Elite Mathematics
Pearson Edexcel International GCSE

IGCSE MODULAR · UNIT 1 · 4WM1

Modular Unit 1 Classified With Answers

Every question followed by Dr Eslam Ahmed's worked solution and final answer.

659	2020–2026	Questions with worked solutions
topic placements	50 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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TOPIC

Number Toolkit

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 36

3 marks | Number Toolkit

36 Here is a list of numbers.

2 9 15 17 21 25 27

From the list, write down

(a) the two prime numbers,

.....

(1)

(b) a square number that is also odd and greater than 10,

.....

(1)

(c) a cube number.

.....

(1)

(Total for Question 36 is 3 marks)

WORKED SOLUTION

Question 36 · EP Specimen

1 **(a) Exactly two factors**

2 and 17 are prime ($15 = 3 \times 5$, $21 = 3 \times 7$).

2 **(b) Square numbers in the list**

$9 = 3^2$ and $25 = 5^2$; the odd one greater than 10 is 25.

3 **(c) Cube numbers**

$27 = 3^3$

✓ **FINAL ANSWER**

(a) $2, 17$ (b) 25 (c) 27

EP SPECIMEN

Question 37

2 marks | Number Toolkit

37 Use the information that $23 \times 47 = 1081$ to write down the value of

(a) 2.3×4.7

.....

(1)

(b) $1081 \div 4.7$

.....

(1)

(Total for Question 37 is 2 marks)

WORKED SOLUTION

Question 37 · EP Specimen

1 **(a) Each number is ten times smaller**

$2.3 \times 4.7 = 1081 \div 100 = 10.81$

2 **(b) Reverse the multiplication**

$1081 \div 47 = 23$, and 4.7 is ten times smaller than 47, so $1081 \div 4.7 = 230$.

✓ **FINAL ANSWER**

(a) 10.81 (b) 230

4MA1/2HR JANUARY 2022

Question 10

3 marks | Number Toolkit

10 The table gives information about the population and the total amount of money, in dollars, spent on healthcare for two countries in 2016

Country	Total population	Total spent on healthcare ($)
Austria	8.7×10^6	4.2×10^{10}
Luxembourg	6.3×10^5	3.7×10^9

Work out how much more was spent **per person** on healthcare in Luxembourg than in Austria.
Give your answer correct to the nearest whole number.

..... dollars

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2022

1 Austria spending per person

Austria spending per person:

$$\frac{4.2 \times 10^{10}}{8.7 \times 10^6} = 4827.586\ldots$$

2 Luxembourg spending per person

Luxembourg spending per person:

$$\frac{3.7 \times 10^9}{6.3 \times 10^5} = 5873.015\ldots$$

3 Difference

Difference:

$$5873.015\ldots - 4827.586\ldots = 1045.429\ldots$$

4 Convert standard form

To the nearest whole number, this is

$$1045$$

FINAL ANSWER

1045 dollars

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TOPIC

Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 2

3 marks | Fractions

2 Show that $5\frac{1}{3} - 2\frac{6}{7} = 2\frac{10}{21}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 Specimen

1 Convert to improper fractions

$$5\frac{1}{3} = \frac{16}{3}, \qquad 2\frac{6}{7} = \frac{20}{7}$$

2 Use a common denominator

$$\frac{16}{3} - \frac{20}{7} = \frac{112}{21} - \frac{60}{21} = \frac{52}{21}$$

3 Convert back to a mixed number

$$\frac{52}{21} = 2\frac{10}{21}$$

FINAL ANSWER

$2\frac{10}{21}$.

4MA1/1HR JANUARY 2020

Question 2

3 marks | Fractions

2 Show that $3\frac{1}{5} \times 2\frac{5}{8} = 8\frac{2}{5}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2020

1 Simplify fraction

Convert the mixed numbers to improper fractions:

$$3\frac{1}{5} = \frac{16}{5}$$

$$2\frac{5}{8} = \frac{21}{8}$$

2 Simplify fraction

Now multiply:

$$\frac{16}{5} \times \frac{21}{8} = \frac{2 \times 21}{5}$$

$$= \frac{42}{5} = 8\frac{2}{5}$$

FINAL ANSWER

$8\frac{2}{5}$

4MA1/2H JANUARY 2020

Question 3

3 marks | Fractions

3 Show that $4\frac{2}{3} + 3\frac{4}{5} = 8\frac{7}{15}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2020

1 Convert to improper fractions

Convert to improper fractions:

$$4\frac{2}{3} = \frac{14}{3}$$

$$3\frac{4}{5} = \frac{19}{5}$$

2 Use a common denominator

Use a common denominator:

$$\frac{14}{3} + \frac{19}{5} = \frac{70}{15} + \frac{57}{15}$$

$$= \frac{127}{15} = 8\frac{7}{15}$$

FINAL ANSWER

$8\frac{7}{15}$

4MA1/1HR MAY/NOVEMBER 2020

Question 1

3 marks | Fractions

1 Show that $3\frac{3}{4} \times \frac{7}{9} = 2\frac{11}{12}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR May/November 2020

1 **Simplify fraction**

$$3\frac{3}{4} = \frac{15}{4}$$

2 **Simplify fraction**

Now multiply:

$$\frac{15}{4} \times \frac{7}{9}$$

3 **Simplify fraction**

Cancel 15 and 9:

$$\frac{5}{4} \times \frac{7}{3} = \frac{35}{12}$$

$$\frac{35}{12} = 2\frac{11}{12}$$

✓ **FINAL ANSWER**

$2\frac{11}{12}$

4MA1/1HR JANUARY 2021

Question 1

3 marks | Fractions

1 Show that $3\frac{1}{5} \times 1\frac{5}{6} = 5\frac{13}{15}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2021

1 Simplify fraction

Convert the mixed numbers to improper fractions:

$$3\frac{1}{5} = \frac{16}{5}$$

$$1\frac{5}{6} = \frac{11}{6}$$

2 Simplify fraction

Now multiply:

$$\frac{16}{5} \times \frac{11}{6} = \frac{176}{30}$$

$$= \frac{88}{15} = 5\frac{13}{15}$$

FINAL ANSWER

$5\frac{13}{15}$

4MA1/2H MAY 2021

Question 2

3 marks | Fractions

2 Show that $2\frac{4}{7} \div 1\frac{1}{8} = 2\frac{2}{7}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H May 2021

1 Simplify fraction

$$2\frac{4}{7} = \frac{18}{7}$$

$$1\frac{1}{8} = \frac{9}{8}$$

2 Divide by multiplying by the

Divide by multiplying by the reciprocal:

$$\frac{18}{7} \div \frac{9}{8} = \frac{18}{7} \times \frac{8}{9}$$

$$= \frac{16}{7} = 2\frac{2}{7}$$

FINAL ANSWER

$2\frac{2}{7}$

4MA1/2H JANUARY 2022

Question 2

3 marks | Fractions

2 Show that $6\frac{3}{4} \div 2\frac{4}{7} = 2\frac{5}{8}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2022

1 Convert to improper fractions

Convert to improper fractions:

$$6\frac{3}{4} = \frac{27}{4}$$

$$2\frac{4}{7} = \frac{18}{7}$$

2 Divide by multiplying by the

Divide by multiplying by the reciprocal:

$$\frac{27}{4} \div \frac{18}{7} = \frac{27}{4} \times \frac{7}{18}$$

$$= \frac{21}{8} = 2\frac{5}{8}$$

FINAL ANSWER

$2\frac{5}{8}$

4MA1/2H JUNE 2022

Question 4

3 marks | Fractions

4 Show that $5\frac{1}{3} - 2\frac{6}{7} = 2\frac{10}{21}$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2022

1 Simplify fraction

$$5\frac{1}{3} = \frac{16}{3}$$

$$2\frac{6}{7} = \frac{20}{7}$$

2 Use a common denominator

Use a common denominator:

$$\frac{16}{3} - \frac{20}{7} = \frac{112}{21} - \frac{60}{21}$$

$$= \frac{52}{21} = 2\frac{10}{21}$$

FINAL ANSWER

$2\frac{10}{21}$

4MA1/1HR JUNE 2022

Question 6

3 marks | Fractions

6 Show that $2\frac{2}{3} + 3\frac{3}{4} = 6\frac{5}{12}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2022

1 Simplify fraction

$$2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{4} = \frac{15}{4}$$

2 Use a common denominator

Use a common denominator:

$$\frac{8}{3} + \frac{15}{4} = \frac{32}{12} + \frac{45}{12}$$

$$= \frac{77}{12} = 6\frac{5}{12}$$

FINAL ANSWER

$6\frac{5}{12}$

4MA1/2H JUNE 2024

Question 4

3 marks | Fractions

4 Show that $2\frac{4}{7} \times 3\frac{1}{9} = 8$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2024

1 Simplify fraction

$$2\frac{4}{7} = \frac{18}{7}$$

$$3\frac{1}{9} = \frac{28}{9}$$

2 Simplify fraction

Now multiply:

$$\frac{18}{7} \times \frac{28}{9}$$

3 Cancel first

Cancel first:

$$\frac{18}{9} \times \frac{28}{7} = 2 \times 4 = 8$$

FINAL ANSWER

8

4WM1/01 MAY 2025

Question 1

3 marks | Fractions

1 Without using a calculator, work out

$$3\frac{2}{3} \times 2\frac{4}{7}$$

Show each stage of your working.
Give your answer as a mixed number in its simplest form.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 May 2025

1 **Simplify fraction**

Convert the mixed numbers to improper fractions.

$$3\frac{2}{3} = \frac{11}{3}, \qquad 2\frac{4}{7} = \frac{18}{7}$$

2 **Simplify fraction**

Then multiply.

$$\frac{11}{3} \times \frac{18}{7} = \frac{198}{21} = \frac{66}{7}$$

3 **Convert to a mixed number**

Convert to a mixed number.

$$\frac{66}{7} = 9\frac{3}{7}$$

✓ **FINAL ANSWER**

$9\frac{3}{7}$

4WM1/01R MAY 2025

Question 2

3 marks | Fractions

2 Show that $2\frac{1}{4} \times 1\frac{5}{7} = 3\frac{6}{7}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01R May 2025

1 **Evaluate fraction**

$$2\frac{1}{4} = \frac{9}{4}, \qquad 1\frac{5}{7} = \frac{12}{7}$$

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{9}{4} \times \frac{12}{7}$$

$$= \frac{108}{28} = \frac{27}{7} = 3\frac{6}{7}$$

✓ **FINAL ANSWER**

$3\frac{6}{7}$.

4WM1/01 NOVEMBER 2025

Question 3

3 marks | Fractions

3 Show that $3\frac{5}{7} + 1\frac{2}{3} = 5\frac{8}{21}$

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4WM1/01 November 2025

1 Evaluate fraction

$$\frac{3}{7} + \frac{1}{3}$$

2 Use denominator 21

Use denominator 21:

$$\frac{3}{7} = \frac{9}{21}, \qquad \frac{1}{3} = \frac{7}{21}$$

$$\frac{9}{21} + \frac{7}{21} = \frac{16}{21}$$

FINAL ANSWER

$\frac{16}{21}$.

4MA1/2H NOVEMBER 2025

Question 3

4 marks | Fractions

3 A box contains only

9 red bricks
43 blue bricks
and some yellow bricks

$\frac{7}{20}$ of the bricks are yellow bricks.

Each brick weighs 35 grams.

Work out the total weight of the yellow bricks.

..... grams

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2025

1 There are red and blue

There are $9 + 43 = 52$ red and blue bricks.

2 Simplify fraction

Let the total number of bricks be T.

3 Simplify fraction

Since $\frac{7}{20}$ of the bricks are yellow, the remaining fraction is

$$1 - \frac{7}{20} = \frac{13}{20}$$

4 Simplify fraction

So

$$\frac{13}{20}T = 52$$

$$T = 52 \times \frac{20}{13} = 80$$

5 **Yellow bricks**

Yellow bricks:

$$\frac{7}{20} \times 80 = 28$$

6 **Simplify fraction**

Each brick weighs 35 grams, so total yellow-brick weight is

$$28 \times 35 = 980$$

✓ **FINAL ANSWER**

980 grams

4MA1/1H JUNE 2026

Question 1

3 marks | Fractions

1 Show that $4\frac{1}{7} - 2\frac{3}{5} = 1\frac{19}{35}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H June 2026

1 **Solution | Step 1**

$$4\frac{1}{7} = \frac{29}{7}, \qquad 2\frac{3}{5} = \frac{13}{5}.$$

Convert each mixed number to an improper fraction.

2 **Solution | Step 2**

$$\frac{29}{7} - \frac{13}{5} = \frac{145 - 91}{35} = \frac{54}{35} = 1\frac{19}{35}.$$

Use the common denominator 35, subtract, then convert back to a mixed number.

✓ **FINAL ANSWER**

$1\frac{19}{35}$

4MA1/1HR JUNE 2026

Question 1

3 marks | Fractions

1 Show that $2\frac{4}{5} \times 3\frac{3}{7} = 9\frac{3}{5}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2026

1 Solution | Step 1

$$2\frac{4}{5} = \frac{14}{5}, \qquad 3\frac{3}{7} = \frac{24}{7}.$$

Convert each mixed number into an improper fraction.

2 Solution | Step 2

$$\frac{14}{5} \times \frac{24}{7} = \frac{2 \times 24}{5} = \frac{48}{5} = 9\frac{3}{5}.$$

Cancel the common factor 7, multiply, then convert back to a mixed number.

✓ FINAL ANSWER

$2\frac{4}{5} \times 3\frac{3}{7} = 9\frac{3}{5}$

4WM1/01 MAY 2026

Question 1

3 marks | Fractions

1 Show that $4\frac{1}{7} - 2\frac{3}{5} = 1\frac{19}{35}$

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 May 2026

1 Improper fractions

$$4\tfrac{1}{7} = \frac{29}{7}, \qquad 2\tfrac{3}{5} = \frac{13}{5}$$

2 Common denominator 35

$$\frac{29}{7} - \frac{13}{5} = \frac{145}{35} - \frac{91}{35} = \frac{54}{35}$$

3 Back to a mixed number

$$\frac{54}{35} = 1\tfrac{19}{35} \quad \text{as required.}$$

✓ **FINAL ANSWER**

$\frac{54}{35} = 1\frac{19}{35}$ (shown)

4MA1/2HR JUNE 2026

Question 17

2 marks | Fractions

17 Use algebra to show that $0.\dot{8}1\dot{4} = \frac{22}{27}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2026

1 **Solution | Step 1**

Let $z = 0.\overline{814} = 0.814814814\ldots$. Then $1000z = 814.814814\ldots$.

Multiplication by 1000 aligns the repeating three-digit tails.

2 **Solution | Step 2**

$$1000z - z = 814 \quad \Rightarrow \quad 999z = 814 \quad \Rightarrow \quad z = \frac{814}{999} = \frac{22}{27}.$$

Subtract to cancel the recurring tails, then cancel the common factor 37.

✓ **FINAL ANSWER**

$0.\overline{814} = \frac{22}{27}$.

TOPIC

Fractions, Decimals & Percentages

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 15

2 marks | Fractions, Decimals & Percentages

15 Write these numbers in order of size. Start with the smallest number.

$0.62 \quad \frac{5}{8} \quad 61\% \quad \frac{2}{3} \quad \frac{3}{5}$

.....

(2)

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · EP Specimen

1 Change every number to a decimal

$0.62,\ \frac{5}{8} = 0.625,\ 61\% = 0.61,\ \frac{2}{3} = 0.666\ldots,\ \frac{3}{5} = 0.6$

2 Order them

$0.6,\ 0.61,\ 0.62,\ 0.625,\ 0.666\ldots$

FINAL ANSWER

$\frac{3}{5},\ 61\%,\ 0.62,\ \frac{5}{8},\ \frac{2}{3}$

EP SPECIMEN

Question 16

2 marks | Fractions, Decimals & Percentages

16 (a) Write $\frac{7}{40}$ as a decimal.

.....

(1)

(b) Write $\frac{7}{40}$ as a percentage.

..... %

(1)

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · EP Specimen

1 **(a) Divide**

$$7 \div 40 = 0.175$$

2 **(b) Multiply by 100**

$$0.175 \times 100 = 17.5\%$$

✓ **FINAL ANSWER**

(a) 0.175 (b) 17.5%

EP SPECIMEN

Question 17

3 marks | Fractions, Decimals & Percentages

17 The students in a class travel to school by walking, by bicycle or by bus.

$\frac{3}{8}$ of the students walk. 0.35 of the students cycle. The rest travel by bus.

11 students travel by bus.

Work out the number of students in the class.

.....

(3)

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · EP Specimen

1 Proportion that walk or cycle

$$\frac{3}{8} + 0.35 = 0.375 + 0.35 = 0.725$$

2 Proportion that travel by bus

$$1 - 0.725 = 0.275$$

3 Number of students

$$0.275 \times n = 11 \Rightarrow n = \frac{11}{0.275} = 40$$

FINAL ANSWER

40

EP SPECIMEN

Question 18

2 marks | Fractions, Decimals & Percentages

18 Which is larger, $\frac{13}{20}$ or 0.66?

You must show your working.

.....

(2)

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · EP Specimen

1 Write both as decimals

$$\frac{13}{20} = 13 \div 20 = 0.65$$

2 Compare

$0.66 > 0.65$, so 0.66 is larger.

FINAL ANSWER

0.66

EP SPECIMEN

Question 19

2 marks | Fractions, Decimals & Percentages

19 Mia took two tests.
She scored 34 out of 40 in the first test.
She scored 87% in the second test.

In which test did Mia get the better score? You must show how you get your answer.

.....

(2)

(Total for Question 19 is 2 marks)

WORKED SOLUTION

Question 19 · EP Specimen

1 Change the first score to a percentage

$$\frac{34}{40} \times 100 = 85\%$$

2 Compare

$87\% > 85\%$, so the second test was better.

FINAL ANSWER

The second test (85% against 87%)

EP SPECIMEN

Question 20

2 marks | Fractions, Decimals & Percentages

20 Write 0.0375 as a fraction in its simplest form.

.....

(2)

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · EP Specimen

1 **Write over a power of 10**

$$0.0375 = \frac{375}{10\,000}$$

2 **Simplify (divide by 125)**

$$\frac{375}{10\,000} = \frac{3}{80}$$

✓ **FINAL ANSWER**

$$\frac{3}{80}$$

EP SPECIMEN

Question 21

3 marks | Fractions, Decimals & Percentages

21 A water tank is $\frac{2}{5}$ full.

120 litres of water are added to the tank.
The tank is now 65% full.

Work out the capacity of the tank.

..... litres

(3)

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · EP Specimen

1 Change both amounts to percentages

$\frac{2}{5} = 40\%$, so the 120 litres fill $65\% - 40\% = 25\%$ of the tank.

2 Find 100%

$$25\% \to 120 \;\Rightarrow\; 100\% \to 120 \times 4 = 480 \text{ litres}$$

FINAL ANSWER

480 litres

TOPIC

Recurring Decimals

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY 2021

Question 16

2 marks | Recurring Decimals

16 Use algebra to show that the recurring decimal $0.28\dot{1}\dot{3} = \frac{557}{1980}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2021

1 Let

Let

$$x = 0.28131313\ldots$$

2 Calculate value

Then

$$100x = 28.131313\ldots$$

3 Calculate value

and

$$10000x = 2813.131313\ldots$$

4 Subtract

Subtract:

$$10000x - 100x = 2813.131313\ldots - 28.131313\ldots$$

$$9900x = 2785$$

$$x = \frac{2785}{9900} = \frac{557}{1980}$$

FINAL ANSWER

$0.28131313\ldots = \frac{557}{1980}$.

4MA1/1H NOVEMBER 2021

Question 18

3 marks | Recurring Decimals

18 $0.4\dot{x}$ is a recurring decimal.
x is a whole number such that $1 \leqslant x \leqslant 9$

Find, in terms of x, the recurring decimal $0.4\dot{x}$ as a fraction.
Give your fraction in its simplest form.
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H November 2021

1 Let

Let

$$y = 0.4\dot{x}$$

2 Simplify fraction

So

$$y = 0.4xxx\ldots$$

3 Simplify fraction

Multiply by 10 and 100:

$$10y = 4.xxx\ldots$$

$$100y = 4x.xxx\ldots$$

4 Subtract

Subtract:

$$100y - 10y = (4x.xxx\ldots) - (4.xxx\ldots)$$

$$90y = x + 36$$

5 Simplify fraction

Therefore

$$y = \frac{x + 36}{90}$$

FINAL ANSWER

$$\frac{x+36}{90}$$

4WM1/01R MAY 2025

Question 14

2 marks | Recurring Decimals

14 Use algebra to show that the recurring decimal $0.6\dot{1}\dot{2} = \frac{101}{165}$

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4WM1/01R May 2025

1 Let

Let

$$x = 0.6121212\ldots$$

2 Calculate value

Then

$$10x = 6.121212\ldots$$

3 Calculate value

and

$$1000x = 612.121212\ldots$$

4 Subtract

Subtract:

$$1000x - 10x = 612.121212\ldots - 6.121212\ldots$$

$$990x = 606$$

$$x = \frac{606}{990} = \frac{101}{165}$$

✓ **FINAL ANSWER**

$0.6121212\ldots = \frac{101}{165}$.

4MA1/1HR JUNE 2025

Question 16

2 marks | Recurring Decimals

16 Use algebra to show that the recurring decimal $0.6\dot{1}\dot{2} = \frac{101}{165}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2025

1 Let

Let

$$x = 0.6121212\ldots$$

2 Calculate value

Then

$$10x = 6.121212\ldots$$

3 Calculate value

and

$$1000x = 612.121212\ldots$$

4 Subtract

Subtract:

$$1000x - 10x = 612.121212\ldots - 6.121212\ldots$$

$$990x = 606$$

$$x = \frac{606}{990} = \frac{101}{165}$$

✓ **FINAL ANSWER**

$0.6121212\ldots = \frac{101}{165}$.

TOPIC

Rounding, Estimation & Bounds

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 1

1 marks | Rounding, Estimation & Bounds

1 The weight of a cake is 2.75 kg, correct to 2 decimal places.

Write down the lower bound of the weight of the cake.

..... kg

(Total for Question 1 is 1 mark)

WORKED SOLUTION

Question 1 · 4WM1/01 Specimen

1 Find half the rounding interval

Correct to 2 decimal places means the nearest 0.01 kg.

$$\frac{0.01}{2} = 0.005$$

2 Subtract from the rounded value

$$2.75 - 0.005 = 2.745$$

FINAL ANSWER

2.745 kg.

Question 15

4 marks | Rounding, Estimation & Bounds

15 Martin and Lucia went on two different car journeys.

For Martin's journey

distance = 80 km correct to the nearest 5 km
time = 2.7 hours correct to 1 decimal place

For Lucia's journey

distance = 33 km correct to 2 significant figures
time = 1 hour correct to the nearest 0.1 hour

Martin says,

"My average speed could have been greater than Lucia's average speed."

By considering bounds, show that Martin is correct.
Show your working clearly.

(Total for Question 15 is 4 marks)

1 Use Martin's greatest possible speed

Martin's greatest possible distance is 82.5 km and his least possible time is 2.65 hours.

$$\frac{82.5}{2.65} = 31.132\ldots$$

2 Use Lucia's least possible speed

Lucia's least possible distance is 32.5 km and her greatest possible time is 1.05 hours.

$$\frac{32.5}{1.05} = 30.952\ldots$$

3 Compare the possible speeds

$$31.132\ldots > 30.952\ldots$$

So Martin's speed could have been greater than Lucia's speed.

FINAL ANSWER

Martin could have been faster because $31.132\ldots > 30.952\ldots$ km/h.

4MA1/1HR JANUARY 2020

Question 11

4 marks | Rounding, Estimation & Bounds

11 The weight of a cat is 4.3 kg correct to 2 significant figures.

(a) Write down the upper bound of the weight of the cat.

..... kg

(1)

(b) Write down the lower bound of the weight of the cat.

..... kg

(1)

$G = e - f$

e = 17 correct to the nearest integer
f = 9.4 correct to one decimal place

(c) Work out the upper bound for the value of G.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2020

1 **Estimate the value**

The weight 4.3 kg correct to 2 significant figures has bounds

$$4.25 \leq w < 4.35$$

2 **Find upper bound**

So the upper bound is 4.35 kg and the lower bound is 4.25 kg.

3 **Estimate the value**

For

$$G = e - f$$

4 **the upper bound uses the**

the upper bound uses the upper bound of e and the lower bound of f.

$$e < 17.5, \quad f \geq 9.35$$

$$G_{\text{upper}} = 17.5 - 9.35 = 8.15$$

✓ **FINAL ANSWER**

(a) 4.35 kg, (b) 4.25 kg, (c) 8.15

4MA1/1H JANUARY 2020

Question 16

3 marks | Rounding, Estimation & Bounds

16 $P = \dfrac{2a - c}{d}$

$a = 58.4$ correct to 3 significant figures.
$c = 20$ correct to 2 significant figures.
$d = 3.6$ correct to 2 significant figures.

Work out the upper bound for the value of P.
Show your working clearly.
Give your answer correct to 2 decimal places.

.....

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2020

1 **Estimate the value**

$$P = \frac{2a - c}{d}$$

2 **Find upper bound**

For the upper bound of P, make the numerator as large as possible and the denominator as small as possible.

$$a < 58.45, \quad c \geq 19.5, \quad d \geq 3.55$$

$$P_{\text{upper}} = \frac{2(58.45) - 19.5}{3.55} = 27.4366\ldots$$

✓ **FINAL ANSWER**

27.44

4MA1/1H MAY/NOVEMBER 2020

Question 17

4 marks | Rounding, Estimation & Bounds

17 A metal block has a mass of 5 kg, correct to the nearest 50 grams.
The block has a volume of $(1.84 \times 10^{-3})\,m^3$, correct to 3 significant figures.

Work out the upper bound for the density of the block.
Give your answer in kg/m^3 correct to 1 decimal place.
Show your working clearly.

..... kg/m^3

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May/November 2020

1 Density is

Density is

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2 To get the upper bound

To get the upper bound for density, use the upper bound for mass and the lower bound for volume.

3 The mass is kg correct

The mass is 5 kg correct to the nearest 50 g.

$$50\text{ g} = 0.05\text{ kg}$$

4 Find upper bound

Half of this is 0.025 kg, so

$$\text{upper bound for mass} = 5 + 0.025 = 5.025$$

5 Find upper bound

The volume is $1.84 \times 10^{-3} = 0.00184\ m^3$, correct to 3 significant figures.

6 The lower bound is

The lower bound is

$$0.00184 - 0.000005 = 0.001835$$

7 **Find upper bound**

So the upper bound for the density is

$$\frac{5.025}{0.001835} = 2738.419\ldots$$

8 **Find upper bound**

Correct to 1 decimal place,

$$2738.4$$

✓ **FINAL ANSWER**

$2738.4\ \mathrm{kg/m^3}$

4MA1/1HR MAY/NOVEMBER 2020

Question 19

3 marks | Rounding, Estimation & Bounds

19 The diagram shows rectangle $ABCD$ with rectangle $EFGH$ cut out to form the shaded region.

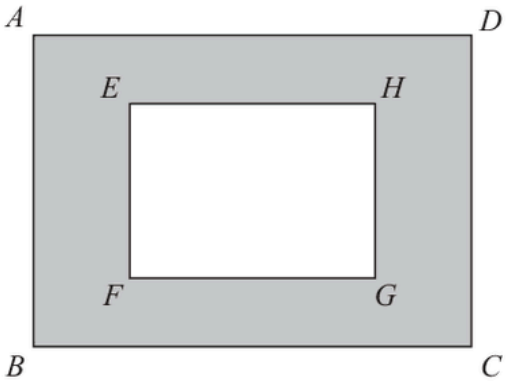


Diagram **NOT** accurately drawn

$AD = 8.3$ cm correct to one decimal place
$DC = 7.2$ cm correct to one decimal place
$EH = 6.2$ cm correct to one decimal place
$HG = 5.3$ cm correct to one decimal place

Work out the upper bound of the area of the shaded region.
Show your working clearly.

..... cm²

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR May/November 2020

1 **Find upper bound**

For the upper bound of the shaded area, use the upper bound of the outer rectangle and subtract the lower bound of the cut-out rectangle.

2 **Outer rectangle**

Outer rectangle:

$$AD < 8.35, \quad DC < 7.25$$

3 Cut out rectangle

Cut-out rectangle:

$$EH \geq 6.15, \quad HG \geq 5.25$$

$$\text{upper shaded area} = 8.35(7.25) - 6.15(5.25)$$

$$= 60.5375 - 32.2875 = 28.25$$

FINAL ANSWER

$28.25\ \text{cm}^2$

4MA1/1H JANUARY 2021

Question 15

3 marks | Rounding, Estimation & Bounds

15

$$a = \frac{v - u}{t}$$

$v = 9.6$ correct to 1 decimal place
$u = 3.8$ correct to 1 decimal place
$t = 1.84$ correct to 2 decimal places

Calculate the upper bound for the value of a.
Give your answer as a decimal correct to 2 decimal places.
Show your working clearly.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2021

1 Estimate the value

$$a = \frac{v - u}{t}$$

2 Find upper bound

For the upper bound, use the upper bound of v, the lower bound of u, and the lower bound of t.

$$v < 9.65, \quad u \geq 3.75, \quad t \geq 1.835$$

$$a_{\text{upper}} = \frac{9.65 - 3.75}{1.835} = 3.2152\ldots$$

FINAL ANSWER

3.22

4MA1/1HR JANUARY 2021

Question 19

3 marks | Rounding, Estimation & Bounds

19 $k = \dfrac{t}{a-h}$

$t = 14$ correct to 2 significant figures
$a = 7.8$ correct to 2 significant figures
$h = 3.4$ correct to 2 significant figures

Work out the lower bound for the value of k.
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2021

1 Estimate the value

$$k = \frac{t}{a-h}$$

2 Find lower bound

For the lower bound of k, use the lower bound of t and make the denominator as large as possible.

$$t \geq 13.5$$

$$a < 7.85, \quad h \geq 3.35$$

$$(a-h)_{\text{upper}} = 7.85 - 3.35 = 4.5$$

$$k_{\text{lower}} = \frac{13.5}{4.5} = 3$$

FINAL ANSWER

3

4MA1/1H MAY 2021

Question 20

3 marks | Rounding, Estimation & Bounds

20 $P = \dfrac{t - w}{y}$

t = 9.7 correct to 1 decimal place

w = 5.9 correct to 1 decimal place

y = 3 correct to 1 significant figure

Calculate the upper bound for the value of P.
Show your working clearly.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2021

1 Estimate the value

$$P = \frac{t - w}{y}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$t < 9.75, \quad w \geq 5.85, \quad y \geq 2.5$$

$$P_{\text{upper}} = \frac{9.75 - 5.85}{2.5} = 1.56$$

FINAL ANSWER

1.56

4MA1/1H NOVEMBER 2021

Question 13

3 marks | Rounding, Estimation & Bounds

13 Here is a triangle XYZ.

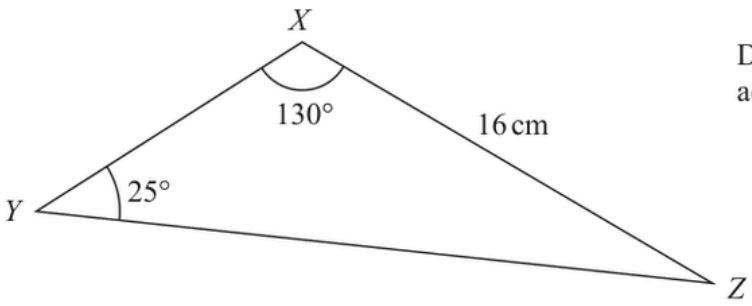


Diagram **NOT** accurately drawn

The length XZ and the angles YXZ and XYZ are each given correct to 2 significant figures.

Calculate the upper bound for the length YZ.
Give your answer correct to one decimal place.
Show your working clearly.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2021

1 Use trigonometry

Using the sine rule,

$$\frac{YZ}{\sin X} = \frac{XZ}{\sin Y}$$

2 Find upper bound

For the upper bound of YZ:

$$XZ < 16.5, \quad X < 25.5°, \quad Y < 135°$$

$$YZ_{\text{upper}} = \frac{16.5 \sin 25.5°}{\sin 135°} = 10.0457\ldots$$

3 Use trigonometry

Correct to one decimal place,

$$10.0457\ldots \approx 10.0$$

FINAL ANSWER

10.0 cm

4MA1/2H JANUARY 2022

Question 4

2 marks | Rounding, Estimation & Bounds

4 Each side of a regular octagon has a length of 18 mm, correct to the nearest 0.5 mm

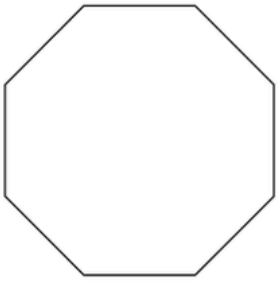


Diagram **NOT** accurately drawn

(a) Write down the lower bound of the length of each side of the octagon.

..... mm

(1)

(b) Write down the upper bound of the length of each side of the octagon.

..... mm

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2022

1 The side length is mm

The side length is 18 mm correct to the nearest 0.5 mm.

2 Half of mm is mm

Half of 0.5 mm is 0.25 mm.

$$18 - 0.25 = 17.75$$

$$18 + 0.25 = 18.25$$

FINAL ANSWER

(a) 17.75 mm, (b) 18.25 mm

4MA1/1H JANUARY 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $A = w - \dfrac{x^2}{y}$

$w = 3.45$ correct to 2 decimal places.
$x = 1.9$ correct to 1 decimal place.
$y = 5$ correct to the nearest whole number.

Work out the lower bound of the value of A
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2022

1 Estimate the value

$$A = w - \frac{x^2}{y}$$

2 Find lower bound

For the lower bound, use the lower bound of w, and make $\frac{x^2}{y}$ as large as possible.

$$w \geq 3.445, \quad x < 1.95, \quad y \geq 4.5$$

$$A_{\text{lower}} = 3.445 - \frac{1.95^2}{4.5}$$

$$A_{\text{lower}} = 2.6$$

FINAL ANSWER

2.6

4MA1/1HR JANUARY 2022

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows triangle ABC

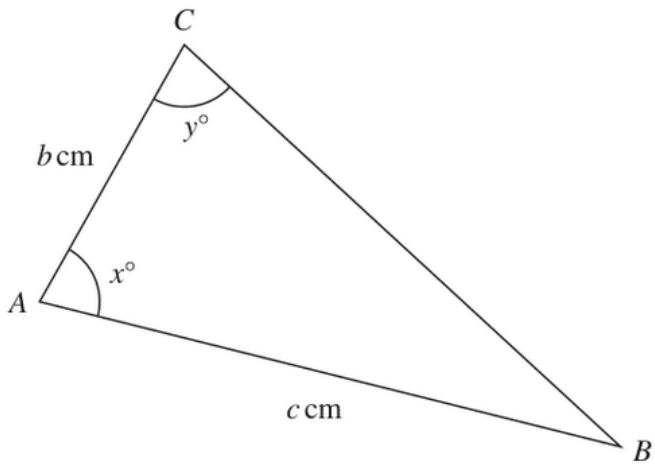


Diagram **NOT** accurately drawn

$c = 11.5$ correct to one decimal place
$x = 80$ correct to the nearest whole number
$y = 75$ correct to the nearest whole number

Calculate the upper bound for the value of b
Show your working clearly.
Give your answer correct to 3 significant figures.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2022

1 Find upper bound

For an upper bound for b, use the upper bound of c:

$c = 11.55$

2 Use sine rule

Use the lower bounds of x and y, because this makes angle B as large as possible and makes $\sin y$ as small as possible.

$x = 79.5°, \quad y = 74.5°$

3 Use trigonometry

So

$\angle B = 180° - 79.5° - 74.5° = 26°$

4 **Use trigonometry**

Using the sine rule,

$$\frac{b}{\sin 26^\circ} = \frac{11.55}{\sin 74.5^\circ}$$

$$b = \frac{11.55 \sin 26^\circ}{\sin 74.5^\circ} = 5.254...$$

✓ **FINAL ANSWER**

5.25 cm to 3 significant figures.

4MA1/2H JUNE 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $X = \dfrac{2a - b}{f}$

$a = 7.5$ correct to 1 decimal place.
$b = 3.42$ correct to 2 decimal places.
$f = 2$ correct to the nearest whole number.

Work out the upper bound of the value of X
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2022

1 Estimate the value

$$X = \frac{2a - b}{f}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$a < 7.55, \quad b \geq 3.415, \quad f \geq 1.5$$

$$X_{\text{upper}} = \frac{2(7.55) - 3.415}{1.5} = 7.79$$

FINAL ANSWER

7.79

4MA1/2HR JUNE 2022

Question 18

4 marks | Rounding, Estimation & Bounds

18 Kaidan and Sonja went on two different car journeys.

For Kaidan's journey
distance = 80 km correct to the nearest 5 km
time = 2.7 hours correct to 1 decimal place

For Sonja's journey
distance = 33 km correct to 2 significant figures
time = 1 hour correct to the nearest 0.1 hour

Kaidan says,
"My average speed could have been greater than Sonja's average speed."

By considering bounds, show that Kaidan is correct.
Show your working clearly.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2022

1 To show Kaidan s average

To show Kaidan's average speed could be greater, compare Kaidan's greatest possible speed with Sonja's smallest possible speed.

2 Estimate the value

For Kaidan:

$$d < 82.5, \quad t \geq 2.65$$

$$\text{greatest speed} = \frac{82.5}{2.65} = 31.13\ldots \text{ km/h}$$

3 Estimate the value

For Sonja:

$$d \geq 32.5, \quad t < 1.05$$

$$\text{smallest speed} = \frac{32.5}{1.05} = 30.95\ldots \text{ km/h}$$

4 **Estimate the value**

Since

$$31.13\ldots > 30.95\ldots$$

5 **Kaidan s average speed could**

Kaidan's average speed could have been greater.

✓ **FINAL ANSWER**

Kaidan is correct.

4MA1/1HR MAY 2023

Question 18

4 marks | Rounding, Estimation & Bounds

18 Diego builds a fence using fence panels.

The total length of the fence is 50 metres, correct to the nearest 5 metres.
The length of each fence panel is 1.3 metres, correct to the nearest 10 cm.

The cost of each fence panel is £8.65

Diego may only buy complete fence panels.
Diego only pays for the number of panels he needs to build the fence.

Work out the greatest difference in the possible amounts that Diego could pay to build the fence.
Show your working clearly.

£.....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR May 2023

1 The fence length m correct

The fence length 50 m correct to the nearest 5 m has bounds

$$47.5 \leq L < 52.5$$

2 The panel length m correct

The panel length 1.3 m correct to the nearest 10 cm has bounds

$$1.25 \leq p < 1.35$$

3 Smallest possible number of panels

Smallest possible number of panels:

$$\left\lceil \frac{47.5}{1.35} \right\rceil = 36$$

4 Greatest possible number of panels

Greatest possible number of panels:

$$\left\lceil \frac{52.5}{1.25} \right\rceil = 42$$

5 **Difference in number of panels**

Difference in number of panels:

$$42 - 36 = 6$$

6 **Greatest difference in cost**

Greatest difference in cost:

$$6 \times 8.65 = 51.90$$

✓ **FINAL ANSWER**

51.90 pounds

4MA1/2H JUNE 2023

Question 25

4 marks | Rounding, Estimation & Bounds

25 A solid sphere has a radius of 2.8 centimetres, correct to 1 decimal place.
The sphere has a mass of $M\pi$ grams, where $M = 260$ correct to 2 significant figures.

Work out the upper bound for the density of the sphere.
Give your answer in g/cm^3 correct to 2 decimal places.
Show your working clearly.

..... g/cm^3

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2023

1 Find upper bound

For the upper bound of density, use the upper bound of the mass and the lower bound of the volume.

2 The radius is cm correct

The radius is 2.8 cm correct to 1 decimal place, so the lower bound is
$r = 2.75$

3 Find upper bound

$M = 260$ correct to 2 significant figures, so the upper bound is
$M = 265$

4 The mass is and the

The mass is $M\pi$ and the volume of the sphere is $\frac{4}{3}\pi r^3$.

5 Rearrange formula

So

$$\text{density} = \frac{M\pi}{\frac{4}{3}\pi r^3} = \frac{3M}{4r^3}$$

6 Upper bound

Upper bound:

$$\frac{3(265)}{4(2.75)^3} = 9.5567\ldots$$

FINAL ANSWER

$9.56\ \mathrm{g/cm^3}$ to 2 decimal places.

4MA1/2H NOVEMBER 2023

Question 17

3 marks | Rounding, Estimation & Bounds

17 $P = a(c + y)$

$a =$ 8.3 correct to 2 significant figures
$c =$ 2 correct to 1 significant figure
$y =$ 15 correct to the nearest 5

Work out the upper bound for the value of P
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2023

1 Estimate the value

$$P = a(c + y)$$

2 Find upper bound

For the upper bound, use the upper bounds of all three values.

$$a < 8.35, \quad c < 2.5, \quad y < 17.5$$

$$P_{\text{upper}} = 8.35(2.5 + 17.5) = 167$$

FINAL ANSWER

167

4MA1/2H JUNE 2024

Question 19

3 marks | Rounding, Estimation & Bounds

19 $G = \dfrac{c}{2f - 3h}$

$c = 8$ correct to the nearest whole number
$f = 6.62$ correct to 2 decimal places
$h = 1.2$ correct to 1 decimal place

Work out the lower bound for the value of G
Give your answer correct to 3 decimal places.
Show your working clearly.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2024

1 Estimate the value

$$G = \frac{c}{2f - 3h}$$

2 Find lower bound

For the lower bound, use the lower bound of c and make the denominator as large as possible.

$$c \geq 7.5$$

$$f < 6.625, \quad h \geq 1.15$$

$$(2f - 3h)_{\text{upper}} = 2(6.625) - 3(1.15) = 9.8$$

$$G_{\text{lower}} = \frac{7.5}{9.8} = 0.765306\ldots$$

✓ **FINAL ANSWER**

0.765

4MA1/2HR JUNE 2024

Question 19

3 marks | Rounding, Estimation & Bounds

19 Aviv goes on a cycle journey.

For the cycle journey

average speed = 19 km/h correct to the nearest whole number

time = 1.5 hours correct to one decimal place

Work out the upper bound for the distance Aviv travels.
Give your answer correct to 3 significant figures.

..... km

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2024

1 Distance is found using

Distance is found using

$$\text{distance} = \text{speed} \times \text{time}$$

2 Find upper bound

For the upper bound,

$$\text{speed} < 19.5, \quad \text{time} < 1.55$$

$$\text{distance}_{\text{upper}} = 19.5 \times 1.55 = 30.225$$

3 Estimate the value

Correct to 3 significant figures,

$$30.225 \approx 30.2$$

FINAL ANSWER

30.2 km

4MA1/2H NOVEMBER 2024

Question 2

2 marks | Rounding, Estimation & Bounds

2 The length of a table is measured as 1.4 metres correct to one decimal place.

(a) Write down the upper bound of the length of the table.

..... metres

(1)

(b) Write down the lower bound of the length of the table.

..... metres

(1)

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2024

1 The length is m correct

The length is 1.4 m correct to one decimal place.

2 Half of is

Half of 0.1 is 0.05.

$$1.4 + 0.05 = 1.45$$

$$1.4 - 0.05 = 1.35$$

FINAL ANSWER

(a) 1.45 m, (b) 1.35 m

4MA1/1H NOVEMBER 2024

Question 21

3 marks | Rounding, Estimation & Bounds

21 $T = \dfrac{x^2 + y^2}{w}$

$x = 28.4$ correct to 1 decimal place.

$y = 17$ correct to 2 significant figures.

$w = 90$ correct to the nearest 5

Calculate the upper bound for the value of T

Give your answer correct to 3 significant figures.
Show your working clearly.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2024

1 Estimate the value

$$T = \frac{x^2 + y^2}{w}$$

2 Find upper bound

For the upper bound, use the upper bounds of x and y, and the lower bound of w.

$$x < 28.45, \quad y < 17.5, \quad w \geq 87.5$$

$$T_{\text{upper}} = \frac{28.45^2 + 17.5^2}{87.5} = 12.7503\ldots$$

3 Estimate the value

Correct to 3 significant figures,

$$12.7503\ldots \approx 12.8$$

FINAL ANSWER

12.8

4WM1/01R MAY 2025

Question 1

2 marks | Rounding, Estimation & Bounds

1 The length of a ship is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the ship.

..... m

(1)

(ii) Write down the upper bound of the length of the ship.

..... m

(1)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4WM1/01R May 2025

1 **Calculate value**

The value is correct to 1 decimal place, so the half-unit is 0.05.

$$142.8 - 0.05 = 142.75, \qquad 142.8 + 0.05 = 142.85$$

✓ **FINAL ANSWER**

lower bound 142.75 m, upper bound 142.85 m.

4MA1/1HR JUNE 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 The length of a ship is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the ship.

..... m

(1)

(ii) Write down the upper bound of the length of the ship.

..... m

(1)

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2025

1 The length is m correct

The length is 142.8 m correct to 1 decimal place.

2 Half of is

Half of 0.1 is 0.05.

$$142.8 - 0.05 = 142.75$$

$$142.8 + 0.05 = 142.85$$

FINAL ANSWER

(i) 142.75 m, (ii) 142.85 m

4WM1/01 MAY 2025

Question 2

4 marks | Rounding, Estimation & Bounds

2 The height of a tree is 17.8 metres correct to one decimal place.

(a) Write down the lower bound for the height of the tree.

..... metres

(1)

The weight of an apple is 145 grams correct to the nearest 5 grams.

(b) Write down the upper bound for the weight of the apple.

..... grams

(1)

$$A = \frac{4.766 \times 815}{0.399}$$

(c) By rounding each number to one significant figure, work out an estimate for the value of A
Show your working clearly.

.....

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 May 2025

1 **Find lower bound**

For 17.8 correct to 1 decimal place, the lower bound is

$$17.8 - 0.05 = 17.75$$

2 **Find upper bound**

For 145 correct to the nearest 5 grams, the upper bound is

$$145 + 2.5 = 147.5$$

3 **Estimate the value**

For the estimate,

$$A = \frac{4.766 \times 815}{0.399}$$

4 **Round to 1 significant figure**

Round to 1 significant figure.

$$4.766 \approx 5, \quad 815 \approx 800, \quad 0.399 \approx 0.4$$

$$A \approx \frac{5 \times 800}{0.4} = 10000$$

✓ **FINAL ANSWER**

17.75 m, 147.5 g, and 10000.

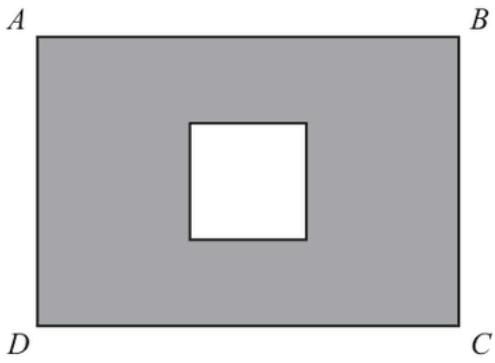
4MA1/2H JUNE 2025

Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows a square inside rectangle $ABCD$

Diagram **NOT** accurately drawn



The total area of the region shown shaded in the diagram is $X\text{cm}^2$

$AB = 11.5$ cm correct to the nearest 0.5 cm
$BC = 9.2$ cm correct to 2 significant figures
side of square $= 4.1$ cm correct to 2 significant figures

By considering bounds, work out the value of X to a suitable degree of accuracy.
Show your working clearly.

$X =$ .....

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2025

1 **The shaded area is**

The shaded area is

$$X = \text{area of rectangle} - \text{area of square}$$

2 **Bounds**

Bounds:

$$11.25 \leq AB < 11.75$$

$$9.15 \leq BC < 9.25$$

$$4.05 \leq \text{square side} < 4.15$$

3 **Lower bound**

Lower bound:

$$11.25(9.15) - 4.15^2 = 85.715$$

4 **Upper bound**

Upper bound:

$$11.75(9.25) - 4.05^2 = 92.285$$

5 **Estimate the value**

So

$$85.715 < X < 92.285$$

6 **Estimate the value**

Both bounds round to 90 to the nearest 10.

✓ **FINAL ANSWER**

$X = 90\ \text{cm}^2$ to a suitable degree of accuracy

4WM1/01 NOVEMBER 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 By rounding each number to one significant figure, work out an estimate for the value of

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}}$$

Show your working clearly.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 November 2025

1 Round each number to one

Round each number to one significant figure:

$$2.11 \approx 2, \quad 58.9 \approx 60, \quad 8.859 \approx 9$$

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}} \approx \frac{2^2 \times 60}{\sqrt{9}}$$

$$= \frac{4 \times 60}{3} = 80$$

FINAL ANSWER

80

4MA1/1H NOVEMBER 2025

Question 2

2 marks | Rounding, Estimation & Bounds

2 By rounding each number to one significant figure, work out an estimate for the value of

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}}$$

Show your working clearly.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2025

1 Round each number to one

Round each number to one significant figure:

$$2.11 \approx 2, \quad 58.9 \approx 60, \quad 8.859 \approx 9$$

$$\frac{2.11^2 \times 58.9}{\sqrt{8.859}} \approx \frac{2^2 \times 60}{\sqrt{9}}$$

$$= \frac{4 \times 60}{3} = 80$$

FINAL ANSWER

80

4WM1/01 NOVEMBER 2025

Question 6

2 marks | Rounding, Estimation & Bounds

6 The weight of a bag of apples is 475 g correct to the nearest g

(a) Write down the lower bound of the weight.

..... g

(1)

The height of a box is 120 cm correct to the nearest 10 cm

(b) Write down the upper bound for the height.

..... cm

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 November 2025

1 Find lower bound

The weight 475 g correct to the nearest g has lower bound

$$475 - 0.5 = 474.5$$

2 Find upper bound

The height 120 cm correct to the nearest 10 cm has upper bound

$$120 + 5 = 125$$

FINAL ANSWER

(a) 474.5 g, (b) 125 cm

4MA1/1H NOVEMBER 2025

Question 7

2 marks | Rounding, Estimation & Bounds

7 The weight of a bag of apples is 475 g correct to the nearest g

(a) Write down the lower bound of the weight.

..... g

(1)

The height of a box is 120 cm correct to the nearest 10 cm

(b) Write down the upper bound for the height.

..... cm

(1)

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2025

1 Find lower bound

The weight 475 g correct to the nearest g has lower bound

$$475 - 0.5 = 474.5$$

2 Find upper bound

The height 120 cm correct to the nearest 10 cm has upper bound

$$120 + 5 = 125$$

FINAL ANSWER

(a) 474.5 g, (b) 125 cm

4MA1/2H NOVEMBER 2025

Question 24

5 marks | Rounding, Estimation & Bounds

24 $D = \dfrac{n}{p - q}$

$n = 10.3$ correct to 1 decimal place
$p = 7.24$ correct to 2 decimal places
$q = 4.39$ correct to 2 decimal places

By considering bounds, work out the value of D to a suitable degree of accuracy.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2025

1 Estimate the value

$$D = \frac{n}{p - q}$$

2 Bounds

Bounds:

$$10.25 \leq n < 10.35$$

$$7.235 \leq p < 7.245$$

$$4.385 \leq q < 4.395$$

3 Lower bound

Lower bound:

$$D_{\text{lower}} = \frac{10.25}{7.245 - 4.385} = 3.5839\ldots$$

4 Upper bound

Upper bound:

$$D_{\text{upper}} = \frac{10.35}{7.235 - 4.395} = 3.6443\ldots$$

5 **Estimate the value**

So

$$3.5839\ldots < D < 3.6443\ldots$$

---

6 **Estimate the value**

Both bounds round to $\mathbf{3.6}$ to 1 decimal place.

✓ **FINAL ANSWER**

$D = \mathbf{3.6}$ to a suitable degree of accuracy

4MA1/1HR JUNE 2026

Question 4

2 marks | Rounding, Estimation & Bounds

4 The weight of a giraffe is 1178 kg, correct to the nearest kg

(i) Write down the lower bound of the weight of the giraffe.

..... kg

(1)

(ii) Write down the upper bound of the weight of the giraffe.

..... kg

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2026

1 Part (i) | Step 1

$$1178 - 0.5 = 1177.5 \text{ kg}.$$

Rounding to the nearest kilogram allows half a kilogram below the stated value.

2 Part (ii) | Step 1

$$1178 + 0.5 = 1178.5 \text{ kg}.$$

The rounding interval is $1177.5 \leq w < 1178.5$; the upper endpoint is the upper bound, not an included weight.

FINAL ANSWER

(i) 1177.5 kg (ii) 1178.5 kg

Question 18

5 marks | Rounding, Estimation & Bounds

18 The final velocity, v m/s, of an object is given by

$$v = 2\left(\frac{s}{t}\right) - u$$

where

$s = 100$ correct to the nearest integer
$t = 1.68$ correct to 2 decimal places
$u = 0.5$ correct to 1 decimal place

By considering bounds, work out the value of v to a suitable degree of accuracy.
Show your working clearly.

.....

(Total for Question 18 is 5 marks)

1 **Bounds**

$99.5 \leq s < 100.5$, $1.675 \leq t < 1.685$, $0.45 \leq u < 0.55$

2 **Upper bound of v**

Big s, small t, small u:

$$v_{\max} = 2\left(\frac{100.5}{1.675}\right) - 0.45 = 119.55$$

3 **Lower bound of v**

Small s, big t, big u:

$$v_{\min} = 2\left(\frac{99.5}{1.685}\right) - 0.55 = 117.55\ldots$$

4 **Suitable accuracy**

Both bounds round to 120 to 2 s.f. (they differ to 3 s.f.), so $v = 120$.

✓ **FINAL ANSWER**

$v = 120$ m/s (2 s.f.)

4MA1/1H JUNE 2026

Question 20

5 marks | Rounding, Estimation & Bounds

20 The final velocity, v m/s, of an object is given by

$$v = 2\left(\frac{s}{t}\right) - u$$

where

$s = 100$ correct to the nearest integer
$t = 1.68$ correct to 2 decimal places
$u = 0.5$ correct to 1 decimal place

By considering bounds, work out the value of v to a suitable degree of accuracy.
Show your working clearly.

.....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H June 2026

1 Solution | Step 1

$$99.5 \leq s < 100.5, \quad 1.675 \leq t < 1.685, \quad 0.45 \leq u < 0.55.$$

Use half a rounding unit on each side of the stated value.

2 Solution | Step 2

For $v = 2s/t - u$, the lower limiting value is

$$v_L = \frac{2(99.5)}{1.685} - 0.55 = 117.550890\ldots \text{ m/s}.$$

Decrease the numerator, increase the positive denominator and subtract the largest u.

3 Solution | Step 3

The upper limiting value is

$$v_U = \frac{2(100.5)}{1.675} - 0.45 = 119.55 \text{ m/s}.$$

Increase the numerator, decrease the denominator and subtract the smallest u.

4

Solution | Step 4

Both limiting values round to $120\ \mathrm{m/s}$ to 2 significant figures. They do not round to the same integer, so

$$v = 120\ \mathrm{m/s} \quad (2\ \mathrm{s.f.}).$$

This is the greatest common significant-figure accuracy justified throughout the interval.

✓ **FINAL ANSWER**

$120\ \mathrm{m/s}$ to 2 significant figures.

4MA1/2HR JUNE 2026

Question 21

3 marks | Rounding, Estimation & Bounds

21 $p = \dfrac{d-e}{f}$

$d = 6.6$ correct to 1 decimal place
$e = 2.8$ correct to 1 decimal place
$f = 0.47$ correct to 2 decimal places

Calculate the lower bound for the value of p
Give your answer correct to 3 significant figures.
Show your working clearly.

.....

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2026

1 Solution | Step 1

$$6.55 \le d < 6.65, \quad 2.75 \le e < 2.85, \quad 0.465 \le f < 0.475.$$

Use half of each rounding unit to obtain the bounds.

2 Solution | Step 2

$$p_{\text{lower}} = \frac{6.55 - 2.85}{0.475} = \frac{3.70}{0.475} = 7.789473684\ldots.$$

Minimize the positive numerator by using the lower bound of d and upper bound of e; maximize the positive denominator.

3 Solution | Step 3

$$p_{\text{lower}} = 7.79 \quad \text{(3 significant figures)}.$$

Round the boundary value to the requested accuracy.

✓ **FINAL ANSWER**

Lower bound $= 7.79$ (3 significant figures).

TOPIC

Powers & Roots

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 33

3 marks | Powers & Roots

33 Find the value of

(a) 5^0

.....

(1)

(b) $8^{2/3}$

.....

(1)

(c) $16^{-1/2}$

.....

(1)

(Total for Question 33 is 3 marks)

WORKED SOLUTION

Question 33 · EP Specimen

1 **(a) Any non-zero number to the power 0**

$5^0 = 1$

2 **(b) Root first, then power**

$$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

3 **(c) Negative power means reciprocal**

$$16^{-1/2} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

✓ **FINAL ANSWER**

(a) 1 (b) 4 (c) $\frac{1}{4}$

EP SPECIMEN

Question 34

2 marks | Powers & Roots

34 Write $\dfrac{3^5 \times 3^{-2}}{3^4}$ as a single power of 3

.....

(2)

(Total for Question 34 is 2 marks)

WORKED SOLUTION

Question 34 · EP Specimen

1 Multiply: add the powers

$3^5 \times 3^{-2} = 3^3$

2 Divide: subtract the powers

$$\frac{3^3}{3^4} = 3^{-1}$$

✓ **FINAL ANSWER**

3^{-1}

EP SPECIMEN

Question 35

3 marks | Powers & Roots

35 $2^n = 8^4 \div 4^3$

Find the value of n.

$n =$ .....

(3)

(Total for Question 35 is 3 marks)

WORKED SOLUTION

Question 35 · EP Specimen

1 Write everything as a power of 2

$8 = 2^3$, so $8^4 = 2^{12}$. $4 = 2^2$, so $4^3 = 2^6$.

2 Divide

$$\frac{2^{12}}{2^6} = 2^6 \Rightarrow n = 6$$

FINAL ANSWER

$n = 6$

Question 3

5 marks | Powers & Roots

3 $20^{70} \times 20^{8} = 20^{n}$

(a) Find the value of n

$n =$ .....

(1)

$36^{80} \div 36^{5} = 36^{m}$

(b) Find the value of m

$m =$ .....

(1)

$\left(52^{4}\right)^{16} = 52^{p}$

(c) Find the value of p

$p =$ .....

(1)

In a race, Dani ran a distance of 100 metres, correct to the nearest metre.

(d) Write down the lower bound for the distance.

..... metres

(1)

Dani ran this race in a time of 16.3 seconds, correct to one decimal place.

(e) Write down the upper bound for the time.

..... seconds

(1)

(Total for Question 3 is 5 marks)

1 **(a) Multiply: add powers**

$$20^{70} \times 20^{8} = 20^{78} \Rightarrow n = 78$$

2 **(b) Divide: subtract powers**

$$36^{80} \div 36^{5} = 36^{75} \Rightarrow m = 75$$

3 **(c) Power of a power: multiply**

$$(52^{4})^{16} = 52^{64} \Rightarrow p = 64$$

4 **(d) Lower bound**

Nearest metre means ± 0.5 m: lower bound $= 99.5$ m.

5 **(e) Upper bound**

One decimal place means ± 0.05 s: upper bound $= 16.35$ s.

✓ **FINAL ANSWER**

(a) 78 (b) 75 (c) 64 (d) 99.5 m (e) 16.35 s

TOPIC

Surds

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 17

4 marks | Surds

17 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show clearly each stage of your working.

.....

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 Specimen

1 Simplify the surds

$$\sqrt{8}=2\sqrt{2}$$

$$(\sqrt{2}-1)^2=2-2\sqrt{2}+1=3-2\sqrt{2}$$

2 Use the conjugate

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

The denominator is

$$9-(2\sqrt{2})^2=9-8=1$$

3 **Expand the numerator**

$$(3+2\sqrt{2})^2 = 9 + 12\sqrt{2} + 8 = 17 + 12\sqrt{2}$$

4 **Write in the requested form**

$$12\sqrt{2} = \sqrt{144 \times 2} = \sqrt{288}$$

✓ **FINAL ANSWER**

$17 + \sqrt{288}$.

4MA1/2H JANUARY 2020

Question 20

3 marks | Surds

20 The area of a rectangle is $18\,\text{cm}^2$

The length of the rectangle is $\left(\sqrt{7}+1\right)$ cm.

Without using a calculator and showing each stage of your working,

find the width of the rectangle.
Give your answer in the form $a\sqrt{b}+c$ where a, b and c are integers.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2020

1 Simplify surd

$$\text{width} = \frac{18}{\sqrt{7}+1}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{18}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{18(\sqrt{7}-1)}{7-1}$$

$$= \frac{18(\sqrt{7}-1)}{6} = 3(\sqrt{7}-1)$$

$$= 3\sqrt{7}-3$$

FINAL ANSWER

$3\sqrt{7}-3$ cm

4MA1/1H MAY/NOVEMBER 2020

Question 19

3 marks | Surds

19 Without using a calculator, rationalise the denominator of $\frac{6}{3-\sqrt{7}}$

Simplify your answer.
You must show each stage of your working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May/November 2020

1 Simplify surd

$$\frac{6}{3-\sqrt{7}}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{6}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{6(3+\sqrt{7})}{9-7}$$

$$= \frac{6(3+\sqrt{7})}{2} = 3(3+\sqrt{7})$$

$$= 9+3\sqrt{7}$$

FINAL ANSWER

$9+3\sqrt{7}$

4MA1/1H JANUARY 2021

Question 18

3 marks | Surds

18 Given that $(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$ where x is a prime number and y is an integer,

find the value of x and the value of y.
Show each stage of your working clearly.

$x =$ .....

$y =$ .....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2021

1 Simplify surd

$$(8 - \sqrt{x})(5 + \sqrt{x}) = 40 + 8\sqrt{x} - 5\sqrt{x} - x$$

$$= 40 + 3\sqrt{x} - x$$

2 Simplify surd

This is equal to

$$y\sqrt{x} + 21$$

3 Compare the rational parts

Compare the rational parts:

$$40 - x = 21$$

$$x = 19$$

4 Compare the surd coefficients

Compare the surd coefficients:

$$y = 3$$

FINAL ANSWER

$x = 19, y = 3$

4MA1/2H NOVEMBER 2021

Question 17

3 marks | Surds

17 Express $\frac{8}{\sqrt{5}-1}$ in the form $\sqrt{a}+b$ where a and b are integers.

Show each stage of your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2021

1 Simplify surd

$$\frac{8}{\sqrt{5}-1}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{8}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{8(\sqrt{5}+1)}{5-1}$$

$$= 2(\sqrt{5}+1) = 2\sqrt{5}+2$$

3 Simplify surd

In the form $\sqrt{a}+b$,

$$2\sqrt{5}+2 = \sqrt{20}+2$$

FINAL ANSWER

$\sqrt{20}+2$

4MA1/1H JUNE 2022

Question 16

3 marks | Surds

16 Without using a calculator, show that $\frac{12}{\sqrt{2}-1} - \left(\sqrt{2}\right)^5 = 2\sqrt{32} + 12$

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2022

1 **Proof**

Rationalise the denominator:

$$\frac{12}{\sqrt{2}-1} = \frac{12(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= 12(\sqrt{2}+1) = 12\sqrt{2}+12$$

Also,

$$(\sqrt{2})^5 = (\sqrt{2})^4\sqrt{2} = 4\sqrt{2}$$

So the left side is

$$12\sqrt{2}+12-4\sqrt{2} = 8\sqrt{2}+12$$

And

$$2\sqrt{32}+12 = 2(4\sqrt{2})+12 = 8\sqrt{2}+12$$

So both sides are equal.

FINAL ANSWER

Shown.

4MA1/1HR JUNE 2022

Question 21

4 marks | Surds

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2022

1 **Simplify surd**

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)^2}$$

2 **Simplify surd**

Since $\sqrt{8}=2\sqrt{2}$,

$$\frac{3+2\sqrt{2}}{(\sqrt{2}-1)^2}$$

3 **Simplify surd**

Also

$$(\sqrt{2}-1)^2=3-2\sqrt{2}$$

4 **Simplify surd**

So

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{(3+2\sqrt{2})^2}{9-8}$$

$$=(3+2\sqrt{2})^2=9+12\sqrt{2}+8$$

$$=17+12\sqrt{2}=17+\sqrt{288}$$

FINAL ANSWER

$17+\sqrt{288}$

4MA1/2H JUNE 2023

Question 23

4 marks | Surds

23 The diagram shows a cuboid with a square cross section.

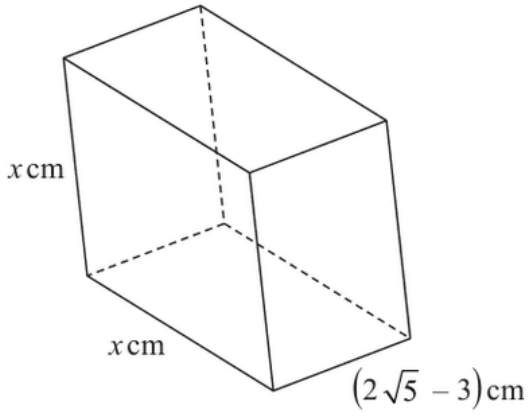


Diagram **NOT** accurately drawn

The volume of the cuboid is $(13 + 6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of x
Give your answer in the form $a + \sqrt{b}$ where a and b are integers.
Show your working clearly.

$x =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2023

1 The volume is

The volume is

$$x^2(2\sqrt{5} - 3) = 13 + 6\sqrt{5}$$

2 Simplify surd

So

$$x^2 = \frac{13 + 6\sqrt{5}}{2\sqrt{5} - 3}$$

3 **Rationalise**

Rationalise:

$$x^2 = \frac{(13+6\sqrt{5})(2\sqrt{5}+3)}{(2\sqrt{5})^2 - 3^2}$$

$$= \frac{26\sqrt{5}+39+60+18\sqrt{5}}{20-9}$$

$$= \frac{99+44\sqrt{5}}{11} = 9+4\sqrt{5}$$

---

4 **Simplify surd**

Since

$$(\sqrt{5}+2)^2 = 5+4+4\sqrt{5} = 9+4\sqrt{5}$$

$$x = \sqrt{5}+2$$

✓ **FINAL ANSWER**

$2+\sqrt{5}$

4WM1/01R MAY 2025

Question 16

4 marks | Surds

16 $\sqrt{96} = 2\sqrt{k}$

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{8}{3-\sqrt{5}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers.

Show each stage of your working.

(3)

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4WM1/01R May 2025

1 **(a)**

(a)

$$\sqrt{96} = 4\sqrt{6} = 2\sqrt{24}$$

2 **Calculate value**

So $k = 24$.

3 **(b)**

(b)

$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{8(3+\sqrt{5})}{9-5} = 2(3+\sqrt{5})$$

$$= 6+2\sqrt{5}$$

✓ **FINAL ANSWER**

$k = 24, 6+2\sqrt{5}$.

4WM1/01 MAY 2025

Question 17

4 marks | Surds

17 $(\sqrt{3})^5 = k\sqrt{3}$ where k is an integer.

(a) Find the value of k

$k =$ .....

(1)

(b) Show that $\dfrac{21}{3-\sqrt{2}}$ can be written in the form $c + \sqrt{d}$

where c and d are integers.
Show each stage of your working clearly.

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 May 2025

1 **Simplify surd**

$$(\sqrt{3})^5 = (\sqrt{3})^4\sqrt{3} = 3^2\sqrt{3} = 9\sqrt{3}$$

2 **Simplify surd**

So $k = 9$.

3 **Split the ratio**

For part (b), rationalise the denominator.

$$\frac{21}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{21(3+\sqrt{2})}{9-2}$$

$$= 3(3+\sqrt{2}) = 9+3\sqrt{2}$$

4 **Simplify surd**

Since $3\sqrt{2} = \sqrt{18}$,

$$9+3\sqrt{2} = 9+\sqrt{18}$$

FINAL ANSWER

$k = 9$, and $9 + \sqrt{18}$.

4WM1/01 NOVEMBER 2025

Question 16

3 marks | Surds

16 Show that $\dfrac{2\sqrt{7}+2}{\sqrt{7}-3}$ can be written in the form $a-\sqrt{b}$ where a and b are integers.

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM1/01 November 2025

1 **Simplify surd**

$$\frac{2\sqrt{7}+2}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}$$

$$=\frac{(2\sqrt{7}+2)(\sqrt{7}+3)}{7-9}$$

$$=\frac{20+8\sqrt{7}}{-2}$$

$$=-10-4\sqrt{7}=-10-\sqrt{112}$$

FINAL ANSWER

$-10-\sqrt{112}$.

Question 16

6 marks | Surds

16 (a) Solve $x^2 - \left(2\sqrt{3}\right)x - 13 = 0$

Give your solutions in the form $\sqrt{a} \pm b$ where a and b are integers.
Show your working clearly.

.....

(3)

(b) Show that $\dfrac{\sqrt{20}}{\sqrt{5}+2}$ can be written in the form $c - \sqrt{d}$ where c and d are integers.

(3)

(Total for Question 16 is 6 marks)

1 **(a) Quadratic formula**

$$x = \frac{2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 + 4(13)}}{2} = \frac{2\sqrt{3} \pm \sqrt{64}}{2}$$

2 **Simplify**

$$x = \frac{2\sqrt{3} \pm 8}{2} = \sqrt{3} \pm 4$$

3 **(b) Rationalise**

$$\frac{\sqrt{20}}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{2\sqrt{5}(\sqrt{5}-2)}{5-4}$$

4 **Simplify**

$$= 10 - 4\sqrt{5} = 10 - \sqrt{80} \quad (c = 10,\ d = 80)$$

✓ **FINAL ANSWER**

(a) $\sqrt{3} \pm 4$ (b) $10 - \sqrt{80}$

4MA1/1H JUNE 2026

Question 17

3 marks | Surds

17 Show that $\left(4-\sqrt{20}\right)\left(5-2\sqrt{5}\right)$ can be written in the form $a-\sqrt{b}$

where a and b are integers.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2026

1 Solution | Step 1

$$\sqrt{20}=2\sqrt{5}, \qquad (4-\sqrt{20})(5-2\sqrt{5})=(4-2\sqrt{5})(5-2\sqrt{5}).$$

Simplify the surd first.

2 Solution | Step 2

$$20-8\sqrt{5}-10\sqrt{5}+20=40-18\sqrt{5}.$$

Expand; the product of the two surd terms is positive 20.

3 Solution | Step 3

$$18\sqrt{5}=\sqrt{18^2\times 5}=\sqrt{1620}, \quad (4-\sqrt{20})(5-2\sqrt{5})=40-\sqrt{1620}.$$

Thus the required integer values are a=40 and b=1620.

FINAL ANSWER

$40-\sqrt{1620}$, so $a=40,\ b=1620$.

TOPIC

Using a Calculator

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 1

2 marks | Using a Calculator

1 Work out $\dfrac{5.7^2 - \sqrt{30.25}}{1.8 \times 0.65}$

Write down all the figures on your calculator display.

.....

(2)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · EP Specimen

1 **Work out the top and the bottom**

$$5.7^2 - \sqrt{30.25} = 32.49 - 5.5 = 26.99, \qquad 1.8 \times 0.65 = 1.17$$

2 **Divide**

$$\frac{26.99}{1.17} = 23.06837607\ldots$$

✓ **FINAL ANSWER**

23.06837607

EP SPECIMEN

Question 2

3 marks | Using a Calculator

2 (a) Work out $4.3^3 - \frac{1}{0.16}$

Write down all the figures on your calculator display.

.....

(2)

(b) Give your answer to part (a) correct to 3 significant figures.

.....

(1)

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · EP Specimen

1 **(a) Use the calculator**

$$4.3^3 - \frac{1}{0.16} = 79.507 - 6.25 = 73.257$$

2 **(b) Round to 3 significant figures**

The fourth figure is 5, so round up: 73.3.

FINAL ANSWER

(a) 73.257 (b) 73.3

EP SPECIMEN

Question 3

2 marks | Using a Calculator

3 Work out $\dfrac{8.4 \times \tan 35°}{\cos 62°}$

Give your answer correct to 3 significant figures.

.....

(2)

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · EP Specimen

1 Check the calculator is in degrees

Then

$$\frac{8.4 \times \tan 35°}{\cos 62°} = \frac{5.8817\ldots}{0.46947\ldots} = 12.528\ldots$$

2 Round

12.5 (3 significant figures)

FINAL ANSWER

12.5

EP SPECIMEN

Question 4

3 marks | Using a Calculator

4 Work out $\dfrac{3.7 \times 10^5 + 2.9 \times 10^4}{8.1 \times 10^{-3}}$

Give your answer in standard form correct to 3 significant figures.

.....

(3)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · EP Specimen

1 Add the top

$$3.7 \times 10^5 + 2.9 \times 10^4 = 370\,000 + 29\,000 = 399\,000$$

2 Divide

$$\frac{399\,000}{8.1 \times 10^{-3}} = 49\,259\,259.26\ldots$$

3 Standard form, 3 significant figures

4.93×10^7

✓ **FINAL ANSWER**

4.93×10^7

EP SPECIMEN

Question 5

2 marks | Using a Calculator

5 $p = 2.6 \quad q = -1.8$

Work out the value of $\dfrac{p^2 - 3q}{\sqrt{(p + q^2)}}$

Give your answer correct to 3 decimal places.

.....

(2)

(Total for Question 5 is 2 marks)

WORKED SOLUTION

Question 5 · EP Specimen

1 **Substitute (brackets round the negative)**

$$p^2 - 3q = 2.6^2 - 3(-1.8) = 6.76 + 5.4 = 12.16$$

$$\sqrt{p + q^2} = \sqrt{2.6 + (-1.8)^2} = \sqrt{5.84} = 2.4166\ldots$$

2 **Divide and round**

$$\frac{12.16}{2.4166\ldots} = 5.03184\ldots \approx 5.032$$

✓ **FINAL ANSWER**

5.032

EP SPECIMEN

Question 6

2 marks | Using a Calculator

6 Work out the value of $4.8^{-0.5} + 1.7^3$

Give your answer correct to 4 significant figures.

.....

(2)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · EP Specimen

1 Evaluate each power

$$4.8^{-0.5} = \frac{1}{\sqrt{4.8}} = 0.45643\ldots, \qquad 1.7^3 = 4.913$$

2 Add and round

$$0.45643\ldots + 4.913 = 5.36943\ldots \approx 5.369$$

✓ **FINAL ANSWER**

5.369

EP SPECIMEN

Question 7

3 marks | Using a Calculator

7 Here is a calculation: $\frac{19.6 \times 0.48}{4.1}$

(a) By rounding each number to 1 significant figure, work out an estimate for the calculation.

.....

(2)

(b) Use your calculator to work out the calculation. Give your answer correct to 3 significant figures.

.....

(1)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · EP Specimen

1 **(a) Round each number to 1 significant figure**

$$\frac{20 \times 0.5}{4} = \frac{10}{4} = 2.5$$

2 **(b) Calculator value**

$$\frac{19.6 \times 0.48}{4.1} = 2.2946\ldots \approx 2.29$$

✓ **FINAL ANSWER**

(a) 2.5 (b) 2.29

TOPIC

Standard & Compound Units

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 5

3 marks | Standard & Compound Units

5 A plane flies from New York City to Los Angeles.

The plane flies a distance of 3980 kilometres in 5 hours 24 minutes.

Work out the average speed of the plane.
Give your answer in kilometres per hour correct to the nearest whole number.

..... kilometres per hour

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 Specimen

1 **Convert the time**

$$24 \text{ minutes} = \frac{24}{60} = 0.4 \text{ hours}$$

$$5 \text{ hours } 24 \text{ minutes} = 5.4 \text{ hours}$$

2 **Calculate the speed**

$$\text{speed} = \frac{3980}{5.4} = 737.037\ldots$$

3 **Round the answer**

$$737.037\ldots \approx 737$$

FINAL ANSWER

737 kilometres per hour.

4WM1/01 SPECIMEN

Question 8

3 marks | Standard & Compound Units

(Total for Question 7 is 4 marks)

8 Change a speed of 81 kilometres per hour to a speed in metres per second.

..... metres per second

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 Specimen

1 Convert kilometres to metres

$$81 \text{ km} = 81000 \text{ m}$$

2 Convert hours to seconds

$$1 \text{ hour} = 3600 \text{ seconds}$$

3 Divide to find metres per second

$$\frac{81000}{3600} = 22.5$$

FINAL ANSWER

22.5 metres per second.

4MA1/2H JANUARY 2020

Question 2

2 marks | Standard & Compound Units

2 Change 32.4 m^3 into cm^3

..... cm^3

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2020

1 Calculate value

$$1\text{ m} = 100\text{ cm}$$

2 Calculate value

So

$$1\text{ m}^3 = 100^3\text{ cm}^3 = 1,000,000\text{ cm}^3$$

$$32.4\text{ m}^3 = 32.4 \times 1,000,000$$

$$= 32,400,000\text{ cm}^3$$

FINAL ANSWER

$32,400,000\text{ cm}^3$.

4MA1/2HR JANUARY 2020

Question 5

3 marks | Standard & Compound Units

5 A plane takes 3 hours 36 minutes to fly from the Cayman Islands to New York.
The plane flies a distance of 2470 km.

Work out the average speed of the plane in km/h.
Give your answer correct to the nearest whole number.

..... km/h

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2020

1 The time is

The time is

$$3\text{ h }36\text{ min} = 3.6\text{ h}$$

2 Average speed

Average speed:

$$2470 \div 3.6 = 686.111\ldots$$

FINAL ANSWER

686 km/h.

4MA1/1HR JANUARY 2020

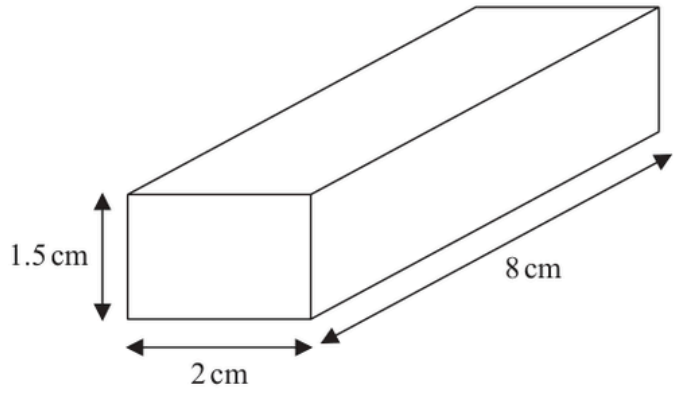
Question 9

5 marks | Standard & Compound Units

9 Pablo made a solid gold statue.

He melted down some gold blocks and used the gold to make the statue.
Each block of gold was a cuboid, as shown below.

Diagram **NOT** accurately drawn



The mass of the statue is 5.73 kg.
The density of gold is 19.32 g/cm^3

Work out the least number of gold blocks Pablo melted down in order to make the statue.
Show your working clearly.

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2020

1 Convert the mass to grams

Convert the mass to grams:

$$5.73 \text{ kg} = 5730 \text{ g}$$

2 Calculate volume

Using

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume of gold} = \frac{5730}{19.32} = 296.58\ldots \text{ cm}^3$$

3 **Volume of one cuboid block**

Volume of one cuboid block is

$$1.5 \times 2 \times 8 = 24 \text{ cm}^3$$

4 **Number of blocks needed is**

Number of blocks needed is

$$\frac{296.58\ldots}{24} = 12.35\ldots$$

5 **Pablo must have a whole**

Pablo must have a whole number of blocks, so he needs at least 13 blocks.

✓ **FINAL ANSWER**

13 blocks.

4MA1/2H JANUARY 2020

Question 9

2 marks | Standard & Compound Units

9 The density of gold is 19.3 g/cm^3
A gold bar has volume 150 cm^3

Work out the mass of the gold bar.

..... g

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2020

1 Calculate volume

$$\text{mass} = \text{density} \times \text{volume}$$

$$\text{mass} = 19.3 \times 150$$

$$\text{mass} = 2895$$

FINAL ANSWER

2895 g.

4MA1/2HR JANUARY 2020

Question 9

3 marks | Standard & Compound Units

9 The diagram shows a box in the shape of a cuboid.

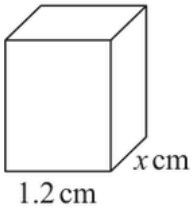


Diagram **NOT** accurately drawn

The box is put on a table.
The face of the box in contact with the table has length 1.2 metres and width x metres.

The force exerted by the box on the table is 27 newtons.
The pressure on the table due to the box is 30 newtons/m^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the value of x.

$x =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2020

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$30 = \frac{27}{\text{area}}$$

$$\text{area} = \frac{27}{30} = 0.9 \text{ m}^2$$

2 **The base area is**

The base area is

$$1.2x = 0.9$$

$$x = \frac{0.9}{1.2} = 0.75$$

✓ **FINAL ANSWER**

$x = 0.75\ \text{m}$.

4MA1/2H JANUARY 2020

Question 10

3 marks | Standard & Compound Units

10 Change a speed of 50 metres per second to a speed in kilometres per hour.

..... kilometres per hour

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2020

1 **Convert m/s to km/h multiply**

To convert m/s to km/h, multiply by $\mathbf{3.6}$.

$$50 \times 3.6 = 180$$

FINAL ANSWER

$180\ \mathrm{km/h}$.

4MA1/1H JANUARY 2020

Question 13

4 marks | Standard & Compound Units

13 The diagram shows a solid cube.
The cube is placed on a table so that the whole of one face of the cube is in contact with the table.

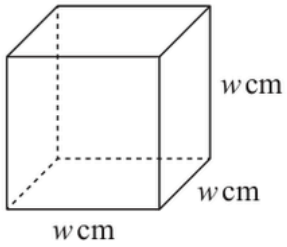


Diagram **NOT** accurately drawn

The cube exerts a force of 56 newtons on the table.
The pressure on the table due to the cube is 0.14 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cube.

..... cm^3

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2020

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$0.14 = \frac{56}{\text{area}}$$

$$\text{area} = \frac{56}{0.14} = 400 \text{ cm}^2$$

2 **The cube rests on one**

The cube rests on one square face, so

$$w^2 = 400$$

$$w = 20$$

3 **Volume of the cube is**

Volume of the cube is

$$20^3 = 8000$$

FINAL ANSWER

$8000\ \text{cm}^3$.

4MA1/1H MAY/NOVEMBER 2020

Question 6

3 marks | Standard & Compound Units

6 A rocket travelled 100 km at an average speed of 28 440 km/h.

Work out how long it took the rocket to travel the 100 km.
Give your answer in seconds, correct to the nearest second.

..... seconds

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May/November 2020

1 Use

Use

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{100}{28440} \text{ hours}$$

2 Convert hours to seconds

Convert hours to seconds:

$$\frac{100}{28440} \times 3600 = 12.658\ldots$$

3 To the nearest second

To the nearest second,

$$12.658\ldots \approx 13$$

FINAL ANSWER

13 seconds.

4MA1/1HR MAY/NOVEMBER 2020

Question 7

4 marks | Standard & Compound Units

7 Pedro drove from Toulouse to Montpellier in 2 hours 42 minutes.
He drove at an average speed of 90 km/hour.

Janine drove from Toulouse to Montpellier along the same route as Pedro.
The journey took her 3 hours.

Work out Janine's average speed for the journey.

..... km/hour

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May/November 2020

1 Pedro s time was

Pedro's time was

$$2\text{ h }42\text{ min} = 2.7\text{ h}$$

2 Distance from Toulouse to Montpellier

Distance from Toulouse to Montpellier:

$$90 \times 2.7 = 243\text{ km}$$

3 Calculate value

Janine took 3 hours, so her average speed was

$$243 \div 3 = 81$$

✓ **FINAL ANSWER**

81 km/hour.

4MA1/1HR MAY/NOVEMBER 2020

Question 9

3 marks | Standard & Compound Units

9 A box is put on a horizontal table.

The face of the box in contact with the table is a square of side 1.5 metres.
The pressure on the table due to the box is 34.8 newtons/m^2

Work out the force exerted by the box on the table.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

..... newtons

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May/November 2020

1 The area of the square

The area of the square face is

$$1.5 \times 1.5 = 2.25 \text{ m}^2$$

2 Use

Use

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

3 Calculate area

So

$$\text{force} = \text{pressure} \times \text{area}$$

$$34.8 \times 2.25 = 78.3$$

FINAL ANSWER

78.3 newtons.

4MA1/2H JANUARY 2021

Question 1

3 marks | Standard & Compound Units

1 A train takes 6 hours 39 minutes to travel from New Delhi to Kanpur.
The train travels a distance of 429 km.

Work out the average speed of the train.
Give your answer in km/h correct to one decimal place.

..... km/h

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2021

1 The time is

The time is

$$6\text{ h }39\text{ min} = 6.65\text{ h}$$

2 Average speed

Average speed:

$$429 \div 6.65 = 64.511\ldots$$

✓ **FINAL ANSWER**

64.5 km/h.

4MA1/1HR JANUARY 2021

Question 4

3 marks | Standard & Compound Units

4 A train journey from Paris to Amsterdam took 3 hours 24 minutes.
The total distance the train travelled was 433.5 km.

Work out the average speed of the train.
Give your answer in kilometres per hour.

..... km/h

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR January 2021

1 The time is

The time is

$$3\text{ h }24\text{ min} = 3.4\text{ h}$$

2 Average speed

Average speed:

$$433.5 \div 3.4 = 127.5$$

✓ **FINAL ANSWER**

127.5 km/h.

4MA1/1H MAY 2021

Question 1

3 marks | Standard & Compound Units

1 A plane flew from Madrid to Dubai.

The distance the plane flew was 5658 km.
The flight time was 8 hours 12 minutes.

Work out the average speed of the plane.

..... km/h

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2021

1 Evaluate fraction

$$8\text{ h }12\text{ min} = 8 + \frac{12}{60} = 8.2\text{ hours}$$

$$\text{average speed} = \frac{5658}{8.2} = 690$$

FINAL ANSWER

690 km/h.

4MA1/1H NOVEMBER 2021

Question 10

5 marks | Standard & Compound Units

10 A solid aluminium cylinder has radius 10 cm and height h cm.

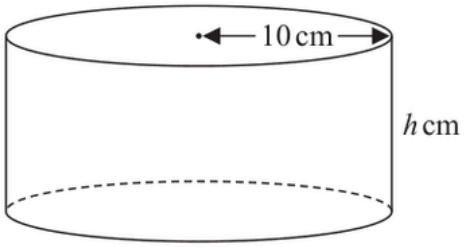


Diagram **NOT** accurately drawn

The mass of the cylinder is 5.4 kg.
The density of aluminium is 0.0027 kg/cm^3

Calculate the value of h.
Give your answer correct to one decimal place.

$h =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2021

1 Calculate volume

The mass is $5.4\ \text{kg}$, and the density is $0.0027\ \text{kg/cm}^3$.

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{5.4}{0.0027} = 2000\ \text{cm}^3$$

2 Evaluate fraction

For the cylinder,

$$V = \pi r^2 h$$

$$2000 = \pi(10)^2 h$$

$$h = \frac{2000}{100\pi} = 6.366\ldots$$

FINAL ANSWER

$6.4\ \text{cm}$.

4MA1/1HR JANUARY 2022

Question 7

4 marks | Standard & Compound Units

7 Milly went on a car journey.
She travelled from Anesey to Breigh to Clando and then to Duckbridge.

For Anesey to Breigh, Milly drove the 245 km in 2.5 hours.
For Breigh to Clando, Milly drove the 220 km at an average speed of 80 km/h
For Clando to Duckbridge, Milly drove at an average speed of 72 km/h in 50 minutes.

Work out Milly's average speed, in km/h, for the journey from Anesey to Duckbridge.
Give your answer correct to one decimal place.

..... km/h

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2022

1 Calculate value

First part:

$$\text{distance} = 245 \text{ km}, \qquad \text{time} = 2.5 \text{ h}$$

2 Second part

Second part:

$$\text{time} = \frac{220}{80} = 2.75 \text{ h}$$

3 Third part

Third part:

$$50 \text{ minutes} = \frac{5}{6} \text{ h}$$

$$\text{distance} = 72 \times \frac{5}{6} = 60 \text{ km}$$

4 Total distance

Total distance:

$$245 + 220 + 60 = 525 \text{ km}$$

5 **Total time**

Total time:

$$2.5 + 2.75 + \frac{5}{6} = 6.0833\ldots\ \text{h}$$

6 **Average speed**

Average speed:

$$\frac{525}{6.0833\ldots} = 86.301\ldots$$

✓ **FINAL ANSWER**

$86.3\ \text{km/h}$.

Question 21

6 marks | Standard & Compound Units

21 The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

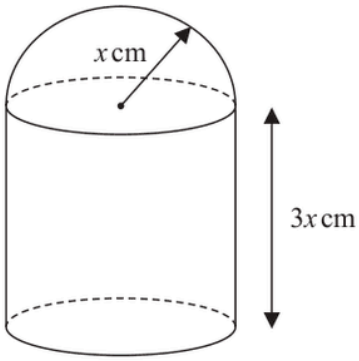


Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is 81π cm^2
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm^3)
Aluminium	2.7
Nickel	8.9
Gold	19.3
Silver	10.5

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

(Total for Question 21 is 6 marks)

1 **The outside surface area is**

The outside surface area is:

2 **curved surface area of cylinder**

curved surface area of cylinder:

$$2\pi x(3x) = 6\pi x^2$$

3 **bottom of cylinder**

bottom of cylinder:

$$\pi x^2$$

4 **curved surface area of hemisphere**

curved surface area of hemisphere:

$$2\pi x^2$$

5 **Calculate value**

So

$$6\pi x^2 + \pi x^2 + 2\pi x^2 = 81\pi$$

$$9\pi x^2 = 81\pi$$

$$x^2 = 9$$

$$x = 3$$

6 **Volume of the cylinder**

Volume of the cylinder:

$$\pi x^2(3x) = 3\pi x^3$$

7 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

8 **Total volume**

Total volume:

$$3\pi(3)^3 + \frac{2}{3}\pi(3)^3$$

$$= 81\pi + 18\pi = 99\pi$$

9 **Density**

Density:

$$\frac{840}{99\pi} = 2.70\ldots$$

10 **Compare density**

This matches aluminium, whose density is $2.7\ \text{g/cm}^3$.

✓ **FINAL ANSWER**

Aluminium

4MA1/2H JUNE 2022

Question 3

3 marks | Standard & Compound Units

3 An aeroplane travelled from New York City to Los Angeles.

The aeroplane travelled a distance of 3980 kilometres in 5 hours 24 minutes.

Work out the average speed of the aeroplane.
Give your answer in kilometres per hour correct to the nearest whole number.

..... kilometres per hour

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2022

1 The time is

The time is

$$5\text{ h }24\text{ min} = 5.4\text{ h}$$

2 Average speed

Average speed:

$$3980 \div 5.4 = 737.037\ldots$$

✓ FINAL ANSWER

737 kilometres per hour.

4MA1/1H JUNE 2022

Question 7

4 marks | Standard & Compound Units

7 A cylinder is placed on the ground.

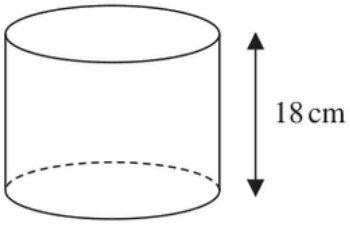


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm.

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2022

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$1.4 = \frac{72}{\text{area}}$$

$$\text{area} = \frac{72}{1.4} = 51.428\ldots \text{ cm}^2$$

2 The base area of the

The base area of the cylinder is $51.428\ldots$ cm^2.

3 **Volume is**

Volume is

$$51.428\ldots \times 18 = 925.714\ldots$$

✓ **FINAL ANSWER**

$926\ \text{cm}^3$ to 3 significant figures.

4MA1/1HR JUNE 2022

Question 7

3 marks | Standard & Compound Units

7 The diagram shows a solid cylinder made from iron.

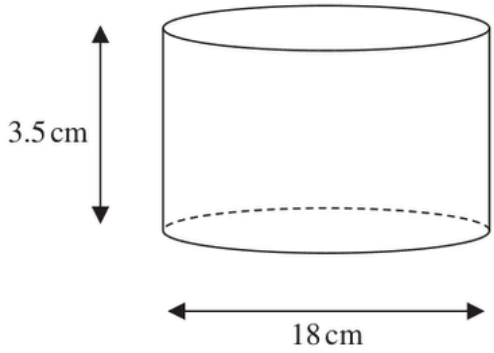


Diagram **NOT** accurately drawn

The cylinder has diameter 18 cm and height 3.5 cm
The mass of the cylinder is 7.04 kg

Work out the density of the iron.
Give your answer in g/cm^3 correct to 2 significant figures.

..... g/cm^3

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR June 2022

1 Radius of the cylinder is

Radius of the cylinder is

$$\frac{18}{2} = 9 \text{ cm}$$

2 Volume is

Volume is

$$\pi r^2 h = \pi(9^2)(3.5) = 283.5\pi$$

$$= 890.64\ldots \text{ cm}^3$$

3 Mass is

Mass is

$$7.04 \text{ kg} = 7040 \text{ g}$$

4 **Density is**

Density is

$$\frac{7040}{890.64\ldots} = 7.904\ldots$$

✓ **FINAL ANSWER**

$7.9\ \mathrm{g/cm^3}$ to 2 significant figures.

4MA1/2H JUNE 2022

Question 7

3 marks | Standard & Compound Units

7 Change a speed of 81 kilometres per hour to a speed in metres per second.

..... metres per second

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2022

1 Convert km/h to m/s divide

To convert km/h to m/s, divide by 3.6.

$$81 \div 3.6 = 22.5$$

FINAL ANSWER

$22.5\ \text{m/s}$.

4MA1/2HR JUNE 2023

Question 3

3 marks | Standard & Compound Units

3 Anjali travels on the Eurostar train from Paris to Amsterdam.

The distance the train travels between Paris and Amsterdam is 515 km.
The time taken by the train to travel between Paris and Amsterdam is 3 hours 18 minutes.

Work out the average speed of the train.
Give your answer in km/h correct to the nearest whole number.

..... km/h

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2023

1 Time hours minutes

Time $= 3$ hours 18 minutes:

$$3 + \frac{18}{60} = 3.3 \text{ hours}$$

$$\text{average speed} = \frac{515}{3.3} = 156.06\ldots$$

FINAL ANSWER

156 km/h.

4MA1/2H JUNE 2023

Question 9

3 marks | Standard & Compound Units

9 Change a speed of 27 kilometres per hour to a speed in metres per second.

..... m/s

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2023

1 Convert km/h to m/s divide

To convert km/h to m/s, divide by 3.6.

$$27 \div 3.6 = 7.5$$

FINAL ANSWER

7.5 m/s.

Question 11

3 marks | Standard & Compound Units

11 The diagram shows a block of iron in the shape of a cuboid.

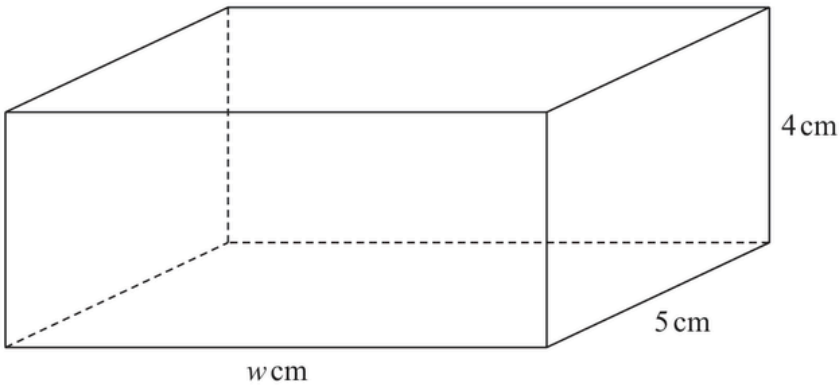


Diagram **NOT** accurately drawn

The block has length w cm, width 5 cm and height 4 cm

The density of iron is 7.8 g/cm^3
The mass of the block is 1950 g

Work out the value of w

$w =$ .....

(Total for Question 11 is 3 marks)

1 Calculate volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$7.8 = \frac{1950}{\text{volume}}$$

$$\text{volume} = \frac{1950}{7.8} = 250 \text{ cm}^3$$

2 Calculate value

For the cuboid,

$$w \times 5 \times 4 = 250$$

$$20w = 250$$

$$w = 12.5$$

FINAL ANSWER

$w = 12.5$ cm.

4MA1/2H JUNE 2024

Question 3

3 marks | Standard & Compound Units

3 A plane takes 9 hours 36 minutes to fly from New Delhi to Perth.

The plane flies at an average speed of 820 km/h.

Work out the total distance the plane flies.

..... km

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2024

1 Evaluate fraction

$$9\text{ h }36\text{ min} = 9 + \frac{36}{60} = 9.6\text{ hours}$$

$$\text{distance} = 820 \times 9.6 = 7872$$

FINAL ANSWER

7872 km.

4MA1/2H JUNE 2024

Question 6

3 marks | Standard & Compound Units

6 Change a speed of w metres per second to a speed in kilometres per hour.
Give your answer in terms of w in its simplest form.

..... kilometres per hour

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2024

1 Convert m/s to km/h multiply

To convert m/s to km/h, multiply by 3.6.

$$w \text{ m/s} = 3.6w \text{ km/h}$$

$$3.6w = \frac{18w}{5}$$

FINAL ANSWER

$\frac{18w}{5}$ km/h.

4MA1/1HR MAY 2024

Question 6

4 marks | Standard & Compound Units

6 The diagram shows a solid wooden cylinder.

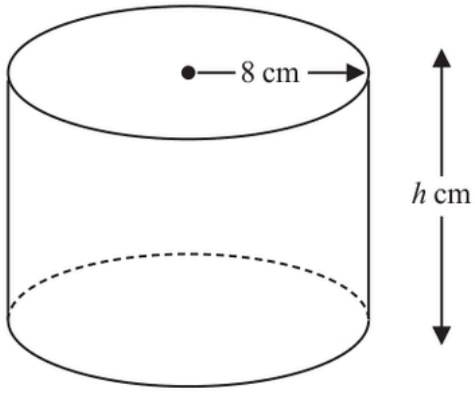


Diagram **NOT** accurately drawn

The cylinder has radius 8 cm and height h cm.
The volume of the cylinder is 1208 cm^3

(a) Work out the value of h
Give your answer correct to the nearest whole number.

$h =$ .....

(2)

The density of the wood is 1.25 g/cm^3

(b) Work out the mass of the cylinder.
Give your answer in kilograms.

..... kilograms

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May 2024

1 **Evaluate fraction**

For the cylinder,

$$V = \pi r^2 h$$

$$1208 = \pi(8)^2 h$$

$$h = \frac{1208}{64\pi} = 6.006\ldots$$

2 Calculate value

So, to the nearest whole number,

$$h = 6$$

3 Calculate volume

For the mass, use the given volume:

$$\text{mass} = \text{density} \times \text{volume}$$

$$1.25 \times 1208 = 1510 \text{ g}$$

$$1510 \text{ g} = 1.51 \text{ kg}$$

FINAL ANSWER

$h = 6$ cm, and the mass is 1.51 kg.

4MA1/1H MAY 2024

Question 8

3 marks | Standard & Compound Units

8 A cylinder is placed on a table.

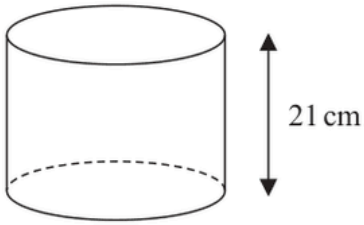


Diagram **NOT** accurately drawn

The volume of the cylinder is $1575\,\text{cm}^3$
The force exerted by the cylinder on the table is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the table due to the cylinder.

..... newtons/cm^2

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2024

1 The volume of a cylinder

The volume of a cylinder is

$$\text{volume} = \text{area of base} \times \text{height}$$

2 Calculate area

So the area in contact with the table is

$$1575 \div 21 = 75\ \text{cm}^2$$

3 Use

Use

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{pressure} = \frac{84}{75} = 1.12$$

FINAL ANSWER

1.12 newtons/cm^2.

4WM1/01 MAY 2025

Question 6

3 marks | Standard & Compound Units

6 The diagram shows a block in the shape of a cuboid.

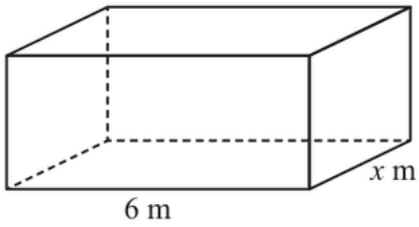


Diagram **NOT** accurately drawn

The block has a horizontal base with length 6 m and width x m

The block is placed on a table so that the whole of its horizontal base is in contact with the table.

The block exerts a force of 702 newtons on the table.
The pressure on the table due to the block is 65 newtons/m²

Work out the value of x

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 May 2025

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$65 = \frac{702}{\text{area}}$$

$$\text{area} = \frac{702}{65} = 10.8$$

2 **Calculate area**

The base area is $6x$, so

$$6x = 10.8$$

$$x = 1.8$$

✓ **FINAL ANSWER**

$x = 1.8$

4MA1/1HR JUNE 2025

Question 7

3 marks | Standard & Compound Units

7 Change a speed of $50x$ metres per second to a speed in kilometres per hour.

..... km/h

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR June 2025

1 Convert m/s to km/h multiply

To convert m/s to km/h, multiply by $\mathbf{3.6}$.

$$50x \times 3.6 = 180x$$

FINAL ANSWER

$180x$ km/h.

4MA1/2HR JUNE 2025

Question 7

2 marks | Standard & Compound Units

7 The mass of a silver coin is 48.3 g
The density of silver is 10.5 g/cm^3

Work out the volume of the silver coin.

..... cm^3

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2025

1 Use

Use

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

2 Calculate volume

So

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{48.3}{10.5} = 4.6$$

FINAL ANSWER

$4.6\ \text{cm}^3$.

4WM1/01R MAY 2025

Question 7

3 marks | Standard & Compound Units

7 Change a speed of 40 metres per second to a speed in kilometres per hour.
Show your working clearly.

..... kilometres per hour

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4WM1/01R May 2025

1 Evaluate fraction

$$40\ \text{m/s} = 40 \times \frac{3600}{1000}\ \text{km/h}$$

$$= 144\ \text{km/h}$$

FINAL ANSWER

144 kilometres per hour.

4WM1/01R MAY 2025

Question 10

5 marks | Standard & Compound Units

10 The diagram shows a rectangular sheet of metal.

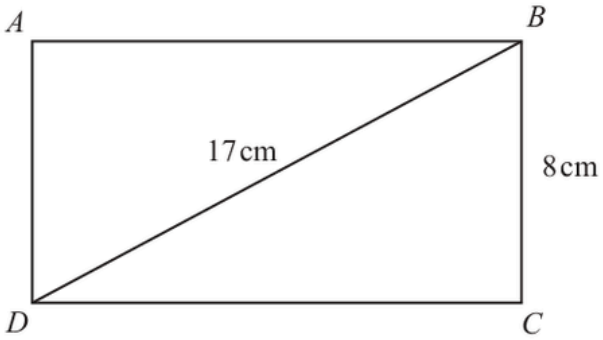


Diagram **NOT** accurately drawn

$BC = 8$ cm $\quad DB = 17$ cm

The sheet of metal lies flat on a table.
The force exerted by the sheet of metal on the table is 150 newtons.

Work out the pressure on the table due to the sheet of metal.
Show your working clearly.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

..... newtons/cm^2

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM1/01R May 2025

1 **Use Pythagoras to find the**

Use Pythagoras to find the other side of the rectangle.

$$DC = \sqrt{17^2 - 8^2} = 15$$

$$\text{area} = 15 \times 8 = 120 \text{ cm}^2$$

$$\text{pressure} = \frac{150}{120} = 1.25$$

FINAL ANSWER

1.25 newtons/cm^2.

4WM1/01 NOVEMBER 2025

Question 10

3 marks | Standard & Compound Units

10 A parcel in the shape of a cube is placed on a table so that the whole of one face of the parcel is in contact with the table.

The parcel has sides of length 7 cm

The pressure on the table due to the parcel is 62 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the force exerted by the parcel on the table.

..... newtons

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4WM1/01 November 2025

1 **The contact area is one**

The contact area is one square face:

$$7 \times 7 = 49 \text{ cm}^2$$

$$\text{force} = \text{pressure} \times \text{area} = 62 \times 49 = 3038$$

FINAL ANSWER

3038 newtons.

4MA1/2H NOVEMBER 2025

Question 26

6 marks | Standard & Compound Units

26 A solid cone is joined to a solid hemisphere to make the solid shown below.

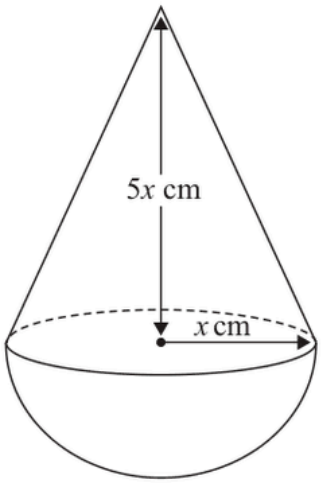


Diagram **NOT** accurately drawn

The cone is made from copper.
The density of copper is $9\,\text{g/cm}^3$

The hemisphere is made from a different metal.

The total mass of the solid is 4752π grams
The total volume of the solid is $504\pi\,\text{cm}^3$

Work out the density of the hemisphere.
Show your working clearly.

..... g/cm^3

(Total for Question 26 is 6 marks)

WORKED SOLUTION

Question 26 · 4MA1/2H November 2025

1 **Volume of cone**

Volume of cone:

$$\frac{1}{3}\pi x^2(5x) = \frac{5}{3}\pi x^3$$

2 **Volume of hemisphere**

Volume of hemisphere:

$$\frac{2}{3}\pi x^3$$

3 **Total volume**

Total volume:

$$\frac{5}{3}\pi x^3 + \frac{2}{3}\pi x^3 = 504\pi$$

$$\frac{7}{3}\pi x^3 = 504\pi$$

$$x^3 = 216$$

$$x = 6$$

4 **Cone volume**

Cone volume:

$$\frac{5}{3}\pi (6)^3 = 360\pi$$

5 **Mass of copper cone**

Mass of copper cone:

$$360\pi \times 9 = 3240\pi$$

6 **Mass of hemisphere**

Mass of hemisphere:

$$4752\pi - 3240\pi = 1512\pi$$

7 **Hemisphere volume**

Hemisphere volume:

$$\frac{2}{3}\pi (6)^3 = 144\pi$$

8 **Density of the hemisphere**

Density of the hemisphere:

$$\frac{1512\pi}{144\pi} = 10.5$$

FINAL ANSWER

$10.5\ \text{g/cm}^3$

4MA1/1H JUNE 2026

Question 4

4 marks | Standard & Compound Units

4 Henri competes in a race.

For the first part of the race Henri cycles 60 km at an average speed of 24 km/h
For the remaining part of the race Henri runs for 1 hour and 15 minutes at an average speed of 12 km/h

Work out Henri's average speed for the whole race.
Give your answer in km/h
Show your working clearly.

..... km/h

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2026

1 Solution | Step 1

Cycling time: $60 \div 24 = 2.5$ hours. Running time: $1 + 15/60 = 1.25$ hours.
Time equals distance divided by speed.

2 Solution | Step 2

Running distance: $12 \times 1.25 = 15$ km. Total distance: $60 + 15 = 75$ km. Total time: $2.5 + 1.25 = 3.75$ hours.
Combine the actual distances and times.

3 Solution | Step 3

$$\text{Average speed} = \frac{75}{3.75} = 20 \text{ km/h}.$$

Average speed is total distance divided by total time, not the mean of the two speeds.

FINAL ANSWER

20 km/h

4MA1/1H JUNE 2026

Question 8

3 marks | Standard & Compound Units

8 Change a speed of 70 metres per second to a speed in kilometres per hour.
Show your working clearly.

..... km/h

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H June 2026

1 Solution | Step 1

$$70\ \text{m/s} = 70 \times 3600 = 252000\ \text{m/h}.$$

There are 3600 seconds in one hour.

2 Solution | Step 2

$$252000 \div 1000 = 252\ \text{km/h}.$$

There are 1000 metres in one kilometre.

FINAL ANSWER

$252\ \text{km/h}$

4WM1/01 MAY 2026

Question 8

3 marks | Standard & Compound Units

8 A block of wood has a mass of 480 kg
The wood has a density of 640 kg/m^3

Work out the volume of the block of wood.

..... m^3

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 May 2026

1 **Volume = mass ÷ density**

$$V = \frac{480}{640} = 0.75 \text{ m}^3$$

FINAL ANSWER

0.75 m^3

4MA1/2HR JUNE 2026

Question 9

3 marks | Standard & Compound Units

9 Change a speed of 198 kilometres per hour to a speed in metres per second.
Show your working clearly.

..... m/s

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2026

1 Solution | Step 1

$198\ \text{km} = 198000\ \text{m}$ and $1\ \text{hour} = 3600\ \text{s}$.

Convert both the distance and the time in the rate.

2 Solution | Step 2

$$198\ \text{km/h} = \frac{198000}{3600} = 55\ \text{m/s}.$$

Metres per second means metres divided by seconds.

✓ FINAL ANSWER

$55\ \text{m/s}$.

4MA1/2HR JUNE 2026

Question 10

3 marks | Standard & Compound Units

10 The diagram shows a solid cylinder.

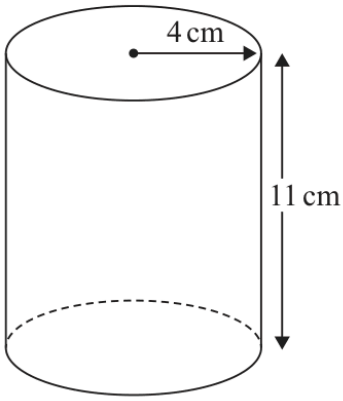


Diagram **NOT** accurately drawn

The cylinder has radius 4 cm and height 11 cm
The cylinder is made from wood with a density of 0.6 g/cm^3

Work out the mass of the cylinder.
Give your answer correct to 3 significant figures.

..... g

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$$V = \pi r^2 h = \pi(4)^2(11) = 176\pi \text{ cm}^3.$$

Use the cylinder volume formula.

2 **Solution | Step 2**

$$m = \rho V = 0.6(176\pi) = 105.6\pi = 331.7521842\ldots \text{ g}.$$

Mass equals density times volume; keep full precision until the end.

3 **Solution | Step 3**

$$m = 332 \text{ g} \quad (3 \text{ significant figures}).$$

Round the final mass as requested.

FINAL ANSWER

332 g (3 significant figures).

TOPIC

Algebra Toolkit

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 22

2 marks | Algebra Toolkit

22 (a) Simplify $5a + 3b - 2a + 7b$

.....

(1)

(b) Simplify $4x \times 3y$

.....

(1)

(Total for Question 22 is 2 marks)

WORKED SOLUTION

Question 22 · EP Specimen

1 **(a) Collect like terms**

$$5a - 2a = 3a, \qquad 3b + 7b = 10b \Rightarrow 3a + 10b$$

2 **(b) Multiply the numbers, then the letters**

$$4 \times 3 = 12 \Rightarrow 12xy$$

✓ **FINAL ANSWER**

(a) $3a + 10b$ (b) $12xy$

EP SPECIMEN

Question 23

2 marks | Algebra Toolkit

23 Work out the value of $3p^2 - 2q$ when $p = -4$ and $q = 5$

.....

(2)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · EP Specimen

1 **Substitute, with brackets round the negative**

$$3(-4)^2 - 2(5) = 3 \times 16 - 10$$

2 **Work it out**

$$48 - 10 = 38$$

✓ **FINAL ANSWER**

38

EP SPECIMEN

Question 24

2 marks | Algebra Toolkit

24 $v = u + at$

Work out the value of v when $u = 12$, $a = -3.5$ and $t = 6$

$v =$ .....

(2)

(Total for Question 24 is 2 marks)

WORKED SOLUTION

Question 24 · EP Specimen

1 Substitute

$$v = 12 + (-3.5) \times 6$$

2 Work it out

$$v = 12 - 21 = -9$$

✓ FINAL ANSWER

$v = -9$

EP SPECIMEN

Question 25

2 marks | Algebra Toolkit

25 Pens cost 35p each. Pencils cost 18p each.

Write down an expression, in terms of x and y, for the total cost, in pence, of x pens and y pencils.

.....

(2)

(Total for Question 25 is 2 marks)

WORKED SOLUTION

Question 25 · EP Specimen

1 **Cost of each item × number bought**

x pens cost $35x$ pence; y pencils cost $18y$ pence.

2 **Add**

$35x + 18y$

✓ **FINAL ANSWER**

$35x + 18y$

EP SPECIMEN

Question 26

2 marks | Algebra Toolkit

26 $T = \dfrac{a^2 + b}{2c}$

Work out the value of T when $a = -3$, $b = 7$ and $c = 0.5$

$T =$ .....

(2)

(Total for Question 26 is 2 marks)

WORKED SOLUTION

Question 26 · EP Specimen

1 Substitute

$$T = \frac{(-3)^2 + 7}{2 \times 0.5} = \frac{9 + 7}{1}$$

2 Work it out

$T = 16$

FINAL ANSWER

$T = 16$

EP SPECIMEN

Question 27

2 marks | Algebra Toolkit

27 Simplify $6x^2y - 2xy^2 + 3x^2y + 5xy^2$

.....

(2)

(Total for Question 27 is 2 marks)

WORKED SOLUTION

Question 27 · EP Specimen

1 Like terms have exactly the same letters and powers

x^2y terms: $6 + 3 = 9$. xy^2 terms: $-2 + 5 = 3$.

2 Write the answer

$9x^2y + 3xy^2$

✓ **FINAL ANSWER**

$9x^2y + 3xy^2$

EP SPECIMEN

Question 28

2 marks | Algebra Toolkit

28 A rectangle has length $(3x + 2)$ cm and width $(x - 1)$ cm.

Show that the perimeter, in cm, of the rectangle is $8x + 2$

.....

(2)

(Total for Question 28 is 2 marks)

WORKED SOLUTION

Question 28 · EP Specimen

1 **Perimeter = 2 lengths + 2 widths**

$$2(3x + 2) + 2(x - 1)$$

2 **Expand and collect**

$$6x + 4 + 2x - 2 = 8x + 2$$

✓ **FINAL ANSWER**

Shown: $8x + 2$

EP SPECIMEN

Question 29

2 marks | Algebra Toolkit

29 Here are five expressions.

$3n$ $\quad n+3$ $\quad n^3$ $\quad 3n^2$ $\quad -n$

When $n = -2$, which expression has the greatest value?
You must show your working.

.....

(2)

(Total for Question 29 is 2 marks)

WORKED SOLUTION

Question 29 · EP Specimen

1 Substitute n = −2 into each expression

$$3n = -6, \quad n+3 = 1, \quad n^3 = -8, \quad 3n^2 = 3 \times 4 = 12, \quad -n = 2$$

2 Pick the greatest

$3n^2 = 12$ is the greatest.

FINAL ANSWER

$3n^2$ (value 12)

TOPIC

Expanding Brackets

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR MAY/NOVEMBER 2020

Question 14

3 marks | Expanding Brackets

14 Expand and simplify $(4x + 1)(x - 3)(5x + 6)$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR May/November 2020

1 Expand brackets

$$(4x + 1)(x - 3)(5x + 6)$$

$$(x - 3)(5x + 6) = 5x^2 - 9x - 18$$

$$(4x + 1)(5x^2 - 9x - 18) = 20x^3 - 31x^2 - 81x - 18$$

FINAL ANSWER

$20x^3 - 31x^2 - 81x - 18$

4MA1/2HR JANUARY 2021

Question 1

8 marks | Expanding Brackets

1 $w = 5y^2 - y^3$

(a) Work out the value of w when $y = -2$

$w =$ .....

(2)

(b) Factorise fully $8p^2 - 2p$

.....

(2)

(c) Expand $4t(3t - 2)$

.....

(2)

(d) Expand and simplify $(5x - 2)(x + 4)$

.....

(2)

(Total for Question 1 is 8 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2021

1 **Expand brackets**

For $w = 5y^2 - y^3$ and $y = -2$:

$$w = 5(-2)^2 - (-2)^3 = 20 + 8 = 28$$

$$8p^2 - 2p = 2p(4p - 1)$$

$$4t(3t - 2) = 12t^2 - 8t$$

$$(5x - 2)(x + 4) = 5x^2 + 18x - 8$$

✓ **FINAL ANSWER**

$28,\ 2p(4p - 1),\ 12t^2 - 8t,\ 5x^2 + 18x - 8$

4MA1/1H JANUARY 2021

Question 12

6 marks | Expanding Brackets

12 (a) Express $\frac{4}{x-2} - \frac{3}{x+1}$ as a single fraction.
Give your answer in its simplest form.

.....

(3)

(b) Expand and simplify $2x(x-5)(x-3)$

.....

(3)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H January 2021

1 **(a)**

(a)

$$\frac{4}{x-2} - \frac{3}{x+1} = \frac{4(x+1) - 3(x-2)}{(x-2)(x+1)}$$

$$= \frac{4x + 4 - 3x + 6}{(x-2)(x+1)} = \frac{x+10}{(x-2)(x+1)}$$

2 **(b)**

(b)

$$2x(x-5)(x-3) = 2x(x^2 - 8x + 15)$$

$$= 2x^3 - 16x^2 + 30x$$

✓ **FINAL ANSWER**

(a) $\frac{x+10}{(x-2)(x+1)}$, (b) $2x^3 - 16x^2 + 30x$

4MA1/1HR JANUARY 2021

Question 16

5 marks | Expanding Brackets

16 (a) Expand and simplify $(x + 4)(x - 2)(x + 1)$

.....

(3)

(b) Express $x^2 - 10x + 40$ in the form $(x + a)^2 + b$, where a and b are integers.

.....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$(x+4)(x-2)(x+1)$$

$$(x-2)(x+1) = x^2 - x - 2$$

$$(x+4)(x^2 - x - 2) = x^3 + 3x^2 - 6x - 8$$

2 **(b)**

(b)

$$x^2 - 10x + 40 = (x-5)^2 + 15$$

✓ **FINAL ANSWER**

(a) $x^3 + 3x^2 - 6x - 8$, (b) $(x-5)^2 + 15$

4MA1/1H MAY 2021

Question 13

3 marks | Expanding Brackets

13 Expand and simplify $4x(3x + 1)(2x - 3)$
Show your working clearly.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2021

1 Expand brackets

$$4x(3x+1)(2x-3) = 4x(6x^2 - 7x - 3)$$

$$= 24x^3 - 28x^2 - 12x$$

FINAL ANSWER

$24x^3 - 28x^2 - 12x$

4MA1/1H NOVEMBER 2021

Question 1

3 marks | Expanding Brackets

1 (a) Simplify $e^8 \div e^2$

.....

(1)

(b) Expand and simplify $(x - 3)(x + 1)$

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2021

1 **(a)**

(a) $e^8 \div e^2 = e^6$.

2 **(b)**

(b) $(x - 3)(x + 1) = x^2 - 2x - 3$.

✓ **FINAL ANSWER**

(a) e^6, (b) $x^2 - 2x - 3$

4MA1/2H NOVEMBER 2021

Question 12

7 marks | Expanding Brackets

12 (a) Simplify $(64p^9q^{12})^{\frac{2}{3}}$

.....

(2)

(b) Write as a single fraction $\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x}$

Give your answer in its simplest form.

.....

(2)

(c) Expand and simplify $4x(x - 5)(2x + 3)$

Show your working clearly.

.....

(3)

(Total for Question 12 is 7 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2021

1 **(a)**

(a)

$$(64p^9q^{12})^{2/3} = 16p^6q^8$$

2 **(b)**

(b)

$$\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x} = \frac{20 + 24 - 27}{30x} = \frac{17}{30x}$$

3 **(c)**

(c)

$$4x(x-5)(2x+3) = 4x(2x^2 - 7x - 15)$$

$$= 8x^3 - 28x^2 - 60x$$

✓ **FINAL ANSWER**

(a) $16p^6q^8$, (b) $\frac{17}{30x}$, (c) $8x^3 - 28x^2 - 60x$

4MA1/2H JANUARY 2022

Question 1

4 marks | Expanding Brackets

1 (a) Expand and simplify $(y + 4)(2 - y)$

.....

(2)

(b) Factorise fully $15b^5c - 35b^3c^9$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2022

1 **(a)**

(a)

$$(y+4)(2-y) = 2y - y^2 + 8 - 4y = -y^2 - 2y + 8$$

2 **(b)**

(b)

$$15b^5c - 35b^3c^9 = 5b^3c(3b^2 - 7c^8)$$

✓ **FINAL ANSWER**

(a) $-y^2 - 2y + 8$, (b) $5b^3c(3b^2 - 7c^8)$

4MA1/1H JANUARY 2022

Question 13

6 marks | Expanding Brackets

13 (a) Expand and simplify $5x(x + 2)(3x - 4)$

.....

(3)

(b) Simplify completely $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

.....

(3)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x(x+2)(3x-4) = 5x(3x^2+2x-8)$$

$$= 15x^3 + 10x^2 - 40x$$

2 **(b)**

(b)

$$\left(\frac{16w^8}{y^{20}}\right)^{-3/4} = 16^{-3/4}w^{-6}y^{15}$$

3 **Expand brackets**

Since $16^{3/4} = 8$,

$$= \frac{y^{15}}{8w^6}$$

✓ **FINAL ANSWER**

(a) $15x^3 + 10x^2 - 40x$, (b) $\frac{y^{15}}{8w^6}$

4MA1/1HR JUNE 2022

Question 16

6 marks | Expanding Brackets

16 (a) Expand and simplify $(3x - 1)(x + 2)(3x + 1)$

.....

(3)

(b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

.....

(3)

(Total for Question 16 is 6 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$(3x - 1)(x + 2)(3x + 1) = (9x^2 - 1)(x + 2)$$

$$= 9x^3 + 18x^2 - x - 2$$

2 **(b)**

(b)

$$\left(\frac{2x^5}{8xy^2}\right)^{-2} = \left(\frac{x^4}{4y^2}\right)^{-2}$$

$$= \left(\frac{4y^2}{x^4}\right)^{2} = \frac{16y^4}{x^8}$$

✓ **FINAL ANSWER**

(a) $9x^3 + 18x^2 - x - 2$, (b) $\frac{16y^4}{x^8}$

4MA1/2HR JUNE 2023

Question 1

4 marks | Expanding Brackets

1 $P = m^2 - 4c$

(a) Work out the value of P when $m = -5$ and $c = 3$

$P =$ .....

(2)

(b) Expand and simplify $(x + 5)(x - 7)$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2023

1 **Expand brackets**

$$P = m^2 - 4c = (-5)^2 - 4(3) = 25 - 12 = 13$$

$$(x + 5)(x - 7) = x^2 - 7x + 5x - 35 = x^2 - 2x - 35$$

FINAL ANSWER

$P = 13$, and $x^2 - 2x - 35$

4MA1/2H NOVEMBER 2023

Question 16

3 marks | Expanding Brackets

16 Expand and simplify $(2x + 3)(x - 5)(x + 4)$

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2023

1 Expand brackets

$$(x - 5)(x + 4) = x^2 - x - 20$$

$$(2x + 3)(x^2 - x - 20) = 2x^3 + x^2 - 43x - 60$$

FINAL ANSWER

$2x^3 + x^2 - 43x - 60$

4MA1/1HR MAY 2024

Question 14

6 marks | Expanding Brackets

14 (a) Expand and simplify $(3x+1)(2-x)(4+x)$

.....

(3)

(b) Simplify fully $\left(\frac{a^3b}{a^9b^5}\right)^{-\frac{1}{2}}$

.....

(3)

(Total for Question 14 is 6 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR May 2024

1 **(a)**

(a)

$$(2-x)(4+x) = 8 - 2x - x^2$$

$$(3x+1)(8-2x-x^2) = -3x^3 - 7x^2 + 22x + 8$$

2 **(b)**

(b)

$$\left(\frac{a^3b}{a^9b^5}\right)^{-1/2} = \left(\frac{1}{a^6b^4}\right)^{-1/2} = a^3b^2$$

✓ **FINAL ANSWER**

(a) $-3x^3 - 7x^2 + 22x + 8$, (b) a^3b^2

4MA1/1H NOVEMBER 2024

Question 15

3 marks | Expanding Brackets

15 Write $3x(2x-1)(5x+4)$ in the form ax^3+bx^2+cx where a, b and c are integers.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2024

1 Expand brackets

$$3x(2x-1)(5x+4)$$

2 Expand brackets

First expand the two brackets:

$$(2x-1)(5x+4) = 10x^2+8x-5x-4$$

$$= 10x^2+3x-4$$

3 Expand brackets

Now multiply by $3x$:

$$3x(10x^2+3x-4) = 30x^3+9x^2-12x$$

FINAL ANSWER

$30x^3+9x^2-12x$

4WM1/01 MAY 2025

Question 11

6 marks | Expanding Brackets

11 (a) Expand and simplify $5x(5x + 4)(2x - 3)$

.....

(3)

(b) Express $\frac{9}{4} + \frac{y - 7}{5y}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4WM1/01 May 2025

1 **Expand brackets**

$$5x(5x + 4)(2x - 3)$$

2 **Expand brackets**

First,

$$(5x + 4)(2x - 3) = 10x^2 - 15x + 8x - 12 = 10x^2 - 7x - 12$$

3 **Expand brackets**

Then multiply by $5x$.

$$5x(10x^2 - 7x - 12) = 50x^3 - 35x^2 - 60x$$

4 **Expand brackets**

For part (b), use denominator $20y$.

$$\frac{9}{4} + \frac{y - 7}{5y} = \frac{45y}{20y} + \frac{4(y - 7)}{20y}$$

$$= \frac{49y - 28}{20y}$$

FINAL ANSWER

$50x^3 - 35x^2 - 60x$ and $\frac{49y - 28}{20y}$.

4MA1/2H JUNE 2025

Question 12

3 marks | Expanding Brackets

12 Expand and simplify $3x(2x+5)(7x-4)$

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2025

1 Expand brackets

$$3x(2x+5)(7x-4) = 3x(14x^2+27x-20)$$

$$= 42x^3+81x^2-60x$$

FINAL ANSWER

$42x^3+81x^2-60x$

4MA1/2HR JUNE 2025

Question 13

5 marks | Expanding Brackets

13 (a) Factorise $4x^2 - 25y^2$

.....

(2)

(b) Show that $4x(x + 3)(2x - 5)$ can be written in the form $ax^3 + bx^2 + cx$ where a, b and c are integers to be found.

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2025

1 **(a)**

(a)

$$4x^2 - 25y^2 = (2x - 5y)(2x + 5y)$$

2 **(b)**

(b)

$$4x(x + 3)(2x - 5)$$

3 **Factorise expression**

First,

$$(x + 3)(2x - 5) = 2x^2 + x - 15$$

4 **Factorise expression**

Then

$$4x(2x^2 + x - 15) = 8x^3 + 4x^2 - 60x$$

5 **Factorise expression**

So $a = 8$, $b = 4$, and $c = -60$.

FINAL ANSWER

(a) $(2x - 5y)(2x + 5y)$, (b) $8x^3 + 4x^2 - 60x$

4WM1/01R MAY 2025

Question 15

6 marks | Expanding Brackets

15 (a) Solve $3x^2 + 4x - 18 = 0$
Show your working clearly.
Give your solutions correct to 3 significant figures.

.....

(3)

(b) Expand and simplify $(2x - 3)(x + 4)(x - 1)$

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4WM1/01R May 2025

1 **Simplify surd**

(a) For $3x^2 + 4x - 18 = 0$,

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-18)}}{2(3)}$$

$$x \approx 1.87 \quad \text{or} \quad x \approx -3.21$$

2 **(b)**

(b)

$$(2x - 3)(x + 4)(x - 1)$$

$$= (2x^2 + 5x - 12)(x - 1)$$

$$= 2x^3 + 3x^2 - 17x + 12$$

FINAL ANSWER

$x = 1.87$ or $x = -3.21$; $2x^3 + 3x^2 - 17x + 12$.

4WM1/01 NOVEMBER 2025

Question 13

6 marks | Expanding Brackets

13 (a) Expand and simplify $7x(3x + 2)(2x - 5)$

.....

(3)

(b) Solve $\frac{9}{2y} + \frac{5}{7} = 5$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4WM1/01 November 2025

1 **(a)**

(a)

$$7x(3x + 2)(2x - 5)$$

$$= 7x(6x^2 - 11x - 10)$$

$$= 42x^3 - 77x^2 - 70x$$

2 **(b)**

(b)

$$\frac{9}{2y} + \frac{5}{7} = 5$$

$$\frac{9}{2y} = \frac{30}{7}$$

$$63 = 60y$$

$$y = \frac{21}{20}$$

FINAL ANSWER

$42x^3 - 77x^2 - 70x$, $y = \frac{21}{20}$.

4MA1/1H NOVEMBER 2025

Question 14

6 marks | Expanding Brackets

14 (a) Expand and simplify $7x(3x + 2)(2x - 5)$

.....

(3)

(b) Solve $\frac{9}{2y} + \frac{5}{7} = 5$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 14 is 6 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H November 2025

1 **(a)**

(a)

$$7x(3x+2)(2x-5) = 7x(6x^2 - 11x - 10)$$

$$= 42x^3 - 77x^2 - 70x$$

2 **(b)**

(b)

$$\frac{9}{2y} + \frac{5}{7} = 5$$

$$\frac{9}{2y} = \frac{30}{7}$$

$$63 = 60y$$

FINAL ANSWER

(a) $42x^3 - 77x^2 - 70x$, (b) $y = \frac{21}{20}$

4MA1/1H JUNE 2026

Question 5

3 marks | Expanding Brackets

5 (a) Expand $4e(2e-3)$

.....

(1)

(b) Expand and simplify $(f+3)(f-6)$

.....

(2)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$4e(2e-3) = 8e^2 - 12e.$$

Multiply 4e by both terms in the bracket.

2 **Part (b) | Step 1**

$$(f+3)(f-6) = f^2 - 6f + 3f - 18 = f^2 - 3f - 18.$$

Multiply each term in the first bracket by each term in the second and collect like terms.

✓ **FINAL ANSWER**

(a) $8e^2 - 12e$ (b) $f^2 - 3f - 18$

4MA1/1H JUNE 2026

Question 12

6 marks | Expanding Brackets

12 $\dfrac{2^3}{8^x} = 64$

(a) Find the value of x
Show clear algebraic working.

$x =$ .....

(3)

(b) Expand and simplify $4t(t + 5)(2t - 3)$

.....

(3)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$\frac{2^{x/3}}{8^x} = \frac{2^{x/3}}{2^{3x}} = 2^{x/3-3x} = 2^{-8x/3}.$$

Write 8 as a power of 2 and subtract exponents when dividing.

2 **Part (a) | Step 2**

$$2^{-8x/3} = 64 = 2^6 \implies -\frac{8x}{3} = 6 \implies x = -\frac{9}{4}.$$

Equal powers of base 2 have equal exponents.

3 **Part (b) | Step 1**

$$(t+5)(2t-3) = 2t^2 - 3t + 10t - 15 = 2t^2 + 7t - 15.$$

Expand the two brackets and collect like terms.

4 **Part (b) | Step 2**

$$4t(t+5)(2t-3) = 4t(2t^2 + 7t - 15) = 8t^3 + 28t^2 - 60t.$$

Multiply every term by 4t.

FINAL ANSWER

(a) $x = -\frac{9}{4}$ (b) $8t^3 + 28t^2 - 60t$

TOPIC

Factorising

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H MAY/NOVEMBER 2020

Question 7

8 marks | Factorising

7 (a) Solve $5(4 - x) = 7 - 3x$
Show clear algebraic working.

$x =$ .....

(3)

(b) Factorise fully $16m^3g^3 + 24m^2g^5$

.....

(2)

(c) (i) Factorise $y^2 - 2y - 48$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 48 = 0$

.....

(1)

(Total for Question 7 is 8 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May/November 2020

1 **(a)**

(a)

$$5(4 - x) = 7 - 3x$$

$$20 - 5x = 7 - 3x$$

$$13 = 2x$$

$$x = \frac{13}{2}$$

2 **(b)**

(b)

$$16m^3g^3 + 24m^2g^5 = 8m^2g^3(2m + 3g^2)$$

3 **(c)(i)**

(c)(i)

$$y^2 - 2y - 48 = (y - 8)(y + 6)$$

4 **Factorise expression**

(c)(ii) $y = 8$ or $y = -6$.

FINAL ANSWER

$x = \frac{13}{2}$, $8m^2g^3(2m + 3g^2)$, $(y - 8)(y + 6)$, $y = 8$ or $y = -6$

4MA1/1H JANUARY 2021

Question 9

5 marks | Factorising

9 (a) Simplify $(2x^3y^5)^4$

.....

(2)

(b) (i) Factorise $x^2 + 5x - 36$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 36 = 0$

.....

(1)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2021

1 **(a)**

(a)

$$(2x^3y^5)^4 = 16x^{12}y^{20}$$

2 **(b)(i)**

(b)(i)

$$x^2 + 5x - 36 = (x + 9)(x - 4)$$

3 **Factorise expression**

(b)(ii) $x = -9$ or $x = 4$.

✓ **FINAL ANSWER**

(a) $16x^{12}y^{20}$, (b)(i) $(x + 9)(x - 4)$, (b)(ii) $x = -9$ or $x = 4$

4MA1/1H MAY 2021

Question 9

5 marks | Factorising

9 (a) Factorise fully $25a^4c^7d + 45a^9c^3h$

.....

(2)

(b) Solve $(2x + 5)^2 = (2x + 3)(2x - 1)$

$x =$ .....

(3)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2021

1 (a)

(a)

$$25a^4c^7d + 45a^9c^3h = 5a^4c^3(5c^4d + 9a^5h)$$

2 (b)

(b)

$$(2x + 5)^2 = (2x + 3)(2x - 1)$$

$$4x^2 + 20x + 25 = 4x^2 + 4x - 3$$

$$16x = -28$$

$$x = -\frac{7}{4}$$

FINAL ANSWER

(a) $5a^4c^3(5c^4d + 9a^5h)$, (b) $x = -\frac{7}{4}$

4MA1/2H MAY 2021

Question 9

3 marks | Factorising

9 (i) Factorise $x^2 + 2x - 24$

.....

(2)

(ii) Hence solve $x^2 + 2x - 24 = 0$

.....

(1)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H May 2021

1 **Factorise expression**

$$x^2 + 2x - 24 = (x + 6)(x - 4)$$

2 **Factorise expression**

So

$$(x + 6)(x - 4) = 0$$

✓ **FINAL ANSWER**

$x = -6$ or $x = 4$

4MA1/2H NOVEMBER 2021

Question 8

4 marks | Factorising

8 (a) Simplify $(3x^2y)^0$

.....

(1)

(b) (i) Factorise $x^2 - 5x - 36$

.....

(2)

(ii) Hence solve $x^2 - 5x - 36 = 0$

.....

(1)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2021

1 (a)

(a) $(3x^2y)^0 = 1$.

2 (b)(i)

(b)(i)

$$x^2 - 5x - 36 = (x - 9)(x + 4)$$

3 Factorise expression

(b)(ii) $x = 9$ or $x = -4$.

FINAL ANSWER

1, $(x - 9)(x + 4)$, $x = 9$ or $x = -4$

4MA1/1HR JANUARY 2022

Question 9

5 marks | Factorising

9 (a) Simplify $x^4 \times x^5$

.....

(1)

(b) Simplify $(4y^2)^3$

.....

(2)

(c) Factorise $n^2 - 7n + 12$

.....

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2022

1 **(a)**

(a) $x^4 \times x^5 = x^9$.

2 **(b)**

(b) $(4y^2)^3 = 64y^6$.

3 **(c)**

(c)

$$n^2 - 7n + 12 = (n - 3)(n - 4)$$

✓ **FINAL ANSWER**

(a) x^9, (b) $64y^6$, (c) $(n - 3)(n - 4)$

4MA1/1HR JUNE 2022

Question 5

5 marks | Factorising

5 (a) Expand $3c^3(c + 4)$

.....

(2)

(b) (i) Factorise $x^2 + 8x - 9$

.....

(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

.....

(1)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR June 2022

1 **(a)**

(a) $3c^3(c+4) = 3c^4 + 12c^3$.

2 **(b)(i)**

(b)(i)

$$x^2 + 8x - 9 = (x+9)(x-1)$$

3 **(b)(ii)**

(b)(ii)

$$(x+9)(x-1) = 0$$

✓ **FINAL ANSWER**

(a) $3c^4 + 12c^3$, (b)(i) $(x+9)(x-1)$, (b)(ii) $x = -9$ or $x = 1$

4MA1/1H JUNE 2022

Question 11

5 marks | Factorising

11 (a) Factorise $9x^2 - 4y^2$

.....

(2)

(b) Express $\frac{7}{8} - \frac{x+3}{4x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2022

1 **(a)**

(a)

$$9x^2 - 4y^2 = (3x)^2 - (2y)^2 = (3x - 2y)(3x + 2y)$$

2 **(b)**

(b)

$$\frac{7}{8} - \frac{x+3}{4x} = \frac{7x}{8x} - \frac{2(x+3)}{8x}$$

$$= \frac{7x - 2x - 6}{8x} = \frac{5x - 6}{8x}$$

FINAL ANSWER

(a) $(3x - 2y)(3x + 2y)$, (b) $\frac{5x-6}{8x}$

4MA1/2H JUNE 2022

Question 11

3 marks | Factorising

11 (i) Factorise $x^2 + 5x - 24$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

.....

(1)

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2022

1 **Factorise expression**

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

2 **Factorise expression**

So

$$(x + 8)(x - 3) = 0$$

✓ **FINAL ANSWER**

$x = -8$ or $x = 3$

4MA1/1H MAY 2023

Question 6

8 marks | Factorising

6 (a) Simplify $(2c^4d^7)^3$

.....

(2)

(b) Find the value of $5y^0$ where $y > 0$

.....

(1)

(c) Factorise fully $16a^2b^3 + 20a^3b$

.....

(2)

(d) (i) Factorise $x^2 + 9x - 22$

.....

(2)

(ii) Hence solve $x^2 + 9x - 22 = 0$

.....

(1)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2023

1 **(a)**

(a)

$$(2c^4d^7)^3 = 8c^{12}d^{21}$$

2 **Factorise expression**

(b) $5y^0 = 5$, since $y^0 = 1$.

3 **(c)**

(c)

$$16a^2b^3 + 20a^3b = 4a^2b(4b^2 + 5a)$$

4 **(d)(i)**

(d)(i)

$$x^2 + 9x - 22 = (x + 11)(x - 2)$$

5 **Factorise expression**

(d)(ii) $x = -11$ or $x = 2$.

✓ **FINAL ANSWER**

$8c^{12}d^{21}$, 5, $4a^2b(4b^2 + 5a)$, $(x + 11)(x - 2)$, $x = -11$ or $x = 2$

4MA1/1HR MAY 2023

Question 7

3 marks | Factorising

7 (a) Expand $4x(x - 5)$

.....

(1)

(b) Factorise $y^2 - 9y + 20$

.....

(2)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR May 2023

1 (a)

(a)

$$4x(x-5) = 4x^2 - 20x$$

2 (b)

(b)

$$y^2 - 9y + 20 = (y-4)(y-5)$$

FINAL ANSWER

(a) $4x^2 - 20x$, (b) $(y-4)(y-5)$

4MA1/1H MAY 2023

Question 14

3 marks | Factorising

14 Factorise fully $50g^2 - 18$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2023

1 Factorise expression

$$50g^2 - 18 = 2(25g^2 - 9)$$

$$= 2(5g - 3)(5g + 3)$$

FINAL ANSWER

$2(5g - 3)(5g + 3)$

4MA1/2H JUNE 2024

Question 11

3 marks | Factorising

11 (i) Factorise $x^2 + 9x - 22$

.....

(2)

(ii) Hence, solve $x^2 + 9x - 22 = 0$

.....

(1)

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2024

1 **Factorise expression**

$$x^2 + 9x - 22 = (x + 11)(x - 2)$$

2 **Factorise expression**

So

$$(x + 11)(x - 2) = 0$$

FINAL ANSWER

$x = -11$ or $x = 2$

4MA1/1H JUNE 2025

Question 6

6 marks | Factorising

6 (a) Solve $x - 4 = \frac{3 + 2x}{6}$

Show clear algebraic working.

$x =$ .....

(3)

(b) (i) Factorise $y^2 - 11y + 30$

.....

(2)

(ii) Hence solve $y^2 - 11y + 30 = 0$

.....

(1)

(Total for Question 6 is 6 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2025

1 **(a)**

(a)

$$x - 4 = \frac{3 + 2x}{6}$$

$$6x - 24 = 3 + 2x$$

$$4x = 27$$

$$x = \frac{27}{4}$$

2 **(b)(i)**

(b)(i)

$$y^2 - 11y + 30 = (y - 5)(y - 6)$$

3 **(b)(ii)**

(b)(ii)

$$y = 5 \quad \text{or} \quad y = 6$$

FINAL ANSWER

$x = \frac{27}{4}$, $(y-5)(y-6)$, $y = 5$ or $y = 6$

4WM1/01 MAY 2025

Question 7

6 marks | Factorising

7 (a) Solve $x - 4 = \dfrac{3 + 2x}{6}$

Show clear algebraic working.

$x =$ .....

(3)

(b) (i) Factorise $y^2 - 11y + 30$

.....

(2)

(ii) Hence solve $y^2 - 11y + 30 = 0$

.....

(1)

(Total for Question 7 is 6 marks)

WORKED SOLUTION

Question 7 · 4WM1/01 May 2025

1 Factorise expression

For part (a),

$$x - 4 = \frac{3 + 2x}{6}$$

2 Multiply by

Multiply by 6.

$$6x - 24 = 3 + 2x$$

$$4x = 27$$

$$x = \frac{27}{4} = 6.75$$

3 Factorise expression

For part (b),

$$y^2 - 11y + 30 = (y - 5)(y - 6)$$

4 Factorise expression

Hence,

$$(y-5)(y-6)=0$$

$$y=5 \quad \text{or} \quad y=6$$

FINAL ANSWER

$x=6.75$, $(y-5)(y-6)$, and $y=5$ or 6.

4MA1/1HR JUNE 2025

Question 8

5 marks | Factorising

8 (a) Simplify $a^6 \times a^{10}$

.....

(1)

(b) Simplify $c^{30} \div c^{12}$

.....

(1)

(c) (i) Factorise $y^2 - 10y + 21$

.....

(2)

(ii) Hence, solve $y^2 - 10y + 21 = 0$

.....

(1)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2025

1 **(a)**

(a) $a^6 \times a^{10} = a^{16}$.

2 **(b)**

(b) $c^{30} \div c^{12} = c^{18}$.

3 **(c)(i)**

(c)(i)

$$y^2 - 10y + 21 = (y - 3)(y - 7)$$

4 **(c)(ii)**

(c)(ii)

$$y = 3 \quad \text{or} \quad y = 7$$

✓ **FINAL ANSWER**

a^{16}, c^{18}, $(y - 3)(y - 7)$, $y = 3$ or $y = 7$

4WM1/01R MAY 2025

Question 9

7 marks | Factorising

9 (a) Simplify $\left(5k^3n^4\right)^2$

.....

(2)

(b) Factorise fully $12m^2y^4 + 18m^3y$

.....

(2)

(c) (i) Factorise $x^2 - 13x + 42$

.....

(2)

(ii) Hence, solve $x^2 - 13x + 42 = 0$

.....

(1)

(Total for Question 9 is 7 marks)

WORKED SOLUTION

Question 9 · 4WM1/01R May 2025

1 **(a)**

(a)

$$(5k^3n^4)^2 = 25k^6n^8$$

2 **(b)**

(b)

$$12m^2y^4 + 18m^3y^3 = 6m^2y^3(2y + 3m)$$

3 **(c)**

(c)

$$x^2 - 13x + 42 = (x - 6)(x - 7)$$

4 **Calculate value**

So

$$x = 6 \quad \text{or} \quad x = 7$$

FINAL ANSWER

$25k^6n^8$, $6m^2y^3(2y+3m)$, $(x-6)(x-7)$, $x=6,7$.

4MA1/2H NOVEMBER 2025

Question 7

3 marks | Factorising

7 (i) Factorise $x^2 + 2x - 48$

.....

(2)

(ii) Hence, solve $x^2 + 2x - 48 = 0$

.....

(1)

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2025

1 Factorise expression

$$x^2 + 2x - 48 = (x + 8)(x - 6)$$

2 Factorise expression

So

$$(x + 8)(x - 6) = 0$$

FINAL ANSWER

$x = -8$ or $x = 6$

4WM1/01 NOVEMBER 2025

Question 8

9 marks | Factorising

8 (a) Expand $5w(3w - 4w^2)$

.....

(2)

(b) Simplify fully $\left(4a^3c^5\right)^3$

.....

(2)

(c) Factorise fully $14m^3p^2 + 21m^2p^4$

.....

(2)

(d) (i) Factorise $x^2 - 10x + 24$

.....

(2)

(ii) Hence solve $x^2 - 10x + 24 = 0$

.....

(1)

(Total for Question 8 is 9 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 November 2025

1 **(a)**

(a)

$$5w(3w - 4w^2) = 15w^2 - 20w^3$$

2 **(b)**

(b)

$$(4a^3c^5)^3 = 64a^9c^{15}$$

3 **(c)**

(c)

$$14m^3p^2 + 21m^2p^4 = 7m^2p^2(2m + 3p^2)$$

4 **(d)**

(d)

$$x^2 - 10x + 24 = (x - 4)(x - 6)$$

---

5 **Calculate value**

So

$$x = 4 \quad \text{or} \quad x = 6$$

✓ **FINAL ANSWER**

$15w^2 - 20w^3$, $64a^9c^{15}$, $7m^2p^2(2m + 3p^2)$, $(x - 4)(x - 6)$, $x = 4, 6$.

Question 10

7 marks | Factorising

10 (a) Simplify $(6m)^0$

.....

(1)

(b) Solve $5y + 20 < 7y + 1$

.....

(2)

(c) Factorise fully $15w^2x^5 + 25w^3x^2$

.....

(2)

(d) Simplify fully $\dfrac{36a^5c^7}{12a^2c^3}$

.....

(2)

(Total for Question 10 is 7 marks)

(a)

(a) $(6m)^0 = 1$.

(b)

(b)

$$5y + 20 < 7y + 1$$

$$19 < 2y$$

$$y > \frac{19}{2}$$

(c)

(c)

$$15w^2x^5 + 25w^3x^2 = 5w^2x^2(3x^3 + 5w)$$

(d)

(d)

$$\frac{36a^5c^7}{12a^2c^3} = 3a^3c^4$$

FINAL ANSWER

1, $y > \frac{19}{2}$, $5w^2x^2(3x^3 + 5w)$, $3a^3c^4$

4MA1/1HR JUNE 2026

Question 8

2 marks | Factorising

8 Factorise $x^2 - 12x + 32$

.....

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2026

1 Solution | Step 1

$$(-4)(-8) = 32, \qquad -4 + (-8) = -12.$$

Choose two numbers with product 32 and sum -12.

2 Solution | Step 2

$$x^2 - 12x + 32 = (x - 4)(x - 8).$$

The chosen numbers give the two constant terms in the factors.

FINAL ANSWER

$(x - 4)(x - 8)$

4MA1/2H JUNE 2026

Question 9

7 marks | Factorising

9 (a) Find the value of $7n^0$ where $n > 0$

.....

(1)

(b) Simplify fully $\left(2m^5p^4\right)^3$

.....

(2)

(c) Factorise fully $18c^5w^2 + 30cw^3$

.....

(2)

(d) Factorise $x^2 + 6x - 27$

.....

(2)

(Total for Question 9 is 7 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

Since $n > 0$, $n^0 = 1$. Hence $7n^0 = 7(1) = 7$.

A nonzero number raised to power zero equals 1; only n is raised to zero.

2 **Part (b) | Step 1**

$$(2m^5p^4)^3 = 2^3m^{5\times3}p^{4\times3} = 8m^{15}p^{12}.$$

Cube the coefficient and multiply each exponent by 3.

3 **Part (c) | Step 1**

The highest common factor of $18c^5w^2$ and $30cw^3$ is $6cw^2$.

Take the numerical HCF and the smallest power of each shared variable.

4 **Part (c) | Step 2**

$$18c^5w^2 + 30cw^3 = 6cw^2(3c^4 + 5w).$$

Divide each original term by the common factor.

5

Part (d) | Step 1

Two numbers with product -27 and sum 6 are 9 and -3.

For a monic quadratic, find a factor pair with the required sum.

6

Part (d) | Step 2

$$x^2 + 6x - 27 = (x + 9)(x - 3).$$

The cross terms are -3x+9x=6x.

✓ **FINAL ANSWER**

(a) 7 (b) $8m^{15}p^{12}$ (c) $6cw^2(3c^4 + 5w)$ (d) $(x + 9)(x - 3)$

Question 9

5 marks | Factorising

9 (a) Factorise $15g^4h - 20gh^3$

.....

(2)

(b) (i) Factorise $x^2 + 3x - 28$

.....

(2)

(ii) Hence, solve $x^2 + 3x - 28 = 0$

.....

(1)

(Total for Question 9 is 5 marks)

1 **(a) Take out the HCF**

HCF of $15g^4h$ and $20gh^3$ is $5gh$:

$$15g^4h - 20gh^3 = 5gh(3g^3 - 4h^2)$$

2 **(b)(i) Two numbers: product −28, sum 3**

7 and -4:

$$x^2 + 3x - 28 = (x + 7)(x - 4)$$

3 **(b)(ii) Each bracket = 0**

$$x = -7 \quad \text{or} \quad x = 4$$

✓ **FINAL ANSWER**

(a) $5gh(3g^3 - 4h^2)$ (b)(i) $(x + 7)(x - 4)$ (ii) $x = -7,\ 4$

TOPIC

Algebraic Fractions

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 12

5 marks | Algebraic Fractions

12 (a) Factorise $9x^2 - 4y^2$

.....

(2)

(b) Express $\frac{7}{8} - \frac{x+3}{4x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 Specimen

1 **Use difference of two squares**

$$9x^2 - 4y^2 = (3x)^2 - (2y)^2$$

$$= (3x - 2y)(3x + 2y)$$

2 **Use a common denominator**

$$\frac{7}{8} - \frac{x+3}{4x} = \frac{7x}{8x} - \frac{2(x+3)}{8x}$$

3 **Simplify the numerator**

$$\frac{7x - 2x - 6}{8x} = \frac{5x - 6}{8x}$$

✓ **FINAL ANSWER**

$(3x - 2y)(3x + 2y)$, and $\frac{5x-6}{8x}$.

Question 21

3 marks | Algebraic Fractions

21 Express $\left(\frac{20}{x^2 - 36} - \frac{2}{x - 6}\right) \times \frac{1}{4 - x}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

1 Factorise the denominator

$$x^2 - 36 = (x - 6)(x + 6)$$

2 Simplify the bracket

$$\frac{20}{(x-6)(x+6)} - \frac{2}{x-6} = \frac{20 - 2(x+6)}{(x-6)(x+6)}$$

$$= \frac{8 - 2x}{(x-6)(x+6)} = \frac{-2(x-4)}{(x-6)(x+6)}$$

3 Cancel the common factor

Since $4 - x = -(x - 4)$,

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \frac{1}{4-x} = \frac{2}{(x-6)(x+6)}$$

FINAL ANSWER

$\frac{2}{x^2-36}$.

4MA1/2H JANUARY 2020

Question 24

4 marks | Algebraic Fractions

24 Express

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H January 2020

1 Use trigonometry

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

2 Use trigonometry

First simplify the bracket:

$$\frac{4}{2x-5}-\frac{3}{2x-3}=\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$

$$=\frac{8x-12-6x+15}{(2x-5)(2x-3)}$$

$$=\frac{2x+3}{(2x-5)(2x-3)}$$

3 Use trigonometry

Now factor the second fraction:

$$9x-4x^3=-x(2x-3)(2x+3)$$

$$6x^2-17x+5=(2x-5)(3x-1)$$

4 **Dividing by a fraction means**

Dividing by a fraction means multiplying by its reciprocal:

$$\frac{2x+3}{(2x-5)(2x-3)} \times \frac{(2x-5)(3x-1)}{-x(2x-3)(2x+3)}$$

5 **Cancel common factors**

Cancel common factors:

$$= -\frac{3x-1}{x(2x-3)^2}$$

$$= \frac{1-3x}{x(2x-3)^2}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{1-3x}{x(2x-3)^2}}$$

4MA1/1H MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Fractions

21 Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May/November 2020

1 Use trigonometry

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

2 Factor

Factor:

$$9x^2-4 = (3x-2)(3x+2)$$

$$3x^2-13x-10 = (3x+2)(x-5)$$

3 Use trigonometry

So the first product becomes

$$\frac{1}{3x-2} \times \frac{(3x-2)(3x+2)}{(3x+2)(x-5)}$$

4 Cancel common factors

Cancel common factors:

$$\frac{1}{x-5}$$

5 Use trigonometry

Now subtract:

$$\frac{1}{x-5} - \frac{7}{x-1}$$

6 Use the common denominator

Use the common denominator $(x-5)(x-1)$:

$$\frac{x-1-7(x-5)}{(x-5)(x-1)}$$

$$=\frac{x-1-7x+35}{(x-5)(x-1)}$$

$$=\frac{34-6x}{(x-5)(x-1)}$$

FINAL ANSWER

$$\boxed{\frac{34-6x}{(x-5)(x-1)}}$$

4MA1/2HR JANUARY 2021

Question 18

5 marks | Algebraic Fractions

18 Solve the equation

$$\frac{5}{x+2}+\frac{3}{x^2+2x}=2$$

Show clear algebraic working.

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2021

1 Solve equation

$$\frac{5}{x+2}+\frac{3}{x^2+2x}=2$$

2 Solve equation

Since

$$x^2+2x=x(x+2)$$

3 multiply by

multiply by $x(x+2)$:

$$5x+3=2x(x+2)$$

$$5x+3=2x^2+4x$$

$$2x^2-x-3=0$$

4 Factorise

Factorise:

$$(2x-3)(x+1)=0$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-1$$

5 **Both values are allowed because**

Both values are allowed because $x \neq 0$ and $x \neq -2$.

✓ **FINAL ANSWER**

$x = \frac{3}{2}$ or $x = -1$.

4MA1/2H JANUARY 2021

Question 21

4 marks | Algebraic Fractions

21 Write $\dfrac{25x^2 - 64}{5x^2 - 13x - 6} \times \dfrac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

as a single fraction in its simplest form.
Show clear algebraic working.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2021

1 Use trigonometry

$$\frac{25x^2 - 64}{5x^2 - 13x - 6} \times \frac{x^2 - 8x + 15}{5x + 8} - (x - 7)$$

2 Factor

Factor:

$$25x^2 - 64 = (5x - 8)(5x + 8)$$

$$5x^2 - 13x - 6 = (5x + 2)(x - 3)$$

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

3 Use trigonometry

So

$$\frac{(5x - 8)(5x + 8)}{(5x + 2)(x - 3)} \times \frac{(x - 3)(x - 5)}{5x + 8} - (x - 7)$$

4 Cancel common factors

Cancel common factors:

$$\frac{(5x - 8)(x - 5)}{5x + 2} - (x - 7)$$

5 **Use the common denominator**

Use the common denominator $5x + 2$:

$$\frac{(5x - 8)(x - 5) - (x - 7)(5x + 2)}{5x + 2}$$

6 **Expand**

Expand:

$$(5x - 8)(x - 5) = 5x^2 - 33x + 40$$

$$(x - 7)(5x + 2) = 5x^2 - 33x - 14$$

7 **Use trigonometry**

So the numerator is

$$5x^2 - 33x + 40 - (5x^2 - 33x - 14) = 54$$

✓ **FINAL ANSWER**

$$\boxed{\frac{54}{5x + 2}}$$

4MA1/1H MAY 2021

Question 21

4 marks | Algebraic Fractions

21 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2021

1 Given

Given

$$x = \frac{5}{9y + 5}$$

2 Use trigonometry

and

$$y = \frac{5}{5a - 2}$$

3 Use cosine rule

Substitute $y = \frac{5}{5a-2}$ into the expression for x:

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5}$$

$$x = \frac{5}{\frac{45}{5a-2} + 5}$$

4 **Write the denominator as a**

Write the denominator as a single fraction:

$$\frac{45}{5a-2}+5=\frac{45+5(5a-2)}{5a-2}$$

$$=\frac{45+25a-10}{5a-2}$$

$$=\frac{25a+35}{5a-2}$$

---

5 **Use trigonometry**

So

$$x=5\div\frac{25a+35}{5a-2}$$

$$x=5\times\frac{5a-2}{25a+35}$$

$$x=\frac{5(5a-2)}{5(5a+7)}$$

$$x=\frac{5a-2}{5a+7}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5a-2}{5a+7}}$$

4MA1/1HR JANUARY 2022

Question 12

5 marks | Algebraic Fractions

12 (a) Expand and simplify $n(n-4)(3n+5)$

.....

(2)

(b) Express

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$n(n-4)(3n+5) = n(3n^2-7n-20) = 3n^3-7n^2-20n$$

2 **(b)**

(b)

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

3 **Use denominator**

Use denominator $4x$:

$$\frac{12}{4x}+\frac{2(x+2)}{4x}+\frac{x}{4x}=\frac{12+2x+4+x}{4x}$$

$$=\frac{3x+16}{4x}$$

FINAL ANSWER

(a) $3n^3-7n^2-20n$, (b) $\frac{3x+16}{4x}$

4MA1/2H JUNE 2022

Question 19

3 marks | Algebraic Fractions

19 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2022

1 Simplify fraction

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 Simplify fraction

Substitute for x:

$$3x - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

3 Simplify fraction

So

$$a = \frac{14}{\frac{42 - 28y}{4y - 3}} = \frac{4y - 3}{3 - 2y}$$

FINAL ANSWER

$a = \frac{4y - 3}{3 - 2y}$

4MA1/2HR JUNE 2022

Question 23

3 marks | Algebraic Fractions

23 Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2022

1 Use trigonometry

$$\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$$

2 Factor

Factor:

$$x^2-36=(x-6)(x+6)$$

3 Use trigonometry

So

$$\frac{20}{(x-6)(x+6)}-\frac{2}{x-6}$$

4 Use the common denominator

Use the common denominator $(x-6)(x+6)$:

$$\frac{20-2(x+6)}{(x-6)(x+6)}$$

$$=\frac{20-2x-12}{(x-6)(x+6)}$$

$$=\frac{8-2x}{(x-6)(x+6)}$$

$$=\frac{-2(x-4)}{(x-6)(x+6)}$$

5 **Use trigonometry**

Now multiply by

$$\frac{1}{4-x} = -\frac{1}{x-4}$$

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \left(-\frac{1}{x-4}\right)$$

---

6 **Cancel**

Cancel $x-4$:

$$\frac{2}{(x-6)(x+6)}$$

$$= \frac{2}{x^2-36}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{2}{x^2-36}}$$

4MA1/1HR MAY 2023

Question 23

4 marks | Algebraic Fractions

23 Simplify $(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$

Give your answer in the form $\frac{ax^2}{bx + c}$ where a, b and c are integers.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May 2023

1 Simplify surd

$$(x^2 - 4) \div \left(\frac{4x^2 - 7x - 2}{x}\right) - 2x$$

2 Change division into multiplication by

Change division into multiplication by the reciprocal:

$$(x^2 - 4) \times \frac{x}{4x^2 - 7x - 2} - 2x$$

3 Factorise

Factorise:

$$x^2 - 4 = (x - 2)(x + 2)$$

$$4x^2 - 7x - 2 = (4x + 1)(x - 2)$$

4 Simplify surd

So

$$\frac{x(x - 2)(x + 2)}{(4x + 1)(x - 2)} - 2x = \frac{x(x + 2)}{4x + 1} - 2x$$

$$\frac{x(x + 2) - 2x(4x + 1)}{4x + 1} = \frac{x^2 + 2x - 8x^2 - 2x}{4x + 1} = \frac{-7x^2}{4x + 1}$$

FINAL ANSWER

$$\frac{-7x^2}{4x + 1}$$

4MA1/1H NOVEMBER 2023

Question 11

3 marks | Algebraic Fractions

11 Express $\frac{3}{4}+\frac{5-x}{6x}$ as a single fraction in its simplest terms.

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H November 2023

1 Simplify fraction

$$\frac{3}{4}+\frac{5-x}{6x}$$

2 The common denominator is

The common denominator is $12x$:

$$\frac{3}{4}=\frac{9x}{12x}$$

$$\frac{5-x}{6x}=\frac{10-2x}{12x}$$

$$\frac{9x+10-2x}{12x}=\frac{7x+10}{12x}$$

FINAL ANSWER

$\frac{7x+10}{12x}$.

4MA1/2H NOVEMBER 2023

Question 24

5 marks | Algebraic Fractions

24 Solve $\frac{45x^3-80x}{3x^2+x-4}\times\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$

Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2023

1 Solve quadratic equation

$$\frac{45x^3-80x}{3x^2+x-4}\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$$

2 Factorise

Factorise:

$$45x^3-80x=5x(3x-4)(3x+4)$$

$$3x^2+x-4=(3x+4)(x-1)$$

3 Solve quadratic equation

Also,

$$\frac{1}{3x-4}+\frac{1}{x}=\frac{x+3x-4}{x(3x-4)}=\frac{4(x-1)}{x(3x-4)}$$

4 Solve quadratic equation

So the left side becomes

$$\frac{5x(3x-4)(3x+4)}{(3x+4)(x-1)}\times\frac{4(x-1)}{x(3x-4)}=20$$

5 **Solve quadratic equation**

Therefore,

$$20 = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4(x+2)$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{7}{4}$$

✓ **FINAL ANSWER**

$x = \frac{7}{4}$

4MA1/1H MAY 2024

Question 12

5 marks | Algebraic Fractions

12 (a) Factorise fully $6y^2 - 5y - 4$

.....

(2)

(b) Express $\frac{2x+1}{4x} + \frac{7-5x}{3x}$ as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May 2024

1 **(a)**

(a)

$$6y^2 - 5y - 4 = (3y - 4)(2y + 1)$$

2 **(b)**

(b)

$$\frac{2x+1}{4x} + \frac{7-5x}{3x}$$

3 **Use denominator**

Use denominator $12x$:

$$\frac{3(2x+1) + 4(7-5x)}{12x}$$

$$= \frac{6x + 3 + 28 - 20x}{12x} = \frac{31 - 14x}{12x}$$

FINAL ANSWER

(a) $(3y - 4)(2y + 1)$, (b) $\frac{31-14x}{12x}$

4MA1/2H JUNE 2024

Question 20

3 marks | Algebraic Fractions

20 Given that $k = x - y$ and $x = \dfrac{1}{4y}$

express $\dfrac{5k}{x+2}$ in the form $\dfrac{a - by^2}{c + dy}$ where a, b, c and d are integers.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2024

1 **Simplify fraction**

$$k = x - y, \quad x = \frac{1}{4y}$$

2 **Simplify fraction**

So

$$k = \frac{1}{4y} - y = \frac{1 - 4y^2}{4y}$$

3 **Simplify fraction**

Also

$$x + 2 = \frac{1}{4y} + 2 = \frac{1 + 8y}{4y}$$

4 **Simplify fraction**

Therefore

$$\frac{5k}{x+2} = \frac{5\left(\frac{1-4y^2}{4y}\right)}{\frac{1+8y}{4y}}$$

5 **Cancel**

Cancel $4y$:

$$\frac{5(1-4y^2)}{1+8y}$$

$$= \frac{5-20y^2}{1+8y}$$

✓ **FINAL ANSWER**

$$\frac{5-20y^2}{1+8y}$$

4MA1/2HR JUNE 2024

Question 26

4 marks | Algebraic Fractions

26 Write $4 - \left[(3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}\right]$ as a single fraction in its simplest form.

(Total for Question 26 is 4 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2024

1 Use trigonometry

$$4 - \left[(3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}\right]$$

2 Factor the quadratic

Factor the quadratic:

$$3x^2 + x - 10 = (3x - 5)(x + 2)$$

3 Use trigonometry

So the expression inside the square brackets is

$$(3x - 5) \div \frac{(3x - 5)(x + 2)}{4x - 1}$$

4 Dividing by a fraction means

Dividing by a fraction means multiplying by its reciprocal:

$$(3x - 5) \times \frac{4x - 1}{(3x - 5)(x + 2)}$$

5 Cancel

Cancel $3x - 5$:

$$\frac{4x - 1}{x + 2}$$

6 Use trigonometry

Therefore the full expression is

$$4 - \frac{4x - 1}{x + 2}$$

7 Use the common denominator

Use the common denominator $x + 2$:

$$\frac{4(x+2) - (4x-1)}{x+2}$$

$$= \frac{4x + 8 - 4x + 1}{x+2}$$

$$= \frac{9}{x+2}$$

FINAL ANSWER

$$\boxed{\frac{9}{x+2}}$$

4MA1/1H JUNE 2025

Question 12

3 marks | Algebraic Fractions

12 Express $\frac{5}{4}+\frac{x-3}{6x}$ as a single fraction in its simplest form.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2025

1 Simplify fraction

$$\frac{5}{4}+\frac{x-3}{6x}$$

2 The common denominator is

The common denominator is $12x$:

$$\frac{5}{4}=\frac{15x}{12x}$$

$$\frac{x-3}{6x}=\frac{2x-6}{12x}$$

$$\frac{15x}{12x}+\frac{2x-6}{12x}=\frac{17x-6}{12x}$$

FINAL ANSWER

$\frac{17x-6}{12x}$.

4WM1/01 MAY 2025

Question 19

3 marks | Algebraic Fractions

19 Simplify fully $\dfrac{3x^2 - 3y^2}{5x + 5y} \div \dfrac{xy^2 - x^2y}{10xy}$

Show clear algebraic working.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM1/01 May 2025

1 **Simplify fraction**

$$\frac{3x^2 - 3y^2}{5x + 5y} \div \frac{xy^2 - x^2y}{10xy}$$

2 **Factorise**

Factorise.

$$= \frac{3(x - y)(x + y)}{5(x + y)} \div \frac{xy(y - x)}{10xy}$$

$$= \frac{3(x - y)}{5} \div \frac{y - x}{10}$$

$$= \frac{3(x - y)}{5} \times \frac{10}{y - x}$$

3 **Simplify fraction**

Since $y - x = -(x - y)$, the expression is

$$-6$$

FINAL ANSWER

-6

4WM1/01 MAY 2025

Question 20

3 marks | Algebraic Fractions

20 Given that $k = m + n$ and $m = \dfrac{3}{4n}$

express $\dfrac{7k}{5-m}$ in the form $\dfrac{a+bn^2}{cn-d}$ where a, b and c are integers and d is prime.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01 May 2025

1 Simplify fraction

$$k = m + n, \qquad m = \frac{3}{4n}$$

2 Simplify fraction

So

$$k = \frac{3}{4n} + n = \frac{3 + 4n^2}{4n}$$

3 Simplify fraction

Then

$$7k = \frac{21 + 28n^2}{4n}$$

4 Simplify fraction

and

$$5 - m = 5 - \frac{3}{4n} = \frac{20n - 3}{4n}$$

5 Simplify fraction

Therefore

$$\frac{7k}{5-m} = \frac{\frac{21+28n^2}{4n}}{\frac{20n-3}{4n}}$$

$$= \frac{21 + 28n^2}{20n - 3}$$

FINAL ANSWER

$$\frac{21+28n^2}{20n-3}$$

4MA1/2H JUNE 2025

Question 23

4 marks | Algebraic Fractions

23 $\dfrac{4x^2-4x-120}{5x^2-180} \div \dfrac{x^2+5x}{10x^2+60x} = p$ where p is an integer.

Find the value of p

Show clear algebraic working.

$p =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2025

1 Split the ratio

$$\frac{4x^2-4x-120}{5x^2-180} \div \frac{x^2+5x}{10x^2+60x}$$

2 Factorise

Factorise:

$$4x^2-4x-120 = 4(x-6)(x+5)$$

$$5x^2-180 = 5(x-6)(x+6)$$

$$x^2+5x = x(x+5)$$

$$10x^2+60x = 10x(x+6)$$

3 Split the ratio

So

$$\frac{4(x-6)(x+5)}{5(x-6)(x+6)} \div \frac{x(x+5)}{10x(x+6)}$$

$$= \frac{4(x+5)}{5(x+6)} \times \frac{10(x+6)}{x+5} = 8$$

FINAL ANSWER

$p = 8$

4WM1/01R MAY 2025

Question 24

4 marks | Algebraic Fractions

24 Write $(2x-1)+\dfrac{2x^2-5x-12}{x^2-16}\times\dfrac{x^2+4x}{6x^2+11x+3}$ as a single fraction in its simplest form.

Show clear algebraic working.

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4WM1/01R May 2025

1 Factor the fractional part first

Factor the fractional part first:

$$\frac{2x^2-5x-12}{x^2-16}\times\frac{x^2+4x}{6x^2+11x+3}$$

$$=\frac{(2x+3)(x-4)}{(x-4)(x+4)}\times\frac{x(x+4)}{(3x+1)(2x+3)}$$

$$=\frac{x}{3x+1}$$

2 Evaluate fraction

So

$$(2x-1)+\frac{x}{3x+1}=\frac{(2x-1)(3x+1)+x}{3x+1}$$

$$=\frac{6x^2-1}{3x+1}$$

FINAL ANSWER

$$\frac{6x^2-1}{3x+1}.$$

4MA1/1H NOVEMBER 2025

Question 23

3 marks | Algebraic Fractions

23 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 25

3 marks | Algebraic Fractions

25 $a = \dfrac{2x+5}{1-x}$ $x = \dfrac{5-2y}{3y}$

Write a in the form $\dfrac{m+ny}{p(y-1)}$ where m, n and p are integers.

Show your working clearly.

$a =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 November 2025

1 Split the ratio

$$a = \frac{2x+5}{1-x}, \quad x = \frac{5-2y}{3y}$$

2 Split the ratio

Substitute for x.

3 Numerator

Numerator:

$$2x+5 = 2\left(\frac{5-2y}{3y}\right)+5$$

$$= \frac{10-4y+15y}{3y} = \frac{10+11y}{3y}$$

4 Denominator

Denominator:

$$1-x = 1-\frac{5-2y}{3y}$$

$$= \frac{3y-5+2y}{3y} = \frac{5y-5}{3y} = \frac{5(y-1)}{3y}$$

5 **Split the ratio**

Therefore

$$a = \frac{\frac{10+11y}{3y}}{\frac{5(y-1)}{3y}} = \frac{10+11y}{5(y-1)}$$

✓ **FINAL ANSWER**

$$\frac{10+11y}{5(y-1)}$$

4WM1/01 NOVEMBER 2025

Question 28

4 marks | Algebraic Fractions

28 Solve $\dfrac{6x^3+5x^2+x}{4x^2-1} \div \dfrac{15x^2-x-2}{8x-4} = 2x$

Show clear algebraic working.

$x =$ .....

(Total for Question 28 is 4 marks)

WORKED SOLUTION

Question 28 · 4WM1/01 November 2025

1 Factor each expression

Factor each expression:

$$\frac{x(3x+1)(2x+1)}{(2x-1)(2x+1)} \div \frac{(5x-2)(3x+1)}{4(2x-1)} = 2x$$

$$\frac{x(3x+1)}{2x-1} \times \frac{4(2x-1)}{(5x-2)(3x+1)} = 2x$$

$$\frac{4x}{5x-2} = 2x$$

$$4x = 2x(5x-2)$$

$$10x^2 - 8x = 0$$

$$2x(5x-4) = 0$$

2 The original fractions do not

The original fractions do not allow $x = 0$, so

$$x = \frac{4}{5}$$

FINAL ANSWER

$x = \frac{4}{5}$.

4MA1/1HR JUNE 2026

Question 16

6 marks | Algebraic Fractions

16 (a) Express $\frac{5}{8} - \frac{x+2}{4x}$ as a single fraction in its simplest form.

.....

(3)

(b) Expand and simplify $3x(2x - 1)(x + 4)$

.....

(3)

(Total for Question 16 is 6 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$\frac{5}{8} - \frac{x+2}{4x} = \frac{5x}{8x} - \frac{2(x+2)}{8x}, \qquad x \neq 0.$$

Use 8x as a common denominator.

2 **Part (a) | Step 2**

$$\frac{5x - 2(x+2)}{8x} = \frac{5x - 2x - 4}{8x} = \frac{3x - 4}{8x}.$$

Subtract the entire numerator of the second fraction and collect like terms.

3 **Part (b) | Step 1**

$$(2x - 1)(x + 4) = 2x^2 + 8x - x - 4 = 2x^2 + 7x - 4.$$

Multiply each term in the first bracket by each term in the second.

4 **Part (b) | Step 2**

$$3x(2x^2 + 7x - 4) = 6x^3 + 21x^2 - 12x.$$

Multiply every term by 3x.

FINAL ANSWER

(a) $\frac{3x-4}{8x}$ (b) $6x^3 + 21x^2 - 12x$

4MA1/1H JUNE 2026

Question 18

3 marks | Algebraic Fractions

18 Simplify fully $\dfrac{4x^2 - 9}{2x^2 - 5x - 12}$

Show clear algebraic working.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2026

1 Solution | Step 1

$$4x^2 - 9 = (2x - 3)(2x + 3), \qquad 2x^2 - 5x - 12 = (2x + 3)(x - 4).$$

Use difference of squares for the numerator and factorise the denominator.

2 Solution | Step 2

$$\frac{4x^2 - 9}{2x^2 - 5x - 12} = \frac{(2x - 3)(2x + 3)}{(2x + 3)(x - 4)} = \frac{2x - 3}{x - 4}.$$

Cancel the common factor, not individual terms.

3 Solution | Step 3

The original fraction requires $x \neq -\frac{3}{2}$ and $x \neq 4$.
Cancellation does not restore excluded values.

✓ **FINAL ANSWER**

$\frac{2x-3}{x-4}$, $x \neq -\frac{3}{2}, 4$.

Question 20

4 marks | Algebraic Fractions

20 Express $\left(\frac{5}{4x-7}-\frac{3}{3x-5}\right)\div\frac{9x^2-16}{6x^2-7x-5}$ as a single fraction in its simplest form.

Show clear algebraic working.

.....

(Total for Question 20 is 4 marks)

1 **Subtract the fractions**

$$\frac{5(3x-5)-3(4x-7)}{(4x-7)(3x-5)} = \frac{3x-4}{(4x-7)(3x-5)}$$

2 **Factorise the second fraction**

$$9x^2-16=(3x-4)(3x+4), \qquad 6x^2-7x-5=(3x-5)(2x+1)$$

3 **Divide = multiply by the reciprocal**

$$\frac{3x-4}{(4x-7)(3x-5)} \times \frac{(3x-5)(2x+1)}{(3x-4)(3x+4)}$$

4 **Cancel**

$$= \frac{2x+1}{(4x-7)(3x+4)}$$

✓ **FINAL ANSWER**

$$\frac{2x+1}{(4x-7)(3x+4)}$$

TOPIC

Algebraic Roots & Indices

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 7

4 marks | Algebraic Roots & Indices

7 (a) Simplify $8 \times (4t)^0$

.....

(1)

$x^6 \div x^{-5} = x^p$

(b) Find the value of p

$p =$ .....

(1)

(c) Simplify fully $(2k^2m^4)^3$

.....

(2)

WORKED SOLUTION

Question 7 · 4WM1/01 Specimen

1 Use the zero power

$$(4t)^0 = 1$$

$$8(4t)^0 = 8$$

2 Subtract the indices

$$x^6 \div x^{-5} = x^{6-(-5)} = x^{11}$$

So $p = 11$.

3 **Raise each factor to the power**

$$(2k^2m^4)^3 = 2^3k^{2\times3}m^{4\times3}$$

$$= 8k^6m^{12}$$

✓ **FINAL ANSWER**

8, $p = 11$, and $8k^6m^{12}$.

Question 13

6 marks | Algebraic Roots & Indices

13 (a) Expand and simplify $(3x-1)(x+2)(3x+1)$

.....

(3)

(b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

.....

(3)

(Total for Question 13 is 6 marks)

1 **Multiply the conjugate factors**

$$(3x-1)(3x+1) = 9x^2 - 1$$

2 **Expand the remaining bracket**

$$(9x^2-1)(x+2) = 9x^3 + 18x^2 - x - 2$$

3 **Simplify inside the bracket**

$$\frac{2x^5}{8xy^2} = \frac{x^4}{4y^2}$$

4 **Apply the negative power**

$$\left(\frac{x^4}{4y^2}\right)^{-2} = \left(\frac{4y^2}{x^4}\right)^{2} = \frac{16y^4}{x^8}$$

✓ **FINAL ANSWER**

$9x^3 + 18x^2 - x - 2$, and $\frac{16y^4}{x^8}$.

Question 22

3 marks | Algebraic Roots & Indices

22 $\dfrac{18 \times (\sqrt{27})^{}}{6 \times 9^{2n+8}} = 3^x$

Express x in terms of n

Show your working clearly and simplify your expression.

$x =$ .....

1 Write everything as powers of 3

$$\sqrt{27} = 3^{3/2}, \qquad 9 = 3^2, \qquad \frac{18}{6} = 3$$

2 Simplify the powers

$$\frac{18(\sqrt{27})^{4n+6}}{6(9)^{2n+8}} = 3 \times \frac{\left(3^{3/2}\right)^{4n+6}}{(3^2)^{2n+8}}$$

$$= 3^1 \times \frac{3^{6n+9}}{3^{4n+16}}$$

3 Subtract the indices

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

So

$$x = 2n - 6$$

FINAL ANSWER

$x = 2n - 6$.

4MA1/2H JANUARY 2020

Question 1

3 marks | Algebraic Roots & Indices

1 (a) Simplify $\dfrac{x^9}{x^2}$

.....

(1)

(b) Write $\dfrac{7^8 \times 7^4}{7^3}$ as a single power of 7

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2020

1 **(a)**

(a)

$$\frac{x^9}{x^2} = x^{9-2} = x^7$$

2 **(b)**

(b)

$$\frac{7^8 \times 7^4}{7^3} = 7^{8+4-3} = 7^9$$

✓ **FINAL ANSWER**

(a) x^7, (b) 7^9.

4MA1/1H JANUARY 2020

Question 17

6 marks | Algebraic Roots & Indices

17 (a) Show that $\left(6+2\sqrt{12}\right)^2 = 12\left(7+4\sqrt{3}\right)$

Show each stage of your working.

(3)

(b) Simplify fully $\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}}$

.....

(3)

(Total for Question 17 is 6 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H January 2020

1 **Simplify surd**

First,

$$\sqrt{12} = 2\sqrt{3}$$

2 **Simplify surd**

So

$$6 + 2\sqrt{12} = 6 + 4\sqrt{3}$$

3 **Simplify surd**

Now square:

$$(6 + 4\sqrt{3})^2 = 36 + 48\sqrt{3} + 48$$

$$= 84 + 48\sqrt{3}$$

4 **Factor out**

Factor out 12:

$$84 + 48\sqrt{3} = 12(7 + 4\sqrt{3})$$

5 **Simplify surd**

For part (b),

$$\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}} = \left(\frac{t^{15}}{27a^{12}}\right)^{\frac{2}{3}}$$

$$= \frac{t^{10}}{9a^{8}}$$

✓ **FINAL ANSWER**

(a) shown, (b) $\frac{t^{10}}{9a^{8}}$

4MA1/2HR JANUARY 2020

Question 21

7 marks | Algebraic Roots & Indices

21 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

.....

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

.....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2020

1 Factorise expression

For part (a), factorise the numerator and denominator:

$$10x^2 + 23x + 12 = (5x + 4)(2x + 3)$$

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

2 Use index laws

So

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} = \frac{(5x + 4)(2x + 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{5x + 4}{2x - 3}$$

3 Use index laws

For part (b),

$$2^{2y} \times 2^{3y+2} = 2^{5y+2}$$

4 **Use index laws**

Also,

$$\frac{8^{5y}}{4^n} = \frac{(2^3)^{5y}}{(2^2)^n} = 2^{15y-2n}$$

5 **Equate powers of 2**

Equate powers of 2:

$$5y + 2 = 15y - 2n$$

$$2n = 10y - 2$$

$$n = 5y - 1$$

✓ **FINAL ANSWER**

(a) $\frac{5x+4}{2x-3}$, (b) $n = 5y - 1$.

4MA1/1HR MAY/NOVEMBER 2020

Question 21

5 marks | Algebraic Roots & Indices

21 Given that $M = \dfrac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2}$

find the values of n for which $M = 2$

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May/November 2020

1 **Use index laws**

$$18^{4n} = (2 \times 3^2)^{4n} = 2^{4n}3^{8n}$$

$$12^2 = (2^2 \times 3)^2 = 2^4 3^2$$

2 **Use index laws**

So

$$M = \frac{2^{4n}3^{8n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{2^4 3^2}$$

3 **Combine powers of 2**

Combine powers of 2:

$$2^{4n+3n^2-18n-4} = 2^{3n^2-14n-4}$$

4 **Combine powers of 3**

Combine powers of 3:

$$3^{8n+2-8n-2} = 3^0 = 1$$

5 **Use index laws**

So

$$M = 2^{3n^2-14n-4}$$

6 **Use index laws**

Given $M = 2$,

$$3n^2 - 14n - 4 = 1$$

$$3n^2 - 14n - 5 = 0$$

$$(3n + 1)(n - 5) = 0$$

✓ **FINAL ANSWER**

$n = -\frac{1}{3}$ or $n = 5$

4MA1/2HR JANUARY 2021

Question 4

4 marks | Algebraic Roots & Indices

4 (a) Simplify $(3k^2)^4$

.....

(2)

(b) Simplify $(21m^4n) \div (3n^{-5})$

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2021

1 Use index laws

For part (a),

$$(3k^2)^4 = 3^4(k^2)^4$$

$$= 81k^8$$

2 Use index laws

For part (b),

$$(21m^4n) \div (3n^{-5}) = \frac{21m^4n}{3n^{-5}}$$

$$= 7m^4n^{1-(-5)}$$

$$= 7m^4n^6$$

FINAL ANSWER

(a) $81k^8$, (b) $7m^4n^6$.

4MA1/2H JANUARY 2021

Question 11

3 marks | Algebraic Roots & Indices

11 Simplify fully $\left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2021

1 Use index laws

$$\left(\frac{9t^4w^9}{18t^6w^{10}}\right)^{-2}$$

2 Simplify inside the brackets

Simplify inside the brackets:

$$\frac{9t^4w^9}{18t^6w^{10}} = \frac{1}{2}t^{-2}w^{-1} = \frac{1}{2t^2w}$$

3 Use index laws

Now apply the power -2:

$$\left(\frac{1}{2t^2w}\right)^{-2} = (2t^2w)^2$$

$$= 4t^4w^2$$

✓ **FINAL ANSWER**

$4t^4w^2$.

4MA1/1HR JANUARY 2021

Question 12

9 marks | Algebraic Roots & Indices

12 (a) Simplify $(16e^{10}f^{6})^{\frac{1}{2}}$

.....

(2)

(b) Write $\frac{2x+1}{4}+\frac{x-2}{3}$ as a single fraction in its simplest form.

.....

(3)

Given that $4^{k+3}=16\times 2^{k}$

(c) find the value of k.
Show your working clearly.

$k =$ .....

(4)

(Total for Question 12 is 9 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2021

1 **(a)**

(a)

$$(16e^{10}f^{6})^{\frac{1}{2}}=4e^{5}f^{3}$$

2 **(b)**

(b)

$$\frac{2x+1}{4}+\frac{x-2}{3}=\frac{3(2x+1)+4(x-2)}{12}$$

$$=\frac{6x+3+4x-8}{12}=\frac{10x-5}{12}=\frac{5(2x-1)}{12}$$

3 **(c)**

(c)

$$4^{k+3} = 16 \times 2^k$$

$$(2^2)^{k+3} = 2^4 \times 2^k$$

$$2^{2k+6} = 2^{k+4}$$

$$2k + 6 = k + 4$$

$$k = -2$$

FINAL ANSWER

(a) $4e^5f^3$, (b) $\frac{5(2x-1)}{12}$, (c) $k = -2$.

4MA1/2HR JANUARY 2021

Question 19

7 marks | Algebraic Roots & Indices

19 (a) Simplify $8^2 \times \sqrt[3]{4^6}$

Give your answer in the form 2^a where a is an integer.

Show each stage of your working clearly.

.....

(3)

Given that $n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$ where $n > 0$

(b) find the value of n.

$n =$ .....

(4)

(Total for Question 19 is 7 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2021

1 **Simplify surd**

$$8^2 = (2^3)^2 = 2^6$$

$$\sqrt[3]{4^6} = \sqrt[3]{(2^2)^6} = \sqrt[3]{2^{12}} = 2^4$$

2 **Simplify surd**

So

$$8^2 \times \sqrt[3]{4^6} = 2^6 \times 2^4 = 2^{10}$$

3 **Use index laws**

For part (b),

$$n^{-4/5} = \left(\frac{1}{2}\right)^4 = 2^{-4}$$

4 **Use index laws**

So

$$n^{4/5} = 2^4$$

5 **Raise both sides to the**

Raise both sides to the power $\frac{5}{4}$:

$$n = (2^4)^{5/4} = 2^5 = 32$$

FINAL ANSWER

(a) 2^{10}, (b) $n = 32$.

4MA1/1H MAY 2021

Question 11

4 marks | Algebraic Roots & Indices

11 $\sqrt{2} \times 16 = 2^x$

(a) Find the value of x.
Show your working clearly.

$x =$ .....

(2)

$$\frac{(11^{-6})^5}{11^4} = 11^n$$

(b) Find the value of n.
Show your working clearly.

$n =$ .....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2021

1 **Simplify surd**

$$\sqrt{2} \times 16 = 2^x$$

2 **Write both factors as powers**

Write both factors as powers of 2:

$$\sqrt{2} = 2^{1/2}, \quad 16 = 2^4$$

3 **Use index laws**

So

$$2^{1/2} \times 2^4 = 2^{9/2}$$

4 **Use index laws**

Therefore

$$x = \frac{9}{2}$$

5 **Use index laws**

For part (b),

$$\frac{(11^{-6})^5}{11^4} = \frac{11^{-30}}{11^4} = 11^{-30-4} = 11^{-34}$$

6 **Use index laws**

So

$$n = -34$$

FINAL ANSWER

(a) $x = \frac{9}{2}$, (b) $n = -34$.

4MA1/2H MAY 2021

Question 14

3 marks | Algebraic Roots & Indices

14 Simplify fully $\left(\frac{9x^4}{16y^{10}}\right)^{-\frac{1}{2}}$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H May 2021

1 Use index laws

$$\left(\frac{9x^4}{16y^{10}}\right)^{-1/2}$$

2 A negative power takes the

A negative power takes the reciprocal:

$$\left(\frac{16y^{10}}{9x^4}\right)^{1/2}$$

3 Simplify surd

Now take the square root of each factor:

$$\frac{\sqrt{16y^{10}}}{\sqrt{9x^4}} = \frac{4y^5}{3x^2}$$

FINAL ANSWER

$\frac{4y^5}{3x^2}$.

4MA1/2H NOVEMBER 2021

Question 1

4 marks | Algebraic Roots & Indices

1 (a) Write down the value of m, given that $3^4 \times 3^5 = 3^m$

$m =$ .....

(1)

(b) Write down the value of n, given that $(5^3)^7 = 5^n$

$n =$ .....

(1)

(c) Find the value of p, given that $\dfrac{7^8 \times 7^2}{7^p} = 7^6$

$p =$ .....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2021

1 **Use index laws**

(a) $3^4 \times 3^5 = 3^9$, so $m = 9$.

2 **Use index laws**

(b) $(5^3)^7 = 5^{21}$, so $n = 21$.

3 **(c)**

(c)

$$\frac{7^8 \times 7^2}{7^p} = 7^{10-p} = 7^6$$

4 **Use index laws**

So $10 - p = 6$, hence $p = 4$.

✓ **FINAL ANSWER**

$m = 9$, $n = 21$, $p = 4$

4MA1/1H JANUARY 2022

Question 1

6 marks | Algebraic Roots & Indices

1 (a) Simplify $a^7 \times a^4$

.....

(1)

(b) Simplify $w^{15} \div w^3$

.....

(1)

(c) Simplify $(8x^5y^3)^2$

.....

(2)

(d) Make t the subject of $c = t^3 - 8v$

.....

(2)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2022

1 **(a)**

(a)

$$a^7 \times a^4 = a^{11}$$

2 **(b)**

(b)

$$w^{15} \div w^3 = w^{12}$$

3 **(c)**

(c)

$$(8x^5y^3)^2 = 64x^{10}y^6$$

4 **(d)**

(d)

$$c = t^3 - 8v$$

$$t^3 = c + 8v$$

FINAL ANSWER

a^{11}, w^{12}, $64x^{10}y^6$, $t = \sqrt[3]{c + 8v}$

4MA1/2HR JANUARY 2022

Question 6

4 marks | Algebraic Roots & Indices

6 (a) Write down the value of x^0

.....

(1)

Given that $2^{-3} \times 2^9 = 2^n$

(b) find the value of n

$n =$ .....

(1)

Given that $\dfrac{7^{206} \times 7^m}{7^{214}} = 7^{-3}$

(c) find the value of m

$m =$ .....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR January 2022

1 **Use index laws**

$$x^0 = 1$$

2 **Use index laws**

For part (b),

$$2^{-3} \times 2^9 = 2^{-3+9} = 2^6$$

3 **Use index laws**

So

$$n = 6$$

4 **Use index laws**

For part (c),

$$\frac{7^{206} \times 7^m}{7^{214}} = 7^{206+m-214}$$

5 **Use index laws**

This equals 7^{-3}, so

$$206 + m - 214 = -3$$

$$m = 5$$

✓ **FINAL ANSWER**

(a) 1, (b) $n = 6$, (c) $m = 5$.

4MA1/2HR JUNE 2022

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2022

1 Use index laws

$$\frac{2^k}{4^n} = 2^x$$

2 Use index laws

Since $4 = 2^2$,

$$4^n = (2^2)^n = 2^{2n}$$

3 Use index laws

So

$$\frac{2^k}{2^{2n}} = 2^{k-2n}$$

4 Use index laws

Therefore

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

4MA1/1H JUNE 2022

Question 9

4 marks | Algebraic Roots & Indices

9 (a) Simplify $8 \times (4t)^0$

.....

(1)

$x^6 \div x^{-5} = x^p$

(b) Find the value of p

$p =$ .....

(1)

(c) Simplify fully $(2k^2m^4)^3$

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2022

1 **Use index laws**

$$8 \times (4t)^0 = 8 \times 1 = 8$$

2 **Use index laws**

For part (b),

$$x^6 \div x^{-5} = x^{6-(-5)} = x^{11}$$

3 **Use index laws**

So

$$p = 11$$

4 **Use index laws**

For part (c),

$$(2k^2m^4)^3 = 2^3k^{2\times3}m^{4\times3}$$

$$= 8k^6m^{12}$$

FINAL ANSWER

(a) 8, (b) $p = 11$, (c) $8k^6m^{12}$.

4MA1/2H JUNE 2022

Question 24

3 marks | Algebraic Roots & Indices

24 $$\frac{18 \times (\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2022

1 Simplify surd

$$\frac{18(\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

2 Use index laws

First,

$$\frac{18}{6} = 3 = 3^1$$

3 Simplify surd

Also,

$$\sqrt{27} = \sqrt{3^3} = 3^{3/2}$$

4 Simplify surd

So

$$(\sqrt{27})^{4n+6} = 3^{\frac{3}{2}(4n+6)} = 3^{6n+9}$$

5 Use index laws

And

$$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$$

6 Use index laws

Therefore the whole expression is

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

7 **Use index laws**

So

$$x = 2n - 6$$

✓ **FINAL ANSWER**

$x = 2n - 6$.

4MA1/1HR MAY 2023

Question 6

4 marks | Algebraic Roots & Indices

6 (a) Simplify $m^{10} \div m^3$

.....

(1)

$k^n \times k^4 = k^{12}$

(b) Write down the value of n

$n =$ .....

(1)

(c) Simplify $(3x^6y^8)^2$

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May 2023

1 **(a)**

(a) $m^{10} \div m^3 = m^{10-3} = m^7$.

2 **Use index laws**

(b) $k^n \times k^4 = k^{n+4} = k^{12}$, so $n + 4 = 12$ and $n = 8$.

3 **(c)**

(c) $(3x^6y^8)^2 = 3^2x^{12}y^{16} = 9x^{12}y^{16}$.

✓ **FINAL ANSWER**

(a) m^7, (b) 8, (c) $9x^{12}y^{16}$

4MA1/2HR JUNE 2023

Question 7

2 marks | Algebraic Roots & Indices

7 (a) Simplify $(4^{-2})^0$

.....

(1)

$3^{-14} \times 3^8 = 3^m$

(b) Find the value of m

$m =$ .....

(1)

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2023

1 Use index laws

Any non-zero expression to the power 0 is 1:

$$(4^{-2})^0 = 1$$

2 Use index laws

For part (b),

$$3^{-14} \times 3^8 = 3^{-14+8} = 3^{-6}$$

3 Use index laws

So

$$m = -6$$

FINAL ANSWER

(a) 1, (b) $m = -6$.

4MA1/2HR JUNE 2023

Question 11

2 marks | Algebraic Roots & Indices

11 Express $\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}}$ in the form $\frac{m^ak^b}{c}$ where a, b and c are integers to be found.

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2023

1 Convert standard form

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{(m^6)^{\frac{3}{2}}(k^{10})^{\frac{3}{2}}}{25^{\frac{3}{2}}}$$

$$(m^6)^{\frac{3}{2}} = m^9$$

$$(k^{10})^{\frac{3}{2}} = k^{15}$$

2 Convert standard form

Also,

$$25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$$

3 Convert standard form

So

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{m^9k^{15}}{125}$$

✓ **FINAL ANSWER**

$$\frac{m^9k^{15}}{125}$$

4MA1/2H JUNE 2023

Question 15

7 marks | Algebraic Roots & Indices

15 (a) Simplify fully $\left(32a^{15}\right)^{\frac{3}{5}}$

.....

(2)

(b) Express $\left(\frac{1}{10x}\right)^{-3}$ in the form px^n where p and n are integers.

.....

(2)

(c) Solve $\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$

Show clear algebraic working.

$y =$ .....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2023

1 **Solve equation**

For part (a),

$$(32a^{15})^{3/5} = 32^{3/5}a^{15\times 3/5}$$

$$= 8a^9$$

2 **Solve equation**

For part (b),

$$\left(\frac{1}{10x}\right)^{-3} = (10x)^3 = 1000x^3$$

3 **Solve equation**

For part (c),

$$\frac{1-2y}{3} = \frac{4}{5} - \frac{2y-1}{2}$$

4 **Multiply by**

Multiply by 30:

$$10(1-2y) = 24 - 15(2y-1)$$

$$10 - 20y = 39 - 30y$$

$$10y = 29$$

$$y = \frac{29}{10}$$

✓ **FINAL ANSWER**

(a) $8a^9$, (b) $1000x^3$, (c) $y = \frac{29}{10}$

4MA1/1HR MAY 2023

Question 24

5 marks | Algebraic Roots & Indices

24 Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for x in terms of n
State any restrictions on n

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR May 2023

1 **Use index laws**

$$16^x = 2^{4x}, \qquad 8 = 2^3$$

2 **Use index laws**

So

$$2^n = 2^{x^2} \times 2^{4x} \times 2^3 = 2^{x^2+4x+3}$$

3 **Hence**

Hence

$$n = x^2 + 4x + 3 = (x+2)^2 - 1$$

$$(x+2)^2 = n + 1$$

4 **Simplify surd**

Since $x > 0$,

$$x + 2 = \sqrt{n+1}$$

$$x = -2 + \sqrt{n+1}$$

5 **Simplify surd**

For $x > 0$, $\sqrt{n+1} > 2$, so $n > 3$.

FINAL ANSWER

$x = -2 + \sqrt{n+1}, n > 3$

4MA1/2H NOVEMBER 2023

Question 4

8 marks | Algebraic Roots & Indices

4 (a) Solve $\frac{2x+5}{6} = 2x - 5$

Show clear algebraic working.

$x =$ .....

(3)

(b) Simplify $h^{15} \div h^3$

.....

(1)

(c) Simplify fully $\left(2g^3k^5\right)^4$

.....

(2)

(d) Given that $\frac{y^5 \times y^n}{y^7} = y^{12}$

work out the value of n

$n =$ .....

(2)

(Total for Question 4 is 8 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2023

1 **(a)**

(a)

$$\frac{2x+5}{6} = 2x - 5$$

$$2x + 5 = 12x - 30$$

$$35 = 10x$$

$$x = \frac{7}{2}$$

2 **(b)**

(b)

$$h^{15} \div h^3 = h^{15-3} = h^{12}$$

---

3 **(c)**

(c)

$$(2g^3k^5)^4 = 2^4 g^{12} k^{20} = 16g^{12}k^{20}$$

---

4 **(d)**

(d)

$$\frac{y^5 \times y^n}{y^7} = y^{5+n-7} = y^{n-2}$$

---

5 **Use index laws**

Given that this equals y^{12},

$$n - 2 = 12$$

$$n = 14$$

✓ **FINAL ANSWER**

$x = \frac{7}{2}$, h^{12}, $16g^{12}k^{20}$, and $n = 14$.

4MA1/2H NOVEMBER 2023

Question 12

4 marks | Algebraic Roots & Indices

12 $2^7 \times 4^5 = 4^x$

(a) Calculate the value of x

$x =$ .....

(2)

(b) Simplify fully $\left(125p^6y^{24}\right)^{\frac{2}{3}}$

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2023

1 **(a)**

(a)

$$2^7 \times 4^5 = 2^7 \times (2^2)^5 = 2^{17}$$

2 **Use index laws**

Also

$$4^x = (2^2)^x = 2^{2x}$$

3 **Use index laws**

So

$$2x = 17$$

$$x = \frac{17}{2}$$

4 **(b)**

(b)

$$(125p^6y^{24})^{2/3} = 125^{2/3}p^4y^{16}$$

$$= 25p^4y^{16}$$

FINAL ANSWER

(a) $x = \frac{17}{2}$, (b) $25p^4y^{16}$

4MA1/2HR JUNE 2024

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{3^{-2}\times 3^{5}}{3^{10}} = 3^{n}$

Find the value of n

$n =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2024

1 Use index laws

$$\frac{3^{-2} \times 3^{5}}{3^{10}} = 3^{n}$$

2 Use index laws

Use index laws:

$$\frac{3^{-2+5}}{3^{10}} = 3^{3-10} = 3^{-7}$$

3 Use index laws

So

$$n = -7$$

FINAL ANSWER

$n = -7$.

4MA1/1H MAY 2024

Question 10

4 marks | Algebraic Roots & Indices

10 (a) Simplify $(2p)^0$ where $p > 0$

.....

(1)

$y^9 \times y^{-3} = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4c^2)^3$

.....

(2)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2024

1 **(a)**

(a) $(2p)^0 = 1$.

2 **Use index laws**

(b) $y^9 \times y^{-3} = y^{9-3} = y^6$, so $n = 6$.

3 **(c)**

(c) $(5a^4c^2)^3 = 5^3a^{12}c^6 = 125a^{12}c^6$.

✓ **FINAL ANSWER**

(a) 1, (b) 6, (c) $125a^{12}c^6$

4MA1/1HR MAY 2024

Question 17

4 marks | Algebraic Roots & Indices

17 (a) $\left(\sqrt[4]{k^{12}}\right)^5 = k^n$

Find the value of n

$n =$ .....

(1)

(b) Express $\dfrac{7}{2-\sqrt{3}}$ in the form $\sqrt{c} + d$ where c and d are integers.

Show your working clearly.

.....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May 2024

1 **(a) so**

(a)

$$\left(\sqrt[4]{k^{12}}\right)^5 = (k^3)^5 = k^{15}$$

so $n = 15$.

2 **(b)**

(b)

$$\frac{7}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{7(2+\sqrt{3})}{4-3} = 14 + 7\sqrt{3}$$

$$7\sqrt{3} = \sqrt{49 \times 3} = \sqrt{147}$$

FINAL ANSWER

(a) 15, (b) $\sqrt{147} + 14$

4MA1/2H JUNE 2024

Question 23

3 marks | Algebraic Roots & Indices

23 Simplify $\dfrac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$

Give your answer in the form 5^w where w is an expression in terms of x
Show each stage of your working clearly.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2024

1 Simplify surd

$$\frac{30 \times 25^{2x+7}}{\sqrt{180} \times (\sqrt{5})^{4x+9}}$$

2 Simplify the number part

Simplify the number part:

$$\sqrt{180} = 6\sqrt{5}$$

$$\frac{30}{6\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5} = 5^{1/2}$$

3 Simplify surd

Now write the powers with base 5:

$$25^{2x+7} = (5^2)^{2x+7} = 5^{4x+14}$$

$$(\sqrt{5})^{4x+9} = 5^{(4x+9)/2} = 5^{2x+9/2}$$

4 Use index laws

So the full expression is

$$5^{1/2+4x+14-(2x+9/2)}$$

$$= 5^{2x+10}$$

FINAL ANSWER

5^{2x+10}.

4MA1/2H NOVEMBER 2024

Question 14

3 marks | Algebraic Roots & Indices

14 Given that $\dfrac{3^{2n+3}}{3^4} = 3^3 \times 3^{1-2n}$

find the value of n
Show your working clearly.

$n =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H November 2024

1 **Use index laws**

$$\frac{3^{2n+3}}{3^4} = 3^{2n-1}$$

$$3^3 \times 3^{1-2n} = 3^{4-2n}$$

2 **Use index laws**

So

$$2n - 1 = 4 - 2n$$

$$4n = 5$$

✓ **FINAL ANSWER**

$n = \frac{5}{4}$

4MA1/1H JUNE 2025

Question 4

4 marks | Algebraic Roots & Indices

4 $x^5 \times x^7 = x^m$

(a) Find the value of m

$m =$ .....

(1)

$y^8 \div y^3 = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4r^2)^3$

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H June 2025

1 **Use index laws**

For part (a),

$$x^5 \times x^7 = x^{12}$$

2 **Use index laws**

So $m = 12$.

3 **Use index laws**

For part (b),

$$y^8 \div y^3 = y^{8-3} = y^5$$

4 **Use index laws**

So $n = 5$.

5 **Use index laws**

For part (c),

$$(5a^4r^2)^3 = 5^3a^{4\times3}r^{2\times3}$$

$$= 125a^{12}r^6$$

✓ **FINAL ANSWER**

(a) $m = 12$, (b) $n = 5$, (c) $125a^{12}r^6$.

4WM1/01 MAY 2025

Question 5

4 marks | Algebraic Roots & Indices

5 $x^5 \times x^7 = x^m$

(a) Find the value of m

$m =$ .....

(1)

$y^8 \div y^3 = y^n$

(b) Find the value of n

$n =$ .....

(1)

(c) Simplify fully $(5a^4r^2)^3$

.....

(2)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 May 2025

1 **Use index laws**

$$x^5 \times x^7 = x^{5+7} = x^{12}$$

2 **Use index laws**

So $m = 12$.

$$y^8 \div y^3 = y^{8-3} = y^5$$

3 **Use index laws**

So $n = 5$.

$$(5a^4r^2)^3 = 5^3(a^4)^3(r^2)^3 = 125a^{12}r^6$$

✓ **FINAL ANSWER**

$m = 12$, $n = 5$, and $125a^{12}r^6$.

Question 5

4 marks | Algebraic Roots & Indices

5 $3^8 \times 3^4 = 3^m$

(a) Find the value of m

$m =$ .....

(1)

$4^{12} \div 4^7 = 4^n$

(b) Find the value of n

$n =$ .....

(1)

$\left(6^2\right)^{10} = 6^p$

(c) Find the value of p

$p =$ .....

(1)

$5^r = 1$

(d) Write down the value of r

$r =$ .....

(1)

(Total for Question 5 is 4 marks)

1 Evaluate fraction

$$3^8 \times 3^4 = 3^{8+4} = 3^{12}$$

$$\frac{4^{12}}{4^7} = 4^{12-7} = 4^5$$

$$(6^2)^{10} = 6^{20}$$

$$5^0 = 1$$

FINAL ANSWER

$m = 12, n = 5, p = 20, r = 0.$

4MA1/2HR JUNE 2025

Question 6

5 marks | Algebraic Roots & Indices

6 (a) Write down the value of 5^0

.....

(1)

$$\frac{5^9 \times 5^{-3}}{5^{-2}} = 5^k$$

(b) Find the value of k

$k =$ .....

(2)

(c) Simplify fully $\left(2d^4e^5\right)^3$

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2025

1 **Use index laws**

$$5^0 = 1$$

2 **Use index laws**

For part (b),

$$\frac{5^9 \times 5^{-3}}{5^{-2}} = 5^{9+(-3)-(-2)} = 5^8$$

3 **Use index laws**

So

$$k = 8$$

4 **Use index laws**

For part (c),

$$(2d^4e^5)^3 = 2^3 d^{4\times 3} e^{5\times 3}$$

$$= 8d^{12}e^{15}$$

FINAL ANSWER

(a) $\mathbf{1}$, (b) $\boldsymbol{k = 8}$, (c) $\boldsymbol{8d^{12}e^{15}}$.

4WM1/01 MAY 2025

Question 9

3 marks | Algebraic Roots & Indices

9 $7^x = 1$

(a) Write down the value of x

$x =$ .....

(1)

$$\frac{8^{-4} \times 8^{11}}{8^{12}} = 8^n$$

(b) Work out the value of n

$n =$ .....

(2)

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4WM1/01 May 2025

1 Use index laws

$$7^x = 1$$

2 Use index laws

Since $7^0 = 1$,

$$x = 0$$

3 Use index laws

For part (b),

$$\frac{8^{-4} \times 8^{11}}{8^{12}} = 8^n$$

$$8^{-4} \times 8^{11} = 8^7$$

$$\frac{8^7}{8^{12}} = 8^{7-12} = 8^{-5}$$

4 Use index laws

So $n = -5$.

FINAL ANSWER

$x = 0, n = -5.$

4MA1/2H JUNE 2025

Question 15

3 marks | Algebraic Roots & Indices

15 Simplify fully $\left(\frac{2a^3}{10a^7c^2}\right)^{-3}$

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2025

1 **Use index laws**

$$\left(\frac{2a^3}{10a^7c^2}\right)^{-3}$$

2 **Use index laws**

First simplify inside the bracket:

$$\frac{2a^3}{10a^7c^2} = \frac{1}{5a^4c^2}$$

3 **A negative power means take**

A negative power means take the reciprocal:

$$\left(\frac{1}{5a^4c^2}\right)^{-3} = (5a^4c^2)^3$$

$$= 125a^{12}c^6$$

✓ **FINAL ANSWER**

$125a^{12}c^6$.

4WM1/01 MAY 2025

Question 16

3 marks | Algebraic Roots & Indices

16 Solve the equation $8^{3y+4} \times 4^{3y} = 2^{5y}$
Show your working clearly.

$y =$ .....

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4WM1/01 May 2025

1 Solve equation

$$8^{3y+4} \times 4^{3y} = 2^{5y}$$

2 Solve equation

Write 8 and 4 as powers of 2.

$$(2^3)^{3y+4}(2^2)^{3y} = 2^{5y}$$

$$2^{9y+12} \times 2^{6y} = 2^{5y}$$

$$2^{15y+12} = 2^{5y}$$

3 Solve equation

So

$$15y + 12 = 5y$$

$$y = -\frac{6}{5}$$

✓ **FINAL ANSWER**

$y = -\frac{6}{5}$

4WM1/01 NOVEMBER 2025

Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{8^{-2} \times 8^{9}}{8^{10}} = 8^{n}$

Work out the value of n

$n =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4WM1/01 November 2025

1 **Use index laws**

$$\frac{8^{-2} \times 8^{9}}{8^{10}} = 8^{-2+9-10} = 8^{-3}$$

FINAL ANSWER

$n = -3$

4MA1/1H NOVEMBER 2025

Question 8

2 marks | Algebraic Roots & Indices

8 $\dfrac{8^{-2} \times 8^{9}}{8^{10}} = 8^{n}$

Work out the value of n

$n =$ .....

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2025

1 **Use index laws**

$$\frac{8^{-2} \times 8^{9}}{8^{10}} = 8^{-2+9-10} = 8^{-3}$$

✓ **FINAL ANSWER**

$n = -3$

4MA1/2H NOVEMBER 2025

Question 13

4 marks | Algebraic Roots & Indices

13 (a) Simplify fully $\left(\frac{125x^6}{y^{15}}\right)^3$

.....

(2)

Given that $4^n = \frac{4^m}{64^p}$

(b) Express n in terms of m and p

$n =$ .....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2025

1 **(a)**

(a)

$$\left(\frac{125x^6}{y^{15}}\right)^{1/3} = \frac{5x^2}{y^5}$$

2 **(b)**

(b)

$$64^p = (4^3)^p = 4^{3p}$$

$$4^n = \frac{4^m}{64^p} = \frac{4^m}{4^{3p}} = 4^{m-3p}$$

✓ **FINAL ANSWER**

(a) $\frac{5x^2}{y^5}$, (b) $n = m - 3p$

4WM1/01 NOVEMBER 2025

Question 15

3 marks | Algebraic Roots & Indices

15 $\dfrac{16^{x-3} \times 8^{x+1}}{32^{x}} = 2^{n}$

Express n in terms of x
Give your answer in its simplest form.

$n =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM1/01 November 2025

1 Evaluate fraction

$$\frac{16^{x-3} \times 8^{x+1}}{32^{x}} = \frac{2^{4x-12} \times 2^{3x+3}}{2^{5x}}$$

$$= 2^{4x-12+3x+3-5x} = 2^{2x-9}$$

FINAL ANSWER

$n = 2x - 9$.

4MA1/1HR JUNE 2026

Question 3

2 marks | Algebraic Roots & Indices

3 (a) Simplify $e^7 \times e^3$

.....

(1)

(b) Simplify $g^5 \div g^2$

.....

(1)

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$e^7 \times e^3 = e^{7+3} = e^{10}.$$

For equal bases, add the indices when multiplying.

2 **Part (b) | Step 1**

$$g^5 \div g^2 = g^{5-2} = g^3 \quad (g \neq 0).$$

For equal bases, subtract the indices when dividing.

✓ **FINAL ANSWER**

(a) e^{10} (b) g^3

4MA1/1H JUNE 2026

Question 6

2 marks | Algebraic Roots & Indices

6 Simplify $\left(4x^5y^6\right)^3$

.....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H June 2026

1 Solution | Step 1

$$(4x^5y^6)^3 = 4^3x^{5\times3}y^{6\times3} = 64x^{15}y^{18}.$$

Raise each factor to the third power and multiply exponents for a power of a power.

FINAL ANSWER

$64x^{15}y^{18}$

4WM1/01 MAY 2026

Question 6

2 marks | Algebraic Roots & Indices

6 Simplify $\left(4x^5y^6\right)^3$

.....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4WM1/01 May 2026

1 Cube every part

$$(4x^5y^6)^3 = 4^3x^{15}y^{18} = 64x^{15}y^{18}$$

✓ **FINAL ANSWER**

$64x^{15}y^{18}$

Question 12

6 marks | Algebraic Roots & Indices

12 $\dfrac{2^{\frac{x}{3}}}{8^x} = 64$

(a) Find the value of x
Show clear algebraic working.

$x =$ .....

(3)

(b) Expand and simplify $4t(t + 5)(2t - 3)$

.....

(3)

(Total for Question 12 is 6 marks)

1 **(a) Write everything as powers of 2**

$$8^x = 2^{3x}, \quad 64 = 2^6 \Rightarrow \frac{2^{x/3}}{2^{3x}} = 2^6$$

2 **Compare powers**

$$\frac{x}{3} - 3x = 6 \Rightarrow -\frac{8x}{3} = 6 \Rightarrow x = -\frac{9}{4}$$

3 **(b) Expand two brackets first**

$$(t+5)(2t-3) = 2t^2 + 7t - 15$$

4 **Multiply by 4t**

$$4t(2t^2 + 7t - 15) = 8t^3 + 28t^2 - 60t$$

✓ **FINAL ANSWER**

(a) $x = -\frac{9}{4}$ (b) $8t^3 + 28t^2 - 60t$

4MA1/2HR JUNE 2026

Question 16

2 marks | Algebraic Roots & Indices

16 $\left(\frac{1}{\sqrt{a}}\right)^6 = a^n$

Work out the value of n

$n =$ .....

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2026

1 Solution | Step 1

$$\frac{1}{\sqrt{a}} = a^{-1/2}, \qquad \left(a^{-1/2}\right)^6 = a^{-3}.$$

A reciprocal changes the sign of the exponent and an outer power multiplies exponents.

FINAL ANSWER

$n = -3$.

4MA1/1HR JUNE 2026

Question 26

3 marks | Algebraic Roots & Indices

26

$$\frac{27^x \times 3^{n+6}}{9^{4-n}} = \frac{6^x}{2^x}$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/1HR June 2026

1 Solution | Step 1

$$27 = 3^3, \qquad 9 = 3^2, \qquad \frac{6^x}{2^x} = \left(\frac{6}{2}\right)^x = 3^x.$$

Write both sides in a common base.

2 Solution | Step 2

$$\frac{27^x \times 3^{n+6}}{9^{4-n}} = 3^{3x+n+6-2(4-n)} = 3^{3x+3n-2}.$$

Multiply powers by adding indices and divide powers by subtracting indices.

3 Solution | Step 3

$$3^{3x+3n-2} = 3^x \quad \Longrightarrow \quad 3x + 3n - 2 = x.$$

The base-3 exponential is one-to-one, so equal powers have equal indices.

4 Solution | Step 4

$$2x = 2 - 3n \quad \Longrightarrow \quad x = 1 - \frac{3n}{2}.$$

Collect the x terms and divide by 2.

FINAL ANSWER

$x = 1 - \frac{3n}{2}$

TOPIC

Completing the Square

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 20

4 marks | Completing the Square

20 Find the values of a, b and c so that

$$7 + 12x - 2x^2$$

is written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 20 is 4 marks)

1 **Factor out -2 from the quadratic terms**

$$7 + 12x - 2x^2 = -2(x^2 - 6x) + 7$$

2 **Complete the square**

$$x^2 - 6x = (x - 3)^2 - 9$$

3 **Simplify the expression**

$$-2((x - 3)^2 - 9) + 7 = -2(x - 3)^2 + 18 + 7$$

$$= 25 - 2(x - 3)^2$$

4 **Read the values**

$$a = 25, \qquad b = 2, \qquad c = 3$$

✓ **FINAL ANSWER**

$a = 25,\ b = 2,\ c = 3.$

4MA1/2HR JANUARY 2020

Question 23

3 marks | Completing the Square

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2020

1 Solve inequality

$$7 - 12x - 2x^2 = -2x^2 - 12x + 7$$

$$= -2(x^2 + 6x) + 7$$

$$= -2\left((x + 3)^2 - 9\right) + 7$$

$$= -2(x + 3)^2 + 18 + 7$$

$$= 25 - 2(x + 3)^2$$

FINAL ANSWER

$25 - 2(x + 3)^2$

4MA1/2H MAY 2021

Question 22

4 marks | Completing the Square

22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a, b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H May 2021

1 Solve inequality

$$g(x) = 2x^2 - 12x + 13$$

$$= 2(x^2 - 6x) + 13$$

$$= 2\left((x - 3)^2 - 9\right) + 13$$

$$= 2(x - 3)^2 - 18 + 13$$

$$= 2(x - 3)^2 - 5$$

2 Calculate value

Starting with $S : y = x^2$:

3 Translate 3 units to the

- Translate 3 units to the right to get $y = (x - 3)^2$.
- Stretch parallel to the y-axis by scale factor 2 to get $y = 2(x - 3)^2$.
- Translate 5 units down to get $y = 2(x - 3)^2 - 5$.

✓ **FINAL ANSWER**

$g(x) = 2(x - 3)^2 - 5$. Transformations: right 3, stretch parallel to the y-axis by scale factor 2, then down 5.

4MA1/1HR JANUARY 2022

Question 19

3 marks | Completing the Square

19 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^m$

(a) find the value of m

$m =$ .....

(1)

Given that a, b and c are integers,

(b) express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$

.....

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$\sqrt[3]{\frac{1}{x}} = x^{-1/3}$$

2 **Evaluate fraction**

So $m = -\frac{1}{3}$.

3 **(b)**

(b)

$$3x^2 + 12x + 19 = 3(x^2 + 4x) + 19$$

$$= 3\left((x + 2)^2 - 4\right) + 19$$

$$= 3(x + 2)^2 + 7$$

FINAL ANSWER

$m = -\frac{1}{3}$, and $3(x + 2)^2 + 7$

4MA1/1H JANUARY 2022

Question 20

4 marks | Completing the Square

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

.....

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$
The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(..... , ...)

(1)

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2022

1 **(a)**

(a)

$$7 + 12x - 3x^2 = -3x^2 + 12x + 7$$

$$= -3(x^2 - 4x) + 7$$

$$= -3\left((x - 2)^2 - 4\right) + 7$$

$$= 19 - 3(x - 2)^2$$

2 **This is the same as**

This is the same as

$$19 - 3(x + (-2))^2$$

3 **(b) The maximum point is**

(b) The maximum point is read from the completed square form.

$$y = 19 - 3(x - 2)^2$$

4 **Calculate value**

The maximum occurs when $x = 2$, giving $y = 19$.

✓ **FINAL ANSWER**

(a) $19 - 3(x - 2)^2$, (b) $A = (2, 19)$

4MA1/1HR JUNE 2022

Question 23

5 marks | Completing the Square

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

(..... , ...)

(2)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$2x^2 - 12x + 3 = 2(x^2 - 6x) + 3$$

$$= 2\left((x - 3)^2 - 9\right) + 3$$

$$= 2(x - 3)^2 - 18 + 3$$

$$= 2(x - 3)^2 - 15$$

2 **(b)**

(b)

$$y = 2(x + 4)^2 - 12(x + 4) + 3$$

3 **Let**

Let $u = x + 4$. From part (a),

$$2u^2 - 12u + 3 = 2(u - 3)^2 - 15$$

4 **Calculate value**

So

$$y = 2((x + 4) - 3)^2 - 15$$

$$y = 2(x + 1)^2 - 15$$

5 **The minimum point is when**

The minimum point is when $x = -1$, giving $y = -15$.

FINAL ANSWER

(a) $2(x - 3)^2 - 15$, (b) $M = (-1, -15)$

4MA1/1H JUNE 2022

Question 24

4 marks | Completing the Square

24 Express each of a, b and c in terms of q so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H June 2022

1 Evaluate fraction

$$q + 12x - qx^2 = -q\left(x^2 - \frac{12}{q}x\right) + q$$

2 Complete the square

Complete the square:

$$x^2 - \frac{12}{q}x = \left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}$$

3 Evaluate fraction

So

$$q + 12x - qx^2 = -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}\right] + q$$

$$= q + \frac{36}{q} - q\left(x - \frac{6}{q}\right)^2$$

4 Compare coefficients

Compare with

$$a - b(x - c)^2$$

FINAL ANSWER

$$a = q + \frac{36}{q},\ b = q,\ c = \frac{6}{q}$$

4MA1/2H JUNE 2023

Question 19

3 marks | Completing the Square

19 Express $3x^2 - 6x + 5$ in the form $a(x - b)^2 + c$

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2023

1 **Calculate value**

$$3x^2 - 6x + 5 = 3(x^2 - 2x) + 5$$

$$= 3\left((x - 1)^2 - 1\right) + 5$$

$$= 3(x - 1)^2 + 2$$

FINAL ANSWER

$3(x - 1)^2 + 2$

Question 20

5 marks | Completing the Square

20 (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where a, b and c are numbers to be found.

.....

(3)

The curve **C** has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point P is the minimum point on **C**

(b) Find the coordinates of P

(..... , ...)

(2)

(Total for Question 20 is 5 marks)

1 **Use Moved Coordinate Geometry Completing**

For part (a),

$$2x^2 - 11x + 9$$

2 **Factor out 2 from the**

Factor out 2 from the quadratic terms:

$$2\left(x^2 - \frac{11}{2}x\right) + 9$$

3 **Complete the square**

Complete the square:

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + 9$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{72}{8}$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

4 **Evaluate fraction**

So

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

5 **Calculate value**

For part (b),

$$y = 2(x-3)^2 - 11(x-3) + 9$$

6 **Let**

Let

$$u = x - 3$$

7 **Evaluate fraction**

From part (a), the minimum occurs when

$$u = \frac{11}{4}$$

8 **Evaluate fraction**

So

$$x - 3 = \frac{11}{4}$$

$$x = \frac{23}{4}$$

9 **The minimum value of is**

The minimum value of y is

$$-\frac{49}{8}$$

✓ **FINAL ANSWER**

$$2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8},\ P = \left(\frac{23}{4}, -\frac{49}{8}\right)$$

4MA1/1H MAY 2024

Question 25

4 marks | Completing the Square

25 $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where a, b and c are constants.

$f(x) =$ .....

(Total for Question 25 is 4 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2024

1 Solve inequality

$$f(x) = 17 - 3x^2 + 12x$$

$$= -3x^2 + 12x + 17$$

$$= -3(x^2 - 4x) + 17$$

$$= -3\left((x - 2)^2 - 4\right) + 17$$

$$= -3(x - 2)^2 + 12 + 17$$

$$= 29 - 3(x - 2)^2$$

FINAL ANSWER

$f(x) = 29 - 3(x - 2)^2$

4WM1/01R MAY 2025

Question 18

4 marks | Completing the Square

18 (a) Express $2x^2 - 12x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.
Show clear algebraic working.

.....

(3)

(b) Hence, state the least value of $2x^2 - 12x + 23$

.....

(1)

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM1/01R May 2025

1 **Calculate value**

$$2x^2 - 12x + 23 = 2(x^2 - 6x) + 23$$

$$= 2\left((x - 3)^2 - 9\right) + 23$$

$$= 2(x - 3)^2 + 5$$

2 **The squared part is at**

The squared part is at least zero.

✓ **FINAL ANSWER**

$2(x - 3)^2 + 5$; least value 5.

Question 25

6 marks | Completing the Square

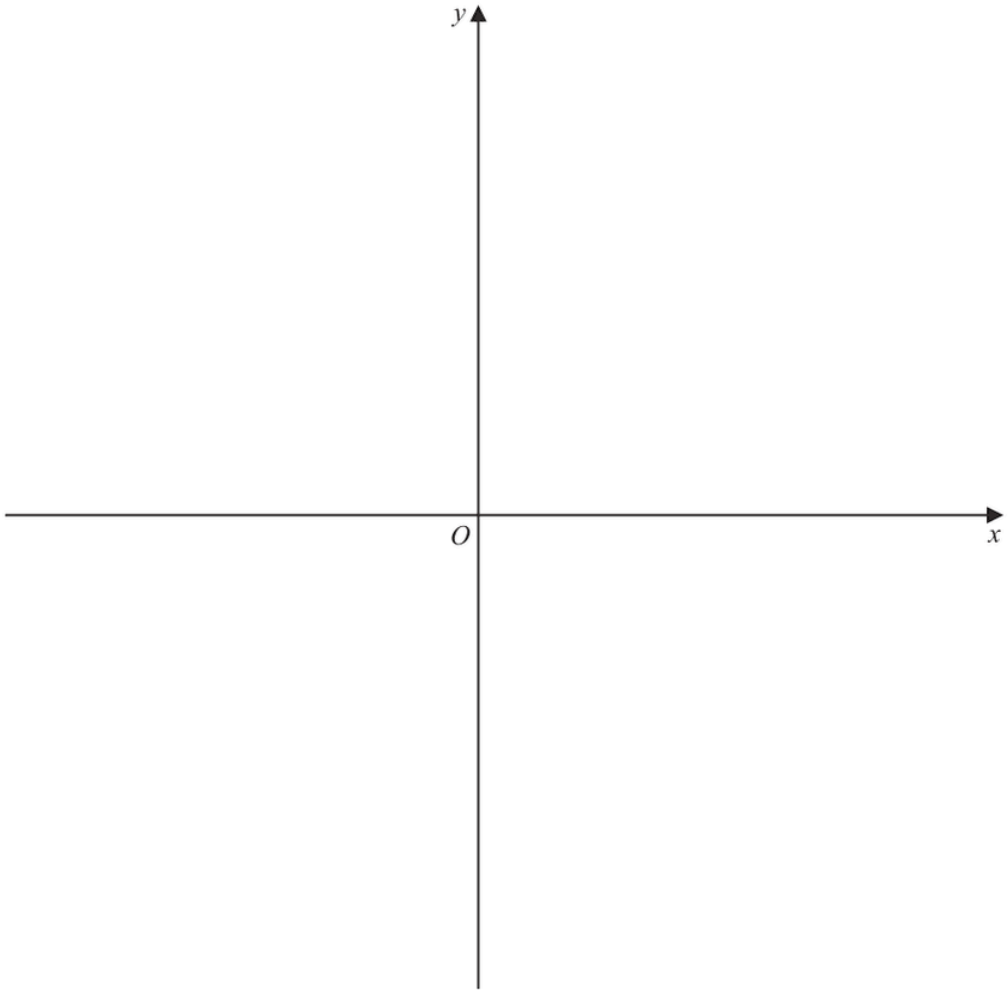
25 (a) Write $28+24x-6x^2$ in the form $a-b(x-c)^2$ where a, b and c are integers.

.....

(3)

(b) On the axes below, sketch the graph of $y=28+24x-6x^2$

Show clearly the coordinates of the turning point and the coordinates of the point of intersection of the graph with the y-axis.



(3)

(Total for Question 25 is 6 marks)

1 **(a)**

(a)

$$28 + 24x - 6x^2 = -6x^2 + 24x + 28$$

$$= -6(x^2 - 4x) + 28$$

$$= -6\left((x-2)^2 - 4\right) + 28$$

$$= 52 - 6(x-2)^2$$

2 **(b)**

(b)

$$y = 52 - 6(x-2)^2$$

3 **The graph is a downward**

The graph is a downward-opening parabola.

4 **The turning point is**

The turning point is

$$(2, 52)$$

5 **Calculate value**

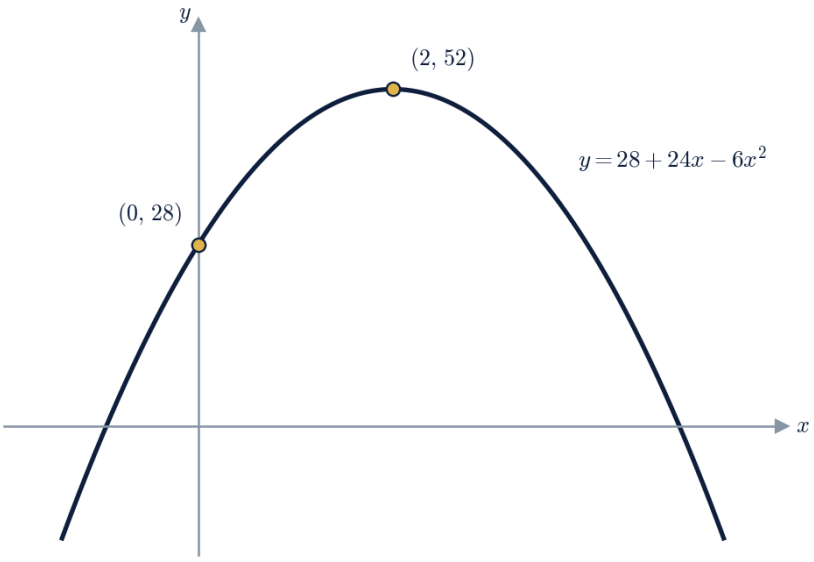
The point of intersection with the y-axis is found by putting $x = 0$:

$$y = 28$$

6 **Find intercepts**

So the y-intercept is

$$(0, 28)$$



FINAL ANSWER

(a) $52 - 6(x - 2)^2$. For the sketch, show a downward parabola with turning point $(2, 52)$ and y-intercept $(0, 28)$.

4WM1/01 MAY 2025

Question 25

5 marks | Completing the Square

25 (a) Express $8 - 12x - 4x^2$ in the form $a - b(x + c)^2$
where a, b and c are constants.

.....

(3)

(b) Hence solve $8 - 12x - 4x^2 = 0$
Show your working clearly.
Give your solutions in surd form.

.....

(2)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4WM1/01 May 2025

1 **Simplify surd**

$$8x^2 - 12x - 4 = 8\left(x^2 - \frac{3}{2}x\right) - 4$$

2 **Complete the square**

Complete the square.

$$x^2 - \frac{3}{2}x = \left(x - \frac{3}{4}\right)^2 - \frac{9}{16}$$

3 **Simplify surd**

So

$$8x^2 - 12x - 4 = 8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$$

4 **Simplify surd**

Now solve

$$8x^2 - 12x - 4 = 0$$

$$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2} = 0$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

✓ **FINAL ANSWER**

$8\left(x - \frac{3}{4}\right)^2 - \frac{17}{2}$, and $x = \frac{3 \pm \sqrt{17}}{4}$.

4MA1/1H NOVEMBER 2025

Question 21

3 marks | Completing the Square

21 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H November 2025

1 Solve inequality

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 - 20 + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$

4WM1/01 NOVEMBER 2025

Question 22

2 marks | Completing the Square

22 $9x^2 - 12x + q$ can be written in the form $(px - r)^2$ where p, q and r are integers.

Find the value of q

$q =$ .....

(Total for Question 22 is 2 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 November 2025

1 **Calculate value**

$$9x^2 - 12x + q = (3x - 2)^2$$

$$(3x - 2)^2 = 9x^2 - 12x + 4$$

FINAL ANSWER

$q = 4$.

4WM1/01 NOVEMBER 2025

Question 24

3 marks | Completing the Square

24 Express $5x^2 - 20x + 23$ in the form $a(x - b)^2 + c$ where a, b and c are integers.

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 November 2025

1 **Calculate value**

$$5x^2 - 20x + 23 = 5(x^2 - 4x) + 23$$

$$= 5\left((x - 2)^2 - 4\right) + 23$$

$$= 5(x - 2)^2 + 3$$

FINAL ANSWER

$5(x - 2)^2 + 3$.

4WM1/01 MAY 2026

Question 24

3 marks | Completing the Square

24 Express $22 - 3x^2 + 30x$ in the form $a - b(x - c)^2$ where a, b and c are integers.

.....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2026

1 Factor out −3 from the x terms

$$22 - 3x^2 + 30x = 22 - 3(x^2 - 10x)$$

2 Complete the square

$$x^2 - 10x = (x - 5)^2 - 25$$

3 Simplify

$$22 - 3\big((x - 5)^2 - 25\big) = 97 - 3(x - 5)^2$$

FINAL ANSWER

$97 - 3(x - 5)^2$

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TOPIC

Quadratic Formula

Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 30

3 marks | Quadratic Formula

30 Solve $2x^2 - 5x - 4 = 0$

Show your working clearly.
Give your solutions correct to 3 significant figures.

.....

(3)

(Total for Question 30 is 3 marks)

WORKED SOLUTION

Question 30 · EP Specimen

1 Identify a, b and c

$a = 2,\ b = -5,\ c = -4$

2 Substitute into the formula

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)} = \frac{5 \pm \sqrt{57}}{4}$$

3 Work out both solutions

$$x = \frac{5 + 7.5498\ldots}{4} = 3.14, \qquad x = \frac{5 - 7.5498\ldots}{4} = -0.637$$

FINAL ANSWER

$x = 3.14$ or $x = -0.637$

EP SPECIMEN

Question 31

3 marks | Quadratic Formula

31 Solve $x^2 + 6x + 2 = 0$

Give your solutions in the form $a \pm \sqrt{b}$ where a and b are integers.

.....

(3)

(Total for Question 31 is 3 marks)

WORKED SOLUTION

Question 31 · EP Specimen

1 **Use the formula**

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2} = \frac{-6 \pm \sqrt{28}}{2}$$

2 **Simplify the surd**

$\sqrt{28} = 2\sqrt{7}$, so

$$x = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

✓ **FINAL ANSWER**

$x = -3 \pm \sqrt{7}$

EP SPECIMEN

Question 32

5 marks | Quadratic Formula

32 A rectangle has length $(x + 3)$ cm and width $(x - 1)$ cm.
The area of the rectangle is 20 cm^2.

(a) Show that $x^2 + 2x - 23 = 0$

.....

(2)

(b) Work out the length of the rectangle.
Give your answer correct to 3 significant figures.

..... cm

(3)

(Total for Question 32 is 5 marks)

WORKED SOLUTION

Question 32 · EP Specimen

1 **(a) Area = length × width**

$$(x+3)(x-1) = 20 \Rightarrow x^2 + 2x - 3 = 20 \Rightarrow x^2 + 2x - 23 = 0$$

2 **(b) Use the formula**

$$x = \frac{-2 \pm \sqrt{4 + 92}}{2} = \frac{-2 \pm \sqrt{96}}{2} = -1 \pm \sqrt{24}$$

The width $x - 1$ must be positive, so $x = -1 + \sqrt{24} = 3.898\ldots$

3 **(b) Length**

$$x + 3 = 6.898\ldots \approx 6.90 \text{ cm}$$

✓ **FINAL ANSWER**

(b) **6.90** cm

TOPIC

Quadratic Equations

Every past-paper question on this topic, 2020 to 2026.

4MA1/1HR JANUARY 2020

Question 6

3 marks | Quadratic Equations

6 Solve $x^2 - 5x - 36 = 0$
Show clear algebraic working.

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2020

1 Solve quadratic equation

$$x^2 - 5x - 36 = 0$$

2 Factorise

Factorise:

$$(x - 9)(x + 4) = 0$$

3 Solve quadratic equation

So

$$x = 9 \quad \text{or} \quad x = -4$$

FINAL ANSWER

$x = 9$ or $x = -4$.

4MA1/2HR JANUARY 2021

Question 9

3 marks | Quadratic Equations

9 Solve $x^2 - 21x + 20 = 0$
Show your working clearly.

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2021

1 Solve quadratic equation

$$x^2 - 21x + 20 = 0$$

2 Factorise

Factorise:

$$(x - 20)(x - 1) = 0$$

3 Solve quadratic equation

So

$$x = 20 \quad \text{or} \quad x = 1$$

FINAL ANSWER

$x = 20$ or $x = 1$.

4WM1/01 MAY 2025

Question 15

3 marks | Quadratic Equations

15 Solve the equation $5x^2 + 9x - 17 = 0$
Show your working clearly.
Give your solutions correct to 3 significant figures.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4WM1/01 May 2025

1 **Solve quadratic equation**

$$5x^2 + 9x - 17 = 0$$

2 **Use quadratic formula**

Use the quadratic formula with $a = 5$, $b = 9$, $c = -17$.

$$x = \frac{-9 \pm \sqrt{9^2 - 4(5)(-17)}}{2(5)}$$

$$x = \frac{-9 \pm \sqrt{421}}{10}$$

$$x = 1.152\ldots \quad \text{or} \quad x = -2.952\ldots$$

3 **Solve quadratic equation**

To 3 significant figures,

$$x = 1.15 \quad \text{or} \quad x = -2.95$$

✓ **FINAL ANSWER**

$x = 1.15$ or $x = -2.95$.

TOPIC

Linear Graphs

Every past-paper question on this topic, 2020 to 2026.

Question 9

7 marks | Linear Graphs

9 (a) Simplify $3a^4b^5 \times 4a^7b^2$

.....

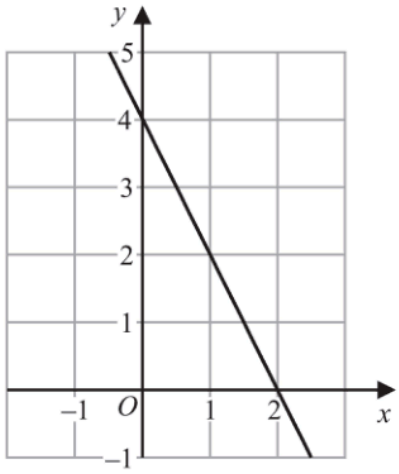
(2)

(b) Factorise fully $14x^2y^4 + 21x^3y^2$

.....

(2)

The diagram shows a straight line drawn on a grid.



(c) Find an equation of the line.

.....

(2)

A different straight line has equation $y = 3x - 5$

(d) Write down the coordinates of the point at which the line crosses the y-axis.

(....., ...)

(1)

(Total for Question 9 is 7 marks)

1 **Simplify the product**

$$3a^4b^5 \times 4a^7b^2 = 12a^{11}b^7$$

2 **Factorise fully**

$$14x^2y^4 + 21x^3y^2 = 7x^2y^2(2y^2 + 3x)$$

3 **Find the gradient**

The line passes through $(0, 4)$ and $(2, 0)$.

$$m = \frac{0 - 4}{2 - 0} = -2$$

4 **Write the equation**

The y-intercept is 4, so

$$y = -2x + 4$$

5 **Read the y-axis crossing**

For $y = 3x - 5$, put $x = 0$:

$$y = -5$$

So the crossing point is $(0, -5)$.

✓ **FINAL ANSWER**

$12a^{11}b^7$, $7x^2y^2(2y^2 + 3x)$, $y = -2x + 4$, and $(0, -5)$.

Question 23

6 marks | Linear Graphs

23 $ABCD$ is a kite.

$AB = AD$ and $CB = CD$

The point B has coordinates $(k, 1)$ where k is a negative constant.
The point D has coordinates $(8, 7)$

The straight line **L** passes through the points B and D and has a gradient of $\frac{3}{5}$

Find an equation of AC
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show your working clearly.

.....

(Total for Question 23 is 6 marks)

1 Find k from the gradient

The gradient of BD is

$$\frac{7-1}{8-k}=\frac{3}{5}$$

$$\frac{6}{8-k}=\frac{3}{5}$$

$$30=24-3k$$

$$k=-2$$

2 Find the midpoint of BD

So $B=(-2,1)$ and $D=(8,7)$.

$$\text{midpoint of } BD=\left(\frac{-2+8}{2},\frac{1+7}{2}\right)=(3,4)$$

3 Use the perpendicular gradient

In this kite, AC is the perpendicular bisector of BD.

The gradient of BD is $\frac{3}{5}$, so the gradient of AC is

$$-\frac{5}{3}$$

4 Form the equation

Using point $(3,4)$:

$$y-4=-\frac{5}{3}(x-3)$$

$$3y-12=-5x+15$$

$$5x+3y=27$$

FINAL ANSWER

$5x+3y=27$.

4MA1/1H JANUARY 2020

Question 1

4 marks | Linear Graphs

1 The point A has coordinates $(5, -4)$
The point B has coordinates $(13, 1)$

(a) Work out the coordinates of the midpoint of AB.

(..... , ...)

(2)

Line **L** has equation $y = 2 - 3x$

(b) Write down the gradient of line **L**.

.....

(1)

Line **L** has equation $y = 2 - 3x$

(c) Does the point with coordinates $(100, -302)$ lie on line **L**?
You must give a reason for your answer.

.....

(1)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2020

1 **Find the midpoint**

(a) The midpoint of $A(5, -4)$ and $B(13, 1)$ is

$$\left(\frac{5+13}{2}, \frac{-4+1}{2}\right)$$

$$= (9, -1.5)$$

2 **(b)**

(b)

$$y = 2 - 3x$$

3 **The gradient is**

The gradient is -3.

4 **Calculate value**

(c) For $x = 100$,

$$y = 2 - 3(100) = -298$$

5 **Calculate value**

The point has $y = -302$, not -298, so it does not lie on the line.

FINAL ANSWER

$(9, -1.5)$, gradient -3, and no.

4MA1/2H JANUARY 2020

Question 19

5 marks | Linear Graphs

19 (a) Write down an equation of a line that is parallel to the line with equation $y = 7 - 4x$

.....

(1)

The line **L** passes through the points with coordinates (–3, 1) and (2, –2)

(b) Find an equation of the line that is perpendicular to **L** and passes through the point with coordinates (–6, 4)
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

.....

(4)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2020

1 **(a)**

(a)

$$y = 7 - 4x$$

2 **A parallel line has the**

A parallel line has the same gradient, -4. One possible answer is

$$y = -4x$$

3 **(b) The gradient of line**

(b) The gradient of line L through $(-3, 1)$ and $(2, -2)$ is

$$\frac{-2-1}{2-(-3)} = -\frac{3}{5}$$

4 **A perpendicular line has gradient**

A perpendicular line has gradient $\frac{5}{3}$.

5 **Evaluate fraction**

Through $(-6, 4)$:

$$y - 4 = \frac{5}{3}(x + 6)$$

6 **Multiply by 3**

Multiply by 3:

$$3y - 12 = 5x + 30$$

$$5x - 3y + 42 = 0$$

✓ **FINAL ANSWER**

For example $y = -4x$, and $5x - 3y + 42 = 0$.

4MA1/1H JANUARY 2020

Question 22

6 marks | Linear Graphs

22 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.

$j =$ .....

$k =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2020

1 Find the midpoint

Since M is the midpoint of JK,

$$M = \left(\frac{j+6}{2}, \frac{15+k}{2}\right)$$

2 Find the gradient

The gradient of HM is 2, and $H = (-4, 1)$.

$$\frac{\frac{15+k}{2} - 1}{\frac{j+6}{2} + 4} = 2$$

$$\frac{k+13}{j+14} = 2$$

$$k = 2j + 15$$

3 **Simplify surd**

Also,

$$JK = \sqrt{80}$$

4 **Calculate value**

So

$$(6 - j)^2 + (k - 15)^2 = 80$$

5 **Substitute**

Substitute $k = 2j + 15$:

$$(6 - j)^2 + (2j)^2 = 80$$

$$j^2 - 12j + 36 + 4j^2 = 80$$

$$5j^2 - 12j - 44 = 0$$

$$(5j + 10)(j - 4.4) = 0$$

6 **Solve inequality**

Since $j < 0$,

$$j = -2$$

7 **Calculate value**

Then

$$k = 2(-2) + 15 = 11$$

✓ **FINAL ANSWER**

$j = -2,\ k = 11$

4MA1/2H JANUARY 2020

Question 22

5 marks | Linear Graphs

22 The line with equation $y = x + 2$ intersects the curve with equation $x^2 + y^2 - 2y = 24$ at the points A and B.

Find the coordinates of A and B.
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2020

1 The line is

The line is

$$y = x + 2$$

2 Substitute this into the circle

Substitute this into the circle equation:

$$x^2 + y^2 - 2y = 24$$

$$x^2 + (x+2)^2 - 2(x+2) = 24$$

$$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$$

$$2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

3 **Calculate value**

So

$$x = -4 \quad \text{or} \quad x = 3$$

---

4 **Calculate value**

Using $y = x + 2$:

$$x = -4 \Rightarrow y = -2$$

$$x = 3 \Rightarrow y = 5$$

✓ **FINAL ANSWER**

$(-4, -2)$ and $(3, 5)$

4MA1/2HR JANUARY 2020

Question 24

5 marks | Linear Graphs

24 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the x-axis at the point A.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR January 2020

1 Evaluate fraction

For L_1,

$$5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

2 Find the gradient

So the gradient of L_1 is $-\frac{1}{2}$.

3 Find the gradient

Since L_2 is perpendicular to L_1, the gradient of L_2 is 2.

4 Calculate value

Using the point $(8, 6)$:

$$y - 6 = 2(x - 8)$$

$$y = 2x - 10$$

5 **Calculate value**

At A, the line crosses the x-axis, so $y = 0$:

$$0 = 2x - 10$$

$$x = 5$$

---

6 **Calculate value**

So $A = (5, 0)$.

---

7 **Calculate value**

At B, $x = -3$:

$$y = 2(-3) - 10 = -16$$

---

8 **Calculate value**

So $B = (-3, -16)$.

---

9 **Triangle has base and perpendicular**

Triangle AOB has base $OA = 5$ and perpendicular height 16.

$$\text{Area} = \frac{1}{2} \times 5 \times 16 = 40$$

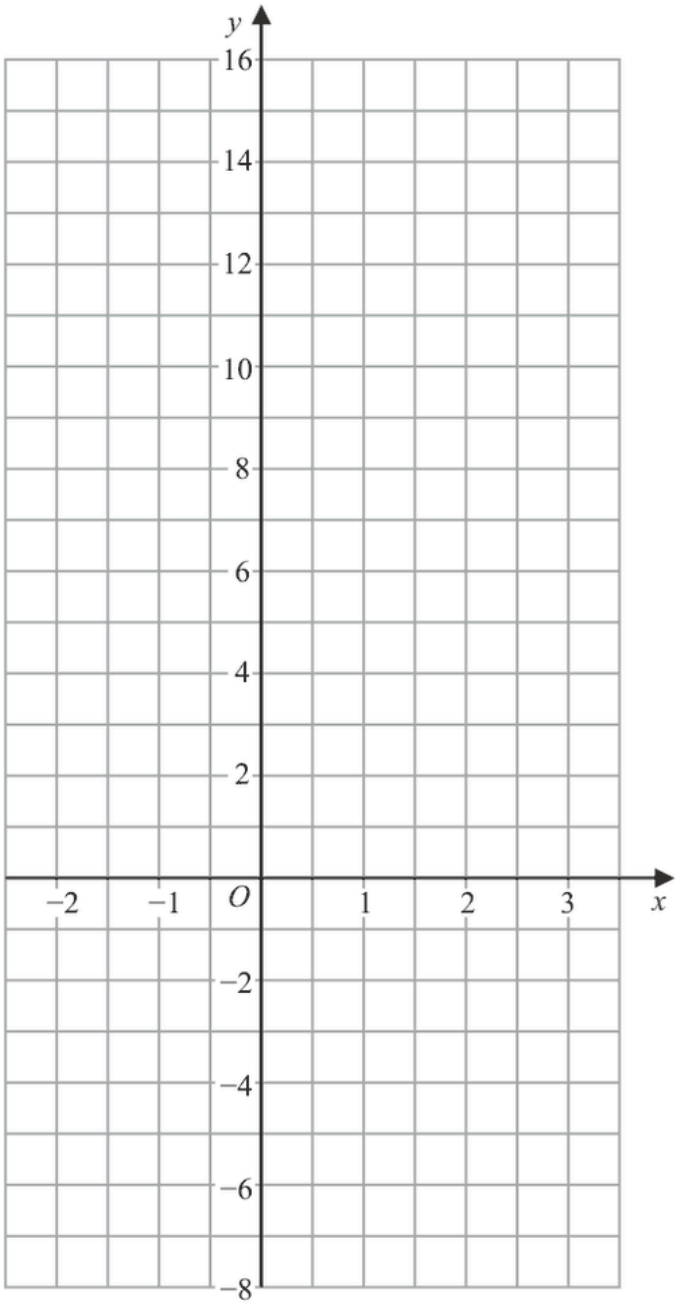
✓ **FINAL ANSWER**

40

Question 3

3 marks | Linear Graphs

3 On the grid, draw the graph of $y = 7 - 4x$ for values of x from -2 to 3



(Total for Question 3 is 3 marks)

1 Read the graph

$$y = 7 - 4x$$

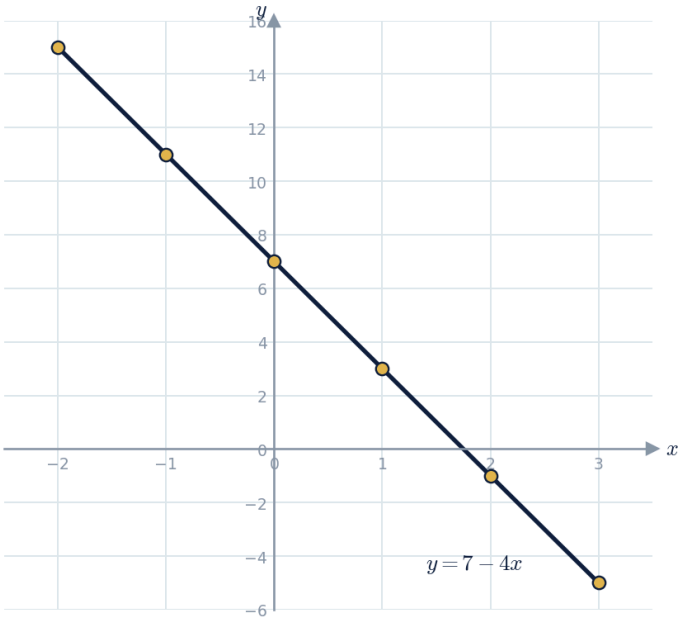
2 Useful points are

Useful points are:

$$(-2, 15), \quad (0, 7), \quad (1, 3), \quad (2, -1), \quad (3, -5)$$

3 Plot these points and join

Plot these points and join them with a straight line.



FINAL ANSWER

The graph is the straight line $y = 7 - 4x$.

4MA1/1HR MAY/NOVEMBER 2020

Question 6

2 marks | Linear Graphs

6 Point A has coordinates $(-3, 11)$
Point B has coordinates $(47, b)$
The midpoint of AB has coordinates $(a, -19)$

Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR May/November 2020

1 Calculate value

$$A = (3, 11), \qquad B = (47, b)$$

2 The midpoint is

The midpoint is $(a, 19)$.

3 Evaluate fraction

For the x-coordinate:

$$a = \frac{3 + 47}{2} = 25$$

4 Evaluate fraction

For the y-coordinate:

$$19 = \frac{11 + b}{2}$$

$$38 = 11 + b$$

$$b = 27$$

FINAL ANSWER

$a = 25, b = 27$.

4MA1/1H MAY/NOVEMBER 2020

Question 22

4 marks | Linear Graphs

22 $ABCD$ is a rhombus.

The diagonals, AC and BD, intersect at the point M.
The coordinates of M are $(6, -11)$

The points A and C both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through B and D intersects the y-axis.

(..... , ...)

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May/November 2020

1 Find the gradient

In a rhombus, the diagonals are perpendicular and bisect each other.

2 The diagonal lies on

The diagonal AC lies on

$$2y + 7x = 20$$

$$2y = -7x + 20$$

$$y = -\frac{7}{2}x + 10$$

3 Find the gradient

So the gradient of AC is

$$-\frac{7}{2}$$

4 Find the gradient

Therefore the gradient of BD is

$$\frac{2}{7}$$

5 **Calculate value**

The line BD passes through

$$M = (6, -11)$$

6 **Evaluate fraction**

So

$$y + 11 = \frac{2}{7}(x - 6)$$

7 **To find where this line**

To find where this line intersects the y-axis, put $x = 0$:

$$y + 11 = \frac{2}{7}(0 - 6)$$

$$y + 11 = -\frac{12}{7}$$

$$y = -11 - \frac{12}{7}$$

$$y = -\frac{89}{7}$$

✓ **FINAL ANSWER**

$$\left(0, -\frac{89}{7}\right)$$

4MA1/1H JANUARY 2021

Question 13

3 marks | Linear Graphs

13 Point A has coordinates (5, 8)
Point B has coordinates (9, –4)

(a) Work out the gradient of AB.

.....

(2)

The straight line **L** has equation $y = -4x + 5$

(b) Write down the gradient of a straight line that is perpendicular to **L**.

.....

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2021

1 **Find the gradient**

(a) The gradient of AB, where $A(5, 8)$ and $B(9, -4)$, is

$$\frac{-4-8}{9-5} = \frac{-12}{4} = -3$$

2 **Find the gradient**

(b) A line perpendicular to a line with gradient -3 has gradient

$$\frac{1}{3}$$

FINAL ANSWER

Gradient of $AB = -3$, perpendicular gradient $\frac{1}{3}$.

4MA1/2H JANUARY 2021

Question 18

7 marks | Linear Graphs

18 A rectangle $ABCD$ is to be drawn on a centimetre grid such that

A has coordinates $(-4, -2)$
B has coordinates $(1, 10)$
C has coordinates $(19, a)$
D has coordinates (b, c)

(a) Work out the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(4)

(b) Calculate the perimeter, in centimetres, of rectangle $ABCD$.

..... cm

(3)

(Total for Question 18 is 7 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H January 2021

1 Calculate value

$$A = (-4, -2), \quad B = (1, 10), \quad C = (19, a), \quad D = (b, c)$$

2 The vector is

The vector AB is

$$(1 - (-4), 10 - (-2)) = (5, 12)$$

3 The vector is

The vector BC is

$$(19 - 1, a - 10) = (18, a - 10)$$

4 **Adjacent sides of a rectangle**

Adjacent sides of a rectangle are perpendicular, so their dot product is zero:

$$5(18) + 12(a - 10) = 0$$

$$90 + 12a - 120 = 0$$

$$a = \frac{5}{2}$$

---

5 **Calculate value**

Now

$$BC = (18, -7.5)$$

---

6 **Calculate value**

So

$$D = A + BC = (-4, -2) + (18, -7.5) = (14, -9.5)$$

$$b = 14, \qquad c = -9.5$$

---

7 **The side lengths are**

The side lengths are

$$AB = \sqrt{5^2 + 12^2} = 13$$

$$BC = \sqrt{18^2 + (-7.5)^2} = 19.5$$

---

8 **Perimeter**

Perimeter:

$$2(13 + 19.5) = 65$$

✓ **FINAL ANSWER**

$a = \frac{5}{2}$, $b = 14$, $c = -9.5$, and perimeter $65\ \text{cm}$.

4MA1/1HR JANUARY 2021

Question 22

5 marks | Linear Graphs

22 ABC is an isosceles triangle with $AB = AC$.

B is the point with coordinates $(-1, 5)$
C is the point with coordinates $(2, 10)$
M is the midpoint of BC.

Find an equation of the line through the points A and M.
Give your answer in the form $py + qx = r$ where p, q and r are integers.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2021

1 Find the gradient

Since $AB = AC$, point A lies on the perpendicular bisector of BC.

2 The midpoint of is

The midpoint of BC is

$$M = \left(\frac{-1+2}{2}, \frac{5+10}{2}\right)$$

$$M = \left(\frac{1}{2}, \frac{15}{2}\right)$$

3 The gradient of is

The gradient of BC is

$$\frac{10-5}{2-(-1)} = \frac{5}{3}$$

4 Find the gradient

So the gradient of the perpendicular bisector is

$$-\frac{3}{5}$$

5 **Find line equation**

Use the line through M:

$$y - \frac{15}{2} = -\frac{3}{5}\left(x - \frac{1}{2}\right)$$

6 **Multiply by 10**

Multiply by 10:

$$10y - 75 = -6x + 3$$

$$10y + 6x = 78$$

7 **Divide by 2**

Divide by 2:

$$5y + 3x = 39$$

✓ **FINAL ANSWER**

$5y + 3x = 39$

Question 17

4 marks | Linear Graphs

17 The straight line **L** passes through the points (4, −1) and (6, 4)

The straight line **M** is perpendicular to **L** and intersects the y-axis at the point (0, 8)

Find the coordinates of the point where **M** intersects the x-axis.

(..... , ...)

(Total for Question 17 is 4 marks)

1 **Use Coordinate geometry tag correct**

Line L passes through $(4, -1)$ and $(6, 4)$.

2 **Its gradient is**

Its gradient is

$$\frac{4-(-1)}{6-4} = \frac{5}{2}$$

3 **Find the gradient**

Line M is perpendicular to L, so its gradient is

$$-\frac{2}{5}$$

4 **Line intersects the y axis**

Line M intersects the y-axis at $(0, 8)$, so

$$y = -\frac{2}{5}x + 8$$

5 **Evaluate fraction**

At the x-axis, $y = 0$:

$$0 = -\frac{2}{5}x + 8$$

$$\frac{2}{5}x = 8$$

$$x = 20$$

✓ **FINAL ANSWER**

$(20, 0)$.

4MA1/2H NOVEMBER 2021

Question 19

6 marks | Linear Graphs

19 The straight line **L** has equation $x - y = 3$
The curve **C** has equation $3x^2 - y^2 + xy = 9$

L and **C** intersect at the points P and Q.

Find the coordinates of the midpoint of PQ.
Show clear algebraic working.

(..... , ...)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2021

1 Line is

Line L is

$$x - y = 3$$

2 Calculate value

So

$$y = x - 3$$

3 The curve is

The curve is

$$3x - y + y^2 = 9$$

4 Substitute

Substitute $y = x - 3$:

$$3x - (x - 3) + (x - 3)^2 = 9$$

$$2x + 3 + x^2 - 6x + 9 = 9$$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

5 **Calculate value**

So $x = 1$ or $x = 3$.

6 **Calculate value**

If $x = 1$, then $y = -2$. If $x = 3$, then $y = 0$.

7 **The midpoint of and is**

The midpoint of $(1, -2)$ and $(3, 0)$ is

$$\left(\frac{1+3}{2}, \frac{-2+0}{2}\right) = (2, -1)$$

FINAL ANSWER

$(2, -1)$.

4MA1/1H NOVEMBER 2021

Question 20

7 marks | Linear Graphs

20 The straight line **L** passes through point A $(-6, 2)$ and point B $(5, 3)$
The straight line **M** is perpendicular to **L** and passes through the midpoint of A and B.
The line **M** intersects the line $x = -1$ at point C.

Calculate the area of triangle ABC.

(Total for Question 20 is 7 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2021

1 **Calculate value**

$$A = (-6, 2), \qquad B = (5, 3)$$

2 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

3 **The gradient of is**

The gradient of AB is

$$\frac{3-2}{5-(-6)} = \frac{1}{11}$$

4 **Find the gradient**

So the perpendicular gradient is

$$-11$$

5 **Evaluate fraction**

Line M passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$, so

$$y - \frac{5}{2} = -11\left(x + \frac{1}{2}\right)$$

6 **Evaluate fraction**

At point C, $x = -1$:

$$y - \frac{5}{2} = -11\left(-\frac{1}{2}\right)$$

$$y - \frac{5}{2} = \frac{11}{2}$$

$$y = 8$$

---

7 **Calculate value**

So

$$C = (-1, 8)$$

---

8 **Use vectors**

Now use vectors from A:

$$\vec{AB} = (11, 1), \qquad \vec{AC} = (5, 6)$$

---

9 **Area of triangle is**

Area of triangle ABC is

$$\frac{1}{2}|11(6) - 1(5)|$$

$$= \frac{1}{2}(61)$$

$$= 30.5$$

✓ **FINAL ANSWER**

30.5

4MA1/1H JANUARY 2022

Question 10

3 marks | Linear Graphs

10 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

.....

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

(..... , ...)

(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2022

1 **The line is**

The line is

$$2y + 7x = 10$$

$$2y = -7x + 10$$

$$y = -\frac{7}{2}x + 5$$

2 **Find the gradient**

So the gradient is

$$-\frac{7}{2}$$

3 **Evaluate fraction**

For the x-axis, $y = 0$:

$$2(0) + 7x = 10$$

$$x = \frac{10}{7}$$

FINAL ANSWER

Gradient $-\frac{7}{2}$, and x-axis intersection $\left(\frac{10}{7}, 0\right)$.

4MA1/2HR JANUARY 2022

Question 18

5 marks | Linear Graphs

18 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...) and (..... , ...)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2022

1 The line is

The line is

$$2y = x + 1$$

2 Calculate value

So

$$x = 2y - 1$$

3 Substitute into the curve

Substitute into the curve

$$3y^2 + 7y + 16 = x^2 - x$$

$$3y^2 + 7y + 16 = (2y - 1)^2 - (2y - 1)$$

$$3y^2 + 7y + 16 = 4y^2 - 6y + 2$$

$$y^2 - 13y - 14 = 0$$

$$(y - 14)(y + 1) = 0$$

4 **Calculate value**

So

$$y = 14 \quad \text{or} \quad y = -1$$

5 **Calculate value**

If $y = 14$, then $x = 27$.

6 **Calculate value**

If $y = -1$, then $x = -3$.

✓ **FINAL ANSWER**

$(27, 14)$ and $(-3, -1)$.

4MA1/2HR JANUARY 2022

Question 22

5 marks | Linear Graphs

22 ABC is a triangle in which angle $ABC = 90°$

p and q are integers such that

the coordinates of A are $(p, 10)$
the coordinates of B are $(-1, -5)$
the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$p =$ .....

$q =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2022

1 The gradient of is

The gradient of AC is $-\frac{6}{7}$.

$$\frac{q-10}{8-p} = -\frac{6}{7}$$

$$7(q-10) = -6(8-p)$$

$$7q - 70 = -48 + 6p$$

$$6p - 7q = -22$$

2 **Find the gradient**

Since $\angle ABC = 90°$, lines AB and BC are perpendicular.

$$\text{gradient of } AB = \frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$$

$$\text{gradient of } BC = \frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$$

---

3 **The product of perpendicular gradients**

The product of perpendicular gradients is -1:

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$5(q+5) = -3(p+1)$$

$$3p + 5q = -28$$

---

4 **Solve**

Now solve

$$6p - 7q = -22$$

$$3p + 5q = -28$$

---

5 **Double the second equation**

Double the second equation:

$$6p + 10q = -56$$

---

6 **Subtract**

Subtract:

$$17q = -34$$

$$q = -2$$

---

7 **Calculate value**

Then

$$3p + 5(-2) = -28$$

$$3p = -18$$

$$p = -6$$

✓ **FINAL ANSWER**

$p = -6,\ q = -2$

4MA1/1H JUNE 2022

Question 10

4 marks | Linear Graphs

10 Two circles, C_1 and C_2, are drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis.

The centre of circle C_1 is at the point with coordinates $(-1, 3)$ and the radius of C_1 is 13 cm.

The centre of circle C_2 is at the point with coordinates $(7, 18)$ and the radius of C_2 is 6 cm.

(a) Work out the distance between the centre of C_1 and the centre of C_2

..... cm

(3)

(b) Explain why circle C_1 intersects circle C_2

.....

.....

(1)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H June 2022

1 The centres are

The centres are

$$C_1 = (-1, 3), \qquad C_2 = (7, 18)$$

2 The distance between the centres

The distance between the centres is

$$\sqrt{(7-(-1))^2 + (18-3)^2}$$

$$= \sqrt{8^2 + 15^2} = \sqrt{289} = 17$$

3 Calculate value

The radii are 3 cm and 15 cm.

$$3 + 15 = 18$$

4 **Use Coordinate geometry tag correct**

Since the distance between the centres is 17 cm, which is less than the sum of the radii, the circles intersect.

✓ **FINAL ANSWER**

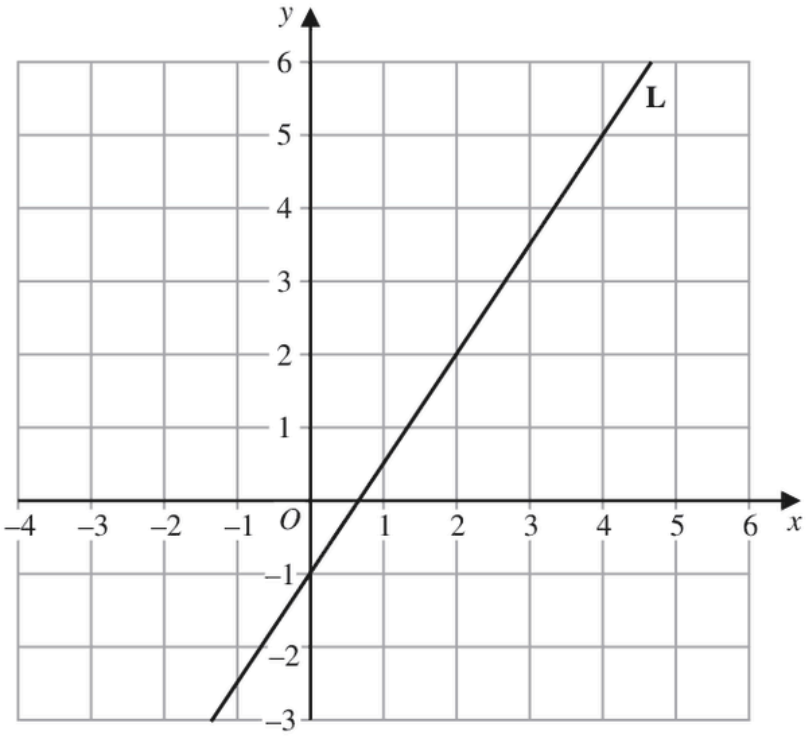
$17\ \text{cm}$, and the circles intersect because $17 < 18$.

4MA1/1HR JUNE 2022

Question 10

3 marks | Linear Graphs

10 Line **L** is drawn on the grid.



Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2022

1 **Solve equation**

From the graph, the line crosses the y-axis at -1, so

$$c = -1$$

2 **Find the gradient**

Using two points on the line, for example $(0, -1)$ and $(2, 2)$, the gradient is

$$\frac{2-(-1)}{2-0} = \frac{3}{2}$$

3 **Solve equation**

So the equation is

$$y = \frac{3}{2}x - 1$$

✓ **FINAL ANSWER**

$y = \frac{3}{2}x - 1$.

4MA1/2HR JUNE 2022

Question 20

4 marks | Linear Graphs

20 The centre O of a circle has coordinates $(4, 7)$

The point A, on the circle, has coordinates $(6, 11)$ and AOP is a diameter of the circle.

Find an equation of the tangent to the circle at the point P

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2022

1 The centre of the circle

The centre of the circle is

$$O = (4, 7)$$

2 The point is

The point A is

$$A = (6, 11)$$

3 Find the midpoint

Since AOP is a diameter, O is the midpoint of AP.

4 Let

Let $P = (x, y)$.

$$\left(\frac{6+x}{2}, \frac{11+y}{2}\right) = (4, 7)$$

$$6 + x = 8, \qquad 11 + y = 14$$

$$P = (2, 3)$$

5 The gradient of is

The gradient of OP is

$$\frac{3-7}{2-4} = 2$$

6 **Use circle theorem**

The tangent is perpendicular to the radius, so its gradient is

$$-\frac{1}{2}$$

---

7 **Evaluate fraction**

Using point $P = (2, 3)$,

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 4$$

✓ **FINAL ANSWER**

$$y = -\frac{1}{2}x + 4$$

4MA1/2H JUNE 2022

Question 22

7 marks | Linear Graphs

22 $ABCD$ is a kite, with diagonals AC and BD, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point M with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the x coordinate of B is positive,

find the coordinates of the points B and D.

(..... , ...)

(..... , ...)

(Total for Question 22 is 7 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2022

1 The midpoint of is

The midpoint of BD is $M = (0, 2)$.

2 Find the gradient

Since $A = (-3, 4)$, the gradient of AM is

$$\frac{2-4}{0-(-3)} = -\frac{2}{3}$$

3 Find the gradient

For the kite, diagonal AC is perpendicular to BD, so the gradient of BD is

$$\frac{3}{2}$$

4 Use direction vector along

Use direction vector $(2, 3)$ along BD.

5 **Let**

Let

$$B = (2t, 2 + 3t)$$

6 **Calculate value**

and

$$D = (-2t, 2 - 3t)$$

7 **Calculate value**

Since $AB = 6.5$,

$$(2t - (-3))^2 + (2 + 3t - 4)^2 = 6.5^2$$

$$(2t + 3)^2 + (3t - 2)^2 = 42.25$$

$$4t^2 + 12t + 9 + 9t^2 - 12t + 4 = 42.25$$

$$13t^2 + 13 = 42.25$$

$$13t^2 = 29.25$$

$$t^2 = 2.25$$

$$t = 1.5$$

8 **Use**

The x-coordinate of B is positive, so use $t = 1.5$.

$$B = (3, 6.5)$$

$$D = (-3, -2.5)$$

✓ **FINAL ANSWER**

$B = (3, 6.5),\ D = (-3, -2.5)$

4MA1/1H MAY 2023

Question 13

3 marks | Linear Graphs

13 A is the point with coordinates $(-5, 12)$

B is the point with coordinates $(19, -48)$

Find an equation of the straight line that passes through the points A and B

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2023

1 Calculate value

$$A = (-5, 12), \qquad B = (19, -48)$$

2 The gradient is

The gradient is

$$\frac{-48 - 12}{19 - (-5)} = \frac{-60}{24} = -\frac{5}{2}$$

3 Evaluate fraction

Using point $(-5, 12)$:

$$y - 12 = -\frac{5}{2}(x + 5)$$

4 Multiply by 2

Multiply by 2:

$$2y - 24 = -5x - 25$$

$$5x + 2y + 1 = 0$$

FINAL ANSWER

$5x + 2y + 1 = 0$.

4MA1/2HR JUNE 2023

Question 21

5 marks | Linear Graphs

21 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2023

1 **The line is**

The line is

$$y - 2x = 1$$

2 **Calculate value**

So

$$y = 2x + 1$$

3 **Calculate value**

Substitute into

$$y^2 + xy = 7$$

$$(2x + 1)^2 + x(2x + 1) = 7$$

$$4x^2 + 4x + 1 + 2x^2 + x = 7$$

$$6x^2 + 5x - 6 = 0$$

4 **Factorise**

Factorise:

$$(3x - 2)(2x + 3) = 0$$

5 **Evaluate fraction**

So

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

---

6 **Evaluate fraction**

Using $y = 2x + 1$:

$$x = \frac{2}{3} \Rightarrow y = \frac{7}{3}$$

$$x = -\frac{3}{2} \Rightarrow y = -2$$

✓ **FINAL ANSWER**

$\left(\frac{2}{3}, \frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -2\right)$

4MA1/2H JUNE 2023

Question 24

6 marks | Linear Graphs

24 $ABCD$ is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of D

(..... , ...)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2023

1 Calculate value

In the kite, $AB = AD$ and $CB = CD$, so AC is the line of symmetry.

2 Apply transformation

Therefore D is the reflection of B in the line AC.

$$A = (-2, 10), \qquad B = \left(-\frac{27}{5}, 4\right), \qquad C = (4, -5)$$

3 A direction vector for is

A direction vector for AC is

$$\begin{pmatrix} 4 - (-2) \\ -5 - 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$$

4 Use the simpler direction vector

Use the simpler direction vector

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

5 **Find the gradient**

Let H be the foot of the perpendicular from B to AC.

$$\vec{AB} = \begin{pmatrix} -\frac{17}{5} \\ -6 \end{pmatrix}$$

6 **The fraction along to is**

The fraction along AC to H is

$$\frac{\vec{AB} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}}{2^2 + (-5)^2} = \frac{-\frac{34}{5} + 30}{29}$$

$$= \frac{\frac{116}{5}}{29} = \frac{4}{5}$$

7 **Use matrices**

So

$$H = A + \frac{4}{5}\begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$H = \left(-2 + \frac{8}{5}, 10 - 4\right)$$

$$H = \left(-\frac{2}{5}, 6\right)$$

8 **Find the midpoint**

Since H is the midpoint of BD,

$$D = 2H - B$$

$$D = \left(-\frac{4}{5}, 12\right) - \left(-\frac{27}{5}, 4\right)$$

$$D = \left(\frac{23}{5}, 8\right)$$

✓ **FINAL ANSWER**

$$D = \left(\frac{23}{5}, 8\right)$$

4MA1/2HR JUNE 2023

Question 25

6 marks | Linear Graphs

25 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

(..... , ...)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2023

1 The perpendicular bisector is

The perpendicular bisector is

$$y - 2x = 7$$

2 Calculate value

so

$$y = 2x + 7$$

3 The gradient of the perpendicular

The gradient of the perpendicular bisector is 2, so the gradient of AB is

$$-\frac{1}{2}$$

4 **Evaluate fraction**

Since $A = (j, 7)$ and $B = (6, k)$,

$$\frac{k-7}{6-j} = -\frac{1}{2}$$

$$2(k-7) = -(6-j)$$

$$j = 2k - 8$$

5 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$$

6 **Substitute**

Substitute $j = 2k - 8$:

$$\left(k-1, \frac{k+7}{2}\right)$$

7 **Find the midpoint**

The midpoint lies on $y = 2x + 7$, so

$$\frac{k+7}{2} = 2(k-1) + 7$$

$$k + 7 = 4k + 10$$

$$k = -1$$

8 **Calculate value**

Then

$$j = 2(-1) - 8 = -10$$

9 **Find the midpoint**

Therefore the midpoint is

$$\left(\frac{-10+6}{2}, \frac{7+(-1)}{2}\right) = (-2, 3)$$

FINAL ANSWER

$$\boxed{(-2, 3)}$$

4MA1/2H NOVEMBER 2023

Question 9

2 marks | Linear Graphs

9 A straight line passes through the points with coordinates (0, –3) and (2, 0)

Find an equation of the line.

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2023

1 Solve equation

The line passes through $(0, -3)$ and $(2, 0)$.

2 The gradient is

The gradient is

$$\frac{0-(-3)}{2-0} = \frac{3}{2}$$

3 Solve equation

The y-intercept is -3, so

$$y = \frac{3}{2}x - 3$$

FINAL ANSWER

$y = \frac{3}{2}x - 3$.

4MA1/2H NOVEMBER 2023

Question 21

5 marks | Linear Graphs

21 The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2023

1 The line is

The line is

$$x + 2y = 5$$

2 Calculate value

So

$$x = 5 - 2y$$

3 Calculate value

Substitute into

$$x^2 + 3y^2 = 13$$

$$(5 - 2y)^2 + 3y^2 = 13$$

$$25 - 20y + 4y^2 + 3y^2 = 13$$

$$7y^2 - 20y + 12 = 0$$

4 **Factorise**

Factorise:

$$(7y - 6)(y - 2) = 0$$

5 **Evaluate fraction**

So

$$y = \frac{6}{7} \quad \text{or} \quad y = 2$$

6 **Evaluate fraction**

Using $x = 5 - 2y$:

$$y = \frac{6}{7} \Rightarrow x = \frac{23}{7}$$

$$y = 2 \Rightarrow x = 1$$

✓ **FINAL ANSWER**

$\left(\frac{23}{7}, \frac{6}{7}\right)$ and $(1, 2)$

4MA1/1H NOVEMBER 2023

Question 22

6 marks | Linear Graphs

22 [In this question 1 cm = 1 unit on the x-axis and 1 cm = 1 unit on the y-axis]

P is a point on a circle with centre $(0, 0)$
The coordinates of P are $(8, -10)$

The line **L** is the tangent to the circle at the point P
L crosses the x-axis at the point Q and crosses the y-axis at the point R

Work out the length of RQ
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2023

1 The centre of the circle

The centre of the circle is $(0, 0)$, and

$$P = (8, -10)$$

2 The gradient of is

The gradient of OP is

$$\frac{-10}{8} = -\frac{5}{4}$$

3 Find the gradient

So the gradient of the tangent is

$$\frac{4}{5}$$

4 The tangent through is

The tangent through P is

$$y + 10 = \frac{4}{5}(x - 8)$$

$$y = \frac{4}{5}x - \frac{82}{5}$$

5 **Evaluate fraction**

At the x-axis, $y = 0$:

$$0 = \frac{4}{5}x - \frac{82}{5}$$

$$x = \frac{41}{2}$$

---

6 **Evaluate fraction**

So

$$Q = \left(\frac{41}{2}, 0\right)$$

---

7 **Evaluate fraction**

At the y-axis, $x = 0$:

$$R = \left(0, -\frac{82}{5}\right)$$

---

8 **Simplify surd**

Therefore

$$RQ = \sqrt{\left(\frac{41}{2}\right)^2 + \left(\frac{82}{5}\right)^2}$$

$$RQ = 26.252\ldots$$

✓ **FINAL ANSWER**

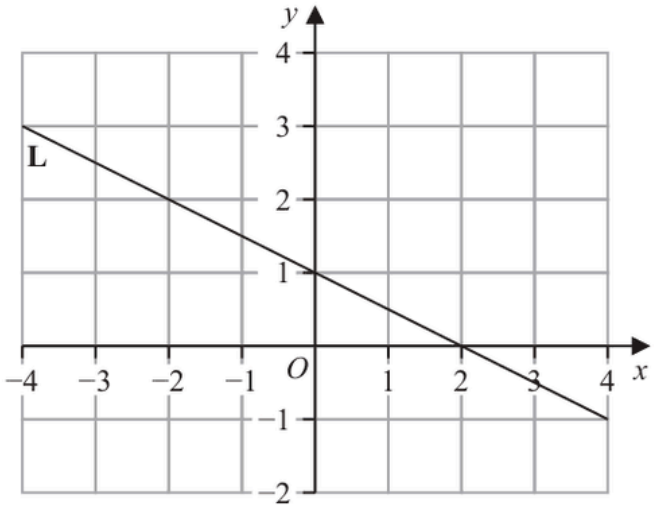
$26.3\ \text{cm}$

4MA1/1HR MAY 2024

Question 9

3 marks | Linear Graphs

9 Line **L** is drawn on the grid.



Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR May 2024

1 Solve equation

From the graph, the line crosses the y-axis at 1, so

$$c = 1$$

2 It also crosses the x

It also crosses the x-axis at 2, so using $(0, 1)$ and $(2, 0)$,

$$m = \frac{0 - 1}{2 - 0} = -\frac{1}{2}$$

3 Solve equation

Therefore

$$y = -\frac{1}{2}x + 1$$

FINAL ANSWER

$y = -\frac{1}{2}x + 1$.

4MA1/2H JUNE 2024

Question 18

2 marks | Linear Graphs

18 The straight line **P** has equation $5y + 2x = 7$

Find the gradient of a straight line that is perpendicular to **P**

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2024

1 The line is

The line is

$$5y + 2x = 7$$

$$5y = -2x + 7$$

$$y = -\frac{2}{5}x + \frac{7}{5}$$

2 Find the gradient

So the gradient of line P is $-\frac{2}{5}$.

3 A line perpendicular to P

A line perpendicular to P has gradient

$$\frac{5}{2}$$

FINAL ANSWER

$\frac{5}{2}$.

4MA1/2HR JUNE 2024

Question 21

4 marks | Linear Graphs

21 $ABCD$ is a square.

The point A has coordinates $(-5, 2)$
The point B has coordinates $(3, 5)$

Find an equation of the line that passes through B and C
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2024

1 Calculate value

$$A = (-5, 2), \qquad B = (3, 5)$$

2 The gradient of is

The gradient of AB is

$$\frac{5-2}{3-(-5)} = \frac{3}{8}$$

3 Find the gradient

Since $ABCD$ is a square, BC is perpendicular to AB.

4 Find the gradient

So the gradient of BC is

$$-\frac{8}{3}$$

5 The line through is

The line through $B = (3, 5)$ is

$$y - 5 = -\frac{8}{3}(x - 3)$$

6 **Multiply by 3**

Multiply by 3:

$$3y - 15 = -8x + 24$$

$$8x + 3y - 39 = 0$$

✓ **FINAL ANSWER**

$8x + 3y - 39 = 0$

4MA1/2H JUNE 2024

Question 22

5 marks | Linear Graphs

22 The straight line **L** has equation $x + y = 5$

The curve **C** has equation $2x^2 + 3y^2 = 210$

Find the coordinates of the points where **L** and **C** intersect.
Show clear algebraic working.

(..... , ...) (..... , ...)

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2024

1 The line is

The line is

$$x + y = 5$$

2 Calculate value

So

$$y = 5 - x$$

3 Substitute into the curve

Substitute into the curve:

$$2x^2 + 3y^2 = 210$$

$$2x^2 + 3(5 - x)^2 = 210$$

$$2x^2 + 3(x^2 - 10x + 25) = 210$$

$$5x^2 - 30x + 75 = 210$$

$$5x^2 - 30x - 135 = 0$$

4 **Divide by 5**

Divide by 5:

$$x^2 - 6x - 27 = 0$$

5 **Factorise**

Factorise:

$$(x - 9)(x + 3) = 0$$

6 **Calculate value**

So

$$x = 9 \quad \text{or} \quad x = -3$$

7 **Calculate value**

Using $y = 5 - x$:

$$x = 9 \Rightarrow y = -4$$

$$x = -3 \Rightarrow y = 8$$

✓ **FINAL ANSWER**

$(9, -4)$ and $(-3, 8)$

4MA1/2H NOVEMBER 2024

Question 18

4 marks | Linear Graphs

18 The straight line **L** is perpendicular to the straight line with equation $2x + y = 9$ and passes through the point with coordinates $(8, 11)$

Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2024

1 The given line is

The given line is

$$2x + y = 9$$

$$y = -2x + 9$$

2 Find the gradient

Its gradient is -2, so a perpendicular line has gradient $\frac{1}{2}$.

3 Find the gradient

Line L passes through $(8, 11)$:

$$y - 11 = \frac{1}{2}(x - 8)$$

$$y = \frac{1}{2}x + 7$$

FINAL ANSWER

$y = \frac{1}{2}x + 7$.

4MA1/2HR JUNE 2025

Question 10

2 marks | Linear Graphs

10 A straight line, **L**, is parallel to the line with equation $y = 2 - 5x$
The line **L** passes through the point $(0, 6)$

Find an equation of the line **L**

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2025

1 The line has gradient

The line

$$y = 2 - 5x$$

has gradient -5.

2 A parallel line has the

A parallel line has the same gradient, so line L has form

$$y = -5x + c$$

3 Find the gradient

Line L passes through $(0, 6)$, so

$$6 = c$$

FINAL ANSWER

$y = -5x + 6$.

4WM1/01R MAY 2025

Question 23

5 marks | Linear Graphs

23 A circle C has centre $(3q, 4q)$ where q is an integer.

The straight line with equation $y = -\frac{5}{4}x + \frac{21}{4}$ is the tangent to C at the point $(p, p+3)$ where p is an integer.

Work out the radius of C
Give your answer in the form $\sqrt{a}$ where a is an integer.
Show your working clearly.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4WM1/01R May 2025

1 The tangent point lies on

The tangent point lies on the tangent line, so

$$p + 3 = -\frac{5}{4}p + \frac{21}{4}$$

$$p = 1$$

2 Find the gradient

The tangent gradient is $-\frac{5}{4}$, so the radius gradient is $\frac{4}{5}$.

3 Use trigonometry

Using centre $(3q, 4q)$ and tangent point $(1, 4)$,

$$\frac{4 - 4q}{1 - 3q} = \frac{4}{5}$$

$$q = 2$$

4

Simplify surd

So the centre is $(6, 8)$. Therefore

$$r = \sqrt{(6-1)^2 + (8-4)^2} = \sqrt{41}$$

FINAL ANSWER

$\sqrt{41}$.

4MA1/2HR JUNE 2025

Question 24

6 marks | Linear Graphs

24 PQ is a straight line drawn on a square grid, with a scale of 1 cm for 1 unit on each axis.

P has coordinates $(-5,\ a)$ and Q has coordinates $(7,\ 3a)$ where $a > 0$

The length of PQ is $4\sqrt{10}$ cm

Find an equation of the perpendicular bisector of PQ
Give your answer in the form $y = mx + c$ where m and c are integers.

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2025

1 Calculate value

$$P = (-5, a), \qquad Q = (7, 3a)$$

2 Simplify surd

The length of PQ is $4\sqrt{10}$, so

$$\sqrt{(7-(-5))^2 + (3a-a)^2} = 4\sqrt{10}$$

$$\sqrt{12^2 + (2a)^2} = 4\sqrt{10}$$

3 Square both sides

Square both sides:

$$144 + 4a^2 = 160$$

$$4a^2 = 16$$

$$a^2 = 4$$

4 Solve inequality

Since $a > 0$,

$$a = 2$$

5 **Calculate value**

So

$$P = (-5, 2), \qquad Q = (7, 6)$$

6 **The midpoint of is**

The midpoint of PQ is

$$\left(\frac{-5+7}{2}, \frac{2+6}{2}\right) = (1, 4)$$

7 **The gradient of is**

The gradient of PQ is

$$\frac{6-2}{7-(-5)} = \frac{4}{12} = \frac{1}{3}$$

8 **Find the gradient**

So the gradient of the perpendicular bisector is

$$-3$$

9 **Calculate value**

Using point $(1, 4)$:

$$y - 4 = -3(x - 1)$$

$$y = -3x + 7$$

✓ **FINAL ANSWER**

$$\boxed{y = -3x + 7}$$

4WM1/01 MAY 2025

Question 24

5 marks | Linear Graphs

24 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4WM1/01 May 2025

1 The diagonals of a square

The diagonals of a square bisect each other and are perpendicular.

2 Find the midpoint

Midpoint of $P(4, 7)$ and $R(8, -5)$:

$$\left(\frac{4+8}{2}, \frac{7-5}{2}\right) = (6, 1)$$

3 Find the gradient

Gradient of PR:

$$\frac{-5-7}{8-4} = -3$$

4 Find the gradient

So the perpendicular gradient is $\frac{1}{3}$.

5 **The line through and passes**

The line through Q and S passes through $(6, 1)$, so

$$y - 1 = \frac{1}{3}(x - 6)$$

$$3y - 3 = x - 6$$

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/1H JUNE 2025

Question 25

5 marks | Linear Graphs

25 $PQRS$ is a square.

PR is a diagonal of the square.

P is the point with coordinates $(4, 7)$
R is the point with coordinates $(8, -5)$

Find an equation of the straight line that passes through the points Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H June 2025

1 Calculate value

$$P = (4, 7), \qquad R = (8, -5)$$

2 The gradient of diagonal is

The gradient of diagonal PR is

$$\frac{-5 - 7}{8 - 4} = -3$$

3 The other diagonal is perpendicular

The other diagonal QS is perpendicular to PR, so its gradient is

$$\frac{1}{3}$$

4 The diagonals of a square

The diagonals of a square bisect each other.

5 The midpoint of is

The midpoint of PR is

$$\left(\frac{4 + 8}{2}, \frac{7 + (-5)}{2}\right) = (6, 1)$$

6 **Find the gradient**

So QS passes through $(6, 1)$ with gradient $\frac{1}{3}$.

$$y - 1 = \frac{1}{3}(x - 6)$$

$$y = \frac{1}{3}x - 1$$

---

7 **Multiply by 3**

Multiply by 3:

$$3y = x - 3$$

✓ **FINAL ANSWER**

$3y = x - 3$

4MA1/2H NOVEMBER 2025

Question 1

2 marks | Linear Graphs

1 The point P has coordinates $(3, 4)$
The point Q has coordinates $(9, 16)$

M is the midpoint of the line PQ

Find the coordinates of M

(..... , ...)

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2025

1 Calculate value

$$P = (-3, 0), \qquad Q = (9, 16)$$

2 The midpoint is

The midpoint is

$$\left(\frac{-3+9}{2}, \frac{0+16}{2}\right)$$

$$= (3, 8)$$

FINAL ANSWER

$(3, 8)$.

4MA1/1H NOVEMBER 2025

Question 20

3 marks | Linear Graphs

20 The straight line **L** has equation $y = 4x + 7$

The straight line **M** is perpendicular to **L** and passes through the point with coordinates (8, 1)

Find an equation for **M**
Give your answer in the form $y = mx + c$

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2025

1 Line has equation

Line L has equation

$$y = 4x + 7$$

2 Find the gradient

So its gradient is

$$4$$

3 Find the gradient

Line M is perpendicular to L, so its gradient is

$$-\frac{1}{4}$$

4 Evaluate fraction

Line M passes through $(8, 1)$.

$$y - 1 = -\frac{1}{4}(x - 8)$$

$$y - 1 = -\frac{1}{4}x + 2$$

$$y = -\frac{1}{4}x + 3$$

FINAL ANSWER

$$y = -\frac{1}{4}x + 3$$

4WM1/01 NOVEMBER 2025

Question 27

5 marks | Linear Graphs

27 $ABCD$ is a rhombus with $AB = BC = CD = DA$

A has coordinates $(16, -9)$

C has coordinates $(24, 15)$

Find an equation of the line BD
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.
Show your working clearly.

(Total for Question 27 is 5 marks)

WORKED SOLUTION

Question 27 · 4WM1/01 November 2025

1 Diagonals of a rhombus bisect

Diagonals of a rhombus bisect each other at right angles.

2 Find the midpoint

Midpoint of AC:

$$\left(\frac{16+24}{2}, \frac{-9+15}{2}\right) = (20, 3)$$

3 Find the gradient

Gradient of AC:

$$\frac{15-(-9)}{24-16} = 3$$

4 Find the gradient

So gradient of BD is $-\frac{1}{3}$.

$$y - 3 = -\frac{1}{3}(x - 20)$$

$$x + 3y - 29 = 0$$

FINAL ANSWER

$x + 3y - 29 = 0$.

4MA1/1H NOVEMBER 2025

Question 28

6 marks | Linear Graphs

28 The curve **C** has equation $x^2 + y^2 = d - 11x$ where d is an integer.
The line **L** has equation $y = 3x + e$ where e is an integer.

C and **L** intersect at the point A and at the point B

The coordinates of A are $(0.2, 2.6)$

Work out the coordinates of B
Show your working clearly.

(..... , ...)

(Total for Question 28 is 6 marks)

WORKED SOLUTION

Question 28 · 4MA1/1H November 2025

1 The line is

The line is

$$y = 3x + e$$

2 Point lies on the line

Point $A = (0.2, 2.6)$ lies on the line.

$$2.6 = 3(0.2) + e$$

$$2.6 = 0.6 + e$$

$$e = 2$$

3 Calculate value

So

$$y = 3x + 2$$

4 **The curve is**

The curve is

$$x^2 + y^2 = d - 11x$$

---

5 **Use point**

Use point $A = (0.2, 2.6)$:

$$0.2^2 + 2.6^2 = d - 11(0.2)$$

$$0.04 + 6.76 = d - 2.2$$

$$d = 9$$

---

6 **Substitute into the curve**

Now substitute $y = 3x + 2$ into the curve:

$$x^2 + (3x + 2)^2 = 9 - 11x$$

$$x^2 + 9x^2 + 12x + 4 = 9 - 11x$$

$$10x^2 + 23x - 5 = 0$$

---

7 **One root is**

One root is $0.2 = \frac{1}{5}$.

---

8 **The product of the roots**

The product of the roots is

$$\frac{-5}{10} = -\frac{1}{2}$$

---

9 **Evaluate fraction**

So the other root is

$$\frac{-\frac{1}{2}}{\frac{1}{5}} = -\frac{5}{2}$$

---

10 **Evaluate fraction**

Then

$$y = 3\left(-\frac{5}{2}\right) + 2$$

$$y = -\frac{11}{2}$$

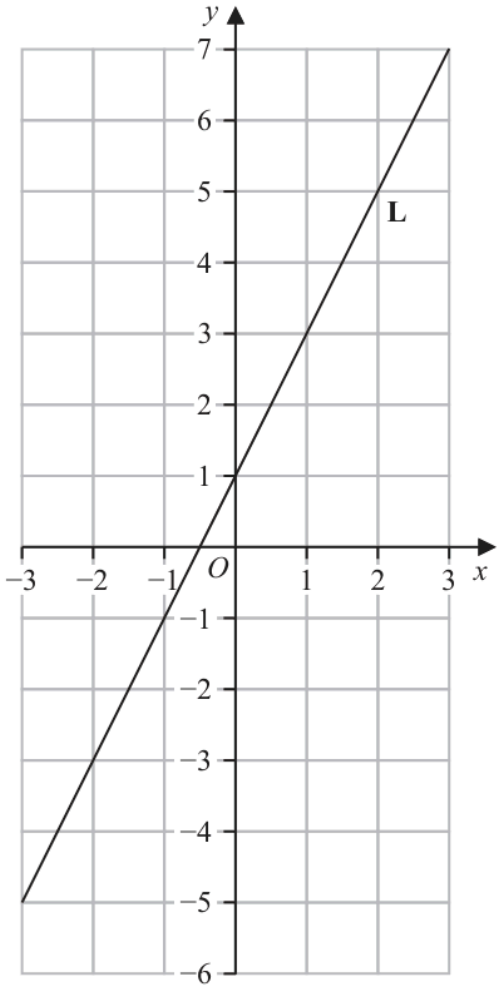
✓ **FINAL ANSWER**

$$B = \left(-\frac{5}{2}, -\frac{11}{2}\right)$$

Question 10

3 marks | Linear Graphs

10 The line **L** is drawn on a grid.



Find an equation for **L**

.....

(Total for Question 10 is 3 marks)

1 **Read two points**

The line passes through $(0, 1)$ and $(2, 5)$.

2 **Gradient and intercept**

$$m = \frac{5 - 1}{2 - 0} = 2, \qquad c = 1$$

3 **Equation**

$$y = 2x + 1$$

✓ **FINAL ANSWER**

$y = 2x + 1$

4MA1/1HR JUNE 2026

Question 13

3 marks | Linear Graphs

13 The straight line **L** has equation $4y - 10x - 5 = 0$

(a) Find the gradient of **L**

.....

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

(..... , ...)

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

$$4y - 10x - 5 = 0 \implies 4y = 10x + 5 \implies y = \frac{5}{2}x + \frac{5}{4}.$$

Rearrange into the form y=mx+c.

2 **Part (a) | Step 2**

The gradient is $\frac{5}{2}$.

The coefficient of x is the gradient.

3 **Part (b) | Step 1**

$$x = 0 \implies 4y - 5 = 0 \implies y = \frac{5}{4}.$$

Every point on the y-axis has x-coordinate zero.

✓ **FINAL ANSWER**

(a) $\frac{5}{2}$ (b) $\left(0, \frac{5}{4}\right)$

4MA1/1H JUNE 2026

Question 15

2 marks | Linear Graphs

15 Point A has coordinates $(-3, 4)$
Point B has coordinates $(2, -6)$

Work out the gradient of the line AB

.....

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H June 2026

1 Solution | Step 1

$$m = \frac{-6 - 4}{2 - (-3)} = \frac{-10}{5} = -2.$$

Use change in y divided by the corresponding change in x.

FINAL ANSWER

-2

Question 17

4 marks | Linear Graphs

17 The line **L** is perpendicular to the line with equation $y = 7 + 4x$

L passes through the point A with coordinates $\left(0, \frac{19}{2}\right)$

L crosses the x-axis at the point B

Find the area of triangle AOB where O is the origin.
Show your working clearly.

.....

(Total for Question 17 is 4 marks)

1 **Perpendicular gradient**

$y = 7 + 4x$ has gradient 4, so $\mathbf{L}$ has gradient $-\frac{1}{4}$.

2 **Equation of L**

$$y = -\tfrac{1}{4}x + \tfrac{19}{2}$$

3 **Point B (y = 0)**

$$0 = -\tfrac{1}{4}x + \tfrac{19}{2} \Rightarrow x = 38,\ B = (38, 0)$$

4 **Area of triangle AOB**

$$\tfrac{1}{2} \times 38 \times \tfrac{19}{2} = 180.5$$

✓ **FINAL ANSWER**

180.5

TOPIC

Linear Equations

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 SPECIMEN

Question 4

5 marks | Linear Equations

4 (a) Expand and simplify $(n - 6)(n + 4)$

.....

(2)

(b) Solve $2x - 3 = \dfrac{3x - 5}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 4 is 5 marks)

1 **Expand the brackets**

$$(n-6)(n+4) = n^2 + 4n - 6n - 24$$

$$= n^2 - 2n - 24$$

2 **Clear the denominator**

$$2x - 3 = \frac{3x - 5}{4}$$

$$4(2x - 3) = 3x - 5$$

3 **Solve the equation**

$$8x - 12 = 3x - 5$$

$$5x = 7$$

$$x = \frac{7}{5}$$

✓ **FINAL ANSWER**

$n^2 - 2n - 24$, and $x = \frac{7}{5}$.

4MA1/1H JANUARY 2020

Question 7

9 marks | Linear Equations

7 (a) Expand and simplify $(m - 8)(m + 5)$

.....

(2)

(b) Factorise fully $5y + 20y^2$

.....

(2)

(c) Simplify $(p^2 + 3)^0$

.....

(1)

(d) Solve $3(2x - 5) = \frac{9 - x}{2}$

Show clear algebraic working.

$x =$ .....

(4)

(Total for Question 7 is 9 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2020

1 **(a)**

(a) $(m-8)(m+5) = m^2 - 3m - 40$.

2 **(b)**

(b) $5y + 20y^2 = 5y(1 + 4y)$.

3 **(c)**

(c) $(p^2+3)^0 = 1$.

4 **(d)**

(d)

$$3(2x-5)=\frac{9-x}{2}$$

$$12x-30=9-x$$

$$13x=39$$

FINAL ANSWER

(a) $m^2-3m-40$, (b) $5y(1+4y)$, (c) 1, (d) $x=3$

4MA1/1HR MAY/NOVEMBER 2020

Question 3

6 marks | Linear Equations

3 (a) Simplify $h^7 \times h^2$

.....

(1)

$G = c^2 - 4c$

(b) Find the value of G when $c = -5$

$G =$ .....

(2)

(c) Solve $\dfrac{5x - 3}{4} = 2x + 3$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR May/November 2020

1 **(a)**

(a)

$$h^7 \times h^2 = h^{7+2} = h^9$$

2 **(b)**

(b)

$$G = c^2 - 4c$$

3 **Solve quadratic equation**

When $c = -5$:

$$G = (-5)^2 - 4(-5) = 25 + 20 = 45$$

4 **(c)**

(c)

$$\frac{5x-3}{4}=2x+3$$

---

5 **Multiply by 4**

Multiply by 4:

$$5x-3=8x+12$$

$$-3x=15$$

$$x=-5$$

FINAL ANSWER

h^9, $G=45$, and $x=-5$.

4MA1/1H JANUARY 2021

Question 5

5 marks | Linear Equations

5 (a) Factorise fully $15y^4 + 20uy^3$

.....

(2)

(b) Solve $4 - 3x = \dfrac{5 - 8x}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2021

1 **(a)**

(a)

$$15y^4 + 20uy^3 = 5y^3(3y + 4u)$$

2 **(b)**

(b)

$$4 - 3x = \frac{5 - 8x}{4}$$

$$16 - 12x = 5 - 8x$$

$$11 = 4x$$

FINAL ANSWER

(a) $5y^3(3y + 4u)$, (b) $x = \frac{11}{4}$

4MA1/2HR JANUARY 2022

Question 3

5 marks | Linear Equations

3 (a) Solve $4y + 5 > 12$

.....

(2)

(b) Solve $6x - 5 = \frac{4x - 7}{2}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2022

1 **(a)**

(a)

$$4y + 5 > 12$$

$$4y > 7$$

$$y > \frac{7}{4}$$

2 **(b)**

(b)

$$6x - 5 = \frac{4x - 7}{2}$$

3 Multiply by 2

Multiply by 2:

$$12x - 10 = 4x - 7$$

$$8x = 3$$

$$x = \frac{3}{8}$$

FINAL ANSWER

$y > \frac{7}{4}$, and $x = \frac{3}{8}$.

4MA1/1H JUNE 2022

Question 5

5 marks | Linear Equations

5 (a) Expand and simplify $(n-6)(n+4)$

.....

(2)

(b) Solve $2x-3=\dfrac{3x-5}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H June 2022

1 **(a)**

(a)

$$(n-6)(n+4)=n^2-2n-24$$

2 **(b)**

(b)

$$2x-3=\frac{3x-5}{4}$$

$$8x-12=3x-5$$

$$5x=7$$

✓ **FINAL ANSWER**

(a) $n^2-2n-24$, (b) $x=\frac{7}{5}$

4MA1/2HR JUNE 2022

Question 6

8 marks | Linear Equations

6 (a) Solve $p = \dfrac{3p - 5}{10}$

Show clear algebraic working.

$p =$ .....

(3)

(b) Simplify a^0 where $a > 0$

.....

(1)

(c) Simplify fully $\dfrac{3xy^3}{6x^2y}$

.....

(2)

(d) Factorise fully $10c^3d^2 + 15cd^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2022

1 **(a)**

(a)

$$p = \frac{3p - 5}{10}$$

$$10p = 3p - 5$$

$$7p = -5$$

$$p = -\frac{5}{7}$$

2 **(b)**

(b) $a^0 = 1$.

---

3 **(c)**

(c)

$$\frac{3xy^3}{6x^2y} = \frac{y^2}{2x}$$

---

4 **(d)**

(d)

$$10c^3d^2 + 15cd^4 = 5cd^2(2c^2 + 3d^2)$$

✓ **FINAL ANSWER**

(a) $p = -\frac{5}{7}$, (b) 1, (c) $\frac{y^2}{2x}$, (d) $5cd^2(2c^2 + 3d^2)$

4MA1/2HR JUNE 2024

Question 6

8 marks | Linear Equations

6 (a) Solve $\frac{2f}{3} = 4f - 17$

Show clear algebraic working.

$f =$ .....

(3)

(b) Simplify $(e + 12)^0$ where $e > 0$

.....

(1)

(c) Simplify fully $\frac{12a^4h^6}{4ah^2}$

.....

(2)

(d) Factorise fully $20x^5y + 12x^3y^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2024

1 (a)

(a)

$$\frac{2f}{3} = 4f - 17$$

$$2f = 12f - 51$$

$$10f = 51$$

$$f = \frac{51}{10}$$

2 **(b)**

(b) $(e+12)^0 = 1$.

---

3 **(c)**

(c)

$$\frac{12a^4h^6}{4ah^2} = 3a^3h^4$$

---

4 **(d)**

(d)

$$20x^5y + 12x^3y^4 = 4x^3y(5x^2 + 3y^3)$$

✓ **FINAL ANSWER**

$f = \frac{51}{10}$, 1, $3a^3h^4$, $4x^3y(5x^2 + 3y^3)$

4MA1/1H MAY 2024

Question 6

5 marks | Linear Equations

6 (a) Expand and simplify $(m + 5)(m - 8)$

.....

(2)

(b) Solve $3n - 4 = \dfrac{5n + 6}{3}$

Show clear algebraic working.

$n =$ .....

(3)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2024

1 **(a)**

(a)

$$(m+5)(m-8) = m^2 - 3m - 40$$

2 **(b)**

(b)

$$3n - 4 = \frac{5n+6}{3}$$

$$9n - 12 = 5n + 6$$

$$4n = 18$$

FINAL ANSWER

(a) $m^2 - 3m - 40$, (b) $n = \frac{9}{2}$

4MA1/1H NOVEMBER 2024

Question 3

6 marks | Linear Equations

3 (a) Simplify $\left(p^3\right)^5$

.....

(1)

(b) Expand and simplify $2n(4n+3)+n(n-4)$

.....

(2)

(c) Solve $\frac{2x+5}{3} = 4-x$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 6 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2024

1 **(a)**

(a) $(p^3)^5 = p^{15}$.

2 **(b)**

(b)

$$2n(4n+3)+n(n-4) = 8n^2+6n+n^2-4n = 9n^2+2n$$

3 **(c)**

(c)

$$\frac{2x+5}{3} = 4-x$$

$$2x+5 = 12-3x$$

$$5x = 7$$

FINAL ANSWER

(a) p^{15}, (b) $9n^2 + 2n$, (c) $x = \frac{7}{5}$

4MA1/2HR JUNE 2025

Question 1

5 marks | Linear Equations

1 (a) Factorise fully $18c - 45cd$

.....

(2)

(b) Solve $\frac{5 - 2x}{6} = 3x - 4$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 1 is 5 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2025

1 **(a)**

(a)

$$18c - 45cd = 9c(2 - 5d)$$

2 **(b)**

(b)

$$\frac{5 - 2x}{6} = 3x - 4$$

$$5 - 2x = 18x - 24$$

$$29 = 20x$$

$$x = \frac{29}{20}$$

FINAL ANSWER

(a) $9c(2 - 5d)$, (b) $x = \frac{29}{20}$

4WM1/01R MAY 2025

Question 12

6 marks | Linear Equations

12 (a) Solve $\frac{5a+8}{3} - \frac{2a+5}{4} = 23$

Show clear algebraic working.

$a =$ .....

(4)

(b) Express $\left(\frac{\sqrt{y}}{3}\right)^{-1}$ in the form cy^n where c and n are numbers to be found.

.....

(2)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4WM1/01R May 2025

1 Solve equation

For part (a),

$$\frac{5a+8}{3} - \frac{2a+5}{4} = 23$$

2 Multiply by

Multiply by 12:

$$4(5a+8) - 3(2a+5) = 276$$

$$20a + 32 - 6a - 15 = 276$$

$$14a + 17 = 276$$

$$14a = 259$$

$$a = \frac{259}{14} = \frac{37}{2}$$

3 Solve equation

For part (b),

$$\left(\frac{\sqrt{y}}{3}\right)^{-1} = \frac{3}{\sqrt{y}}$$

$$= 3y^{-1/2}$$

FINAL ANSWER

(a) $a = \frac{37}{2}$, (b) $3y^{-1/2}$

4MA1/1HR JUNE 2025

Question 15

6 marks | Linear Equations

15 (a) Solve $\frac{5a+8}{3} - \frac{2a+5}{4} = 23$

Show clear algebraic working.

$a =$ .....

(4)

(b) Express $\left(\frac{\sqrt{y}}{3}\right)^{-1}$ in the form cy^n where c and n are numbers to be found.

.....

(2)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR June 2025

1 Solve equation

For part (a),

$$\frac{5a+8}{3} - \frac{2a+5}{4} = 23$$

2 Multiply by

Multiply by 12:

$$4(5a+8) - 3(2a+5) = 276$$

$$20a + 32 - 6a - 15 = 276$$

$$14a + 17 = 276$$

$$14a = 259$$

$$a = \frac{259}{14} = \frac{37}{2}$$

3 **Solve equation**

For part (b),

$$\left(\frac{\sqrt{y}}{3}\right)^{-1} = \frac{3}{\sqrt{y}}$$

$$= 3y^{-1/2}$$

✓ **FINAL ANSWER**

(a) $a = \frac{37}{2}$, (b) $3y^{-1/2}$

4MA1/2HR JUNE 2026

Question 4

6 marks | Linear Equations

4 (a) Solve $3(x + 2) = 1 - 5x$
Show clear algebraic working.

$x =$ .....

(3)

(c) Simplify m^0

.....

(1)

(d) Simplify $(4r^2t)^3$

.....

(2)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2026

1 **Part (a) | Step 1**

$$3x + 6 = 1 - 5x.$$

Expand the bracket.

2 **Part (a) | Step 2**

$$8x = -5 \quad \Rightarrow \quad x = -\frac{5}{8}.$$

Add 5x and subtract 6 on both sides, then divide by 8.

3 **Part (c) | Step 1**

$m^0 = 1$ for $m \neq 0$.

Any nonzero base to the zero power equals 1.

4 **Part (d) | Step 1**

$$(4r^2t)^3 = 4^3(r^2)^3t^3 = 64r^6t^3.$$

Cube each factor and multiply the exponents in the power of a power.

✓ **FINAL ANSWER**

(a) $x = -\frac{5}{8}$. (c) 1. (d) $64r^6t^3$.

TOPIC

Graphs of Functions

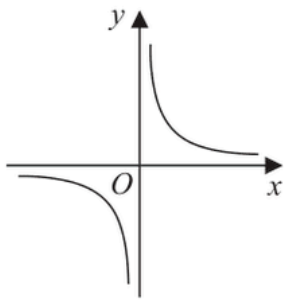
Every past-paper question on this topic, 2020 to 2026.

Question 15

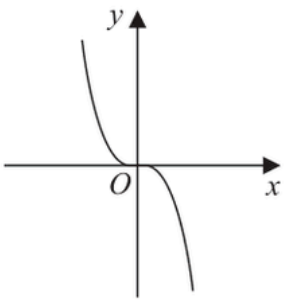
3 marks | Graphs of Functions

15 Here are six graphs.

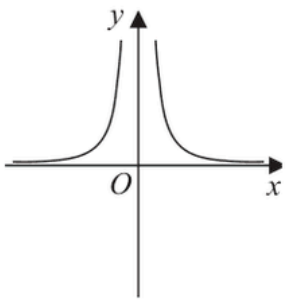
Graph A



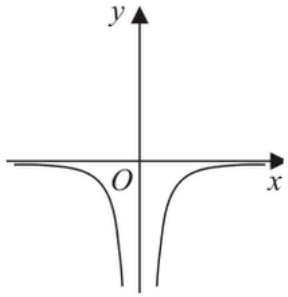
Graph B



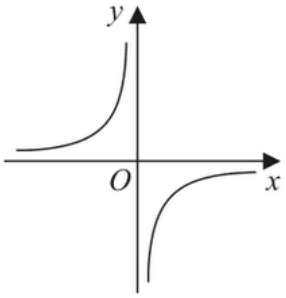
Graph C



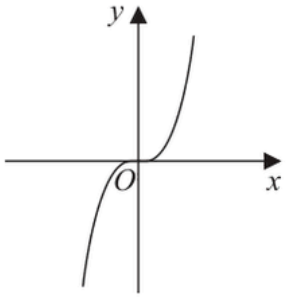
Graph D



Graph E



Graph F



Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \frac{2}{x^2}$	.....
$y = -\frac{1}{2}x^3$	.....
$y = -\frac{5}{x}$	.....

(Total for Question 15 is 3 marks)

1 **Solve equation**

For

$$y = -\frac{2}{x^2}$$

the graph is negative on both sides of the y-axis and is symmetrical in the y-axis.

2 **This is Graph D**

This is Graph D.

3 **Solve equation**

For

$$y = -\frac{1}{2}x^3$$

the graph is a decreasing cubic through the origin.

4 **This is Graph B**

This is Graph B.

5 **Solve equation**

For

$$y = \frac{5}{x}$$

the graph is a positive reciprocal graph in quadrants 1 and 3.

6 **This is Graph A**

This is Graph A.

✓ **FINAL ANSWER**

D, B, A.

Question 15

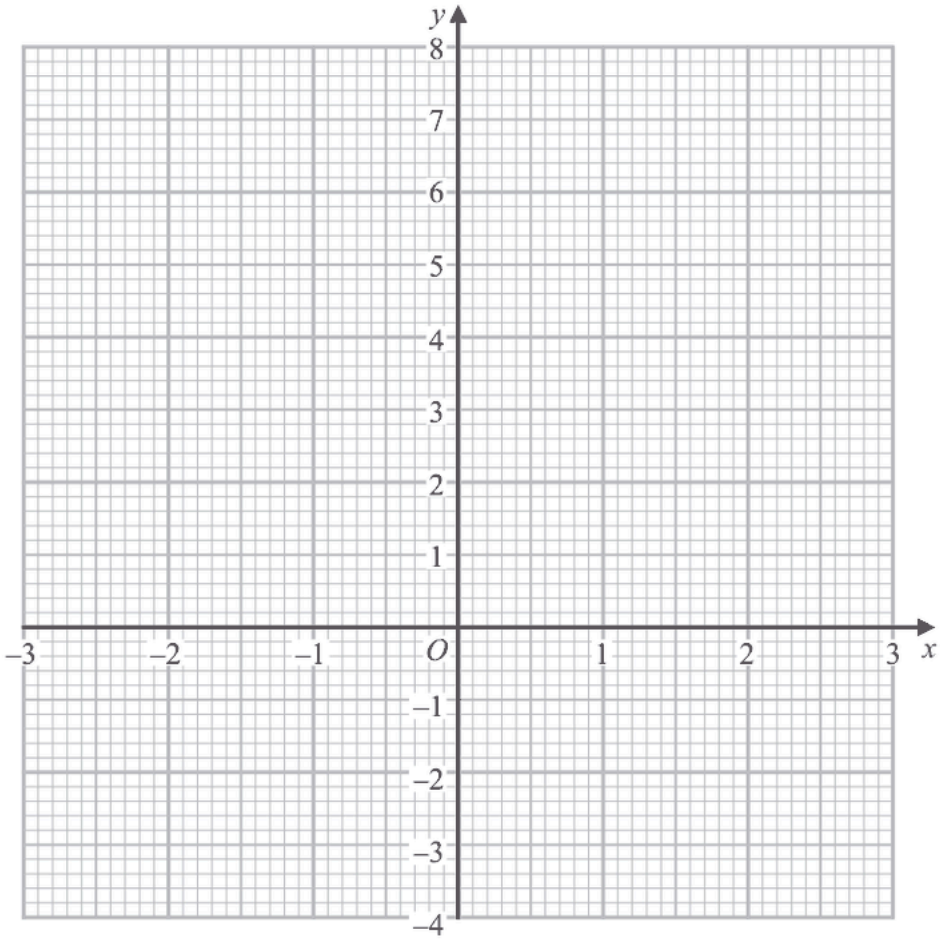
4 marks | Graphs of Functions

15 (a) Complete the table of values for $y = x^2 - \frac{x}{2} - 3$

x	–3	–2	–1	0	1	2	3
y	7.5				–2.5		4.5

(2)

(b) On the grid, draw the graph of $y = x^2 - \frac{x}{2} - 3$ for values of x from –3 to 3



(2)

(Total for Question 15 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - \frac{x}{2} - 3$$

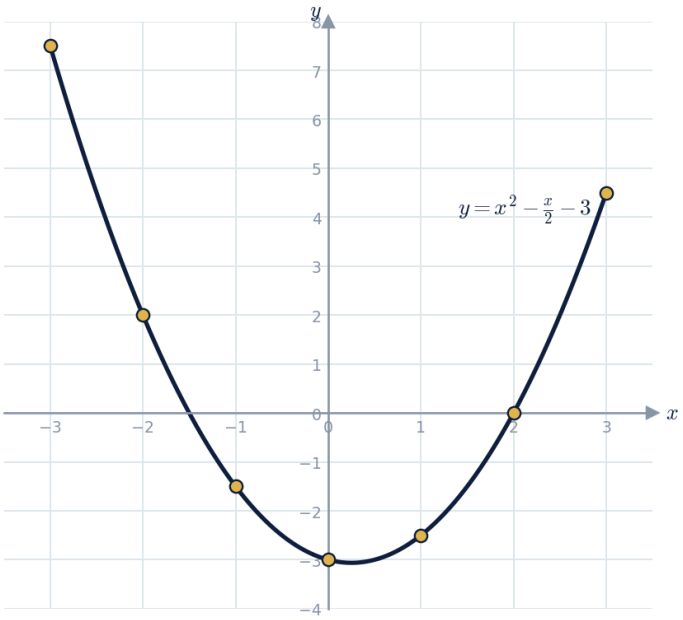
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	7.5	2	-1.5	-3	-2.5	0	4.5

3 Plot these points and join

Plot these points and join them with a smooth U-shaped curve.



FINAL ANSWER

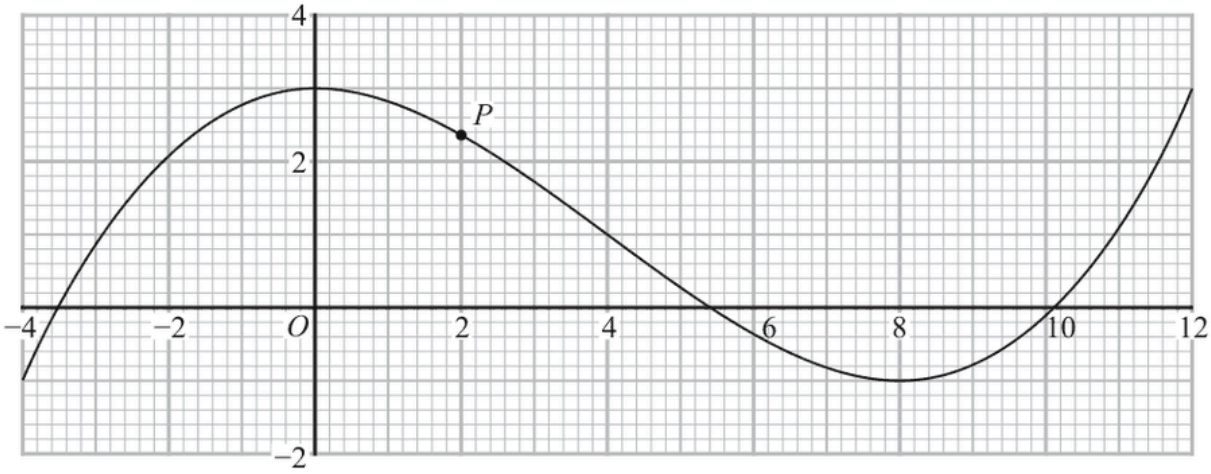
Missing values are $2, -1.5, -3, 0$.

4MA1/1HR JANUARY 2020

Question 18

7 marks | Graphs of Functions

18 The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$



The point P on the curve has x coordinate 2

(a) (i) Use the graph to find an estimate for the gradient of the curve at P.

.....

(3)

(ii) Hence find an equation of the tangent to the curve at P.
Give your answer in the form $y = mx + c$

.....

(2)

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

(b) Use the graph to find the two possible values of k.

..... , ...

(2)

(Total for Question 18 is 7 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2020

1 **Draw a tangent to the**

Draw a tangent to the curve at P.

2 **A reasonable tangent gives an**

A reasonable tangent gives an estimated gradient of about

$$m \approx -0.6$$

3 **The point is approximately**

The point P is approximately

$$(2, 2.4)$$

4 **Find the gradient**

Using $y = mx + c$,

$$2.4 \approx -0.6(2) + c$$

$$c \approx 3.6$$

5 **Find the gradient**

So an approximate tangent equation is

$$y \approx -0.6x + 3.6$$

6 **The equation has exactly two**

The equation $f(x) = k$ has exactly two different solutions when the horizontal line touches one turning level.

7 **Find the gradient**

From the graph, these levels are approximately

$$k = 3 \quad \text{and} \quad k = -1$$

✓ **FINAL ANSWER**

Gradient ≈ -0.6, tangent $y \approx -0.6x + 3.6$, and $k \approx 3, -1$.

4MA1/1H JANUARY 2020

Question 20

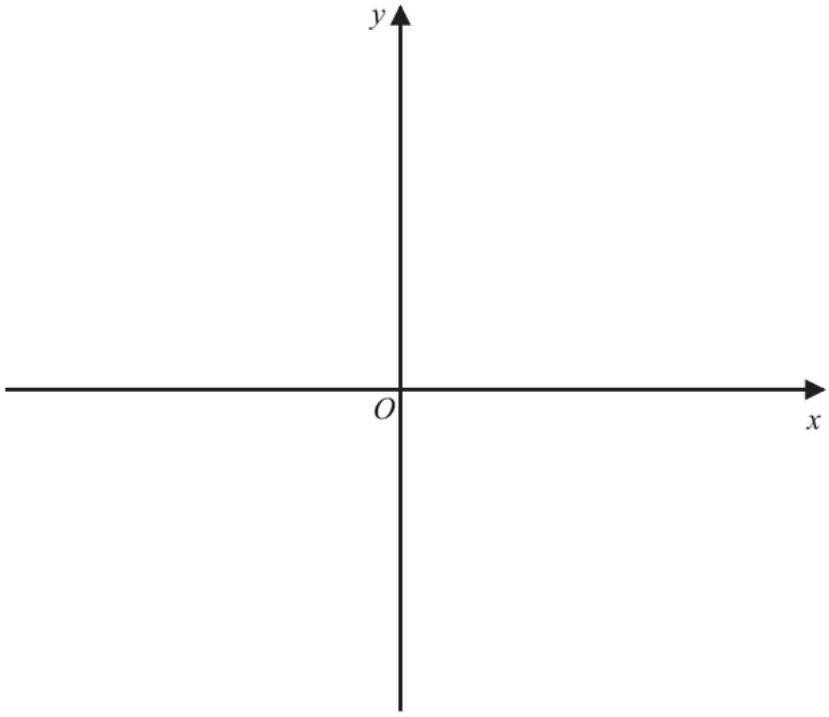
4 marks | Graphs of Functions

20 The curve **C** has equation $y = 4(x - 1)^2 - a$ where $a > 4$

Using the axes below, sketch the curve **C**.
On your sketch show clearly, in terms of a,

(i) the coordinates of any points of intersection of **C** with the coordinate axes,

(ii) the coordinates of the turning point.



(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2020

1 **Calculate value**

$$y = 4(x - 1)^2 - a$$

2 **The turning point is read**

The turning point is read directly from the completed square form:

$$(1, -a)$$

3 **Find intercepts**

For the y-intercept, put $x = 0$:

$$y = 4(0-1)^2 - a$$

$$y = 4 - a$$

4 **Find intercepts**

So the y-intercept is

$$(0, 4-a)$$

5 **Find intercepts**

For the x-intercepts, put $y = 0$:

$$4(x-1)^2 - a = 0$$

$$4(x-1)^2 = a$$

$$(x-1)^2 = \frac{a}{4}$$

$$x - 1 = \pm\frac{\sqrt{a}}{2}$$

$$x = 1 \pm \frac{\sqrt{a}}{2}$$

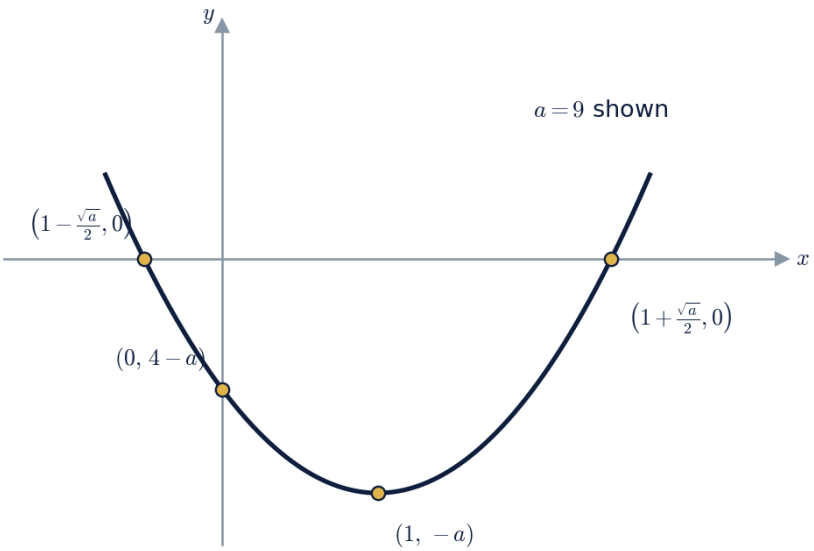
6 **Find intercepts**

So the x-intercepts are

$$\left(1 - \frac{\sqrt{a}}{2}, 0\right) \quad \text{and} \quad \left(1 + \frac{\sqrt{a}}{2}, 0\right)$$

7 **Solve inequality**

Since $a > 4$, the turning point is below the x-axis and the curve crosses the x-axis twice.



FINAL ANSWER

turning point $(1, -a)$, y-intercept $(0, 4-a)$, x-intercepts $\left(1-\frac{\sqrt{a}}{2}, 0\right)$ and $\left(1+\frac{\sqrt{a}}{2}, 0\right)$

Question 15

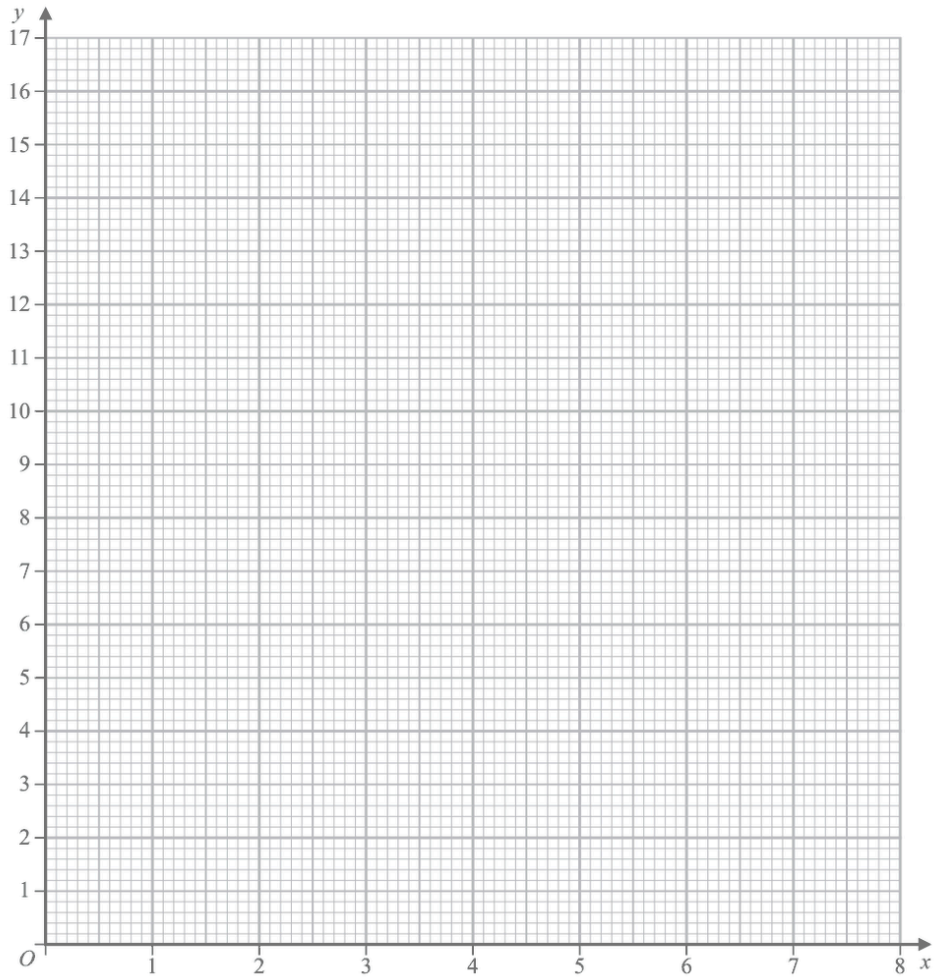
4 marks | Graphs of Functions

15 (a) Complete the table of values for $y = \frac{1}{x}(x^2 + 4)$

x	0.25	0.5	1	2	4	8
y	16.25					8.5

(2)

(b) On the grid, draw the graph of $y = \frac{1}{x}(x^2 + 4)$ for $0.25 \leqslant x \leqslant 8$



(2)

(Total for Question 15 is 4 marks)

1 Read the graph

$$y = \frac{1}{x}(x^2 + 4) = x + \frac{4}{x}$$

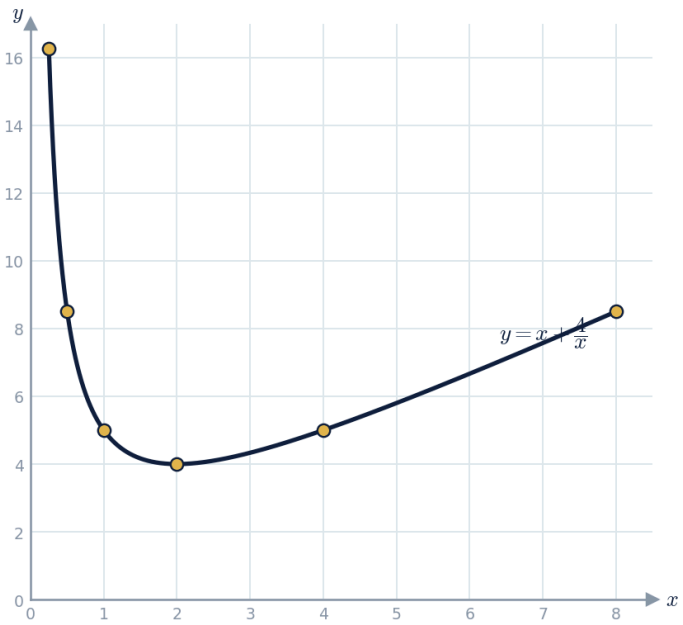
2 The completed table is

The completed table is:

x	0.25	0.5	1	2	4	8
y	16.25	8.5	5	4	5	8.5

3 Plot the points and join

Plot the points and join them with a smooth curve.



FINAL ANSWER

Missing values are $8.5, 5, 4, 5$.

Question 4

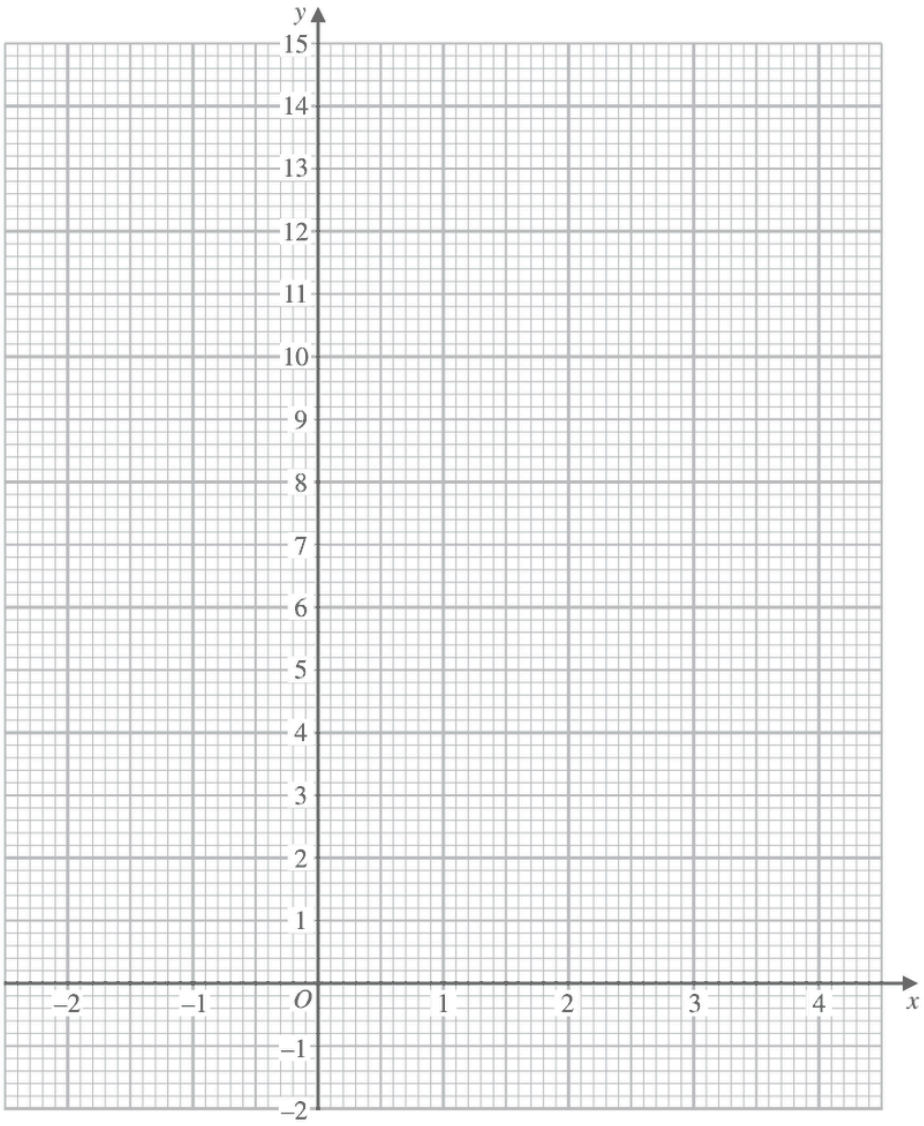
4 marks | Graphs of Functions

4 (a) Complete the table of values for $y = x^2 - 4x + 3$

x	–2	–1	0	1	2	3	4
y		8	3			0	

(2)

(b) On the grid, draw the graph of $y = x^2 - 4x + 3$ for values of x from –2 to 4



(2)

(Total for Question 4 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - 4x + 3$$

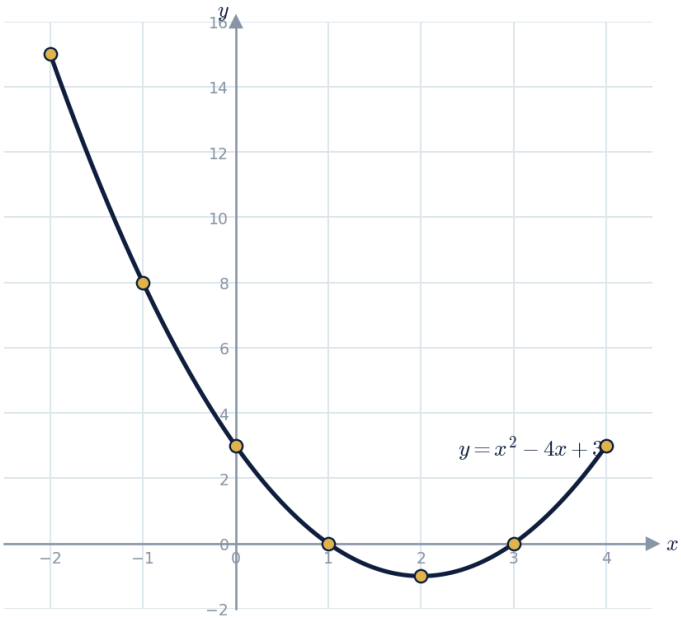
2 The completed table is

The completed table is:

x	-2	-1	0	1	2	3	4
y	15	8	3	0	-1	0	3

3 Plot the points and join

Plot the points and join them with a smooth U-shaped curve.



FINAL ANSWER

Missing values are $15, 0, -1, 3$.

Question 11

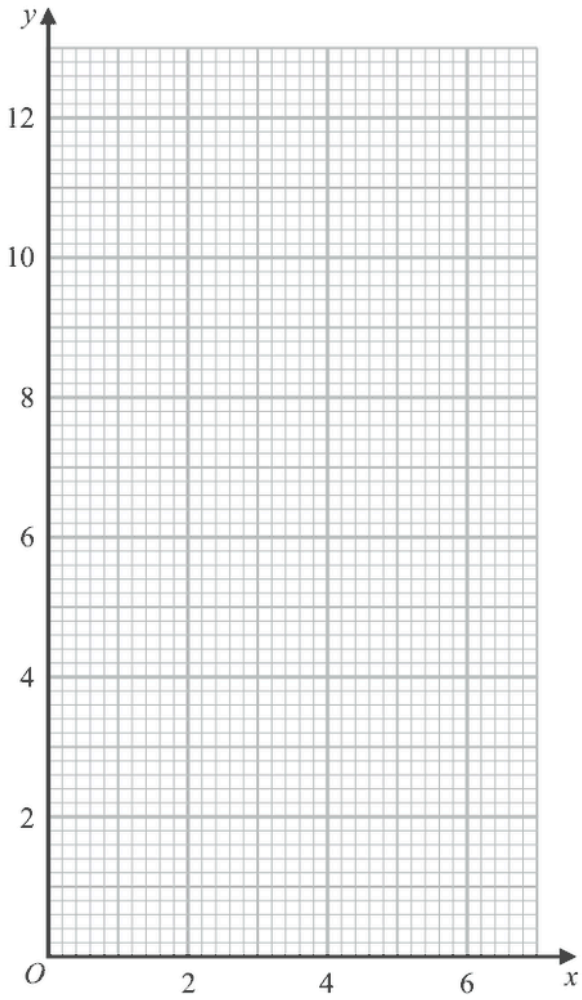
4 marks | Graphs of Functions

11 (a) Complete the table of values for $y = \frac{6}{x}$

x	0.5	1	2	3	4	5	6
y		6		2			1

(2)

(b) On the grid, draw the graph of $y = \frac{6}{x}$ for $0.5 \leqslant x \leqslant 6$



(2)

(Total for Question 11 is 4 marks)

1 Read the graph

$$y = \frac{6}{x}$$

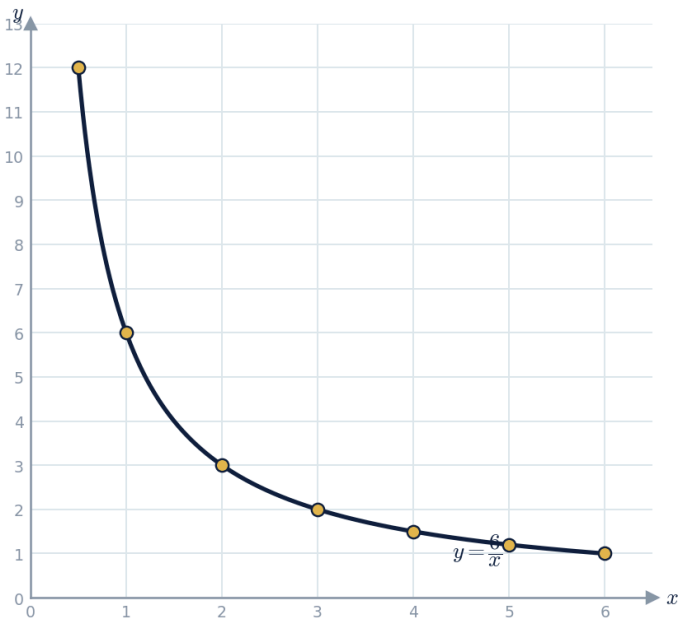
2 The completed table is

The completed table is:

x	0.5	1	2	3	4	5	6
y	12	6	3	2	1.5	1.2	1

3 Plot the points and join

Plot the points and join them with a smooth decreasing reciprocal curve.



FINAL ANSWER

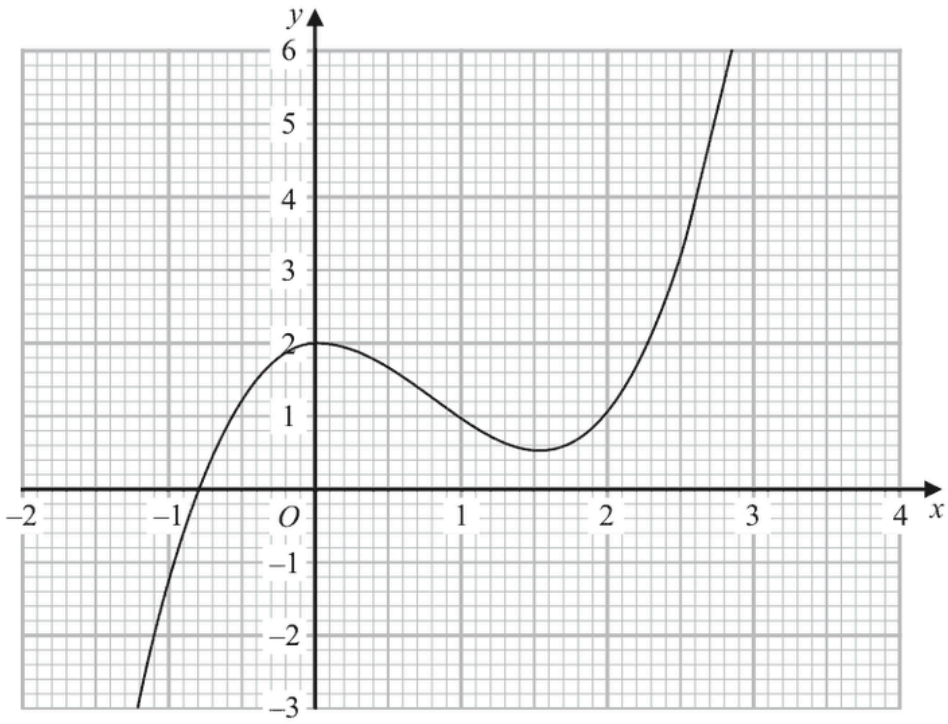
Missing values are $12, 3, 1.5, 1.2$.

4MA1/2HR JANUARY 2022

Question 17

3 marks | Graphs of Functions

17 Part of the curve with equation $y = f(x)$ is shown on the grid.



Find an estimate for the gradient of the curve at the point where $x = 2$
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR January 2022

1 Draw a tangent to the

Draw a tangent to the curve at the point where

$$x = 2$$

2 Find the gradient

Using two clear points on the tangent, for example approximately $(1.5, 0.6)$ and $(2.3, 2.2)$,

$$\text{gradient} \approx \frac{2.2 - 0.6}{2.3 - 1.5}$$

$$\text{gradient} \approx 2.0$$

FINAL ANSWER

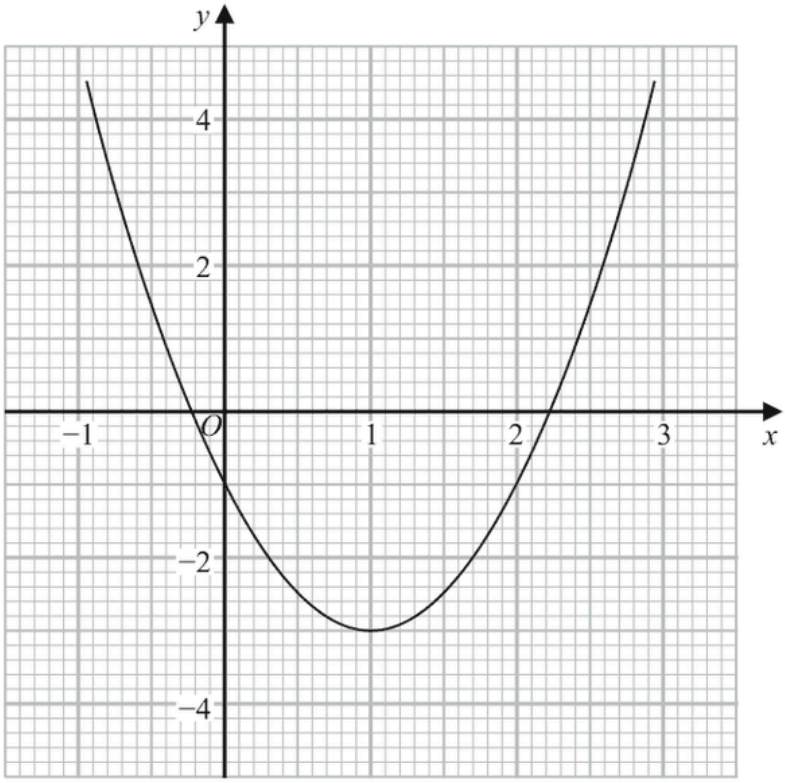
The gradient is approximately $\mathbf{2}$.

4MA1/2H JANUARY 2022

Question 21

5 marks | Graphs of Functions

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

.....

(2)

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

.....

(3)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2022

1 **Read the graph**

For part (a), solve

$$2x^2 - 4x - 1 = 0$$

2 **Read the graph**

Using the graph gives approximately

$$x = -0.2, \quad x = 2.2$$

3 **Read the graph**

For part (b), we need to solve

$$x^2 - x - 1 = 0$$

4 **Multiply by 2**

Multiply by 2:

$$2x^2 - 2x - 2 = 0$$

5 **The given graph is**

The given graph is

$$y = 2x^2 - 4x - 1$$

6 **Read the graph**

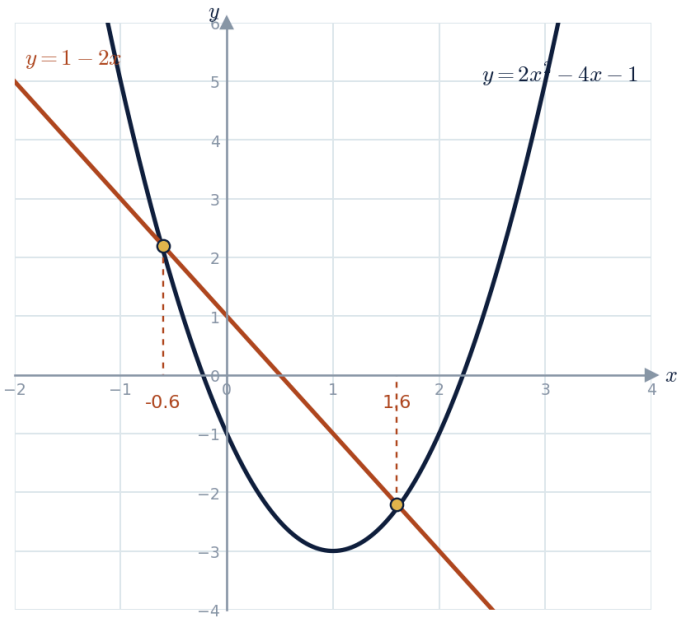
Now

$$2x^2 - 2x - 2 = (2x^2 - 4x - 1) + 2x - 1$$

7 **Read the graph**

So draw the straight line

$$y = 1 - 2x$$



8 **The intersections of this line**

The intersections of this line with the curve give approximately

$$x = -0.6, \quad x = 1.6$$

✓ **FINAL ANSWER**

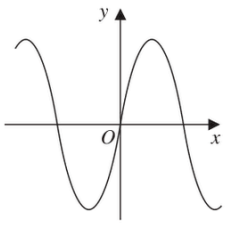
(a) $-0.2,\ 2.2$; (b) draw $y = 1 - 2x$, giving $-0.6,\ 1.6$

Question 16

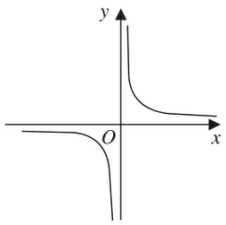
3 marks | Graphs of Functions

16 Here are nine graphs.

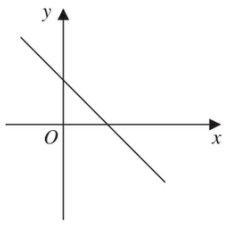
Graph A



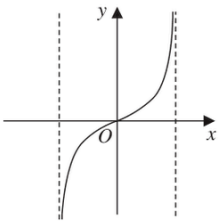
Graph B



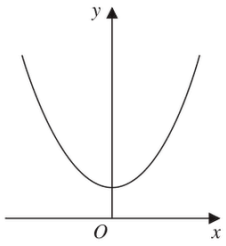
Graph C



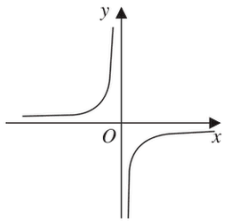
Graph D



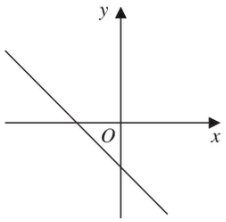
Graph E



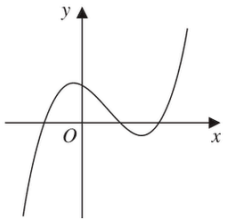
Graph F



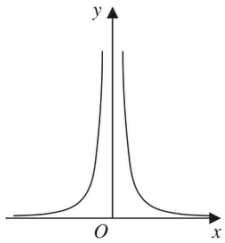
Graph G



Graph H



Graph I



Complete the table below with the letter of the graph that could represent each given equation.
Write each answer on the dotted line.

Equation	Graph
$y = -2x + 3$	.....
$y = -\frac{1}{x}$	.....
$y = \tan x°$	.....
$y = (x + 1)(x - 1)(x - 2)$	.....

(Total for Question 16 is 3 marks)

1 **is a negative straight line**

$$y = -2x + 3$$

is a negative straight line with positive y-intercept, so it is **Graph C**.

2 **Read the graph**

$$y = \frac{1}{x}$$

is the reciprocal graph in quadrants I and III, so it is **Graph B**.

3 **has repeating branches with vertical**

$$y = \tan x$$

has repeating branches with vertical asymptotes and passes through the origin, so it is **Graph D**.

4 **Read the graph**

$$y = (x + 1)(x - 1)(x - 2)$$

is a positive cubic with three roots, so it is **Graph H**.

✓ **FINAL ANSWER**

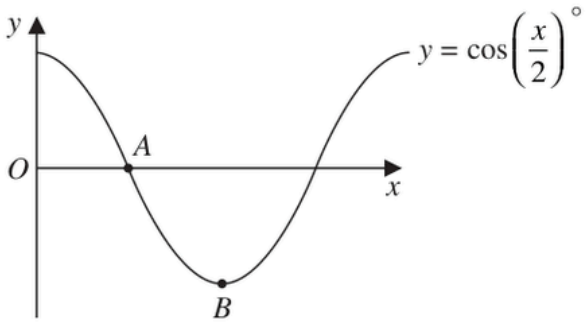
C, B, D, H.

4MA1/2H JUNE 2022

Question 23

2 marks | Graphs of Functions

23 The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$



(i) Find the coordinates of the point A

(.....,.....)

(1)

(ii) Find the coordinates of the point B

(.....,.....)

(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2022

1 **The graph is**

The graph is

$$y = \cos\left(\frac{x}{2}\right)^\circ$$

2 **Point is the first positive**

Point A is the first positive x-intercept, so $y = 0$.

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180$$

3 **Read the graph**

So

$$A = (180, 0)$$

---

4 **Point is the minimum point**

Point B is the minimum point.

---

5 **The minimum value of cosine**

The minimum value of cosine is -1, which occurs when

$$\frac{x}{2} = 180°$$

$$x = 360$$

---

6 **Read the graph**

So

$$B = (360, -1)$$

✓ **FINAL ANSWER**

$A = (180, 0),\ B = (360, -1)$

Question 4

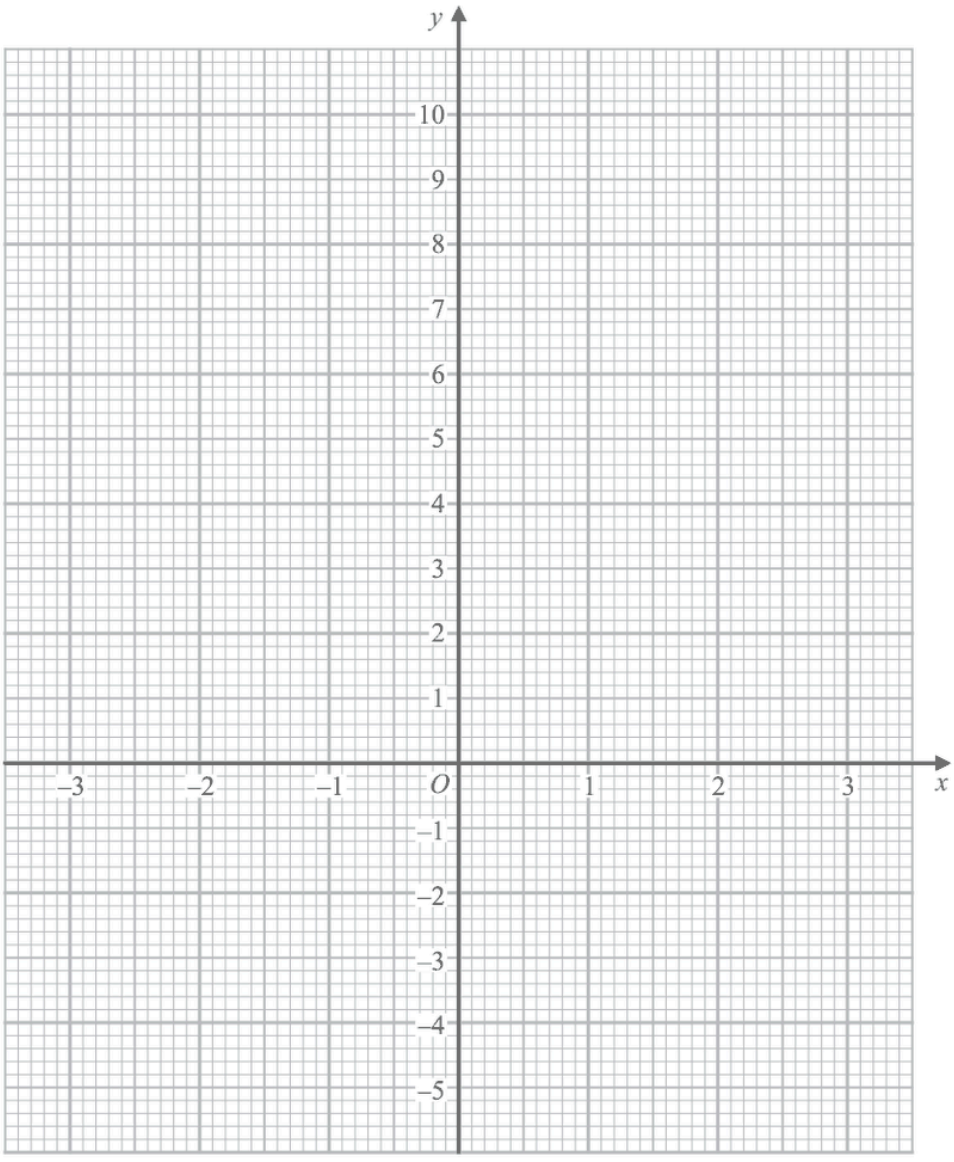
4 marks | Graphs of Functions

4 (a) Complete the table of values for $y = x^2 - x - 4$

x	–3	–2	–1	0	1	2	3
y		2			–4		

(2)

(b) On the grid below, draw the graph of $y = x^2 - x - 4$ for values of x from –3 to 3



(2)

(Total for Question 4 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - x - 4$$

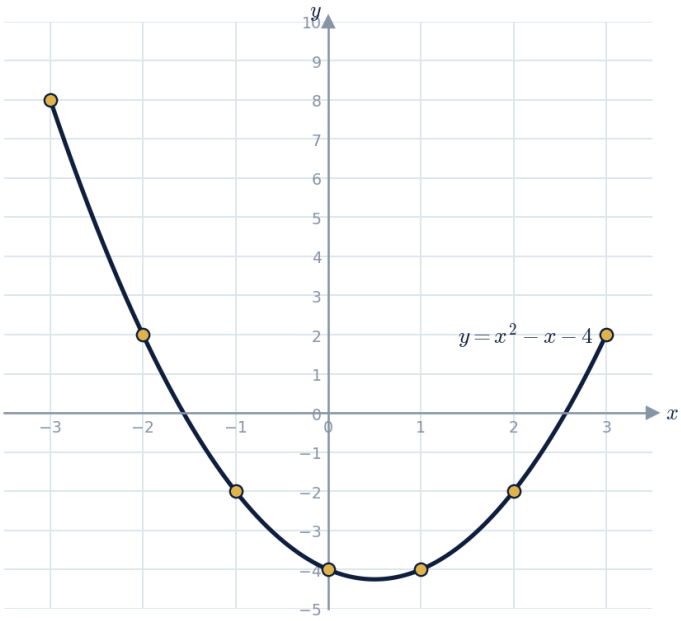
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2

3 Plot the points and join

Plot the points and join them with a smooth U-shaped curve.



FINAL ANSWER

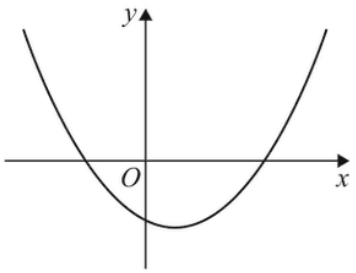
Missing values are $8, -2, -4, -2, 2$.

Question 12

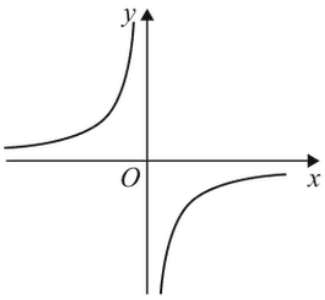
3 marks | Graphs of Functions

12 Here are six graphs.

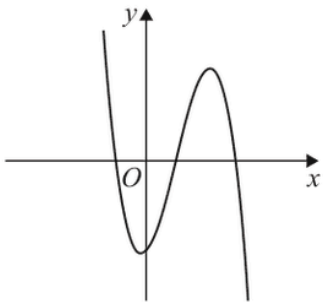
Graph **A**



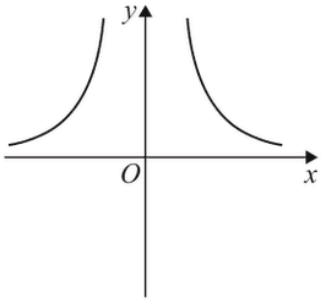
Graph **B**



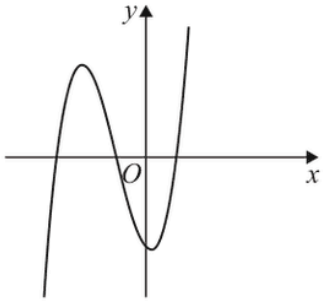
Graph **C**



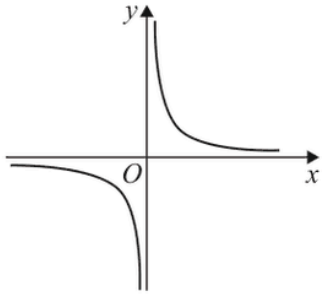
Graph **D**



Graph **E**



Graph **F**



Write down the letter of the graph of

(a) $y = \frac{10}{x^2}$

.....

(1)

(b) $y = x - 3 + 3x^2 - x^3$

.....

(1)

(c) $y = -\frac{3}{x}$

.....

(1)

(Total for Question 12 is 3 marks)

1 **Solve equation**

For

$$y = \frac{10}{x^2}$$

the graph is positive on both sides and symmetrical in the y-axis.

2 **This is Graph D**

This is Graph D.

3 **Solve equation**

For

$$y = x - 3 + 3x^2 - x^3$$

the leading term is $-x^3$, so it is a cubic falling to the right with two turning points.

4 **This is Graph C**

This is Graph C.

5 **Solve equation**

For

$$y = -\frac{3}{x}$$

the graph is a negative reciprocal graph in quadrants 2 and 4.

6 **This is Graph B**

This is Graph B.

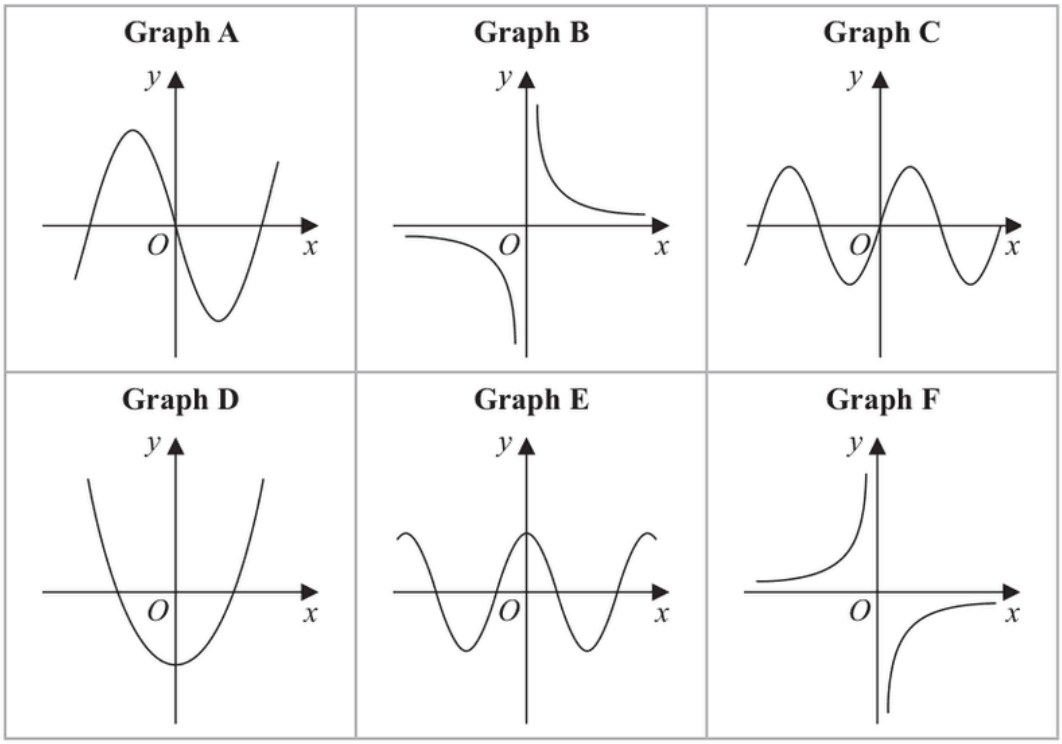
✓ **FINAL ANSWER**

D, C, B.

Question 18

3 marks | Graphs of Functions

18 Here are 6 graphs.



Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \sin x$	.....
$y = -\frac{3}{x}$	.....
$y = 4x^3 - 5x$	.....

(Total for Question 18 is 3 marks)

1 **is the periodic sine shaped**

$$y = \sin x$$

is the periodic sine-shaped graph, so it is **Graph C**.

2 **Read the graph**

$$y = \frac{3}{x}$$

is the positive reciprocal graph, so it is **Graph B**.

3 **Find turning point**

$$y = 4x^3 - 5x$$

is a cubic graph with two turning points, so it is **Graph A**.

✓ **FINAL ANSWER**

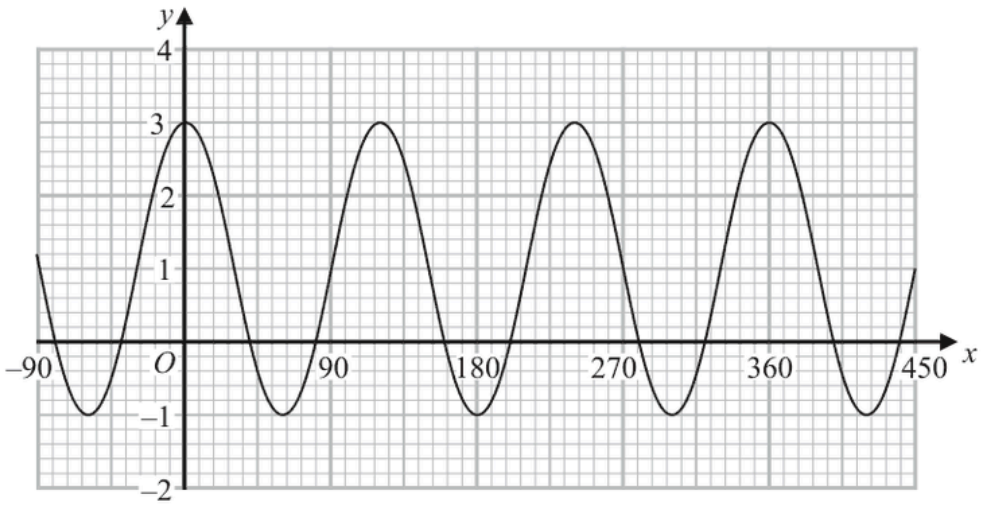
C, B, A.

4MA1/2HR JUNE 2023

Question 26

3 marks | Graphs of Functions

26 Here is a sketch of the curve with equation $y = a\cos bx° + c$ where $-90 \leqslant x \leqslant 450$



Find the value of a, the value of b and the value of c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2023

1 The curve is

The curve is

$$y = a\cos bx° + c$$

2 Use trigonometry

From the graph, the maximum value is

$$3$$

3 Use trigonometry

and the minimum value is

$$-1$$

4 **The amplitude is half the**

The amplitude is half the difference:

$$a = \frac{3 - (-1)}{2} = 2$$

5 **The midline is the average**

The midline is the average:

$$c = \frac{3 + (-1)}{2} = 1$$

6 **The distance between consecutive maxima**

The distance between consecutive maxima is $120°$, so the period is

$$120°$$

7 **Use trigonometry**

For $y = a\cos bx° + c$, the period is

$$\frac{360°}{b}$$

8 **Use trigonometry**

So

$$\frac{360}{b} = 120$$

$$b = 3$$

✓ **FINAL ANSWER**

$$\boxed{a = 2,\ b = 3,\ c = 1}$$

Question 12

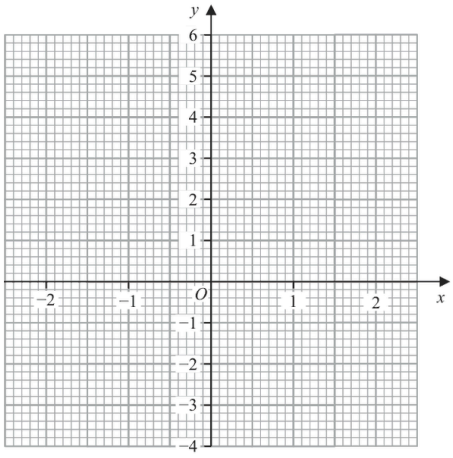
5 marks | Graphs of Functions

12 (a) Complete the table of values for $y = x^3 - 3x + 1$

x	−2	−1	0	1	2
y				−1	

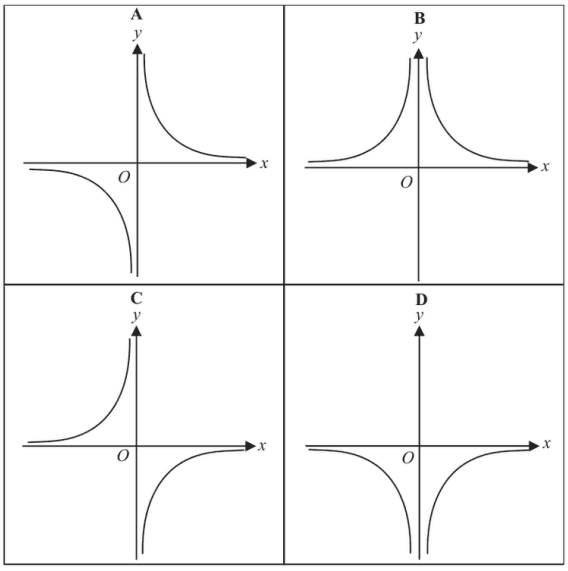
(2)

(b) On the grid, draw the graph of $y = x^3 - 3x + 1$ for values of x from − 2 to 2



(2)

Here are four graphs.



(c) Write down the letter of the graph that could have the equation $y = -\frac{1}{x^2}$

.....

(1)

(Total for Question 12 is 5 marks)

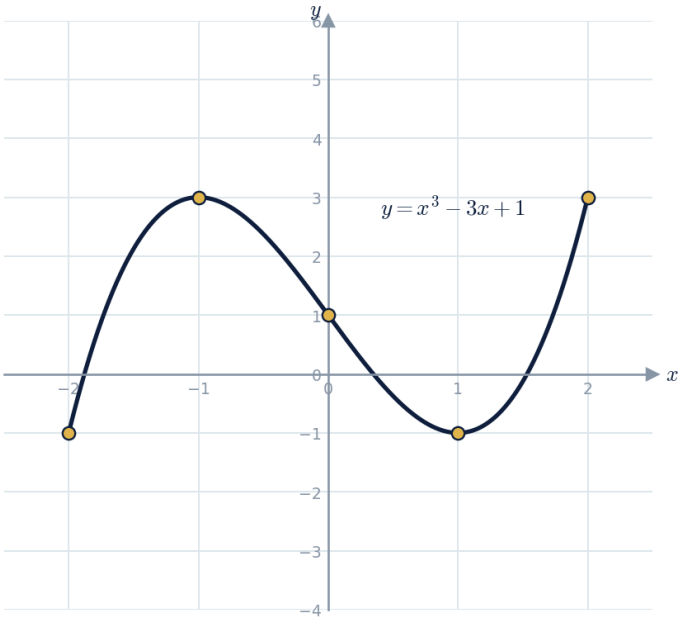
1 Read the graph

$$y = x^3 - 3x + 1$$

2 The completed table is

The completed table is:

x	-2	-1	0	1	2
y	-1	3	1	-1	3



3 The graph of is below

The graph of

$$y = -\frac{1}{x^2}$$

is below the x-axis on both sides and is symmetrical in the y-axis.

FINAL ANSWER

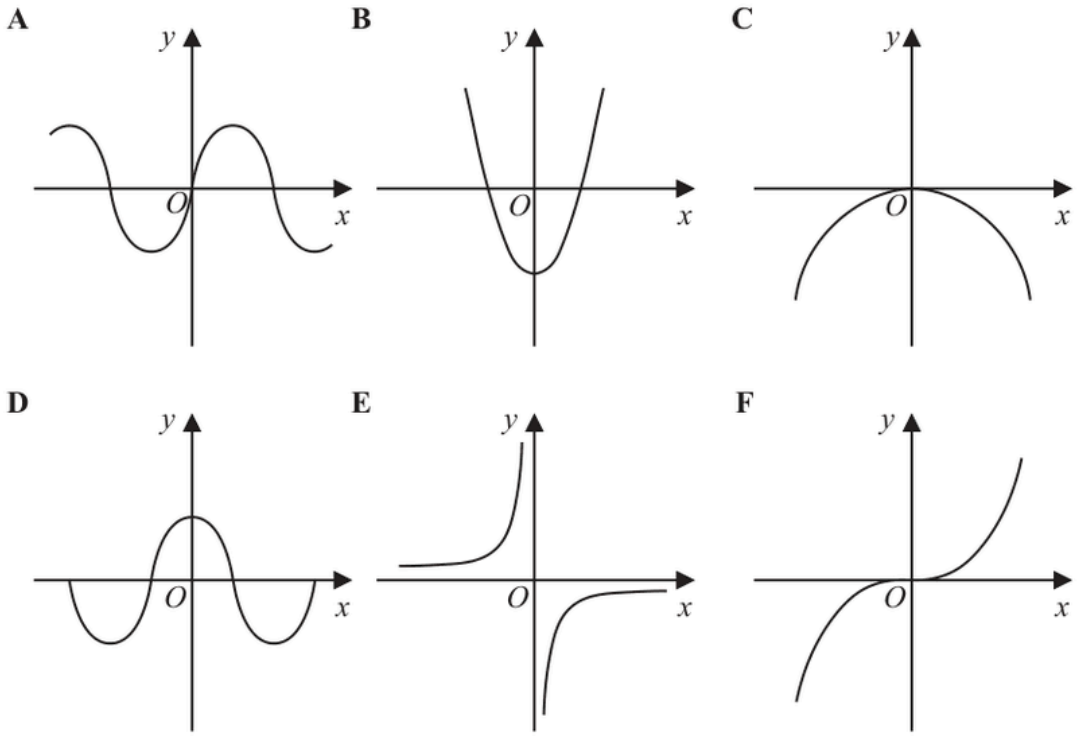
The graph letter is D.

4MA1/2HR JUNE 2024

Question 16

2 marks | Graphs of Functions

16 Here are six graphs.



Write down the letter of the graph that could have the equation

(i) $y = -\frac{1}{x}$

.....

(1)

(ii) $y = \sin x°$

.....

(1)

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2024

1 **(i) has branches in quadrants**

(i)

$$y = -\frac{1}{x}$$

has branches in quadrants II and IV, so it is **Graph E**.

2 **(ii) is the sine shaped**

(ii)

$$y = \sin x^\circ$$

is the sine-shaped graph through the origin, so it is **Graph A**.

✓ **FINAL ANSWER**

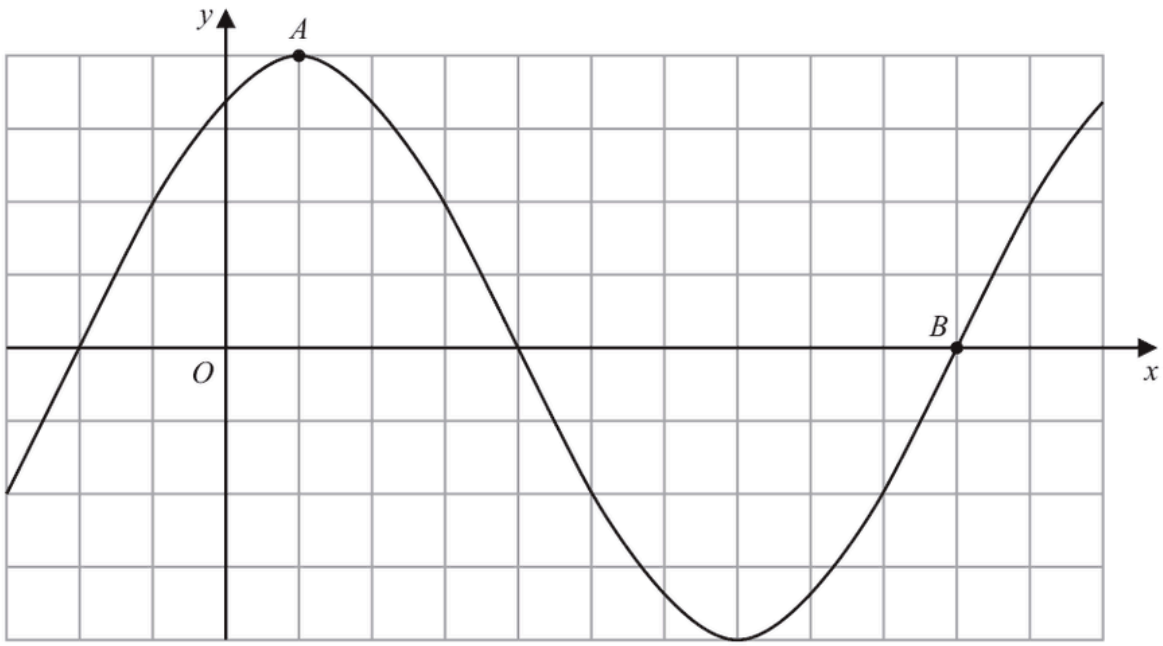
(i) E, (ii) A.

4MA1/2H JUNE 2024

Question 25

2 marks | Graphs of Functions

25 The diagram shows a sketch of the graph of $y = 2\sin(x + 60)°$



(i) Find the coordinates of the point A

(..... , ...)

(1)

(ii) Find the coordinates of the point B

(..... , ...)

(1)

(Total for Question 25 is 2 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H June 2024

1 **Use trigonometry**

$$y = 2\sin(x + 60)°$$

2 **Point is the maximum point**

Point A is the maximum point.

3 **The maximum value is**

The maximum value is

$$y = 2$$

4 **This happens when**

This happens when

$$x + 60 = 90$$

$$x = 30$$

5 **Read the graph**

So

$$A = (30, 2)$$

6 **Point is the positive intercept**

Point B is the positive x-intercept after the minimum.

$$2\sin(x + 60)° = 0$$

$$\sin(x + 60)° = 0$$

7 **Read the graph**

For this point,

$$x + 60 = 360$$

$$x = 300$$

8 **Read the graph**

So

$$B = (300, 0)$$

✓ **FINAL ANSWER**

$A = (30, 2),\ B = (300, 0)$

Question 11

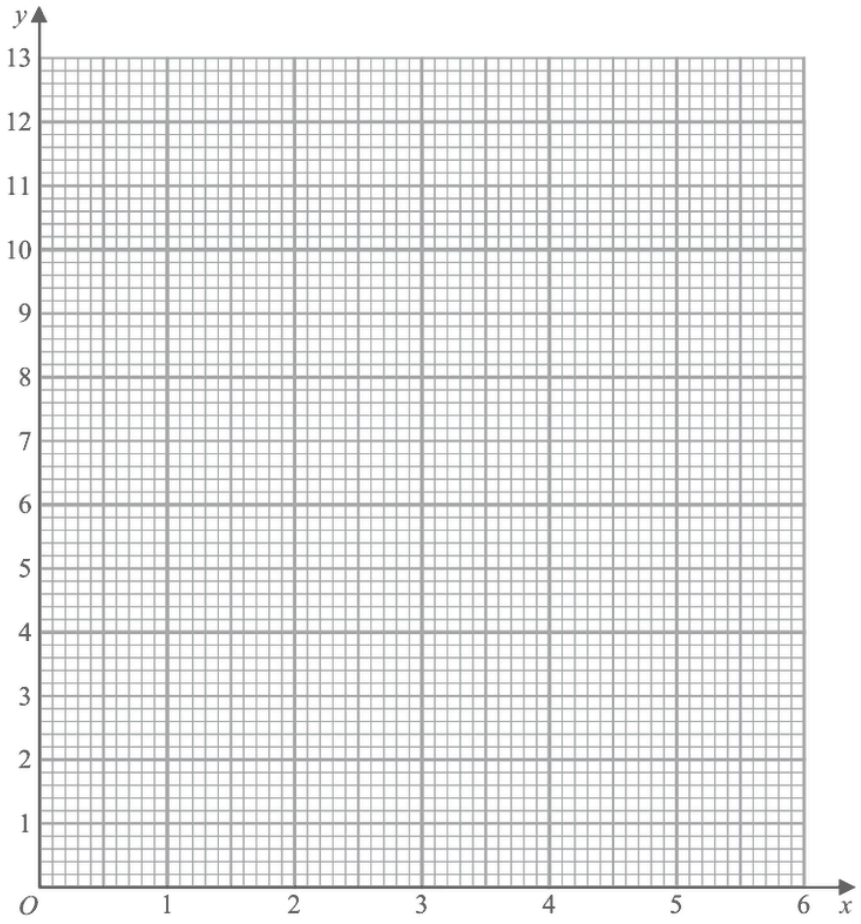
4 marks | Graphs of Functions

11 (a) Complete the table of values for $y = 2\left(x + \frac{1}{x}\right)$

x	0.5	1	2	3	4	5	6
y			5	6.7			12.3

(2)

(b) On the grid, draw the graph of $y = 2\left(x + \frac{1}{x}\right)$ for $0.5 \leqslant x \leqslant 6$



(2)

(Total for Question 11 is 4 marks)

1 Read the graph

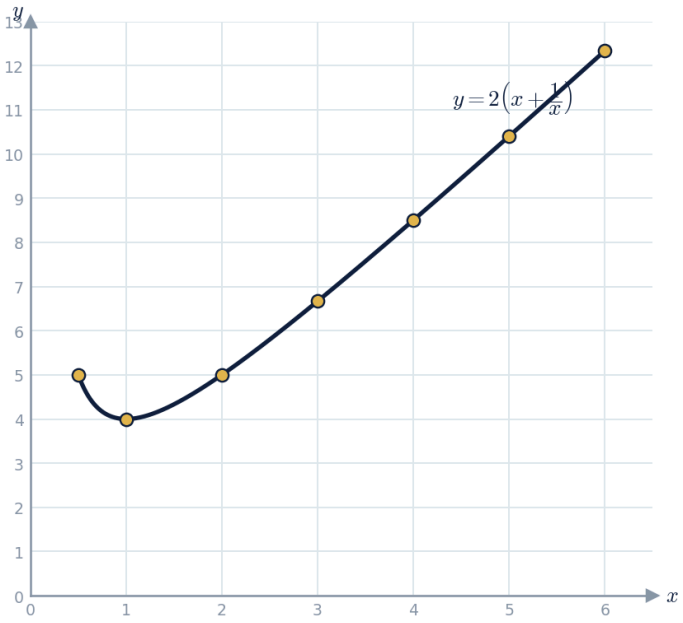
$$y = 2\left(x + \frac{1}{x}\right)$$

2 The completed table is

The completed table is:

x	0.5	1	2	3	4	5	6
y	5	4	5	6.7	8.5	10.4	12.3

3 Plot the points and join them with a smooth curve.



FINAL ANSWER

Missing values are $5, 4, 8.5, 10.4$.

4MA1/1H NOVEMBER 2024

Question 23

5 marks | Graphs of Functions

23 The curve **C** has equation $y = x^2 - 8x - 9$
The straight line **L** has equation $y = k$ where k is an integer.

C and **L** intersect at the points A and B

The coordinates of point A are (p, k)
The coordinates of point B are (q, k)

Given that $p - q = 14$

find the value of k
Show clear algebraic working.

$k =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H November 2024

1 **The curve is**

The curve is

$$y = x^2 - 8x - 9$$

2 **The line is**

The line is

$$y = k$$

3 **Read the graph**

At the intersection points,

$$x^2 - 8x - 9 = k$$

$$x^2 - 8x - (9 + k) = 0$$

4 **Read the graph**

The two roots are p and q.

5 **Solve quadratic equation**

For this quadratic,

$$p + q = 8$$

6 **We are also given**

We are also given

$$p - q = 14$$

7 **Solve the two equations**

Solve the two equations:

$$2p = 22$$

$$p = 11$$

$$q = -3$$

8 **Read the graph**

Use one root in $y = x^2 - 8x - 9$:

$$k = 11^2 - 8(11) - 9$$

$$k = 121 - 88 - 9$$

$$k = 24$$

✓ **FINAL ANSWER**

$k = 24$

Question 10

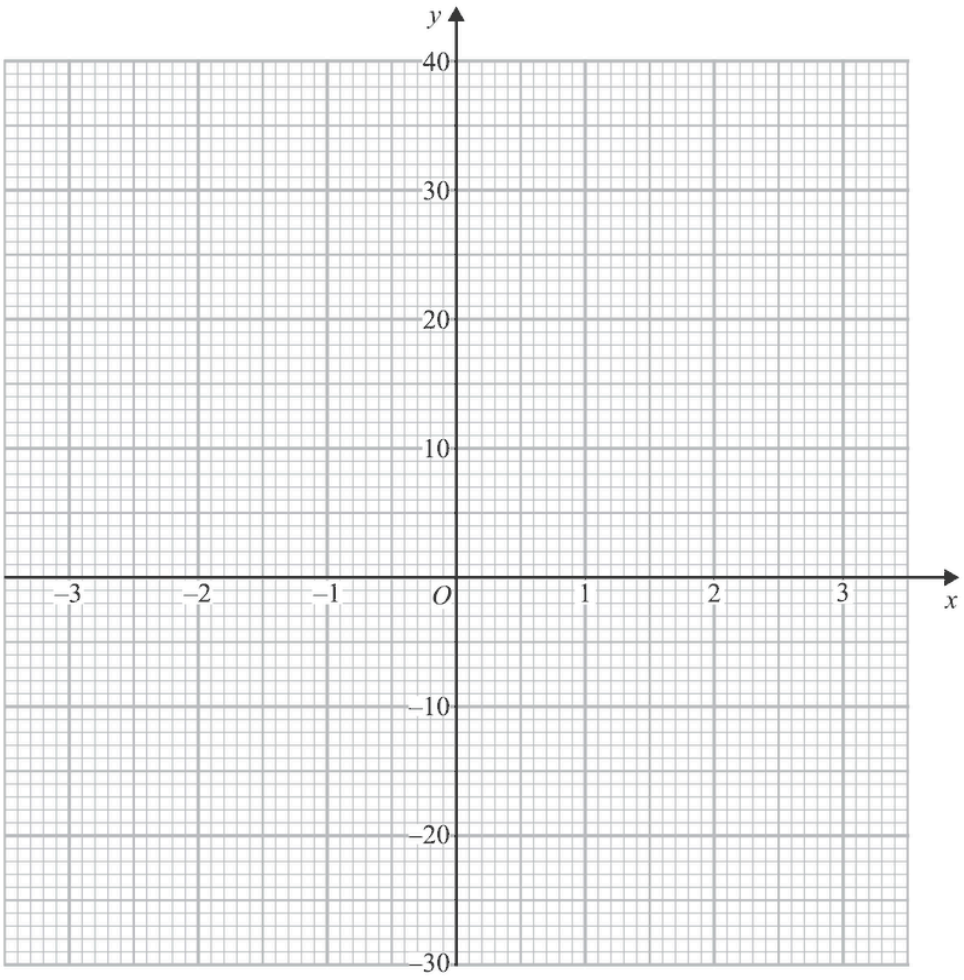
3 marks | Graphs of Functions

10 (a) Complete the table of values for $y = x^3 + 2x + 3$

x	–3	–2	–1	0	1	2	3
y		–9	0	3	6		36

(1)

(b) On the grid, draw the graph of $y = x^3 + 2x + 3$ for $-3 \leqslant x \leqslant 3$



(2)

(Total for Question 10 is 3 marks)

1 Read the graph

$$y = x^3 + 2x + 3$$

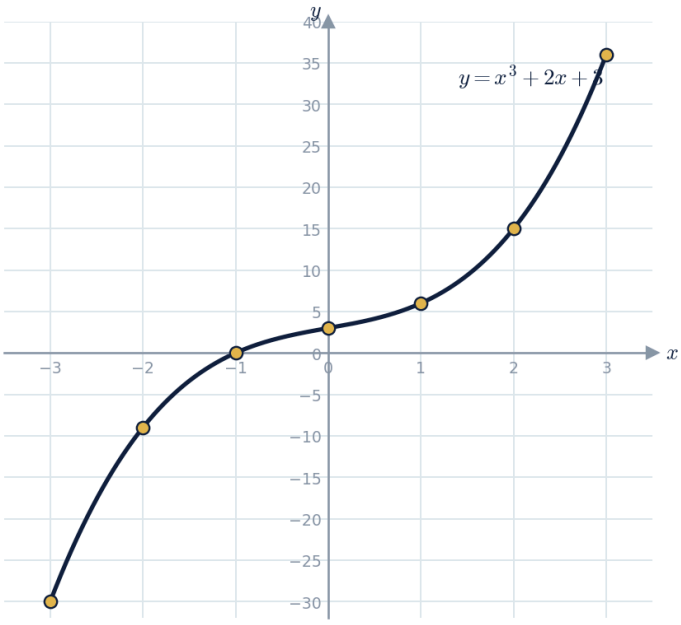
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	-30	-9	0	3	6	15	36

3 Plot the points and join

Plot the points and join them with a smooth cubic curve.



FINAL ANSWER

Missing values are $-30, 15$.

Question 13

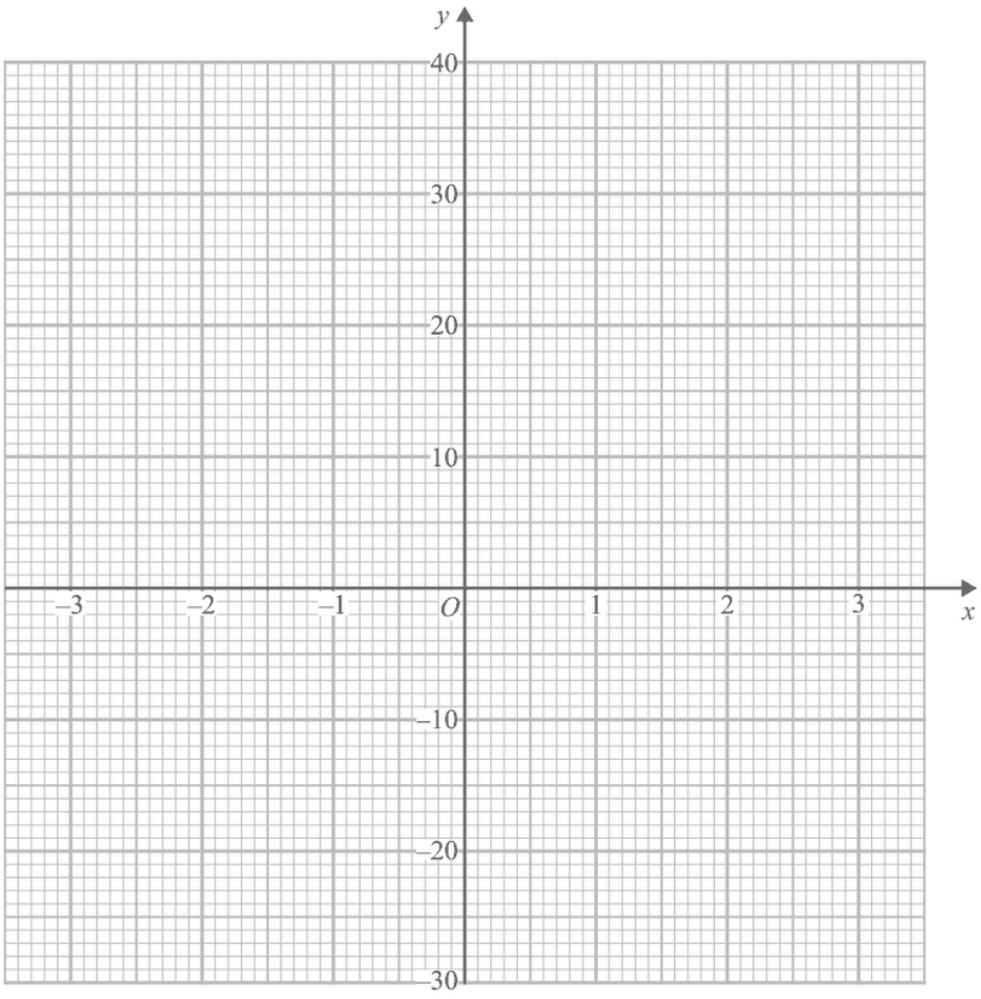
3 marks | Graphs of Functions

13 (a) Complete the table of values for $y = x^3 + 2x + 3$

x	–3	–2	–1	0	1	2	3
y		–9	0	3	6		36

(1)

(b) On the grid, draw the graph of $y = x^3 + 2x + 3$ for $-3 \leqslant x \leqslant 3$



(2)

(Total for Question 13 is 3 marks)

1 **Use**

Use $y = x^3 + 2x + 3$.

2 **Read the graph**

For $x = -3$,

$$y = (-3)^3 + 2(-3) + 3 = -30$$

3 **Read the graph**

For $x = 2$,

$$y = 2^3 + 2(2) + 3 = 15$$

4 **Read the graph**

So the table is

x	-3	-2	-1	0	1	2	3
y	-30	-9	0	3	6	15	36

5 **Plot these points and draw**

Plot these points and draw a smooth cubic curve through them.



✓ **FINAL ANSWER**

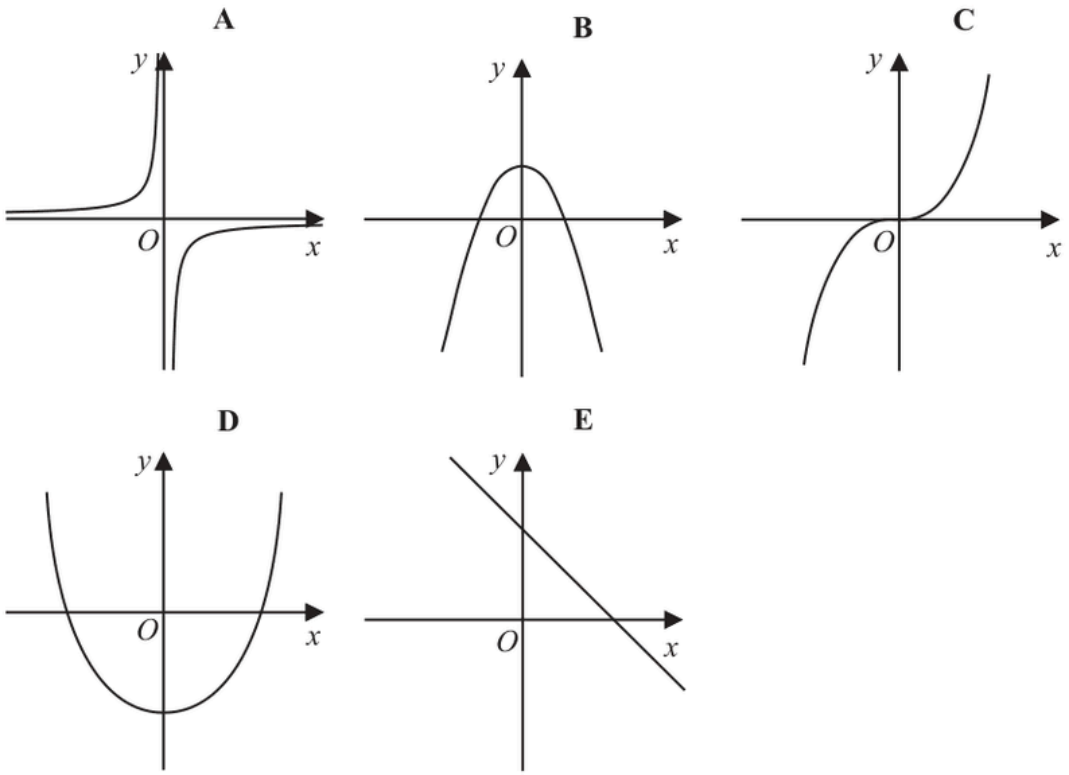
Missing values -30 and 15; draw the smooth curve through the table points.

4MA1/2H JUNE 2025

Question 14

2 marks | Graphs of Functions

14 Here are five graphs.



(a) Write down the letter of the graph that could have the equation $y = 5 - x^2$

.....

(1)

(b) Write down the letter of the graph that could have the equation $y = 2x^3$

.....

(1)

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2025

1 **(a)**

(a)

$$y = 5 - x^2$$

2 **This is a quadratic graph**

This is a quadratic graph with a negative coefficient of x^2, so it is a downward-opening parabola.

3 **That is graph B**

That is graph **B**.

4 **(b)**

(b)

$$y = 2x^3$$

5 **This is an increasing cubic**

This is an increasing cubic graph passing through the origin.

6 **That is graph C**

That is graph **C**.

✓ **FINAL ANSWER**

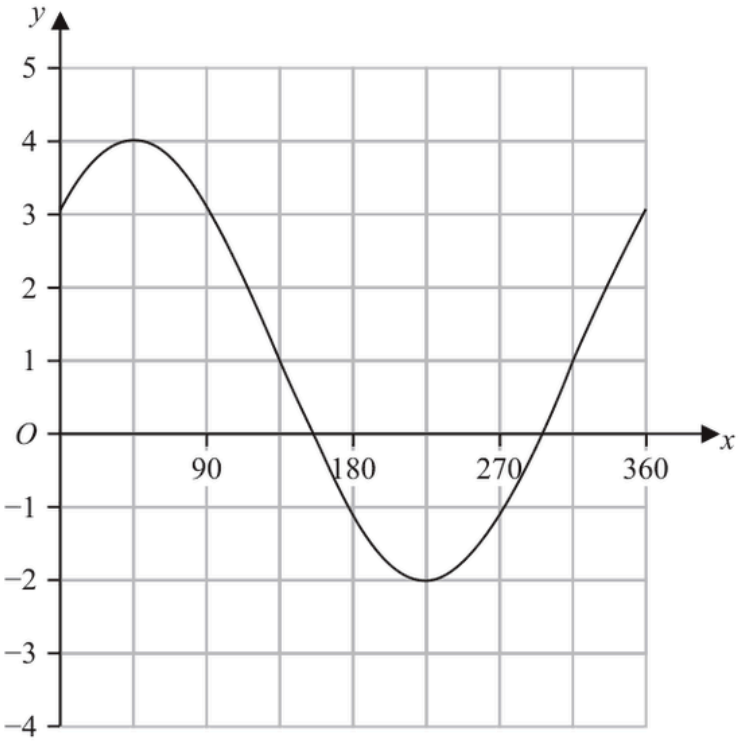
(a) B, (b) C.

4MA1/2HR JUNE 2025

Question 25

3 marks | Graphs of Functions

25 The graph of $y = a\sin(x + b)^\circ + c$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Find a suitable value for a, for b and for c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2025

1 **Read the graph**

From the graph,

$$\text{maximum} = 4$$

$$\text{minimum} = -2$$

2 **The amplitude is**

The amplitude is

$$a = \frac{4 - (-2)}{2} = 3$$

3 **The centre line is**

The centre line is

$$c = \frac{4 + (-2)}{2} = 1$$

4 **Use trigonometry**

The maximum is about $45°$. For a sine graph, a maximum happens when the angle inside the sine is $90°$, so

$$45 + b = 90$$

$$b = 45$$

✓ **FINAL ANSWER**

$a = 3,\ b = 45,\ c = 1$

Question 12

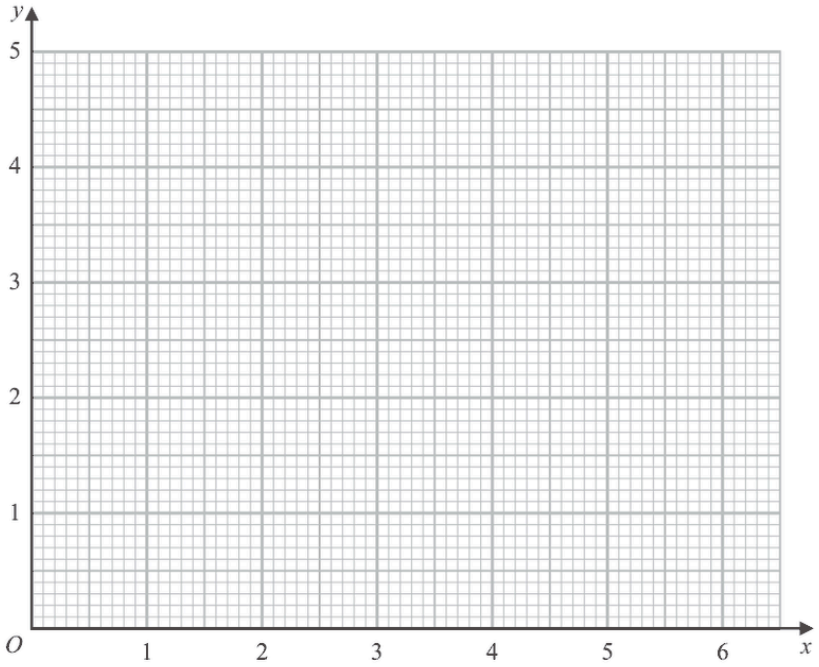
5 marks | Graphs of Functions

12 (a) Complete the table of values for $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$

x	0.5	1	2	3	4	5	6
y	4.25			2.17	2.5	2.9	3.33

(1)

(b) Draw the graph of $y = \frac{1}{2}\left(x + \frac{4}{x}\right)$ for $0.5 \leqslant x \leqslant 6$



(2)

(c) By drawing a suitable line on the grid, find estimates for the solutions of the equation $x + \frac{4}{x} = 6$

Give your answers correct to one decimal place.

.....

(2)

(Total for Question 12 is 5 marks)

1 **Read the graph**

$$y = \frac{1}{2}\left(x + \frac{4}{x}\right)$$

2 **The completed table is**

The completed table is:

x	0.5	1	2	3	4	5	6
y	4.25	2.5	2	2.17	2.5	2.9	3.33

3 **Read the graph**

For

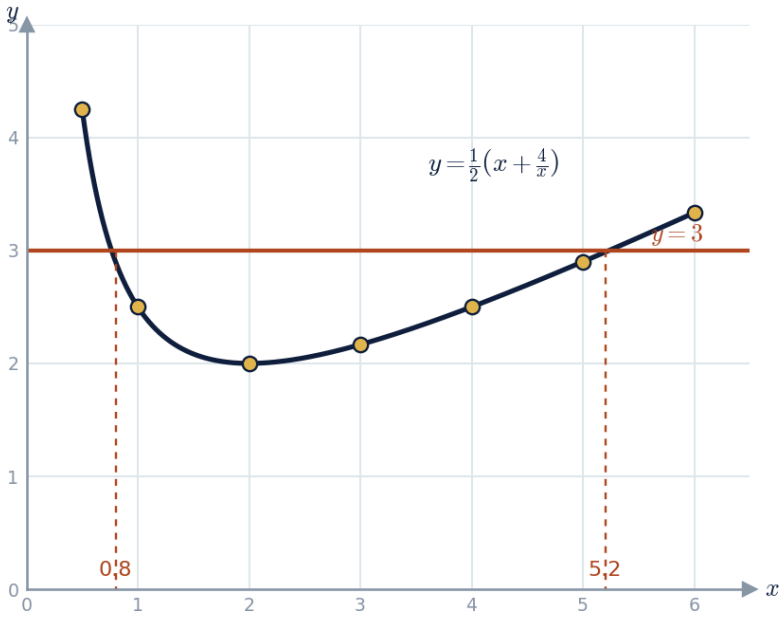
$$x + \frac{4}{x} = 6$$

we need

$$\frac{1}{2}\left(x + \frac{4}{x}\right) = 3$$

4 **Read the graph**

So draw the horizontal line $y = 3$ on the graph.



5 **The intersections give approximately**

The intersections give approximately

$$x = 0.8 \quad \text{and} \quad x = 5.2$$

FINAL ANSWER

Table missing values $2.5, 2$. Estimates 0.8 and 5.2.

Question 13

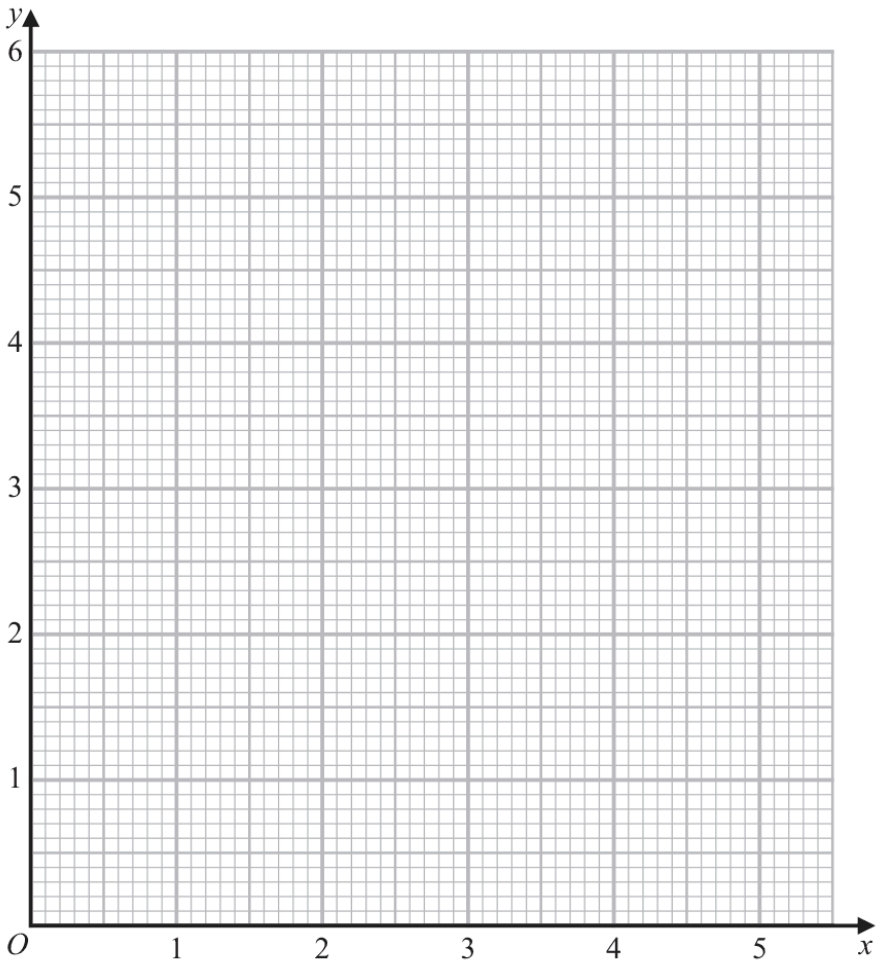
4 marks | Graphs of Functions

13 (a) Complete the table of values for $y = x + \frac{1}{x}$

x	0.25	0.5	1	2	3	4	5
y	4.25			2.5	3.33		

(2)

(b) On the grid, draw the graph of $y = x + \frac{1}{x}$ for $0.25 \leqslant x \leqslant 5$



(2)

(Total for Question 13 is 4 marks)

1

Part (a) | Step 1

Use $y = x + 1/x$:

$$y(0.5) = 0.5 + 2 = 2.5, \quad y(1) = 1 + 1 = 2, \quad y(4) = 4 + 0.25 = 4.25, \quad y(5) = 5 + 0.2 = 5.2.$$

Substitute each required x-value into the formula.

2

Part (b) | Step 1

Plot

$$(0.25, 4.25),\ (0.5, 2.5),\ (1, 2),\ (2, 2.5),\ (3, 10/3),\ (4, 4.25),\ (5, 5.2).$$

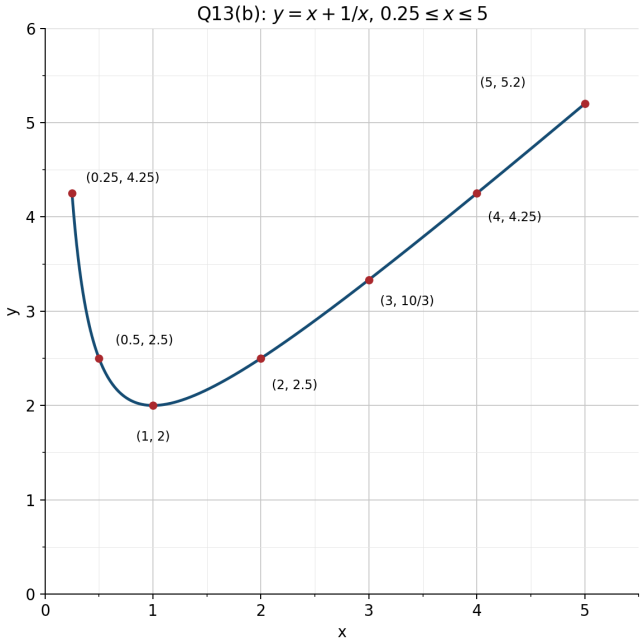
Use x horizontally and y vertically on the supplied grid; 10/3 is approximately 3.33.

3

Part (b) | Step 2

Join the points with one smooth curve for $0.25 \le x \le 5$, descending to its minimum $(1, 2)$ and then ascending.

The graph is curved, not a set of straight segments, and includes both endpoints.



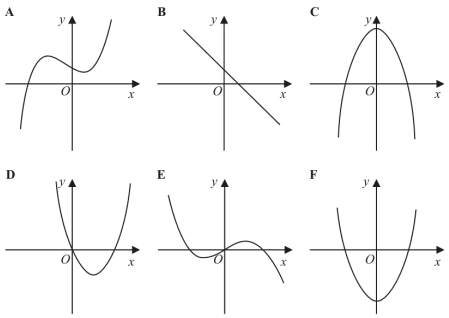
✓ **FINAL ANSWER**

(a) Table entries, in order: $2.5, 2, 4.25, 5.2$. (b) The smooth curve $y = x + 1/x$ through the stated coordinates on $[0.25, 5]$.

Question 14

6 marks | Graphs of Functions

14 Here are six graphs.



(a) Write down the letter of the graph that could have the equation

(i) $y = x^2 - 3x$

.....

(1)

(ii) $y = -x^3 + 9x$

.....

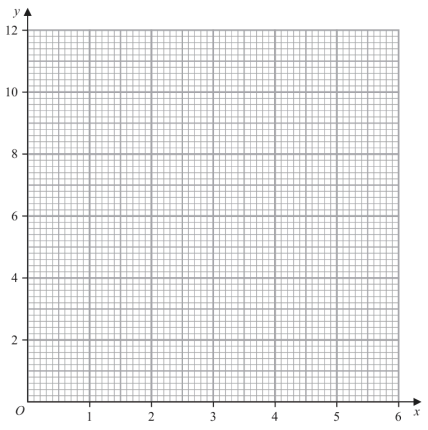
(1)

(b) Complete the table of values for $y = \frac{12}{x^2}$

x	1	1.5	2	3	4	5	6
y		5.33		1.33			0.33

(2)

(c) On the grid, draw the graph of $y = \frac{12}{x^2}$ for $1 \leqslant x \leqslant 6$



(2)

(Total for Question 14 is 6 marks)

1 **(a)(i) Positive quadratic**

$y = x^2 - 3x = x(x-3)$: U-shape through O and $x = 3$ → graph D.

2 **(a)(ii) Negative cubic**

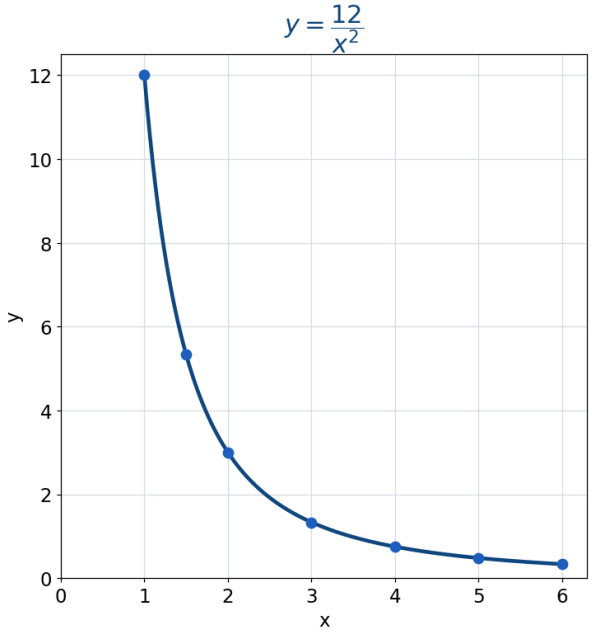
$y = -x^3 + 9x$: negative cubic through the origin → graph E.

3 **(b) Substitute**

$$x = 1 :\ 12, \quad x = 2 :\ 3, \quad x = 4 :\ 0.75, \quad x = 5 :\ 0.48$$

4 **(c) Plot and join with a smooth curve**

Plot all 7 points and draw a smooth curve (no straight segments).



✓ **FINAL ANSWER**

(a)(i) D (ii) E (b) $12,\ 3,\ 0.75,\ 0.48$ (c) smooth curve through the points

TOPIC

Kinematic Graphs

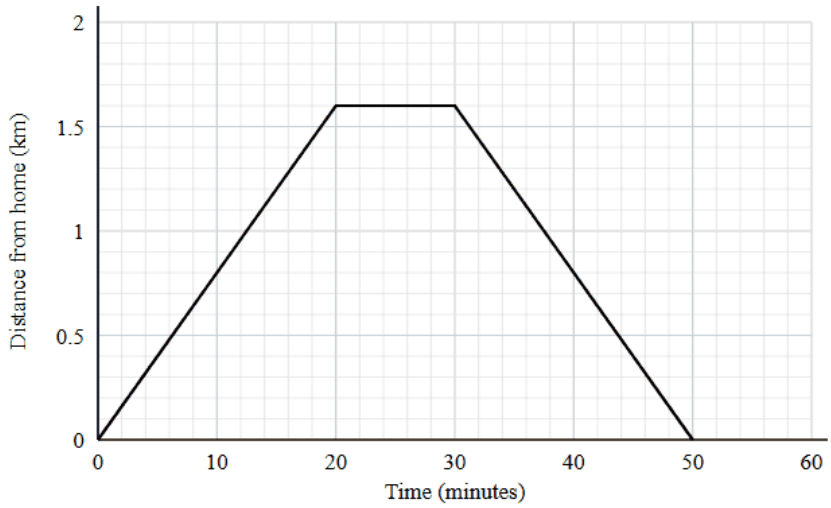
Every past-paper question on this topic, 2020 to 2026.

EP SPECIMEN

Question 8

4 marks | Kinematic Graphs

8 Ali walked from his home to a shop. He stayed at the shop and then walked back home.
Here is the distance-time graph for his journey.



(a) How far was Ali from home after 10 minutes?

..... km

(1)

(b) How many minutes did Ali stay at the shop?

..... minutes

(1)

(c) Work out Ali's speed, in kilometres per hour, on his walk back home.

..... km/h

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · EP Specimen

1 **(a) Read the graph at 10 minutes**

The line from $(0, 0)$ to $(20, 1.6)$ is halfway up at 10 minutes: 0.8 km.

2 **(b) The flat section**

From 20 to 30 minutes: 10 minutes.

3 **(c) Speed = distance ÷ time**

He walks 1.6 km in $50 - 30 = 20$ minutes $= \frac{1}{3}$ hour.

$$1.6 \div \frac{1}{3} = 4.8 \text{ km/h}$$

✓ **FINAL ANSWER**

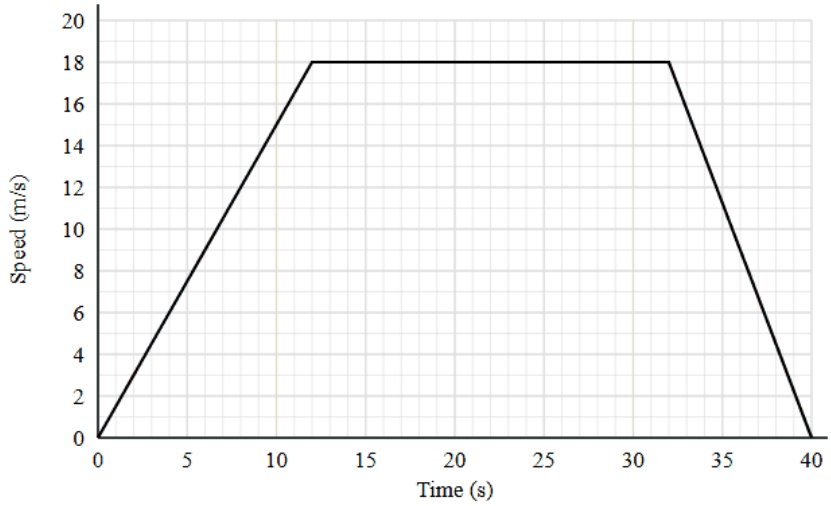
(a) 0.8 km (b) 10 minutes (c) 4.8 km/h

EP SPECIMEN

Question 9

5 marks | Kinematic Graphs

9 Here is the speed-time graph for the first 40 seconds of a car's journey.



(a) Work out the acceleration of the car during the first 12 seconds.

..... m/s 2

(2)

(b) Work out the total distance the car travels in the 40 seconds.

..... m

(3)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · EP Specimen

1 **(a) Acceleration = gradient**

$$\frac{18-0}{12} = 1.5\ \text{m/s}^2$$

2 **(b) Distance = area under the graph**

Split into a triangle, a rectangle and a triangle:

$$\tfrac{1}{2} \times 12 \times 18 + 20 \times 18 + \tfrac{1}{2} \times 8 \times 18 = 108 + 360 + 72 = 540\ \text{m}$$

✓ **FINAL ANSWER**

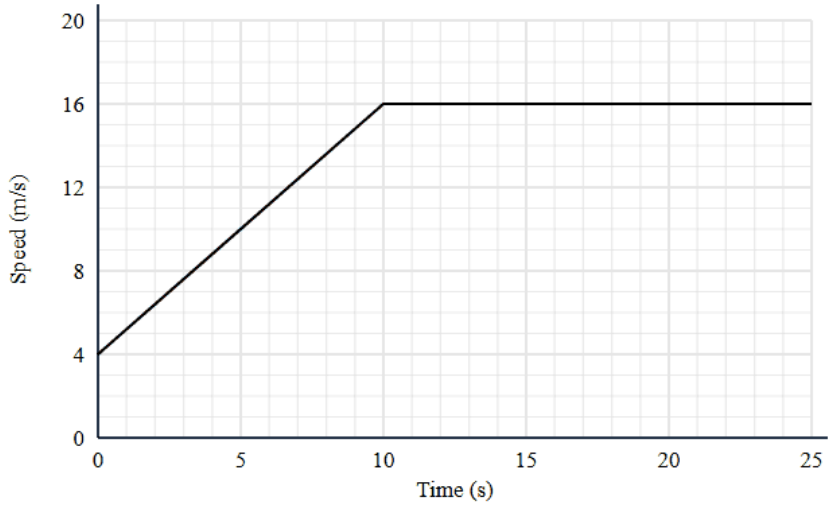
(a) 1.5 m/s^2 (b) 540 m

EP SPECIMEN

Question 10

4 marks | Kinematic Graphs

10 Here is the speed-time graph for 25 seconds of a cyclist's journey.



(a) Work out the distance the cyclist travels in the 25 seconds.

..... m

(3)

(b) Work out the average speed of the cyclist for the 25 seconds.

..... m/s

(1)

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · EP Specimen

1 **(a) Area of the trapezium (first 10 s)**

$$\frac{4+16}{2} \times 10 = 100 \text{ m}$$

2 **(a) Area of the rectangle (last 15 s)**

$$16 \times 15 = 240 \text{ m}$$

Total: $100 + 240 = 340$ m.

3 **(b) Average speed = total distance ÷ total time**

$$340 \div 25 = 13.6 \text{ m/s}$$

FINAL ANSWER

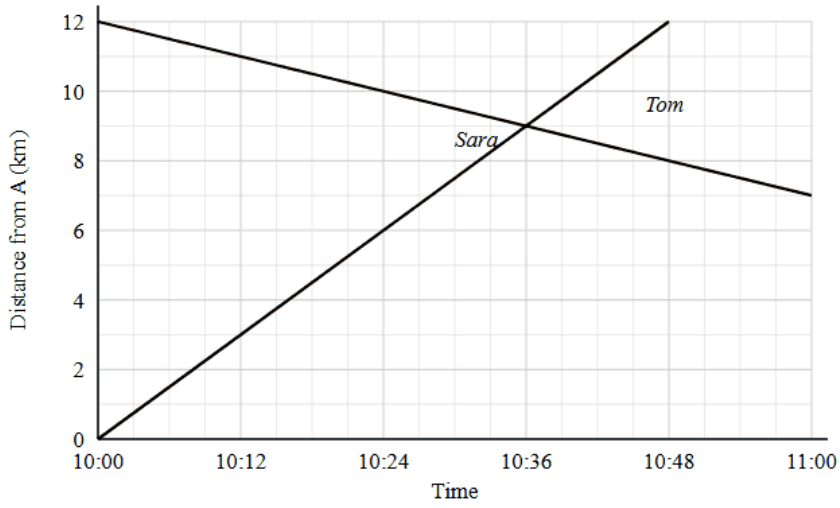
(a) $\mathbf{340}$ m (b) $\mathbf{13.6}$ m/s

EP SPECIMEN

Question 11

4 marks | Kinematic Graphs

11 At 10:00 Sara leaves town A and cycles towards town B.
At 10:00 Tom leaves town B and walks towards town A.
The graph shows their distances from town A.



(a) At what time do Sara and Tom pass each other?

.....

(1)

(b) How far from town A are they when they pass each other?

..... km

(1)

(c) Work out Tom's speed in kilometres per hour.

..... km/h

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · EP Specimen

1 **(a) Where the lines cross**

The lines cross at 36 minutes after 10:00, so at $10{:}36$.

2 **(b) Read across**

At the crossing point the distance from A is 9 km.

3 (c) Tom's speed

Tom goes from 12 km to 7 km from A in 1 hour:

$$\frac{12-7}{1} = 5\ \text{km/h}$$

FINAL ANSWER

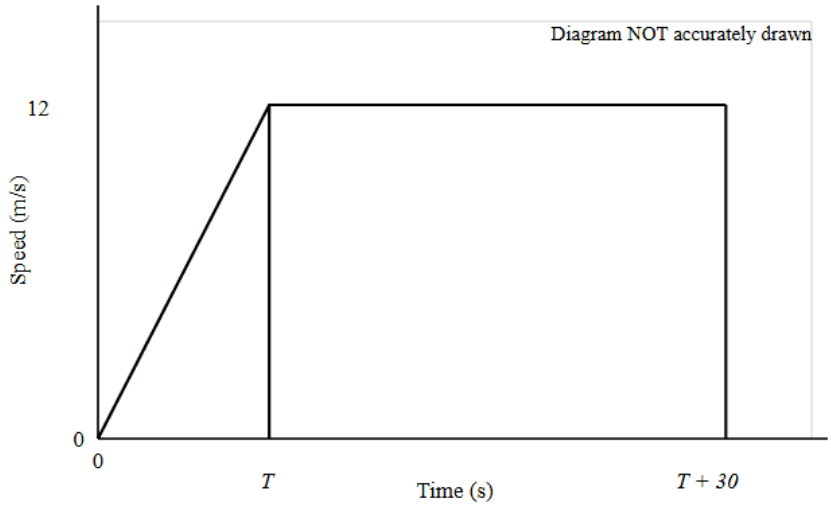
(a) $10{:}36$ (b) 9 km (c) 5 km/h

EP SPECIMEN

Question 12

3 marks | Kinematic Graphs

12 A particle starts from rest. It speeds up at a constant rate for T seconds until its speed is 12 m/s.
It then travels at 12 m/s for 30 seconds.
The total distance it travels is 450 m.



Work out the value of T.

$T =$ .....

(3)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · EP Specimen

1 **Distance = area under the graph**

Triangle: $\frac{1}{2} \times T \times 12 = 6T$. Rectangle: $30 \times 12 = 360$.

2 **Form and solve the equation**

$$6T + 360 = 450 \Rightarrow 6T = 90 \Rightarrow T = 15$$

✓ **FINAL ANSWER**

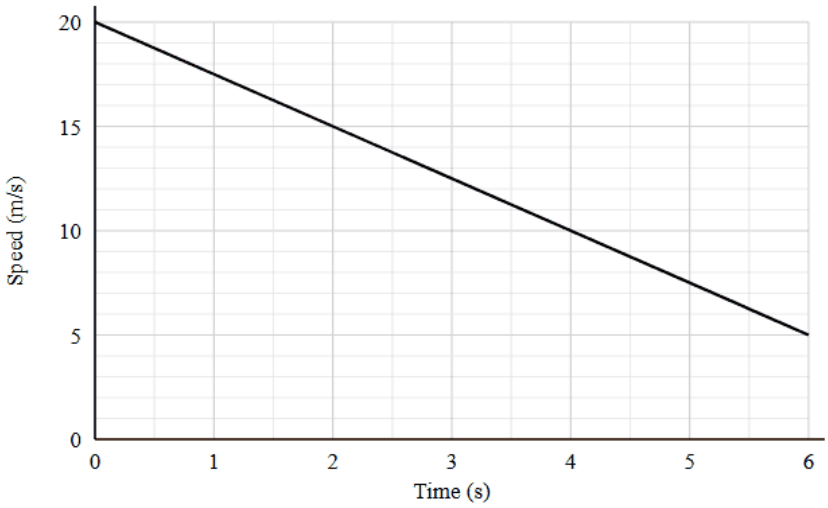
$T = 15$

EP SPECIMEN

Question 13

4 marks | Kinematic Graphs

13 A car is travelling at 20 m/s. The driver brakes and the car slows down at a constant rate. Here is the speed-time graph for the next 6 seconds.



(a) Work out the deceleration of the car.

..... m/s 2

(2)

(b) Work out the distance the car travels in the 6 seconds.

..... m

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · EP Specimen

1 **(a) Gradient of the line**

$$\frac{5-20}{6} = -2.5$$

So the deceleration is 2.5 m/s^2.

2 **(b) Area of the trapezium**

$$\frac{20+5}{2} \times 6 = 75 \text{ m}$$

FINAL ANSWER

(a) 2.5 m/s^2 (b) 75 m

EP SPECIMEN

Question 14

4 marks | Kinematic Graphs

14 A train starts from rest and speeds up at a constant rate for 50 seconds until its speed is 30 m/s.
It then travels at 30 m/s for t seconds.
It then slows down at a constant rate and stops 40 seconds later.
The total distance the train travels is 6 km.

Work out the value of t.

$t =$ .....

(4)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · EP Specimen

1 Sketch the speed-time graph

A trapezium: up for 50 s, flat at 30 m/s for t s, down for 40 s.

2 Distance = area (in metres)

$$\tfrac{1}{2} \times 50 \times 30 + 30t + \tfrac{1}{2} \times 40 \times 30 = 750 + 30t + 600$$

3 Form and solve the equation

6 km $= 6000$ m:

$$1350 + 30t = 6000 \Rightarrow 30t = 4650 \Rightarrow t = 155$$

FINAL ANSWER

$t = 155$

TOPIC

Perimeter & Area

Every past-paper question on this topic, 2020 to 2026.

Question 6

4 marks | Perimeter & Area

6 The diagram shows an 8-sided shape *ABCDEFGH*

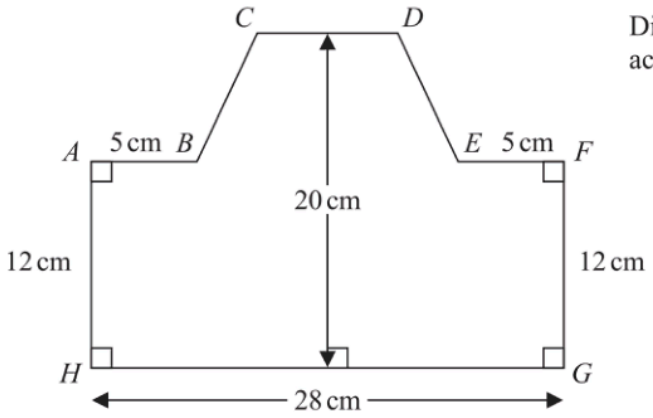


Diagram **NOT** accurately drawn

$HG = 28\,\text{cm}$ $AH = FG = 12\,\text{cm}$ $AB = EF = 5\,\text{cm}$
The height of the shape is 20 cm
CD is parallel to *HG*

The area of shape *ABCDEFGH* is $434\,\text{cm}^2$

Find the length of *CD*

1 Find the rectangular part

The rectangle at the bottom has width 28 cm and height 12 cm.

$$28 \times 12 = 336$$

2 Find the top area

$$434 - 336 = 98$$

3 Set up the trapezium

The top trapezium has height

$$20 - 12 = 8$$

Its lower parallel side is

$$28 - 5 - 5 = 18$$

4 Solve for the missing length

Let $CD = x$.

$$\frac{1}{2}(x + 18) \times 8 = 98$$

$$4(x + 18) = 98$$

$$x = 6.5$$

FINAL ANSWER

$CD = 6.5$ cm.

4MA1/1H JANUARY 2020

Question 3

4 marks | Perimeter & Area

3 The diagram shows a shape.

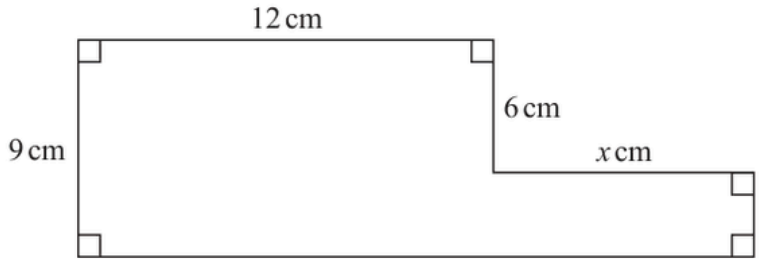


Diagram **NOT** accurately drawn

The shape has area 129 cm^2

Work out the value of x.

$x =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2020

1 Split the shape into two

Split the shape into two rectangles.

2 The left rectangle has area

The left rectangle has area

$$12 \times 9 = 108 \text{ cm}^2$$

3 The right rectangle has height

The right rectangle has height

$$9 - 6 = 3 \text{ cm}$$

4 Calculate area

So its area is

$$3x$$

5 **Total area**

Total area:

$$108 + 3x = 129$$

$$3x = 21$$

$$x = 7$$

✓ **FINAL ANSWER**

$7\ \text{cm}$.

4MA1/1HR JANUARY 2020

Question 5

3 marks | Perimeter & Area

5 The diagram shows Yuen's garden.

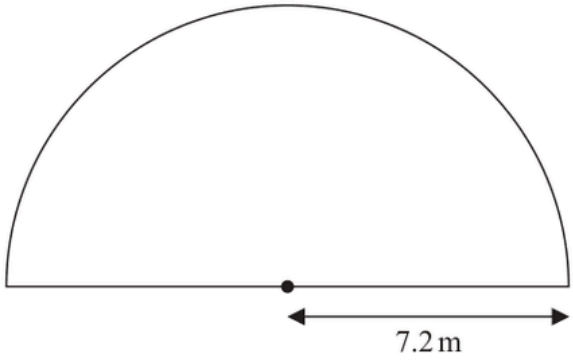


Diagram **NOT** accurately drawn

The garden is in the shape of a semicircle of radius 7.2 m.
Yuen is going to cover his garden with grass seed.

Yuen has 12 boxes of grass seed.
Each box of grass seed contains enough seed to cover 6 m² of the garden.

Has Yuen enough grass seed for his garden?
Show your working clearly.

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2020

1 The garden is a semicircle

The garden is a semicircle of radius $7.2\ \text{m}$.

$$\text{area} = \frac{1}{2}\pi r^2$$

$$\text{area} = \frac{1}{2}\pi(7.2)^2 = 81.43\ldots\ \text{m}^2$$

2 Calculate area

Yuen has 12 boxes, and each box covers $6\ \text{m}^2$:

$$12 \times 6 = 72\ \text{m}^2$$

3 Calculate area

Since

$$72 < 81.43\ldots$$

4

Yuen does not have enough

Yuen does not have enough grass seed.

✓ **FINAL ANSWER**

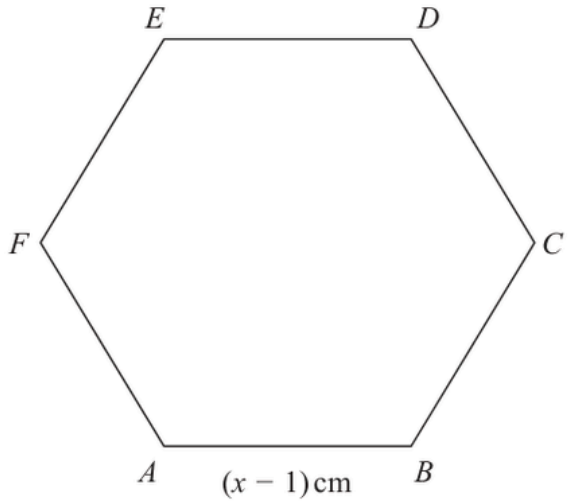
No, he does not have enough grass seed.

4MA1/1HR JANUARY 2020

Question 10

5 marks | Perimeter & Area

10 The diagram shows a regular hexagon, $ABCDEF$, and an isosceles triangle, GHJ.



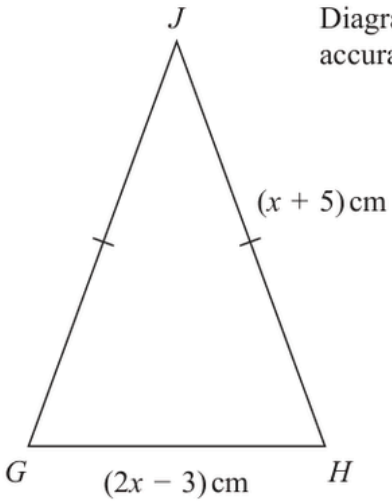


Diagram **NOT** accurately drawn

The perimeter of the hexagon is equal to the perimeter of the triangle.

Find the length of each side of the hexagon.
Show clear algebraic working.

..... cm

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2020

1 **The regular hexagon has side**

The regular hexagon has side length $(x - 1)$ cm, so its perimeter is

$$6(x - 1)$$

2 **The isosceles triangle has perimeter**

The isosceles triangle has perimeter

$$(x + 5) + (x + 5) + (2x - 3) = 4x + 7$$

3 The perimeters are equal

The perimeters are equal:

$$6(x-1) = 4x + 7$$

$$6x - 6 = 4x + 7$$

$$2x = 13$$

$$x = 6.5$$

---

4 Each side of the hexagon

Each side of the hexagon is

$$x - 1 = 6.5 - 1 = 5.5$$

FINAL ANSWER

$5.5\ \text{cm}$.

4MA1/2H JANUARY 2020

Question 11

5 marks | Perimeter & Area

11 The diagram shows a shaded shape $ABCD$ made from a semicircle ABC and a right-angled triangle ACD.

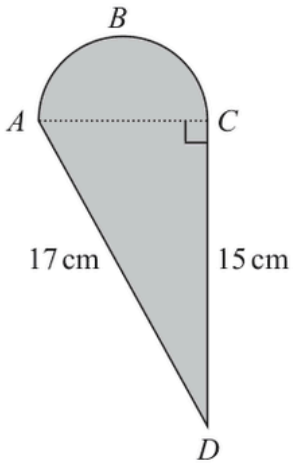


Diagram **NOT** accurately drawn

AC is the diameter of the semicircle ABC.

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2020

1 Use trigonometry

Triangle ACD is right-angled, with

$$AD = 17 \text{ cm}, \qquad CD = 15 \text{ cm}$$

2 Use Pythagoras to find

Use Pythagoras to find AC:

$$AC^2 = 17^2 - 15^2 = 289 - 225 = 64$$

$$AC = 8 \text{ cm}$$

3 **Calculate area**

So the semicircle has radius

$$r = 4 \text{ cm}$$

4 **The arc length of the**

The arc length of the semicircle is

$$\pi r = 4\pi$$

5 **The perimeter of the shaded**

The perimeter of the shaded shape is

$$AD + CD + \text{semicircle arc} = 17 + 15 + 4\pi = 44.566\ldots$$

✓ **FINAL ANSWER**

44.6 cm.

4MA1/1H MAY/NOVEMBER 2020

Question 11

6 marks | Perimeter & Area

11 The diagram shows trapezium $ABCD$ in which BC and AD are parallel.

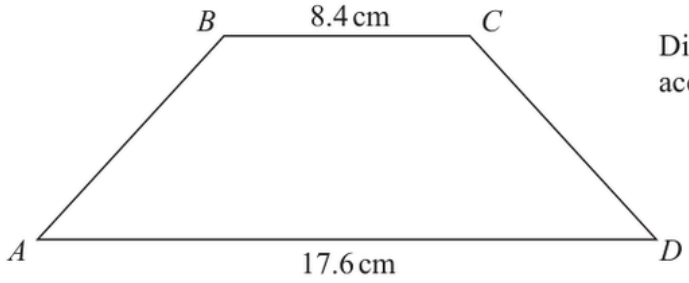


Diagram **NOT** accurately drawn

The trapezium has exactly one line of symmetry.

$BC = 8.4\,\text{cm}$
$AD = 17.6\,\text{cm}$

The trapezium has area $179.4\,\text{cm}^2$

Work out the size of angle ABC.
Give your answer correct to 1 decimal place.

.....°

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May/November 2020

1 **The average of the parallel**

The average of the parallel sides is

$$\frac{8.4 + 17.6}{2} = 13$$

2 **Find the gradient**

So the perpendicular height is

$$179.4 \div 13 = 13.8$$

3 **The horizontal offset on each**

The horizontal offset on each side is

$$\frac{17.6 - 8.4}{2} = 4.6$$

4 **Use trigonometry**

The acute angle between AB and the horizontal is

$$\tan^{-1}\left(\frac{13.8}{4.6}\right) = 71.565\ldots^\circ$$

5 **Use trigonometry**

So the interior angle ABC is

$$180^\circ - 71.565\ldots^\circ = 108.434\ldots^\circ$$

✓ **FINAL ANSWER**

108.4°.

4MA1/1HR MAY/NOVEMBER 2020

Question 22

6 marks | Perimeter & Area

22 The diagram shows a regular octagon *ABCDEFGH*.

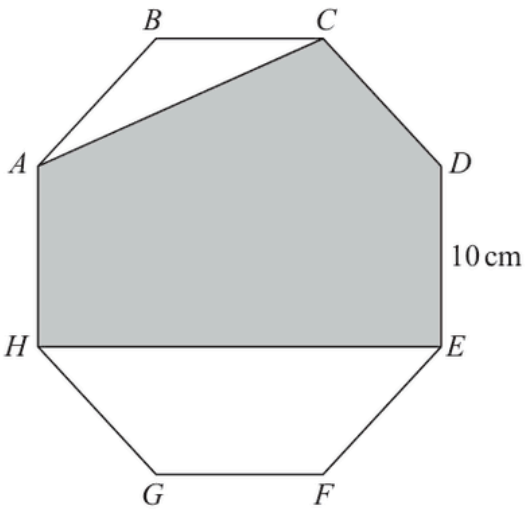


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May/November 2020

1 **Each side of the regular**

Each side of the regular octagon is 10 cm.

2 **Calculate area**

For a regular octagon with side length s,

$$\text{area} = 2(1+\sqrt{2})s^2$$

3 **Calculate area**

So the area of the whole octagon is

$$2(1+\sqrt{2})(10)^2 = 200 + 200\sqrt{2}$$

4 **The unshaded triangle has**

The unshaded triangle ABC has

$$AB = 10, \qquad BC = 10, \qquad \angle ABC = 135^\circ$$

5 **Use trigonometry**

So

$$\text{area of triangle } ABC = \frac{1}{2}(10)(10)\sin 135^\circ = 25\sqrt{2}$$

6 **Find the gradient**

At the bottom, the unshaded trapezium $HEFG$ has parallel sides

$$HE = 10 + 10\sqrt{2}, \qquad GF = 10$$

7 **Simplify surd**

and height

$$5\sqrt{2}$$

8 **Calculate area**

So

$$\text{area of trapezium } HEFG = \frac{1}{2}(10 + 10 + 10\sqrt{2})(5\sqrt{2})$$

$$= 50 + 50\sqrt{2}$$

9 **The shaded area is**

The shaded area is

$$(200 + 200\sqrt{2}) - 25\sqrt{2} - (50 + 50\sqrt{2})$$

$$= 150 + 125\sqrt{2}$$

$$= 326.7766\ldots$$

FINAL ANSWER

$\boxed{327\ \text{cm}^2}$

4MA1/2HR JANUARY 2021

Question 2

3 marks | Perimeter & Area

2 The diagram shows a rectangle $ABCD$ and a semicircle with diameter AB where $AB = 12\,\text{cm}$. The point E lies on DC and also on the semicircle.

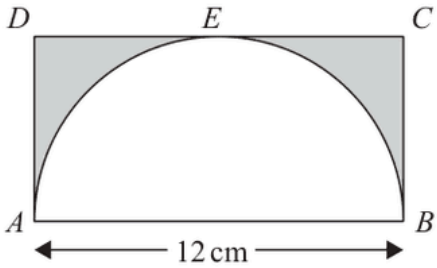


Diagram **NOT** accurately drawn

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR January 2021

1 **Calculate area**

The semicircle has diameter $12\ \text{cm}$, so its radius is $6\ \text{cm}$.

2 **The rectangle has area**

The rectangle has area

$$12 \times 6 = 72$$

3 **Area of the semicircle**

Area of the semicircle:

$$\frac{1}{2}\pi(6)^2 = 18\pi$$

4 **Shaded area**

Shaded area:

$$72 - 18\pi = 15.451\ldots$$

FINAL ANSWER

$15.5\ \text{cm}^2$.

4MA1/2H JANUARY 2021

Question 8

5 marks | Perimeter & Area

8 A, B and C are points on a circle with centre O.

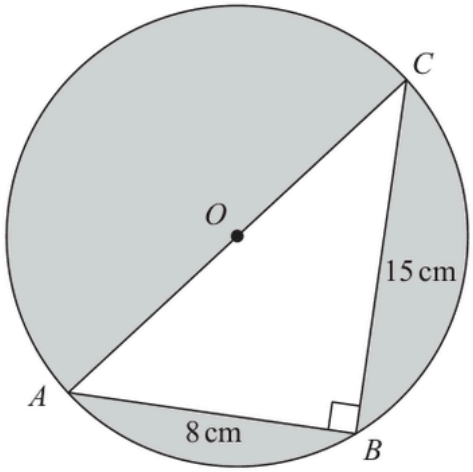


Diagram **NOT** accurately drawn

AOC is a diameter of the circle.

$AB = 8\,\text{cm}$ $BC = 15\,\text{cm}$

Angle $ABC = 90°$

Work out the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2021

1 **Triangle is right angled**

Triangle ABC is right-angled:

$$AC^2 = 8^2 + 15^2 = 64 + 225 = 289$$

$$AC = 17\,\text{cm}$$

2 **Calculate area**

Since AOC is a diameter, the radius of the circle is

$$17 \div 2 = 8.5 \text{ cm}$$

3 **Area of the circle**

Area of the circle:

$$\pi(8.5)^2 = 226.98\ldots \text{ cm}^2$$

4 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$$

5 **The shaded area is**

The shaded area is

$$226.98\ldots - 60 = 166.98\ldots$$

✓ **FINAL ANSWER**

167 cm^2.

4MA1/1HR JANUARY 2021

Question 10

5 marks | Perimeter & Area

10 The diagram shows one face of a wall.
This face is in the shape of a pentagon with exactly one line of symmetry.

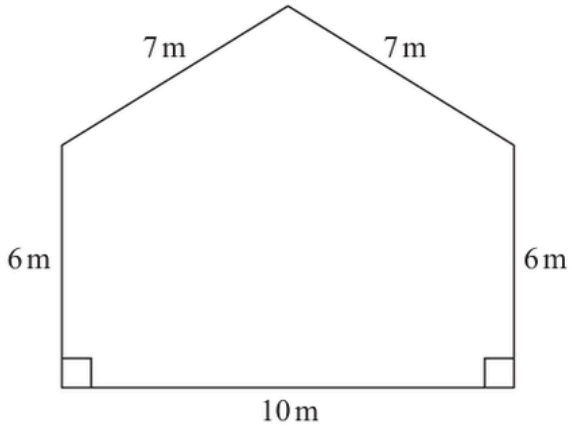


Diagram **NOT** accurately drawn

Omondi is going to paint this face of the wall once.
He has to buy all the paint that he needs to use.

The paint in each tin of paint Omondi is going to buy will cover $16\,m^2$ of the face of the wall.

Work out the least number of tins of paint Omondi will need to buy.
Show your working clearly.

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2021

1 **The wall is a rectangle**

The wall is a rectangle plus an isosceles triangle.

2 **Rectangle area**

Rectangle area:

$$10 \times 6 = 60$$

3 **Use trigonometry**

For the triangle, half the base is 5 m.

$$h^2 = 7^2 - 5^2 = 24$$

$$h = \sqrt{24}$$

4 **Triangle area**

Triangle area:

$$\frac{1}{2} \times 10 \times \sqrt{24} = 24.494\ldots$$

5 **Total area**

Total area:

$$60 + 24.494\ldots = 84.494\ldots$$

6 **Each tin covers**

Each tin covers $16\ \mathrm{m}^2$:

$$\frac{84.494\ldots}{16} = 5.28\ldots$$

7 **Calculate area**

Omondi must buy whole tins, so he needs 6 tins.

✓ **FINAL ANSWER**

6 tins.

4MA1/2H JANUARY 2021

Question 20

4 marks | Perimeter & Area

20

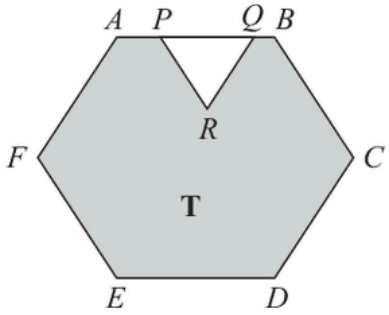


Diagram **NOT** accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle PQR from a regular hexagon $ABCDEF$.

The points P and Q lie on AB such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3}$ cm^2

work out the length of PQ.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2021

1 Let

Let

$$PQ = p$$

2 Given

Given

$$AB = 1.5PQ$$

3 Find the gradient

so the side length of the regular hexagon is

$$1.5p = \frac{3}{2}p$$

4 **Area of a regular hexagon**

Area of a regular hexagon with side length s is

$$\frac{3\sqrt{3}}{2}s^2$$

5 **Find the gradient**

So the area of the hexagon is

$$\frac{3\sqrt{3}}{2}\left(\frac{3}{2}p\right)^2$$

$$=\frac{27\sqrt{3}}{8}p^2$$

6 **Find the gradient**

The removed triangle PQR is equilateral, so its area is

$$\frac{\sqrt{3}}{4}p^2$$

7 **The shaded area is**

The shaded area is

$$\frac{27\sqrt{3}}{8}p^2-\frac{\sqrt{3}}{4}p^2$$

$$=\frac{25\sqrt{3}}{8}p^2$$

8 **Find the gradient**

This is $72\sqrt{3}$, so

$$\frac{25\sqrt{3}}{8}p^2=72\sqrt{3}$$

$$\frac{25}{8}p^2=72$$

$$p^2=\frac{576}{25}$$

$$p=4.8$$

FINAL ANSWER

$PQ = 4.8\ \text{cm}$

4MA1/1H MAY 2021

Question 3

4 marks | Perimeter & Area

3

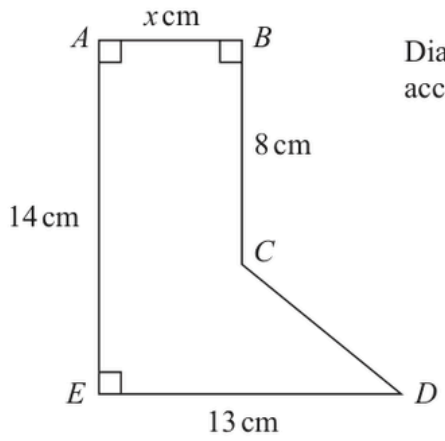


Diagram **NOT** accurately drawn

The diagram shows the shape $ABCDE$.

The area of the shape is $91.8\,\text{cm}^2$

Work out the value of x.

$x =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2021

1 Split the shape into a

Split the shape into a rectangle and a triangle.

2 Rectangle area

Rectangle area:

$$14x$$

3 Use trigonometry

The triangle has base $13 - x$ and height 6, so its area is

$$\frac{1}{2}(13 - x)(6) = 39 - 3x$$

4 **Total area**

Total area:

$$14x + 39 - 3x = 91.5$$

$$11x = 52.5$$

$$x = 4.772\ldots$$

✓ **FINAL ANSWER**

$4.77\ \text{cm}$.

4MA1/1H NOVEMBER 2021

Question 4

5 marks | Perimeter & Area

4 The diagram shows the front of a wooden door with a semicircular glass window.

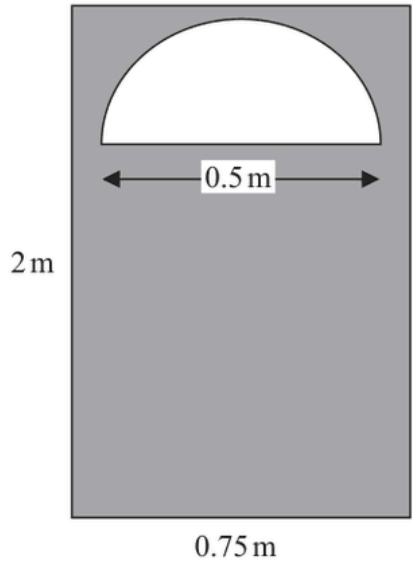


Diagram **NOT** accurately drawn

Julie wants to apply 2 coats of wood varnish to the front of the door, shown shaded in the diagram.

250 millilitres of wood varnish covers $4\,m^2$ of the wood.

Work out how many millilitres of wood varnish Julie will need.
Give your answer correct to the nearest millilitre.

..... millilitres

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2021

1 **Area of the rectangular door**

Area of the rectangular door:

$$0.75 \times 2 = 1.5 \text{ m}^2$$

2 **The semicircular window has diameter**

The semicircular window has diameter 0.5 m, so

$$r = 0.25 \text{ m}$$

3 Area of the semicircular window

Area of the semicircular window:

$$\frac{1}{2}\pi(0.25)^2 = 0.098174\ldots \text{ m}^2$$

4 Shaded area for one coat

Shaded area for one coat:

$$1.5 - 0.098174\ldots = 1.401825\ldots \text{ m}^2$$

5 Calculate area

For 2 coats:

$$2 \times 1.401825\ldots = 2.80365\ldots \text{ m}^2$$

6 Calculate area

Since 250 ml covers 4 m^2, varnish needed is

$$\frac{2.80365\ldots}{4} \times 250 = 175.22\ldots$$

FINAL ANSWER

175 millilitres.

4MA1/2H NOVEMBER 2021

Question 10

4 marks | Perimeter & Area

10 ABC is an isosceles triangle with $BA = BC$.

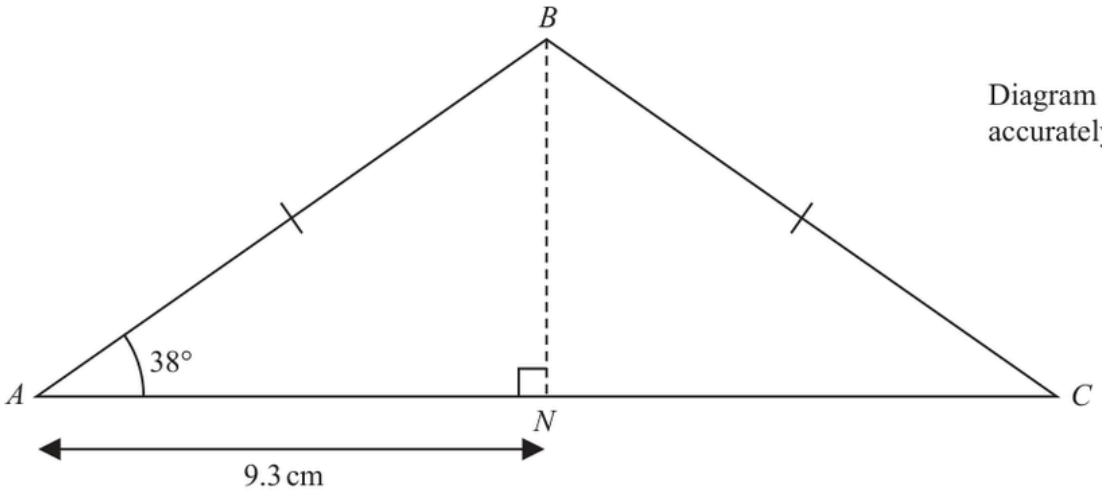


Diagram **NOT** accurately drawn

N is the point on AC such that $AN = 9.3$ cm and BN is perpendicular to AC.

Work out the perimeter of triangle ABC.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2021

1 Use trigonometry

In right-angled triangle ABN,

$$\cos 38° = \frac{9.3}{AB}$$

$$AB = \frac{9.3}{\cos 38°} = 11.802\ldots$$

2 Calculate area

Since ABC is isosceles,

$$BC = AB$$

3 **Calculate area**

Also,

$$AC = 2 \times 9.3 = 18.6$$

---

4 **Perimeter**

Perimeter:

$$11.802\ldots + 11.802\ldots + 18.6 = 42.204\ldots$$

✓ **FINAL ANSWER**

$42.2\ \text{cm}$.

4MA1/2H JUNE 2022

Question 5

4 marks | Perimeter & Area

5 The diagram shows an 8-sided shape *ABCDEFGH*.

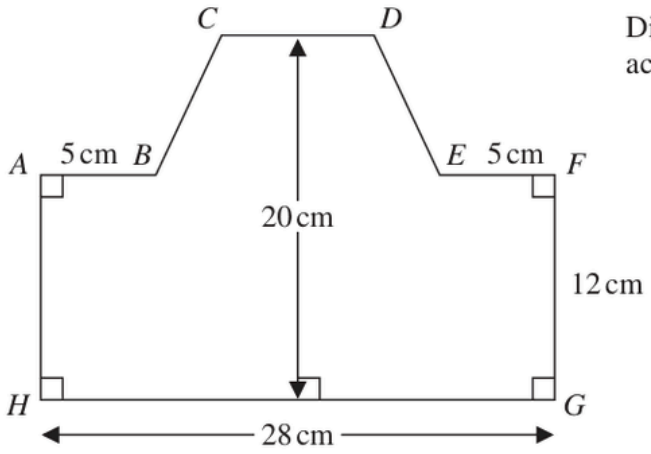


Diagram **NOT** accurately drawn

$HG = 28\,\text{cm}$ $FG = 12\,\text{cm}$ $AB = EF = 5\,\text{cm}$
The height of the shape is 20 cm
CD is parallel to *HG*

The area of shape *ABCDEFGH* is $434\,\text{cm}^2$

Find the length of *CD*.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2022

1 **The lower rectangle has width**

The lower rectangle has width $28\ \text{cm}$ and height $12\ \text{cm}$:

$$28 \times 12 = 336\ \text{cm}^2$$

2 **The top part is a**

The top part is a trapezium of height

$$20 - 12 = 8\ \text{cm}$$

3 **Find the gradient**

The lower parallel side of this trapezium is

$$28 - 5 - 5 = 18\ \text{cm}$$

4 **Area of the trapezium**

Area of the trapezium:

$$434 - 336 = 98 \text{ cm}^2$$

---

5 **Use**

Use

$$\text{area} = \frac{1}{2}(a+b)h$$

$$98 = \frac{1}{2}(18+CD)(8)$$

$$98 = 4(18+CD)$$

$$18+CD = 24.5$$

$$CD = 6.5$$

✓ **FINAL ANSWER**

6.5 cm.

4MA1/2HR JUNE 2022

Question 5

4 marks | Perimeter & Area

5 The diagram shows a shape made from a square $ABCD$ and 4 identical semicircles.

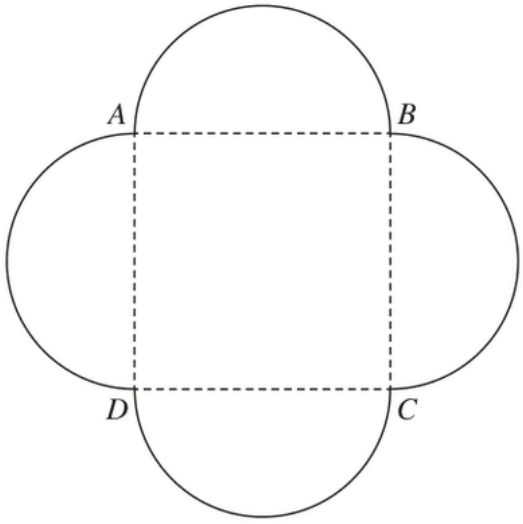


Diagram **NOT** accurately drawn

As shown in the diagram, the semicircles have AB, BC, CD and DA as diameters.

The area of the square is $36\,\text{cm}^2$

Calculate the total area of the shape.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2022

1 **Calculate area**

The square has area $36\ \text{cm}^2$, so its side length is

$$\sqrt{36} = 6\ \text{cm}$$

2 **Calculate area**

Each semicircle has diameter $6\ \text{cm}$, so radius

$$r = 3\ \text{cm}$$

3 Area of 4 semicircles

Area of 4 semicircles:

$$4 \times \frac{1}{2}\pi(3)^2 = 18\pi$$

4 Total area

Total area:

$$36 + 18\pi = 92.548\ldots$$

FINAL ANSWER

$92.5\ \text{cm}^2$.

Question 11

6 marks | Perimeter & Area

11 The diagram shows a quadrilateral $ABCD$

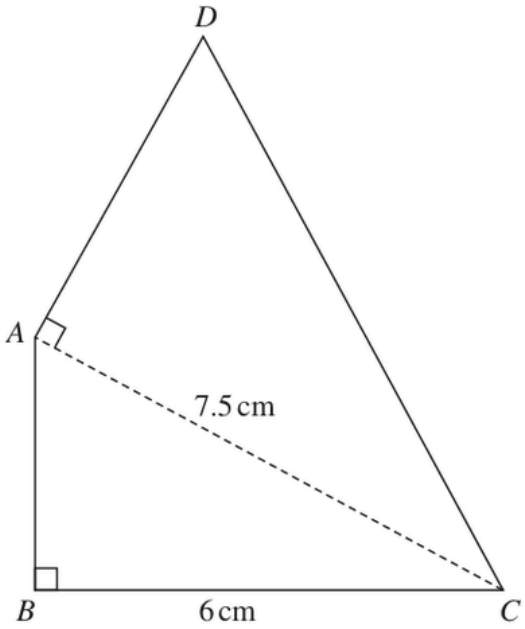


Diagram **NOT** accurately drawn

In the diagram, ABC and DAC are right-angled triangles.

$BC = 6\,\text{cm}$ $AC = 7.5\,\text{cm}$

The area of quadrilateral $ABCD$ is $31.5\,\text{cm}^2$

Work out the length of AD

.....cm

(Total for Question 11 is 6 marks)

1 **Use trigonometry**

In right-angled triangle ABC,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 6^2 = 7.5^2$$

$$AB^2 = 56.25 - 36 = 20.25$$

$$AB = 4.5 \text{ cm}$$

2 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 4.5 = 13.5 \text{ cm}^2$$

3 **Area of triangle**

Area of triangle ADC:

$$31.5 - 13.5 = 18 \text{ cm}^2$$

4 **Use trigonometry**

Triangle ADC is right-angled, so

$$\frac{1}{2} \times AD \times 7.5 = 18$$

$$AD = \frac{36}{7.5} = 4.8$$

✓ **FINAL ANSWER**

4.8 cm.

4MA1/2HR JUNE 2023

Question 5

4 marks | Perimeter & Area

5 A field is in the shape of a trapezium.

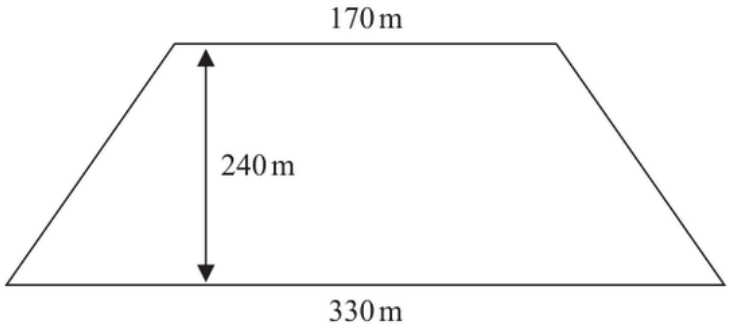


Diagram **NOT** accurately drawn

The field is sold for a price of \$49 650

Given that 1 hectare = 10 000 m²

work out the average price of the field per hectare.

\$.....

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2023

1 Calculate area

$$\text{area} = \frac{1}{2}(170 + 330) \times 240 = 60000 \text{ m}^2$$

$$60000 \text{ m}^2 = 6 \text{ hectares}$$

$$\text{price per hectare} = 49650 \div 6 = 8275$$

FINAL ANSWER

\$8275.

4MA1/2H NOVEMBER 2023

Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

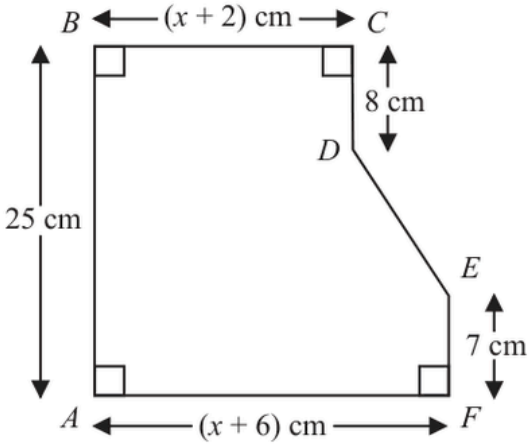


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad BC = (x + 2)$ cm $\quad CD = 8$ cm $\quad EF = 7$ cm $\quad AF = (x + 6)$ cm

The area of hexagon $ABCDEF$ is 258 cm²

Work out the value of x

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2023

1 The area of the hexagon

The area of the hexagon can be found from coordinates or by splitting the shape.

2 Calculate area

It simplifies to

$$\text{area} = 25x + 98$$

3 **Calculate area**

Given the area is $258\ \text{cm}^2$,

$$25x + 98 = 258$$

$$25x = 160$$

$$x = 6.4$$

FINAL ANSWER

$x = 6.4$.

4MA1/2H JUNE 2024

Question 7

4 marks | Perimeter & Area

7 The diagram shows a 6-sided shape $ABCDEF$

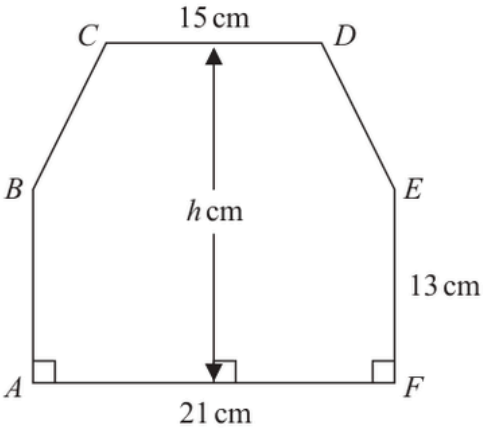


Diagram **NOT** accurately drawn

$AF = 21$ cm $\quad CD = 15$ cm $\quad AB = FE = 13$ cm

The perpendicular height of the shape is h cm
CD is parallel to AF

The area of the shape is $390\,\text{cm}^2$

Work out the value of h

$h =$ .....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2024

1 The bottom rectangle has area

The bottom rectangle has area

$$21 \times 13 = 273$$

2 **The top trapezium has height**

The top trapezium has height $h - 13$, with parallel sides 21 and 15.

$$273 + \frac{1}{2}(21 + 15)(h - 13) = 390$$

$$273 + 18(h - 13) = 390$$

$$18h + 39 = 390$$

$$h = 19.5$$

✓ **FINAL ANSWER**

$19.5\ \text{cm}$.

4MA1/1HR MAY 2024

Question 8

6 marks | Perimeter & Area

8 The diagram shows a trapezium, $ABCD$

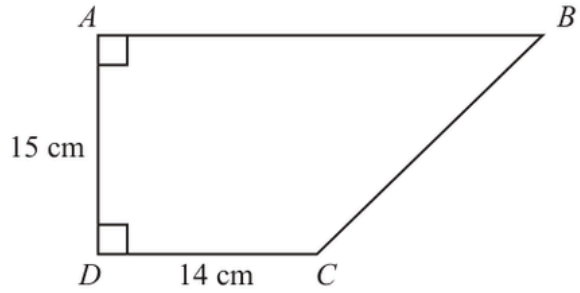


Diagram **NOT** accurately drawn

DAB and ADC are right angles.

$AD = 15$ cm $DC = 14$ cm

The area of the trapezium is 360 cm²

Work out the perimeter of the trapezium.

..... cm

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR May 2024

1 **Use the trapezium area formula**

Use the trapezium area formula:

$$360 = \frac{1}{2}(AB + 14)(15)$$

$$AB + 14 = 48$$

$$AB = 34$$

2 **Use trigonometry**

Triangle ABC is right-angled with horizontal length

$$34 - 14 = 20$$

3 Calculate area

So

$$BC = \sqrt{20^2 + 15^2} = 25$$

4 Perimeter

Perimeter:

$$15 + 14 + 25 + 34 = 88$$

FINAL ANSWER

$88\ \text{cm}$.

Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

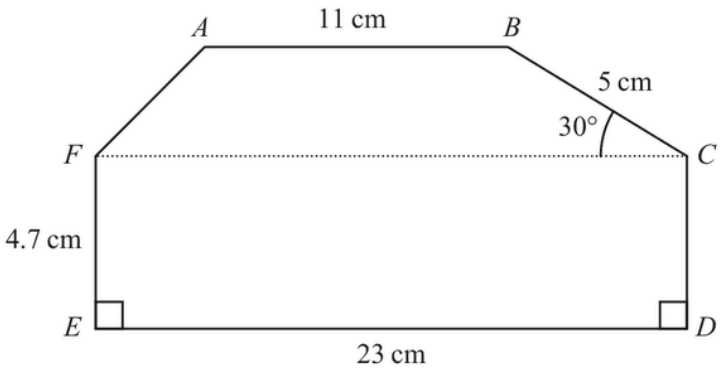


Diagram **NOT** accurately drawn

Angle $BCF = 30°$
AB, FC and ED are parallel.

Calculate the area of $ABCDEF$
Show your working clearly.

..... cm^2

(Total for Question 10 is 5 marks)

1 **The rectangle has area**

The rectangle $EDCF$ has area

$$23 \times 4.7 = 108.1$$

2 **The height of the top**

The height of the top trapezium is

$$5 \sin 30^\circ = 2.5$$

3 **Area of the top trapezium**

Area of the top trapezium:

$$\frac{1}{2}(11 + 23) \times 2.5 = 42.5$$

4 **Total area**

Total area:

$$108.1 + 42.5 = 150.6$$

✓ **FINAL ANSWER**

$150.6\ \text{cm}^2$.

4WM1/01 MAY 2025

Question 4

3 marks | Perimeter & Area

4 The diagram shows triangle ABC and a semicircle.

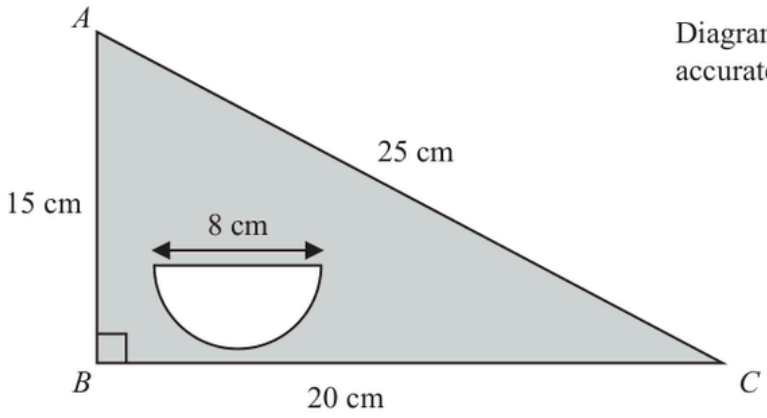


Diagram **NOT** accurately drawn

$AB = 15$ cm $\quad BC = 20$ cm $\quad AC = 25$ cm $\quad$ angle $ABC = 90°$

The diameter of the semicircle is 8 cm

Work out the area of the region shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4WM1/01 May 2025

1 **The shaded area is the**

The shaded area is the triangle area minus the semicircle area.

$$\text{triangle area} = \frac{1}{2} \times 15 \times 20 = 150$$

2 **Calculate area**

The semicircle has diameter 8, so radius 4.

$$\text{semicircle area} = \frac{1}{2}\pi(4)^2 = 8\pi$$

$$\text{shaded area} = 150 - 8\pi = 124.867\ldots$$

3 **Calculate area**

To 3 significant figures,

$$125$$

FINAL ANSWER

$125\ \text{cm}^2$

4MA1/1H JUNE 2025

Question 9

5 marks | Perimeter & Area

9

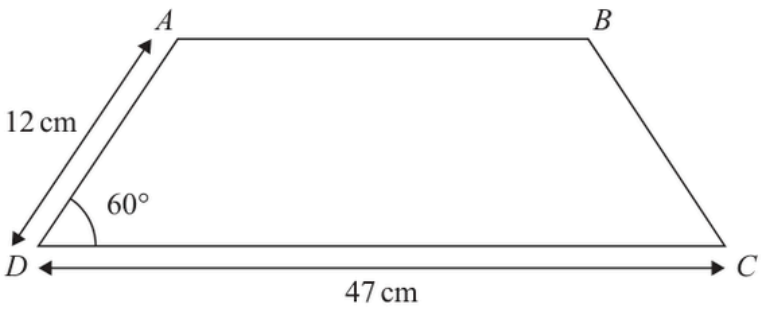


Diagram **NOT** accurately drawn

$ABCD$ is a trapezium with one line of symmetry.

angle $ADC = 60°$ $AD = 12\,\text{cm}$ $DC = 47\,\text{cm}$

Work out the area of the trapezium.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H June 2025

1 Use trigonometry

Because the trapezium has one line of symmetry, each sloping side makes the same $60°$ angle with the base.

2 Height

Height:

$$12\sin 60° = 10.392\ldots$$

3 The top parallel side is

The top parallel side is

$$47 - 2(12\cos 60°) = 35$$

4 **Area**

Area:

$$\frac{1}{2}(47+35)(10.392\ldots) = 426.084\ldots$$

✓ **FINAL ANSWER**

$426\ \text{cm}^2$.

4WM1/01 MAY 2025

Question 10

5 marks | Perimeter & Area

10

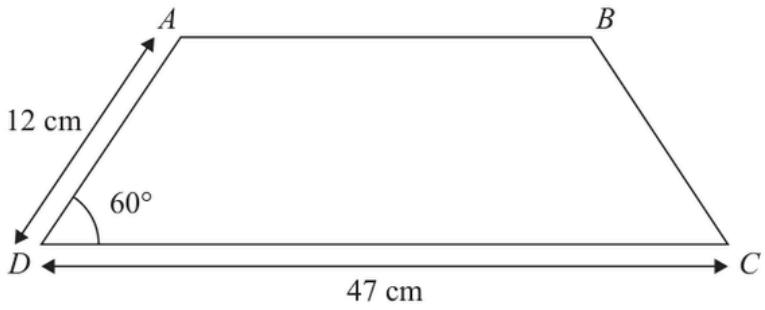


Diagram **NOT** accurately drawn

ABCD is a trapezium with one line of symmetry.

angle $ADC = 60°$ $AD = 12\,\text{cm}$ $DC = 47\,\text{cm}$

Work out the area of the trapezium.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4WM1/01 May 2025

1 The trapezium is symmetrical. The

The trapezium is symmetrical. The horizontal shift from each sloping side is

$$12\cos 60° = 6$$

2 Find the gradient

So the top parallel side is

$$47 - 6 - 6 = 35$$

3 The height is

The height is

$$12\sin 60° = 10.392\ldots$$

4 Area of the trapezium

Area of the trapezium:

$$\frac{1}{2}(47+35)(12\sin 60°) = 426.084\ldots$$

5 Calculate area

To 3 significant figures,

$$426$$

FINAL ANSWER

$426\ \text{cm}^2$

4WM1/01R MAY 2025

Question 19

5 marks | Perimeter & Area

19 The diagram shows a pentagon $ABCDE$

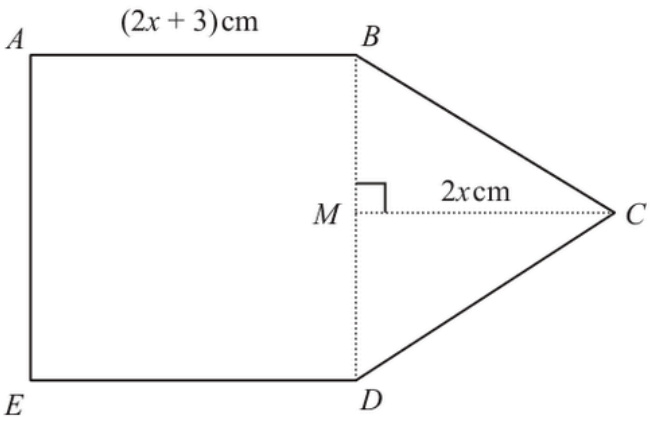


Diagram **NOT** accurately drawn

$ABCDE$ is made from a square $ABDE$ and a triangle BCD
M is the midpoint of BD

$AB = (2x + 3)$ cm $MC = 2x$ cm angle $BMC = 90°$

The area of pentagon $ABCDE$ is 84 cm^2

Work out the area of square $ABDE$
Show clear algebraic working.

..... cm^2

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4WM1/01R May 2025

1 **Square area**

Square area:

$$(2x + 3)^2$$

2 **Triangle area**

Triangle area:

$$\frac{1}{2}(2x + 3)(2x)$$

3 Evaluate fraction

So

$$(2x+3)^2+\frac{1}{2}(2x+3)(2x)=84$$

$$6x^2+15x-75=0$$

$$2x^2+5x-25=0$$

$$(2x-5)(x+5)=0$$

4 Calculate value

The positive solution is $x=2.5$, so

$$(2x+3)^2=8^2=64$$

FINAL ANSWER

$64\ \text{cm}^2$.

4WM1/01 NOVEMBER 2025

Question 1

2 marks | Perimeter & Area

1 The diagram shows a semicircle.

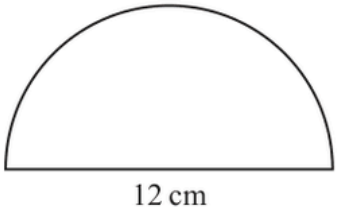


Diagram **NOT** accurately drawn

The semicircle has diameter 12 cm

Work out the perimeter of the semicircle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 1 is 2 marks)

WORKED SOLUTION

Question 1 · 4WM1/01 November 2025

1 **Radius cm**

Radius $= 6$ cm.

$$\text{perimeter} = \pi r + 12 = 6\pi + 12 \approx 30.8496$$

FINAL ANSWER

30.8 cm to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 18

4 marks | Perimeter & Area

18 The diagram shows a rectangle.

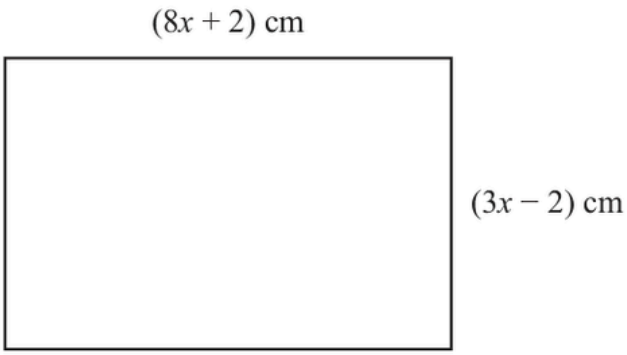


Diagram **NOT** accurately drawn

The rectangle has length $(8x + 2)$ cm and width $(3x - 2)$ cm
The area of the rectangle is $500\ \text{cm}^2$

(a) Show that $12x^2 - 5x - 252 = 0$

(1)

(b) Hence find the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(3)

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4WM1/01 November 2025

1 Calculate value

$$(8x + 2)(3x - 2) = 500$$

$$24x^2 - 10x - 4 = 500$$

$$12x^2 - 5x - 252 = 0$$

2 Solve quadratic equation

Using the quadratic formula,

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(12)(-252)}}{24}$$

3 **The positive solution is**

The positive solution is

$$x \approx 4.79564$$

✓ **FINAL ANSWER**

$x = 4.80$ to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 21

4 marks | Perimeter & Area

21 The diagram shows an equilateral triangle and a circle.

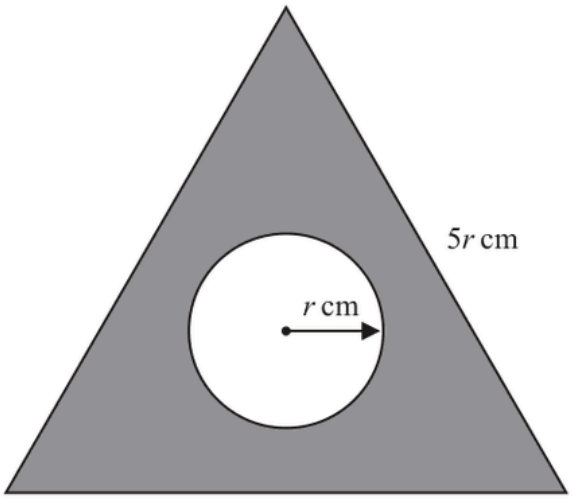


Diagram **NOT** accurately drawn

The equilateral triangle has sides of length $5r$ cm
The circle has radius r cm

The area of the shaded region is 610π cm^2
Work out the value of r

Give your answer correct to 3 significant figures.

$r =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 November 2025

1 **Triangle area**

Triangle area:

$$\frac{\sqrt{3}}{4}(5r)^2 = \frac{25\sqrt{3}}{4}r^2$$

2 **Circle area**

Circle area:

$$\pi r^2$$

3 **Simplify surd**

So

$$\left(\frac{25\sqrt{3}}{4} - \pi\right) r^2 = 610\pi$$

$$r \approx 15.7926$$

FINAL ANSWER

$r = 15.8$ to 3 significant figures.

Question 4

5 marks | Perimeter & Area

4 The diagram shows a shaded shape $ABCD$ made from a right-angled triangle ABD and a semicircle BCD

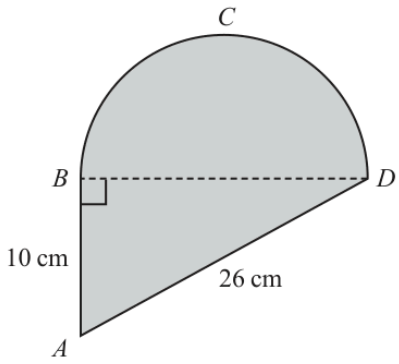


Diagram **NOT** accurately drawn

$AB = 10$ cm $AD = 26$ cm angle $ABD = 90°$

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 4 is 5 marks)

1 **Find BD with Pythagoras**

$$BD = \sqrt{26^2 - 10^2} = \sqrt{576} = 24 \text{ cm}$$

2 **Semicircle arc**

Diameter $BD = 24$, so the arc is

$$\tfrac{1}{2}\pi \times 24 = 12\pi = 37.699\ldots \text{ cm}$$

3 **Perimeter (BD is inside, not counted)**

$$10 + 26 + 12\pi = 73.699\ldots$$

✓ **FINAL ANSWER**

73.7 cm

4MA1/1H JUNE 2026

Question 11

5 marks | Perimeter & Area

11 The diagram shows a shaded shape $ABCDEO$ made from a semicircle BCD and two identical right-angled triangles ABO and EDO

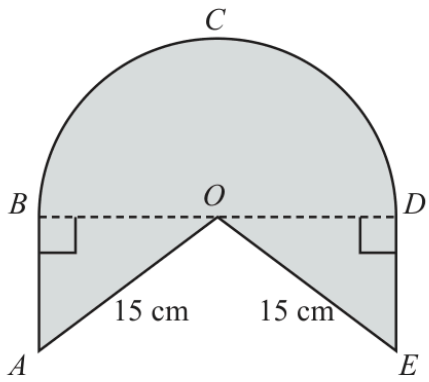


Diagram **NOT** accurately drawn

BD is the diameter of the semicircle.
O is the midpoint of BD

angle ABO = angle EDO = 90° $\quad AO = EO = 15$ cm

The area of semicircle BCD is 227 cm^2

Work out the perimeter of the shaded shape.
Show your working clearly.
Give your answer correct to one decimal place.

..... cm

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2026

1 **Solution | Step 1**

Let the radius be $r = OB = OD$. Then $\frac{1}{2}\pi r^2 = 227$, so $r = \sqrt{454/\pi} = 12.021343\ldots$ cm.
The given area belongs to the semicircle.

2 **Solution | Step 2**

$$AB = DE = \sqrt{15^2 - r^2} = 8.971472\ldots \text{ cm.}$$

The identical triangles have hypotenuse 15 cm and horizontal leg r.

3 Solution | Step 3

$$P = \pi r + AB + DE + AO + EO = \pi r + 2\sqrt{225 - r^2} + 30 = 85.709107\ldots \text{ cm}.$$

The boundary contains the semicircular arc and the four outer triangle sides, but not the internal diameter.

4 Solution | Step 4

$$P = 85.7 \text{ cm} \quad (1 \text{ d.p.}).$$

Retain unrounded intermediate values.

FINAL ANSWER

85.7 cm

TOPIC

Circles, Arcs & Sectors

Every past-paper question on this topic, 2020 to 2026.

Question 18

4 marks | Circles, Arcs & Sectors

18 The diagram shows two identical circles drawn inside a rectangle.

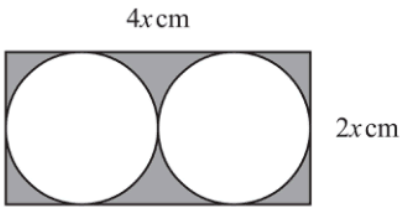


Diagram **NOT** accurately drawn

The length of the rectangle is $4x$ cm and the width of the rectangle is $2x$ cm
Each circle has a radius of x cm

The circles touch each other and each circle touches three sides of the rectangle.

The region inside the rectangle that is outside the circles, shown shaded in the diagram, has a total area of $20\,\text{cm}^2$

Work out the perimeter of the rectangle.
Give your answer correct to 3 significant figures.

..... cm

1 Write the shaded area

Rectangle area:

$$4x \times 2x = 8x^2$$

Area of the two circles:

$$2\pi x^2$$

So

$$8x^2 - 2\pi x^2 = 20$$

2 Solve for x

$$x^2(8 - 2\pi) = 20$$

$$x = \sqrt{\frac{20}{8 - 2\pi}} = 3.413\ldots$$

3 Find the perimeter

$$P = 2(4x + 2x) = 12x$$

$$12(3.413\ldots) = 40.96\ldots$$

4 Round to 3 significant figures

$$40.96\ldots \approx 41.0$$

FINAL ANSWER

41.0 cm.

4MA1/1H JANUARY 2021

Question 8

3 marks | Circles, Arcs & Sectors

8 The region, shown shaded in the diagram, is a path.

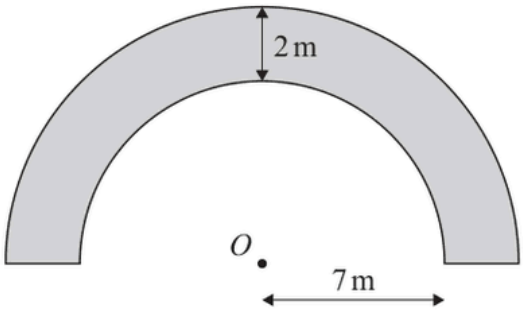


Diagram **NOT** accurately drawn

The boundary of the path is formed by two semicircles, with the same centre O, and two straight lines.

The inner semicircle has a radius of 7 metres.
The path has a width of 2 metres.

Work out the perimeter of the path.
Give your answer correct to one decimal place.

..... m

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2021

1 **The inner radius is m**

The inner radius is 7 m.

2 **Calculate value**

The path is 2 m wide, so the outer radius is

$$7 + 2 = 9 \text{ m}$$

3 Perimeter of the path

Perimeter of the path:

$$\text{inner semicircle} + \text{outer semicircle} + 2 \text{ straight ends}$$

$$= 7\pi + 9\pi + 2 + 2$$

$$= 16\pi + 4$$

$$= 54.265\ldots$$

FINAL ANSWER

$54.3\ \text{m}$.

4MA1/2HR JANUARY 2021

Question 11

4 marks | Circles, Arcs & Sectors

11 A circle centre O has radius 9 cm.

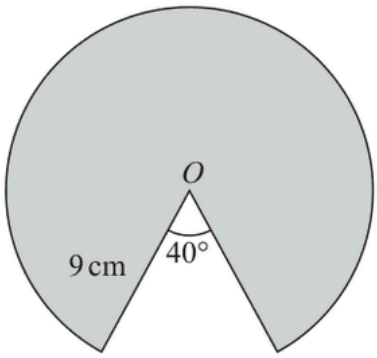


Diagram **NOT** accurately drawn

Calculate the perimeter of the shaded sector of the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR January 2021

1 **The shaded sector is the**

The shaded sector is the major sector:

$$360° - 40° = 320°$$

2 **Arc length**

Arc length:

$$\frac{320}{360} \times 2\pi(9) = 16\pi$$

3 **Perimeter**

Perimeter:

$$16\pi + 2(9) = 68.265\ldots$$

✓ **FINAL ANSWER**

68.3 cm.

4MA1/2HR JANUARY 2021

Question 20

5 marks | Circles, Arcs & Sectors

20 A, B and C are points on a circle with centre O.

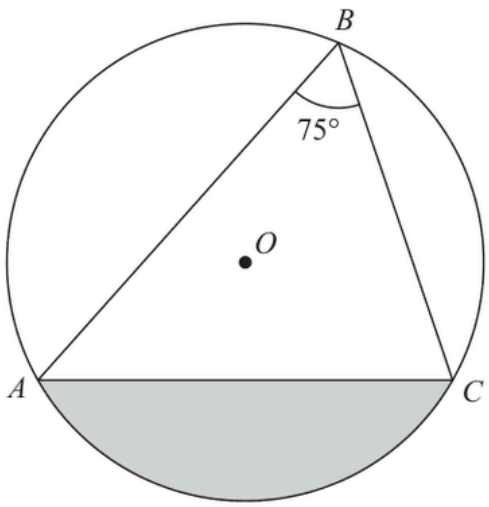


Diagram **NOT** accurately drawn

Angle $ABC = 75°$

The area of the shaded segment is $200\,\text{cm}^2$

Calculate the radius of the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2021

1 **Use trigonometry**

$$\angle ABC = 75°$$

2 **The angle at the centre**

The angle at the centre is twice the angle at the circumference, so

$$\angle AOC = 2(75°) = 150°$$

3 **The shaded segment is**

The shaded segment is:

$$\text{sector } AOC - \text{triangle } AOC$$

4 **Let the radius be**

Let the radius be r.

5 **Sector area**

Sector area:

$$\frac{150}{360}\pi r^2$$

6 **Triangle area**

Triangle area:

$$\frac{1}{2}r^2 \sin 150°$$

7 **Use trigonometry**

So

$$\frac{150}{360}\pi r^2 - \frac{1}{2}r^2 \sin 150° = 200$$

$$r^2\left(\frac{5\pi}{12} - \frac{1}{4}\right) = 200$$

$$r^2 = \frac{200}{\frac{5\pi}{12} - \frac{1}{4}}$$

$$r = 13.7426\ldots$$

✓ **FINAL ANSWER**

$\boxed{13.7 \text{ cm}}$

4MA1/2H JANUARY 2021

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows a sector OBC of a circle with centre O and radius $(6 + x)$ cm.

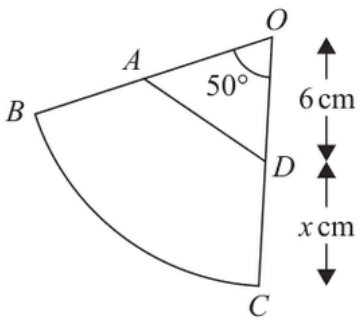


Diagram **NOT** accurately drawn

A is the point on OB and D is the point on OC such that $OA = OD = 6$ cm

Angle $BOC = 50°$

Given that

the perimeter of sector $OBC = 2 \times$ the perimeter of triangle OAD

find the value of x.
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2021

1 The radius of sector is

The radius of sector OBC is

$$6 + x$$

2 Use trigonometry

The angle is $50°$, so the arc length of sector OBC is

$$\frac{50}{360} \times 2\pi(6 + x)$$

3 **Evaluate fraction**

Therefore the perimeter of sector OBC is

$$2(6+x)+\frac{50}{360}\times 2\pi(6+x)$$

$$=(6+x)\left(2+\frac{5\pi}{18}\right)$$

4 **Use trigonometry**

In triangle OAD,

$$OA=OD=6, \qquad \angle AOD=50^\circ$$

5 **Use trigonometry**

So

$$AD=2(6)\sin 25^\circ=12\sin 25^\circ$$

6 **The perimeter of triangle is**

The perimeter of triangle OAD is

$$12+12\sin 25^\circ$$

7 **Given that**

Given that

$$\text{perimeter of sector } OBC=2\times\text{perimeter of triangle } OAD$$

$$(6+x)\left(2+\frac{5\pi}{18}\right)=2(12+12\sin 25^\circ)$$

$$x=\frac{2(12+12\sin 25^\circ)}{2+\frac{5\pi}{18}}-6$$

$$x=5.8854\ldots$$

✓ **FINAL ANSWER**

$\boxed{5.89}$

4MA1/1H MAY 2021

Question 12

2 marks | Circles, Arcs & Sectors

12 The diagram shows a sector of a circle with radius 7 cm.

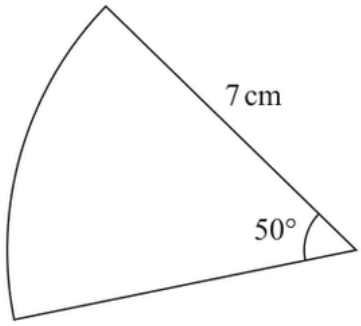


Diagram **NOT** accurately drawn

Work out the length of the arc of the sector.
Give your answer correct to one decimal place.

..... cm

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May 2021

1 **Evaluate fraction**

$$\text{arc length} = \frac{50}{360} \times 2\pi(7)$$

$$= 6.108\ldots$$

FINAL ANSWER

6.1 cm.

4MA1/1H MAY 2021

Question 25

6 marks | Circles, Arcs & Sectors

25 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

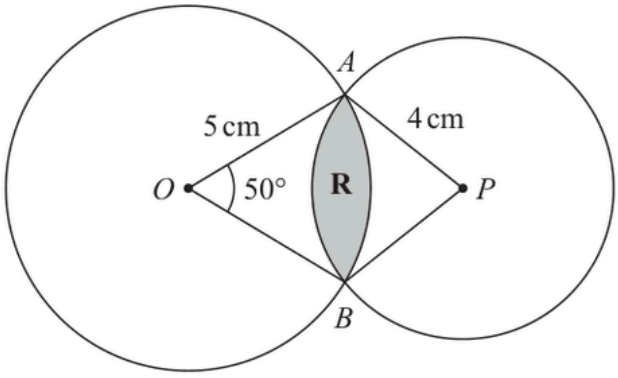


Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2021

1 **Use trigonometry**

For the circle with centre O:

$$\angle AOB = 50°, \qquad OA = OB = 5$$

2 **The half chord length is**

The half chord length is

$$\frac{AB}{2} = 5\sin 25°$$

3 **Use trigonometry**

For the circle with centre P, let

$$\angle APB = \theta$$

---

4 **Use trigonometry**

The radius is 4, so

$$4\sin\left(\frac{\theta}{2}\right) = 5\sin 25°$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{5\sin 25°}{4}$$

$$\theta = 63.7777\ldots°$$

---

5 **The common region is made**

The common region is made from two minor segments.

---

6 **Use trigonometry**

For the circle with centre O:

$$\text{segment area} = \frac{50}{360}\pi(5)^2 - \frac{1}{2}(5)^2\sin 50° = 1.3328\ldots$$

---

7 **Use trigonometry**

For the circle with centre P:

$$\text{segment area} = \frac{\theta}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin\theta$$

$$= 1.7284\ldots$$

---

8 **Total area of region**

Total area of region R:

$$1.3328\ldots + 1.7284\ldots = 3.0611\ldots$$

✓ **FINAL ANSWER**

$\boxed{3.06\ \text{cm}^2}$

4MA1/2HR JANUARY 2022

Question 9

5 marks | Circles, Arcs & Sectors

9 In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc.

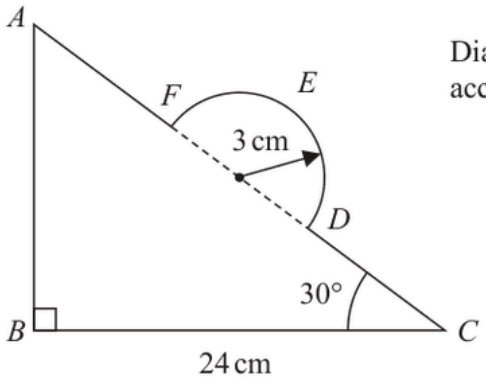


Diagram **NOT** accurately drawn

In triangle ABC

$BC = 24\,\text{cm}$ angle $ABC = 90°$ angle $BCA = 30°$

The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF
The radius of the semicircular arc is 3 cm.

Work out the length of $AFEDC$
Give your answer correct to 2 significant figures.

..... cm

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2022

1 **Use trigonometry**

In triangle ABC,

$$\cos 30° = \frac{24}{AC}$$

$$AC = \frac{24}{\cos 30°} = 27.712\ldots$$

2 **Calculate value**

The semicircle has radius $3\ \text{cm}$, so its diameter is $6\ \text{cm}$.

$$AF + DC = AC - DF = 27.712\ldots - 6$$

3 **Arc is a semicircle**

Arc FED is a semicircle:

$$\text{arc length} = 3\pi$$

4 **Calculate value**

Therefore

$$AFEDC = 27.712\ldots - 6 + 3\pi = 31.137\ldots$$

✓ **FINAL ANSWER**

$31\ \text{cm}$.

4MA1/1HR JANUARY 2022

Question 11

2 marks | Circles, Arcs & Sectors

11 The diagram shows sector OPQ of a circle, centre O

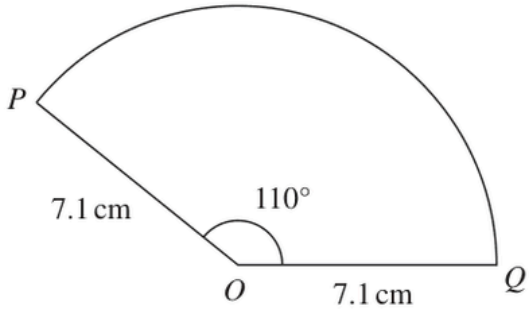


Diagram **NOT** accurately drawn

$OP = OQ = 7.1$ cm
Angle $POQ = 110°$

Calculate the area of sector OPQ
Give your answer correct to one decimal place.

..... cm²

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2022

1 Calculate area

$$\text{area} = \frac{110}{360} \times \pi(7.1)^2$$

$$= 48.390\ldots$$

FINAL ANSWER

$48.4\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 16

4 marks | Circles, Arcs & Sectors

16 The diagram shows a circle with centre O

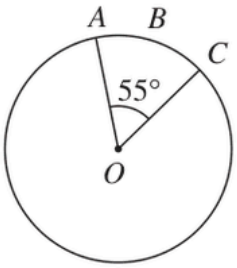


Diagram **NOT** accurately drawn

A, B and C are points on the circle so that the length of the arc ABC is 5 cm.

Given that angle $AOC = 55°$

work out the area of the circle.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2022

1 Use trigonometry

The arc length is $5\ \text{cm}$, and the angle is $55°$.

$$5 = \frac{55}{360} \times 2\pi r$$

$$r = 5.208\ldots$$

2 Area of the circle

Area of the circle:

$$\pi r^2 = \pi(5.208\ldots)^2 = 85.233\ldots$$

FINAL ANSWER

$85.2\ \text{cm}^2$.

4MA1/2HR JUNE 2022

Question 13

3 marks | Circles, Arcs & Sectors

13 The diagram shows a sector AOB of a circle with centre O

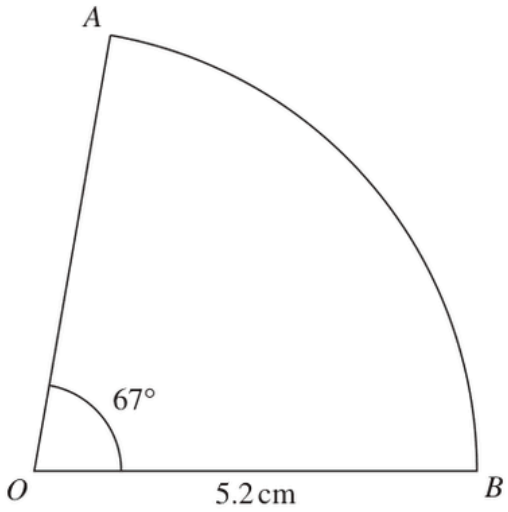


Diagram **NOT** accurately drawn

Angle $AOB = 67°$
$OA = OB = 5.2\,\text{cm}$

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2022

1 Arc length

Arc length:

$$\frac{67}{360} \times 2\pi(5.2) = 6.080\ldots$$

2 Perimeter

Perimeter:

$$5.2 + 5.2 + 6.080\ldots = 16.480\ldots$$

FINAL ANSWER

$16.5\ \text{cm}$.

4MA1/1H JUNE 2022

Question 19

5 marks | Circles, Arcs & Sectors

19

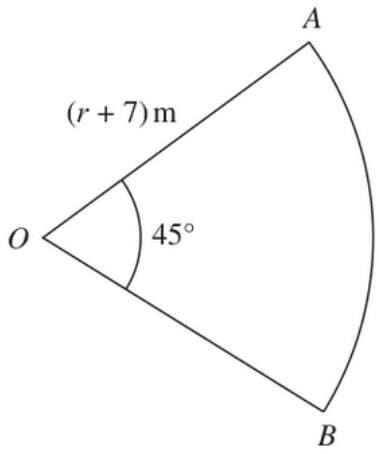


Diagram **NOT** accurately drawn

OAB is a sector **S** of a circle with centre *O* and radius $(r + 7)$ metres.
Angle $AOB = 45°$

A circle **C** has radius $(r - 2)$ metres.

The area of sector **S** is twice the area of circle **C**

Find the value of r
Show your working clearly.

Question 19 continued.

$r =$ .....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H June 2022

1 **Area of sector**

Area of sector S:

$$\frac{45}{360}\pi(r+7)^2 = \frac{1}{8}\pi(r+7)^2$$

2 **Area of circle**

Area of circle C:

$$\pi(r-2)^2$$

---

3 **The sector area is twice**

The sector area is twice the circle area:

$$\frac{1}{8}\pi(r+7)^2 = 2\pi(r-2)^2$$

$$(r+7)^2 = 16(r-2)^2$$

---

4 **Solve inequality**

Since $r > 2$,

$$r+7 = 4(r-2)$$

$$r+7 = 4r-8$$

$$r = 5$$

✓ **FINAL ANSWER**

$r = 5$.

4MA1/2H JUNE 2022

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows four identical circles drawn inside a square.

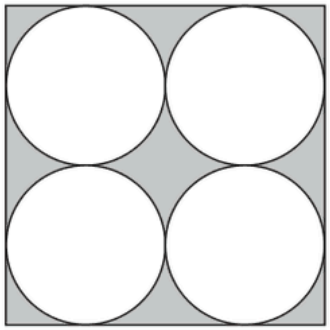


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of $40\,\text{cm}^2$

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2022

1 Let the radius of each

Let the radius of each circle be r.

2 There are two circles across

There are two circles across the width of the square, so the side length of the square is

$$4r$$

3 Area of the square

Area of the square:

$$(4r)^2 = 16r^2$$

4 Area of the four circles

Area of the four circles:

$$4\pi r^2$$

5 **Calculate area**

The shaded area is $40\ \text{cm}^2$, so

$$16r^2 - 4\pi r^2 = 40$$

$$r^2(16 - 4\pi) = 40$$

$$r = \sqrt{\frac{40}{16 - 4\pi}} = 3.4131\ldots$$

---

6 **The perimeter of the square**

The perimeter of the square is

$$4(4r) = 16r$$

$$16r = 54.6101\ldots$$

✓ **FINAL ANSWER**

$\boxed{54.6\ \text{cm}}$

4MA1/1HR MAY 2023

Question 4

5 marks | Circles, Arcs & Sectors

4 The diagram shows a shape made up of three semicircles, enclosing a right-angled triangle.

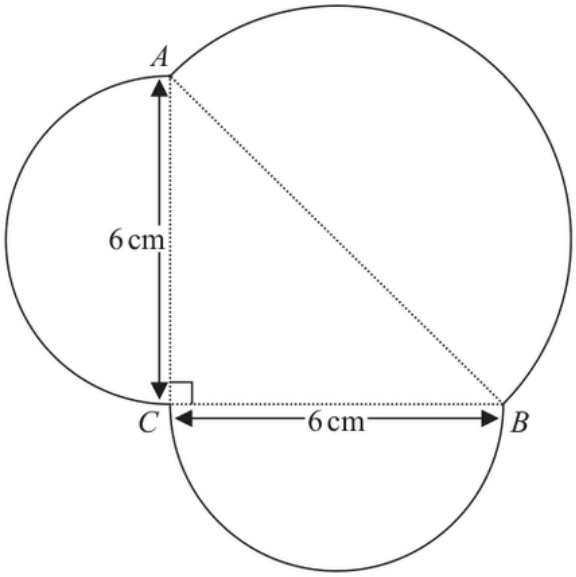


Diagram **NOT** accurately drawn

AB, BC and CA are each the diameter of a semicircle.

$BC = CA = 6$ cm.

Work out the perimeter of the shape.
Give your answer correct to one decimal place.

..... cm

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR May 2023

1 **Simplify surd**

$$AB = \sqrt{6^2 + 6^2} = \sqrt{72} = 8.485\ldots$$

2 **The perimeter is the sum**

The perimeter is the sum of three semicircular arcs:

$$\frac{\pi}{2}(AB + BC + CA) = \frac{\pi}{2}(8.485\ldots + 6 + 6) = 32.178\ldots$$

FINAL ANSWER

$32.2\ \text{cm}$.

4MA1/1H MAY 2023

Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

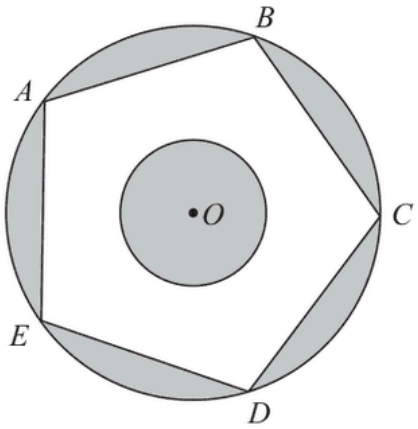


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2023

1 **The regular pentagon has side**

The regular pentagon has side length 8 cm.

2 **Use trigonometry**

For a regular pentagon,

$$R = \frac{s}{2\sin 36°}$$

3 **Use trigonometry**

where R is the radius of the larger circle.

$$R = \frac{8}{2\sin 36^\circ} = 6.8052\ldots$$

4 **Area of the larger circle**

Area of the larger circle:

$$\pi R^2 = 145.4898\ldots$$

5 **Area of the regular pentagon**

Area of the regular pentagon:

$$\frac{5s^2}{4\tan 36^\circ} = \frac{5(8)^2}{4\tan 36^\circ} = 110.1106\ldots$$

6 **Let the radius of the**

Let the radius of the smaller circle be r.

7 **The shaded area is**

The shaded area is:

$$\text{larger circle area} - \text{pentagon area} + \pi r^2$$

8 **The unshaded area is**

The unshaded area is:

$$\text{pentagon area} - \pi r^2$$

9 **Simplify surd**

These are equal, so

$$145.4898 - 110.1106 + \pi r^2 = 110.1106 - \pi r^2$$

$$2\pi r^2 = 2(110.1106) - 145.4898$$

$$\pi r^2 = 37.3657\ldots$$

$$r = \sqrt{\frac{37.3657\ldots}{\pi}} = 3.4487\ldots$$

✓ **FINAL ANSWER**

$$\boxed{3.45\text{ cm}}$$

4MA1/1H NOVEMBER 2023

Question 5

3 marks | Circles, Arcs & Sectors

5 The diagram shows a shaded shape made from three identical semicircles, X, Y and Z

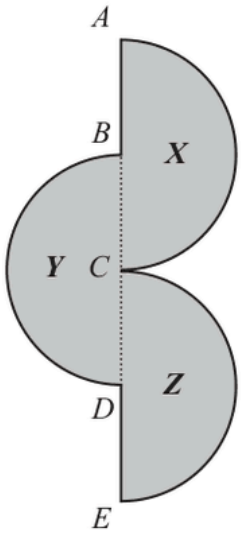


Diagram **NOT** accurately drawn

$ABCDE$ is a straight line.

AC is the diameter of semicircle X and B is the centre of semicircle X
BD is the diameter of semicircle Y and C is the centre of semicircle Y
CE is the diameter of semicircle Z and D is the centre of semicircle Z

$AC = BD = CE = 20$ cm

Work out the perimeter of the shaded shape.
Give your answer correct to the nearest whole number.

..... cm

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2023

1 Each semicircle has diameter

Each semicircle has diameter 20 cm, so each has radius 10 cm.

2 The total curved perimeter is

The total curved perimeter is three semicircles:

$$3 \times \pi r = 3 \times 10\pi = 30\pi$$

3 The two exposed straight parts

The two exposed straight parts are

$$AB = 10, \qquad DE = 10$$

4 Calculate value

So the perimeter is

$$30\pi + 20 = 114.2\ldots$$

FINAL ANSWER

$114\ \text{cm}$ to the nearest whole number.

4MA1/2H NOVEMBER 2023

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows equilateral triangle ABC with sides of length 10 cm.
A circle is drawn inside the triangle.

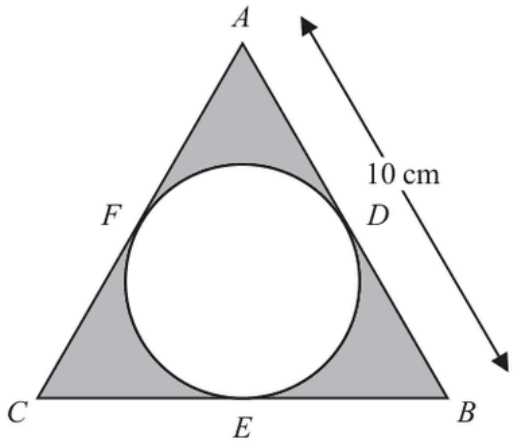


Diagram **NOT** accurately drawn

D, E and F are points on the circle.

ADB, BEC and CFA are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2023

1 Use trigonometry

The triangle is equilateral with side length 10 cm.

2 Area of the triangle

Area of the triangle:

$$\frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$$

3 Use trigonometry

For an equilateral triangle, the inradius is

$$r = \frac{s\sqrt{3}}{6}$$

4 **Simplify surd**

So

$$r = \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

5 **Area of the circle**

Area of the circle:

$$\pi r^2 = \pi\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{25\pi}{3}$$

6 **The shaded area is**

The shaded area is

$$25\sqrt{3} - \frac{25\pi}{3} = 17.1213\ldots$$

✓ **FINAL ANSWER**

$\boxed{17.1\ \text{cm}^2}$

4MA1/2HR JUNE 2024

Question 5

4 marks | Circles, Arcs & Sectors

5 The diagram shows the design for a badge, which will be made using wire.
The design is a circle inside a square $ABCD$

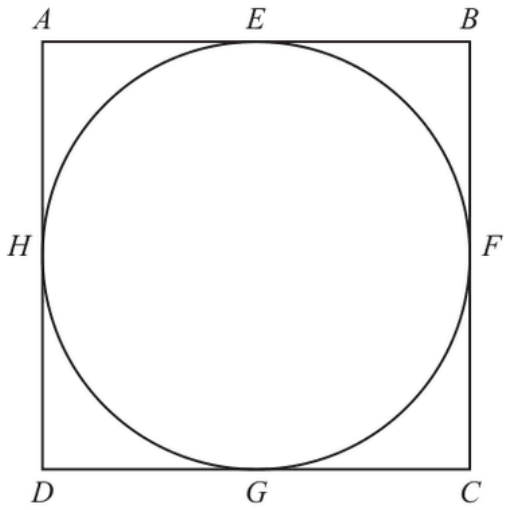


Diagram **NOT** accurately drawn

The circle touches the square at the points E, F, G and H

The area of the square is $81\ \text{cm}^2$

Calculate the total length of wire that will be needed to make the square and the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2024

1 **Calculate area**

The square has area $81\ \text{cm}^2$, so its side length is

$$\sqrt{81} = 9\ \text{cm}$$

2 **Perimeter of the square**

Perimeter of the square:

$$4 \times 9 = 36$$

3 **Calculate value**

The circle has diameter $9\ \text{cm}$, so its circumference is

$$\pi d = 9\pi$$

4 **Total wire length**

Total wire length:

$$36 + 9\pi = 64.274\ldots$$

✓ **FINAL ANSWER**

$64.3\ \text{cm}$ to 3 significant figures.

4MA1/2HR JUNE 2024

Question 13

3 marks | Circles, Arcs & Sectors

13 OAB is a sector of a circle with centre O and radius r cm.

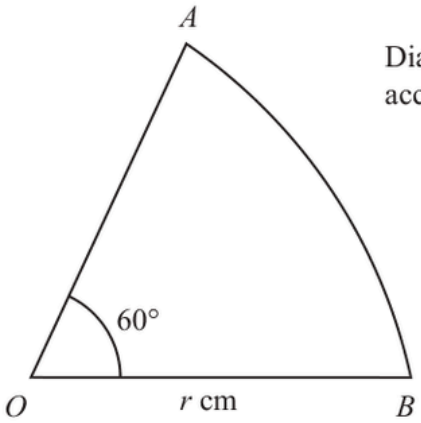


Diagram **NOT** accurately drawn

Angle $AOB = 60°$
The perimeter of the sector is P cm.

Find a formula for P in terms of r
Give your answer in the form $P = r(c\pi + k)$ where c and k are values to be found.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2024

1 The arc length is

The arc length is

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

2 The perimeter is two radii

The perimeter is two radii plus the arc:

$$P = 2r + \frac{\pi r}{3}$$

$$P = r\left(\frac{\pi}{3} + 2\right)$$

FINAL ANSWER

$P = r\left(\frac{1}{3}\pi + 2\right)$.

4MA1/1H MAY 2024

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows a sector $OABC$ of a circle centre O

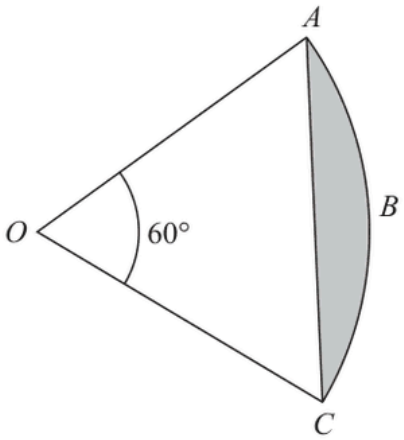


Diagram **NOT** accurately drawn

Angle $AOC = 60°$

The area of the shaded segment ABC is $38\,\text{cm}^2$

Work out the perimeter of the shaded segment ABC
Give your answer correct to one decimal place.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2024

1 **Let the radius of the**

Let the radius of the sector be r.

2 **The shaded segment is**

The shaded segment is

$$\text{sector area} - \text{triangle area}$$

3 **Use trigonometry**

The angle is $60°$, so

$$38 = \frac{60}{360}\pi r^2 - \frac{1}{2}r^2 \sin 60°$$

$$38 = r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$r = 20.4815\ldots$$

4 **The perimeter of the shaded**

The perimeter of the shaded segment is the arc AC plus the chord AC.

5 **Arc length**

Arc length:

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

6 **Chord length**

Chord length:

$$2r \sin 30° = r$$

7 **Evaluate fraction**

So the perimeter is

$$\frac{\pi r}{3} + r$$

$$= \frac{\pi(20.4815\ldots)}{3} + 20.4815\ldots$$

$$= 41.9296\ldots$$

✓ **FINAL ANSWER**

$\boxed{41.9\text{ cm}}$

4MA1/2H JUNE 2024

Question 21

5 marks | Circles, Arcs & Sectors

21 The diagram shows a square $ABCD$ and a circle.

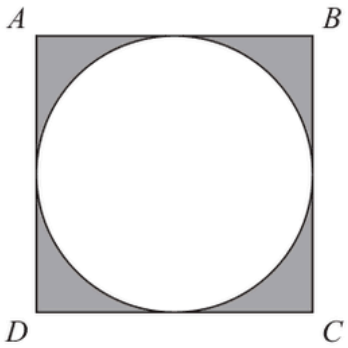


Diagram **NOT** accurately drawn

The sides of the square are tangents to the circle.

The total area of the shaded regions is $80\,\text{cm}^2$

Work out the length of AC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2024

1 **Let the side length**

Let the side length of the square be s.

2 **Square**

The circle is inscribed in the square, so its radius is

$$\frac{s}{2}$$

3 **The shaded area is the**

The shaded area is the area of the square minus the area of the circle:

$$s^2 - \pi\left(\frac{s}{2}\right)^2 = 80$$

$$s^2 - \frac{\pi s^2}{4} = 80$$

$$s^2\left(1 - \frac{\pi}{4}\right) = 80$$

$$s = \sqrt{\frac{80}{1 - \frac{\pi}{4}}} = 19.3076\ldots$$

---

4 **The diagonal of the square**

The diagonal of the square is

$$AC = s\sqrt{2}$$

$$AC = 19.3076\ldots\sqrt{2} = 27.3051\ldots$$

✓ **FINAL ANSWER**

$\boxed{27.3\text{ cm}}$

4MA1/1H NOVEMBER 2024

Question 16

4 marks | Circles, Arcs & Sectors

16 $OAPB$ is a sector of a circle, centre O

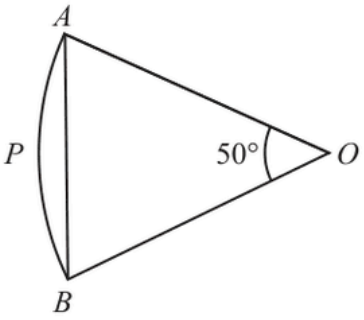


Diagram **NOT** accurately drawn

Angle $AOB = 50°$

Area of triangle $OAB = 120\ \text{cm}^2$

Work out the area of the sector $OAPB$
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H November 2024

1 Use trigonometry

For triangle OAB,

$$120 = \frac{1}{2} r^2 \sin 50°$$

$$r^2 = \frac{240}{\sin 50°}$$

2 Area of sector

Area of sector $OAPB$:

$$\frac{50}{360}\pi r^2 = \frac{50}{360}\pi \left(\frac{240}{\sin 50°}\right) = 136.701\ldots$$

FINAL ANSWER

$137\ \text{cm}^2$.

4MA1/2H NOVEMBER 2024

Question 25

5 marks | Circles, Arcs & Sectors

25 The diagram shows an equilateral triangle ABC and a circle with centre O

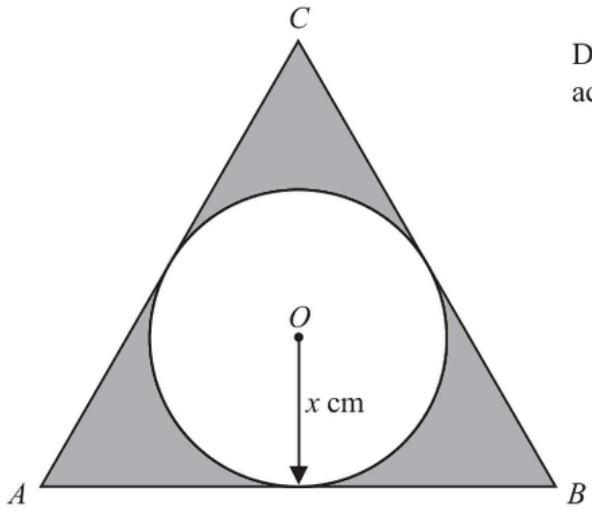


Diagram **NOT** accurately drawn

AB, BC and CA are tangents to the circle.

The radius of the circle is x cm

The total area, in cm^2, of the regions shown shaded in the diagram is nx^2

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2H November 2024

1 **Use trigonometry**

The circle has radius x, so the inradius of the equilateral triangle is x.

2 **Use trigonometry**

For an equilateral triangle with side length a,

$$\text{inradius} = \frac{a\sqrt{3}}{6}$$

3 **Simplify surd**

So

$$x = \frac{a\sqrt{3}}{6}$$

$$a = \frac{6x}{\sqrt{3}} = 2\sqrt{3}x$$

4 **Area of the triangle**

Area of the triangle:

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(2\sqrt{3}x)^2$$

$$= \frac{\sqrt{3}}{4}(12x^2) = 3\sqrt{3}x^2$$

5 **Area of the circle**

Area of the circle:

$$\pi x^2$$

6 **The shaded area is**

The shaded area is

$$3\sqrt{3}x^2 - \pi x^2$$

$$= (3\sqrt{3} - \pi)x^2$$

7 **Simplify surd**

Therefore

$$n = 3\sqrt{3} - \pi$$

✓ **FINAL ANSWER**

$$\boxed{3\sqrt{3} - \pi}$$

4MA1/1H JUNE 2025

Question 11

2 marks | Circles, Arcs & Sectors

11 $OABC$ is a sector of a circle, centre O

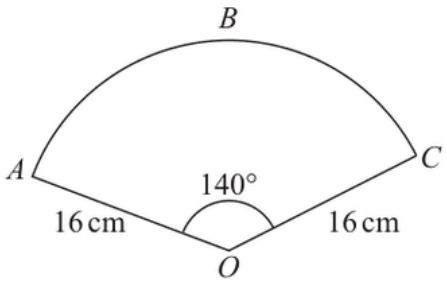


Diagram **NOT** accurately drawn

angle $AOC = 140°$
$OA = OC = 16\,\text{cm}$

Calculate the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H June 2025

1 Calculate area

$$\text{area} = \frac{140}{360} \times \pi(16)^2$$

$$= 312.763\ldots$$

FINAL ANSWER

$313\ \text{cm}^2$.

Question 14

4 marks | Circles, Arcs & Sectors

14 The diagram shows sector OAB of a circle, centre O

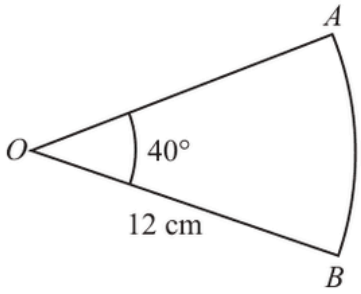


Diagram **NOT** accurately drawn

$OA = OB = 12$ cm angle $AOB = 40°$

(a) Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

The diagram shows triangle XYZ

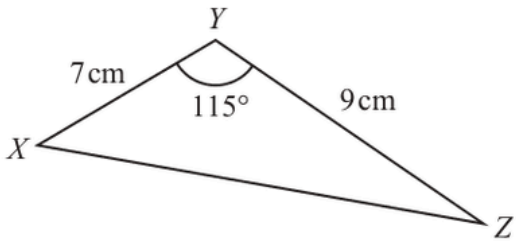


Diagram **NOT** accurately drawn

$XY = 7$ cm $YZ = 9$ cm angle $XYZ = 115°$

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 14 is 4 marks)

1 **Sector area**

Sector area:

$$\frac{40}{360}\pi(12)^2 = 16\pi = 50.265\ldots$$

2 **To 3 significant figures**

To 3 significant figures,

$$50.3\ \text{cm}^2$$

3 **Triangle area**

Triangle area:

$$\frac{1}{2}(7)(9)\sin 115^\circ = 28.548\ldots$$

4 **To 3 significant figures**

To 3 significant figures,

$$28.5\ \text{cm}^2$$

✓ **FINAL ANSWER**

$50.3\ \text{cm}^2$ and $28.5\ \text{cm}^2$.

4WM1/01R MAY 2025

Question 22

6 marks | Circles, Arcs & Sectors

22

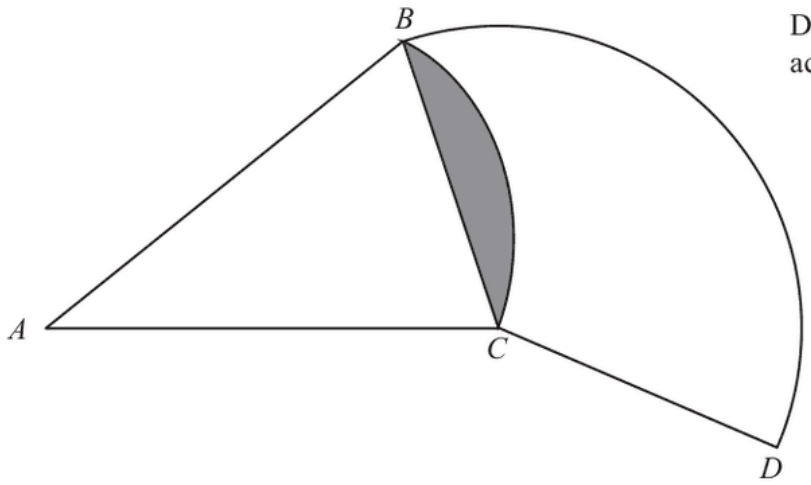


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre A

BCD is a sector of a circle, centre C

Angle $BAC = 40°$
Angle $BCD = 130°$
Area of shaded segment $= 28\,\text{cm}^2$

Find the length of the arc BD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4WM1/01R May 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/1HR JUNE 2025

Question 25

6 marks | Circles, Arcs & Sectors

25

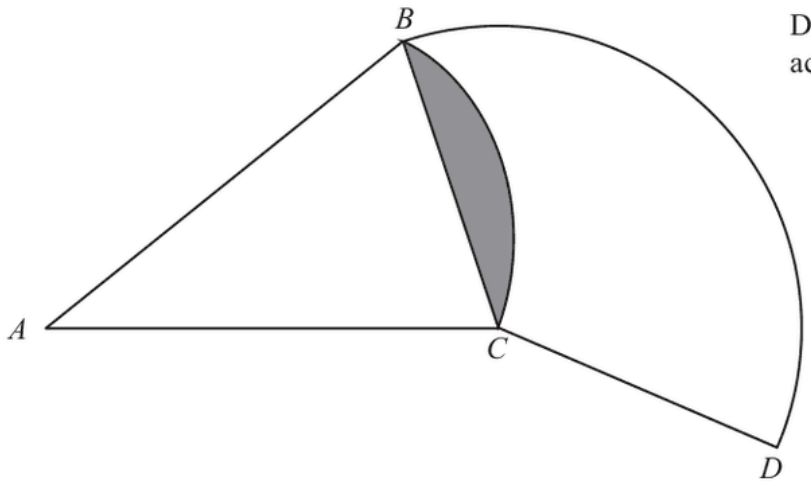


Diagram **NOT** accurately drawn

BAC is a sector of a circle, centre *A*

BCD is a sector of a circle, centre *C*

Angle *BAC* = 40°
Angle *BCD* = 130°
Area of shaded segment = 28 cm^2

Find the length of the arc *BD*
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2025

1 Let the radius of sector

Let the radius of sector BAC be R.

2 The shaded segment is the

The shaded segment is the segment cut off by chord BC in the sector with centre A.

3 **Its area is**

Its area is

$$\text{sector area} - \text{triangle area}$$

$$28 = \frac{40}{360}\pi R^2 - \frac{1}{2}R^2 \sin 40°$$

$$28 = R^2\left(\frac{\pi}{9} - \frac{1}{2}\sin 40°\right)$$

$$R = 31.8096\ldots$$

---

4 **Use trigonometry**

Now find the chord BC:

$$BC = 2R\sin 20°$$

$$BC = 2(31.8096\ldots)\sin 20° = 21.7591\ldots$$

---

5 **Calculate value**

In sector BCD, the centre is C, so the radius is

$$CB = CD = 21.7591\ldots$$

---

6 **Use trigonometry**

The arc BD has angle $130°$, so

$$\text{arc } BD = \frac{130}{360} \times 2\pi(21.7591\ldots)$$

$$= 49.3697\ldots$$

✓ **FINAL ANSWER**

$\boxed{49.4 \text{ cm}}$

4MA1/2H NOVEMBER 2025

Question 6

5 marks | Circles, Arcs & Sectors

6 A large circle with centre O contains 3 identical small circles.
Each circle touches two other circles.

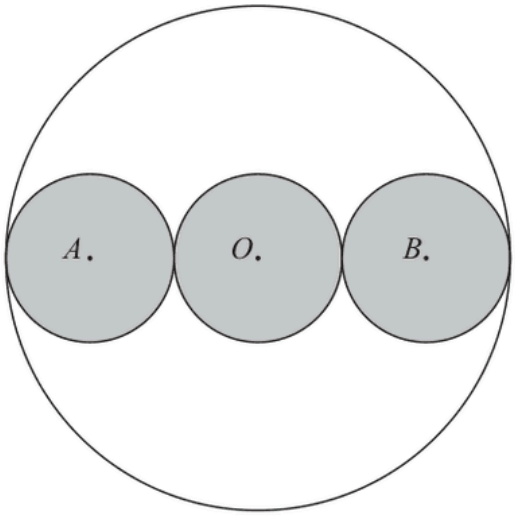


Diagram **NOT** accurately drawn

A, O and B are the centres of the small circles.
AOB is a straight line.

The circumference of the large circle is 160 cm

Work out the area of one small circle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2025

1 **The circumference of the large**

The circumference of the large circle is $160\ \text{cm}$, so

$$2\pi R = 160$$

$$R = \frac{80}{\pi}$$

2 The three equal small circles

The three equal small circles fit along the diameter of the large circle, so

$$2R = 6r$$

$$r = \frac{R}{3} = \frac{80}{3\pi}$$

3 Area of one small circle

Area of one small circle is

$$\pi r^2 = \pi\left(\frac{80}{3\pi}\right)^2$$

$$= 226.35\ldots$$

FINAL ANSWER

$226\ \text{cm}^2$ to 3 significant figures.

4WM1/01 NOVEMBER 2025

Question 14

2 marks | Circles, Arcs & Sectors

14 AOB is a sector of a circle with centre O and radius 12 cm

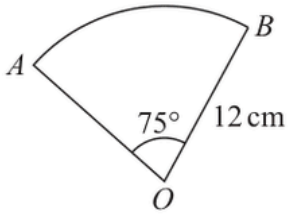


Diagram **NOT** accurately drawn

Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4WM1/01 November 2025

1 Calculate area

$$\text{area} = \frac{75}{360}\pi(12)^2$$

$$= 30\pi \approx 94.2478$$

FINAL ANSWER

$94.2\ \text{cm}^2$ to 3 significant figures.

4MA1/1H NOVEMBER 2025

Question 15

2 marks | Circles, Arcs & Sectors

15 AOB is a sector of a circle with centre O and radius 12 cm

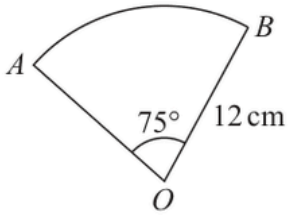


Diagram **NOT** accurately drawn

Work out the area of the sector.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 2 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H November 2025

1 Calculate area

$$\text{area} = \frac{75}{360} \times \pi(12)^2$$

$$= 94.247\ldots$$

FINAL ANSWER

$94.2\ \text{cm}^2$.

4MA1/2H JUNE 2026

Question 15

4 marks | Circles, Arcs & Sectors

15 The diagram shows a sector $OABC$ of a circle, centre O

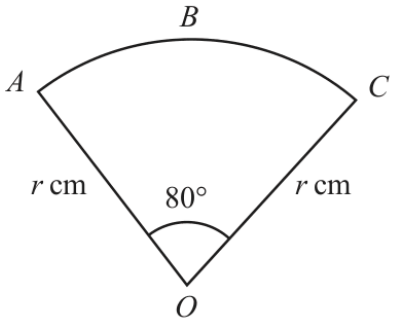


Diagram **NOT** accurately drawn

$OA = OC = r$ cm angle $AOC = 80°$

The area of the sector $OABC$ is 32π cm^2

Work out the perimeter of the sector $OABC$
Give your answer correct to one decimal place.

..... cm

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2026

1 **Solution | Step 1**

$$\frac{80}{360}\pi r^2 = 32\pi \quad \Rightarrow \quad r^2 = 32\frac{360}{80} = 144 \quad \Rightarrow \quad r = 12 \text{ cm}.$$

Use the fraction of the full circle and choose the positive radius.

2 **Solution | Step 2**

$$\text{Arc length} = \frac{80}{360}(2\pi \times 12) = \frac{16\pi}{3} \text{ cm}.$$

The same central-angle fraction applies to the circumference.

3 **Solution | Step 3**

$$P = 12 + 12 + \frac{16\pi}{3} = 40.7551608\ldots \text{ cm}.$$

The sector perimeter contains both radii as well as the arc.

FINAL ANSWER

$40.8\ \text{cm}$ (1 decimal place).

Question 23

6 marks | Circles, Arcs & Sectors

23 OPQ and OAB are sectors of a circle, centre O

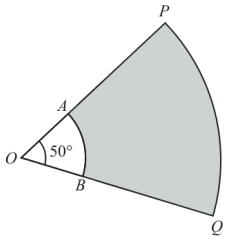


Diagram **NOT** accurately drawn

Angle $POQ = 50°$

A is the point on OP such that $OA = \frac{1}{4}OP$

The area of the shaded region is $\frac{243}{16}\pi\ \text{cm}^2$

Work out the perimeter of the shaded region.
Give your answer in the form $a\pi + b$ where a and b are constants.
Show your working clearly.

.....cm

(Total for Question 23 is 6 marks)

1 **Let OP = r**

Then $OA = \frac{r}{4}$. Shaded area:

$$\frac{50}{360}\pi\left(r^2 - \frac{r^2}{16}\right) = \frac{243}{16}\pi$$

2 **Solve for r**

$$\frac{75}{576}r^2 = \frac{243}{16} \Rightarrow r^2 = 116.64 \Rightarrow r = 10.8,\ OA = 2.7$$

3 **Arcs**

$$\text{arc } PQ = \frac{50}{360} \times 2\pi(10.8) = 3\pi, \qquad \text{arc } AB = \frac{50}{360} \times 2\pi(2.7) = \tfrac{3}{4}\pi$$

4 **Straight edges and total**

$AP = BQ = 10.8 - 2.7 = 8.1$:

$$3\pi + \tfrac{3}{4}\pi + 2(8.1) = \tfrac{15}{4}\pi + 16.2$$

FINAL ANSWER

$\frac{15}{4}\pi + 16.2$ cm

TOPIC

Right-Angled Triangles - Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

Question 10

6 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 The diagram shows a quadrilateral $ABCD$

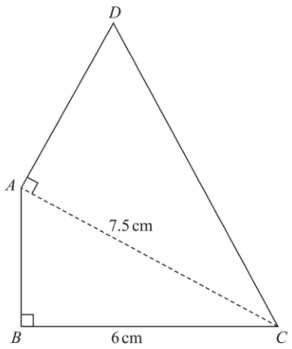


Diagram **NOT** accurately drawn

In the diagram, ABC and DAC are right-angled triangles.

$BC = 6$ cm $\quad AC = 7.5$ cm

The area of quadrilateral $ABCD$ is 31.5 cm^2

(a) Work out the length of AB

..... cm

(2)

(b) Work out the length of AD

..... cm

(4)

(Total for Question 10 is 6 marks)

1 **Use Pythagoras for AB**

In triangle ABC,

$$AB^2 + 6^2 = 7.5^2$$

$$AB^2 = 56.25 - 36 = 20.25$$

$$AB = 4.5$$

2 **Find the area of triangle ABC**

$$\frac{1}{2} \times 6 \times 4.5 = 13.5$$

3 **Find the area of triangle DAC**

$$31.5 - 13.5 = 18$$

4 **Use the triangle area formula**

In triangle DAC, AD and AC are perpendicular.

$$\frac{1}{2} \times AD \times 7.5 = 18$$

$$AD = \frac{36}{7.5} = 4.8$$

✓ **FINAL ANSWER**

$AB = 4.5$ cm, $AD = 4.8$ cm.

4MA1/2HR JANUARY 2020

Question 4

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

4 The diagram shows a rectangle and a diagonal of the rectangle.

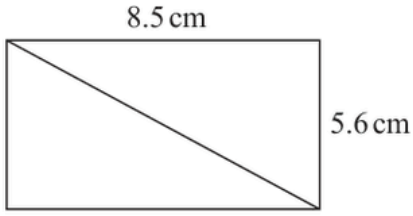


Diagram **NOT** accurately drawn

Work out the length of the diagonal of the rectangle.
Give your answer correct to 1 decimal place.

..... cm

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2020

1 Use trigonometry

$$d^2 = 8.5^2 + 5.6^2$$

$$d = \sqrt{103.61} = 10.178\ldots$$

FINAL ANSWER

10.2 cm.

Question 9

3 marks

Right-Angled Triangles - Pythagoras & Trigonometry

9 Here is a right-angled triangle.

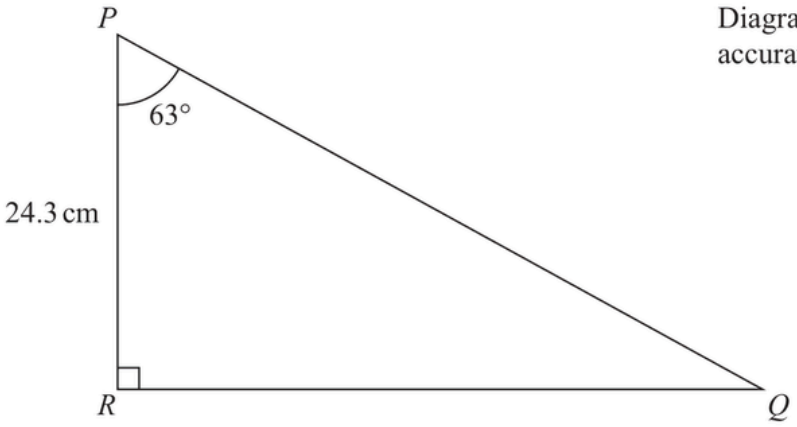


Diagram **NOT** accurately drawn

Calculate the length of PQ.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 9 is 3 marks)

1 Use trigonometry

$$\cos 63^\circ = \frac{24.3}{PQ}$$

$$PQ = \frac{24.3}{\cos 63^\circ} = 53.525\ldots$$

FINAL ANSWER

53.5 cm.

4MA1/1HR JANUARY 2020

Question 15

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

15 The diagram shows two right-angled triangles, DEF and EFG.

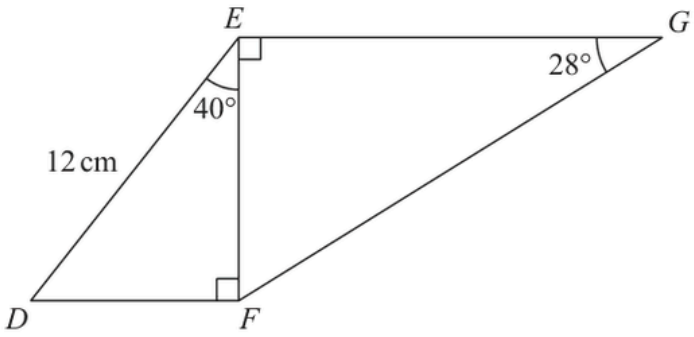


Diagram **NOT** accurately drawn

Work out the length of EG.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2020

1 Use trigonometry

In triangle DEF,

$$EF = 12\cos 40^\circ = 9.192\ldots$$

2 Use trigonometry

In triangle EFG,

$$\sin 28^\circ = \frac{EF}{EG}$$

$$EG = \frac{9.192\ldots}{\sin 28^\circ} = 19.580\ldots$$

FINAL ANSWER

19.6 cm.

4MA1/1H JANUARY 2021

Question 10

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 Here is isosceles triangle ABC.

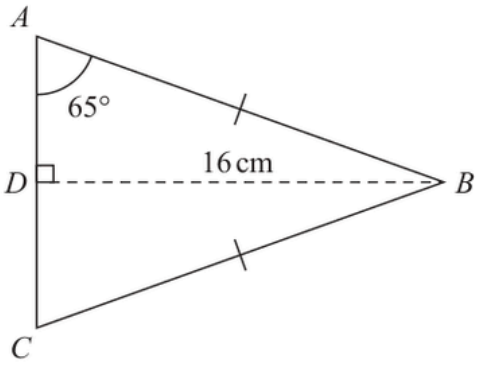


Diagram **NOT** accurately drawn

D is the midpoint of AC and $DB = 16$ cm.

Angle $DAB = 65°$

Work out the perimeter of triangle ABC.
Give your answer correct to one decimal place.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2021

1 Find the gradient

Since ABC is isosceles and D is the midpoint of AC, BD is perpendicular to AC.

2 Use trigonometry

In right triangle ABD,

$$BD = 16, \qquad \angle DAB = 65°$$

$$\sin 65° = \frac{16}{AB}$$

$$AB = \frac{16}{\sin 65°}$$

3 **Use trigonometry**

Also,

$$\tan 65^\circ = \frac{16}{AD}$$

$$AD = \frac{16}{\tan 65^\circ}$$

---

4 **The perimeter is**

The perimeter is

$$AB + BC + AC = 2AB + 2AD$$

$$= 2\left(\frac{16}{\sin 65^\circ}\right) + 2\left(\frac{16}{\tan 65^\circ}\right)$$

$$= 50.2299\ldots$$

✓ **FINAL ANSWER**

$50.2\ \text{cm}$.

4MA1/2HR JANUARY 2021

Question 13

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

13 The diagram shows two hot air balloons.
A is a point on the base of one of the balloons and B is a point on the base of the other balloon.

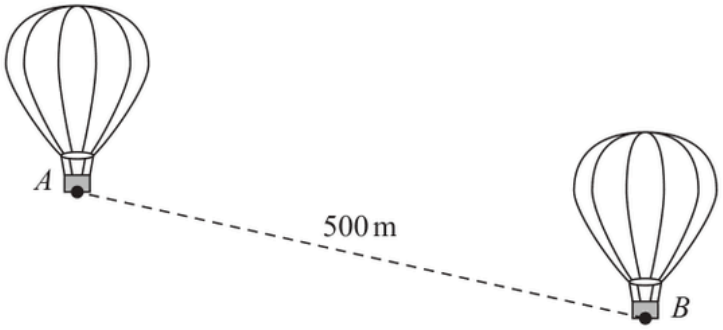


Diagram **NOT** accurately drawn

The distance between A and B is 500 metres.
The angle of depression of B from A is 23°

Calculate the vertical height of A above B.
Give your answer correct to one decimal place.

..... metres

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2021

1 **The vertical height is opposite**

The vertical height is opposite the $23°$ angle, and $AB = 500\ \text{m}$ is the hypotenuse.

$$\sin 23° = \frac{h}{500}$$

$$h = 500 \sin 23° = 195.365\ldots$$

FINAL ANSWER

$195.4\ \text{m}$.

4MA1/2H MAY 2021

Question 7

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

7 Here is a right-angled triangle.

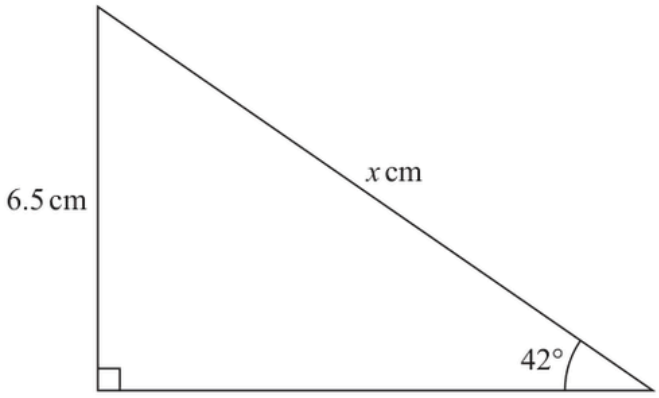


Diagram **NOT** accurately drawn

Work out the value of x.
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H May 2021

1 The side cm is opposite

The side 6.5 cm is opposite the $42°$ angle and x is the hypotenuse.

$$\sin 42° = \frac{6.5}{x}$$

$$x = \frac{6.5}{\sin 42°} = 9.714\ldots$$

FINAL ANSWER

$x = 9.7$ cm.

4MA1/1H NOVEMBER 2021

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 Here is a right-angled triangle.

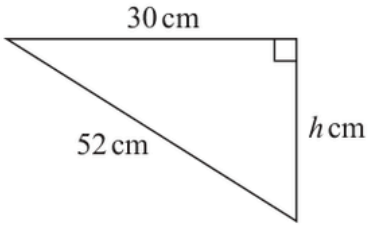


Diagram **NOT** accurately drawn

Calculate the value of h.
Give your answer correct to 3 significant figures.

h = ...

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2021

1 Use trigonometry

$$h^2 + 30^2 = 52^2$$

$$h^2 = 52^2 - 30^2 = 1804$$

$$h = \sqrt{1804} = 42.473\ldots$$

FINAL ANSWER

42.5 cm to 3 significant figures.

Question 12

6 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows a vertical cliff with a vertical radio mast on top of the cliff and a buoy in the sea.

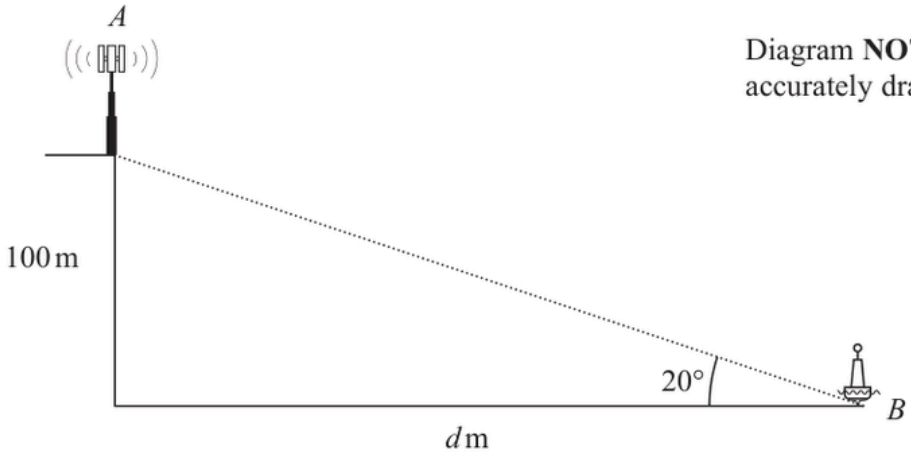


Diagram **NOT** accurately drawn

The height of the cliff is 100 metres.
The buoy is at the point B that is d metres from the base of the cliff.
The angle of elevation from B to the top of the cliff is 20°

(a) Calculate the value of d.
Give your answer correct to 3 significant figures.

$d =$ .....

(3)

The point A at the top of the radio mast is vertically above the top of the cliff.
The angle of elevation from B to A is 25°

(b) Calculate the height of the radio mast.
Give your answer correct to 3 significant figures.

..... m

(3)

(Total for Question 12 is 6 marks)

1 (a)

(a)

$$\tan 20° = \frac{100}{d}$$

$$d = \frac{100}{\tan 20°} = 274.747\ldots$$

2 Use trigonometry

So

$$d = 275 \text{ m}$$

3 (b) The angle of elevation

(b) The angle of elevation to the top of the radio mast is $25°$.

$$\tan 25° = \frac{100 + h}{274.747\ldots}$$

$$100 + h = 274.747\ldots \tan 25° = 128.116\ldots$$

$$h = 28.116\ldots$$

FINAL ANSWER

(a) 275 m, (b) 28.1 m.

4MA1/1H JANUARY 2022

Question 3

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The diagram shows triangle ABC

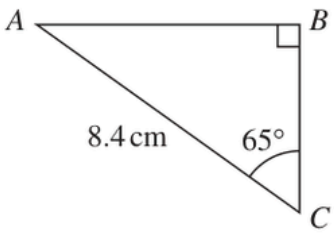


Diagram **NOT** accurately drawn

Work out the length of the side AB
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2022

1 **Use trigonometry**

Side AB is opposite the $65°$ angle, and $AC = 8.4$ cm is the hypotenuse.

$$\sin 65° = \frac{AB}{8.4}$$

$$AB = 8.4 \sin 65° = 7.613\ldots$$

✓ **FINAL ANSWER**

7.61 cm.

4MA1/1HR JANUARY 2022

Question 3

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The shaded shape is made using three identical right-angled triangles and a square.

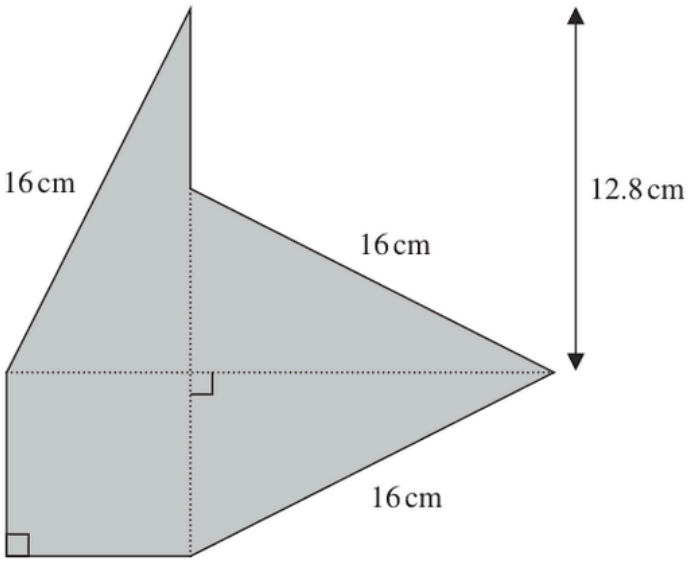


Diagram **NOT** accurately drawn

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2022

1 **Use trigonometry**

Each triangle has hypotenuse $16\ \text{cm}$ and one side $12.8\ \text{cm}$.

2 **Find the other side**

Find the other side:

$$\sqrt{16^2 - 12.8^2} = 9.6\ \text{cm}$$

3 **The exposed short vertical side**

The exposed short vertical side is

$$12.8 - 9.6 = 3.2\ \text{cm}$$

4 **Perimeter**

Perimeter:

$$3(16) + 9.6 + 9.6 + 3.2 = 70.4$$

✓ **FINAL ANSWER**

$70.4\ \text{cm}$.

4MA1/2HR JANUARY 2022

Question 12

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows two vertical phone masts, AB and CD, on horizontal ground.

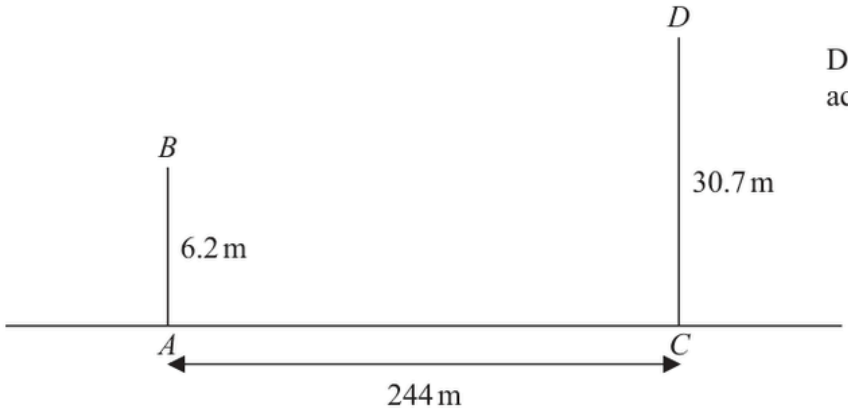


Diagram **NOT** accurately drawn

$AB = 6.2\,\text{m}$ $AC = 244\,\text{m}$ $CD = 30.7\,\text{m}$

Work out the size of the angle of depression of B from D
Give your answer correct to one decimal place.

.....°

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2022

1 **The vertical difference between the**

The vertical difference between the tops of the masts is

$$30.7 - 6.2 = 24.5\ \text{m}$$

2 **The horizontal distance is**

The horizontal distance is $244\ \text{m}$.

$$\tan\theta = \frac{24.5}{244}$$

$$\theta = 5.733\ldots^\circ$$

✓ **FINAL ANSWER**

5.7°.

4MA1/2H JANUARY 2022

Question 14

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

14 A zip wire is shown as the dashed line AC in the diagram.

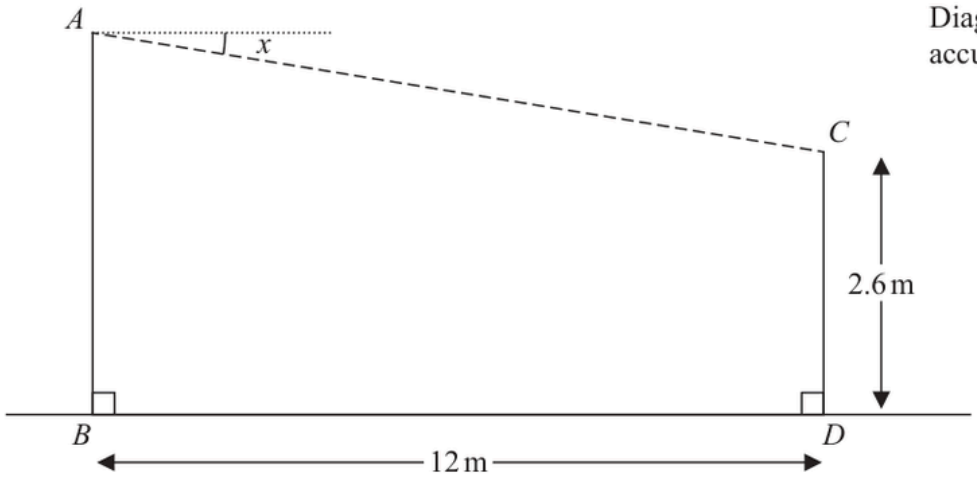


Diagram **NOT** accurately drawn

The zip wire is supported by two vertical posts AB and CD standing on horizontal ground.

$$CD = 2.6\,\text{m} \qquad BD = 12\,\text{m}$$

The zip wire makes an angle x with the horizontal, as shown in the diagram.
The design of the zip wire requires the angle x to be at least 5°

Work out the least possible height of the post AB
Give your answer correct to 3 significant figures.

..... m

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2022

1 **Use trigonometry**

The horizontal distance is $12\ \text{m}$ and the zip wire is $13\ \text{m}$.

2 **Vertical difference**

Vertical difference:

$$\sqrt{13^2 - 12^2} = 5$$

3 **Use trigonometry**

Since the shorter post is $2.6\ \text{m}$,

$$AB = 2.6 + 5 = 7.6$$

FINAL ANSWER

$7.60\ \text{m}$.

4MA1/2H JUNE 2022

Question 6

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

6 The diagram shows triangle PQR.

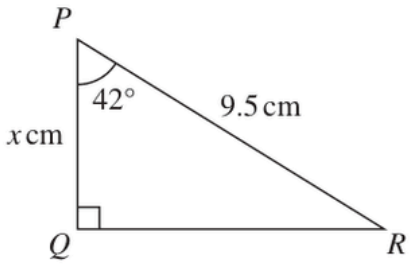


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2022

1 **Use trigonometry**

In the right-angled triangle, x is adjacent to the $42°$ angle and 9.5 cm is the hypotenuse.

$$\cos 42° = \frac{x}{9.5}$$

$$x = 9.5 \cos 42° = 7.059\ldots$$

✓ **FINAL ANSWER**

7.1 cm.

4MA1/2HR JUNE 2022

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 The diagram shows triangle ABP inside the regular hexagon $ABCDEF$

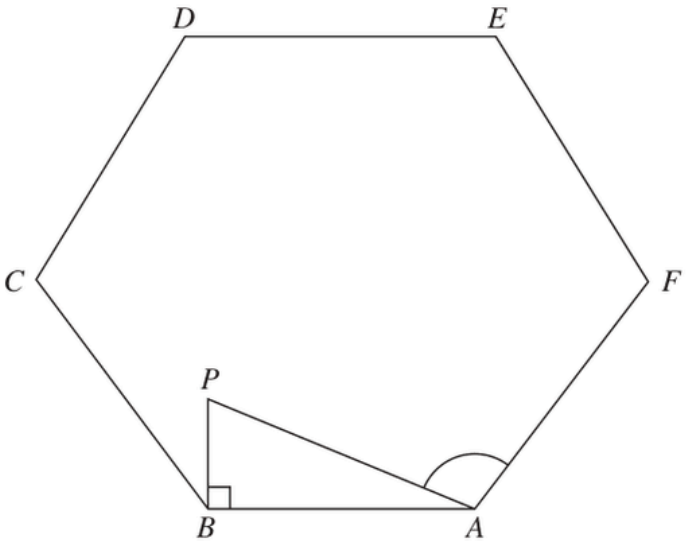


Diagram **NOT** accurately drawn

$AB = 5\,\text{cm}$ $BP = 2\,\text{cm}$ Angle $ABP = 90°$

Work out the size of angle PAF
Give your answer correct to 3 significant figures.

.....°

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2022

1 **A regular hexagon has interior**

A regular hexagon has interior angle

$$120°$$

2 **Use trigonometry**

In right-angled triangle ABP,

$$\tan(\angle PAB) = \frac{BP}{AB} = \frac{2}{5}$$

$$\angle PAB = \tan^{-1}\left(\frac{2}{5}\right) = 21.801\ldots°$$

3 Use trigonometry

Therefore

$$\angle PAF = 120° - 21.801\ldots° = 98.198\ldots°$$

FINAL ANSWER

$98.2°$.

4MA1/1H MAY 2023

Question 5

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

5 The diagram shows a shaded shape $AEBCD$ made by removing triangle AEB from rectangle $ABCD$

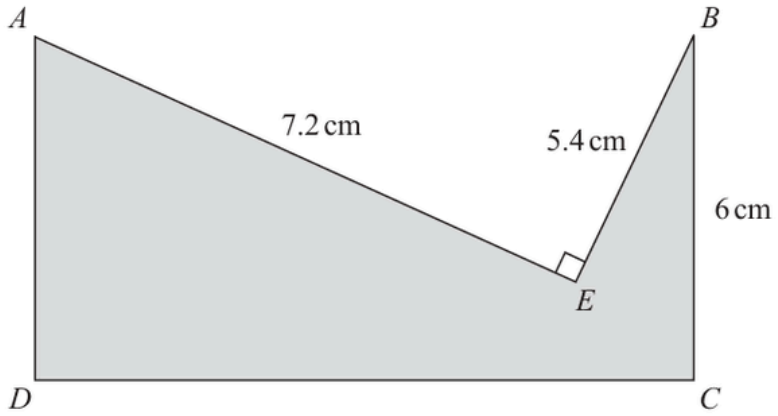


Diagram **NOT** accurately drawn

$AE = 7.2\,\text{cm}$ $BE = 5.4\,\text{cm}$ $BC = 6\,\text{cm}$ angle $AEB = 90°$

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2023

1 Triangle is right angled

Triangle AEB is right-angled:

$$AB^2 = 7.2^2 + 5.4^2 = 81$$

$$AB = 9$$

2 Use trigonometry

For rectangle $ABCD$,

$$DC = 9, \qquad AD = BC = 6$$

3 Perimeter of the shaded shape

Perimeter of the shaded shape:

$$7.2 + 5.4 + 6 + 9 + 6 = 33.6$$

FINAL ANSWER

$33.6\ \text{cm}$.

4MA1/2H JUNE 2023

Question 12

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows right-angled triangle ABD

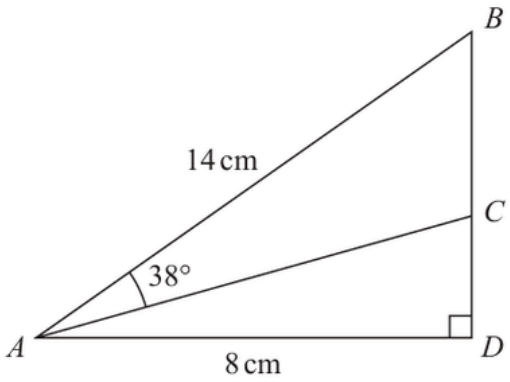


Diagram **NOT** accurately drawn

$AB = 14\,\text{cm}$ $AD = 8\,\text{cm}$

C is the point on BD such that angle $BAC = 38°$

Work out the length of CD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2023

1 Use trigonometry

First find angle DAB:

$$\cos(\angle DAB) = \frac{8}{14}$$

$$\angle DAB = 55.150\ldots°$$

2 Given

Given

$$\angle BAC = 38°$$

3 Use trigonometry

So

$$\angle CAD = 55.150\ldots° - 38° = 17.150\ldots°$$

4 **Use trigonometry**

In right-angled triangle ACD,

$$\tan(17.150\ldots^\circ) = \frac{CD}{8}$$

$$CD = 2.468\ldots$$

✓ **FINAL ANSWER**

$2.47\ \text{cm}$.

4MA1/2H NOVEMBER 2023

Question 2

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 The diagram shows triangle ABC and triangle ECD

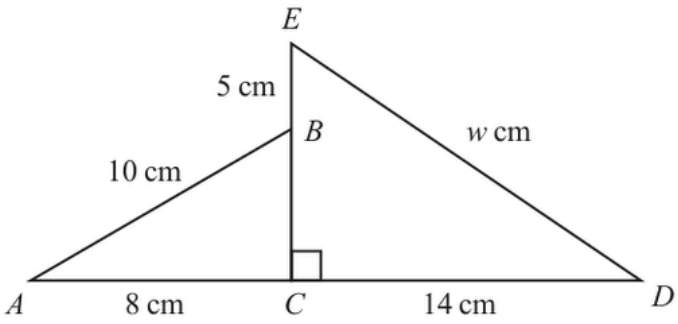


Diagram **NOT** accurately drawn

ACD and EBC are straight lines.

$AB = 10$ cm $\quad AC = 8$ cm $\quad EB = 5$ cm $\quad CD = 14$ cm $\quad ED = w$ cm

Work out the value of w
Give your answer correct to one decimal place.

$w =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2023

1 **Use trigonometry**

In triangle ABC,

$$BC^2 = 10^2 - 8^2$$

$$BC^2 = 36$$

$$BC = 6$$

2 **Use trigonometry**

So

$$EC = EB + BC = 5 + 6 = 11$$

3 **Use trigonometry**

In triangle ECD,

$$w^2 = 11^2 + 14^2$$

$$w^2 = 317$$

$$w = 17.804\ldots$$

✓ **FINAL ANSWER**

$w = 17.8\ \text{cm}$ to 1 decimal place.

4MA1/1H NOVEMBER 2023

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 Here is triangle ABC

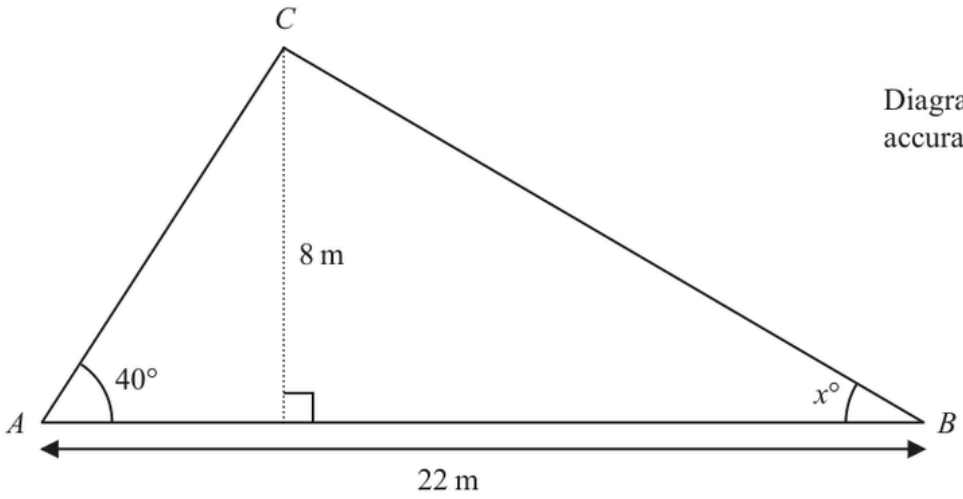


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.
Show your working clearly.

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2023

1 **Let the perpendicular from meet**

Let the perpendicular from C meet AB at F.

$$AF = \frac{8}{\tan 40^\circ} = 9.534\ldots$$

$$FB = 22 - 9.534\ldots = 12.465\ldots$$

2 **Use trigonometry**

In right-angled triangle CFB,

$$\tan x = \frac{8}{12.465\ldots}$$

$$x = 32.7^\circ$$

FINAL ANSWER

$32.7°$.

4MA1/2H JUNE 2024

Question 5

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

5 The diagram shows triangle ABC

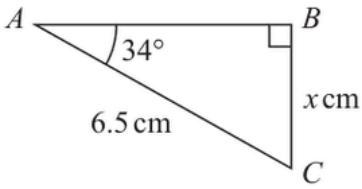


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2024

1 Use trigonometry

The hypotenuse is 6.5 cm, and x is opposite the $34°$ angle.

$$\sin 34° = \frac{x}{6.5}$$

$$x = 6.5 \sin 34° = 3.634\ldots$$

FINAL ANSWER

3.6 cm.

4MA1/1H MAY 2024

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a roof support.

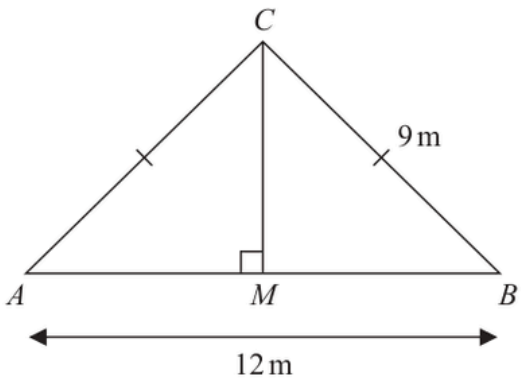


Diagram **NOT** accurately drawn

The roof support is made from four lengths of wood, AB, AC, BC and MC

$AC = BC = 9\,\text{m}$ $\quad AB = 12\,\text{m}$

angle $AMC = 90°$

Lewis is going to buy lengths of wood to make the roof support.

The wood costs 21.50 euros per metre.
Each length of wood he buys has to be a whole number of metres.

Work out the total cost of the wood Lewis needs to buy.
Show your working clearly.

..... euros

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2024

1 **Find the gradient**

Since $AC = BC$, the perpendicular support from C meets AB at its midpoint.

$$AM = \frac{12}{2} = 6$$

2 **Use trigonometry**

In right-angled triangle AMC,

$$AC^2 = AM^2 + MC^2$$

$$9^2 = 6^2 + MC^2$$

$$MC^2 = 81 - 36 = 45$$

$$MC = \sqrt{45} = 6.708\ldots$$

---

3 **A whole number of metres**

A whole number of metres must be bought, so buy 7 m for MC.

---

4 **Total length needed**

Total length needed:

$$12 + 9 + 9 + 7 = 37$$

---

5 **Cost**

Cost:

$$37 \times 21.50 = 795.50$$

✓ **FINAL ANSWER**

795.50 euros.

4MA1/2H NOVEMBER 2024

Question 3

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The diagram shows triangle PQR

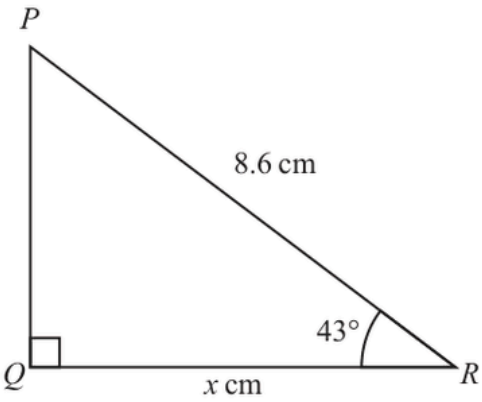


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2024

1 Use trigonometry

In the right-angled triangle, x is adjacent to the $43°$ angle.

$$\cos 43° = \frac{x}{8.6}$$

$$x = 8.6 \cos 43° = 6.289\ldots$$

FINAL ANSWER

6.3 cm.

4MA1/1H NOVEMBER 2024

Question 12

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 ABC is a right-angled triangle.
D is a point on BC

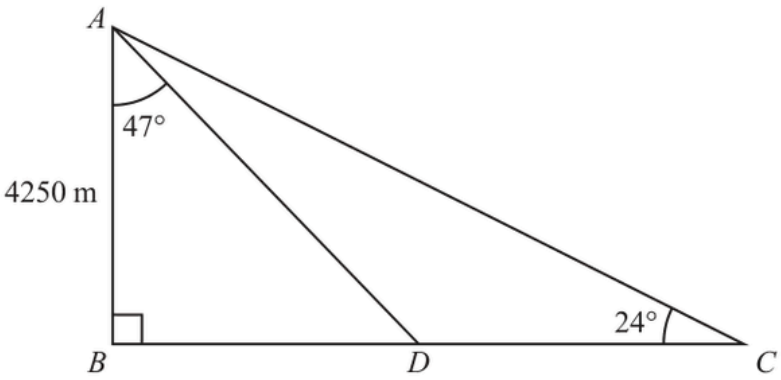


Diagram **NOT** accurately drawn

$AB = 4250$ m angle $BAD = 47°$ angle $BCA = 24°$

Work out the length of DC
Give your answer correct to the nearest integer.

..... m

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H November 2024

1 **Use trigonometry**

In triangle ABD,

$$\tan 47° = \frac{BD}{4250}$$

$$BD = 4250 \tan 47° = 4557.567\ldots$$

2 **Use trigonometry**

In triangle ABC,

$$\tan 24^\circ = \frac{4250}{BC}$$

$$BC = \frac{4250}{\tan 24^\circ} = 9545.656\ldots$$

$$DC = BC - BD = 4988.089\ldots$$

FINAL ANSWER

$4988\ \text{m}$.

4WM1/01R MAY 2025

Question 8

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

8

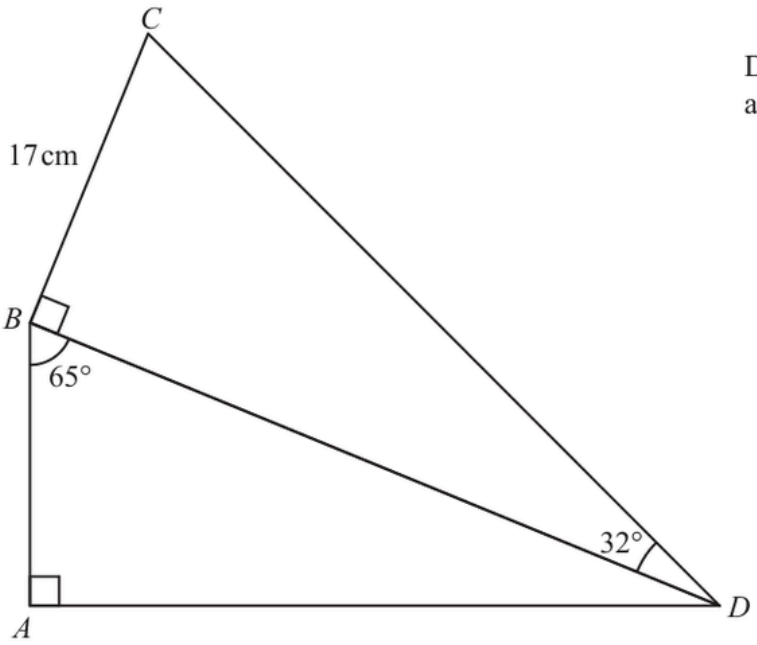


Diagram **NOT** accurately drawn

ABD and *BCD* are right-angled triangles.

angle $ABD = 65°$ angle $CDB = 32°$ $BC = 17$ cm

Work out the length of *AD*
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4WM1/01R May 2025

1 **Use trigonometry**

In right triangle BCD,

$$\tan 32° = \frac{17}{BD}$$

$$BD = \frac{17}{\tan 32°}$$

2 **Use trigonometry**

In right triangle ABD,

$$\sin 65^\circ = \frac{AD}{BD}$$

$$AD = \frac{17}{\tan 32^\circ} \sin 65^\circ \approx 24.6567$$

✓ **FINAL ANSWER**

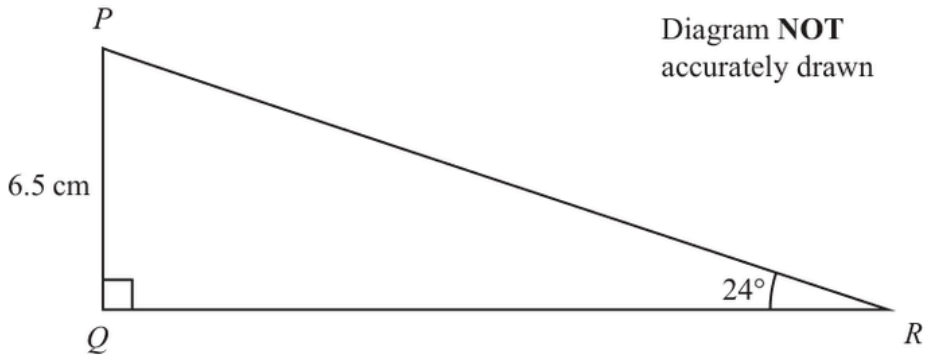
$AD = 24.7$ cm to 3 significant figures.

4MA1/1HR JUNE 2025

Question 9

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

9 The diagram shows triangle PQR



Work out the length of QR
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2025

1 Use trigonometry

$$\tan 24^\circ = \frac{6.5}{QR}$$

$$QR = \frac{6.5}{\tan 24^\circ} = 14.599\ldots$$

FINAL ANSWER

14.6 cm.

4MA1/2H JUNE 2025

Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 ABD and ABC are right-angled triangles.

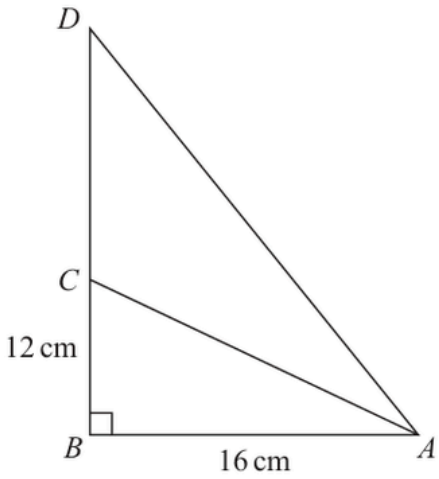


Diagram **NOT** accurately drawn

$AB = 16\,\text{cm}$ $BC = 12\,\text{cm}$ $AD = 1.5 \times AC$

Find the length of CD
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2025

1 **Use trigonometry**

In triangle ABC,

$$AC^2 = 16^2 + 12^2 = 400$$

$$AC = 20$$

2 **Use trigonometry**

Given $AD = 1.5AC$,

$$AD = 30$$

3 **Use trigonometry**

In triangle ABD,

$$BD^2 = 30^2 - 16^2 = 644$$

$$BD = 25.377\ldots$$

---

4 **Use trigonometry**

So

$$CD = BD - BC = 25.377\ldots - 12 = 13.377\ldots$$

✓ **FINAL ANSWER**

$13.4\ \text{cm}$.

4MA1/2HR JUNE 2025

Question 11

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows two triangles, ADE and CDB

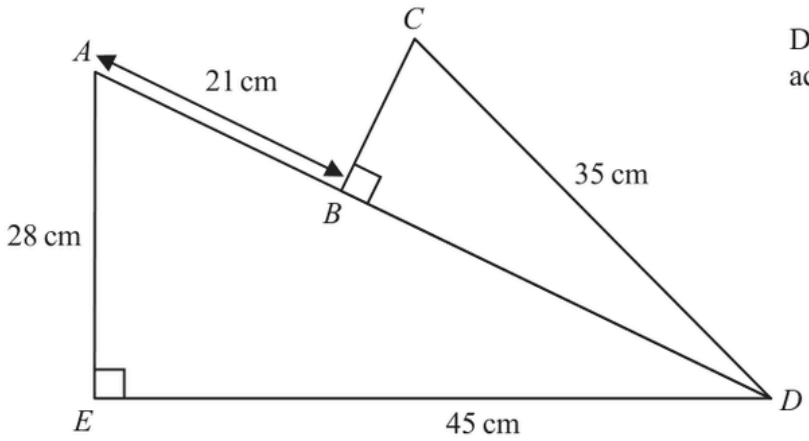


Diagram **NOT** accurately drawn

ABD is a straight line.

$AE = 28\,\text{cm}$ $ED = 45\,\text{cm}$ $AB = 21\,\text{cm}$ $CD = 35\,\text{cm}$

angle AED = angle $CBD = 90°$

Work out the area of triangle CDB
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2025

1 **Use trigonometry**

In right-angled triangle AFD,

$$AD^2 = 28^2 + 45^2$$

$$AD^2 = 2809$$

$$AD = 53$$

2 **Use trigonometry**

Since $AB = 21$,

$$BD = 53 - 21 = 32$$

3 **Use trigonometry**

In right-angled triangle CBD,

$$CB^2 = 35^2 - 32^2$$

$$CB^2 = 201$$

$$CB = \sqrt{201}$$

---

4 **Area of triangle is**

Area of triangle CDB is

$$\frac{1}{2} \times BD \times CB$$

$$\text{Area} = \frac{1}{2} \times 32 \times \sqrt{201} = 226.9\ldots$$

✓ **FINAL ANSWER**

$227\ \text{cm}^2$ to 3 significant figures.

Question 13

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

13 In the diagram, AB represents a vertical tower and CD represents a vertical building.

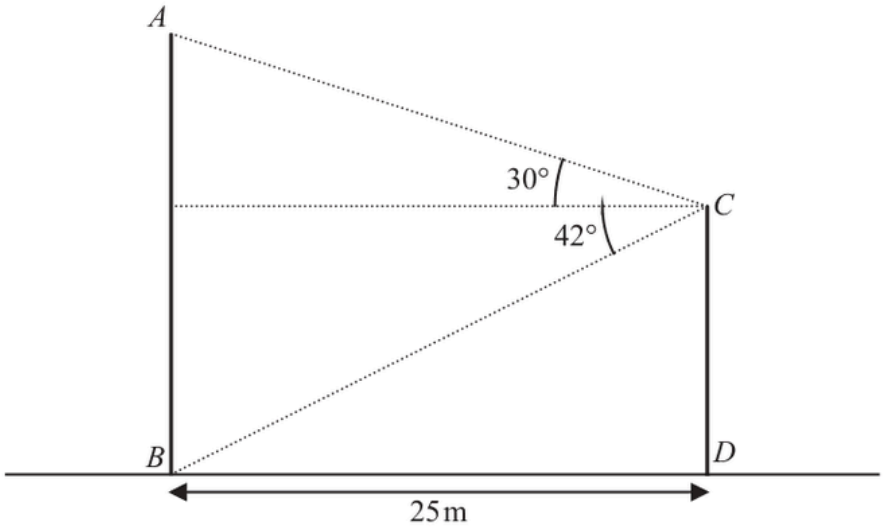


Diagram **NOT** accurately drawn

The bottom of the tower, B, and the bottom of the building, D, are on horizontal ground such that $BD = 25$ m

The angle of elevation of A from C is $30°$
The angle of depression of B from C is $42°$

(a) Work out the height of the tower AB
Give your answer correct to one decimal place.

..... m

(3)

(b) Work out the angle of elevation of A from D
Give your answer correct to one decimal place.

..... °

(2)

(Total for Question 13 is 5 marks)

1 **The height of the building**

The height of the building is

$$CD = 25\tan 42°$$

2 **Use trigonometry**

The extra height from C to A is

$$25\tan 30°$$

3 **Use trigonometry**

So

$$AB = 25\tan 42° + 25\tan 30° \approx 36.9439$$

4 **Use trigonometry**

For the angle of elevation θ from D,

$$\tan\theta = \frac{AB}{25}$$

$$\theta \approx 55.9137°$$

✓ **FINAL ANSWER**

$AB = 36.9$ m, angle of elevation $55.9°$.

4WM1/01 NOVEMBER 2025

Question 9

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

9 The diagram shows a sketch of a right-angled triangle, ABC

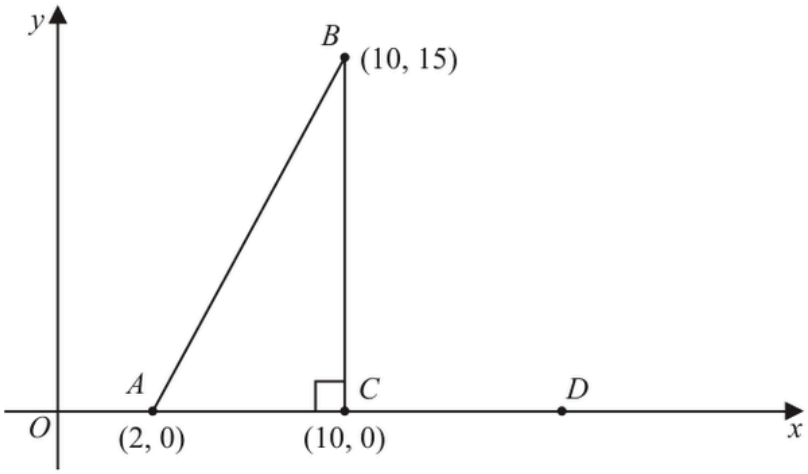


Diagram **NOT** accurately drawn

A is the point with coordinates $(2, 0)$
B is the point with coordinates $(10, 15)$
C is the point with coordinates $(10, 0)$

D is a point on the x-axis such that $AB = AD$

Find the coordinates of D

(..... , ...)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4WM1/01 November 2025

1 **Simplify surd**

$$AB = \sqrt{(10-2)^2 + (15-0)^2}$$

$$= \sqrt{64 + 225} = 17$$

2 **Calculate value**

Since D is on the positive x-axis and $AD = 17$,

$$D = (2 + 17, 0) = (19, 0)$$

FINAL ANSWER

$(19, 0)$.

4WM1/01 NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 ABC is a right-angled triangle.

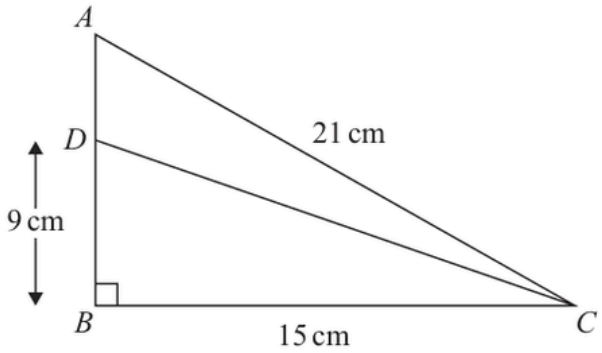


Diagram **NOT** accurately drawn

$AC = 21$ cm $BC = 15$ cm angle $ABC = 90°$

The point D lies on AB such that $DB = 9$ cm

Work out the size of angle ACD
Give your answer correct to one decimal place.

.....°

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4WM1/01 November 2025

1 Use trigonometry

First find AB:

$$AB^2 = 21^2 - 15^2 = 216$$

$$AB = 14.696\ldots$$

$$AD = AB - DB = 14.696\ldots - 9 = 5.696\ldots$$

2 Angle

Angle ACB:

$$\cos ACB = \frac{15}{21}$$

$$ACB = 44.415\ldots°$$

3 **Angle**

Angle DCB:

$$\tan DCB = \frac{9}{15}$$

$$DCB = 30.963\ldots^\circ$$

$$ACD = 44.415\ldots - 30.963\ldots = 13.451\ldots^\circ$$

✓ **FINAL ANSWER**

13.5°.

4MA1/1H NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 ABC is a right-angled triangle.

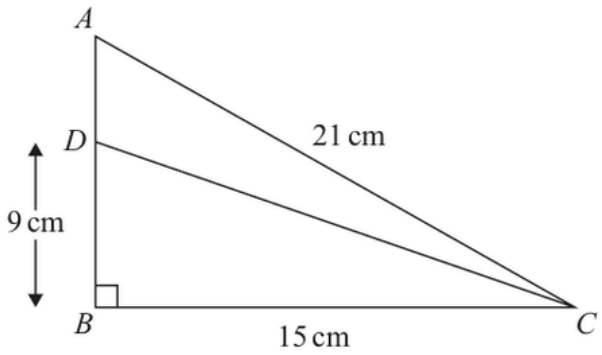


Diagram **NOT** accurately drawn

$AC = 21\,\text{cm}$ $BC = 15\,\text{cm}$ angle $ABC = 90°$

The point D lies on AB such that $DB = 9\,\text{cm}$

Work out the size of angle ACD
Give your answer correct to one decimal place.

.....°

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H November 2025

1 **Use trigonometry**

First find AB:

$$AB^2 = 21^2 - 15^2 = 216$$

$$AB = 14.696\ldots$$

$$AD = AB - DB = 14.696\ldots - 9 = 5.696\ldots$$

2 **Angle**

Angle ACB:

$$\cos ACB = \frac{15}{21}$$

$$ACB = 44.415\ldots^\circ$$

---

3 **Angle**

Angle DCB:

$$\tan DCB = \frac{9}{15}$$

$$DCB = 30.963\ldots^\circ$$

$$ACD = 44.415\ldots - 30.963\ldots = 13.451\ldots^\circ$$

✓ **FINAL ANSWER**

13.5°.

4MA1/2H NOVEMBER 2025

Question 11

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a right-angled triangle ABC

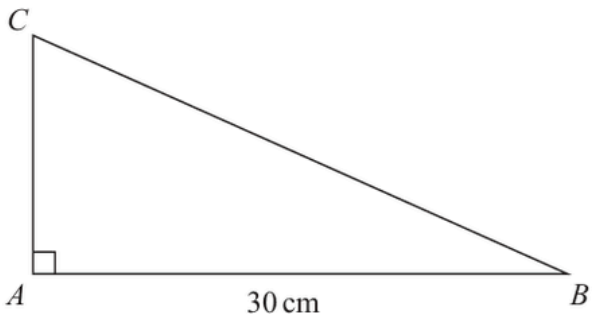


Diagram **NOT** accurately drawn

Area of triangle $ABC = 240\,\text{cm}^2$

Work out the perimeter of triangle ABC

..... cm

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2025

1 Use the area to find

Use the area to find AC:

$$\frac{1}{2} \times 30 \times AC = 240$$

$$15AC = 240$$

$$AC = 16$$

2 Use trigonometry

Then

$$BC^2 = 30^2 + 16^2 = 1156$$

$$BC = 34$$

3 **Perimeter**

Perimeter:

$$30 + 16 + 34 = 80$$

✓ **FINAL ANSWER**

$80\ \text{cm}$.

4MA1/2HR JUNE 2026

Question 1

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

1 The diagram shows a right-angled triangle.

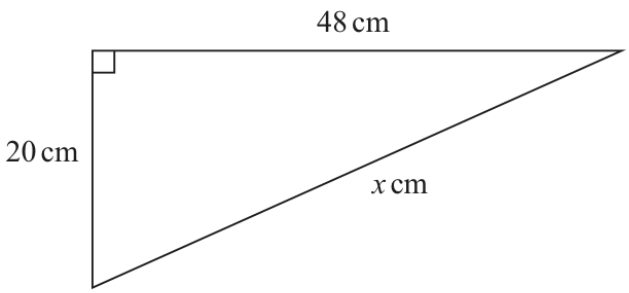


Diagram **NOT** accurately drawn

Work out the value of x

$x =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2026

1 Solution | Step 1

$$x^2 = 20^2 + 48^2 = 400 + 2304 = 2704.$$

Use Pythagoras because the 20 cm and 48 cm sides meet at the right angle.

2 Solution | Step 2

$$x = \sqrt{2704} = 52.$$

Take the positive square root because x is a length.

FINAL ANSWER

$x = 52$, so the hypotenuse is 52 cm.

4MA1/1H JUNE 2026

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2

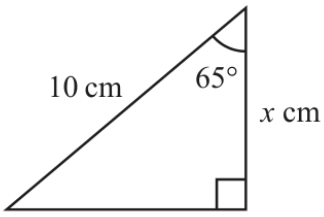


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2026

1 Solution | Step 1

$$\cos 65^\circ = \frac{x}{10}.$$

The side of length 10 cm is the hypotenuse and x is adjacent to the given angle.

2 Solution | Step 2

$$x = 10\cos 65^\circ = 4.22618\ldots = 4.23 \text{ cm} \quad (3 \text{ s.f.}).$$

Multiply by 10 and round only the final value.

✓ FINAL ANSWER

$x = 4.23$ cm

4WM1/01 MAY 2026

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2

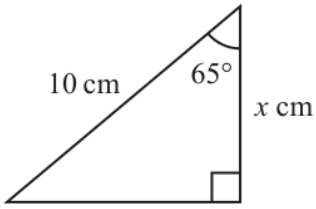


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4WM1/01 May 2026

1 Choose the ratio

x is next to the $65°$ angle and 10 cm is the hypotenuse, so use cosine.

2 Calculate

$$x = 10\cos 65° = 4.2261\ldots$$

FINAL ANSWER

$x = 4.23$

4MA1/2HR JUNE 2026

Question 11

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

11 The diagram shows a square $ABCD$ and a right-angled triangle EFG

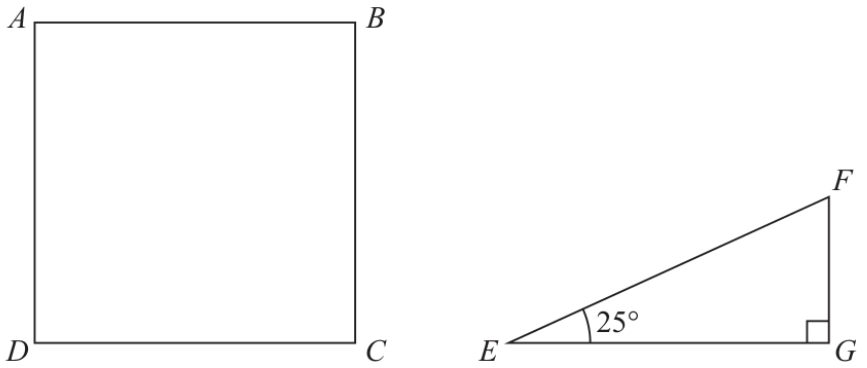


Diagram **NOT** accurately drawn

The area of square $ABCD$ is $36\,\text{cm}^2$
$DC = EG$

Work out the area of triangle EFG
Give your answer correct to one decimal place.
Show your working clearly.

..... cm^2

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$$DC = \sqrt{36} = 6\text{ cm}, \qquad EG = 6\text{ cm}.$$

All sides of the square have length equal to the square root of its area, and DC=EG.

2 **Solution | Step 2**

$$\tan 25^\circ = \frac{FG}{EG} \quad \Rightarrow \quad FG = 6\tan 25^\circ = 2.7978459\ldots\text{ cm}.$$

Relative to angle E, FG is opposite and EG is adjacent.

3 **Solution | Step 3**

$$\text{Area} = \frac{1}{2}(EG)(FG) = \frac{1}{2}(6)(6\tan 25^\circ) = 8.3935378\ldots\text{ cm}^2.$$

The base EG and height FG are perpendicular.

FINAL ANSWER

$8.4\ \text{cm}^2$ (1 decimal place).

TOPIC

3D Pythagoras & Trigonometry

Every past-paper question on this topic, 2020 to 2026.

Question 19

4 marks | 3D Pythagoras & Trigonometry

19 The diagram shows a triangular prism, $ABCDEF$, with a rectangular base $ABCD$

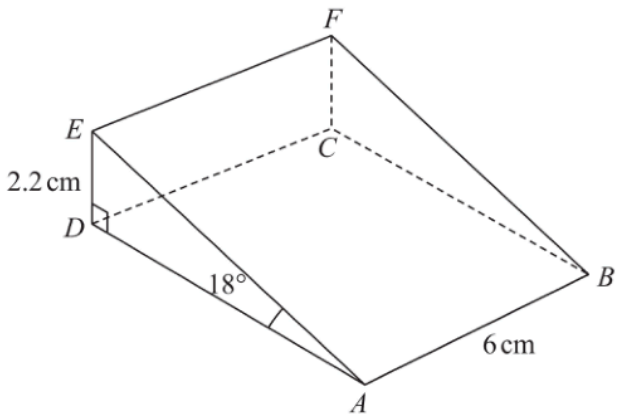


Diagram **NOT** accurately drawn

$AB = 6\,\text{cm}$ $DE = 2.2\,\text{cm}$ angle $DAE = 18°$ angle $ADE = 90°$

Work out the angle that BE makes with the plane $ABCD$
Give your answer correct to one decimal place.

1 Find AD

In triangle ADE,

$$\tan 18^\circ = \frac{DE}{AD}$$

$$AD = \frac{2.2}{\tan 18^\circ} = 6.77\ldots$$

2 Find DB in the base

The base $ABCD$ is a rectangle, so

$$DB = \sqrt{6^2 + 6.77\ldots^2} = 9.04\ldots$$

3 Use the projection triangle

The projection of BE on the plane is BD. Let the required angle be θ.

$$\tan\theta = \frac{DE}{DB} = \frac{2.2}{9.04\ldots}$$

4 Calculate the angle

$$\theta = 13.7^\circ$$

FINAL ANSWER

13.7°.

4MA1/1H JANUARY 2020

Question 19

4 marks | 3D Pythagoras & Trigonometry

19 The diagram shows a cuboid $ABCDEFGH$.

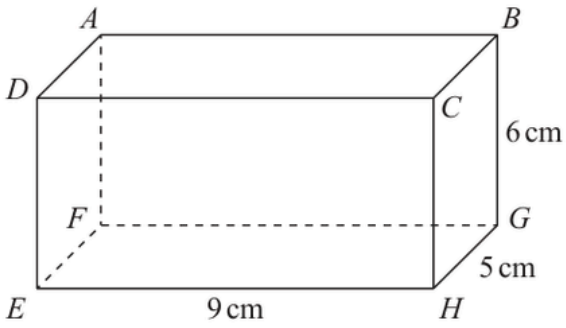


Diagram **NOT** accurately drawn

$EH = 9\,\text{cm}$, $HG = 5\,\text{cm}$ and $GB = 6\,\text{cm}$.

Work out the size of the angle between AH and the plane $EFGH$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2020

1 **The projection of onto the**

The projection of AH onto the base plane is the diagonal of the base rectangle:

$$\sqrt{9^2 + 5^2} = \sqrt{106}$$

2 **Use trigonometry**

The vertical height is $6\ \text{cm}$, so

$$\tan\theta = \frac{6}{\sqrt{106}}$$

$$\theta = 30.232\ldots^\circ$$

FINAL ANSWER

30.2°.

4MA1/1HR MAY/NOVEMBER 2020

Question 17

6 marks | 3D Pythagoras & Trigonometry

17 The diagram shows a prism $ABCDEFGH$ with a horizontal base.

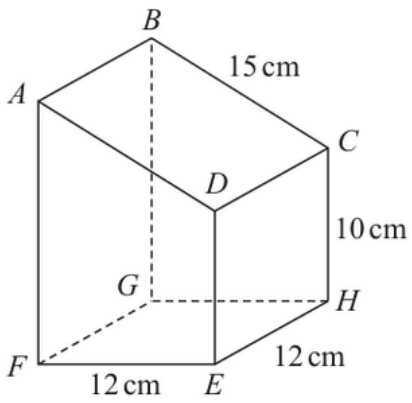


Diagram **NOT** accurately drawn

The base of the prism, $EFGH$, is a square of side 12 cm.
Trapezium $ADEF$ is a cross section of the prism where AF and DE are vertical edges.

$DE = CH = 10$ cm
$AD = BC = 15$ cm

(a) Work out the size of the angle between CF and the base $EFGH$.
Give your answer correct to one decimal place.

..... °

(3)

(b) Work out the length of BE.
Give your answer correct to one decimal place.

..... cm

(3)

(Total for Question 17 is 6 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR May/November 2020

1 **Use trigonometry**

(a) C is vertically above H, so the vertical height from F to C is 10 cm.

2 **The projection of on the**

The projection of CF on the base is FH:

$$FH = 12\sqrt{2}$$

$$\tan\theta = \frac{10}{12\sqrt{2}}$$

$$\theta = 30.508\ldots^\circ$$

---

3 **Answer**

Answer: 30.5°.

---

4 **(b) In the trapezium cross**

(b) In the trapezium cross-section, $AD = 15$ and the horizontal distance is 12, so the difference in vertical heights is

$$\sqrt{15^2 - 12^2} = 9$$

---

5 **Use trigonometry**

Since $DE = 10$, the height $BG = 19\ \text{cm}$.

---

6 **The projection of on the**

The projection of BE on the base is $GE = 12\sqrt{2}$, so

$$BE = \sqrt{19^2 + (12\sqrt{2})^2} = 25.475\ldots$$

✓ **FINAL ANSWER**

$25.5\ \text{cm}$.

4MA1/2HR JANUARY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 ABC is an isosceles triangle in a horizontal plane.
The point T is vertically above B.

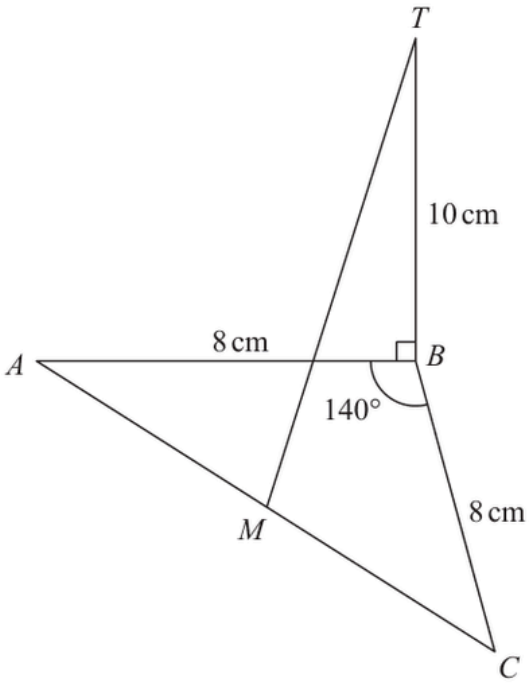


Diagram **NOT** accurately drawn

Angle $ABC = 140°$
$AB = BC = 8$ cm
$TB = 10$ cm
M is the midpoint of AC.

Calculate the size of the angle between MT and the horizontal plane ABC.
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2021

1 **Use trigonometry**

Since $AB = BC = 8$, triangle ABC is isosceles.

2 **The line from to the**

The line from B to the midpoint M bisects angle ABC.

$$\angle ABM = \frac{140^\circ}{2} = 70^\circ$$

3 **Use trigonometry**

In right triangle ABM,

$$\cos 70^\circ = \frac{BM}{8}$$

$$BM = 8\cos 70^\circ$$

4 **Use trigonometry**

Since T is vertically above B, the projection of MT onto the horizontal plane is MB.

5 **Let the required angle be**

Let the required angle be θ.

$$\tan\theta = \frac{TB}{BM}$$

$$\tan\theta = \frac{10}{8\cos 70^\circ}$$

$$\theta = 74.7^\circ$$

FINAL ANSWER

74.7°

4MA1/1H MAY 2021

Question 22

4 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

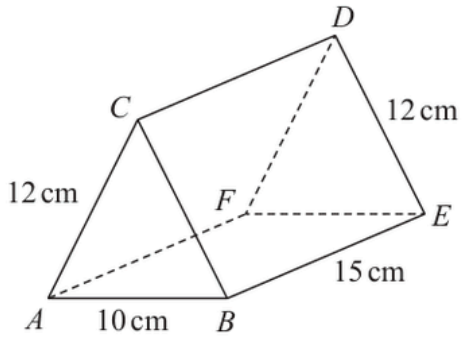


Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between AD and the base $ABEF$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2021

1 Calculate value

The triangular end is isosceles, with

$$AC = BC = 12, \qquad AB = 10$$

2 The perpendicular height of the

The perpendicular height of the triangle is found by halving AB:

$$AM = 5$$

$$CM^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$CM = \sqrt{119}$$

3 The projection of on the

The projection of AD on the horizontal base goes from A to the midpoint of FE. Its length is

$$\sqrt{15^2 + 5^2} = \sqrt{250}$$

4 Use trigonometry

So, for the angle θ between AD and the base,

$$\tan\theta = \frac{\sqrt{119}}{\sqrt{250}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.602\ldots$$

FINAL ANSWER

$34.6°$

4MA1/2HR JANUARY 2022

Question 19

3 marks | 3D Pythagoras & Trigonometry

19 $ABCD$ is a horizontal rectangular field.

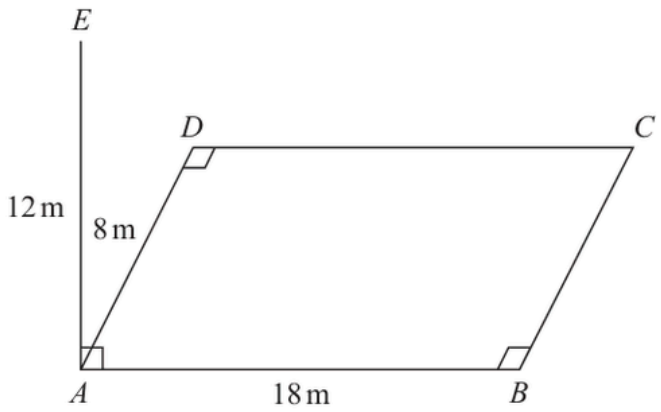


Diagram **NOT** accurately drawn

A vertical pole, AE, is placed at the corner A of the field.

$AE = 12\,\text{m}$ $AB = 18\,\text{m}$ $AD = 8\,\text{m}$

Calculate the size of the angle between EC and the plane $ABCD$
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2022

1 The projection of onto the

The projection of EC onto the field is AC.

$$AC = \sqrt{18^2 + 8^2} = \sqrt{388}$$

2 Use trigonometry

The vertical height is $AE = 12\ \text{m}$, so

$$\tan\theta = \frac{12}{\sqrt{388}}$$

$$\theta = 31.350\ldots^\circ$$

FINAL ANSWER

$31.4°$.

4MA1/1H JANUARY 2022

Question 21

5 marks | 3D Pythagoras & Trigonometry

21 The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

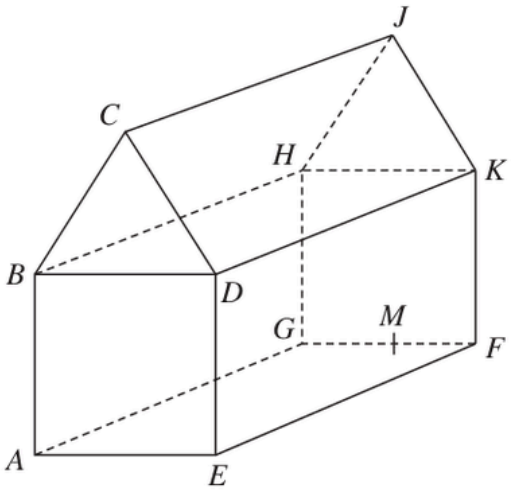


Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of T, giving your answer in the form $\dfrac{\sqrt{p} + \sqrt{q}}{17}$ where p and q are integers.

$T =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2022

1 **Let**

Let

$$AE = x$$

2 **Use trigonometry**

Since $ABDE$ is a square,

$$AB = BD = DE = AE = x$$

3 **Use trigonometry**

Since BCD is an equilateral triangle, the height from C to BD is

$$\frac{\sqrt{3}}{2}x$$

4 **Use trigonometry**

Therefore the vertical height from the base to J is

$$JM = x + \frac{\sqrt{3}}{2}x$$

$$JM = \frac{x(2+\sqrt{3})}{2}$$

5 **Use trigonometry**

Also,

$$EF = 2AE = 2x$$

6 **Find the midpoint**

Since M is the midpoint of GF, the horizontal distance from A to M is

$$AM = \sqrt{\left(\frac{x}{2}\right)^2 + (2x)^2}$$

$$AM = \sqrt{\frac{x^2}{4} + 4x^2}$$

$$AM = \frac{x\sqrt{17}}{2}$$

7 **Use trigonometry**

In right triangle AMJ,

$$\tan y^\circ = \frac{JM}{AM}$$

$$T = \frac{\frac{x(2+\sqrt{3})}{2}}{\frac{x\sqrt{17}}{2}}$$

$$T = \frac{2+\sqrt{3}}{\sqrt{17}}$$

---

8 **Rationalise the denominator**

Rationalise the denominator:

$$T = \frac{(2+\sqrt{3})\sqrt{17}}{17}$$

$$T = \frac{2\sqrt{17}+\sqrt{51}}{17}$$

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

✓ **FINAL ANSWER**

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

4MA1/1HR JUNE 2022

Question 18

4 marks | 3D Pythagoras & Trigonometry

18 The diagram shows a cube $ABCDEFGH$ with sides of length 6 cm.

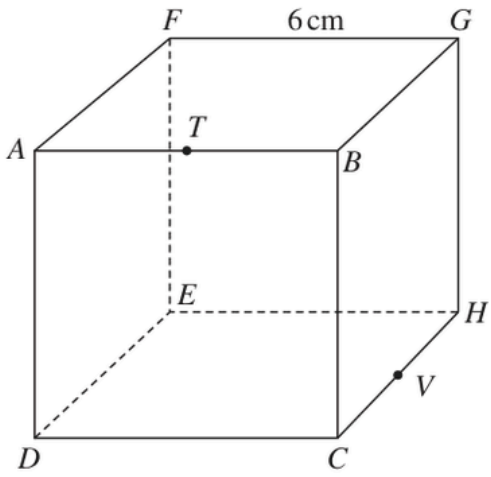


Diagram **NOT** accurately drawn

T is the midpoint of AB and V is the midpoint of CH

Work out the distance from T to V in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where a is an integer.

..... cm

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2022

1 The cube has side length

The cube has side length $\mathbf{6}$ cm.

2 Find the midpoint

Let T be the midpoint of AB and V be the midpoint of CH.

3 The changes in the three

The changes in the three perpendicular directions from T to V are

$$3, \quad 3, \quad 6$$

4 **Use trigonometry**

So by 3D Pythagoras,

$$TV = \sqrt{3^2 + 3^2 + 6^2}$$

$$TV = \sqrt{9 + 9 + 36} = \sqrt{54}$$

✓ **FINAL ANSWER**

$\sqrt{54}\ \text{cm}$.

Question 22

5 marks | 3D Pythagoras & Trigonometry

22 Here is a cuboid *ABCDEFGH*

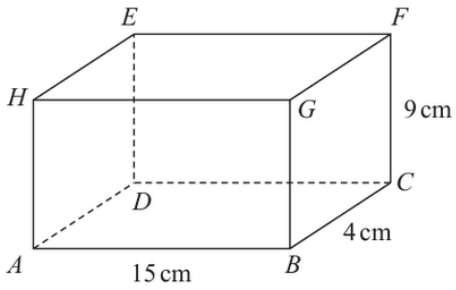


Diagram **NOT** accurately drawn

$AB = 15\,\text{cm}$ $BC = 4\,\text{cm}$ $CF = 9\,\text{cm}$

(a) Work out the length of *BE*
Give your answer correct to 3 significant figures.

..... cm

(2)

Here is a cuboid *PQRSTUVW*

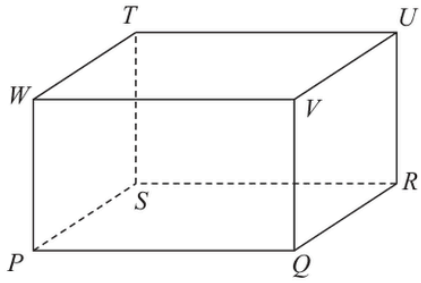


Diagram **NOT** accurately drawn

$PR = 42\,\text{cm}$

The size of the angle between *PU* and the plane *PQRS* is 30°

M is the midpoint of *PR*

(b) Work out the size of angle *UMR*
Give your answer correct to 3 significant figures.

.....°

(3)

(Total for Question 22 is 5 marks)

1 **Find the gradient**

For part (a), BE is the space diagonal using the three perpendicular lengths:

$$AB = 15, \qquad BC = 4, \qquad CF = 9$$

$$BE^2 = 15^2 + 4^2 + 9^2$$

$$BE^2 = 225 + 16 + 81 = 322$$

$$BE = \sqrt{322} = 17.9 \text{ cm}$$

2 **Use trigonometry**

For part (b), the projection of PU onto the plane $PQRS$ is PR.

$$PR = 42$$

3 **The angle between and the**

The angle between PU and the plane is $30°$, so

$$\tan 30° = \frac{RU}{PR}$$

$$RU = 42 \tan 30°$$

4 **Find the midpoint**

Since M is the midpoint of PR,

$$MR = 21$$

5 **Use trigonometry**

In right triangle MRU,

$$\tan \angle UMR = \frac{RU}{MR}$$

$$\tan \angle UMR = \frac{42 \tan 30°}{21}$$

$$\tan \angle UMR = 2 \tan 30°$$

$$\angle UMR = 49.1°$$

FINAL ANSWER

$BE = 17.9\ \text{cm}$, $\angle UMR = 49.1°$

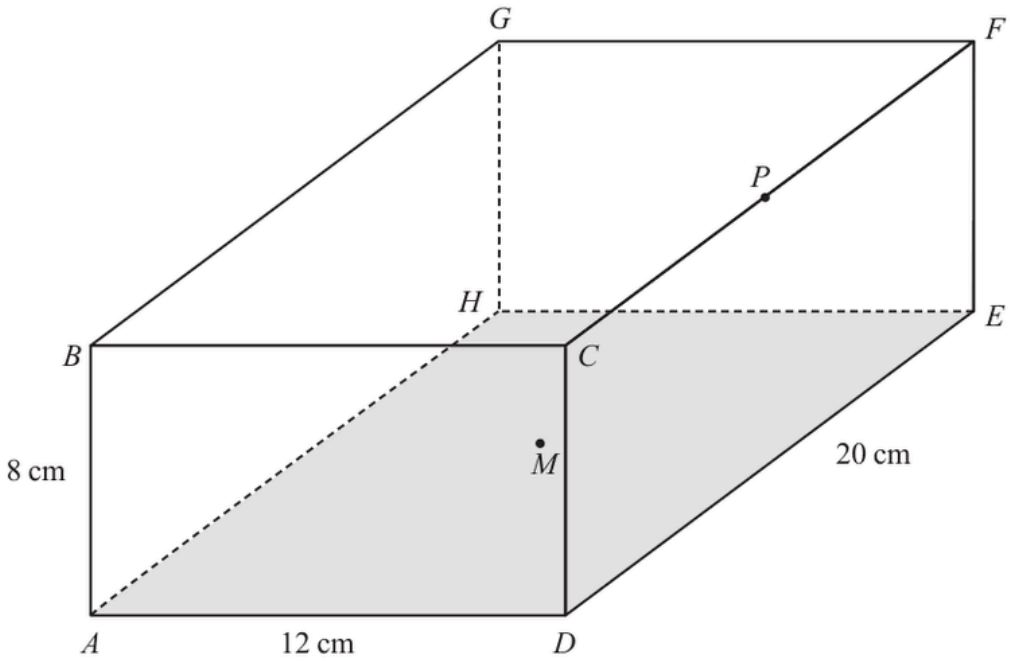
4MA1/1HR MAY 2024

Question 22

6 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a cuboid $ABCDEFGH$ with horizontal base $ADEH$

Diagram **NOT** accurately drawn



$AB = 8$ cm $\quad AD = 12$ cm $\quad DE = 20$ cm

M is the midpoint of the base $ADEH$ and P is the midpoint of the edge CF

Work out the size of angle BMP
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR May 2024

1 **Use trigonometry**

Use coordinates with

$$A = (0, 0, 0)$$

2 **Use trigonometry**

Since

$$AD = 12, \qquad DE = 20, \qquad AB = 8$$

3 **take**

take

$$B = (0, 0, 8)$$

4 **The centre of the rectangular**

The centre of the rectangular base $ADEH$ is

$$M = (6, 10, 0)$$

5 **Find the midpoint**

Point P is the midpoint of CF.

$$C = (12, 0, 8), \qquad F = (12, 20, 8)$$

$$P = (12, 10, 8)$$

6 **Use trigonometry**

Now

$$\vec{MB} = B - M = (-6, -10, 8)$$

$$\vec{MP} = P - M = (6, 0, 8)$$

7 Use the scalar product formula

Use the scalar product formula:

$$\vec{MB} \cdot \vec{MP} = (-6)(6) + (-10)(0) + (8)(8)$$

$$\vec{MB} \cdot \vec{MP} = 28$$

$$|\vec{MB}| = \sqrt{(-6)^2 + (-10)^2 + 8^2} = \sqrt{200}$$

$$|\vec{MP}| = \sqrt{6^2 + 0^2 + 8^2} = 10$$

$$\cos \angle BMP = \frac{28}{10\sqrt{200}}$$

$$\angle BMP = 78.6°$$

FINAL ANSWER

$78.6°$

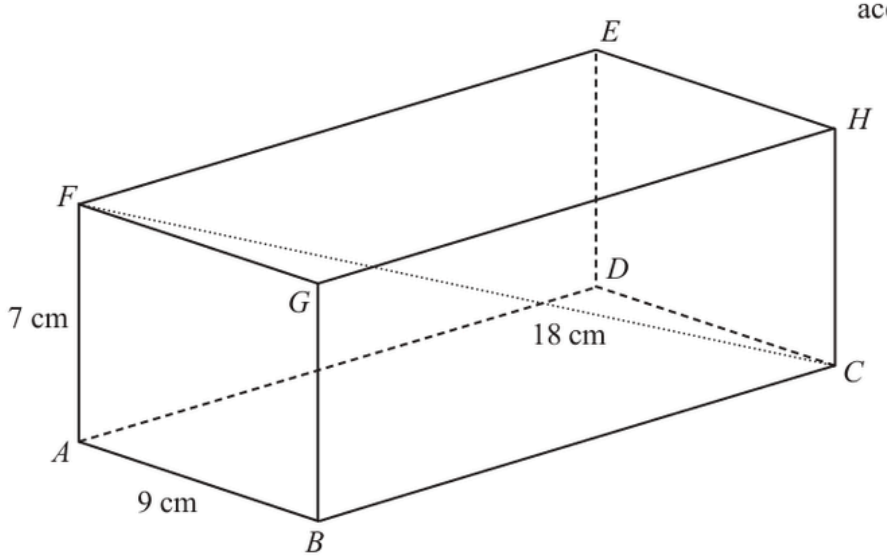
4MA1/1HR JUNE 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2025

1 **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

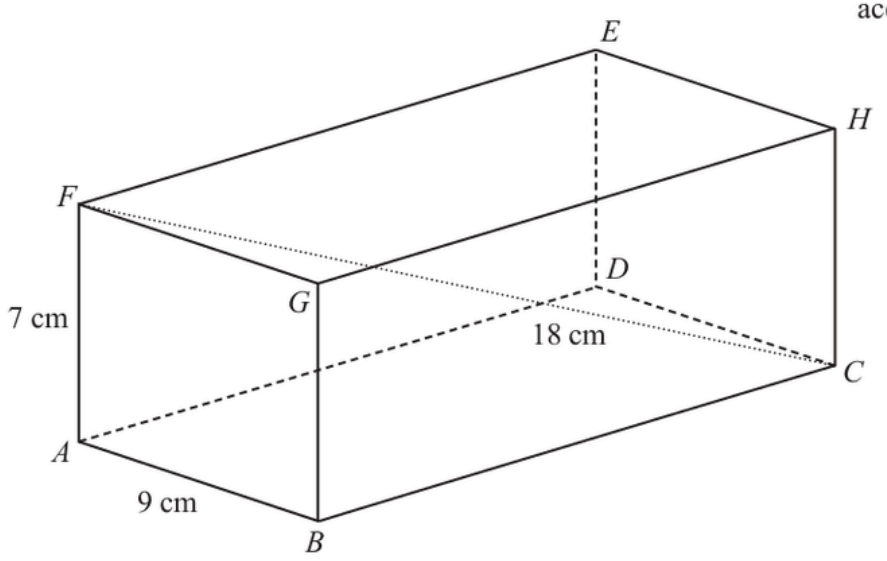
4WM1/01R MAY 2025

Question 20

3 marks | 3D Pythagoras & Trigonometry

20 The diagram shows cuboid $ABCDEFGH$

Diagram **NOT** accurately drawn



$AB = 9\,\text{cm}$ $AF = 7\,\text{cm}$ $FC = 18\,\text{cm}$

Calculate the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4WM1/01R May 2025

1 **Use trigonometry**

In the rectangle $ABCF$, use Pythagoras with the diagonal FC:

$$AC^2 = FC^2 - AF^2$$

$$AC^2 = 18^2 - 7^2 = 324 - 49 = 275$$

2 **Use trigonometry**

Now use Pythagoras in triangle ABC:

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 275 - 9^2 = 275 - 81 = 194$$

$$BC = \sqrt{194} = 13.928\ldots$$

✓ **FINAL ANSWER**

$\mathbf{13.9\ cm}$ to 3 significant figures

4MA1/2HR JUNE 2025

Question 23

3 marks | 3D Pythagoras & Trigonometry

23 The diagram shows a triangular prism, $ABCDEF$, with a horizontal rectangular base $ABCD$

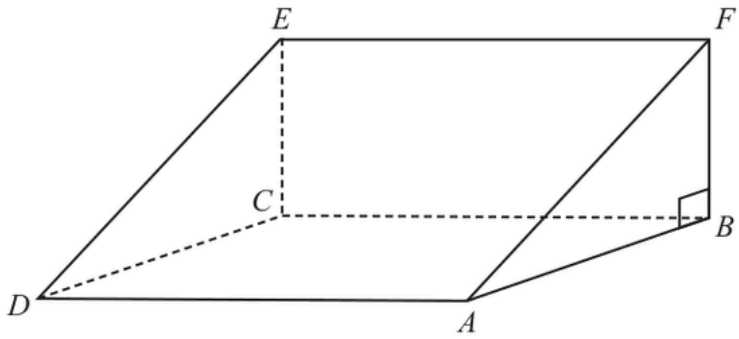


Diagram **NOT** accurately drawn

M is the midpoint of the line AC

$AC = 40\,\text{cm}$ angle $CAE = 35°$ angle $ABF = 90°$

Work out the size of angle CME
Give your answer correct to 3 significant figures.

.....°

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2025

1 Use trigonometry

$$AC = 40$$

2 The point is vertically above

The point E is vertically above C, so triangle ACE is right-angled at C.

$$\angle CAE = 35°$$

$$\tan 35° = \frac{CE}{AC}$$

$$CE = 40 \tan 35°$$

3 **Find the midpoint**

Since M is the midpoint of AC,

$$CM = 20$$

---

4 **Use trigonometry**

In right triangle CME,

$$\tan \angle CME = \frac{CE}{CM}$$

$$\tan \angle CME = \frac{40 \tan 35°}{20}$$

$$\tan \angle CME = 2 \tan 35°$$

$$\angle CME = 54.5°$$

✓ **FINAL ANSWER**

$54.5°$

4WM1/01 MAY 2025

Question 23

6 marks | 3D Pythagoras & Trigonometry

23 The diagram shows cuboid $ABCDEFGH$

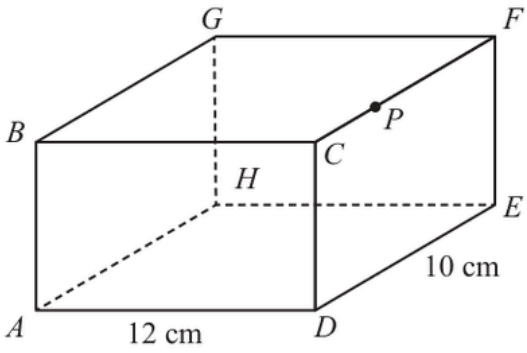


Diagram **NOT** accurately drawn

The point P lies on CF

$AD = 12$ cm $\quad DE = 10$ cm $\quad CP = 3$ cm

The angle of elevation of P from A is $24°$

Calculate the size of angle APG
Give your answer correct to the nearest degree.
Show your working clearly.

.....°

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 May 2025

1 **Point is cm from along**

Point P is 3 cm from C along CF. The horizontal distance from A to the point directly below P is

$$\sqrt{12^2 + 3^2} = \sqrt{153}$$

2 **Use trigonometry**

Using the angle of elevation,

$$h = \sqrt{153}\tan 24° = 5.507\ldots$$

3 **Use trigonometry**

So

$$AP = \frac{\sqrt{153}}{\cos 24^\circ} = 13.5399\ldots$$

4 **Use trigonometry**

On the top face,

$$PG = \sqrt{12^2 + 7^2} = \sqrt{193} = 13.8924\ldots$$

5 **Use trigonometry**

Also,

$$AG = \sqrt{12^2 + 10^2 + h^2} = 16.561\ldots$$

6 **Use cosine rule**

Use the cosine rule in triangle APG.

$$\cos \angle APG = \frac{AP^2 + PG^2 - AG^2}{2(AP)(PG)}$$

$$\angle APG = 49.163\ldots^\circ$$

7 **Use trigonometry**

To the nearest degree,

$$49^\circ$$

✓ **FINAL ANSWER**

49°

4MA1/2H JUNE 2025

Question 24

4 marks | 3D Pythagoras & Trigonometry

24 The diagram shows a square-based pyramid $ABCDE$

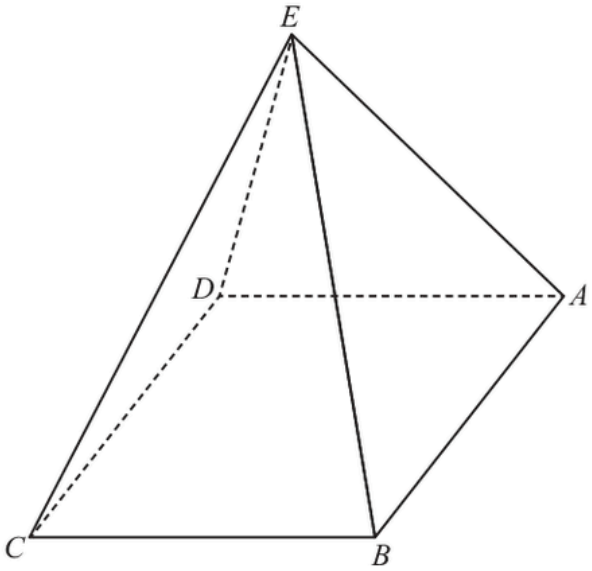


Diagram **NOT** accurately drawn

$EA = EB = EC = ED$

M is the centre of the horizontal square base $ABCD$
Q is the midpoint of AB
Angle $EQM = 80°$

$EA : AB = n : 1$

Find the value of n
Give your answer correct to 3 significant figures.

$n =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2025

1 Let

Let

$$AB = s$$

2 **Find the midpoint**

Since M is the centre of the square base and Q is the midpoint of AB,

$$QM = \frac{s}{2}$$

3 **Use trigonometry**

Since $EA = EB = EC = ED$, the point E is vertically above M.

4 **Use trigonometry**

In right triangle EQM,

$$\tan 80° = \frac{EM}{QM}$$

$$EM = \frac{s}{2}\tan 80°$$

5 **The distance from the centre**

The distance from the centre of a square to a vertex is half the diagonal:

$$AM = \frac{s\sqrt{2}}{2}$$

6 **Use trigonometry**

In right triangle EMA,

$$EA^2 = EM^2 + AM^2$$

$$EA^2 = \left(\frac{s}{2}\tan 80°\right)^2 + \left(\frac{s\sqrt{2}}{2}\right)^2$$

$$EA = s\sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

7 **Use trigonometry**

So

$$\frac{EA}{AB} = \sqrt{\frac{\tan^2 80°}{4} + \frac{1}{2}}$$

$$n = 2.92$$

FINAL ANSWER

$n = 2.92$

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 The diagram shows prism $ABCDEFGH$

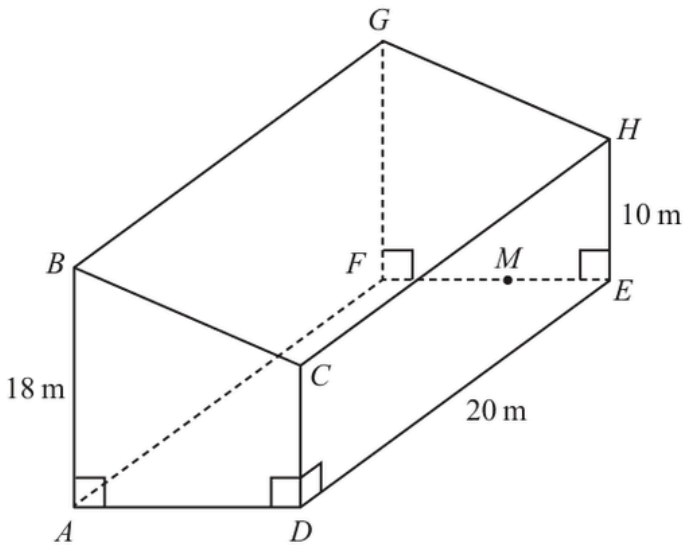


Diagram **NOT** accurately drawn

$$AB = FG = 18\,\text{m} \qquad DC = EH = 10\,\text{m} \qquad AD = FE$$
$$DE = CH = AF = BG = 20\,\text{m}$$
$$\text{angle } BAD = \text{angle } ADC = \text{angle } CDE = 90°$$

M is the point on FE such that $FM = \frac{3}{5}FE$

Given that angle $GMF = 60°$

calculate the angle of elevation of G from D
Give your answer correct to one decimal place.
Show your working clearly.

.....°

(Total for Question 26 is 5 marks)

1 **Use trigonometry**

From triangle GMF,

$$\tan 60^\circ = \frac{FG}{FM} = \frac{18}{FM}$$

$$FM = \frac{18}{\tan 60^\circ} = 6\sqrt{3}$$

2 **Simplify surd**

Since $FM = \frac{3}{5}FE$,

$$FE = 10\sqrt{3}$$

3 **Simplify surd**

The horizontal distance from D to the point below G is

$$\sqrt{20^2 + (10\sqrt{3})^2} = 10\sqrt{7}$$

4 **Use trigonometry**

If θ is the angle of elevation,

$$\tan\theta = \frac{18}{10\sqrt{7}}$$

$$\theta \approx 34.2289^\circ$$

✓ **FINAL ANSWER**

34.2°.

4MA1/1H NOVEMBER 2025

Question 26

5 marks | 3D Pythagoras & Trigonometry

26 Here is a prism $ABCDEFGH$ with a horizontal, rectangular base $ABGF$

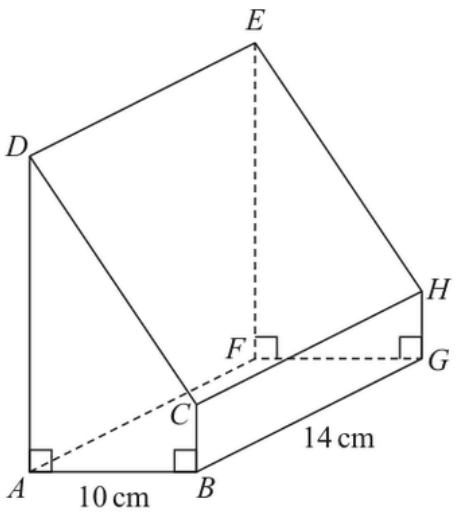


Diagram **NOT** accurately drawn

$AB = 10\,\text{cm}$ $BG = 14\,\text{cm}$

angle DAB = angle ABC = angle EFG = angle $FGH = 90°$

$BC = \frac{1}{5}AD$

The angle of elevation of E from B is $50°$

Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^3

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H November 2025

1 **The horizontal base is a**

The horizontal base $ABGF$ is a rectangle.

$$AB = 10, \qquad BG = 14$$

2 **Use trigonometry**

So the horizontal distance from B to F is

$$BF = \sqrt{10^2 + 14^2} = \sqrt{296}$$

3 **Use trigonometry**

The angle of elevation of E from B is $50°$.

4 **Use trigonometry**

Since E is vertically above F,

$$\tan 50° = \frac{FE}{BF}$$

$$FE = \sqrt{296} \tan 50°$$

5 **The prism has the same**

The prism has the same cross section throughout, so

$$AD = FE = \sqrt{296} \tan 50°$$

6 **Use trigonometry**

Also

$$BC = \frac{1}{5} AD$$

7 **Area of the trapezium cross**

Area of the trapezium cross section $ABCD$:

$$\frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}\left(AD + \frac{1}{5}AD\right) \times 10$$

$$= 6AD$$

8 **Volume of the prism**

Volume of the prism:

$$6AD \times 14 = 84AD$$

$$= 84\sqrt{296}\tan 50°$$

$$= 1722.311\ldots$$

9 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1720$$

✓ **FINAL ANSWER**

$$\boxed{1720\ \text{cm}^3}$$

Question 19

3 marks | 3D Pythagoras & Trigonometry

19 The diagram shows cuboid $PQRSTUVW$

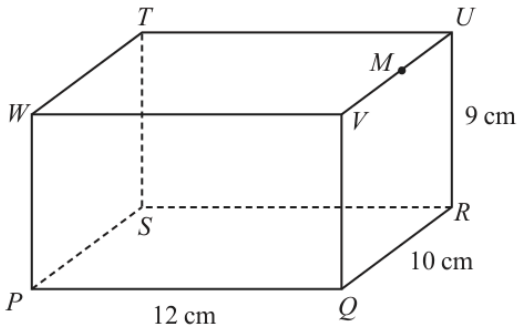


Diagram **NOT** accurately drawn

$PQ = 12$ cm $\quad QR = 10$ cm $\quad RU = 9$ cm

M is the midpoint of VU

Work out the size of the angle between line MP and plane $PQRS$
Give your answer correct to one decimal place.

..... °

(Total for Question 19 is 3 marks)

1 **Drop M to the base**

M is above the midpoint N of QR, so $QN = 5$ and $MN = 9$.

2 **Length PN**

$$PN = \sqrt{12^2 + 5^2} = 13$$

3 **Angle**

$$\tan\theta = \frac{9}{13} \Rightarrow \theta = 34.69\ldots^\circ$$

✓ **FINAL ANSWER**

34.7°

4MA1/1H JUNE 2026

Question 25

5 marks | 3D Pythagoras & Trigonometry

25 The diagram shows a triangular prism $ABCDEF$ with horizontal rectangular base $ABCD$
F is vertically above A

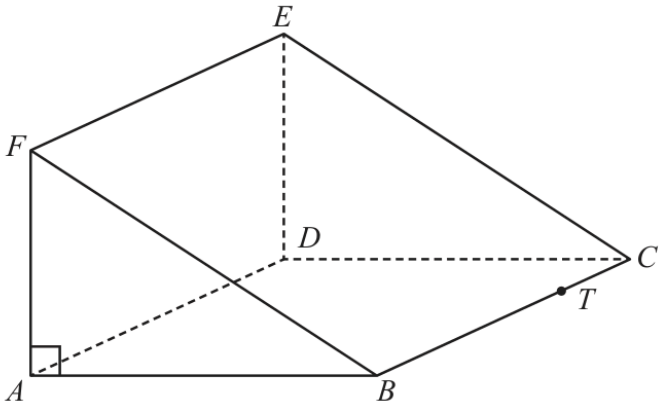


Diagram **NOT** accurately drawn

$AF = 12\,\text{cm}$ angle $ABF = 30°$ angle $BAF = 90°$

T is the point on the line BC such that $BF : BT = 3 : 2$

Work out the size of the angle between the line FT and the plane $ABCD$
Give your answer correct to one decimal place.
Show your working clearly.

..... °

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H June 2026

1 Solution | Step 1

$$BF = \frac{12}{\sin 30°} = 24\,\text{cm}, \qquad AB = \frac{12}{\tan 30°} = 12\sqrt{3}\,\text{cm}.$$

Use right triangle ABF.

2 Solution | Step 2

$$BT = \frac{2}{3}BF = 16\,\text{cm}.$$

Apply the given BF:BT ratio.

3 **Solution | Step 3**

$$AT = \sqrt{AB^2 + BT^2} = \sqrt{432 + 256} = \sqrt{688} = 4\sqrt{43}\ \text{cm}.$$

AB is perpendicular to BC in the rectangular base, so triangle ABT is right-angled at B.

4 **Solution | Step 4**

The projection of FT onto plane ABCD is AT because F lies vertically above A. Therefore the required angle is $\theta = \angle FTA$.

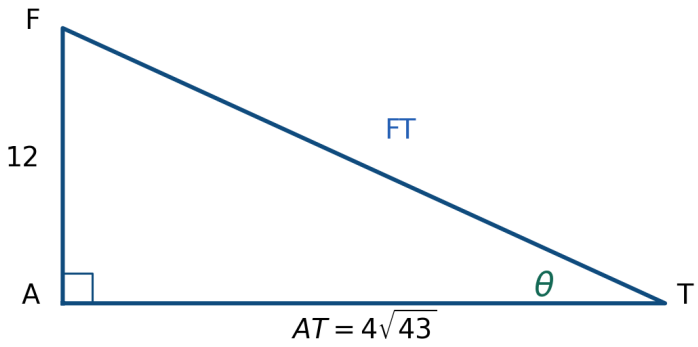
A line-plane angle is the angle between the line and its perpendicular projection onto the plane.

5 **Solution | Step 5**

$$\tan\theta = \frac{AF}{AT} = \frac{12}{4\sqrt{43}} = \frac{3}{\sqrt{43}}, \quad \theta = 24.583891\ldots^\circ = 24.6^\circ.$$

Use right triangle FAT and round to one decimal place.

FT projects onto AT in the base plane



✓ **FINAL ANSWER**

24.6°

TOPIC

Sine & Cosine Rules

Every past-paper question on this topic, 2020 to 2026.

4MA1/1H JANUARY 2020

Question 14

5 marks | Sine & Cosine Rules

14 The diagram shows parallelogram $EFGH$.

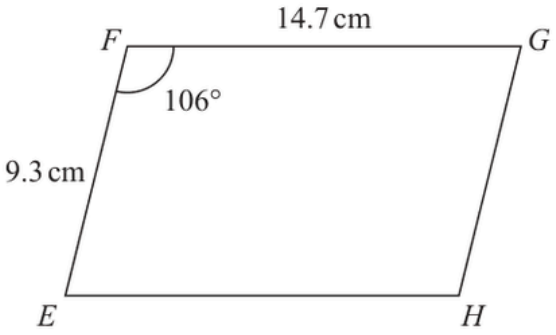


Diagram **NOT** accurately drawn

$EF = 9.3$ cm
$FG = 14.7$ cm
Angle $EFG = 106°$

(a) Work out the area of the parallelogram.
Give your answer correct to 3 significant figures.

..... cm^2
(2)

(b) Work out the length of the diagonal EG of the parallelogram.
Give your answer correct to 3 significant figures.

..... cm
(3)

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2020

1 **(a)**

(a)

$$\text{area} = 9.3 \times 14.7 \times \sin 106° = 131.414\ldots$$

2 **Answer**

Answer: $131\ cm^2$.

3 **Use trigonometry**

(b) In triangle EFG,

$$EG^2 = 9.3^2 + 14.7^2 - 2(9.3)(14.7)\cos 106°$$

$$EG = 19.440\ldots$$

✓ **FINAL ANSWER**

$19.4\ \text{cm}$.

4MA1/1HR JANUARY 2020

Question 21

6 marks | Sine & Cosine Rules

21 The diagram shows the prism $ABCDEF$ with cross section triangle ABC.

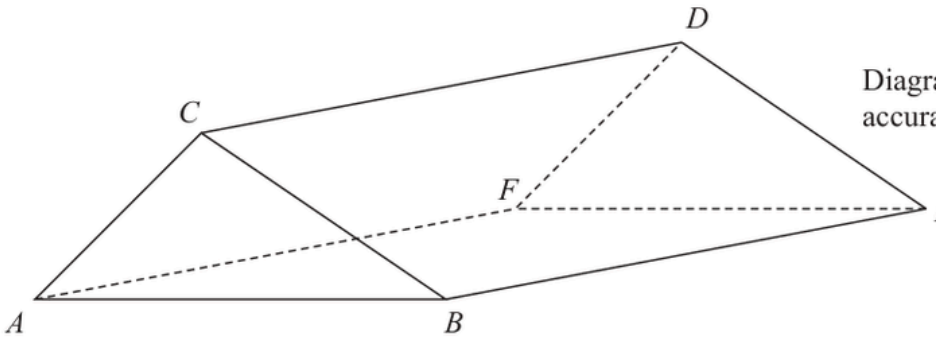


Diagram **NOT** accurately drawn

Angle $BEC = 40°$ and angle ACB is obtuse.
$AC = 6\,\text{cm}$ and $CE = 13\,\text{cm}$

The area of triangle ABC is $22\,\text{cm}^2$

Calculate the length of AB.
Give your answer correct to one decimal place.

..... cm

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2020

1 Use trigonometry

In triangle BEC, $CE = 13$ and $\angle BEC = 40°$.

2 Find the gradient

Since this is a prism, BE is perpendicular to the cross section, so triangle BEC is right-angled at B.

$$BC = 13 \sin 40°$$

$$BC = 8.356238\ldots$$

3 The area of triangle is

The area of triangle ABC is $22\ \text{cm}^2$.

4 **Use trigonometry**

Using

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$22 = \frac{1}{2}(AC)(BC)\sin(\angle ACB)$$

$$22 = \frac{1}{2}(6)(8.356238\ldots)\sin(\angle ACB)$$

$$\sin(\angle ACB) = 0.877587\ldots$$

---

5 **Use trigonometry**

The angle ACB is obtuse, so

$$\angle ACB = 180° - \sin^{-1}(0.877587\ldots)$$

$$\angle ACB = 118.647\ldots°$$

---

6 **Use the cosine rule**

Use the cosine rule:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos(\angle ACB)$$

$$AB^2 = 6^2 + (8.356238\ldots)^2 - 2(6)(8.356238\ldots)\cos(118.647\ldots°)$$

$$AB = 12.4056\ldots$$

---

7 **Correct to one decimal place**

Correct to one decimal place:

$$12.4$$

✓ **FINAL ANSWER**

$\boxed{12.4\text{ cm}}$

4MA1/2H NOVEMBER 2021

Question 18

5 marks | Sine & Cosine Rules

18 Here is a quadrilateral $ABCD$.

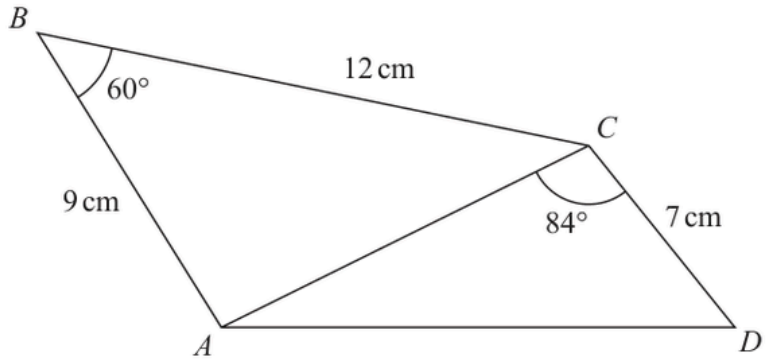


Diagram **NOT** accurately drawn

Calculate the area of quadrilateral $ABCD$.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2021

1 Area of triangle

Area of triangle ABC:

$$\frac{1}{2}(9)(12)\sin 60° = 46.765\ldots$$

2 Find using the cosine rule

Find AC using the cosine rule:

$$AC^2 = 9^2 + 12^2 - 2(9)(12)\cos 60°$$

$$AC = 10.816\ldots$$

3 Area of triangle

Area of triangle ACD:

$$\frac{1}{2}(10.816\ldots)(7)\sin 84° = 37.642\ldots$$

4 **Total area**

Total area:

$$46.765\ldots + 37.642\ldots = 84.407\ldots$$

✓ **FINAL ANSWER**

$84.4\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 9

4 marks | Sine & Cosine Rules

9 The diagram shows isosceles triangle ABC

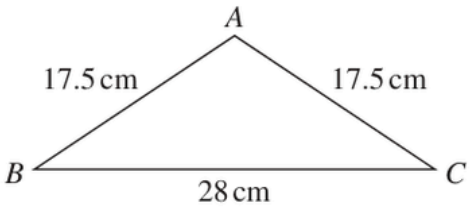


Diagram **NOT** accurately drawn

$AB = AC = 17.5$ cm $\quad BC = 28$ cm

Calculate the area of triangle ABC

..... cm^2

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2022

1 Use cosine rule

Split the isosceles triangle into two right-angled triangles.

$$\frac{BC}{2} = 14$$

$$h^2 = 17.5^2 - 14^2 = 110.25$$

$$h = 10.5$$

$$\text{area} = \frac{1}{2} \times 28 \times 10.5 = 147$$

FINAL ANSWER

$147\ \text{cm}^2$.

4MA1/2HR JANUARY 2022

Question 16

5 marks | Sine & Cosine Rules

16 The diagram shows quadrilateral $ABCD$

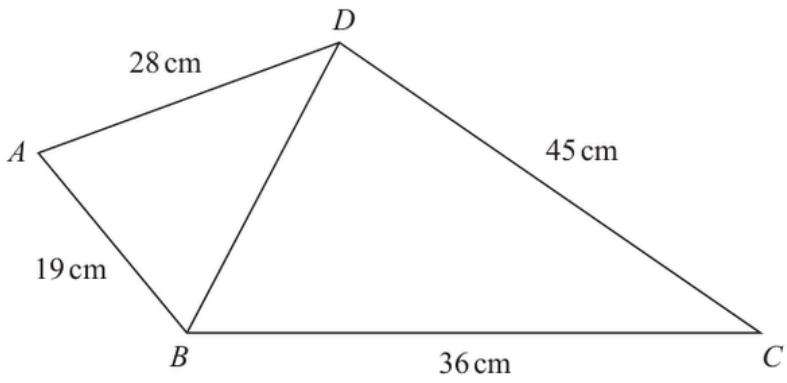


Diagram **NOT** accurately drawn

The angle BCD is acute.

Given that the area of triangle $BCD = 405\,\text{cm}^2$

work out the size of angle ABD
Give your answer correct to one decimal place.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR January 2022

1 **Use trigonometry**

For triangle BCD,

$$405 = \frac{1}{2}(36)(45)\sin BCD$$

$$\sin BCD = 0.5$$

2 **Use trigonometry**

The angle BCD is acute, so

$$BCD = 30°$$

3 **Find**

Find BD:

$$BD^2 = 36^2 + 45^2 - 2(36)(45)\cos 30°$$

$$BD = 22.695\ldots$$

---

4 **Use trigonometry**

Now use cosine rule in triangle ABD:

$$\cos ABD = \frac{19^2 + BD^2 - 28^2}{2(19)(BD)}$$

$$ABD = 83.871\ldots°$$

✓ **FINAL ANSWER**

$83.9°$.

4MA1/2H JANUARY 2022

Question 23

6 marks | Sine & Cosine Rules

23 The diagram shows triangle PQR

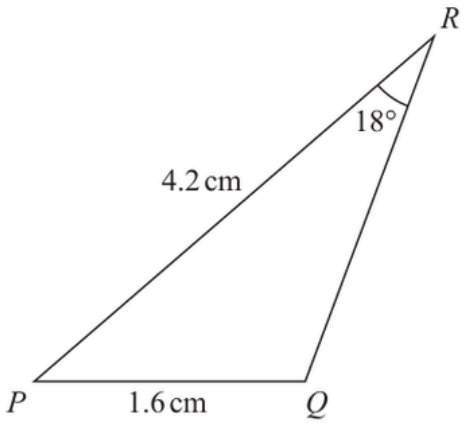


Diagram **NOT** accurately drawn

$PQ = 1.6\,\text{cm}$ $PR = 4.2\,\text{cm}$ Angle $PRQ = 18°$

Given that angle PQR is obtuse,

work out the area of triangle PQR
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H January 2022

1 **Use trigonometry**

In triangle PQR,

$$PQ = 1.6, \quad PR = 4.2, \quad \angle PRQ = 18°$$

2 **Use the sine rule**

Use the sine rule:

$$\frac{\sin Q}{PR} = \frac{\sin R}{PQ}$$

$$\frac{\sin Q}{4.2} = \frac{\sin 18^\circ}{1.6}$$

$$\sin Q = \frac{4.2 \sin 18^\circ}{1.6}$$

$$\sin Q = 0.811169\ldots$$

---

3 **Use trigonometry**

Since angle PQR is obtuse,

$$Q = 180^\circ - \sin^{-1}(0.811169\ldots)$$

$$Q = 125.7896\ldots^\circ$$

---

4 **Use trigonometry**

Now find angle P:

$$P = 180^\circ - 18^\circ - 125.7896\ldots^\circ$$

$$P = 36.2103\ldots^\circ$$

---

5 **Area of triangle**

Area of triangle PQR:

$$\frac{1}{2}(PQ)(PR)\sin P$$

$$= \frac{1}{2}(1.6)(4.2)\sin(36.2103\ldots^\circ)$$

$$= 1.984925\ldots$$

---

6 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1.98$$

FINAL ANSWER

$\boxed{1.98\ \text{cm}^2}$

4MA1/2HR JUNE 2022

Question 15

3 marks | Sine & Cosine Rules

15 The diagram shows a kite $ABCD$

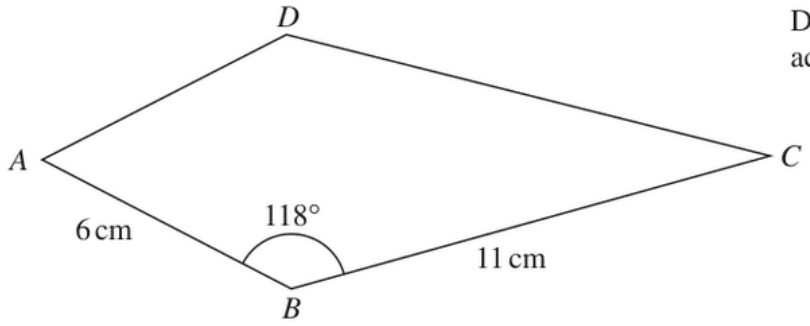


Diagram **NOT** accurately drawn

$AB = 6\,\text{cm}$ $BC = 11\,\text{cm}$ Angle $ABC = 118°$

Calculate the area of the kite.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2022

1 **A kite can be split**

A kite can be split into two congruent triangles.

2 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 11 \times \sin 118°$$

3 **Use trigonometry**

So the area of the kite is

$$2 \times \frac{1}{2} \times 6 \times 11 \times \sin 118°$$

$$= 6 \times 11 \times \sin 118° = 58.274\ldots$$

FINAL ANSWER

$58.3\ \text{cm}^2$.

4MA1/1HR JUNE 2022

Question 17

5 marks | Sine & Cosine Rules

17 Here is a parallelogram $PQRS$, in which angle SPQ is acute.

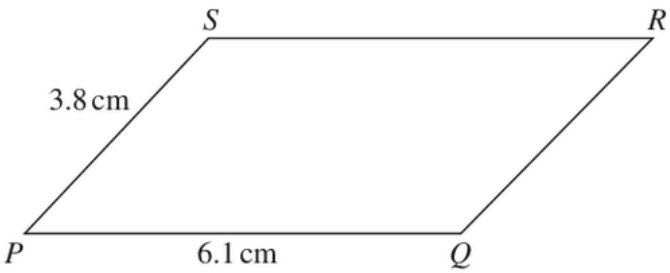


Diagram **NOT** accurately drawn

$PQ = 6.1\,\text{cm}$ $PS = 3.8\,\text{cm}$

The area of the parallelogram is $18\,\text{cm}^2$

Work out the length of QS
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2022

1 Height of the parallelogram

Height of the parallelogram:

$$h = \frac{18}{6.1} = 2.9508\ldots$$

2 Use cosine rule

The horizontal offset from P to S is

$$\sqrt{3.8^2 - h^2} = \sqrt{3.8^2 - 2.9508\ldots^2} = 2.394\ldots$$

3 Use trigonometry

So the horizontal distance from S to Q is

$$6.1 - 2.394\ldots = 3.705\ldots$$

$$QS = \sqrt{3.705\ldots^2 + 2.9508\ldots^2} = 4.737\ldots$$

FINAL ANSWER

4.74 cm.

4MA1/1H JUNE 2022

Question 18

5 marks | Sine & Cosine Rules

18 Here is triangle ABC

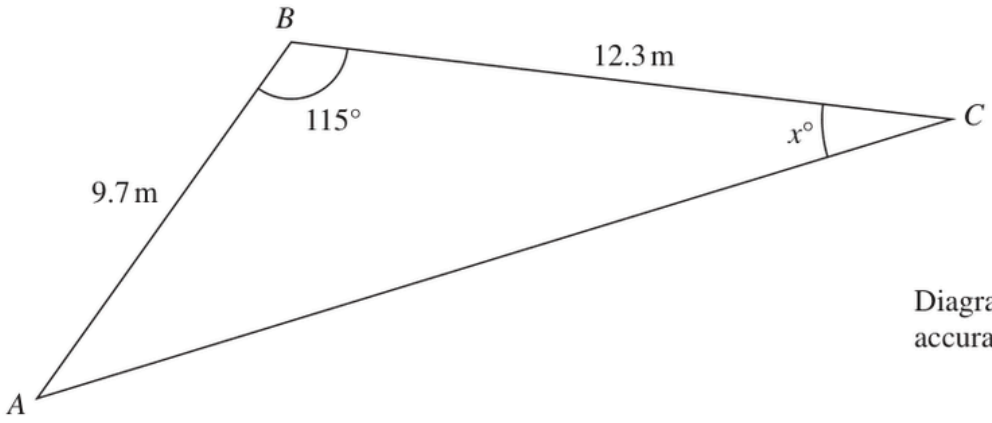


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H June 2022

1 Use trigonometry

First use the cosine rule to find AC:

$$AC^2 = 9.7^2 + 12.3^2 - 2(9.7)(12.3)\cos 115°$$

$$AC = 18.607\ldots$$

2 Use trigonometry

Now use the sine rule:

$$\frac{\sin x}{9.7} = \frac{\sin 115°}{18.607\ldots}$$

$$\sin x = \frac{9.7 \sin 115°}{18.607\ldots}$$

$$x = 28.194\ldots°$$

FINAL ANSWER

$28.2°$.

4MA1/2H JUNE 2023

Question 22

5 marks | Sine & Cosine Rules

22 The diagram shows a triangle ABC and a flagpole BF

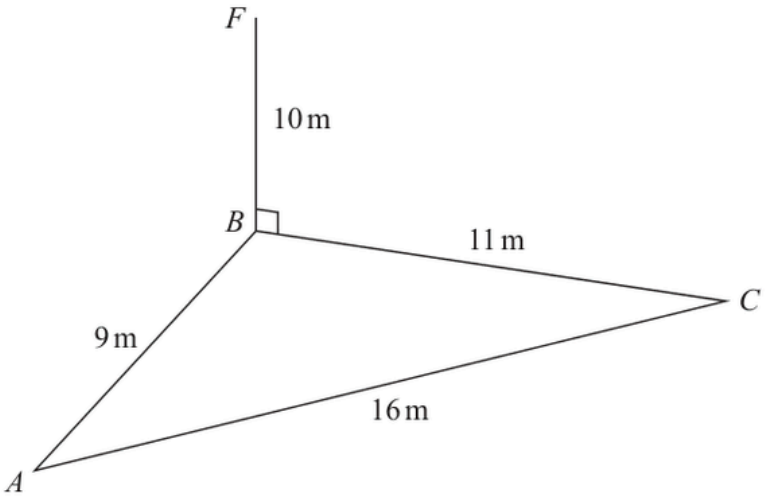


Diagram **NOT** accurately drawn

A, B and C are points on horizontal ground.

BF is vertical.

$AB = 9\,\text{m}$ $BC = 11\,\text{m}$ $AC = 16\,\text{m}$ $BF = 10\,\text{m}$

D is the point on AC such that angle $BDC = 90°$

Work out the size of the angle of elevation of the point F from the point D
Give your answer correct to one decimal place.

.....°

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2023

1 **Use trigonometry**

In triangle ABC,

$$AB = 9, \qquad BC = 11, \qquad AC = 16$$

2 **Use Heron s formula**

Use Heron's formula.

$$s = \frac{9 + 11 + 16}{2} = 18$$

$$\text{area} = \sqrt{18(18-9)(18-11)(18-16)}$$

$$= \sqrt{18 \cdot 9 \cdot 7 \cdot 2} = 47.6235\ldots$$

---

3 **is the foot of the**

D is the foot of the perpendicular from B to AC, so BD is the height of triangle ABC.

$$\text{area} = \frac{1}{2} \times AC \times BD$$

$$47.6235\ldots = \frac{1}{2} \times 16 \times BD$$

$$BD = 5.9529\ldots$$

---

4 **Use trigonometry**

In right-angled triangle BDF,

$$\tan\theta = \frac{BF}{BD}$$

$$\tan\theta = \frac{10}{5.9529\ldots}$$

$$\theta = \tan^{-1}\left(\frac{10}{5.9529\ldots}\right) = 59.2349\ldots^\circ$$

✓ **FINAL ANSWER**

$\boxed{59.2^\circ}$

4MA1/1HR MAY 2023

Question 25

6 marks | Sine & Cosine Rules

25 Here is a triangle ABC

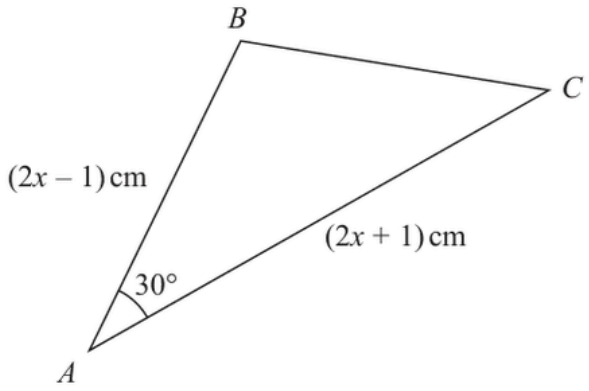


Diagram **NOT** accurately drawn

The area of the triangle is $(x^2 + x - 3.75)\,\text{cm}^2$

Find the size of the largest angle in triangle ABC
Give your answer correct to the nearest degree.

.....°

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR May 2023

1 **The two sides around the**

The two sides around the $30°$ angle are

$$2x - 1 \quad \text{and} \quad 2x + 1$$

2 **Use trigonometry**

Using the area formula,

$$\text{area} = \frac{1}{2}ab\sin C$$

$$x^2 + x - 3.75 = \frac{1}{2}(2x - 1)(2x + 1)\sin 30°$$

3 **Use trigonometry**

Since $\sin 30° = \frac{1}{2}$,

$$x^2 + x - 3.75 = \frac{1}{4}(2x - 1)(2x + 1)$$

$$x^2 + x - 3.75 = \frac{1}{4}(4x^2 - 1)$$

$$x^2 + x - 3.75 = x^2 - 0.25$$

$$x = 3.5$$

---

4 **Use trigonometry**

So

$$AB = 2x - 1 = 6, \qquad AC = 2x + 1 = 8$$

---

5 **Use cosine rule**

Use the cosine rule to find BC:

$$BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 30°$$

$$BC = 4.1063\ldots$$

---

6 **Use trigonometry**

The largest side is $AC = 8$, so the largest angle is $\angle B$.

---

7 **Use trigonometry**

Using the cosine rule again:

$$\cos B = \frac{6^2 + BC^2 - 8^2}{2(6)(BC)}$$

$$B = 103.0643\ldots°$$

✓ **FINAL ANSWER**

$\boxed{103°}$

4MA1/2HR JUNE 2024

Question 15

3 marks | Sine & Cosine Rules

15

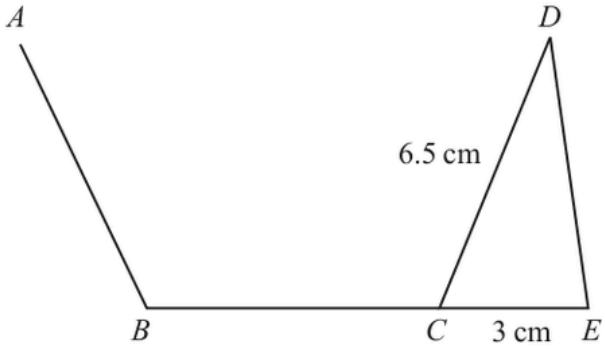


Diagram **NOT** accurately drawn

AB, *BC* and *CD* are three sides of a regular pentagon and *CDE* is a triangle.
BCE is a straight line.

$CD = 6.5\,\text{cm}$ $CE = 3\,\text{cm}$

Work out the area of triangle *CDE*
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2024

1 A regular pentagon has interior

A regular pentagon has interior angle

$$\frac{(5-2)\times 180}{5} = 108^\circ$$

2 Use trigonometry

Since BCE is a straight line,

$$\angle DCE = 180^\circ - 108^\circ = 72^\circ$$

3 **Area of triangle is**

Area of triangle CDE is

$$\frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}(6.5)(3)\sin 72°$$

$$\text{Area} = 9.27\ldots$$

✓ **FINAL ANSWER**

$9.27\ \text{cm}^2$ to 3 significant figures.

4MA1/1HR MAY 2024

Question 15

3 marks | Sine & Cosine Rules

15 The diagram shows isosceles triangle EFG

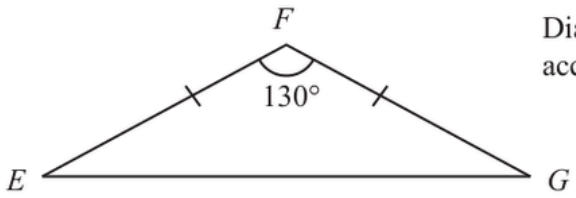


Diagram **NOT** accurately drawn

$EF = GF$
Angle $EFG = 130°$
The area of triangle EFG is 74 cm²

Work out the length of EF
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR May 2024

1 Let

Let

$$EF = FG = x$$

2 Use the area formula

Use the area formula:

$$74 = \frac{1}{2}x^2 \sin 130°$$

$$x^2 = \frac{148}{\sin 130°}$$

$$x = 13.899\ldots$$

FINAL ANSWER

13.9 cm.

4MA1/1H MAY 2024

Question 19

5 marks | Sine & Cosine Rules

19 Here is quadrilateral $ABCD$

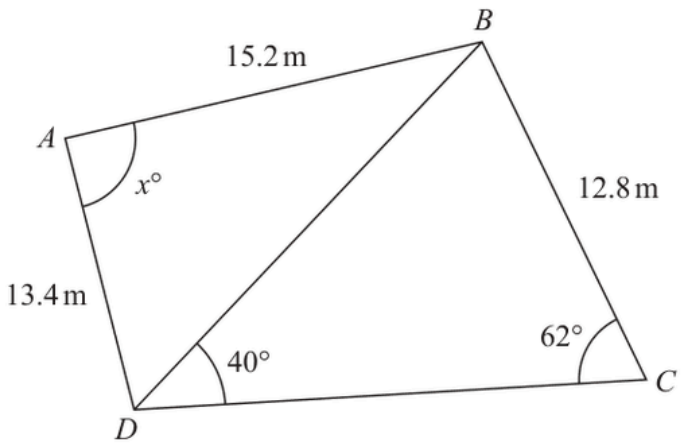


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

$x =$ .....

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2024

1 **Use trigonometry**

In triangle BCD,

$$\angle DBC = 180° - 40° - 62° = 78°$$

2 **Use trigonometry**

By the sine rule,

$$\frac{DB}{\sin 62°} = \frac{12.8}{\sin 40°}$$

$$DB = 17.582\ldots$$

3 Use trigonometry

Now use cosine rule in triangle ABD:

$$\cos x = \frac{15.2^2 + 13.4^2 - 17.582\ldots^2}{2(15.2)(13.4)}$$

$$x = 75.577\ldots^\circ$$

FINAL ANSWER

75.6°.

Question 20

5 marks | Sine & Cosine Rules

20

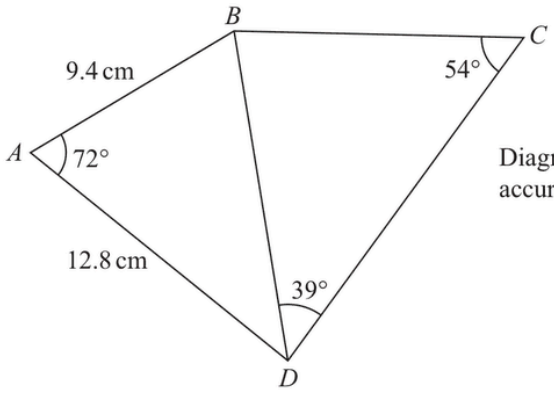


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 5 marks)

1 **Use trigonometry**

First use triangle ABD.

$$AB = 9.4, \quad AD = 12.8, \quad \angle BAD = 72°$$

2 **Use cosine rule**

Use the cosine rule to find BD:

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

3 **Use trigonometry**

In triangle BCD,

$$\angle BDC = 39°, \quad \angle BCD = 54°$$

4 **Use the sine rule**

Use the sine rule:

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = \frac{13.3356\ldots \sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

5 **Use trigonometry**

Correct to 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

10.4 cm

4WM1/01 MAY 2025

Question 21

5 marks | Sine & Cosine Rules

21

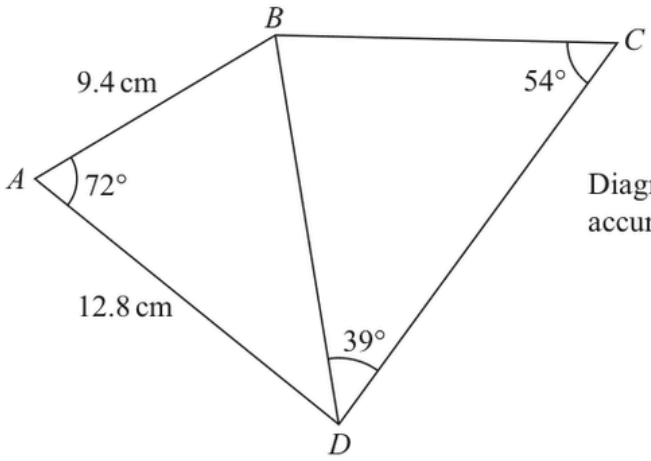


Diagram **NOT** accurately drawn

Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4WM1/01 May 2025

1 **Use trigonometry**

First find BD in triangle ABD using the cosine rule.

$$BD^2 = 9.4^2 + 12.8^2 - 2(9.4)(12.8)\cos 72°$$

$$BD = 13.3356\ldots$$

2 **Use trigonometry**

In triangle BCD, use the sine rule.

$$\frac{BC}{\sin 39°} = \frac{BD}{\sin 54°}$$

$$BC = 13.3356\ldots \times \frac{\sin 39°}{\sin 54°}$$

$$BC = 10.3735\ldots$$

3 **Use trigonometry**

To 3 significant figures,

$$BC = 10.4$$

✓ **FINAL ANSWER**

$10.4\ \text{cm}$

4WM1/01 NOVEMBER 2025

Question 19

3 marks | Sine & Cosine Rules

19 The diagram shows triangle ABC

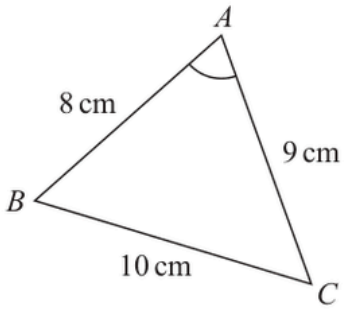


Diagram **NOT** accurately drawn

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$ $CA = 9\,\text{cm}$

Work out the size of angle BAC
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4WM1/01 November 2025

1 Use cosine rule

Use the cosine rule for angle BAC:

$$\cos A = \frac{8^2 + 9^2 - 10^2}{2(8)(9)}$$

$$\cos A = \frac{45}{144}$$

$$A = 71.790\ldots^\circ$$

FINAL ANSWER

71.8°.

4MA1/1H NOVEMBER 2025

Question 19

3 marks | Sine & Cosine Rules

19 The diagram shows triangle ABC

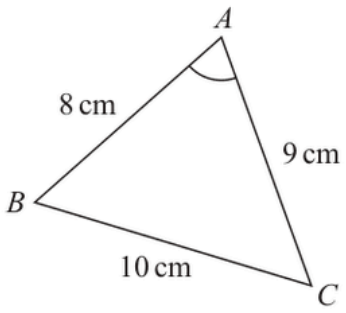


Diagram **NOT** accurately drawn

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$ $CA = 9\,\text{cm}$

Work out the size of angle BAC
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H November 2025

1 Use cosine rule

Use the cosine rule for angle BAC:

$$\cos A = \frac{8^2 + 9^2 - 10^2}{2(8)(9)}$$

$$\cos A = \frac{45}{144}$$

$$A = 71.790\ldots^\circ$$

FINAL ANSWER

71.8°.

Question 15

5 marks | Sine & Cosine Rules

15 The diagram shows triangle ABC

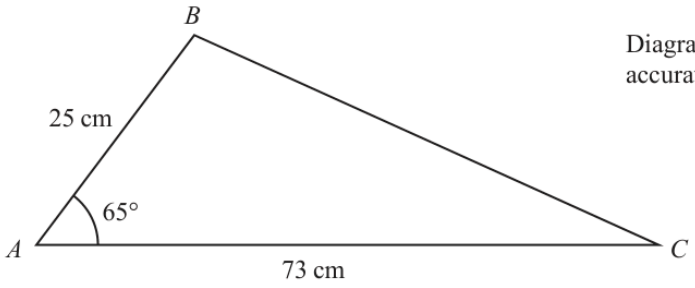


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad AC = 73$ cm $\quad$ angle $BAC = 65°$

(a) Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(3)

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 15 is 5 marks)

1 **(a) Cosine rule**

$$BC^2 = 25^2 + 73^2 - 2(25)(73)\cos 65° = 4411.4\ldots$$

2 **Square root**

$$BC = 66.418\ldots$$

3 **(b) Area formula**

$$\tfrac{1}{2}(25)(73)\sin 65° = 827.005\ldots$$

✓ **FINAL ANSWER**

(a) 66.4 cm (b) 827 cm^2

4MA1/1H JUNE 2026

Question 16

5 marks | Sine & Cosine Rules

16 The diagram shows triangle ABC

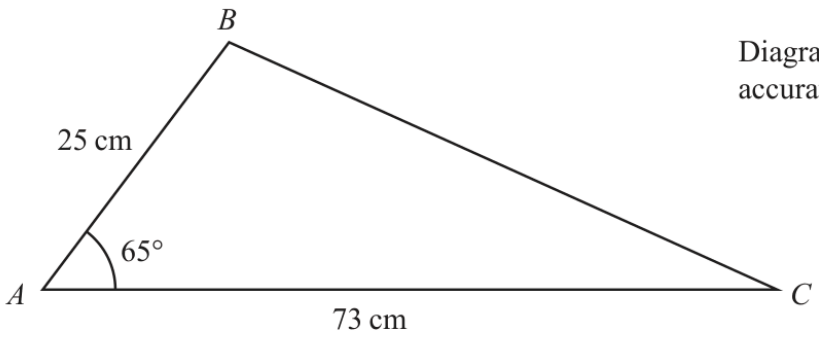


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad AC = 73$ cm $\quad$ angle $BAC = 65°$

(a) Work out the length of BC
Give your answer correct to 3 significant figures.

..... cm

(3)

(b) Work out the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

$$BC^2 = 25^2 + 73^2 - 2(25)(73)\cos 65°.$$

The included angle is between the two given sides.

2 **Part (a) | Step 2**

$$BC^2 = 4411.443344\ldots, \quad BC = \sqrt{4411.443344\ldots} = 66.418697\ldots = 66.4 \text{ cm} \quad (3 \text{ s.f.}).$$

Take the positive square root for a length.

3 **Part (b) | Step 1**

$$\text{Area} = \frac{1}{2}(25)(73)\sin 65^\circ = 827.005855\ldots = 827\ \text{cm}^2 \quad (3\ \text{s.f.}).$$

Use half the product of the two sides and the sine of their included angle.

✓ **FINAL ANSWER**

(a) $66.4\ \text{cm}$ (b) $827\ \text{cm}^2$

4MA1/1HR JUNE 2026

Question 22

4 marks | Sine & Cosine Rules

22 The diagram shows triangle ABC

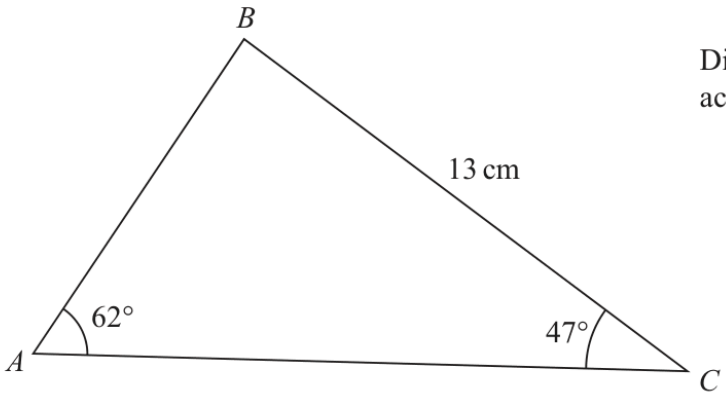


Diagram **NOT** accurately drawn

Calculate the area of the triangle.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR June 2026

1 **Solution | Step 1**

$$\angle ABC = 180^\circ - 62^\circ - 47^\circ = 71^\circ.$$

The three triangle angles sum to 180 degrees.

2 **Solution | Step 2**

$$\frac{AB}{\sin 47^\circ} = \frac{13}{\sin 62^\circ} \quad \Longrightarrow \quad AB = \frac{13 \sin 47^\circ}{\sin 62^\circ} = 10.7680208859\ldots \text{ cm}.$$

Match each side to its opposite angle in the sine rule.

3 **Solution | Step 3**

$$\text{Area} = \frac{1}{2}(13)\left(\frac{13 \sin 47^\circ}{\sin 62^\circ}\right)\sin 71^\circ = 66.1788645053\ldots \text{ cm}^2.$$

Use half the product of two sides times the sine of their included angle; retain unrounded values.

4 **Solution | Step 4**

$$\text{Area} = 66.2\ \text{cm}^2 \quad (3\ \text{significant figures}).$$

Round only the final area.

✓ **FINAL ANSWER**

$66.2\ \text{cm}^2$

TOPIC

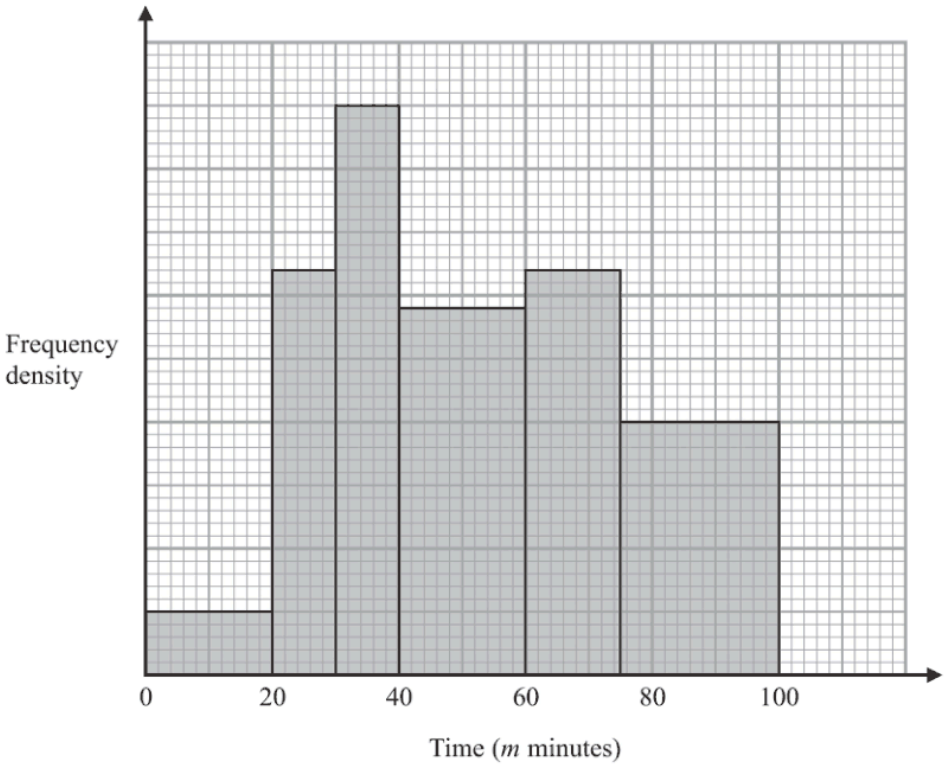
Histograms

Every past-paper question on this topic, 2020 to 2026.

Question 16

3 marks | Histograms

16 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$
Work out an estimate for the number of children for whom $50 < m \leqslant 80$

.....

(Total for Question 16 is 3 marks)

1 Set the frequency-density scale

For $0 < m \leq 20$, the frequency is 10, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

2 Read the needed bar heights

Using the same scale from the histogram:

$$50 < m \leq 60 : \ 10 \times 2.9 = 29$$

$$60 < m \leq 75 : \ 15 \times 3.2 = 48$$

$$75 < m \leq 80 : \ 5 \times 2 = 10$$

3 Add the estimates

$$19 + 36 + 32 = 87$$

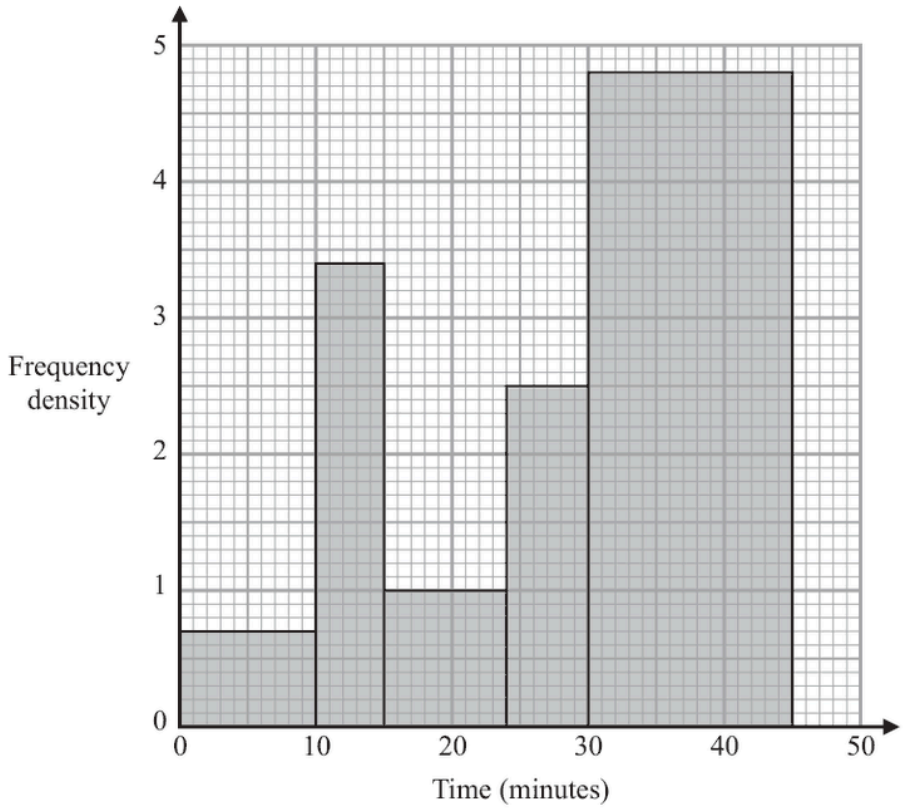
FINAL ANSWER

87.

Question 18

5 marks | Histograms

18 The histogram gives information about the times, in minutes, that some customers spent in a supermarket.



(a) Work out an estimate for the proportion of these customers who spent between 17 minutes and 35 minutes in the supermarket.

.....

(3)

One of the customers is selected at random.

Given that this customer had spent more than 30 minutes in the supermarket,

(b) find the probability that this customer spent more than 36 minutes in the supermarket.

.....

(2)

(Total for Question 18 is 5 marks)

1 Total frequency

Total frequency:

$$10(0.7) + 5(3.4) + 9(1) + 6(2.5) + 15(4.8) = 120$$

2 Between 17 and 35 minutes

Between 17 and 35 minutes:

$$7(1) + 6(2.5) + 5(4.8) = 46$$

$$\text{proportion} = \frac{46}{120} = \frac{23}{60}$$

3 Given the customer spent more

Given the customer spent more than 30 minutes, they are in the 30 to 45 minute class.

4 More than 36 minutes is

More than 36 minutes is 9 minutes out of the 15-minute class:

$$\frac{9}{15} = \frac{3}{5}$$

FINAL ANSWER

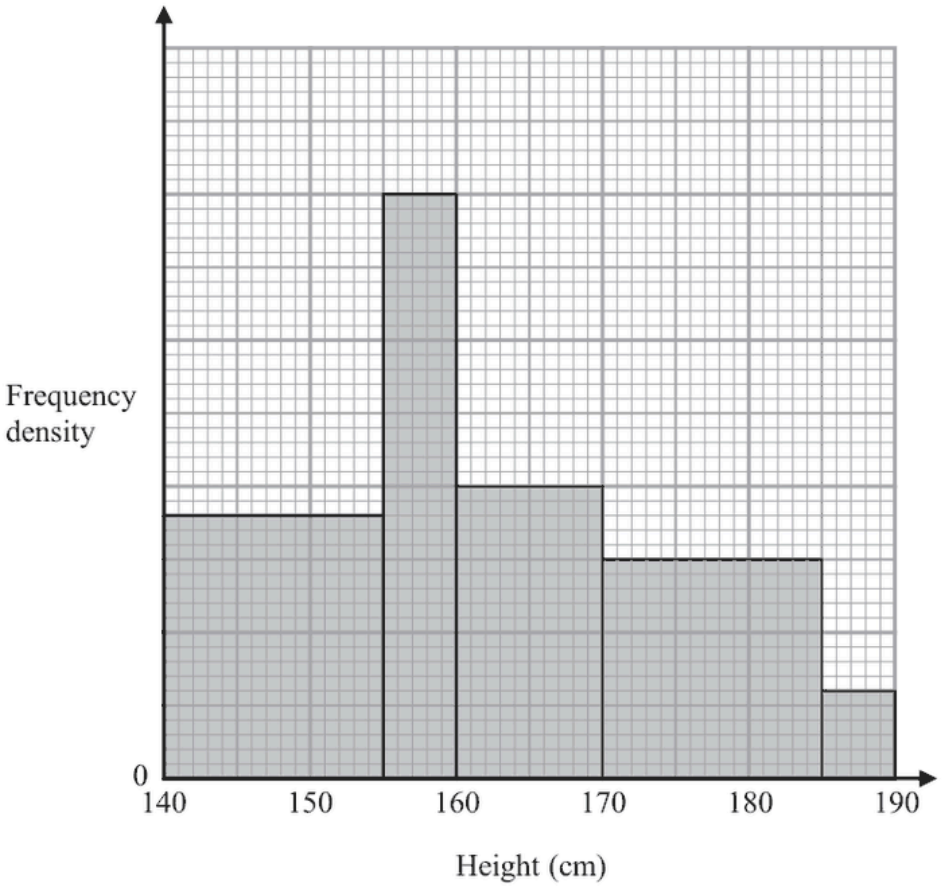
$\frac{23}{60}$, then $\frac{3}{5}$.

4MA1/1HR JANUARY 2020

Question 19

4 marks | Histograms

19



The histogram gives information about the heights of all the Year 11 students at a school.

There are 160 students in Year 11 with a height between 155 cm and 170 cm.

Work out the total number of students in Year 11 at the school.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2020

1 Use the relative histogram heights

Use the relative histogram heights.

2 Calculate area

For $155 < h \leq 170$, the relative area is

$$5(40) + 10(20) = 400$$

3 Calculate area

This represents 160 students, so each relative area unit represents

$$\frac{160}{400} = 0.4$$

4 Total relative area

Total relative area:

$$15(18) + 5(40) + 10(20) + 15(15) + 5(6) = 925$$

5 Estimated total

Estimated total:

$$925 \times 0.4 = 370$$

FINAL ANSWER

370 students.

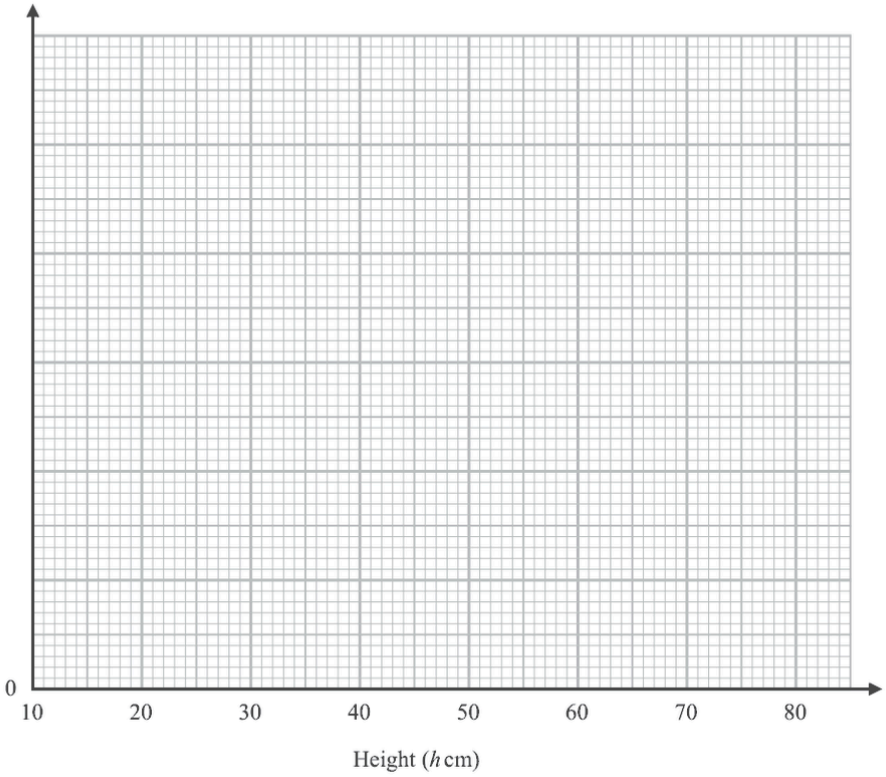
Question 18

5 marks | Histograms

18 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$10 < h \leqslant 20$	35
$20 < h \leqslant 35$	45
$35 < h \leqslant 50$	75
$50 < h \leqslant 70$	40
$70 < h \leqslant 80$	8

(a) On the grid, draw a histogram for this information.



(3)

(b) Work out an estimate for the number of these plants with a height greater than 40 cm.

.....

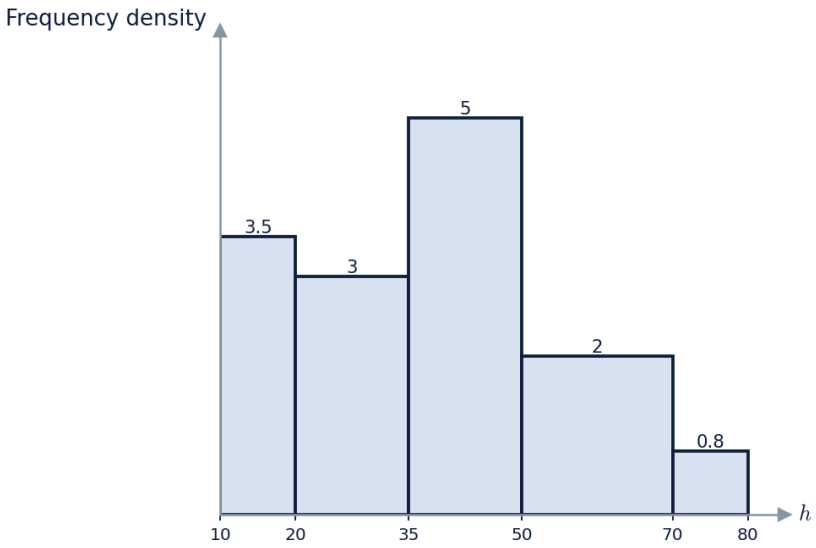
(2)

(Total for Question 18 is 5 marks)

1 **The histogram densities are**

The histogram densities are

$$\frac{35}{10} = 3.5, \quad \frac{45}{15} = 3, \quad \frac{75}{15} = 5, \quad \frac{40}{20} = 2, \quad \frac{8}{10} = 0.8$$



2 **Use histogram**

For plants taller than $40\ \text{cm}$:

3 **Use histogram**

From $40 < h \leq 50$:

$$10(5) = 50$$

4 **Use histogram**

From $50 < h \leq 70$:

$$40$$

5 **Use histogram**

From $70 < h \leq 80$:

$$8$$

$$50 + 40 + 8 = 98$$

FINAL ANSWER

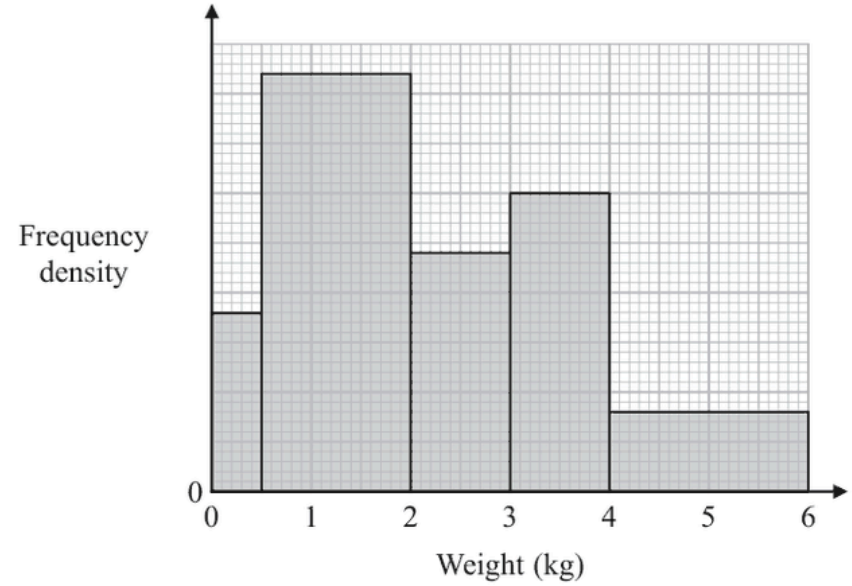
draw bars with densities $3.5, 3, 5, 2, 0.8$, and 98 plants.

Question 15

6 marks | Histograms

15 A postman records the weight of each parcel that he delivers.

The histogram shows information about the weights of all the parcels that the postman delivered last Monday. No parcels weighed more than 6 kg.



63 of the parcels that the postman delivered last Monday each had a weight between 0.5 kg and 2 kg.

(a) Work out the total number of parcels the postman delivered last Monday.

.....

(3)

The postman picks at random two of the records of the parcels he delivered last Monday.

(b) Work out an estimate for the probability that each parcel weighed more than 2.25 kg.

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H January 2021

1 **Use histogram**

For $0.5 < w \leq 2$, the frequency is 63, so the density is

$$\frac{63}{1.5} = 42$$

2 **Use histogram**

Using the histogram heights, the class frequencies are

$$9,\ 63,\ 24,\ 30,\ 16$$

---

3 **Use histogram**

So the total number of parcels is

$$9 + 63 + 24 + 30 + 16 = 142$$

---

4 **Use histogram**

For weight greater than $2.25\ \text{kg}$:

$$0.75(24) + 30 + 16 = 64$$

---

5 **The probability that both randomly**

The probability that both randomly chosen records are greater than $2.25\ \text{kg}$ is

$$\frac{64}{142} \times \frac{63}{141} = 0.201\ldots$$

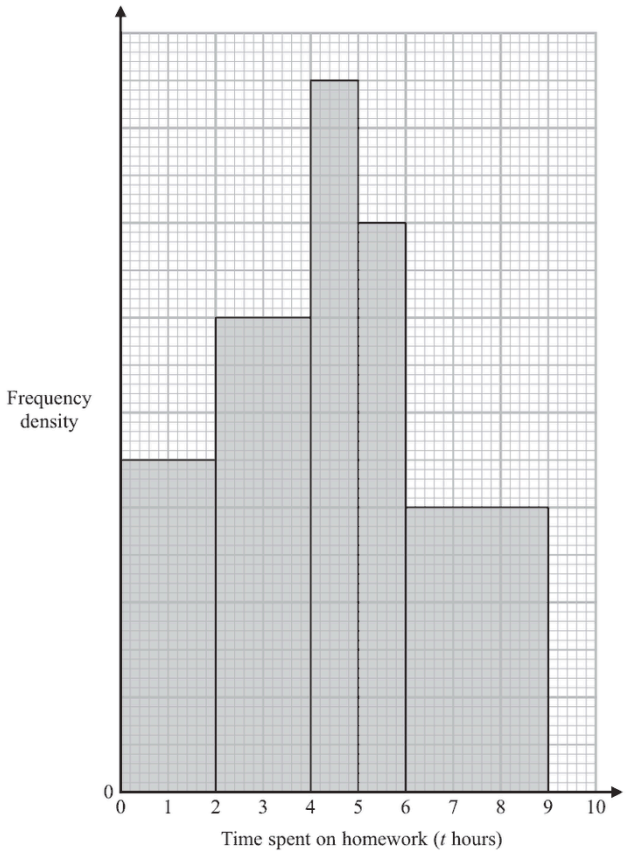
✓ **FINAL ANSWER**

142, then 0.201 approximately.

Question 18

5 marks | Histograms

18 The histogram and the table give some information about the amounts of time, in hours, that Year 11 students at Bergdesh Academy spent, in total, on their homework last week.
No student in Year 11 spent longer than 9 hours on their homework.



Time spent on homework (t hours)	Frequency
$0 < t \leqslant 2$	28
$2 < t \leqslant 4$	
$4 < t \leqslant 5$	
$5 < t \leqslant 6$	
$6 < t \leqslant 9$	

Using the information in the histogram and in the table, work out an estimate for the mean amount of time the Year 11 students spent on their homework last week.
Give your answer in hours correct to 3 significant figures.

..... hours

(Total for Question 18 is 5 marks)

1 **Use histogram**

The $0 < t \leq 2$ class has frequency 28. From the histogram, the relative heights are

$$1,\ 4,\ 9,\ 6,\ 1$$

2 **Use histogram**

So the frequencies are

$$28,\ 112,\ 126,\ 84,\ 42$$

3 **Use midpoints**

Use midpoints $1, 3, 4.5, 5.5, 7.5$.

$$\text{total frequency} = 28 + 112 + 126 + 84 + 42 = 392$$

$$\text{estimated total} = 1(28) + 3(112) + 4.5(126) + 5.5(84) + 7.5(42) = 1708$$

$$\text{mean} = \frac{1708}{392} = 4.357\ldots$$

FINAL ANSWER

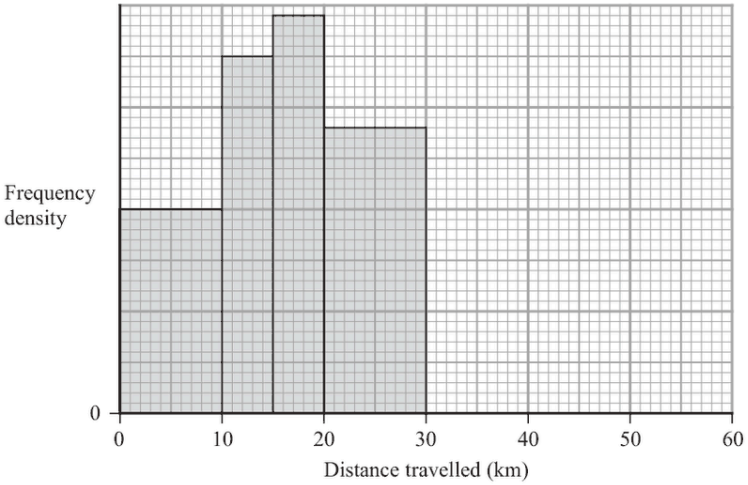
4.36 hours.

Question 18

5 marks | Histograms

18 The table and histogram give information about the distance travelled, in order to get to work, by each person working in a large store.

Distance (d km)	Frequency
$0 \leqslant d < 10$	40
$10 \leqslant d < 15$	
$15 \leqslant d < 20$	
$20 \leqslant d < 30$	
$30 \leqslant d < 60$	30



Using the information in the table and in the histogram,

(a) complete the table,

(2)

(b) complete the histogram.

(1)

One of the people working in the store is chosen at random.

(c) Work out an estimate for the probability that the distance travelled by this person, in order to get to work, was greater than 25 km.

.....

(2)

(Total for Question 18 is 5 marks)

1 **Use histogram**

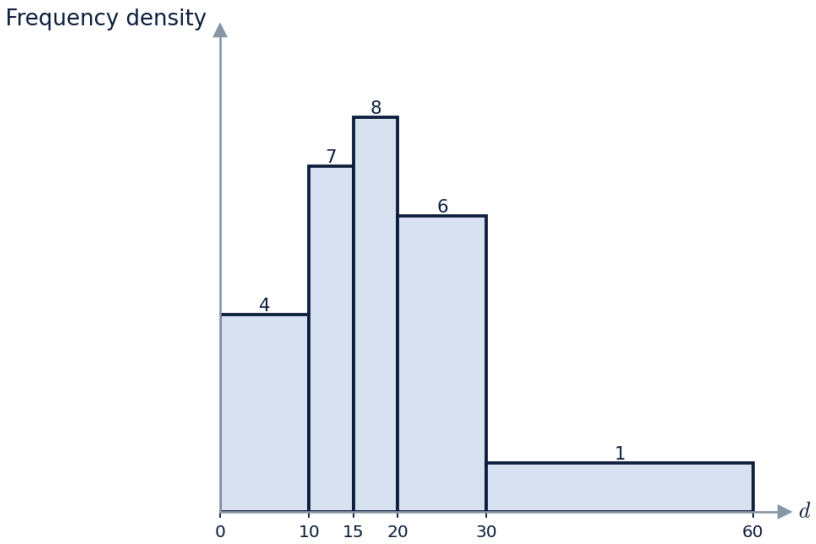
The $0 \leq d < 10$ class has frequency 40, so its density is

$$\frac{40}{10} = 4$$

2 **Use histogram**

From the histogram, the missing frequencies are

$$10 \leq d < 15 : \ 35, \qquad 15 \leq d < 20 : \ 40, \qquad 20 \leq d < 30 : \ 60$$



3 **Together with the table**

Together with the table:

$$40 + 35 + 40 + 60 + 30 = 205$$

4 **Use histogram**

For $d > 25$:

$$5(6) + 30 = 60$$

$$P(d > 25) = \frac{60}{205} = \frac{12}{41}$$

FINAL ANSWER

completed frequencies $35, 40, 60$, missing histogram density 1, and probability $\frac{12}{41}$.

Question 14

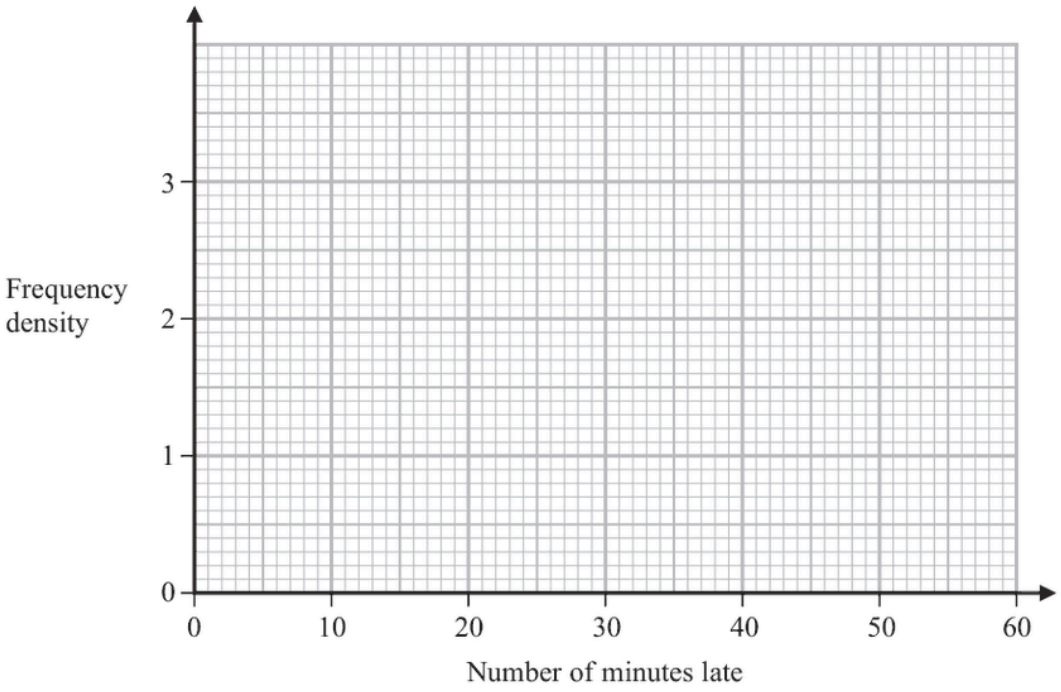
5 marks | Histograms

14 Manuel collected information about the flights that arrived late at an airport last month.

The table gives information about the number of minutes that these flights were late.

Minutes late (L minutes)	Frequency
$0 < L \leqslant 10$	8
$10 < L \leqslant 15$	13
$15 < L \leqslant 25$	19
$25 < L \leqslant 40$	24
$40 < L \leqslant 60$	6

(a) On the grid, draw a histogram for this information.



(3)

Manuel selected at random a flight that was late by 25 minutes or less from his results.

(b) Work out an estimate for the probability that this flight was late by 5 minutes or less.

.....

(2)

(Total for Question 14 is 5 marks)

1 **The histogram densities are**

The histogram densities are

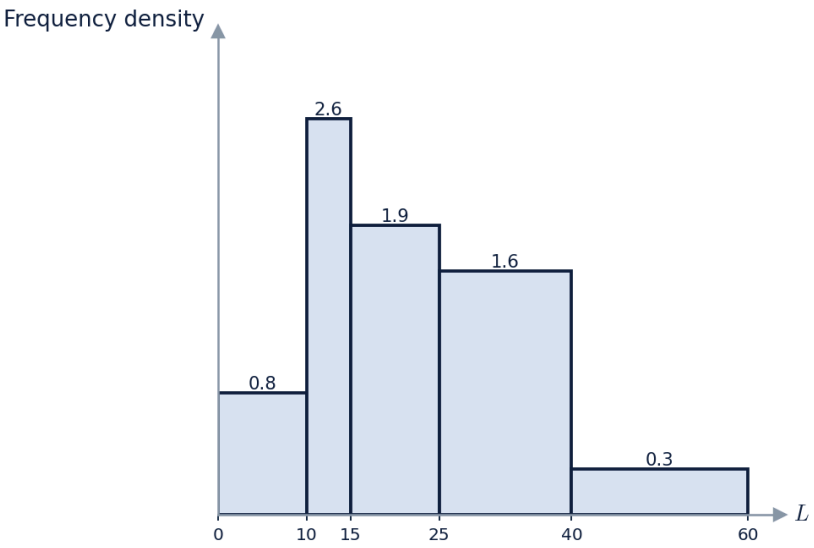
$$0 < L \leq 10 : \frac{8}{10} = 0.8$$

$$10 < L \leq 15 : \frac{13}{5} = 2.6$$

$$15 < L \leq 25 : \frac{19}{10} = 1.9$$

$$25 < L \leq 40 : \frac{24}{15} = 1.6$$

$$40 < L \leq 60 : \frac{6}{20} = 0.3$$



2 **Use histogram**

Given $L \leq 25$, the total frequency is

$$8 + 13 + 19 = 40$$

3 **Use histogram**

For $L \leq 5$, estimate half of the first class:

$$5(0.8) = 4$$

$$P(L \leq 5 \mid L \leq 25) = \frac{4}{40} = \frac{1}{10}$$

FINAL ANSWER

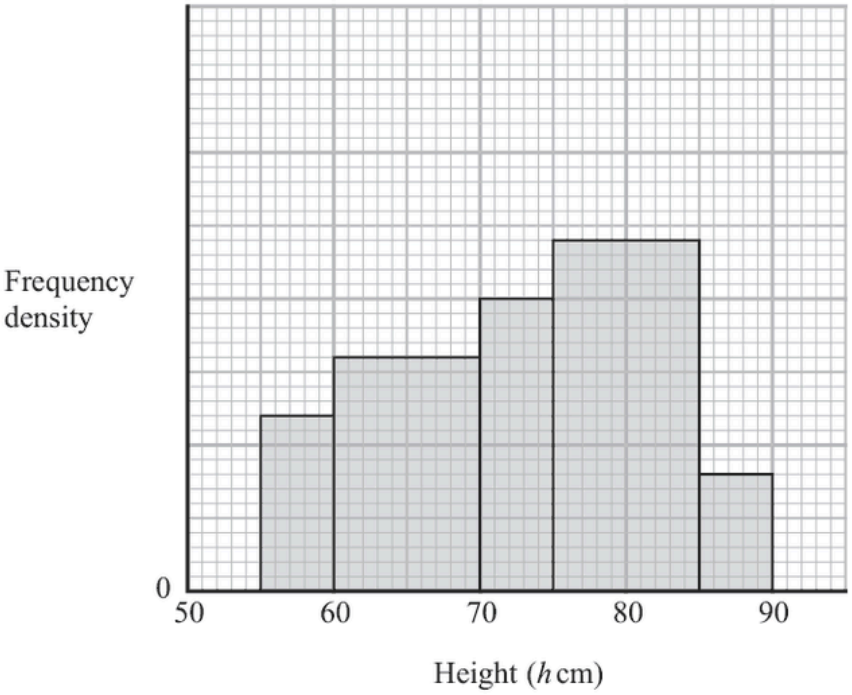
histogram densities $0.8, 2.6, 1.9, 1.6, 0.3$, and probability $\frac{1}{10}$.

4MA1/2H JANUARY 2022

Question 20

4 marks | Histograms

20 The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $75 < h \leq 85$, the frequency is 12 and the class width is 10, so

$$\text{frequency density} = \frac{12}{10} = 1.2$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$0.6,\ 0.8,\ 1.0,\ 1.2,\ 0.4$$

4 The total number of tomato

The total number of tomato plants is

$$(5)(0.6) + (10)(0.8) + (5)(1.0) + (10)(1.2) + (5)(0.4)$$

$$= 3 + 8 + 5 + 12 + 2 = 30$$

5 Solve inequality

For $h > 82.5$:

$$(2.5)(1.2) + (5)(0.4) = 3 + 2 = 5$$

6 Calculate probability

So the probability is

$$\frac{5}{30} = \frac{1}{6}$$

FINAL ANSWER

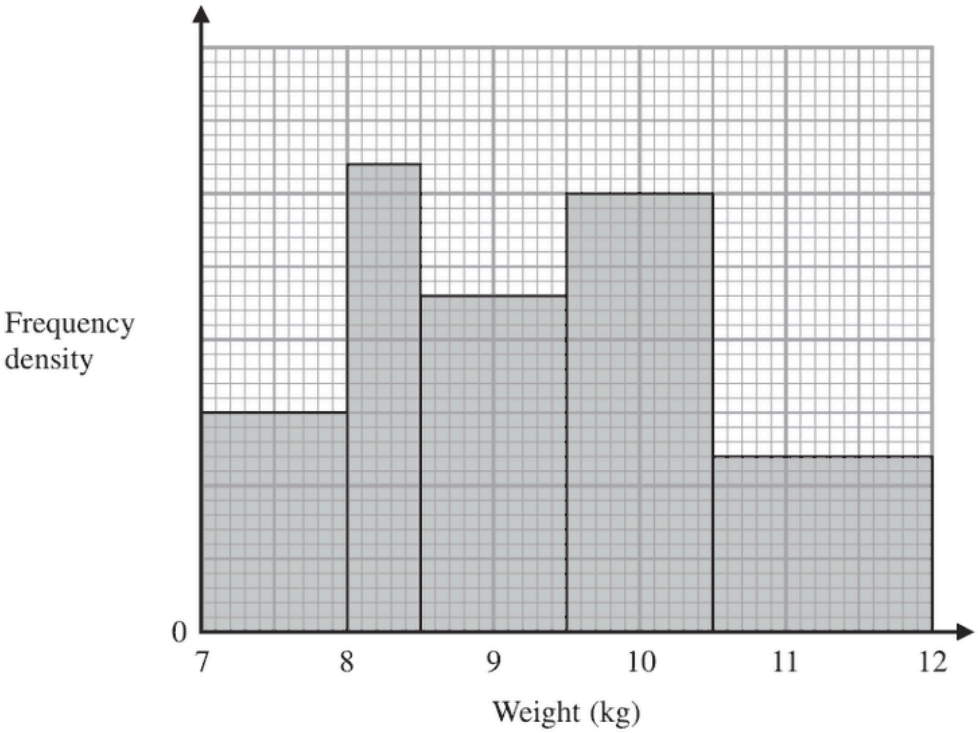
$\frac{1}{6}$

4MA1/1HR JANUARY 2022

Question 21

3 marks | Histograms

21



The histogram gives information about the weights, in kg, of all the watermelons in a field.

There are 16 watermelons with a weight between 8 kg and 8.5 kg

Work out the total number of watermelons in the field.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $8 \leq w < 8.5$, the frequency is 16 and the class width is 0.5, so

$$\text{frequency density} = \frac{16}{0.5} = 32$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$15,\ 32,\ 23,\ 30,\ 12$$

4 The total frequency is

The total frequency is

$$(1)(15) + (0.5)(32) + (1)(23) + (1)(30) + (1.5)(12)$$

$$= 15 + 16 + 23 + 30 + 18$$

$$= 102$$

FINAL ANSWER

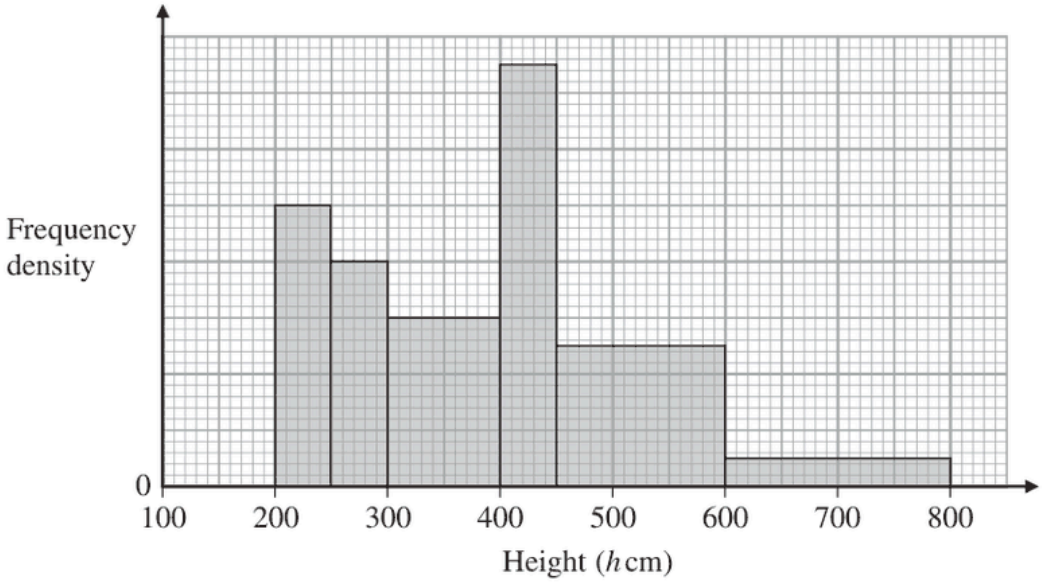
102 watermelons

4MA1/1HR JUNE 2022

Question 19

3 marks | Histograms

19 The histogram gives information about the height, h cm, of each tree in part of a forest.



There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2022

1 **Use relative densities from the**

Use relative densities from the histogram:

$$10,\ 8,\ 6,\ 15,\ 5,\ 1$$

2 **Use histogram**

The number of trees with $300 < h \leq 400$ is

$$100(6s) = 600s$$

3 **Use histogram**

The number of trees with $400 < h \leq 500$ is

$$50(15s) + 50(5s) = 1000s$$

4 **The difference is**

The difference is 8:

$$1000s - 600s = 8$$

$$400s = 8$$

$$s = 0.02$$

5 **Use histogram**

For height greater than 500:

$$100(5s) + 200(1s) = 700s$$

$$700(0.02) = 14$$

✓ **FINAL ANSWER**

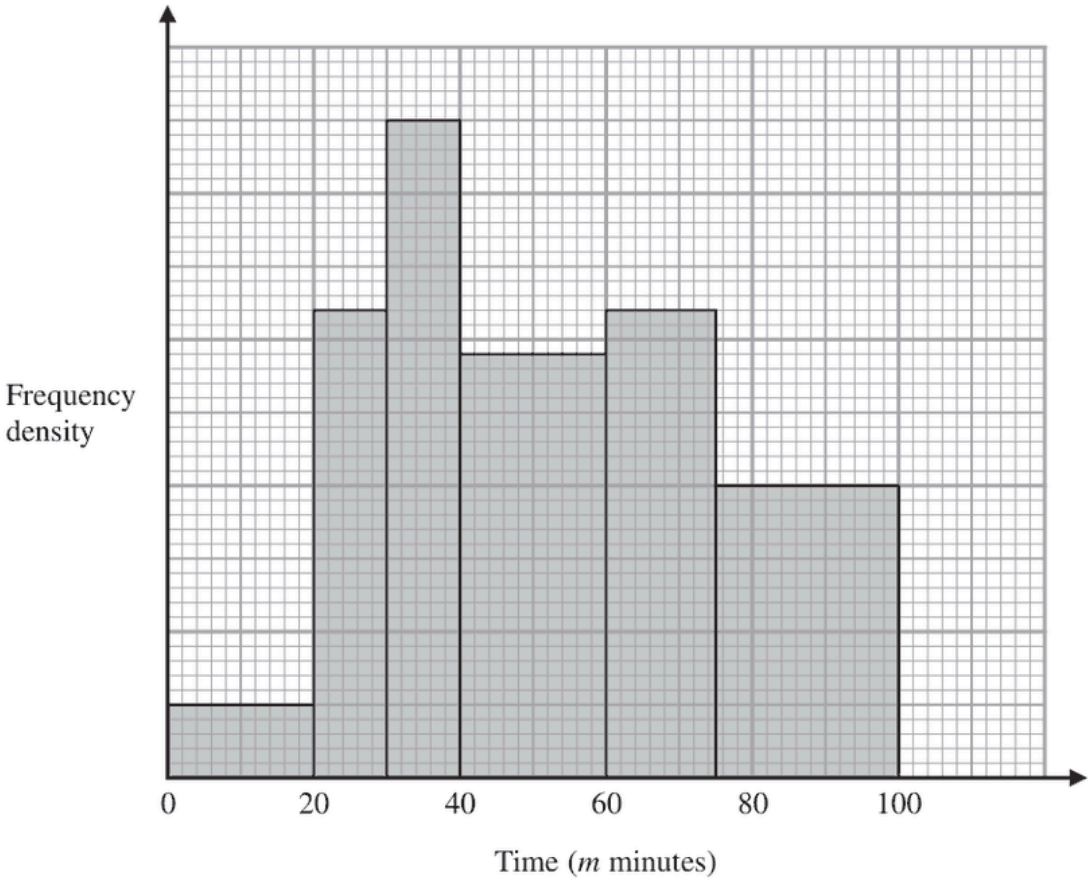
14 trees.

4MA1/1H JUNE 2022

Question 21

3 marks | Histograms

21 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.



There are no children for whom $m > 100$
There are 10 children for whom $m \leqslant 20$

Work out an estimate for the number of children for whom $50 < m \leqslant 80$

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2022

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $m \leq 20$, the frequency is 10 and the class width is 20, so

$$\text{frequency density} = \frac{10}{20} = 0.5$$

---

3 **Use histogram**

Using this scale from the histogram:

$$40 < m \leq 60 \quad \text{has frequency density } 2.9$$

$$60 < m \leq 75 \quad \text{has frequency density } 3.2$$

$$75 < m \leq 100 \quad \text{has frequency density } 2.0$$

---

4 **Use part of each relevant**

For $50 < m \leq 80$, use part of each relevant bar:

$$(10)(2.9) + (15)(3.2) + (5)(2.0)$$

$$= 29 + 48 + 10$$

$$= 87$$

✓ **FINAL ANSWER**

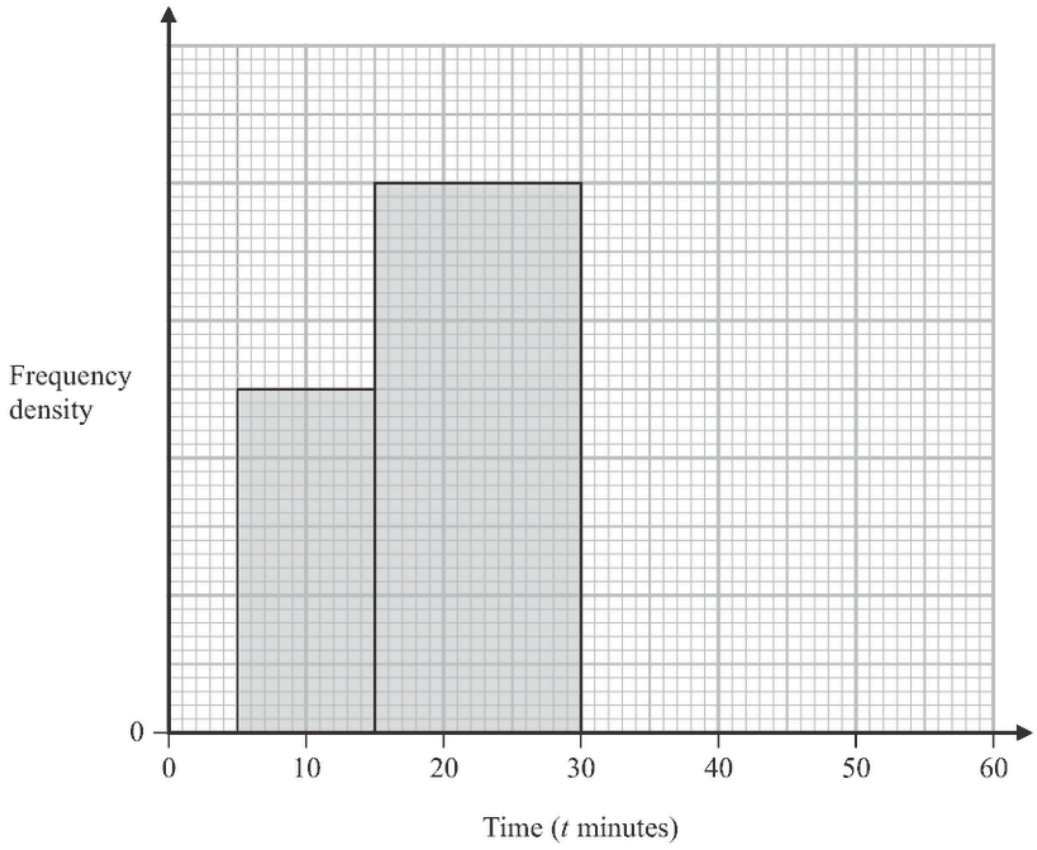
87 children

Question 18

4 marks | Histograms

18 The incomplete table and incomplete histogram give information about the times, in minutes, that 140 people waited at a station for a train.

Time (t minutes)	Frequency
$0 < t \leqslant 5$	23
$5 < t \leqslant 15$	
$15 < t \leqslant 30$	
$30 < t \leqslant 40$	18
$40 < t \leqslant 60$	14



Complete the table and the histogram.

(Total for Question 18 is 4 marks)

1 **The total frequency is**

The total frequency is 140.

2 **The known frequencies are**

The known frequencies are

$$23 + 18 + 14 = 55$$

3 **Use histogram**

So the two missing frequencies have total

$$140 - 55 = 85$$

4 **Use histogram**

From the histogram, the missing frequencies are

$$5 < t \leq 15 : \ 25, \qquad 15 < t \leq 30 : \ 60$$

5 **The missing histogram densities are**

The missing histogram densities are

$$0 < t \leq 5 : \frac{23}{5} = 4.6$$

$$30 < t \leq 40 : \frac{18}{10} = 1.8$$

$$40 < t \leq 60 : \frac{14}{20} = 0.7$$

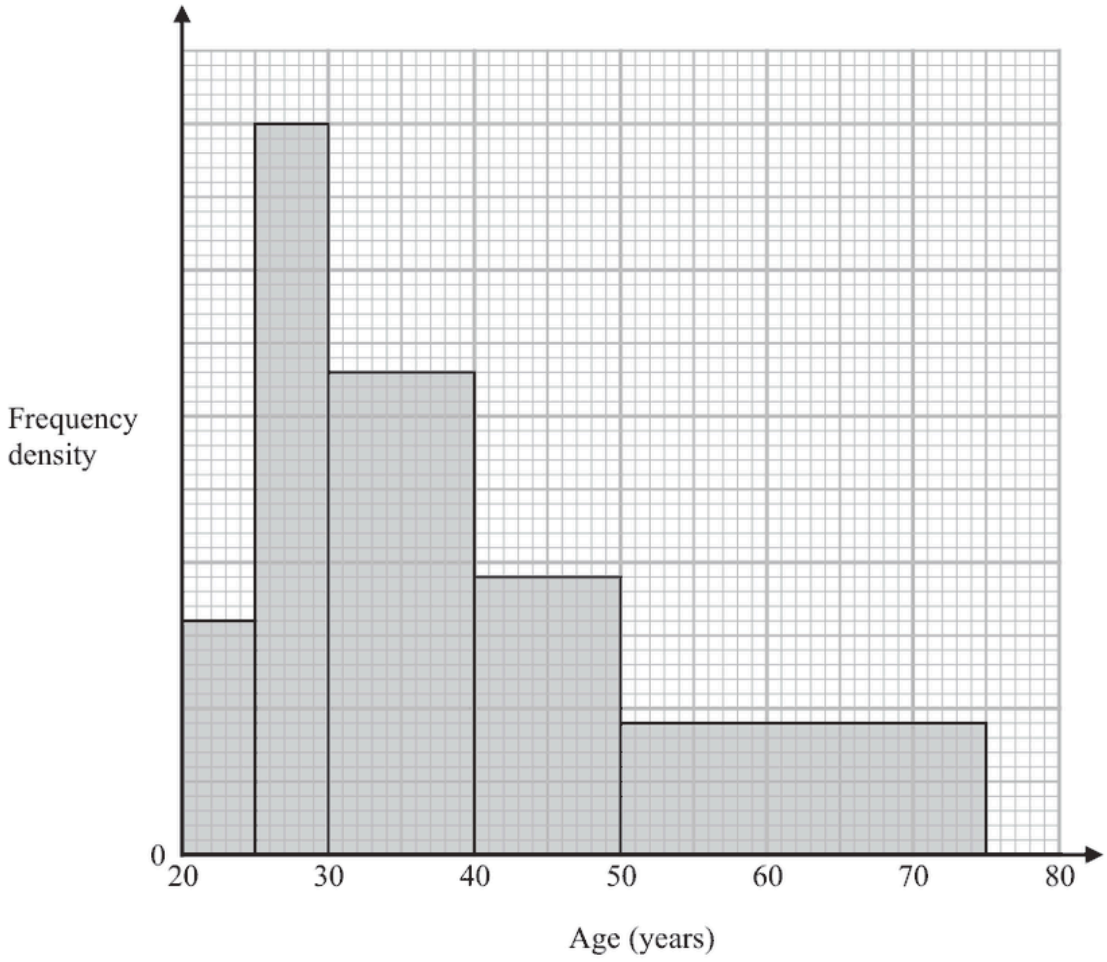
FINAL ANSWER

missing frequencies $25, 60$, and missing densities $4.6, 1.8, 0.7$.

Question 22

4 marks | Histograms

22 Some people attend a concert.
The histogram shows information about the ages of these people.



Work out an estimate for the percentage of these people who are aged more than 55 years.
Give your answer correct to one decimal place.

.....%

(Total for Question 22 is 4 marks)

1 The scale of the frequency

The scale of the frequency density axis is not needed because the percentage is found from areas.

2 Calculate area

Using the bar heights from the histogram, the relative areas are:

$$(5)(277) + (5)(866) + (10)(571) + (10)(329) + (25)(155)$$

$$= 1385 + 4330 + 5710 + 3290 + 3875$$

$$= 18590$$

3 Calculate value

For ages more than 55, use the part of the last bar from 55 to 75:

$$(20)(155) = 3100$$

4 Calculate percentage

So the estimated percentage is

$$\frac{3100}{18590} \times 100 = 16.675\ldots$$

FINAL ANSWER

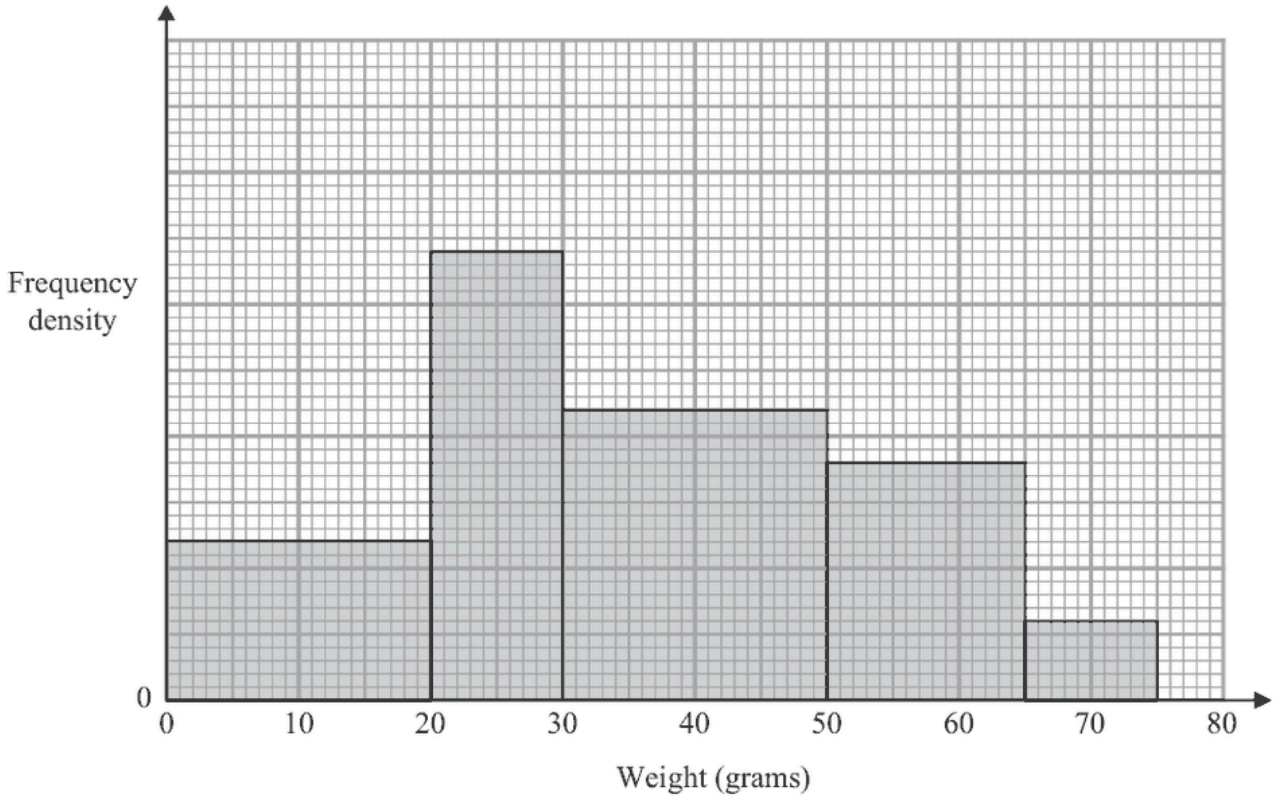
16.7%

4MA1/1H NOVEMBER 2023

Question 17

3 marks | Histograms

17 The histogram gives information about the weights, in grams, of some oranges in a box.



24 of these oranges weigh less than 20 grams.

Medium oranges weigh between 35 grams and 55 grams.

Work out an estimate for the number of medium oranges in the box.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2023

1 **The to gram class has**

The 0 to 20 gram class has frequency 24, so its density is

$$\frac{24}{20} = 1.2$$

2 **Use histogram**

From the histogram scale, the density for $30 < w \leq 50$ is about 2.2, and the density for $50 < w \leq 65$ is about 1.8.

3 **Medium oranges weigh between g**

Medium oranges weigh between 35 g and 55 g:

$$15(2.2) + 5(1.8) = 33 + 9 = 42$$

✓ **FINAL ANSWER**

about 42 medium oranges.

4MA1/1HR MAY 2024

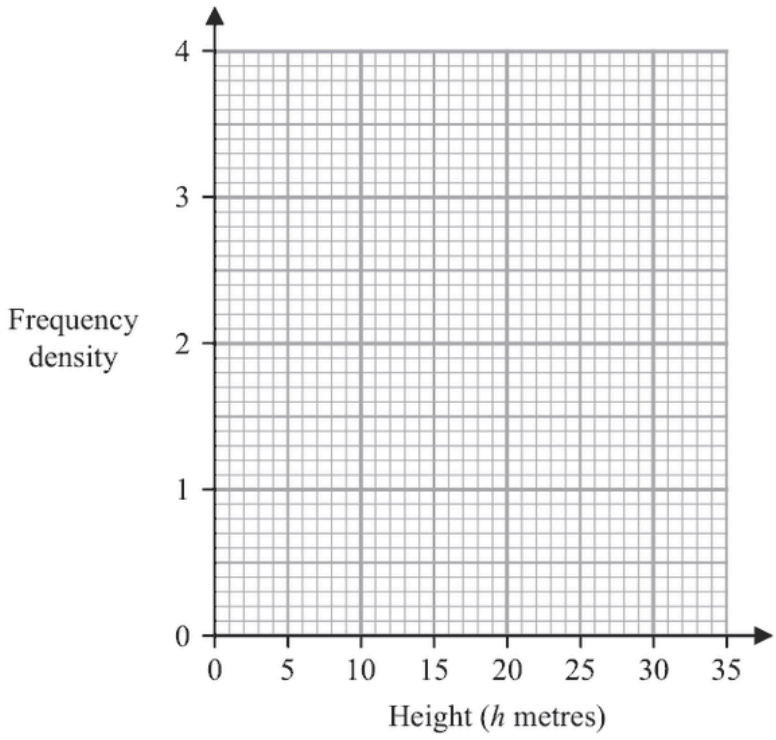
Question 16

3 marks | Histograms

16 The table gives information about the heights, in metres, of the trees in a park.

Height (h metres)	Frequency
$0 < h \leqslant 2$	5
$2 < h \leqslant 5$	12
$5 < h \leqslant 10$	18
$10 < h \leqslant 20$	14
$20 < h \leqslant 35$	9

On the grid, draw a histogram for this information.



(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR May 2024

1 **Use histogram**

For a histogram,

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

2 **The bar heights are**

The bar heights are:

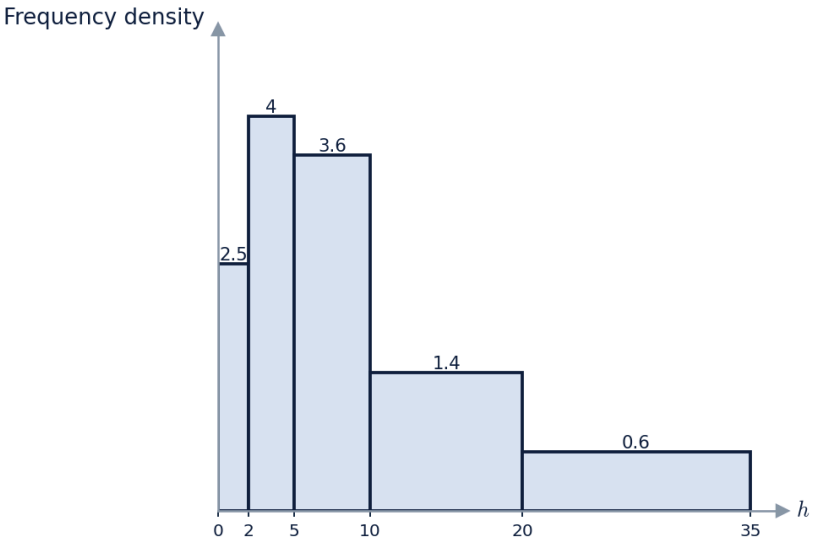
$$0 < h \leq 2 : \frac{5}{2} = 2.5$$

$$2 < h \leq 5 : \frac{12}{3} = 4$$

$$5 < h \leq 10 : \frac{18}{5} = 3.6$$

$$10 < h \leq 20 : \frac{14}{10} = 1.4$$

$$20 < h \leq 35 : \frac{9}{15} = 0.6$$



FINAL ANSWER

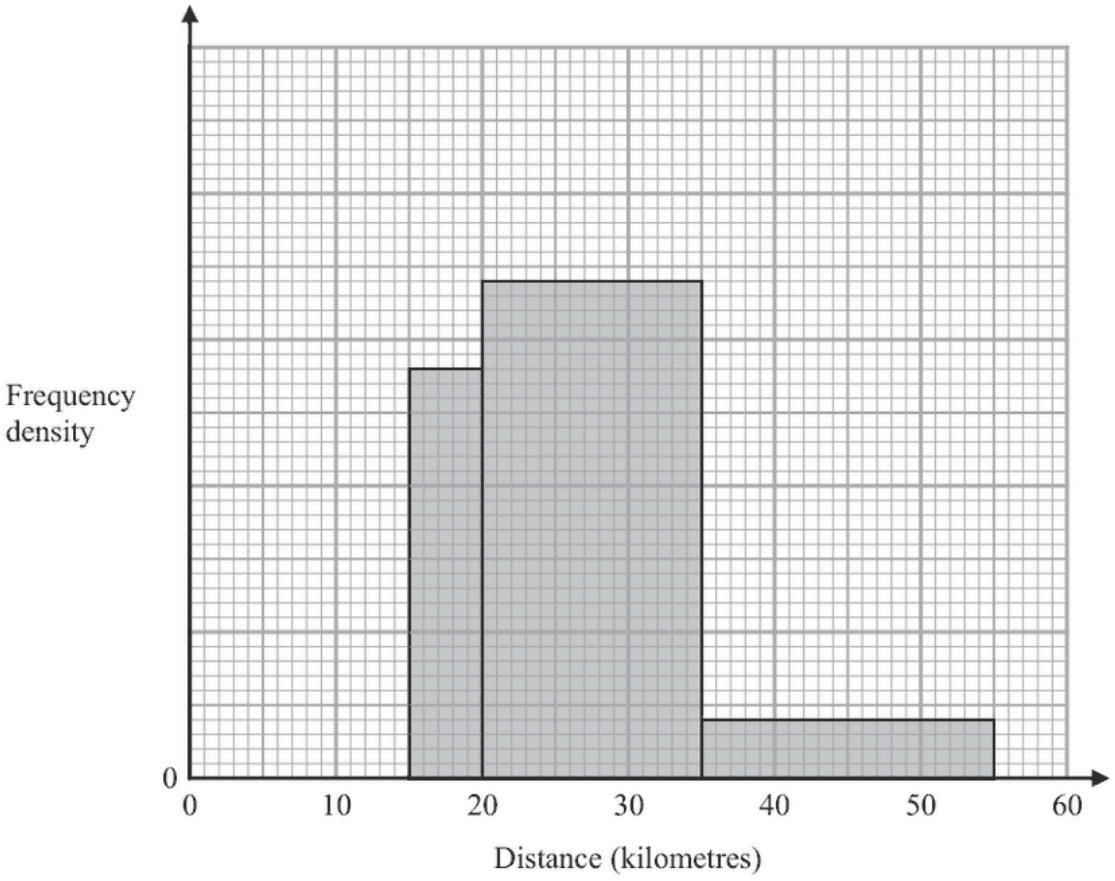
draw bars with heights $2.5, 4, 3.6, 1.4, 0.6$.

4MA1/1H MAY 2024

Question 22

3 marks | Histograms

22 The incomplete histogram shows some information about the distances, in kilometres, that 100 adults ran last week.



All of the adults ran at least 5 kilometres.
None of the adults ran more than 55 kilometres.
14 adults ran between 15 kilometres and 20 kilometres.

Complete the histogram.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2024

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $15 \leq d < 20$, the frequency is 14 and the class width is 5, so

$$\text{frequency density} = \frac{14}{5} = 2.8$$

3 **Use histogram**

Using this scale from the histogram:

$$20 \leq d < 35 \quad \text{has frequency density } 3.4$$

$$35 \leq d \leq 55 \quad \text{has frequency density } 0.4$$

4 **Calculate value**

So the known frequencies are

$$14 + (15)(3.4) + (20)(0.4)$$

$$= 14 + 51 + 8 = 73$$

5 **Use frequency table**

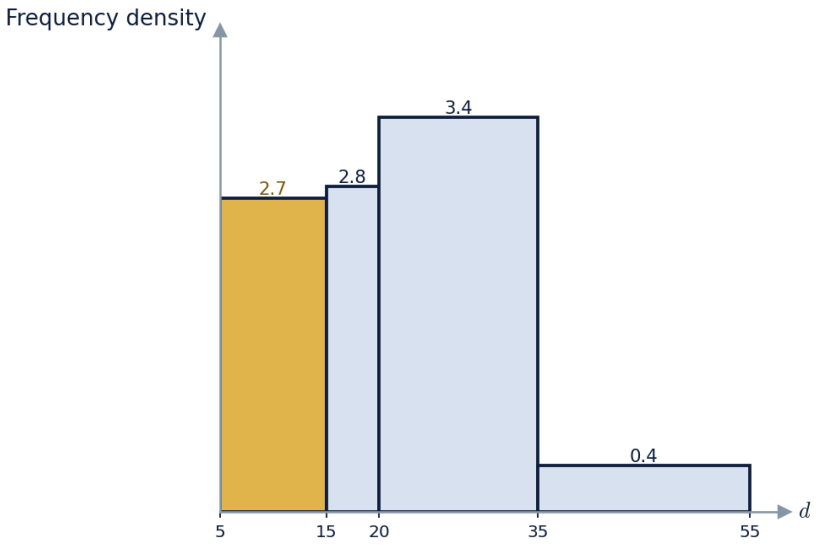
There are 100 adults in total, so the missing frequency from 5 km to 15 km is

$$100 - 73 = 27$$

6 **The missing class width is**

The missing class width is 10, so its frequency density is

$$\frac{27}{10} = 2.7$$



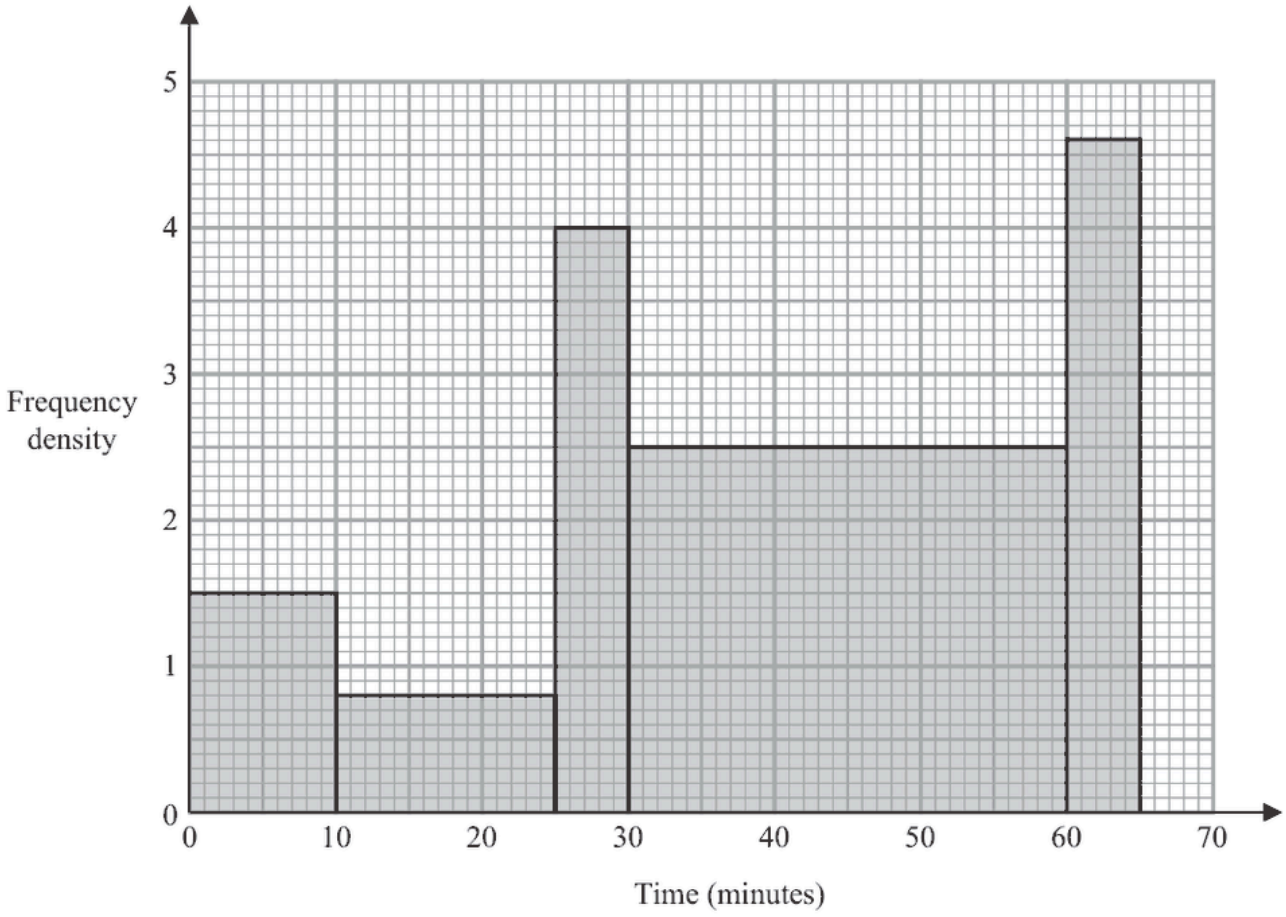
FINAL ANSWER

draw the missing bar from 5 to 15 with frequency density 2.7

Question 18

4 marks | Histograms

18 The histogram shows information about the times some students took to complete a puzzle.



Work out an estimate for the fraction of these students who took between 20 minutes and 60 minutes to complete the puzzle.

(Total for Question 18 is 4 marks)

1 Estimate the frequencies from each

Estimate the frequencies from each bar:

$$0 \text{ to } 10: \ 10(1.5) = 15$$

$$10 \text{ to } 25: \ 15(0.8) = 12$$

$$25 \text{ to } 30: \ 5(4) = 20$$

$$30 \text{ to } 60: \ 30(2.5) = 75$$

$$60 \text{ to } 65: \ 5(4.6) = 23$$

2 Total frequency

Total frequency:

$$15 + 12 + 20 + 75 + 23 = 145$$

3 Between and minutes

Between 20 and 60 minutes:

$$5(0.8) + 20 + 75 = 99$$

$$\text{fraction} = \frac{99}{145}$$

FINAL ANSWER

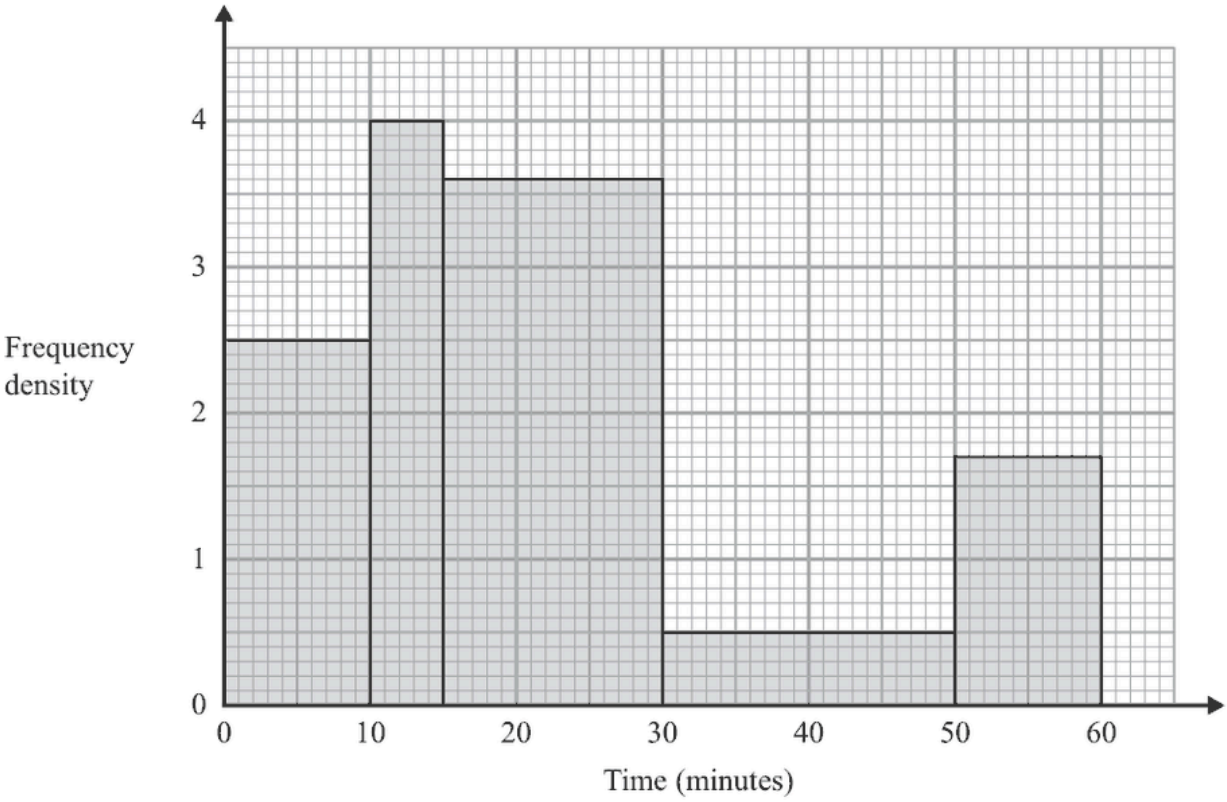
$\frac{99}{145}$.

4MA1/1H JUNE 2025

Question 17

3 marks | Histograms

17 The histogram shows information about the times, in minutes, that some people were in a shop.



Work out an estimate for the proportion of these people who were in the shop for more than 40 minutes.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H June 2025

1 Total estimated frequency

Total estimated frequency:

$$10(2.5) + 5(4) + 15(3.6) + 20(0.5) + 10(1.7) = 126$$

2 More than minutes

More than 40 minutes:

$$10(0.5) + 10(1.7) = 22$$

3 Use histogram

So the proportion is

$$\frac{22}{126} = \frac{11}{63}$$

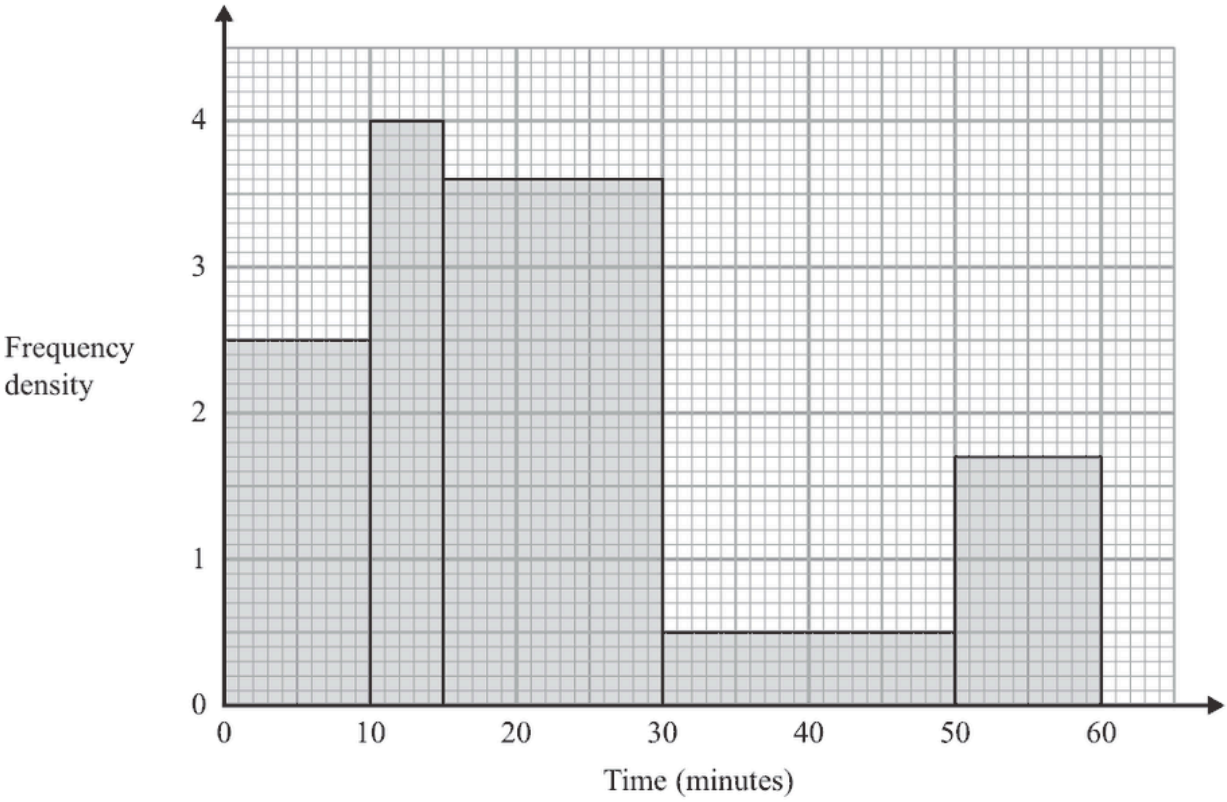
FINAL ANSWER

$\frac{11}{63}$.

Question 18

3 marks | Histograms

18 The histogram shows information about the times, in minutes, that some people were in a shop.



Work out an estimate for the proportion of these people who were in the shop for more than 40 minutes.

(Total for Question 18 is 3 marks)

1 **Frequency is class width multiplied**

Frequency is class width multiplied by frequency density.

$$0\text{--}10 : 10 \times 2.5 = 25$$

$$10\text{--}15 : 5 \times 4 = 20$$

$$15\text{--}30 : 15 \times 3.6 = 54$$

$$30\text{--}50 : 20 \times 0.5 = 10$$

$$50\text{--}60 : 10 \times 1.7 = 17$$

2 **Total**

Total:

$$25 + 20 + 54 + 10 + 17 = 126$$

3 **More than minutes is the**

More than 40 minutes is the part from 40 to 50, plus 50 to 60.

$$10 \times 0.5 + 17 = 22$$

4 **Use histogram**

So the proportion is

$$\frac{22}{126} = \frac{11}{63}$$

✓ **FINAL ANSWER**

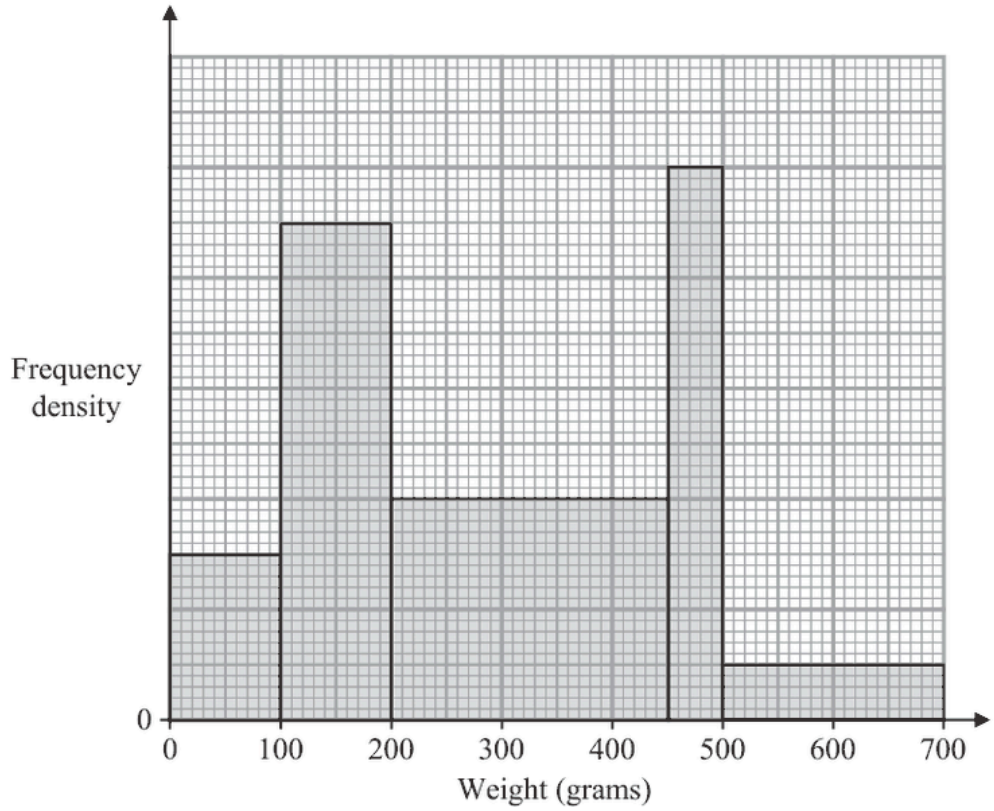
$\frac{11}{63}$

4MA1/2HR JUNE 2025

Question 20

4 marks | Histograms

20 The histogram gives some information about the weights, in grams, of some books.



75 books weigh less than 100 grams.

A book is chosen at random.

Find an estimate for the probability that this book weighs between 400 grams and 600 grams.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For books weighing less than 100 g, the frequency is 75 and the class width is 100, so

$$\text{frequency density} = \frac{75}{100} = 0.75$$

3 **Use histogram**

Using this scale from the histogram, the frequency densities are:

$$0.75,\ 2.25,\ 1.0,\ 2.5,\ 0.25$$

4 **The total number of books**

The total number of books is

$$(100)(0.75) + (100)(2.25) + (250)(1.0) + (50)(2.5) + (200)(0.25)$$

$$= 75 + 225 + 250 + 125 + 50 = 725$$

5 **Calculate value**

For weights between 400 g and 600 g:

$$(50)(1.0) + (50)(2.5) + (100)(0.25)$$

$$= 50 + 125 + 25 = 200$$

6 **Calculate probability**

So the probability is

$$\frac{200}{725} = \frac{8}{29}$$

✓ **FINAL ANSWER**

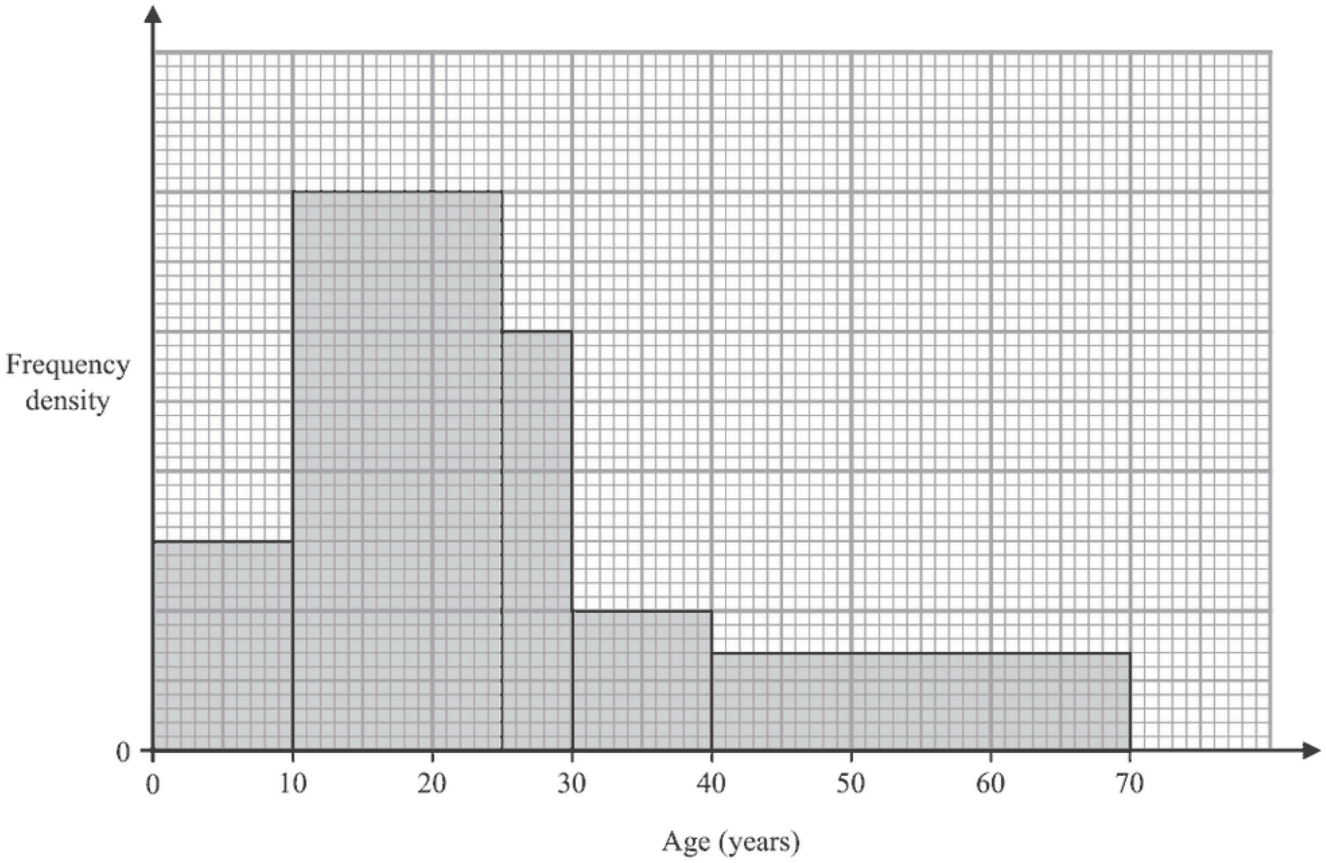
$\frac{8}{29}$ or 0.276 to 3 significant figures

4WM1/01R MAY 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the ages of the members of a chess club.



There are 60 members aged between 10 and 25
There are no members older than 70

Work out an estimate for the number of members who are over 35 years of age.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4WM1/01R May 2025

1 **The interval to has width**

The interval 10 to 25 has width 15 and frequency 60, so its frequency density is

$$\frac{60}{15} = 4$$

2 **Use histogram**

Using that scale from the histogram:

$$35 \text{ to } 40 : 5 \times 1 = 5$$

$$40 \text{ to } 70 : 30 \times 0.75 = 22.5$$

$$5 + 22.5 = 27.5$$

✓ **FINAL ANSWER**

about 28 members.

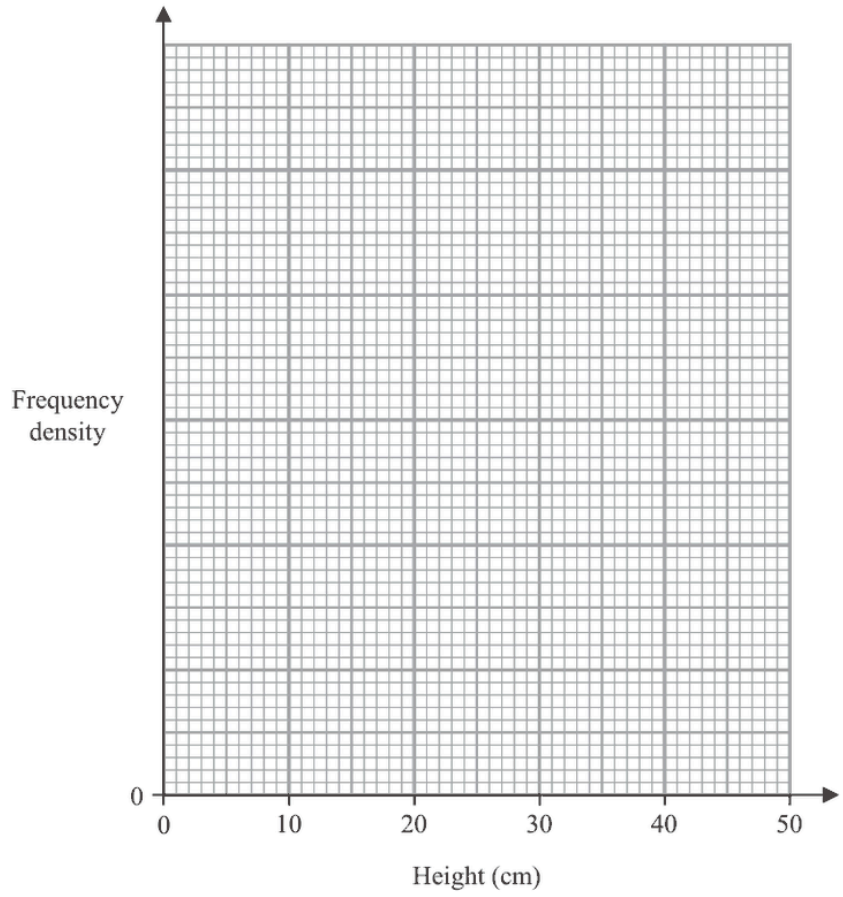
Question 20

3 marks | Histograms

20 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$0 < h \leqslant 5$	12
$5 < h \leqslant 15$	34
$15 < h \leqslant 30$	45
$30 < h \leqslant 50$	28

On the grid, draw a histogram for this information.
You must include a scale on the vertical axis.



(Total for Question 20 is 3 marks)

1 Frequency density

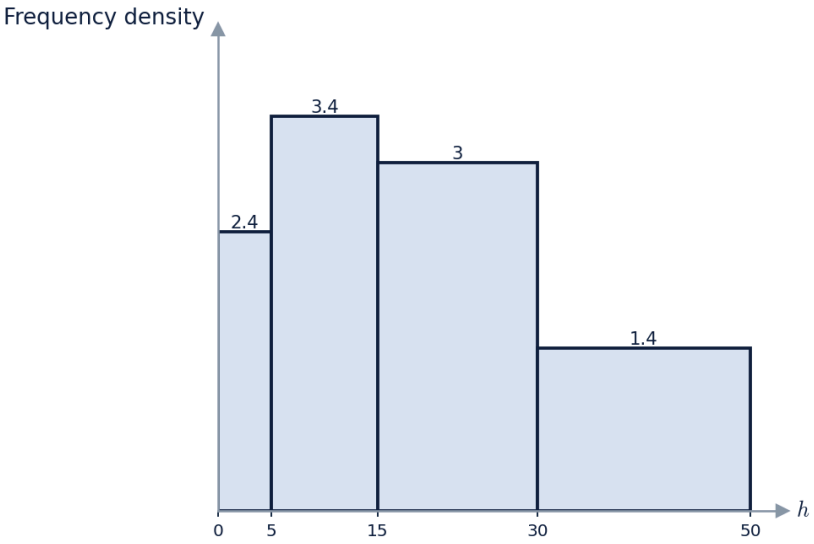
Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$.

$$0 < h \leq 5 : \frac{12}{5} = 2.4$$

$$5 < h \leq 15 : \frac{34}{10} = 3.4$$

$$15 < h \leq 30 : \frac{45}{15} = 3$$

$$30 < h \leq 50 : \frac{28}{20} = 1.4$$



FINAL ANSWER

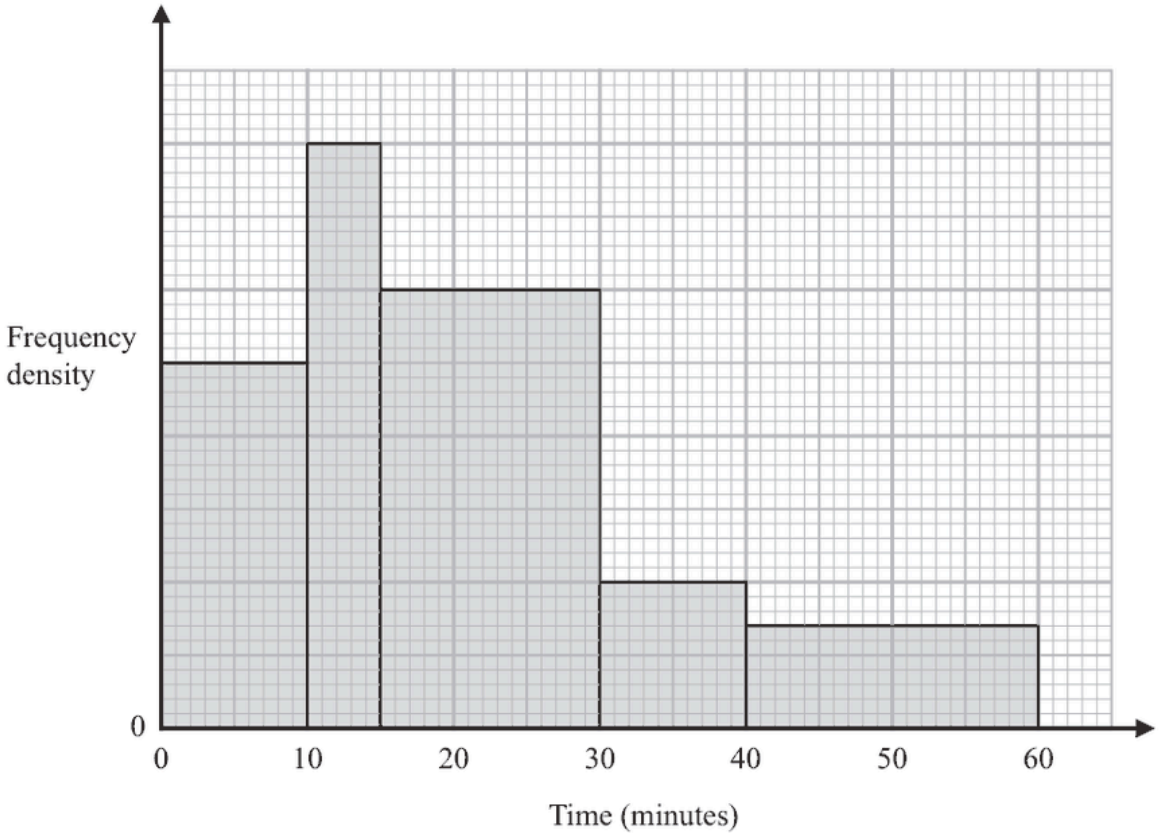
draw bars with densities $2.4, 3.4, 3, 1.4$.

4MA1/2H NOVEMBER 2025

Question 21

3 marks | Histograms

21 The histogram shows information about the times, in minutes, that some trains arrived late at a station one day.



20 of these trains arrived between 10 minutes late and 15 minutes late.

No trains arrived more than 60 minutes late.

Work out an estimate for the number of these trains that arrived at least 25 minutes late.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2025

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $10 < t \leq 15$, the frequency is 20 and the class width is 5, so

$$\text{frequency density} = \frac{20}{5} = 4$$

---

3 **Use histogram**

Using this scale from the histogram:

$$15 < t \leq 30 \quad \text{has frequency density } 3$$

$$30 < t \leq 40 \quad \text{has frequency density } 1$$

$$40 < t \leq 60 \quad \text{has frequency density } 0.7$$

---

4 **Calculate value**

For trains at least 25 minutes late:

$$(5)(3) + (10)(1) + (20)(0.7)$$

$$= 15 + 10 + 14$$

$$= 39$$

✓ **FINAL ANSWER**

39 trains

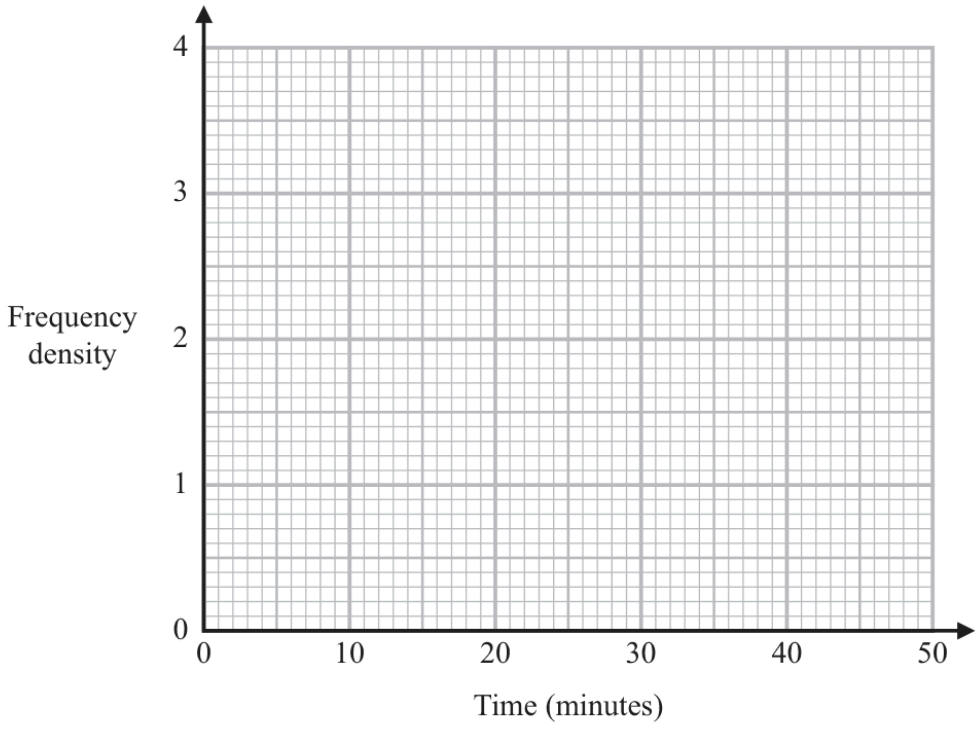
Question 19

3 marks | Histograms

19 The table gives information about the times, in minutes, some people spent waiting to go on a rollercoaster.

Time (t minutes)	Frequency
$0 < t \leqslant 20$	14
$20 < t \leqslant 25$	12
$25 < t \leqslant 35$	32
$35 < t \leqslant 50$	30

On the grid, draw a histogram for this information.



(Total for Question 19 is 3 marks)

1 Solution | Step 1

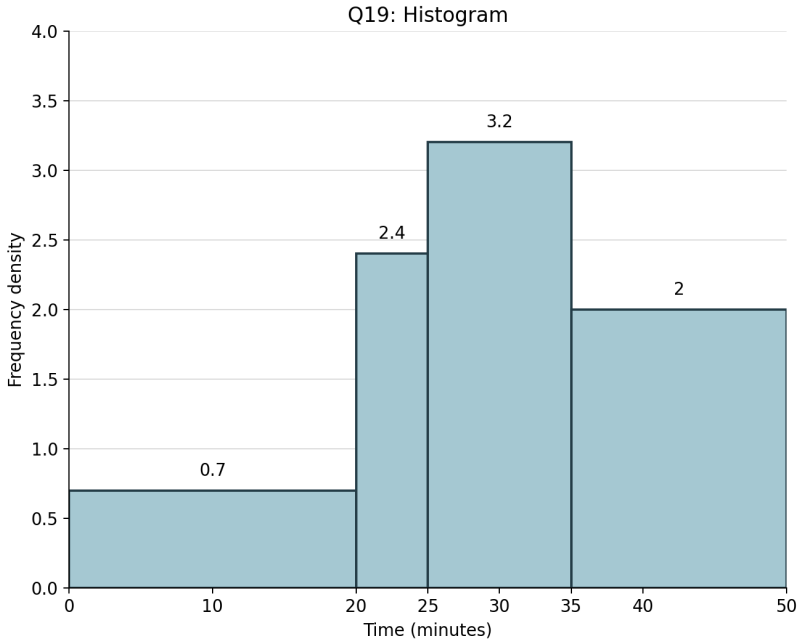
Use $\textbf{frequency density} = \textbf{frequency}/\textbf{class width}$:

$$14/20 = 0.7, \quad 12/5 = 2.4, \quad 32/10 = 3.2, \quad 30/15 = 2.$$

Histogram areas, rather than heights alone, represent frequencies.

2 Solution | Step 2

Draw four adjoining rectangles on the supplied time and frequency-density axes: $[0, 20]$ at height 0.7, $[20, 25]$ at height 2.4, $[25, 35]$ at height 3.2, and $[35, 50]$ at height 2.
Each rectangle starts at density 0 and has the stated class boundaries as its vertical edges; leave no gaps between classes.



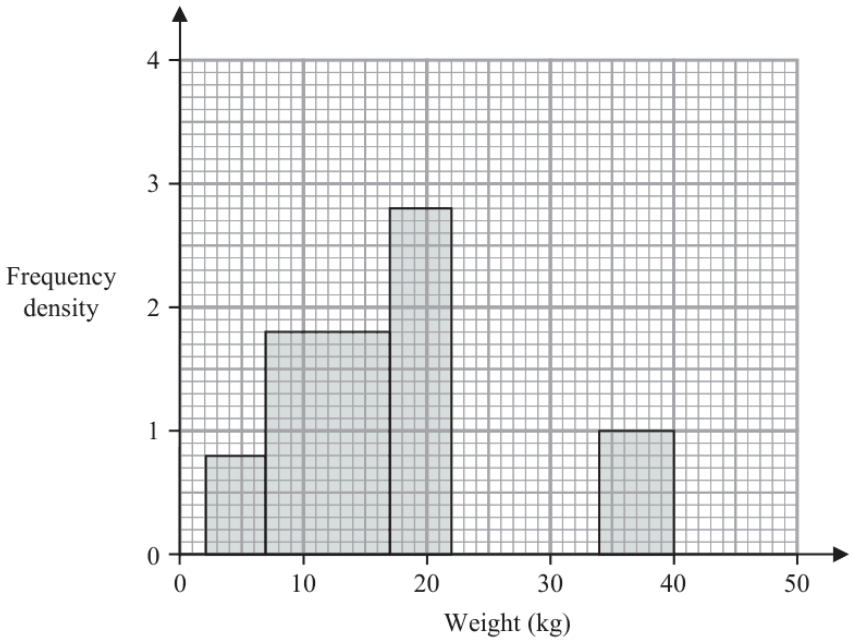
FINAL ANSWER

Bar intervals/heights: $[0, 20] : 0.7;\ [20, 25] : 2.4;\ [25, 35] : 3.2;\ [35, 50] : 2$.

Question 21

3 marks | Histograms

21 The incomplete histogram shows information about the weights, in kg, of some parcels.



No parcels weigh less than 2 kg
No parcels weigh more than 40 kg
Half of the parcels weigh between 22 kg and 34 kg

Complete the histogram by drawing a bar to represent the parcels that weigh between 22 kg and 34 kg

(Total for Question 21 is 3 marks)

1 **Frequency = width × height**

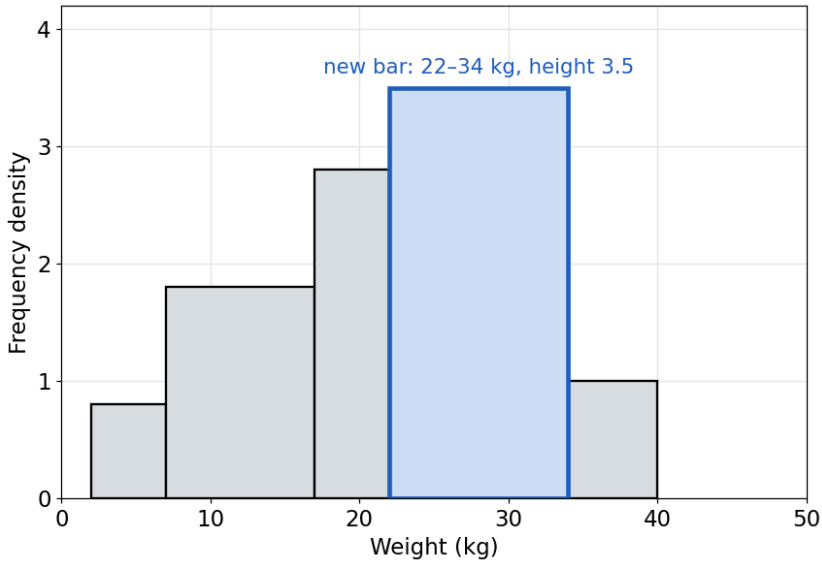
$$5(0.8) + 10(1.8) + 5(2.8) + 6(1) = 4 + 18 + 14 + 6 = 42$$

2 **Half are in 22–34**

So the other half is also 42 parcels.

3 **Height of the new bar**

$$\text{f.d.} = \frac{42}{34 - 22} = 3.5$$



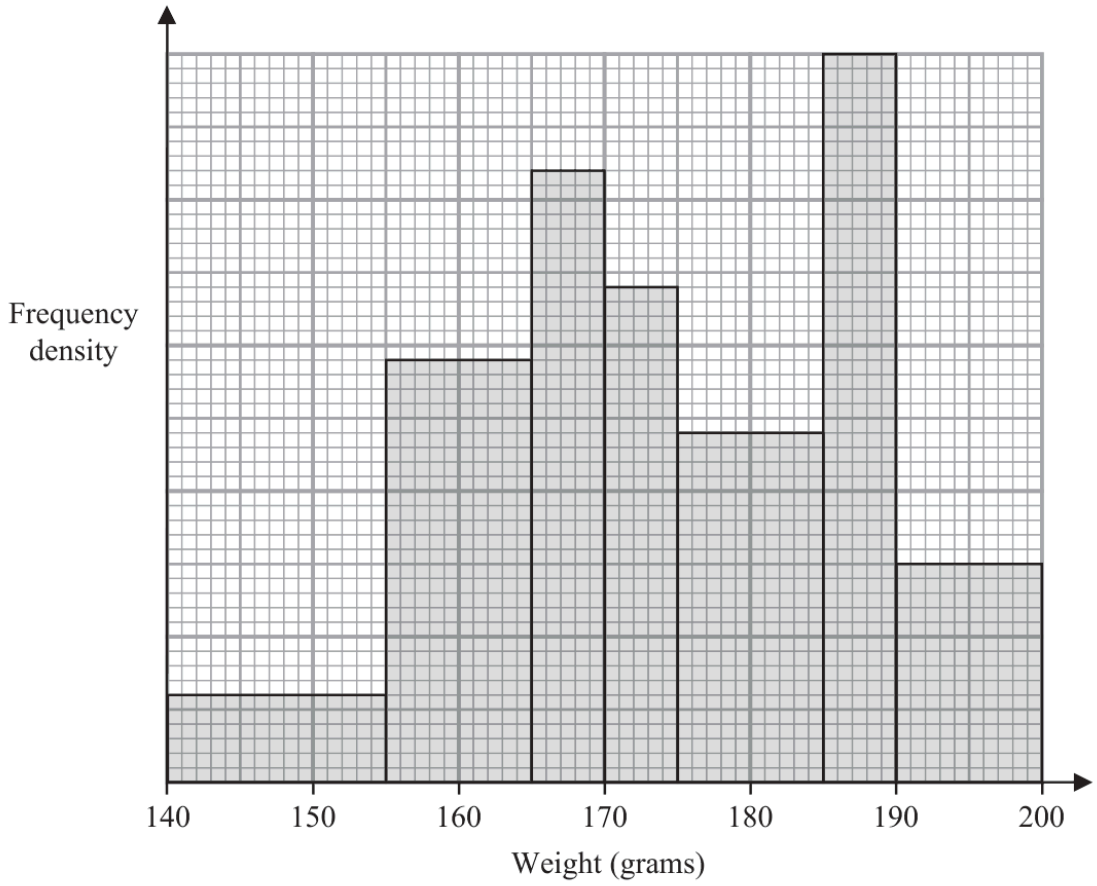
FINAL ANSWER

Bar from 22 to 34 kg, height 3.5

Question 22

5 marks | Histograms

22 The histogram gives information about the weights, in grams, of all the apples in a shop.



There are 9 apples with a weight between 140 grams and 155 grams.

(a) Work out the number of apples with a weight less than 175 grams.

.....

(3)

An apple is classified as a large apple if it has a weight of more than 180 grams.
One of the large apples is chosen at random.

(b) Work out the probability that it has a weight of more than 190 grams.

.....

(2)

(Total for Question 22 is 5 marks)

1 **Part (a) | Step 1**

The first class has width 15 g and frequency 9, so its density is $9/15 = 0.6$. Its height is 6 small grid squares, so each vertical square represents density 0.1.

Frequency is class width multiplied by frequency density.

2 **Part (a) | Step 2**

The classes below 175 g have densities $0.6, 2.9, 4.2, 3.4$ and widths $15, 10, 5, 5$, respectively.

Read the bar heights using the calibrated vertical scale.

3 **Part (a) | Step 3**

$$N = 15(0.6) + 10(2.9) + 5(4.2) + 5(3.4) = 9 + 29 + 21 + 17 = 76.$$

Add the frequencies, not the heights.

4 **Part (b) | Step 1**

Above 180 g, the frequency from 180 to 185 is $5(2.4) = 12$, from 185 to 190 is $5(5) = 25$, and from 190 to 200 is $10(1.5) = 15$.

Use only half of the 175-185 bar and all of the two heavier classes.

5 **Part (b) | Step 2**

$$P(W > 190 \mid W > 180) = \frac{15}{12 + 25 + 15} = \frac{15}{52}.$$

Restrict the denominator to the large apples.

✓ **FINAL ANSWER**

(a) 76 apples (b) $15/52$

TOPIC

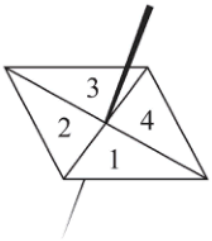
Probability Toolkit

Every past-paper question on this topic, 2020 to 2026.

Question 3

4 marks | Probability Toolkit

3 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

Number	1	2	3	4
Probability	0.26		0.18	

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4

Priya is going to spin the spinner 250 times.

Work out an estimate for the number of times the spinner will land on a 1 or a 2

.....

(Total for Question 3 is 4 marks)

1 Find the remaining probability

$$1 - 0.26 - 0.18 = 0.56$$

2 Split the equal probabilities

The probabilities for 2 and 4 are equal, so

$$P(2) = P(4) = \frac{0.56}{2} = 0.28$$

3 Find the probability of 1 or 2

$$P(1 \text{ or } 2) = 0.26 + 0.28 = 0.54$$

4 Calculate the estimate

$$0.54 \times 250 = 135$$

FINAL ANSWER

135.

4MA1/2H JANUARY 2020

Question 6

4 marks | Probability Toolkit

6 A bag contains only red beads, blue beads, green beads and yellow beads.

The table gives the probabilities that, when a bead is taken at random from the bag, the bead will be blue or the bead will be yellow.

Colour	red	blue	green	yellow
Probability		0.24		0.31

The probability that the bead will be green is twice the probability that the bead will be red.

Sofia takes at random a bead from the bag.
She writes down the colour of the bead and puts the bead back into the bag.

She does this 180 times.

Work out an estimate for the number of times she takes a red bead from the bag.

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2020

1 Use the fact that probabilities add to 1

Let $P(\text{red}) = r$, so $P(\text{green}) = 2r$.

$$r + 0.24 + 2r + 0.31 = 1 \Rightarrow 3r = 0.45 \Rightarrow r = 0.15$$

2 Expected number of red beads

$$180 \times 0.15 = 27$$

FINAL ANSWER

27

4MA1/1H JANUARY 2020

Question 18

4 marks | Probability Toolkit

18 There are 16 sweets in a bowl.

4 of the sweets are blackcurrant.
5 of the sweets are lemon.
7 of the sweets are orange.

Anna, Ravi and Sam each take at random one sweet from the bowl.

Work out the probability that the 5 lemon sweets are still in the bowl.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2020

1 Count the sweets that are not lemon

There are 16 sweets and 5 are lemon, so 11 are not lemon.

2 Three picks without replacement, none lemon

All 5 lemon sweets stay in the bowl only if Anna, Ravi and Sam each take a non-lemon sweet:

$$\frac{11}{16} \times \frac{10}{15} \times \frac{9}{14} = \frac{990}{3360} = \frac{33}{112}$$

FINAL ANSWER

$$\frac{33}{112}$$

4MA1/1H MAY/NOVEMBER 2020

Question 2

4 marks | Probability Toolkit

2 Toy cars are made in a factory.
The toy cars are made for 15 hours each day.
5 toy cars are made every 12 seconds.

For the toy cars made each day, the probability of a toy car being faulty is 0.002

Work out an estimate of the number of faulty toy cars that are made each day.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May/November 2020

1 Seconds in a working day

$$15 \times 60 \times 60 = 54\,000 \text{ seconds}$$

2 Toy cars made each day

$$\frac{54\,000}{12} \times 5 = 22\,500$$

3 Expected number of faulty cars

$$22\,500 \times 0.002 = 45$$

✓ **FINAL ANSWER**

45

4MA1/1HR MAY/NOVEMBER 2020

Question 23

4 marks | Probability Toolkit

23 In a bag, there are only

3 blue beads
4 white beads
and x orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

(Total for Question 23 is 4 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR May/November 2020

1 The total number of beads

The total number of beads is

$$x + 7$$

2 The probability of choosing two

The probability of choosing two beads of the same colour is

$$\frac{\binom{3}{2} + \binom{4}{2} + \binom{x}{2}}{\binom{x+7}{2}}$$

3 **Evaluate fraction**

This is equal to $\frac{3}{8}$, so

$$\frac{3+6+\frac{x(x-1)}{2}}{\frac{(x+7)(x+6)}{2}}=\frac{3}{8}$$

$$\frac{x^2-x+18}{(x+7)(x+6)}=\frac{3}{8}$$

$$8(x^2-x+18)=3(x+7)(x+6)$$

$$8x^2-8x+144=3x^2+39x+126$$

$$5x^2-47x+18=0$$

$$(x-9)(5x-2)=0$$

4 **Calculate value**

Since x must be a whole number,

$$x=9$$

5 **Calculate value**

So the total number of beads is

$$9+7=16$$

FINAL ANSWER

16 beads

4MA1/2HR JANUARY 2021

Question 21

6 marks | Probability Toolkit

21 A bag contains n beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of n.

$n =$ .....

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2021

1 There are beads altogether

There are n beads altogether.

2 Red beads

Red beads:

$$6$$

3 Blue beads

Blue beads:

$$n - 6$$

4 The probability of taking two

The probability of taking two beads of the same colour is

$$P(RR) + P(BB) = \frac{9}{17}$$

5 **Use trigonometry**

So

$$\frac{6}{n} \cdot \frac{5}{n-1} + \frac{n-6}{n} \cdot \frac{n-7}{n-1} = \frac{9}{17}$$

$$\frac{30 + (n-6)(n-7)}{n(n-1)} = \frac{9}{17}$$

6 **Expand**

Expand:

$$(n-6)(n-7) = n^2 - 13n + 42$$

7 **Use trigonometry**

So

$$\frac{n^2 - 13n + 72}{n(n-1)} = \frac{9}{17}$$

8 **Cross multiply**

Cross multiply:

$$17(n^2 - 13n + 72) = 9n(n-1)$$

$$17n^2 - 221n + 1224 = 9n^2 - 9n$$

$$8n^2 - 212n + 1224 = 0$$

9 **Divide by 4**

Divide by 4:

$$2n^2 - 53n + 306 = 0$$

10 **Factor**

Factor:

$$(2n - 17)(n - 18) = 0$$

11 **Use trigonometry**

So

$$n = \frac{17}{2} \quad \text{or} \quad n = 18$$

12 **The number of beads must**

The number of beads must be a whole number, so

$$n = 18$$

✓ **FINAL ANSWER**

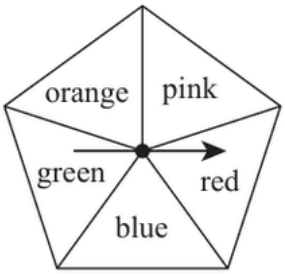
$\boxed{18}$

4MA1/1H MAY 2021

Question 6

5 marks | Probability Toolkit

6 Grace has a biased 5-sided spinner.



Grace is going to spin the arrow on the spinner once.

The table below gives the probabilities that the spinner will land on red or on blue or on green.

Colour	Red	Blue	Green	Orange	Pink
Probability	0.20	0.12	0.08		

The probability that the spinner will land on orange is 3 times the probability that the spinner will land on pink.

(a) Work out the probability that the spinner will land on orange.

.....

(3)

Grace spins the arrow on the spinner 150 times.

(b) Work out an estimate for the number of times the spinner lands on blue.

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2021

1 **(a) Set up with the missing probabilities**

Let $P(\text{pink}) = x$, so $P(\text{orange}) = 3x$.

$$0.20 + 0.12 + 0.08 + 3x + x = 1 \Rightarrow 4x = 0.60 \Rightarrow x = 0.15$$

2 **(a) Probability of orange**

$$3x = 0.45$$

3 (b) Estimate for blue

$$150 \times 0.12 = 18$$

FINAL ANSWER

(a) 0.45 (b) 18

4MA1/2H MAY 2021

Question 23

5 marks | Probability Toolkit

23 Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H May 2021

1 Let the number of red

Let the number of red pens be r.

2 List probability cases

Then the number of black pens is

$$r + 3$$

3 Calculate value

and

$$N = 2r + 3$$

4 Evaluate fraction

So

$$r = \frac{N - 3}{2}$$

5 **Evaluate fraction**

and

$$r+3=\frac{N+3}{2}$$

6 **The probability of black followed**

The probability of black followed by red is

$$\frac{(N+3)/2}{N}\cdot\frac{(N-3)/2}{N-1}=\frac{9}{35}$$

$$\frac{N^2-9}{4N(N-1)}=\frac{9}{35}$$

$$35(N^2-9)=36N(N-1)$$

$$35N^2-315=36N^2-36N$$

$$N^2-36N+315=0$$

$$(N-15)(N-21)=0$$

✓ **FINAL ANSWER**

$N=15$ or $N=21$

4MA1/1H NOVEMBER 2021

Question 3

4 marks | Probability Toolkit

3 There are 54 fish in a tank.
Some of the fish are white and the rest of the fish are red.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is $\frac{4}{9}$

(a) Work out the number of white fish originally in the tank.

.....

(2)

Jeevan puts the fish he took out, back into the tank.
He puts some more white fish into the tank.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is now $\frac{1}{2}$

(b) Work out the number of white fish Jeevan put into the tank.

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2021

1 Calculate probability

(a) Let the number of white fish be w.

$$\frac{w}{54} = \frac{4}{9}$$

$$w = 54 \times \frac{4}{9} = 24$$

2 (b) There are still 54

(b) There are still 54 fish, and now

$$\frac{\text{white fish}}{54} = \frac{1}{2}$$

$$\text{white fish} = 27$$

3 **Jeevan added**

Jeevan added

$$27 - 24 = 3$$

✓ **FINAL ANSWER**

(a) 24 white fish, (b) 3 white fish.

4MA1/2HR JANUARY 2022

Question 1

4 marks | Probability Toolkit

1 A tin contains tea bags with a choice of four different flavours of tea.
The four flavours of tea are Assam or Darjeeling or Nilgiri or Rize.

Sara takes at random a tea bag from the tin.

The table shows each of the probabilities that the flavour of the tea Sara takes is Assam or Darjeeling or Rize.

Flavour of tea	Assam	Darjeeling	Nilgiri	Rize
Probability	0.38	0.24		0.16

(a) Work out the probability that the flavour of the tea Sara takes is Nilgiri.

.....

(2)

(b) Work out the probability that the flavour of the tea Sara takes is either Darjeeling or Rize.

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2022

1 **(a) Probabilities add to 1**

$$P(\text{Nilgiri}) = 1 - (0.38 + 0.24 + 0.16) = 1 - 0.78 = 0.22$$

2 **(b) Darjeeling or Rize**

$$0.24 + 0.16 = 0.40$$

✓ **FINAL ANSWER**

(a) 0.22 (b) 0.4

4MA1/1HR JANUARY 2022

Question 2

2 marks | Probability Toolkit

2 Each time John plays a game, the probability that he wins the game is 0.65

John is going to play the game 300 times.

Work out an estimate for the number of games that John wins.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2022

1 **Expected number = probability × number of games**

$$0.65 \times 300 = 195$$

FINAL ANSWER

195

4MA1/2H JANUARY 2022

Question 6

5 marks | Probability Toolkit

6 A bag contains only pink sweets, white sweets, green sweets and red sweets.

The table gives each of the probabilities that, when a sweet is taken at random from the bag, the sweet will be green or the sweet will be red.

Sweet	pink	white	green	red
Probability			0.2	0.35

The ratio
number of pink sweets : number of white sweets = 2 : 1

There are 28 red sweets in the bag.

Work out the number of white sweets in the bag.

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2022

1 **Split the ratio**

The probabilities for green and red are

$$0.20 + 0.35 = 0.55$$

2 **Split the ratio**

So

$$P(\text{pink or white}) = 1 - 0.55 = 0.45$$

3 **Split the ratio**

Pink : white $= 2 : 1$, so white is $\frac{1}{3}$ of 0.45.

$$P(\text{white}) = \frac{1}{3} \times 0.45 = 0.15$$

4 **Split the ratio**

The red probability is 0.35, and there are 28 red sweets.

$$\text{total sweets} = \frac{28}{0.35} = 80$$

5 **Number of white sweets**

Number of white sweets:

$$0.15 \times 80 = 12$$

✓ **FINAL ANSWER**

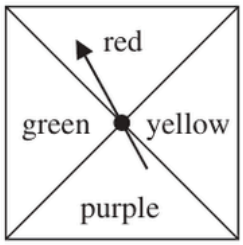
12

4MA1/1HR JUNE 2022

Question 1

4 marks | Probability Toolkit

1 Here is a biased spinner.



When the spinner is spun once, the probabilities that it lands on red or on yellow or on green are given in the table.

Colour	red	yellow	purple	green
Probability	0.25	0.2		0.2

(a) Work out the probability that the spinner lands on red or on yellow.

.....

(1)

Yang is going to spin the spinner 300 times.

(b) Work out an estimate for the number of times the spinner will land on purple.

.....

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2022

1 **(a) Red or yellow**

$$0.25 + 0.2 = 0.45$$

2 **(b) Probability of purple**

$$1 - (0.25 + 0.2 + 0.2) = 0.35$$

3 **(b) Estimate for 300 spins**

$$300 \times 0.35 = 105$$

FINAL ANSWER

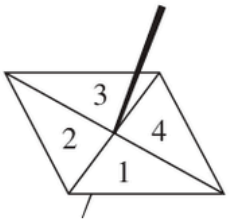
(a) 0.45 (b) 105

4MA1/1H JUNE 2022

Question 2

4 marks | Probability Toolkit

2 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

Number	1	2	3	4
Probability	0.26		0.18	

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4

Ravina is going to spin the spinner a number of times.

Ravina works out that an estimate for the number of times the spinner will land on 3 is 45

Work out an estimate for the number of times the spinner will land on 4

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H June 2022

1 **Probabilities of landing on 2 and on 4**

The probabilities add to 1, and $P(2) = P(4)$:

$$1 - (0.26 + 0.18) = 0.56, \qquad P(4) = 0.56 \div 2 = 0.28$$

2 **Number of spins**

An estimate of 45 for the number 3 means

$$0.18 \times n = 45 \Rightarrow n = 250$$

3 **Estimate for landing on 4**

$$250 \times 0.28 = 70$$

FINAL ANSWER

70

4MA1/1H JUNE 2022

Question 15

4 marks | Probability Toolkit

15 Abraham is going to play a computer game.
Abraham can win the game, draw the game or lose the game.

For any game that Abraham plays

the probability that he wins the game is 0.3
the probability that he draws the game is 0.5
the probability that he loses the game is 0.2

When Abraham wins a game, he scores +10 points.
When Abraham draws a game, he scores 0 points.
When Abraham loses a game, he scores −5 points.

Abraham plays 3 games and the points he scores in each of the 3 games are added together to get his total score.

Work out the probability that when he has played 3 games his total score is 0 points.

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H June 2022

1 Ways to score a total of 0

Three draws ($0+0+0$), or one win and two losses ($10-5-5=0$) in any order.

2 Three draws

$$0.5^3 = 0.125$$

3 One win and two losses (3 orders)

$$3 \times 0.3 \times 0.2 \times 0.2 = 0.036$$

4 Add the cases

$$0.125 + 0.036 = 0.161$$

FINAL ANSWER

0.161

4MA1/1HR JUNE 2022

Question 24

5 marks | Probability Toolkit

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR June 2022

1 There are counters. Initially 5

There are x counters. Initially 5 show red, so $x - 5$ show green.

2 List probability cases

After two turns, there are still 5 red faces if:

3 the same counter is picked

- the same counter is picked twice, or
- one red counter and one green counter are picked.

4 The probability of picking the

The probability of picking the same counter twice is

$$\frac{1}{x}$$

5 **The probability of picking one**

The probability of picking one red and one green in either order is

$$\frac{5}{x} \cdot \frac{x-5}{x} + \frac{x-5}{x} \cdot \frac{5}{x}$$

$$= \frac{10(x-5)}{x^2}$$

---

6 **Evaluate fraction**

So

$$\frac{1}{x} + \frac{10(x-5)}{x^2} = \frac{19}{32}$$

$$\frac{11x-50}{x^2} = \frac{19}{32}$$

$$32(11x-50) = 19x^2$$

$$19x^2 - 352x + 1600 = 0$$

$$(x-8)(19x-200) = 0$$

---

7 **Calculate value**

Since x must be a whole number,

$$x = 8$$

✓ **FINAL ANSWER**

$x = 8$

4MA1/2H JUNE 2023

Question 2

3 marks | Probability Toolkit

2 A biased spinner can land on green or on yellow or on brown or on pink.

The table gives the probabilities that, when the spinner is spun, it will land on green or on yellow or on brown.

Colour	green	yellow	brown	pink
Probability	0.32	0.13	0.28	

Timucin spins the spinner 200 times.

Work out an estimate for the number of times the spinner lands on pink.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2023

1 Probability of pink

$$1 - (0.32 + 0.13 + 0.28) = 1 - 0.73 = 0.27$$

2 Estimate for 200 spins

$$200 \times 0.27 = 54$$

FINAL ANSWER

54

4MA1/1HR MAY 2023

Question 5

2 marks | Probability Toolkit

5 Each time Evie plays a game against her computer, she will win or lose.

For each game, the probability that Evie will win is 0.74
Evie is going to play 300 games against her computer.

Work out an estimate for the number of games that Evie will lose.

(Total for Question 5 is 2 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR May 2023

1 Probability of losing

$$1 - 0.74 = 0.26$$

2 Estimate for 300 games

$$300 \times 0.26 = 78$$

FINAL ANSWER

78

4MA1/2HR JUNE 2023

Question 15

3 marks | Probability Toolkit

15 Osvaldo has a biased coin.
He spins the coin three times.

The probability that the coin lands on a head three times is $\frac{27}{64}$

Work out the probability that the coin will land on a tail three times.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2023

1 Find the probability of a head

$$P(H)^3 = \frac{27}{64} \Rightarrow P(H) = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

2 Three tails

$P(T) = 1 - \frac{3}{4} = \frac{1}{4}$, so

$$P(TTT) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

FINAL ANSWER

$$\frac{1}{64}$$

4MA1/2HR JUNE 2023

Question 19

4 marks | Probability Toolkit

19 30 adults booked to stay in a hotel.

19 adults booked breakfast
15 adults booked dinner
4 adults did not book breakfast or dinner

Some adults booked breakfast **and** dinner.

Meihui chooses at random two of the 30 adults.

Work out the probability that these two adults each booked breakfast **and** dinner.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2023

1 **Adults who booked breakfast and dinner**

$30 - 4 = 26$ adults booked at least one meal.

$$n(B \cap D) = 19 + 15 - 26 = 8$$

2 **Choose two of them (without replacement)**

$$\frac{8}{30} \times \frac{7}{29} = \frac{56}{870} = \frac{28}{435}$$

✓ **FINAL ANSWER**

$$\frac{28}{435}$$

4MA1/1H NOVEMBER 2023

Question 4

4 marks | Probability Toolkit

4 A biased spinner has three sections each of a different colour.

The table shows the probability that, when the spinner is spun once, it will land on blue or on orange or on white.

Colour	blue	orange	white
Probability	0.58	$2x$	x

(a) Work out the value of x

$x =$ .....

(2)

The spinner is spun 250 times.

(b) Work out an estimate for the number of times the spinner will land on blue.

.....

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2023

1 **(a) Probabilities add to 1**

$$0.58 + 2x + x = 1 \Rightarrow 3x = 0.42 \Rightarrow x = 0.14$$

2 **(b) Estimate for blue in 250 spins**

$$250 \times 0.58 = 145$$

✓ **FINAL ANSWER**

(a) $x = 0.14$ (b) 145

4MA1/1H NOVEMBER 2023

Question 20

4 marks | Probability Toolkit

20 A bag contains only 10 cent coins and 20 cent coins.

Josip takes at random a coin from the bag, records its value and replaces it in the bag. He then takes at random a second coin from the bag, records its value and replaces it in the bag.

Josip finds the mean value of the two coins.

The probability that the two coins have a mean value of 10 cents is $\frac{49}{121}$

Work out the probability that the two coins have a mean value of 15 cents.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H November 2023

1 Let the probability of taking

Let the probability of taking a 10 cent coin be p.

2 The mean is 10 cents

The mean is 10 cents only when both coins are 10 cent coins.

$$p^2 = \frac{49}{121}$$

$$p = \frac{7}{11}$$

3 Calculate probability

So the probability of taking a 20 cent coin is

$$1 - \frac{7}{11} = \frac{4}{11}$$

4 The mean is 15 cents

The mean is 15 cents when one coin is 10 cents and the other is 20 cents.

5 **There are two possible orders**

There are two possible orders:

$$P(10, 20) + P(20, 10)$$

$$= \frac{7}{11} \cdot \frac{4}{11} + \frac{4}{11} \cdot \frac{7}{11}$$

$$= \frac{56}{121}$$

✓ **FINAL ANSWER**

$\frac{56}{121}$

4MA1/1H MAY 2024

Question 2

4 marks | Probability Toolkit

2 450 students were asked how they travelled to school on Monday.
Each student walked or travelled by bus or travelled by car or travelled by bicycle.
Each student used just one method of travel.

One of these students is chosen at random.
The table shows information about the probability of each method of travel.

Method of travel	walk	bus	car	bicycle
Probability	0.20	x	$2x$	0.26

Work out how many of the 450 students travelled by car.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2024

1 Find x

$$0.20 + x + 2x + 0.26 = 1 \Rightarrow 3x = 0.54 \Rightarrow x = 0.18$$

2 Probability of travelling by car

$$2x = 0.36$$

3 Number of students

$$450 \times 0.36 = 162$$

✓ **FINAL ANSWER**

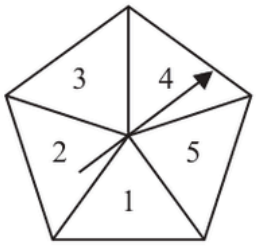
162

4MA1/1HR MAY 2024

Question 2

4 marks | Probability Toolkit

2 Here is a biased spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4	5
Probability	$2x$	0.27	0.04	x	0.12

Alexis is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on an odd number.

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR May 2024

1 Find x

$$2x + 0.27 + 0.04 + x + 0.12 = 1 \Rightarrow 3x = 0.57 \Rightarrow x = 0.19$$

2 Probability of an odd number (1, 3 or 5)

$$2x + 0.04 + 0.12 = 0.38 + 0.04 + 0.12 = 0.54$$

3 Estimate for 400 spins

$$400 \times 0.54 = 216$$

FINAL ANSWER

216

4MA1/2HR JUNE 2024

Question 14

3 marks | Probability Toolkit

14 Adriana is going to roll a biased dice and spin a biased coin.

The probability that the coin will land on Heads is 0.8
The probability that the dice will land on 6 and the coin will land on Heads is 0.24

Work out the probability that the dice will land on 6 and the coin will land on Tails.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2024

1 Find the probability of a 6

The dice and the coin are independent:

$$P(6) \times 0.8 = 0.24 \Rightarrow P(6) = 0.3$$

2 6 and Tails

$P(\text{Tails}) = 1 - 0.8 = 0.2$, so

$$0.3 \times 0.2 = 0.06$$

FINAL ANSWER

0.06

4MA1/1HR MAY 2024

Question 21

5 marks | Probability Toolkit

21 There are 25 counters in a bag such that

6 counters are blue
x counters are orange, where $x > 9$
the rest of the counters are pink

Maalam takes at random two of the counters from the bag.

The probability that Maalam takes one orange counter and one pink counter is $\frac{22}{75}$

Calculate the probability that Maalam takes 2 pink counters from the bag.
Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR May 2024

1 **There are 25 counters**

There are 25 counters.

2 **Blue counters**

Blue counters:

$$6$$

3 **Orange counters**

Orange counters:

$$x$$

4 **Pink counters**

Pink counters:

$$25 - 6 - x = 19 - x$$

5 **The probability of taking one**

The probability of taking one orange and one pink in either order is

$$\frac{x}{25} \cdot \frac{19-x}{24} + \frac{19-x}{25} \cdot \frac{x}{24}$$

$$= \frac{2x(19-x)}{25 \cdot 24}$$

6 **Evaluate fraction**

This is equal to $\frac{22}{75}$, so

$$\frac{2x(19-x)}{600} = \frac{22}{75}$$

$$\frac{x(19-x)}{300} = \frac{22}{75}$$

$$x(19-x) = 88$$

$$19x - x^2 = 88$$

$$x^2 - 19x + 88 = 0$$

$$(x-8)(x-11) = 0$$

7 **Solve inequality**

Since $x > 9$,

$$x = 11$$

8 **Calculate value**

So the number of pink counters is

$$19 - 11 = 8$$

9 **The probability of taking 2**

The probability of taking 2 pink counters is

$$\frac{8}{25} \cdot \frac{7}{24} = \frac{56}{600}$$

$$= \frac{7}{75}$$

FINAL ANSWER

$$\frac{7}{75}$$

4MA1/2H NOVEMBER 2024

Question 5

4 marks | Probability Toolkit

5 Cody has a biased 5-sided spinner, numbered 1, 2, 3, 4, 5

The table gives the probabilities that when the spinner is spun it will land on 2 or on 3 or on 5

Number	1	2	3	4	5
Probability		0.14	0.17		0.21

The probability that the spinner will land on 1 is the same as the probability that the spinner will land on 4

Cody is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on 4

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2024

1 Probabilities of 1 and 4 together

$$1 - (0.14 + 0.17 + 0.21) = 1 - 0.52 = 0.48$$

2 They are equal

$$P(4) = 0.48 \div 2 = 0.24$$

3 Estimate for 400 spins

$$400 \times 0.24 = 96$$

✓ **FINAL ANSWER**

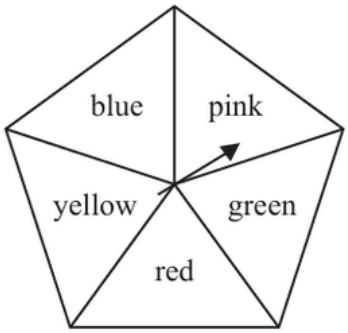
96

4WM1/01 MAY 2025

Question 3

4 marks | Probability Toolkit

3 Here is a biased spinner.
The spinner can land on pink or on green or on red or on yellow or on blue.



The table gives information about the probability that, when the spinner is spun, it will land on each colour.

Colour	pink	green	red	yellow	blue
Probability	0.14	0.23	$2x$	0.18	$7x$

Jim is going to spin the spinner 500 times.

Work out the expected number of times the spinner will land on blue.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4WM1/01 May 2025

1 Calculate probability

The probabilities add to 1.

$$0.14 + 0.23 + 2x + 0.18 + 7x = 1$$

$$0.55 + 9x = 1$$

$$x = 0.05$$

2 The probability of blue is

The probability of blue is

$$7x = 7(0.05) = 0.35$$

3 Expected number of blue results

Expected number of blue results:

$$500 \times 0.35 = 175$$

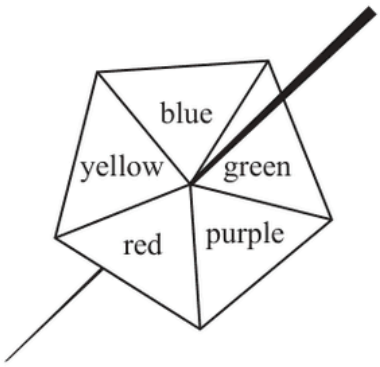
FINAL ANSWER

175

Question 3

5 marks | Probability Toolkit

3 Here is a biased 5-sided spinner.
When the spinner is spun, it can land on blue or on green or on purple or on red or on yellow.



The table gives information about the probability of the spinner landing on each colour.

Colour	blue	green	purple	red	yellow
Probability	0.12	0.20	0.38	$4x$	x

Sophie spins the spinner once.

(a) Work out the probability that the spinner lands on blue or on green or on purple.

.....

(1)

Max spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on red.

.....

(4)

(Total for Question 3 is 5 marks)

1 Let the yellow probability be

Let the yellow probability be x. Then the red probability is $4x$.

$$0.12 + 0.20 + 0.30 + 4x + x = 1$$

$$5x = 0.38$$

$$x = 0.076$$

2 Calculate probability

So

$$P(\text{red}) = 4x = 0.304$$

$$P(\text{blue, green or purple}) = 0.12 + 0.20 + 0.30 = 0.62$$

3 Expected red

Expected red:

$$350 \times 0.304 = 106.4$$

FINAL ANSWER

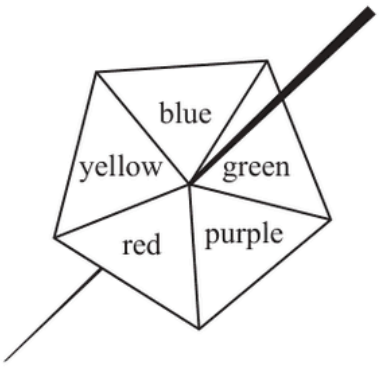
0.62, and about 106.

4MA1/1HR JUNE 2025

Question 4

5 marks | Probability Toolkit

4 Here is a biased 5-sided spinner.
When the spinner is spun, it can land on blue or on green or on purple or on red or on yellow.



The table gives information about the probability of the spinner landing on each colour.

Colour	blue	green	purple	red	yellow
Probability	0.12	0.20	0.38	$4x$	x

Sophie spins the spinner once.

(a) Work out the probability that the spinner lands on blue or on green or on purple.

.....

(1)

Max spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on red.

.....

(4)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2025

1 **(a) Blue, green or purple**

$$0.12 + 0.20 + 0.38 = 0.70$$

2 **(b) Find x**

Red and yellow share the rest: $4x + x = 1 - 0.70 = 0.30$, so $5x = 0.30$ and $x = 0.06$.

3 (b) Probability of red

$$4x = 4 \times 0.06 = 0.24$$

4 (b) Estimate for 350 spins

$$350 \times 0.24 = 84$$

FINAL ANSWER

(a) 0.7 (b) 84

4MA1/2H JUNE 2025

Question 11

3 marks | Probability Toolkit

11 Khalid has a box of counters.

17 of the counters are red
28 of the counters are blue
the rest of the counters are orange

Khalid is going to take at random a counter from the box.

The probability that Khalid will take an orange counter is $\frac{4}{9}$

Work out the number of orange counters that are in the box.

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2025

1 Counters that are not orange

$17 + 28 = 45$ counters are not orange, and $P(\text{not orange}) = 1 - \frac{4}{9} = \frac{5}{9}$.

2 Total number of counters

$$\frac{5}{9} \times T = 45 \Rightarrow T = 81$$

3 Orange counters

$$81 - 45 = 36$$

FINAL ANSWER

36

4WM1/01R MAY 2025

Question 17

3 marks | Probability Toolkit

17 Kannika has 9 counters.
There is a number on each counter.



Kannika puts the 9 counters in a bag.
She takes at random a counter from the bag and does not replace the counter.
She then takes at random a second counter from the bag.

Work out the probability that the sum of the numbers on the two counters is less than 5

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4WM1/01R May 2025

1 The counters are

The counters are

$$1,\ 1,\ 2,\ 3,\ 3,\ 3,\ 5,\ 6,\ 6$$

2 There are

There are

$$\binom{9}{2} = 36$$

3 ways to choose two counters

ways to choose two counters.

4 Calculate probability

For a sum less than 5, the pairs are:

$$(1,1),\quad (1,2),\quad (1,3)$$

5 Counting the repeated counters gives

Counting the repeated counters gives

$$1+2+6=9$$

6 Calculate probability

So

$$P(\text{sum} < 5) = \frac{9}{36} = \frac{1}{4}$$

FINAL ANSWER

$\frac{1}{4}$.

4MA1/1HR JUNE 2025

Question 19

3 marks | Probability Toolkit

19 Kannika has 9 counters.
There is a number on each counter.



Kannika puts the 9 counters in a bag.
She takes at random a counter from the bag and does not replace the counter.
She then takes at random a second counter from the bag.

Work out the probability that the sum of the numbers on the two counters is less than 5

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2025

1 Pairs with a sum less than 5

The counters are $1, 1, 2, 3, 3, 3, 5, 6, 6$. The only possible sums under 5 are $1+1$, $1+2$ and $1+3$.

2 First counter, then second (no replacement)

$$P(1,1) = \frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$$

$$P(1 \text{ and } 2) = 2 \times \frac{2}{9} \times \frac{1}{8} = \frac{4}{72}, \qquad P(1 \text{ and } 3) = 2 \times \frac{2}{9} \times \frac{3}{8} = \frac{12}{72}$$

3 Add the cases

$$\frac{2+4+12}{72} = \frac{18}{72} = \frac{1}{4}$$

✓ **FINAL ANSWER**

$$\frac{1}{4}$$

4MA1/1H JUNE 2025

Question 21

3 marks | Probability Toolkit

21 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H June 2025

1 There are

There are

$$\binom{20}{3} = 1140$$

2 ways to choose 3 counters

ways to choose 3 counters.

3 Exactly two counters are the

Exactly two counters are the same colour:

$$\binom{9}{2}(11) + \binom{7}{2}(13) + \binom{4}{2}(16)$$

$$= 36(11) + 21(13) + 6(16)$$

$$= 396 + 273 + 96 = 765$$

4 Calculate probability

So the probability is

$$\frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$$\frac{51}{76}$$

4WM1/01 MAY 2025

Question 22

3 marks | Probability Toolkit

22 A box contains 20 counters.

9 of the counters are red
7 of the counters are yellow
4 of the counters are green

Alex takes at random three counters from the box.

Work out the probability that exactly two of the three counters are the same colour.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4WM1/01 May 2025

1 Total ways to choose counters

Total ways to choose 3 counters:

$$\binom{20}{3} = 1140$$

2 Exactly two red

Exactly two red:

$$\binom{9}{2}\binom{11}{1} = 396$$

3 Exactly two yellow

Exactly two yellow:

$$\binom{7}{2}\binom{13}{1} = 273$$

4 Exactly two green

Exactly two green:

$$\binom{4}{2}\binom{16}{1} = 96$$

5 Favourable outcomes

Favourable outcomes:

$$396 + 273 + 96 = 765$$

$$P = \frac{765}{1140} = \frac{51}{76}$$

FINAL ANSWER

$\frac{51}{76}$

4WM1/01 NOVEMBER 2025

Question 5

5 marks | Probability Toolkit

5 All students at a school are asked which sport they prefer from football, tennis, rugby and cricket.

The table gives information about the probabilities that, when a student is chosen at random, the student prefers football or tennis or rugby or cricket.

Favourite sport	football	tennis	rugby	cricket
Probability	$2x$	0.2	x	0.35

One of the students at the school is chosen at random.

(a) Work out the probability that this student prefers tennis or cricket.

.....

(1)

There are 700 students at the school.

(b) Work out how many students prefer football.

.....

(4)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4WM1/01 November 2025

1 **Calculate value**

$$P(\text{tennis or cricket}) = 0.2 + 0.35 = 0.55$$

2 **Calculate value**

Also,

$$2x + 0.2 + x + 0.35 = 1$$

$$3x = 0.45$$

$$x = 0.15$$

3 Calculate value

So

$$P(\text{football}) = 2x = 0.30$$

$$700 \times 0.30 = 210$$

FINAL ANSWER

0.55, 210 students.

Question 9

3 marks | Probability Toolkit

9 Mark has three bags, bag A, bag B and bag C

Bag A contains 120 counters.
The probability of taking at random a red counter from bag A is 0.45

Bag B contains 80 counters.
The probability of taking at random a red counter from bag B is 0.3

Bag C is empty.

Mark puts all the counters from bag A and bag B into bag C

Mark takes at random a counter from bag C

Work out the probability that he takes a red counter.

.....

(Total for Question 9 is 3 marks)

1 Red counters in each bag

Bag A: $120 \times 0.45 = 54$. Bag B: $80 \times 0.3 = 24$.

2 Probability of red from bag C

Bag C has $54 + 24 = 78$ red counters out of $120 + 80 = 200$.

$$\frac{78}{200} = 0.39$$

FINAL ANSWER

0.39

4MA1/1HR JUNE 2026

Question 5

4 marks | Probability Toolkit

5 A bag contains only blue counters, green counters, red counters and yellow counters.

Danny takes a counter at random from the bag.

The table gives information about the probabilities that he takes a blue counter or a green counter or a red counter or a yellow counter.

Colour	blue	green	red	yellow
Probability	0.45	0.31	$5x$	x

There are 400 counters in the bag.

Work out the number of red counters in the bag.

.....

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR June 2026

1 Solution | Step 1

$$0.45 + 0.31 + 5x + x = 1.$$

The four colours are mutually exclusive and exhaust all counters, so their probabilities sum to 1.

2 Solution | Step 2

$$6x = 1 - 0.76 = 0.24, \qquad x = 0.04.$$

Collect the unknown probabilities and solve the linear equation.

3 Solution | Step 3

$$P(\text{red}) = 5(0.04) = 0.20, \qquad N_{\text{red}} = 400(0.20) = 80.$$

Multiply the red proportion by the total number of counters.

FINAL ANSWER

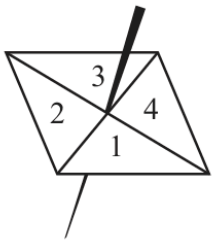
80 red counters

4MA1/1H JUNE 2026

Question 7

4 marks | Probability Toolkit

7 Here is a biased 4-sided spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4
Probability	0.12	x	$2x$	0.16

Nevan spins the spinner 500 times.

Work out the expected number of times that the spinner lands on an even number.

.....

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H June 2026

1 Solution | Step 1

$$0.12 + x + 2x + 0.16 = 1 \implies 3x = 0.72 \implies x = 0.24.$$

The four exhaustive outcome probabilities sum to 1.

2 Solution | Step 2

$$P(\text{even}) = P(2) + P(4) = 0.24 + 0.16 = 0.40.$$

The events 2 and 4 are mutually exclusive.

3 Solution | Step 3

$$\text{Expected number} = 500 \times 0.40 = 200.$$

Multiply the number of spins by the probability of an even result.

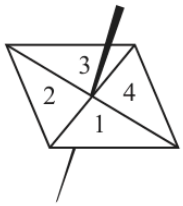
FINAL ANSWER

200

Question 7

4 marks | Probability Toolkit

7 Here is a biased 4-sided spinner.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4
Probability	0.12	x	$2x$	0.16

Nevan spins the spinner 500 times.

Work out the expected number of times that the spinner lands on an even number.

.....

(Total for Question 7 is 4 marks)

1 **Probabilities add to 1**

$$0.12 + x + 2x + 0.16 = 1 \Rightarrow 3x = 0.72 \Rightarrow x = 0.24$$

2 **Even numbers are 2 and 4**

$$P(\text{even}) = 0.24 + 0.16 = 0.40$$

3 **Expected number**

$$500 \times 0.40 = 200$$

✓ **FINAL ANSWER**

200

Question 11

3 marks | Probability Toolkit

11 There are 40 counters in a bag.

25 of the counters are green
7 of the counters are blue
8 of the counters are red

Ben puts some extra blue counters and some extra red counters into the bag.
The number of blue counters added to the bag is twice as many as the number of red counters added to the bag.

Ben takes at random a counter from the bag.

The probability that he takes a green counter is $\frac{5}{11}$

How many of the extra counters that Ben put into the bag were blue?

.....

(Total for Question 11 is 3 marks)

1 **Let r red be added**

Then $2r$ blue are added and the total is $40 + 3r$.

2 **Use P(green)**

$$\frac{25}{40+3r} = \frac{5}{11} \Rightarrow 275 = 200 + 15r \Rightarrow r = 5$$

3 **Blue added**

$$2r = 10$$

✓ **FINAL ANSWER**

10

4MA1/2H JUNE 2026

Question 20

3 marks | Probability Toolkit

20 A box only contains green counters and red counters.

Nuno chooses at random a counter from the box.
He puts the counter back into the box.

Nuno then chooses at random a second counter from the box.

The probability that Nuno chooses at least one red counter is $\frac{39}{64}$

Work out the probability that Nuno chooses exactly one red counter.

.....

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2026

1 Solution | Step 1

Let $g = P(G)$. With replacement, the draws are independent.

$$P(GG) = 1 - P(\text{at least one red}) = 1 - \frac{39}{64} = \frac{25}{64}.$$

The complementary event to at least one red is two greens.

2 Solution | Step 2

$$g^2 = \frac{25}{64} \quad \Rightarrow \quad g = \frac{5}{8}, \qquad P(R) = 1 - \frac{5}{8} = \frac{3}{8}.$$

Multiply identical independent green probabilities and take the nonnegative root.

3 Solution | Step 3

$$P(\text{exactly one red}) = P(RG) + P(GR) = \frac{3}{8}\frac{5}{8} + \frac{5}{8}\frac{3}{8} = \frac{30}{64} = \frac{15}{32}.$$

The two possible orders are mutually exclusive and must both be included.

✓ **FINAL ANSWER**

$$\frac{15}{32}$$

TOPIC

Tree Diagrams & Conditional Probability

Every past-paper question on this topic, 2020 to 2026.

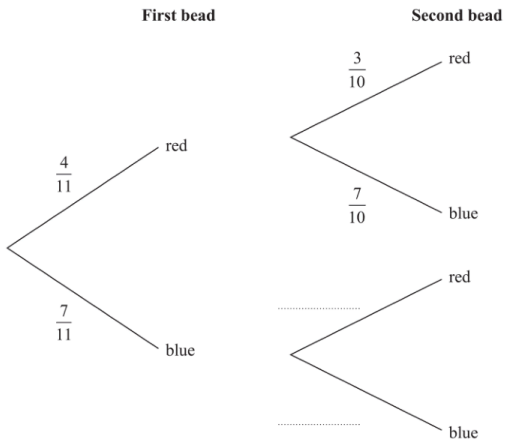
Question 11

6 marks | Tree Diagrams & Conditional Probability

11 Tess has a bag containing 11 beads.
4 of the beads are red.
7 of the beads are blue.

Tess takes at random a bead from the bag and keeps it.
Tess takes at random another bead from the bag and keeps it.

(a) Complete the probability tree diagram.



(1)

(b) Work out the probability that both of the beads are red.

.....

(2)

(c) Work out the probability that the two beads are of different colours.

.....

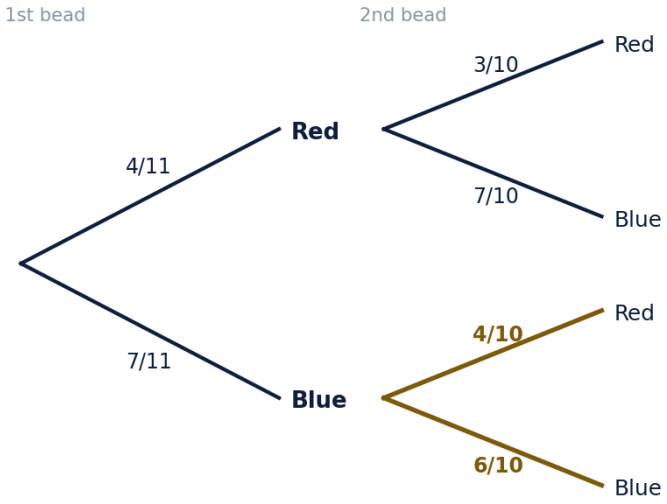
(3)

(Total for Question 11 is 6 marks)

1 Complete the blue-first branch

After a blue bead is taken first, there are 4 red and 6 blue beads left out of 10.

$$P(\text{red second after blue}) = \frac{4}{10}, \qquad P(\text{blue second after blue}) = \frac{6}{10}$$



2 Find both red

$$P(RR) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110} = \frac{6}{55}$$

3 Find different colours

$$P(RB) = \frac{4}{11} \times \frac{7}{10}$$

$$P(BR) = \frac{7}{11} \times \frac{4}{10}$$

$$P(\text{different}) = \frac{28}{110} + \frac{28}{110} = \frac{28}{55}$$

FINAL ANSWER

Missing probabilities $\frac{4}{10}$ and $\frac{6}{10}$; $P(RR) = \frac{6}{55}$; $P(\text{different}) = \frac{28}{55}$.

4MA1/2H JANUARY 2020

Question 14

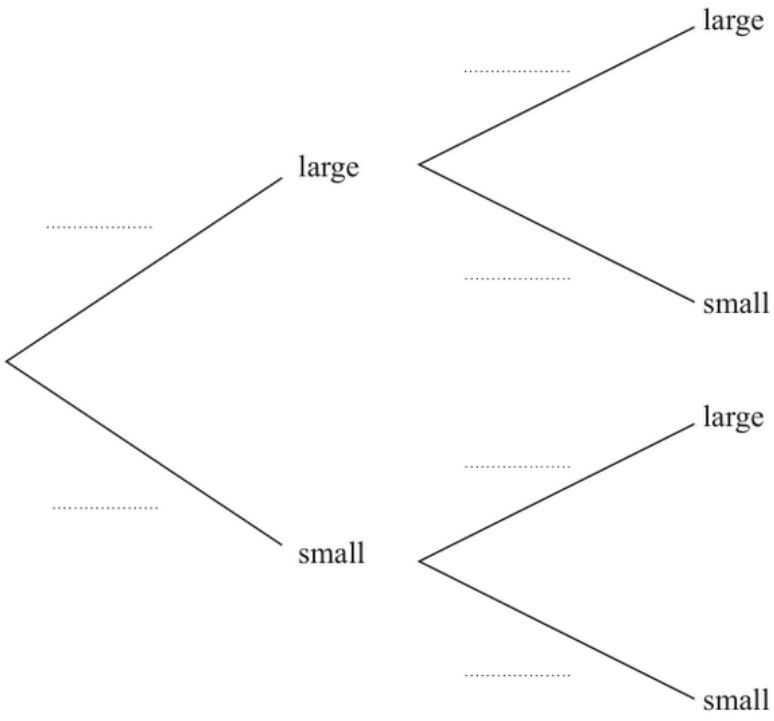
4 marks | Tree Diagrams & Conditional Probability

14 There are 20 glasses in a cupboard.

13 of the glasses are large
7 of the glasses are small

Roberto takes at random two glasses from the cupboard.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Roberto takes two small glasses.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2020

1 **(a) First glass**

$$P(\text{large}) = \frac{13}{20}, \qquad P(\text{small}) = \frac{7}{20}$$

2 **(a) Second glass (one fewer glass left)**

After a large glass: large $\frac{12}{19}$, small $\frac{7}{19}$.
After a small glass: large $\frac{13}{19}$, small $\frac{6}{19}$.

---

3 **(b) Two small glasses**

$$\frac{7}{20} \times \frac{6}{19} = \frac{42}{380} = \frac{21}{190}$$

✓ **FINAL ANSWER**

(a) $\frac{13}{20}, \frac{7}{20}$; $\frac{12}{19}, \frac{7}{19}$; $\frac{13}{19}, \frac{6}{19}$ (b) $\frac{21}{190}$

4MA1/1HR JANUARY 2020

Question 16

4 marks | Tree Diagrams & Conditional Probability

16 Steffi is going to play one game of tennis and one game of chess.

The probability that she will win the game of tennis is 0.6
The probability that she will win **both** games is 0.42

Work out the probability that she will **not** win either game.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2020

1 Find the probability of winning chess

Treat the two games as independent:

$$0.6 \times P(\text{win chess}) = 0.42 \Rightarrow P(\text{win chess}) = 0.42 \div 0.6 = 0.7$$

2 Probabilities of losing

$$P(\text{not win tennis}) = 1 - 0.6 = 0.4, \qquad P(\text{not win chess}) = 1 - 0.7 = 0.3$$

3 Win neither game

$$0.4 \times 0.3 = 0.12$$

FINAL ANSWER

0.12

Question 16

8 marks | Tree Diagrams & Conditional Probability

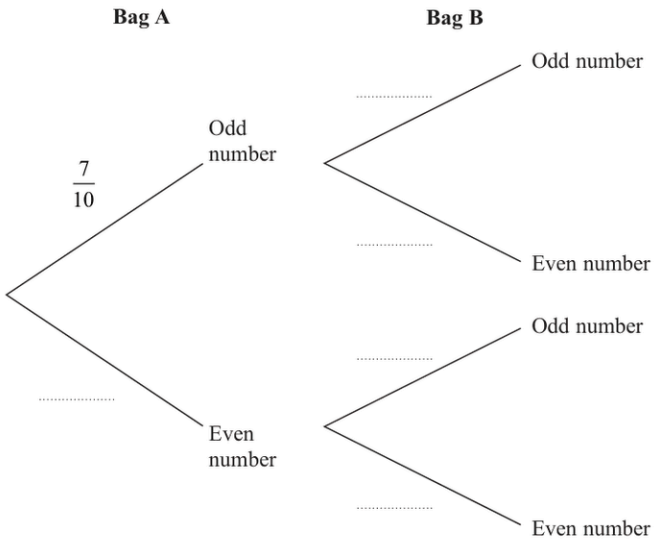
16 Cody has two bags of counters, bag **A** and bag **B**.

Each of the counters has either an odd number or an even number written on it.

There are 10 counters in bag **A** and 7 of these counters have an **odd** number written on them.
There are 12 counters in bag **B** and 7 of these counters have an **odd** number written on them.

Cody is going to take at random a counter from bag **A** and a counter from bag **B**.

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that the total of the numbers on the two counters will be an odd number.

.....

(3)

Harriet also has a bag of counters.
Each of her counters also has either an odd number or an even number written on it.

Harriet is going to take at random a counter from her bag of counters.

The probability that the number on each of Cody's two counters **and** the number on Harriet's counter will all be even is $\frac{3}{100}$

(c) Find the least number of counters that Harriet has in her bag.
Show your working clearly.

.....

(3)

(Total for Question 16 is 8 marks)

1 **(a) Complete the tree**

Bag A: odd $\frac{7}{10}$, even $\frac{3}{10}$.

Bag B (on both branches): odd $\frac{7}{12}$, even $\frac{5}{12}$.

2 **(b) Odd total = odd + even or even + odd**

$$\frac{7}{10} \times \frac{5}{12} + \frac{3}{10} \times \frac{7}{12} = \frac{35}{120} + \frac{21}{120} = \frac{56}{120} = \frac{7}{15}$$

3 **(c) Probability Harriet takes an even counter**

Cody takes two even counters with probability $\frac{3}{10} \times \frac{5}{12} = \frac{1}{8}$.

$$\frac{1}{8} \times p = \frac{3}{100} \Rightarrow p = \frac{24}{100} = \frac{6}{25}$$

4 **(c) Least number of counters**

$\frac{6}{25}$ is in its simplest form, so the number of counters must be a multiple of 25. The least number is 25.

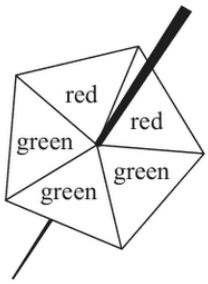
✓ **FINAL ANSWER**

(a) $\frac{3}{10}$; $\frac{7}{12}$, $\frac{5}{12}$ on both branches (b) $\frac{7}{15}$ (c) 25

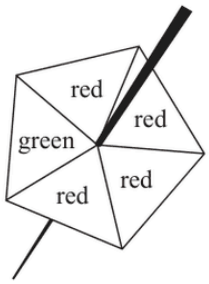
Question 15

5 marks | Tree Diagrams & Conditional Probability

15 Harry has two fair 5-sided spinners.



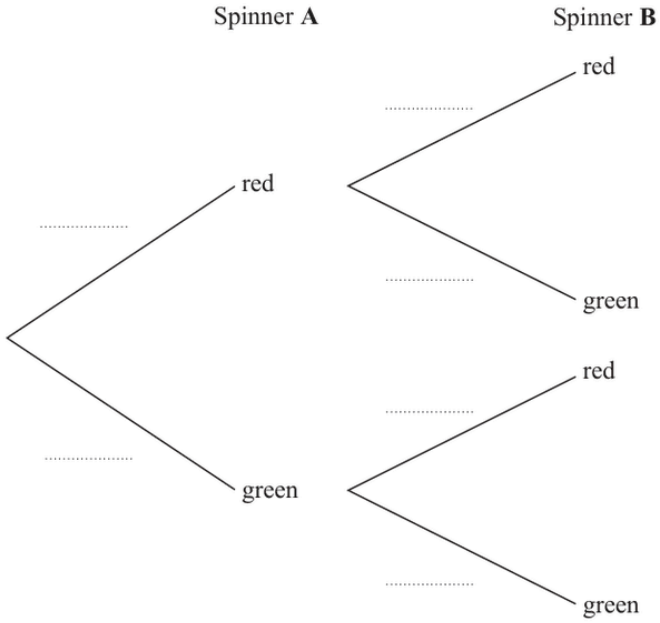
Spinner **A**



Spinner **B**

Harry is going to spin each spinner once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that at least one of the spinners will land on green.

.....

(3)

(Total for Question 15 is 5 marks)

1

(a) Complete the tree

Spinner A has 2 red and 3 green sections: red $\frac{2}{5}$, green $\frac{3}{5}$.

Spinner B has 4 red and 1 green (on both branches): red $\frac{4}{5}$, green $\frac{1}{5}$.

2

(b) Use the complement

"At least one green" is the opposite of "both red":

$$1 - \frac{2}{5} \times \frac{4}{5} = 1 - \frac{8}{25} = \frac{17}{25}$$

FINAL ANSWER

(a) $\frac{2}{5}, \frac{3}{5}$; $\frac{4}{5}, \frac{1}{5}$ on both branches (b) $\frac{17}{25}$

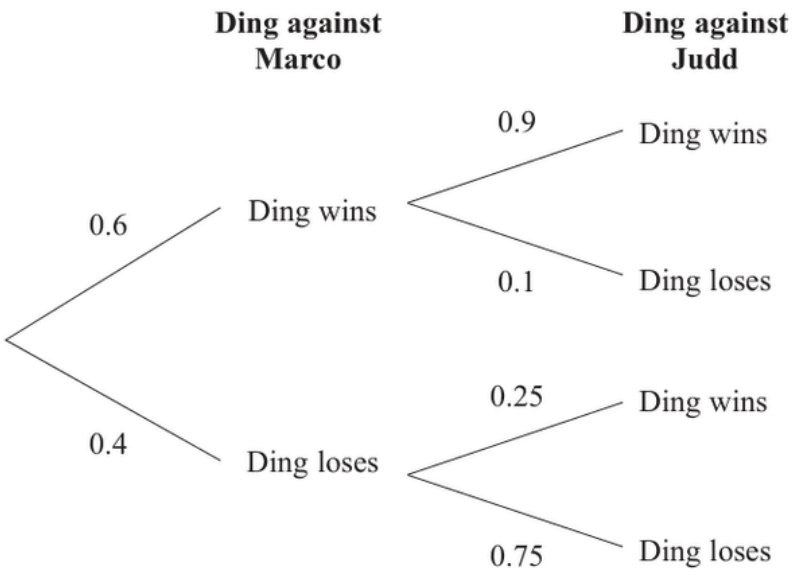
4MA1/1H JANUARY 2021

Question 14

5 marks | Tree Diagrams & Conditional Probability

14 Ding is going to play one game of snooker against each of two of his friends, Marco and Judd.

The probability tree diagram gives information about the probabilities that Ding will win or lose each of these two games.



(a) Work out the probability that Ding will win both games.

.....

(2)

(b) Work out the probability that Ding will win exactly one of the games.

.....

(3)

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2021

1 **(a) Win both games**

Multiply along the "win, win" branch:

$$0.6 \times 0.9 = 0.54$$

2 (b) Win exactly one game

Win then lose, or lose then win:

$$0.6 \times 0.1 + 0.4 \times 0.25 = 0.06 + 0.10 = 0.16$$

FINAL ANSWER

(a) 0.54 (b) 0.16

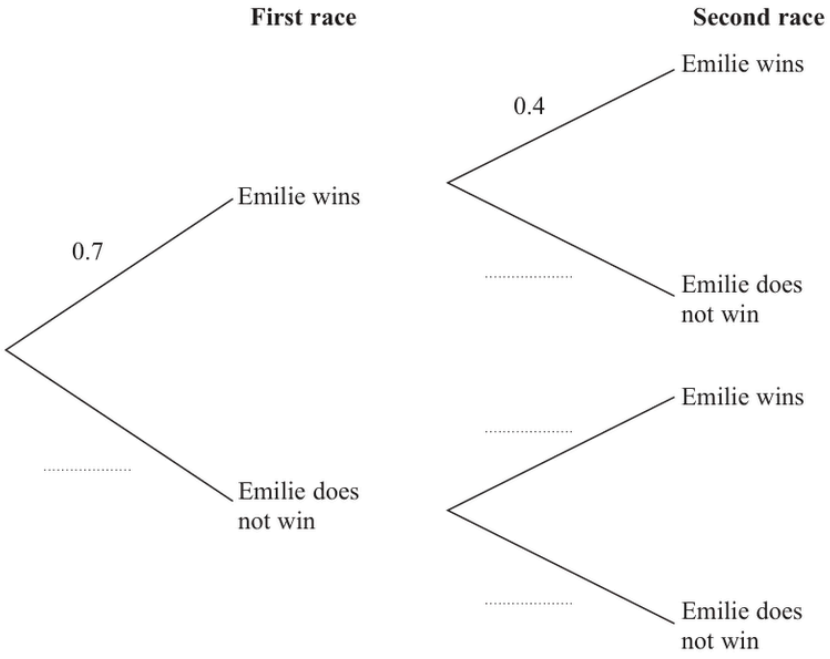
Question 13

8 marks | Tree Diagrams & Conditional Probability

13 Emilie takes part in two races.

The probability that she wins the first race is 0.7
The probability that she wins the second race is 0.4
The outcomes of the two races are independent.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Emilie wins exactly one of the two races.

.....

(3)

Emilie is going to take part in a third race.

If she wins both of the first two races, the probability that she will win the third race is 0.6

If she wins exactly one of the first two races, the probability that she will win the third race is 0.3

(c) Work out the probability that Emilie will win exactly two of the three races.

.....

(3)

(Total for Question 13 is 8 marks)

1 **(a) Complete the tree**

First race: does not win 0.3.

Second race: after a win, does not win 0.6; after not winning, wins 0.4 and does not win 0.6.

2 **(b) Exactly one win**

$$0.7 \times 0.6 + 0.3 \times 0.4 = 0.42 + 0.12 = 0.54$$

3 **(c) Win the first two, lose the third**

$P(\text{win both}) = 0.7 \times 0.4 = 0.28$, then lose the third with $1 - 0.6 = 0.4$:

$$0.28 \times 0.4 = 0.112$$

4 **(c) Exactly one of the first two, then win the third**

$$0.54 \times 0.3 = 0.162$$

5 **(c) Add the cases**

$$0.112 + 0.162 = 0.274$$

✓ **FINAL ANSWER**

(a) 0.3; 0.6; $0.4, 0.6$ (b) 0.54 (c) 0.274

Question 15

8 marks | Tree Diagrams & Conditional Probability

15 Magnus and Garry play 2 games of chess against each other.

The probability that Magnus beats Garry in any game is $\frac{2}{9}$

The probability that any game between Magnus and Garry is drawn is $\frac{4}{9}$

The result of any game is independent of the result of any other game.

(a) Complete the probability tree diagram.



(2)

For each game of chess,

the winner gets 2 points and the loser gets 0 points,
when the game is drawn, each player gets 1 point.

(b) Work out the probability that, after 2 games, Magnus and Garry have the same number of points.

.....

(3)

Magnus and Garry now play a third game of chess.

(c) Work out the probability that, after 3 games, Magnus and Garry have the same number of points.

.....

(3)

(Total for Question 15 is 8 marks)

1 **(a) Complete the tree**

Magnus loses: $1 - \frac{2}{9} - \frac{4}{9} = \frac{3}{9}$.

Every second-game branch repeats $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ (the games are independent).

2 **(b) Equal points after 2 games**

Two draws, or one win and one loss (either order):

$$\left(\frac{4}{9}\right)^2 + 2 \times \frac{2}{9} \times \frac{3}{9} = \frac{16}{81} + \frac{12}{81} = \frac{28}{81}$$

3 **(c) Equal points after 3 games**

Each player needs 3 of the 6 points: three draws, or one win, one draw and one loss ($3! = 6$ orders).

4 **(c) Add the cases**

$$\left(\frac{4}{9}\right)^3 + 6 \times \frac{2}{9} \times \frac{4}{9} \times \frac{3}{9} = \frac{64}{729} + \frac{144}{729} = \frac{208}{729}$$

✓ **FINAL ANSWER**

(a) $\frac{3}{9}$; $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ on every branch (b) $\frac{28}{81}$ (c) $\frac{208}{729}$

4MA1/2H NOVEMBER 2021

Question 16

3 marks | Tree Diagrams & Conditional Probability

16 A box contains 15 counters.

There are 4 red counters, 5 green counters and the rest are yellow counters.

Niklas takes at random a counter from the box and writes down the colour of his counter.
He then puts the counter back into the box.

Sasha then takes at random a counter from the box and writes down the colour of her counter.

Work out the probability that the counters taken by Niklas and Sasha both have the same colour.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2021

1 Count the colours

4 red, 5 green and $15 - 9 = 6$ yellow. The counter is replaced, so the second pick has the same probabilities.

2 Both the same colour

$$\left(\frac{4}{15}\right)^2 + \left(\frac{5}{15}\right)^2 + \left(\frac{6}{15}\right)^2 = \frac{16 + 25 + 36}{225} = \frac{77}{225}$$

FINAL ANSWER

$$\frac{77}{225}$$

Question 13

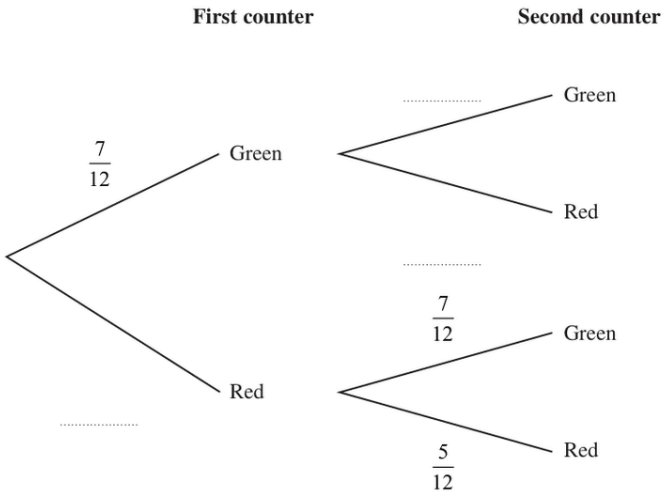
7 marks | Tree Diagrams & Conditional Probability

13 Hector has a bag that contains 12 counters.
There are 7 green counters and 5 red counters in the bag.

Hector takes at random a counter from the bag.
He looks at the counter and puts the counter back into the bag.

Hector then takes at random a second counter from the bag.
He looks at the counter and puts the counter back into the bag.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters are red.

.....

(2)

Meghan has a jar containing 15 counters.
There are only blue counters, green counters and red counters in the jar.

Hector is going to take at random one of the counters from his bag of 12 counters.
He will look at the counter and put the counter back into the bag.

Hector is then going to take at random a second counter from his bag.
He will look at the counter and put the counter back into the bag.

Meghan is then going to take at random one of the counters from her jar of counters.
She will look at the counter and put the counter back into the jar.

The probability that the 3 counters each have a different colour is $\frac{7}{24}$

(c) Work out how many blue counters there are in the jar.

.....

(3)

(Total for Question 13 is 7 marks)

1 (a) Complete the tree (counter is replaced)

First counter red: $\frac{5}{12}$.

After green, second counter: green $\frac{7}{12}$, red $\frac{5}{12}$.

2 (b) Both red

$$\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$$

3 (c) What "three different colours" needs

Hector only has green and red, so he must take one green and one red (either order), and Meghan must take blue.

$$P(\text{one green, one red}) = 2 \times \frac{7}{12} \times \frac{5}{12} = \frac{35}{72}$$

4 (c) Solve for the number of blue counters

Let b be the number of blue counters out of 15.

$$\frac{35}{72} \times \frac{b}{15} = \frac{7}{24} \Rightarrow \frac{b}{15} = \frac{7}{24} \times \frac{72}{35} = \frac{3}{5} \Rightarrow b = 9$$

FINAL ANSWER

(a) $\frac{5}{12}$; $\frac{7}{12}$, $\frac{5}{12}$ (b) $\frac{25}{144}$ (c) 9

4MA1/1H JANUARY 2022

Question 14

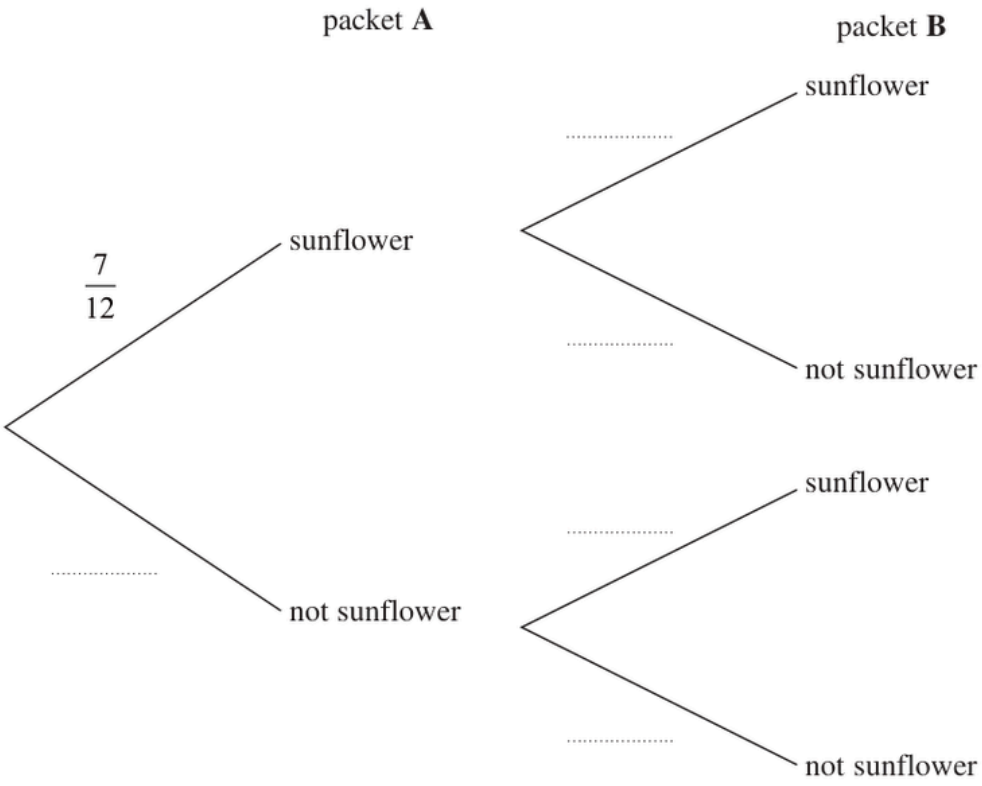
4 marks | Tree Diagrams & Conditional Probability

14 Aika has 2 packets of seeds, packet **A** and packet **B**

There are 12 seeds in packet **A** and 7 of these are sunflower seeds.
There are 15 seeds in packet **B** and 8 of these are sunflower seeds.

Aika is going to take at random a seed from packet **A** and a seed from packet **B**

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that Aika will take two sunflower seeds.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2022

1 **(a) Complete the tree**

Packet A: not sunflower $1 - \frac{7}{12} = \frac{5}{12}$.

Packet B (on both branches): sunflower $\frac{8}{15}$, not sunflower $\frac{7}{15}$.

2 **(b) Two sunflower seeds**

$$\frac{7}{12} \times \frac{8}{15} = \frac{56}{180} = \frac{14}{45}$$

✓ **FINAL ANSWER**

(a) $\frac{5}{12}$; $\frac{8}{15}$, $\frac{7}{15}$ on both branches (b) $\frac{14}{45}$

4MA1/1H JUNE 2022

Question 12

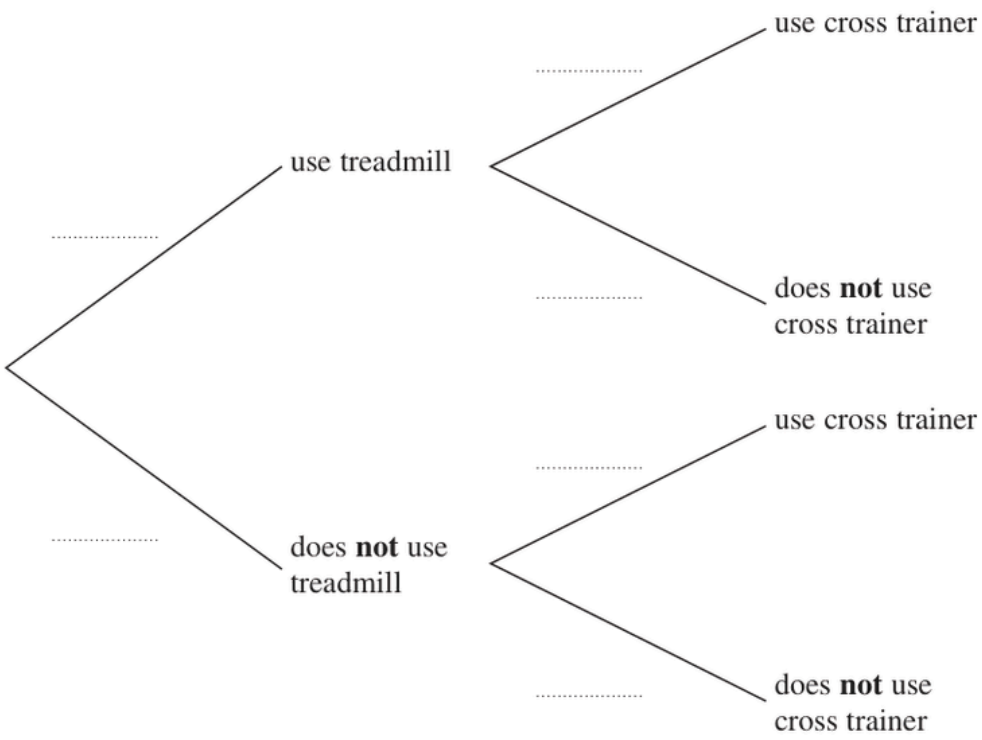
4 marks | Tree Diagrams & Conditional Probability

12 Rudolf goes to the gym.

The probability that he will use the treadmill is 0.8

When he uses the treadmill, the probability that he will use the cross trainer is 0.3
When he does **not** use the treadmill, the probability that he will use the cross trainer is 0.6

(a) Complete the probability tree diagram for this information.



(2)

(b) Work out the probability that Rudolf uses both the treadmill and the cross trainer.

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H June 2022

1 **(a) Complete the tree**

Treadmill: use 0.8, not use 0.2.

After using the treadmill: cross trainer 0.3, not 0.7.

After not using the treadmill: cross trainer 0.6, not 0.4.

2 (b) Treadmill and cross trainer

$$0.8 \times 0.3 = 0.24$$

FINAL ANSWER

(a) $0.8, 0.2$; $0.3, 0.7$; $0.6, 0.4$ (b) 0.24

4MA1/2HR JUNE 2022

Question 14

3 marks | Tree Diagrams & Conditional Probability

14 Ciara throws **four** fair six-sided dice.

The faces of each dice are labelled with the numbers 1, 2, 3, 4, 5, 6

Work out the probability that at least one of the dice lands on an even number.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2022

1 Use the complement

"At least one even" is the opposite of "all four dice are odd". Each die is odd with probability $\frac{1}{2}$.

2 All four dice odd

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

3 At least one even

$$1 - \frac{1}{16} = \frac{15}{16}$$

FINAL ANSWER

$$\frac{15}{16}$$

Question 12

5 marks | Tree Diagrams & Conditional Probability

12 Moeen has a biased 6-sided dice.

The table gives information about the probability that, when the dice is thrown, it will land on each number.

Number	1	2	3	4	5	6
Probability	x	0.15	0.5	y	0.13	0.03

(a) Show that $x + y = 0.19$

(2)

Given that $3x - y = 0.09$

and $x + y = 0.19$

(b) work out the value of x and the value of y
Show clear algebraic working.

$x =$ .....

$y =$ .....

(3)

(Total for Question 12 is 5 marks)

1 **(a) Probabilities add to 1**

$$x + 0.15 + 0.5 + y + 0.13 + 0.03 = 1 \Rightarrow x + y + 0.81 = 1 \Rightarrow x + y = 0.19$$

2 **(b) Add the two equations**

$$(3x - y) + (x + y) = 0.09 + 0.19 \Rightarrow 4x = 0.28 \Rightarrow x = 0.07$$

3 **(b) Find y**

$$y = 0.19 - 0.07 = 0.12$$

✓ **FINAL ANSWER**

(b) $x = 0.07,\ y = 0.12$

4MA1/2H JUNE 2023

Question 13

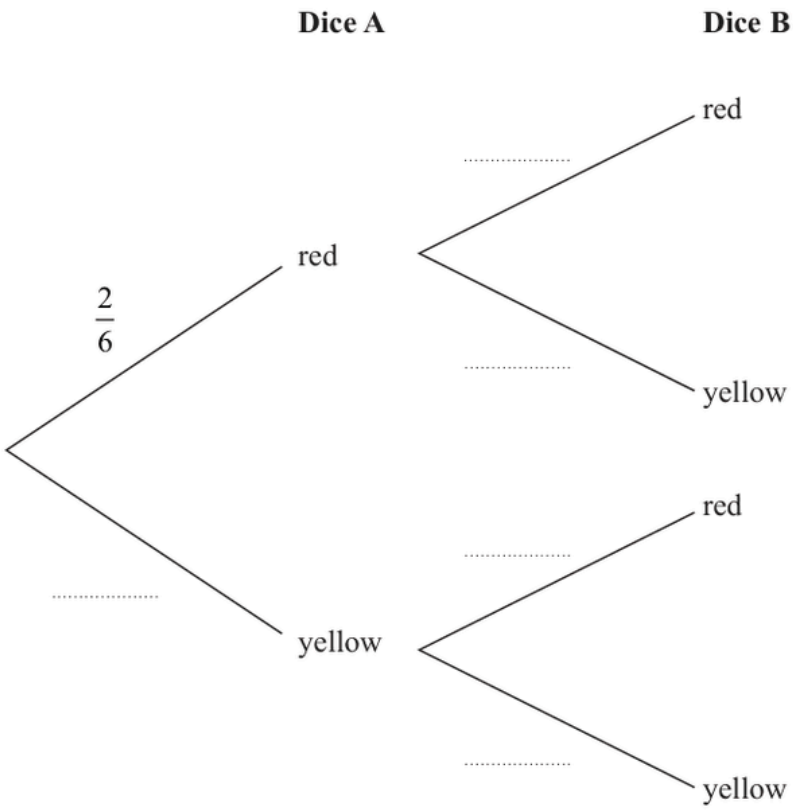
4 marks | Tree Diagrams & Conditional Probability

13 Narin has two fair 6-sided dice.

Dice **A** has 2 red faces and 4 yellow faces.
Dice **B** has 1 red face and 5 yellow faces.

Narin is going to throw each dice once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both dice land on yellow.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H June 2023

1 **(a) Complete the tree**

Dice A: yellow $\frac{4}{6}$.

Dice B (on both branches): red $\frac{1}{6}$, yellow $\frac{5}{6}$.

2 **(b) Both yellow**

$$\frac{4}{6} \times \frac{5}{6} = \frac{20}{36} = \frac{5}{9}$$

✓ **FINAL ANSWER**

(a) $\frac{4}{6}$; $\frac{1}{6}, \frac{5}{6}$ on both branches (b) $\frac{5}{9}$

Question 13

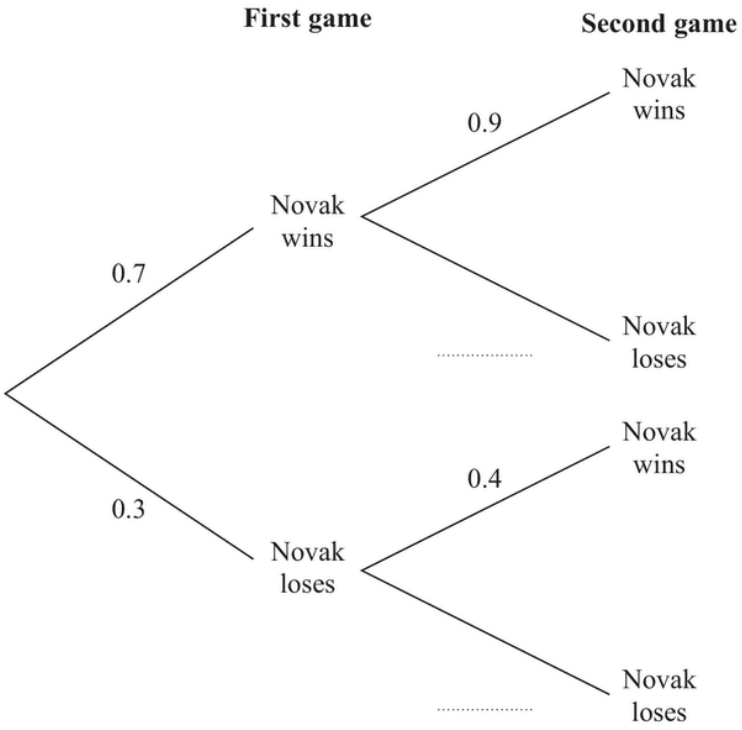
3 marks | Tree Diagrams & Conditional Probability

13 Novak is going to play two games of tennis.

The probability that he wins the first game is 0.7

If he wins the first game, the probability that he wins the second game is 0.9
If he loses the first game, the probability that he wins the second game is 0.4

(a) Complete the probability tree diagram.



(1)

(b) Work out the probability that Novak wins both games of tennis.

.....

(2)

(Total for Question 13 is 3 marks)

1 (a) Complete the tree

After a win: loses $1 - 0.9 = 0.1$. After a loss: loses $1 - 0.4 = 0.6$.

2 (b) Wins both games

$$0.7 \times 0.9 = 0.63$$

FINAL ANSWER

(a) 0.1, 0.6 (b) 0.63

4MA1/2H JUNE 2023

Question 20

3 marks | Tree Diagrams & Conditional Probability

20 There are 12 counters in a bag.

3 of the counters are red
9 of the counters are green

Ameya, Jack and Ella each take at random one counter from the bag.

Work out the probability that at least one red counter is still in the bag.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2023

1 There are 3 red counters

There are 3 red counters.

2 At least one red counter

At least one red counter is still in the bag unless all 3 red counters are taken.

3 Use the complement

So use the complement:

$$P(\text{all 3 red are taken}) = \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$$

$$= \frac{6}{1320}$$

$$= \frac{1}{220}$$

4 Evaluate fraction

Therefore,

$$P(\text{at least one red is still in the bag}) = 1 - \frac{1}{220}$$

$$= \frac{219}{220}$$

FINAL ANSWER

$$\frac{219}{220}$$

4MA1/2H NOVEMBER 2023

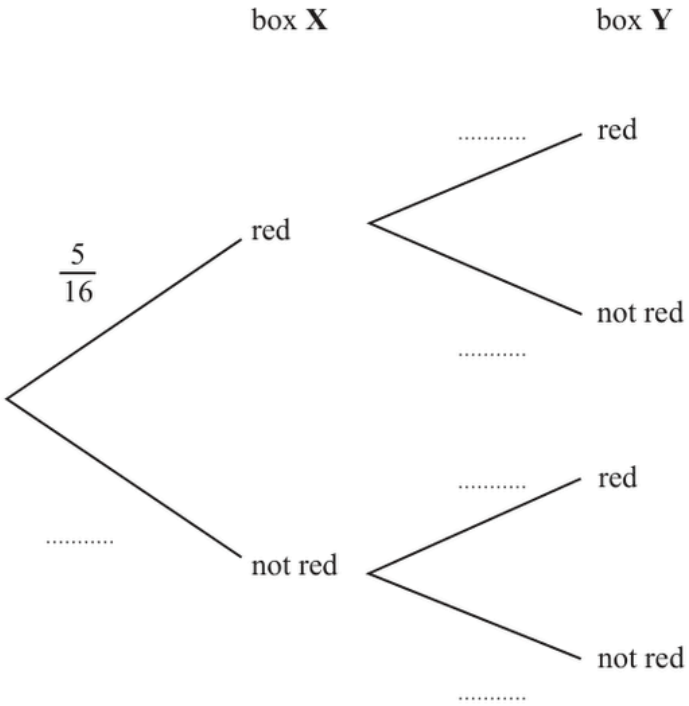
Question 11

5 marks | Tree Diagrams & Conditional Probability

11 Sid has 2 boxes of crayons, box **X** and box **Y**

5 of the 16 crayons in box **X** are red.
7 of the 20 crayons in box **Y** are red.
Sid takes at random one crayon from box **X** and one crayon from box **Y**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Sid takes two crayons that are red or two crayons that are not red.

.....

(3)

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2023

1 **(a) Complete the tree**

Box X: not red $\frac{11}{16}$.

Box Y (on both branches): red $\frac{7}{20}$, not red $\frac{13}{20}$.

2 **(b) Both red or both not red**

$$\frac{5}{16} \times \frac{7}{20} + \frac{11}{16} \times \frac{13}{20} = \frac{35}{320} + \frac{143}{320} = \frac{178}{320} = \frac{89}{160}$$

✓ **FINAL ANSWER**

(a) $\frac{11}{16}$; $\frac{7}{20}$, $\frac{13}{20}$ on both branches (b) $\frac{89}{160}$

4MA1/1H MAY 2024

Question 13

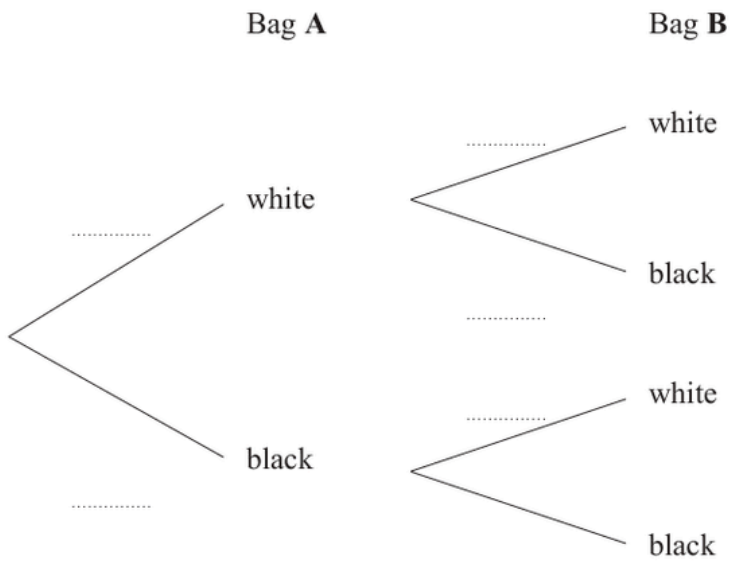
5 marks | Tree Diagrams & Conditional Probability

13 Harman has two bags of beads.

In bag **A**, there are 3 white beads and 7 black beads.
In bag **B**, there are 5 white beads and 4 black beads.

Harman takes at random a bead from bag **A** and a bead from bag **B**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Harman takes two beads of the same colour.

.....

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2024

1 **(a) Complete the tree**

Bag A: white $\frac{3}{10}$, black $\frac{7}{10}$.
Bag B (on both branches): white $\frac{5}{9}$, black $\frac{4}{9}$.

2 **(b) Same colour = both white or both black**

$$\frac{3}{10} \times \frac{5}{9} + \frac{7}{10} \times \frac{4}{9} = \frac{15}{90} + \frac{28}{90} = \frac{43}{90}$$

FINAL ANSWER

(a) $\frac{3}{10}, \frac{7}{10}$; $\frac{5}{9}, \frac{4}{9}$ on both branches (b) $\frac{43}{90}$

4MA1/1H MAY 2024

Question 16

4 marks | Tree Diagrams & Conditional Probability

16 There are 20 sweets in a box.

15 of the sweets are red
5 of the sweets are yellow

Fred takes at random 3 sweets from the box.

Work out the probability that Fred takes at least one sweet of each colour from the box.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2024

1 Use the complement

"At least one of each colour" fails only if all three sweets are red or all three are yellow.

2 All red, all yellow (no replacement)

$$P(RRR) = \frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} = \frac{2730}{6840}, \qquad P(YYY) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{60}{6840}$$

3 At least one of each

$$1 - \frac{2730 + 60}{6840} = \frac{4050}{6840} = \frac{45}{76}$$

FINAL ANSWER

$$\frac{45}{76}$$

4MA1/1H NOVEMBER 2024

Question 13

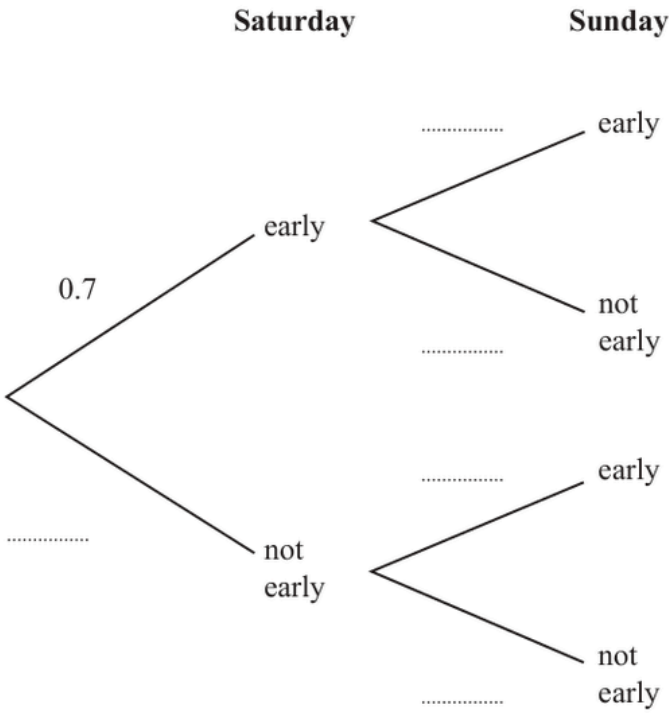
4 marks | Tree Diagrams & Conditional Probability

13 The probability that Thomas gets to work early on Saturday is 0.7

If Thomas gets to work early on Saturday, the probability that he will get to work early on Sunday is 0.9

If Thomas does **not** get to work early on Saturday, the probability that he will get to work early on Sunday is 0.6

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that Thomas gets to work early on both Saturday and Sunday.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2024

1 **(a) Complete the tree**

Saturday: not early 0.3.

After early: Sunday early 0.9, not early 0.1. After not early: Sunday early 0.6, not early 0.4.

2 (b) Early on both days

$$0.7 \times 0.9 = 0.63$$

FINAL ANSWER

(a) 0.3; $0.9, 0.1$; $0.6, 0.4$ (b) 0.63

Question 11

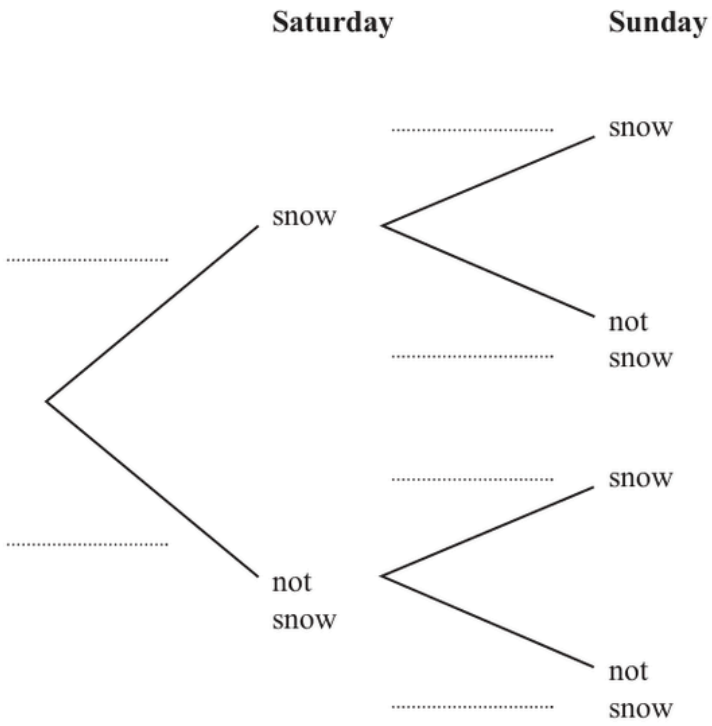
4 marks | Tree Diagrams & Conditional Probability

11 The probability that it will snow on Saturday is 0.2

If it snows on Saturday, the probability that it will snow on Sunday is 0.35

If it does not snow on Saturday, the probability that it will snow on Sunday is 0.4

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that it will snow on both Saturday and Sunday.

.....

(2)

(Total for Question 11 is 4 marks)

1 **The missing branch probabilities are**

The missing branch probabilities are complements.

$$P(\text{not snow on Saturday}) = 0.8$$

$$P(\text{not snow on Sunday} \mid \text{snow Saturday}) = 0.65$$

$$P(\text{not snow on Sunday} \mid \text{not snow Saturday}) = 0.6$$

2 **Calculate value**

For snow on both days,

$$0.2 \times 0.35 = 0.07$$

✓ **FINAL ANSWER**

tree labels $0.2, 0.8, 0.35, 0.65, 0.4, 0.6$; probability 0.07.

4WM1/01 MAY 2025

Question 12

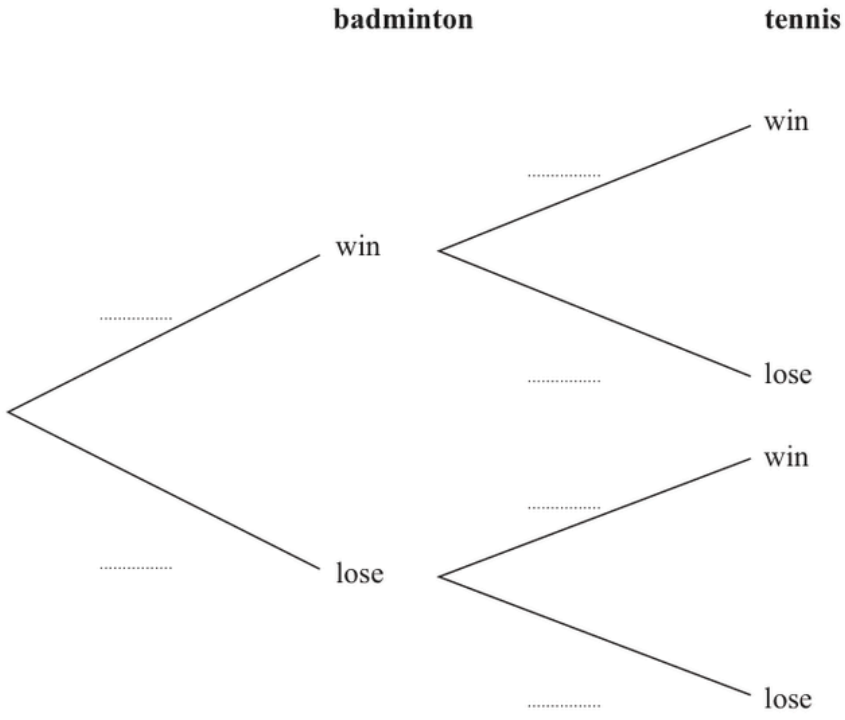
4 marks | Tree Diagrams & Conditional Probability

12 Ricardo is going to play one game of badminton and one game of tennis.

He can either win or lose each game.

The probability that he will win the game of badminton is 0.7
The probability that he will win the game of tennis is 0.4

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Ricardo loses both games.

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 May 2025

1 **Calculate probability**

For badminton,

$$P(\text{win}) = 0.7, \qquad P(\text{lose}) = 0.3$$

2 **Calculate probability**

For tennis,

$$P(\text{win}) = 0.4, \qquad P(\text{lose}) = 0.6$$

3 **The probability of losing both**

The probability of losing both games is

$$0.3 \times 0.6 = 0.18$$

✓ **FINAL ANSWER**

Tennis branches are 0.4 and 0.6 on each branch; probability of losing both is 0.18.

4MA1/1H JUNE 2025

Question 14

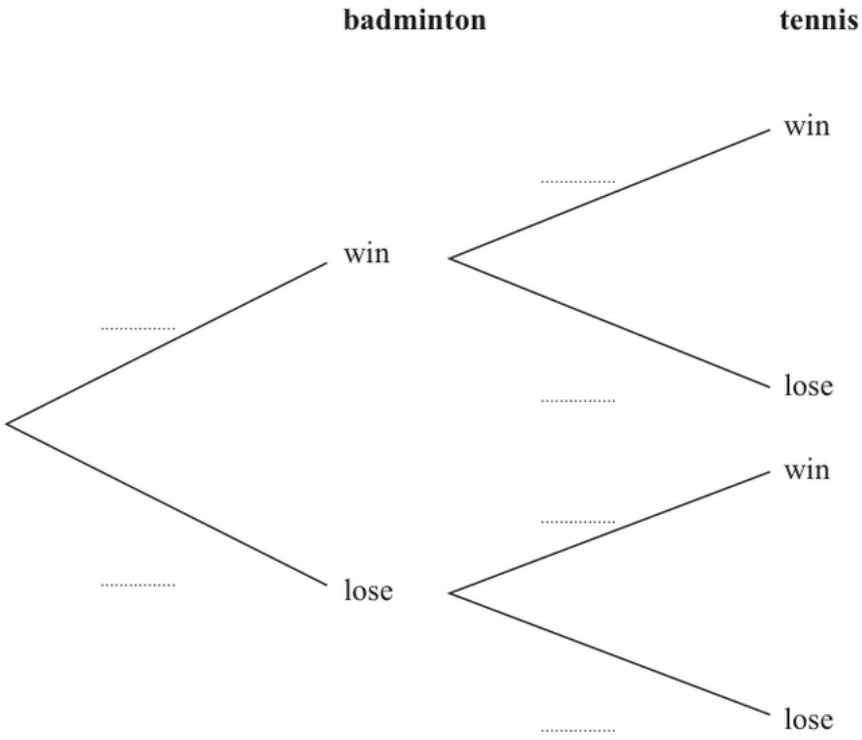
4 marks | Tree Diagrams & Conditional Probability

14 Ricardo is going to play one game of badminton and one game of tennis.

He can either win or lose each game.

The probability that he will win the game of badminton is 0.7
The probability that he will win the game of tennis is 0.4

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Ricardo loses both games.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H June 2025

1 **(a) Complete the tree**

Badminton: win 0.7, lose 0.3.

Tennis (on both branches): win 0.4, lose 0.6.

2 **(b) Lose both**

$$0.3 \times 0.6 = 0.18$$

✓ **FINAL ANSWER**

(a) $0.7, 0.3$; $0.4, 0.6$ on both branches (b) 0.18

Question 14

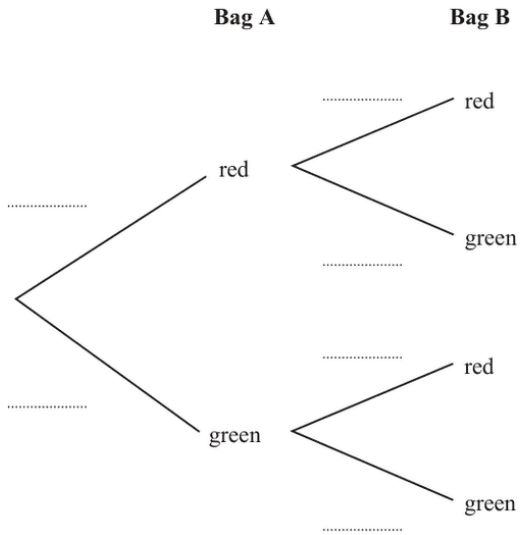
7 marks | Tree Diagrams & Conditional Probability

14 Sara has two bags, **A** and **B**

In bag **A**, there are only 5 red beads and 4 green beads.
In bag **B**, there are only 7 red beads and 3 green beads.

Sara takes at random a bead from bag **A**
She then takes at random a bead from bag **B**

(a) Use this information to complete the probability tree diagram.



(2)

(b) Work out the probability that Sara takes two red beads.

.....

(2)

Sara puts the beads back into their original bags.

Sara also has a box of beads.
In the box, there are only red beads and green beads.

When a bead is taken at random from the box, the probability that it is

a green bead is $\frac{2}{11}$

Sara takes at random a bead from bag **A**
She then takes at random a bead from bag **B**
She then takes at random a bead from the box.

(c) Work out the probability that Sara takes more red beads than green beads.

.....

(3)

(Total for Question 14 is 7 marks)

1 **(a) Complete the tree**

Bag A: red $\frac{5}{9}$, green $\frac{4}{9}$.

Bag B (on both branches): red $\frac{7}{10}$, green $\frac{3}{10}$.

2 **(b) Two red beads**

$$\frac{5}{9} \times \frac{7}{10} = \frac{35}{90} = \frac{7}{18}$$

3 **(c) More red than green with three beads**

She needs at least two reds: RRR, RRG, RGR or GRR. The box gives red $\frac{9}{11}$, green $\frac{2}{11}$.

4 **(c) Add the four cases**

$$\frac{5\cdot7\cdot9 + 5\cdot7\cdot2 + 5\cdot3\cdot9 + 4\cdot7\cdot9}{9\cdot10\cdot11} = \frac{315+70+135+252}{990} = \frac{772}{990} = \frac{386}{495}$$

✓ **FINAL ANSWER**

(a) $\frac{5}{9}, \frac{4}{9}; \frac{7}{10}, \frac{3}{10}$ on both branches (b) $\frac{7}{18}$ (c) $\frac{386}{495}$

4WM1/01 NOVEMBER 2025

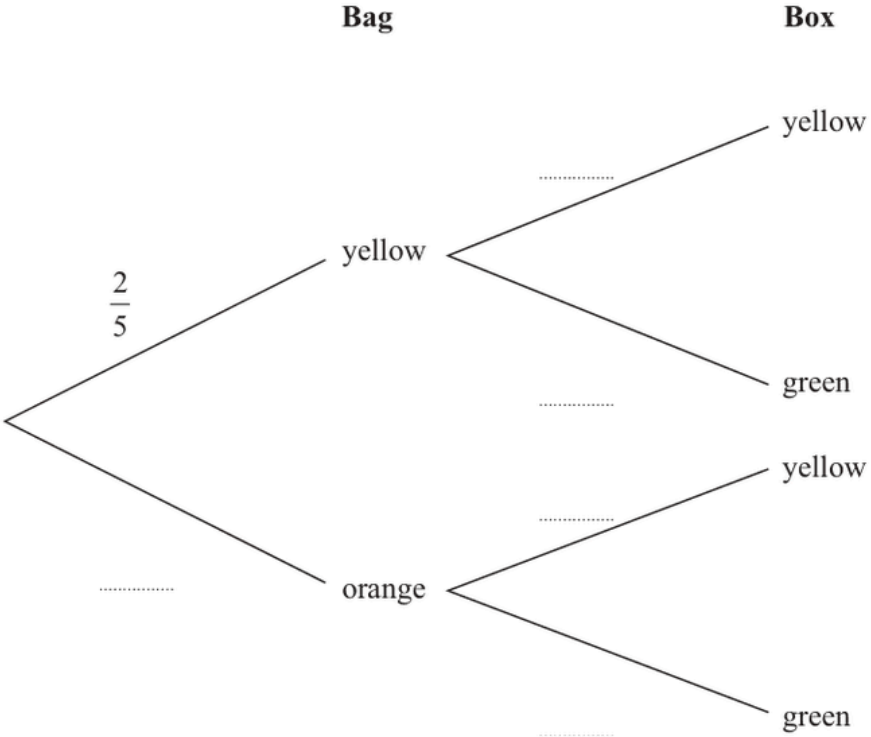
Question 12

5 marks | Tree Diagrams & Conditional Probability

12 In a bag, there are only 2 yellow beads and 3 orange beads.
In a box, there are only 3 yellow beads and 5 green beads.

Chi takes at random a bead from the bag and a bead from the box.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Chi takes two beads of different colours.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4WM1/01 November 2025

1 **Bag 1**

Bag 1:

$$P(Y) = \frac{2}{5}, \qquad P(O) = \frac{3}{5}$$

2 **Bag 2**

Bag 2:

$$P(Y) = \frac{3}{8}, \qquad P(G) = \frac{5}{8}$$

---

3 **The only way to get**

The only way to get the same colour is yellow then yellow.

$$P(\text{different}) = 1 - \frac{2}{5} \times \frac{3}{8}$$

$$= 1 - \frac{3}{20} = \frac{17}{20}$$

✓ **FINAL ANSWER**

$\frac{17}{20}$.

4MA1/1H NOVEMBER 2025

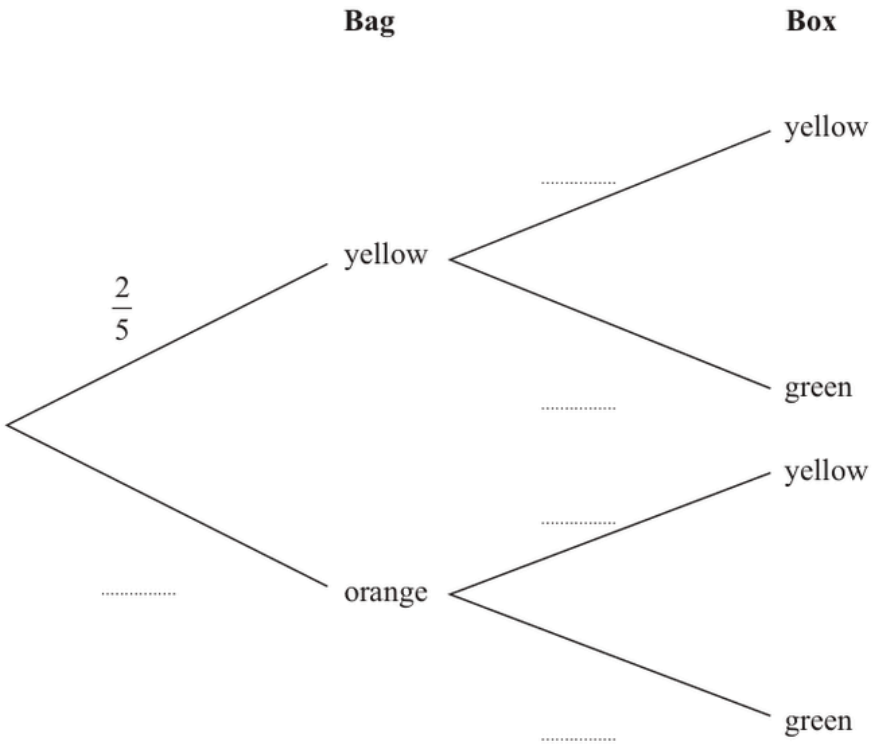
Question 13

5 marks | Tree Diagrams & Conditional Probability

13 In a bag, there are only 2 yellow beads and 3 orange beads.
In a box, there are only 3 yellow beads and 5 green beads.

Chi takes at random a bead from the bag and a bead from the box.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Chi takes two beads of different colours.

.....

(3)

(Total for Question 13 is 5 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2025

1 **(a) Complete the tree**

Bag: orange $\frac{3}{5}$.

Box (on both branches): yellow $\frac{3}{8}$, green $\frac{5}{8}$.

2 **(b) Use the complement**

The only way to get the same colour is yellow from both, so

$$P(\text{different}) = 1 - \frac{2}{5} \times \frac{3}{8} = 1 - \frac{6}{40} = \frac{17}{20}$$

✓ **FINAL ANSWER**

(a) $\frac{3}{5}$; $\frac{3}{8}, \frac{5}{8}$ on both branches (b) $\frac{17}{20}$

4MA1/1H NOVEMBER 2025

Question 22

3 marks | Tree Diagrams & Conditional Probability

22 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

4WM1/01 NOVEMBER 2025

Question 23

3 marks | Tree Diagrams & Conditional Probability

23 There are 15 buttons in a box.

3 of the buttons are red
2 of the buttons are pink
10 of the buttons are blue

Pete takes at random three buttons from the box.

Work out the probability that there is still at least one pink button in the box.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4WM1/01 November 2025

1 There are 2 pink buttons

There are 2 pink buttons.

2 At least one pink button

At least one pink button is still in the box unless both pink buttons are taken.

3 Use the complement

So use the complement.

4 The probability that both pink

The probability that both pink buttons are taken is

$$\frac{\binom{2}{2}\binom{13}{1}}{\binom{15}{3}}$$

$$= \frac{13}{455}$$

$$= \frac{1}{35}$$

5 **Evaluate fraction**

Therefore,

$$P(\text{at least one pink is still in the box}) = 1 - \frac{1}{35}$$

$$= \frac{34}{35}$$

✓ **FINAL ANSWER**

$\frac{34}{35}$

4MA1/1H JUNE 2026

Question 13

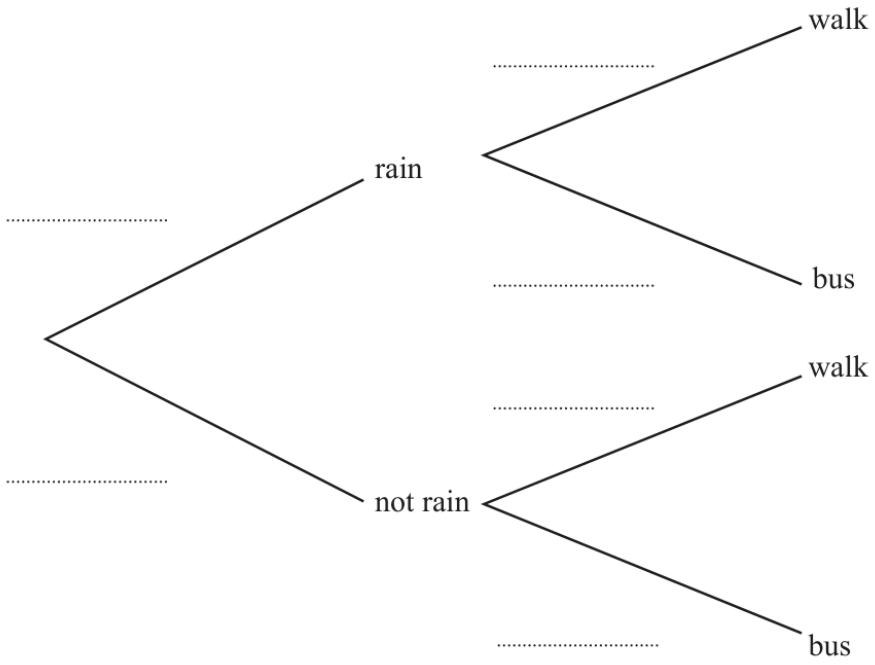
4 marks | Tree Diagrams & Conditional Probability

13 On any school day, Paulo either walks to school or travels by bus to school.

The probability that it rains on any school day is 0.15

If it rains on a school day, the probability that Paulo walks to school is 0.2
If it does not rain on a school day, the probability that Paulo walks to school is 0.75

(a) Complete the probability tree diagram for this information.



(2)

(b) Work out the probability that on a school day, it rains and Paulo walks to school.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H June 2026

1 **Part (a) | Step 1**

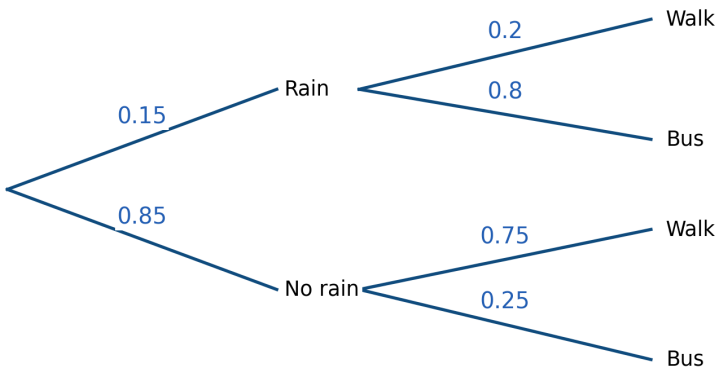
First split: rain 0.15, not rain 0.85.

Complementary branches sum to 1.

2 Part (a) | Step 2

After rain: walk 0.20, bus 0.80. After not rain: walk 0.75, bus 0.25.

Use the stated conditional probabilities and their complements on the corresponding branches.



3 Part (b) | Step 1

$$P(\text{rain and walk}) = P(\text{rain})P(\text{walk} \mid \text{rain}) = 0.15 \times 0.2 = 0.03.$$

Multiply the probabilities along the rain-then-walk path; independence is not assumed.

FINAL ANSWER

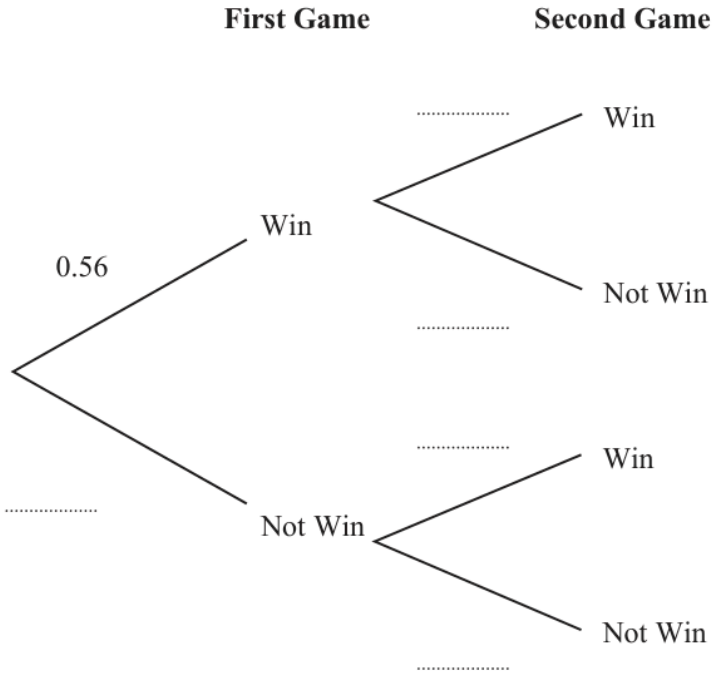
(a) Rain/not rain: $0.15, 0.85$; walk/bus after rain: $0.2, 0.8$; walk/bus after no rain: $0.75, 0.25$. (b) 0.03

Question 13

4 marks | Tree Diagrams & Conditional Probability

13 A team plays two games of football.
For each game, the probability that the team wins is 0.56

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that the team wins both games.

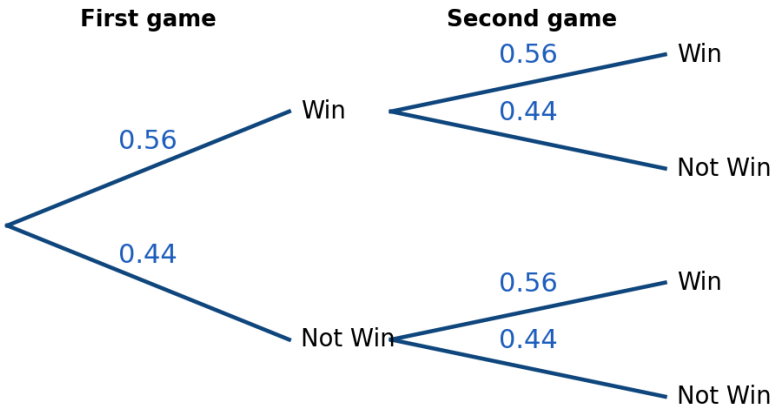
.....

(2)

(Total for Question 13 is 4 marks)

1 (a) Branches add to 1

Not win $= 1 - 0.56 = 0.44$ on every pair of branches.



2 (b) Win and win

$$0.56 \times 0.56 = 0.3136$$

FINAL ANSWER

(a) see diagram (b) 0.3136

Question 14

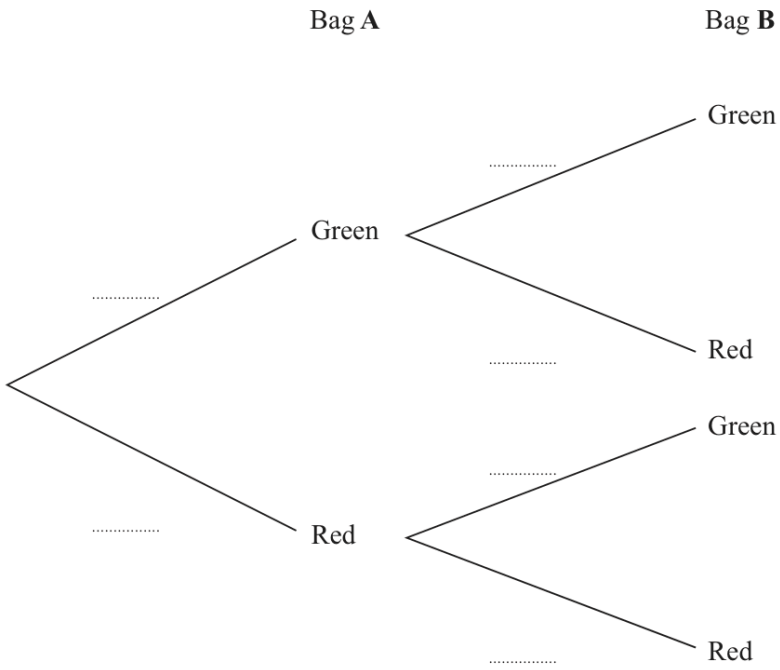
8 marks | Tree Diagrams & Conditional Probability

14 Pras has 2 bags of counters, bag **A** and bag **B**

In bag **A**, there are 4 green counters and 3 red counters.
In bag **B**, there are 5 green counters and 1 red counter.

Pras is going to take at random a counter from bag **A** and a counter from bag **B**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Pras will take two green counters.

.....

(2)

Pras also has a third bag of counters, bag **C**
In bag **C**, there are only green counters and red counters.
All the original counters are back in bag **A** and bag **B**

Pras is going to take at random a counter from bag **A**, a counter from bag **B** and a counter from bag **C**

The probability that Pras will take three red counters is $\frac{3}{70}$

(c) Work out the probability that Pras will take 1 green counter and 2 red counters.

.....

(4)

(Total for Question 14 is 8 marks)

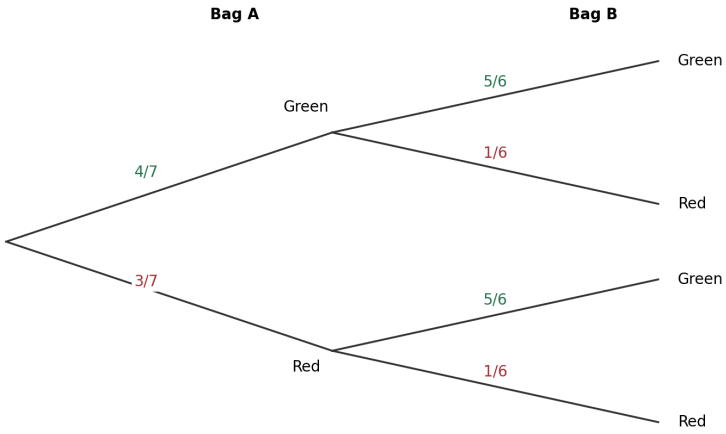
1 **Part (a) | Step 1**

From the start, label the Bag A branches $P(G_A) = 4/7$ and $P(R_A) = 3/7$.
Bag A has 7 counters in total.

2 **Part (a) | Step 2**

From each Bag A outcome, label the Bag B green branch $5/6$ and red branch $1/6$.
Bag B has 6 counters and is unaffected by the result from bag A.

Q14(a): Completed probability tree



3 **Part (b) | Step 1**

$$P(G_A G_B) = \frac{4}{7} \times \frac{5}{6} = \frac{20}{42} = \frac{10}{21}.$$

Both events must occur, so multiply along the green-green path.

4 **Part (c) | Step 1**

Let $r = P(R_C)$. Then

$$\frac{3}{7} \times \frac{1}{6} \times r = \frac{3}{70} \quad \Rightarrow \quad r = \frac{3}{5}, \qquad P(G_C) = \frac{2}{5}.$$

Use the given probability of three reds to find the bag C probabilities.

5 **Part (c) | Step 2**

Exactly one green means the disjoint outcomes $G_A R_B R_C$, $R_A G_B R_C$, or $R_A R_B G_C$.
The single green counter can come from A, B or C.

6 **Part (c) | Step 3**

$$P = \frac{4}{7}\frac{1}{6}\frac{3}{5} + \frac{3}{7}\frac{5}{6}\frac{3}{5} + \frac{3}{7}\frac{1}{6}\frac{2}{5} = \frac{2}{35} + \frac{3}{14} + \frac{1}{35} = \frac{3}{10}.$$

Multiply within each outcome and add the three mutually exclusive probabilities.

FINAL ANSWER

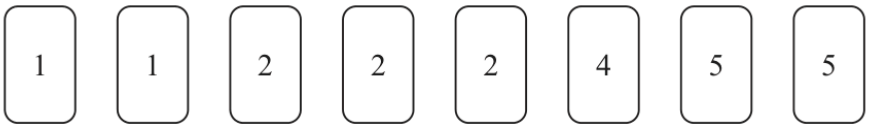
(a) Branch labels from top to bottom: $4/7, 3/7$ for A; $5/6, 1/6$ for B after either A result. (b) $\frac{10}{21}$. (c) $\frac{3}{10}$.

4MA1/1HR JUNE 2026

Question 18

3 marks | Tree Diagrams & Conditional Probability

18 Here are 8 cards.
There is a number on each card.



Gordon puts the 8 cards in a bag.
He takes at random a card from the bag and does not replace it.
Gordon then takes at random a second card from the bag.

Calculate the probability that the number on the second card is 3 more than the number on the first card.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2026

1 Solution | Step 1

The only possible ordered number pairs are $(1, 4)$ and $(2, 5)$.
The second number must be exactly 3 more than the first, not merely differ by 3.

2 Solution | Step 2

$$P(1 \text{ then } 4) = \frac{2}{8} \times \frac{1}{7}, \qquad P(2 \text{ then } 5) = \frac{3}{8} \times \frac{2}{7}.$$

There are 7 cards remaining for the second draw; there are two 1s, three 2s, one 4 and two 5s.

3 Solution | Step 3

$$P = \frac{2}{56} + \frac{6}{56} = \frac{8}{56} = \frac{1}{7}.$$

The two ordered events are mutually exclusive, so add their probabilities.

FINAL ANSWER

$\frac{1}{7}$

Question 22

5 marks | Tree Diagrams & Conditional Probability

22 A bag contains x marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 22 is 5 marks)

1 **Pink then yellow, or yellow then pink**

$$2 \times \frac{12}{x} \times \frac{x-12}{x-1} = \frac{18}{47}$$

2 **Cross-multiply**

$$1128(x-12) = 18x(x-1) \Rightarrow 18x^2 - 1146x + 13536 = 0$$

3 **Simplify (÷6) and solve**

$$3x^2 - 191x + 2256 = 0 \Rightarrow (x-48)(3x-47) = 0$$

4 **x must be a whole number**

$$x = 48$$

✓ **FINAL ANSWER**

$x = 48$

4MA1/1H JUNE 2026

Question 23

5 marks | Tree Diagrams & Conditional Probability

23 A bag contains x marbles.
12 of the marbles are pink and the rest are yellow.

Greta takes at random 2 marbles from the bag.

The probability that Greta takes a pink marble and a yellow marble is $\frac{18}{47}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H June 2026

1 Solution | Step 1

$$P(\text{one pink, one yellow}) = \frac{12}{x}\frac{x-12}{x-1} + \frac{x-12}{x}\frac{12}{x-1} = \frac{24(x-12)}{x(x-1)}.$$

Include pink-then-yellow and yellow-then-pink, without replacement.

2 Solution | Step 2

$$\frac{24(x-12)}{x(x-1)} = \frac{18}{47} \implies 1128(x-12) = 18x(x-1).$$

Equate to the given probability and cross-multiply.

3 Solution | Step 3

$$18x^2 - 1146x + 13536 = 0 \implies 3x^2 - 191x + 2256 = 0.$$

Rearrange and divide by 6.

4 Solution | Step 4

$$(x-48)(3x-47) = 0 \implies x = 48 \text{ or } x = \frac{47}{3}.$$

Factorise the quadratic.

5 **Solution | Step 5**

The number of marbles must be an integer, so $x = 48$.

Reject the non-integer root.

✓ **FINAL ANSWER**

$x = 48$

TOPIC

Set Notation & Venn Diagrams

Every past-paper question on this topic, 2020 to 2026.

Question 14

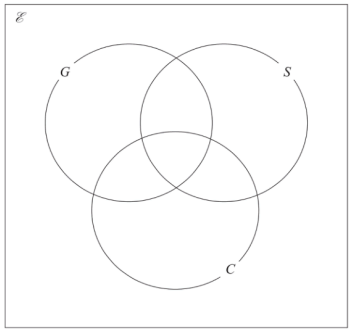
8 marks | Set Notation & Venn Diagrams

14 100 farmers are asked if they have goats (G), sheep (S) or chickens (C) on their farms.

Of these farmers

31 have sheep
53 have chickens
6 have goats, sheep and chickens
11 have sheep and goats
17 have sheep and chickens
18 have goats and chickens
20 do not have any goats, sheep or chickens

(a) Using this information, complete the Venn diagram to show the number of farmers in each appropriate subset.



(3)

(b) Find

(i) $n(G)$

.....

(1)

(ii) $n([G \cup S]')$

.....

(1)

(iii) $n(G' \cap C)$

.....

(1)

One of the farmers who has chickens is chosen at random.

(c) Find the probability that this farmer also has goats.

.....

(2)

(Total for Question 14 is 8 marks)

1 **Place the centre value**

The number with goats, sheep and chickens is 6.

2 **Fill the pair-only regions**

$$G \cap S \text{ only} = 11 - 6 = 5$$

$$S \cap C \text{ only} = 17 - 6 = 11$$

$$G \cap C \text{ only} = 18 - 6 = 12$$

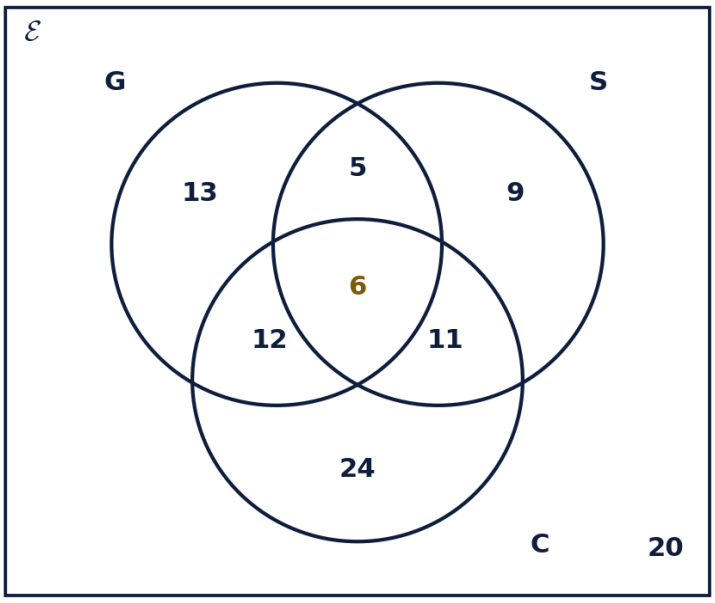
3 **Fill the single-set regions**

$$S \text{ only} = 31 - 5 - 11 - 6 = 9$$

$$C \text{ only} = 53 - 11 - 12 - 6 = 24$$

Since 20 have none, the total inside the circles is 80.

$$G \text{ only} = 80 - (5 + 11 + 12 + 6 + 9 + 24) = 13$$



4 Read the set values

$$n(G) = 13 + 5 + 12 + 6 = 36$$

$$n((G \cup S)') = 24 + 20 = 44$$

$$n(G' \cap C) = 24 + 11 = 35$$

5 Find the conditional probability

Out of the 53 farmers with chickens, 18 also have goats.

$$P(G \mid C) = \frac{18}{53}$$

FINAL ANSWER

Venn regions: G only 13, S only 9, C only 24, $G \cap S$ only 5, $S \cap C$ only 11, $G \cap C$ only 12, all three 6, none 20. Also 36, 44, 35, and $\frac{18}{53}$.

4MA1/1HR JANUARY 2020

Question 4

3 marks | Set Notation & Venn Diagrams

4 $B = \{b, l, u, e\}$

$G = \{g, r, e, y\}$

$W = \{w, h, i, t, e\}$

(a) List all the members of the set

(i) $B \cup G$

(ii) $W \cap G'$

.....

(2)

Serena writes down the statement $B \cap G \cap W = \varnothing$

(b) Is Serena's statement correct?
You must give a reason for your answer.

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR January 2020

1 Calculate value

$$B = \{b, l, u, e\}, \quad G = \{g, r, e, y\}, \quad W = \{w, h, i, t, e\}$$

2 means all the letters that

$B \cup G$ means all the letters that are in B or in G.

$$B \cup G = \{b, l, u, e, g, r, y\}$$

3 Calculate statistic

$W \cap G'$ means letters that are in W, but not in G. Since e is in G, it is not included.

$$W \cap G' = \{w, h, i, t\}$$

4 **Serena is not correct because**

Serena is not correct because e is in all three sets.

$$B \cap G \cap W = \{e\}$$

✓ **FINAL ANSWER**

(a)(i) $\{b, l, u, e, g, r, y\}$, (a)(ii) $\{w, h, i, t\}$. Serena is incorrect.

Question 17

6 marks | Set Notation & Venn Diagrams

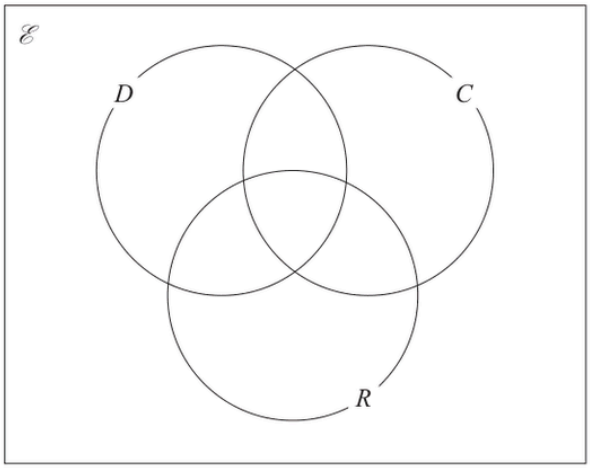
17 Some students in a school were asked the following question.

"Do you have a dog (D), a cat (C) or a rabbit (R)?"

Of these students

28 have a dog
18 have a cat
20 have a rabbit
8 have both a cat and a rabbit
9 have both a dog and a rabbit
x have both a dog and a cat
6 have a dog, a cat and a rabbit
5 have not got a dog or a cat or a rabbit

(a) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.
Give the numbers in terms of x where necessary.



(3)

Given that a total of 50 students answered the question,

(b) work out the value of x.

$x =$ .....

(2)

(c) Find $n(C' \cap D')$

.....

(1)

(Total for Question 17 is 6 marks)

1 **Let the sets dogs**

Let the sets be D for dogs, C for cats, and R for rabbits.

2 **The number in all three**

The number in all three sets is 6.

$$D \cap R \text{ only} = 9 - 6 = 3$$

$$C \cap R \text{ only} = 8 - 6 = 2$$

$$D \cap C \text{ only} = x - 6$$

3 **Calculate value**

So the remaining one-set regions are

$$D \text{ only} = 25 - x, \quad C \text{ only} = 16 - x, \quad R \text{ only} = 9$$

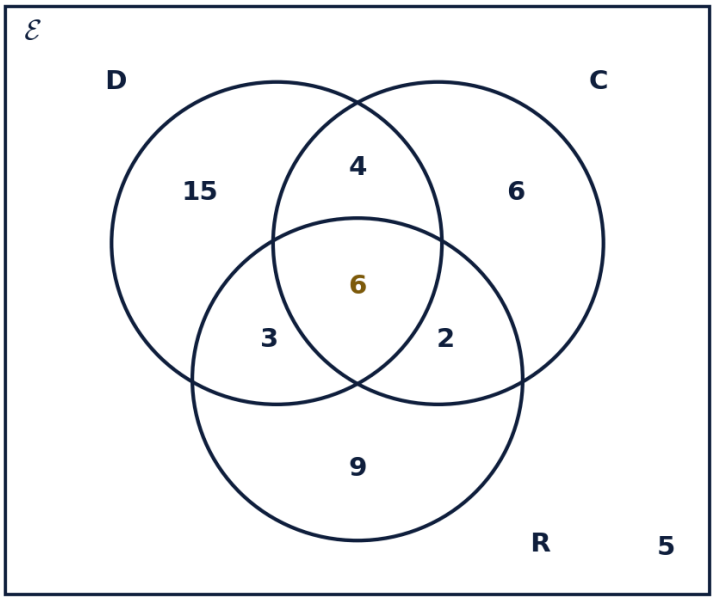
4 **There are students outside all**

There are 5 students outside all three sets. The total number of students is 50, so

$$(25 - x) + (16 - x) + 9 + (x - 6) + 3 + 2 + 6 + 5 = 50$$

$$60 - x = 50$$

$$x = 10$$



5 **Calculate value**

For $C' \cap D'$, we need students who are not in C and not in D. These are the rabbit-only students and the students outside all three sets.

$$9 + 5 = 14$$

✓ **FINAL ANSWER**

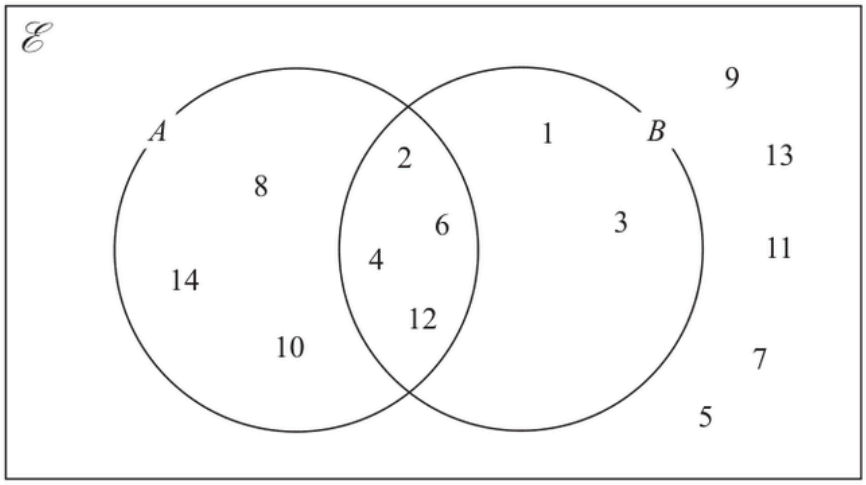
$x = 10$ and $n(C' \cap D') = 14$

4MA1/1H MAY/NOVEMBER 2020

Question 1

4 marks | Set Notation & Venn Diagrams

1 The numbers from 1 to 14 are shown in the Venn diagram.



(a) List the members of the set $A \cap B$

.....

(1)

(b) List the members of the set B'

.....

(1)

A number is picked at random from the numbers in the Venn diagram.

(c) Find the probability that this number is in set A but is **not** in set B.

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May/November 2020

1 **(a) In both A and B**

$A \cap B = \{2, 4, 6, 12\}$

2 **(b) Not in B**

Everything outside circle B: $B' = \{5, 7, 8, 9, 10, 11, 13, 14\}$

3 **(c) In A but not in B**

$\{8, 10, 14\}$: 3 of the 14 numbers.

$$\frac{3}{14}$$

✓ **FINAL ANSWER**

(a) $2, 4, 6, 12$ (b) $5, 7, 8, 9, 10, 11, 13, 14$ (c) $\frac{3}{14}$

4MA1/2HR JANUARY 2021

Question 3

3 marks | Set Notation & Venn Diagrams

3 $\mathscr{E} = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$
$A = \{22, 24, 26, 28, 30\}$
$B = \{21, 24, 27, 30\}$

(a) List the members of the set

(i) $A \cap B$

(ii) A'

.....

(2)

$C = \{23, 25, 29\}$

(b) Using set notation, find an expression for C in terms of A and B.

.....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2021

1 Calculate value

$$A \cap B = \{24, 30\}$$

2 Set is

The complement of A, using the given universal set, is

$$A' = \{21, 23, 25, 27, 29\}$$

3 The set

The set

$$C = \{23, 25, 29\}$$

4 Calculate value

is everything not in A and not in B, so

$$C = A' \cap B'$$

5 **Calculate value**

Equivalently,

$$C = (A \cup B)'$$

FINAL ANSWER

$A \cap B = \{24, 30\}$, $A' = \{21, 23, 25, 27, 29\}$, and $C = A' \cap B'$.

4MA1/1H JANUARY 2021

Question 4

2 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$A = \{\text{odd numbers}\}$
$B = \{\text{multiples of 3}\}$

List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2021

1 Calculate value

$$\mathcal{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$$

2 The odd numbers are

The odd numbers are

$$A = \{21, 23, 25, 27, 29\}$$

3 The multiples of are

The multiples of 3 are

$$B = \{21, 24, 27\}$$

4 The numbers in both sets

The numbers in both sets are

$$A \cap B = \{21, 27\}$$

5 The numbers in either set

The numbers in either set are

$$A \cup B = \{21, 23, 24, 25, 27, 29\}$$

FINAL ANSWER

(i) $\{21, 27\}$, (ii) $\{21, 23, 24, 25, 27, 29\}$

Question 16

6 marks | Set Notation & Venn Diagrams

16 Some students were asked the following question.

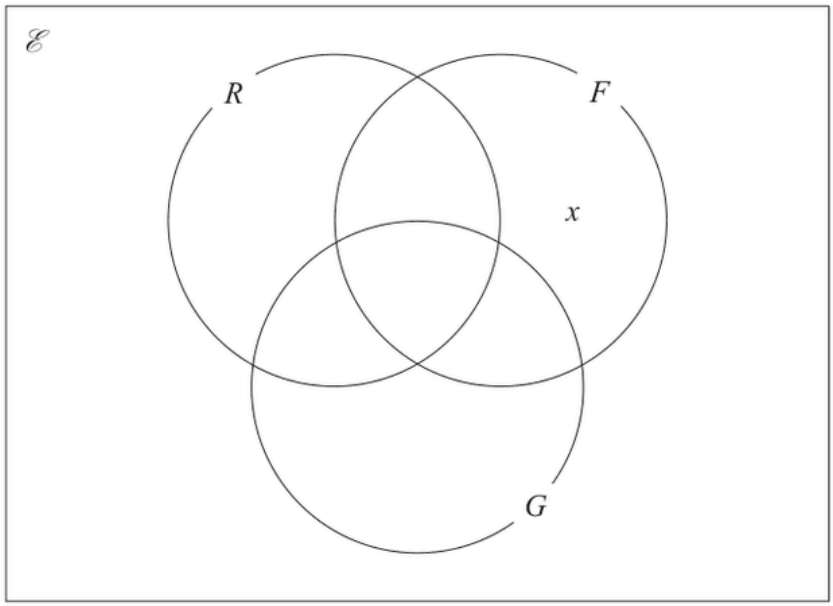
"Which of the subjects Russian (R), French (F) and German (G) do you study?"

Of these students

4 study all three of Russian, French and German
10 study Russian and French
13 study French and German
6 study Russian and German
24 study German
11 study none of the three subjects
the number who study Russian only is twice the number who study French only.

Let x be the number of students who study French only.

(a) Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of x where necessary.



(3)

Given that the number of students who were asked the question was 80

(b) work out the number of these students that study Russian.

.....

(3)

(Total for Question 16 is 6 marks)

1 **The number studying all three**

The number studying all three languages is 4.

2 **Calculate value**

So the two-language-only regions are

$$R \cap F \text{ only} = 10 - 4 = 6$$

$$F \cap G \text{ only} = 13 - 4 = 9$$

$$R \cap G \text{ only} = 6 - 4 = 2$$

3 **The German only region is**

The German-only region is

$$24 - 9 - 2 - 4 = 9$$

4 **The French only region is**

The French-only region is x and the Russian-only region is $2x$. There are 11 students outside all three sets.

5 **Calculate value**

Using the total 80,

$$2x + x + 9 + 6 + 9 + 2 + 4 + 11 = 80$$

$$3x + 41 = 80$$

$$x = 13$$

6 **The number studying Russian is**

The number studying Russian is

$$2x + 6 + 2 + 4 = 26 + 6 + 2 + 4 = 38$$

✓ **FINAL ANSWER**

38

4MA1/2HR JANUARY 2021

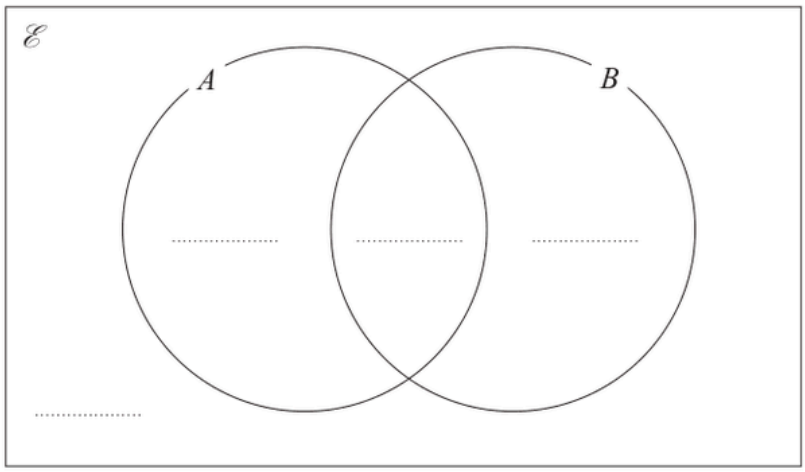
Question 16

5 marks | Set Notation & Venn Diagrams

16 Two events A and B are such that $n(A) = 62$ $n(B) = 30$ and $n(A \cup B) = 68$

Given that $n(\mathscr{E}) = 80$

(a) complete the Venn diagram to show the number of elements in each region.



(2)

An element is chosen at random from $\mathscr{E}$.

(b) Using the Venn diagram, find the probability that this element is in

(i) $A \cap B$

.....

(1)

(ii) $A \cup B'$

.....

(2)

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR January 2021

1 **(a) Find the intersection**

$$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 62 + 30 - 68 = 24$$

2 **(a) Fill every region**

A only $= 62 - 24 = 38$, B only $= 30 - 24 = 6$, outside $= 80 - 68 = 12$.

3 **(b)(i) In both A and B**

$$P(A \cap B) = \frac{24}{80} = \frac{3}{10}$$

4 **(b)(ii) In A or not in B**

$A \cup B'$ is everything except "B only": $80 - 6 = 74$.

$$P(A \cup B') = \frac{74}{80} = \frac{37}{40}$$

✓ **FINAL ANSWER**

(a) $38, 24, 6$, outside 12 (b)(i) $\frac{3}{10}$ (ii) $\frac{37}{40}$

4MA1/2H MAY 2021

Question 4

3 marks | Set Notation & Venn Diagrams

4 $\mathscr{E}$ = {letters of the alphabet}
B = {b, r, a, z, i, l}
I = {i, r, e, l, a, n, d}

(a) List the members of the set

(i) $B \cup I$

(ii) $B \cap I'$

.....

(2)

K = {k, e, n, y, a}

Cody writes down the statement $B \cap K = \varnothing$
Cody's statement is wrong.

(b) Explain why.

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H May 2021

1 Calculate value

$$B = \{b, r, a, z, i, l\}$$

$$I = \{i, r, e, l, a, n, d\}$$

2 The union contains every letter

The union contains every letter that is in B or I:

$$B \cup I = \{a, b, d, e, i, l, n, r, z\}$$

3 Calculate statistic

$B \cap I'$ means letters in B, but not in I:

$$B \cap I' = \{b, z\}$$

4 **Calculate value**

For $K = \{k, e, n, y, a\}$, Cody is not correct because the letter a is in both B and K.

$$B \cap K = \{a\}$$

✓ **FINAL ANSWER**

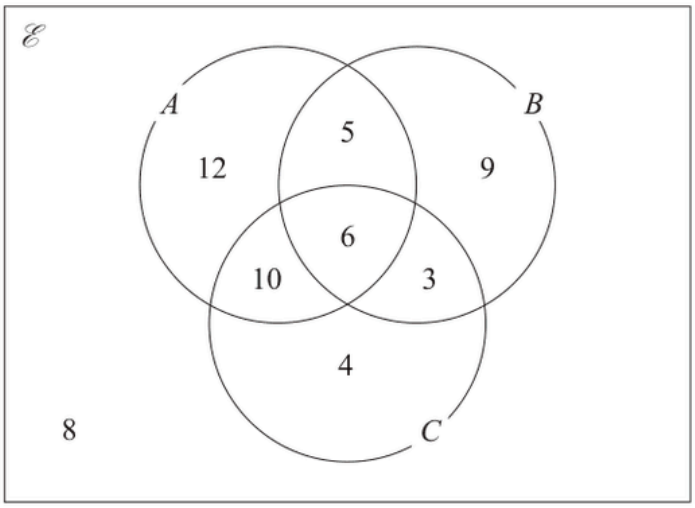
(a)(i) $\{a, b, d, e, i, l, n, r, z\}$, (a)(ii) $\{b, z\}$. Cody is incorrect.

4MA1/1H MAY 2021

Question 19

3 marks | Set Notation & Venn Diagrams

19 The Venn diagram shows a universal set, $\mathscr{E}$ and sets A, B and C.



12, 5, 9, 10, 6, 3, 4 and 8 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A' \cap B')$

.....

(1)

(iii) $n([A \cap B] \cup C)$

.....

(1)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2021

1 **From the Venn diagram**

From the Venn diagram:

2 **only is only is only**

- A only is 12
- $A \cap B$ only is 5
- B only is 9
- $A \cap C$ only is 10
- all three is 6
- $B \cap C$ only is 3
- C only is 4
- outside is 8

---

3 **Add every region inside**

For $A \cup B$, add every region inside A or B:

$$12 + 5 + 9 + 10 + 6 + 3 = 45$$

---

4 **Add the regions outside both**

For $A' \cap B'$, add the regions outside both A and B:

$$4 + 8 = 12$$

---

5 **Add and all**

For $(A \cap B) \cup C$, add $A \cap B$ and all of C:

$$5 + 6 + 10 + 3 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 45, (ii) 12, (iii) 28

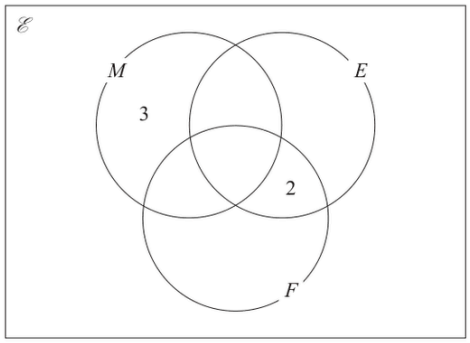
Question 16

5 marks | Set Notation & Venn Diagrams

16 There are 32 students in a class.
In one term these 32 students each took a test in Maths (M), in English (E) and in French (F).

25 students passed the test in Maths.
20 students passed the test in English.
14 students passed the test in French.
18 students passed the tests in Maths and English.
11 students passed the tests in Maths and French.
4 students failed all three tests.
x students passed all three tests.

The incomplete Venn diagram gives some more information about the results of the 32 students.

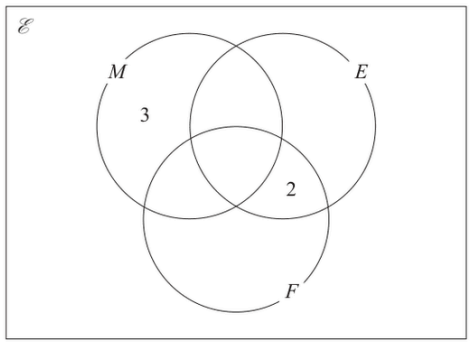


(a) Use all the given information about the results of students who passed the test in Maths to find the value of x.

$x =$ .....

(2)

(b) Use your value of x to complete the Venn diagram to show the number of students in each subset.



(2)

A student who passed the test in Maths is chosen at random.

(c) Find the probability that this student failed the test in French.

.....

(1)

(Total for Question 16 is 5 marks)

1 **(a) Use the Maths circle**

Maths only $= 3$, Maths and English only $= 18 - x$, Maths and French only $= 11 - x$, all three $= x$.

$$3 + (18 - x) + (11 - x) + x = 25 \;\Rightarrow\; 32 - x = 25 \;\Rightarrow\; x = 7$$

2 **(b) Fill every region**

$M \cap E$ only $= 11$, $M \cap F$ only $= 4$, centre $= 7$, $E \cap F$ only $= 2$.

$$E \text{ only} = 20 - 11 - 7 - 2 = 0, \qquad F \text{ only} = 14 - 4 - 7 - 2 = 1$$

Outside: 4. (Check: $3 + 11 + 4 + 7 + 0 + 2 + 1 + 4 = 32$.)

3 **(c) Passed Maths, failed French**

Maths only and Maths-and-English only: $3 + 11 = 14$ of the 25.

$$\frac{14}{25}$$

✓ **FINAL ANSWER**

(a) $x = 7$ (b) M only 3, $M \cap E$ only 11, $M \cap F$ only 4, centre 7, $E \cap F$ only 2, E only 0, F only 1, outside 4 (c) $\frac{14}{25}$

4MA1/1H JANUARY 2022

Question 7

3 marks | Set Notation & Venn Diagrams

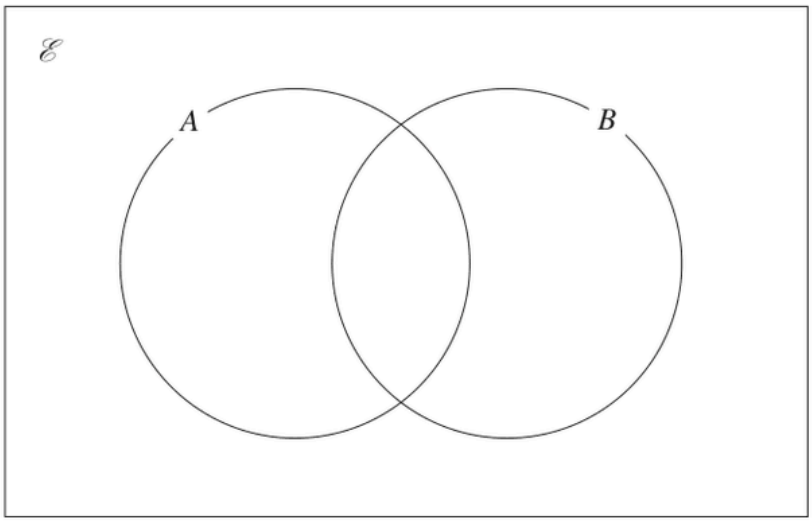
7 $\mathscr{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A \cap B = \{5, 10, 15\}$

$B' = \{7, 8, 9, 11, 12, 13, 14\}$

$A' = \{4, 6, 7, 8, 14\}$

Complete the Venn diagram for this information.



(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2022

1 Calculate value

$$\mathcal{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

2 We are given

We are given

$$A \cap B = \{5, 10, 15\}$$

3 Calculate value

Also,

$$B' = \{7, 8, 9, 11, 12, 13, 14\}$$

4 **Calculate value**

so

$$B = \{4, 5, 6, 10, 15\}$$

5 **Calculate value**

And

$$A' = \{4, 6, 7, 8, 14\}$$

6 **Calculate value**

so

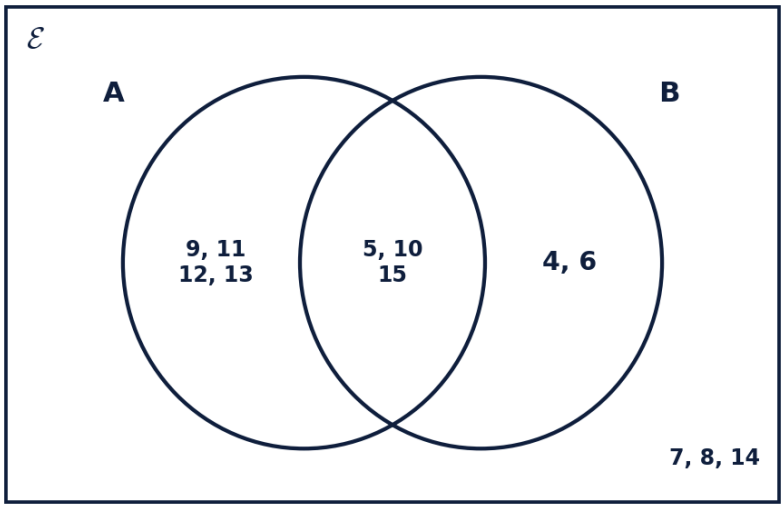
$$A = \{5, 9, 10, 11, 12, 13, 15\}$$

7 **Use Set notation Venn diagrams**

Therefore the Venn diagram regions are:

8 **only only outside both sets**

- $A \cap B$: $\{5, 10, 15\}$
- A only: $\{9, 11, 12, 13\}$
- B only: $\{4, 6\}$
- outside both sets: $\{7, 8, 14\}$



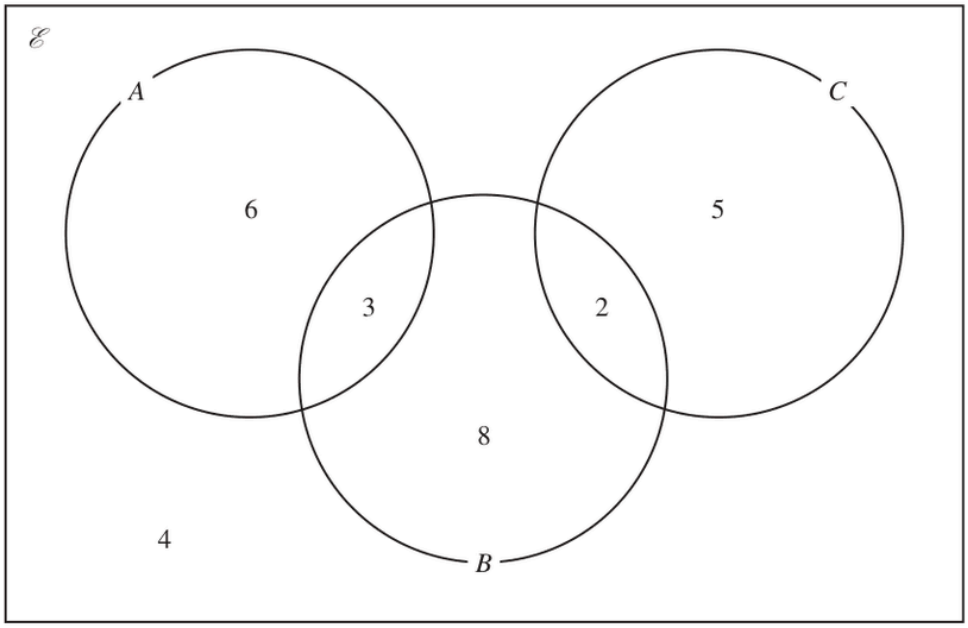
FINAL ANSWER

Put $\{5, 10, 15\}$ in the overlap, $\{9, 11, 12, 13\}$ in A only, $\{4, 6\}$ in B only, and $\{7, 8, 14\}$ outside.

Question 17

4 marks | Set Notation & Venn Diagrams

17 The Venn diagram shows a universal set $\mathscr{E}$ and three sets A, B and C.



6, 3, 8, 2, 5 and 4 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A \cap C)$

.....

(1)

(iii) $n(B \cap C')$

.....

(1)

(iv) $n(A' \cup B' \cup C')$

.....

(1)

(Total for Question 17 is 4 marks)

1 **From the Venn diagram**

From the Venn diagram:

$$A \text{ only} = 6, \quad A \cap B = 3, \quad B \text{ only} = 8, \quad B \cap C = 2, \quad C \text{ only} = 5, \quad \text{outside} = 4$$

2 **Calculate value**

There is no region in both A and C, and there is no triple intersection.

$$n(A \cup B) = 6 + 3 + 8 + 2 = 19$$

$$n(A \cap C) = 0$$

$$n(B \cap C') = 3 + 8 = 11$$

3 **Use**

For $A' \cup B' \cup C'$, use

$$A' \cup B' \cup C' = (A \cap B \cap C)'$$

4 **Set**

Since $A \cap B \cap C$ is empty, this includes everyone in the universal set.

$$6 + 3 + 8 + 2 + 5 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 19, (ii) 0, (iii) 11, (iv) 28

4MA1/2HR JUNE 2022

Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
$A = \{\text{multiples of } 3\}$
$B = \{\text{odd numbers}\}$

(a) List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(b) Is it true that $24 \in A$?

Tick one of the boxes below.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

Set C has 4 members such that $C \cap B' = \{10, 18\}$

(c) List the members of one possible set C

.....

(2)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2022

1 **Calculate value**

$$\mathcal{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

2 **The multiples of in are**

The multiples of 3 in $\mathcal{E}$ are

$$A = \{9, 12, 15, 18\}$$

3 **The odd numbers in are**

The odd numbers in $\mathcal{E}$ are

$$B = \{9, 11, 13, 15, 17, 19\}$$

4 **Calculate value**

So

$$A \cap B = \{9, 15\}$$

5 **Calculate value**

and

$$A \cup B = \{9, 11, 12, 13, 15, 17, 18, 19\}$$

6 **The statement is false because**

The statement $24 \in A$ is false because 24 is not in the universal set $\mathcal{E}$.

7 **Calculate value**

For C, we need $C \cap B' = \{10, 18\}$ and C has 4 members. One valid choice is

$$C = \{9, 10, 11, 18\}$$

✓ **FINAL ANSWER**

(a)(i) $\{9, 15\}$, (a)(ii) $\{9, 11, 12, 13, 15, 17, 18, 19\}$, (b) No, (c) one possible set is $\{9, 10, 11, 18\}$

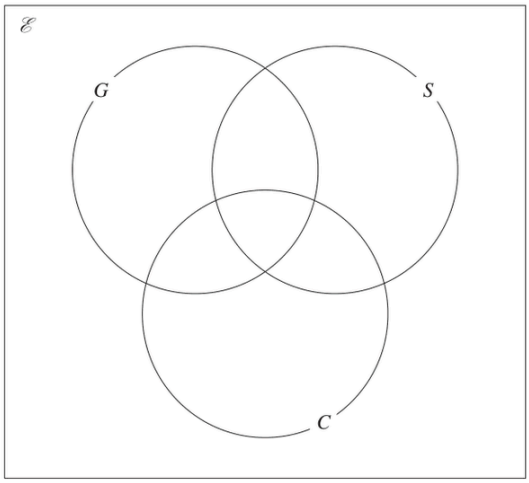
Question 16

8 marks | Set Notation & Venn Diagrams

16 100 farmers are asked if they have goats (G), sheep (S) or chickens (C) on their farms.

Of these farmers
- 31 have sheep
- 53 have chickens
- 6 have goats, sheep and chickens
- 11 have sheep and goats
- 17 have sheep and chickens
- 18 have goats and chickens
- 20 do not have any goats, sheep or chickens

(a) Using this information, complete the Venn diagram to show the number of farmers in each appropriate subset.



(3)

(b) Find

(i) n(G)

.....

(1)

(ii) n($[G \cup S]'$)

.....

(1)

(iii) n($G' \cap C$)

.....

(1)

One of the farmers who has chickens is chosen at random.

(c) Find the probability that this farmer also has goats.

.....

(2)

(Total for Question 16 is 8 marks)

1 **(a) Start from the centre**

All three: 6. Then the "two only" regions:

$$G \cap S \text{ only} = 11 - 6 = 5, \quad S \cap C \text{ only} = 17 - 6 = 11, \quad G \cap C \text{ only} = 18 - 6 = 12$$

2 **(a) Single-animal regions**

$$S \text{ only} = 31 - 5 - 11 - 6 = 9, \qquad C \text{ only} = 53 - 11 - 12 - 6 = 24$$

80 farmers are inside the circles ($100 - 20$), so

$$G \text{ only} = 80 - (9 + 24 + 5 + 11 + 12 + 6) = 13$$

Outside the circles: 20.

3 **(b)(i) n(G)**

$$13 + 5 + 12 + 6 = 36$$

4 **(b)(ii) Not in G or S**

Only "C only" and the outside region: $24 + 20 = 44$.

5 **(b)(iii) In C but not in G**

"C only" and "$S \cap C$ only": $24 + 11 = 35$.

6 **(c) Chickens farmers who also have goats**

53 farmers have chickens; $12 + 6 = 18$ of them also have goats.

$$\frac{18}{53}$$

FINAL ANSWER

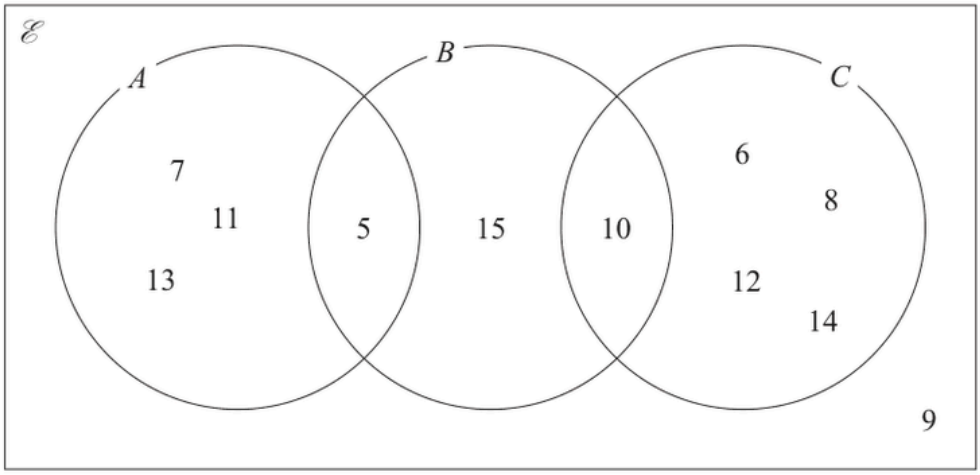
(a) G only 13, S only 9, C only 24, $G \cap S$ only 5, $S \cap C$ only 11, $G \cap C$ only 12, centre 6, outside 20
(b)(i) 36 (ii) 44 (iii) 35 (c) $\frac{18}{53}$

4MA1/1HR MAY 2023

Question 2

3 marks | Set Notation & Venn Diagrams

2 Here is a Venn diagram.



(a) Write down the numbers that are in the set

(i) A

.....

(1)

(ii) $B \cup C$

.....

(1)

Dominic writes down $9 \notin C$

(b) Explain why Dominic is correct.

.....

.....

(1)

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR May 2023

1 **The numbers inside are the**

The numbers inside A are the numbers in the A-only region and the overlap with B:

$$A = \{5, 7, 11, 13\}$$

2 **Calculate statistic**

$B \cup C$ means every number inside B or C:

$$B \cup C = \{5, 6, 8, 10, 12, 14, 15\}$$

---

3 **Dominic writes . This is**

Dominic writes $9 \notin C$. This is correct because 9 is outside circle C in the Venn diagram.

✓ **FINAL ANSWER**

(a)(i) $\{5, 7, 11, 13\}$, (a)(ii) $\{5, 6, 8, 10, 12, 14, 15\}$. Dominic is correct because 9 is not in C.

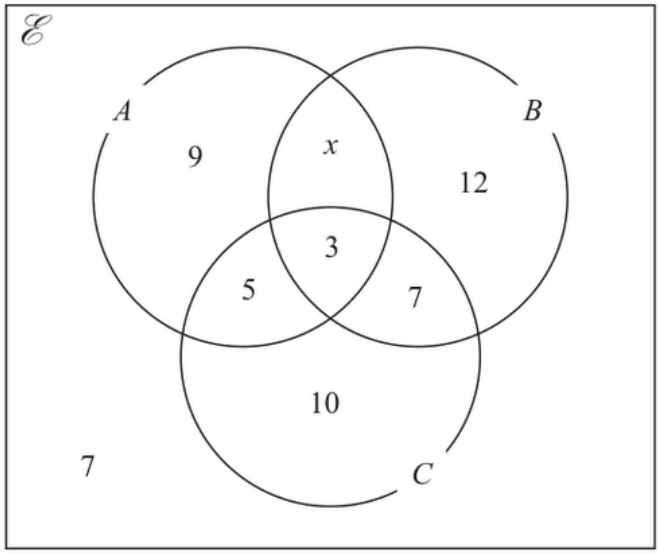
4MA1/1H MAY 2023

Question 16

3 marks | Set Notation & Venn Diagrams

16 The Venn diagram shows a universal set $\mathscr{E}$ and sets A, B and C

The numbers and the letter x represent **numbers** of elements.



Given that $n(A \cup B) = 42$

(a) find the value of x

$x =$ .....

(1)

(b) Find $n(A')$

.....

(1)

(c) Find $n(B' \cap C)$

.....

(1)

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2023

1 **The regions inside are**

The regions inside $A \cup B$ are

$$9 + x + 12 + 5 + 3 + 7$$

2 **We are told that**

We are told that

$$n(A \cup B) = 42$$

3 **Calculate value**

So

$$36 + x = 42$$

$$x = 6$$

4 **Add every region not inside**

For A', add every region not inside A:

$$12 + 7 + 10 + 7 = 36$$

5 **Add the regions**

For $B' \cap C$, add the regions that are in C but not in B:

$$5 + 10 = 15$$

✓ **FINAL ANSWER**

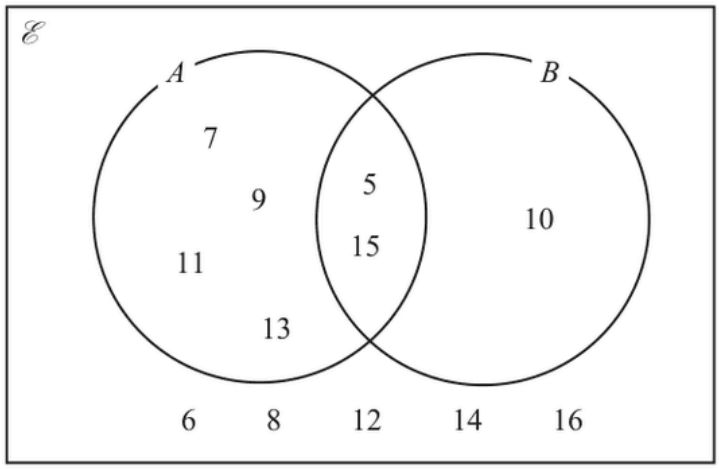
(a) $x = 6$, (b) 36, (c) 15

4MA1/1H NOVEMBER 2023

Question 1

3 marks | Set Notation & Venn Diagrams

1 Here is a Venn diagram.



List the members of the set

(a) A

.....

(1)

(b) $A \cap B$

.....

(1)

(c) $(A \cup B)'$

.....

(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2023

1 **The members of are all**

The members of A are all the numbers inside circle A, including the overlap:

$$A = \{5, 7, 9, 11, 13, 15\}$$

2 **The members of are the**

The members of $A \cap B$ are the numbers in both circles:

$$A \cap B = \{5, 15\}$$

3 **means outside both circles**

$(A \cup B)'$ means outside both circles:

$$(A \cup B)' = \{6, 8, 12, 14, 16\}$$

✓ **FINAL ANSWER**

(a) $\{5, 7, 9, 11, 13, 15\}$, (b) $\{5, 15\}$, (c) $\{6, 8, 12, 14, 16\}$

Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{factors of } 6\}$
$B = \{\text{prime numbers}\}$

(a) List the members of the set

(i) $A \cup B$

.....

(1)

(ii) A'

.....

(1)

Harpreet states that $A \cap B = \varnothing$

Harpreet is incorrect.

(b) Explain why.

.....

(1)

C is a set with 4 members such that

the set $A \cap C$ has 2 members
the set $B \cap C$ has 2 members

Set $A \cap C$ and set $B \cap C$ have no members in common.

(c) List the 4 members of set C

.....

(2)

(Total for Question 4 is 5 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 6\}$$

$$B = \{2, 3, 5, 7\}$$

2 **Calculate value**

So

$$A \cup B = \{1, 2, 3, 5, 6, 7\}$$

3 **Calculate value**

and

$$A' = \{4, 5, 7, 8, 9, 10\}$$

4 **Harpreet is not correct because**

Harpreet is not correct because 2 and 3 are in both A and B.

$$A \cap B = \{2, 3\}$$

5 **Calculate value**

For C, it has 4 members, $n(A \cap C) = 2$, $n(B \cap C) = 2$, and no member is in both of those overlaps. So choose the two elements of A only and the two elements of B only:

$$C = \{1, 5, 6, 7\}$$

✓ **FINAL ANSWER**

(a)(i) $\{1, 2, 3, 5, 6, 7\}$, (a)(ii) $\{4, 5, 7, 8, 9, 10\}$, (b) Harpreet is incorrect, (c) $C = \{1, 5, 6, 7\}$

Question 7

5 marks | Set Notation & Venn Diagrams

7 $\mathscr{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$
$A = \{\text{even numbers}\}$
$B = \{23, 29, 31\}$
$C = \{\text{multiples of 3}\}$

(a) List the members of the set

(i) $B \cup C$

.....

(1)

(ii) $A' \cap C$

.....

(1)

(b) Is it true that $B \cap C = \varnothing$?

Tick (✓) one of the boxes below.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

The set D has 4 members and is such that $D \cap (A \cup C) = \varnothing$

(c) List the members of set D

.....

(2)

(Total for Question 7 is 5 marks)

1 **Calculate value**

$$\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$$

$$A = \{24, 26, 28, 30, 32, 34\}$$

$$B = \{23, 29, 31\}$$

$$C = \{24, 27, 30, 33\}$$

2 **Calculate value**

So

$$B \cup C = \{23, 24, 27, 29, 30, 31, 33\}$$

3 **Set of odd numbers**

Also A' is the set of odd numbers in $\mathcal{E}$, so

$$A' \cap C = \{27, 33\}$$

4 **The statement is true because**

The statement $B \cap C = \emptyset$ is true because none of $23, 29, 31$ is a multiple of 3.

5 **Calculate value**

For D, we need $D \cap (A \cup C) = \emptyset$. So D must contain the four numbers outside $A \cup C$:

$$D = \{23, 25, 29, 31\}$$

✓ **FINAL ANSWER**

(a)(i) $\{23, 24, 27, 29, 30, 31, 33\}$, (a)(ii) $\{27, 33\}$, (b) Yes, (c) $\{23, 25, 29, 31\}$

Question 16

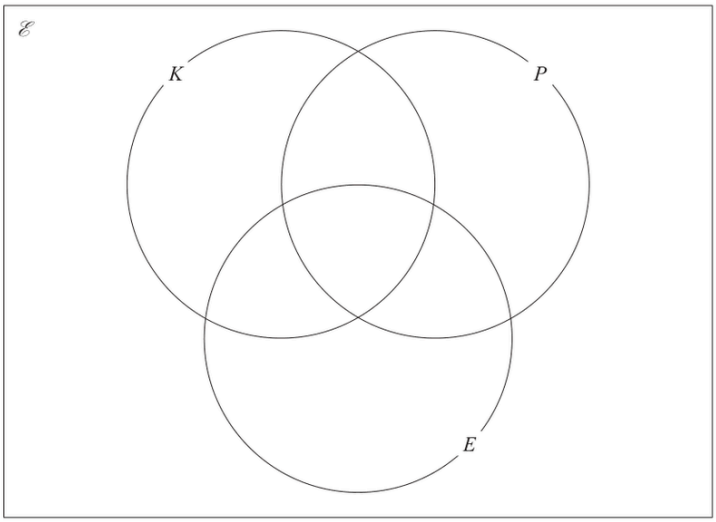
7 marks | Set Notation & Venn Diagrams

16 60 art students were asked if they would like to attend workshops for knitting (K), for photography (P) or for embroidery (E)

Of these students

9 chose knitting, photography and embroidery
17 chose knitting and photography
16 chose photography and embroidery
20 chose knitting and embroidery
28 chose photography
39 chose embroidery
2 chose none of the workshops

(a) Using this information, complete the Venn diagram to show the numbers of students in each subset.



(3)

One of the students is chosen at random.

Given that this student chose photography,

(b) find the probability that this student also chose knitting.

(2)

(c) Find $n(P \cap K')$

(1)

(d) Find $n([P \cup E] \cap K)$

(1)

(Total for Question 16 is 7 marks)

1 **(a) Start from the centre**

All three: 9.

$$K \cap P \text{ only} = 17 - 9 = 8, \quad P \cap E \text{ only} = 16 - 9 = 7, \quad K \cap E \text{ only} = 20 - 9 = 11$$

2 **(a) Single regions**

$$P \text{ only} = 28 - 8 - 7 - 9 = 4, \qquad E \text{ only} = 39 - 11 - 7 - 9 = 12$$

58 students are inside the circles ($60 - 2$):

$$K \text{ only} = 58 - (8 + 7 + 11 + 9 + 4 + 12) = 7$$

Outside: 2.

3 **(b) Knitting, given photography**

28 chose photography; $8 + 9 = 17$ of them also chose knitting.

$$\frac{17}{28}$$

4 **(c) Photography but not knitting**

P only and $P \cap E$ only: $4 + 7 = 11$.

5 **(d) Knitting and (photography or embroidery)**

$8 + 9 + 11 = 28$

FINAL ANSWER

(a) K only 7, P only 4, E only 12, $K \cap P$ only 8, $P \cap E$ only 7, $K \cap E$ only 11, centre 9, outside 2

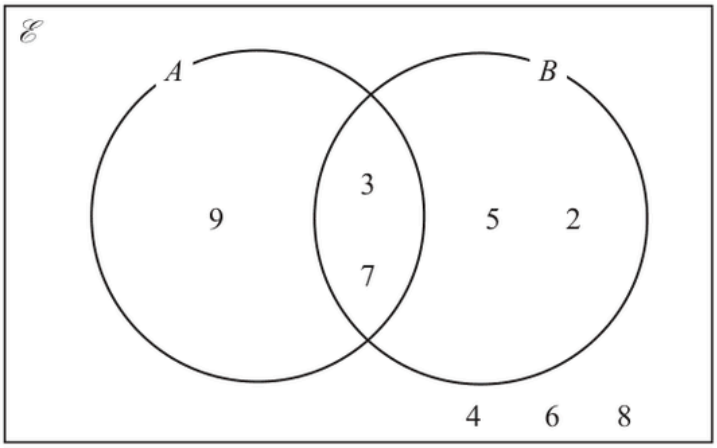
(b) $\frac{17}{28}$ (c) 11 (d) 28

4MA1/1H NOVEMBER 2024

Question 4

3 marks | Set Notation & Venn Diagrams

4 Here is a Venn diagram.



(a) List the members of the set B

.....

(1)

(b) List the members of the set $A \cap B$

.....

(1)

(c) List the members of the set A'

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2024

1 **The members of are all**

The members of B are all the numbers inside circle B, including the overlap:

$$B = \{2, 3, 5, 7\}$$

2 **The members of are the**

The members of $A \cap B$ are the numbers in both circles:

$$A \cap B = \{3, 7\}$$

3 Calculate statistic

A' means all the numbers in the universal set that are not in A:

$$A' = \{2, 4, 5, 6, 8\}$$

FINAL ANSWER

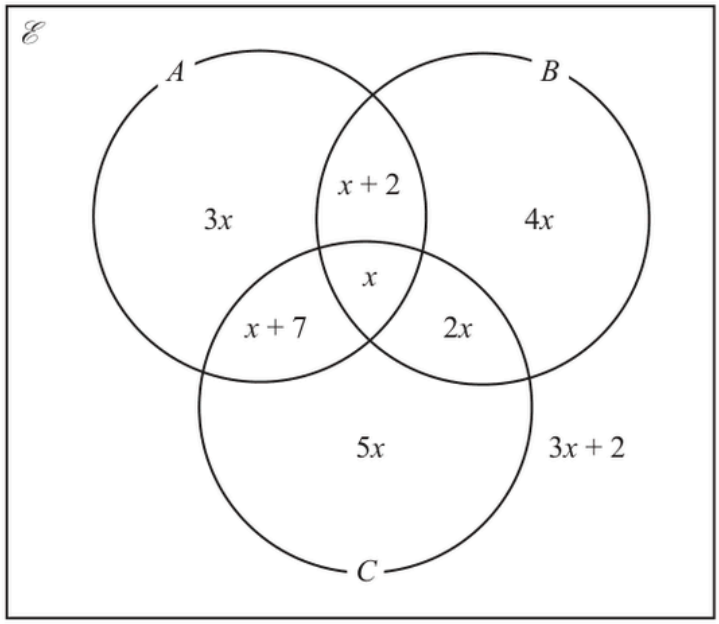
(a) $\{2, 3, 5, 7\}$, (b) $\{3, 7\}$, (c) $\{2, 4, 5, 6, 8\}$

4MA1/2H NOVEMBER 2024

Question 21

4 marks | Set Notation & Venn Diagrams

21 The Venn diagram shows information about the numbers of items in set A, set B and set C, where x is an integer.



Given that $\text{n}(A \cup B)' = 26$

find $\text{n}(A' \cap C)$

$\text{n}(A' \cap C) =$ .....

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2024

1 The complement of contains the

The complement of $A \cup B$ contains the regions outside both A and B. These are $5x$ and $3x + 2$.

2 Calculate value

So $5x + (3x + 2) = 26$, hence $8x + 2 = 26$ and $x = 3$.

3 Calculate statistic

Now $A' \cap C$ means inside C, but not inside A. These are the regions $5x$ and $2x$.

4 Calculate value

So $n(A' \cap C) = 5x + 2x = 7x = 7(3) = 21$.

FINAL ANSWER

21

Question 4

4 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

(a) List the members of the set

(i) $A \cup B$

.....

(ii) B'

.....

(2)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

$\mathscr{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

(b) Write a symbol from the box on each dotted line to make each of the following a true statement.

(i) $A \cap C =$ .....

(ii) 13 ..... $\mathscr{E}$

(2)

(Total for Question 4 is 4 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 4, 6, 8, 10, 12\}$$

$$B = \{3, 6, 9, 12\}$$

$$C = \{1, 3, 5, 7, 9, 11\}$$

2 **Calculate value**

Therefore

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$$

3 **Calculate value**

and

$$B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$$

4 **No number is both even**

No number is both even and odd, so

$$A \cap C = \emptyset$$

5 **Set**

Also $13 \notin \mathcal{E}$ because 13 is not in the universal set.

✓ **FINAL ANSWER**

(a)(i) $\{2, 3, 4, 6, 8, 9, 10, 12\}$, (a)(ii) $\{1, 2, 4, 5, 7, 8, 10, 11\}$, (b)(i) $\emptyset$, (b)(ii) $13 \notin \mathcal{E}$

Question 5

4 marks | Set Notation & Venn Diagrams

5 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

(a) List the members of the set

(i) $A \cup B$

.....

(ii) B'

.....

(2)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 6, 9, 12\}$

$C = \{1, 3, 5, 7, 9, 11\}$

$\mathscr{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

(b) Write a symbol from the box on each dotted line to make each of the following a true statement.

(i) $A \cap C =$ .....

(ii) 13 ..... $\mathscr{E}$

(2)

(Total for Question 5 is 4 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 4, 6, 8, 10, 12\}$$

$$B = \{3, 6, 9, 12\}$$

$$C = \{1, 3, 5, 7, 9, 11\}$$

2 **Calculate value**

Therefore

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$$

3 **Calculate value**

and

$$B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$$

4 **No number is both even**

No number is both even and odd, so

$$A \cap C = \emptyset$$

5 **Set**

Also $13 \notin \mathcal{E}$ because 13 is not in the universal set.

✓ **FINAL ANSWER**

(a)(i) $\{2, 3, 4, 6, 8, 9, 10, 12\}$, (a)(ii) $\{1, 2, 4, 5, 7, 8, 10, 11\}$, (b)(i) $\emptyset$, (b)(ii) $13 \notin \mathcal{E}$

4WM1/01 MAY 2025

Question 8

5 marks | Set Notation & Venn Diagrams

8 $\mathscr{E} = \{18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{\text{even numbers}\}$

$B = \{\text{multiples of 3}\}$

$C = \{\text{multiples of 5}\}$

(a) List the members of the set $(A \cup B)'$

.....

(2)

Sophie writes down the statement $B \cap C = \emptyset$

(b) Explain why Sophie's statement is wrong.

.....

(1)

D is a set such that $A \cap D = \{18, 26\}$
The set D has exactly 5 members.

(c) List the members of one possible set D

.....

(2)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4WM1/01 May 2025

1 **Calculate value**

$$A = \{18, 20, 22, 24, 26, 28, 30\}$$

$$B = \{18, 21, 24, 27, 30\}$$

$$A \cup B = \{18, 20, 21, 22, 24, 26, 27, 28, 30\}$$

2 **Calculate value**

So

$$(A \cup B)' = \{19, 23, 25, 29\}$$

3 **Sophie is wrong because is**

Sophie is wrong because 30 is both a multiple of 3 and a multiple of 5, so $30 \in B \cap C$.

---

4 **Set is**

For D, include 18 and 26, but no other even numbers. One possible set is

$$D = \{18, 19, 23, 25, 26\}$$

✓ **FINAL ANSWER**

$\{19, 23, 25, 29\}$; Sophie is wrong because $30 \in B \cap C$; one possible D is $\{18, 19, 23, 25, 26\}$.

Question 20

6 marks | Set Notation & Venn Diagrams

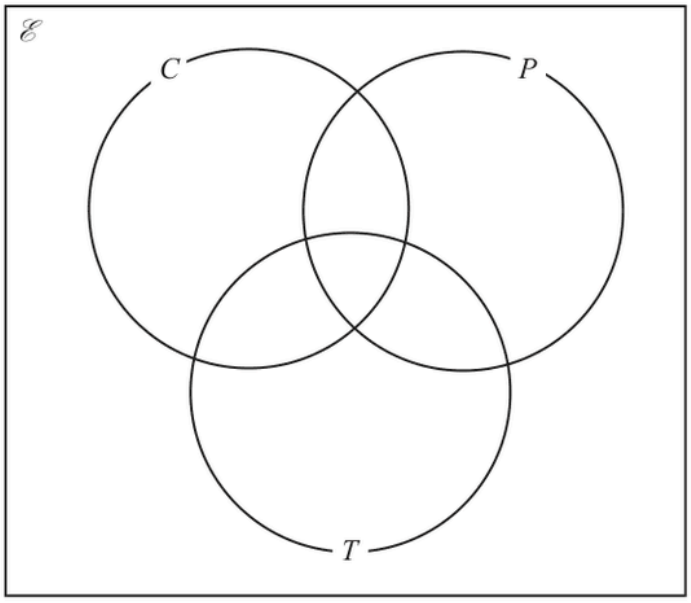
20 120 gardeners were asked if they grow carrots (C) or potatoes (P) or tomatoes (T)

Of these gardeners

43 grow carrots
12 grow carrots and potatoes and tomatoes
18 grow carrots and potatoes
27 grow carrots and tomatoes
32 grow potatoes and tomatoes
29 do not grow carrots or potatoes or tomatoes

The number of these gardeners who grow only potatoes is equal to the number of these gardeners who grow only tomatoes.

(a) Complete the Venn diagram to show this information.



(3)

(b) Find $n(T' \cap C)$

.....

(1)

One of the gardeners who grows carrots is chosen at random.

(c) Calculate the probability that this gardener also grows potatoes.

.....

(2)

(Total for Question 20 is 6 marks)

1 **Calculate value**

There are 120 gardeners in total and 29 are outside all three sets, so

$$n(C \cup P \cup T) = 120 - 29 = 91$$

2 **The triple intersection is**

The triple intersection is

$$n(C \cap P \cap T) = 12$$

3 **Calculate value**

So the pair-only regions are:

$$C \cap P \text{ only} = 18 - 12 = 6$$

$$C \cap T \text{ only} = 27 - 12 = 15$$

$$P \cap T \text{ only} = 32 - 12 = 20$$

4 **Calculate value**

Since 43 grow carrots,

$$C \text{ only} = 43 - 6 - 15 - 12 = 10$$

5 **Let the number who grow**

Let the number who grow only potatoes be y. The number who grow only tomatoes is also y.

6 **Add the regions union**

Now add the regions in the union:

$$10 + y + y + 6 + 15 + 20 + 12 = 91$$

$$63 + 2y = 91$$

$$2y = 28$$

$$y = 14$$

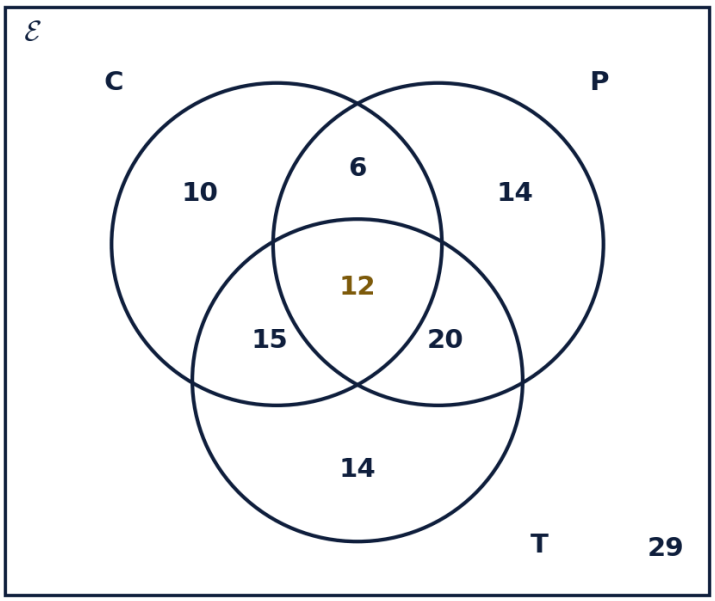
7 **Calculate value**

So the Venn regions are:

$$C \text{ only} = 10, \quad P \text{ only} = 14, \quad T \text{ only} = 14$$

$$C \cap P \text{ only} = 6, \quad C \cap T \text{ only} = 15, \quad P \cap T \text{ only} = 20$$

$$C \cap P \cap T = 12, \quad \text{outside} = 29$$



8 **Calculate value**

For part (b),

$$n(T' \cap C) = C \text{ only} + C \cap P \text{ only}$$

$$= 10 + 6 = 16$$

9 **Evaluate fraction**

For part (c), one gardener who grows carrots is chosen.

$$P(\text{also grows potatoes}) = \frac{n(C \cap P)}{n(C)}$$

$$= \frac{18}{43}$$

✓ **FINAL ANSWER**

(b) 16, (c) $\frac{18}{43}$

Question 4

4 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$
$A = \{\text{odd numbers}\}$
$B = \{\text{multiples of 3}\}$
$C = \{\text{prime numbers}\}$

(a) List the members of the set $A \cup C$

.....

(1)

(b) List the members of the set $(A \cap B)'$

.....

(1)

(c) Is it true that $15 \in (B \cap C)$?

Tick (✓) the correct box.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

D is a set such that $D \cup C = A$
D has 2 members.

(d) List the members of D

.....

(1)

(Total for Question 4 is 4 marks)

1 **Calculate value**

$$A = \{9, 11, 13, 15, 17\}, \quad B = \{9, 12, 15, 18\}, \quad C = \{11, 13, 17\}$$

$$A \cup C = \{9, 11, 13, 15, 17\}$$

$$A \cap B' = \{11, 13, 17\}$$

2 **Check prime condition**

$15 \notin B \cap C$ because 15 is not prime.

3 **Calculate value**

Since $D \cup C = A$ and D has two members,

$$D = \{9, 15\}$$

FINAL ANSWER

$\{9, 11, 13, 15, 17\}$, $\{11, 13, 17\}$, No, $D = \{9, 15\}$.

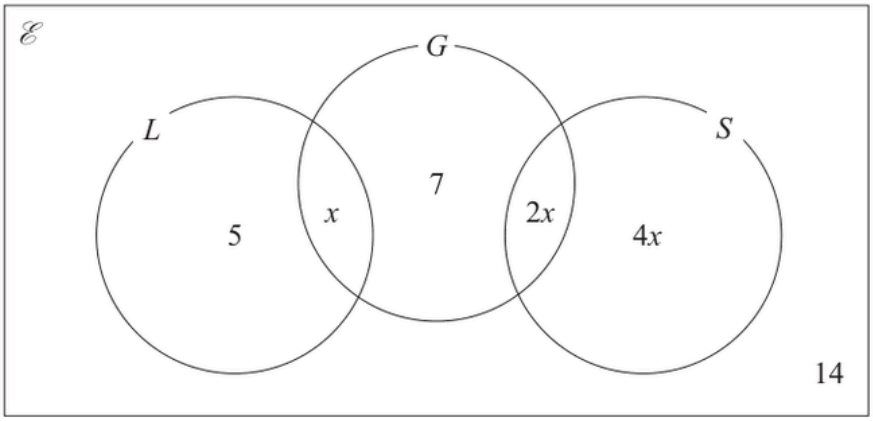
4MA1/2H NOVEMBER 2025

Question 18

3 marks | Set Notation & Venn Diagrams

18 Bianca asked 40 students which of the languages Latin (L), Greek (G) and Sanskrit (S) they study.

The Venn diagram gives some information about her results.
It shows the number of students in each subset.



One of these students is selected at random.
Work out the probability that this student studies Latin or Sanskrit.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2025

1 **Find x (all regions add to 40)**

$$5 + x + 7 + 2x + 4x + 14 = 40 \Rightarrow 26 + 7x = 40 \Rightarrow x = 2$$

2 **Latin or Sanskrit**

Regions in L or S: $5 + x + 2x + 4x = 5 + 2 + 4 + 8 = 19$.

$$\frac{19}{40}$$

✓ **FINAL ANSWER**

$$\frac{19}{40}$$

4MA1/1H JUNE 2026

Question 3

3 marks | Set Notation & Venn Diagrams

3 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

$A = \{\text{factors of } 20\}$

$B = \{\text{multiples of } 5\}$

(a) List the members of the set

(i) $A \cup B$

.....

(1)

(ii) $A \cap B'$

.....

(1)

$C = \{\text{prime numbers}\}$

Ana writes down the statement $A \cap C = \varnothing$
Ana's statement is wrong.

(b) Explain why.

.....

.....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H June 2026

1 **Part (a(i)) | Step 1**

$A = \{1, 2, 4, 5, 10\}$ and $B = \{5, 10, 15\}$, so $A \cup B = \{1, 2, 4, 5, 10, 15\}$.

Include every member of either set once; 20 lies outside the universal set.

2 **Part (a(ii)) | Step 1**

$A \cap B' = \{1, 2, 4\}$.

Retain the factors of 20 in the universal set which are not multiples of 5.

3 **Part (b) | Step 1**

Both 2 and 5 are prime factors of 20, so $A \cap C = \{2, 5\} \neq \varnothing$.

A shared member is enough to disprove the claim that the intersection is empty.

FINAL ANSWER

(a(i)) $\{1, 2, 4, 5, 10, 15\}$ (a(ii)) $\{1, 2, 4\}$ (b) $A \cap C = \{2, 5\}$, not the empty set.

4MA1/2HR JUNE 2026

Question 3

1 marks | Set Notation & Venn Diagrams

3 $\mathscr{E}$ = {integers from 1 to 12}
A = {factors of 12}

List the members of the set A'

.....

(Total for Question 3 is 1 mark)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2026

1 **Solution | Step 1**

$A = \{1, 2, 3, 4, 6, 12\}$. Remove these factors from the integers 1 to 12.
The complement contains precisely the universal-set elements not in A.

FINAL ANSWER

$A' = \{5, 7, 8, 9, 10, 11\}$.

4WM1/01 MAY 2026

Question 5

4 marks | Set Notation & Venn Diagrams

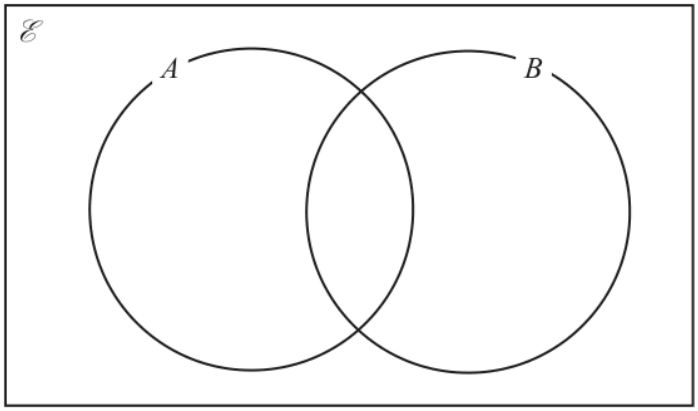
5 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 3, 5, 7, 11\}$

$A \cap B = \{2, 5\}$

$B' = \{3, 4, 6, 7, 9, 11\}$

(a) Complete the Venn diagram for this information.



(3)

A number is chosen at random from the universal set $\mathscr{E}$

(b) Write down the probability that this number is in the set $(A \cup B)'$

.....

(1)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

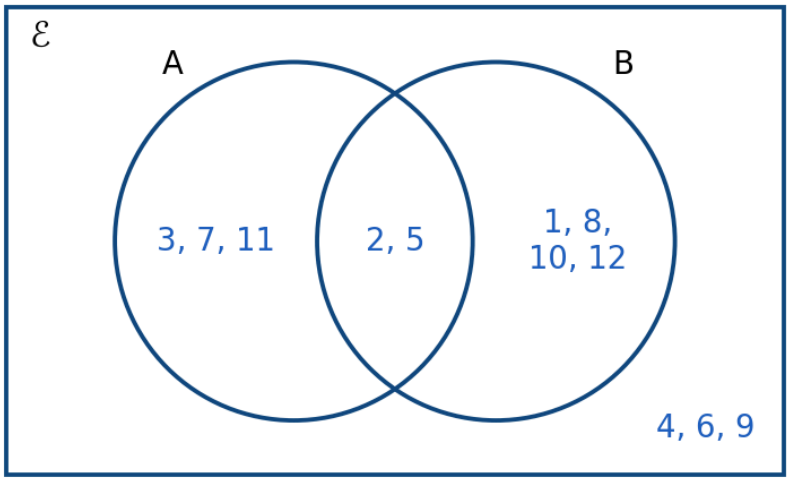
Question 5 · 4WM1/01 May 2026

1 **Find set B**

$B' = \{3, 4, 6, 7, 9, 11\}$, so $B = \{1, 2, 5, 8, 10, 12\}$.

2 **Fill each region**

$A \cap B = \{2, 5\}$; A only $= \{3, 7, 11\}$; B only $= \{1, 8, 10, 12\}$; outside both $= \{4, 6, 9\}$.



3 **(b) Outside both circles**

$$P\big((A \cup B)'\big) = \frac{3}{12} = \frac{1}{4}$$

FINAL ANSWER

(a) see diagram (b) $\frac{3}{12} = \frac{1}{4}$

4MA1/2H JUNE 2026

Question 12

2 marks | Set Notation & Venn Diagrams

12 $\mathscr{E} = \{\text{integers } x \text{ such that } 20 \leqslant x \leqslant 32\}$

$A = \{x \text{ such that } x < 27\}$

$B = \{x \text{ such that } 25 \leqslant x \leqslant 30\}$

(a) List the members of the set $A \cap B$

.....

(1)

(b) List the members of the set $(A \cup B)'$

.....

(1)

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2026

1 **Part (a) | Step 1**

Within the universal set, $A = \{20, 21, 22, 23, 24, 25, 26\}$ and $B = \{25, 26, 27, 28, 29, 30\}$. Their common members are 25 and 26.

Intersection consists of elements in both sets.

2 **Part (b) | Step 1**

$A \cup B = \{20, 21, \ldots, 30\}$. The members of $\mathcal{E} = \{20, 21, \ldots, 32\}$ outside this union are 31 and 32.

Take the complement relative to the stated universal set, not all integers.

✓ **FINAL ANSWER**

(a) $A \cap B = \{25, 26\}$ (b) $(A \cup B)' = \{31, 32\}$

4MA1/1HR JUNE 2026

Question 21

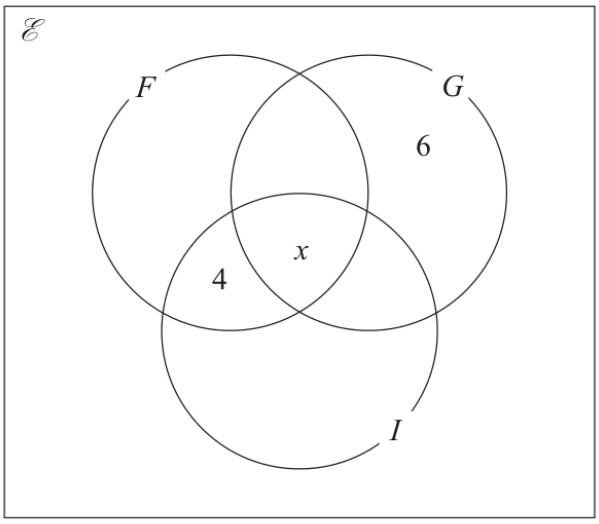
5 marks | Set Notation & Venn Diagrams

21 60 students were asked if they study French (F), German (G) or Italian (I)

Of these students

32 students study French
24 students study German
27 students study Italian
16 students study French and German
15 students study German and Italian

The incomplete Venn diagram gives some more information about the languages the 60 students study.



(a) Work out the value of x

$x =$ .....

(3)

(b) Work out how many of the 60 students study none of French or German or Italian.

.....

(2)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2026

1 **Part (a) | Step 1**

The French-and-German-only region is $16 - x$, and the German-and-Italian-only region is $15 - x$. The totals 16 and 15 each include the triple intersection x.

2 **Part (a) | Step 2**

$$6 + (16 - x) + (15 - x) + x = 24.$$

The four disjoint regions in the German set must total 24.

---

3 **Part (a) | Step 3**

$$37 - x = 24 \quad \Longrightarrow \quad x = 13.$$

Solve for the common triple-intersection count.

---

4 **Part (b) | Step 1**

$$FG \text{ only} = 16 - 13 = 3, \qquad GI \text{ only} = 15 - 13 = 2.$$

Use the triple-intersection value from part (a).

---

5 **Part (b) | Step 2**

$$F \text{ only} = 32 - (3 + 4 + 13) = 12, \qquad I \text{ only} = 27 - (4 + 2 + 13) = 8.$$

The French-and-Italian-only count is the given 4.

---

6 **Part (b) | Step 3**

$$n(F \cup G \cup I) = 12 + 6 + 8 + 3 + 4 + 2 + 13 = 48.$$

Sum all seven mutually exclusive interior regions.

---

7 **Part (b) | Step 4**

$$n(\text{none}) = 60 - 48 = 12.$$

Subtract the union from the total number of students.

✓ **FINAL ANSWER**

(a) $x = 13$ (b) 12 students.

TOPIC

Forming & Solving Equations

Every past-paper question on this topic, 2020 to 2026.

Question 6

3 marks | Forming & Solving Equations

$\left(6^2\right)^{10} = 6^p$

(c) Find the value of p

$p =$ .....

(1)

$5^r = 1$

(d) Write down the value of r

$r =$ .....

(1)

(Total for Question 5 is 4 marks)

6 The diagram shows triangle ABC

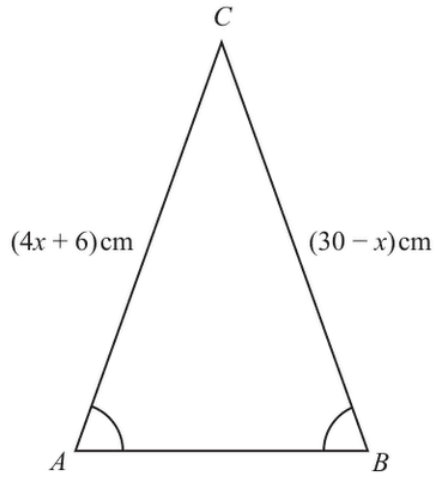


Diagram **NOT** accurately drawn

$CA = (4x + 6)$ cm $\quad CB = (30 - x)$ cm $\quad$ angle CAB = angle CBA

Work out the value of x

$x =$ .....

(Total for Question 6 is 3 marks)

1 **Use trigonometry**

The base angles are equal, so the opposite sides are equal.

$$4x + 6 = 30 - x$$

$$5x = 24$$

$$x = 4.8$$

FINAL ANSWER

$x = 4.8$.

TOPIC

Algebraic Proof

Every past-paper question on this topic, 2020 to 2026.

4WM1/01 NOVEMBER 2025

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.95\dot{4}\dot{5} = \frac{21}{22}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4WM1/01 November 2025

1 Let

Let

$$x = 0.95454545\ldots$$

2 Calculate value

Then

$$10x = 9.5454545\ldots$$

3 Calculate value

and

$$1000x = 954.5454545\ldots$$

4 Subtract

Subtract:

$$990x = 945$$

$$x = \frac{945}{990} = \frac{21}{22}$$

FINAL ANSWER

$0.954545\ldots = \frac{21}{22}$.

4MA1/2H JUNE 2026

Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.5\dot{3}\dot{9} = \frac{89}{165}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2026

1 Solution | Step 1

Let $x = 0.5\overline{39} = 0.5393939\ldots$. Then $10x = 5.393939\ldots$ and $1000x = 539.393939\ldots$.

Shift past the non-recurring digit and then one complete two-digit recurring block.

2 Solution | Step 2

Subtract:

$$1000x - 10x = 539.393939\ldots - 5.393939\ldots = 534.$$

$$990x = 534 \quad \Rightarrow \quad x = \frac{534}{990} = \frac{89}{165}.$$

Identical infinite decimal tails cancel exactly; reduce the resulting fraction by 6.

FINAL ANSWER

$0.5\overline{39} = \frac{89}{165}$.