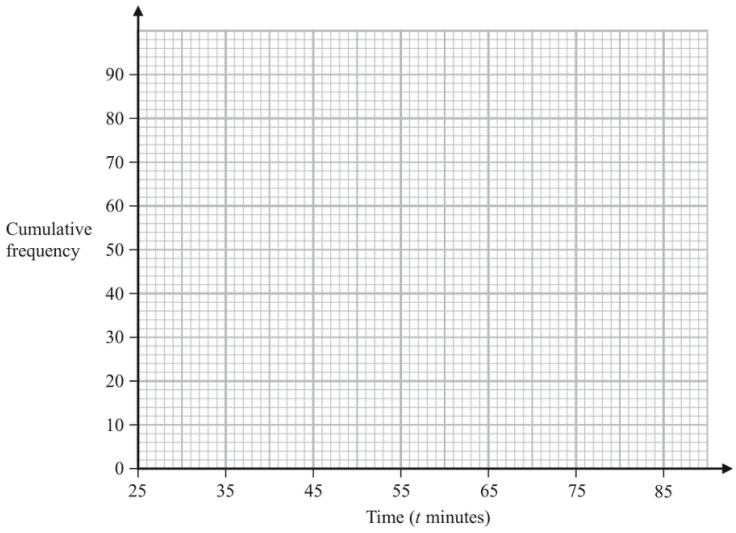
Time (t minutes)	Frequency
$25 < t \leqslant 35$	12
$35 < t \leqslant 45$	24
$45 < t \leqslant 55$	28
$55 < t \leqslant 65$	12
$65 < t \leqslant 75$	10
$75 < t \leqslant 85$	4

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$25 < t \leqslant 35$	12
$25 < t \leqslant 45$	
$25 < t \leqslant 55$	
$25 < t \leqslant 65$	
$25 < t \leqslant 75$	
$25 < t \leqslant 85$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

Any runner who completed the race in a time T minutes such that $42 < T \leqslant 52$ minutes was awarded a silver medal.

(c) Use your graph to find an estimate for the number of runners who were awarded a silver medal.

..... runners

(2)

(Total for Question 11 is 5 marks)

1 **(a) Running totals**

$$12,\ 36,\ 64,\ 76,\ 86,\ 90$$

2 **(b) Plot at the upper class boundaries**

Plot $(35, 12), (45, 36), (55, 64), (65, 76), (75, 86), (85, 90)$, start at $(25, 0)$, and join with a smooth curve.

3 **(c) Read the curve at 52 and 42 minutes**

At 52 minutes: about 56. At 42 minutes: about 29.

4 **(c) Silver medals**

$$56 - 29 \approx 27 \text{ runners}$$

✓ **FINAL ANSWER**

(a) $12, 36, 64, 76, 86, 90$ (c) about 27

Question 12

6 marks | Cumulative Frequency Diagrams

12 The table gives information about the times, in minutes, taken by 80 customers to do their shopping in a supermarket.

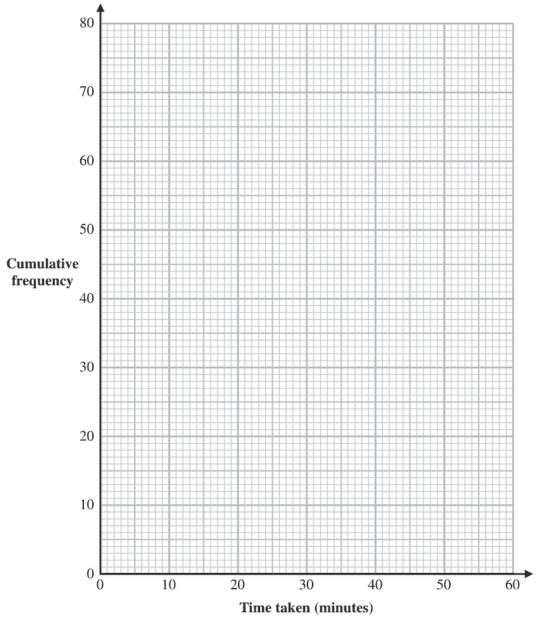
Time taken (t minutes)	Frequency
$0 < t \leqslant 10$	7
$10 < t \leqslant 20$	26
$20 < t \leqslant 30$	24
$30 < t \leqslant 40$	14
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	2

(a) Complete the cumulative frequency table.

Time taken (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	
$0 < t \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time taken.

..... minutes

(1)

One of the 80 customers is chosen at random.

(d) Use your graph to find an estimate for the probability that the time taken by this customer was more than 42 minutes.

.....

(2)

(Total for Question 12 is 6 marks)

1 Complete the cumulative frequencies

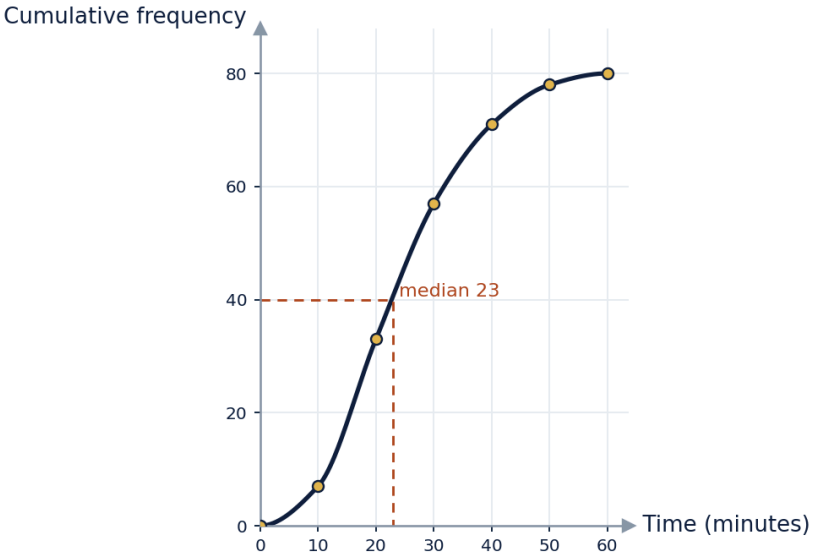
Complete the cumulative frequencies:

$$7,\ 33,\ 57,\ 71,\ 78,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (10, 7), (20, 33), (30, 57), (40, 71), (50, 78), (60, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(20, 33)$ and $(30, 57)$:

$$20 + \frac{40 - 33}{57 - 33} \times 10 = 22.9$$

5 Use cumulative frequency

So the median is about 23 minutes.

6 At minutes

At 42 minutes:

$$71 + \frac{2}{10}(78 - 71) = 72.4$$

7 Number taking more than minutes

Number taking more than 42 minutes:

$$80 - 72.4 = 7.6$$

$$P(t > 42) \approx \frac{7.6}{80} = 0.095$$

FINAL ANSWER

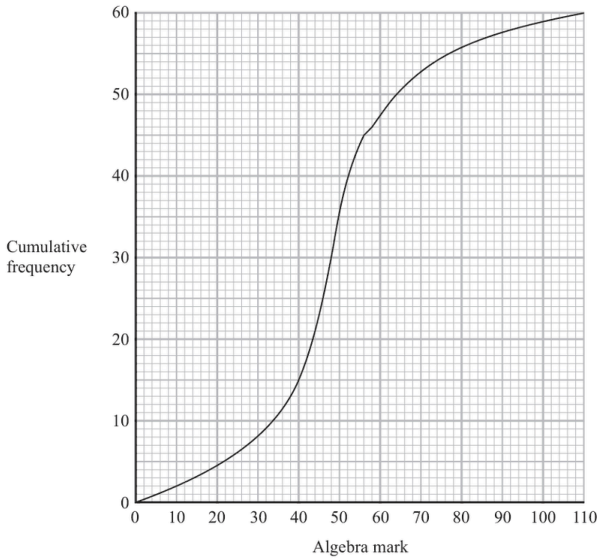
cumulative frequencies $7, 33, 57, 71, 78, 80$, median about 23 minutes, probability about 0.095.

Question 14

8 marks | Cumulative Frequency Diagrams

14 A group of 60 students each sat an algebra test and a geometry test.
Each test was marked out of 110

The cumulative frequency graph gives information about the marks gained by the 60 students in the algebra test.



(a) Use the graph to find an estimate for the median mark in the algebra test.

.....

(1)

(b) Use the graph to find an estimate for the number of students who gained 58 marks or less in the algebra test.

.....

(1)

(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

.....

(2)

The interquartile range of the marks gained in the geometry test is 9

Luis says

"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Luis correct?
Give a reason for your answer.

.....

(1)

To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64

Two students are to be selected at random from the 60 students in the group.

(e) Use the graph to find an estimate for the probability that both of these students were awarded a grade A in the algebra test.

.....

(3)

(Total for Question 14 is 8 marks)

1 **There are students**

There are 60 students.

2 **The median is at cumulative**

The median is at cumulative frequency 30. From the graph,

$$\text{median} \approx 49$$

3 **Use cumulative frequency**

At 58 marks, the cumulative frequency is about 45, so about 45 students gained 58 marks or less.

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 40, \qquad Q_3 \approx 57$$

$$\text{IQR} \approx 57 - 40 = 17$$

6 **Use cumulative frequency**

The geometry IQR is 9, so Luis is correct because the algebra IQR is larger.

7 **Use cumulative frequency**

For grade A, use mark 64. From the graph, about 50 students gained 64 or less, so about

$$60 - 50 = 10$$

8 **students gained more than**

students gained more than 64.

9 **Probability both selected students were**

Probability both selected students were grade A:

$$\frac{10}{60} \times \frac{9}{59} = 0.0254\ldots$$

FINAL ANSWER

median about 49, about 45 students, IQR about 17, Luis is correct, probability about 0.025.

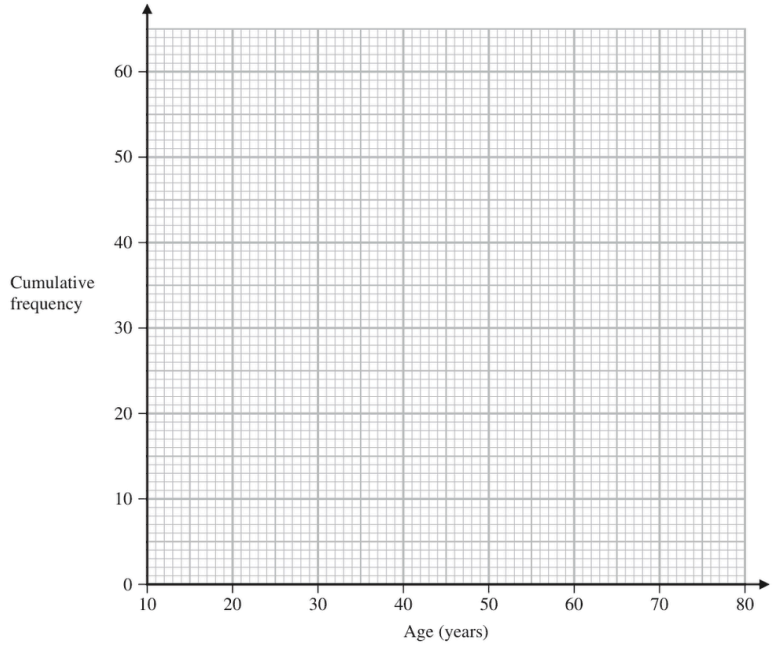
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

Question 11 continued

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

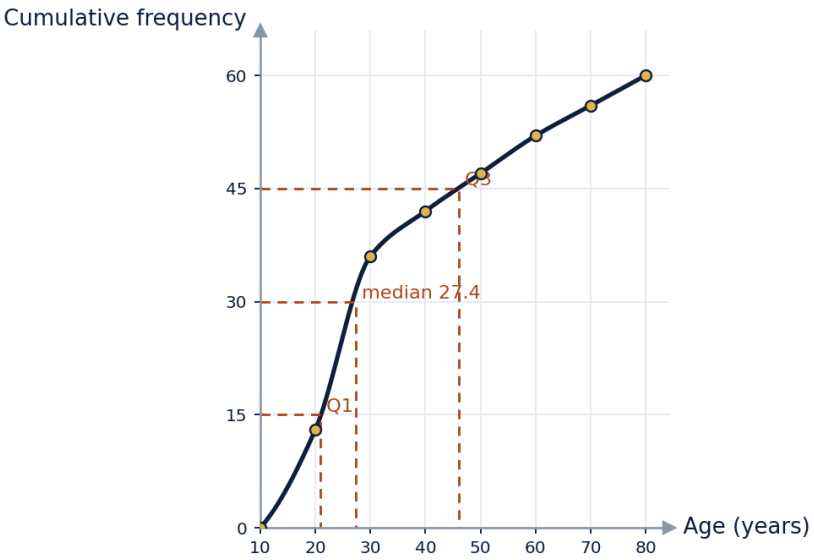
(2)

(Total for Question 11 is 7 marks)

1 Plot the points

Plot the points

$$(10, 0), (20, 13), (30, 36), (40, 42), (50, 47), (60, 52), (70, 56), (80, 60)$$



2 The median is at cumulative

The median is at cumulative frequency 30.

3 Convert standard form

This is between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 = 27.39\ldots$$

4 Median age is about years

Median age is about 27.4 years.

5 Convert standard form

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

$$Q_1 \approx 20 + \frac{15 - 13}{36 - 13} \times 10 = 20.9$$

$$Q_3 \approx 40 + \frac{45 - 42}{47 - 42} \times 10 = 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 Use cumulative frequency

For people older than 55:

$$F(55) \approx 47 + \frac{5}{10}(52 - 47) = 49.5$$

$$60 - 49.5 = 10.5$$

FINAL ANSWER

median about 27.4 years, IQR about 25.1 years, about 11 people older than 55.

Question 13

7 marks | Cumulative Frequency Diagrams

13 The table gives information about the ages, in years, of 80 people in a train carriage.

Age (a years)	Frequency
$0 < a \leqslant 20$	7
$20 < a \leqslant 30$	25
$30 < a \leqslant 40$	20
$40 < a \leqslant 50$	14
$50 < a \leqslant 60$	8
$60 < a \leqslant 70$	6

(a) Complete the cumulative frequency table.

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	
$0 < a \leqslant 30$	
$0 < a \leqslant 40$	
$0 < a \leqslant 50$	
$0 < a \leqslant 60$	
$0 < a \leqslant 70$	

(1)

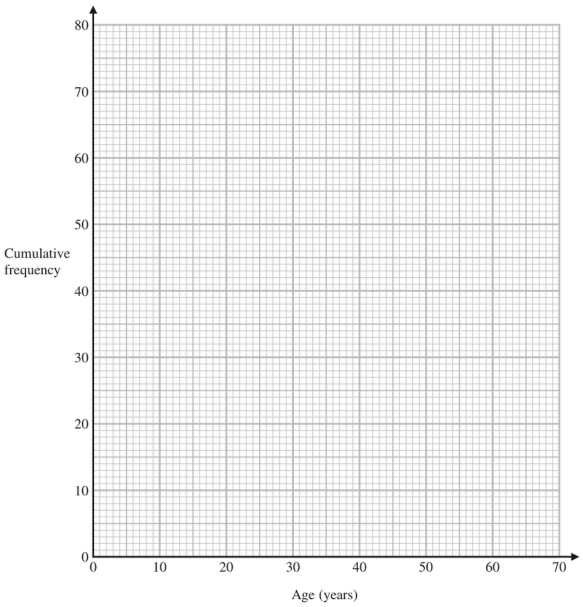
(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)

(c) Use your graph to find an estimate for the median age of the 80 people.

..... years

(1)



Of the people in the train carriage, 60% of those who are aged between 18 and 65 are going to work. None of the other people in the train carriage are going to work.

(d) Use your graph to find an estimate for the number of people in the train carriage who are going to work.

.....

(3)

(Total for Question 13 is 7 marks)

1 Complete the cumulative frequencies

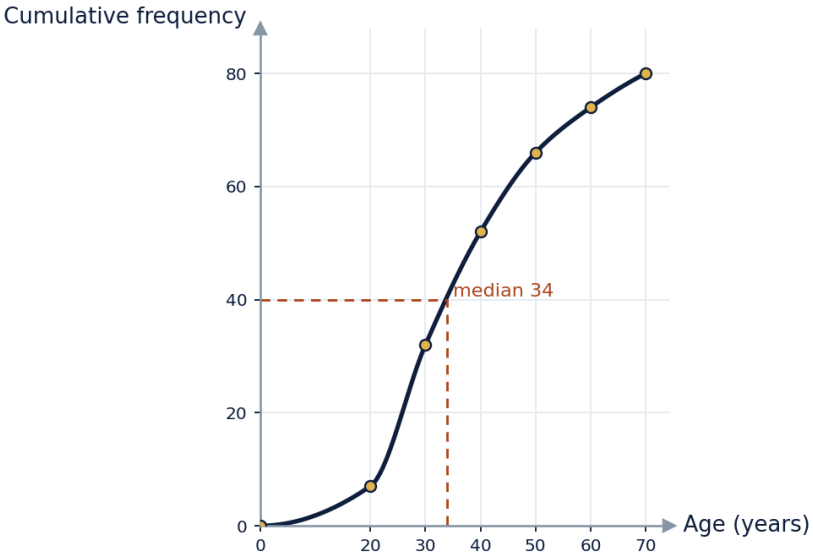
Complete the cumulative frequencies:

$$7,\ 32,\ 52,\ 66,\ 74,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (20, 7), (30, 32), (40, 52), (50, 66), (60, 74), (70, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(30, 32)$ and $(40, 52)$:

$$30 + \frac{40 - 32}{52 - 32} \times 10 = 34$$

5 Use cumulative frequency

So the median age is about 34 years.

6 Estimate the number aged between

Estimate the number aged between 18 and 65:

$$F(18) \approx \frac{18}{20} \times 7 = 6.3$$

$$F(65) \approx 74 + \frac{5}{10}(80 - 74) = 77$$

$$77 - 6.3 = 70.7$$

7 60% are going to work

60% are going to work:

$$0.6(70.7) = 42.42$$

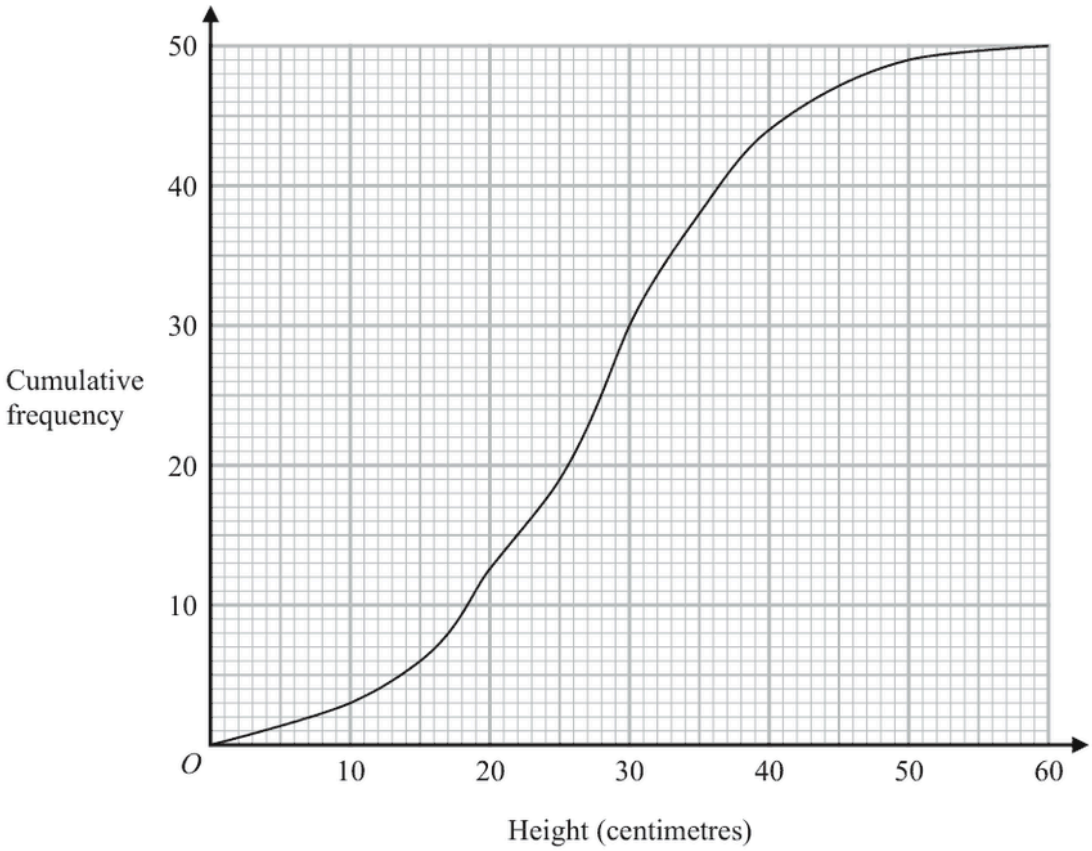
FINAL ANSWER

cumulative frequencies $7, 32, 52, 66, 74, 80$, median about 34 years, about 42 people going to work.

Question 12

4 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph shows information about the heights, in centimetres, of 50 plants in a flowerbed.



(a) Use the graph to find an estimate for the median height of these plants.

..... centimetres

(1)

(b) Use the graph to find the frequency for the class interval $30 < \text{Height} \leqslant 40$

.....

(1)

(c) Use the graph to find an estimate for the number of plants with a height greater than 35 centimetres.

.....

(2)

(Total for Question 12 is 4 marks)

1

(a) Median

There are 50 plants, so read across from 25: the curve reaches 25 at about 28.5 cm.

2

(b) Frequency for 30 < height ≤ 40

Read the curve at both ends of the class: about 44 at 40 cm and 30 at 30 cm.

$$44 - 30 = 14$$

3

(c) Taller than 35 cm

At 35 cm the curve reads about 37, so about $50 - 37 = 13$ plants.

✓ **FINAL ANSWER**

(a) about 28.5 cm (b) 14 (c) about 13

Question 14

6 marks | Cumulative Frequency Diagrams

14 The table gives information about the times taken by 80 people to run a race.

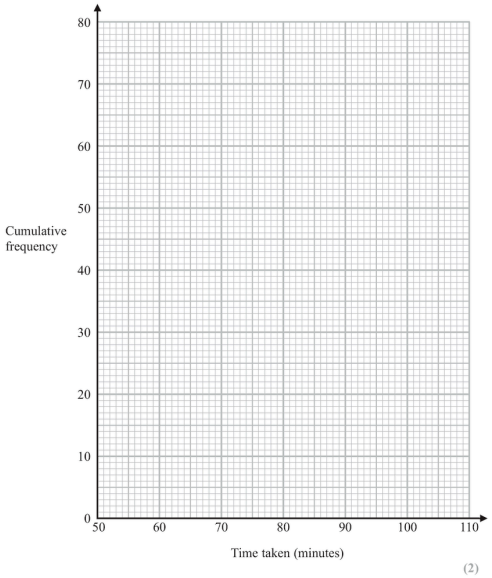
Time taken (t minutes)	Frequency
$50 < t \leqslant 60$	15
$60 < t \leqslant 70$	16
$70 < t \leqslant 80$	21
$80 < t \leqslant 90$	14
$90 < t \leqslant 100$	8
$100 < t \leqslant 110$	6

(a) Complete the cumulative frequency table.

Time taken (t minutes)	Cumulative frequency
$50 < t \leqslant 60$	
$50 < t \leqslant 70$	
$50 < t \leqslant 80$	
$50 < t \leqslant 90$	
$50 < t \leqslant 100$	
$50 < t \leqslant 110$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time taken.

..... minutes

(1)

(d) Use your graph to find an estimate for the interquartile range of the times taken.

..... minutes

(2)

(Total for Question 14 is 6 marks)

1

(a) Running totals

$$15,\ 31,\ 52,\ 66,\ 74,\ 80$$

2

(b) Plot at the upper class boundaries

Plot $(60, 15), (70, 31), (80, 52), (90, 66), (100, 74), (110, 80)$, start at $(50, 0)$, and join with a smooth curve.

3

(c) Median

Read across from 40: about 74 minutes.

4

(d) Interquartile range

Lower quartile at 20: about 63 minutes. Upper quartile at 60: about 86 minutes.

$$\text{IQR} \approx 86 - 63 = 23 \text{ minutes}$$

✓ **FINAL ANSWER**

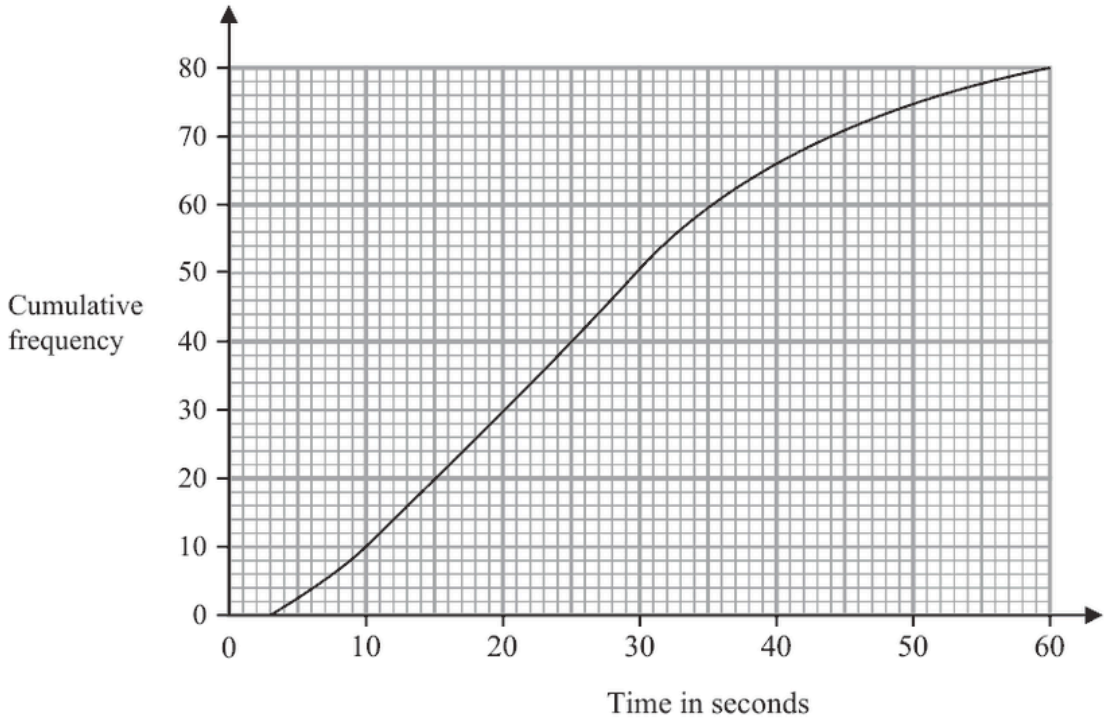
(a) $15, 31, 52, 66, 74, 80$ (c) about 74 minutes (d) about 23 minutes

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Question 13

4 marks | Cumulative Frequency Diagrams

13 The cumulative frequency graph gives information about the times, in seconds, that 80 adults took to log in to an online bank.



(a) Find an estimate for the median time.

..... seconds

(1)

(b) Work out the percentage of these adults that took longer than 50 seconds to log in.
Show your working clearly.

.....%

(3)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2023

1 **(a) Median**

Read across from $\frac{80}{2} = 40$: about 25 seconds.

2 **(b) Adults who took longer than 50 seconds**

At 50 seconds the curve reads about 75, so $80 - 75 = 5$ adults.

3 (b) Percentage

$$\frac{5}{80} \times 100 = 6.25\%$$

FINAL ANSWER

(a) about 25 seconds (b) about 6.25%

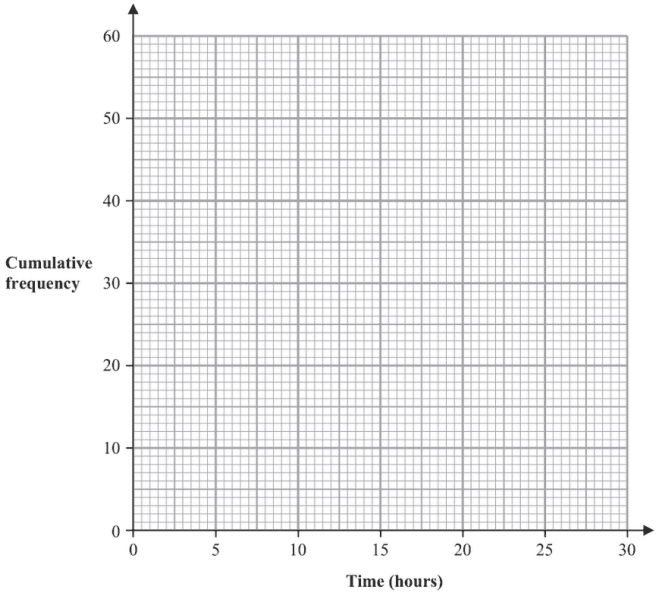
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table gives information about the time, in hours, that each of 60 workers spent working from home in one week.

Time (t hours)	Cumulative frequency
$0 < t \leqslant 5$	6
$0 < t \leqslant 10$	17
$0 < t \leqslant 15$	27
$0 < t \leqslant 20$	42
$0 < t \leqslant 25$	53
$0 < t \leqslant 30$	60

(a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the interquartile range of the times.

..... hours

(2)

25 workers spent more than W hours working from home.

(c) Use your graph to find an estimate for the value of W

$W =$ .....

(2)

One of the 60 workers is chosen at random.
This worker spent H hours working from home.

(d) Find the probability that $5 < H \leqslant 10$

.....

(1)

(Total for Question 11 is 7 marks)

1

(a) Plot the table

Plot $(5, 6), (10, 17), (15, 27), (20, 42), (25, 53), (30, 60)$, start at $(0, 0)$, and join with a smooth curve.

2

(b) Read the quartiles

Lower quartile at $\frac{60}{4} = 15$: about 9 hours. Upper quartile at 45: about 21.4 hours.

$$\text{IQR} \approx 21.4 - 9.1 \approx 12.3 \text{ hours}$$

(Answers from about 11 to 14 are accepted, depending on the curve.)

3

(c) 25 workers above W

So $60 - 25 = 35$ workers spent W hours or less. Reading across from 35: $W \approx 17.7$.

4

(d) Between 5 and 10 hours

$17 - 6 = 11$ workers, so

$$P(5 < H \leq 10) = \frac{11}{60}$$

✓ **FINAL ANSWER**

(b) about 12.3 hours (c) $W \approx 17.7$ (d) $\frac{11}{60}$

Question 13

7 marks | Cumulative Frequency Diagrams

13 The table gives information about the distances, in km, that 70 teachers travel to school.

Distance (d km)	Frequency
$0 < d \leqslant 10$	7
$10 < d \leqslant 20$	17
$20 < d \leqslant 30$	18
$30 < d \leqslant 40$	14
$40 < d \leqslant 50$	10
$50 < d \leqslant 60$	4

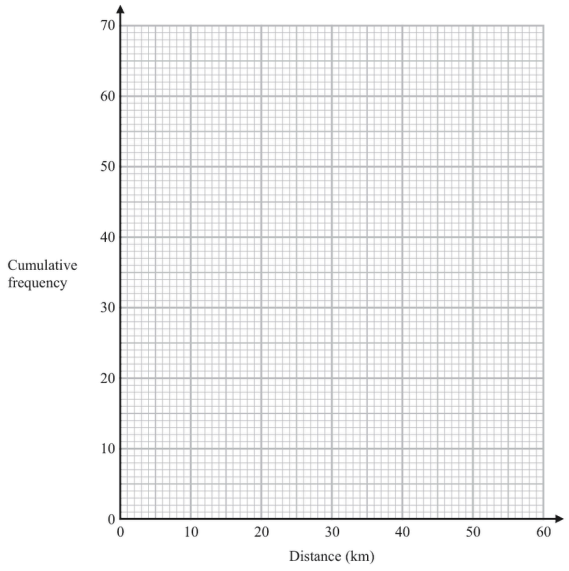
(a) Complete the cumulative frequency table.

Distance (d km)	Cumulative frequency
$0 < d \leqslant 10$	
$0 < d \leqslant 20$	
$0 < d \leqslant 30$	
$0 < d \leqslant 40$	
$0 < d \leqslant 50$	
$0 < d \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)



(c) Use your graph to find an estimate for the interquartile range of the distances.

..... km

(2)

(d) Use your graph to find an estimate for the number of teachers who travel more than 46 km.

.....

(2)

(Total for Question 13 is 7 marks)

1

(a) Running totals

$$7,\ 24,\ 42,\ 56,\ 66,\ 70$$

2

(b) Plot at the upper class boundaries

Plot $(10, 7), (20, 24), (30, 42), (40, 56), (50, 66), (60, 70)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Read the quartiles

Lower quartile at $\frac{70}{4} = 17.5$: about 16 km. Upper quartile at $\frac{3}{4} \times 70 = 52.5$: about 37.5 km.

$$\text{IQR} \approx 37.5 - 16 = 21.5 \text{ km}$$

(Answers from about 19 to 23 km are accepted, depending on the curve.)

4

(d) More than 46 km

At 46 km the curve reads about 62, so

$$70 - 62 = 8 \text{ teachers}$$

✓ **FINAL ANSWER**

(a) $7, 24, 42, 56, 66, 70$ (c) about 21 km (d) about 8

Question 13

6 marks | Cumulative Frequency Diagrams

13 The frequency table gives information about the weights, in kilograms, of 60 parcels in a delivery van.

Weight (w kilograms)	Frequency
$0 < w \leqslant 1$	4
$1 < w \leqslant 2$	15
$2 < w \leqslant 3$	20
$3 < w \leqslant 4$	11
$4 < w \leqslant 5$	6
$5 < w \leqslant 6$	4

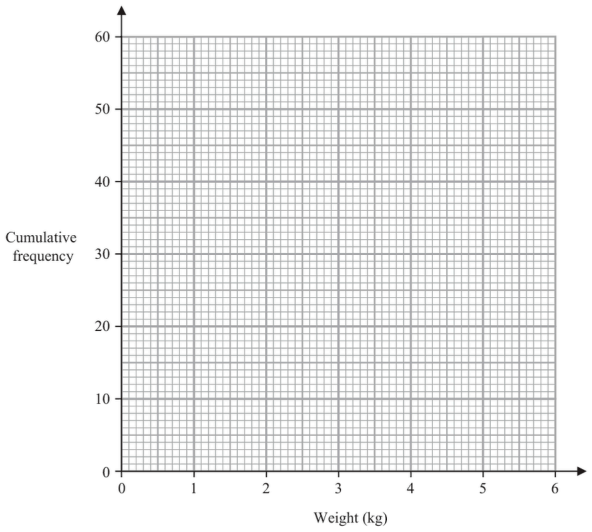
(a) Complete the cumulative frequency table.

Weight (w kilograms)	Cumulative frequency
$0 < w \leqslant 1$	
$0 < w \leqslant 2$	
$0 < w \leqslant 3$	
$0 < w \leqslant 4$	
$0 < w \leqslant 5$	
$0 < w \leqslant 6$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)



(c) Use your graph to find an estimate for the median weight of the 60 parcels.

..... kilograms

(1)

(d) Use your graph to find an estimate for the number of these parcels that weigh more than 3.7 kilograms.

.....

(2)

(Total for Question 13 is 6 marks)

1

(a) Running totals

$$4,\ 19,\ 39,\ 50,\ 56,\ 60$$

2

(b) Plot at the upper class boundaries

Plot $(1, 4), (2, 19), (3, 39), (4, 50), (5, 56), (6, 60)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Median

Read across from $\frac{60}{2} = 30$: about 2.55 kg.

4

(d) Heavier than 3.7 kg

At 3.7 kg the curve reads about 47, so about $60 - 47 = 13$ parcels.

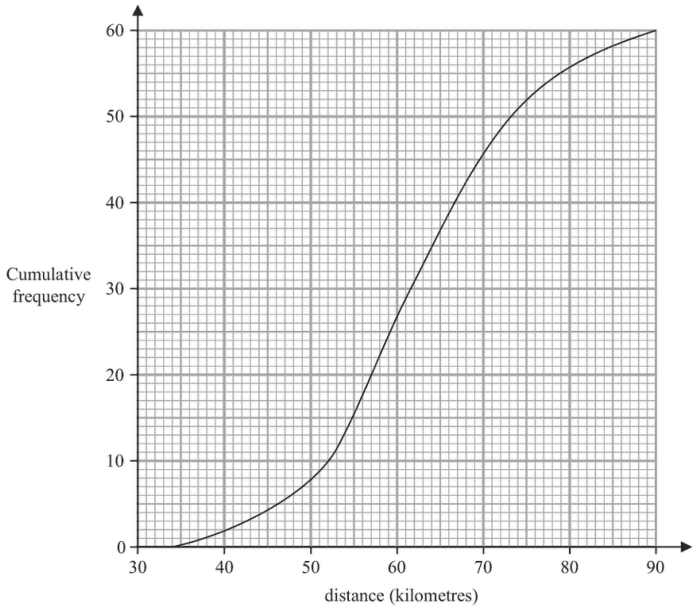
✓ **FINAL ANSWER**

(a) $4, 19, 39, 50, 56, 60$ (c) about 2.55 kg (d) about 13

Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency graph shows information about the distance, in kilometres, Ehrick drives each day for 60 days.



(a) Use the graph to find an estimate for the median of the distances.

..... kilometres
(1)

(b) Use the graph to find an estimate for the interquartile range of the distances.

..... kilometres
(2)

(c) Work out the percentage of days that Ehrick drives for more than 75 kilometres.
Give your answer correct to one decimal place.
Show your working clearly.

..... %
(3)

Tomas and Alison record the number of kilometres that they drive each day for 60 days.
The table gives information about their results.

	Interquartile range
Tomas	28 kilometres
Alison	15 kilometres

(d) Which of Tomas or Alison is more consistent in the number of kilometres that they drive.
Give a reason for your answer.

.....
(1)

(Total for Question 11 is 7 marks)

1 **Required**

a Median(b) IQR(c) % days \> 75 km(d) More consistent?

2 **Median at the th value**

Median at the $frac602 = 30$th value — read from graph at cf = 30:

3 **Answer**

Median ≈ 62 km

4 **Read the graph**

*Q*₁ at cf = 15, *Q*₃ at cf = 45. Read from graph:

*Q*₁ ≈ 56 km, *Q*₃ ≈ 74 km

IQR = *Q*₃ − *Q*₁ = 69 − 55 = 14 km

5 **Answer**

IQR ≈ 14 km

6 **Note**

Graph-reading answers are accepted within ± 2 km of the model answer.

7 **Use cumulative frequency**

From the graph, read the cumulative frequency at *d* = 75 km:

cf at 75 km ≈ 52

8 **Number of days with distance**

Number of days with distance \> 75 km:

60 − 52 = 8 days

9 **Percentage**

Percentage:

$$frac860times100 = 13.3$$

10 **Answer**

13.3% of days Ehrick drives more than 75 km

FINAL ANSWER

Alison is more consistent — her IQR (15 km) is smaller than Tomas's (28 km), meaning less variability in daily distances.

Question 12

6 marks | Cumulative Frequency Diagrams

12 The table gives information about the ages of 80 people in a cinema.

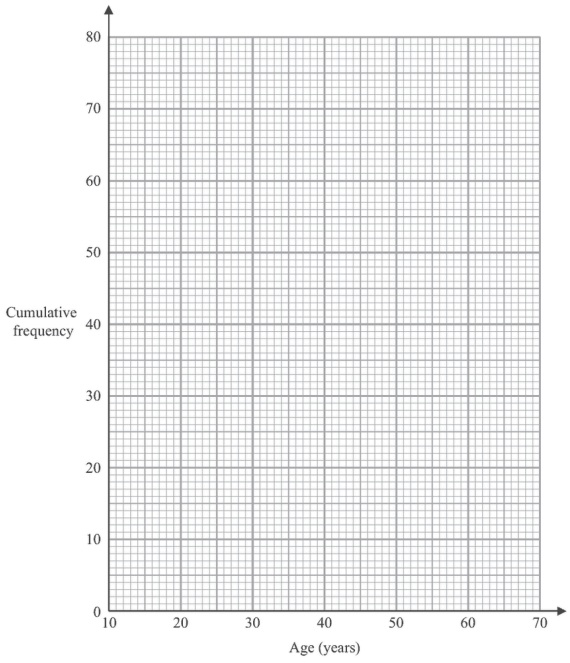
Age (n years)	Frequency
$10 < n \leqslant 20$	12
$20 < n \leqslant 30$	15
$30 < n \leqslant 40$	20
$40 < n \leqslant 50$	18
$50 < n \leqslant 60$	9
$60 < n \leqslant 70$	6

(a) Complete the cumulative frequency table.

Age (n years)	Cumulative frequency
$10 < n \leqslant 20$	
$10 < n \leqslant 30$	
$10 < n \leqslant 40$	
$10 < n \leqslant 50$	
$10 < n \leqslant 60$	
$10 < n \leqslant 70$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the percentage of the 80 people who are more than 46 years of age.
Give your answer correct to the nearest whole number.

.....%

(3)

(Total for Question 12 is 6 marks)

1 **(a) Running totals**

$$12,\ 27,\ 47,\ 65,\ 74,\ 80$$

2 **(b) Plot at the upper class boundaries**

Plot $(20, 12), (30, 27), (40, 47), (50, 65), (60, 74), (70, 80)$, start at $(10, 0)$, and join with a smooth curve.

3 **(c) People aged 46 or under**

Read up from 46: about 58 people.

4 **(c) Percentage older than 46**

$80 - 58 = 22$ people.

$$\frac{22}{80} \times 100 \approx 28\%$$

(About 26% to 30% is accepted, depending on the curve.)

✓ **FINAL ANSWER**

(a) $12, 27, 47, 65, 74, 80$ (c) about 28%

Question 13

6 marks | Cumulative Frequency Diagrams

13 The frequency table gives information about the times, in minutes, that 60 people took to complete a puzzle.

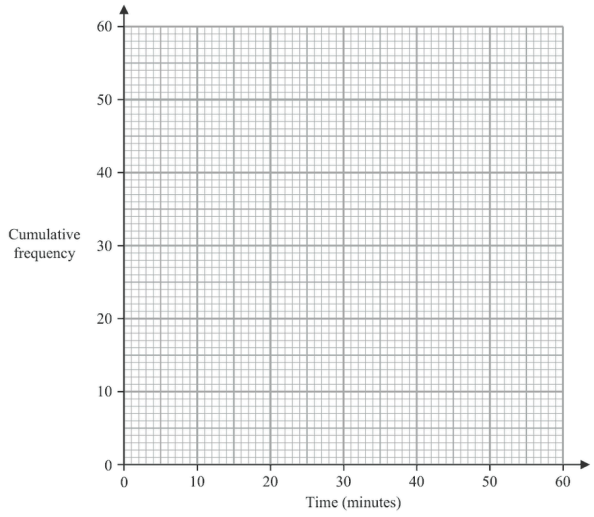
Time (t minutes)	Frequency
$0 < t \leqslant 10$	8
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	17
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	
$0 < t \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time.

..... minutes

(1)

(d) Use your graph to find an estimate for the number of these people who took more than 45 minutes to complete the puzzle.

.....

(2)

(Total for Question 13 is 6 marks)

1

(a) Running totals

$$8,\ 21,\ 33,\ 50,\ 57,\ 60$$

2

(b) Plot at the upper class boundaries

Plot $(10, 8), (20, 21), (30, 33), (40, 50), (50, 57), (60, 60)$, start at $(0, 0)$, and join with a smooth curve.

3

(c) Median

Read across from $\frac{60}{2} = 30$: about 27.5 minutes.

4

(d) More than 45 minutes

At 45 minutes the curve reads about 54, so about $60 - 54 = 6$ people. (Answers from about 5 to 8 are accepted, depending on the curve.)

✓ **FINAL ANSWER**

(a) $8, 21, 33, 50, 57, 60$ (c) about 27.5 minutes (d) about 6

Question 14

6 marks | Cumulative Frequency Diagrams

14 The frequency table gives information about the times, in minutes, that 60 people took to complete a puzzle.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	8
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	17
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

(a) Complete the cumulative frequency table.

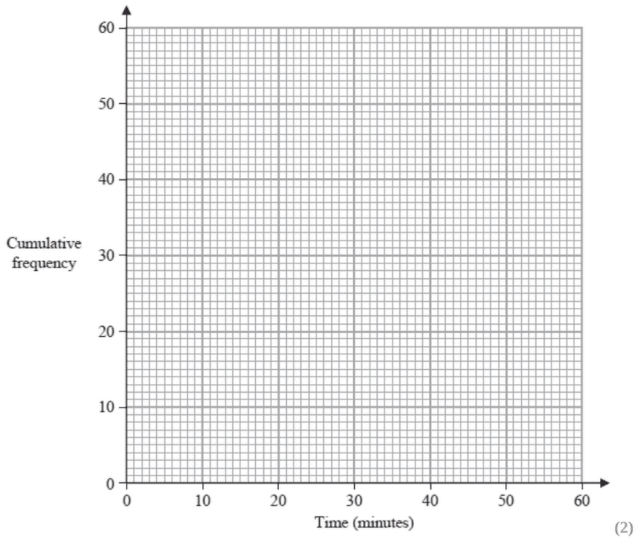
Time (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$10 < t \leqslant 20$	
$20 < t \leqslant 30$	
$30 < t \leqslant 40$	
$40 < t \leqslant 50$	
$50 < t \leqslant 60$	

(1)

(b) On the grid on the next page, draw a cumulative frequency graph for your table.

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(2)

(c) Use your graph to find an estimate for the median time.

..... minutes

(1)

(d) Use your graph to find an estimate for the number of these people who took more than 45 minutes to complete the puzzle.

.....

(2)

(Total for Question 14 is 6 marks)

1 Cumulative frequencies

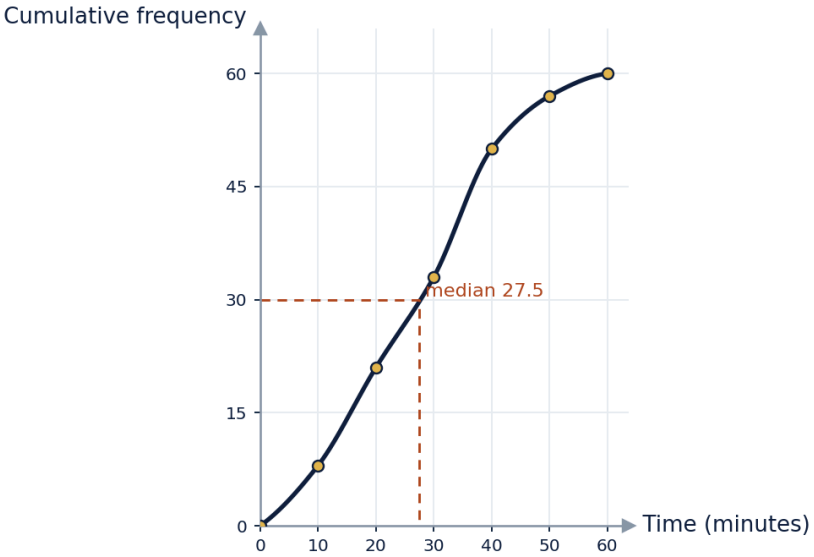
Cumulative frequencies:

$$8,\ 21,\ 33,\ 50,\ 57,\ 60$$

2 Plot the points

Plot the points

$$(10, 8), (20, 21), (30, 33), (40, 50), (50, 57), (60, 60)$$



3 The median is the th

The median is the 30th value. Interpolating in the 20 to 30 interval gives about 27.5 minutes.

4 Estimate the value

For more than 45 minutes, the estimate is about 7 people.

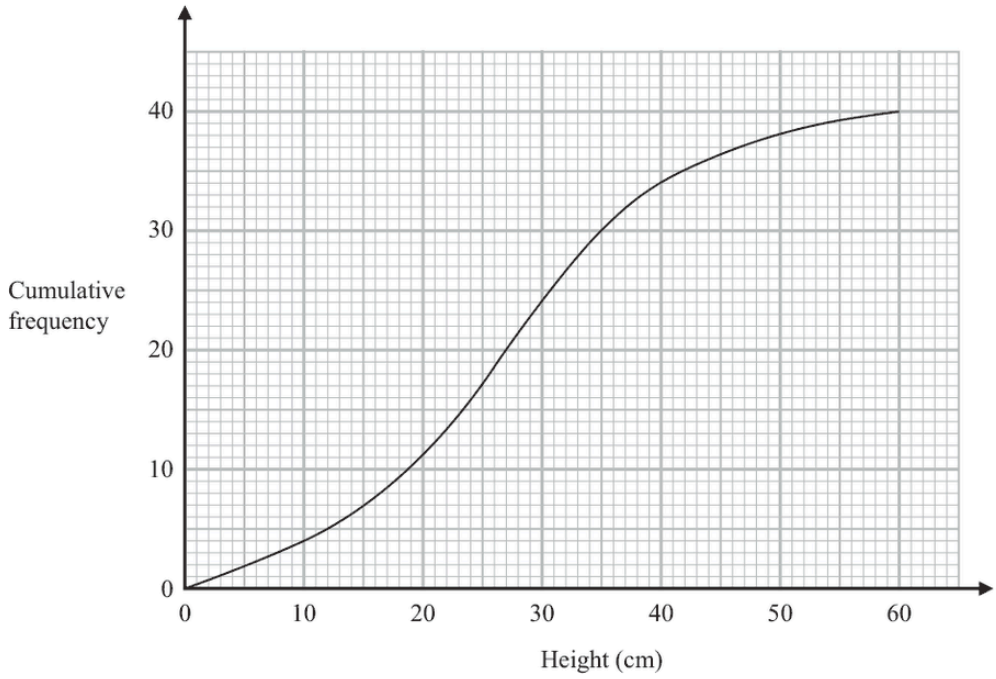
FINAL ANSWER

cumulative frequencies $8, 21, 33, 50, 57, 60$; median about 27.5 minutes; more than 45 minutes about 7 people.

Question 14

5 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph shows information about the heights of 40 plants that Greta has grown.



(a) Use the graph to find an estimate for the median height.

..... cm

(1)

(b) Use the graph to find an estimate for the interquartile range of the heights.

..... cm

(2)

Plants with a height greater than 40 cm are premium plants.
Greta sells all the premium plants for 30 euros each.

(c) Work out the total amount of money Greta receives for the premium plants.

..... euros

(2)

(Total for Question 14 is 5 marks)

1 **Part (a) Median Total plants**

Part (a) Median: Total plants = 40. The median position is at cumulative frequency = 40/2 = 20. Reading across from CF = 20 to the curve and down to the x-axis gives:

2 **Answer**

a 30 cm

3 **Part (b) Interquartile Range (IQR)**

Lower Quartile (Q1) is at CF = 10. Reading the graph ⟹ Height = 20 cm.
Upper Quartile (Q3) is at CF = 30. Reading the graph ⟹ Height = 40 cm.
IQR = *Q*3 − *Q*1 = 40 − 20 = 20 cm

4 **Answer**

b 20 cm

Part (c) Premium plants revenue: Premium plants have height \> 40 cm. From the graph, cumulative frequency at 40 cm is 30. This means 30 plants are ≤ 40 cm.
Number of premium plants = 40 − 30 = 10 plants
Total Money = 10 plants × 30 euros = 300 euros

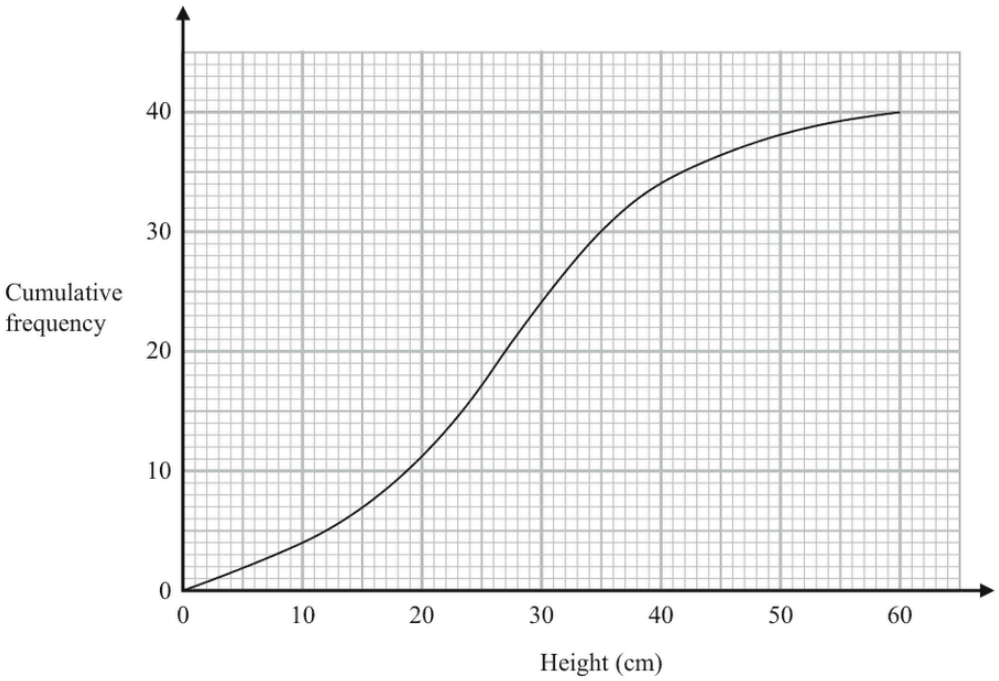
✓ **FINAL ANSWER**

c 300 euros

Question 14

5 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph shows information about the heights of 40 plants that Greta has grown.



(a) Use the graph to find an estimate for the median height.

..... cm

(1)

(b) Use the graph to find an estimate for the interquartile range of the heights.

..... cm

(2)

Plants with a height greater than 40 cm are premium plants.
Greta sells all the premium plants for 30 euros each.

(c) Work out the total amount of money Greta receives for the premium plants.

..... euros

(2)

(Total for Question 14 is 5 marks)

1

(a) Median

Read across from $\frac{40}{2} = 20$: about 27 cm.

2

(b) Interquartile range

Lower quartile at 10: about 19 cm. Upper quartile at 30: about 35 cm.

$$\text{IQR} \approx 35 - 19 = 16 \text{ cm}$$

3

(c) Premium plants

At 40 cm the curve reads about 34, so $40 - 34 = 6$ plants are taller than 40 cm.

$$6 \times 30 = 180 \text{ euros}$$

✓ **FINAL ANSWER**

(a) about 27 cm (b) about 16 cm (c) 180 euros

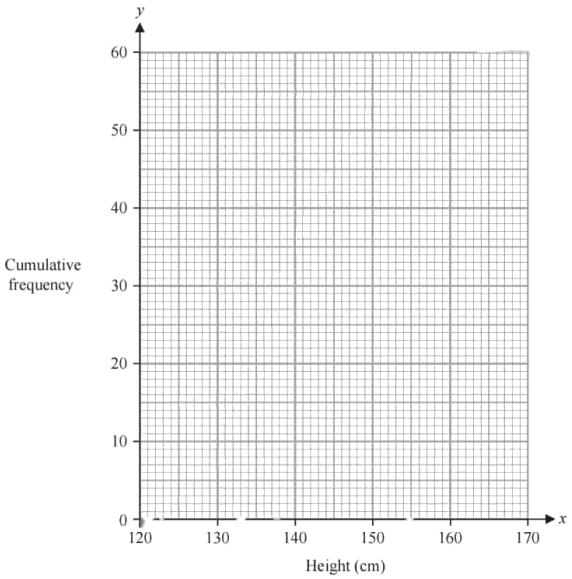
Question 13

6 marks | Cumulative Frequency Diagrams

13 The cumulative frequency table shows information about the heights, in centimetres, of 60 plants.

Height (h cm)	Cumulative frequency
$120 \leqslant h < 130$	8
$120 \leqslant h < 140$	35
$120 \leqslant h < 150$	52
$120 \leqslant h < 160$	58
$120 \leqslant h < 170$	60

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use your graph to find an estimate for the median.

..... cm

(1)

(c) Use your graph to find an estimate for the lower quartile.

..... cm

(1)

(d) Use your graph to find an estimate for the number of plants with a height greater than 155 cm

.....

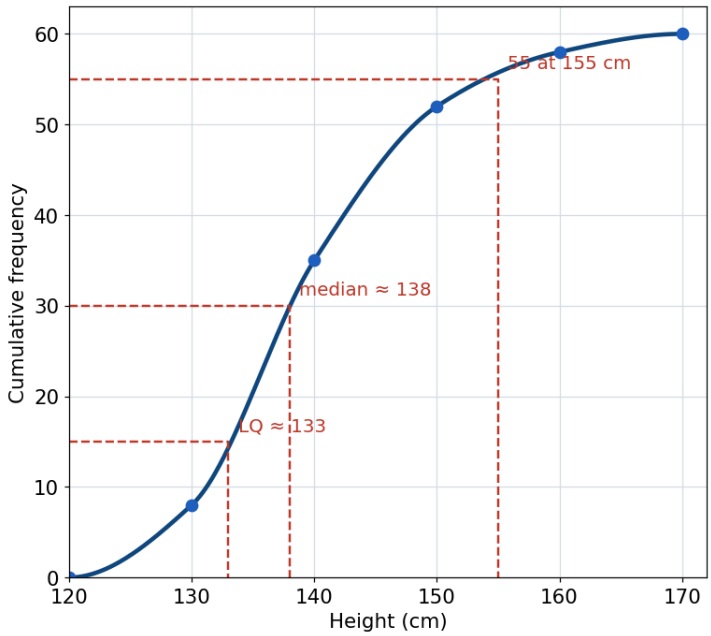
(2)

(Total for Question 13 is 6 marks)

1

(a) Plot at the upper class boundaries

Plot $(130, 8), (140, 35), (150, 52), (160, 58), (170, 60)$, start at $(120, 0)$ and join with a smooth curve.



2

(b) Median

$\frac{1}{2} \times 60 = 30$: read across from 30 → about 138 cm.

3

(c) Lower quartile

$\frac{1}{4} \times 60 = 15$: read across from 15 → about 133 cm.

4

(d) Taller than 155 cm

Up from 155 cm the curve reads about 55, so $60 - 55 = 5$ plants.

✓ **FINAL ANSWER**

(b) 138 cm (c) 133 cm (d) about 5 plants

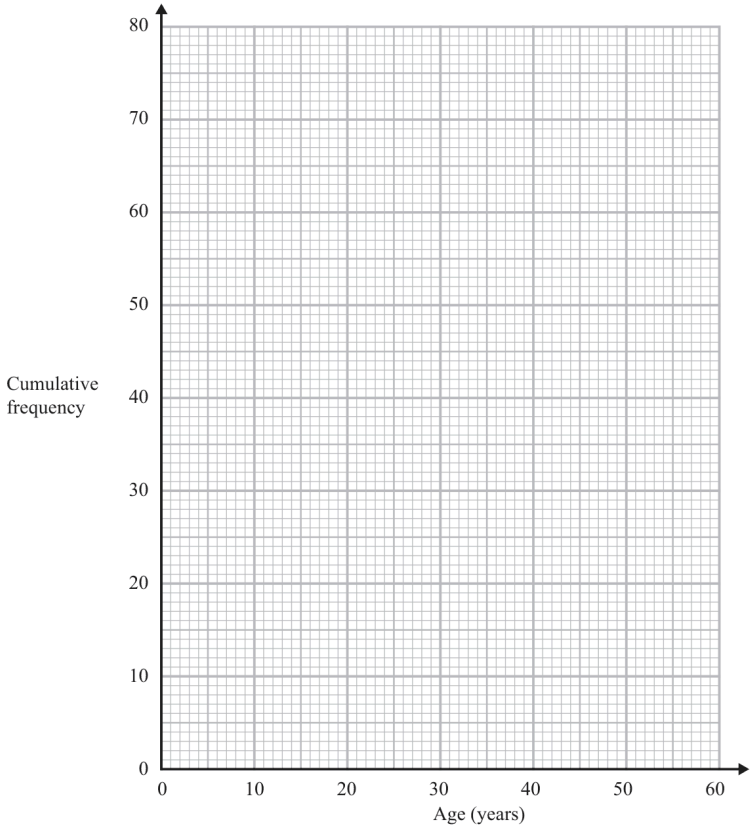
Question 14

4 marks | Cumulative Frequency Diagrams

14 The cumulative frequency table gives information about the ages, in years, of 80 people who went to a cinema on Saturday.

Age (a years)	Cumulative frequency
$0 < a \leqslant 10$	19
$0 < a \leqslant 20$	37
$0 < a \leqslant 30$	52
$0 < a \leqslant 40$	63
$0 < a \leqslant 50$	73
$0 < a \leqslant 60$	80

(a) On the grid below, draw a cumulative frequency graph for this information.



(2)

(b) Use your graph to find an estimate for the median age of the 80 people.

..... years

(1)

(c) Use your graph to find an estimate for the number of the 80 people aged 16 and under.

.....

(1)

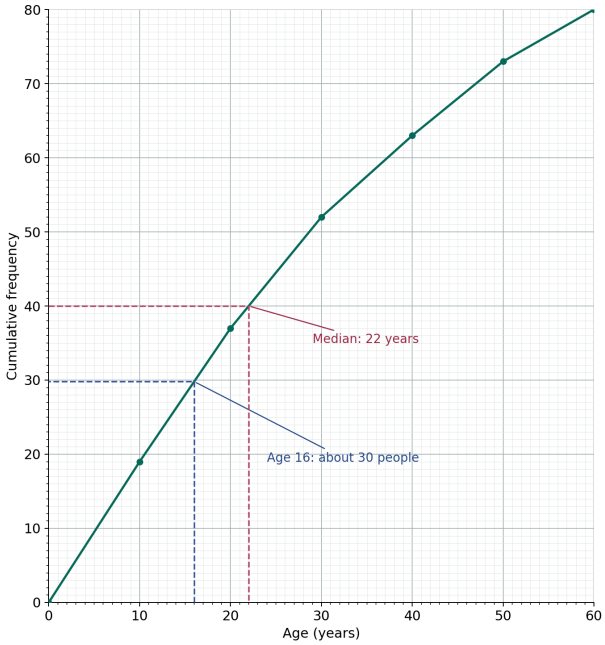
(Total for Question 14 is 4 marks)

1 **Part (a) | Step 1**

Use age (years) on the horizontal axis from 0 to 60, and cumulative frequency on the vertical axis from 0 to 80. Plot $(0, 0), (10, 19), (20, 37), (30, 52), (40, 63), (50, 73), (60, 80)$.
Each cumulative total is plotted at its upper age boundary.

2 **Part (a) | Step 2**

Join the points in order with an increasing curve or straight line segments. Do not draw bars or a staircase.
This represents the accumulated number up to each age boundary.



3 **Part (b) | Step 1**

$$\text{median position} = \frac{80}{2} = 40.$$

Read horizontally from cumulative frequency 40 to the graph, then vertically down to the age axis.

4 **Part (b) | Step 2**

Using straight segments,

$$20 + \frac{40 - 37}{52 - 37} \times 10 = 22 \text{ years}.$$

The median is estimated between ages 20 and 30; a smooth graph gives a nearby estimate.

5 **Part (c) | Step 1**

Read vertically from age 16 to the graph, then horizontally to the cumulative-frequency axis.
The ordinate is the estimated number aged 16 and under.

6

Part (c) | Step 2

Using straight segments,

$$19 + \frac{16 - 10}{20 - 10}(37 - 19) = 29.8 \approx 30.$$

Round the estimated count to a whole number of people.

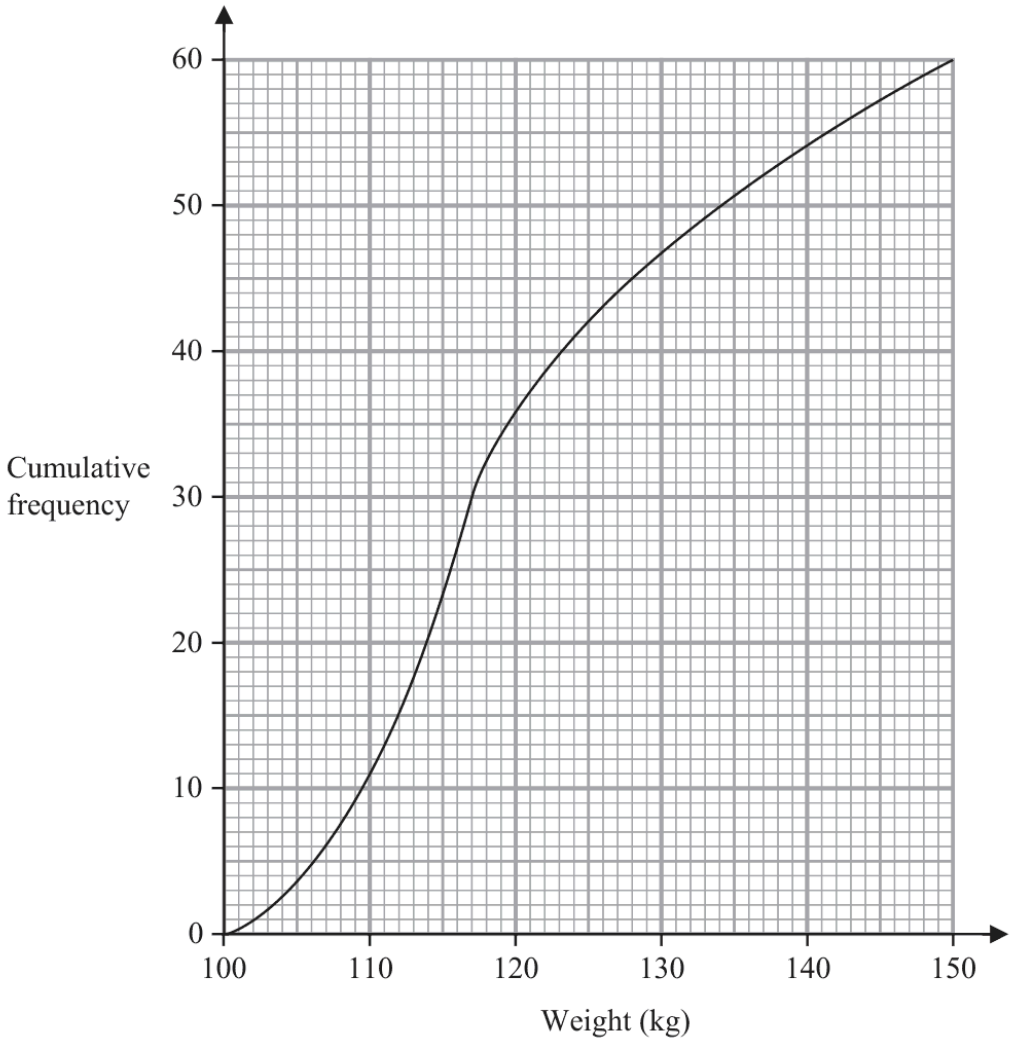
FINAL ANSWER

(a) Increasing cumulative-frequency graph through $(0, 0), (10, 19), (20, 37), (30, 52), (40, 63), (50, 73), (60, 80)$. (b) Approximately 22 years. (c) Approximately 30 people.

Question 14

3 marks | Cumulative Frequency Diagrams

14 The cumulative frequency graph gives information about the weights, in kg, of 60 crates.



(a) Use the graph to find an estimate for the median weight.

..... kg

(1)

(b) Use the graph to find an estimate for the interquartile range.

..... kg

(2)

(Total for Question 14 is 3 marks)

1

Part (a) | Step 1

The median is the $60/2 = 30$th value. Draw horizontally from cumulative frequency 30 to the curve, then vertically to the weight axis: about $117\ \text{kg}$.

The median splits the 60 observations in half.

2

Part (b) | Step 1

Read the lower quartile at cumulative frequency $60/4 = 15$, giving $Q_1 \approx 112\ \text{kg}$. Read the upper quartile at cumulative frequency $3(60)/4 = 45$, giving $Q_3 \approx 128\ \text{kg}$.

Quartiles occur at one-quarter and three-quarters of the total frequency.

3

Part (b) | Step 2

$$\text{IQR} = Q_3 - Q_1 \approx 128 - 112 = 16\ \text{kg}.$$

Interquartile range is upper quartile minus lower quartile.

✓ **FINAL ANSWER**

(a) $117\ \text{kg}$ approximately. (b) $16\ \text{kg}$ approximately.