Elite Mathematics
Pearson Edexcel International GCSE

IGCSE MODULAR · UNIT 1 · 4WM1

Modular Unit 1 Expertise With Answers

Q20+ questions with Dr Eslam Ahmed's worked solutions.

136	2020–2026	Questions with worked solutions
topic placements	50 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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TOPIC

Rounding, Estimation & Bounds

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY 2021

Question 20

3 marks | Rounding, Estimation & Bounds

20 $P = \dfrac{t - w}{y}$

t = 9.7 correct to 1 decimal place

w = 5.9 correct to 1 decimal place

y = 3 correct to 1 significant figure

Calculate the upper bound for the value of P.
Show your working clearly.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2021

1 Estimate the value

$$P = \frac{t - w}{y}$$

2 Find upper bound

For the upper bound of P, make the numerator as large as possible and the denominator as small as possible.

$t = 9.75, \quad w = 5.85, \quad y = 2.5$

$$P = \frac{9.75 - 5.85}{2.5}$$

$$P = \frac{3.90}{2.5} = 1.56$$

✓ **FINAL ANSWER**

1.56

4MA1/1HR JANUARY 2022

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows triangle ABC

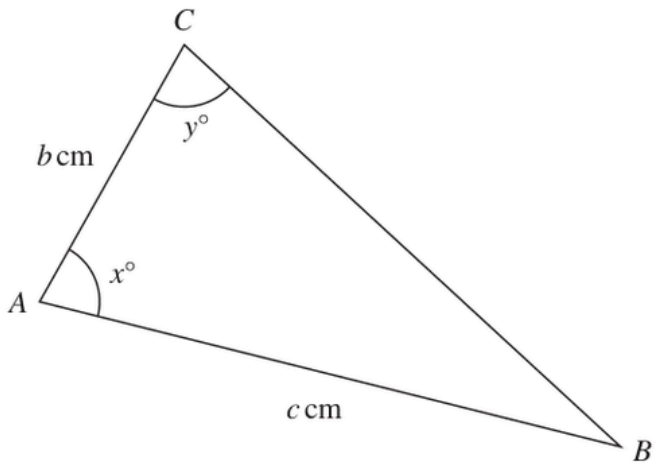


Diagram **NOT** accurately drawn

$c = 11.5$ correct to one decimal place
$x = 80$ correct to the nearest whole number
$y = 75$ correct to the nearest whole number

Calculate the upper bound for the value of b
Show your working clearly.
Give your answer correct to 3 significant figures.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2022

1 Find upper bound

For an upper bound for b, use the upper bound of c:

$c = 11.55$

2 Use sine rule

Use the lower bounds of x and y, because this makes angle B as large as possible and makes $\sin y$ as small as possible.

$x = 79.5°, \quad y = 74.5°$

3 Use trigonometry

So

$\angle B = 180° - 79.5° - 74.5° = 26°$

4 **Use trigonometry**

Using the sine rule,

$$\frac{b}{\sin 26^\circ} = \frac{11.55}{\sin 74.5^\circ}$$

$$b = \frac{11.55 \sin 26^\circ}{\sin 74.5^\circ} = 5.254...$$

✓ **FINAL ANSWER**

5.25 cm to 3 significant figures.

4MA1/2H JUNE 2023

Question 25

4 marks | Rounding, Estimation & Bounds

25 A solid sphere has a radius of 2.8 centimetres, correct to 1 decimal place.
The sphere has a mass of $M\pi$ grams, where $M = 260$ correct to 2 significant figures.

Work out the upper bound for the density of the sphere.
Give your answer in g/cm^3 correct to 2 decimal places.
Show your working clearly.

..... g/cm^3

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2023

1 Find upper bound

For the upper bound of density, use the upper bound of the mass and the lower bound of the volume.

2 The radius is cm correct

The radius is 2.8 cm correct to 1 decimal place, so the lower bound is
$r = 2.75$

3 Find upper bound

$M = 260$ correct to 2 significant figures, so the upper bound is
$M = 265$

4 The mass is given as

The mass is given as $M\pi$ grams, where $M = 265$ for the upper bound.

5 The volume of the sphere

The volume of the sphere is

$$\frac{4}{3}\pi r^3$$

6 Rearrange formula

So

$$\text{density} = \frac{M\pi}{\frac{4}{3}\pi r^3}$$

7 **The cancels**

The π cancels:

$$\text{density} = \frac{3M}{4r^3}$$

8 **Upper bound**

Upper bound:

$$\frac{3(265)}{4(2.75)^3} = 9.5567\ldots$$

✓ **FINAL ANSWER**

$9.56\ \text{g/cm}^3$ to 2 decimal places.

4MA1/1H NOVEMBER 2024

Question 21

3 marks | Rounding, Estimation & Bounds

21 $T = \dfrac{x^2 + y^2}{w}$

$x = 28.4$ correct to 1 decimal place.

$y = 17$ correct to 2 significant figures.

$w = 90$ correct to the nearest 5

Calculate the upper bound for the value of T

Give your answer correct to 3 significant figures.
Show your working clearly.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2024

1 Estimate the value

$$T = \frac{x^2 + y^2}{w}$$

2 Find upper bound

For the upper bound of T, use the upper bounds of x and y, and the lower bound of w.

$x = 28.45, \quad y = 17.5, \quad w = 87.5$

$$T = \frac{28.45^2 + 17.5^2}{87.5}$$

$$T = 12.7503\ldots$$

FINAL ANSWER

12.8 to 3 significant figures.

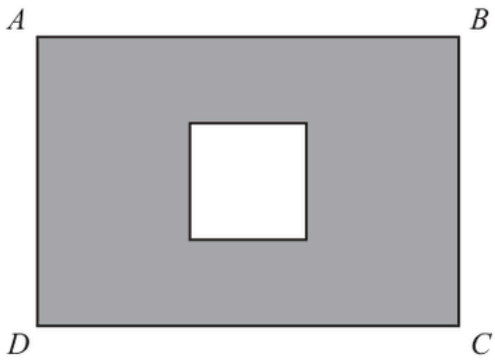
4MA1/2H JUNE 2025

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows a square inside rectangle $ABCD$

Diagram **NOT** accurately drawn



The total area of the region shown shaded in the diagram is X cm²

$AB = 11.5$ cm correct to the nearest 0.5 cm
$BC = 9.2$ cm correct to 2 significant figures
side of square = 4.1 cm correct to 2 significant figures

By considering bounds, work out the value of X to a suitable degree of accuracy.
Show your working clearly.

$X =$ .....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2025

1 The shaded area is

The shaded area is

$$X = \text{area of rectangle} - \text{area of square}$$

2 Bounds

Bounds:

$$11.25 \leq AB < 11.75$$
$$9.15 \leq BC < 9.25$$
$$4.05 \leq \text{side of square} < 4.15$$

3 Find lower bound

Lower bound for X:

$$11.25(9.15) - 4.15^2 = 85.715$$

4 **Find upper bound**

Upper bound for X:

$11.75(9.25) - 4.05^2 = 92.285$

5 **Estimate the value**

So $85.715 \leq X < 92.285$. Both bounds round to 90 to the nearest 10.

✓ **FINAL ANSWER**

$X = 90\ \text{cm}^2$ to a suitable degree of accuracy.

4MA1/2H NOVEMBER 2025

Question 24

5 marks | Rounding, Estimation & Bounds

24 $D = \dfrac{n}{p - q}$

$n = 10.3$ correct to 1 decimal place
$p = 7.24$ correct to 2 decimal places
$q = 4.39$ correct to 2 decimal places

By considering bounds, work out the value of D to a suitable degree of accuracy.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2025

1 Estimate the value

$$D = \frac{n}{p - q}$$

2 Bounds

Bounds:

$10.25 \leq n < 10.35$

$7.235 \leq p < 7.245$

$4.385 \leq q < 4.395$

3 Lower bound

Lower bound:

$$\frac{10.25}{7.245 - 4.385} = 3.5839...$$

4 Upper bound

Upper bound:

$$\frac{10.35}{7.235 - 4.395} = 3.6443...$$

5 Estimate the value

So $3.5839... \leq D < 3.6443...$. Both bounds round to 3.6 to 1 decimal place.

FINAL ANSWER

$D = 3.6$ to a suitable degree of accuracy.

TOPIC

Surds

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2020

Question 20

3 marks | Surds

20 The area of a rectangle is $18\,\text{cm}^2$

The length of the rectangle is $\left(\sqrt{7}+1\right)$ cm.

Without using a calculator and showing each stage of your working,

find the width of the rectangle.
Give your answer in the form $a\sqrt{b}+c$ where a, b and c are integers.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2020

1 **Area of rectangle**

Area of rectangle:

$$18\ \text{cm}^2$$

2 **Length**

Length:

$$\sqrt{7}+1$$

3 **Width**

Width:

$$\frac{18}{\sqrt{7}+1}$$

4 Rationalise the denominator

Rationalise the denominator:

$$\frac{18}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1}$$

$$= \frac{18(\sqrt{7}-1)}{7-1}$$

$$= \frac{18(\sqrt{7}-1)}{6}$$

$$= 3(\sqrt{7}-1)$$

$$= 3\sqrt{7}-3$$

FINAL ANSWER

$$\boxed{3\sqrt{7}-3 \text{ cm}}$$

4MA1/1HR JUNE 2022

Question 21

4 marks | Surds

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2022

1 Simplify surd

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)^2}$$

2 Simplify surd

First simplify:

$$\sqrt{8}=2\sqrt{2}$$

3 Simplify surd

and

$$(\sqrt{2}-1)^2=2-2\sqrt{2}+1=3-2\sqrt{2}$$

4 Simplify surd

So

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

5 Rationalise the denominator

Rationalise the denominator:

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{(3+2\sqrt{2})^2}{9-8}$$

$$(3+2\sqrt{2})^2=9+12\sqrt{2}+8=17+12\sqrt{2}$$

FINAL ANSWER

$17 + 12\sqrt{2}$

4MA1/2H JUNE 2023

Question 23

4 marks | Surds

23 The diagram shows a cuboid with a square cross section.

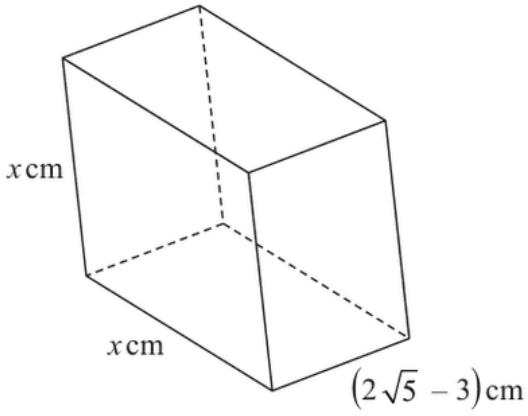


Diagram **NOT** accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of x
Give your answer in the form $a + \sqrt{b}$ where a and b are integers.
Show your working clearly.

$x =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2023

1 **The cuboid has square cross**

The cuboid has square cross section, so its volume is

$$x^2(2\sqrt{5} - 3)$$

2 **Given**

Given

$$x^2(2\sqrt{5} - 3) = 13 + 6\sqrt{5}$$

3 **Use trigonometry**

So

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3}$$

4 **Rationalise the denominator**

Rationalise the denominator:

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3} \cdot \frac{2\sqrt{5} + 3}{2\sqrt{5} + 3}$$

$$x^2 = \frac{(13 + 6\sqrt{5})(2\sqrt{5} + 3)}{20 - 9}$$

$$x^2 = \frac{99 + 44\sqrt{5}}{11}$$

$$x^2 = 9 + 4\sqrt{5}$$

5 **Use trigonometry**

Now

$$(2 + \sqrt{5})^2 = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

6 **Use trigonometry**

Therefore

$$x = 2 + \sqrt{5}$$

✓ **FINAL ANSWER**

$$\boxed{2 + \sqrt{5}}$$

TOPIC

Standard & Compound Units

Every past-paper question on this topic, 2020 to 2026.

Question 21

6 marks | Standard & Compound Units

21 The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

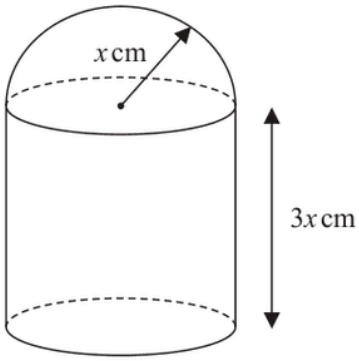


Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is 81π cm^2
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm^3)
Aluminium	2.7
Nickel	8.9
Gold	19.3
Silver	10.5

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

(Total for Question 21 is 6 marks)

1 **The outside surface area is**

The outside surface area is:

2 **curved surface area of cylinder**

curved surface area of cylinder:

$$2\pi x(3x) = 6\pi x^2$$

3 **bottom of cylinder**

bottom of cylinder:

$$\pi x^2$$

4 **curved surface area of hemisphere**

curved surface area of hemisphere:

$$2\pi x^2$$

5 **Calculate value**

So

$$6\pi x^2 + \pi x^2 + 2\pi x^2 = 81\pi$$

$$9\pi x^2 = 81\pi$$

$$x^2 = 9$$

$$x = 3$$

6 **Volume of the cylinder**

Volume of the cylinder:

$$\pi x^2(3x) = 3\pi x^3$$

7 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

8 **Total volume**

Total volume:

$$3\pi(3)^3 + \frac{2}{3}\pi(3)^3$$

$$= 81\pi + 18\pi = 99\pi$$

---

9 **Density**

Density:

$$\frac{840}{99\pi} = 2.70\ldots$$

---

10 **Compare density**

This matches aluminium, whose density is $2.7\ \text{g/cm}^3$.

✓ **FINAL ANSWER**

Aluminium

4MA1/2H NOVEMBER 2025

Question 26

6 marks | Standard & Compound Units

26 A solid cone is joined to a solid hemisphere to make the solid shown below.

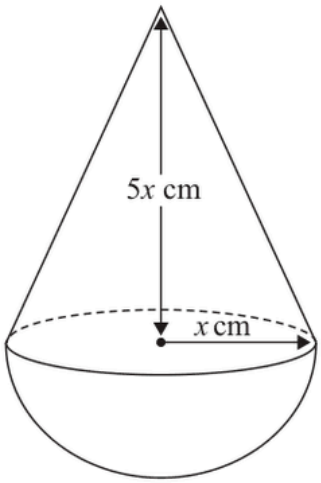


Diagram **NOT** accurately drawn

The cone is made from copper.
The density of copper is $9\,\text{g/cm}^3$

The hemisphere is made from a different metal.

The total mass of the solid is 4752π grams
The total volume of the solid is $504\pi\,\text{cm}^3$

Work out the density of the hemisphere.
Show your working clearly.

..... g/cm^3

(Total for Question 26 is 6 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H November 2025

1 **Volume of cone**

Volume of cone:

$$\frac{1}{3}\pi x^2(5x) = \frac{5}{3}\pi x^3$$

2 **Volume of hemisphere**

Volume of hemisphere:

$$\frac{2}{3}\pi x^3$$

3 **Total volume**

Total volume:

$$\frac{5}{3}\pi x^3 + \frac{2}{3}\pi x^3 = 504\pi$$

$$\frac{7}{3}\pi x^3 = 504\pi$$

$$x^3 = 216$$

$$x = 6$$

4 **Cone volume**

Cone volume:

$$\frac{5}{3}\pi (6)^3 = 360\pi$$

5 **Mass of copper cone**

Mass of copper cone:

$$360\pi \times 9 = 3240\pi$$

6 **Mass of hemisphere**

Mass of hemisphere:

$$4752\pi - 3240\pi = 1512\pi$$

7 **Hemisphere volume**

Hemisphere volume:

$$\frac{2}{3}\pi (6)^3 = 144\pi$$

8 **Density of the hemisphere**

Density of the hemisphere:

$$\frac{1512\pi}{144\pi} = 10.5$$

FINAL ANSWER

$10.5\ \text{g/cm}^3$

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TOPIC

Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 21

3 marks | Algebraic Fractions

21 Express $\left(\frac{20}{x^2 - 36} - \frac{2}{x - 6}\right) \times \frac{1}{4 - x}$ as a single fraction in its simplest form.

Show clear algebraic working.

1 Factorise the denominator

$$x^2 - 36 = (x - 6)(x + 6)$$

2 Simplify the bracket

$$\frac{20}{(x-6)(x+6)} - \frac{2}{x-6} = \frac{20 - 2(x+6)}{(x-6)(x+6)}$$

$$= \frac{8 - 2x}{(x-6)(x+6)} = \frac{-2(x-4)}{(x-6)(x+6)}$$

3 Cancel the common factor

Since $4 - x = -(x - 4)$,

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \frac{1}{4-x} = \frac{2}{(x-6)(x+6)}$$

FINAL ANSWER

$\frac{2}{x^2-36}$.

4MA1/2H JANUARY 2020

Question 24

4 marks | Algebraic Fractions

24 Express

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2020

1 Use trigonometry

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

2 Use trigonometry

First simplify the bracket:

$$\frac{4}{2x-5}-\frac{3}{2x-3}=\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$

$$=\frac{8x-12-6x+15}{(2x-5)(2x-3)}$$

$$=\frac{2x+3}{(2x-5)(2x-3)}$$

3 Use trigonometry

Now factor the second fraction:

$$9x-4x^3=-x(2x-3)(2x+3)$$

$$6x^2-17x+5=(2x-5)(3x-1)$$

4 **Dividing by a fraction means**

Dividing by a fraction means multiplying by its reciprocal:

$$\frac{2x+3}{(2x-5)(2x-3)} \times \frac{(2x-5)(3x-1)}{-x(2x-3)(2x+3)}$$

5 **Cancel common factors**

Cancel common factors:

$$= -\frac{3x-1}{x(2x-3)^2}$$

$$= \frac{1-3x}{x(2x-3)^2}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{1-3x}{x(2x-3)^2}}$$

4MA1/1H MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Fractions

21 Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May/November 2020

1 Use trigonometry

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

2 Factor

Factor:

$$9x^2-4 = (3x-2)(3x+2)$$

$$3x^2-13x-10 = (3x+2)(x-5)$$

3 Use trigonometry

So the first product becomes

$$\frac{1}{3x-2} \times \frac{(3x-2)(3x+2)}{(3x+2)(x-5)}$$

4 Cancel common factors

Cancel common factors:

$$\frac{1}{x-5}$$

5 Use trigonometry

Now subtract:

$$\frac{1}{x-5} - \frac{7}{x-1}$$

6 Use the common denominator

Use the common denominator $(x-5)(x-1)$:

$$\frac{x-1-7(x-5)}{(x-5)(x-1)}$$

$$= \frac{x-1-7x+35}{(x-5)(x-1)}$$

$$= \frac{34-6x}{(x-5)(x-1)}$$

FINAL ANSWER

$$\boxed{\frac{34-6x}{(x-5)(x-1)}}$$

4MA1/2H JANUARY 2021

Question 21

4 marks | Algebraic Fractions

21 Write $\frac{25x^2 - 64}{5x^2 - 13x - 6} \times \frac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

as a single fraction in its simplest form.
Show clear algebraic working.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2021

1 Use trigonometry

$$\frac{25x^2 - 64}{5x^2 - 13x - 6} \times \frac{x^2 - 8x + 15}{5x + 8} - (x - 7)$$

2 Factor

Factor:

$$25x^2 - 64 = (5x - 8)(5x + 8)$$

$$5x^2 - 13x - 6 = (5x + 2)(x - 3)$$

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

3 Use trigonometry

So

$$\frac{(5x - 8)(5x + 8)}{(5x + 2)(x - 3)} \times \frac{(x - 3)(x - 5)}{5x + 8} - (x - 7)$$

4 Cancel common factors

Cancel common factors:

$$\frac{(5x - 8)(x - 5)}{5x + 2} - (x - 7)$$

5 **Use the common denominator**

Use the common denominator $5x + 2$:

$$\frac{(5x - 8)(x - 5) - (x - 7)(5x + 2)}{5x + 2}$$

6 **Expand**

Expand:

$$(5x - 8)(x - 5) = 5x^2 - 33x + 40$$

$$(x - 7)(5x + 2) = 5x^2 - 33x - 14$$

7 **Use trigonometry**

So the numerator is

$$5x^2 - 33x + 40 - (5x^2 - 33x - 14) = 54$$

✓ **FINAL ANSWER**

$$\boxed{\frac{54}{5x + 2}}$$

4MA1/1H MAY 2021

Question 21

4 marks | Algebraic Fractions

21 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2021

1 Given

Given

$$x = \frac{5}{9y + 5}$$

2 Use trigonometry

and

$$y = \frac{5}{5a - 2}$$

3 Use cosine rule

Substitute $y = \frac{5}{5a-2}$ into the expression for x:

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5}$$

$$x = \frac{5}{\frac{45}{5a-2} + 5}$$

4 **Write the denominator as a**

Write the denominator as a single fraction:

$$\frac{45}{5a-2}+5=\frac{45+5(5a-2)}{5a-2}$$

$$=\frac{45+25a-10}{5a-2}$$

$$=\frac{25a+35}{5a-2}$$

---

5 **Use trigonometry**

So

$$x=5\div\frac{25a+35}{5a-2}$$

$$x=5\times\frac{5a-2}{25a+35}$$

$$x=\frac{5(5a-2)}{5(5a+7)}$$

$$x=\frac{5a-2}{5a+7}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5a-2}{5a+7}}$$

4MA1/2HR JUNE 2022

Question 23

3 marks | Algebraic Fractions

23 Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2022

1 Use trigonometry

$$\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$$

2 Factor

Factor:

$$x^2-36=(x-6)(x+6)$$

3 Use trigonometry

So

$$\frac{20}{(x-6)(x+6)}-\frac{2}{x-6}$$

4 Use the common denominator

Use the common denominator $(x-6)(x+6)$:

$$\frac{20-2(x+6)}{(x-6)(x+6)}$$

$$=\frac{20-2x-12}{(x-6)(x+6)}$$

$$=\frac{8-2x}{(x-6)(x+6)}$$

$$=\frac{-2(x-4)}{(x-6)(x+6)}$$

5 **Use trigonometry**

Now multiply by

$$\frac{1}{4-x} = -\frac{1}{x-4}$$

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \left(-\frac{1}{x-4}\right)$$

---

6 **Cancel**

Cancel $x-4$:

$$\frac{2}{(x-6)(x+6)}$$

$$= \frac{2}{x^2-36}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{2}{x^2-36}}$$

4MA1/1HR MAY 2023

Question 23

4 marks | Algebraic Fractions

23 Simplify $(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$

Give your answer in the form $\frac{ax^2}{bx + c}$ where a, b and c are integers.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2023

1 Simplify surd

$$(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$$

2 Change division into multiplication by

Change division into multiplication by the reciprocal:

$$(x^2 - 4) \times \frac{x}{4x^2 - 7x - 2} - 2x$$

3 Factorise

Factorise:

$$x^2 - 4 = (x - 2)(x + 2)$$

$$4x^2 - 7x - 2 = (4x + 1)(x - 2)$$

4 Simplify surd

So

$$\frac{x(x - 2)(x + 2)}{(4x + 1)(x - 2)} - 2x = \frac{x(x + 2)}{4x + 1} - 2x$$

$$\frac{x(x + 2) - 2x(4x + 1)}{4x + 1} = \frac{x^2 + 2x - 8x^2 - 2x}{4x + 1} = \frac{-7x^2}{4x + 1}$$

✓ **FINAL ANSWER**

$$\frac{-7x^2}{4x + 1}$$

4MA1/2H NOVEMBER 2023

Question 24

5 marks | Algebraic Fractions

24 Solve $\dfrac{45x^3-80x}{3x^2+x-4}\times\left(\dfrac{1}{3x-4}+\dfrac{1}{x}\right)=\dfrac{4(x+2)}{5x-8}$

Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2023

1 Solve quadratic equation

$$\frac{45x^3-80x}{3x^2+x-4}\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$$

2 Factorise

Factorise:

$$45x^3-80x=5x(3x-4)(3x+4)$$

$$3x^2+x-4=(3x+4)(x-1)$$

3 Solve quadratic equation

Also,

$$\frac{1}{3x-4}+\frac{1}{x}=\frac{x+3x-4}{x(3x-4)}=\frac{4(x-1)}{x(3x-4)}$$

4 Solve quadratic equation

So the left side becomes

$$\frac{5x(3x-4)(3x+4)}{(3x+4)(x-1)}\times\frac{4(x-1)}{x(3x-4)}=20$$

5 **Solve quadratic equation**

Therefore,

$$20 = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4(x+2)$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{7}{4}$$

✓ **FINAL ANSWER**

$x = \frac{7}{4}$

4MA1/2H JUNE 2024

Question 20

3 marks | Algebraic Fractions

20 Given that $k = x - y$ and $x = \dfrac{1}{4y}$

express $\dfrac{5k}{x+2}$ in the form $\dfrac{a - by^2}{c + dy}$ where a, b, c and d are integers.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2024

1 **Simplify fraction**

$$k = x - y, \quad x = \frac{1}{4y}$$

2 **Simplify fraction**

So

$$k = \frac{1}{4y} - y = \frac{1 - 4y^2}{4y}$$

3 **Simplify fraction**

Also

$$x + 2 = \frac{1}{4y} + 2 = \frac{1 + 8y}{4y}$$

4 **Simplify fraction**

Therefore

$$\frac{5k}{x+2} = \frac{5\left(\frac{1-4y^2}{4y}\right)}{\frac{1+8y}{4y}}$$

5 **Cancel**

Cancel $4y$:

$$\frac{5(1-4y^2)}{1+8y}$$

$$= \frac{5-20y^2}{1+8y}$$

✓ **FINAL ANSWER**

$$\frac{5-20y^2}{1+8y}$$

4MA1/2HR JUNE 2024

Question 26

4 marks | Algebraic Fractions

26 Write $4 - \left[(3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}\right]$ as a single fraction in its simplest form.

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2024

1 Use trigonometry

$$4 - \left[(3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}\right]$$

2 Factor the quadratic

Factor the quadratic:

$$3x^2 + x - 10 = (3x - 5)(x + 2)$$

3 Use trigonometry

So the expression inside the square brackets is

$$(3x - 5) \div \frac{(3x - 5)(x + 2)}{4x - 1}$$

4 Dividing by a fraction means

Dividing by a fraction means multiplying by its reciprocal:

$$(3x - 5) \times \frac{4x - 1}{(3x - 5)(x + 2)}$$

5 Cancel

Cancel $3x - 5$:

$$\frac{4x - 1}{x + 2}$$

6 Use trigonometry

Therefore the full expression is

$$4 - \frac{4x - 1}{x + 2}$$

7 Use the common denominator

Use the common denominator $x + 2$:

$$\frac{4(x+2) - (4x-1)}{x+2}$$

$$= \frac{4x + 8 - 4x + 1}{x+2}$$

$$= \frac{9}{x+2}$$

FINAL ANSWER

$$\boxed{\frac{9}{x+2}}$$

4WM1/01 MAY 2025

Question 20

3 marks | Algebraic Fractions

20 Given that $k = m + n$ and $m = \dfrac{3}{4n}$

express $\dfrac{7k}{5-m}$ in the form $\dfrac{a+bn^2}{cn-d}$ where a, b and c are integers and d is prime.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01 May 2025

1 Simplify fraction

$$k = m + n, \qquad m = \frac{3}{4n}$$

2 Simplify fraction

So

$$k = \frac{3}{4n} + n = \frac{3 + 4n^2}{4n}$$

3 Simplify fraction

Then

$$7k = \frac{21 + 28n^2}{4n}$$

4 Simplify fraction

and

$$5 - m = 5 - \frac{3}{4n} = \frac{20n - 3}{4n}$$

5 Simplify fraction

Therefore

$$\frac{7k}{5-m} = \frac{\frac{21+28n^2}{4n}}{\frac{20n-3}{4n}}$$

$$= \frac{21 + 28n^2}{20n - 3}$$

FINAL ANSWER

$$\frac{21+28n^2}{20n-3}$$

4MA1/2H JUNE 2025

Question 23

4 marks | Algebraic Fractions

23 $\dfrac{4x^2-4x-120}{5x^2-180} \div \dfrac{x^2+5x}{10x^2+60x} = p$ where p is an integer.

Find the value of p

Show clear algebraic working.

$p =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2025

1 **Split the ratio**

$$\frac{4x^2-4x-120}{5x^2-180} \div \frac{x^2+5x}{10x^2+60x}$$

2 **Factorise**

Factorise:

$$4x^2-4x-120 = 4(x-6)(x+5)$$

$$5x^2-180 = 5(x-6)(x+6)$$

$$x^2+5x = x(x+5)$$

$$10x^2+60x = 10x(x+6)$$

3 **Split the ratio**

So

$$\frac{4(x-6)(x+5)}{5(x-6)(x+6)} \div \frac{x(x+5)}{10x(x+6)}$$

$$= \frac{4(x+5)}{5(x+6)} \times \frac{10(x+6)}{x+5} = 8$$

FINAL ANSWER

$p = 8$

4WM1/01R MAY 2025

Question 24

4 marks | Algebraic Fractions

24 Write $(2x-1)+\dfrac{2x^2-5x-12}{x^2-16}\times\dfrac{x^2+4x}{6x^2+11x+3}$ as a single fraction in its simplest form.

Show clear algebraic working.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM1/01R May 2025

1 Factor the fractional part first

Factor the fractional part first:

$$\frac{2x^2-5x-12}{x^2-16}\times\frac{x^2+4x}{6x^2+11x+3}$$

$$=\frac{(2x+3)(x-4)}{(x-4)(x+4)}\times\frac{x(x+4)}{(3x+1)(2x+3)}$$

$$=\frac{x}{3x+1}$$

2 Evaluate fraction

So

$$(2x-1)+\frac{x}{3x+1}=\frac{(2x-1)(3x+1)+x}{3x+1}$$

$$=\frac{6x^2-1}{3x+1}$$

FINAL ANSWER

$$\frac{6x^2-1}{3x+1}.$$

4MA1/1H NOVEMBER 2025

Question 23

3 marks | Algebraic Fractions

23 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 25

3 marks | Algebraic Fractions

25 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 28

4 marks | Algebraic Fractions

28 Solve $\dfrac{6x^3+5x^2+x}{4x^2-1} \div \dfrac{15x^2-x-2}{8x-4} = 2x$

Show clear algebraic working.

$x =$ .....

(Total for Question 28 is 4 marks)

WORKED SOLUTION

Question 28 · 4WM1/01 November 2025

1 Factor each expression

Factor each expression:

$$\frac{x(3x+1)(2x+1)}{(2x-1)(2x+1)} \div \frac{(5x-2)(3x+1)}{4(2x-1)} = 2x$$

$$\frac{x(3x+1)}{2x-1} \times \frac{4(2x-1)}{(5x-2)(3x+1)} = 2x$$

$$\frac{4x}{5x-2} = 2x$$

$$4x = 2x(5x-2)$$

$$10x^2 - 8x = 0$$

$$2x(5x-4) = 0$$

2 The original fractions do not

The original fractions do not allow $x = 0$, so

$$x = \frac{4}{5}$$

FINAL ANSWER

$x = \frac{4}{5}$.

Question 20

4 marks | Algebraic Fractions

20 Express $\left(\frac{5}{4x-7}-\frac{3}{3x-5}\right)\div\frac{9x^2-16}{6x^2-7x-5}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question 20 is 4 marks)

1 **Subtract the fractions**

$$\frac{5(3x-5)-3(4x-7)}{(4x-7)(3x-5)} = \frac{3x-4}{(4x-7)(3x-5)}$$

2 **Factorise the second fraction**

$$9x^2-16=(3x-4)(3x+4), \qquad 6x^2-7x-5=(3x-5)(2x+1)$$

3 **Divide = multiply by the reciprocal**

$$\frac{3x-4}{(4x-7)(3x-5)} \times \frac{(3x-5)(2x+1)}{(3x-4)(3x+4)}$$

4 **Cancel**

$$= \frac{2x+1}{(4x-7)(3x+4)}$$

✓ **FINAL ANSWER**

$$\frac{2x+1}{(4x-7)(3x+4)}$$

TOPIC

Algebraic Roots & Indices

Every past-paper question on this topic, 2020 to 2026.

Question 22

3 marks | Algebraic Roots & Indices

22 $\dfrac{18 \times (\sqrt{27})^{}}{6 \times 9^{2n+8}} = 3^x$

Express x in terms of n

Show your working clearly and simplify your expression.

$x =$ .....

1 Write everything as powers of 3

$$\sqrt{27} = 3^{3/2}, \qquad 9 = 3^2, \qquad \frac{18}{6} = 3$$

2 Simplify the powers

$$\frac{18(\sqrt{27})^{4n+6}}{6(9)^{2n+8}} = 3 \times \frac{\left(3^{3/2}\right)^{4n+6}}{(3^2)^{2n+8}}$$

$$= 3^1 \times \frac{3^{6n+9}}{3^{4n+16}}$$

3 Subtract the indices

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

So

$$x = 2n - 6$$

FINAL ANSWER

$x = 2n - 6$.

4MA1/2HR JANUARY 2020

Question 21

7 marks | Algebraic Roots & Indices

21 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

.....

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

.....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2020

1 **(a)**

(a)

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} = \frac{(5x + 4)(2x + 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{5x + 4}{2x - 3}$$

2 **(b)**

(b)

$$2^{2y} \times 2^{3y+2} = 2^{5y+2}$$

3 **Evaluate fraction**

Also,

$$\frac{8^{5y}}{4^n} = \frac{(2^3)^{5y}}{(2^2)^n} = 2^{15y-2n}$$

4 **Equate powers**

Equate powers:

$$5y + 2 = 15y - 2n$$

$$2n = 10y - 2$$

$$n = 5y - 1$$

✓ **FINAL ANSWER**

(a) $\dfrac{5x + 4}{2x - 3}$, (b) $n = 5y - 1$

4MA1/1HR MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Roots & Indices

21 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of n for which $M = 2$

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May/November 2020

1 Evaluate fraction

$$M = \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$$

2 Use index laws

Write everything in powers of 2 and 3:

$$18^{4n} = (2 \cdot 3^2)^{4n} = 2^{4n}3^{8n}$$

$$2^{3(n^2-6n)} = 2^{3n^2-18n}$$

$$3^{2(1-4n)} = 3^{2-8n}$$

$$12^2 = (2^2 \cdot 3)^2 = 2^4 3^2$$

3 Calculate value

So

$$M = 2^{4n+3n^2-18n-4} \times 3^{8n+2-8n-2}$$

$$M = 2^{3n^2-14n-4}$$

4 **Calculate value**

Given $M = 2$,

$$2^{3n^2-14n-4} = 2^1$$

$$3n^2 - 14n - 4 = 1$$

$$3n^2 - 14n - 5 = 0$$

$$(3n + 1)(n - 5) = 0$$

✓ **FINAL ANSWER**

$n = 5$ or $n = -\frac{1}{3}$

4MA1/2H JUNE 2022

Question 24

3 marks | Algebraic Roots & Indices

24 $$\frac{18 \times (\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2022

1 Simplify surd

$$\frac{18(\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

2 Use powers of 3

Use powers of 3:

$$18 \div 6 = 3$$

$$\sqrt{27} = (3^3)^{1/2} = 3^{3/2}$$

$$(\sqrt{27})^{4n+6} = 3^{6n+9}$$

$$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$$

3 Evaluate fraction

So

$$\frac{3 \cdot 3^{6n+9}}{3^{4n+16}} = 3^{1+6n+9-4n-16} = 3^{2n-6}$$

4 Calculate value

Therefore,

$$x = 2n - 6$$

FINAL ANSWER

$x = 2n - 6$

4MA1/1HR MAY 2023

Question 24

5 marks | Algebraic Roots & Indices

24 Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for x in terms of n
State any restrictions on n

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR May 2023

1 **Calculate value**

$$2^n = 2^{x^2} \times 16^x \times 8$$

$$16^x = (2^4)^x = 2^{4x}$$

$$8 = 2^3$$

2 **Calculate value**

So

$$2^n = 2^{x^2+4x+3}$$

$$n = x^2 + 4x + 3$$

3 **Complete the square**

Complete the square:

$$n = (x+2)^2 - 1$$

$$(x+2)^2 = n + 1$$

4 **Simplify surd**

Since $x > 0$, take the positive root:

$$x + 2 = \sqrt{n+1}$$

$$x = \sqrt{n+1} - 2$$

---

5 **Simplify surd**

For $x > 0$,

$$\sqrt{n+1} > 2$$

$$n > 3$$

✓ **FINAL ANSWER**

$x = \sqrt{n+1} - 2$, with $n > 3$

4MA1/2H JUNE 2024

Question 23

3 marks | Algebraic Roots & Indices

23 Simplify $\dfrac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$

Give your answer in the form 5^w where w is an expression in terms of x
Show each stage of your working clearly.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2024

1 Simplify surd

$$\frac{30 \times 25^{2x+7}}{\sqrt{180} \times (\sqrt{5})^{4x+9}}$$

$$\frac{30}{\sqrt{180}} = \frac{30}{6\sqrt{5}} = \sqrt{5} = 5^{1/2}$$

$$25^{2x+7} = (5^2)^{2x+7} = 5^{4x+14}$$

$$(\sqrt{5})^{4x+9} = 5^{2x+\frac{9}{2}}$$

2 Evaluate fraction

So

$$5^{1/2} \times 5^{4x+14} \div 5^{2x+\frac{9}{2}} = 5^{\frac{1}{2}+4x+14-2x-\frac{9}{2}}$$

$$= 5^{2x+10}$$

FINAL ANSWER

5^{2x+10}, so $w = 2x + 10$

TOPIC

Completing the Square

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 20

4 marks | Completing the Square

20 Find the values of a, b and c so that

$$7 + 12x - 2x^2$$

is written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 20 is 4 marks)

1 **Factor out -2 from the quadratic terms**

$$7 + 12x - 2x^2 = -2(x^2 - 6x) + 7$$

2 **Complete the square**

$$x^2 - 6x = (x - 3)^2 - 9$$

3 **Simplify the expression**

$$-2((x - 3)^2 - 9) + 7 = -2(x - 3)^2 + 18 + 7$$

$$= 25 - 2(x - 3)^2$$

4 **Read the values**

$$a = 25, \qquad b = 2, \qquad c = 3$$

✓ **FINAL ANSWER**

$a = 25,\ b = 2,\ c = 3.$

4MA1/2HR JANUARY 2020

Question 23

3 marks | Completing the Square

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2020

1 Solve inequality

$$7 - 12x - 2x^2 = -2x^2 - 12x + 7$$

$$= -2(x^2 + 6x) + 7$$

$$= -2\left((x + 3)^2 - 9\right) + 7$$

$$= -2(x + 3)^2 + 18 + 7$$

$$= 25 - 2(x + 3)^2$$

FINAL ANSWER

$25 - 2(x + 3)^2$

4MA1/2H MAY 2021

Question 22

4 marks | Completing the Square

22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a, b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H May 2021

1 Solve inequality

$$g(x) = 2x^2 - 12x + 13$$

$$= 2(x^2 - 6x) + 13$$

$$= 2\left((x - 3)^2 - 9\right) + 13$$

$$= 2(x - 3)^2 - 18 + 13$$

$$= 2(x - 3)^2 - 5$$

2 Calculate value

Starting with $S : y = x^2$:

3 Translate 3 units to the

- Translate 3 units to the right to get $y = (x - 3)^2$.
- Stretch parallel to the y-axis by scale factor 2 to get $y = 2(x - 3)^2$.
- Translate 5 units down to get $y = 2(x - 3)^2 - 5$.

✓ FINAL ANSWER

$g(x) = 2(x - 3)^2 - 5$. Transformations: right 3, stretch parallel to the y-axis by scale factor 2, then down 5.

4MA1/1H JANUARY 2022

Question 20

4 marks | Completing the Square

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

.....

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$
The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(..... , ...)

(1)

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2022

1 **(a)**

(a)

$$7 + 12x - 3x^2 = -3x^2 + 12x + 7$$

$$= -3(x^2 - 4x) + 7$$

$$= -3\left((x - 2)^2 - 4\right) + 7$$

$$= 19 - 3(x - 2)^2$$

2 **This is the same as**

This is the same as

$$19 - 3(x + (-2))^2$$

3 **(b) The maximum point is**

(b) The maximum point is read from the completed square form.

$$y = 19 - 3(x - 2)^2$$

4 **Calculate value**

The maximum occurs when $x = 2$, giving $y = 19$.

✓ **FINAL ANSWER**

(a) $19 - 3(x - 2)^2$, (b) $A = (2, 19)$

4MA1/1HR JUNE 2022

Question 23

5 marks | Completing the Square

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

(..... , ...)

(2)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$2x^2 - 12x + 3 = 2(x^2 - 6x) + 3$$

$$= 2\left((x - 3)^2 - 9\right) + 3$$

$$= 2(x - 3)^2 - 18 + 3$$

$$= 2(x - 3)^2 - 15$$

2 **(b)**

(b)

$$y = 2(x + 4)^2 - 12(x + 4) + 3$$

3 **Let**

Let $u = x + 4$. From part (a),

$$2u^2 - 12u + 3 = 2(u - 3)^2 - 15$$

4 **Calculate value**

So

$$y = 2((x + 4) - 3)^2 - 15$$

$$y = 2(x + 1)^2 - 15$$

---

5 **The minimum point is when**

The minimum point is when $x = -1$, giving $y = -15$.

✓ **FINAL ANSWER**

(a) $2(x - 3)^2 - 15$, (b) $M = (-1, -15)$

4MA1/1H JUNE 2022

Question 24

4 marks | Completing the Square

24 Express each of a, b and c in terms of q so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2022

1 Evaluate fraction

$$q + 12x - qx^2 = -q\left(x^2 - \frac{12}{q}x\right) + q$$

2 Complete the square

Complete the square:

$$x^2 - \frac{12}{q}x = \left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}$$

3 Evaluate fraction

So

$$q + 12x - qx^2 = -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}\right] + q$$

$$= q + \frac{36}{q} - q\left(x - \frac{6}{q}\right)^2$$

4 Compare coefficients

Compare with

$$a - b(x - c)^2$$

FINAL ANSWER

$$a = q + \frac{36}{q},\ b = q,\ c = \frac{6}{q}$$

Question 20

5 marks | Completing the Square

20 (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where a, b and c are numbers to be found.

.....

(3)

The curve **C** has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point P is the minimum point on **C**

(b) Find the coordinates of P

(..... , ...)

(2)

(Total for Question 20 is 5 marks)

1 **Use Moved Coordinate Geometry Completing**

For part (a),

$$2x^2 - 11x + 9$$

2 **Factor out 2 from the**

Factor out 2 from the quadratic terms:

$$2\left(x^2 - \frac{11}{2}x\right) + 9$$

3 **Complete the square**

Complete the square:

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{72}{8}$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

4 **Evaluate fraction**

So

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

5 **Calculate value**

For part (b),

$$y = 2(x-3)^2 - 11(x-3) + 9$$

6 **Let**

Let

$$u = x - 3$$

7 **Evaluate fraction**

From part (a), the minimum occurs when

$$u = \frac{11}{4}$$

8 **Evaluate fraction**

So

$$x - 3 = \frac{11}{4}$$

$$x = \frac{23}{4}$$

9 **The minimum value of is**

The minimum value of y is

$$-\frac{49}{8}$$

✓ **FINAL ANSWER**

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8},\ P = \left(\frac{23}{4}, -\frac{49}{8}\right)$$

4MA1/1H MAY 2024

Question 25

4 marks | Completing the Square

25 $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where a, b and c are constants.

$f(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2024

1 Solve inequality

$$f(x) = 17 - 3x^2 + 12x$$

$$= -3x^2 + 12x + 17$$

$$= -3(x^2 - 4x) + 17$$

$$= -3\left((x-2)^2 - 4\right) + 17$$

$$= -3(x-2)^2 + 12 + 17$$

$$= 29 - 3(x-2)^2$$

FINAL ANSWER

$f(x) = 29 - 3(x-2)^2$

Question 25

6 marks | Completing the Square

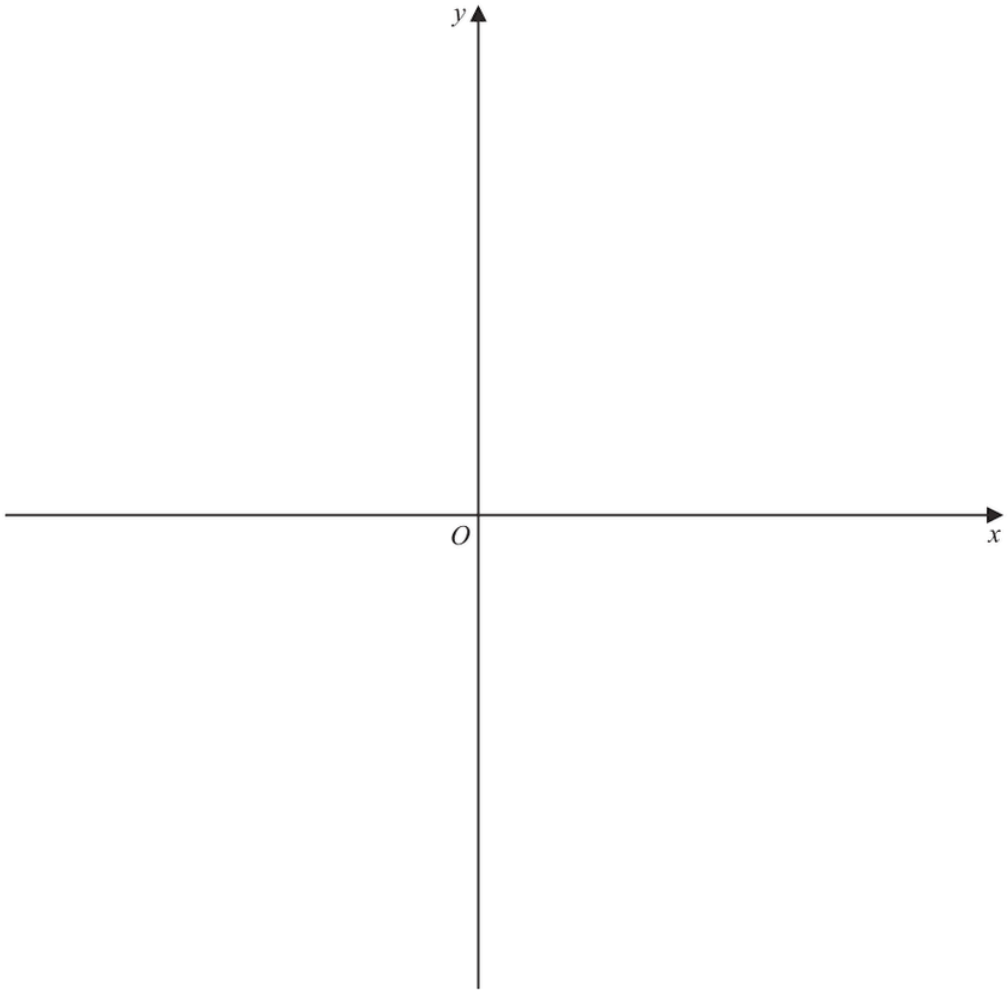
25 (a) Write $28+24x-6x^2$ in the form $a-b(x-c)^2$ where a, b and c are integers.

.....

(3)

(b) On the axes below, sketch the graph of $y=28+24x-6x^2$

Show clearly the coordinates of the turning point and the coordinates of the point of intersection of the graph with the y-axis.



(3)

(Total for Question 25 is 6 marks)

1 **(a)**

(a)

$$28 + 24x - 6x^2 = -6x^2 + 24x + 28$$

$$= -6(x^2 - 4x) + 28$$

$$= -6\left((x-2)^2 - 4\right) + 28$$

$$= 52 - 6(x-2)^2$$

2 **(b)**

(b)

$$y = 52 - 6(x-2)^2$$

3 **The graph is a downward**

The graph is a downward-opening parabola.

4 **The turning point is**

The turning point is

$$(2, 52)$$

5 **Calculate value**

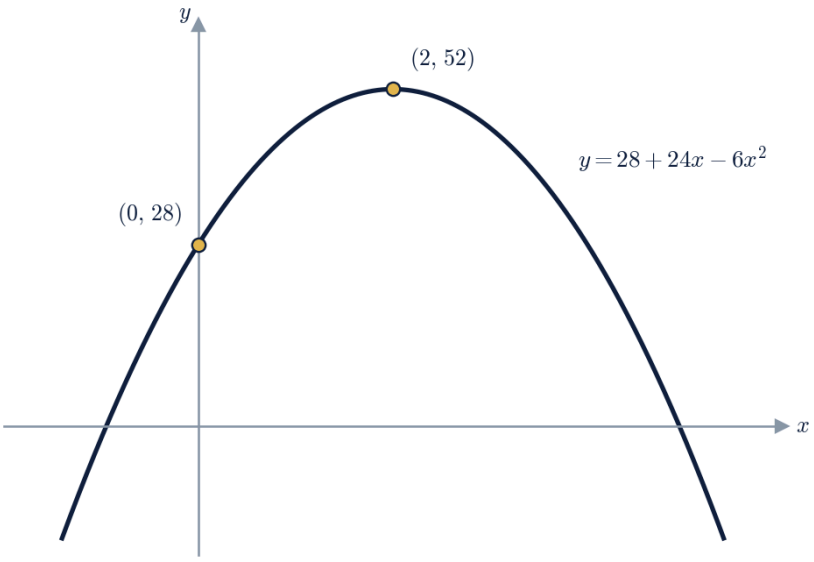
The point of intersection with the y-axis is found by putting $x = 0$:

$$y = 28$$

6 **Find intercepts**

So the y-intercept is

$$(0, 28)$$



FINAL ANSWER

(a) $52 - 6(x - 2)^2$. For the sketch, show a downward parabola with turning point $(2, 52)$ and y-intercept $(0, 28)$.

4WM1/01 MAY 2025

Question 25

5 marks | Completing the Square

25 (a) Express $8 - 12x - 4x^2$ in the form $a - b(x + c)^2$ where a, b and c are constants.

.....

(3)

(b) Hence solve $8 - 12x - 4x^2 = 0$
Show your working clearly.
Give your solutions in surd form.

.....

(2)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 May 2025

1 **Simplify surd**

$$8x^2 - 12x - 4 = 8\left(x^2 - \frac{3}{2}x\right) - 4$$

2 **Complete the square**

Complete the square.

$$x^2 - \frac{3}{2}x = \left(x - \frac{3}{4}\right)^2 - \frac{9}{16}$$

3 **Simplify surd**

So

$$8x^2 - 12x - 4 = 8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$$

4 **Simplify surd**

Now solve

$$8x^2 - 12x - 4 = 0$$

$$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2} = 0$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

✓ **FINAL ANSWER**

$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$, and $x = \frac{3 \pm \sqrt{17}}{4}$.

4MA1/1H NOVEMBER 2025

Question 21

3 marks | Completing the Square

21 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2025

1 Solve inequality

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 - 20 + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$

4WM1/01 NOVEMBER 2025

Question 22

2 marks | Completing the Square

22 $9x^2 - 12x + q$ can be written in the form $(px - r)^2$ where p, q and r are integers.

Find the value of q

$q =$ .....

(Total for Question 22 is 2 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 November 2025

1 **Calculate value**

$$9x^2 - 12x + q = (3x - 2)^2$$

$$(3x - 2)^2 = 9x^2 - 12x + 4$$

FINAL ANSWER

$q = 4$.

4WM1/01 NOVEMBER 2025

Question 24

3 marks | Completing the Square

24 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 November 2025

1 **Calculate value**

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$.

4WM1/01 MAY 2026

Question 24

3 marks | Completing the Square

24 Express $22 - 3x^2 + 30x$ in the form $a - b(x - c)^2$ where a, b and c are integers.

.....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2026

1 Factor out −3 from the x terms

$$22 - 3x^2 + 30x = 22 - 3(x^2 - 10x)$$

2 Complete the square

$$x^2 - 10x = (x - 5)^2 - 25$$

3 Simplify

$$22 - 3\big((x - 5)^2 - 25\big) = 97 - 3(x - 5)^2$$

FINAL ANSWER

$97 - 3(x - 5)^2$

Dr Eslam Ahmed · eliteigcse.com

TOPIC

Linear Graphs

Every past-paper question on this topic, 2020 to 2026.

Question 23

6 marks | Linear Graphs

23 $ABCD$ is a kite.

$AB = AD$ and $CB = CD$

The point B has coordinates $(k, 1)$ where k is a negative constant.
The point D has coordinates $(8, 7)$

The straight line **L** passes through the points B and D and has a gradient of $\frac{3}{5}$

Find an equation of AC
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show your working clearly.

.....

(Total for Question 23 is 6 marks)

1 Find k from the gradient

The gradient of BD is

$$\frac{7-1}{8-k}=\frac{3}{5}$$

$$\frac{6}{8-k}=\frac{3}{5}$$

$$30=24-3k$$

$$k=-2$$

2 Find the midpoint of BD

So $B=(-2,1)$ and $D=(8,7)$.

$$\text{midpoint of } BD=\left(\frac{-2+8}{2},\frac{1+7}{2}\right)=(3,4)$$

3 Use the perpendicular gradient

In this kite, AC is the perpendicular bisector of BD.

The gradient of BD is $\frac{3}{5}$, so the gradient of AC is

$$-\frac{5}{3}$$

4 Form the equation

Using point $(3,4)$:

$$y-4=-\frac{5}{3}(x-3)$$

$$3y-12=-5x+15$$

$$5x+3y=27$$

FINAL ANSWER

$5x+3y=27$.

4MA1/1H JANUARY 2020

Question 22

6 marks | Linear Graphs

22 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.

$j =$ .....

$k =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2020

1 Find the midpoint

Since M is the midpoint of JK,

$$M = \left(\frac{j+6}{2}, \frac{15+k}{2}\right)$$

2 Find the gradient

The gradient of HM is 2, and $H = (-4, 1)$.

$$\frac{\frac{15+k}{2} - 1}{\frac{j+6}{2} + 4} = 2$$

$$\frac{k+13}{j+14} = 2$$

$$k = 2j + 15$$

3 **Simplify surd**

Also,

$$JK = \sqrt{80}$$

4 **Calculate value**

So

$$(6 - j)^2 + (k - 15)^2 = 80$$

5 **Substitute**

Substitute $k = 2j + 15$:

$$(6 - j)^2 + (2j)^2 = 80$$

$$j^2 - 12j + 36 + 4j^2 = 80$$

$$5j^2 - 12j - 44 = 0$$

$$(5j + 10)(j - 4.4) = 0$$

6 **Solve inequality**

Since $j < 0$,

$$j = -2$$

7 **Calculate value**

Then

$$k = 2(-2) + 15 = 11$$

✓ **FINAL ANSWER**

$j = -2,\ k = 11$

4MA1/2H JANUARY 2020

Question 22

5 marks | Linear Graphs

22 The line with equation $y = x + 2$ intersects the curve with equation $x^2 + y^2 - 2y = 24$ at the points A and B.

Find the coordinates of A and B.
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2020

1 **The line is**

The line is

$$y = x + 2$$

2 **Substitute this into the circle**

Substitute this into the circle equation:

$$x^2 + y^2 - 2y = 24$$

$$x^2 + (x+2)^2 - 2(x+2) = 24$$

$$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

3 **Calculate value**

So

$$x = -4 \quad \text{or} \quad x = 3$$

---

4 **Calculate value**

Using $y = x + 2$:

$$x = -4 \Rightarrow y = -2$$

$$x = 3 \Rightarrow y = 5$$

✓ **FINAL ANSWER**

$(-4, -2)$ and $(3, 5)$

4MA1/2HR JANUARY 2020

Question 24

5 marks | Linear Graphs

24 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the x-axis at the point A.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR January 2020

1 Evaluate fraction

For L_1,

$$5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

2 Find the gradient

So the gradient of L_1 is $-\frac{1}{2}$.

3 Find the gradient

Since L_2 is perpendicular to L_1, the gradient of L_2 is 2.

4 Calculate value

Using the point $(8, 6)$:

$$y - 6 = 2(x - 8)$$

$$y = 2x - 10$$

5 **Calculate value**

At A, the line crosses the x-axis, so $y = 0$:

$$0 = 2x - 10$$

$$x = 5$$

---

6 **Calculate value**

So $A = (5, 0)$.

---

7 **Calculate value**

At B, $x = -3$:

$$y = 2(-3) - 10 = -16$$

---

8 **Calculate value**

So $B = (-3, -16)$.

---

9 **Triangle has base and perpendicular**

Triangle AOB has base $OA = 5$ and perpendicular height 16.

$$\text{Area} = \frac{1}{2} \times 5 \times 16 = 40$$

✓ **FINAL ANSWER**

40

4MA1/1H MAY/NOVEMBER 2020

Question 22

4 marks | Linear Graphs

22 $ABCD$ is a rhombus.

The diagonals, AC and BD, intersect at the point M.
The coordinates of M are $(6, -11)$

The points A and C both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through B and D intersects the y-axis.

(..... , ...)

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May/November 2020

1 Find the gradient

In a rhombus, the diagonals are perpendicular and bisect each other.

2 The diagonal lies on

The diagonal AC lies on

$$2y + 7x = 20$$

$$2y = -7x + 20$$

$$y = -\frac{7}{2}x + 10$$

3 Find the gradient

So the gradient of AC is

$$-\frac{7}{2}$$

4 Find the gradient

Therefore the gradient of BD is

$$\frac{2}{7}$$

5 **Calculate value**

The line BD passes through

$$M = (6, -11)$$

6 **Evaluate fraction**

So

$$y + 11 = \frac{2}{7}(x - 6)$$

7 **To find where this line**

To find where this line intersects the y-axis, put $x = 0$:

$$y + 11 = \frac{2}{7}(0 - 6)$$

$$y + 11 = -\frac{12}{7}$$

$$y = -11 - \frac{12}{7}$$

$$y = -\frac{89}{7}$$

✓ **FINAL ANSWER**

$$\left(0, -\frac{89}{7}\right)$$

4MA1/1HR JANUARY 2021

Question 22

5 marks | Linear Graphs

22 ABC is an isosceles triangle with $AB = AC$.

B is the point with coordinates $(-1, 5)$
C is the point with coordinates $(2, 10)$
M is the midpoint of BC.

Find an equation of the line through the points A and M.
Give your answer in the form $py + qx = r$ where p, q and r are integers.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2021

1 Find the gradient

Since $AB = AC$, point A lies on the perpendicular bisector of BC.

2 The midpoint of is

The midpoint of BC is

$$M = \left(\frac{-1+2}{2}, \frac{5+10}{2}\right)$$

$$M = \left(\frac{1}{2}, \frac{15}{2}\right)$$

3 The gradient of is

The gradient of BC is

$$\frac{10-5}{2-(-1)} = \frac{5}{3}$$

4 Find the gradient

So the gradient of the perpendicular bisector is

$$-\frac{3}{5}$$

5 **Find line equation**

Use the line through M:

$$y - \frac{15}{2} = -\frac{3}{5}\left(x - \frac{1}{2}\right)$$

6 **Multiply by 10**

Multiply by 10:

$$10y - 75 = -6x + 3$$

$$10y + 6x = 78$$

7 **Divide by 2**

Divide by 2:

$$5y + 3x = 39$$

✓ **FINAL ANSWER**

$5y + 3x = 39$

4MA1/1H NOVEMBER 2021

Question 20

7 marks | Linear Graphs

20 The straight line **L** passes through point A (– 6, 2) and point B (5, 3)
The straight line **M** is perpendicular to **L** and passes through the midpoint of A and B.
The line **M** intersects the line $x = -1$ at point C.

Calculate the area of triangle ABC.

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2021

1 **Calculate value**

$$A = (-6, 2), \qquad B = (5, 3)$$

2 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

3 **The gradient of is**

The gradient of AB is

$$\frac{3-2}{5-(-6)} = \frac{1}{11}$$

4 **Find the gradient**

So the perpendicular gradient is

$$-11$$

5 **Evaluate fraction**

Line M passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$, so

$$y - \frac{5}{2} = -11\left(x + \frac{1}{2}\right)$$

6 **Evaluate fraction**

At point C, $x = -1$:

$$y - \frac{5}{2} = -11\left(-\frac{1}{2}\right)$$

$$y - \frac{5}{2} = \frac{11}{2}$$

$$y = 8$$

---

7 **Calculate value**

So

$$C = (-1, 8)$$

---

8 **Use vectors**

Now use vectors from A:

$$\vec{AB} = (11, 1), \qquad \vec{AC} = (5, 6)$$

---

9 **Area of triangle is**

Area of triangle ABC is

$$\frac{1}{2}|11(6) - 1(5)|$$

$$= \frac{1}{2}(61)$$

$$= 30.5$$

✓ **FINAL ANSWER**

30.5

4MA1/2HR JANUARY 2022

Question 22

5 marks | Linear Graphs

22 ABC is a triangle in which angle $ABC = 90°$

p and q are integers such that

the coordinates of A are $(p, 10)$
the coordinates of B are $(-1, -5)$
the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$p =$ .....

$q =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2022

1 The gradient of is

The gradient of AC is $-\frac{6}{7}$.

$$\frac{q-10}{8-p} = -\frac{6}{7}$$

$$7(q-10) = -6(8-p)$$

$$7q - 70 = -48 + 6p$$

$$6p - 7q = -22$$

2 **Find the gradient**

Since $\angle ABC = 90°$, lines AB and BC are perpendicular.

$$\text{gradient of } AB = \frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$$

$$\text{gradient of } BC = \frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$$

---

3 **The product of perpendicular gradients**

The product of perpendicular gradients is -1:

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$5(q+5) = -3(p+1)$$

$$3p + 5q = -28$$

---

4 **Solve**

Now solve

$$6p - 7q = -22$$

$$3p + 5q = -28$$

---

5 **Double the second equation**

Double the second equation:

$$6p + 10q = -56$$

---

6 **Subtract**

Subtract:

$$17q = -34$$

$$q = -2$$

---

7 **Calculate value**

Then

$$3p + 5(-2) = -28$$

$$3p = -18$$

$$p = -6$$

FINAL ANSWER

$p = -6,\ q = -2$

4MA1/2HR JUNE 2022

Question 20

4 marks | Linear Graphs

20 The centre O of a circle has coordinates $(4, 7)$

The point A, on the circle, has coordinates $(6, 11)$ and AOP is a diameter of the circle.

Find an equation of the tangent to the circle at the point P

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2022

1 The centre of the circle

The centre of the circle is

$$O = (4, 7)$$

2 The point is

The point A is

$$A = (6, 11)$$

3 Find the midpoint

Since AOP is a diameter, O is the midpoint of AP.

4 Let

Let $P = (x, y)$.

$$\left(\frac{6+x}{2}, \frac{11+y}{2}\right) = (4, 7)$$

$$6 + x = 8, \qquad 11 + y = 14$$

$$P = (2, 3)$$

5 The gradient of is

The gradient of OP is

$$\frac{3-7}{2-4} = 2$$

6 **Use circle theorem**

The tangent is perpendicular to the radius, so its gradient is

$$-\frac{1}{2}$$

---

7 **Evaluate fraction**

Using point $P = (2, 3)$,

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 4$$

✓ **FINAL ANSWER**

$$y = -\frac{1}{2}x + 4$$

4MA1/2H JUNE 2022

Question 22

7 marks | Linear Graphs

22 $ABCD$ is a kite, with diagonals AC and BD, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point M with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the x coordinate of B is positive,

find the coordinates of the points B and D.

(..... , ...)

(..... , ...)

(Total for Question 22 is 7 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2022

1 The midpoint of is

The midpoint of BD is $M = (0, 2)$.

2 Find the gradient

Since $A = (-3, 4)$, the gradient of AM is

$$\frac{2-4}{0-(-3)} = -\frac{2}{3}$$

3 Find the gradient

For the kite, diagonal AC is perpendicular to BD, so the gradient of BD is

$$\frac{3}{2}$$

4 Use direction vector along

Use direction vector $(2, 3)$ along BD.

5 **Let**

Let

$$B = (2t, 2 + 3t)$$

6 **Calculate value**

and

$$D = (-2t, 2 - 3t)$$

7 **Calculate value**

Since $AB = 6.5$,

$$(2t - (-3))^2 + (2 + 3t - 4)^2 = 6.5^2$$

$$(2t + 3)^2 + (3t - 2)^2 = 42.25$$

$$4t^2 + 12t + 9 + 9t^2 - 12t + 4 = 42.25$$

$$13t^2 + 13 = 42.25$$

$$13t^2 = 29.25$$

$$t^2 = 2.25$$

$$t = 1.5$$

8 **Use**

The x-coordinate of B is positive, so use $t = 1.5$.

$$B = (3, 6.5)$$

$$D = (-3, -2.5)$$

✓ **FINAL ANSWER**

$B = (3, 6.5),\ D = (-3, -2.5)$

4MA1/2HR JUNE 2023

Question 21

5 marks | Linear Graphs

21 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2023

1 **The line is**

The line is

$$y - 2x = 1$$

2 **Calculate value**

So

$$y = 2x + 1$$

3 **Calculate value**

Substitute into

$$y^2 + xy = 7$$

$$(2x + 1)^2 + x(2x + 1) = 7$$

$$4x^2 + 4x + 1 + 2x^2 + x = 7$$

$$6x^2 + 5x - 6 = 0$$

4 **Factorise**

Factorise:

$$(3x - 2)(2x + 3) = 0$$

5 **Evaluate fraction**

So

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

---

6 **Evaluate fraction**

Using $y = 2x + 1$:

$$x = \frac{2}{3} \Rightarrow y = \frac{7}{3}$$

$$x = -\frac{3}{2} \Rightarrow y = -2$$

✓ **FINAL ANSWER**

$\left(\frac{2}{3}, \frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -2\right)$

4MA1/2H JUNE 2023

Question 24

6 marks | Linear Graphs

24 $ABCD$ is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of D

(..... , ...)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2023

1 Calculate value

In the kite, $AB = AD$ and $CB = CD$, so AC is the line of symmetry.

2 Apply transformation

Therefore D is the reflection of B in the line AC.

$$A = (-2, 10), \qquad B = \left(-\frac{27}{5}, 4\right), \qquad C = (4, -5)$$

3 A direction vector for is

A direction vector for AC is

$$\begin{pmatrix} 4 - (-2) \\ -5 - 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$$

4 Use the simpler direction vector

Use the simpler direction vector

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

5 **Find the gradient**

Let H be the foot of the perpendicular from B to AC.

$$\vec{AB} = \begin{pmatrix} -\frac{17}{5} \\ -6 \end{pmatrix}$$

6 **The fraction along to is**

The fraction along AC to H is

$$\frac{\vec{AB} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}}{2^2 + (-5)^2} = \frac{-\frac{34}{5} + 30}{29}$$

$$= \frac{\frac{116}{5}}{29} = \frac{4}{5}$$

7 **Use matrices**

So

$$H = A + \frac{4}{5}\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$H = \left(-2 + \frac{8}{5}, 10 - 4\right)$$

$$H = \left(-\frac{2}{5}, 6\right)$$

8 **Find the midpoint**

Since H is the midpoint of BD,

$$D = 2H - B$$

$$D = \left(-\frac{4}{5}, 12\right) - \left(-\frac{27}{5}, 4\right)$$

$$D = \left(\frac{23}{5}, 8\right)$$

✓ **FINAL ANSWER**

$$D = \left(\frac{23}{5}, 8\right)$$

4MA1/2HR JUNE 2023

Question 25

6 marks | Linear Graphs

25 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

(..... , ...)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2023

1 The perpendicular bisector is

The perpendicular bisector is

$$y - 2x = 7$$

2 Calculate value

so

$$y = 2x + 7$$

3 The gradient of the perpendicular

The gradient of the perpendicular bisector is 2, so the gradient of AB is

$$-\frac{1}{2}$$

4 **Evaluate fraction**

Since $A = (j, 7)$ and $B = (6, k)$,

$$\frac{k-7}{6-j} = -\frac{1}{2}$$

$$2(k-7) = -(6-j)$$

$$j = 2k - 8$$

5 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$$

6 **Substitute**

Substitute $j = 2k - 8$:

$$\left(k-1, \frac{k+7}{2}\right)$$

7 **Find the midpoint**

The midpoint lies on $y = 2x + 7$, so

$$\frac{k+7}{2} = 2(k-1) + 7$$

$$k + 7 = 4k + 10$$

$$k = -1$$

8 **Calculate value**

Then

$$j = 2(-1) - 8 = -10$$

9 **Find the midpoint**

Therefore the midpoint is

$$\left(\frac{-10+6}{2}, \frac{7+(-1)}{2}\right) = (-2, 3)$$

FINAL ANSWER

$$\boxed{(-2, 3)}$$

4MA1/2H NOVEMBER 2023

Question 21

5 marks | Linear Graphs

21 The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2023

1 The line is

The line is

$$x + 2y = 5$$

2 Calculate value

So

$$x = 5 - 2y$$

3 Calculate value

Substitute into

$$x^2 + 3y^2 = 13$$

$$(5 - 2y)^2 + 3y^2 = 13$$

$$25 - 20y + 4y^2 + 3y^2 = 13$$

$$7y^2 - 20y + 12 = 0$$

4 **Factorise**

Factorise:

$$(7y - 6)(y - 2) = 0$$

5 **Evaluate fraction**

So

$$y = \frac{6}{7} \quad \text{or} \quad y = 2$$

6 **Evaluate fraction**

Using $x = 5 - 2y$:

$$y = \frac{6}{7} \Rightarrow x = \frac{23}{7}$$

$$y = 2 \Rightarrow x = 1$$

✓ **FINAL ANSWER**

$\left(\frac{23}{7}, \frac{6}{7}\right)$ and $(1, 2)$

4MA1/1H NOVEMBER 2023

Question 22

6 marks | Linear Graphs

22 [In this question 1 cm = 1 unit on the x-axis and 1 cm = 1 unit on the y-axis]

P is a point on a circle with centre $(0, 0)$
The coordinates of P are $(8, -10)$

The line **L** is the tangent to the circle at the point P
L crosses the x-axis at the point Q and crosses the y-axis at the point R

Work out the length of RQ
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2023

1 The centre of the circle

The centre of the circle is $(0, 0)$, and

$$P = (8, -10)$$

2 The gradient of is

The gradient of OP is

$$\frac{-10}{8} = -\frac{5}{4}$$

3 Find the gradient

So the gradient of the tangent is

$$\frac{4}{5}$$

4 The tangent through is

The tangent through P is

$$y + 10 = \frac{4}{5}(x - 8)$$

$$y = \frac{4}{5}x - \frac{82}{5}$$

5 **Evaluate fraction**

At the x-axis, $y = 0$:

$$0 = \frac{4}{5}x - \frac{82}{5}$$

$$x = \frac{41}{2}$$

---

6 **Evaluate fraction**

So

$$Q = \left(\frac{41}{2}, 0\right)$$

---

7 **Evaluate fraction**

At the y-axis, $x = 0$:

$$R = \left(0, -\frac{82}{5}\right)$$

---

8 **Simplify surd**

Therefore

$$RQ = \sqrt{\left(\frac{41}{2}\right)^2 + \left(\frac{82}{5}\right)^2}$$

$$RQ = 26.252\ldots$$

✓ **FINAL ANSWER**

$26.3\ \text{cm}$

4MA1/2HR JUNE 2024

Question 21

4 marks | Linear Graphs

21 $ABCD$ is a square.

The point A has coordinates $(-5, 2)$
The point B has coordinates $(3, 5)$

Find an equation of the line that passes through B and C
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2024

1 Calculate value

$$A = (-5, 2), \qquad B = (3, 5)$$

2 The gradient of is

The gradient of AB is

$$\frac{5-2}{3-(-5)} = \frac{3}{8}$$

3 Find the gradient

Since $ABCD$ is a square, BC is perpendicular to AB.

4 Find the gradient

So the gradient of BC is

$$-\frac{8}{3}$$

5 The line through is

The line through $B = (3, 5)$ is

$$y - 5 = -\frac{8}{3}(x - 3)$$

6 Multiply by 3

Multiply by 3:

$$3y - 15 = -8x + 24$$

$$8x + 3y - 39 = 0$$

FINAL ANSWER

$$8x + 3y - 39 = 0$$

4MA1/2H JUNE 2024

Question 22

5 marks | Linear Graphs

22 The straight line **L** has equation $x + y = 5$

The curve **C** has equation $2x^2 + 3y^2 = 210$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , ...) (..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2024

1 The line is

The line is

$$x + y = 5$$

2 Calculate value

So

$$y = 5 - x$$

3 Substitute into the curve

Substitute into the curve:

$$2x^2 + 3y^2 = 210$$

$$2x^2 + 3(5 - x)^2 = 210$$

$$2x^2 + 3(x^2 - 10x + 25) = 210$$

$$5x^2 - 30x + 75 = 210$$

$$5x^2 - 30x - 135 = 0$$

4 **Divide by 5**

Divide by 5:

$$x^2 - 6x - 27 = 0$$

5 **Factorise**

Factorise:

$$(x - 9)(x + 3) = 0$$

6 **Calculate value**

So

$$x = 9 \quad \text{or} \quad x = -3$$

7 **Calculate value**

Using $y = 5 - x$:

$$x = 9 \Rightarrow y = -4$$

$$x = -3 \Rightarrow y = 8$$

✓ **FINAL ANSWER**

$(9, -4)$ and $(-3, 8)$

4WM1/01R MAY 2025

Question 23

5 marks | Linear Graphs

23 A circle C has centre $(3q, 4q)$ where q is an integer.

The straight line with equation $y = -\frac{5}{4}x + \frac{21}{4}$ is the tangent to C at the point $(p, p+3)$ where p is an integer.

Work out the radius of C
Give your answer in the form $\sqrt{a}$ where a is an integer.
Show your working clearly.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4WM1/01R May 2025

1 The tangent point lies on

The tangent point lies on the tangent line, so

$$p + 3 = -\frac{5}{4}p + \frac{21}{4}$$

$$p = 1$$

2 Find the gradient

The tangent gradient is $-\frac{5}{4}$, so the radius gradient is $\frac{4}{5}$.

3 Use trigonometry

Using centre $(3q, 4q)$ and tangent point $(1, 4)$,

$$\frac{4 - 4q}{1 - 3q} = \frac{4}{5}$$

$$q = 2$$

4 **Simplify surd**

So the centre is $(6, 8)$. Therefore

$$r = \sqrt{(6-1)^2 + (8-4)^2} = \sqrt{41}$$

✓ **FINAL ANSWER**

$\sqrt{41}$.

4MA1/2HR JUNE 2025

Question 24

6 marks | Linear Graphs

24 PQ is a straight line drawn on a square grid, with a scale of 1 cm for 1 unit on each axis.

P has coordinates $(-5,\ a)$ and Q has coordinates $(7,\ 3a)$ where $a > 0$

The length of PQ is $4\sqrt{10}$ cm

Find an equation of the perpendicular bisector of PQ
Give your answer in the form $y = mx + c$ where m and c are integers.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2025

1 Calculate value

$$P = (-5, a), \qquad Q = (7, 3a)$$

2 Simplify surd

The length of PQ is $4\sqrt{10}$, so

$$\sqrt{(7-(-5))^2 + (3a-a)^2} = 4\sqrt{10}$$

$$\sqrt{12^2 + (2a)^2} = 4\sqrt{10}$$

3 Square both sides

Square both sides:

$$144 + 4a^2 = 160$$

$$4a^2 = 16$$

$$a^2 = 4$$

4 Solve inequality

Since $a > 0$,

$$a = 2$$

5 **Calculate value**

So

$$P = (-5, 2), \qquad Q = (7, 6)$$

6 **The midpoint of is**

The midpoint of PQ is

$$\left(\frac{-5+7}{2}, \frac{2+6}{2}\right) = (1, 4)$$

7 **The gradient of is**

The gradient of PQ is

$$\frac{6-2}{7-(-5)} = \frac{4}{12} = \frac{1}{3}$$

8 **Find the gradient**

So the gradient of the perpendicular bisector is

$$-3$$

9 **Calculate value**

Using point $(1, 4)$:

$$y - 4 = -3(x - 1)$$

$$y = -3x + 7$$

✓ **FINAL ANSWER**

$$\boxed{y = -3x + 7}$$

4WM1/01 MAY 2025

Question 24

5 marks | Linear Graphs

24 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2025

1 The diagonals of a square

The diagonals of a square bisect each other and are perpendicular.

2 Find the midpoint

Midpoint of $P(4, 7)$ and $R(8, -5)$:

$$\left(\frac{4+8}{2}, \frac{7-5}{2}\right) = (6, 1)$$

3 Find the gradient

Gradient of PR:

$$\frac{-5-7}{8-4} = -3$$

4 Find the gradient

So the perpendicular gradient is $\frac{1}{3}$.

5 **The line through and passes**

The line through Q and S passes through $(6, 1)$, so

$$y - 1 = \frac{1}{3}(x - 6)$$

$$3y - 3 = x - 6$$

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H JUNE 2025

Question 25

5 marks | Linear Graphs

25 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H June 2025

1 Calculate value

$$P = (4, 7), \qquad R = (8, -5)$$

2 The gradient of diagonal is

The gradient of diagonal PR is

$$\frac{-5 - 7}{8 - 4} = -3$$

3 The other diagonal is perpendicular

The other diagonal QS is perpendicular to PR, so its gradient is

$$\frac{1}{3}$$

4 The diagonals of a square

The diagonals of a square bisect each other.

5 The midpoint of is

The midpoint of PR is

$$\left(\frac{4 + 8}{2}, \frac{7 + (-5)}{2}\right) = (6, 1)$$

6 **Find the gradient**

So QS passes through $(6, 1)$ with gradient $\frac{1}{3}$.

$$y - 1 = \frac{1}{3}(x - 6)$$

$$y = \frac{1}{3}x - 1$$

7 **Multiply by 3**

Multiply by 3:

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H NOVEMBER 2025

Question 20

3 marks | Linear Graphs

20 The straight line **L** has equation $y = 4x + 7$

The straight line **M** is perpendicular to **L** and passes through the point with coordinates (8, 1)

Find an equation for **M**
Give your answer in the form $y = mx + c$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2025

1 Line has equation

Line L has equation

$$y = 4x + 7$$

2 Find the gradient

So its gradient is

$$4$$

3 Find the gradient

Line M is perpendicular to L, so its gradient is

$$-\frac{1}{4}$$

4 Evaluate fraction

Line M passes through $(8, 1)$.

$$y - 1 = -\frac{1}{4}(x - 8)$$

$$y - 1 = -\frac{1}{4}x + 2$$

$$y = -\frac{1}{4}x + 3$$

FINAL ANSWER

$$y = -\frac{1}{4}x + 3$$

4WM1/01 NOVEMBER 2025

Question 27

5 marks | Linear Graphs

27 $ABCD$ is a rhombus with $AB = BC = CD = DA$

A has coordinates $(16, -9)$

C has coordinates $(24, 15)$

Find an equation of the line BD
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.
Show your working clearly.

(Total for Question 27 is 5 marks)

WORKED SOLUTION

Question 27 · 4WM1/01 November 2025

1 Diagonals of a rhombus bisect

Diagonals of a rhombus bisect each other at right angles.

2 Find the midpoint

Midpoint of AC:

$$\left(\frac{16+24}{2}, \frac{-9+15}{2}\right) = (20, 3)$$

3 Find the gradient

Gradient of AC:

$$\frac{15-(-9)}{24-16} = 3$$

4 Find the gradient

So gradient of BD is $-\frac{1}{3}$.

$$y - 3 = -\frac{1}{3}(x - 20)$$

$$x + 3y - 29 = 0$$

FINAL ANSWER

$x + 3y - 29 = 0$.

4MA1/1H NOVEMBER 2025

Question 28

6 marks | Linear Graphs

28 The curve **C** has equation $x^2 + y^2 = d - 11x$ where d is an integer.
The line **L** has equation $y = 3x + e$ where e is an integer.

C and **L** intersect at the point A and at the point B

The coordinates of A are $(0.2, 2.6)$

Work out the coordinates of B
Show your working clearly.

(..... , ...)

(Total for Question 28 is 6 marks)

WORKED SOLUTION

Question 28 · 4MA1/1H November 2025

1 The line is

The line is

$$y = 3x + e$$

2 Point lies on the line

Point $A = (0.2, 2.6)$ lies on the line.

$$2.6 = 3(0.2) + e$$

$$2.6 = 0.6 + e$$

$$e = 2$$

3 Calculate value

So

$$y = 3x + 2$$

4 **The curve is**

The curve is

$$x^2 + y^2 = d - 11x$$

---

5 **Use point**

Use point $A = (0.2, 2.6)$:

$$0.2^2 + 2.6^2 = d - 11(0.2)$$

$$0.04 + 6.76 = d - 2.2$$

$$d = 9$$

---

6 **Substitute into the curve**

Now substitute $y = 3x + 2$ into the curve:

$$x^2 + (3x + 2)^2 = 9 - 11x$$

$$x^2 + 9x^2 + 12x + 4 = 9 - 11x$$

$$10x^2 + 23x - 5 = 0$$

---

7 **One root is**

One root is $0.2 = \frac{1}{5}$.

---

8 **The product of the roots**

The product of the roots is

$$\frac{-5}{10} = -\frac{1}{2}$$

---

9 **Evaluate fraction**

So the other root is

$$\frac{-\frac{1}{2}}{\frac{1}{5}} = -\frac{5}{2}$$

---

10 **Evaluate fraction**

Then

$$y = 3\left(-\frac{5}{2}\right) + 2$$

$$y = -\frac{11}{2}$$

✓ **FINAL ANSWER**

$$B = \left(-\frac{5}{2}, -\frac{11}{2}\right)$$

TOPIC

Graphs of Functions

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 20

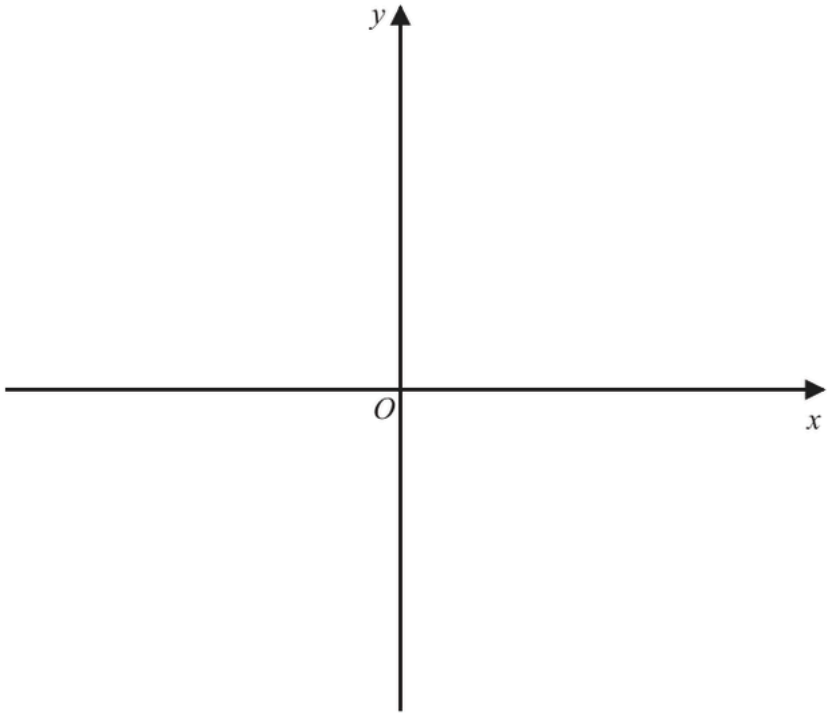
4 marks | Graphs of Functions

20 The curve **C** has equation $y = 4(x - 1)^2 - a$ where $a > 4$

Using the axes below, sketch the curve **C**.
On your sketch show clearly, in terms of a,

(i) the coordinates of any points of intersection of **C** with the coordinate axes,

(ii) the coordinates of the turning point.



(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2020

1 **Calculate value**

$$y = 4(x - 1)^2 - a$$

2 **The turning point is read**

The turning point is read directly from the completed square form:

$$(1, -a)$$

3 **Find intercepts**

For the y-intercept, put $x = 0$:

$$y = 4(0-1)^2 - a$$

$$y = 4 - a$$

4 **Find intercepts**

So the y-intercept is

$$(0, 4-a)$$

5 **Find intercepts**

For the x-intercepts, put $y = 0$:

$$4(x-1)^2 - a = 0$$

$$4(x-1)^2 = a$$

$$(x-1)^2 = \frac{a}{4}$$

$$x - 1 = \pm\frac{\sqrt{a}}{2}$$

$$x = 1 \pm \frac{\sqrt{a}}{2}$$

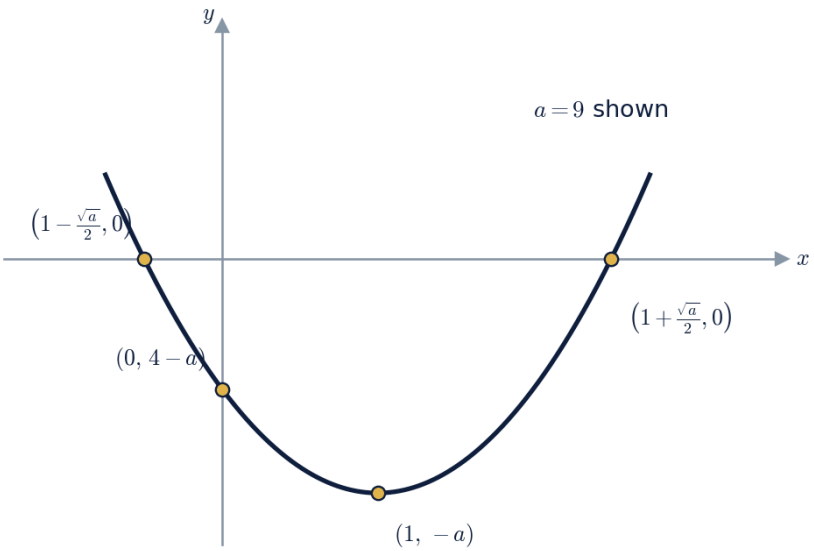
6 **Find intercepts**

So the x-intercepts are

$$\left(1 - \frac{\sqrt{a}}{2}, 0\right) \quad \text{and} \quad \left(1 + \frac{\sqrt{a}}{2}, 0\right)$$

7 **Solve inequality**

Since $a > 4$, the turning point is below the x-axis and the curve crosses the x-axis twice.



FINAL ANSWER

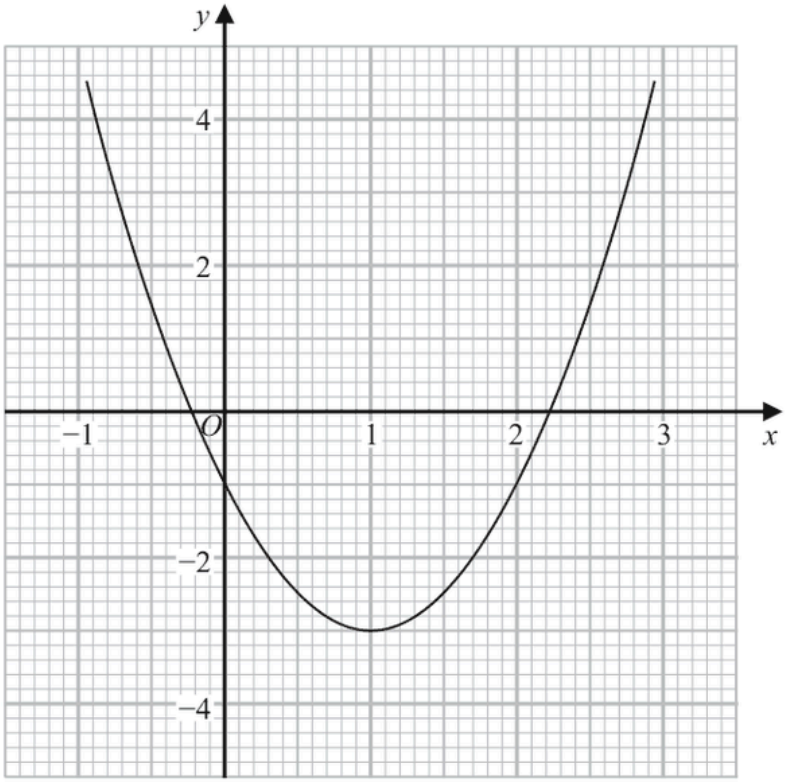
turning point $(1, -a)$, y-intercept $(0, 4-a)$, x-intercepts $\left(1-\frac{\sqrt{a}}{2}, 0\right)$ and $\left(1+\frac{\sqrt{a}}{2}, 0\right)$

4MA1/2H JANUARY 2022

Question 21

5 marks | Graphs of Functions

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

.....

(2)

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

.....

(3)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2022

1 **Read the graph**

For part (a), solve

$$2x^2 - 4x - 1 = 0$$

2 **Read the graph**

Using the graph gives approximately

$$x = -0.2, \quad x = 2.2$$

3 **Read the graph**

For part (b), we need to solve

$$x^2 - x - 1 = 0$$

4 **Multiply by 2**

Multiply by 2:

$$2x^2 - 2x - 2 = 0$$

5 **The given graph is**

The given graph is

$$y = 2x^2 - 4x - 1$$

6 **Read the graph**

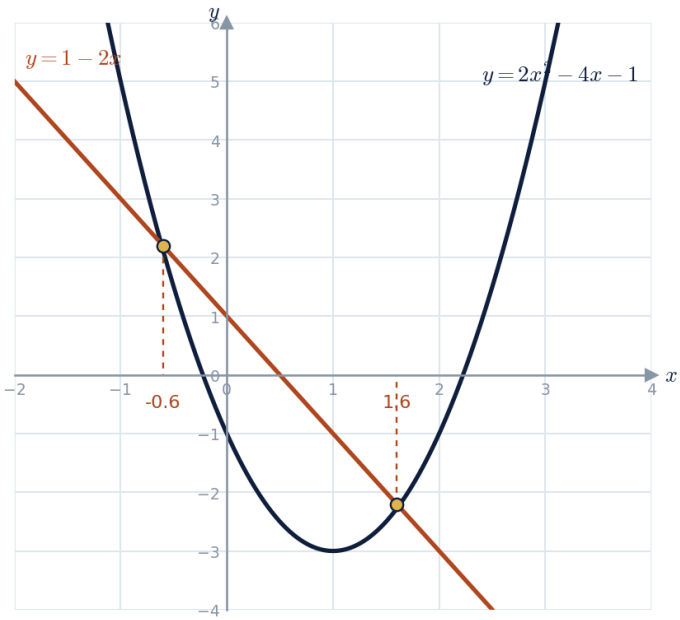
Now

$$2x^2 - 2x - 2 = (2x^2 - 4x - 1) + 2x - 1$$

7 **Read the graph**

So draw the straight line

$$y = 1 - 2x$$



8 **The intersections of this line**

The intersections of this line with the curve give approximately

$$x = -0.6, \quad x = 1.6$$

✓ **FINAL ANSWER**

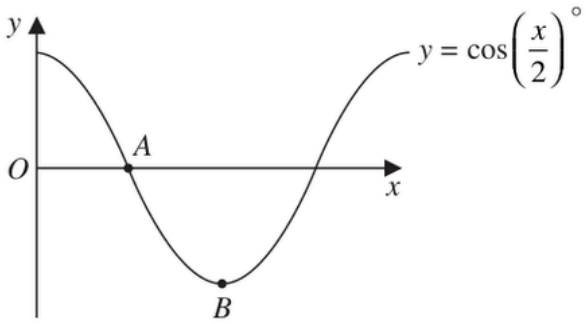
(a) $-0.2,\ 2.2$; (b) draw $y = 1 - 2x$, giving $-0.6,\ 1.6$

4MA1/2H JUNE 2022

Question 23

2 marks | Graphs of Functions

23 The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$



(i) Find the coordinates of the point A

(.....,.....)

(1)

(ii) Find the coordinates of the point B

(.....,.....)

(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2022

1 **The graph is**

The graph is

$$y = \cos\left(\frac{x}{2}\right)^\circ$$

2 **Point is the first positive**

Point A is the first positive x-intercept, so $y = 0$.

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180$$

3 **Read the graph**

So

$$A = (180, 0)$$

---

4 **Point is the minimum point**

Point B is the minimum point.

---

5 **The minimum value of cosine**

The minimum value of cosine is -1, which occurs when

$$\frac{x}{2} = 180°$$

$$x = 360$$

---

6 **Read the graph**

So

$$B = (360, -1)$$

✓ **FINAL ANSWER**

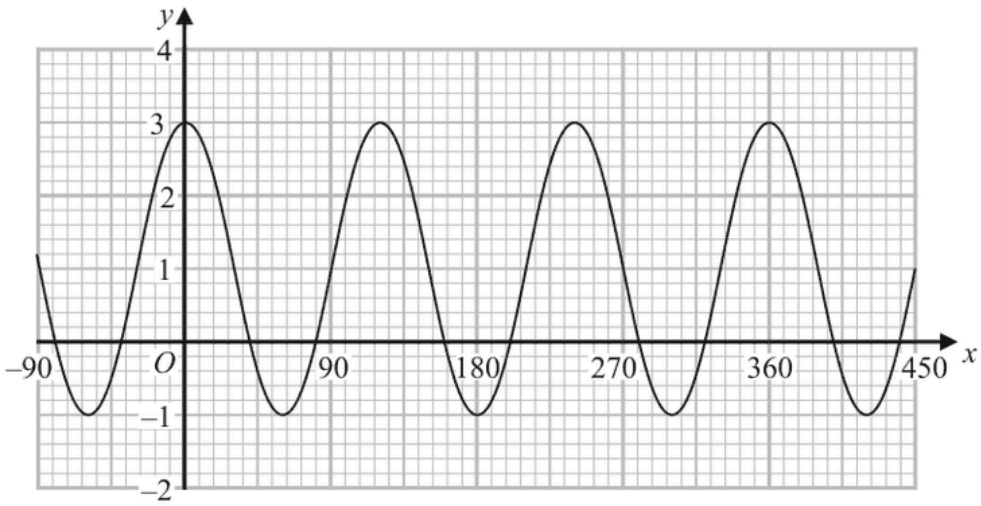
$A = (180, 0),\ B = (360, -1)$

4MA1/2HR JUNE 2023

Question 26

3 marks | Graphs of Functions

26 Here is a sketch of the curve with equation $y = a\cos bx° + c$ where $-90 \leqslant x \leqslant 450$



Find the value of a, the value of b and the value of c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2023

1 The curve is

The curve is

$$y = a\cos bx° + c$$

2 Use trigonometry

From the graph, the maximum value is

$$3$$

3 Use trigonometry

and the minimum value is

$$-1$$

4 **The amplitude is half the**

The amplitude is half the difference:

$$a = \frac{3 - (-1)}{2} = 2$$

5 **The midline is the average**

The midline is the average:

$$c = \frac{3 + (-1)}{2} = 1$$

6 **The distance between consecutive maxima**

The distance between consecutive maxima is $120°$, so the period is

$$120°$$

7 **Use trigonometry**

For $y = a\cos bx° + c$, the period is

$$\frac{360°}{b}$$

8 **Use trigonometry**

So

$$\frac{360}{b} = 120$$

$$b = 3$$

✓ **FINAL ANSWER**

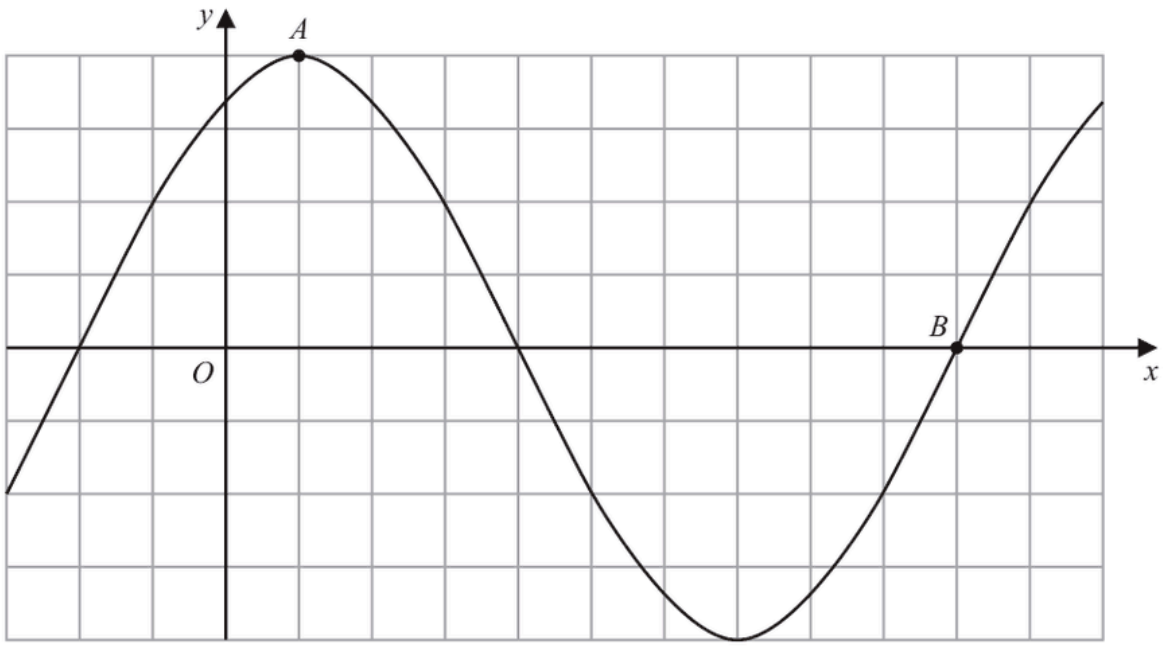
$$\boxed{a = 2,\ b = 3,\ c = 1}$$

4MA1/2H JUNE 2024

Question 25

2 marks | Graphs of Functions

25 The diagram shows a sketch of the graph of $y = 2\sin(x + 60)°$



(i) Find the coordinates of the point A

(..... , ...)

(1)

(ii) Find the coordinates of the point B

(..... , ...)

(1)

(Total for Question 25 is 2 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2024

1 **Use trigonometry**

$$y = 2\sin(x + 60)°$$

2 **Point is the maximum point**

Point A is the maximum point.

3 **The maximum value is**

The maximum value is

$$y = 2$$

4 **This happens when**

This happens when

$$x + 60 = 90$$

$$x = 30$$

5 **Read the graph**

So

$$A = (30, 2)$$

6 **Point is the positive intercept**

Point B is the positive x-intercept after the minimum.

$$2\sin(x + 60)° = 0$$

$$\sin(x + 60)° = 0$$

7 **Read the graph**

For this point,

$$x + 60 = 360$$

$$x = 300$$

8 **Read the graph**

So

$$B = (300, 0)$$

✓ **FINAL ANSWER**

$A = (30, 2),\ B = (300, 0)$

4MA1/1H NOVEMBER 2024

Question 23

5 marks | Graphs of Functions

23 The curve **C** has equation $y = x^2 - 8x - 9$
The straight line **L** has equation $y = k$ where k is an integer.

C and **L** intersect at the points A and B

The coordinates of point A are (p, k)
The coordinates of point B are (q, k)

Given that $p - q = 14$

find the value of k
Show clear algebraic working.

$k =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2024

1 The curve is

The curve is

$$y = x^2 - 8x - 9$$

2 The line is

The line is

$$y = k$$

3 Read the graph

At the intersection points,

$$x^2 - 8x - 9 = k$$

$$x^2 - 8x - (9 + k) = 0$$

4 **Read the graph**

The two roots are p and q.

5 **Solve quadratic equation**

For this quadratic,

$$p + q = 8$$

6 **We are also given**

We are also given

$$p - q = 14$$

7 **Solve the two equations**

Solve the two equations:

$$2p = 22$$

$$p = 11$$

$$q = -3$$

8 **Read the graph**

Use one root in $y = x^2 - 8x - 9$:

$$k = 11^2 - 8(11) - 9$$

$$k = 121 - 88 - 9$$

$$k = 24$$

✓ **FINAL ANSWER**

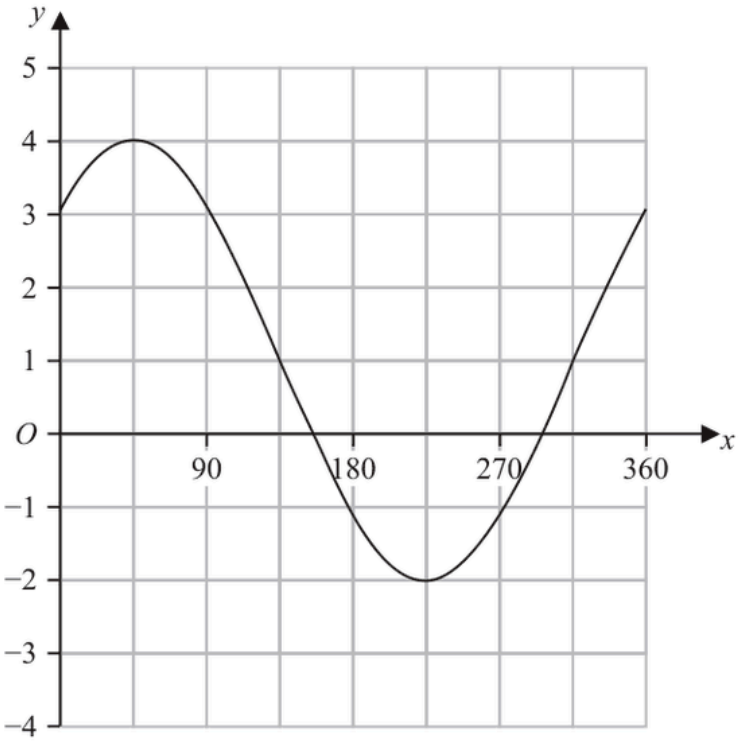
$k = 24$

4MA1/2HR JUNE 2025

Question 25

3 marks | Graphs of Functions

25 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2025

1 **Read the graph**

From the graph,

$$\text{maximum} = 4$$

$$\text{minimum} = -2$$

2 **The amplitude is**

The amplitude is

$$a = \frac{4 - (-2)}{2} = 3$$

3 **The centre line is**

The centre line is

$$c = \frac{4 + (-2)}{2} = 1$$

4 **Use trigonometry**

The maximum is about $45°$. For a sine graph, a maximum happens when the angle inside the sine is $90°$, so

$$45 + b = 90$$

$$b = 45$$

✓ **FINAL ANSWER**

$a = 3,\ b = 45,\ c = 1$

TOPIC

Perimeter & Area

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

Question 22

6 marks | Perimeter & Area

22 The diagram shows a regular octagon *ABCDEFGH*.

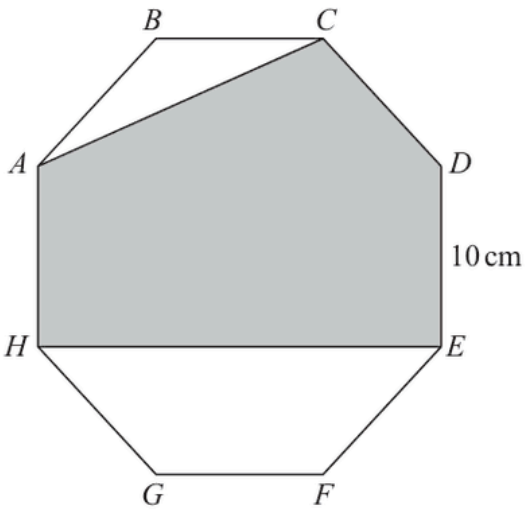


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May/November 2020

1 **Each side of the regular**

Each side of the regular octagon is 10 cm.

2 **Calculate area**

For a regular octagon with side length s,

$$\text{area} = 2(1+\sqrt{2})s^2$$

3 **Calculate area**

So the area of the whole octagon is

$$2(1+\sqrt{2})(10)^2 = 200 + 200\sqrt{2}$$

---

4 **The unshaded triangle has**

The unshaded triangle ABC has

$$AB = 10, \qquad BC = 10, \qquad \angle ABC = 135^\circ$$

---

5 **Use trigonometry**

So

$$\text{area of triangle } ABC = \frac{1}{2}(10)(10)\sin 135^\circ = 25\sqrt{2}$$

---

6 **Find the gradient**

At the bottom, the unshaded trapezium $HEFG$ has parallel sides

$$HE = 10 + 10\sqrt{2}, \qquad GF = 10$$

---

7 **Simplify surd**

and height

$$5\sqrt{2}$$

---

8 **Calculate area**

So

$$\text{area of trapezium } HEFG = \frac{1}{2}(10 + 10 + 10\sqrt{2})(5\sqrt{2})$$

$$= 50 + 50\sqrt{2}$$

---

9 **The shaded area is**

The shaded area is

$$(200 + 200\sqrt{2}) - 25\sqrt{2} - (50 + 50\sqrt{2})$$

$$= 150 + 125\sqrt{2}$$

$$= 326.7766\ldots$$

FINAL ANSWER

$\boxed{327\ \text{cm}^2}$

4MA1/2H JANUARY 2021

Question 20

4 marks | Perimeter & Area

20

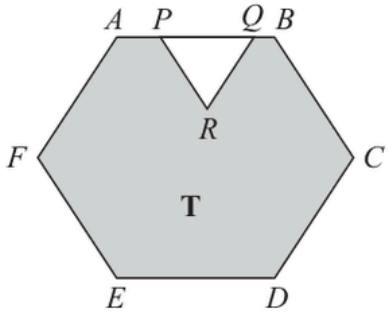


Diagram **NOT** accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle PQR from a regular hexagon $ABCDEF$.

The points P and Q lie on AB such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3}$ cm^2

work out the length of PQ.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2021

1 Let

Let

$$PQ = p$$

2 Given

Given

$$AB = 1.5PQ$$

3 Find the gradient

so the side length of the regular hexagon is

$$1.5p = \frac{3}{2}p$$

4 **Area of a regular hexagon**

Area of a regular hexagon with side length s is

$$\frac{3\sqrt{3}}{2}s^2$$

5 **Find the gradient**

So the area of the hexagon is

$$\frac{3\sqrt{3}}{2}\left(\frac{3}{2}p\right)^2$$

$$=\frac{27\sqrt{3}}{8}p^2$$

6 **Find the gradient**

The removed triangle PQR is equilateral, so its area is

$$\frac{\sqrt{3}}{4}p^2$$

7 **The shaded area is**

The shaded area is

$$\frac{27\sqrt{3}}{8}p^2-\frac{\sqrt{3}}{4}p^2$$

$$=\frac{25\sqrt{3}}{8}p^2$$

8 **Find the gradient**

This is $72\sqrt{3}$, so

$$\frac{25\sqrt{3}}{8}p^2=72\sqrt{3}$$

$$\frac{25}{8}p^2=72$$

$$p^2=\frac{576}{25}$$

$$p=4.8$$

FINAL ANSWER

$PQ = 4.8\ \text{cm}$

4WM1/01 NOVEMBER 2025

Question 21

4 marks | Perimeter & Area

21 The diagram shows an equilateral triangle and a circle.

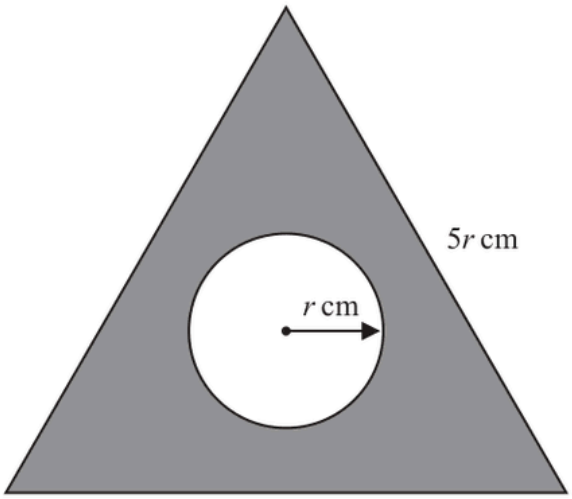


Diagram **NOT** accurately drawn

The equilateral triangle has sides of length $5r$ cm
The circle has radius r cm

The area of the shaded region is 610π cm^2
Work out the value of r

Give your answer correct to 3 significant figures.

$r =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 November 2025

1 **Triangle area**

Triangle area:

$$\frac{\sqrt{3}}{4}(5r)^2 = \frac{25\sqrt{3}}{4}r^2$$

2 **Circle area**

Circle area:

$$\pi r^2$$

3 **Simplify surd**

So

$$\left(\frac{25\sqrt{3}}{4} - \pi\right) r^2 = 610\pi$$

$$r \approx 15.7926$$

FINAL ANSWER

$r = 15.8$ to 3 significant figures.

TOPIC

Circles, Arcs & Sectors

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2021

Question 20

5 marks | Circles, Arcs & Sectors

20 A, B and C are points on a circle with centre O.

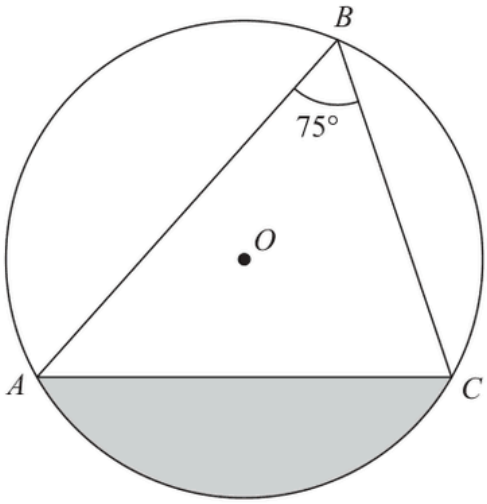


Diagram **NOT** accurately drawn

Angle $ABC = 75°$

The area of the shaded segment is $200\,\text{cm}^2$

Calculate the radius of the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2021

1 **Use trigonometry**

$$\angle ABC = 75°$$

2 **The angle at the centre**

The angle at the centre is twice the angle at the circumference, so

$$\angle AOC = 2(75°) = 150°$$

3 **The shaded segment is**

The shaded segment is:

$$\text{sector } AOC - \text{triangle } AOC$$

4 **Let the radius be**

Let the radius be r.

5 **Sector area**

Sector area:

$$\frac{150}{360}\pi r^2$$

6 **Triangle area**

Triangle area:

$$\frac{1}{2}r^2 \sin 150°$$

7 **Use trigonometry**

So

$$\frac{150}{360}\pi r^2 - \frac{1}{2}r^2 \sin 150° = 200$$

$$r^2\left(\frac{5\pi}{12} - \frac{1}{4}\right) = 200$$

$$r^2 = \frac{200}{\frac{5\pi}{12} - \frac{1}{4}}$$

$$r = 13.7426\ldots$$

✓ **FINAL ANSWER**

$\boxed{13.7 \text{ cm}}$

4MA1/2H JANUARY 2021

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows a sector OBC of a circle with centre O and radius $(6 + x)$ cm.

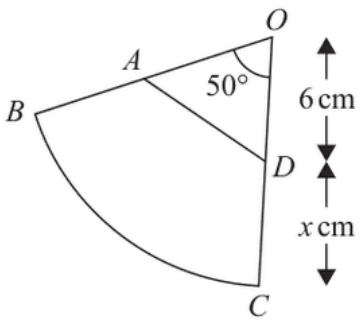


Diagram **NOT** accurately drawn

A is the point on OB and D is the point on OC such that $OA = OD = 6$ cm

Angle $BOC = 50°$

Given that

the perimeter of sector $OBC = 2 \times$ the perimeter of triangle OAD

find the value of x.
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2021

1 The radius of sector is

The radius of sector OBC is

$$6 + x$$

2 Use trigonometry

The angle is $50°$, so the arc length of sector OBC is

$$\frac{50}{360} \times 2\pi(6 + x)$$

3 **Evaluate fraction**

Therefore the perimeter of sector OBC is

$$2(6+x)+\frac{50}{360}\times 2\pi(6+x)$$

$$=(6+x)\left(2+\frac{5\pi}{18}\right)$$

4 **Use trigonometry**

In triangle OAD,

$$OA=OD=6, \qquad \angle AOD=50^\circ$$

5 **Use trigonometry**

So

$$AD=2(6)\sin 25^\circ=12\sin 25^\circ$$

6 **The perimeter of triangle is**

The perimeter of triangle OAD is

$$12+12\sin 25^\circ$$

7 **Given that**

Given that

$$\text{perimeter of sector } OBC=2\times\text{perimeter of triangle } OAD$$

$$(6+x)\left(2+\frac{5\pi}{18}\right)=2(12+12\sin 25^\circ)$$

$$x=\frac{2(12+12\sin 25^\circ)}{2+\frac{5\pi}{18}}-6$$

$$x=5.8854\ldots$$

✓ **FINAL ANSWER**

$\boxed{5.89}$

4MA1/1H MAY 2021

Question 25

6 marks | Circles, Arcs & Sectors

25 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

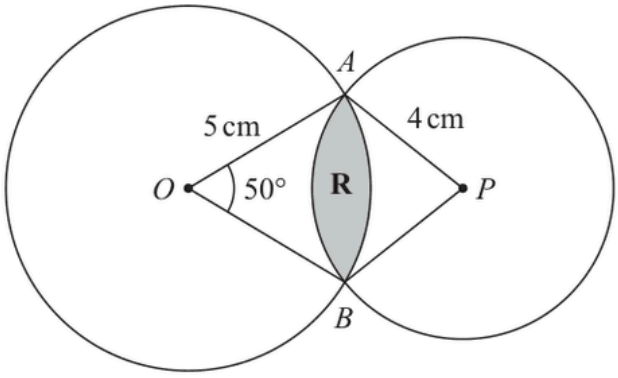


Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2021

1 **Use trigonometry**

For the circle with centre O:

$$\angle AOB = 50°, \qquad OA = OB = 5$$

2 **The half chord length is**

The half chord length is

$$\frac{AB}{2} = 5\sin 25°$$

3 **Use trigonometry**

For the circle with centre P, let

$$\angle APB = \theta$$

---

4 **Use trigonometry**

The radius is 4, so

$$4\sin\left(\frac{\theta}{2}\right) = 5\sin 25°$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{5\sin 25°}{4}$$

$$\theta = 63.7777\ldots°$$

---

5 **The common region is made**

The common region is made from two minor segments.

---

6 **Use trigonometry**

For the circle with centre O:

$$\text{segment area} = \frac{50}{360}\pi(5)^2 - \frac{1}{2}(5)^2\sin 50° = 1.3328\ldots$$

---

7 **Use trigonometry**

For the circle with centre P:

$$\text{segment area} = \frac{\theta}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin\theta$$

$$= 1.7284\ldots$$

---

8 **Total area of region**

Total area of region R:

$$1.3328\ldots + 1.7284\ldots = 3.0611\ldots$$

✓ **FINAL ANSWER**

$\boxed{3.06\ \text{cm}^2}$

4MA1/2H JUNE 2022

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows four identical circles drawn inside a square.

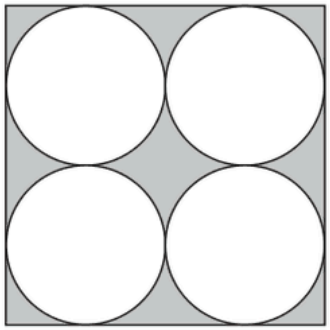


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of $40\,\text{cm}^2$

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2022

1 Let the radius of each

Let the radius of each circle be r.

2 There are two circles across

There are two circles across the width of the square, so the side length of the square is

$$4r$$

3 Area of the square

Area of the square:

$$(4r)^2 = 16r^2$$

4 Area of the four circles

Area of the four circles:

$$4\pi r^2$$

5 **Calculate area**

The shaded area is $40\ \text{cm}^2$, so

$$16r^2 - 4\pi r^2 = 40$$

$$r^2(16 - 4\pi) = 40$$

$$r = \sqrt{\frac{40}{16 - 4\pi}} = 3.4131\ldots$$

---

6 **The perimeter of the square**

The perimeter of the square is

$$4(4r) = 16r$$

$$16r = 54.6101\ldots$$

✓ **FINAL ANSWER**

$\boxed{54.6\ \text{cm}}$

4MA1/1H MAY 2023

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

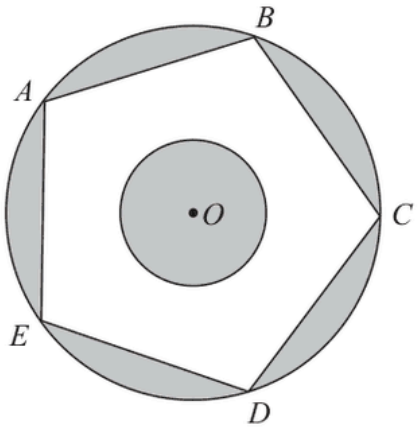


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2023

1 **The regular pentagon has side**

The regular pentagon has side length 8 cm.

2 **Use trigonometry**

For a regular pentagon,

$$R = \frac{s}{2\sin 36°}$$

3 **Use trigonometry**

where R is the radius of the larger circle.

$$R = \frac{8}{2\sin 36^\circ} = 6.8052\ldots$$

4 **Area of the larger circle**

Area of the larger circle:

$$\pi R^2 = 145.4898\ldots$$

5 **Area of the regular pentagon**

Area of the regular pentagon:

$$\frac{5s^2}{4\tan 36^\circ} = \frac{5(8)^2}{4\tan 36^\circ} = 110.1106\ldots$$

6 **Let the radius of the**

Let the radius of the smaller circle be r.

7 **The shaded area is**

The shaded area is:

$$\text{larger circle area} - \text{pentagon area} + \pi r^2$$

8 **The unshaded area is**

The unshaded area is:

$$\text{pentagon area} - \pi r^2$$

9 **Simplify surd**

These are equal, so

$$145.4898 - 110.1106 + \pi r^2 = 110.1106 - \pi r^2$$

$$2\pi r^2 = 2(110.1106) - 145.4898$$

$$\pi r^2 = 37.3657\ldots$$

$$r = \sqrt{\frac{37.3657\ldots}{\pi}} = 3.4487\ldots$$

✓ **FINAL ANSWER**

$$\boxed{3.45\text{ cm}}$$

4MA1/2H NOVEMBER 2023

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows equilateral triangle ABC with sides of length 10 cm.
A circle is drawn inside the triangle.

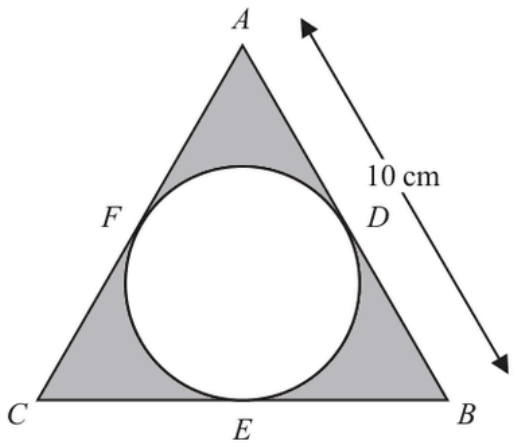


Diagram **NOT** accurately drawn

D, E and F are points on the circle.

ADB, BEC and CFA are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2023

1 Use trigonometry

The triangle is equilateral with side length 10 cm.

2 Area of the triangle

Area of the triangle:

$$\frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$$

3 Use trigonometry

For an equilateral triangle, the inradius is

$$r = \frac{s\sqrt{3}}{6}$$

4 **Simplify surd**

So

$$r = \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

5 **Area of the circle**

Area of the circle:

$$\pi r^2 = \pi\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{25\pi}{3}$$

6 **The shaded area is**

The shaded area is

$$25\sqrt{3} - \frac{25\pi}{3} = 17.1213\ldots$$

✓ **FINAL ANSWER**

$\boxed{17.1\ \text{cm}^2}$

4MA1/1H MAY 2024

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows a sector $OABC$ of a circle centre O

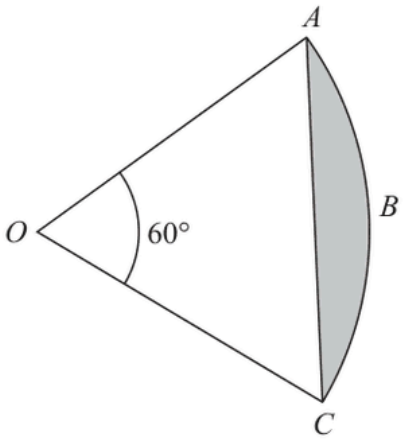


Diagram **NOT** accurately drawn

Angle $AOC = 60°$

The area of the shaded segment ABC is $38\,\text{cm}^2$

Work out the perimeter of the shaded segment ABC
Give your answer correct to one decimal place.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2024

1 **Let the radius of the**

Let the radius of the sector be r.

2 **The shaded segment is**

The shaded segment is

$$\text{sector area} - \text{triangle area}$$

3 **Use trigonometry**

The angle is $60°$, so

$$38 = \frac{60}{360}\pi r^2 - \frac{1}{2}r^2 \sin 60°$$

$$38 = r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$r = 20.4815\ldots$$

4 **The perimeter of the shaded**

The perimeter of the shaded segment is the arc AC plus the chord AC.

5 **Arc length**

Arc length:

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

6 **Chord length**

Chord length:

$$2r \sin 30° = r$$

7 **Evaluate fraction**

So the perimeter is

$$\frac{\pi r}{3} + r$$

$$= \frac{\pi(20.4815\ldots)}{3} + 20.4815\ldots$$

$$= 41.9296\ldots$$

✓ **FINAL ANSWER**

$\boxed{41.9\text{ cm}}$

4MA1/2H JUNE 2024

Question 21

5 marks | Circles, Arcs & Sectors

21 The diagram shows a square $ABCD$ and a circle.

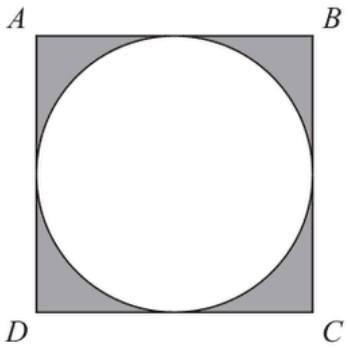


Diagram **NOT** accurately drawn

The sides of the square are tangents to the circle.

The total area of the shaded regions is $80\,\text{cm}^2$

Work out the length of AC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2024

1 **Let the side length**

Let the side length of the square be s.

2 **Square**

The circle is inscribed in the square, so its radius is

$$\frac{s}{2}$$

3 **The shaded area is the**

The shaded area is the area of the square minus the area of the circle:

$$s^2 - \pi\left(\frac{s}{2}\right)^2 = 80$$

$$s^2 - \frac{\pi s^2}{4} = 80$$

$$s^2\left(1 - \frac{\pi}{4}\right) = 80$$

$$s = \sqrt{\frac{80}{1 - \frac{\pi}{4}}} = 19.3076\ldots$$

---

4 **The diagonal of the square**

The diagonal of the square is

$$AC = s\sqrt{2}$$

$$AC = 19.3076\ldots\sqrt{2} = 27.3051\ldots$$

✓ **FINAL ANSWER**

$\boxed{27.3\text{ cm}}$

4MA1/2H NOVEMBER 2024

Question 25

5 marks | Circles, Arcs & Sectors

25 The diagram shows an equilateral triangle ABC and a circle with centre O

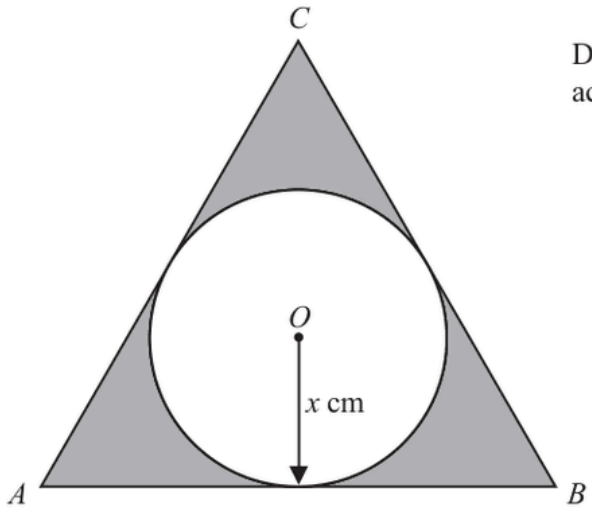


Diagram **NOT** accurately drawn

AB, BC and CA are tangents to the circle.

The radius of the circle is x cm

The total area, in cm^2, of the regions shown shaded in the diagram is nx^2

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2024

1 **Use trigonometry**

The circle has radius x, so the inradius of the equilateral triangle is x.

2 **Use trigonometry**

For an equilateral triangle with side length a,

$$\text{inradius} = \frac{a\sqrt{3}}{6}$$

3 **Simplify surd**

So

$$x = \frac{a\sqrt{3}}{6}$$

$$a = \frac{6x}{\sqrt{3}} = 2\sqrt{3}x$$

4 **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(2\sqrt{3}x)^2$$

$$= \frac{\sqrt{3}}{4}(12x^2) = 3\sqrt{3}x^2$$

5 **Area of the circle**

Area of the circle:

$$\pi x^2$$

6 **The shaded area is**

The shaded area is

$$3\sqrt{3}x^2 - \pi x^2$$

$$= (3\sqrt{3} - \pi)x^2$$

7 **Simplify surd**

Therefore

$$n = 3\sqrt{3} - \pi$$

✓ **FINAL ANSWER**

$$\boxed{3\sqrt{3} - \pi}$$

4WM1/01R MAY 2025

Question 22

6 marks | Circles, Arcs & Sectors

22

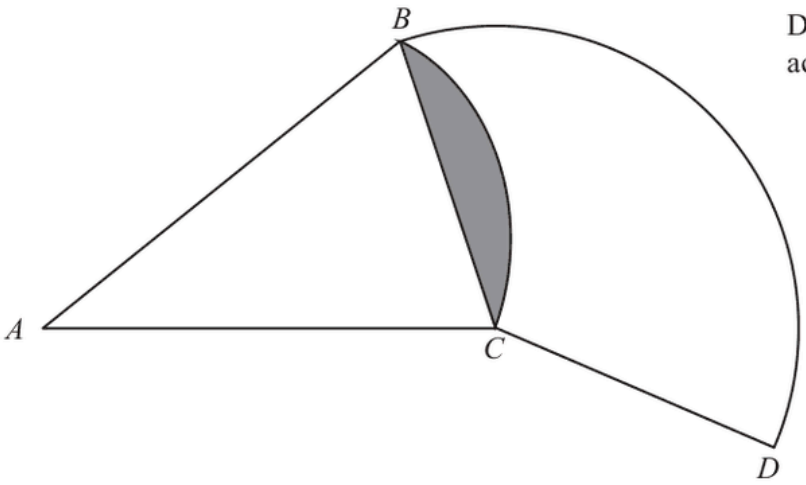


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre A

BCD is a sector of a circle, centre C

Angle $BAC = 40°$
Angle $BCD = 130°$
Area of shaded segment $= 28\,\text{cm}^2$

Find the length of the arc BD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM1/01R May 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/1HR JUNE 2025

Question 25

6 marks | Circles, Arcs & Sectors

25

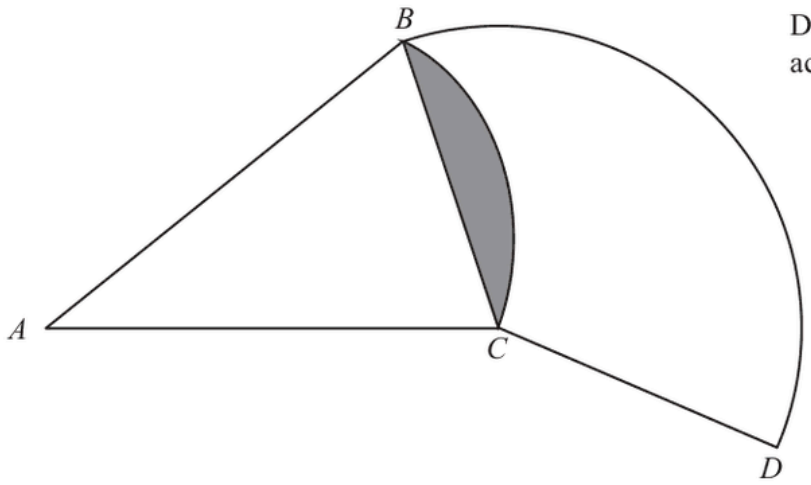


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre *A*

BCD is a sector of a circle, centre *C*

Angle *BAC* = 40°
Angle *BCD* = 130°
Area of shaded segment = 28 cm^2

Find the length of the arc *BD*
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

Question 23

6 marks | Circles, Arcs & Sectors

23 OPQ and OAB are sectors of a circle, centre O

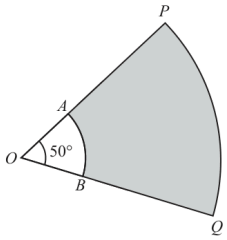


Diagram **NOT** accurately drawn

Angle $POQ = 50°$

A is the point on OP such that $OA = \frac{1}{4}OP$

The area of the shaded region is $\frac{243}{16}\pi\ \text{cm}^2$

Work out the perimeter of the shaded region.
Give your answer in the form $a\pi + b$ where a and b are constants.
Show your working clearly.

.....cm

(Total for Question 23 is 6 marks)

1 **Let OP = r**

Then $OA = \frac{r}{4}$. Shaded area:

$$\frac{50}{360}\pi\left(r^2 - \frac{r^2}{16}\right) = \frac{243}{16}\pi$$

2 **Solve for r**

$$\frac{75}{576}r^2 = \frac{243}{16} \Rightarrow r^2 = 116.64 \Rightarrow r = 10.8,\ OA = 2.7$$

3 **Arcs**

$$\text{arc } PQ = \frac{50}{360} \times 2\pi(10.8) = 3\pi, \qquad \text{arc } AB = \frac{50}{360} \times 2\pi(2.7) = \tfrac{3}{4}\pi$$

4 **Straight edges and total**

$AP = BQ = 10.8 - 2.7 = 8.1$:

$$3\pi + \tfrac{3}{4}\pi + 2(8.1) = \tfrac{15}{4}\pi + 16.2$$

FINAL ANSWER

$\frac{15}{4}\pi + 16.2$ cm

TOPIC

3D Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

4MA1/2HR JANUARY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 ABC is an isosceles triangle in a horizontal plane.
The point T is vertically above B.

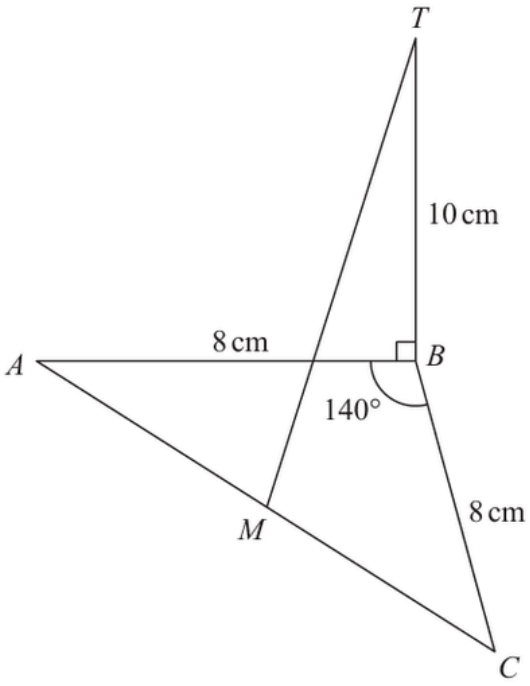


Diagram **NOT** accurately drawn

Angle $ABC = 140°$
$AB = BC = 8$ cm
$TB = 10$ cm
M is the midpoint of AC.

Calculate the size of the angle between MT and the horizontal plane ABC.
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2021

1 **Use trigonometry**

Since $AB = BC = 8$, triangle ABC is isosceles.

2 **The line from to the**

The line from B to the midpoint M bisects angle ABC.

$$\angle ABM = \frac{140^\circ}{2} = 70^\circ$$

3 **Use trigonometry**

In right triangle ABM,

$$\cos 70^\circ = \frac{BM}{8}$$

$$BM = 8\cos 70^\circ$$

4 **Use trigonometry**

Since T is vertically above B, the projection of MT onto the horizontal plane is MB.

5 **Let the required angle be**

Let the required angle be θ.

$$\tan\theta = \frac{TB}{BM}$$

$$\tan\theta = \frac{10}{8\cos 70^\circ}$$

$$\theta = 74.7^\circ$$

FINAL ANSWER

74.7°

4MA1/1H MAY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

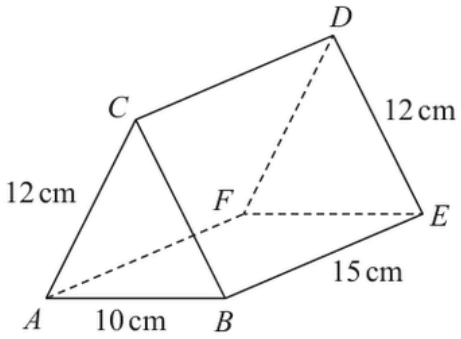


Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between AD and the base $ABEF$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2021

1 Calculate value

The triangular end is isosceles, with

$$AC = BC = 12, \qquad AB = 10$$

2 The perpendicular height of the

The perpendicular height of the triangle is found by halving AB:

$$AM = 5$$

$$CM^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$CM = \sqrt{119}$$

3 The projection of on the

The projection of AD on the horizontal base goes from A to the midpoint of FE. Its length is

$$\sqrt{15^2 + 5^2} = \sqrt{250}$$

---

4 Use trigonometry

So, for the angle θ between AD and the base,

$$\tan\theta = \frac{\sqrt{119}}{\sqrt{250}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.602\ldots$$

FINAL ANSWER

$34.6°$

4MA1/1H JANUARY 2022

Question 21

5 marks | 3D Pythagoras & Trigonometry

21 The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

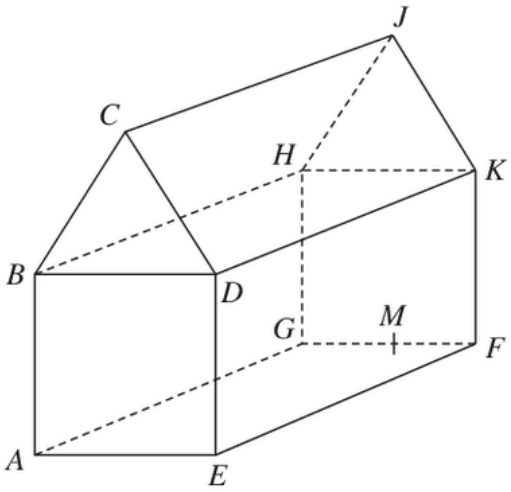


Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of T, giving your answer in the form $\frac{\sqrt{p} + \sqrt{q}}{17}$ where p and q are integers.

$T =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2022

1 Let

Let

$$AE = x$$

2 **Use trigonometry**

Since $ABDE$ is a square,

$$AB = BD = DE = AE = x$$

3 **Use trigonometry**

Since BCD is an equilateral triangle, the height from C to BD is

$$\frac{\sqrt{3}}{2}x$$

4 **Use trigonometry**

Therefore the vertical height from the base to J is

$$JM = x + \frac{\sqrt{3}}{2}x$$

$$JM = \frac{x(2+\sqrt{3})}{2}$$

5 **Use trigonometry**

Also,

$$EF = 2AE = 2x$$

6 **Find the midpoint**

Since M is the midpoint of GF, the horizontal distance from A to M is

$$AM = \sqrt{\left(\frac{x}{2}\right)^2 + (2x)^2}$$

$$AM = \sqrt{\frac{x^2}{4} + 4x^2}$$

$$AM = \frac{x\sqrt{17}}{2}$$

7 **Use trigonometry**

In right triangle AMJ,

$$\tan y^\circ = \frac{JM}{AM}$$

$$T = \frac{\frac{x(2+\sqrt{3})}{2}}{\frac{x\sqrt{17}}{2}}$$

$$T = \frac{2+\sqrt{3}}{\sqrt{17}}$$

---

8 **Rationalise the denominator**

Rationalise the denominator:

$$T = \frac{(2+\sqrt{3})\sqrt{17}}{17}$$

$$T = \frac{2\sqrt{17}+\sqrt{51}}{17}$$

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

✓ **FINAL ANSWER**

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

Question 22

5 marks | 3D Pythagoras & Trigonometry

22 Here is a cuboid $ABCDEFGH$

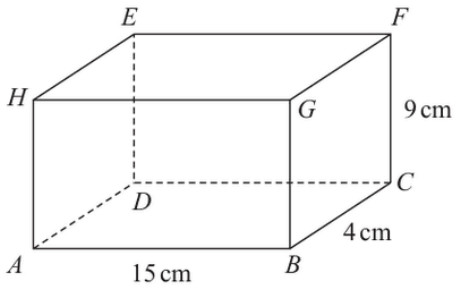


Diagram **NOT** accurately drawn

$AB = 15\,\text{cm}$ $BC = 4\,\text{cm}$ $CF = 9\,\text{cm}$

(a) Work out the length of BE
Give your answer correct to 3 significant figures.

..... cm

(2)

Here is a cuboid $PQRSTUVW$

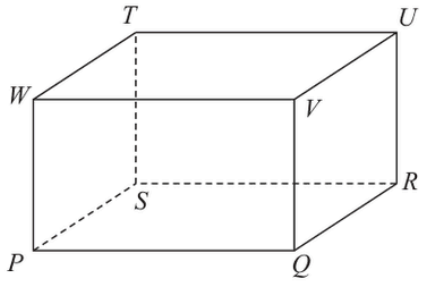


Diagram **NOT** accurately drawn

$PR = 42\,\text{cm}$

The size of the angle between PU and the plane $PQRS$ is 30°

M is the midpoint of PR

(b) Work out the size of angle UMR
Give your answer correct to 3 significant figures.

.....°

(3)

(Total for Question 22 is 5 marks)

1 **Find the gradient**

For part (a), BE is the space diagonal using the three perpendicular lengths:

$$AB = 15, \qquad BC = 4, \qquad CF = 9$$

$$BE^2 = 15^2 + 4^2 + 9^2$$

$$BE^2 = 225 + 16 + 81 = 322$$

$$BE = \sqrt{322} = 17.9 \text{ cm}$$

2 **Use trigonometry**

For part (b), the projection of PU onto the plane $PQRS$ is PR.

$$PR = 42$$

3 **The angle between and the**

The angle between PU and the plane is $30°$, so

$$\tan 30° = \frac{RU}{PR}$$

$$RU = 42 \tan 30°$$

4 **Find the midpoint**

Since M is the midpoint of PR,

$$MR = 21$$

5 **Use trigonometry**

In right triangle MRU,

$$\tan \angle UMR = \frac{RU}{MR}$$

$$\tan \angle UMR = \frac{42 \tan 30°}{21}$$

$$\tan \angle UMR = 2 \tan 30°$$

$$\angle UMR = 49.1°$$

FINAL ANSWER

$BE = 17.9\ \text{cm}$, $\angle UMR = 49.1°$

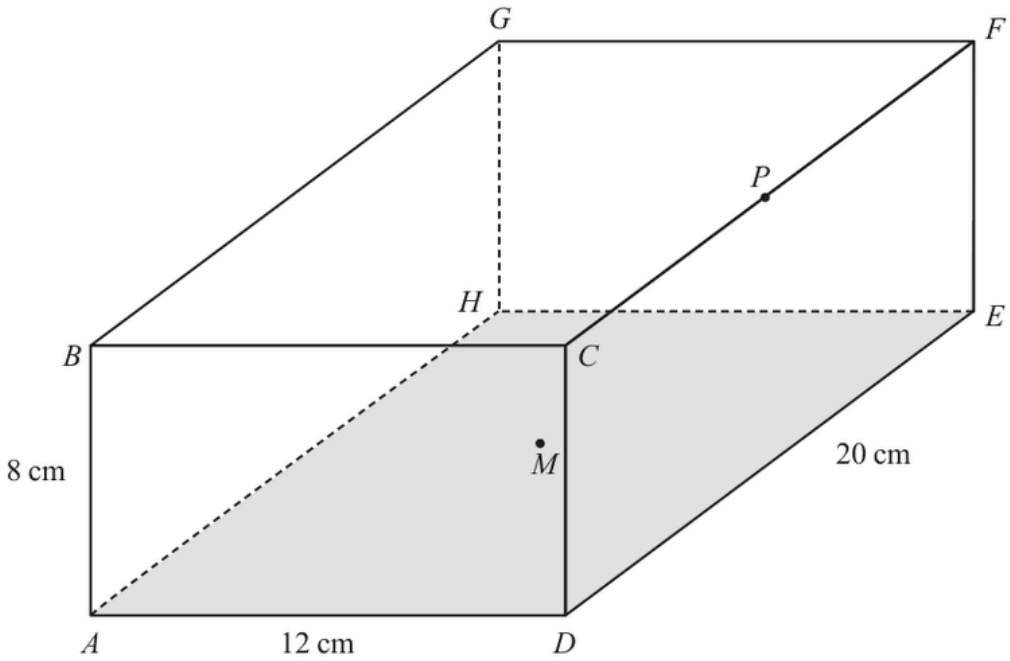
4MA1/1HR MAY 2024

Question 22

6 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a cuboid $ABCDEFGH$ with horizontal base $ADEH$

Diagram **NOT** accurately drawn



$AB = 8$ cm $\quad AD = 12$ cm $\quad DE = 20$ cm

M is the midpoint of the base $ADEH$ and P is the midpoint of the edge CF

Work out the size of angle BMP
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May 2024

1 **Use trigonometry**

Use coordinates with

$$A = (0, 0, 0)$$

2 **Use trigonometry**

Since

$$AD = 12, \qquad DE = 20, \qquad AB = 8$$

3 **take**

take

$$B = (0, 0, 8)$$

4 **The centre of the rectangular**

The centre of the rectangular base $ADEH$ is

$$M = (6, 10, 0)$$

5 **Find the midpoint**

Point P is the midpoint of CF.

$$C = (12, 0, 8), \qquad F = (12, 20, 8)$$

$$P = (12, 10, 8)$$

6 **Use trigonometry**

Now

$$\vec{MB} = B - M = (-6, -10, 8)$$

$$\vec{MP} = P - M = (6, 0, 8)$$

7 Use the scalar product formula

Use the scalar product formula:

$$\vec{MB} \cdot \vec{MP} = (-6)(6) + (-10)(0) + (8)(8)$$

$$\vec{MB} \cdot \vec{MP} = 28$$

$$|\vec{MB}| = \sqrt{(-6)^2 + (-10)^2 + 8^2} = \sqrt{200}$$

$$|\vec{MP}| = \sqrt{6^2 + 0^2 + 8^2} = 10$$

$$\cos \angle BMP = \frac{28}{10\sqrt{200}}$$

$$\angle BMP = 78.6^\circ$$

FINAL ANSWER

78.6°

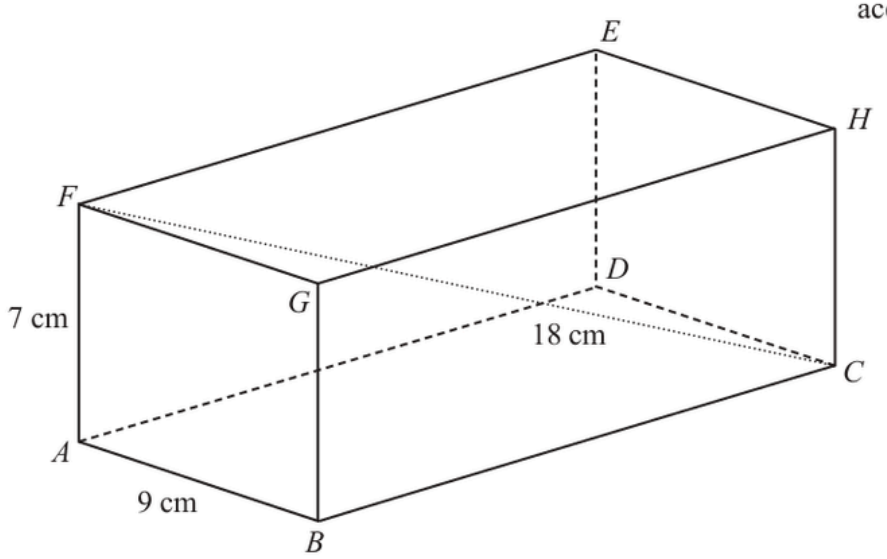
4MA1/1HR JUNE 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2025

1 **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

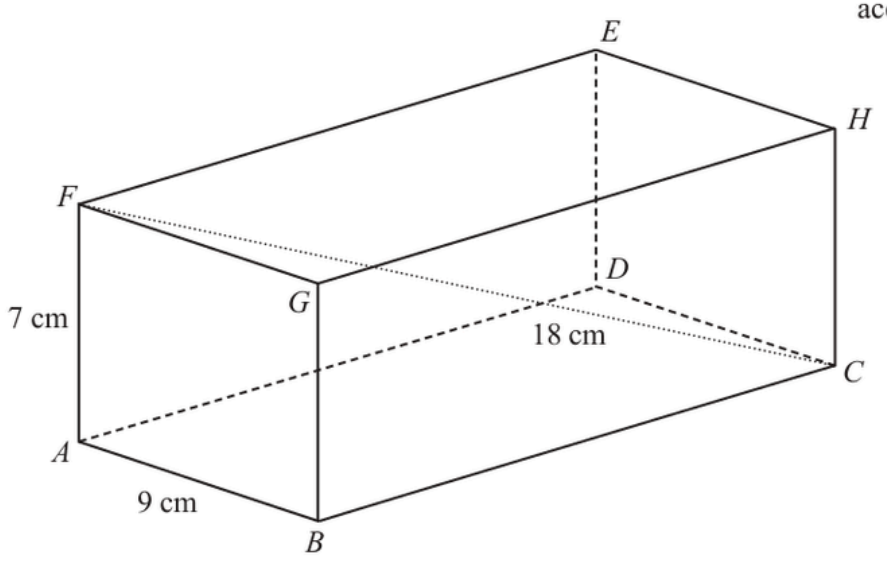
4WM1/01R MAY 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01R May 2025

1 **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

4MA1/2HR JUNE 2025

Question 23

3 marks | 3D Pythagoras & Trigonometry

23 The diagram shows a triangular prism, $ABCDEF$, with a horizontal rectangular base $ABCD$

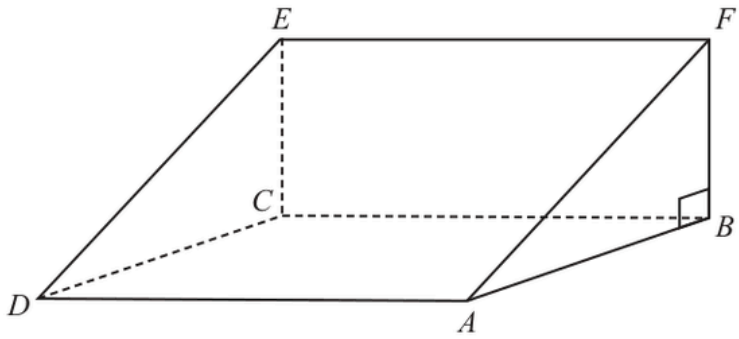


Diagram **NOT** accurately drawn

M is the midpoint of the line AC

$AC = 40\,\text{cm}$ angle $CAE = 35°$ angle $ABF = 90°$

Work out the size of angle CME
Give your answer correct to 3 significant figures.

.....°

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2025

1 Use trigonometry

$$AC = 40$$

2 The point is vertically above

The point E is vertically above C, so triangle ACE is right-angled at C.

$$\angle CAE = 35°$$

$$\tan 35° = \frac{CE}{AC}$$

$$CE = 40 \tan 35°$$

3 **Find the midpoint**

Since M is the midpoint of AC,

$$CM = 20$$

---

4 **Use trigonometry**

In right triangle CME,

$$\tan \angle CME = \frac{CE}{CM}$$

$$\tan \angle CME = \frac{40 \tan 35°}{20}$$

$$\tan \angle CME = 2 \tan 35°$$

$$\angle CME = 54.5°$$

✓ **FINAL ANSWER**

$54.5°$

4WM1/01 MAY 2025

Question 23

6 marks | 3D Pythagoras & Trigonometry

23 The diagram shows cuboid $ABCDEFGH$

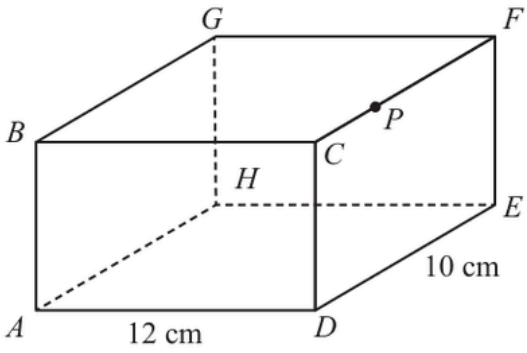


Diagram **NOT** accurately drawn

The point P lies on CF

$AD = 12$ cm $\quad DE = 10$ cm $\quad CP = 3$ cm

The angle of elevation of P from A is $24°$

Calculate the size of angle APG
Give your answer correct to the nearest degree.
Show your working clearly.

.....°

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 May 2025

1 **Point is cm from along**

Point P is 3 cm from C along CF. The horizontal distance from A to the point directly below P is

$$\sqrt{12^2 + 3^2} = \sqrt{153}$$

2 **Use trigonometry**

Using the angle of elevation,

$$h = \sqrt{153}\tan 24° = 5.507\ldots$$

3 **Use trigonometry**

So

$$AP = \frac{\sqrt{153}}{\cos 24^\circ} = 13.5399\ldots$$

4 **Use trigonometry**

On the top face,

$$PG = \sqrt{12^2 + 7^2} = \sqrt{193} = 13.8924\ldots$$

5 **Use trigonometry**

Also,

$$AG = \sqrt{12^2 + 10^2 + h^2} = 16.561\ldots$$

6 **Use cosine rule**

Use the cosine rule in triangle APG.

$$\cos \angle APG = \frac{AP^2 + PG^2 - AG^2}{2(AP)(PG)}$$

$$\angle APG = 49.163\ldots^\circ$$

7 **Use trigonometry**

To the nearest degree,

$$49^\circ$$

✓ **FINAL ANSWER**

49°

4MA1/2H JUNE 2025

Question 24

4 marks | 3D Pythagoras & Trigonometry

24 The diagram shows a square-based pyramid $ABCDE$

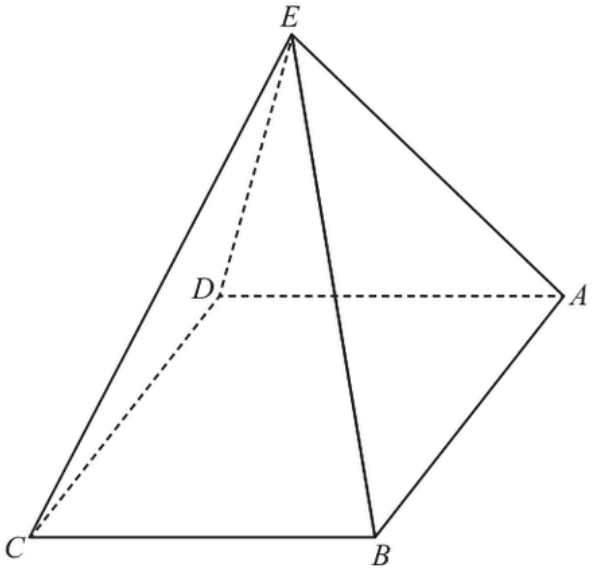


Diagram **NOT** accurately drawn

$EA = EB = EC = ED$

M is the centre of the horizontal square base $ABCD$
Q is the midpoint of AB
Angle $EQM = 80°$

$EA : AB = n : 1$

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2025

1 **Let**

Let

$$AB = s$$

2 **Find the midpoint**

Since M is the centre of the square base and Q is the midpoint of AB,

$$QM = \frac{s}{2}$$

3 **Use trigonometry**

Since $EA = EB = EC = ED$, the point E is vertically above M.

4 **Use trigonometry**

In right triangle EQM,

$$\tan 80° = \frac{EM}{QM}$$

$$EM = \frac{s}{2}\tan 80°$$

5 **The distance from the centre**

The distance from the centre of a square to a vertex is half the diagonal:

$$AM = \frac{s\sqrt{2}}{2}$$

6 **Use trigonometry**

In right triangle EMA,

$$EA^2 = EM^2 + AM^2$$

$$EA^2 = \left(\frac{s}{2}\tan 80°\right)^2 + \left(\frac{s\sqrt{2}}{2}\right)^2$$

$$EA = s\sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

7 **Use trigonometry**

So

$$\frac{EA}{AB} = \sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

$$n = 2.92$$

FINAL ANSWER

$n = 2.92$

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 The diagram shows prism $ABCDEFGH$

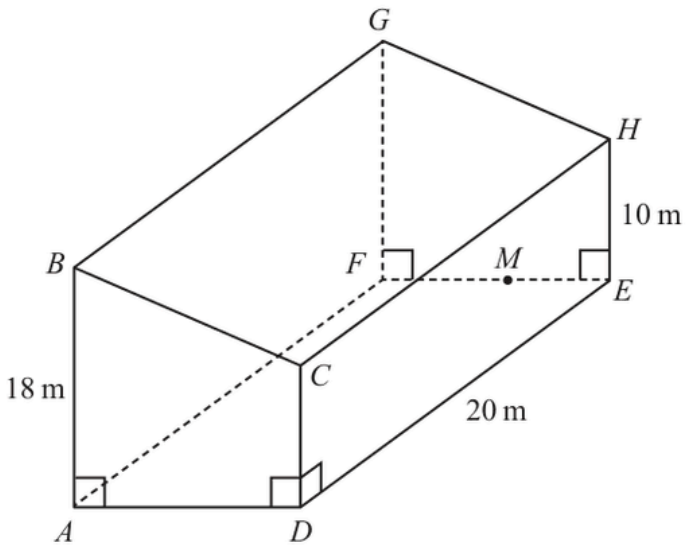


Diagram **NOT** accurately drawn

$$AB = FG = 18\,\text{m} \qquad DC = EH = 10\,\text{m} \qquad AD = FE$$
$$DE = CH = AF = BG = 20\,\text{m}$$
$$\text{angle } BAD = \text{angle } ADC = \text{angle } CDE = 90°$$

M is the point on FE such that $FM = \frac{3}{5}FE$

Given that angle $GMF = 60°$

calculate the angle of elevation of G from D
Give your answer correct to one decimal place.
Show your working clearly.

.....°

(Total for Question 26 is 5 marks)

1 **Use trigonometry**

From triangle GMF,

$$\tan 60^\circ = \frac{FG}{FM} = \frac{18}{FM}$$

$$FM = \frac{18}{\tan 60^\circ} = 6\sqrt{3}$$

2 **Simplify surd**

Since $FM = \frac{3}{5}FE$,

$$FE = 10\sqrt{3}$$

3 **Simplify surd**

The horizontal distance from D to the point below G is

$$\sqrt{20^2 + (10\sqrt{3})^2} = 10\sqrt{7}$$

4 **Use trigonometry**

If θ is the angle of elevation,

$$\tan\theta = \frac{18}{10\sqrt{7}}$$

$$\theta \approx 34.2289^\circ$$

✓ **FINAL ANSWER**

34.2°.

4MA1/1H NOVEMBER 2025

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 Here is a prism $ABCDEFGH$ with a horizontal, rectangular base $ABGF$

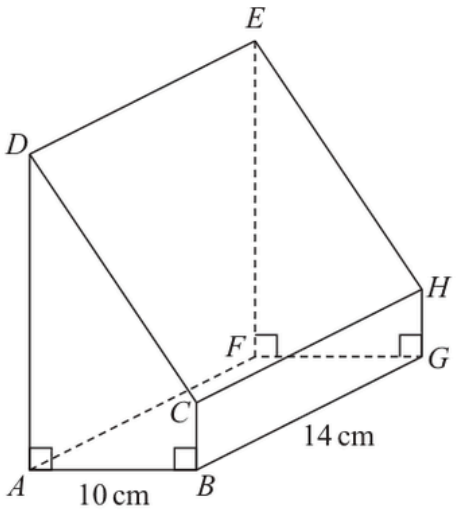


Diagram **NOT** accurately drawn

$AB = 10\,\text{cm}$ $BG = 14\,\text{cm}$

angle DAB = angle ABC = angle EFG = angle $FGH = 90°$

$BC = \frac{1}{5}AD$

The angle of elevation of E from B is $50°$

Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^3

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H November 2025

1 **The horizontal base is a**

The horizontal base $ABGF$ is a rectangle.

$$AB = 10, \qquad BG = 14$$

2 **Use trigonometry**

So the horizontal distance from B to F is

$$BF = \sqrt{10^2 + 14^2} = \sqrt{296}$$

3 **Use trigonometry**

The angle of elevation of E from B is $50°$.

4 **Use trigonometry**

Since E is vertically above F,

$$\tan 50° = \frac{FE}{BF}$$

$$FE = \sqrt{296} \tan 50°$$

5 **The prism has the same**

The prism has the same cross section throughout, so

$$AD = FE = \sqrt{296} \tan 50°$$

6 **Use trigonometry**

Also

$$BC = \frac{1}{5} AD$$

7 **Area of the trapezium cross**

Area of the trapezium cross section $ABCD$:

$$\frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}\left(AD + \frac{1}{5}AD\right) \times 10$$

$$= 6AD$$

8 **Volume of the prism**

Volume of the prism:

$$6AD \times 14 = 84AD$$

$$= 84\sqrt{296}\tan 50°$$

$$= 1722.311\ldots$$

9 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1720$$

✓ **FINAL ANSWER**

$$\boxed{1720\ \text{cm}^3}$$

TOPIC

Sine & Cosine Rules

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 21

6 marks | Sine & Cosine Rules

21 The diagram shows the prism $ABCDEF$ with cross section triangle ABC.

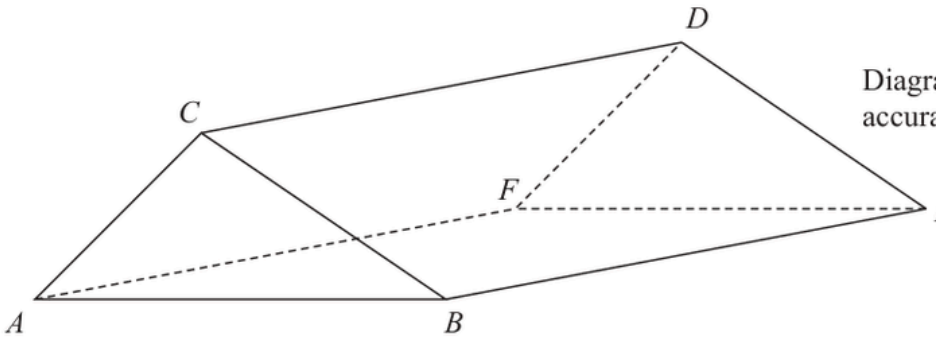


Diagram **NOT** accurately drawn

Angle $BEC = 40°$ and angle ACB is obtuse.
$AC = 6\,\text{cm}$ and $CE = 13\,\text{cm}$

The area of triangle ABC is $22\,\text{cm}^2$

Calculate the length of AB.
Give your answer correct to one decimal place.

..... cm

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2020

1 Use trigonometry

In triangle BEC, $CE = 13$ and $\angle BEC = 40°$.

2 Find the gradient

Since this is a prism, BE is perpendicular to the cross section, so triangle BEC is right-angled at B.

$$BC = 13 \sin 40°$$

$$BC = 8.356238\ldots$$

3 The area of triangle is

The area of triangle ABC is $22\ \text{cm}^2$.

4 **Use trigonometry**

Using

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$22 = \frac{1}{2}(AC)(BC)\sin(\angle ACB)$$

$$22 = \frac{1}{2}(6)(8.356238\ldots)\sin(\angle ACB)$$

$$\sin(\angle ACB) = 0.877587\ldots$$

---

5 **Use trigonometry**

The angle ACB is obtuse, so

$$\angle ACB = 180° - \sin^{-1}(0.877587\ldots)$$

$$\angle ACB = 118.647\ldots°$$

---

6 **Use the cosine rule**

Use the cosine rule:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos(\angle ACB)$$

$$AB^2 = 6^2 + (8.356238\ldots)^2 - 2(6)(8.356238\ldots)\cos(118.647\ldots°)$$

$$AB = 12.4056\ldots$$

---

7 **Correct to one decimal place**

Correct to one decimal place:

$$12.4$$

✓ **FINAL ANSWER**

$\boxed{12.4\text{ cm}}$

4MA1/2H JANUARY 2022

Question 23

6 marks | Sine & Cosine Rules

23 The diagram shows triangle PQR

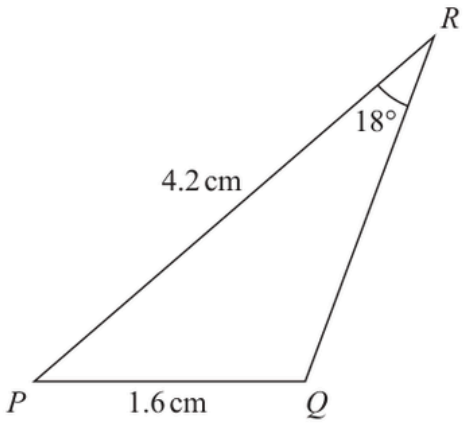


Diagram **NOT** accurately drawn

$PQ = 1.6\,\text{cm}$ $PR = 4.2\,\text{cm}$ Angle $PRQ = 18°$

Given that angle PQR is obtuse,

work out the area of triangle PQR
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2022

1 **Use trigonometry**

In triangle PQR,

$$PQ = 1.6, \qquad PR = 4.2, \qquad \angle PRQ = 18°$$

2 **Use the sine rule**

Use the sine rule:

$$\frac{\sin Q}{PR} = \frac{\sin R}{PQ}$$

$$\frac{\sin Q}{4.2} = \frac{\sin 18^\circ}{1.6}$$

$$\sin Q = \frac{4.2 \sin 18^\circ}{1.6}$$

$$\sin Q = 0.811169\ldots$$

---

3 **Use trigonometry**

Since angle PQR is obtuse,

$$Q = 180^\circ - \sin^{-1}(0.811169\ldots)$$

$$Q = 125.7896\ldots^\circ$$

---

4 **Use trigonometry**

Now find angle P:

$$P = 180^\circ - 18^\circ - 125.7896\ldots^\circ$$

$$P = 36.2103\ldots^\circ$$

---

5 **Area of triangle**

Area of triangle PQR:

$$\frac{1}{2}(PQ)(PR)\sin P$$

$$= \frac{1}{2}(1.6)(4.2)\sin(36.2103\ldots^\circ)$$

$$= 1.984925\ldots$$

---

6 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1.98$$

FINAL ANSWER

$\boxed{1.98\ \text{cm}^2}$

4MA1/2H JUNE 2023

Question 22

5 marks | Sine & Cosine Rules

22 The diagram shows a triangle ABC and a flagpole BF

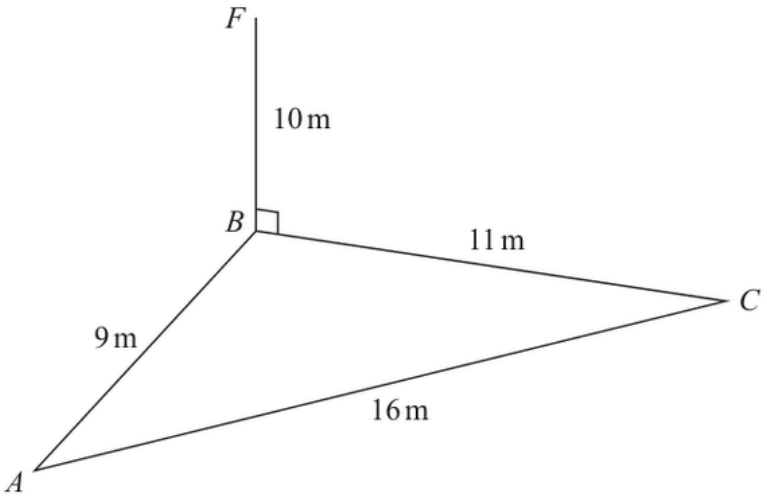


Diagram **NOT** accurately drawn

A, B and C are points on horizontal ground.

BF is vertical.

$AB = 9\,\text{m}$ $BC = 11\,\text{m}$ $AC = 16\,\text{m}$ $BF = 10\,\text{m}$

D is the point on AC such that angle $BDC = 90°$

Work out the size of the angle of elevation of the point F from the point D
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2023

1 **Use trigonometry**

In triangle ABC,

$$AB = 9, \qquad BC = 11, \qquad AC = 16$$

2 **Use Heron s formula**

Use Heron's formula.

$$s = \frac{9 + 11 + 16}{2} = 18$$

$$\text{area} = \sqrt{18(18-9)(18-11)(18-16)}$$

$$= \sqrt{18 \cdot 9 \cdot 7 \cdot 2} = 47.6235\ldots$$

---

3 **is the foot of the**

D is the foot of the perpendicular from B to AC, so BD is the height of triangle ABC.

$$\text{area} = \frac{1}{2} \times AC \times BD$$

$$47.6235\ldots = \frac{1}{2} \times 16 \times BD$$

$$BD = 5.9529\ldots$$

---

4 **Use trigonometry**

In right-angled triangle BDF,

$$\tan\theta = \frac{BF}{BD}$$

$$\tan\theta = \frac{10}{5.9529\ldots}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.9529\ldots}\right) = 59.2349\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{59.2^\circ}$

4MA1/1HR MAY 2023

Question 25

6 marks | Sine & Cosine Rules

25 Here is a triangle ABC

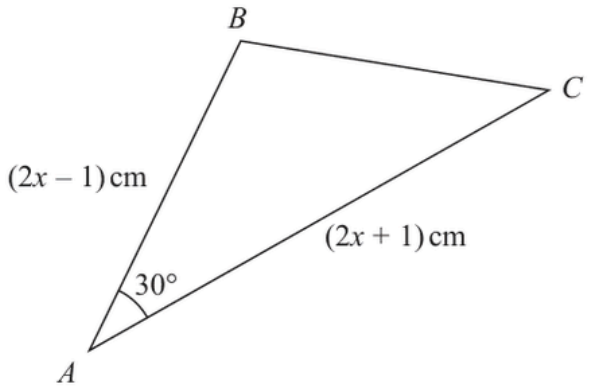


Diagram **NOT** accurately drawn

The area of the triangle is $(x^2 + x - 3.75)\,\text{cm}^2$

Find the size of the largest angle in triangle ABC
Give your answer correct to the nearest degree.

.....°

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR May 2023

1 The two sides around the

The two sides around the $30°$ angle are

$$2x - 1 \quad \text{and} \quad 2x + 1$$

2 Use trigonometry

Using the area formula,

$$\text{area} = \frac{1}{2}ab\sin C$$

$$x^2 + x - 3.75 = \frac{1}{2}(2x - 1)(2x + 1)\sin 30°$$

3 **Use trigonometry**

Since $\sin 30° = \frac{1}{2}$,

$$x^2 + x - 3.75 = \frac{1}{4}(2x - 1)(2x + 1)$$

$$x^2 + x - 3.75 = \frac{1}{4}(4x^2 - 1)$$

$$x^2 + x - 3.75 = x^2 - 0.25$$

$$x = 3.5$$

---

4 **Use trigonometry**

So

$$AB = 2x - 1 = 6, \qquad AC = 2x + 1 = 8$$

---

5 **Use cosine rule**

Use the cosine rule to find BC:

$$BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 30°$$

$$BC = 4.1063\ldots$$

---

6 **Use trigonometry**

The largest side is $AC = 8$, so the largest angle is $\angle B$.

---

7 **Use trigonometry**

Using the cosine rule again:

$$\cos B = \frac{6^2 + BC^2 - 8^2}{2(6)(BC)}$$

$$B = 103.0643\ldots°$$

✓ **FINAL ANSWER**

$\boxed{103°}$

Question 20

5 marks | Sine & Cosine Rules

20

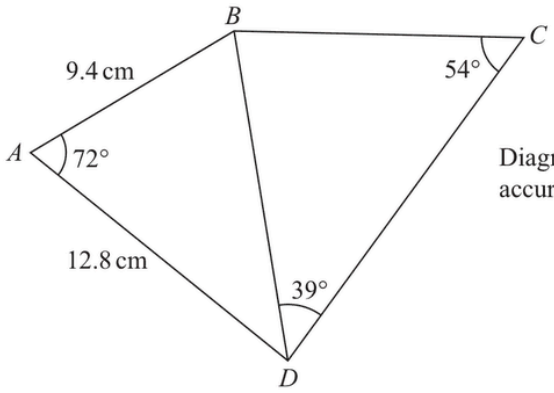


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

1 **Use trigonometry**

First use triangle ABD.

$$AB = 9.4, \quad AD = 12.8, \quad \angle BAD = 72°$$

2 **Use cosine rule**

Use the cosine rule to find BD:

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

3 **Use trigonometry**

In triangle BCD,

$$\angle BDC = 39°, \quad \angle BCD = 54°$$

4 **Use the sine rule**

Use the sine rule:

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = \frac{13.3356\ldots \sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

5 **Use trigonometry**

Correct to 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

10.4 cm

4WM1/01 MAY 2025

Question 21

5 marks | Sine & Cosine Rules

21

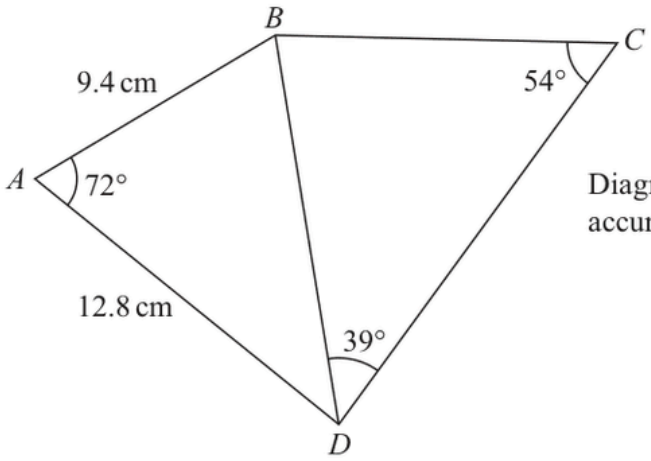


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 May 2025

1 **Use trigonometry**

First find BD in triangle ABD using the cosine rule.

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

2 **Use trigonometry**

In triangle BCD, use the sine rule.

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = 13.3356\ldots \times \frac{\sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

3 **Use trigonometry**

To 3 significant figures,

$$BC = 10.4$$

FINAL ANSWER

$10.4\ \text{cm}$

TOPIC

Histograms

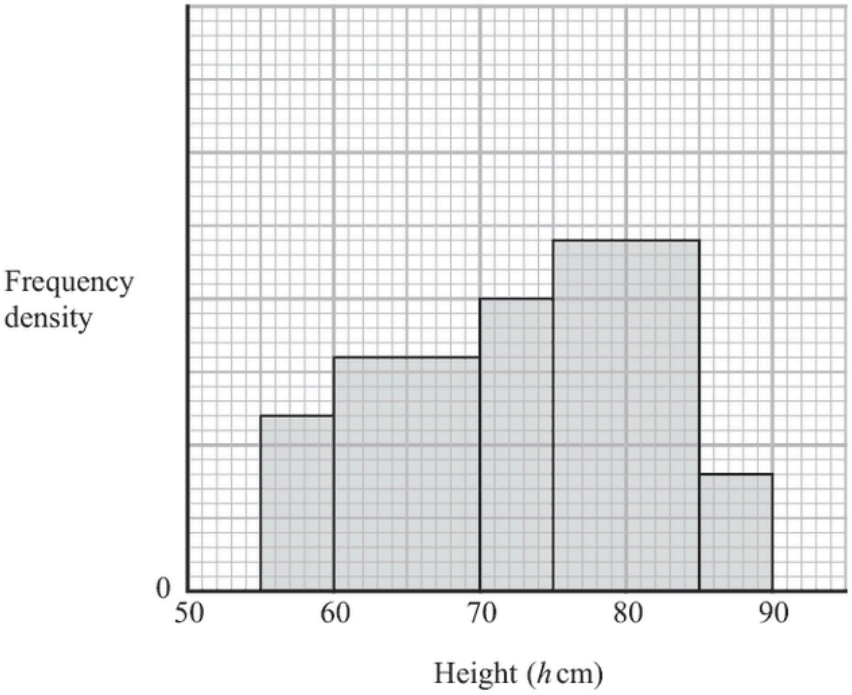
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JANUARY 2022

Question 20

4 marks | Histograms

20 The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $75 < h \leq 85$, the frequency is 12 and the class width is 10, so

$$\text{frequency density} = \frac{12}{10} = 1.2$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$0.6,\ 0.8,\ 1.0,\ 1.2,\ 0.4$$

4 The total number of tomato

The total number of tomato plants is

$$(5)(0.6) + (10)(0.8) + (5)(1.0) + (10)(1.2) + (5)(0.4)$$

$$= 3 + 8 + 5 + 12 + 2 = 30$$

5 Solve inequality

For $h > 82.5$:

$$(2.5)(1.2) + (5)(0.4) = 3 + 2 = 5$$

6 Calculate probability

So the probability is

$$\frac{5}{30} = \frac{1}{6}$$

FINAL ANSWER

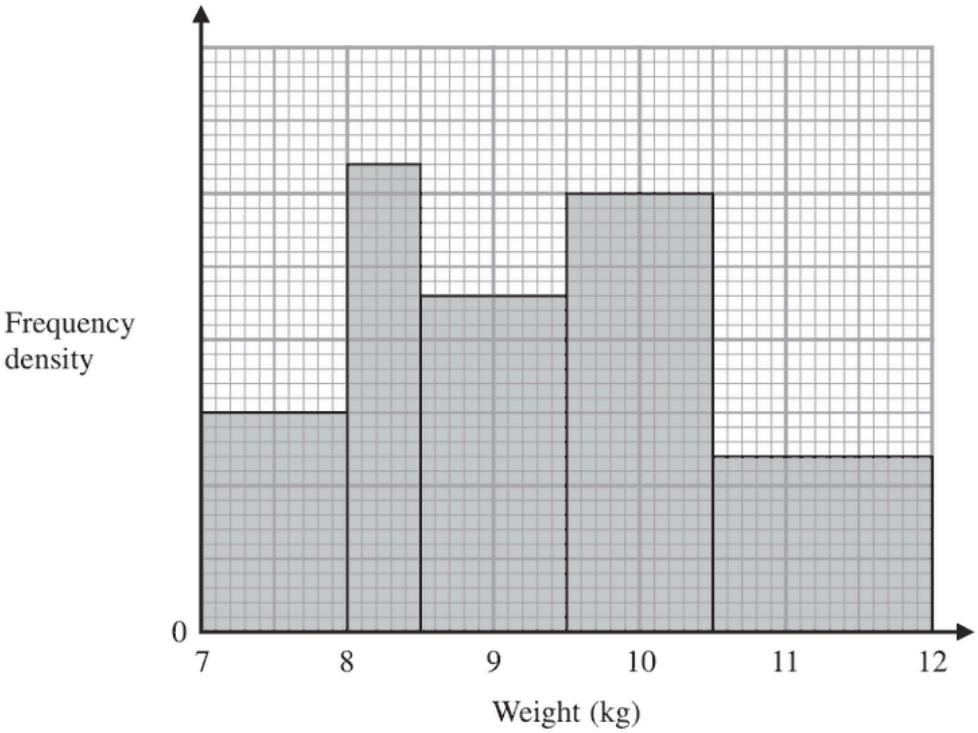
$\frac{1}{6}$

4MA1/1HR JANUARY 2022

Question 21

3 marks | Histograms

21



The histogram gives information about the weights, in kg, of all the watermelons in a field.

There are 16 watermelons with a weight between 8 kg and 8.5 kg

Work out the total number of watermelons in the field.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $8 \leq w < 8.5$, the frequency is 16 and the class width is 0.5, so

$$\text{frequency density} = \frac{16}{0.5} = 32$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$15,\ 32,\ 23,\ 30,\ 12$$

4 The total frequency is

The total frequency is

$$(1)(15) + (0.5)(32) + (1)(23) + (1)(30) + (1.5)(12)$$

$$= 15 + 16 + 23 + 30 + 18$$

$$= 102$$

FINAL ANSWER

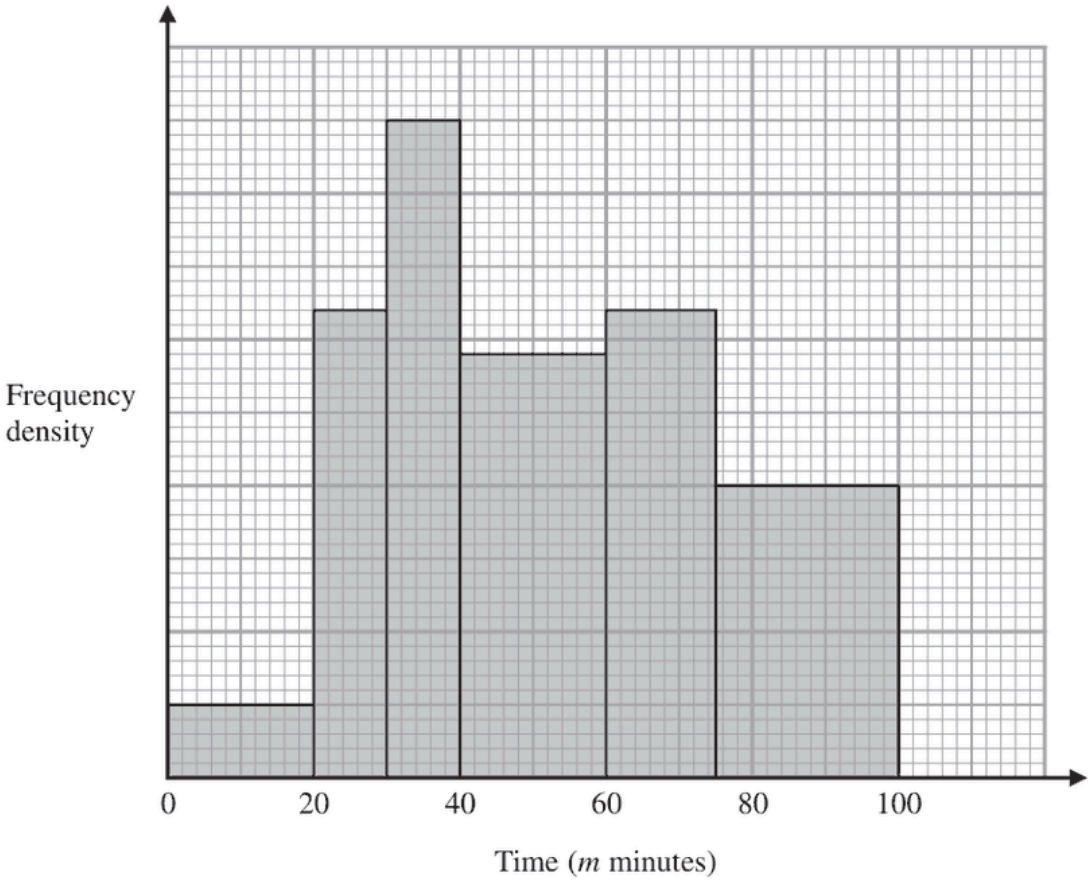
102 watermelons

4MA1/1H JUNE 2022

Question 21

3 marks | Histograms

21 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$

Work out an estimate for the number of children for whom $50 < m \leqslant 80$

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2022

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $m \leq 20$, the frequency is 10 and the class width is 20, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

---

3 **Use histogram**

Using this scale from the histogram:

$$40 < m \leq 60 \quad \text{has frequency density } 2.9$$

$$60 < m \leq 75 \quad \text{has frequency density } 3.2$$

$$75 < m \leq 100 \quad \text{has frequency density } 2.0$$

---

4 **Use part of each relevant**

For $50 < m \leq 80$, use part of each relevant bar:

$$(10)(2.9) + (15)(3.2) + (5)(2.0)$$

$$= 29 + 48 + 10$$

$$= 87$$

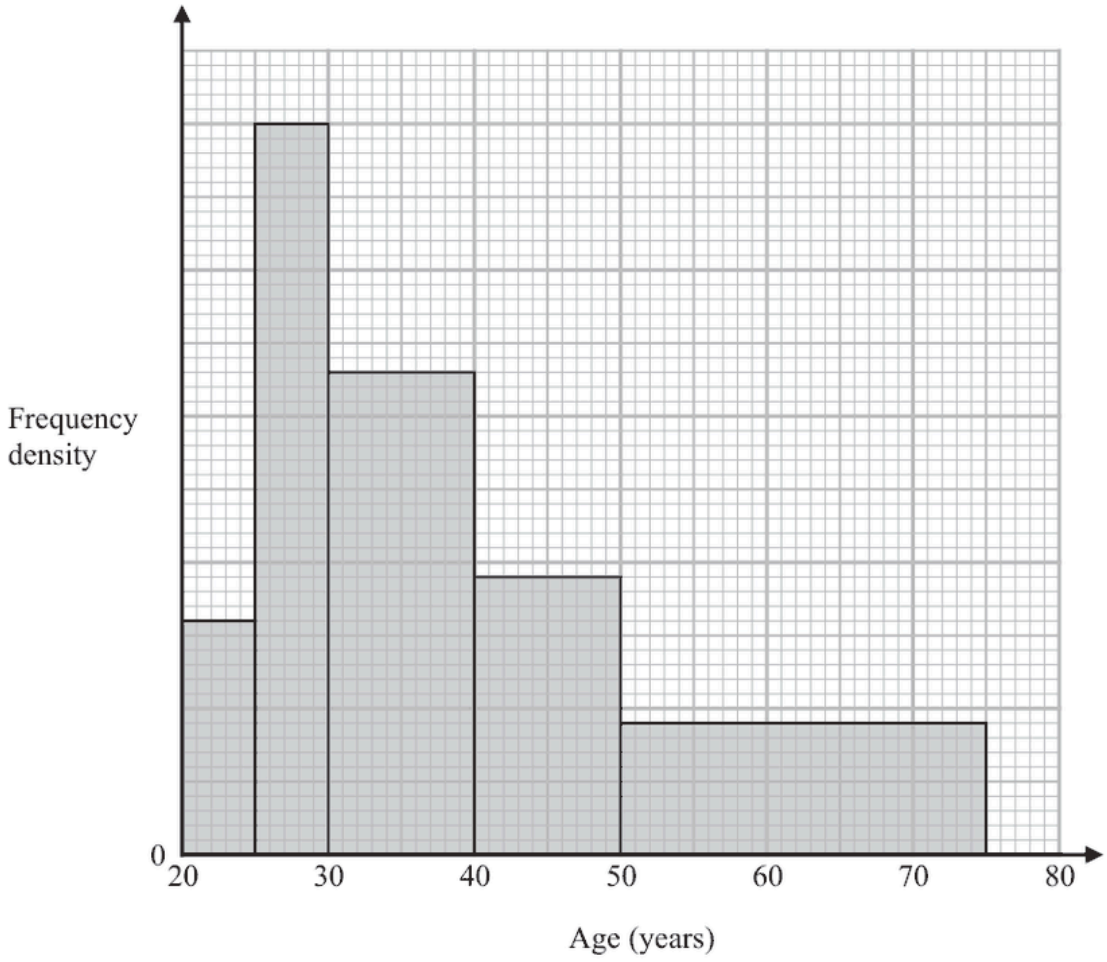
✓ **FINAL ANSWER**

87 children

Question 22

4 marks | Histograms

22 Some people attend a concert.
The histogram shows information about the ages of these people.



Work out an estimate for the percentage of these people who are aged more than 55 years.
Give your answer correct to one decimal place.

.....%

(Total for Question 22 is 4 marks)

1 The scale of the frequency

The scale of the frequency density axis is not needed because the percentage is found from areas.

2 Calculate area

Using the bar heights from the histogram, the relative areas are:

$$(5)(277) + (5)(866) + (10)(571) + (10)(329) + (25)(155)$$

$$= 1385 + 4330 + 5710 + 3290 + 3875$$

$$= 18590$$

3 Calculate value

For ages more than 55, use the part of the last bar from 55 to 75:

$$(20)(155) = 3100$$

4 Calculate percentage

So the estimated percentage is

$$\frac{3100}{18590} \times 100 = 16.675\ldots$$

FINAL ANSWER

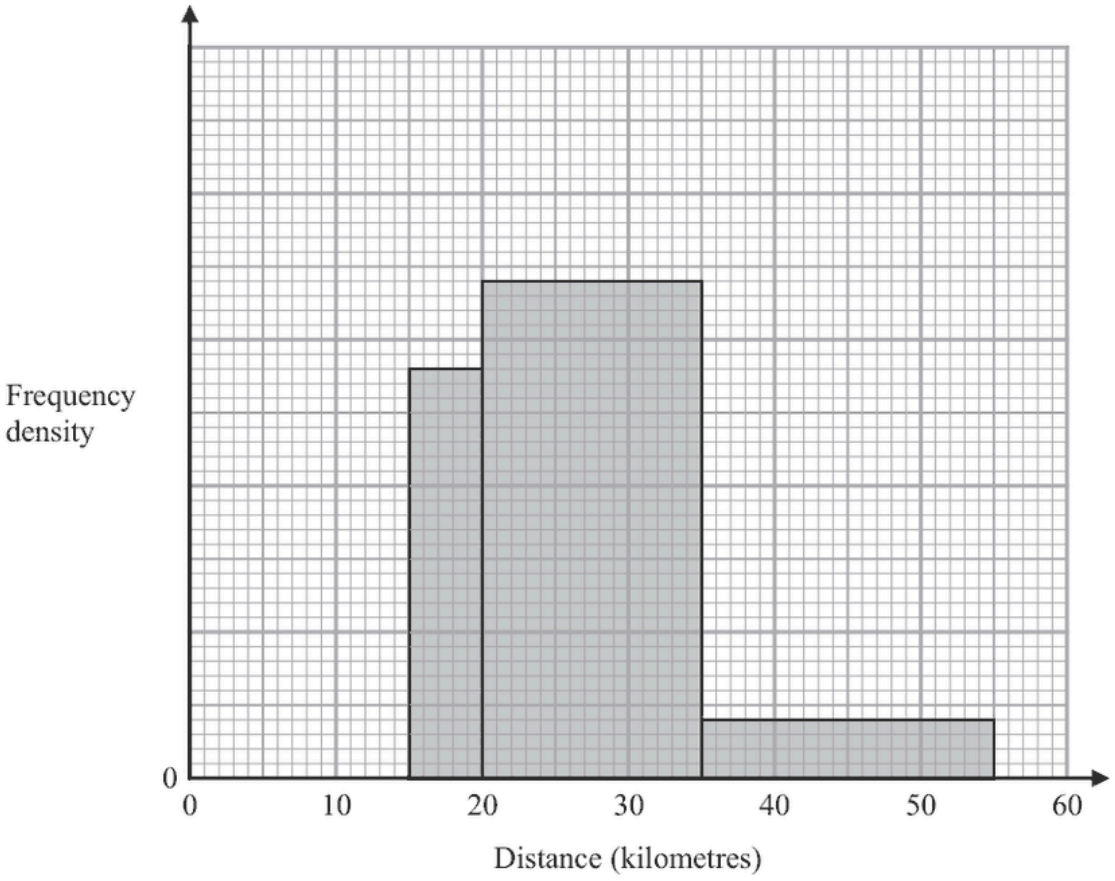
16.7%

4MA1/1H MAY 2024

Question 22

3 marks | Histograms

22 The incomplete histogram shows some information about the distances, in kilometres, that 100 adults ran last week.



All of the adults ran at least 5 kilometres.
None of the adults ran more than 55 kilometres.
14 adults ran between 15 kilometres and 20 kilometres.

Complete the histogram.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2024

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $15 \leq d < 20$, the frequency is 14 and the class width is 5, so

$$\text{frequency density} = \frac{14}{5} = 2.8$$

3 **Use histogram**

Using this scale from the histogram:

$$20 \leq d < 35 \quad \text{has frequency density } 3.4$$

$$35 \leq d \leq 55 \quad \text{has frequency density } 0.4$$

4 **Calculate value**

So the known frequencies are

$$14 + (15)(3.4) + (20)(0.4)$$

$$= 14 + 51 + 8 = 73$$

5 **Use frequency table**

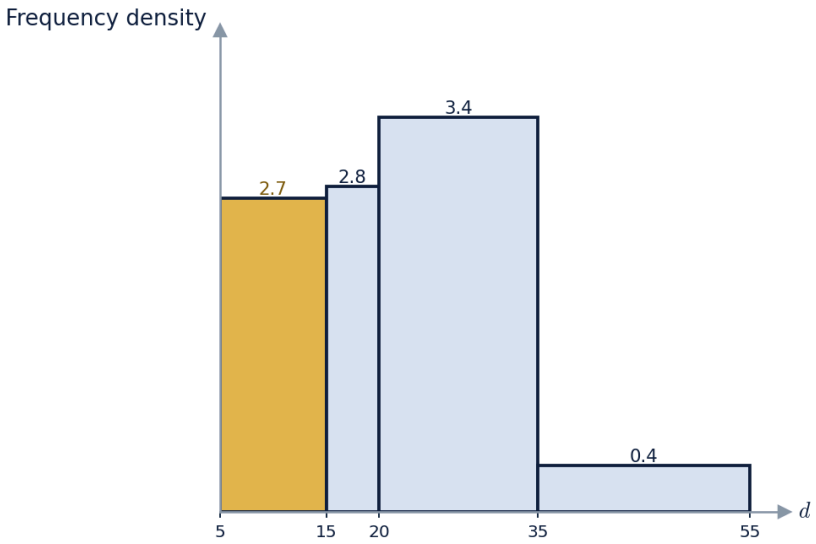
There are 100 adults in total, so the missing frequency from 5 km to 15 km is

$$100 - 73 = 27$$

6 **The missing class width is**

The missing class width is 10, so its frequency density is

$$\frac{27}{10} = 2.7$$



FINAL ANSWER

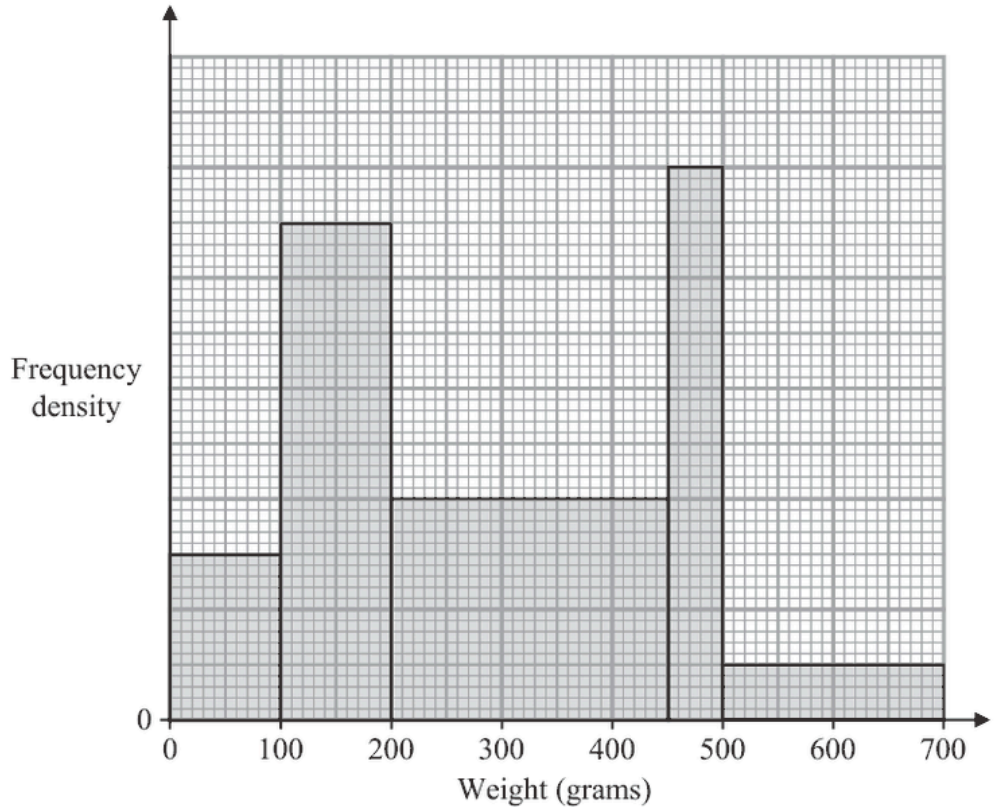
draw the missing bar from 5 to 15 with frequency density 2.7

4MA1/2HR JUNE 2025

Question 20

4 marks | Histograms

20 The histogram gives some information about the weights, in grams, of some books.



75 books weigh less than 100 grams.

A book is chosen at random.

Find an estimate for the probability that this book weighs between 400 grams and 600 grams.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For books weighing less than 100 g, the frequency is 75 and the class width is 100, so

$$\text{frequency density} = \frac{75}{100} = 0.75$$

3 **Use histogram**

Using this scale from the histogram, the frequency densities are:

$$0.75,\ 2.25,\ 1.0,\ 2.5,\ 0.25$$

4 **The total number of books**

The total number of books is

$$(100)(0.75) + (100)(2.25) + (250)(1.0) + (50)(2.5) + (200)(0.25)$$

$$= 75 + 225 + 250 + 125 + 50 = 725$$

5 **Calculate value**

For weights between 400 g and 600 g:

$$(50)(1.0) + (50)(2.5) + (100)(0.25)$$

$$= 50 + 125 + 25 = 200$$

6 **Calculate probability**

So the probability is

$$\frac{200}{725} = \frac{8}{29}$$

✓ **FINAL ANSWER**

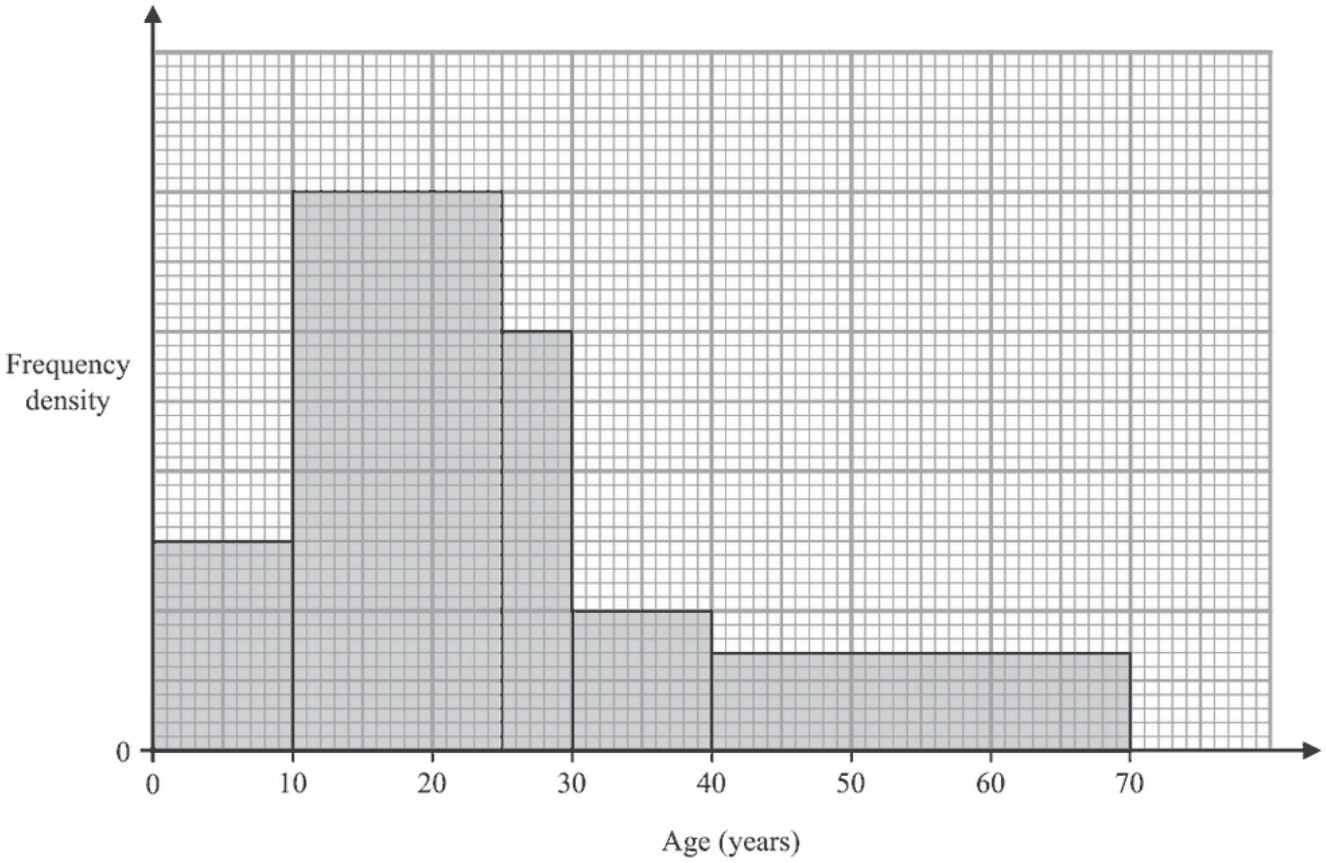
$\frac{8}{29}$ or 0.276 to 3 significant figures

4WM1/01R MAY 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the ages of the members of a chess club.



There are 60 members aged between 10 and 25
There are no members older than 70

Work out an estimate for the number of members who are over 35 years of age.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4WM1/01R May 2025

1 **The interval to has width**

The interval 10 to 25 has width 15 and frequency 60, so its frequency density is

$$\frac{60}{15} = 4$$

2 **Use histogram**

Using that scale from the histogram:

$$35 \text{ to } 40 : 5 \times 1 = 5$$

$$40 \text{ to } 70 : 30 \times 0.75 = 22.5$$

$$5 + 22.5 = 27.5$$

✓ **FINAL ANSWER**

about 28 members.

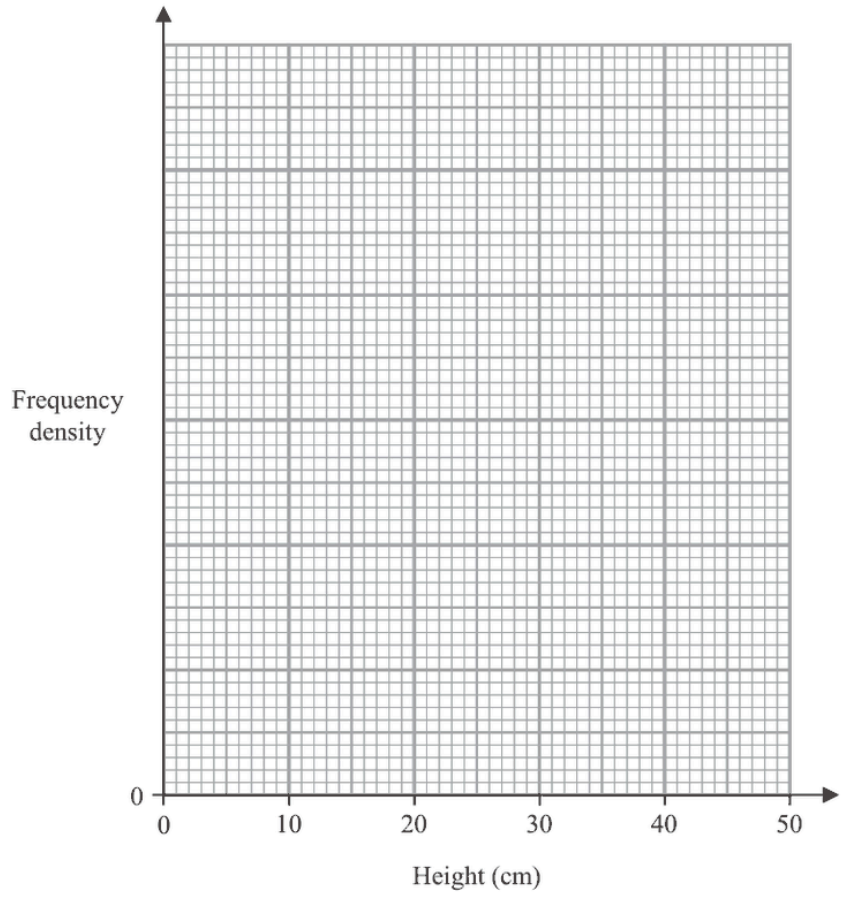
Question 20

3 marks | Histograms

20 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$0 < h \leqslant 5$	12
$5 < h \leqslant 15$	34
$15 < h \leqslant 30$	45
$30 < h \leqslant 50$	28

On the grid, draw a histogram for this information.
You must include a scale on the vertical axis.



(Total for Question 20 is 3 marks)

1 Frequency density

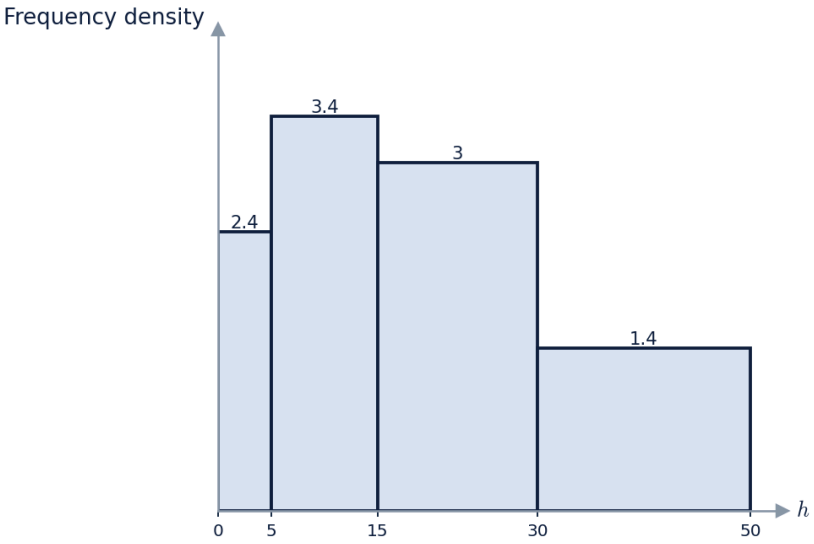
Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$.

$$0 < h \leq 5 : \frac{12}{5} = 2.4$$

$$5 < h \leq 15 : \frac{34}{10} = 3.4$$

$$15 < h \leq 30 : \frac{45}{15} = 3$$

$$30 < h \leq 50 : \frac{28}{20} = 1.4$$



FINAL ANSWER

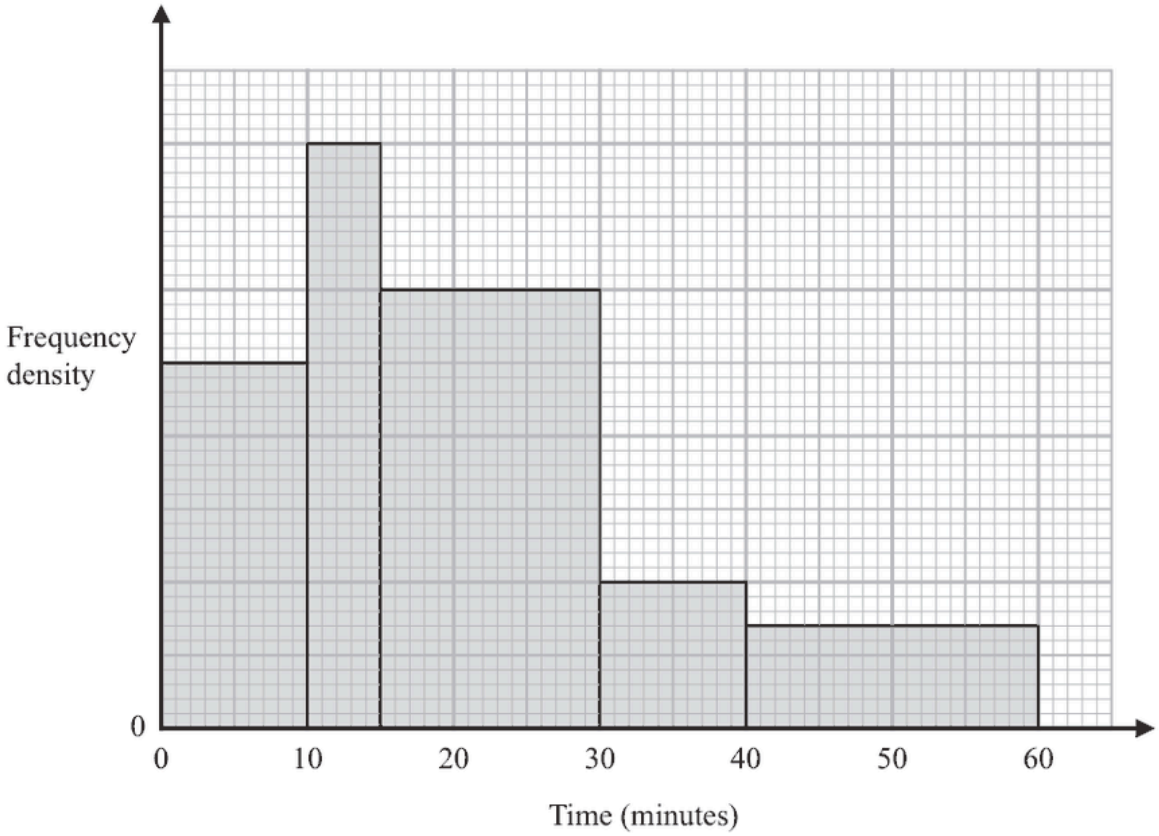
draw bars with densities $2.4, 3.4, 3, 1.4$.

4MA1/2H NOVEMBER 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the times, in minutes, that some trains arrived late at a station one day.



20 of these trains arrived between 10 minutes late and 15 minutes late.

No trains arrived more than 60 minutes late.

Work out an estimate for the number of these trains that arrived at least 25 minutes late.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $10 < t \leq 15$, the frequency is 20 and the class width is 5, so

$$\text{frequency density} = \frac{20}{5} = 4$$

---

3 **Use histogram**

Using this scale from the histogram:

$$15 < t \leq 30 \quad \text{has frequency density } 3$$

$$30 < t \leq 40 \quad \text{has frequency density } 1$$

$$40 < t \leq 60 \quad \text{has frequency density } 0.7$$

---

4 **Calculate value**

For trains at least 25 minutes late:

$$(5)(3) + (10)(1) + (20)(0.7)$$

$$= 15 + 10 + 14$$

$$= 39$$

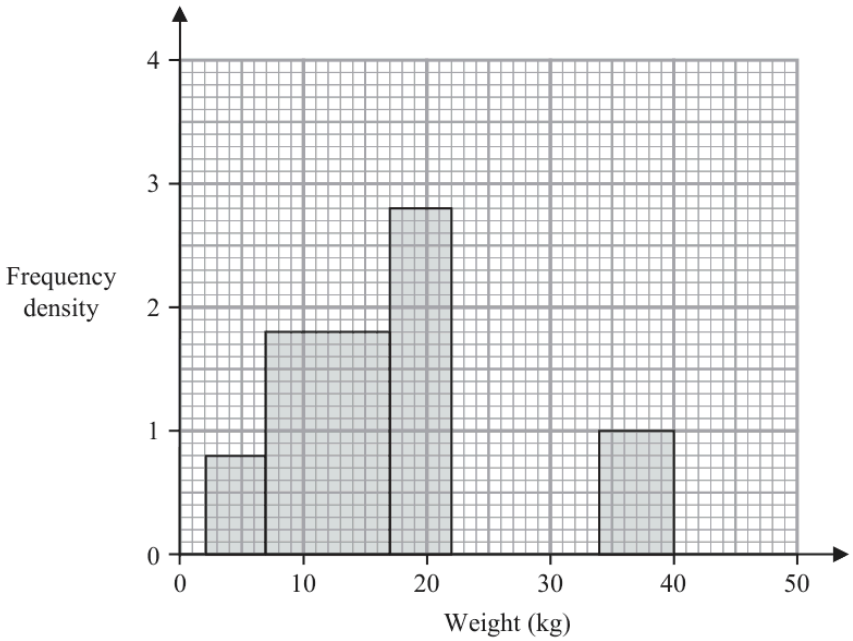
✓ **FINAL ANSWER**

39 trains

Question 21

3 marks | Histograms

21 The incomplete histogram shows information about the weights, in kg, of some parcels.



No parcels weigh less than 2 kg
No parcels weigh more than 40 kg
Half of the parcels weigh between 22 kg and 34 kg

Complete the histogram by drawing a bar to represent the parcels that weigh between 22 kg and 34 kg

(Total for Question 21 is 3 marks)

1 **Frequency = width × height**

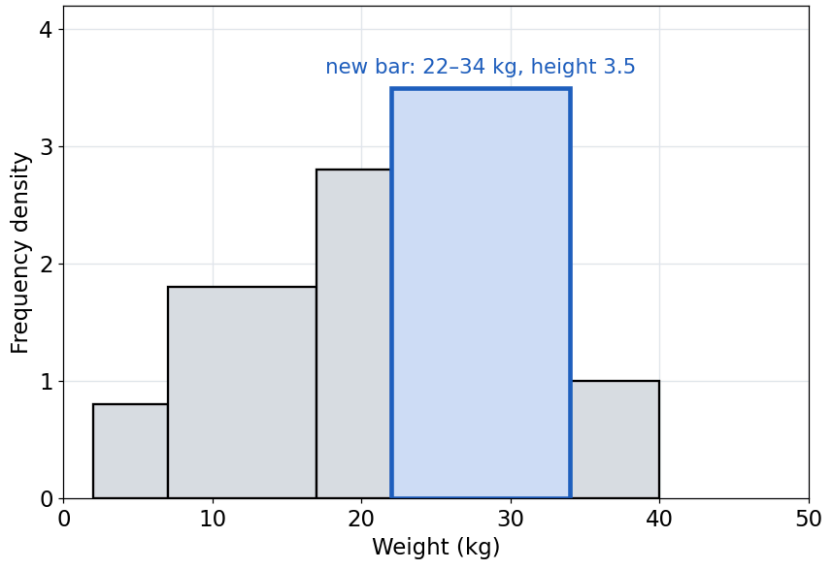
$$5(0.8) + 10(1.8) + 5(2.8) + 6(1) = 4 + 18 + 14 + 6 = 42$$

2 **Half are in 22–34**

So the other half is also 42 parcels.

3 **Height of the new bar**

$$\text{f.d.} = \frac{42}{34 - 22} = 3.5$$



FINAL ANSWER

Bar from 22 to 34 kg, height 3.5

TOPIC

Probability Toolkit

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

Question 23

4 marks | Probability Toolkit

23 In a bag, there are only

3 blue beads
4 white beads
and x orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May/November 2020

1 The total number of beads

The total number of beads is

$$x + 7$$

2 The probability of choosing two

The probability of choosing two beads of the same colour is

$$\frac{\binom{3}{2} + \binom{4}{2} + \binom{x}{2}}{\binom{x+7}{2}}$$

3 **Evaluate fraction**

This is equal to $\frac{3}{8}$, so

$$\frac{3+6+\frac{x(x-1)}{2}}{\frac{(x+7)(x+6)}{2}}=\frac{3}{8}$$

$$\frac{x^2-x+18}{(x+7)(x+6)}=\frac{3}{8}$$

$$8(x^2-x+18)=3(x+7)(x+6)$$

$$8x^2-8x+144=3x^2+39x+126$$

$$5x^2-47x+18=0$$

$$(x-9)(5x-2)=0$$

4 **Calculate value**

Since x must be a whole number,

$$x=9$$

5 **Calculate value**

So the total number of beads is

$$9+7=16$$

FINAL ANSWER

16 beads

4MA1/2HR JANUARY 2021

Question 21

6 marks | Probability Toolkit

21 A bag contains n beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of n.

$n =$ .....

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2021

1 There are beads altogether

There are n beads altogether.

2 Red beads

Red beads:

$$6$$

3 Blue beads

Blue beads:

$$n - 6$$

4 The probability of taking two

The probability of taking two beads of the same colour is

$$P(RR) + P(BB) = \frac{9}{17}$$

5 **Use trigonometry**

So

$$\frac{6}{n} \cdot \frac{5}{n-1} + \frac{n-6}{n} \cdot \frac{n-7}{n-1} = \frac{9}{17}$$

$$\frac{30 + (n-6)(n-7)}{n(n-1)} = \frac{9}{17}$$

6 **Expand**

Expand:

$$(n-6)(n-7) = n^2 - 13n + 42$$

7 **Use trigonometry**

So

$$\frac{n^2 - 13n + 72}{n(n-1)} = \frac{9}{17}$$

8 **Cross multiply**

Cross multiply:

$$17(n^2 - 13n + 72) = 9n(n-1)$$

$$17n^2 - 221n + 1224 = 9n^2 - 9n$$

$$8n^2 - 212n + 1224 = 0$$

9 **Divide by 4**

Divide by 4:

$$2n^2 - 53n + 306 = 0$$

10 **Factor**

Factor:

$$(2n - 17)(n - 18) = 0$$

11 **Use trigonometry**

So

$$n = \frac{17}{2} \quad \text{or} \quad n = 18$$

12 **The number of beads must**

The number of beads must be a whole number, so

$$n = 18$$

✓ **FINAL ANSWER**

$\boxed{18}$

4MA1/2H MAY 2021

Question 23

5 marks | Probability Toolkit

23 Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H May 2021

1 Let the number of red

Let the number of red pens be r.

2 List probability cases

Then the number of black pens is

$$r + 3$$

3 Calculate value

and

$$N = 2r + 3$$

4 Evaluate fraction

So

$$r = \frac{N - 3}{2}$$

5 **Evaluate fraction**

and

$$r+3=\frac{N+3}{2}$$

6 **The probability of black followed**

The probability of black followed by red is

$$\frac{(N+3)/2}{N}\cdot\frac{(N-3)/2}{N-1}=\frac{9}{35}$$

$$\frac{N^2-9}{4N(N-1)}=\frac{9}{35}$$

$$35(N^2-9)=36N(N-1)$$

$$35N^2-315=36N^2-36N$$

$$N^2-36N+315=0$$

$$(N-15)(N-21)=0$$

✓ **FINAL ANSWER**

$N=15$ or $N=21$

4MA1/1HR JUNE 2022

Question 24

5 marks | Probability Toolkit

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2022

1 There are counters. Initially 5

There are x counters. Initially 5 show red, so $x - 5$ show green.

2 List probability cases

After two turns, there are still 5 red faces if:

3 the same counter is picked

- the same counter is picked twice, or
- one red counter and one green counter are picked.

4 The probability of picking the

The probability of picking the same counter twice is

$$\frac{1}{x}$$

5 The probability of picking one

The probability of picking one red and one green in either order is

$$\frac{5}{x}\cdot\frac{x-5}{x}+\frac{x-5}{x}\cdot\frac{5}{x}$$

$$=\frac{10(x-5)}{x^2}$$

6 Evaluate fraction

So

$$\frac{1}{x}+\frac{10(x-5)}{x^2}=\frac{19}{32}$$

$$\frac{11x-50}{x^2}=\frac{19}{32}$$

$$32(11x-50)=19x^2$$

$$19x^2-352x+1600=0$$

$$(x-8)(19x-200)=0$$

7 Calculate value

Since x must be a whole number,

$$x=8$$

FINAL ANSWER

$x=8$

4MA1/1H NOVEMBER 2023

Question 20

4 marks | Probability Toolkit

20 A bag contains only 10 cent coins and 20 cent coins.

Josip takes at random a coin from the bag, records its value and replaces it in the bag. He then takes at random a second coin from the bag, records its value and replaces it in the bag.

Josip finds the mean value of the two coins.

The probability that the two coins have a mean value of 10 cents is $\frac{49}{121}$

Work out the probability that the two coins have a mean value of 15 cents.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2023

1 Let the probability of taking

Let the probability of taking a 10 cent coin be p.

2 The mean is 10 cents

The mean is 10 cents only when both coins are 10 cent coins.

$$p^2 = \frac{49}{121}$$

$$p = \frac{7}{11}$$

3 Calculate probability

So the probability of taking a 20 cent coin is

$$1 - \frac{7}{11} = \frac{4}{11}$$

4 The mean is 15 cents

The mean is 15 cents when one coin is 10 cents and the other is 20 cents.

5 **There are two possible orders**

There are two possible orders:

$$P(10, 20) + P(20, 10)$$

$$= \frac{7}{11} \cdot \frac{4}{11} + \frac{4}{11} \cdot \frac{7}{11}$$

$$= \frac{56}{121}$$

✓ **FINAL ANSWER**

$\frac{56}{121}$

4MA1/1HR MAY 2024

Question 21

5 marks | Probability Toolkit

21 There are 25 counters in a bag such that

6 counters are blue
x counters are orange, where $x > 9$
the rest of the counters are pink

Maalam takes at random two of the counters from the bag.

The probability that Maalam takes one orange counter and one pink counter is $\frac{22}{75}$

Calculate the probability that Maalam takes 2 pink counters from the bag.
Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2024

1 **There are 25 counters**

There are 25 counters.

2 **Blue counters**

Blue counters:

$$6$$

3 **Orange counters**

Orange counters:

$$x$$

4 **Pink counters**

Pink counters:

$$25 - 6 - x = 19 - x$$

5 **The probability of taking one**

The probability of taking one orange and one pink in either order is

$$\frac{x}{25} \cdot \frac{19-x}{24} + \frac{19-x}{25} \cdot \frac{x}{24}$$

$$= \frac{2x(19-x)}{25 \cdot 24}$$

6 **Evaluate fraction**

This is equal to $\frac{22}{75}$, so

$$\frac{2x(19-x)}{600} = \frac{22}{75}$$

$$\frac{x(19-x)}{300} = \frac{22}{75}$$

$$x(19-x) = 88$$

$$19x - x^2 = 88$$

$$x^2 - 19x + 88 = 0$$

$$(x-8)(x-11) = 0$$

7 **Solve inequality**

Since $x > 9$,

$$x = 11$$

8 **Calculate value**

So the number of pink counters is

$$19 - 11 = 8$$

9 **The probability of taking 2**

The probability of taking 2 pink counters is

$$\frac{8}{25} \cdot \frac{7}{24} = \frac{56}{600}$$

$$= \frac{7}{75}$$

FINAL ANSWER

$$\frac{7}{75}$$

4MA1/1H JUNE 2025

Question 21

3 marks | Probability Toolkit

21 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2025

1 There are

There are

$$\binom{20}{3} = 1140$$

2 ways to choose 3 counters

ways to choose 3 counters.

3 Exactly two counters are the

Exactly two counters are the same colour:

$$\binom{9}{2}(11) + \binom{7}{2}(13) + \binom{4}{2}(16)$$

$$= 36(11) + 21(13) + 6(16)$$

$$= 396 + 273 + 96 = 765$$

4 Calculate probability

So the probability is

$$\frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$$\frac{51}{76}$$

4WM1/01 MAY 2025

Question 22

3 marks | Probability Toolkit

22 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 May 2025

1 Total ways to choose counters

Total ways to choose 3 counters:

$$\binom{20}{3} = 1140$$

2 Exactly two red

Exactly two red:

$$\binom{9}{2}\binom{11}{1} = 396$$

3 Exactly two yellow

Exactly two yellow:

$$\binom{7}{2}\binom{13}{1} = 273$$

4 Exactly two green

Exactly two green:

$$\binom{4}{2}\binom{16}{1} = 96$$

5 **Favourable outcomes**

Favourable outcomes:

$$396 + 273 + 96 = 765$$

$$P = \frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$\frac{51}{76}$

TOPIC

Tree Diagrams & Conditional Probability

Every past-paper question on this topic, 2020 to 2026.

4MA1/2H JUNE 2023

Question 20

3 marks | Tree Diagrams & Conditional Probability

20 There are 12 counters in a bag.

3 of the counters are red
9 of the counters are green

Ameya, Jack and Ella each take at random one counter from the bag.

Work out the probability that at least one red counter is still in the bag.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2023

1 There are 3 red counters

There are 3 red counters.

2 At least one red counter

At least one red counter is still in the bag unless all 3 red counters are taken.

3 Use the complement

So use the complement:

$$P(\text{all 3 red are taken}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$$

$$= \frac{6}{1320}$$

$$= \frac{1}{220}$$

4 Evaluate fraction

Therefore,

$$P(\text{at least one red is still in the bag}) = 1 - \frac{1}{220}$$

$$= \frac{219}{220}$$

FINAL ANSWER

$$\frac{219}{220}$$

4MA1/1H NOVEMBER 2025

Question 22

3 marks | Tree Diagrams & Conditional Probability

22 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

4WM1/01 NOVEMBER 2025

Question 23

3 marks | Tree Diagrams & Conditional Probability

23 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

Question 22

5 marks | Tree Diagrams & Conditional Probability

22 A bag contains x marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 22 is 5 marks)

1 **Pink then yellow, or yellow then pink**

$$2 \times \frac{12}{x} \times \frac{x-12}{x-1} = \frac{18}{47}$$

2 **Cross-multiply**

$$1128(x-12) = 18x(x-1) \Rightarrow 18x^2 - 1146x + 13536 = 0$$

3 **Simplify (÷6) and solve**

$$3x^2 - 191x + 2256 = 0 \Rightarrow (x-48)(3x-47) = 0$$

4 **x must be a whole number**

$$x = 48$$

✓ **FINAL ANSWER**

$x = 48$

TOPIC

Set Notation & Venn Diagrams

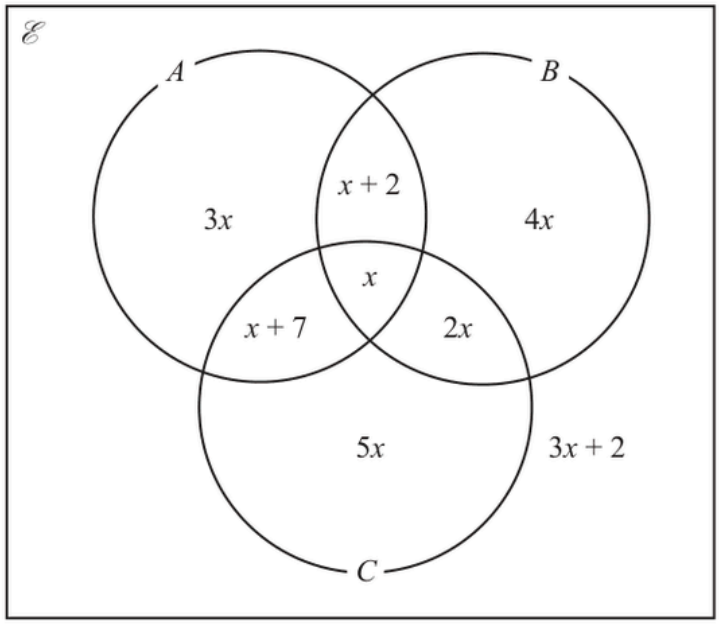
Every past-paper question on this topic, 2020 to 2026.

4MA1/2H NOVEMBER 2024

Question 21

4 marks | Set Notation & Venn Diagrams

21 The Venn diagram shows information about the numbers of items in set A, set B and set C, where x is an integer.



Given that $\text{n}(A \cup B)' = 26$

find $\text{n}(A' \cap C)$

$\text{n}(A' \cap C) =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2024

1 The complement of contains the

The complement of $A \cup B$ contains the regions outside both A and B. These are $5x$ and $3x + 2$.

2 Calculate value

So $5x + (3x + 2) = 26$, hence $8x + 2 = 26$ and $x = 3$.

3 Calculate statistic

Now $A' \cap C$ means inside C, but not inside A. These are the regions $5x$ and $2x$.

4 Calculate value

So $n(A' \cap C) = 5x + 2x = 7x = 7(3) = 21$.

FINAL ANSWER

21

Question 20

6 marks | Set Notation & Venn Diagrams

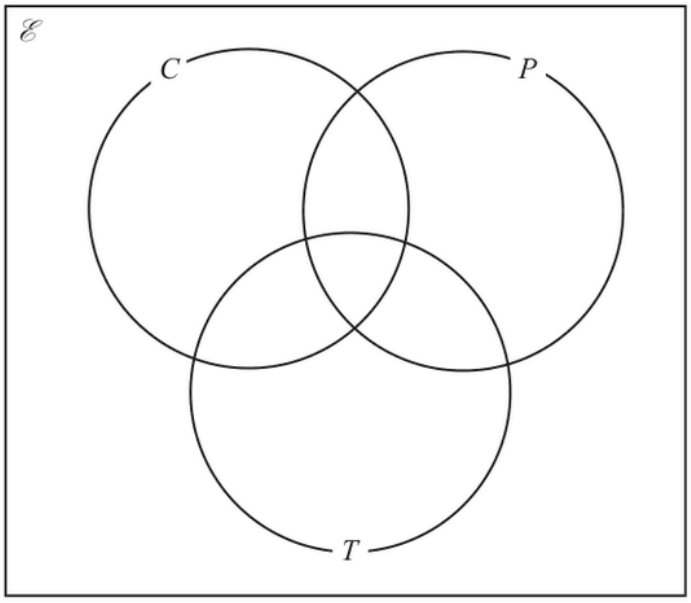
20 120 gardeners were asked if they grow carrots (C) or potatoes (P) or tomatoes (T)

Of these gardeners

43 grow carrots
12 grow carrots and potatoes and tomatoes
18 grow carrots and potatoes
27 grow carrots and tomatoes
32 grow potatoes and tomatoes
29 do not grow carrots or potatoes or tomatoes

The number of these gardeners who grow only potatoes is equal to the number of these gardeners who grow only tomatoes.

(a) Complete the Venn diagram to show this information.



(3)

(b) Find $n(T' \cap C)$

.....

(1)

One of the gardeners who grows carrots is chosen at random.

(c) Calculate the probability that this gardener also grows potatoes.

.....

(2)

(Total for Question 20 is 6 marks)

1 **Calculate value**

There are 120 gardeners in total and 29 are outside all three sets, so

$$n(C \cup P \cup T) = 120 - 29 = 91$$

2 **The triple intersection is**

The triple intersection is

$$n(C \cap P \cap T) = 12$$

3 **Calculate value**

So the pair-only regions are:

$$C \cap P \text{ only} = 18 - 12 = 6$$

$$C \cap T \text{ only} = 27 - 12 = 15$$

$$P \cap T \text{ only} = 32 - 12 = 20$$

4 **Calculate value**

Since 43 grow carrots,

$$C \text{ only} = 43 - 6 - 15 - 12 = 10$$

5 **Let the number who grow**

Let the number who grow only potatoes be y. The number who grow only tomatoes is also y.

6 **Add the regions union**

Now add the regions in the union:

$$10 + y + y + 6 + 15 + 20 + 12 = 91$$

$$63 + 2y = 91$$

$$2y = 28$$

$$y = 14$$

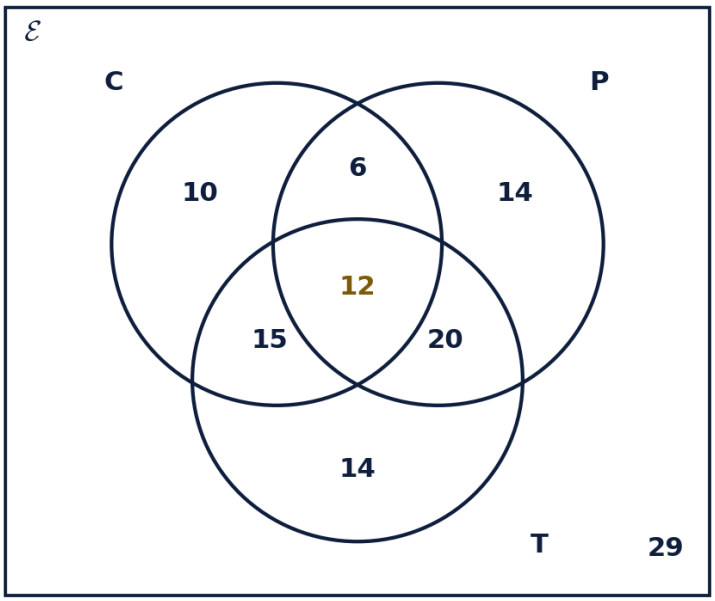
7 **Calculate value**

So the Venn regions are:

$$C \text{ only} = 10, \quad P \text{ only} = 14, \quad T \text{ only} = 14$$

$$C \cap P \text{ only} = 6, \quad C \cap T \text{ only} = 15, \quad P \cap T \text{ only} = 20$$

$$C \cap P \cap T = 12, \quad \text{outside} = 29$$



8 **Calculate value**

For part (b),

$$n(T' \cap C) = C \text{ only} + C \cap P \text{ only}$$

$$= 10 + 6 = 16$$

9 **Evaluate fraction**

For part (c), one gardener who grows carrots is chosen.

$$P(\text{also grows potatoes}) = \frac{n(C \cap P)}{n(C)}$$

$$= \frac{18}{43}$$

✓ **FINAL ANSWER**

(b) 16, (c) $\frac{18}{43}$